SYK models and black holes

Black Hole Initiative Colloquium
Harvard, October 25, 2016

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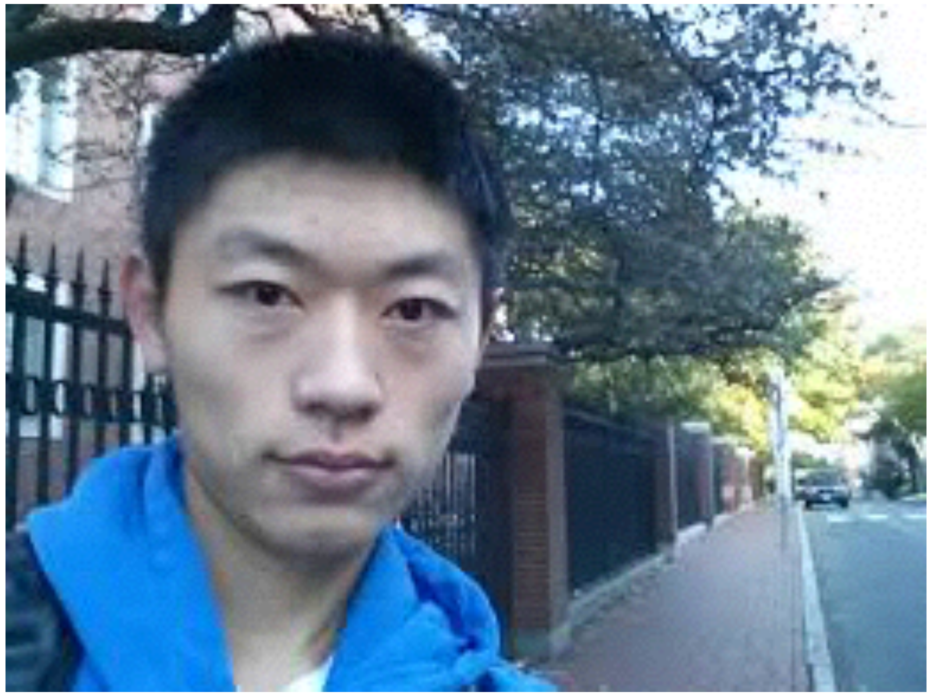
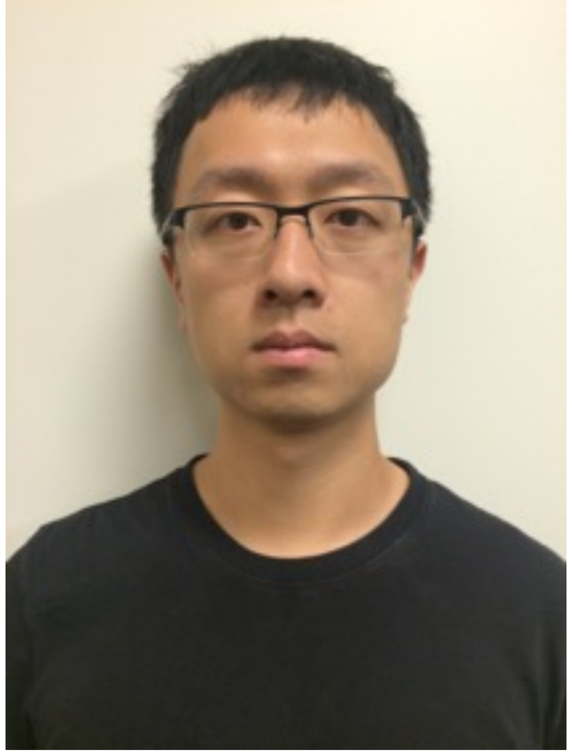


Wenbo Fu, Harvard



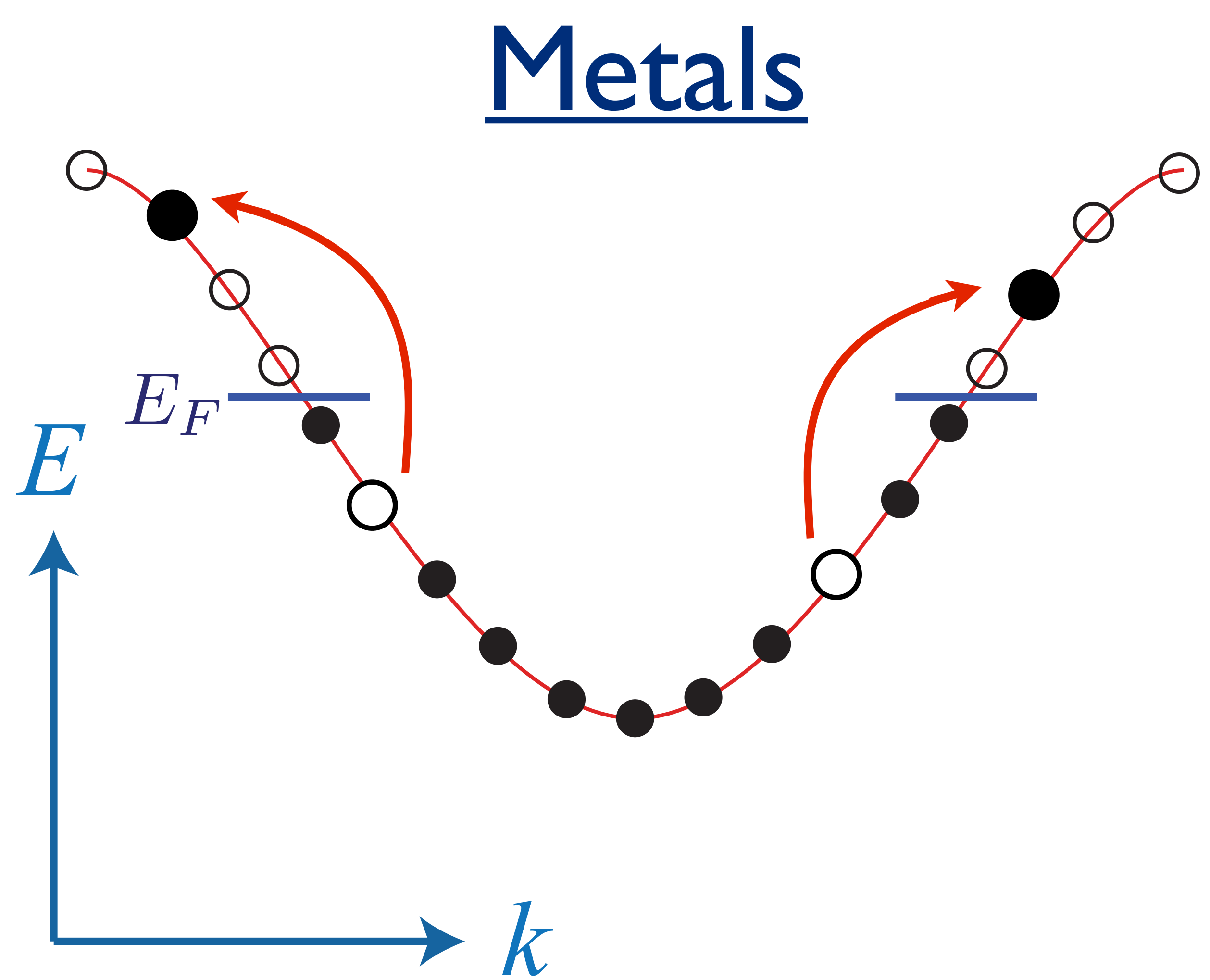
Yingfei Gu, Stanford

Richard Davison, Harvard

Conventional quantum matter:

1. Ground states connected adiabatically to independent electron states

2. Boltzmann-Landau theory of quasiparticles



Topological quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. Boltzmann-Landau theory of quasiparticles

The fractional quantum Hall effect: the ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Quantum matter without quasiparticles:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

Strange metals:

Such metals are found, most prominently, near optimal doping in the the cuprate high temperature superconductors.

Quantum matter without quasiparticles:

1. Ground states disconnected from independent electron states: many-particle entanglement

2. No quasiparticles

Strange metals:

Such metals are found, most prominently, near optimal doping in the the cuprate high temperature superconductors.

But how can we be sure that no quasiparticles exist in a given system? Perhaps there are some exotic quasiparticles inaccessible to current experiments……..

