Deconfined quantum criticality

Leon Balents (UCSB)
Lorenz Bartosch (Frankfurt)
Anton Burkov (Harvard)
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Talks online at http://sachdev.physics.harvard.edu

Outline

I. Magnetic quantum phase transitions in "dimerized" Mott insulators:
Landau-Ginzburg-Wilson (LGW) theory

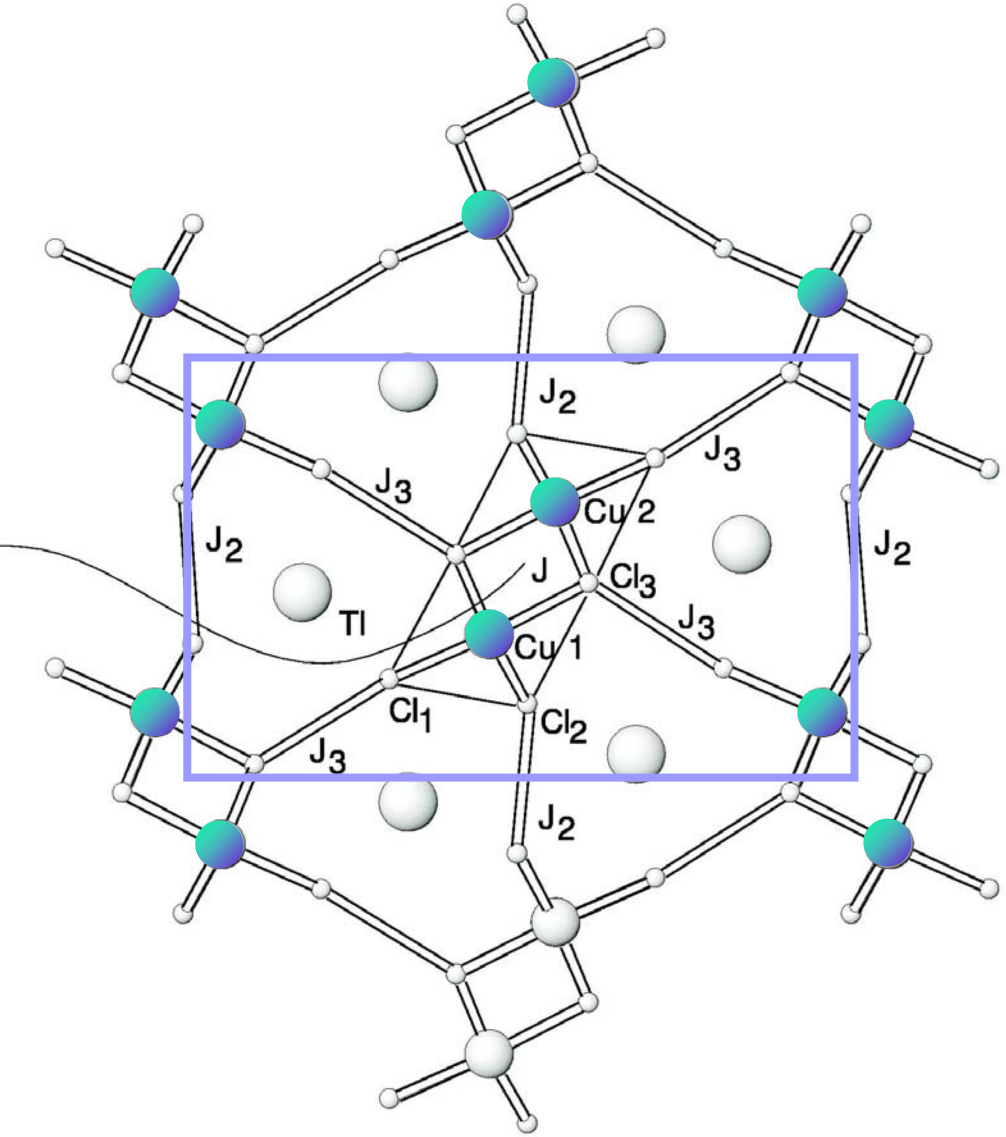
II. Magnetic quantum phase transitions of Mott insulators on the square lattice
A. Breakdown of LGW theory
B. Berry phases
C. Spinor formulation and deconfined criticality

