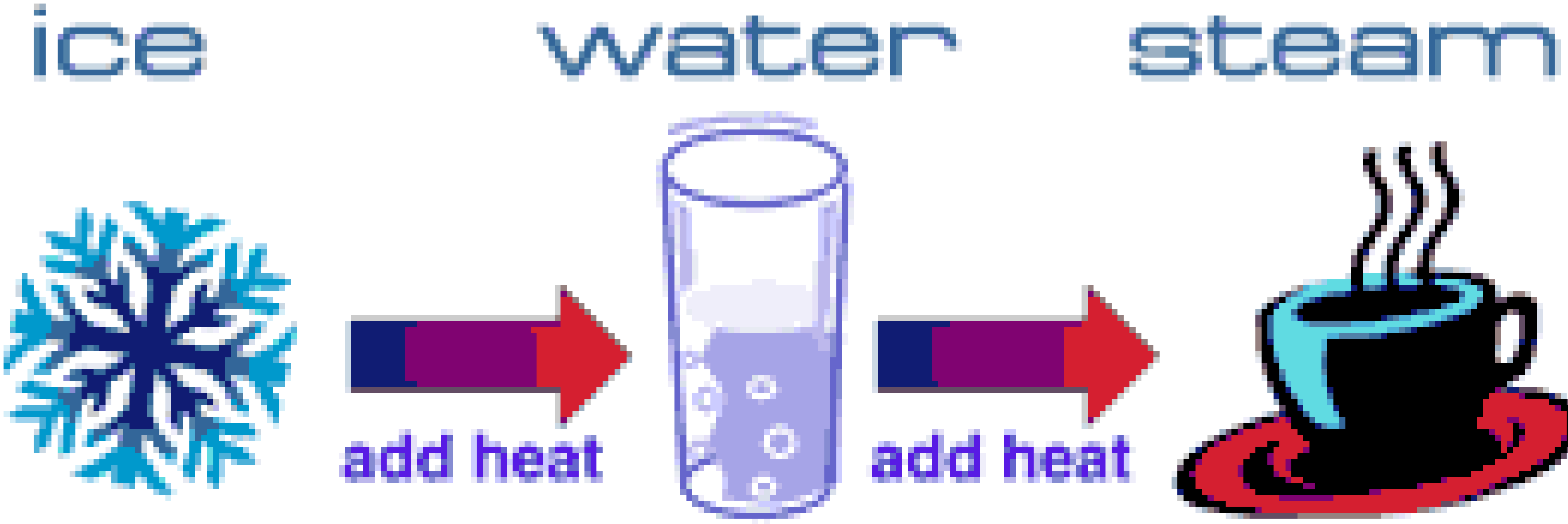
Quantum Phase Transitions

Subir Sachdev

Talks online at http://sachdev.physics.harvard.edu

What is a “phase transition” ?



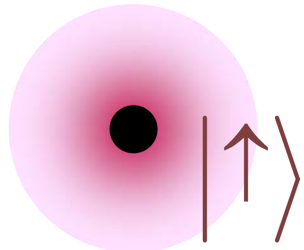
A change in the collective properties of a macroscopic number of atoms

What is a "*quantum phase transition*" ?

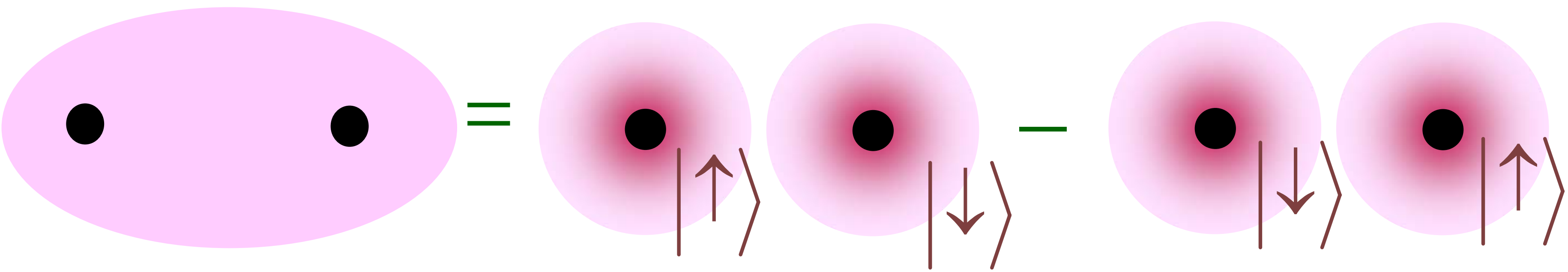
Change in the nature of entanglement in a macroscopic quantum system.

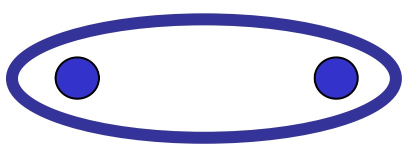
Entanglement

Hydrogen atom:



Hydrogen molecule:





$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Superposition of two electron states leads to non-local correlations between spins

Outline

Quantum phase transitions

1. Spin ordering in “Han purple”

2. Entanglement at the critical point: physical consequences at non-zero temperatures
   (a) Double-layer antiferromagnet
   (b) Superfluid-insulator transition
   (c) Hydrodynamics via mapping to quantum theory of black holes.

3. Entanglement of valence bonds

4. Conclusions

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Chinese Terracotta warriors (479-221 BC)

Han Purple – $BaCuSi_2O_6$

M. Jaime *et al.*, *Phys. Rev. Lett.* **93**, 087203 (2004)

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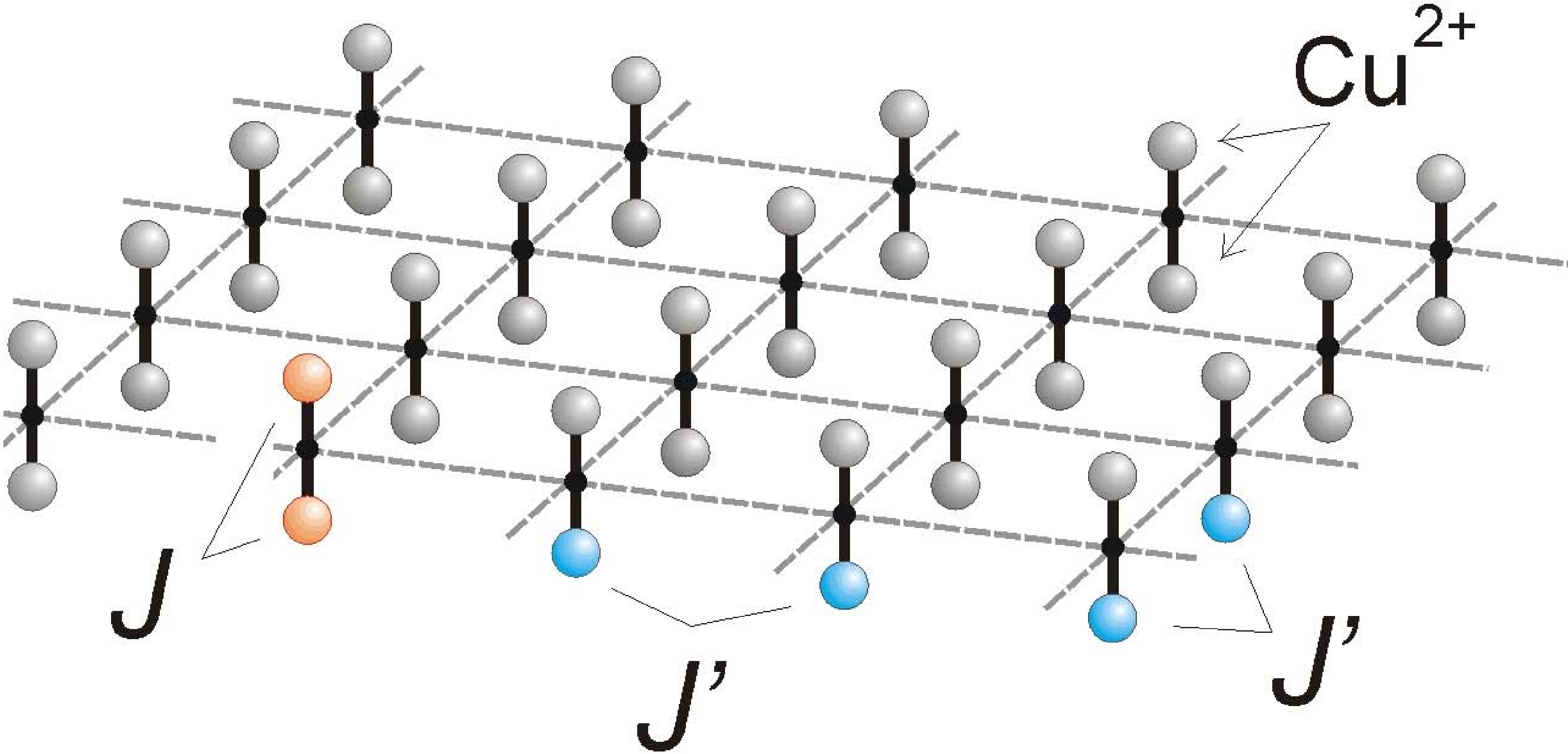

Each Cu^{2+} has a single free electron spin

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Weak magnetic field

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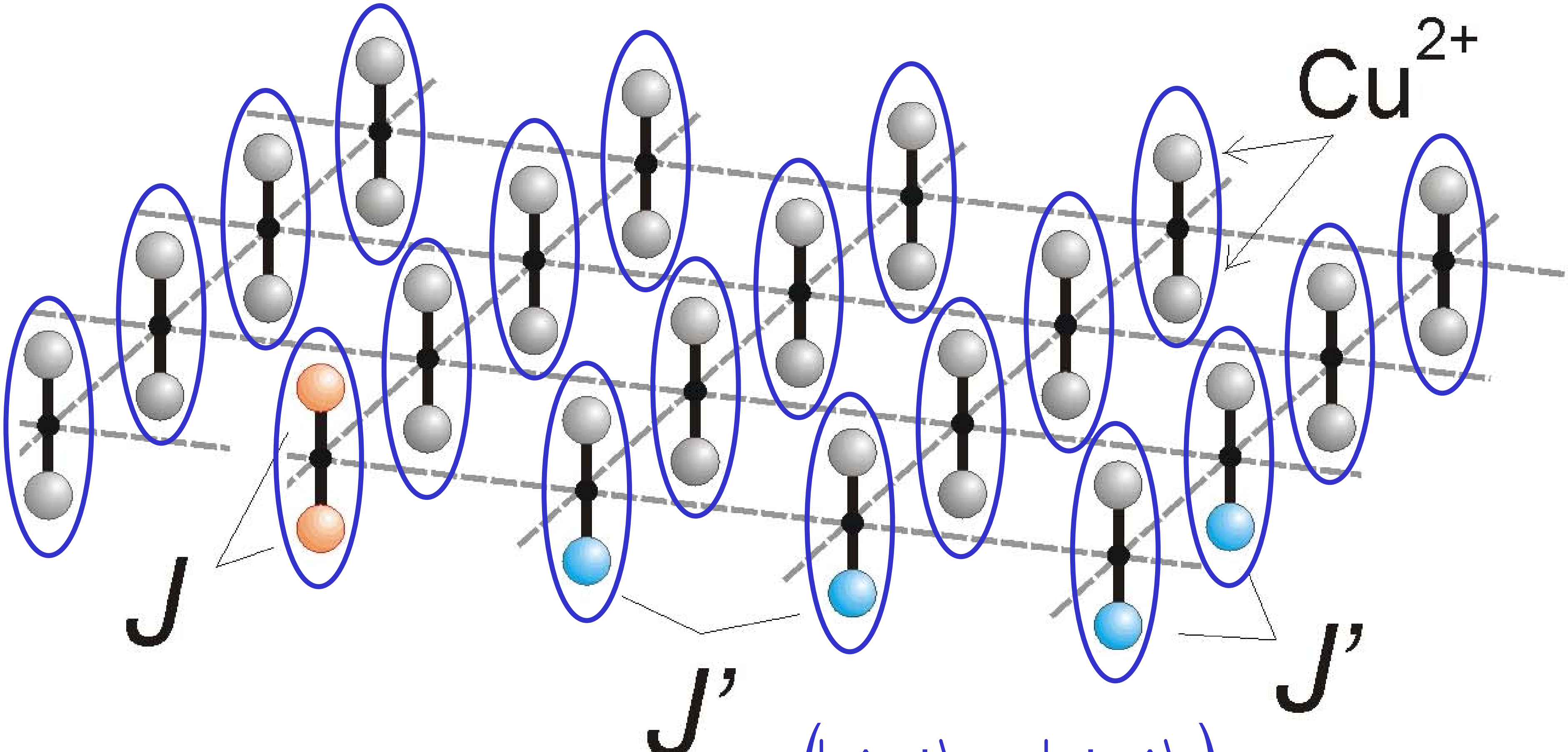

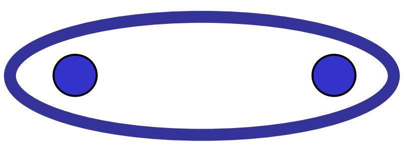

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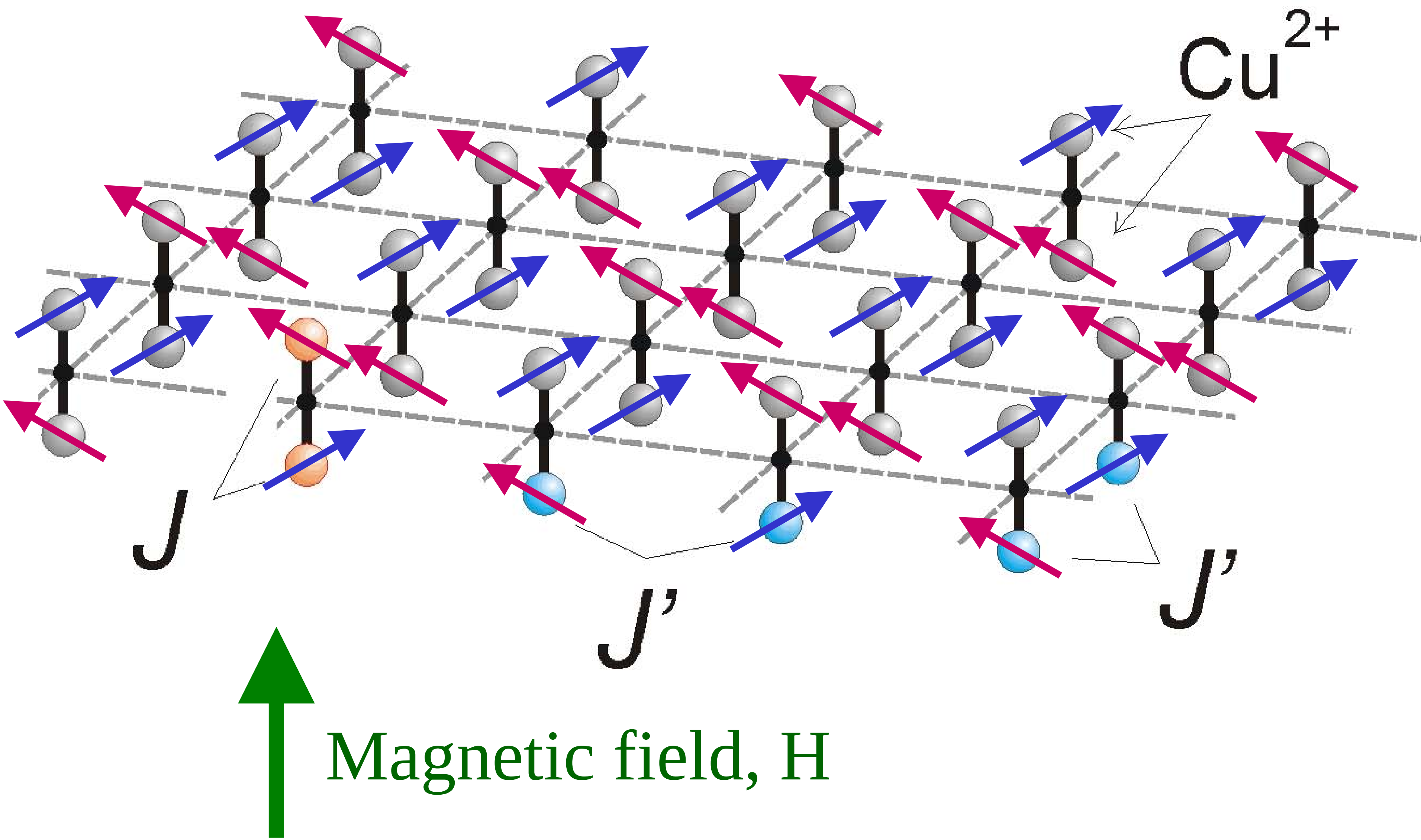

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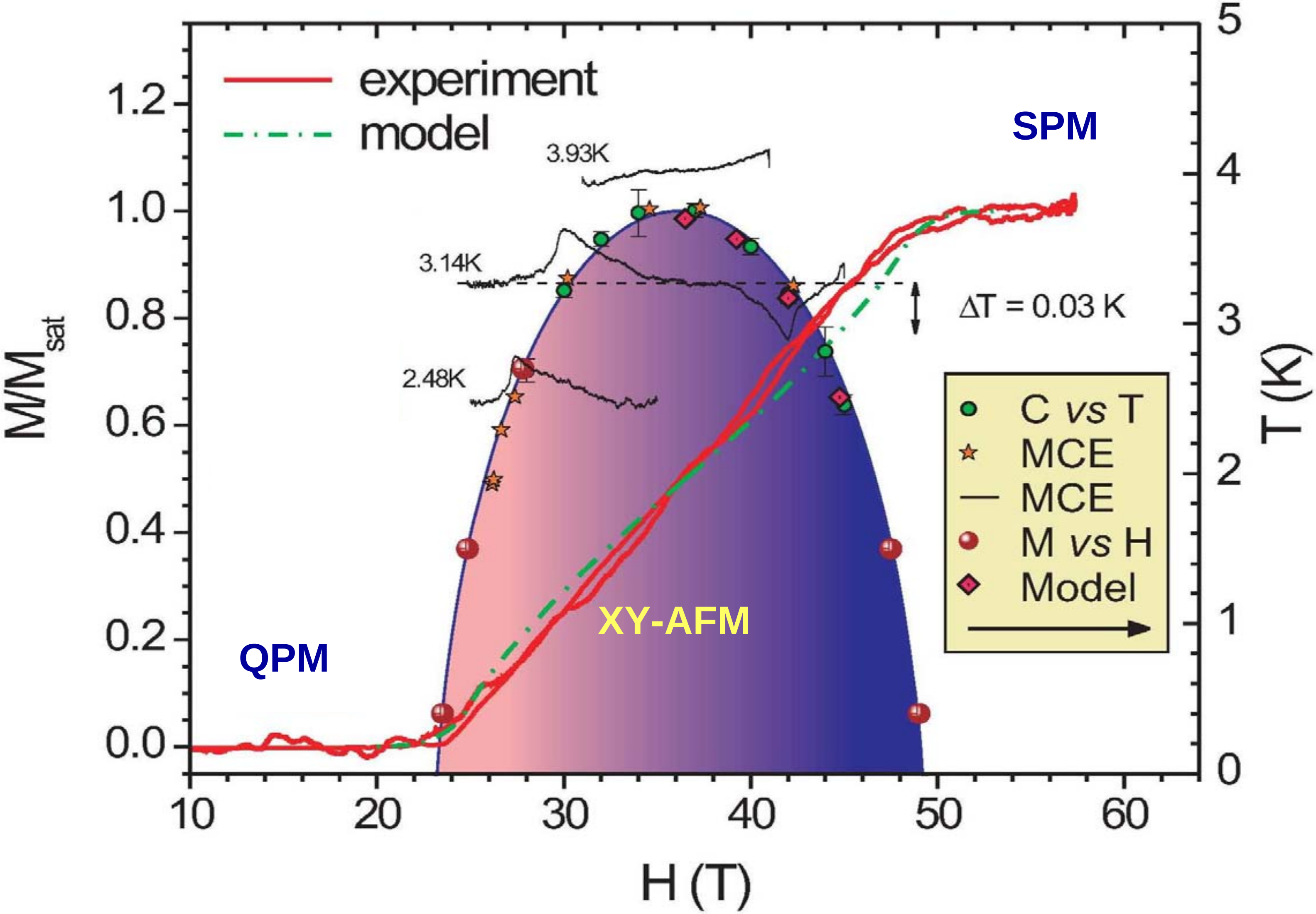

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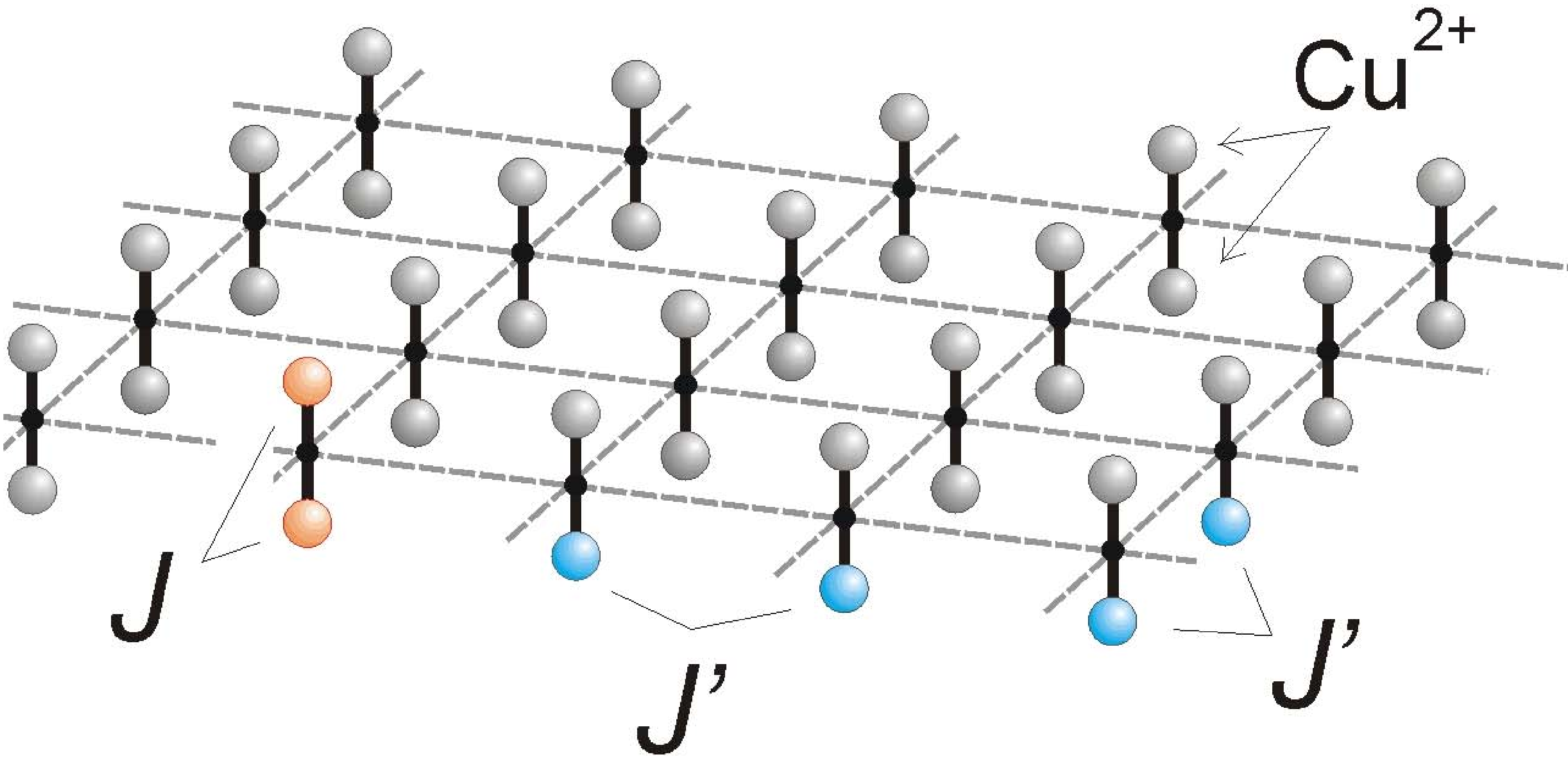
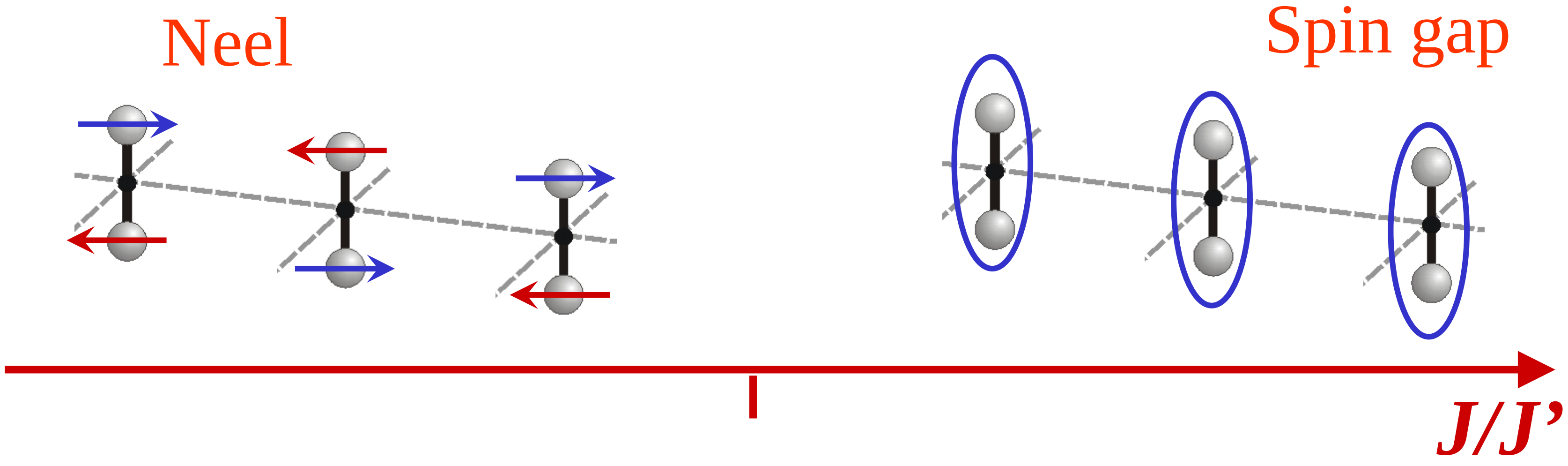


Each Cu^{2+} has a single free electron spin.

Vary the ratio ***J/J'***

M. Jaime *et al.*, *Phys. Rev. Lett.* **93**, 087203 (2004)



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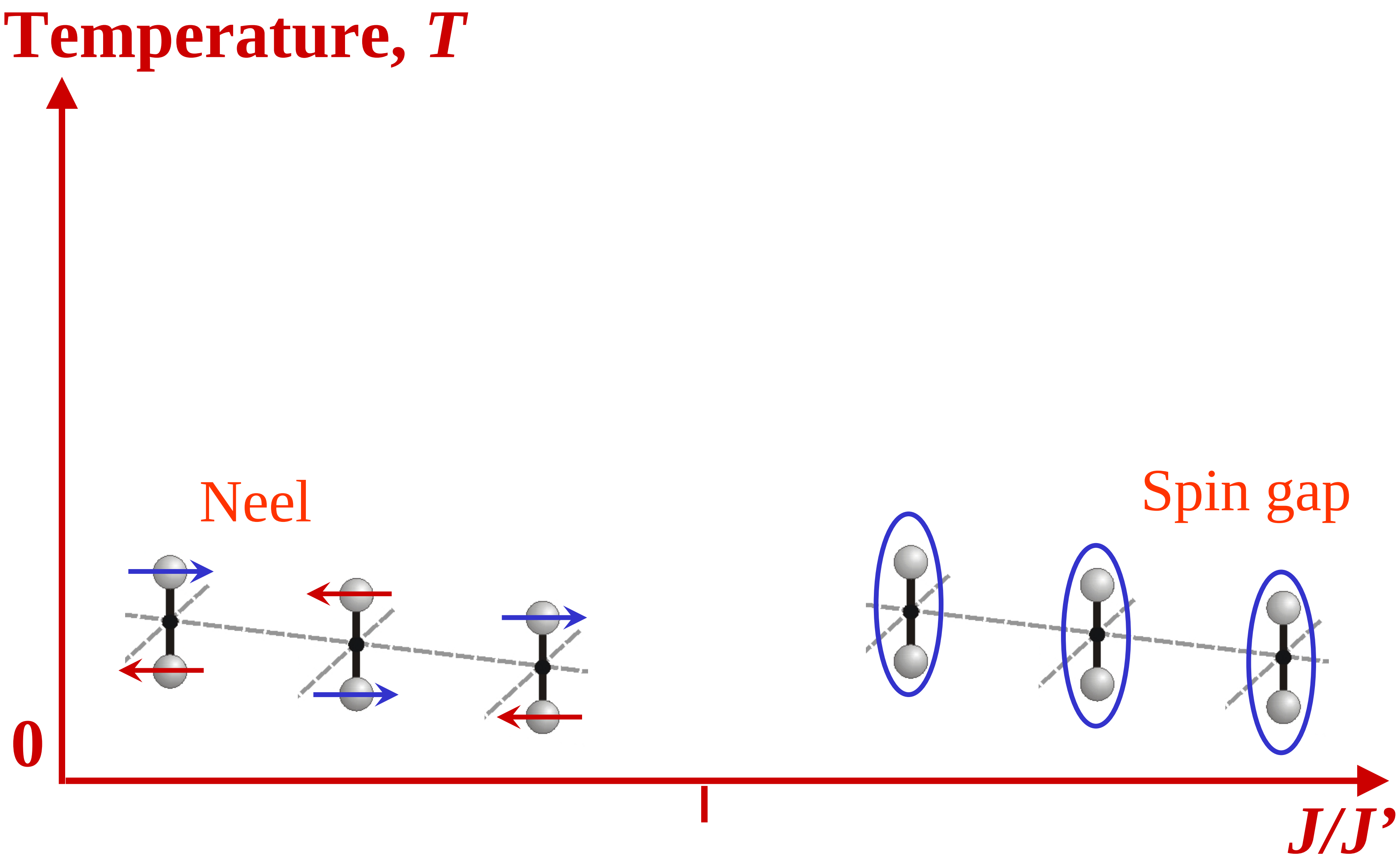
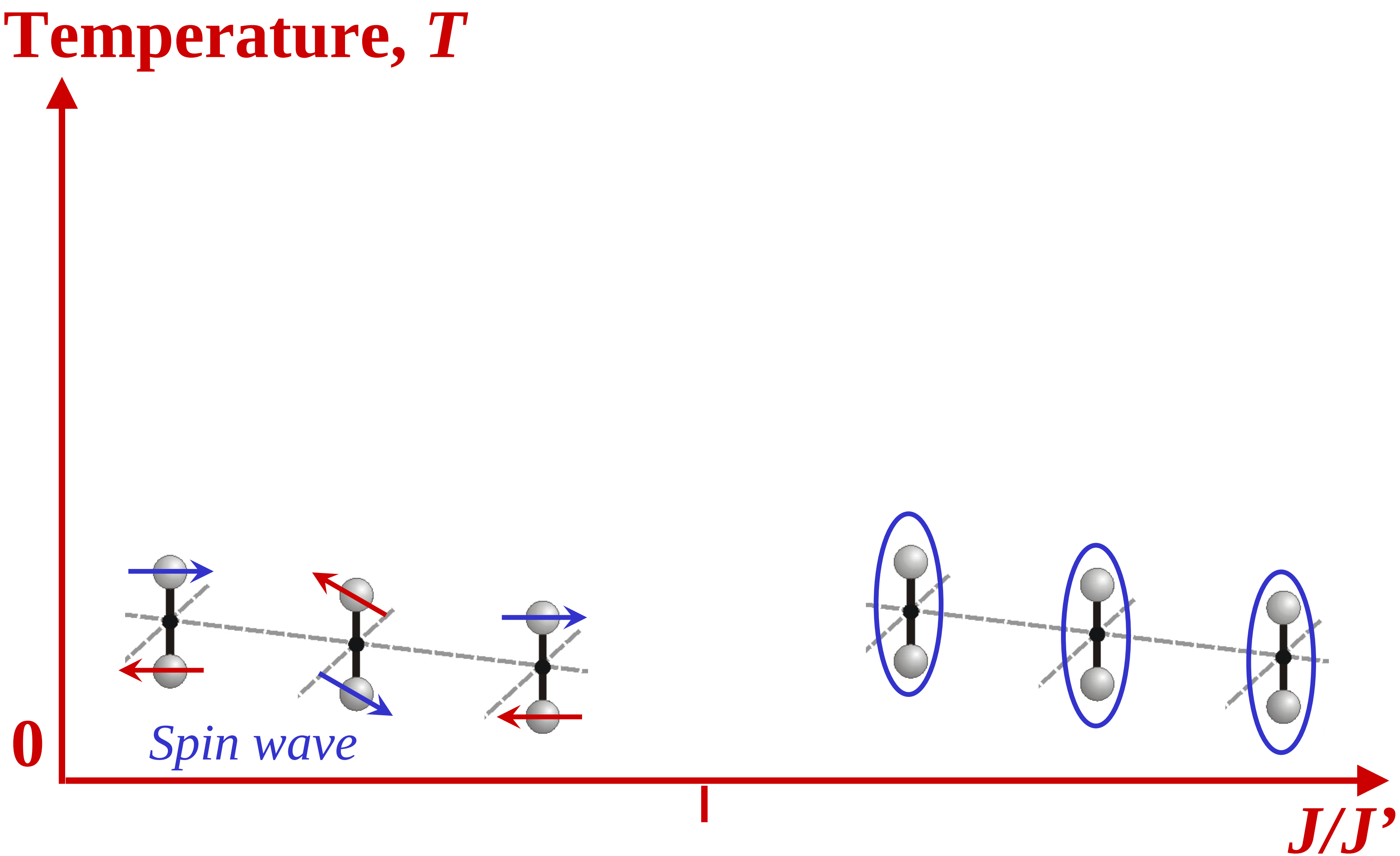


Vary the ratio *J/J'*



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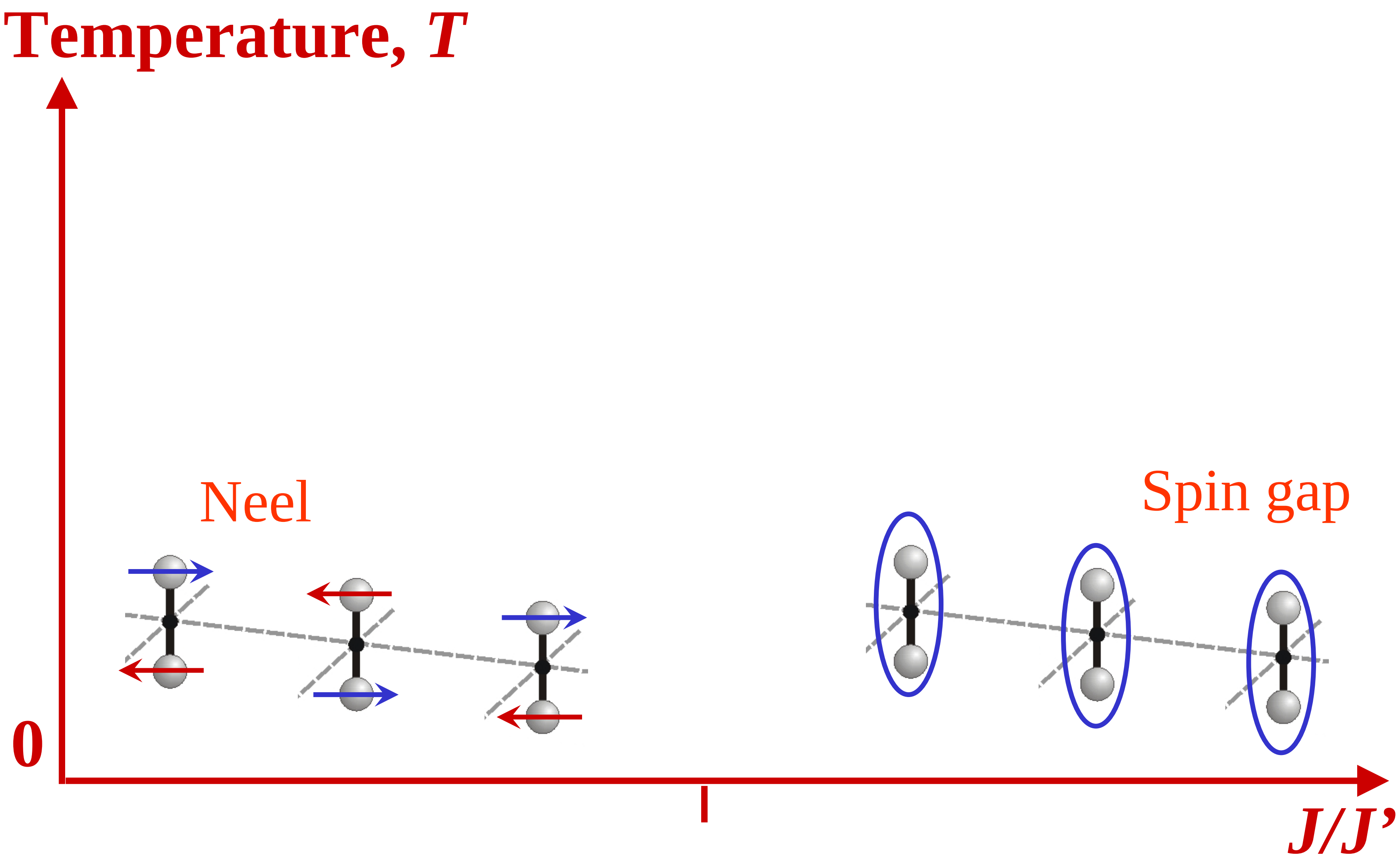