Local thermal equilibration or phase coherence time, τ_φ:

- There is an *lower bound* on τ_φ in all many-body quantum systems of order $\hbar/(k_B T)$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

  and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, τ_φ is parametrically larger at low T;
  e.g. in Fermi liquids $\tau_\varphi \sim 1/T^2$,
  and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where Δ is the energy gap.

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

A bound on quantum chaos:

- The time over which a many-body quantum system becomes "chaotic" is given by $\tau_L = 1/\lambda_L$, where λ_L is the "Lyapunov exponent" determining memory of initial conditions. This LYAPUNOV TIME obeys the rigorous lower bound

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

A. I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)

J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

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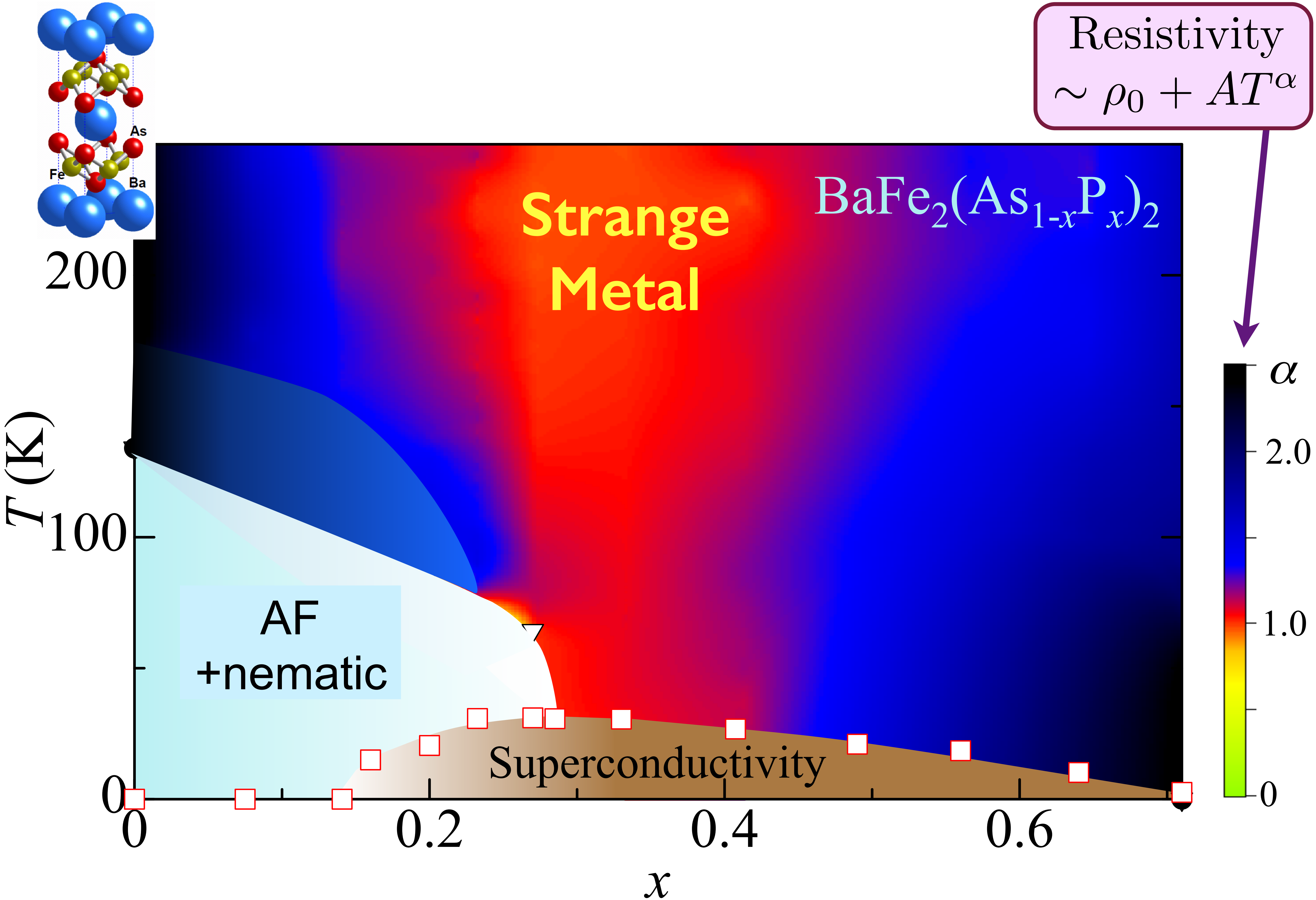
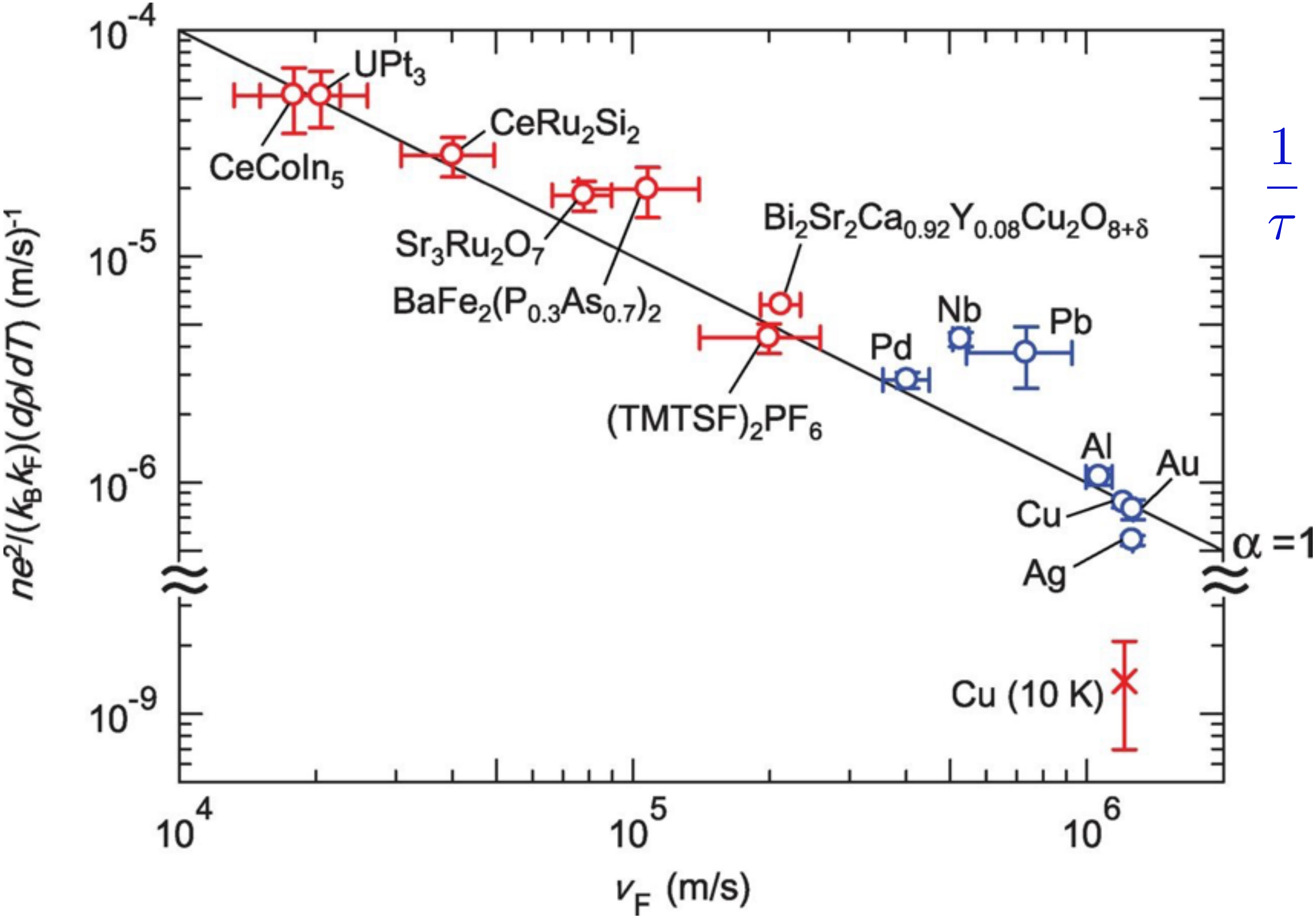
Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos



