The quantum mechanics of vortices in superfluids near a Mott transition

cond-mat/0408329, cond-mat/0409470, and to appear

Leon Balents (UCSB)
Lorenz Bartosch (Yale)
Anton Burkov (UCSB)
Predrag Nikolic (Yale)
Subir Sachdev (Yale)
Krishnendu Sengupta (Toronto)

Talk online: Google Sachdev

The quantum order of superfluids: why all superfluids are not the same

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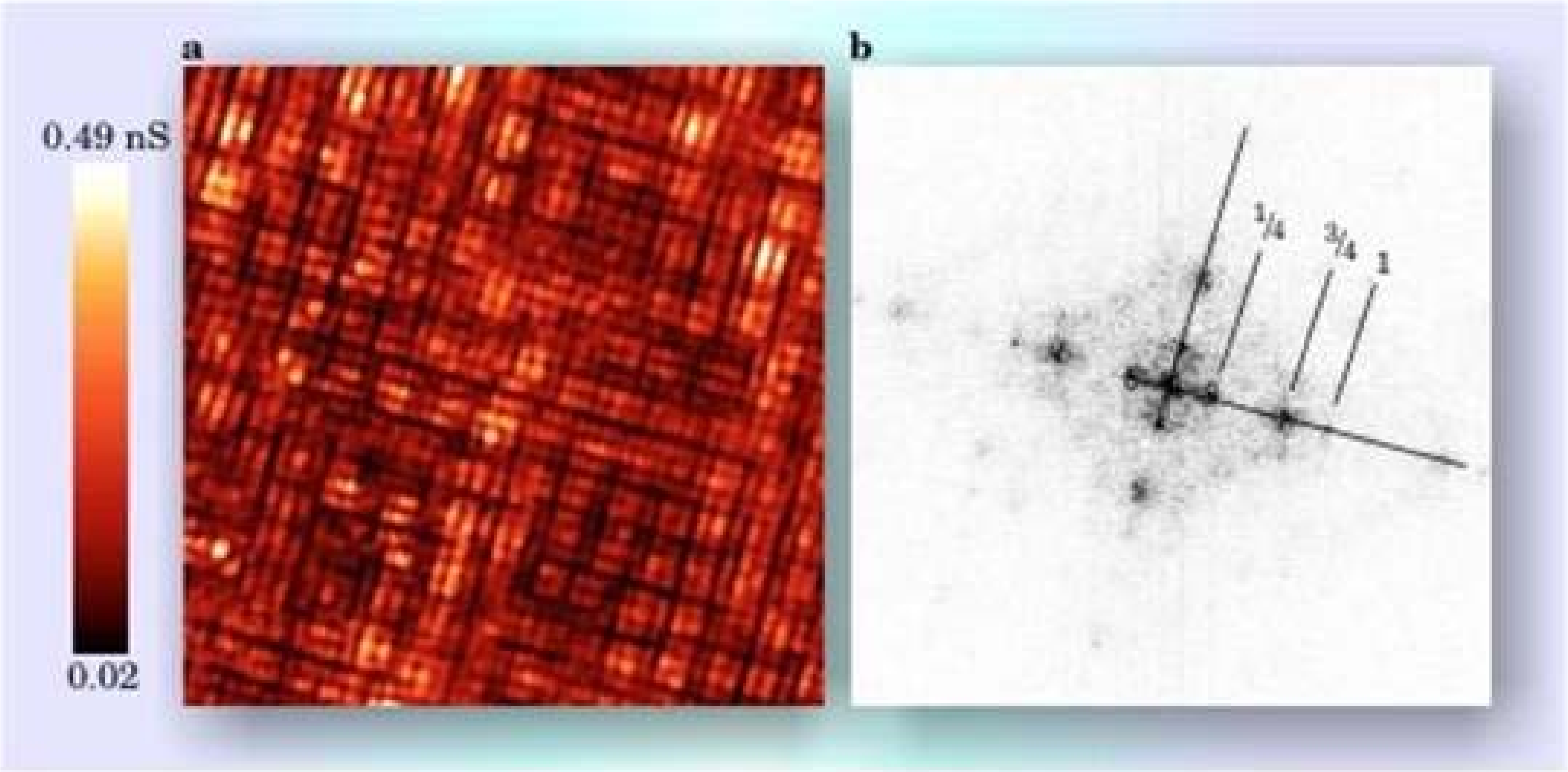
Recent experiments on the cuprate superconductors show:

- Proximity to insulating ground states with density wave order at carrier density $\delta=1/8$

- Vortex/anti-vortex fluctuations for a wide temperature range in the normal state

The cuprate superconductor $Ca_{2-x}Na_xCuO_2Cl_2$

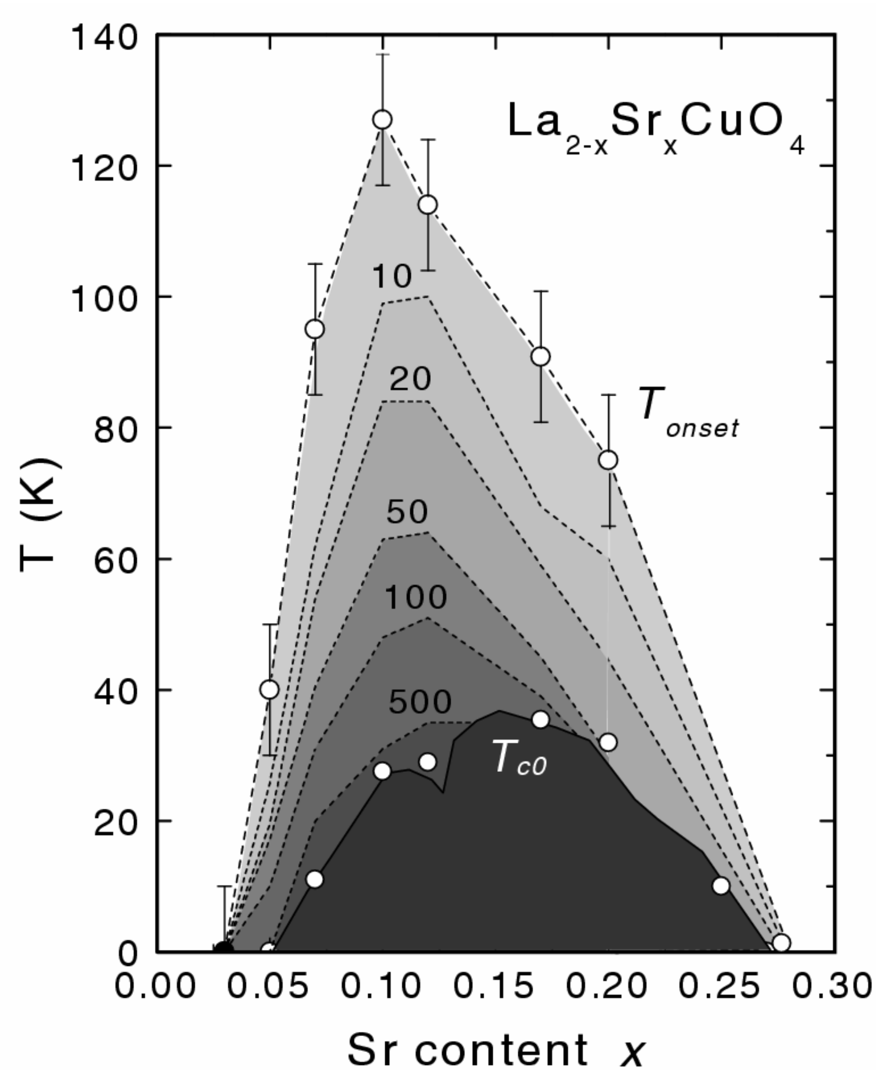


Multiple order parameters: superfluidity and density wave.
Phases: Superconductors, Mott insulators, and/or supersolids

T. Hanaguri, C. Lupien, Y. Kohsaka, D.-H. Lee, M. Azuma, M. Takano, H. Takagi, and J. C. Davis, *Nature* **430**, 1001 (2004).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

Measurements of Nernst effect are well explained by a model of a liquid of vortices and anti-vortices