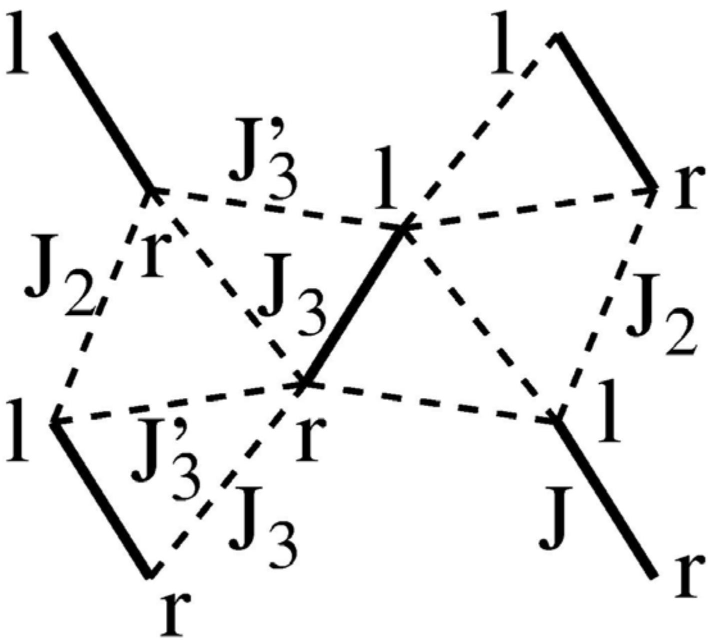
I. Magnetic quantum phase transitions in "dimerized" Mott insulators:

Landau-Ginzburg-Wilson (LGW) theory:

Second-order phase transitions described by fluctuations of an ***order parameter*** *associated with a* ***broken symmetry***