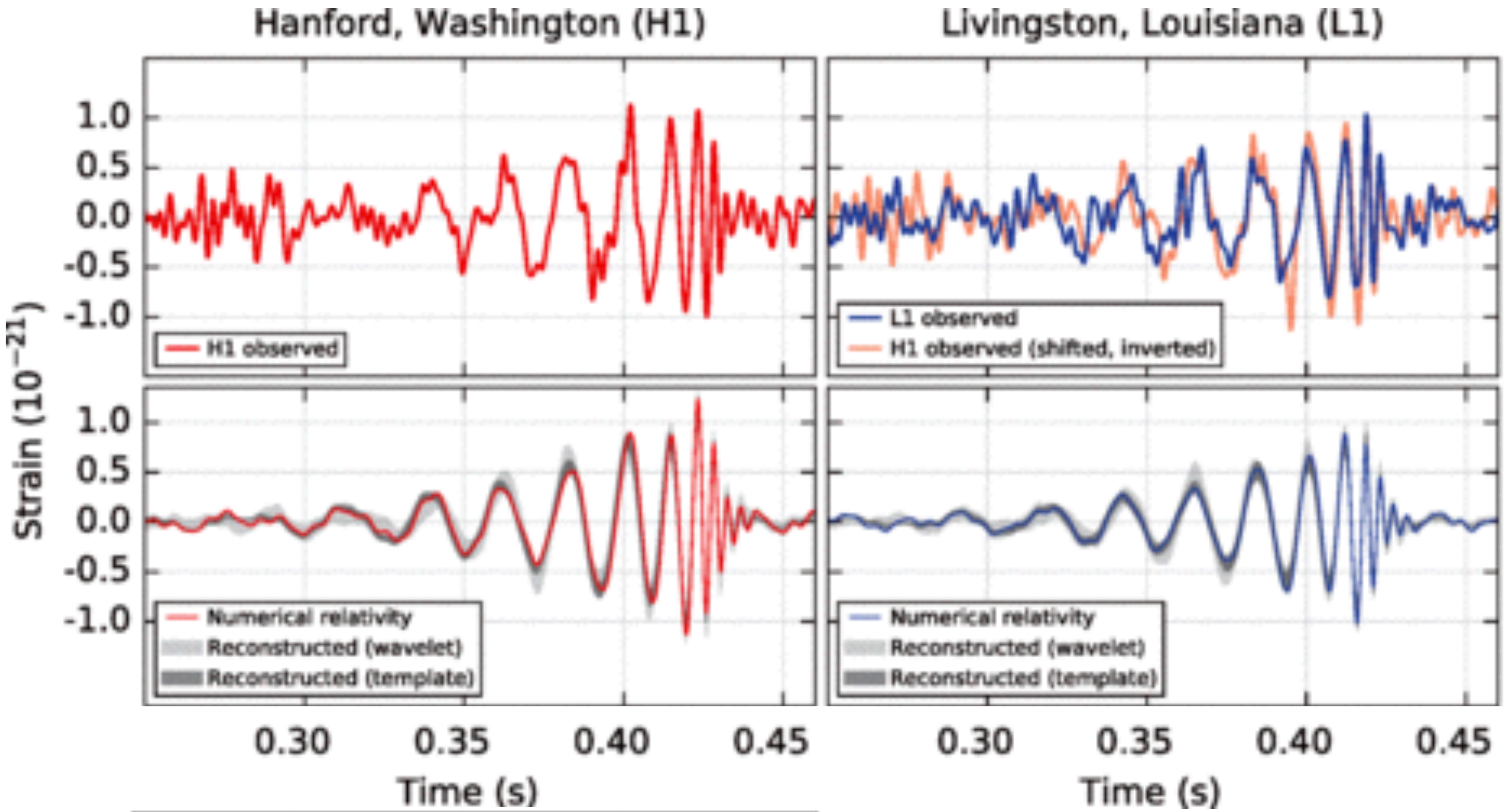
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)

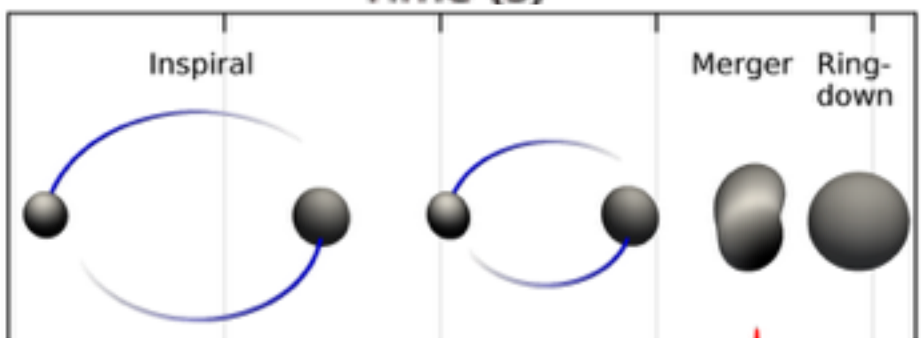
Strange metals



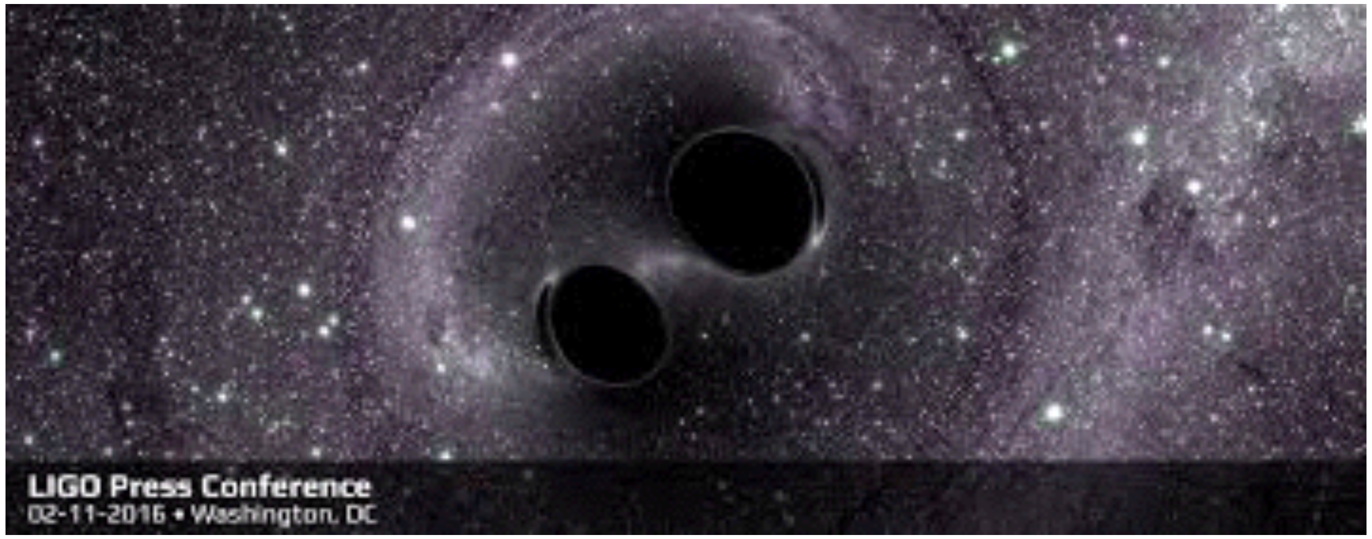
$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