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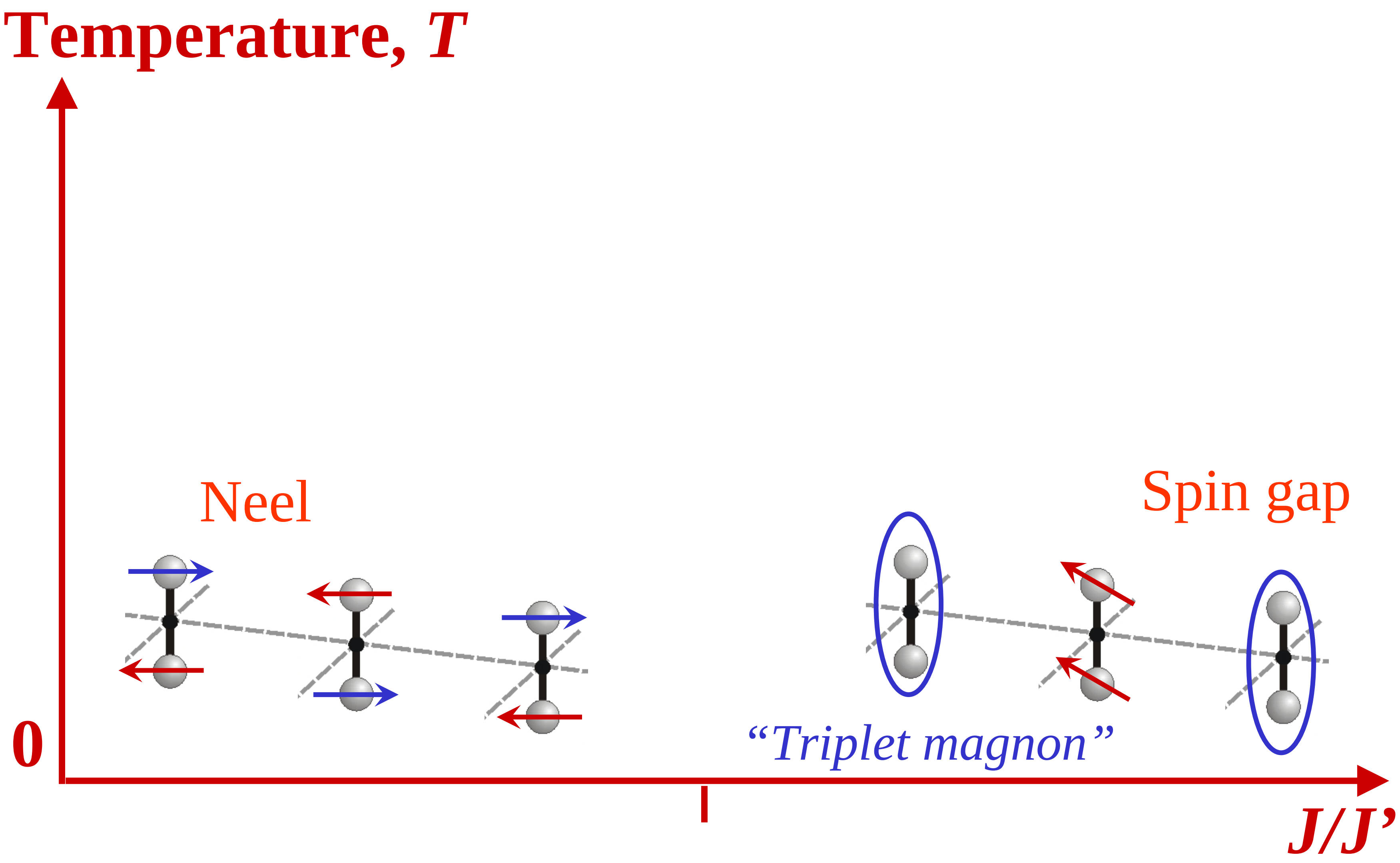

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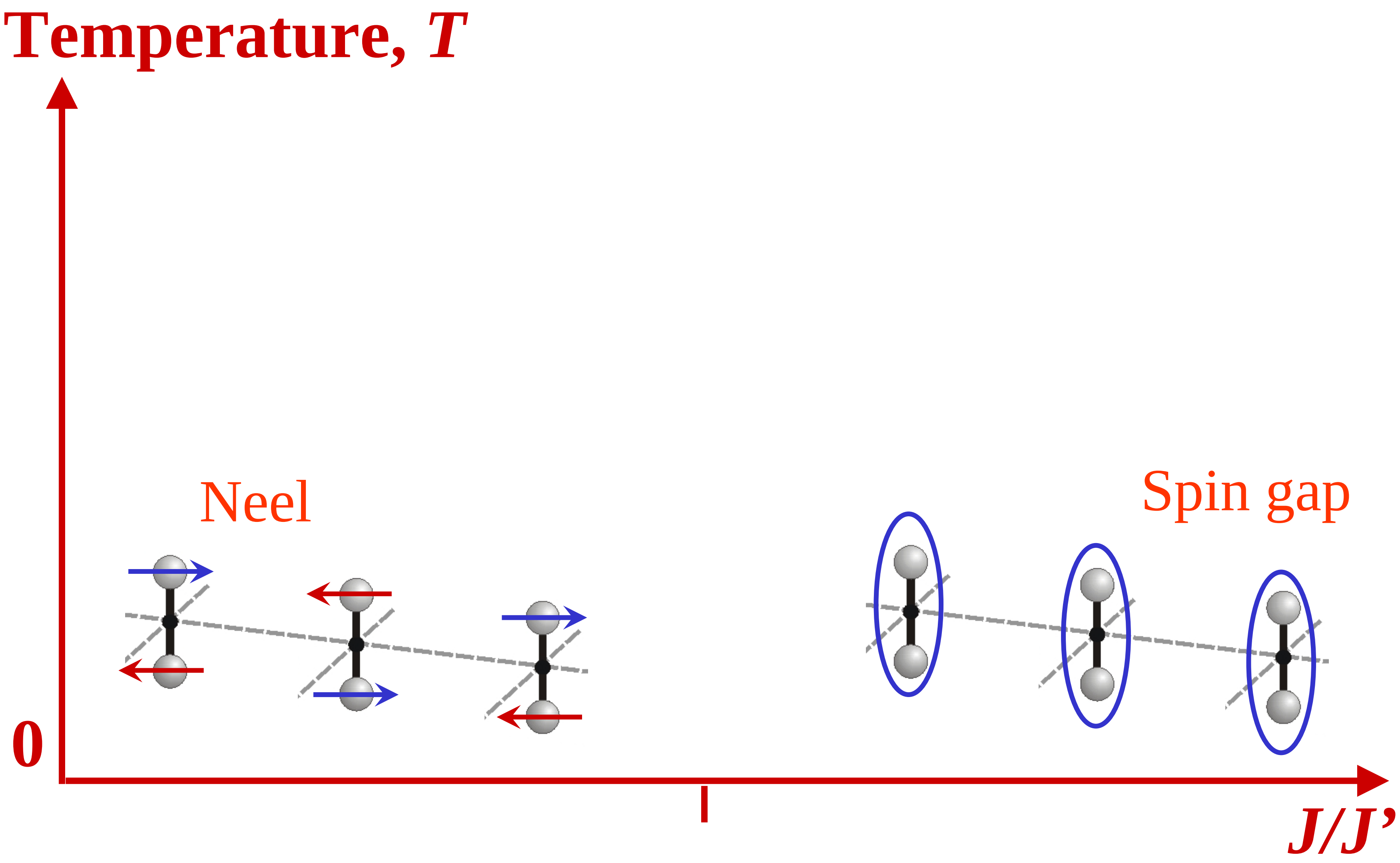

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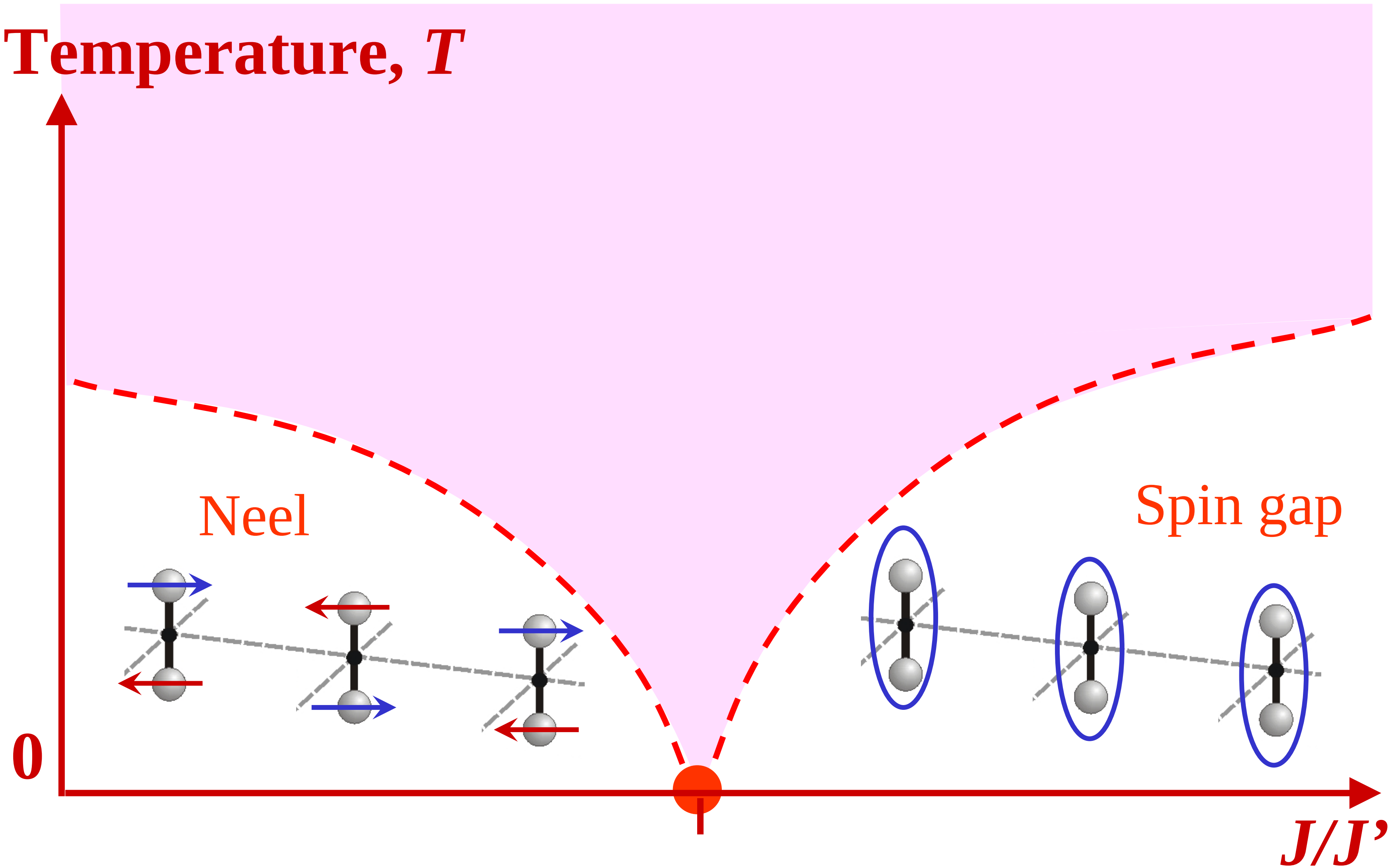

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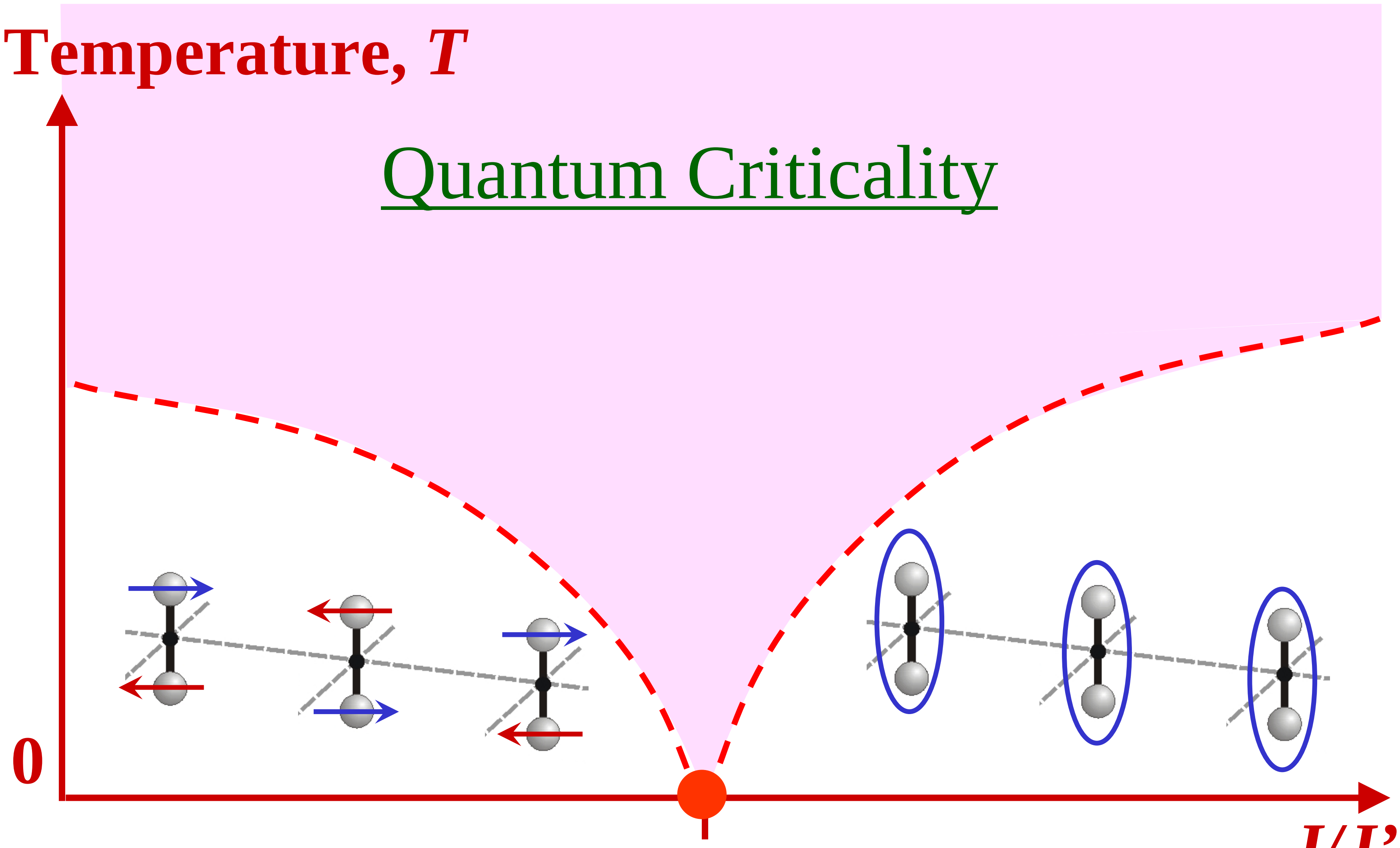
S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)



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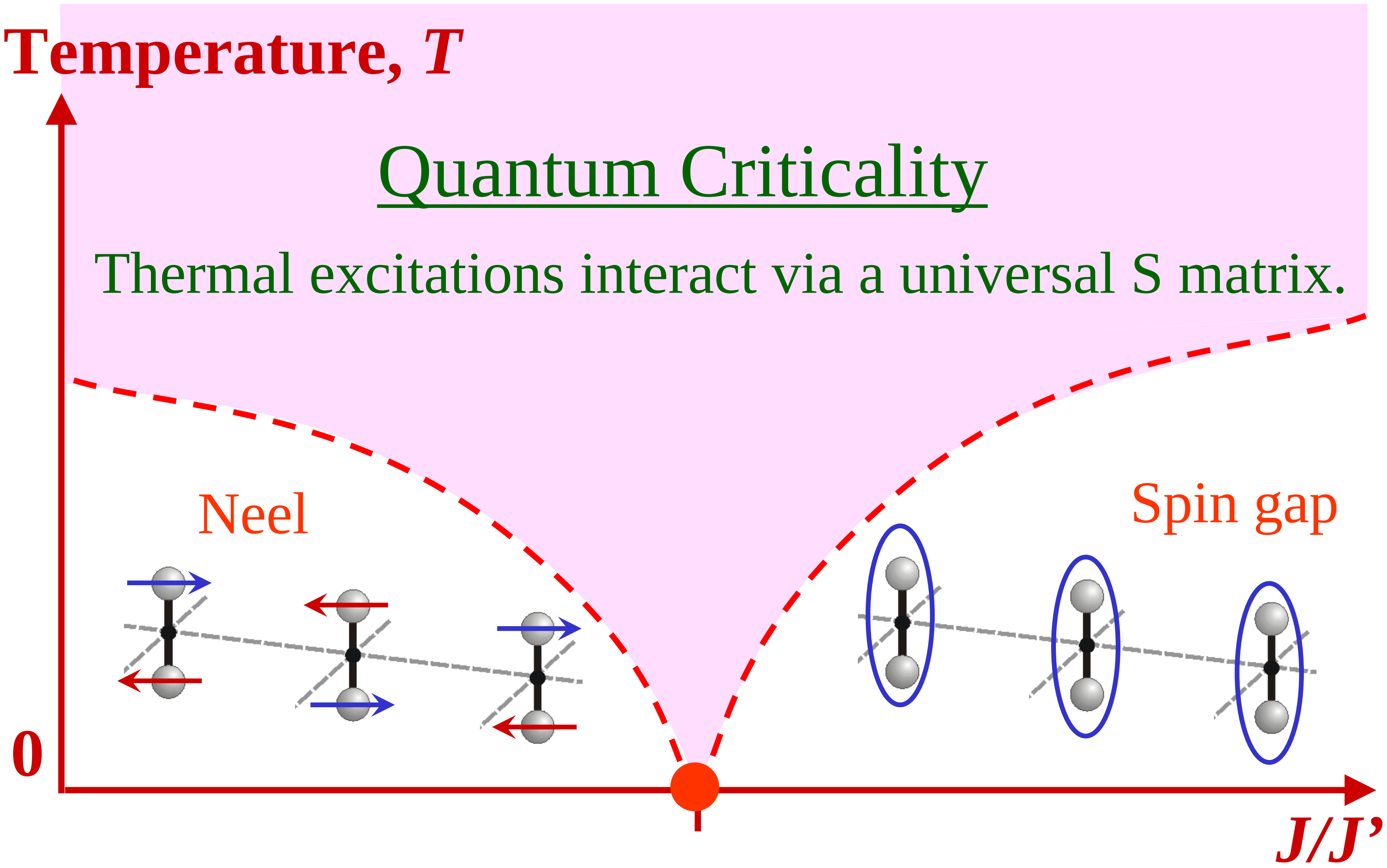

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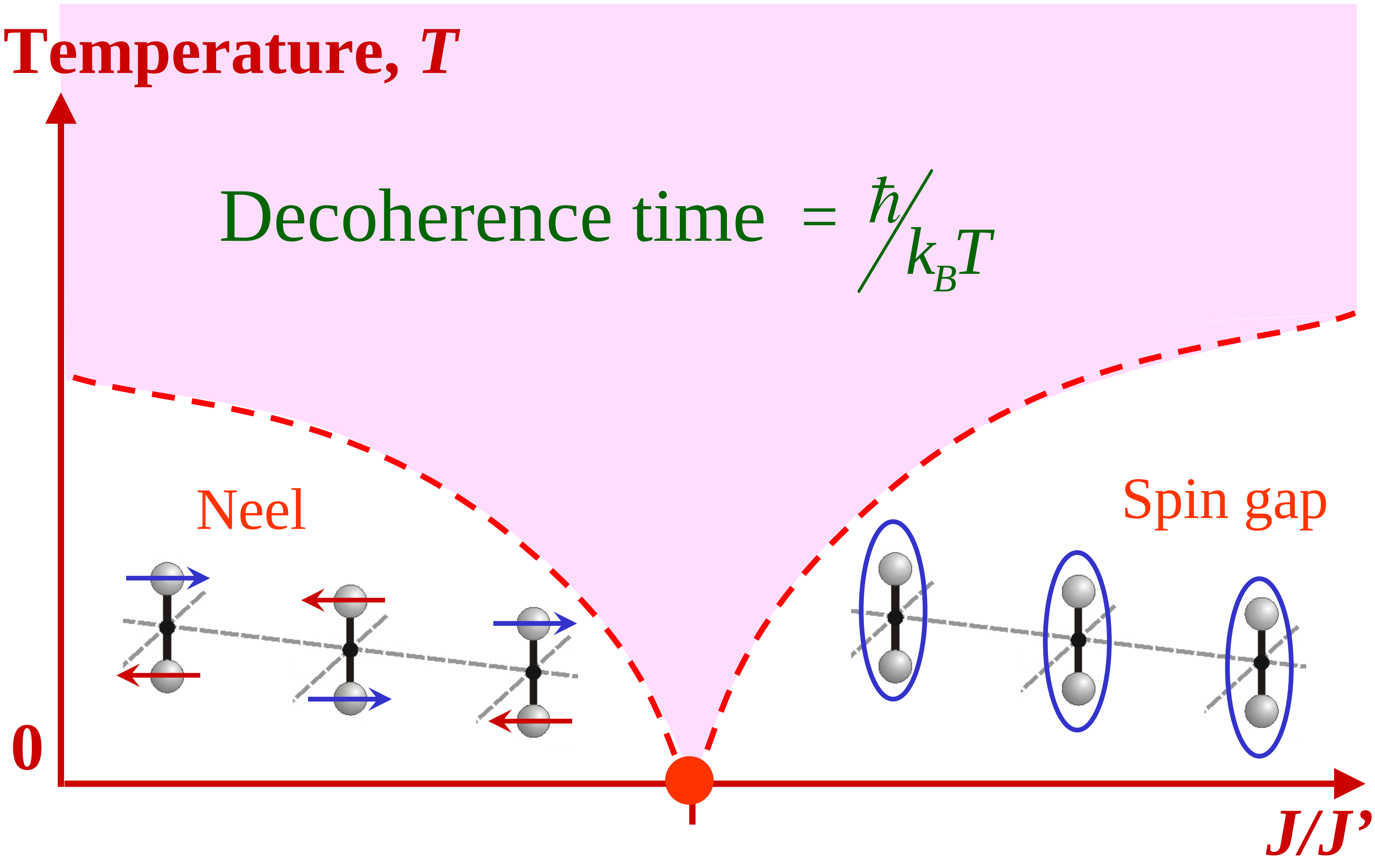

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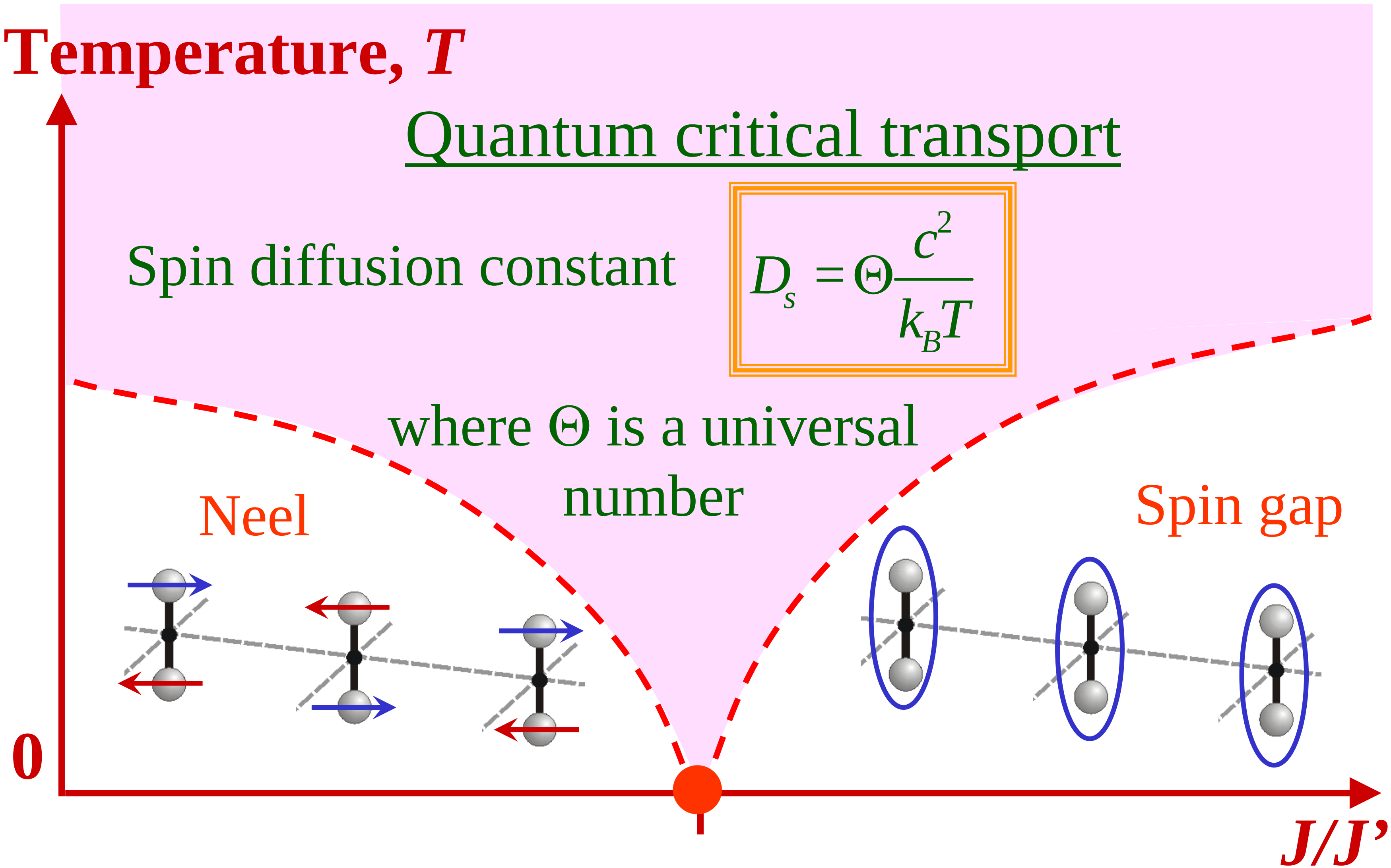

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A.V. Chubukov, S. Sachdev and J. Ye, *Phys. Rev. B* **49**, 11919 (1994).

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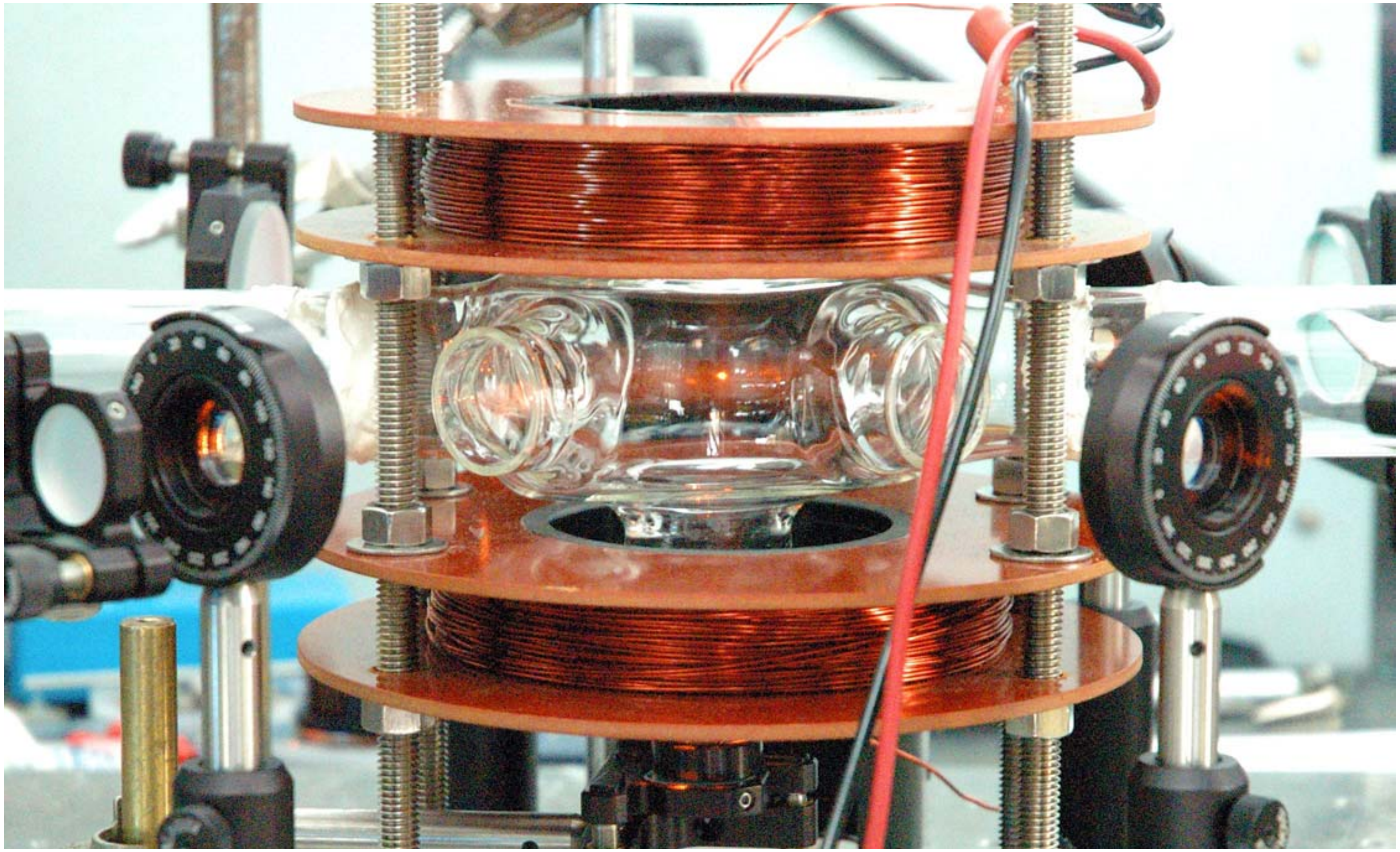
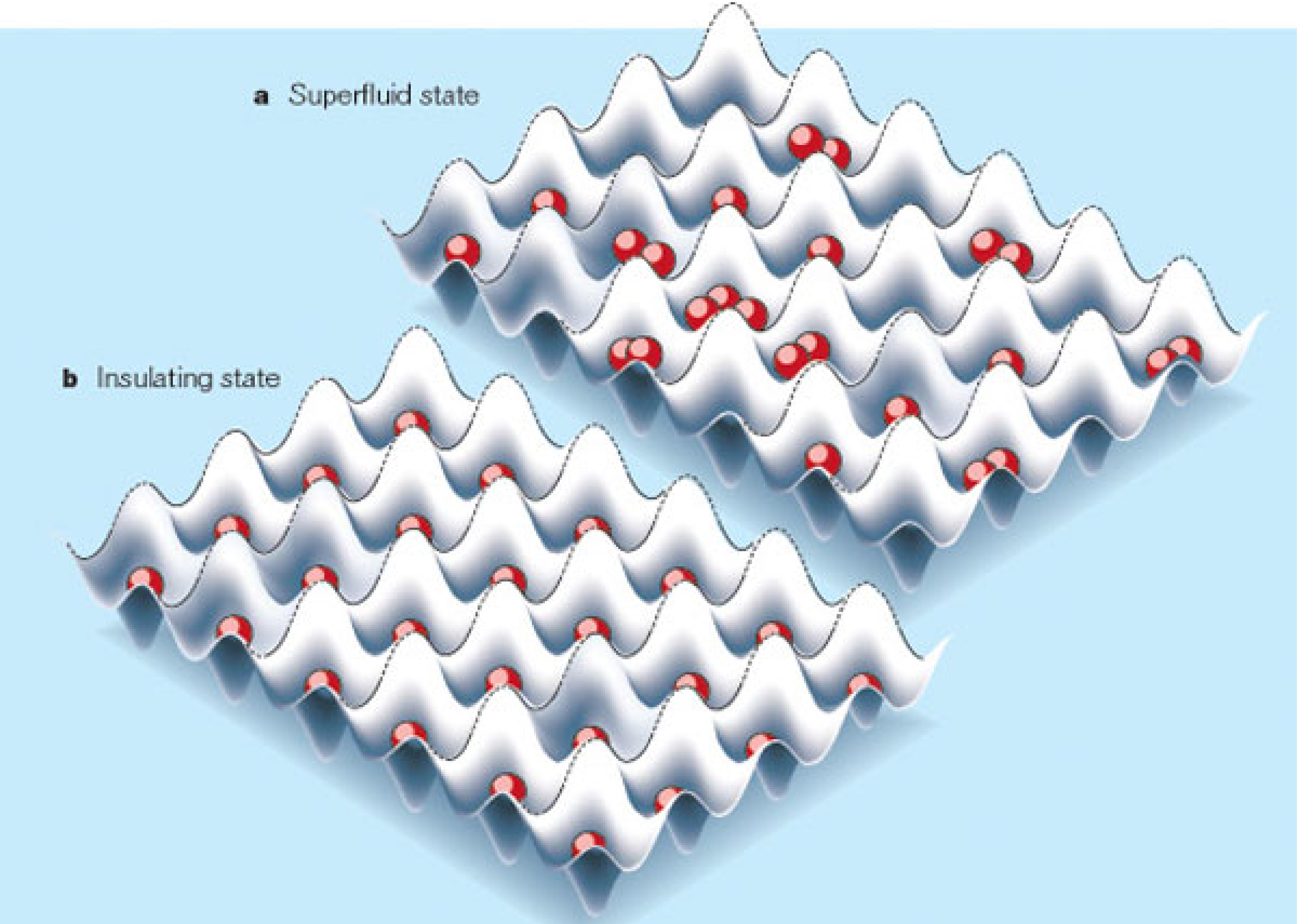
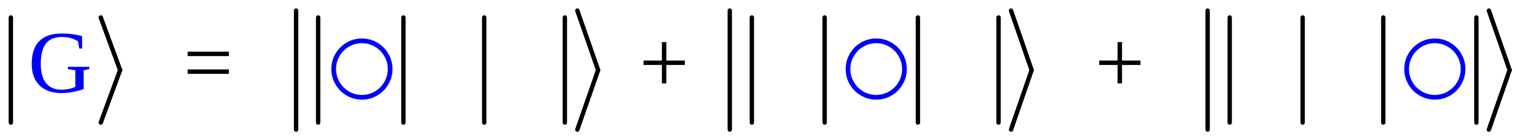


Trap for ultracold ^{87}Rb atoms



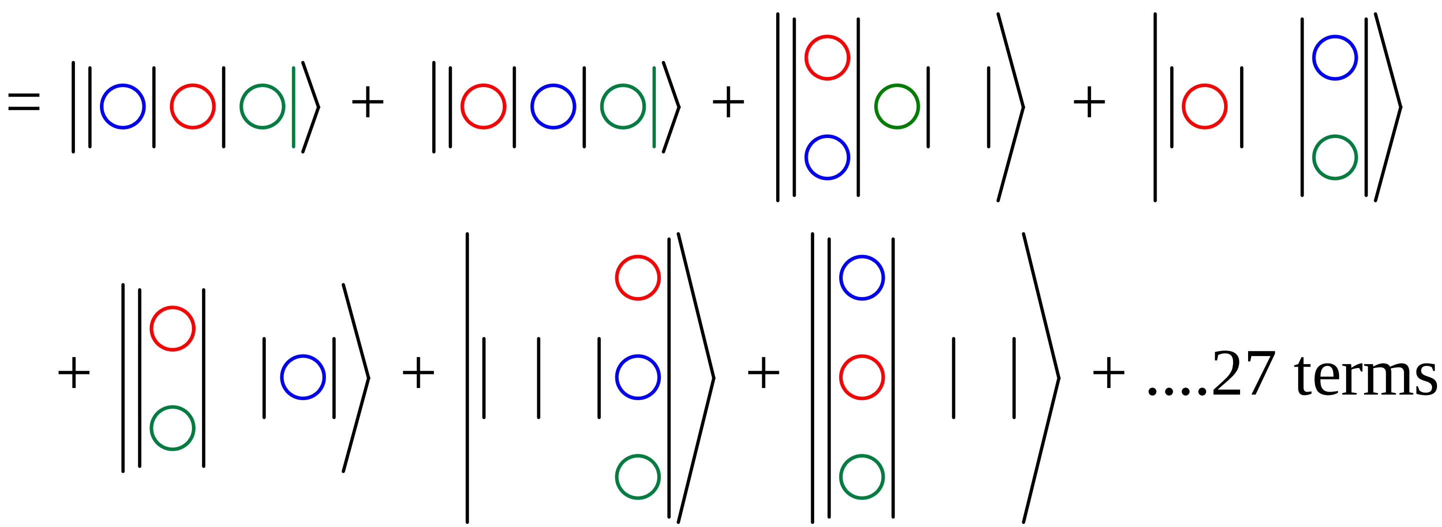
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Bose-Einstein condensate in a periodic potential