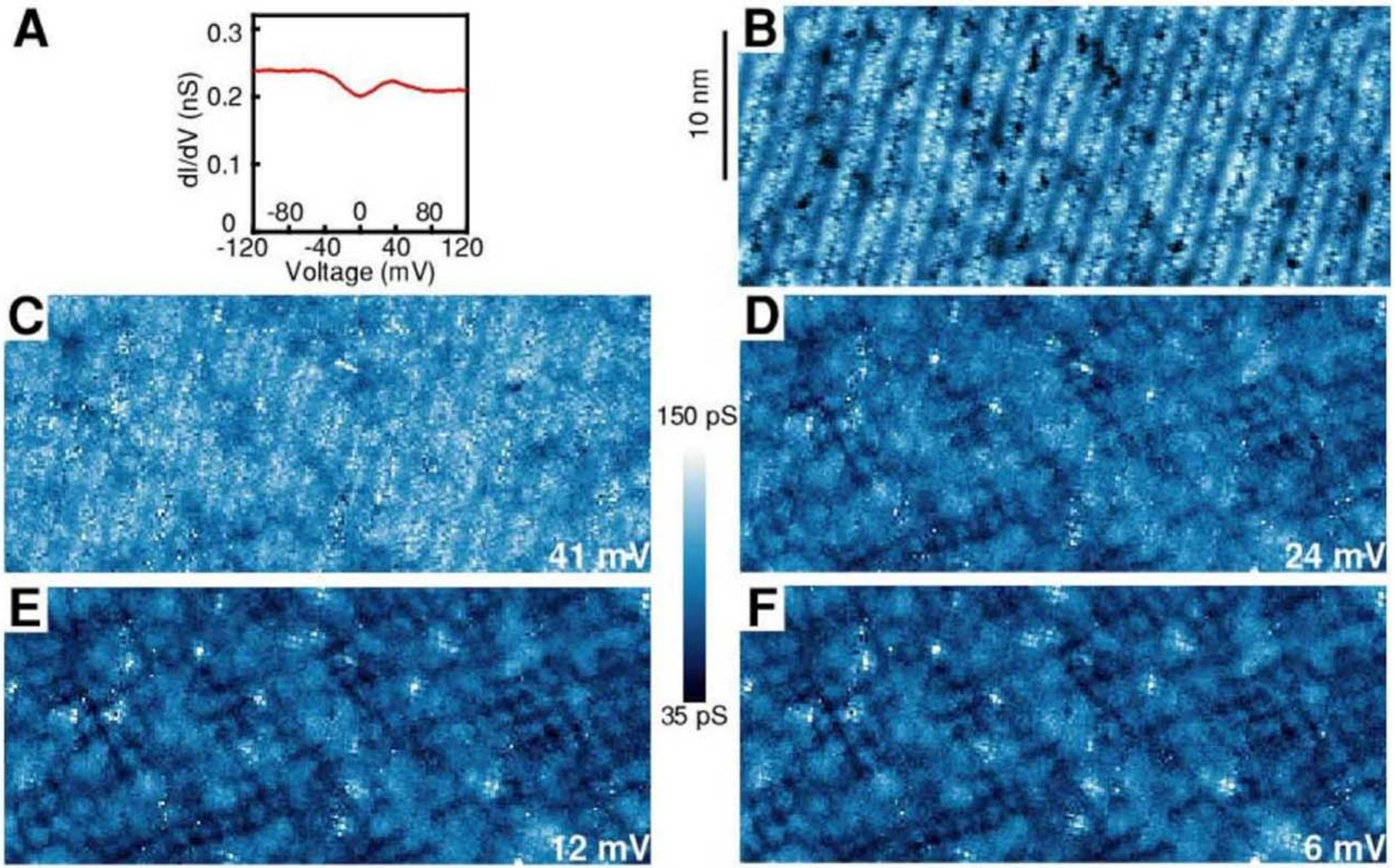


N. P. Ong, Y. Wang, S. Ono, Y. Ando, and S. Uchida, *Annalen der Physik* **13**, 9 (2004).

Y. Wang, S. Ono, Y. Onose, G. Gu, Y. Ando, Y. Tokura, S. Uchida, and N. P. Ong, *Science* **299**, 86 (2003).

Distinct experimental charcteristics of underdoped cuprates at $T > T_c$

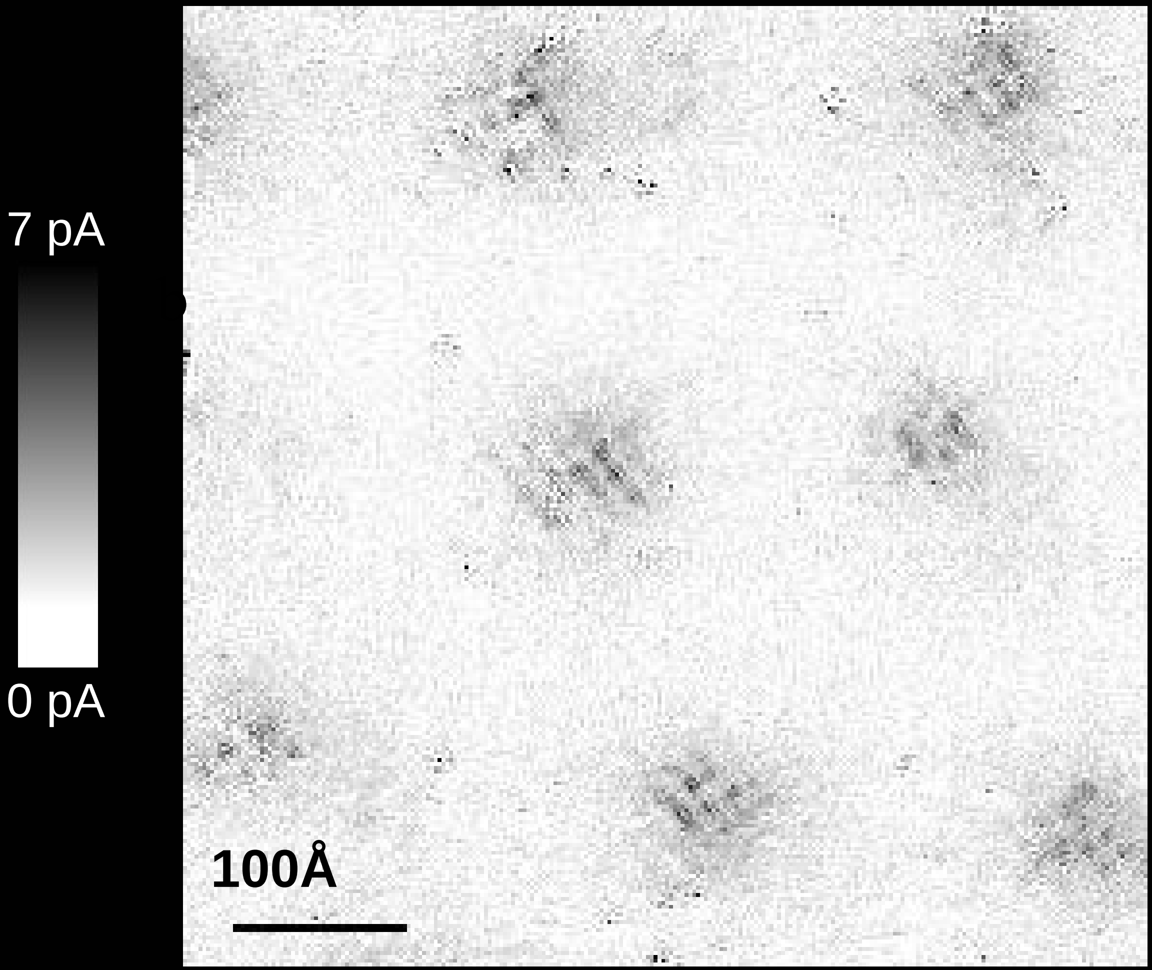
STM measurements observe "density" modulations with a period of ≈ 4 lattice spacings



LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ at 100 K.

M. Vershinin, S. Misra, S. Ono, Y. Abe, Y. Ando, and A. Yazdani, *Science*, **303**, 1995 (2004).

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

Recent experiments on the cuprate superconductors show:

- Proximity to insulating ground states with density wave order at carrier density $\delta=1/8$

- Vortex/anti-vortex fluctuations for a wide temperature range in the normal state

Needed: A quantum theory of transitions between superfluid/supersolid/insulating phases at fractional filling, and a deeper understanding of the role of vortices

Outline

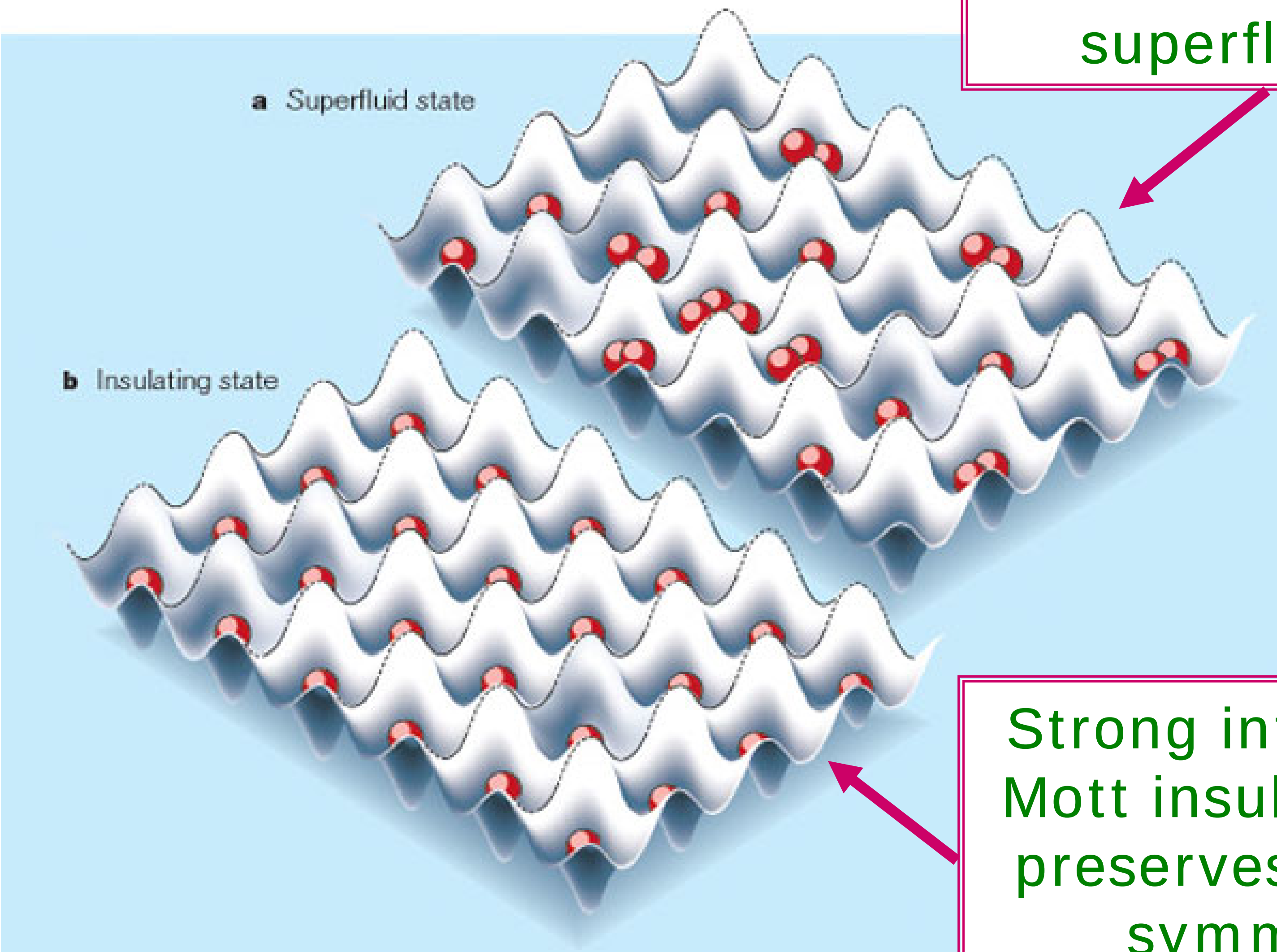
A. Superfluid-insulator transitions of bosons
on the square lattice at filling fraction f
Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator

B. Extension to electronic models for the cuprate superconductors
Dual vortex theories of the doped
(1) Quantum dimer model
(2)"Staggered flux" spin liquid

A. Superfluid-insulator transitions of bosons on the square lattice at filling fraction f

Quantum mechanics of vortices in a superfluid proximate to a commensurate Mott insulator

Bosons at density $f = 1$



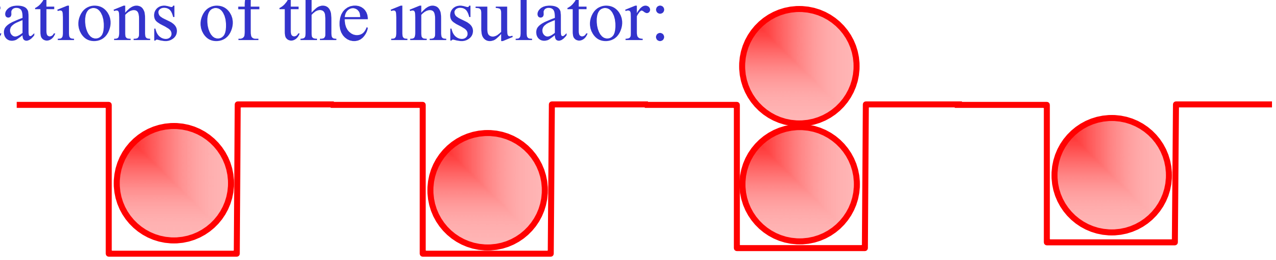
Weak interactions: superfluidity

Strong interactions: Mott insulator which preserves all lattice symmetries

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Approaching the transition from the insulator (f=1)

Excitations of the insulator:



Particles $\sim \psi^\dagger$

Holes $\sim \psi$

Density of particles = density of holes $\Rightarrow$

"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + |\nabla_r \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$

Approaching the transition from the superfluid (f=1)

Excitations of the superfluid: (A) **Superflow ("spin waves")**

With $\psi \sim e^{i\theta}$, the action for fluctuations of the superfluid velocity $\sim \nabla\theta$ is

$$\mathcal{S}_{sw} = \frac{\rho_s}{2} \int d^3x (\partial_\mu \theta)^2$$

Dual form: After a Hubbard-Stratonovich transformation, write

$$\mathcal{S}_{sw} = \int d^3x \left[\frac{1}{2\rho_s} J_\mu^2 + i J_\mu \partial_\mu \theta \right]$$

Integrating over θ yields $\partial_\mu J_\mu = 0$. Solve, by writing

$$J_\mu = \epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda$$

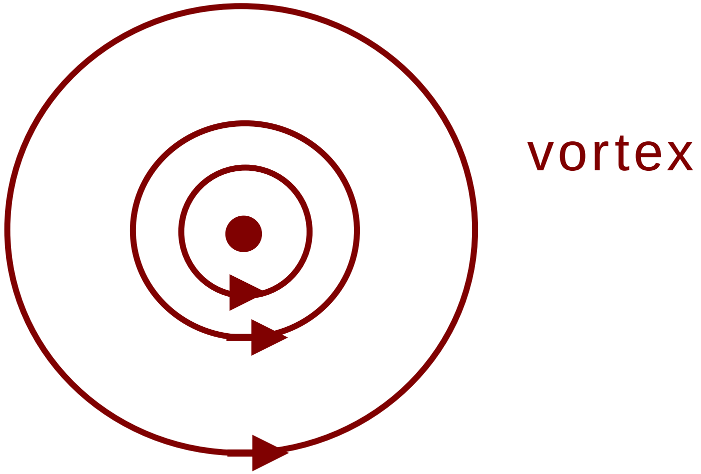
leading to

$$\mathcal{S}_{sw} = \int d^3x \left[\frac{1}{2\rho_s} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 \right]$$

Phase ("spin wave") fluctuations are dual to a U(1) gauge theory in 2+1 dimensions

Approaching the transition from the superfluid (f=1)

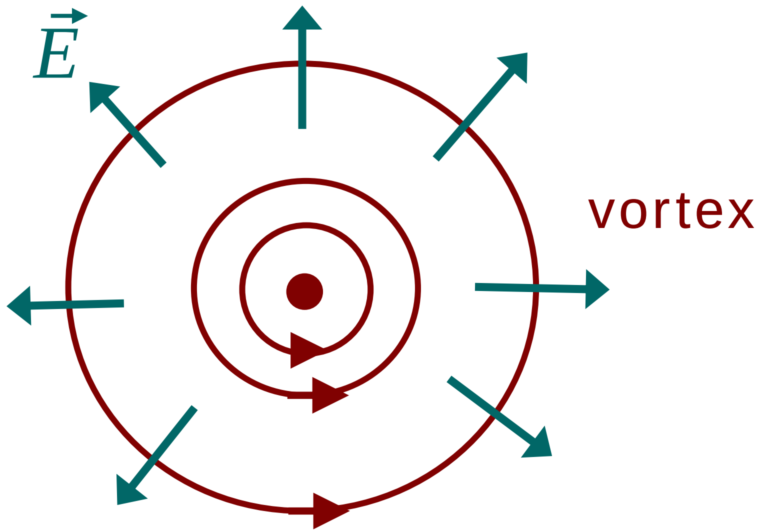
Excitations of the superfluid: (B) **Vortices**



A vortex is a point-like object. We can therefore define a local field operator, φ, which annihilates a vortex.