$TlCuCl_3$



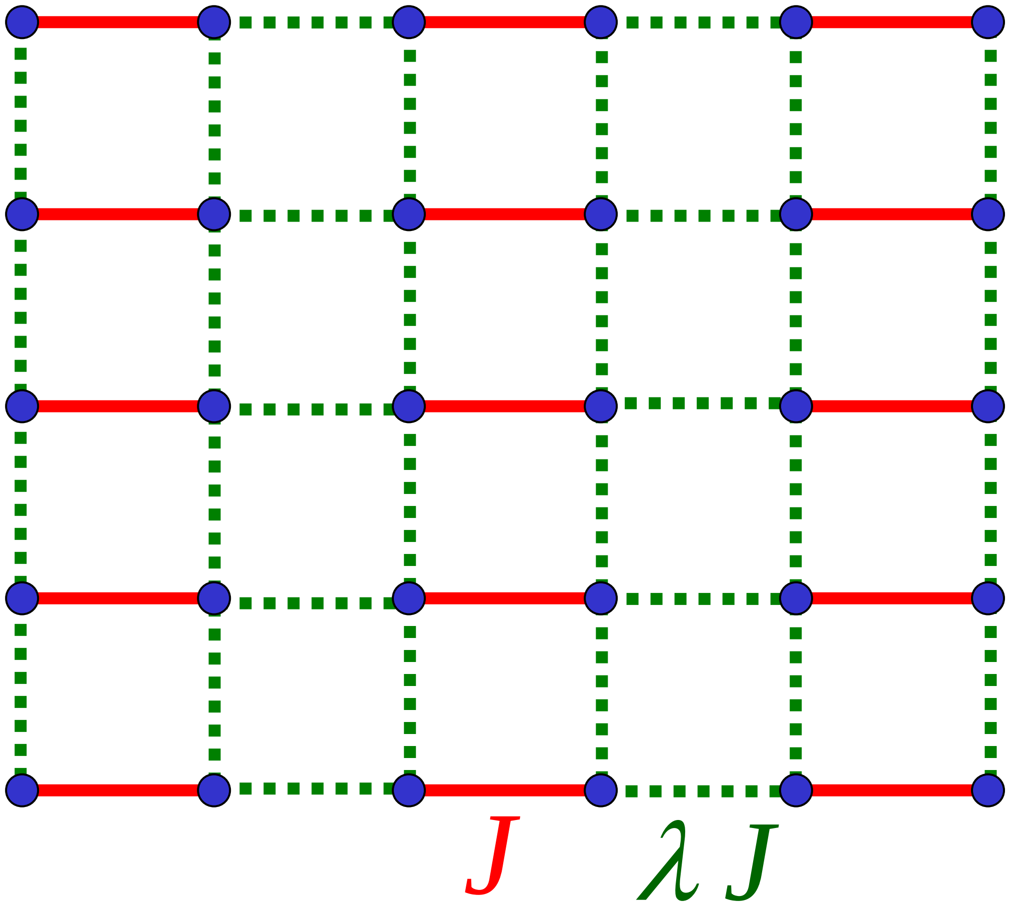


M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, cond-mat/0309440.

Coupled Dimer Antiferromagnet

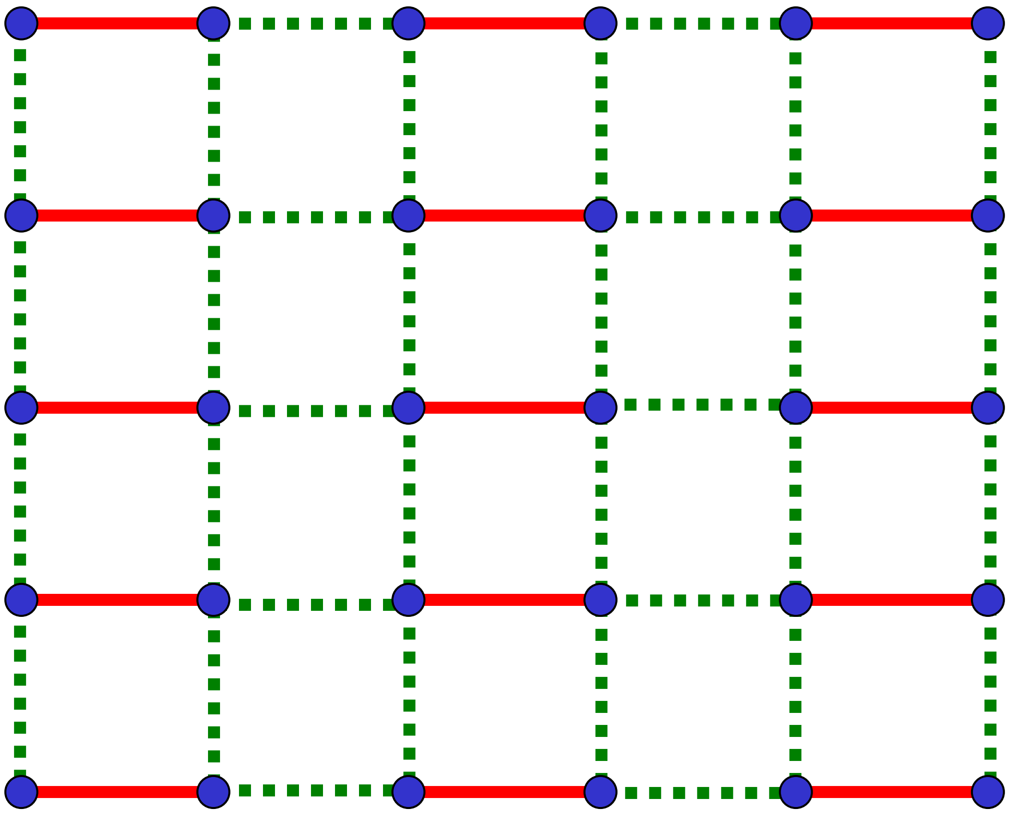
M. P. Gelfand, R. R. P. Singh, and D. A. Huse, *Phys. Rev. B* **40**, 10801-10809 (1989).
N. Katoh and M. Imada, *J. Phys. Soc. Jpn.* **63**, 4529 (1994).
J. Tworzydlo, O. Y. Osman, C. N. A. van Duin, J. Zaanen, *Phys. Rev.* B **59**, 115 (1999).
M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).