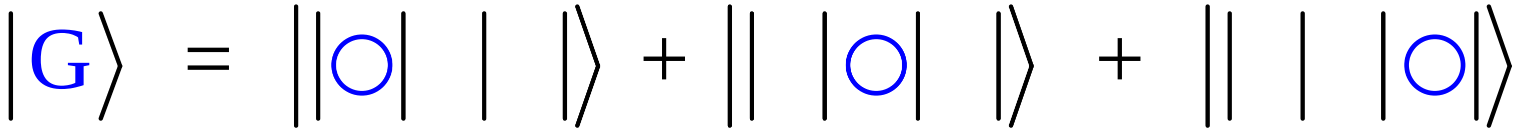
Lowest energy state for many atoms

$$|BEC\rangle = |G\rangle|G\rangle|G\rangle$$



Large fluctuations in number of atoms in each potential well – *superfluidity* (atoms can “flow” without dissipation)

Breaking up the Bose-Einstein condensate



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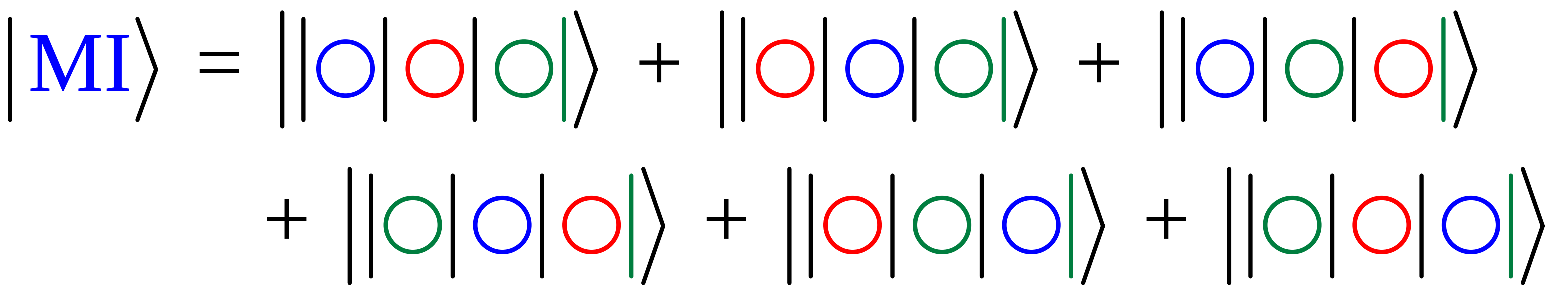
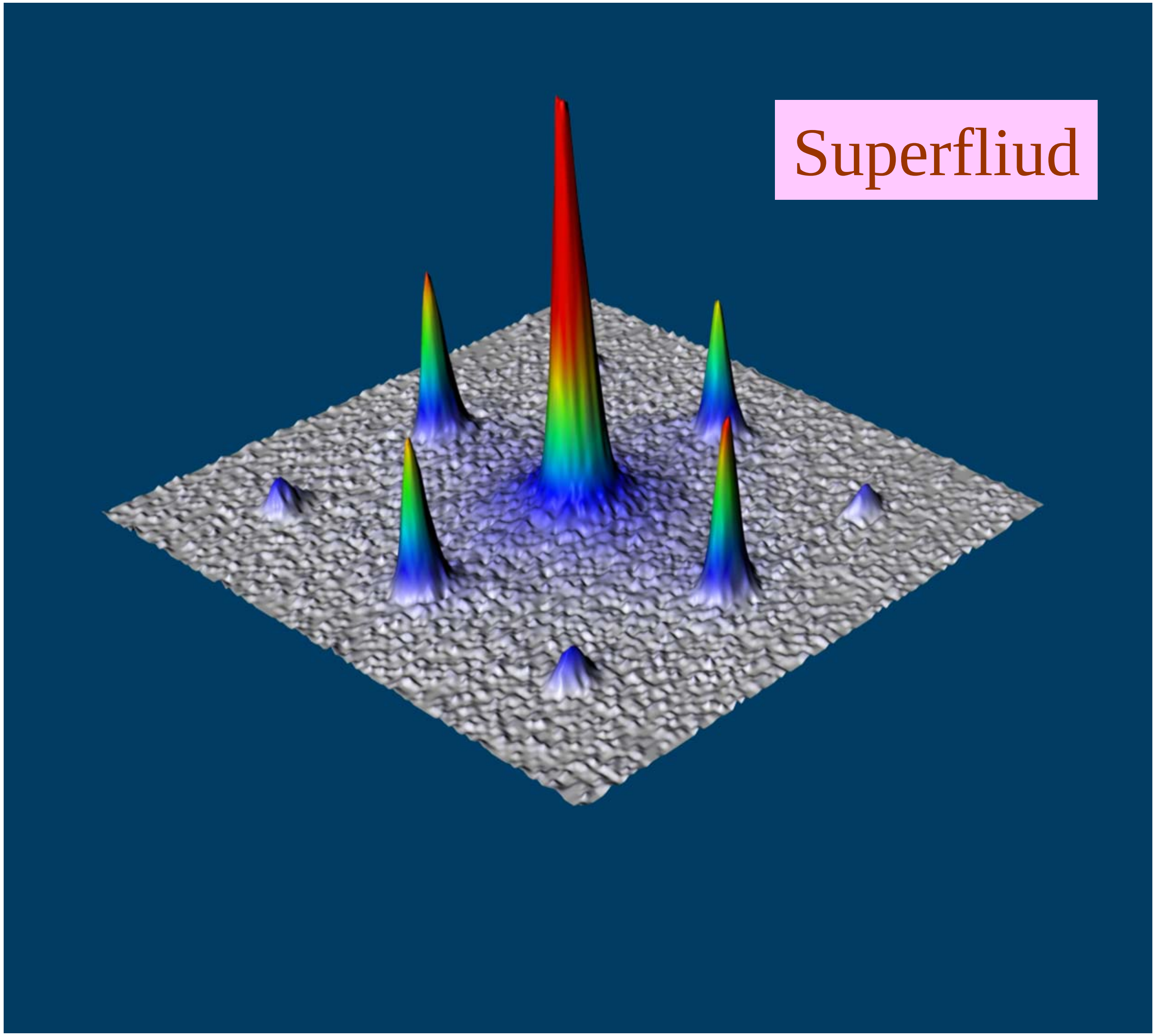


By tuning repulsive interactions between the atoms, states with multiple atoms in a potential well can be suppressed. The lowest energy state is then a *Mott insulator* – it has negligible number fluctuations, and atoms cannot “flow”

Velocity distribution of ^{87}Rb atoms



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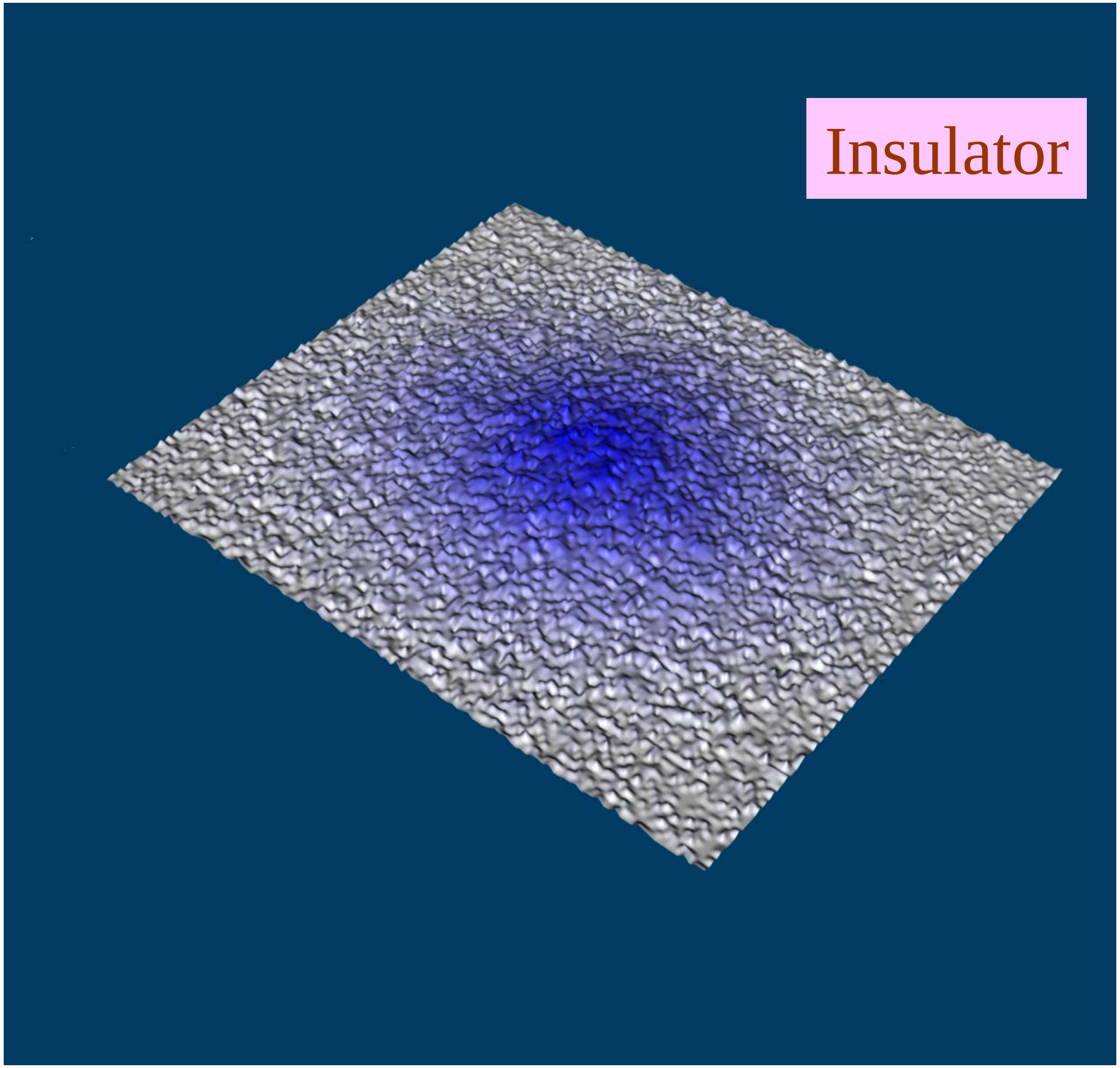

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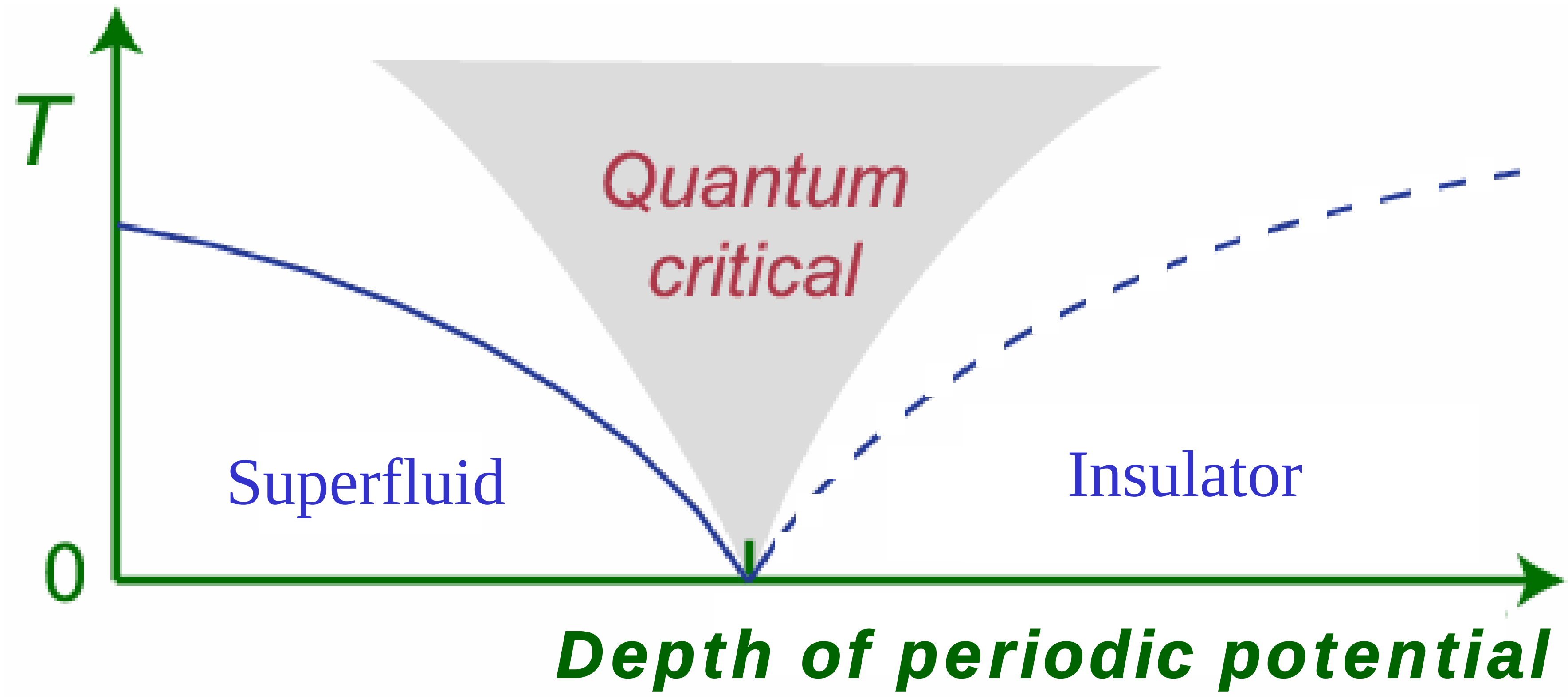
Non-zero temperature phase diagram



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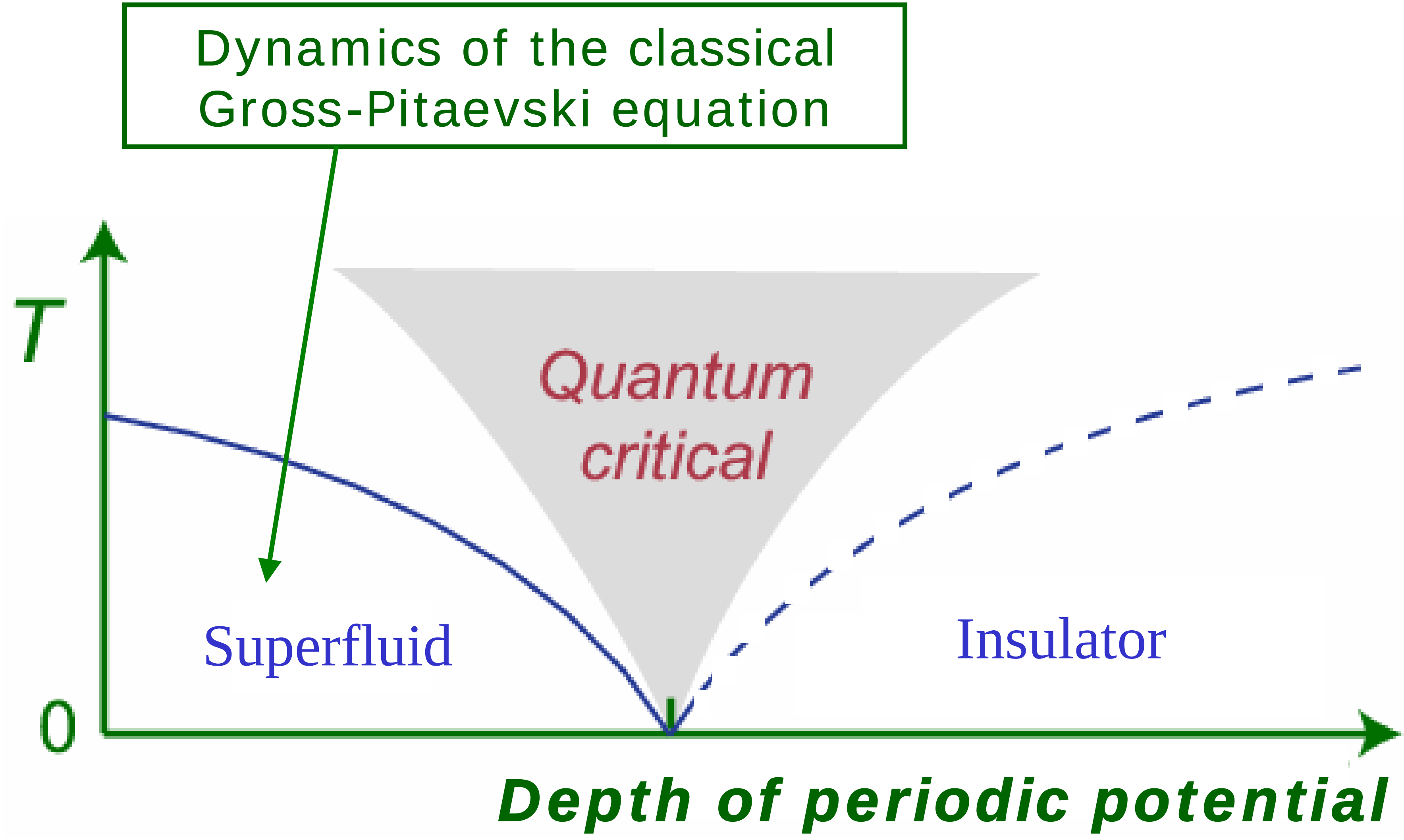

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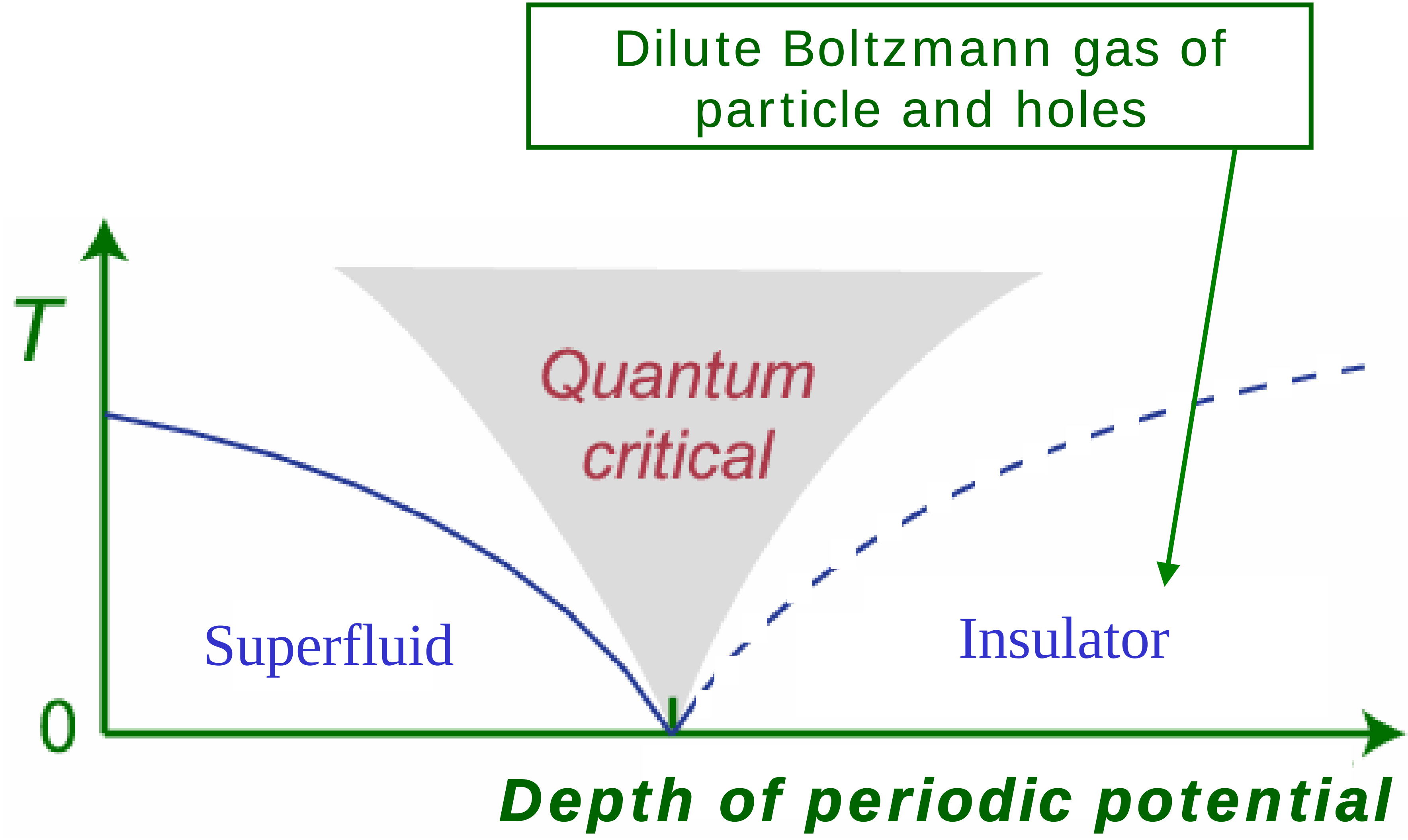

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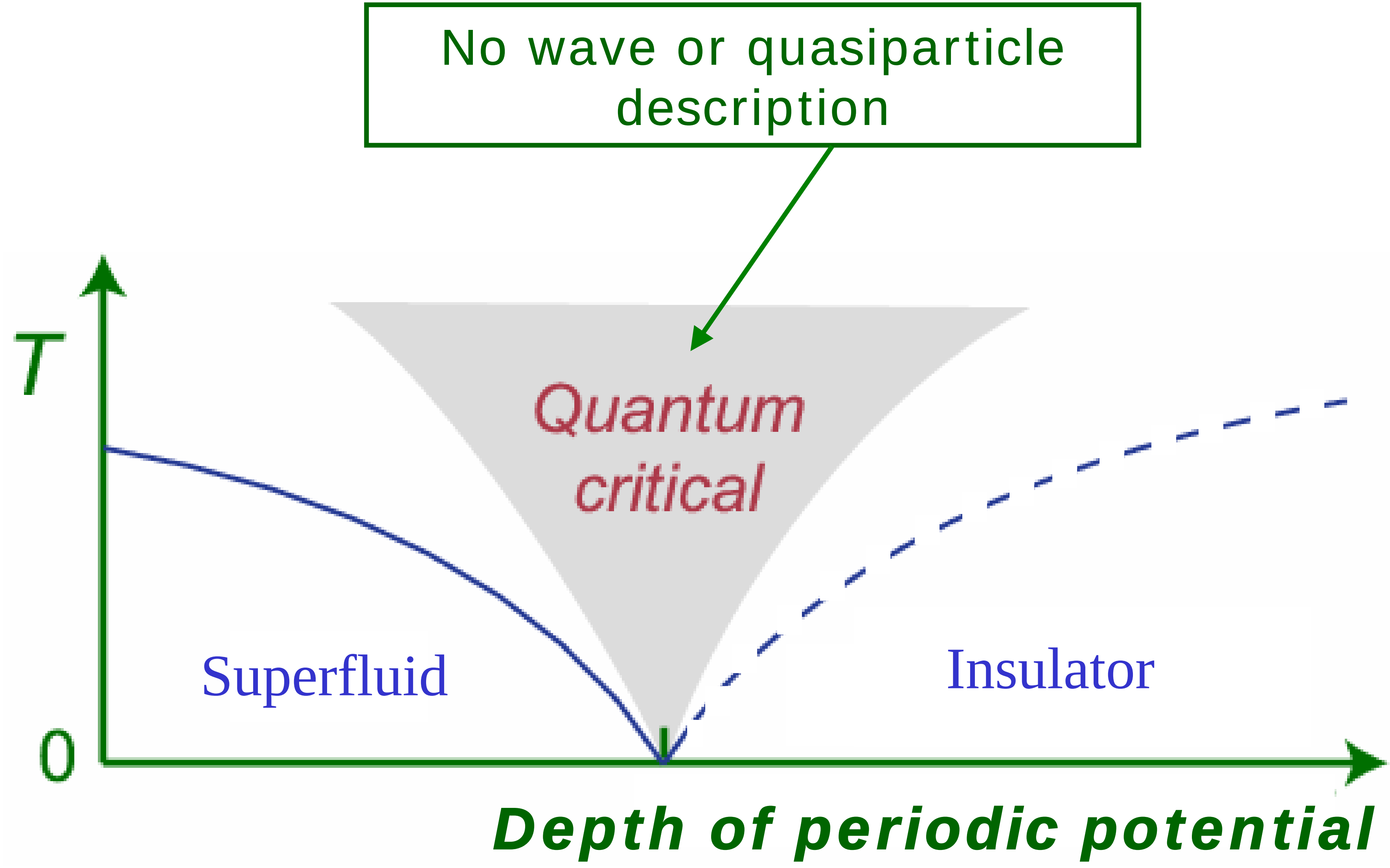


Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}\left(T\rightarrow 0\right)=\infty$$

$$\sigma_{\text{Insulator}}\left(T\rightarrow 0\right)=0$$

$$\sigma_{\text{Quantum critical point}}\left(T\rightarrow 0\right)\approx\frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

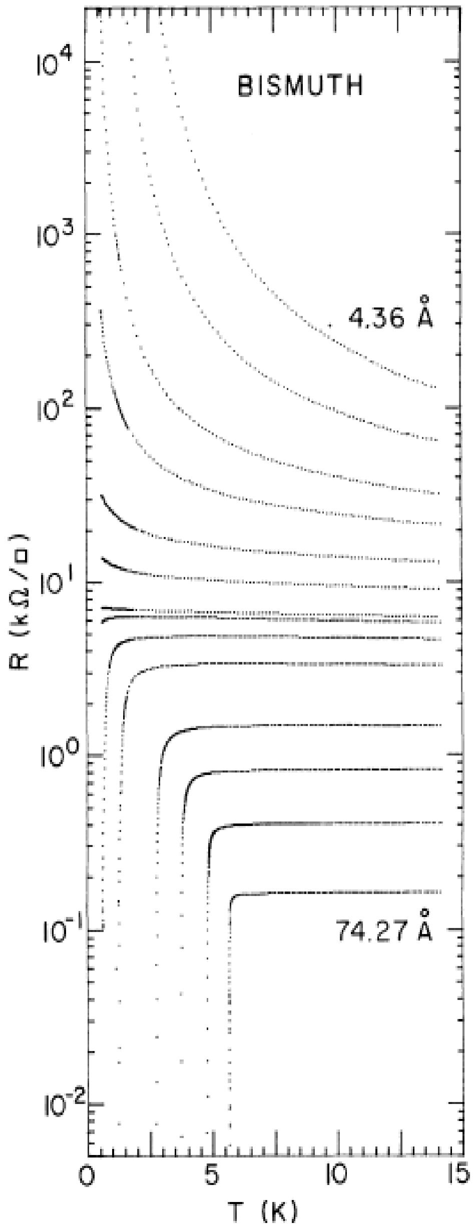
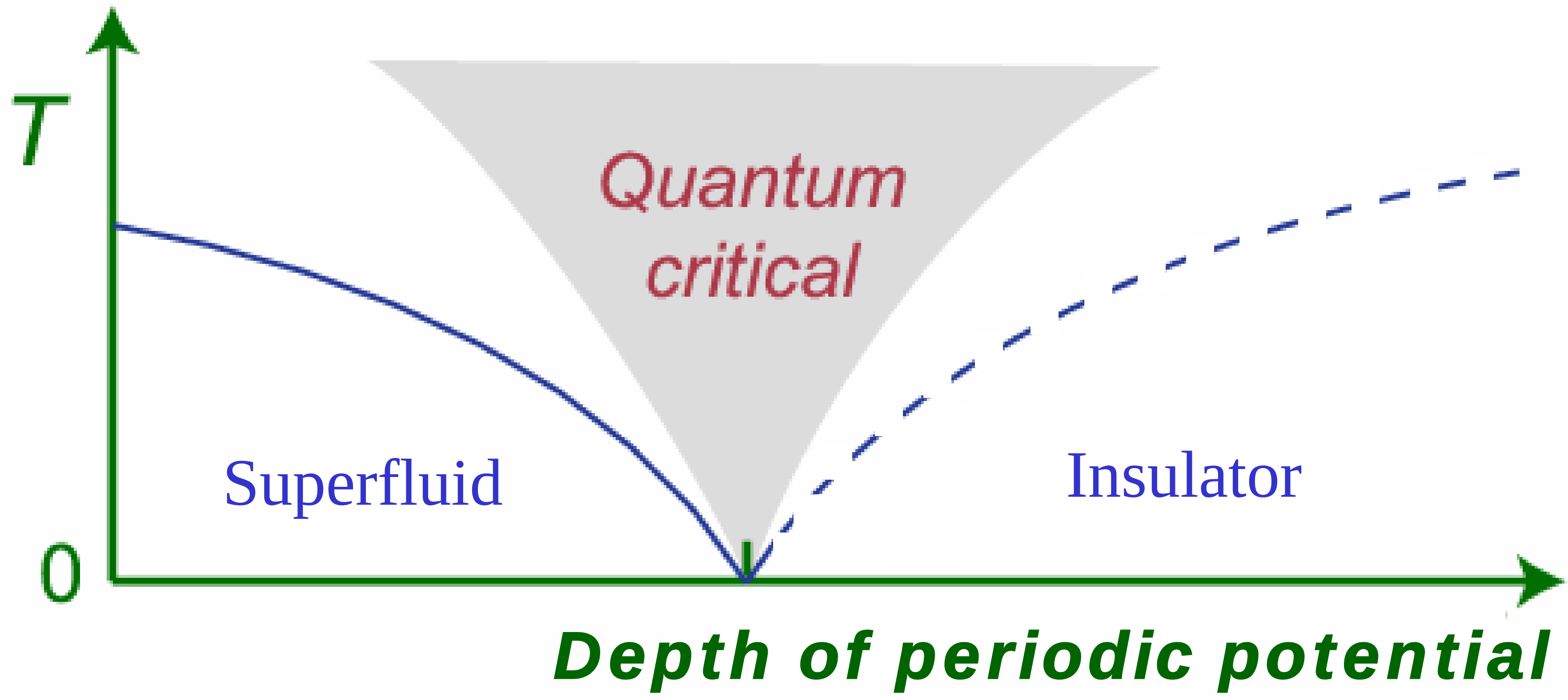


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Non-zero temperature phase diagram



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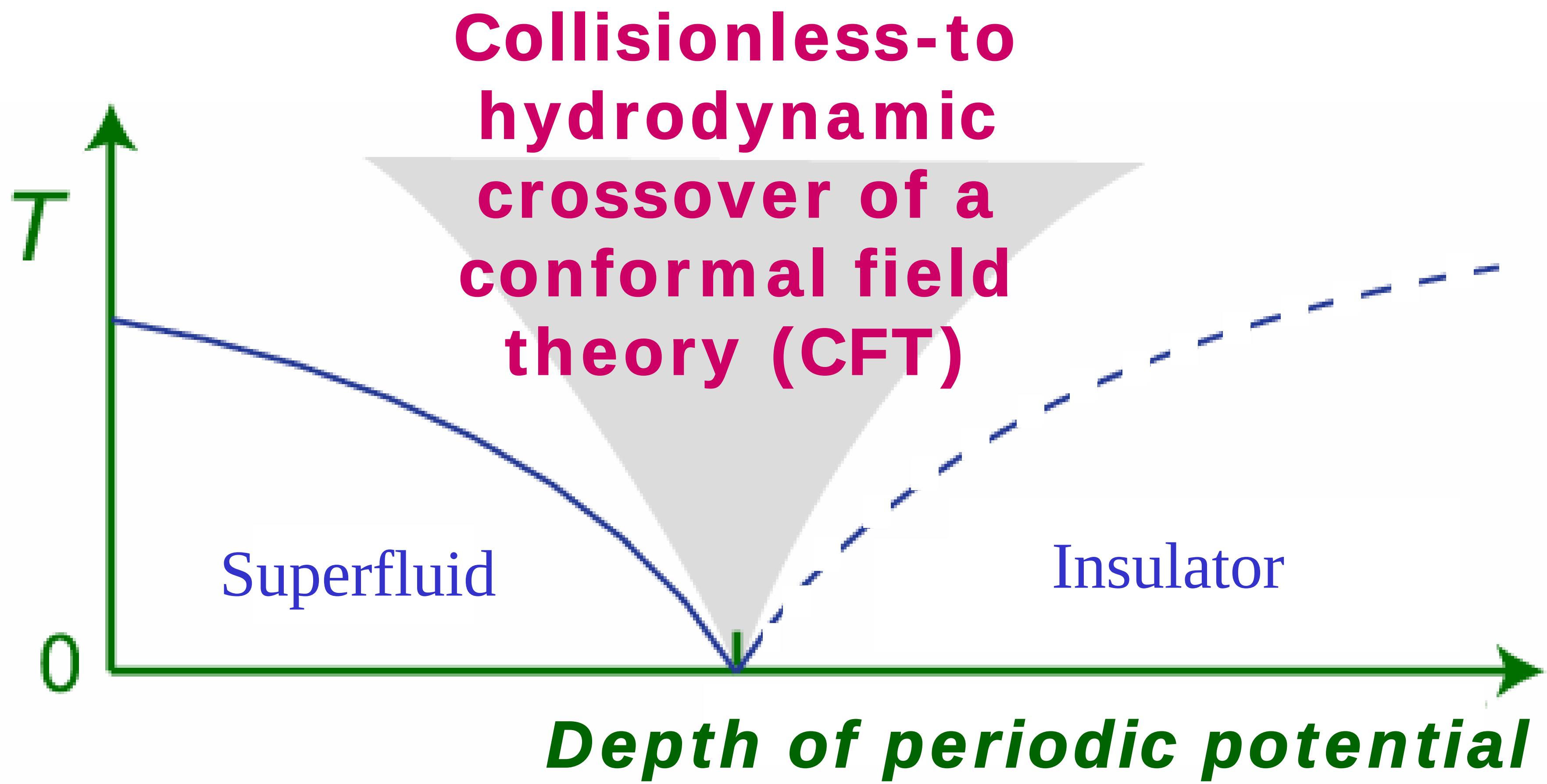
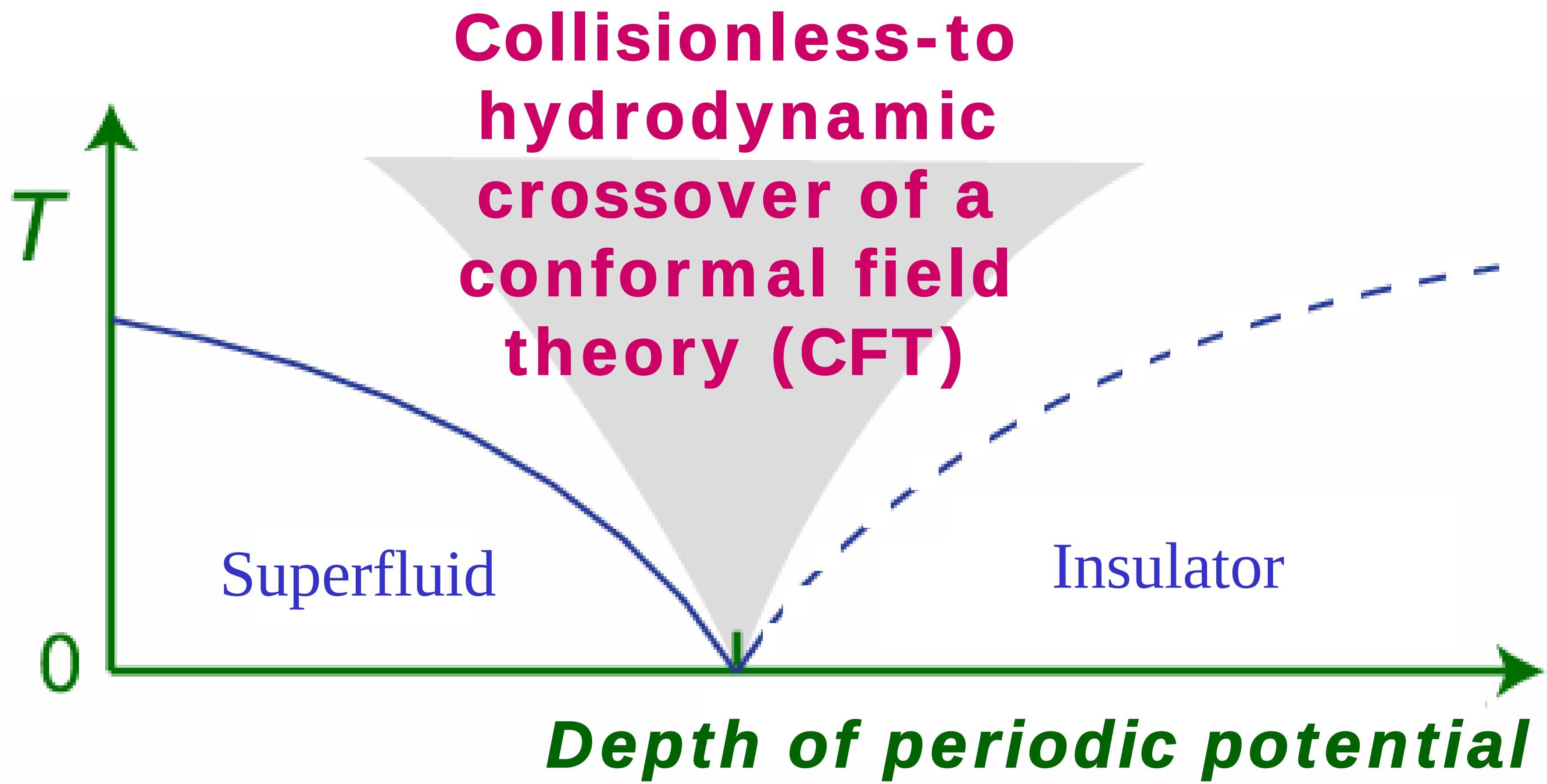