Approaching the transition from the superfluid (f=1)

Excitations of the superfluid: (B) **Vortices**



A vortex is a point-like object. We can therefore define a local field operator, φ, which annihilates a vortex.

Each vortex is the source of an 'electric field' $\vec{E}$ associated with the U(1) gauge field A_μ.

Consequently, φ carries +1 U(1) gauge charge.

Approaching the transition from the superfluid (f=1)

Excitations of the superfluid: **Superflow and vortices**

φ: vortex annihilation operator.

$\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$: boson current $\sim i\psi^*\partial_\mu\psi - i\partial_\mu\psi^*\psi$.

Density of vortices = density of antivortices $\Rightarrow$

"relativistic" field theory for φ:

$$\mathcal{S}_{\text{dual}} = \int d^3x \Big[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2\rho_s}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

$$\text{Superfluid} \Leftrightarrow \langle\varphi\rangle = 0$$

$$\text{Insulator} \Leftrightarrow \langle\varphi\rangle \neq 0$$

Dual theories of the superfluid-insulator transition (f=1)

Using the boson quasiparticle excitations, $\sim \psi$, of the insulator

$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + |\nabla_r \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$

<u>is dual to</u>

Using the vortex quasiparticle, $\sim \varphi$, and superfluid velocity, $\sim \epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$, excitations of the superfluid

$$\mathcal{S}_{\text{dual}} = \int d^3x \left[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2\rho_s}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

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C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981);

A vortex in the vortex field is the original boson

A vortex in φ carries 2π flux in the 'magnetic field' $B = \epsilon_{\tau\mu\nu}\partial_\mu A_\nu$. But this is just the original boson number operator. Consequently, in the path integral viewpoint, the world line of the vortex in φ is just the world line of the original boson.

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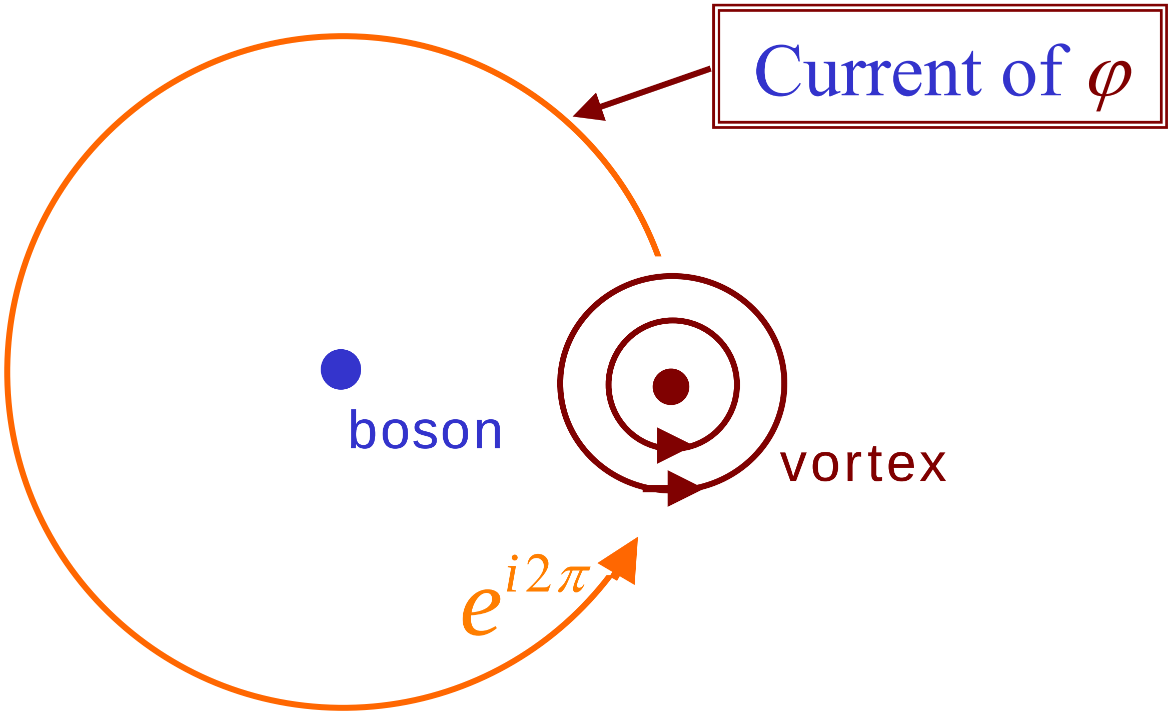
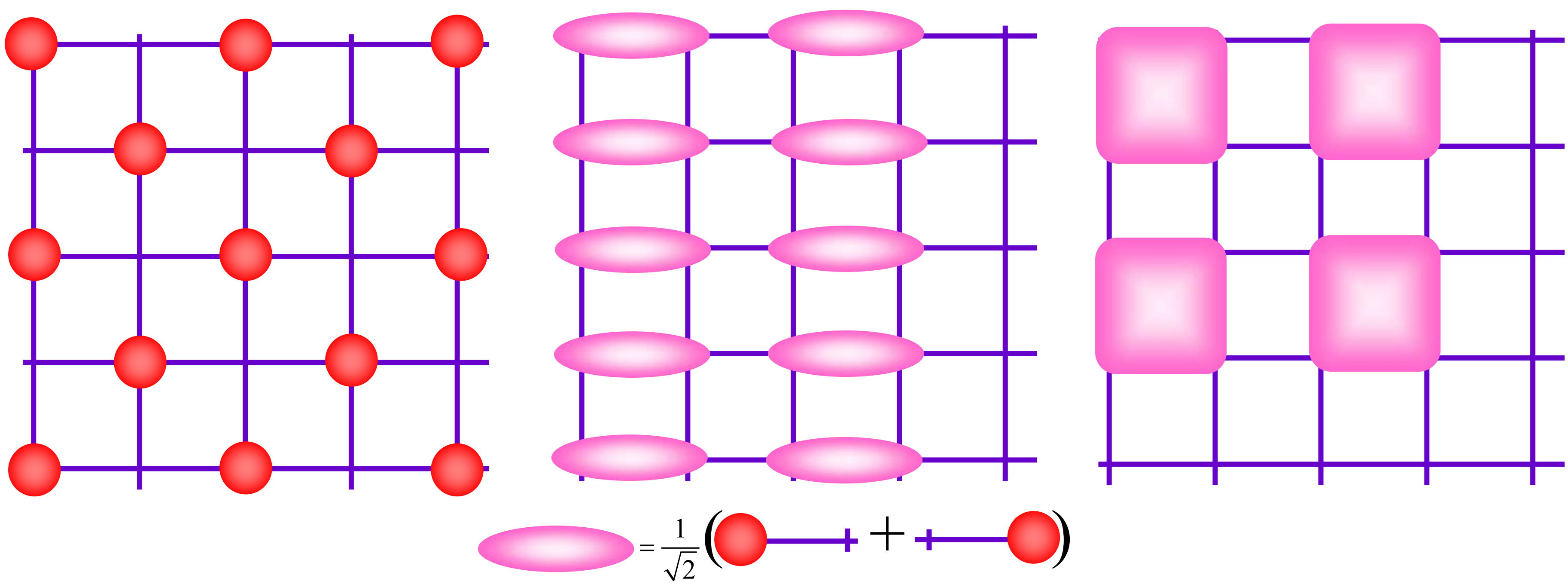


The wavefunction of a vortex acquires a phase of 2π each time the vortex encircles a boson

Bosons at density $f = 1/2$ (equivalent to S=1/2 AFMs)

Weak interactions: superfluidity $\langle \psi \rangle \neq 0$

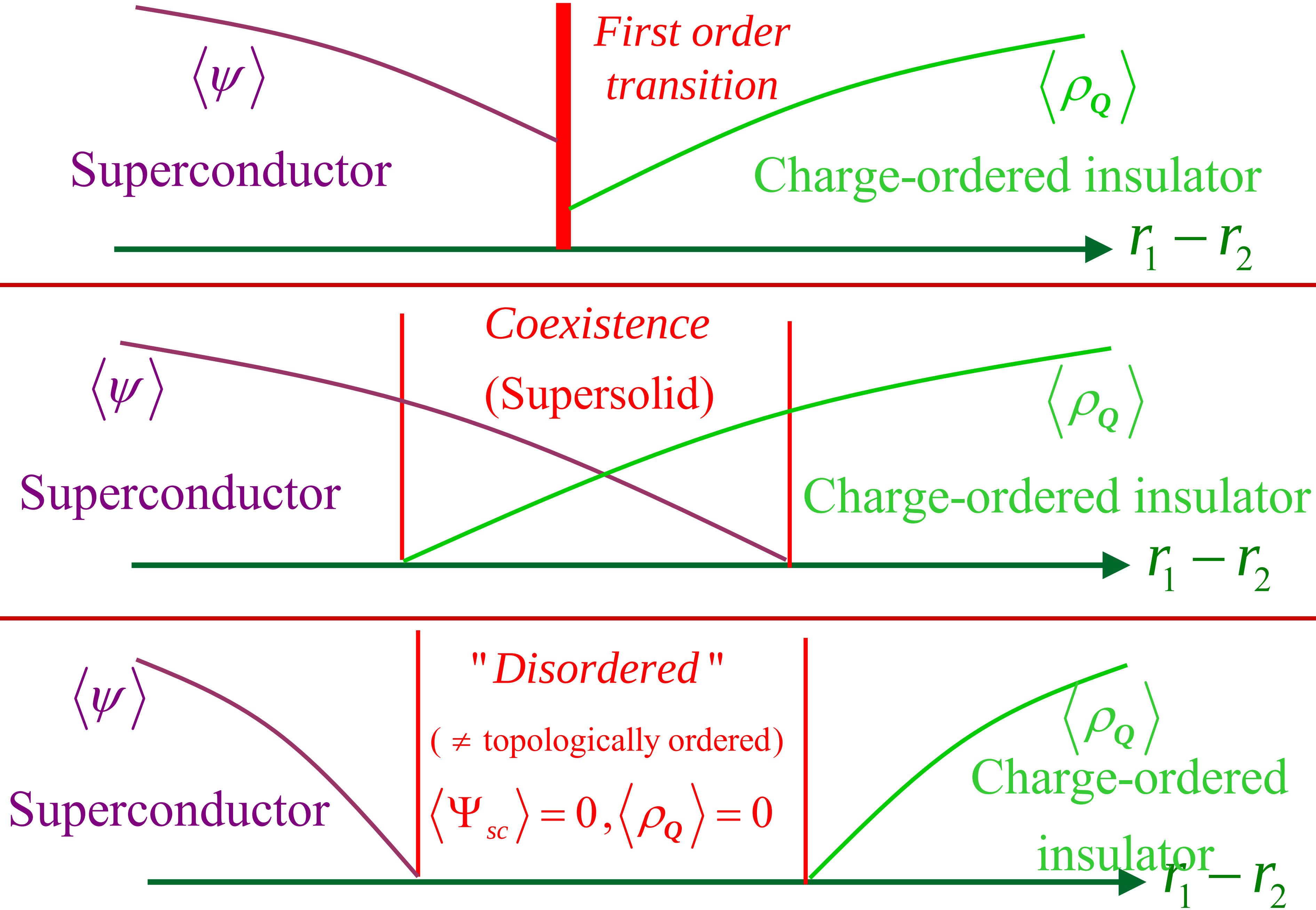
Strong interactions: Candidate insulating states