S=1/2 spins on coupled dimers



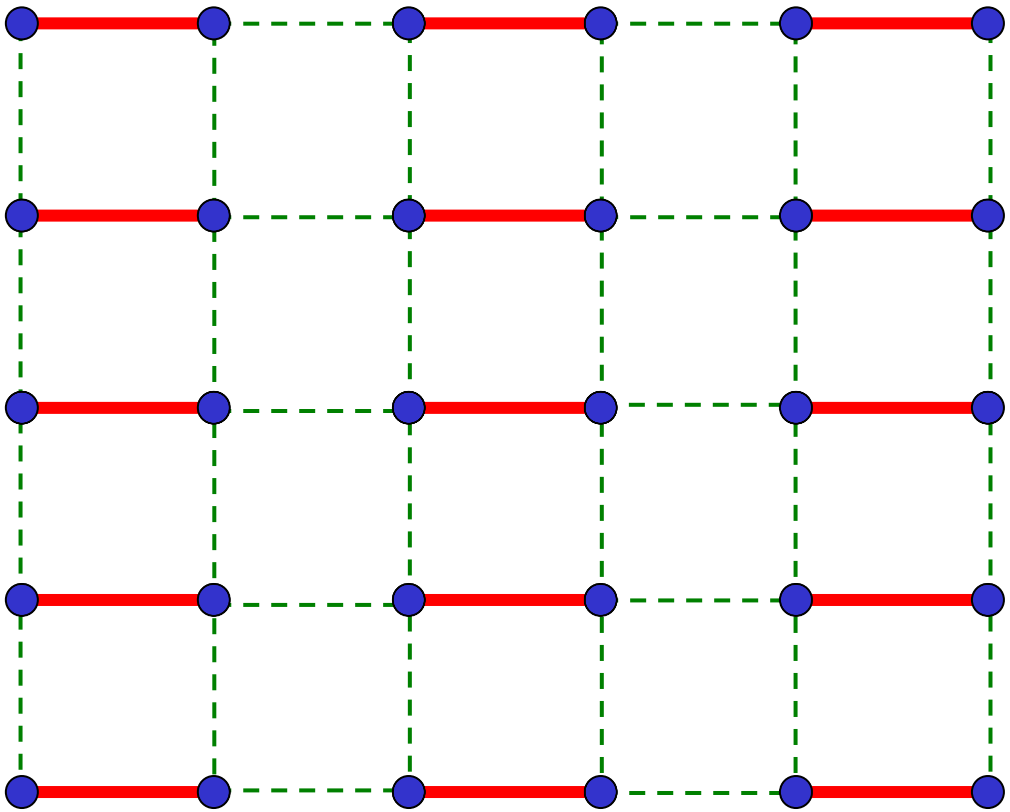
$$H = \sum_{<ij>} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$0 \le \lambda \le 1$$