K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Non-zero temperature phase diagram

Needed: Cold atom experiments in this regime



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Hydrodynamics of a conformal field theory (CFT)

Maldacena's AdS/CFT correspondence relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

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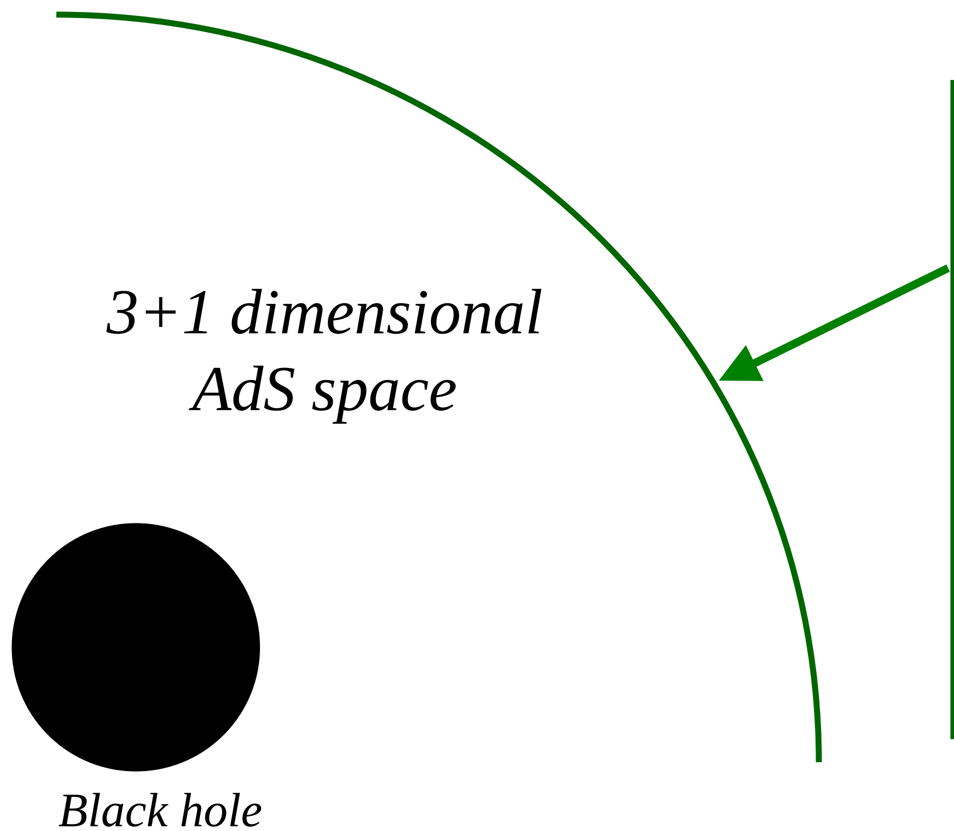
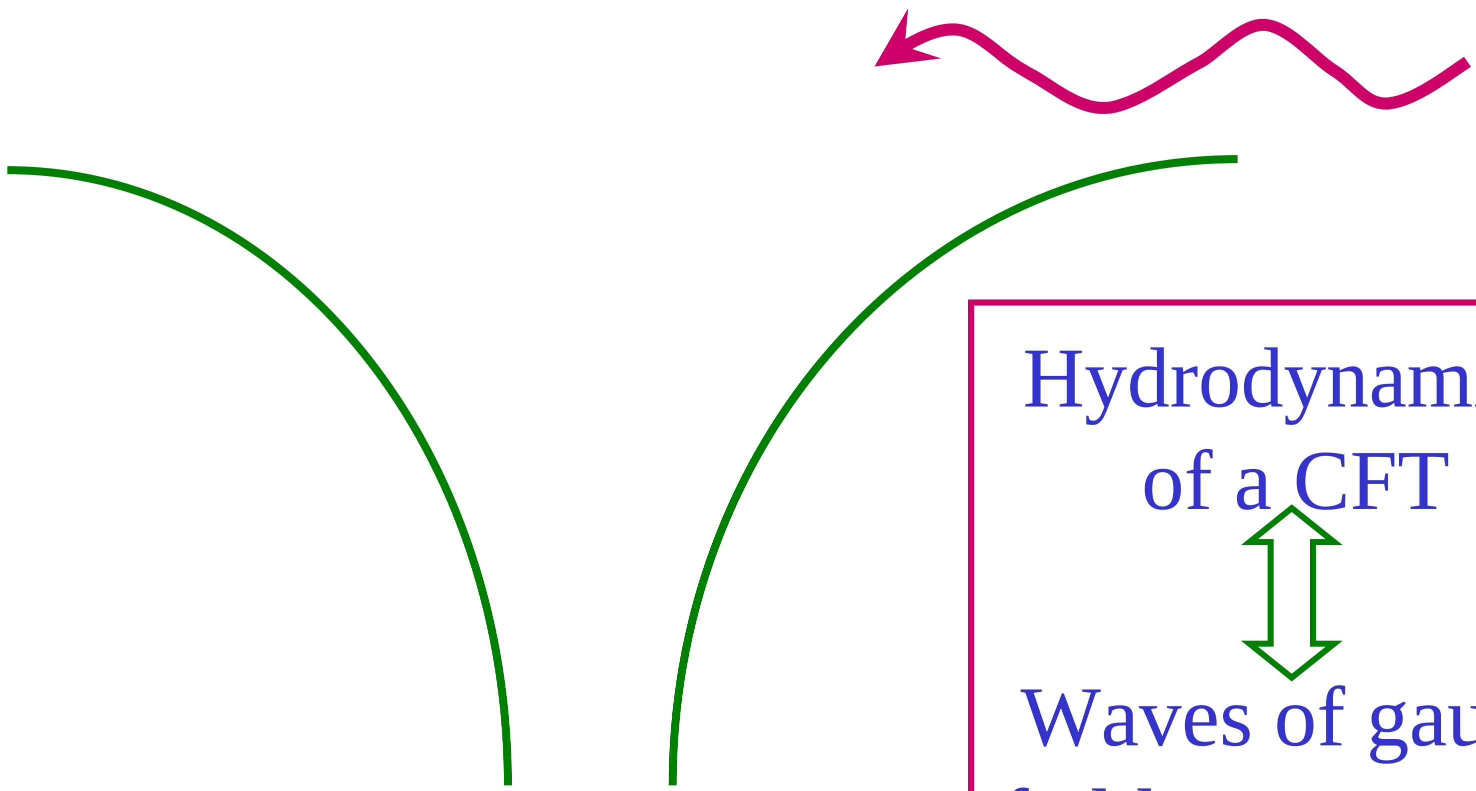


Holographic representation of black hole physics in a 2+1 dimensional CFT at a temperature equal to the Hawking temperature of the black hole.

Hydrodynamics of a conformal field theory (CFT)



Hydrodynamics of a CFT

⇕

Waves of gauge fields in a curved background

Hydrodynamics of a conformal field theory (CFT)

The scattering cross-section of the thermal excitations is universal and so transport co-efficients are universally determined by $k_B T$

Charge diffusion constant $$D_c = \Theta \frac{c^2}{k_B T}$$

Conductivity $$\sigma = \bar{\Theta} \frac{4e^2}{h}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Hydrodynamics of a conformal field theory (CFT)

For the (unique) CFT with a SU(N) gauge field and 16 supercharges, we know the exact diffusion constant associated with a global SO(8) symmetry:

Spin diffusion constant
$$D_s = \frac{3}{4\pi}\frac{c^2}{k_B T}$$

Spin conductivity
$$\sigma = \frac{N^{3/2}}{3\sqrt{2}\pi}$$

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, hep-th/0701036

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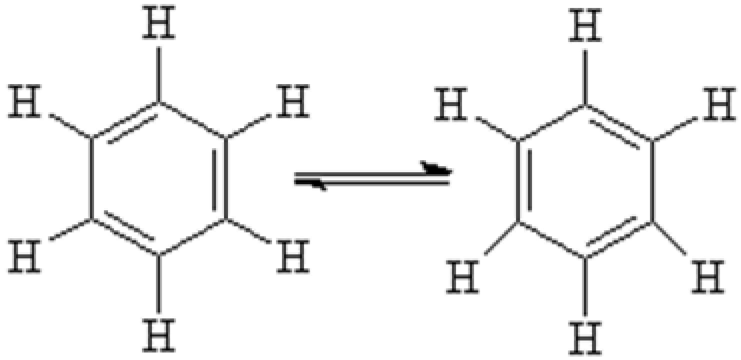
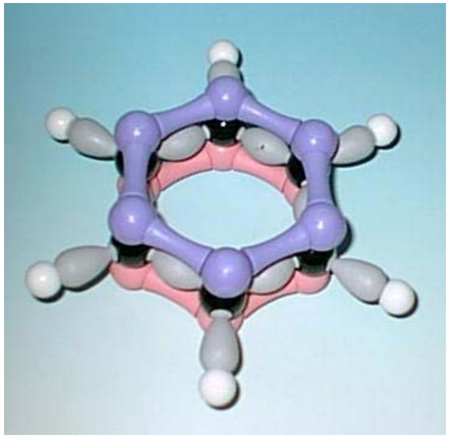
Valence bonds in benzene

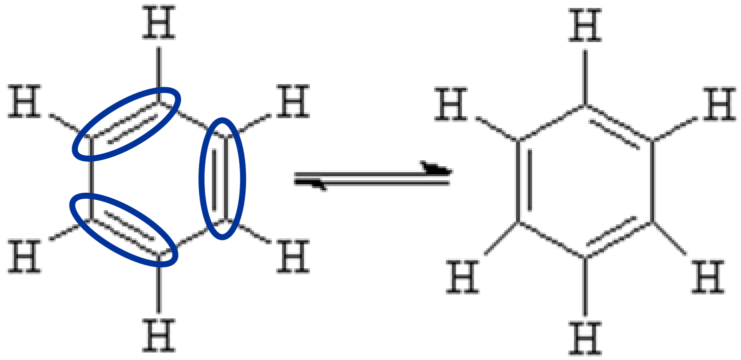


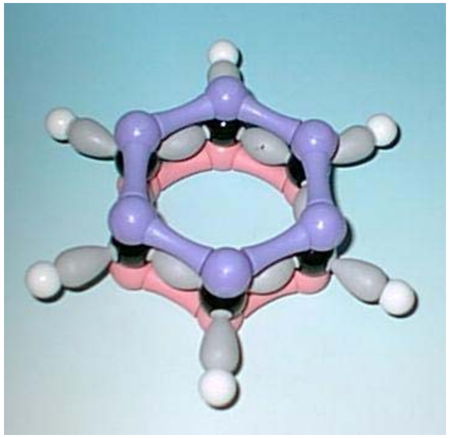


Resonance in benzene leads to a symmetric configuration of valence bonds

(*F. Kekulé, L. Pauling*)

Valence bonds in benzene





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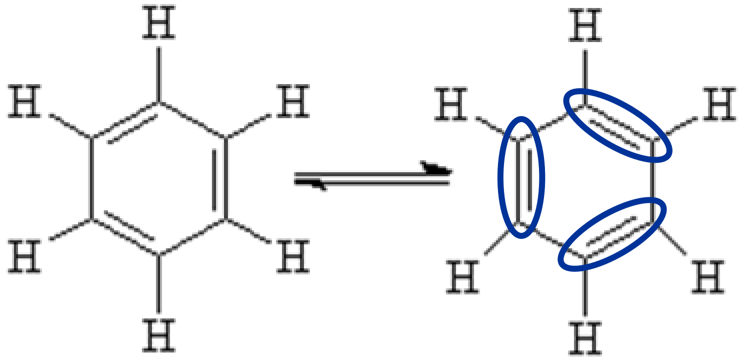

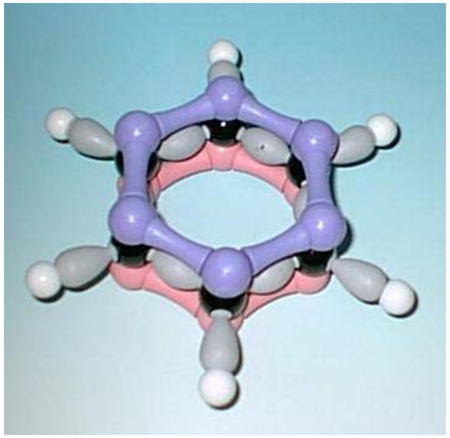

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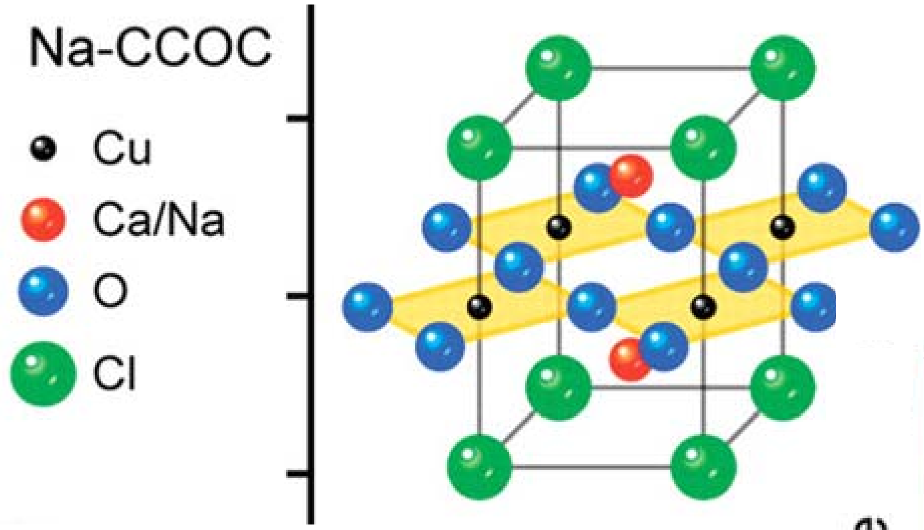
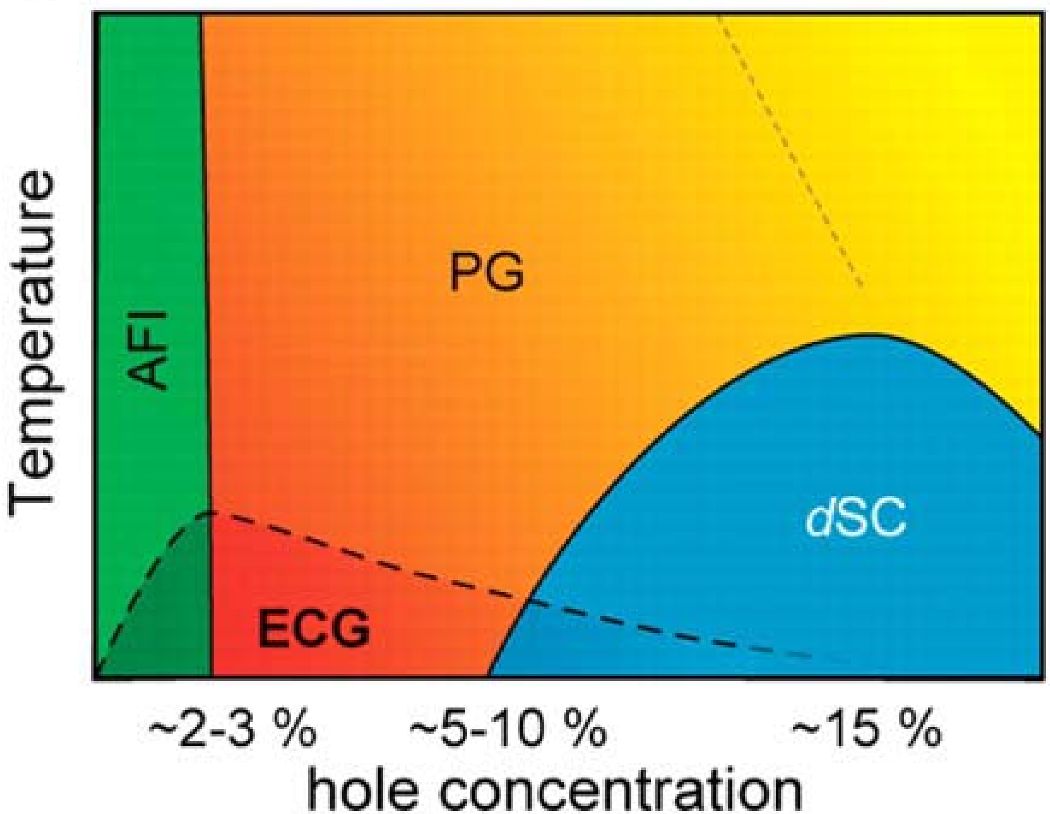
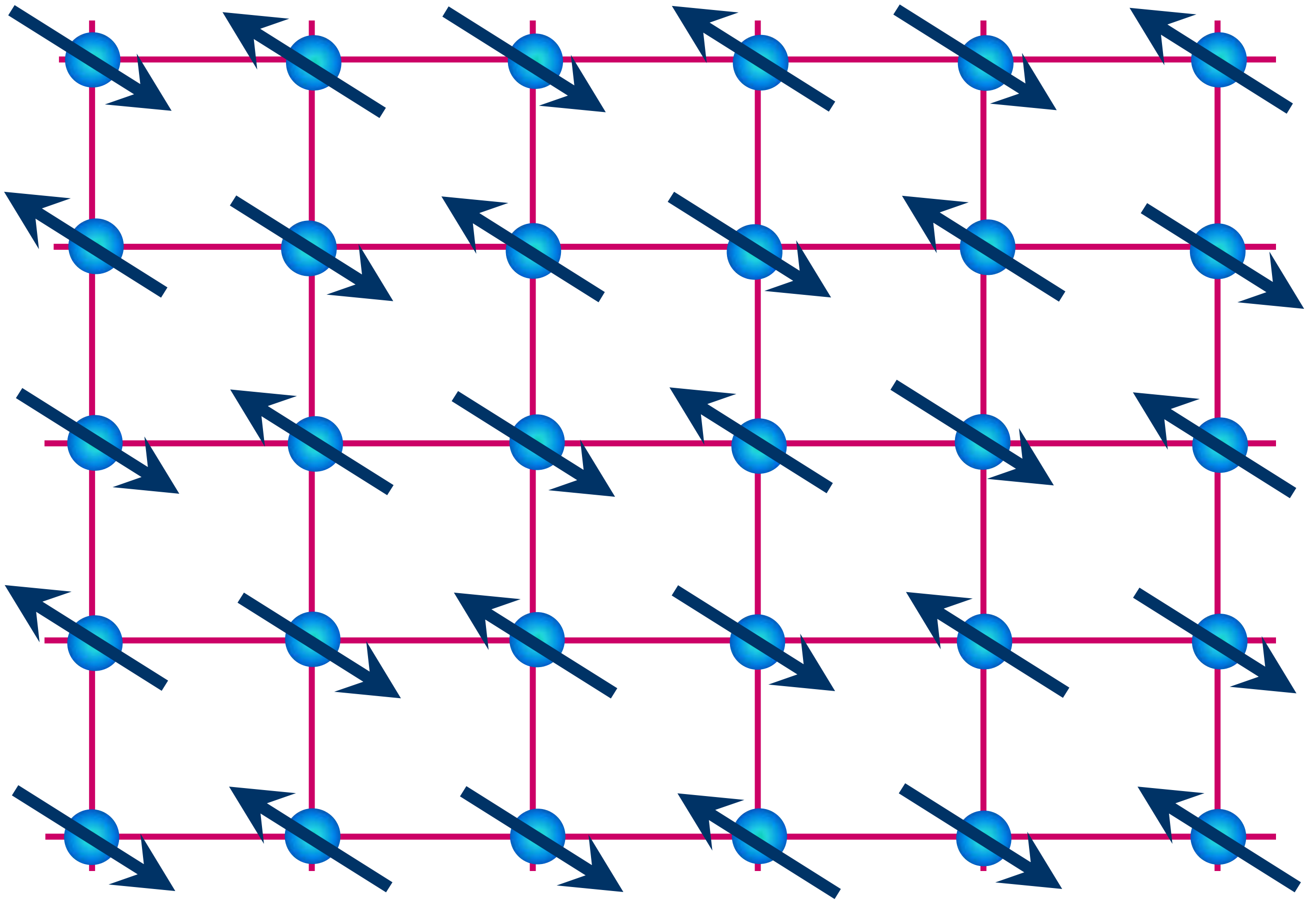
Temperature-doping phase diagram of the cuprate superconductors



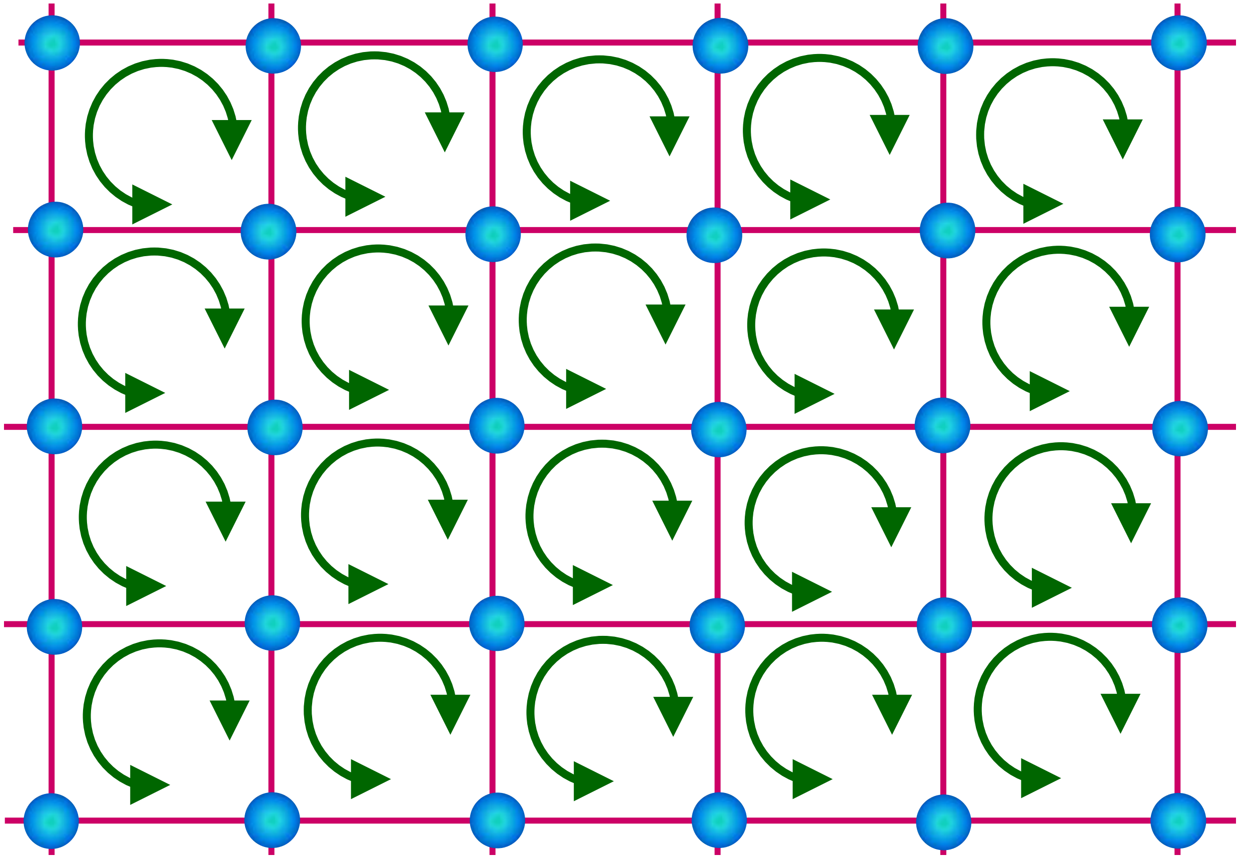


Antiferromagnetic (Neel) order in the insulator



$$H = J\sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{ spin operator with } S\text{=1/2}$$

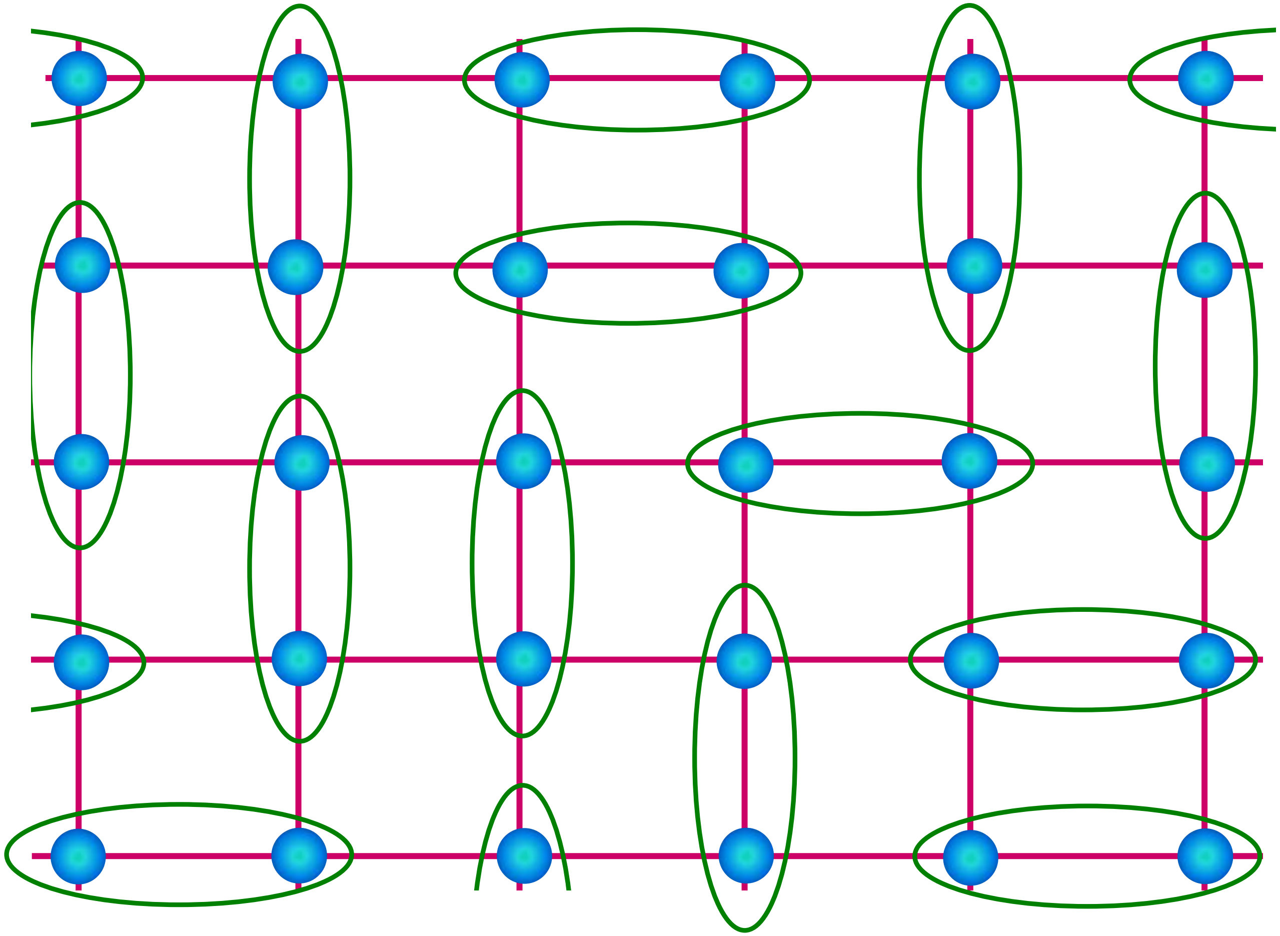
Induce formation of valence bonds by *e.g.* ring-exchange interactions

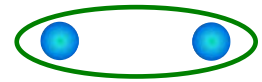


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \bullet \vec{S}_j \; + \; K \sum_{\square} \text{4-spin exchange}$$

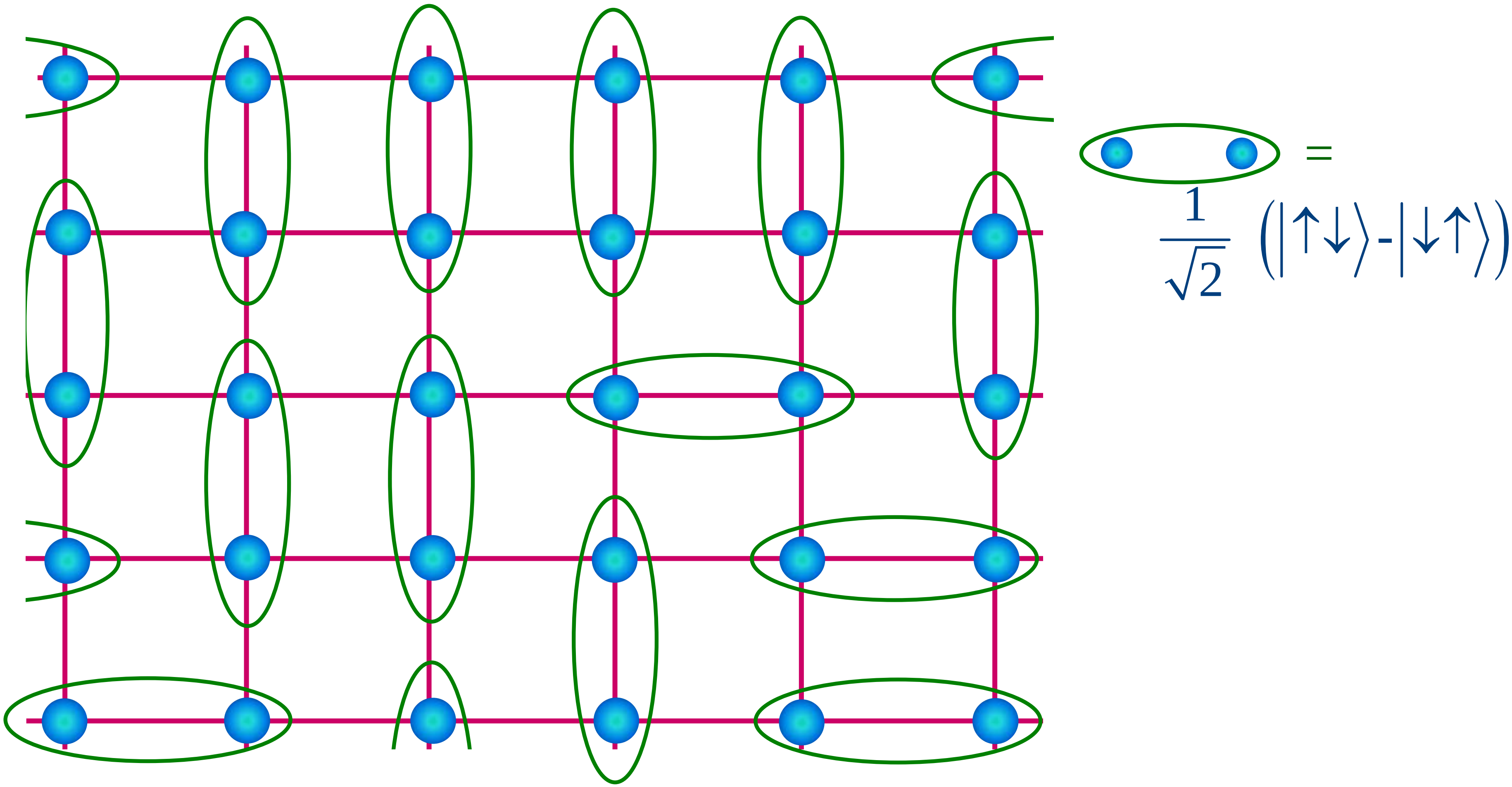
A. W. Sandvik, cond-mat/0611343

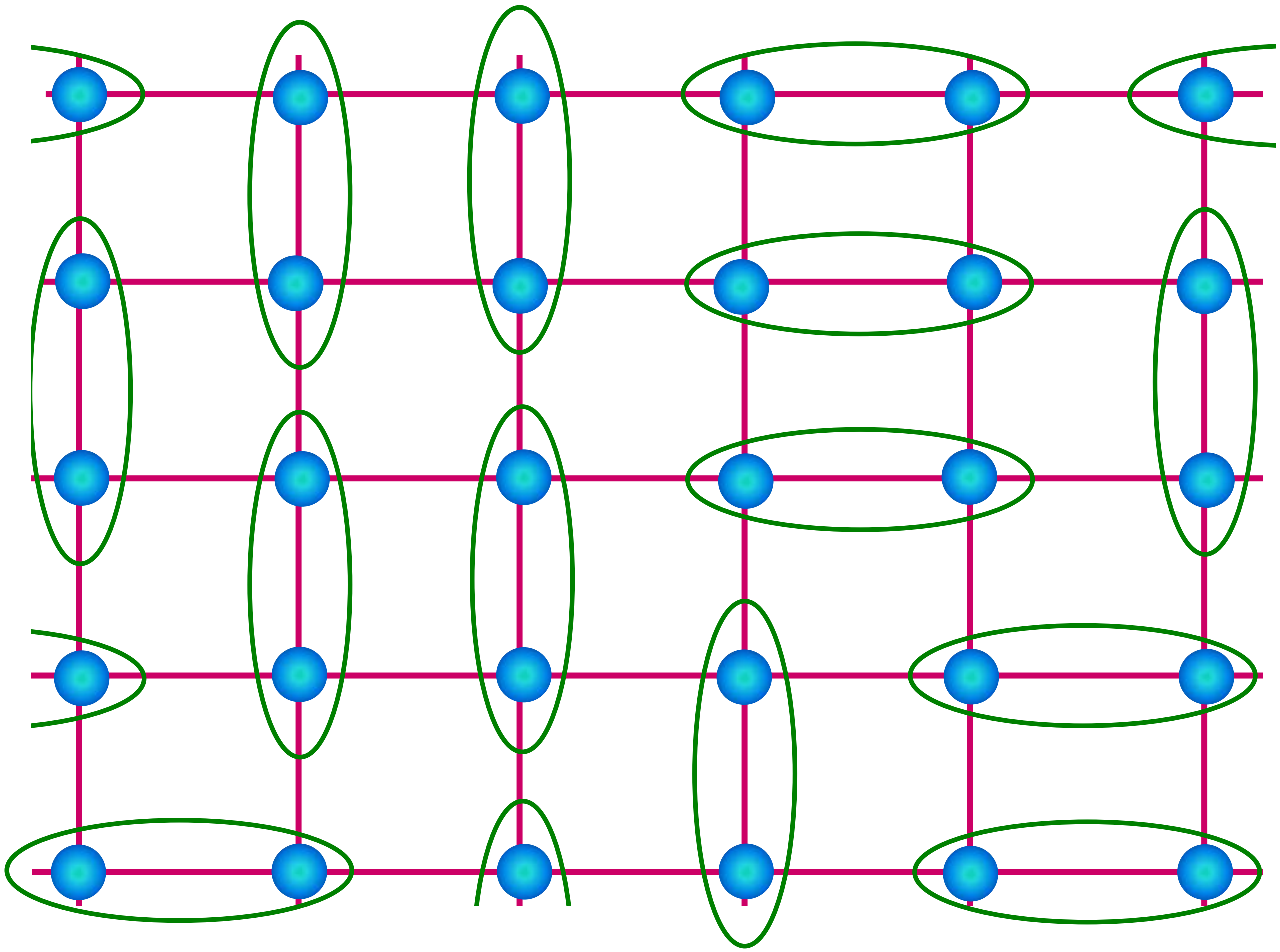
As in H_2 and benzene, each electron wants to pair up with another electron and form a valence bond