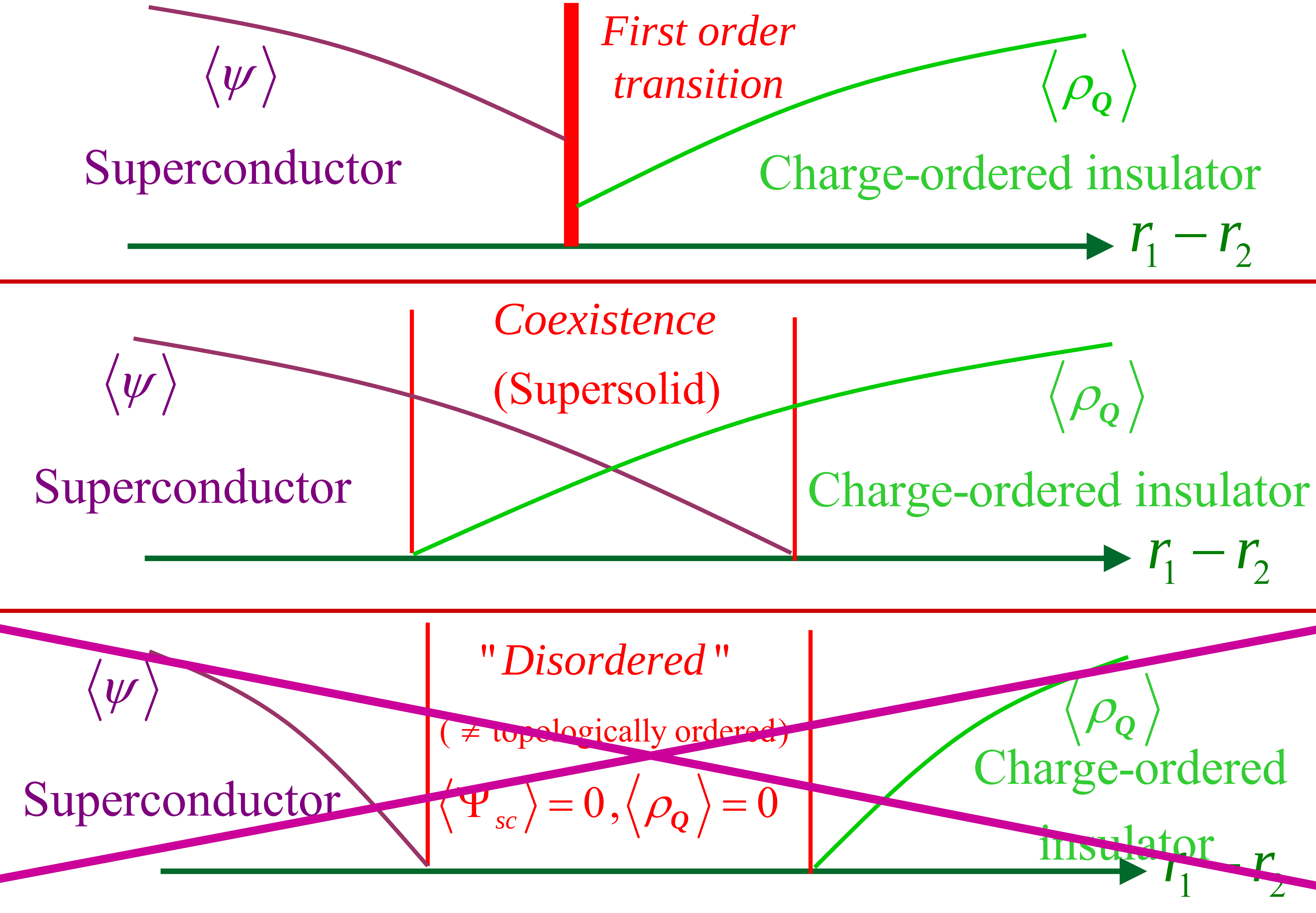
All insulating phases have density-wave order $\rho(\mathbf{r}) = \sum_{\mathbf{Q}} \rho_{\mathbf{Q}} e^{i\mathbf{Q}.\mathbf{r}}$ with $\langle \rho_{\mathbf{Q}} \rangle \neq 0$

C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001)
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

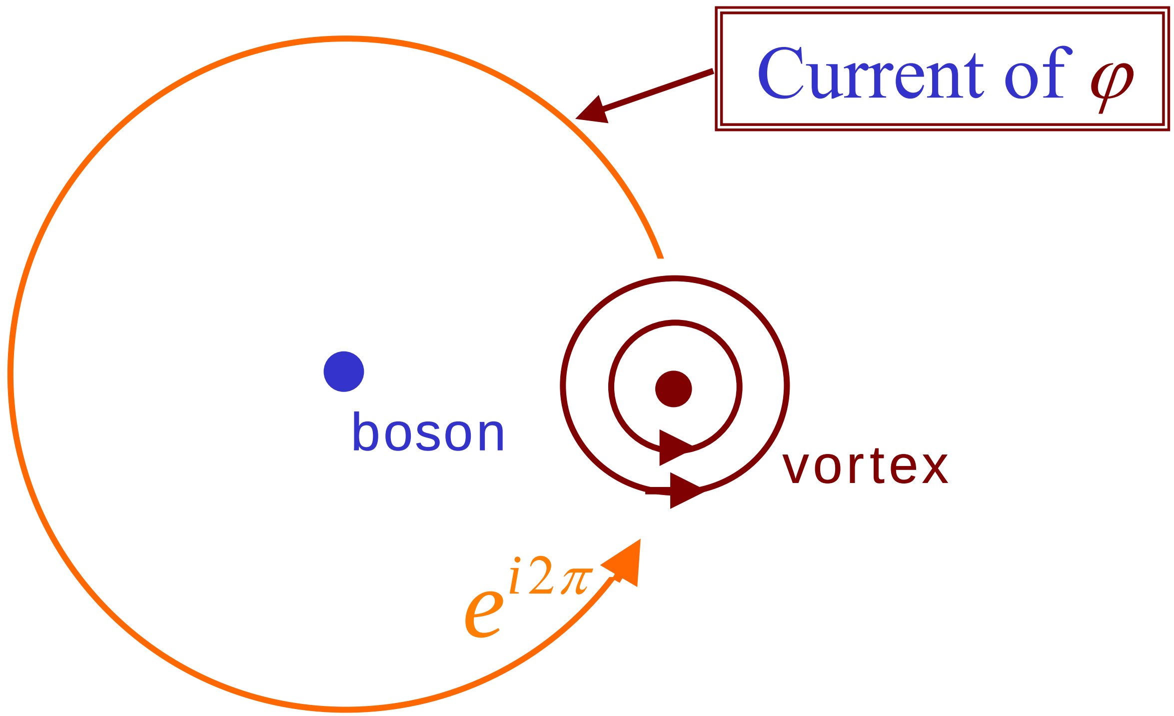
Predictions of LGW theory



Predictions of LGW theory



Boson-vortex duality



The wavefunction of a vortex acquires a phase of 2π each time the vortex encircles a boson

Strength of "magnetic" field on vortex field φ = density of bosons = f flux quanta per plaquette

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989);

Boson-vortex duality

Quantum mechanics of the vortex "particle" φ is invariant under the square lattice space group:

T_x, T_y : Translations by a lattice spacing in the x, y directions

R : Rotation by 90 degrees.

Magnetic space group:

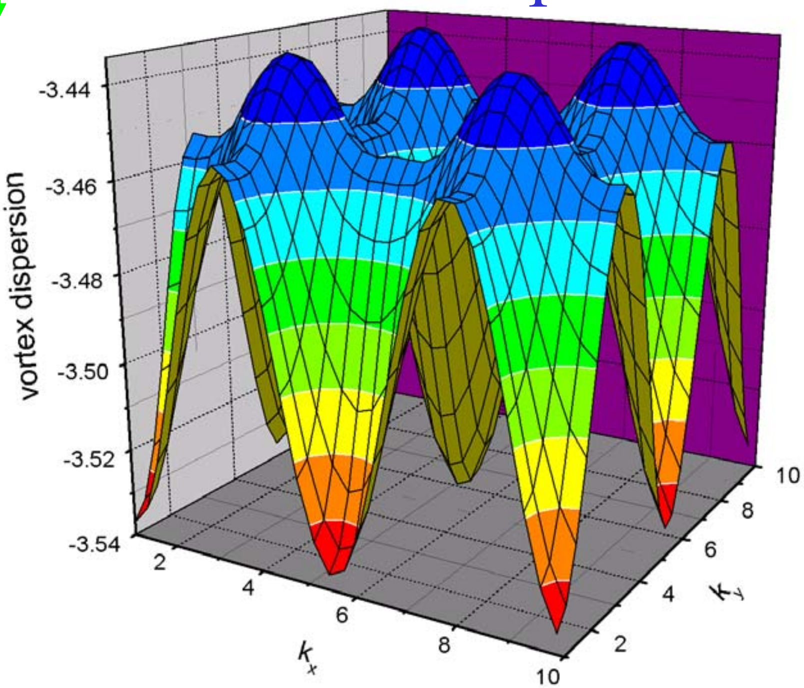
$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

$$R^{-1} T_y R = T_x \quad ; \quad R^{-1} T_x R = T_y^{-1} \quad ; \quad R^4 = 1$$

Strength of "magnetic" field on vortex field φ
= density of bosons = f flux quanta per plaquette

Boson-vortex duality

Hofstadter spectrum of the quantum vortex "particle" φ



At density $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

Magnetic space group:

$$T_x T_y = e^{2\pi i f} T_y T_x \quad ;$$

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Boson-vortex duality

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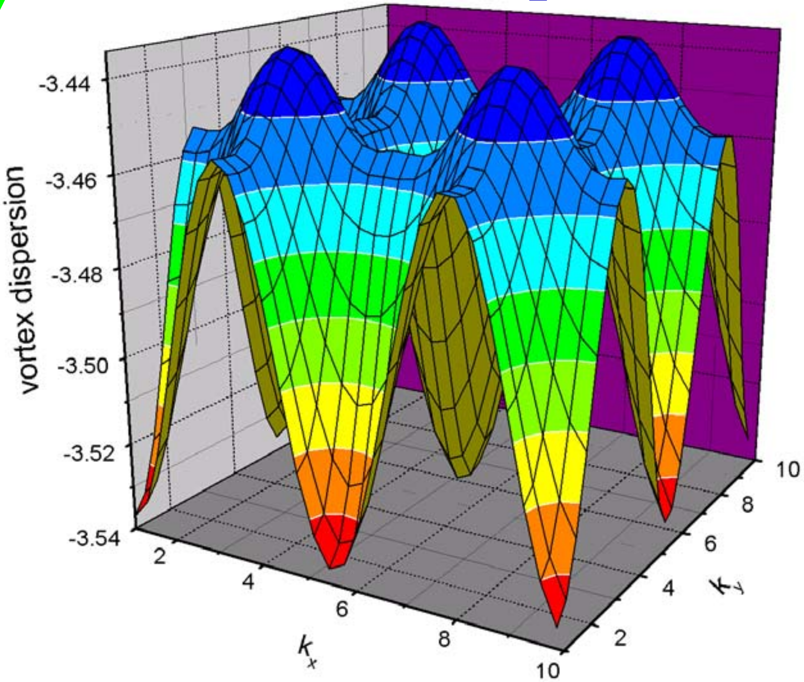


At density $f = p/q$ (p, q relatively prime integers) there are q species of vortices, φ_ℓ (with $\ell = 1 \ldots q$), associated with q gauge-equivalent regions of the Brillouin zone

The q vortices form a *projective* representation of the space group

$$T_x : \varphi_\ell \to \varphi_{\ell+1} \quad ; \quad T_y : \varphi_\ell \to e^{2\pi i \ell f} \varphi_\ell$$

$$R : \varphi_\ell \to \frac{1}{\sqrt{q}} \sum_{m=1}^{q} \varphi_m e^{2\pi i \ell m f}$$

See also X.-G. Wen, *Phys. Rev.* B **65**, 165113 (2002)

Boson-vortex duality

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

$$\underline{\text{Superconductor/insulator}} : \left\langle \varphi_\ell \right\rangle = 0 / \left\langle \varphi_\ell \right\rangle \neq 0$$

Boson-vortex duality

The q φ_ℓ vortices characterize *both* superconducting and density wave orders

Density wave order:

Status of space group symmetry determined by density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell mf}$$

$$T_x : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{x}} \quad ; \quad T_y : \rho_{\mathbf{Q}} \rightarrow \rho_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\hat{y}}$$

$$R : \rho(\mathbf{Q}) \rightarrow \rho(R\mathbf{Q})$$

Field theory with projective symmetry

Degrees of freedom:

q complex φ_ℓ vortex fields

1 non-compact U(1) gauge field A_μ

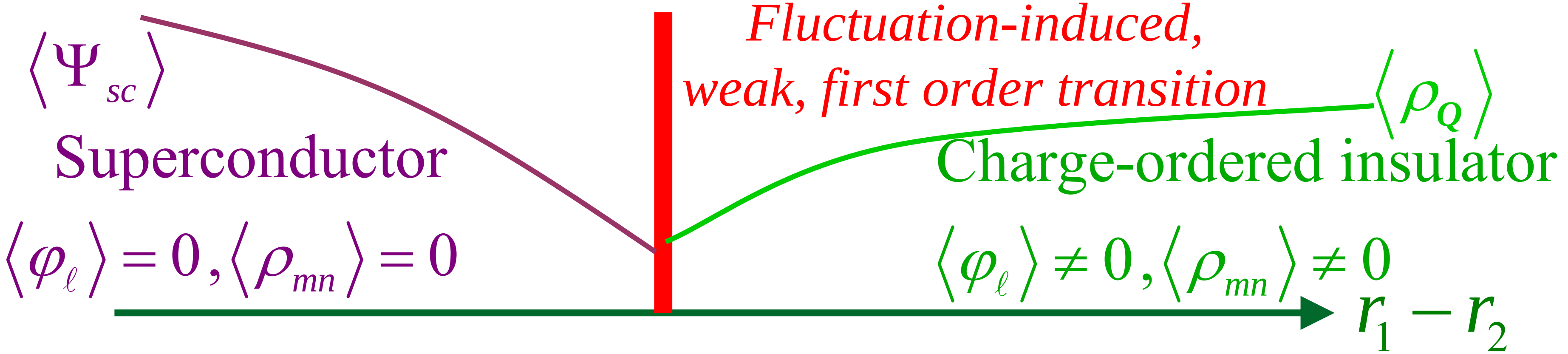
$$\mathcal{S} = \int d^2x d\tau \Bigg[\sum_\ell \left\{ |(\partial_\mu - iA_\mu)\varphi_\ell|^2 + s|\varphi_\ell|^2 \right\} + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \sum_{\ell m n} \gamma_{mn} \varphi_\ell^* \varphi_{\ell+m}^* \varphi_{\ell+n} \varphi_{\ell+m-n} \Bigg]$$

The projective symmetries constrain the couplings γ_{mn} to obey

$$\gamma_{mn} = \gamma_{-m,-n} \; ; \; \gamma_{mn} = \gamma_{m,m-n} \; ; \; \gamma_{mn} = \gamma_{m-2n,-n}$$

$$\gamma_{\bar{m}\bar{n}} = \frac{1}{q} \sum_{mn} \gamma_{mn} e^{-2\pi i f [n(\bar{m}-\bar{n}) + \bar{n}(m-n)]}$$

Field theory with projective symmetry



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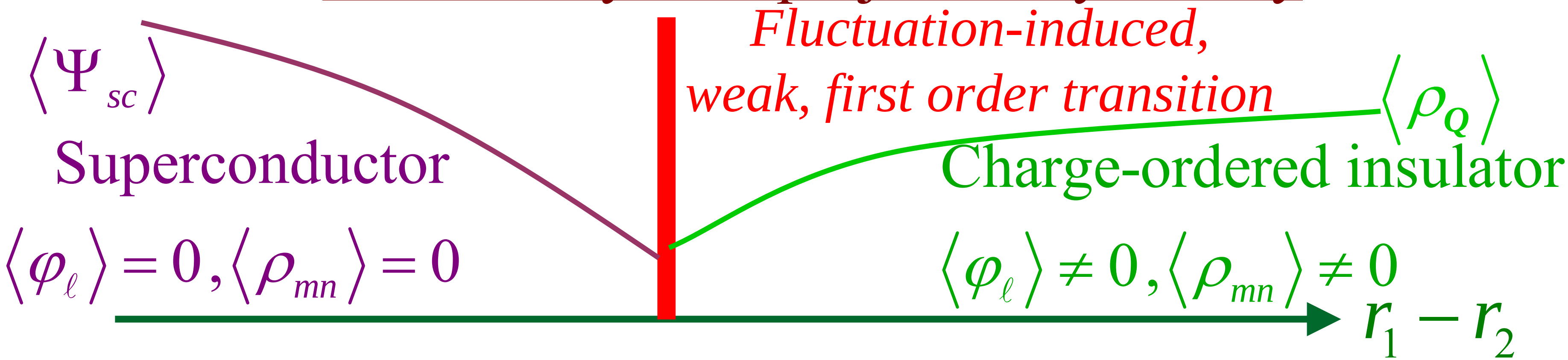
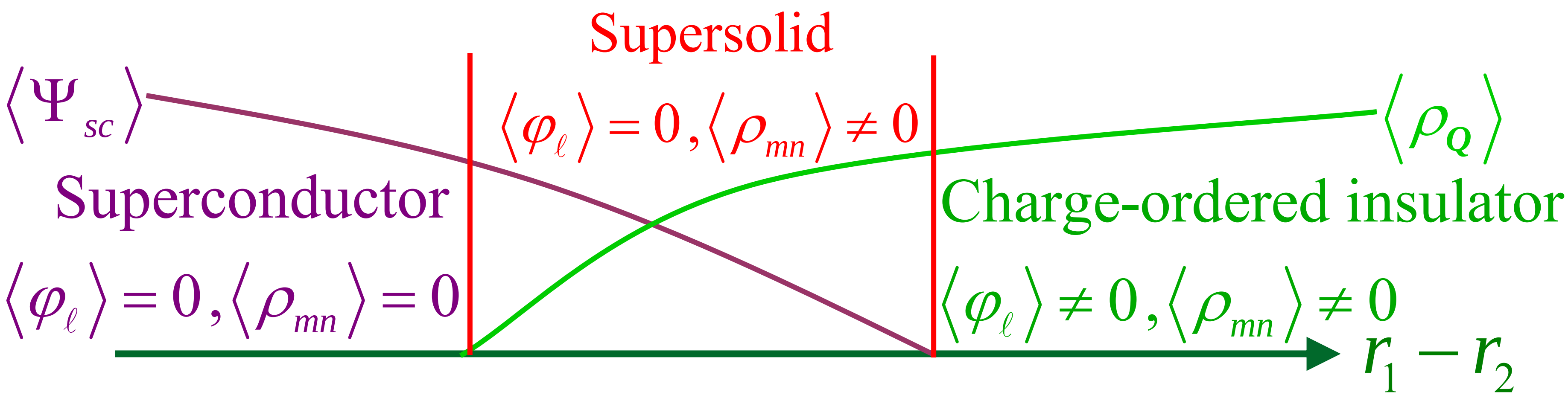
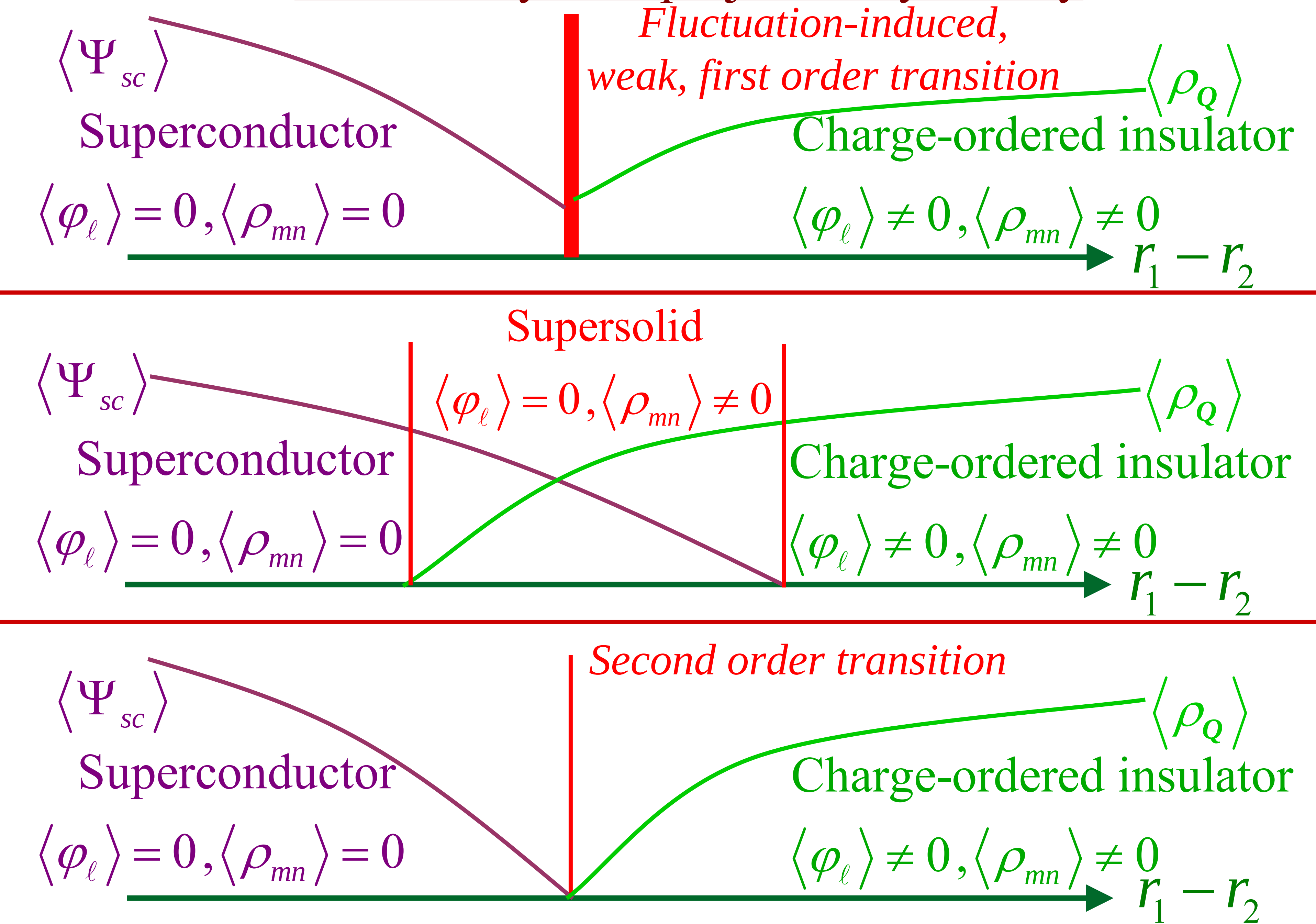


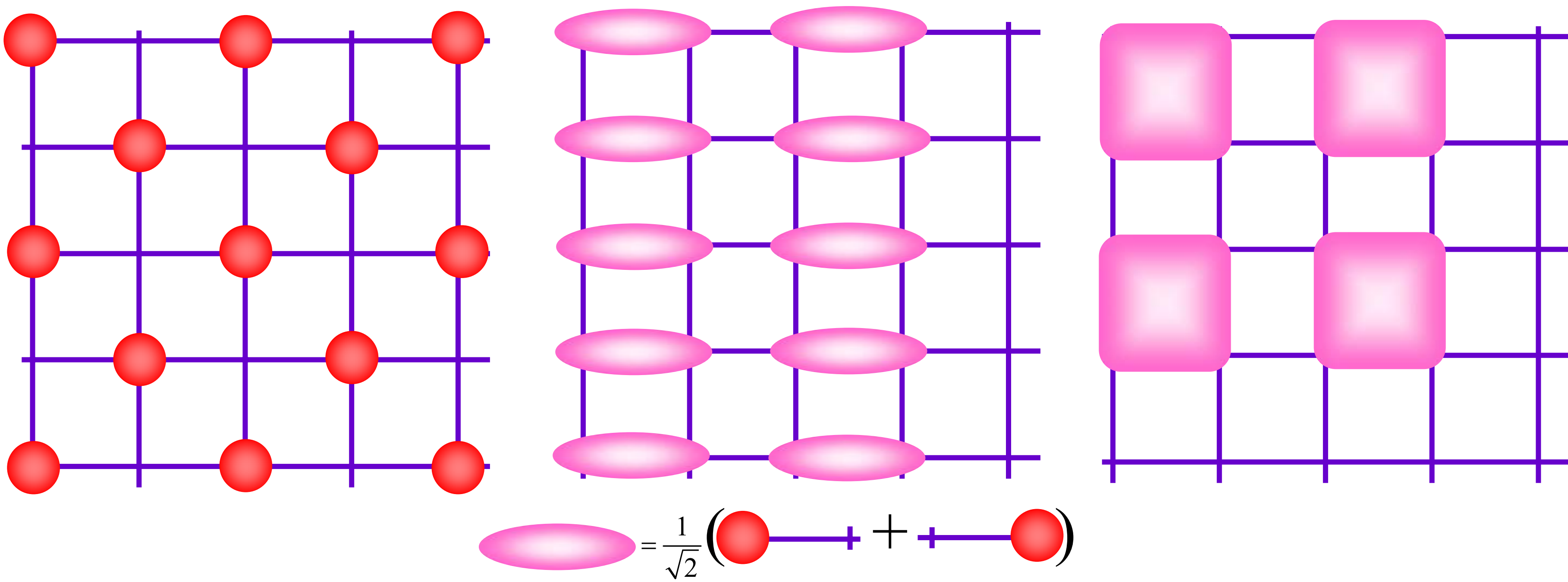


Field theory with projective symmetry



Field theory with projective symmetry

Spatial structure of insulators for $q=2$ ($f=1/2$)