λ close to 0

Weakly coupled dimers



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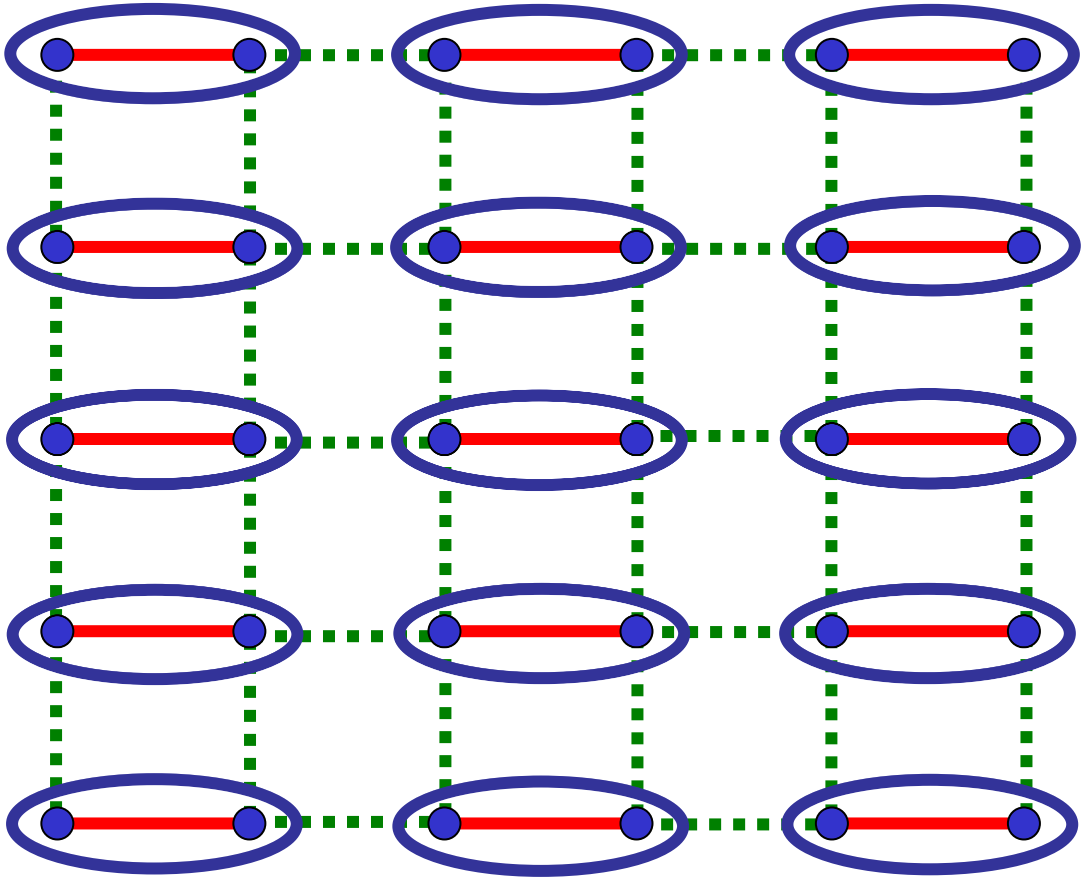
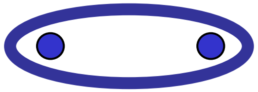