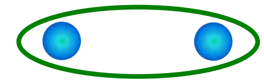


 =

$$\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$


= $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$



 $=$

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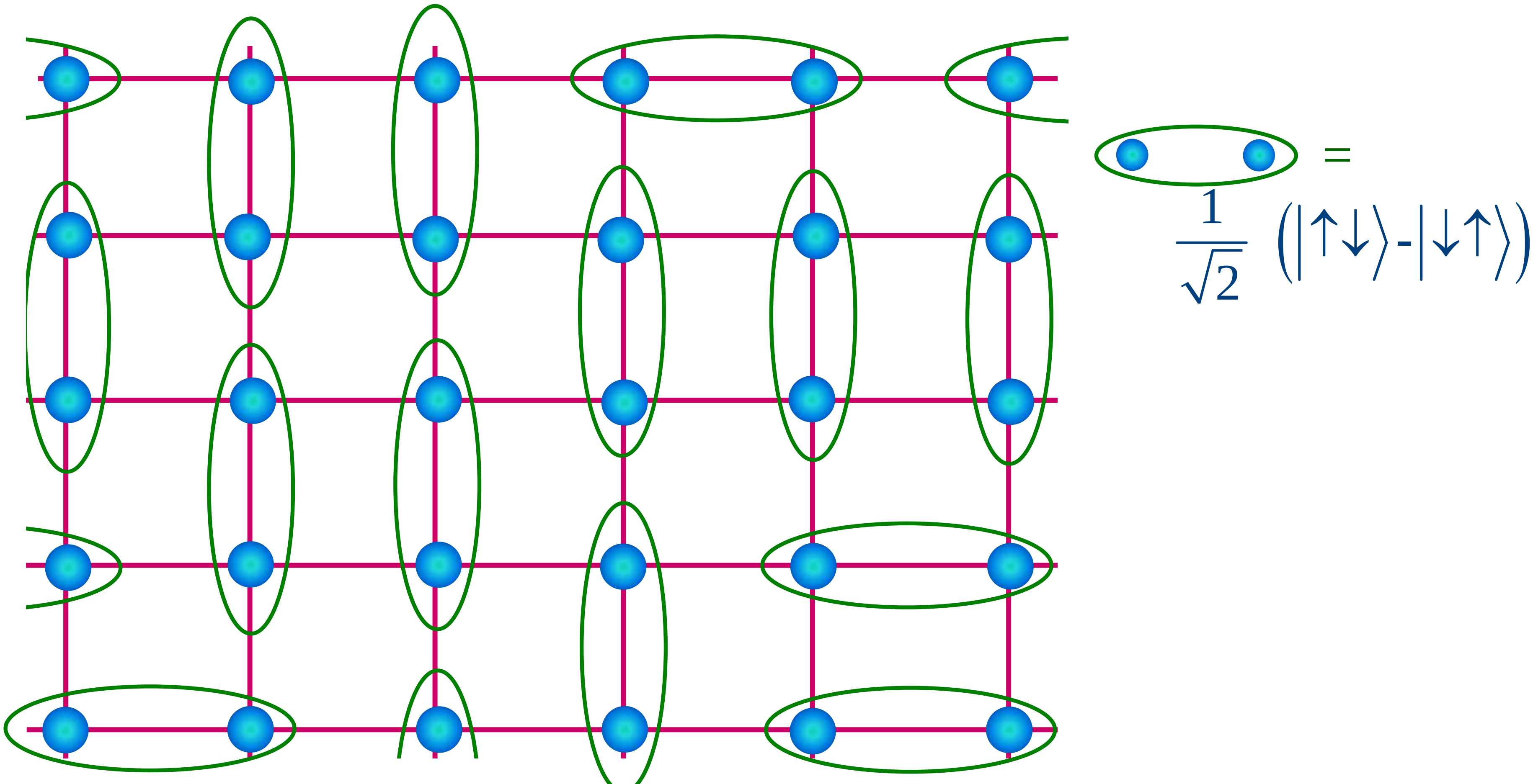
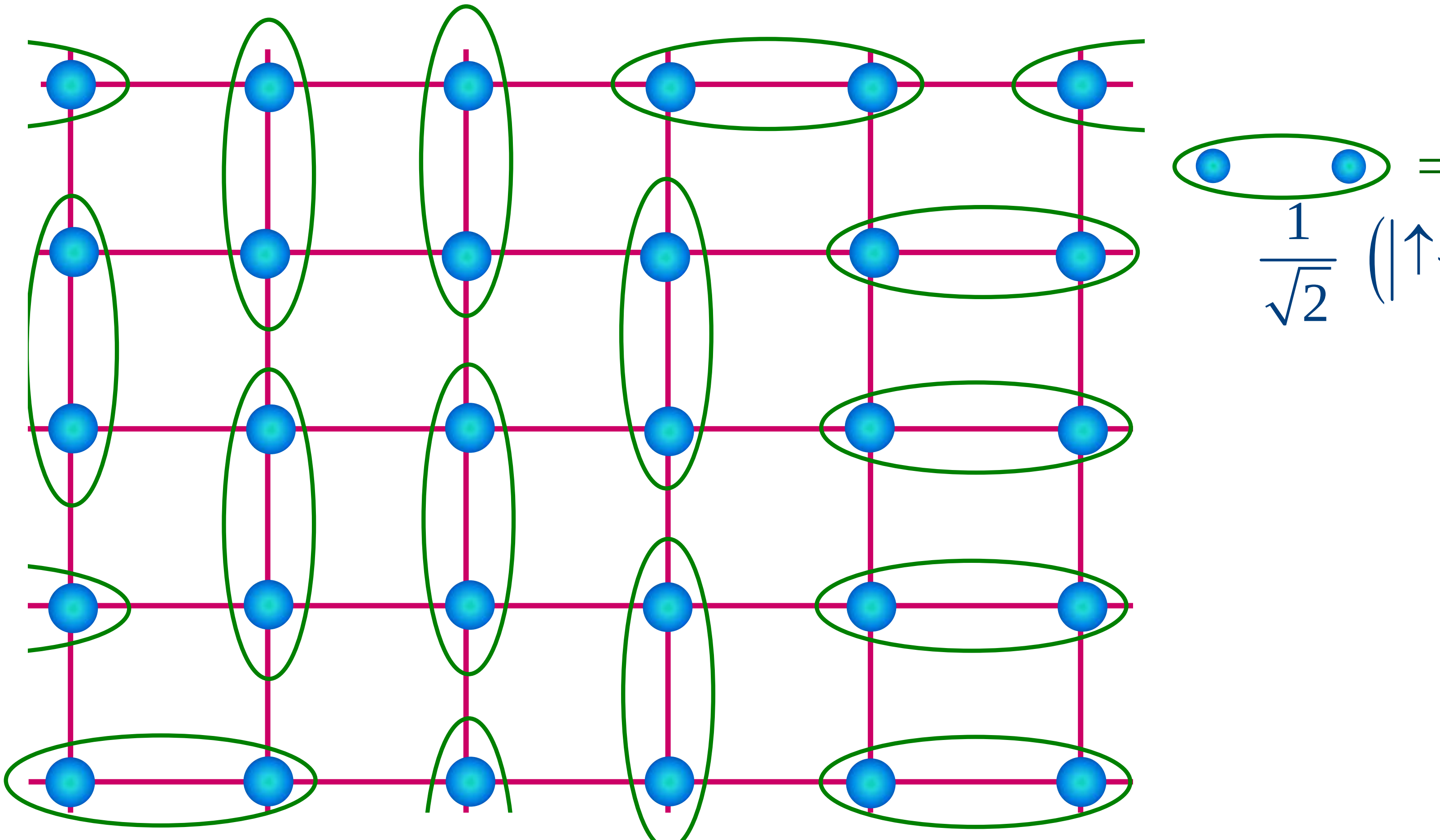
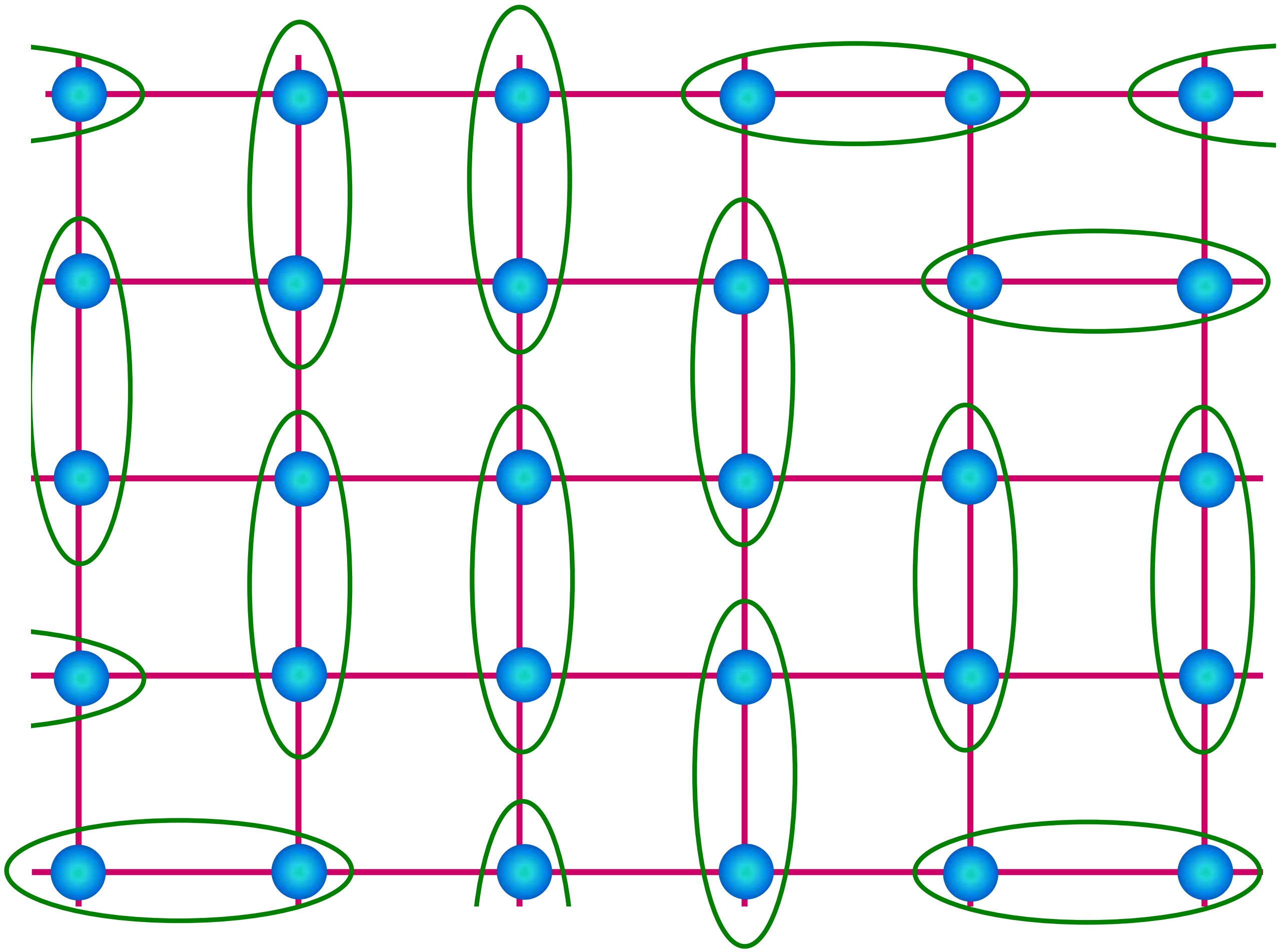
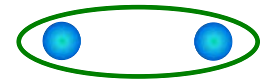

=
$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Entangled liquid of valence bonds
(Resonating valence bonds – RVB)

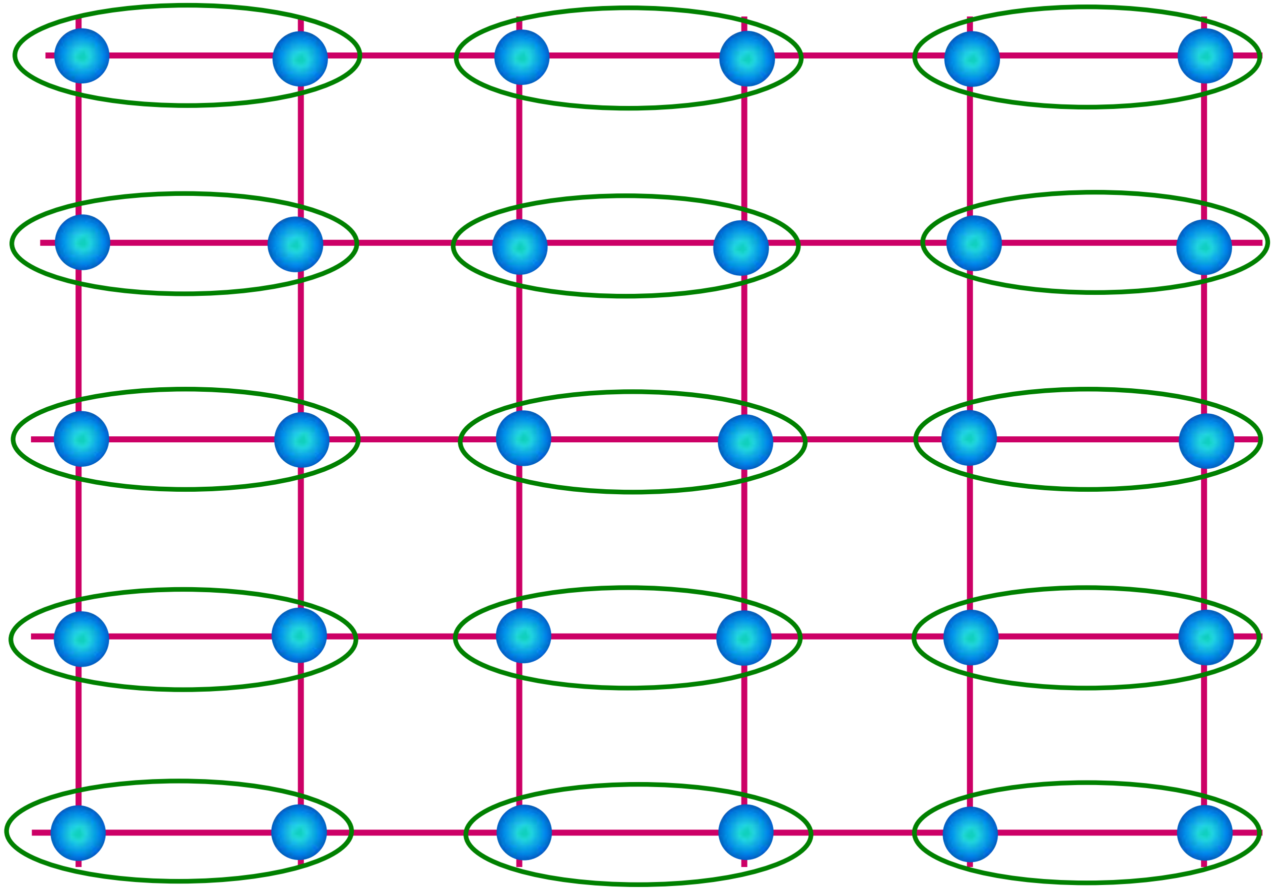


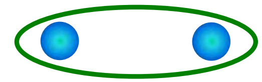


$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

Valence bond solid (VBS)





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N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).

R. Moessner and S. L. Sondhi, *Phys. Rev.* B **63**, 224401 (2001).

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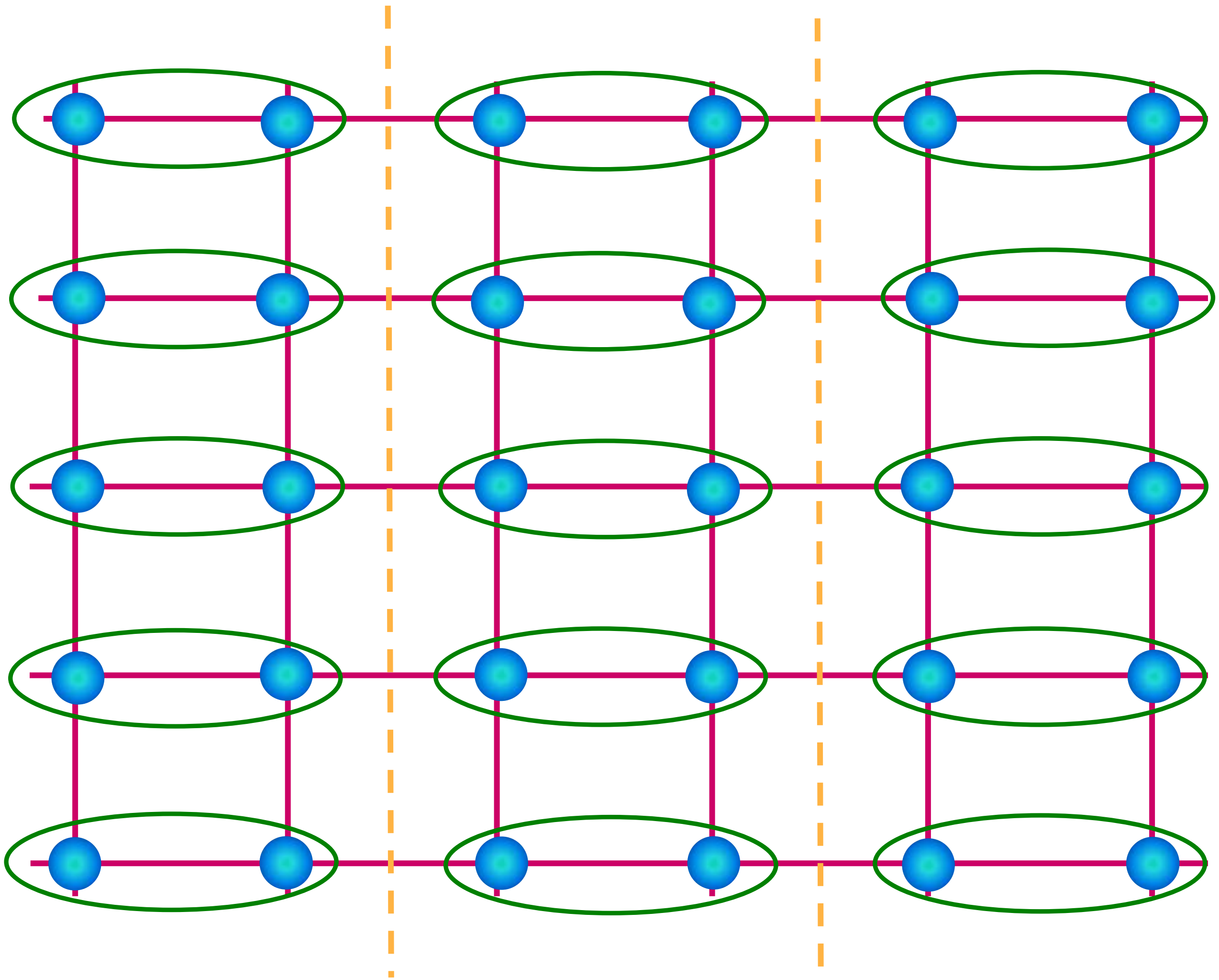

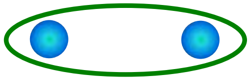 =

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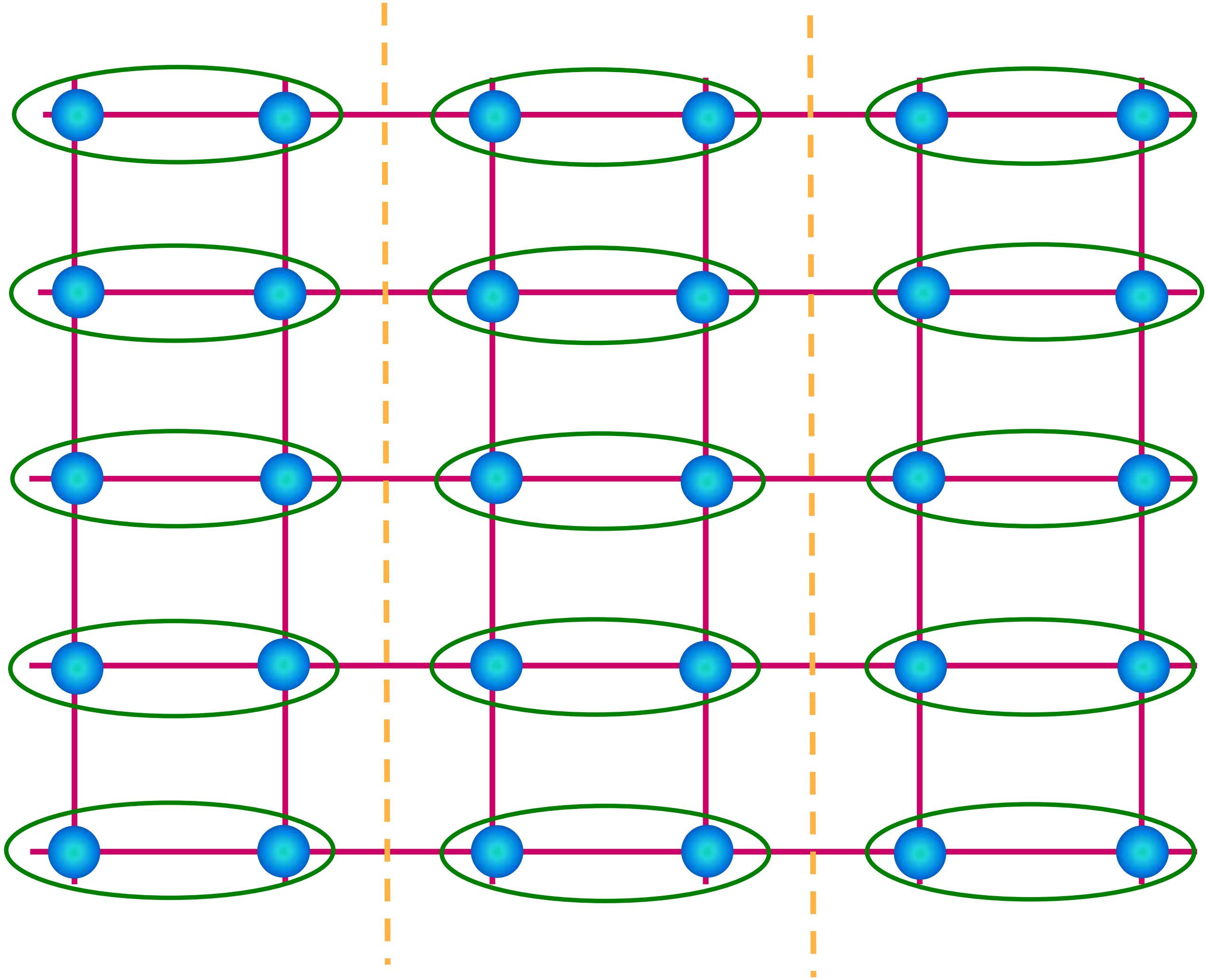
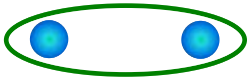
Valence bond solid (VBS)

More possibilities for entanglement with nearby states





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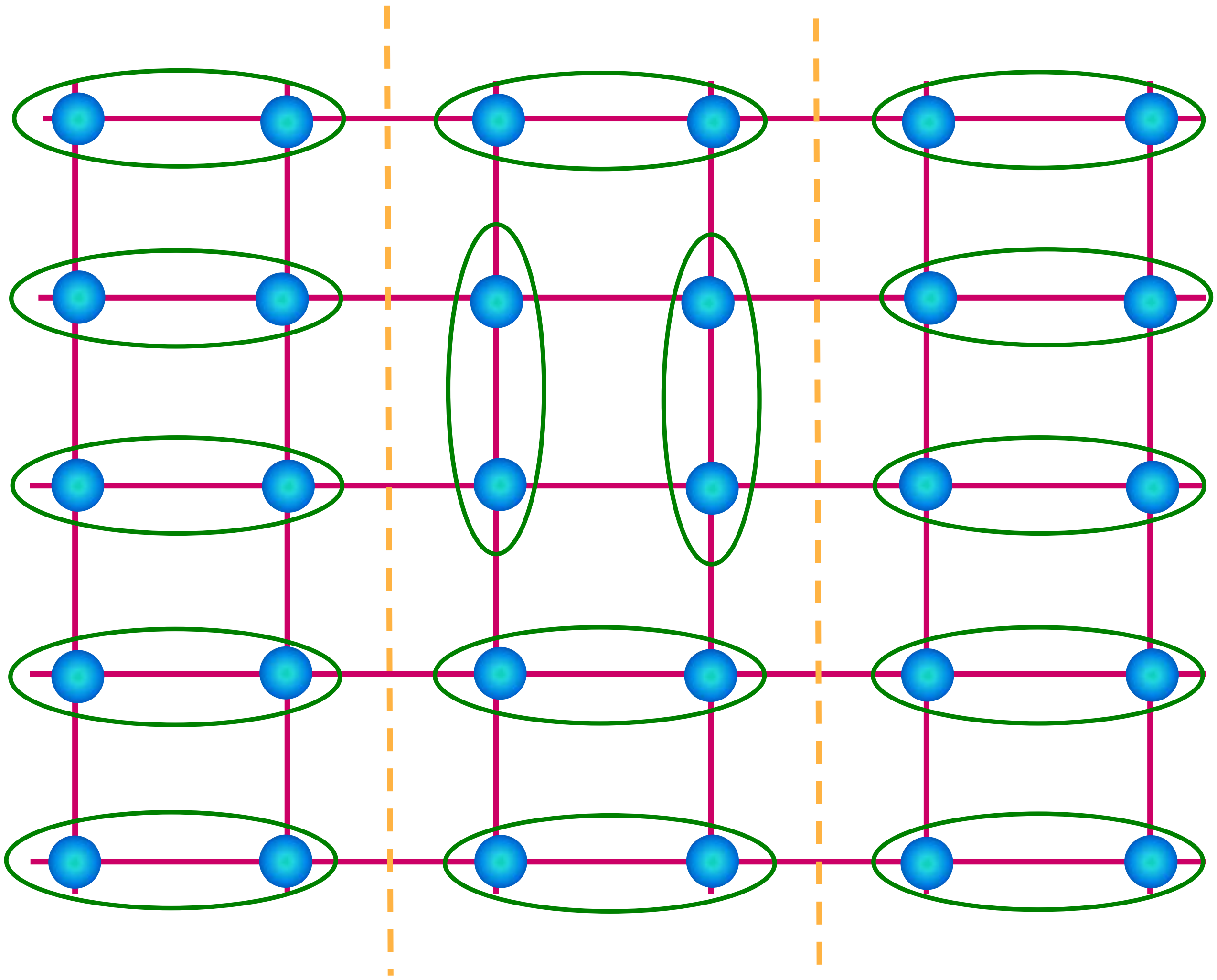

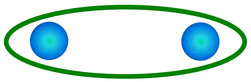 =

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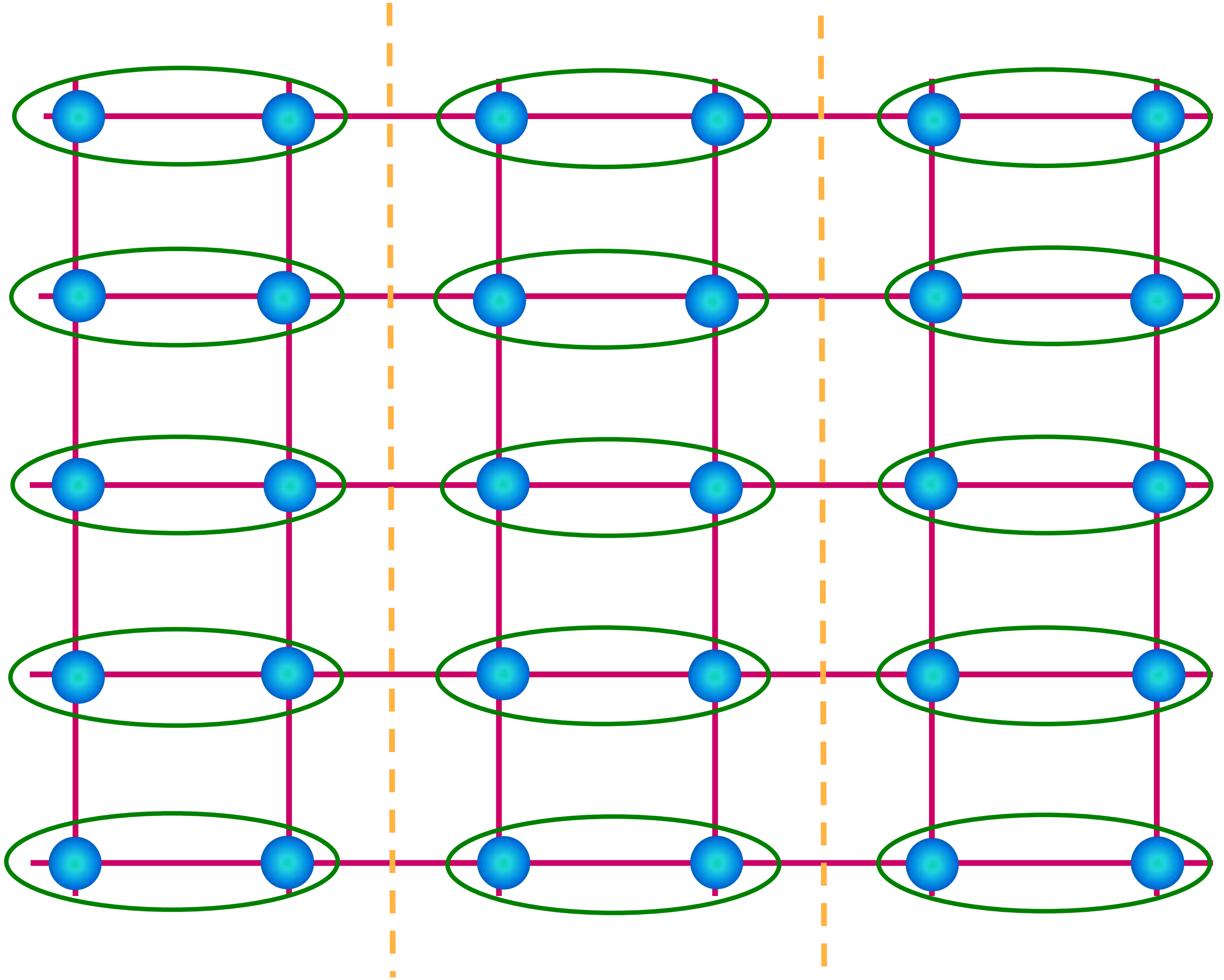

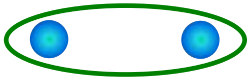

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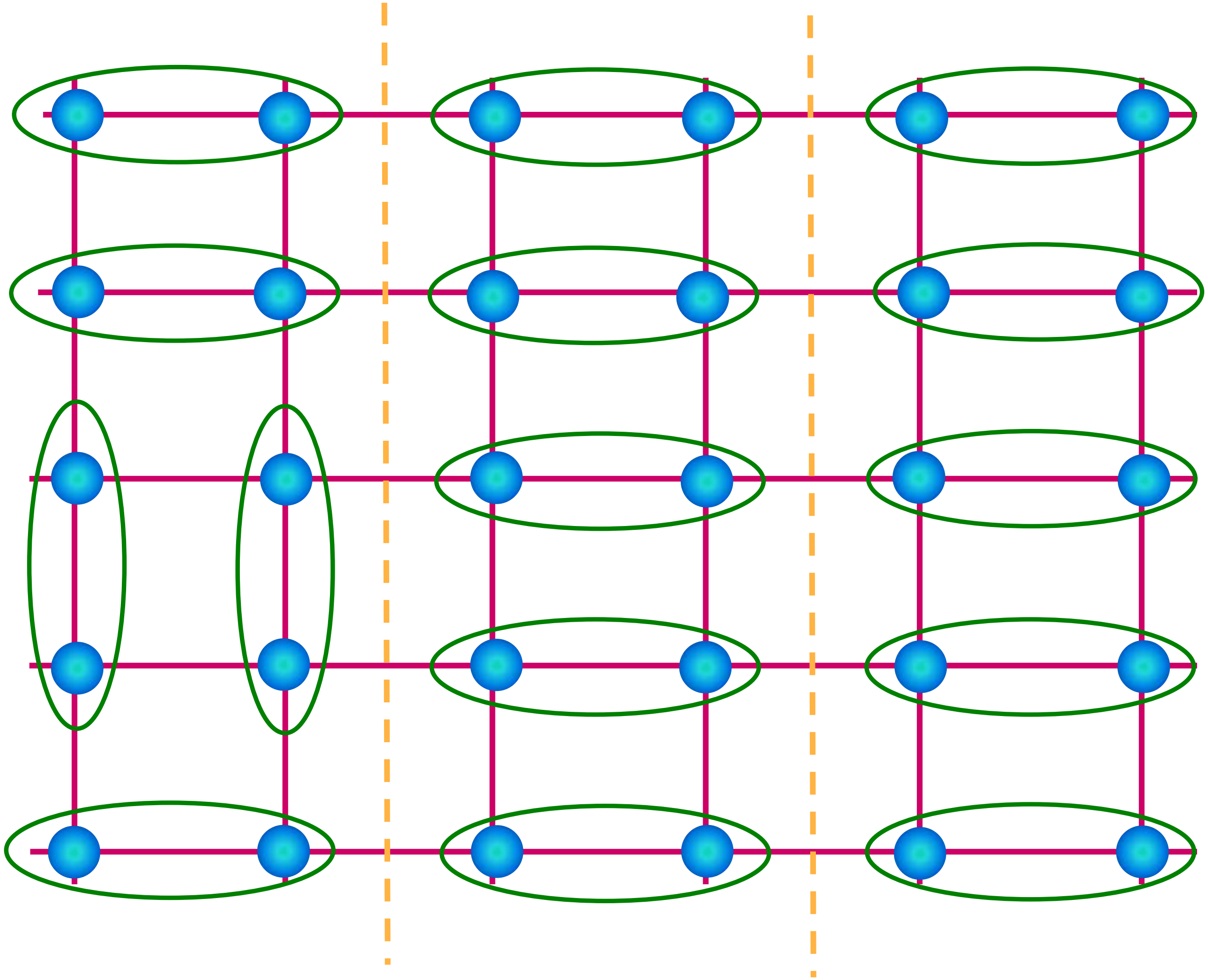

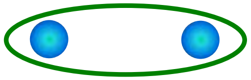

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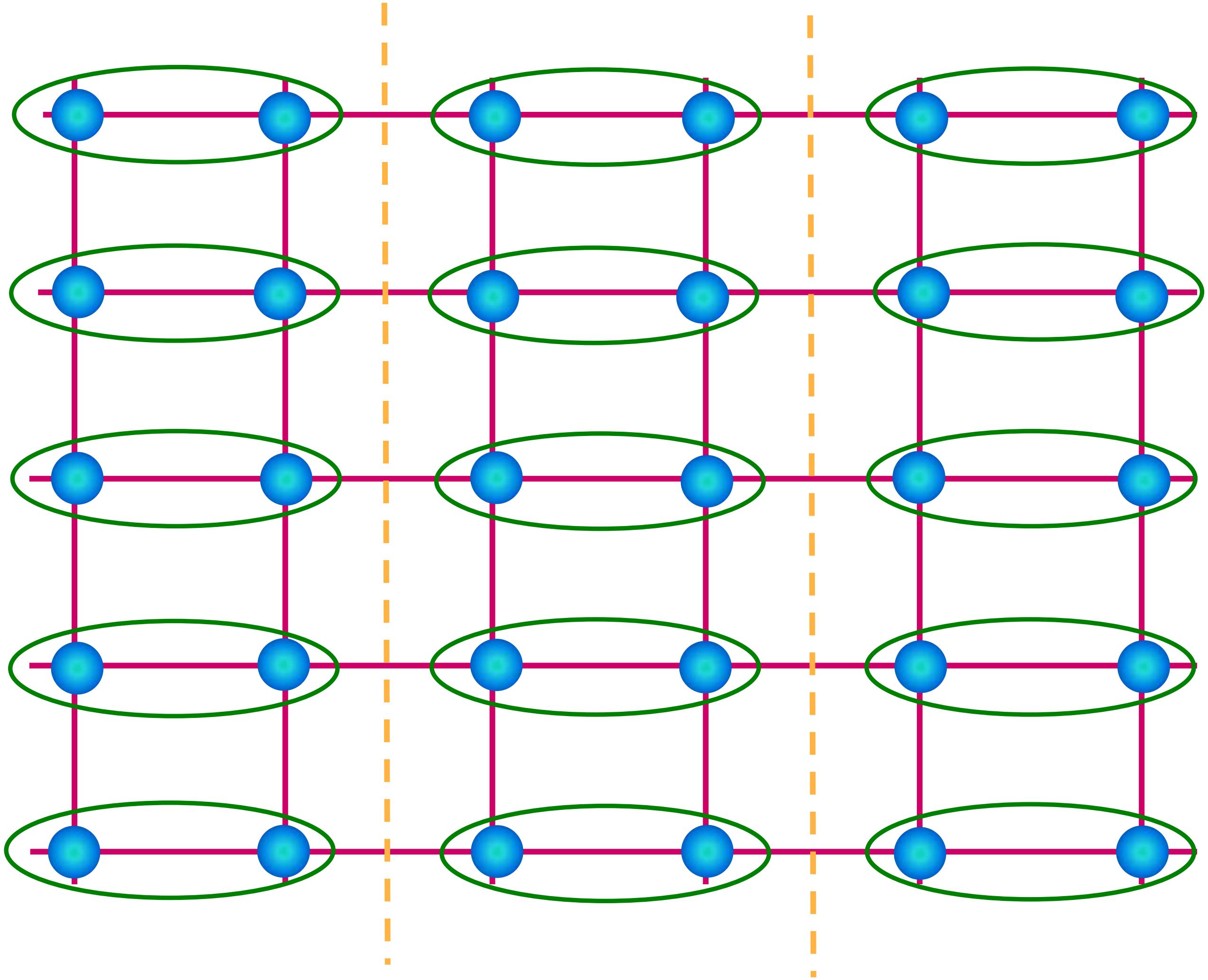