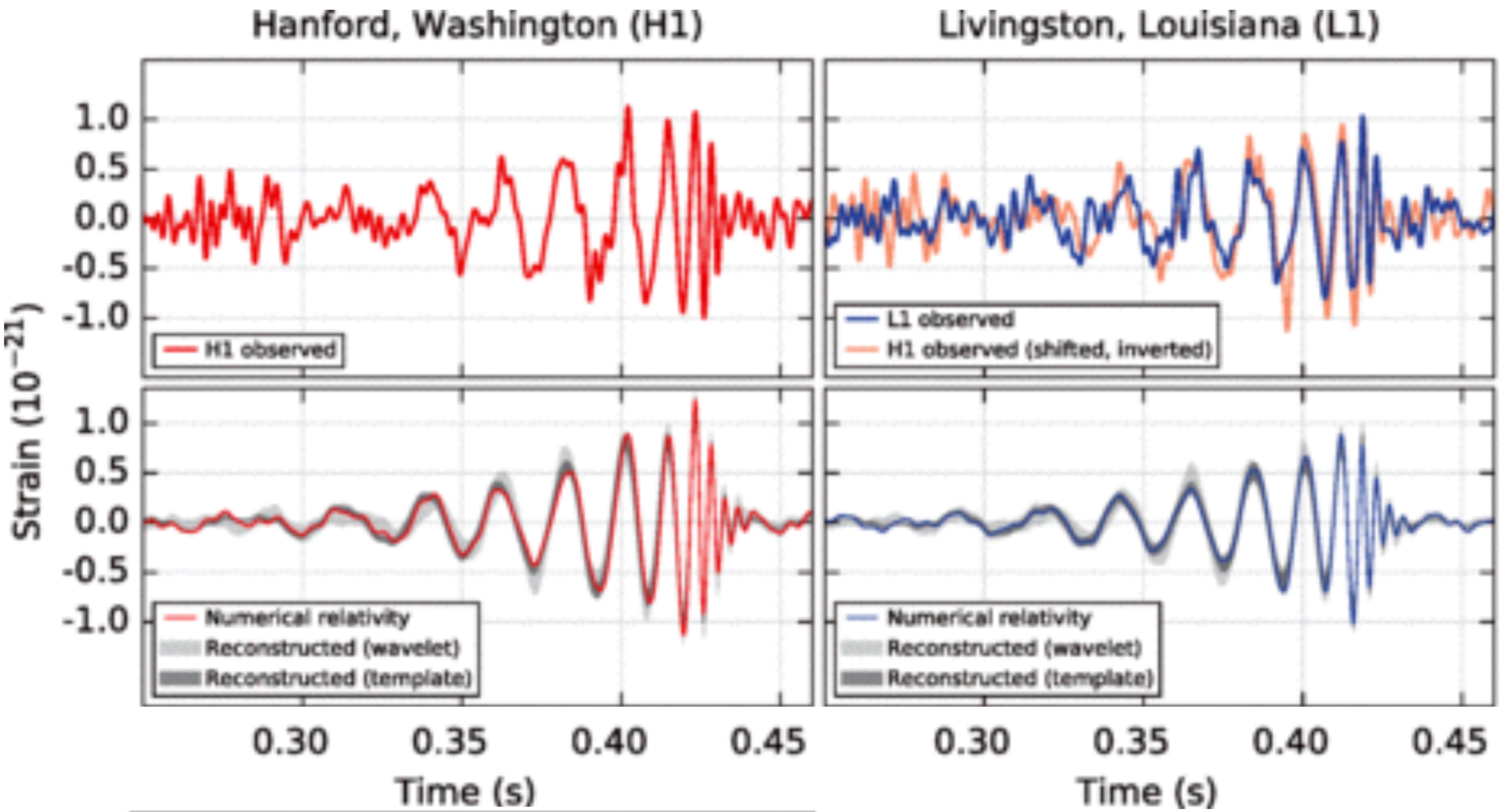
J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie, *Science*. **339,** 804 (2013)

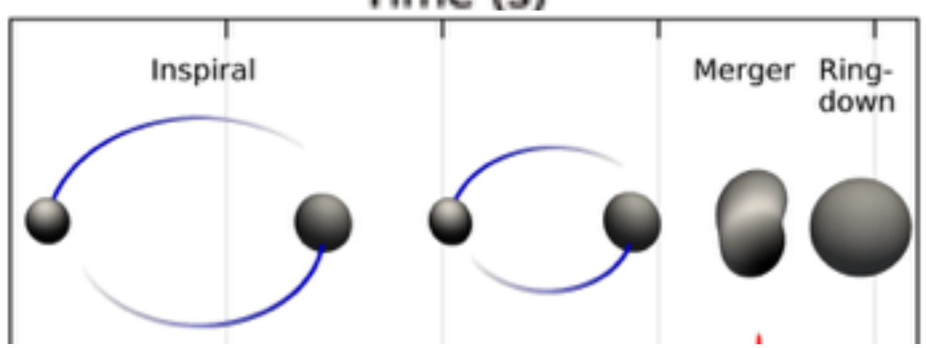




LIGO
September 14, 2015







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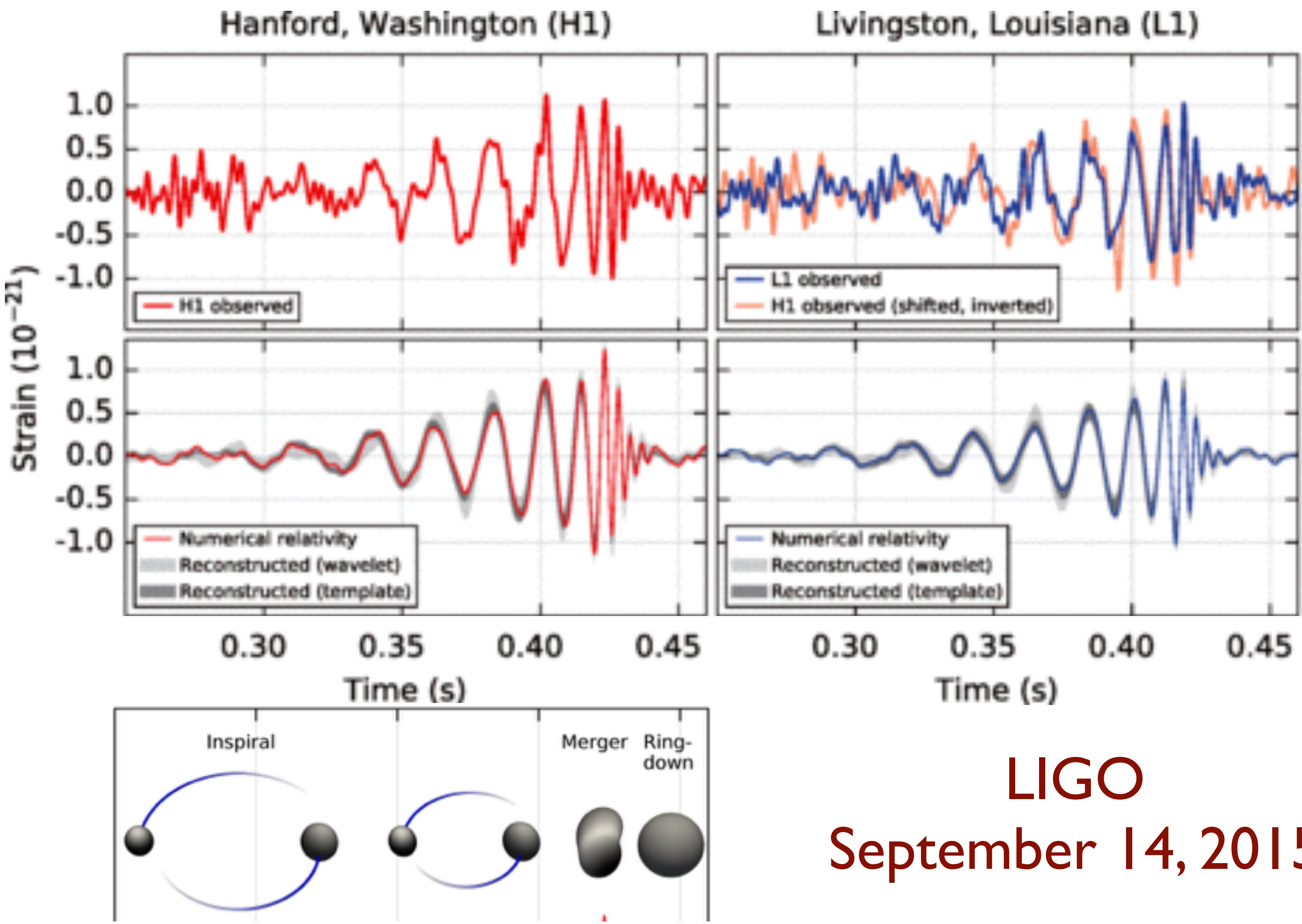
- Black holes have a "ring-down" time, τ_r, in which they radiate energy, and stabilize to a 'featureless' spherical object. This time can be computed in Einstein's general relativity theory.

- For this black hole $\tau_r = 7.7$ milliseconds. (Radius of black hole $= 183$ km; Mass of black hole $= 62$ solar masses.)