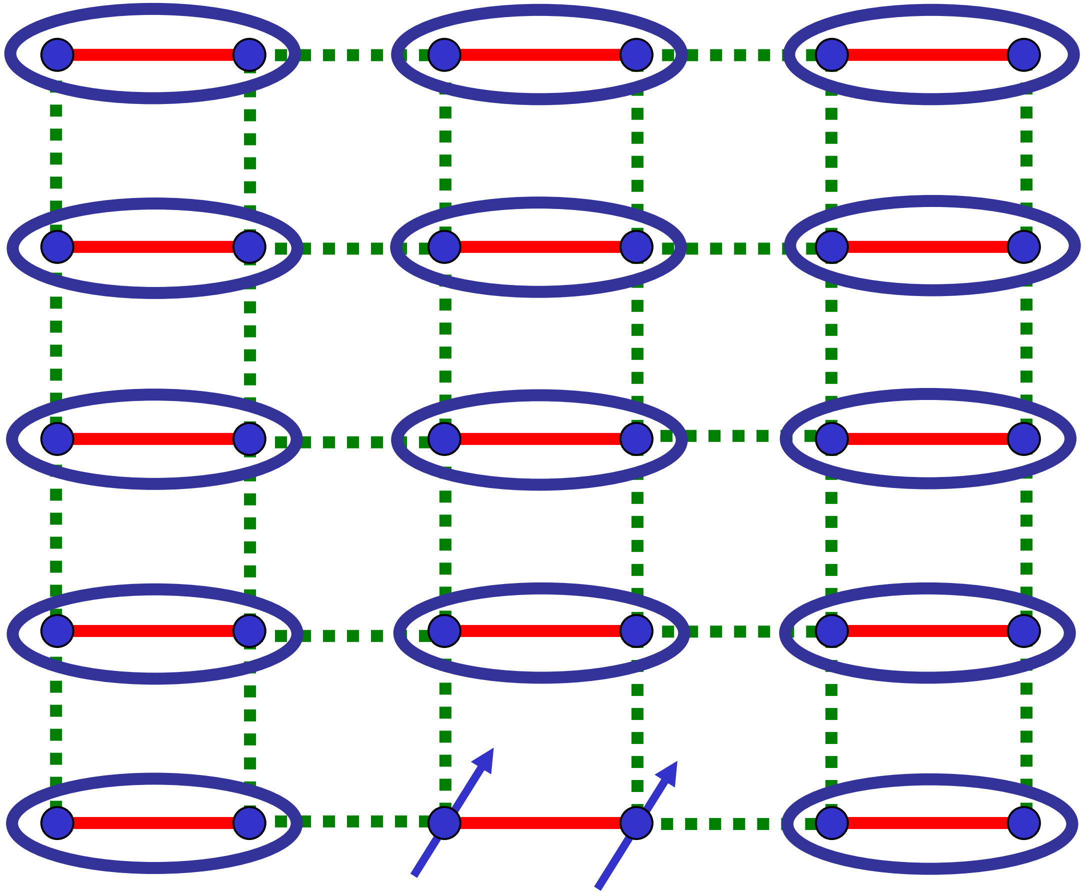
Paramagnetic ground state

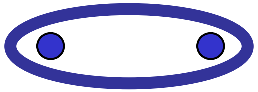
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$