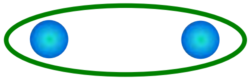 =

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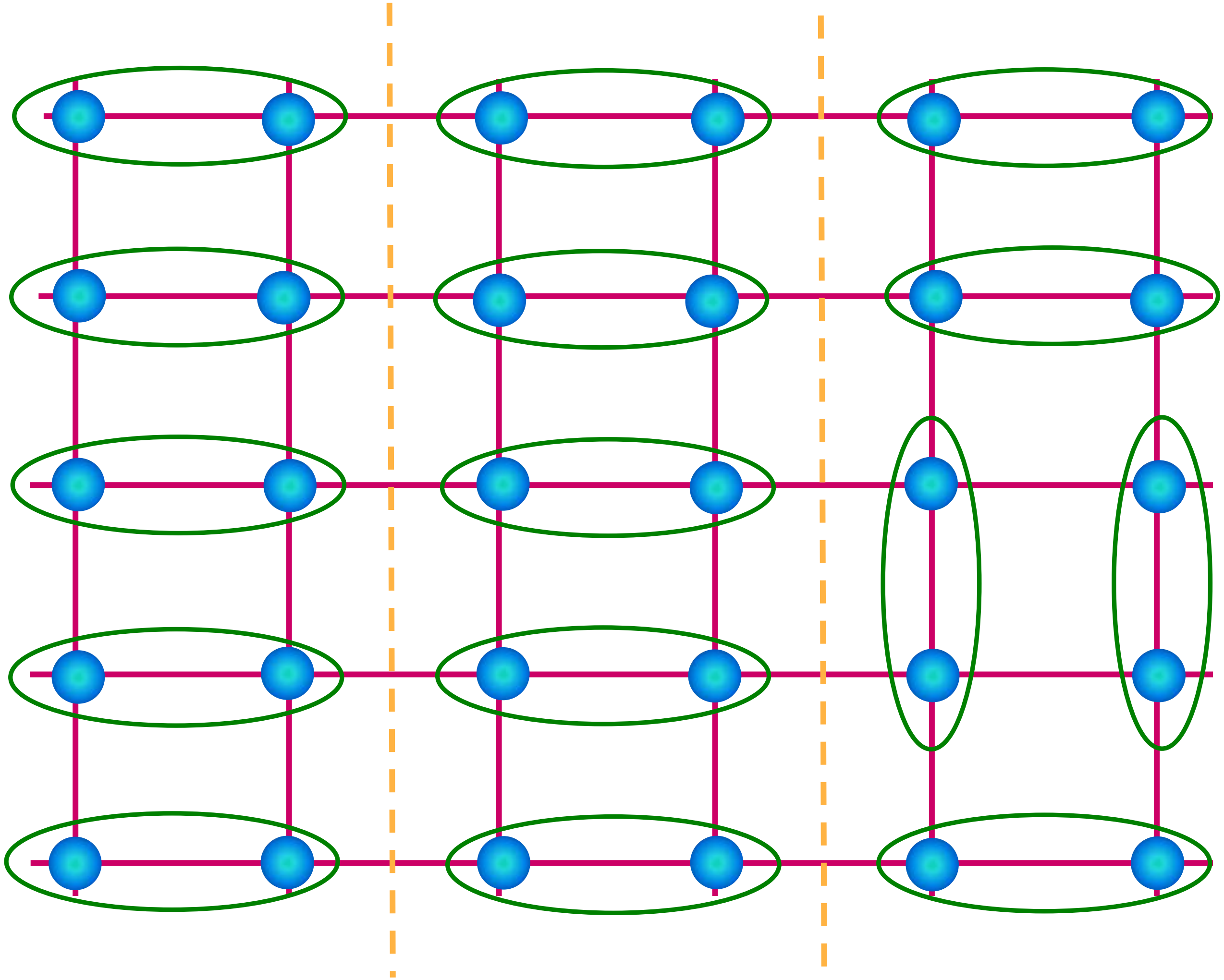
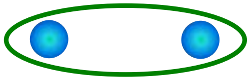
Valence bond solid (VBS)

More possibilities for entanglement with nearby states





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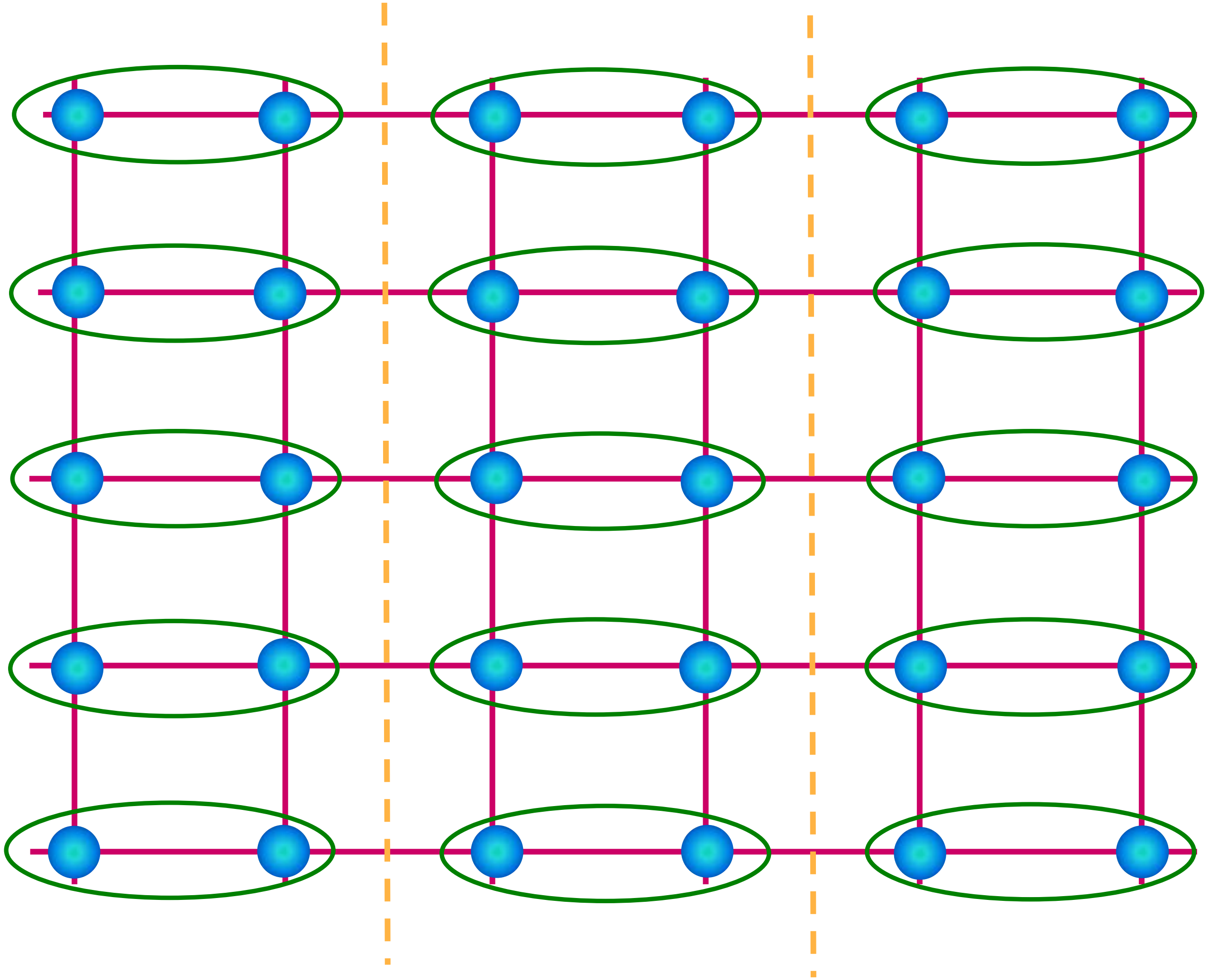

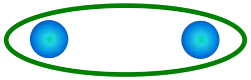

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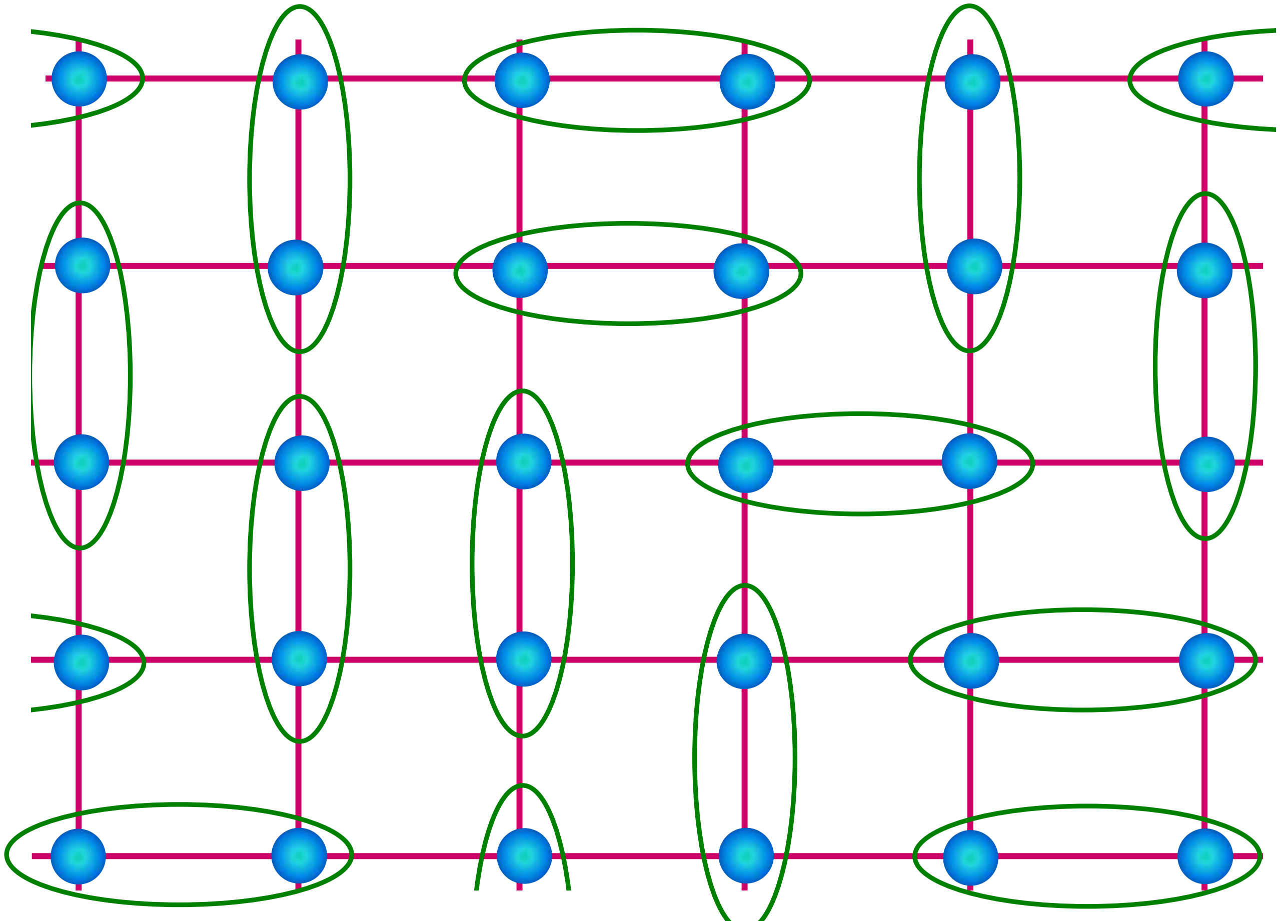
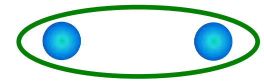
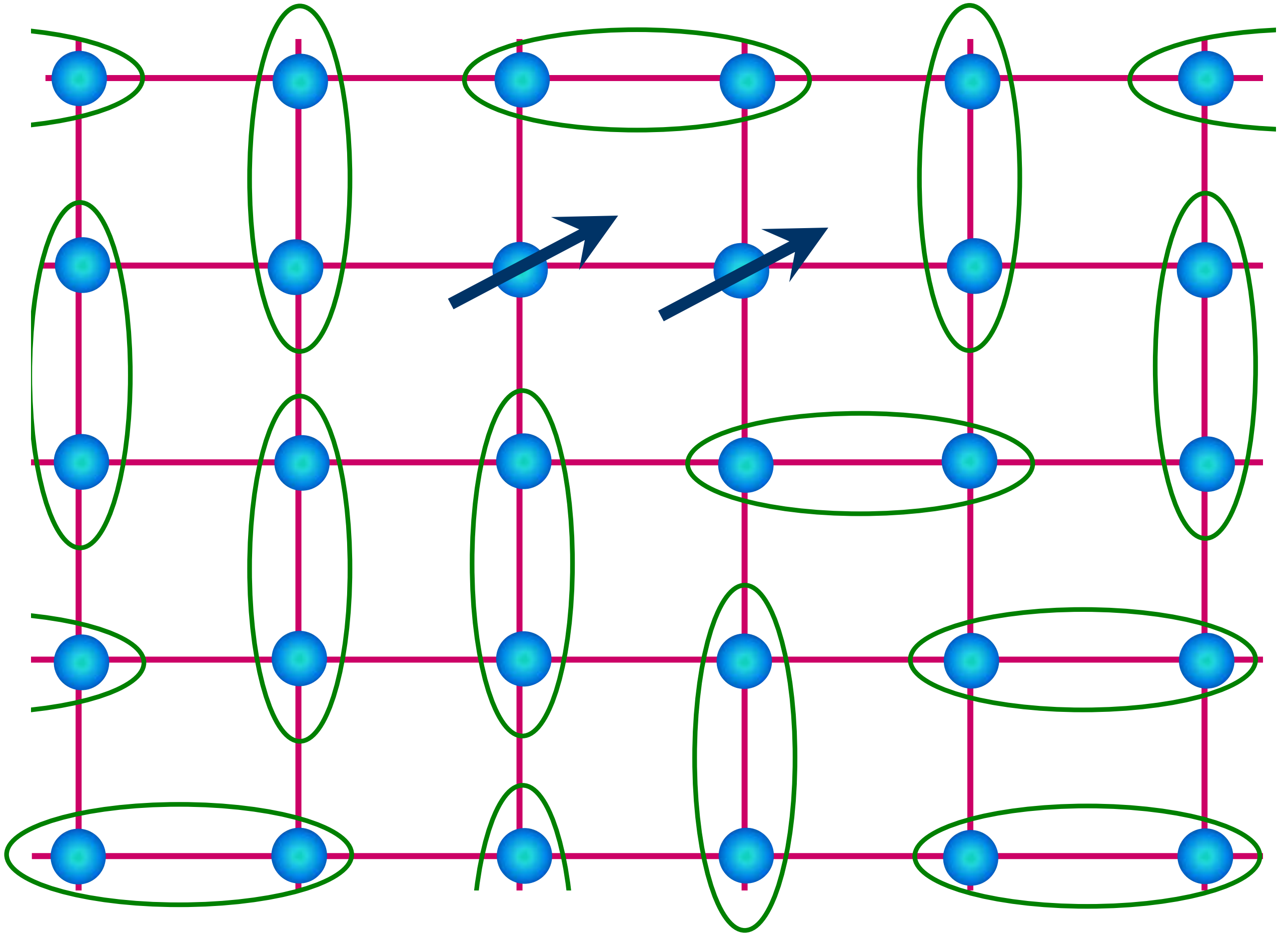
Excitations of the RVB liquid

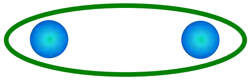


 =

$$\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Excitations of the RVB liquid





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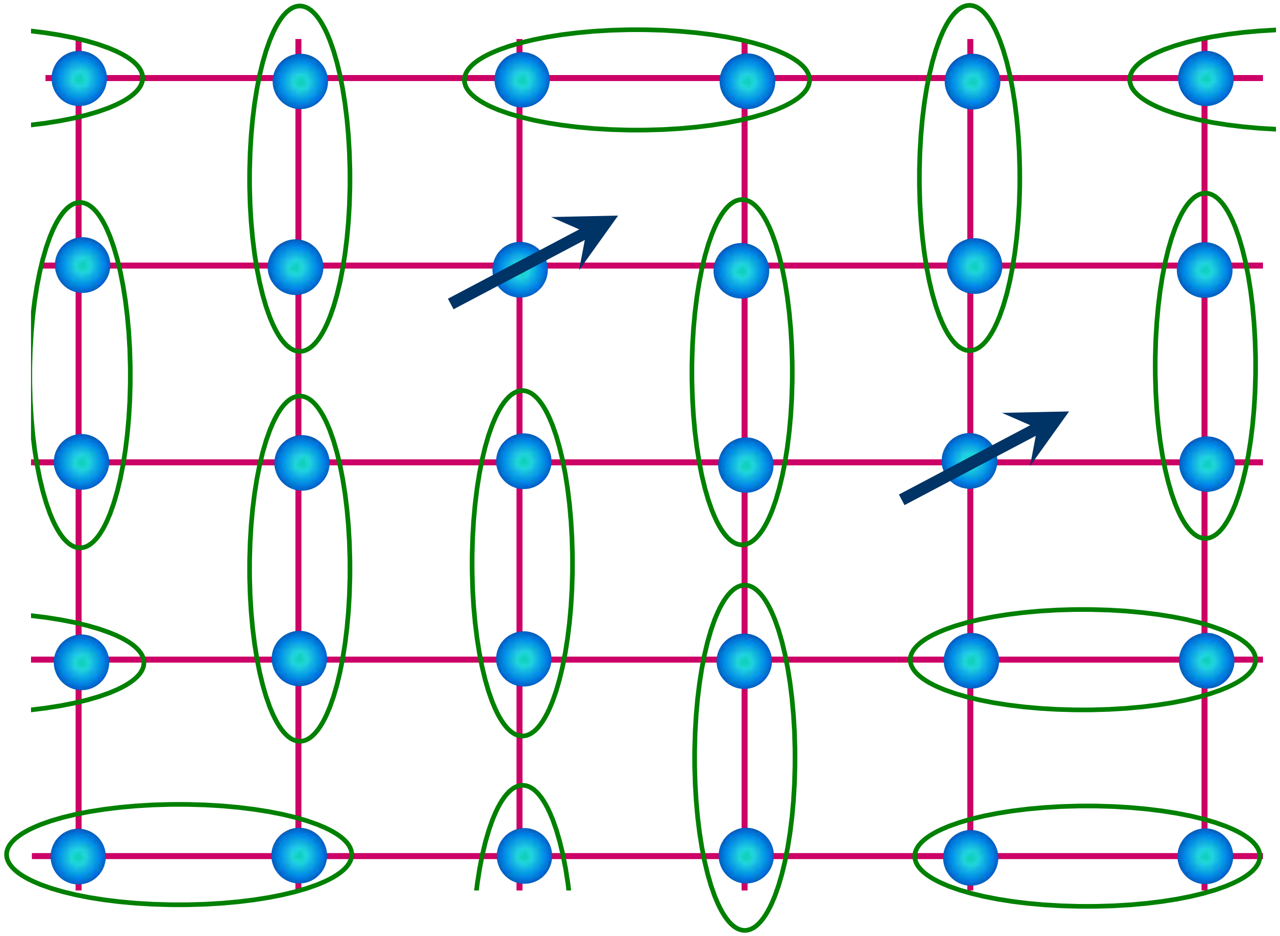

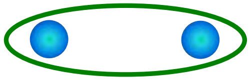
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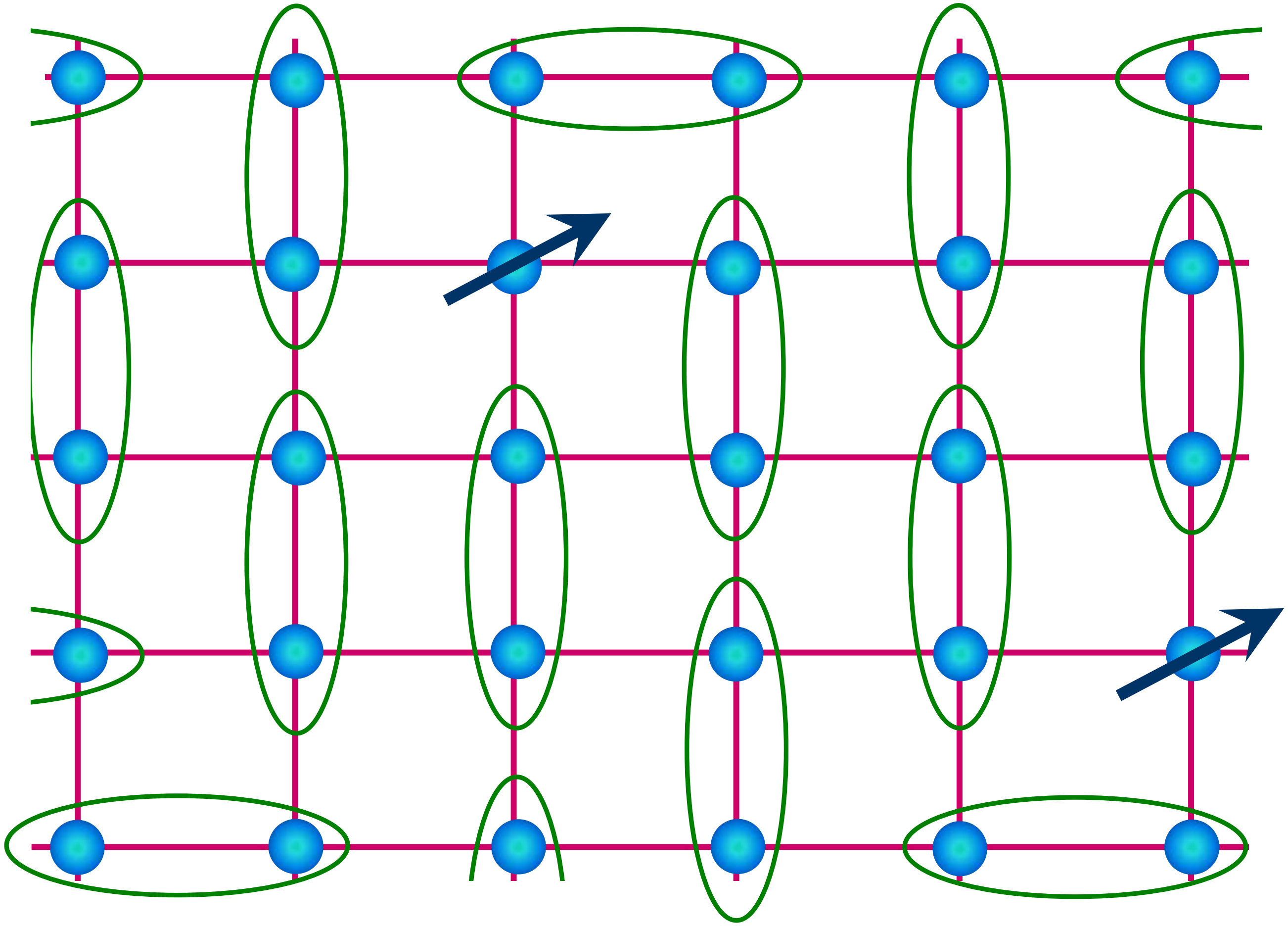
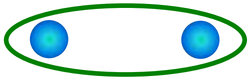
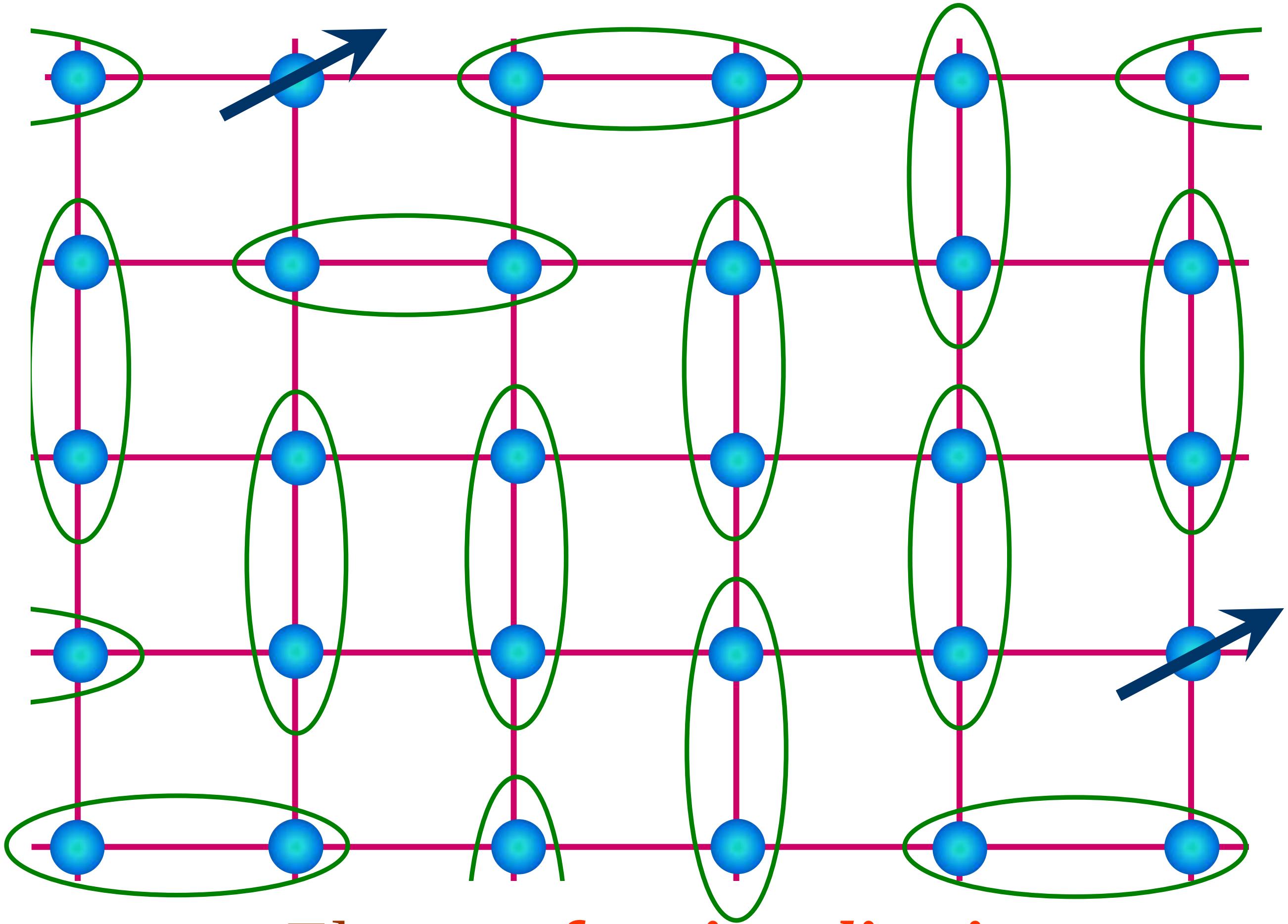
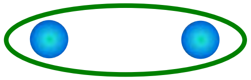
Excitations of the RVB liquid



 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Excitations of the RVB liquid

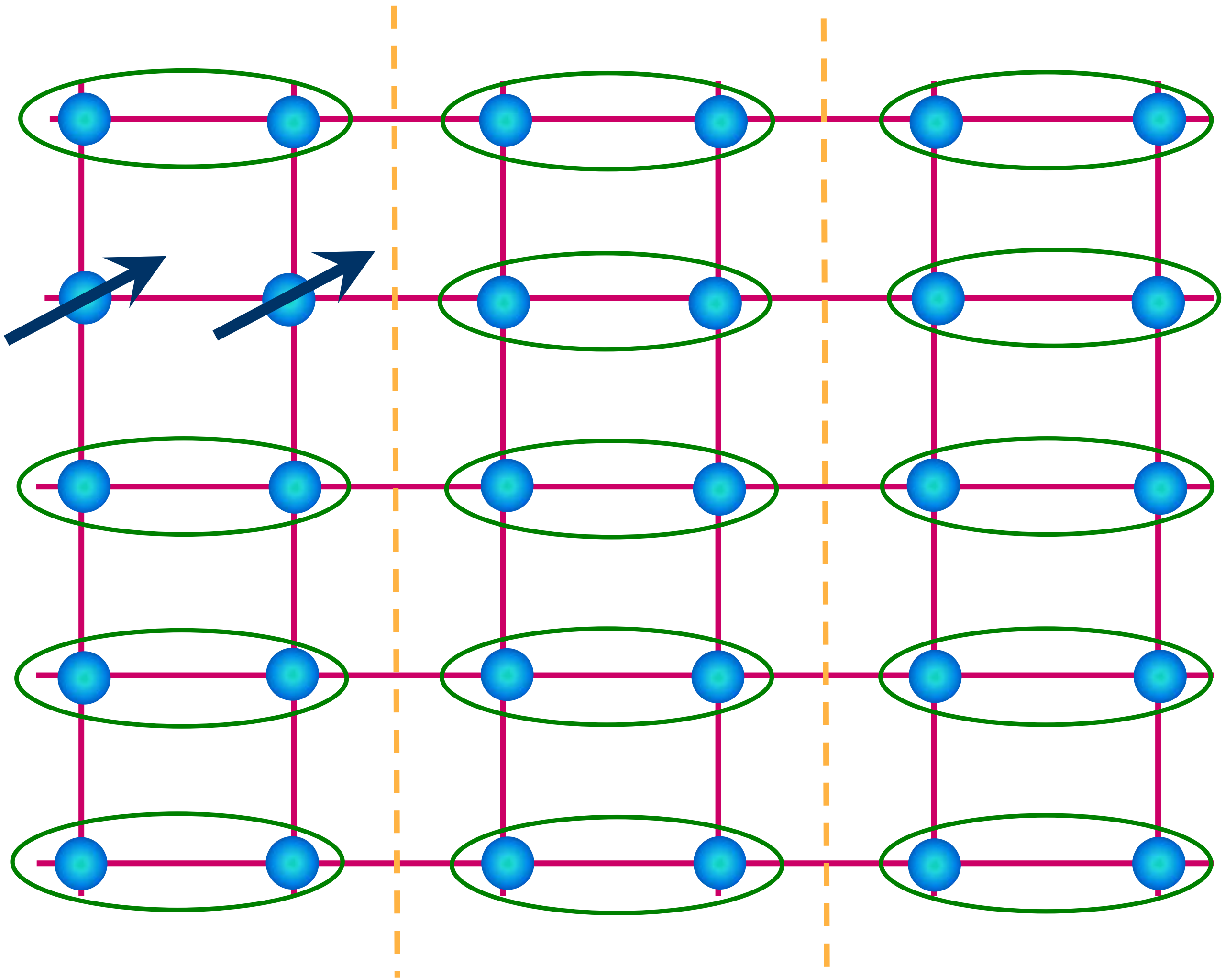


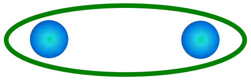


Electron ***fractionalization***:

Excitations carry spin S=1/2 but no charge

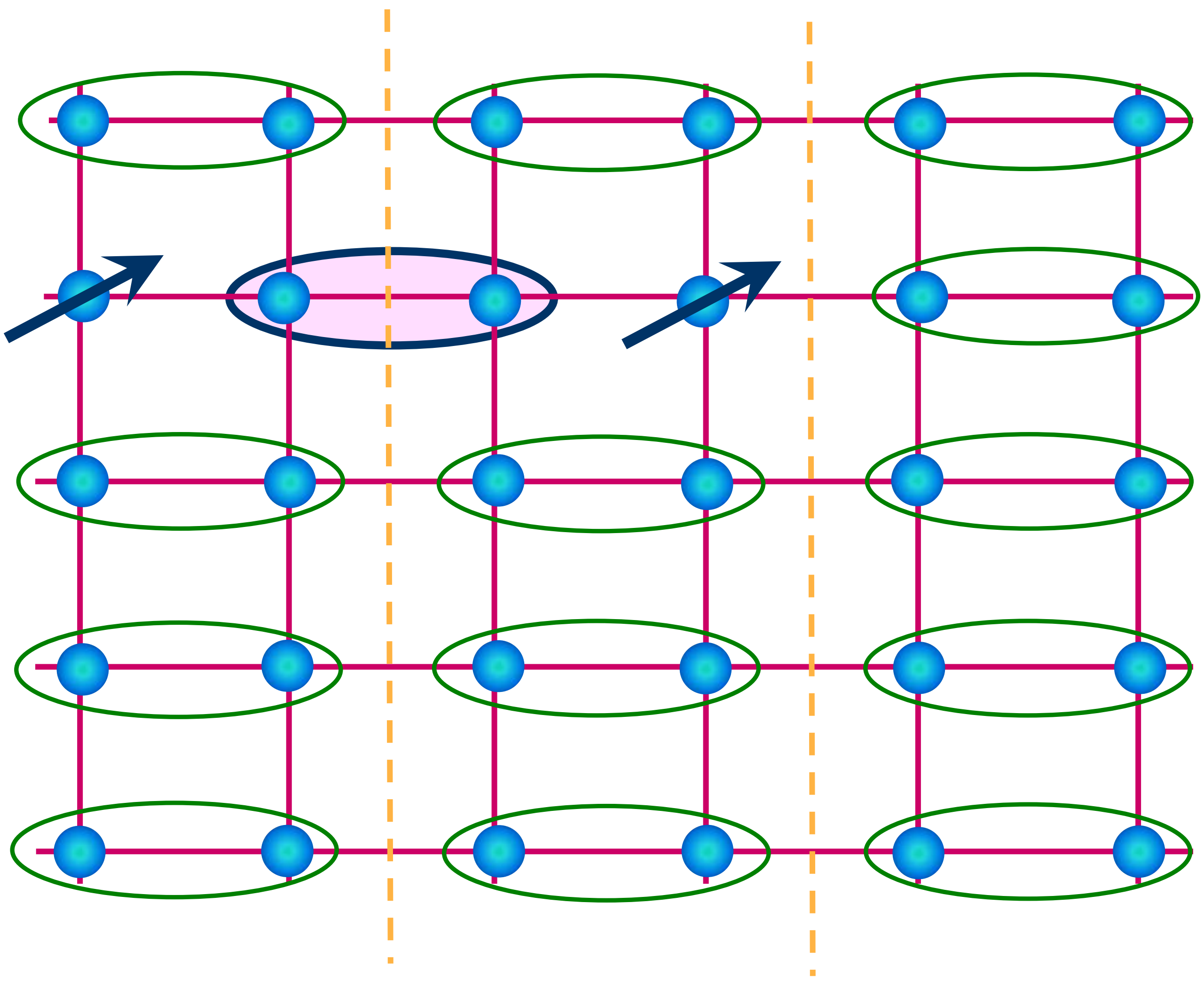
Excitations of the VBS

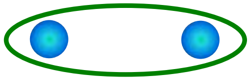




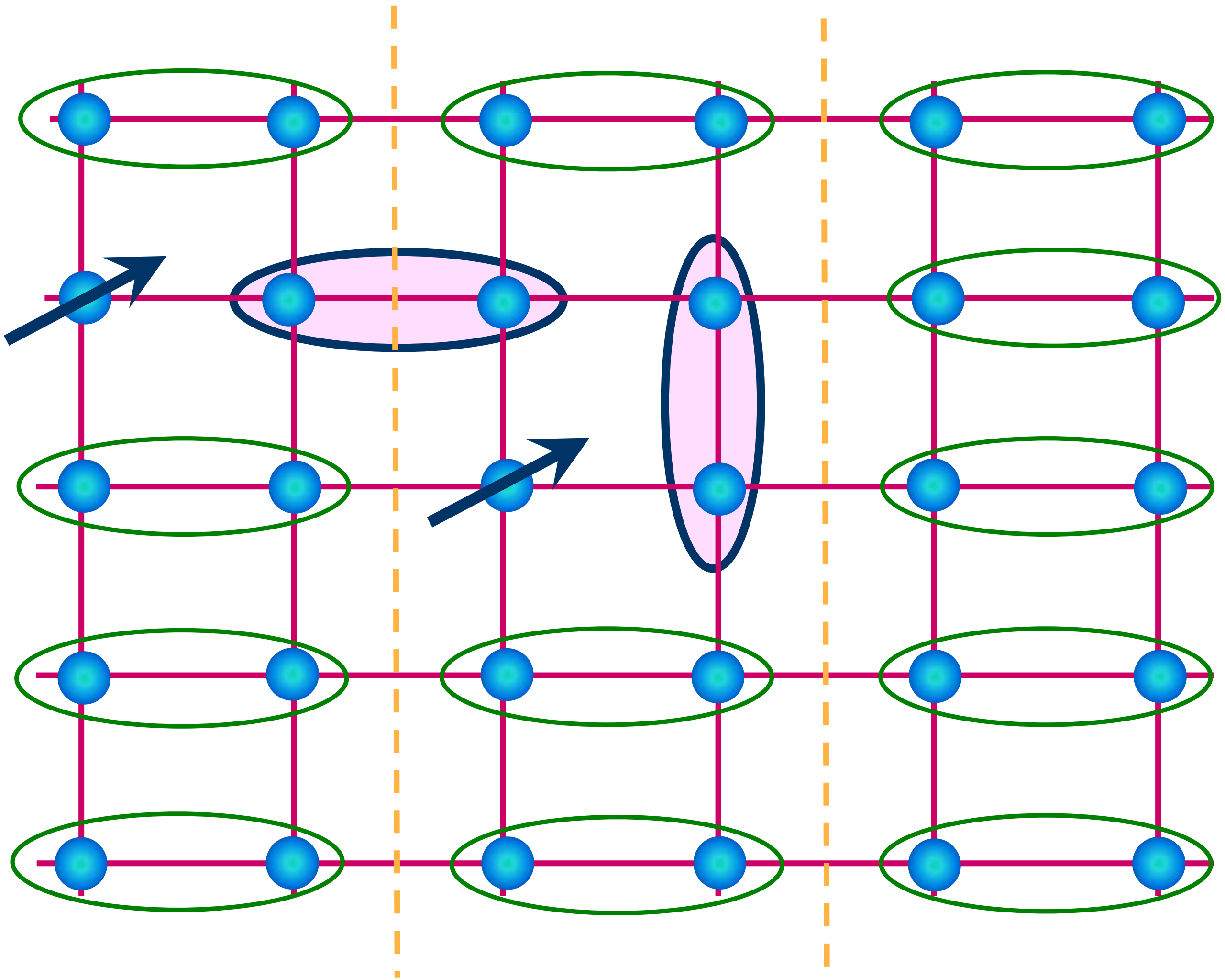
$$= \frac{1}{\sqrt{2}} \left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