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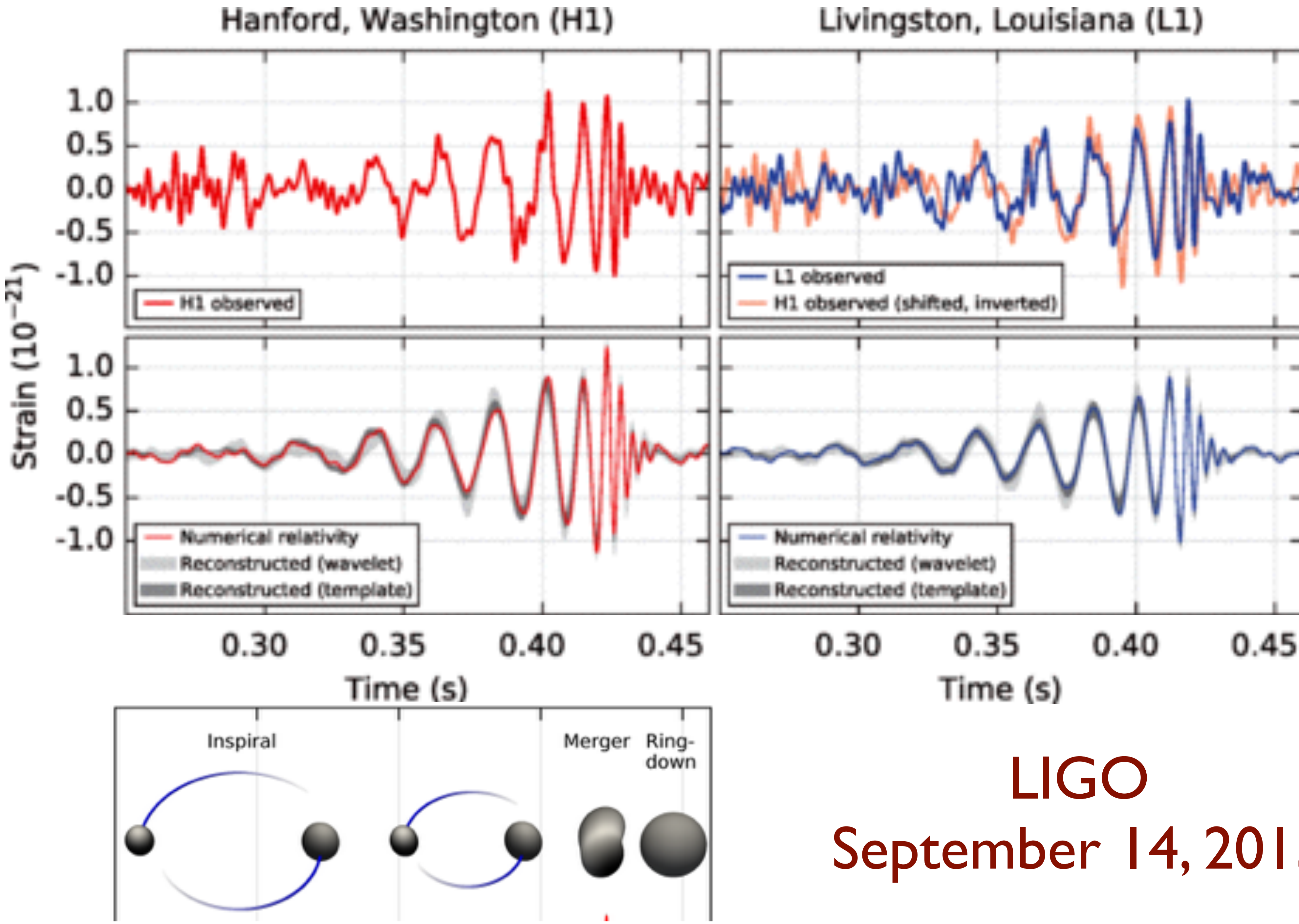
- 'Featureless' black holes have a Bekenstein-Hawking entropy, and a Hawking temperature, T_H.



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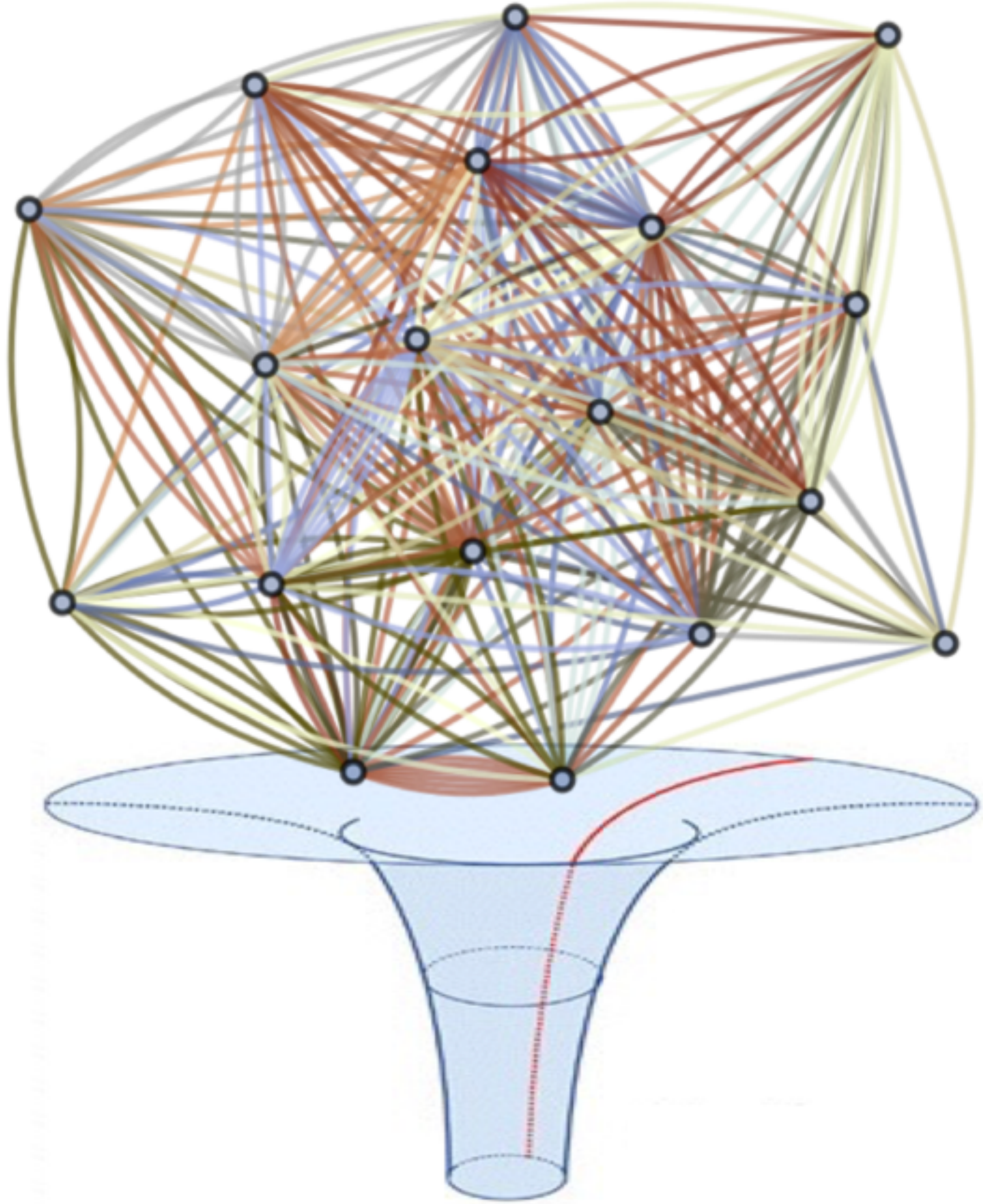
- Expressed in terms of the Hawking temperature, the ring-down time is $\tau_r \sim \hbar/(k_B T_H)$!

- For this black hole $T_H \approx 1$ nK.