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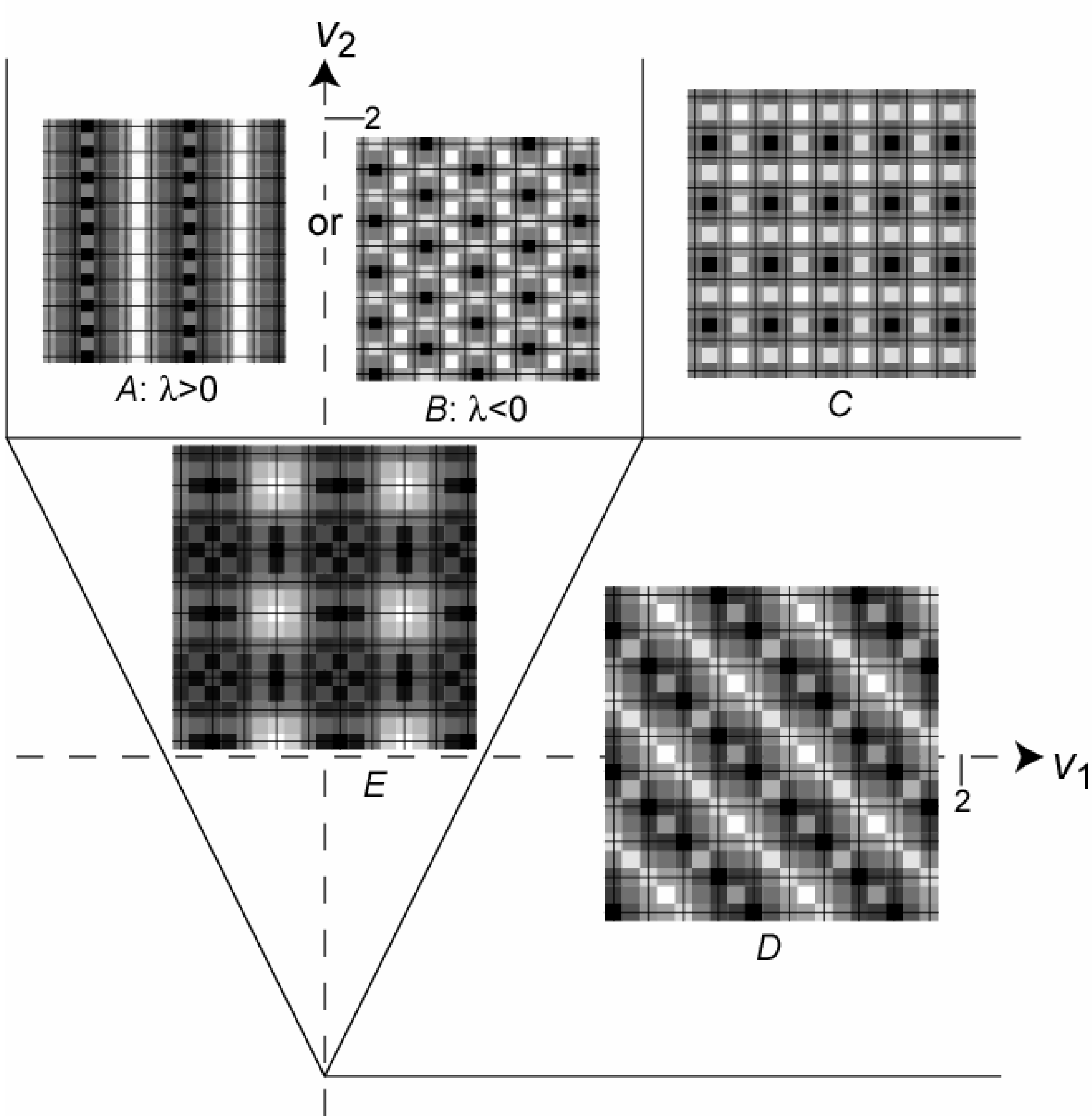
Field theory with projective symmetry

Spatial structure of insulators for q=4 (f=1/4 or 3/4)



$a \times b$ unit cells;

q/a, q/b, ab/q

all integers

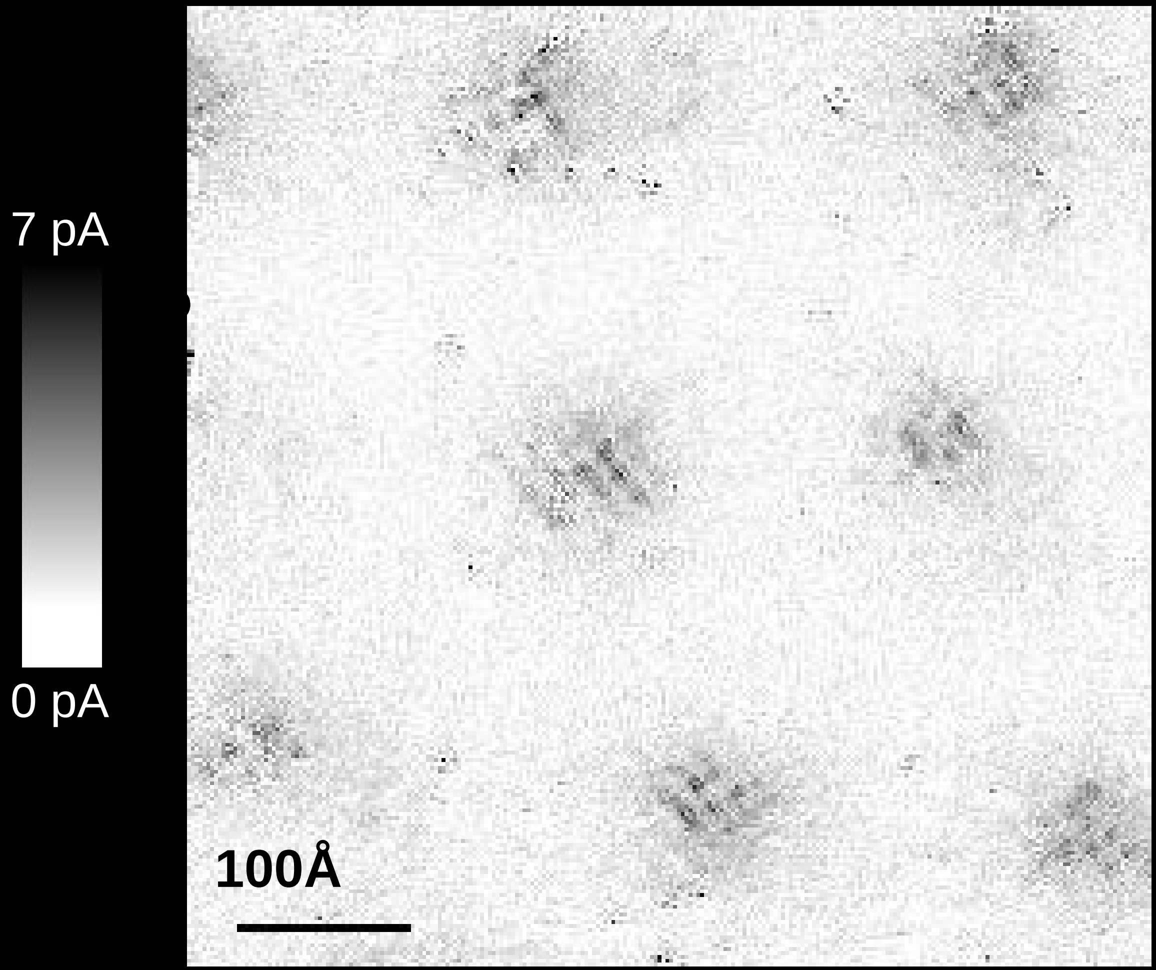
Field theory with projective symmetry

Density operators $\rho_{\mathbf{Q}}$ at wavevectors $\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m,n)$

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_{\ell}^{*} \varphi_{\ell+n} e^{2\pi i \ell mf}$$

Each pinned vortex in the superfluid has a halo of density wave order over a length scale ≈ the zero-point quantum motion of the vortex. This scale diverges upon approaching the insulator

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



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Prediction of VBS order near vortices: K. Park and S. Sachdev, Phys. Rev. B **64**, 184510 (2001).

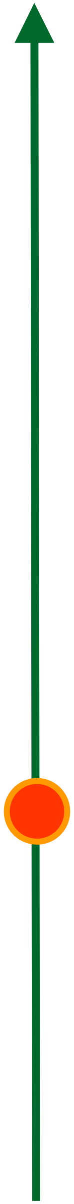
B. Extension to electronic models for the cuprate superconductors

Dual vortex theories of the doped

(1) Quantum dimer model

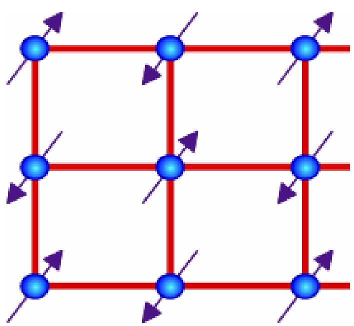
(2)"Staggered flux" spin liquid

(B.1) Phase diagram of doped antiferromagnets



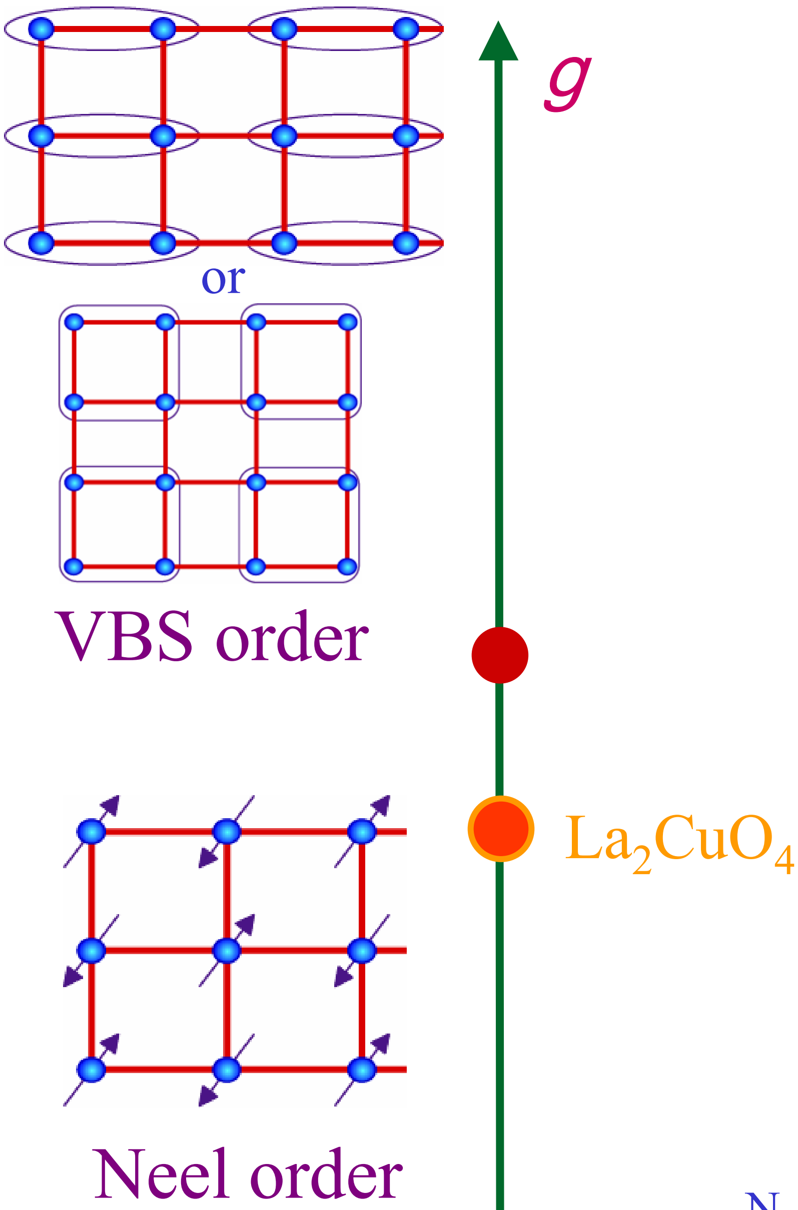
g = parameter controlling strength of quantum fluctuations in a semiclassical theory of the destruction of Neel order