$$\left\langle \vec{S}_i \right\rangle = 0$$

λ close to 0

Weakly coupled dimers

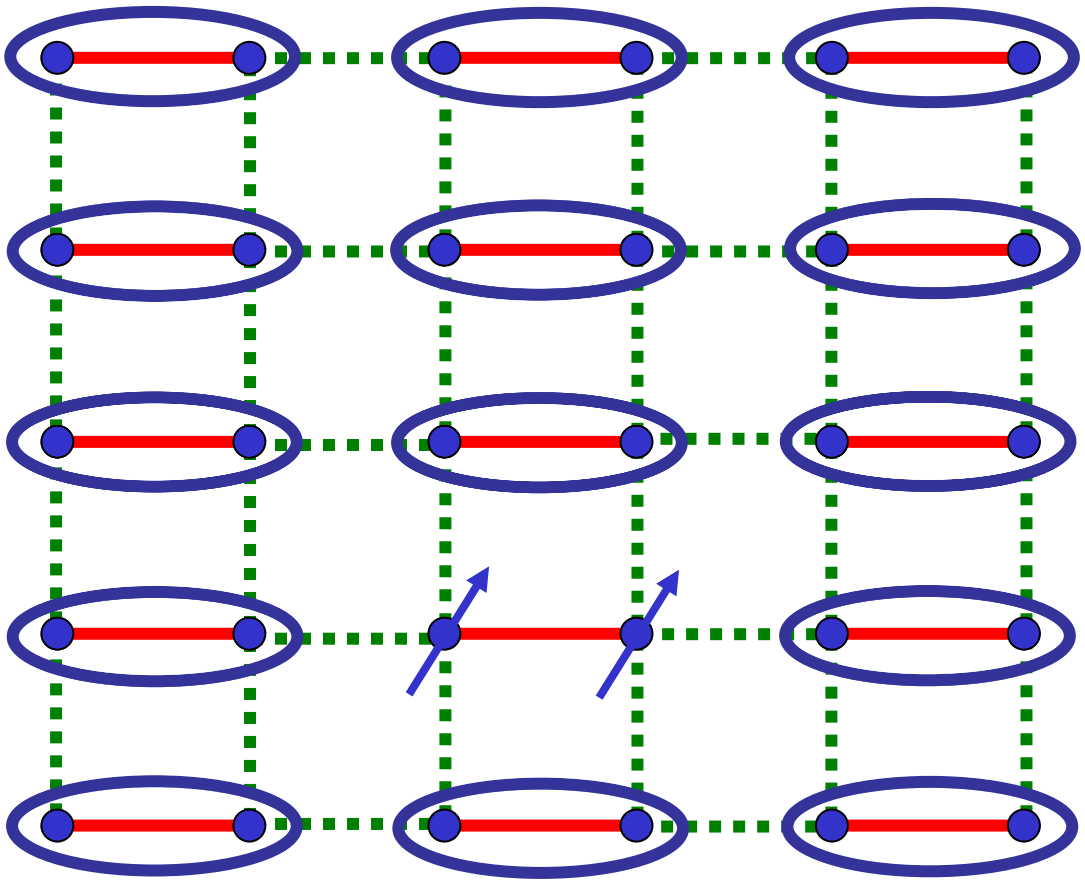


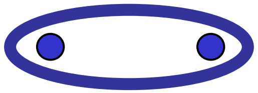
 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

Excitation:
S=1 quasipartcle

λ close to 0

Weakly coupled dimers





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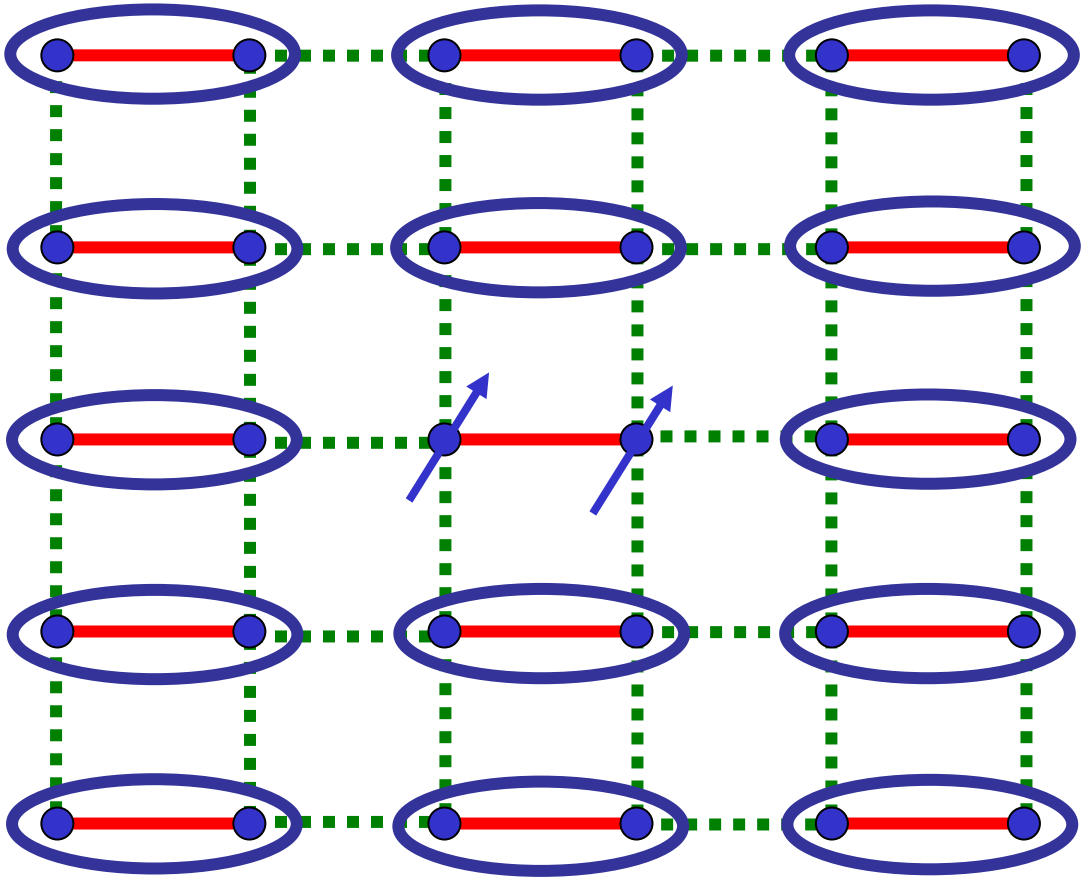