The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2

Figure credit: L. Balents



The Sachdev-Ye-Kitaev (SYK) model:

- A theory of a strange metal
- Dual theory of gravity on AdS_2
- Fastest possible quantum chaos with $\tau_L = \dfrac{\hbar}{2\pi k_B T}$

A. Kitaev, unpublished
J. Maldacena and D. Stanford, arXiv:1604.07818

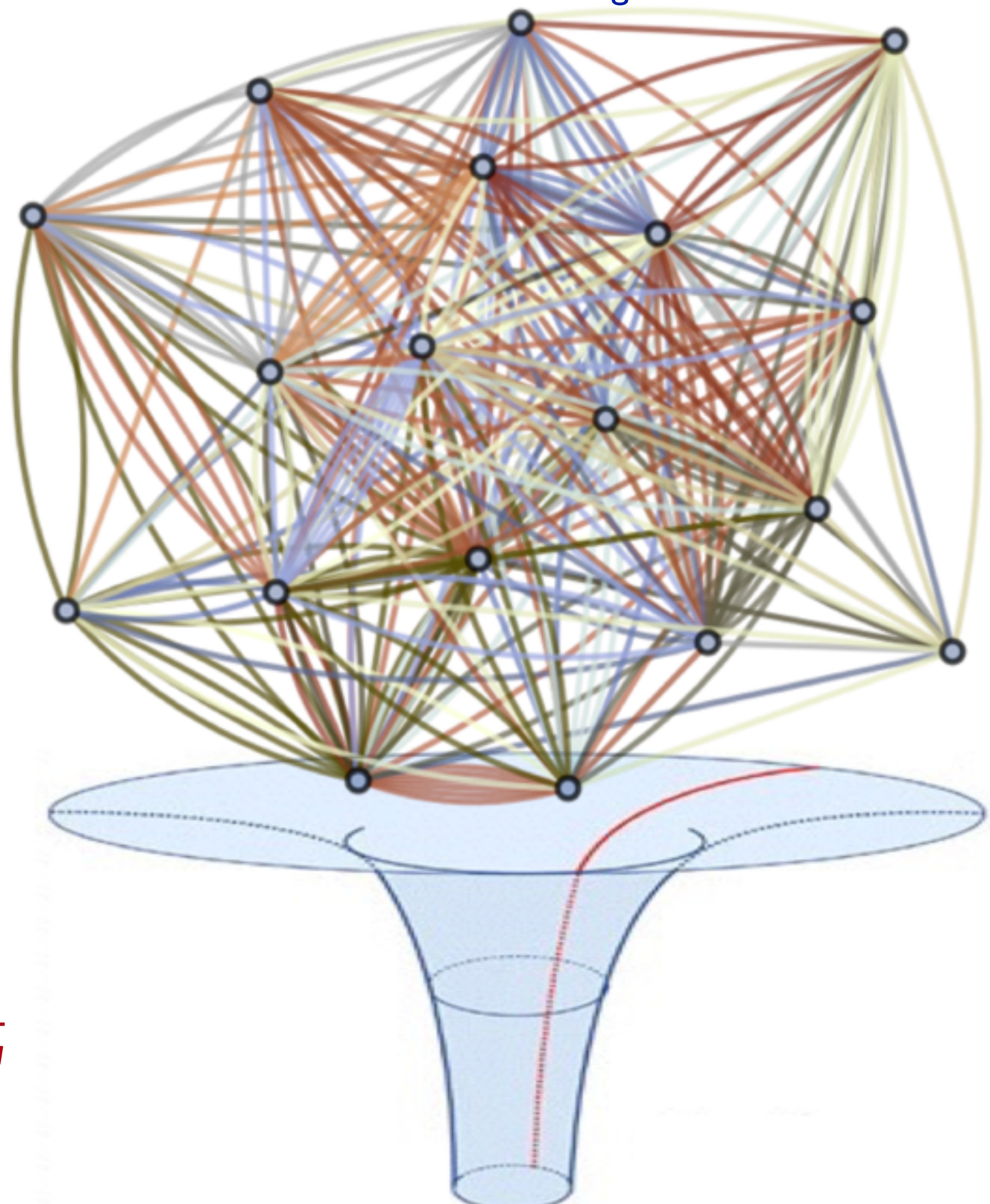


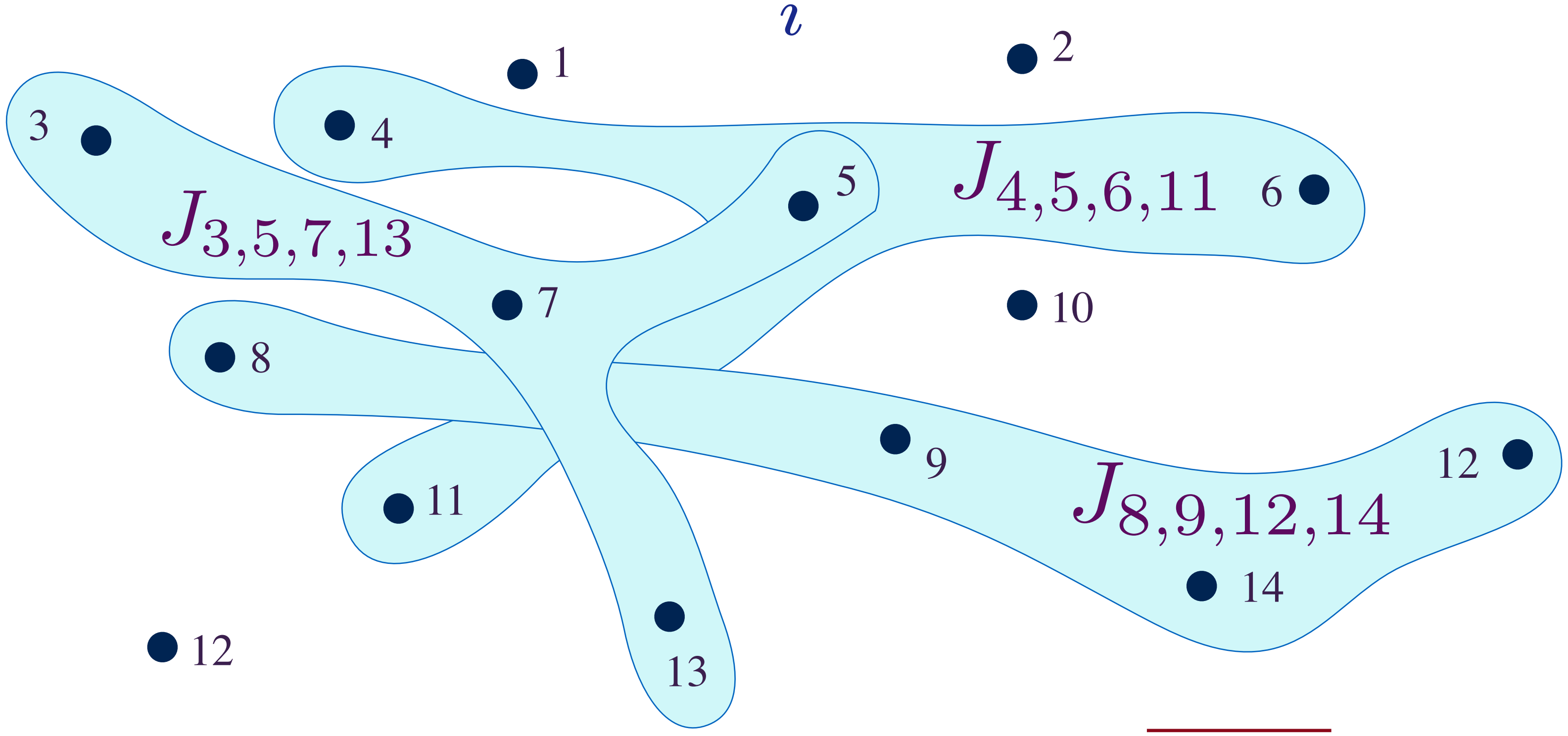
Figure credit: L. Balents

SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$



$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields critical strange metal.