La_2CuO_4



Neel order

(B.1) Phase diagram of doped antiferromagnets



N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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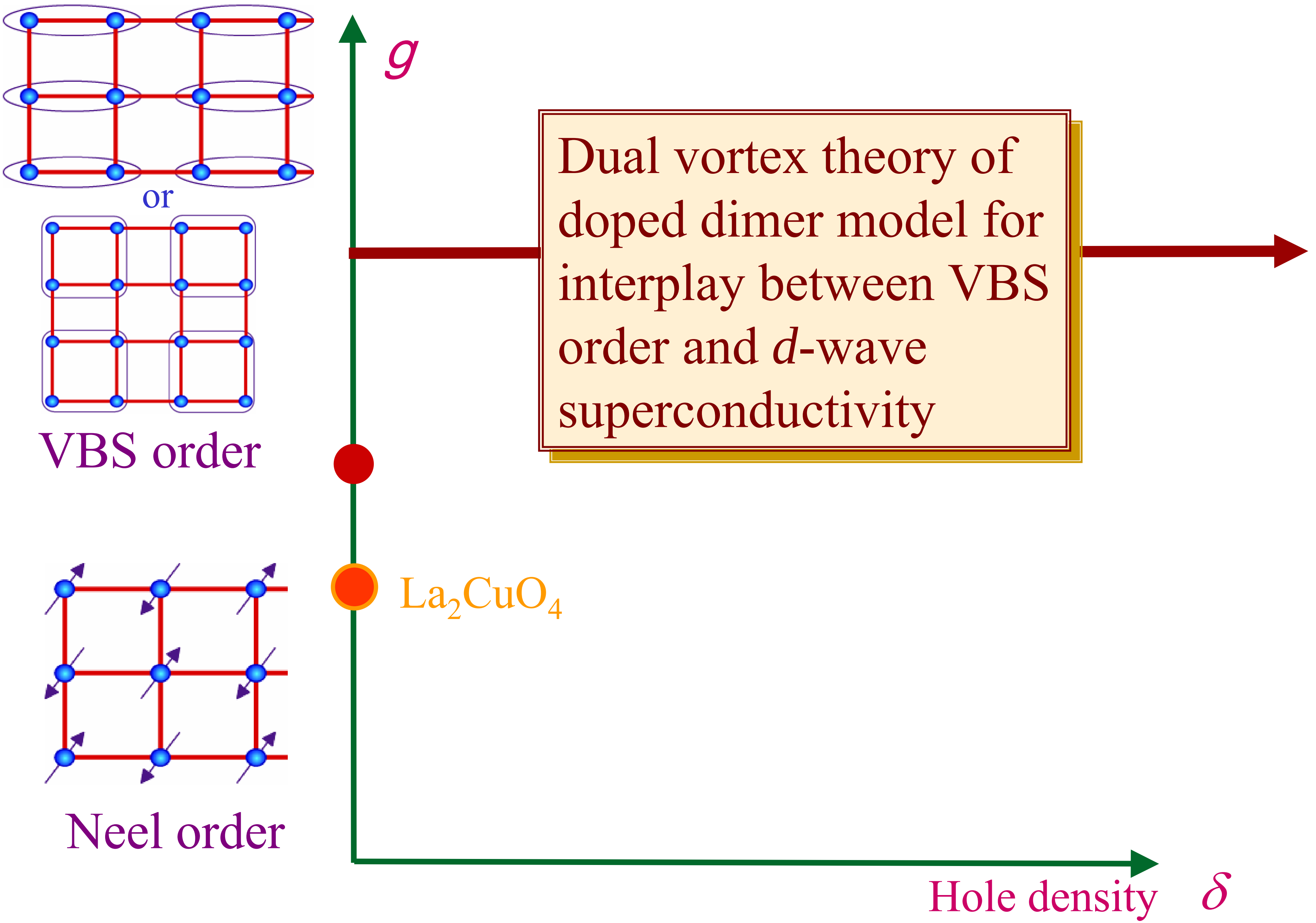
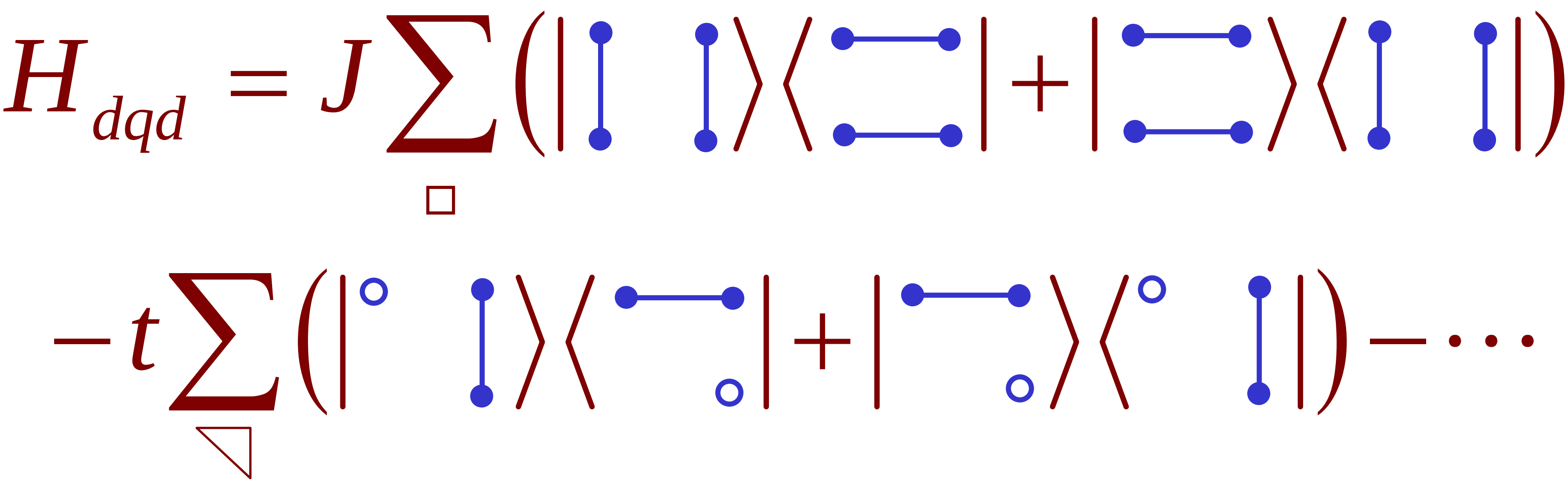


(**B.1**) Doped quantum dimer model



Density of holes = δ

E. Fradkin and S. A. Kivelson, *Mod. Phys. Lett.* B **4**, 225 (1990).

(**B.1**) Duality mapping of doped quantum dimer model shows:

Vortices in the superconducting state obey the magnetic translation algebra

$$T_x T_y = e^{2\pi i f} T_y T_x$$

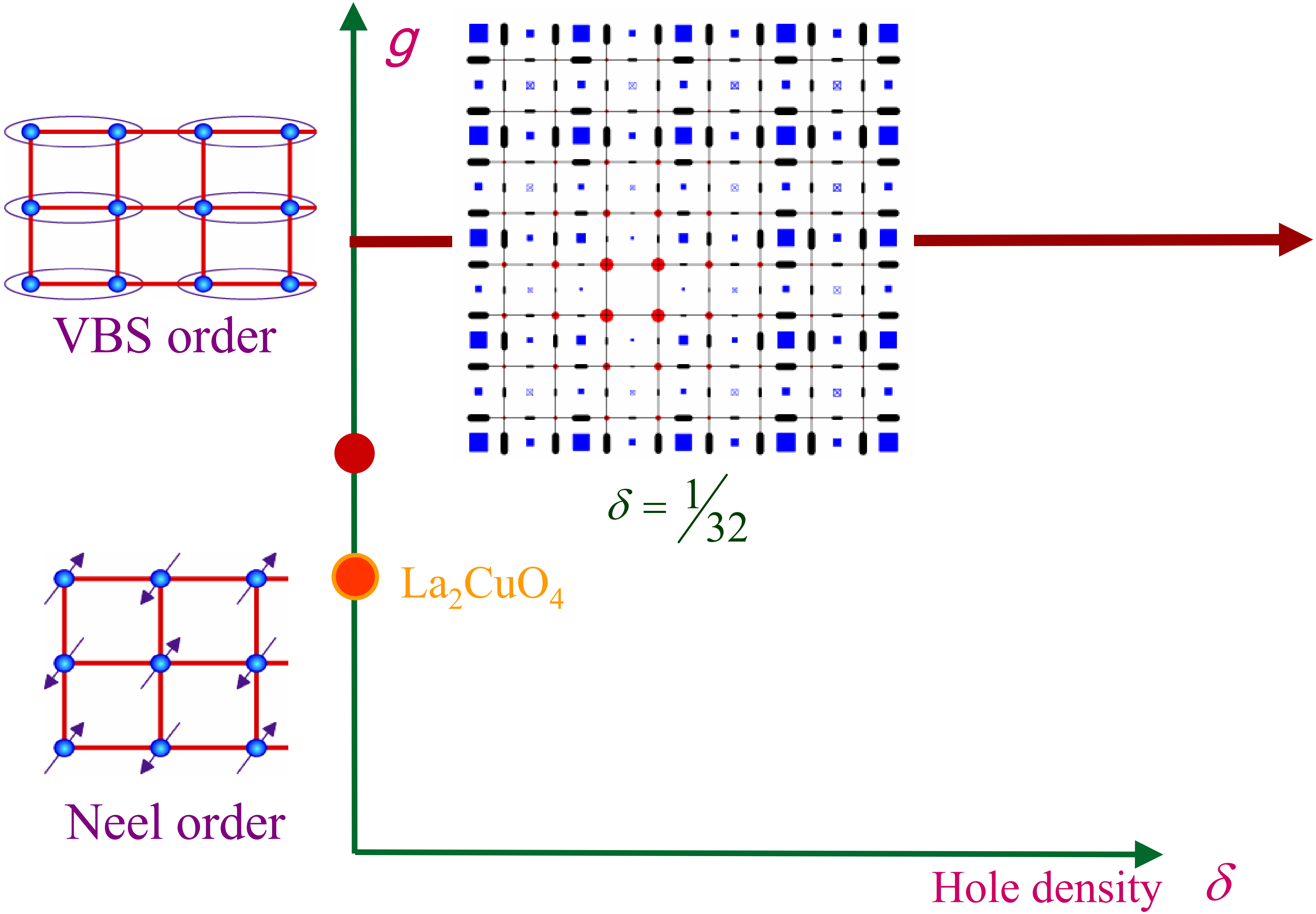
$$\text{with} \quad f = \frac{p}{q} = \frac{1-\delta_{MI}}{2}$$

where δ_{MI} is the density of holes in the proximate Mott insulator (for $\delta_{MI} = 1/8$, $f = 7/16 \Rightarrow q = 16$)

Note: f = density of Cooper pairs

Most results of Part A on bosons can be applied unchanged with q as determined above

(B.1) Phase diagram of doped antiferromagnets



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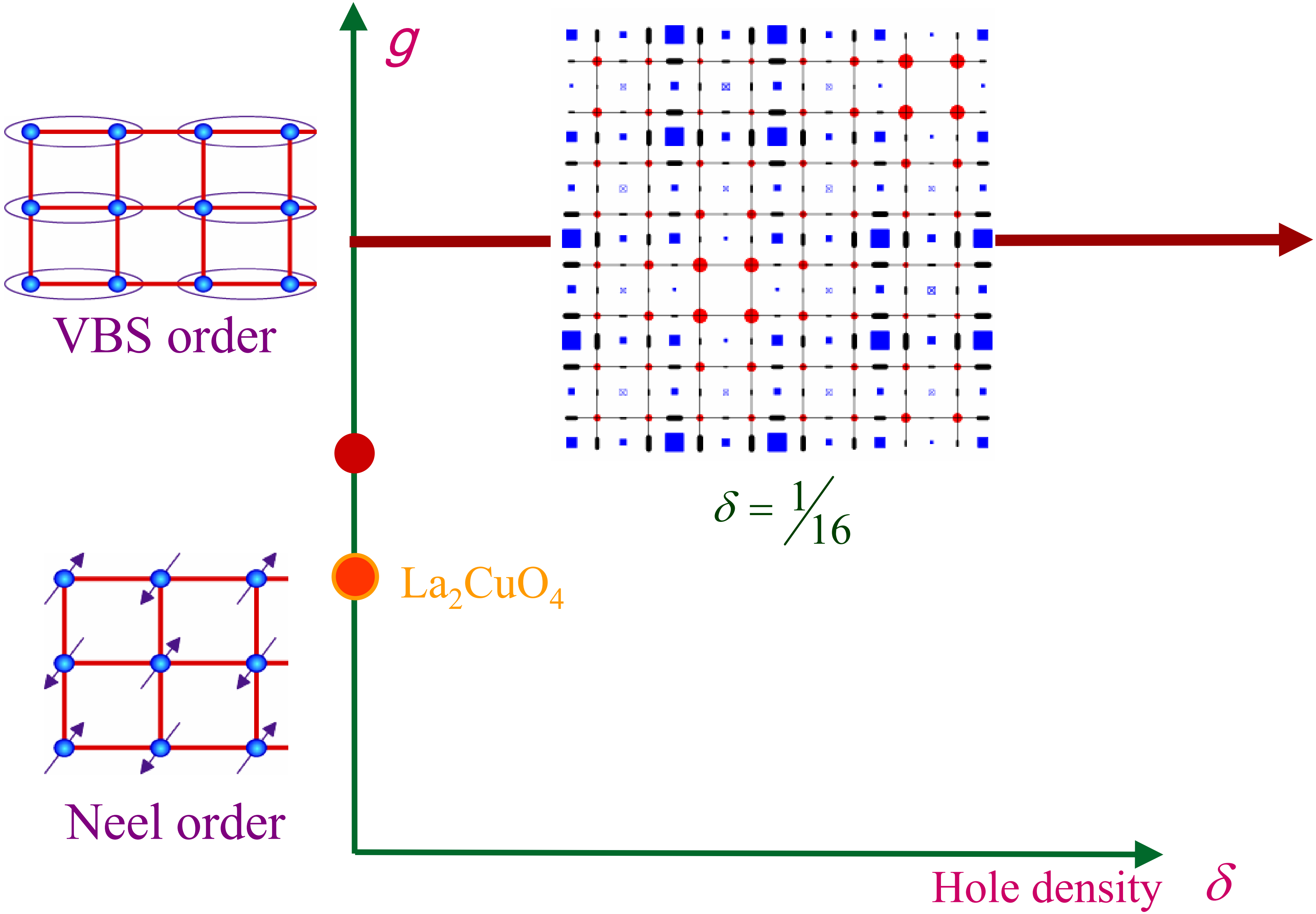