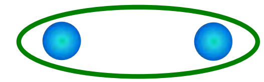
Excitations of the VBS



 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Excitations of the VBS





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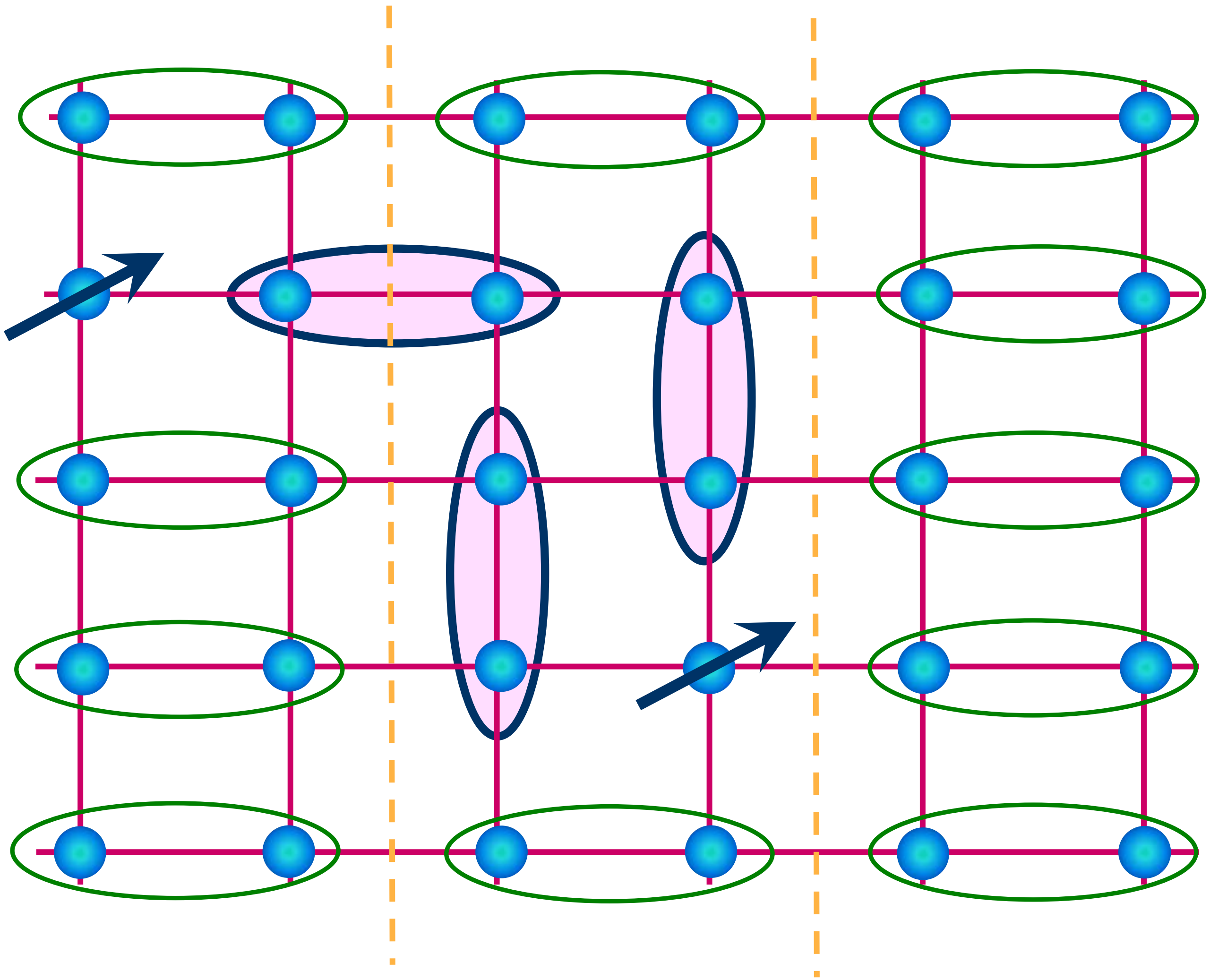

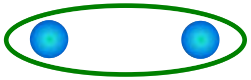

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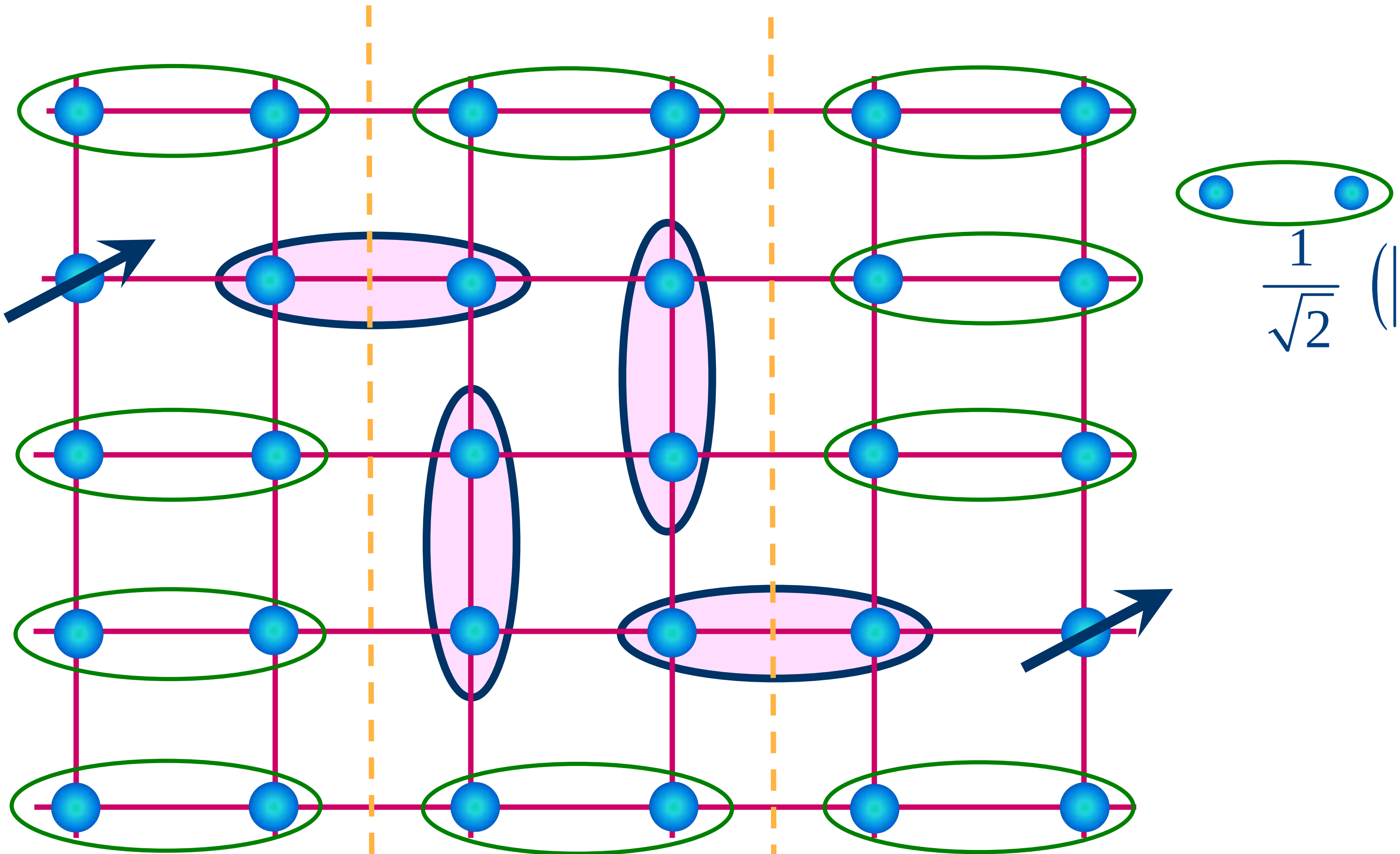
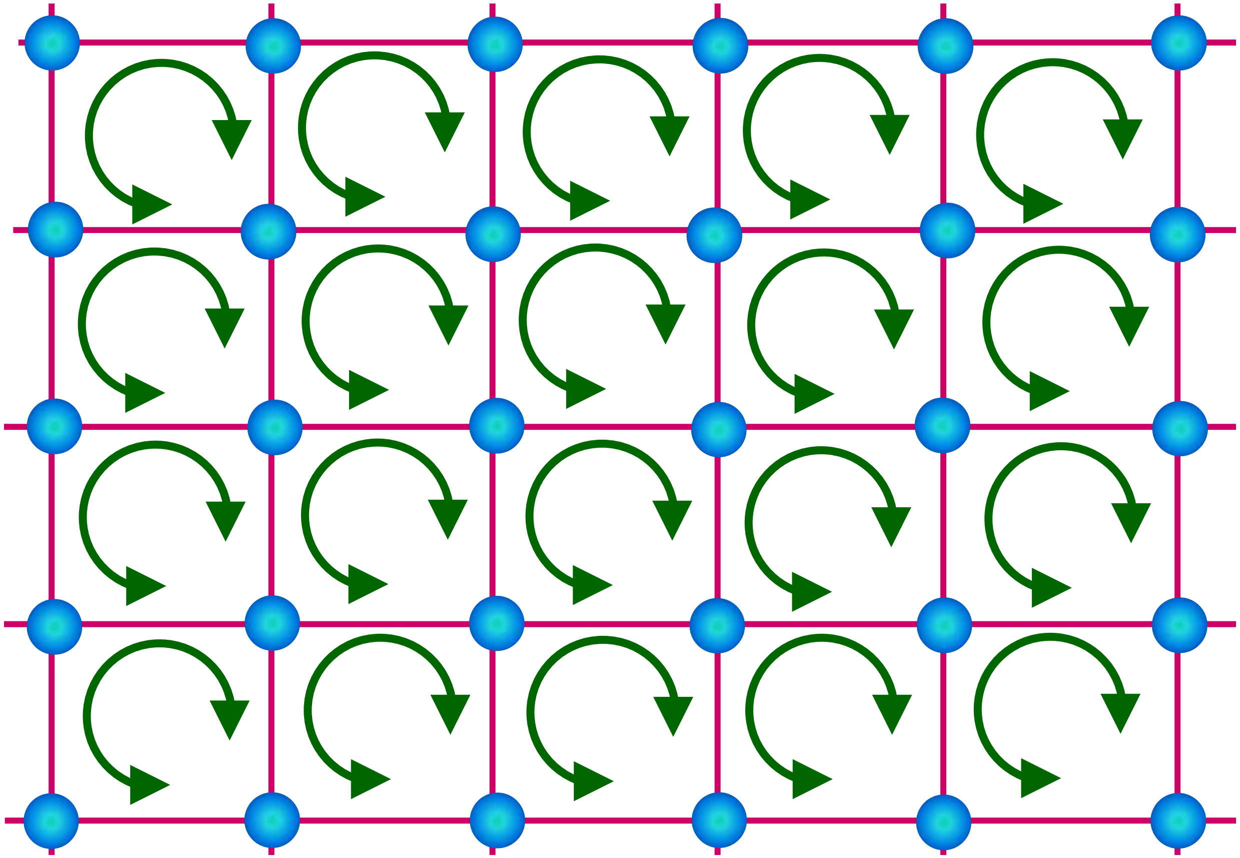


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Free spins are unable to move apart:
no fractionalization, but ***confinement***

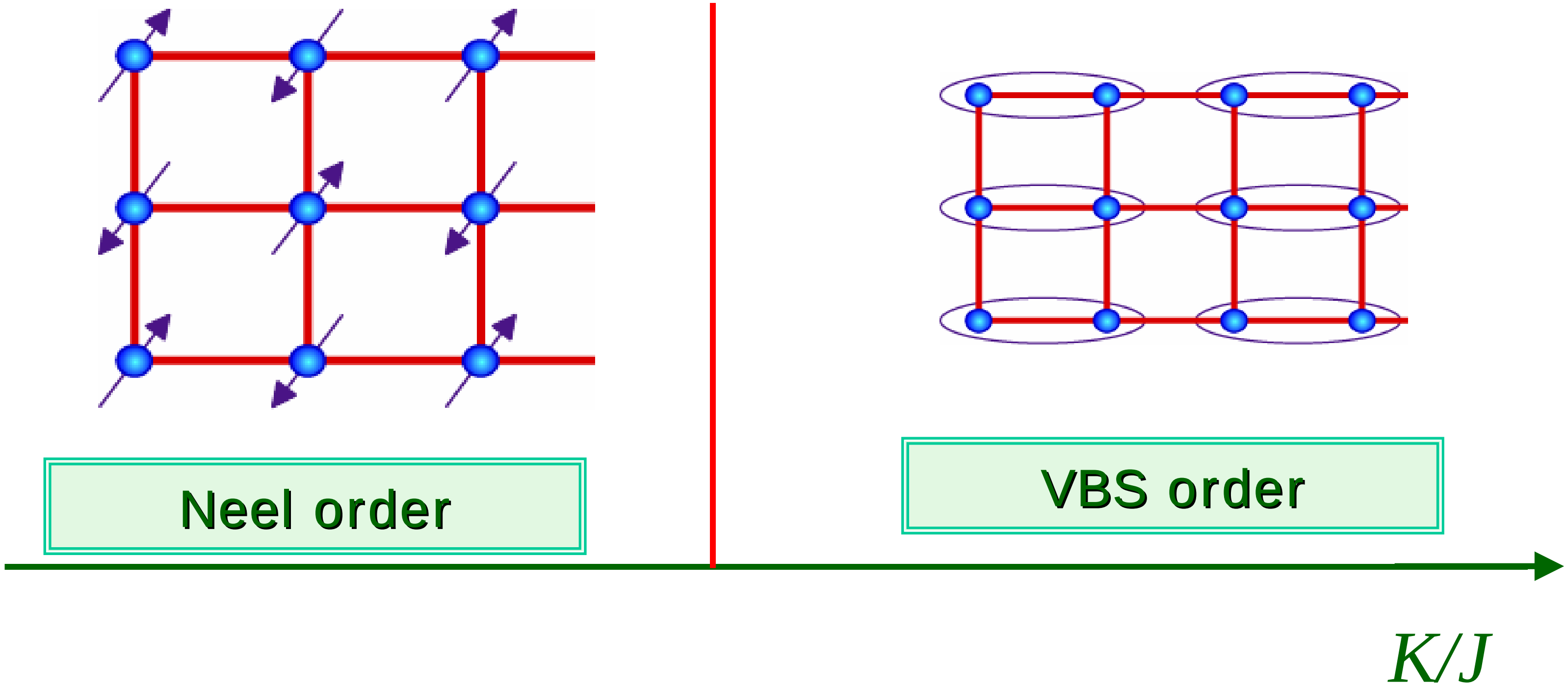
Phase diagram of square lattice antiferromagnet



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \; + \; K \sum_{\Box} \text{4-spin exchange}$$

A. W. Sandvik, cond-mat/0611343

Phase diagram of square lattice antiferromagnet



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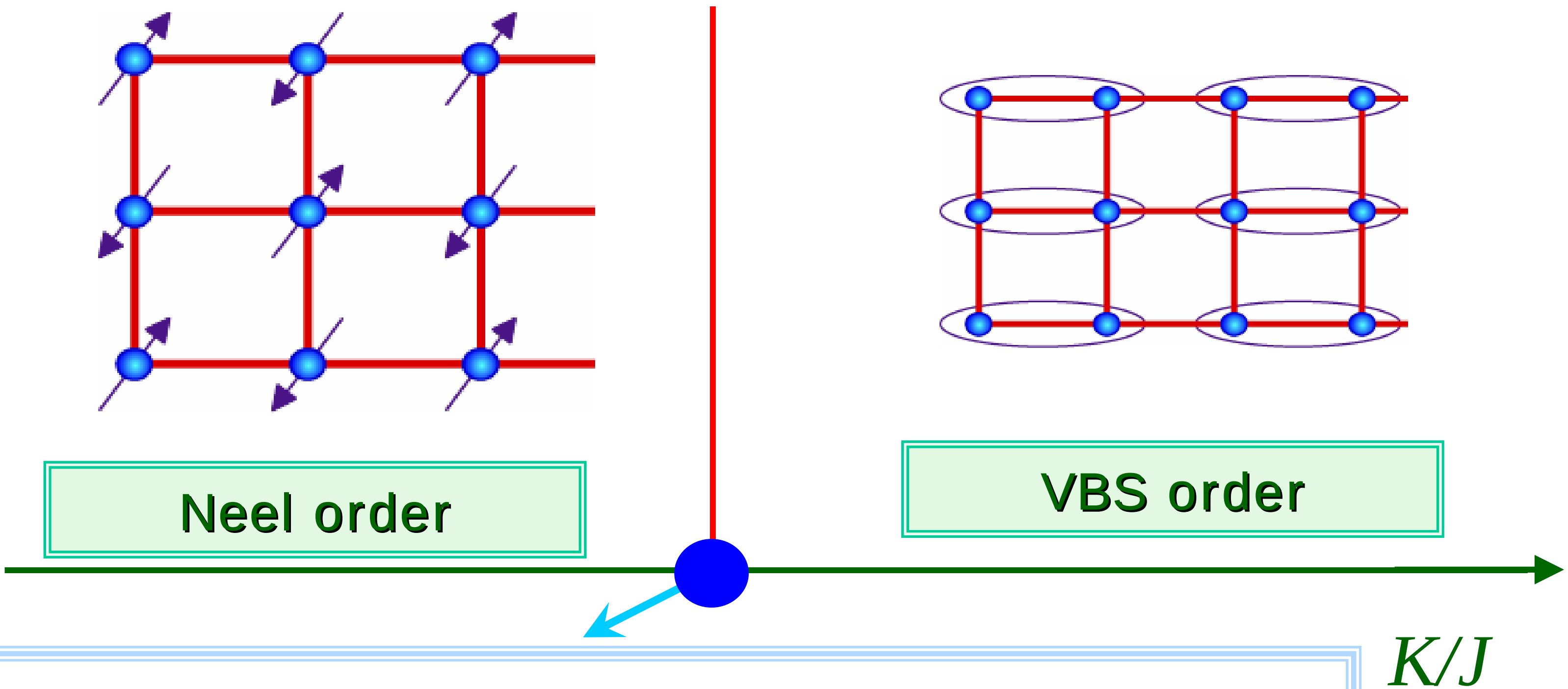
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Phase diagram of square lattice antiferromagnet



RVB physics appears at the quantum critical point which has fractionalized excitations: "*deconfined criticality*"

$$H = J\sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j + K\sum_{\Box} \text{4-spin exchange}$$

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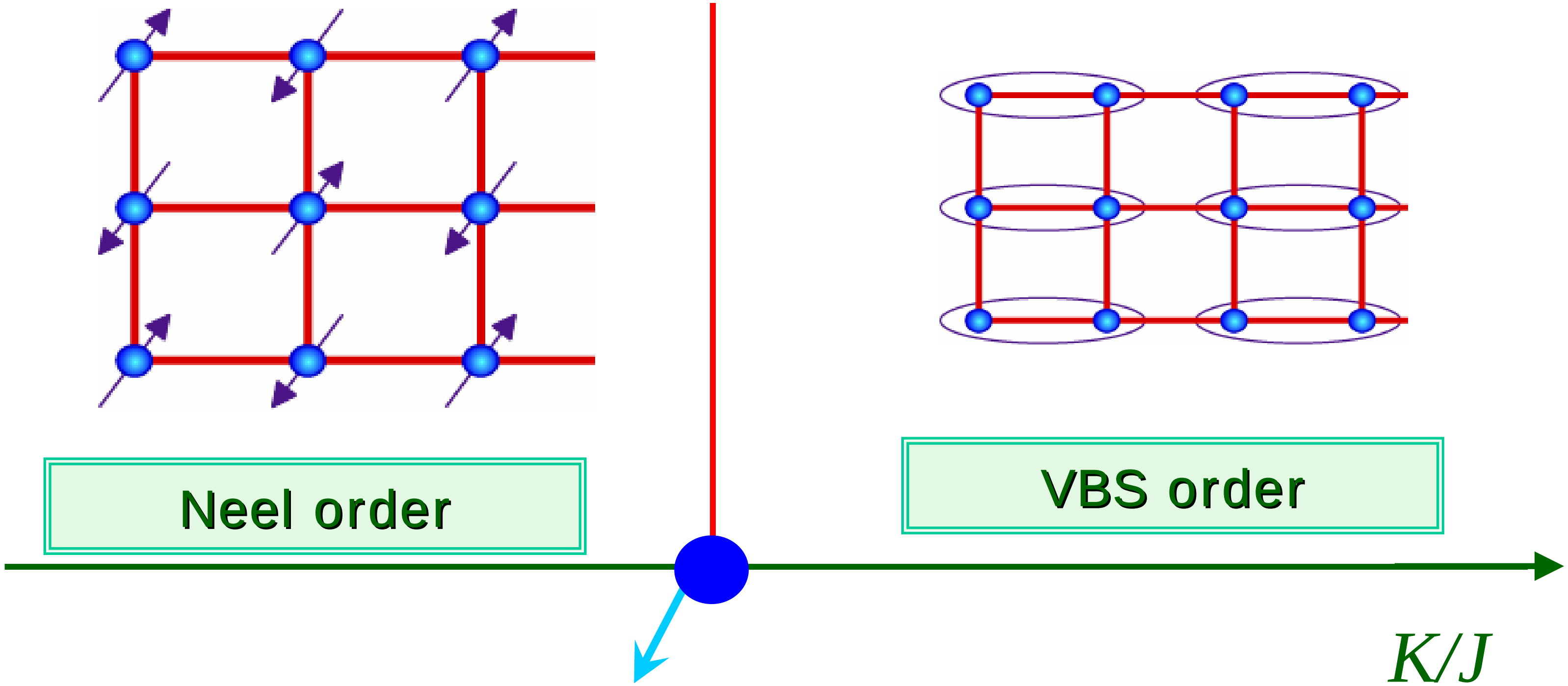
Phase diagram of square lattice antiferromagnet

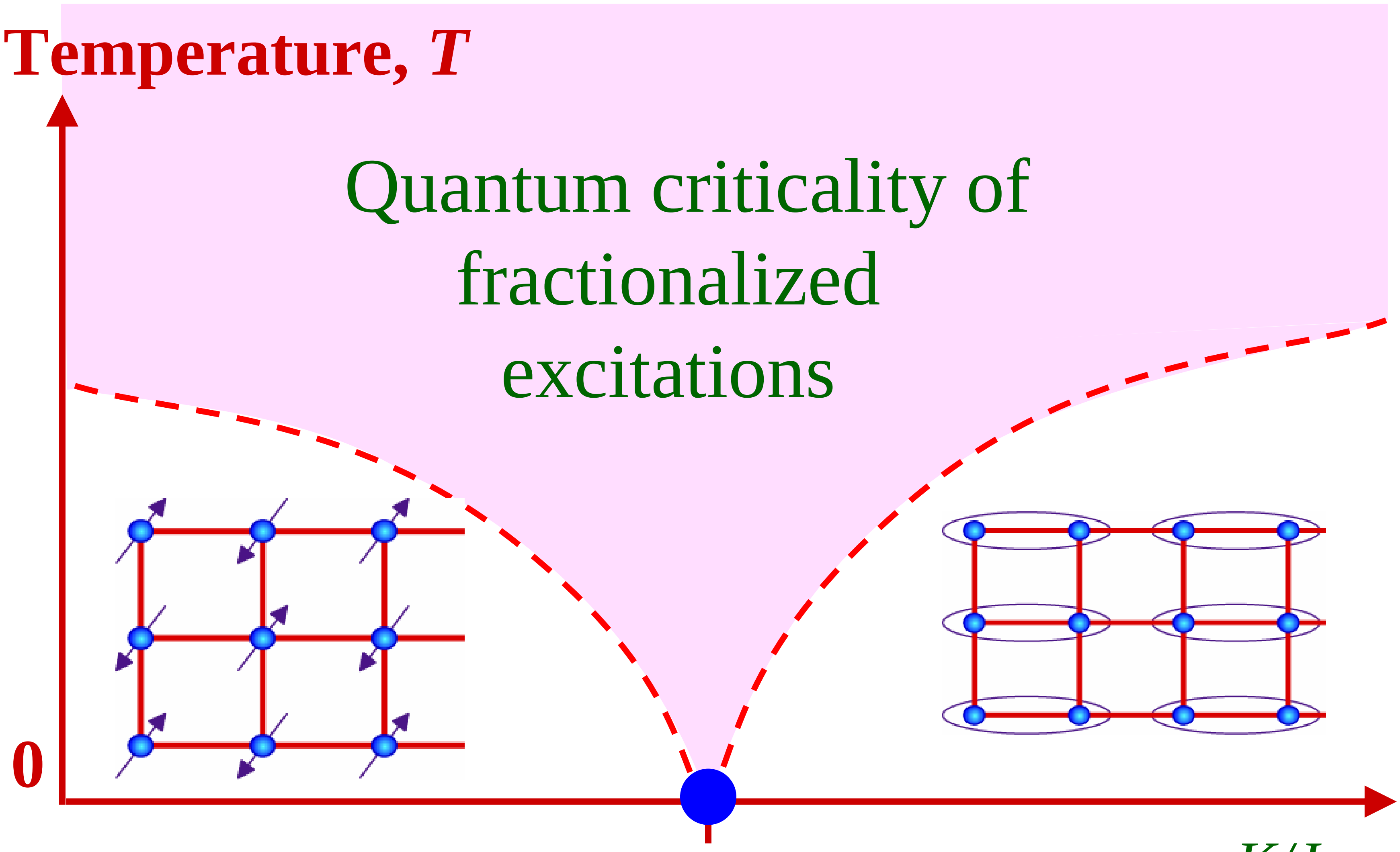


Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + r\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

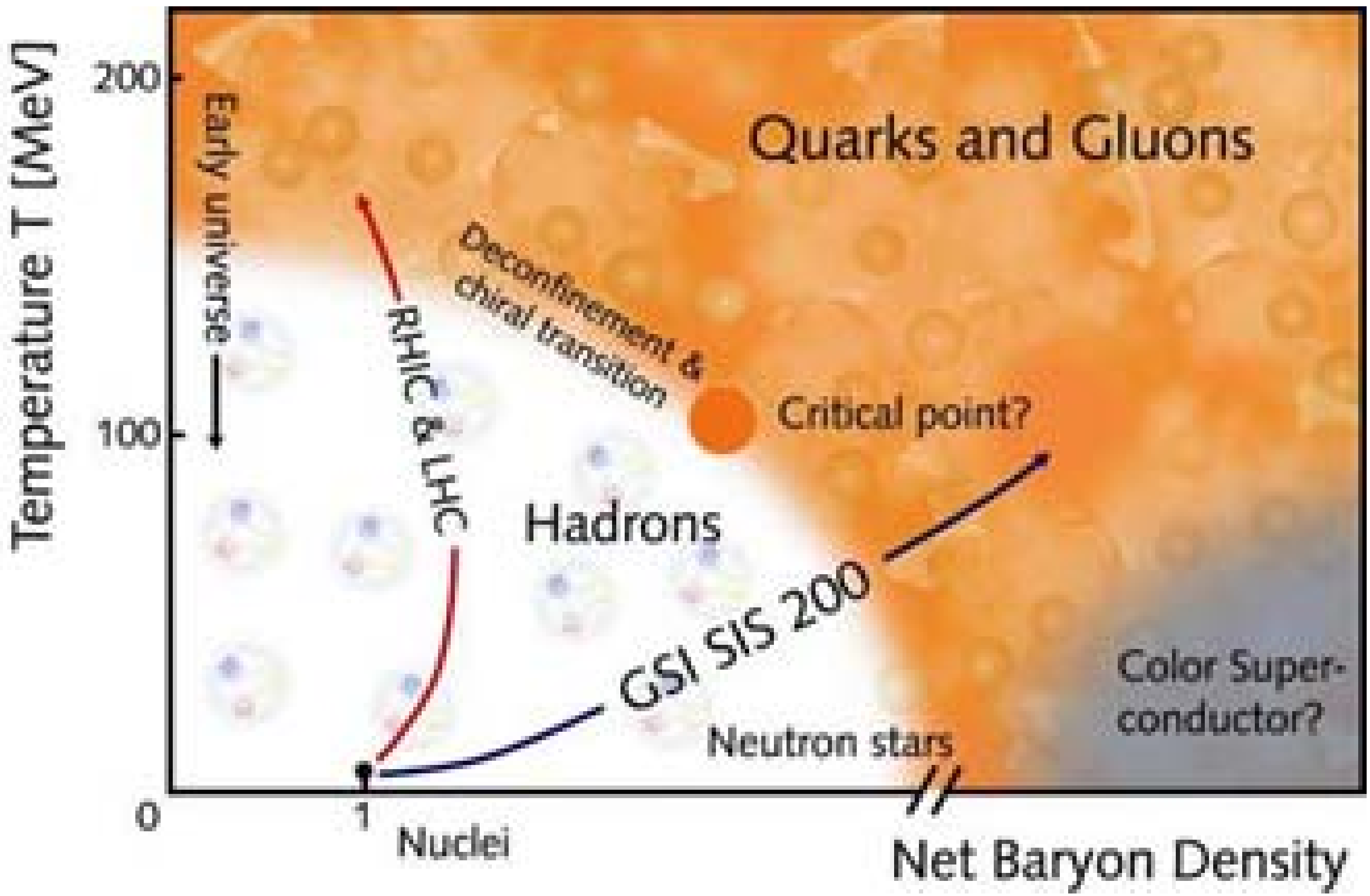
at its critical point $r = r_c$, where z_α are the neutral $S = 1/2$ spinons and A_μ is a *non-compact* U(1) gauge field.

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).



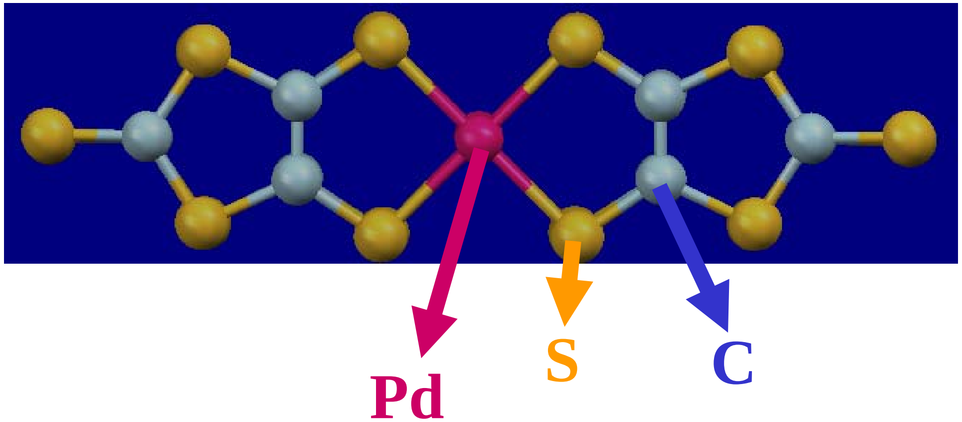
Temperature, *T*
Quantum criticality of fractionalized excitations
0
K/J

Phases of nuclear matter

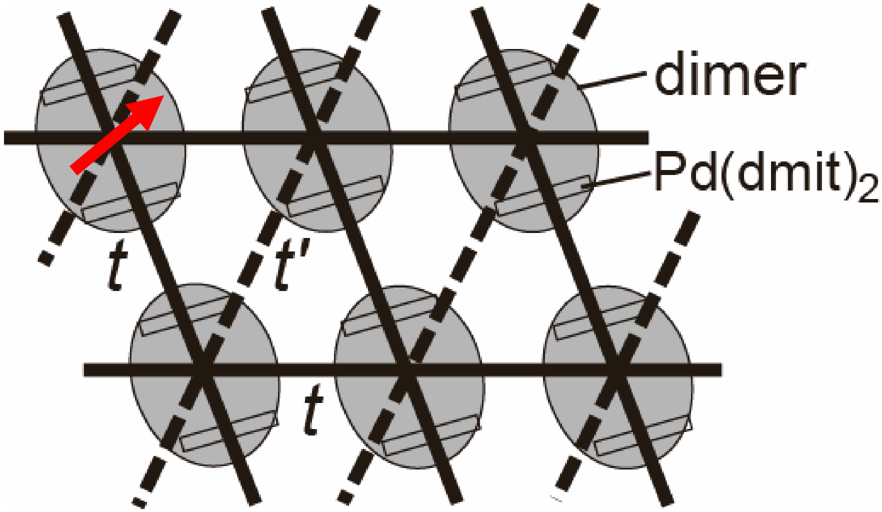


Observation of a valence bond solid (VBS)

$X[Pd(dmit)_2]_2$

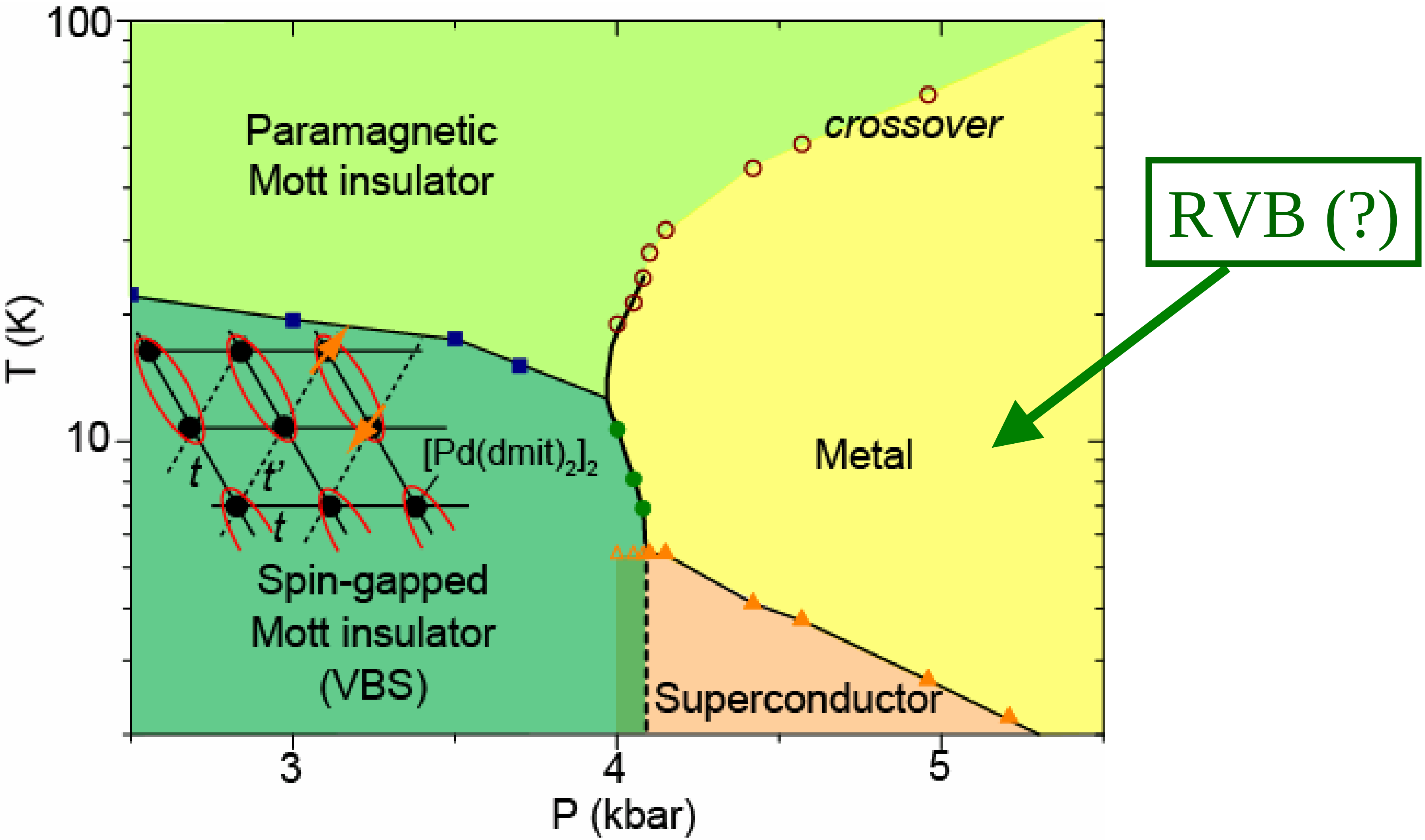


One free electron spin on each vertex of a triangular lattice



M. Tamura *et al.*, *J. Phys. Soc. Jpn.* **75**, 093701 (2006)

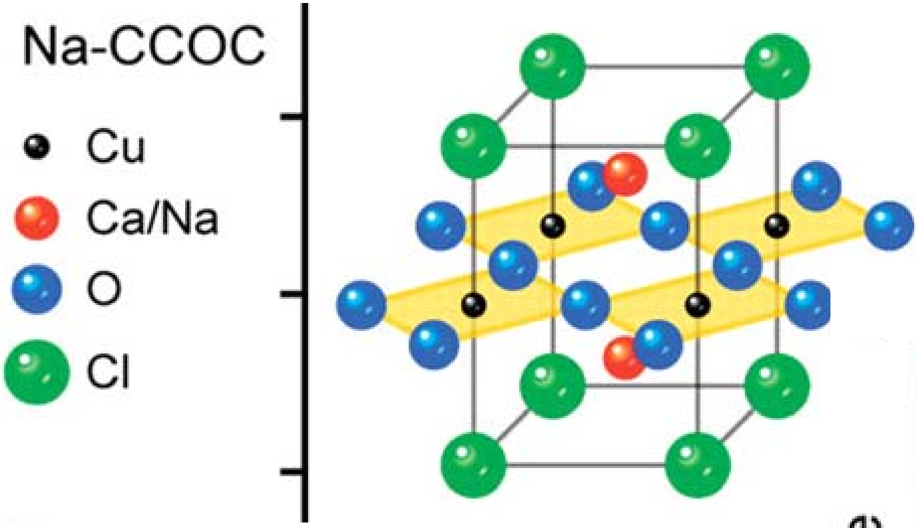
Observation of a valence bond solid (VBS)