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

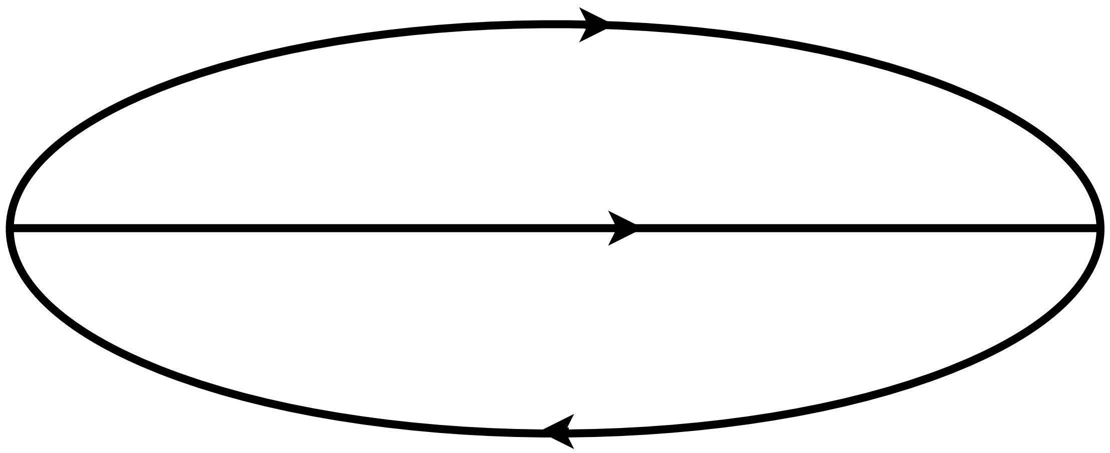
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK model

- $T = 0$ Green's function $G \sim \begin{cases} -1/\sqrt{\tau} & \text{for } \tau > 0 \\ e^{-2\pi\mathcal{E}}/\sqrt{\tau} & \text{for } \tau < 0 \end{cases}$

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- $T > 0$ Green's function has conformal invariance
  $G \sim e^{-2\pi\mathcal{E}T\tau}/(\sin(\pi T\tau))^{1/2}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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- Non-zero GPS entropy as $T \to 0$, $S(T \to 0) = NS_0 + \ldots$

A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

SYK and AdS_2

- $T = 0$ Green's function $G \sim \begin{cases} -1/\sqrt{\tau} & \text{for } \tau > 0 \\ e^{-2\pi\mathcal{E}}/\sqrt{\tau} & \text{for } \tau < 0 \end{cases}$

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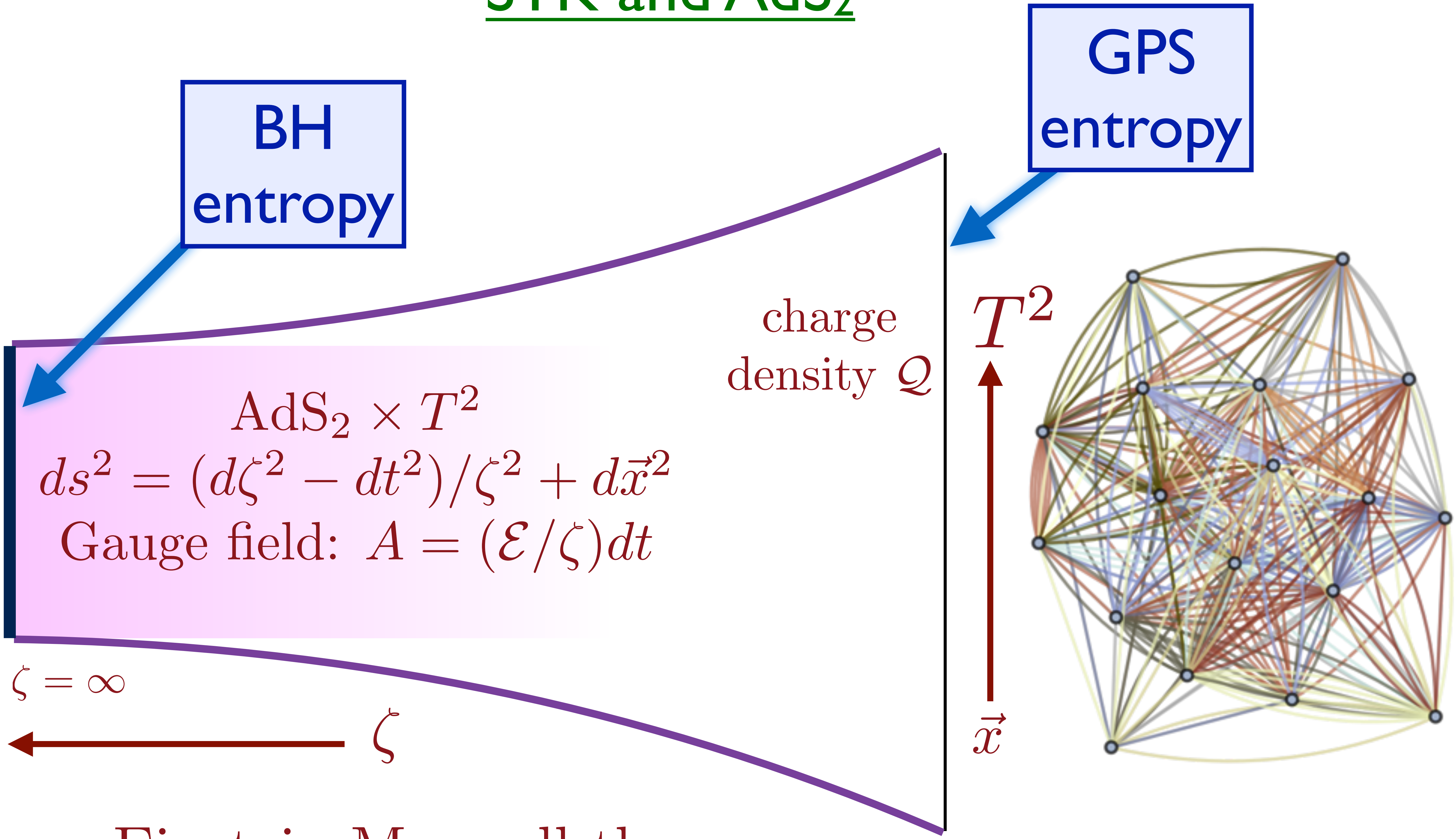
- SYK models are "are states of matter at non-zero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nördstrom black holes". The Bekenstein-Hawking entropy is NS_0 (GPS = BH).

S. Sachdev, PRL **105**, 151602 (2010)

- $\mathcal{E}$ is identified with the electric field on AdS_2. The relationship $(\partial S/\partial \mathcal{Q})_T = 2\pi\mathcal{E}$ is obeyed both by the GPS entropy, and by the BH entropy of AdS_2 horizons in a large class of gravity theories.

S. Sachdev, PRX **5**, 041025 (2015)

SYK and AdS$_2$



Einstein-Maxwell theory
+ cosmological constant

Mapping to SYK applies when temperature $\ll 1/(\text{size of } T^2)$

SYK and AdS_2

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

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$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

At frequencies $\ll J$, the $i\omega + \mu$ can be dropped, and without it equations are invariant under the reparametrization and gauge transformations

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = [f'(\sigma_1) f'(\sigma_2)]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

A. Kitaev, unpublished

S. Sachdev, PRX **5**, 041025 (2015)

SYK and AdS_2

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

A. Kitaev, unpublished

SYK and AdS_2

Let us write the large N saddle point solutions of S as

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Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism

- SL(2,R) is the isometry group of AdS_2.

A. Kitaev, unpublished

SYK and AdS_2

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So the (approximate) reparametrization symmetry is spontaneously broken.

Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action must vanish for $f(\tau) \in$ SL(2,R).

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

SYK and AdS_2

With $g(\tau) = e^{-i\phi(\tau)}$, the action for $\phi(\tau)$ and $f(\tau) = \dfrac{1}{\pi T}\tan(\pi T(\tau + \epsilon(\tau))$ fluctuations is

$$S_{\phi,f} = \frac{K}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau \epsilon)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \, \{f, \tau\},$$

where $\{f, \tau\}$ is the Schwarzian:

$$\{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 .$$

The couplings are given by thermodynamics: Ω is the grand potential, S_0 is the GPS entropy, and $\mathcal{Q}$ is the density.

$$K = -\left(\frac{\partial^2 \Omega}{\partial \mu^2}\right)_T \quad , \quad \gamma + 4\pi^2 \mathcal{E}^2 K = -\left(\frac{\partial^2 \Omega}{\partial T^2}\right)_\mu$$

$$2\pi\mathcal{E} = \frac{\partial S_0}{\partial \mathcal{Q}}$$

In holography: the γ term in the action has been obtained from theories on AdS_2; $\mathcal{E}$ is the electric field, and has the same relationship to S_0.