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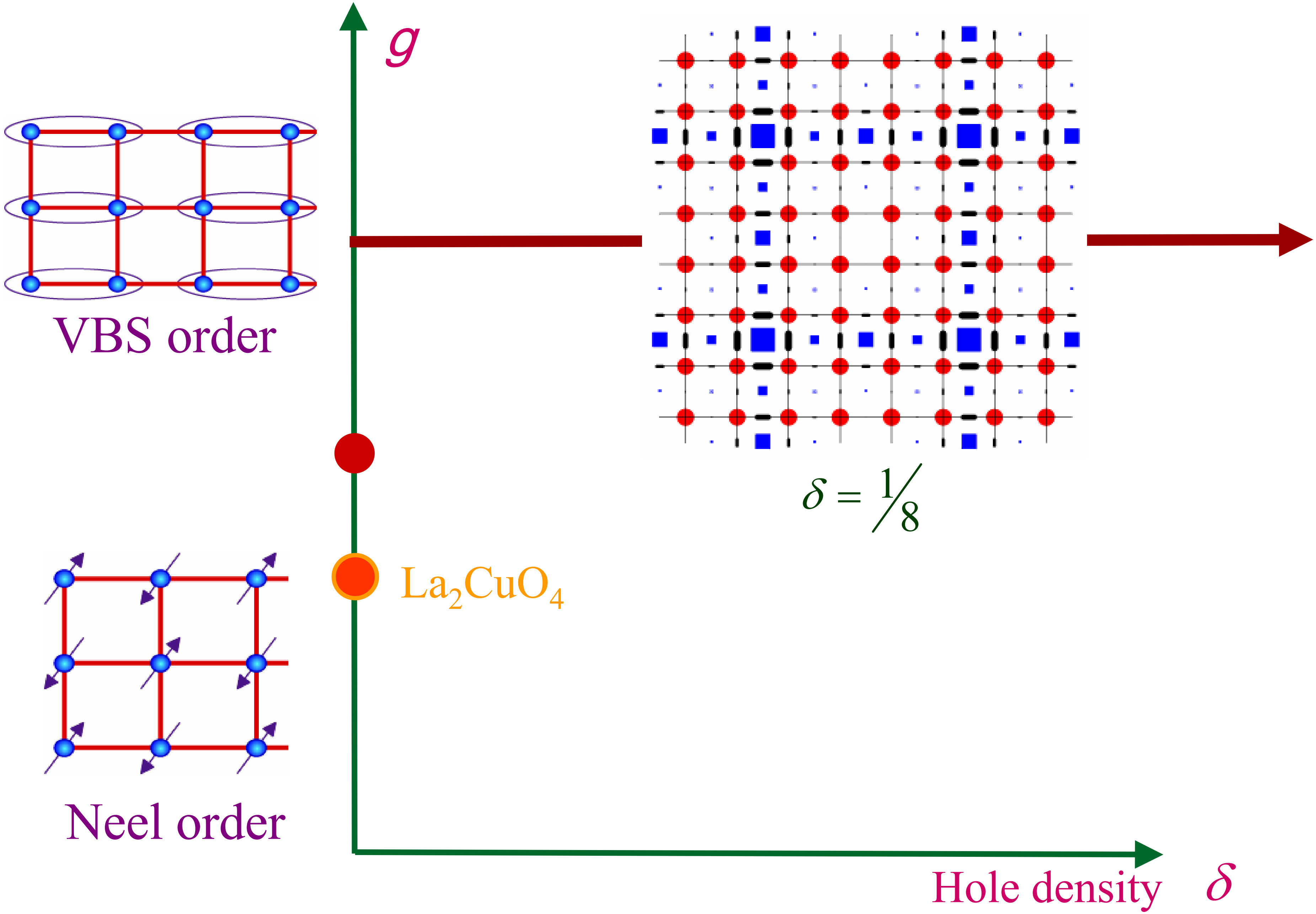

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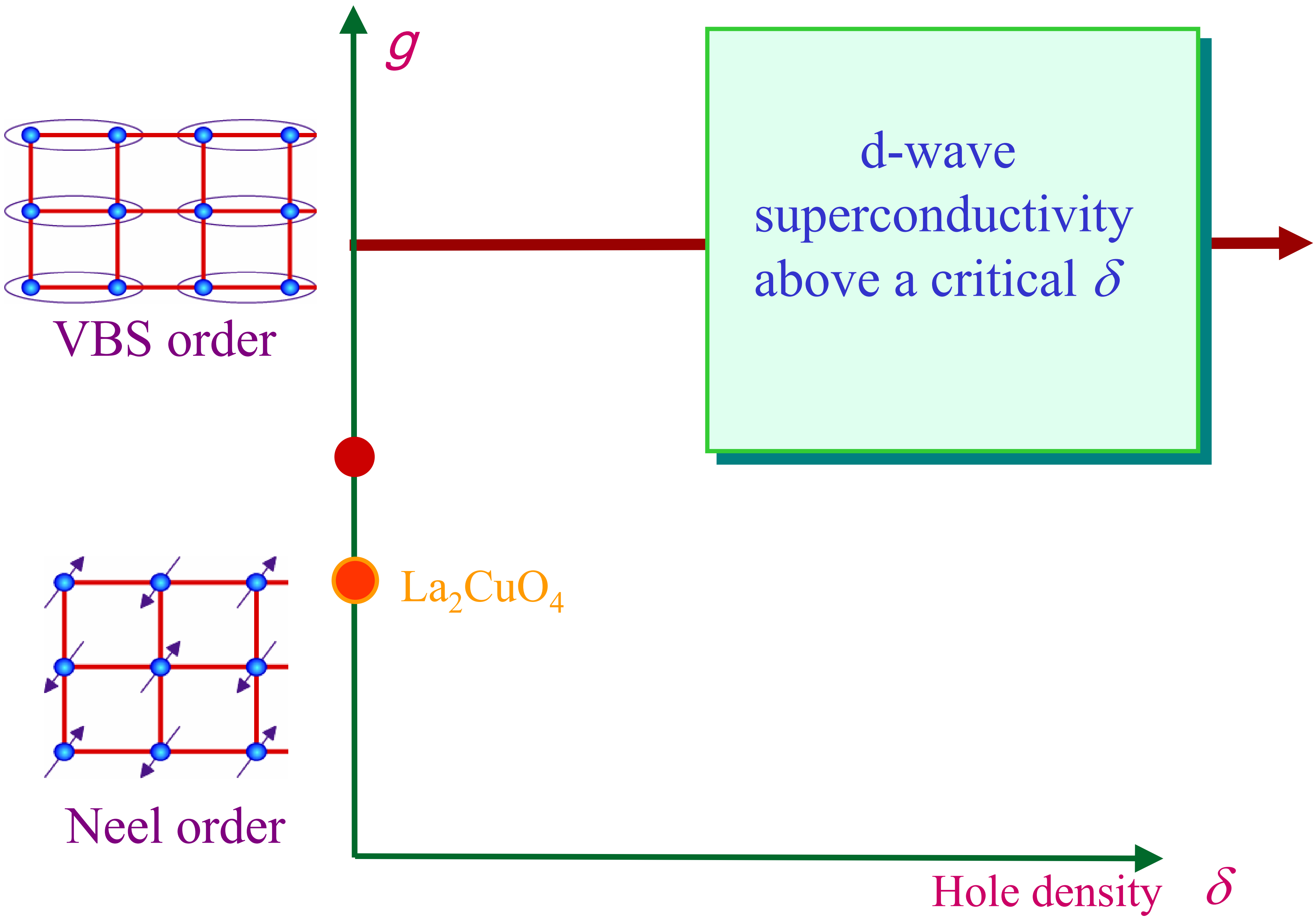

(B.2) Dual vortex theory of doped "staggered flux" spin liquid

We consider a d-wave superconductor described as a doped "staggered flux" spin liquid in the SU(2) gauge theory formulation. We wish to describe quantum fluctuations in such a superconductor near a transition to a Mott insulator. The Mott insulator has hole density δ_{MI}, with

$$\frac{\delta_{MI}}{2} = \frac{p}{q},$$

with p, q relatively prime integers.

The dual theory shows that there are a pair of q complex vortex fields $\varphi_{1\ell}$ and $\varphi_{2\ell}$, which are dual to the two species of bosons, b_1, b_2 of the SU(2) gauge theory. These are coupled to 2 non-compact U(1) gauge fields: A_μ (whose flux represents the superflow), and B_μ (whose Chern-Simons dual is coupled to the nodal fermions).

(B.2) Dual vortex theory of doped "staggered flux" spin liquid

The effective action for the theory is:

$$
\begin{aligned}
\mathcal{S}_{sf} &= \mathcal{S}_v + \mathcal{S}_A \\
\mathcal{S}_v &= \int d^2r d\tau \sum_{\ell=0}^{q-1} \Bigg[h_s (-1)^\ell \Bigg\{ \varphi^*_{1,\ell+q/2} \left(\frac{\partial}{\partial \tau} - iA_\tau - iB_\tau \right) \varphi_{1\ell} \\
&\qquad - \varphi^*_{2,\ell+q/2} \left(\frac{\partial}{\partial \tau} - iA_\tau + iB_\tau \right) \varphi_{2\ell} \Bigg\} \\
&\qquad + \left| (\partial_i - iA_i - iB_i) \varphi_{1\ell} \right|^2 + s |\varphi_{1\ell}|^2 \\
&\qquad + \left| (\partial_i - iA_i + iB_i) \varphi_{2\ell} \right|^2 + s |\varphi_{2\ell}|^2 \Bigg] \\
\mathcal{S}_A &= \int \frac{d^3k}{8\pi^3} \Bigg[\frac{k^2}{2e^2} A_\mu(-k) A_\nu(k) \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \\
&\qquad + \frac{|k|}{2\lambda} B_\mu(-k) B_\nu(k) \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Bigg]
\end{aligned}
$$

There are also additional "monopole" terms which are not shown.

(B.2) Dual vortex theory of doped "staggered flux" spin liquid

Main (preliminary) results:

- Formation of vortex-anti-vortex bound states implies transitions occurs first into a *supersolid*.
- Density wave order in the supersolid is enhanced by the "staggered flux" at wavevectors

$$\mathbf{Q}_{mn} = \frac{2\pi p}{q}(m, n), \text{ with } m + n \ \underline{odd}$$

Conclusions

I. Superfluids near commensurate insulators with "boson" density *p/q* have *q* species of vortices. The projective transformations of these vortices under the lattice space group defines a "quantum order" which distinguishes superfluids from each other. (Note: only the density of the insulator, and *not* the superfluid, is exactly *p/q*).

II. Vortices carry the quantum numbers of *both* superconductivity *and* the square lattice space group (in a projective representation).

III. Vortices carry halo of density wave order, and pinning of vortices/anti-vortices leads to a unified theory of STM modulations in zero and finite magnetic fields.

IV. Field theory of vortices with projective symmetries describes superfluids with precursor fluctuations of density wave order and its transitions to supersolids and insulators.