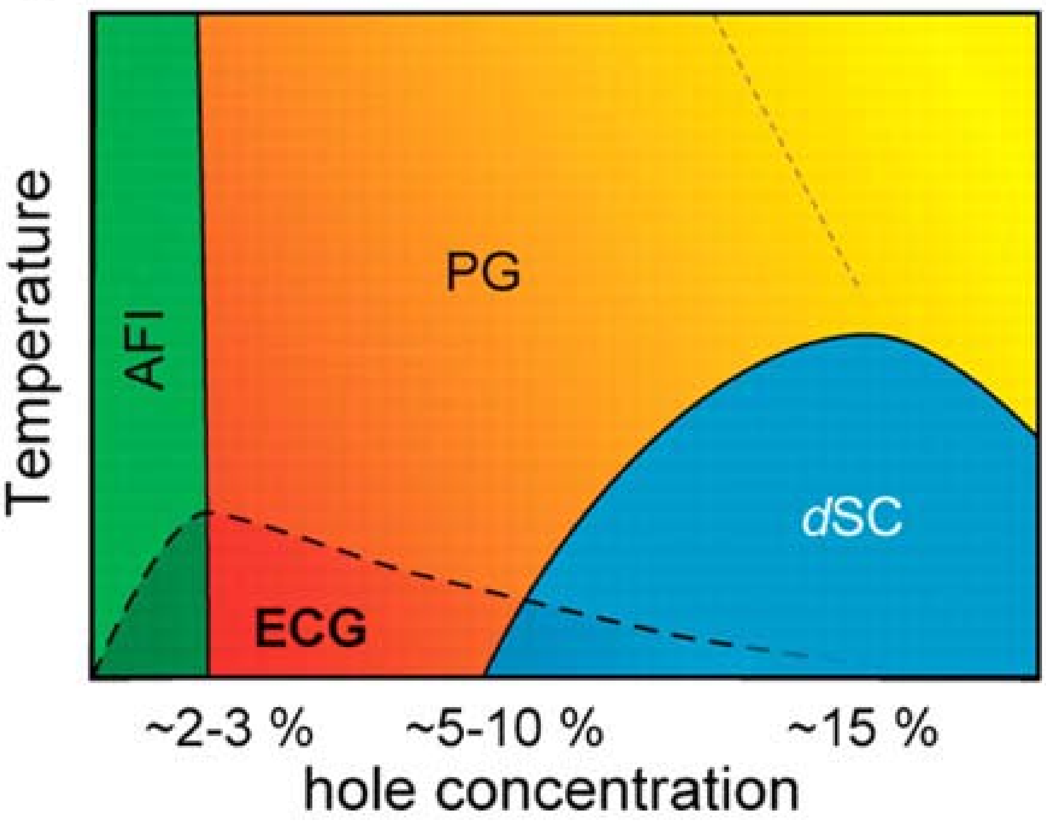


Pressure-temperature phase diagram of $ETMe_3P[Pd(dmit)_2]_2$

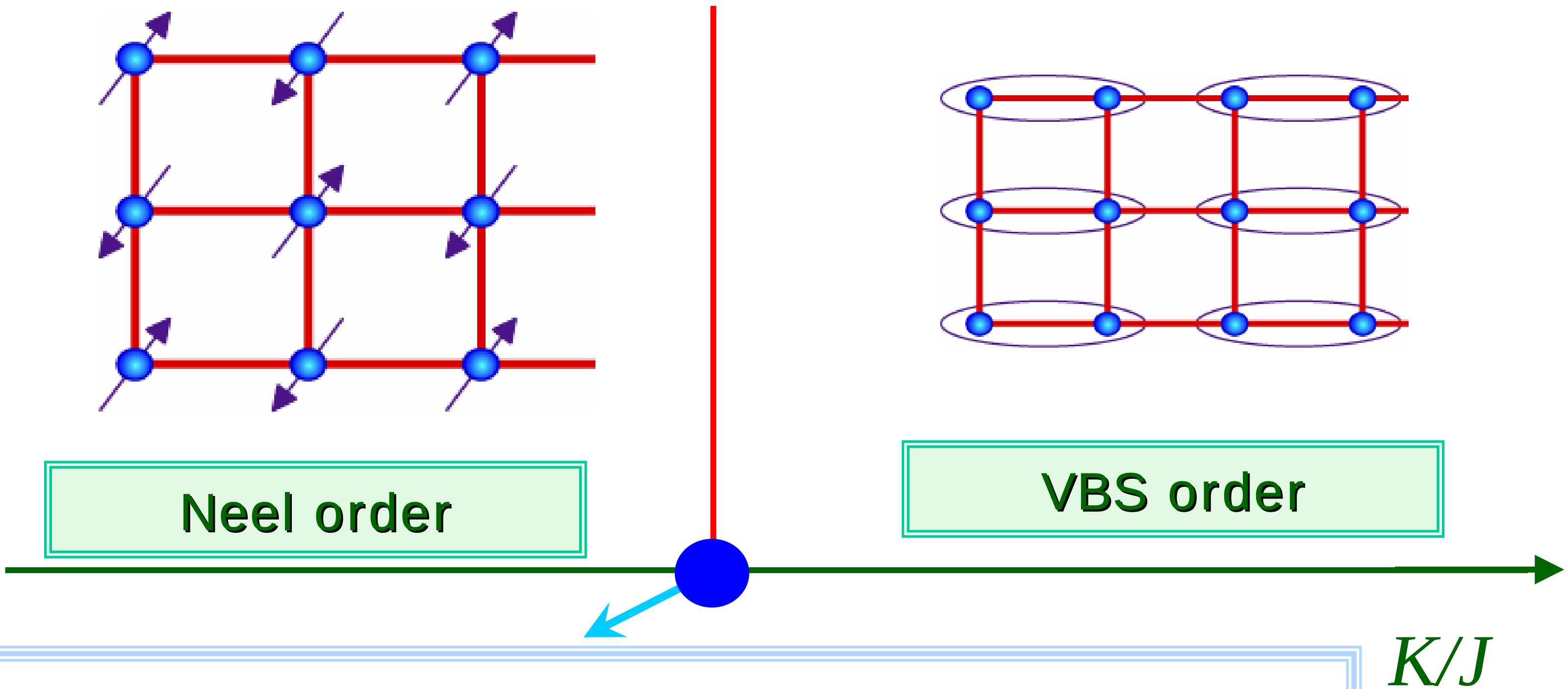
Y. Shimizu *et al*. cond-mat/0612545

Temperature-doping phase diagram of the cuprate superconductors



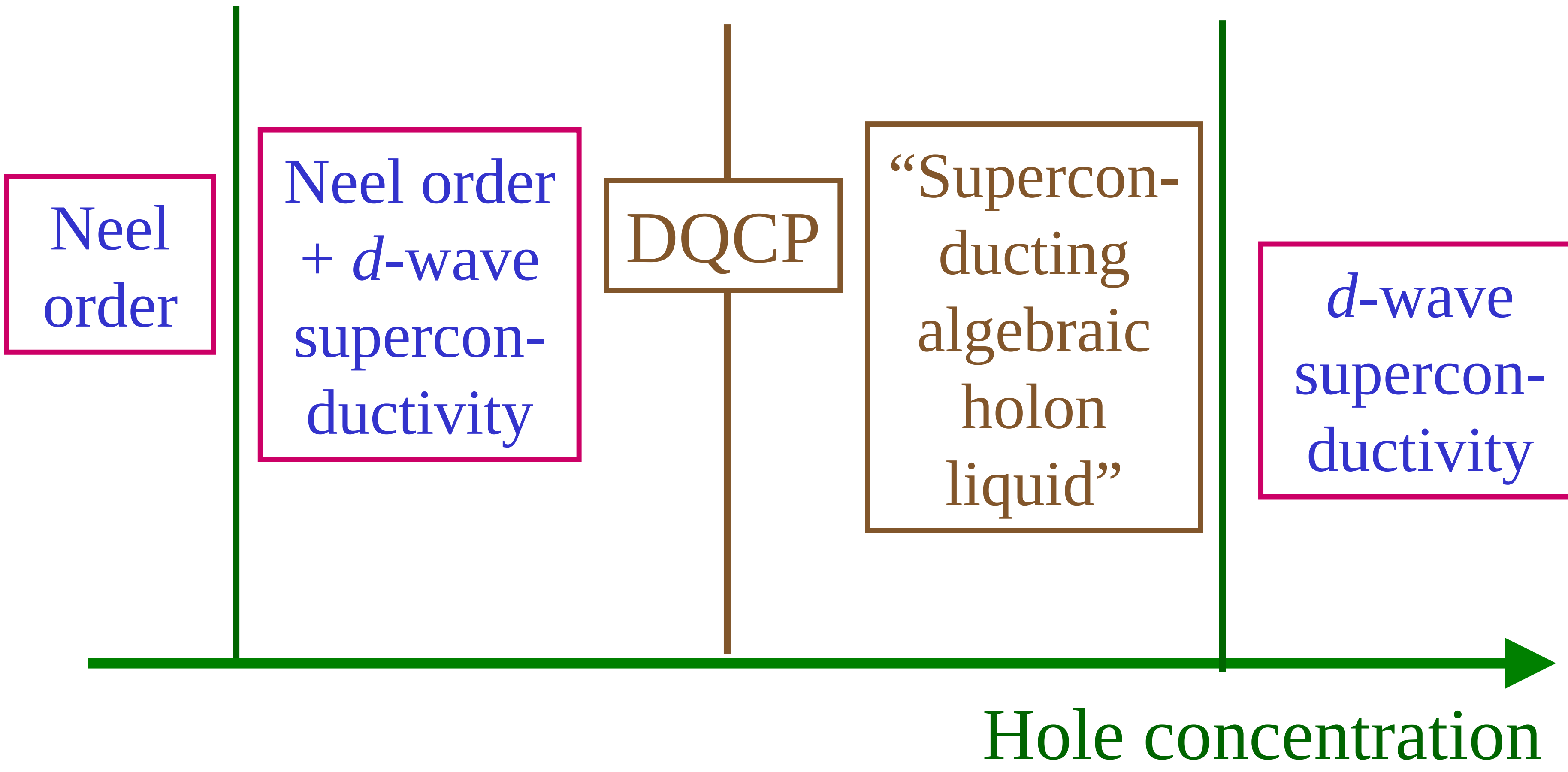


Temperature-doping phase diagram of the cuprate superconductors



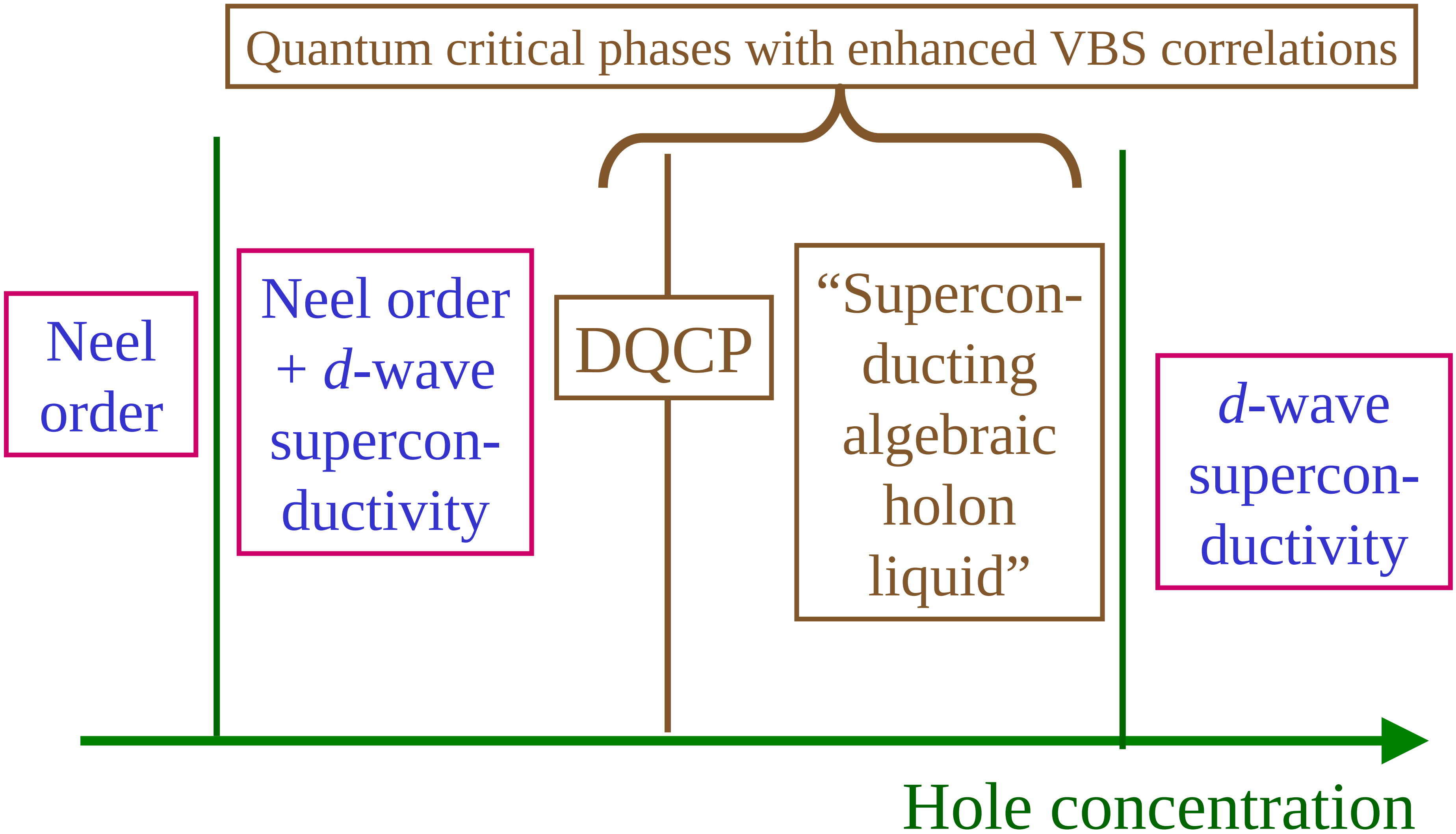
Deconfined quantum critical point (DQCP)

Temperature-doping phase diagram of the cuprate superconductors



R.K. Kaul, Y.-B. Kim, S. Sachdev and T. Senthil, to appear

Temperature-doping phase diagram of the cuprate superconductors



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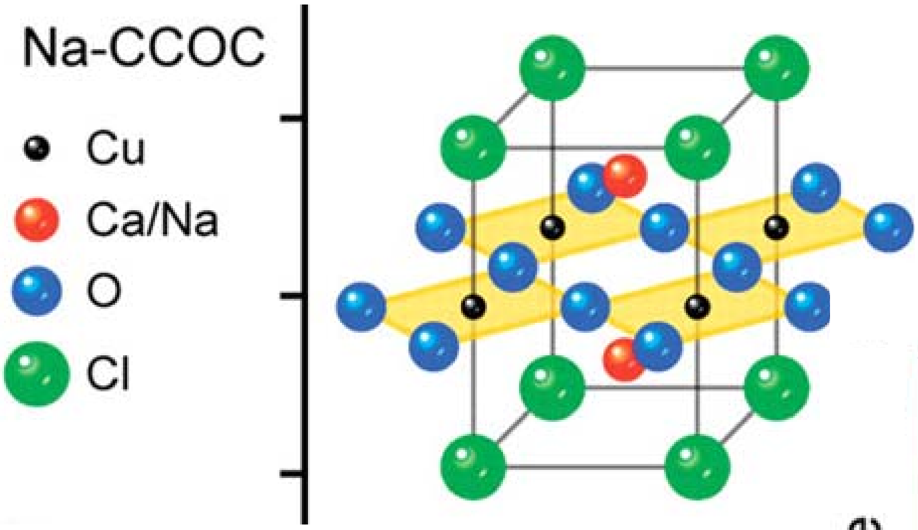
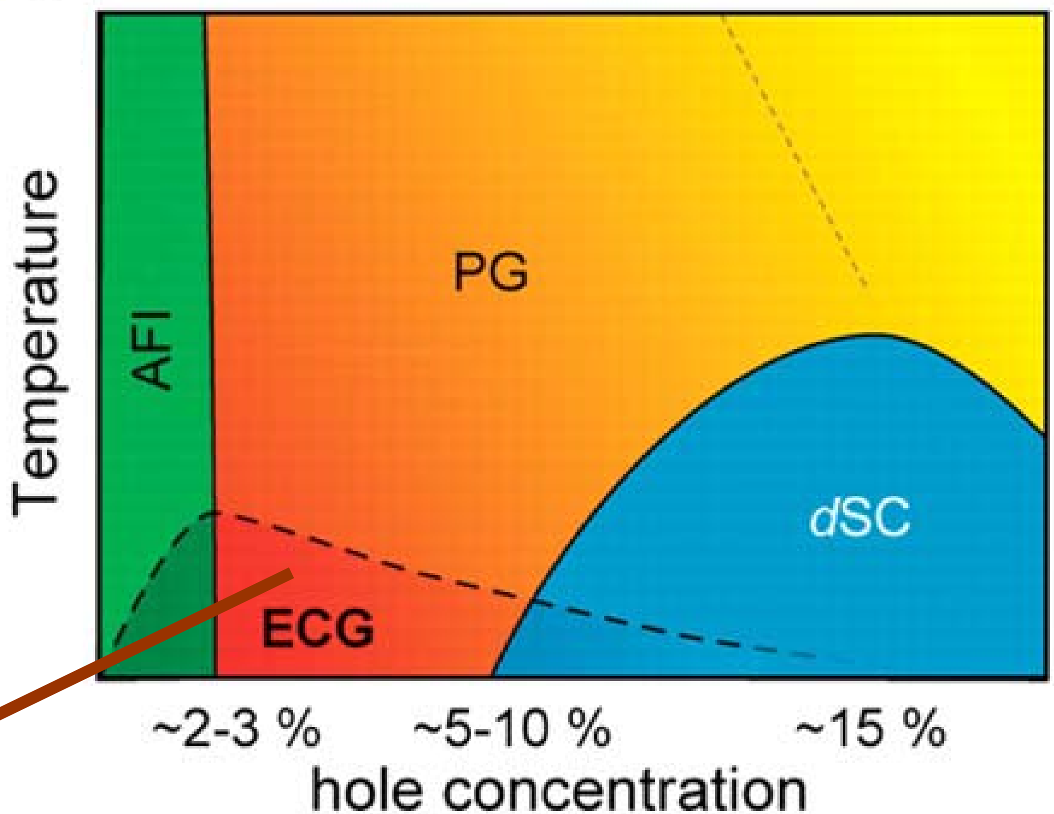
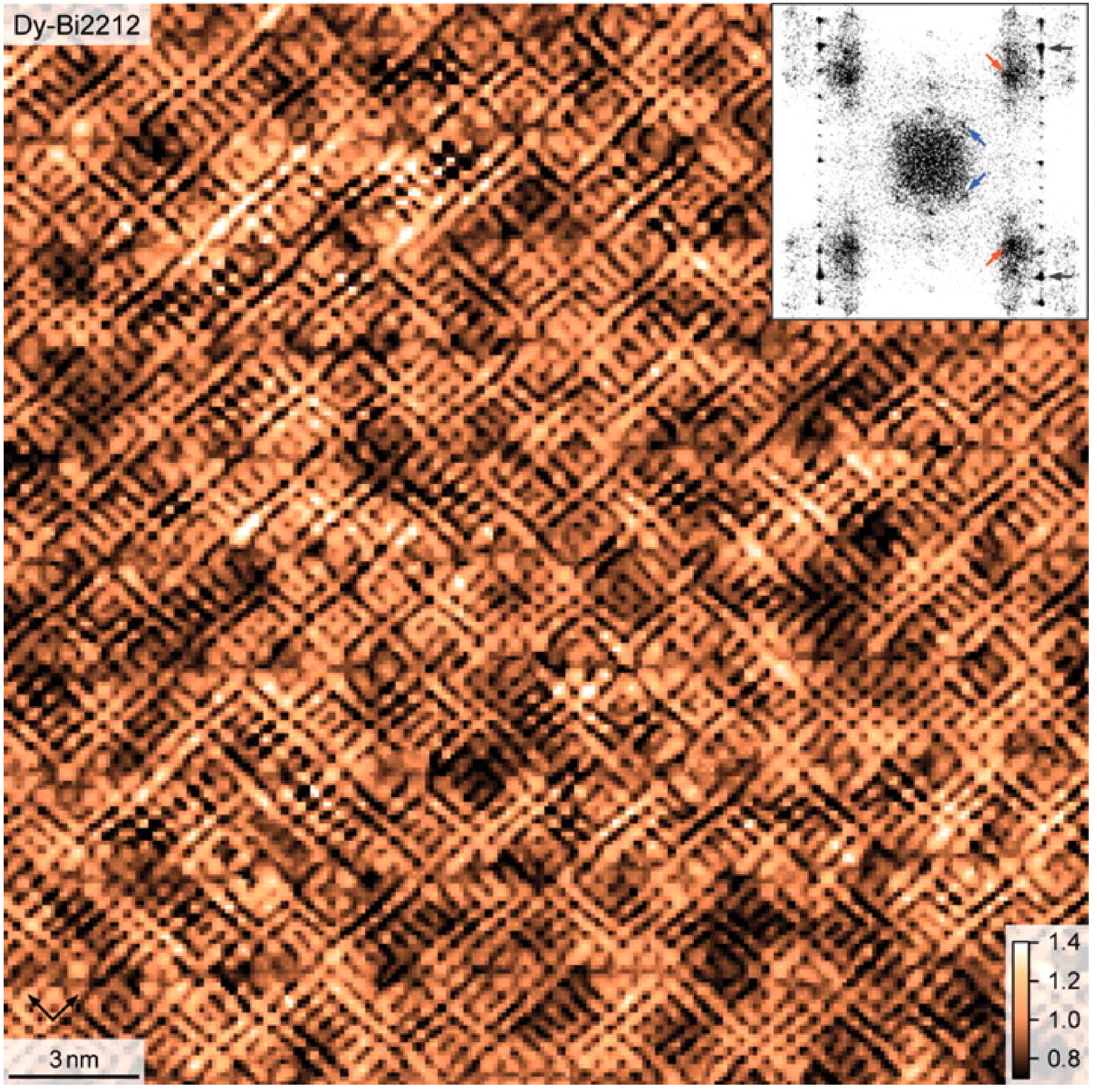
Temperature-doping phase diagram of the cuprate superconductors

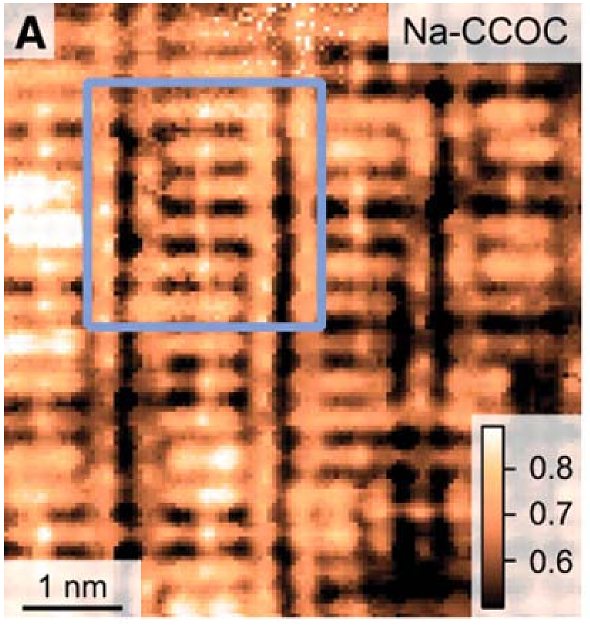




STM in zero field



Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



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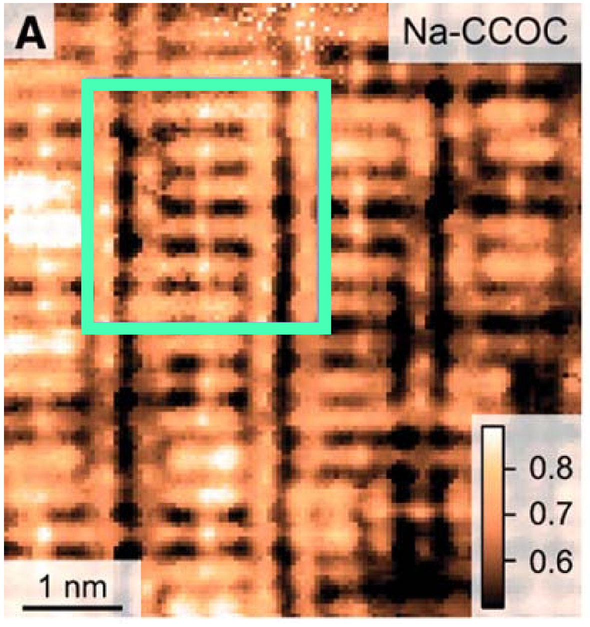

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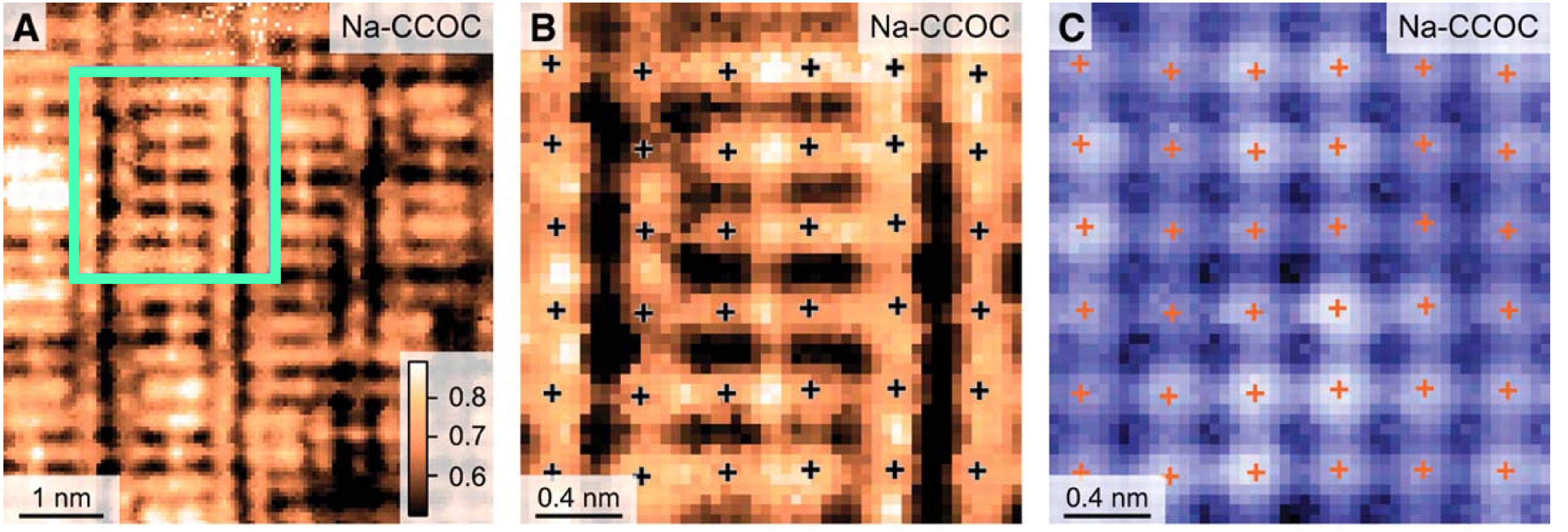

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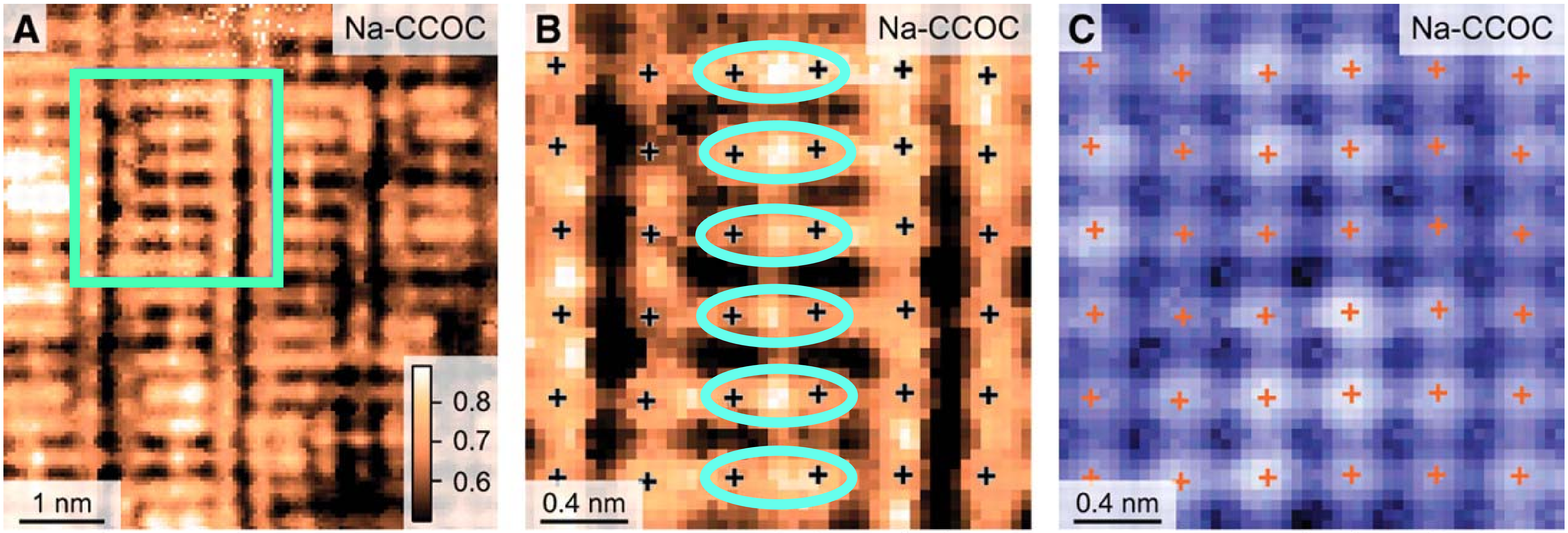

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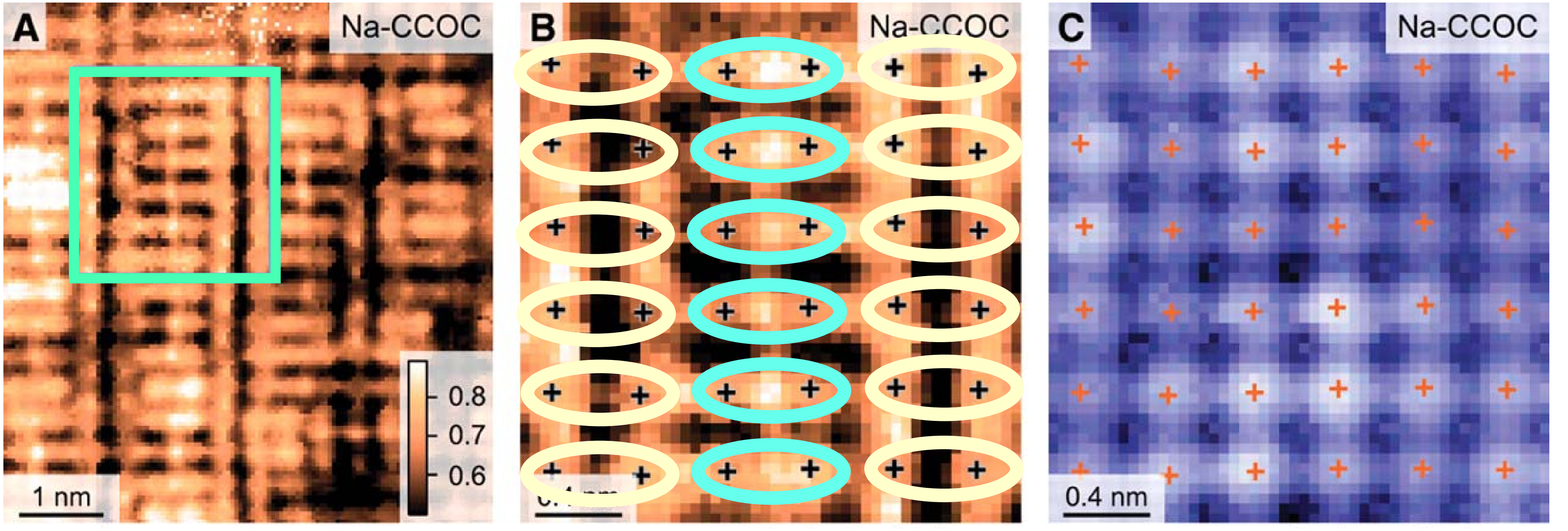
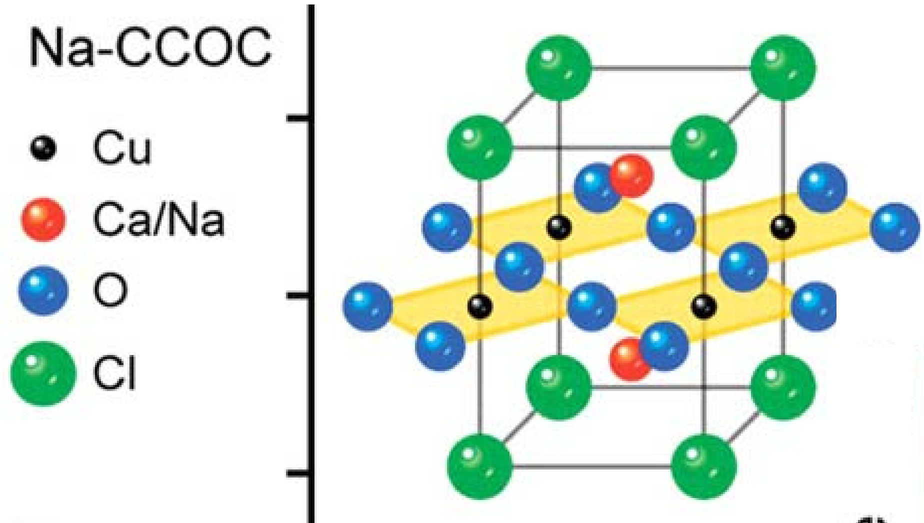
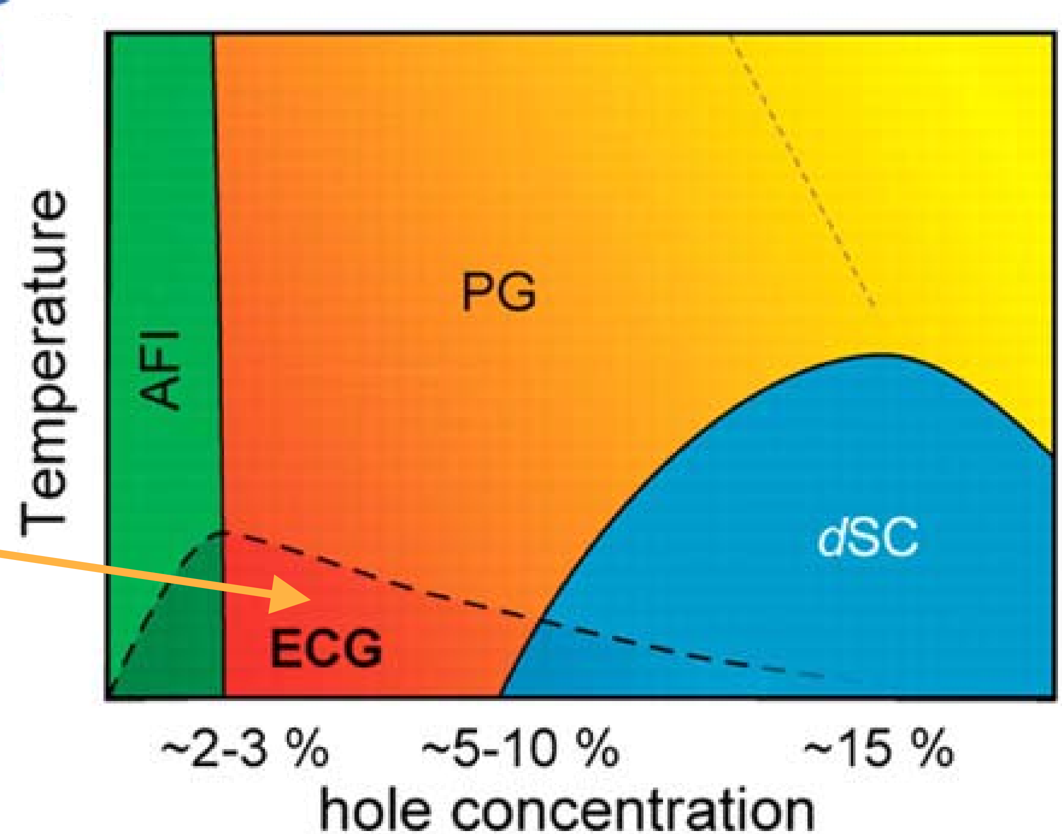


"Glassy" Valence Bond Solid (VBS)

Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)

Temperature-doping phase diagram of the cuprate superconductors





"Glassy" Valence Bond Solid (VBS)

Outline

Quantum phase transitions

1. Spin ordering in “Han purple”
2. Entanglement at the critical point: physical consequences at non-zero temperatures
   (a) Double-layer antiferromagnet
   (b) Superfluid-insulator transition
   (c) Hydrodynamics via mapping to quantum theory of black holes.
3. Entanglement of valence bonds
4. Conclusions

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Conclusions

- Studies of new materials and trapped ultracold atoms are yielding new quantum phases, with novel forms of quantum entanglement.
- Some materials are of technological importance: *e.g.* high temperature superconductors.
- Real-world studies on the entanglement of large numbers of qubits: insights may be important for quantum cryptography and quantum computing.
- Tabletop "laboratories for the entire universe": quantum mechanics of black holes, quark-gluon plasma, neutrons stars, and big-bang physics.