J. Maldacena and D. Stanford, arXiv:1604.07818; R. Davison, Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished;
S. Sachdev, PRX **5**, 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

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The correlators of the density fluctuations, $\delta\mathcal{Q}(\tau)$, and the energy fluctuations $\delta E - \mu\delta\mathcal{Q}(\tau)$ are time independent and given by

$$\left(\begin{array}{cc} \langle \delta\mathcal{Q}(\tau)\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle /T \\ \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))(\delta E(0) - \mu\delta\mathcal{Q}(0))\rangle /T \end{array} \right) = T\chi_s$$

where χ_s is the static susceptibility matrix given by

$$\chi_s \equiv \left(\begin{array}{cc} -(\partial^2\Omega/\partial\mu^2)_T & -\partial^2\Omega/(\partial T\partial\mu) \\ -T\partial^2\Omega/(\partial T\partial\mu) & -T(\partial^2\Omega/\partial T^2)_\mu \end{array} \right).$$

R. Davison, Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished

Coupled SYK models

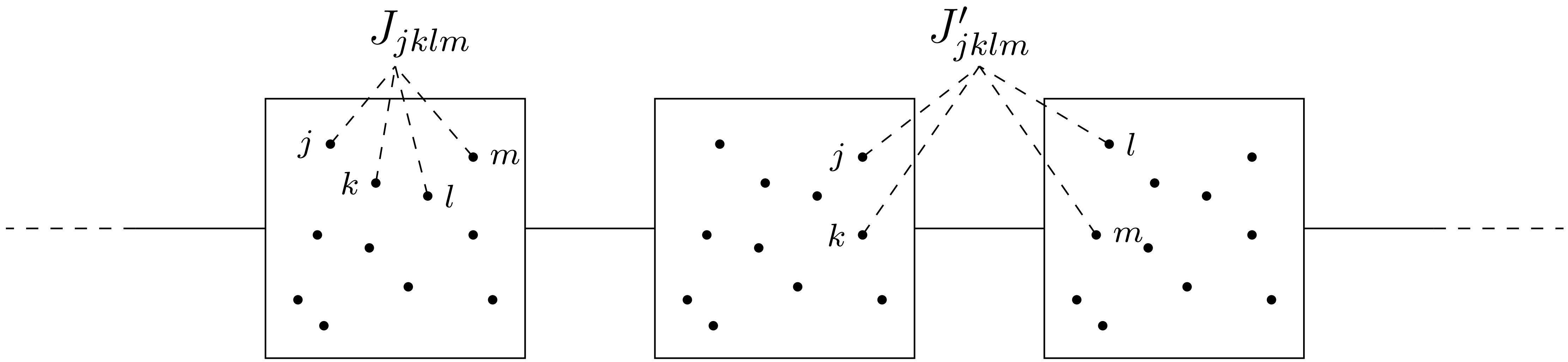

Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832

SYK models

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Coupled SYK models

$$\left(\begin{array}{cc} \langle \mathcal{Q};\mathcal{Q}\rangle_{k,\omega} & \langle E - \mu\mathcal{Q};\mathcal{Q}\rangle_{k,\omega}/T \\ \langle E - \mu\mathcal{Q};\mathcal{Q}\rangle_{k,\omega} & \langle E - \mu\mathcal{Q}; E - \mu\mathcal{Q}\rangle_{k,\omega}/T \end{array} \right) = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \left(\begin{array}{cc} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{array} \right) \chi_s^{-1}.$$

R. Davison, Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished

Coupled SYK models

$$\begin{pmatrix} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} /T \\ \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; E - \mu \mathcal{Q} \rangle_{k,\omega} /T \end{pmatrix} = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

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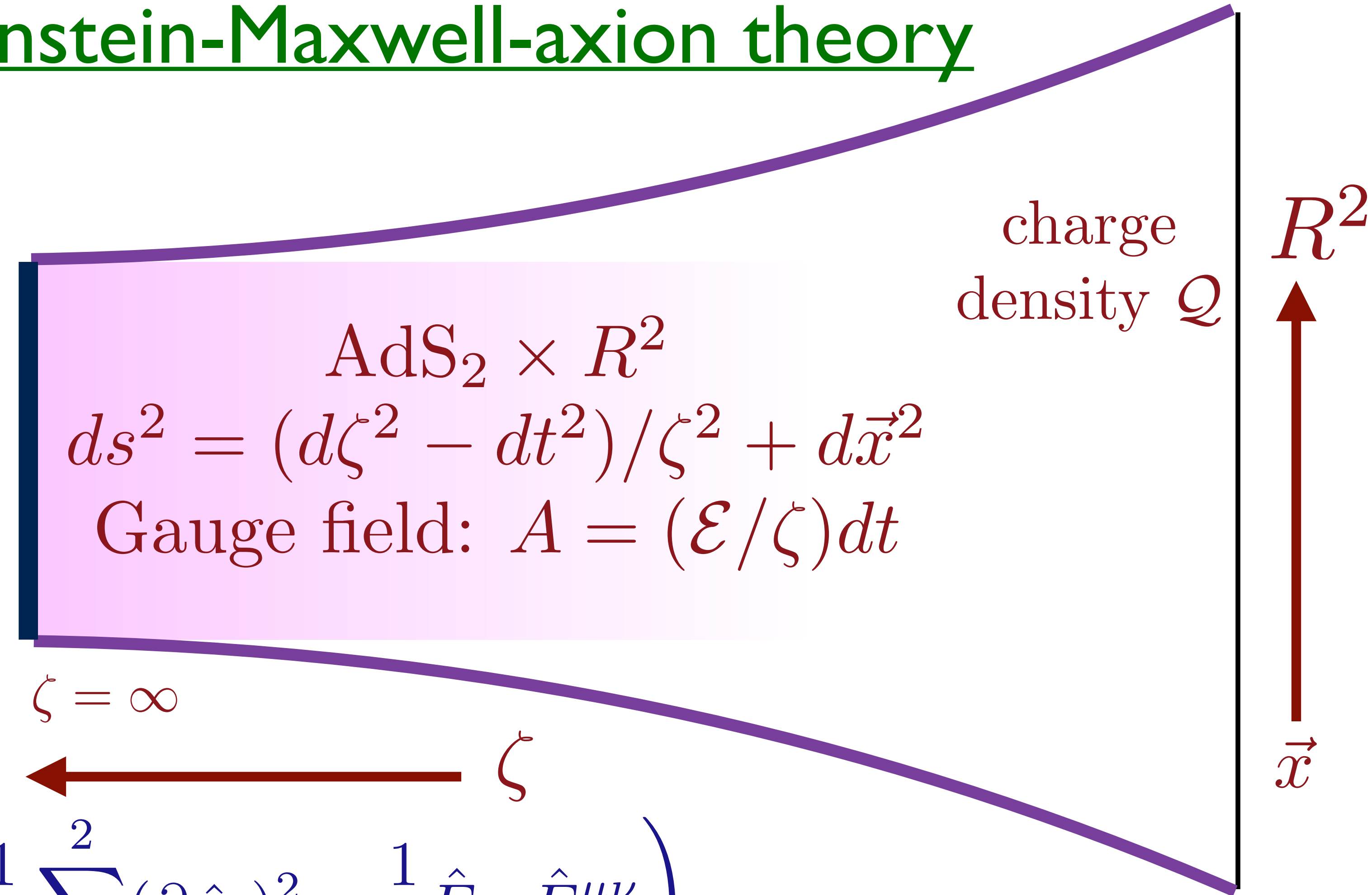
$$\alpha = \sigma \frac{\partial S_0}{\partial \mathcal{Q}}$$

The coupled SYK models realize a diffusive metal with no quasiparticle excitations.
(a "strange metal")

R. Davison, Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished

Holography: Einstein-Maxwell-axion theory



$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

- For $\hat{\varphi}_i = 0$, we obtain the Reissner-Nördstrom-AdS charged black hole, with a near-horizon $\mathrm{AdS}_2 \times R^2$ near-horizon geometry.

- For $\hat{\varphi}_i = k x_i$, we obtain a similar solution but with momentum dissipation (a bulk massive graviton). This yields the same diffusive metal correlators as the coupled SYK models, and the same relationship between α and σ.

Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054; D. Vegh, arXiv:1301.0537;
R.A. Davison, PRD 88 (2013) 086003; M. Blake and D. Tong, PRD 88 (2013), 106004;
T. Andrade and B. Withers, JHEP 05 (2014) 101; M. Blake, PRL **117**, 091601 (2016);
R. Davison, Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished

Entangled quantum matter without quasiparticles

- Is there a connection between strange metals and black holes?
  Yes, *e.g.* the SYK model.

- Why do they have the same equilibration time $\sim \hbar/(k_B T)$?
  Strange metals don't have quasiparticles and thermalize rapidly;
  Black holes are "fast scramblers".

- Theoretical predictions for strange metal transport in graphene agree well with experiments