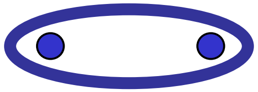

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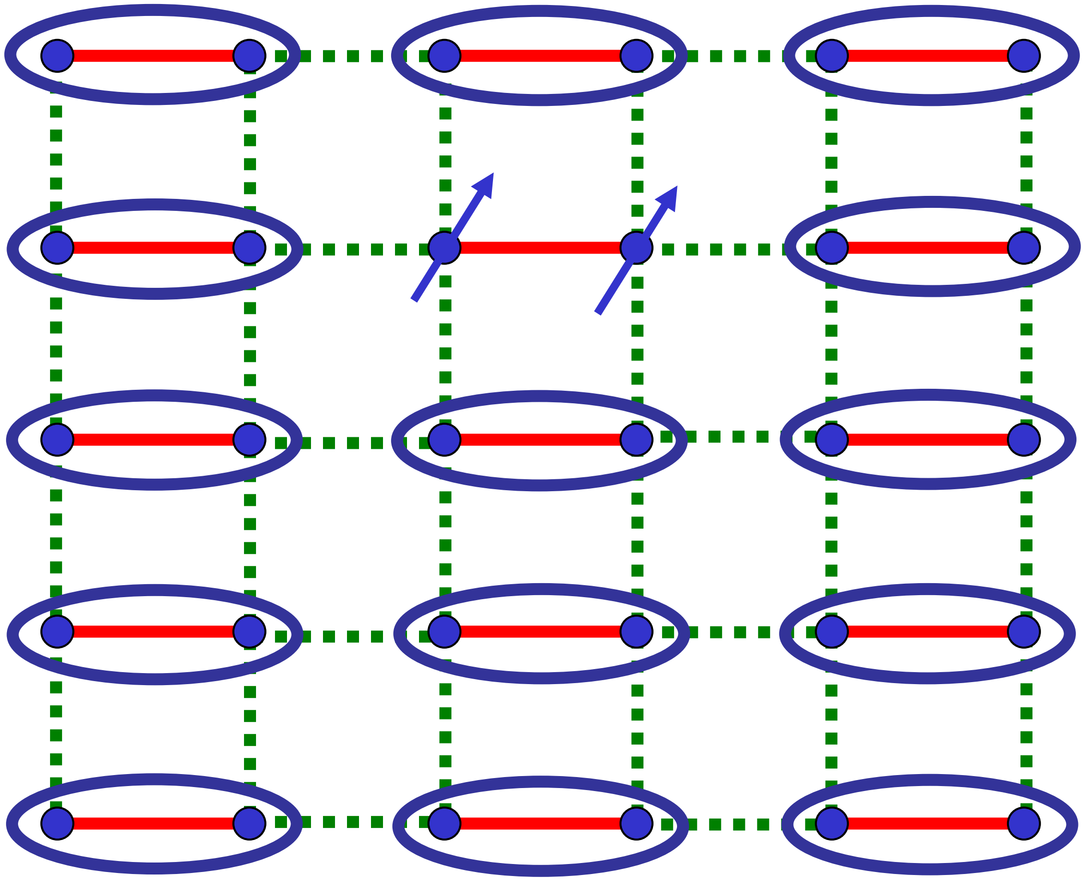

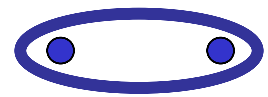

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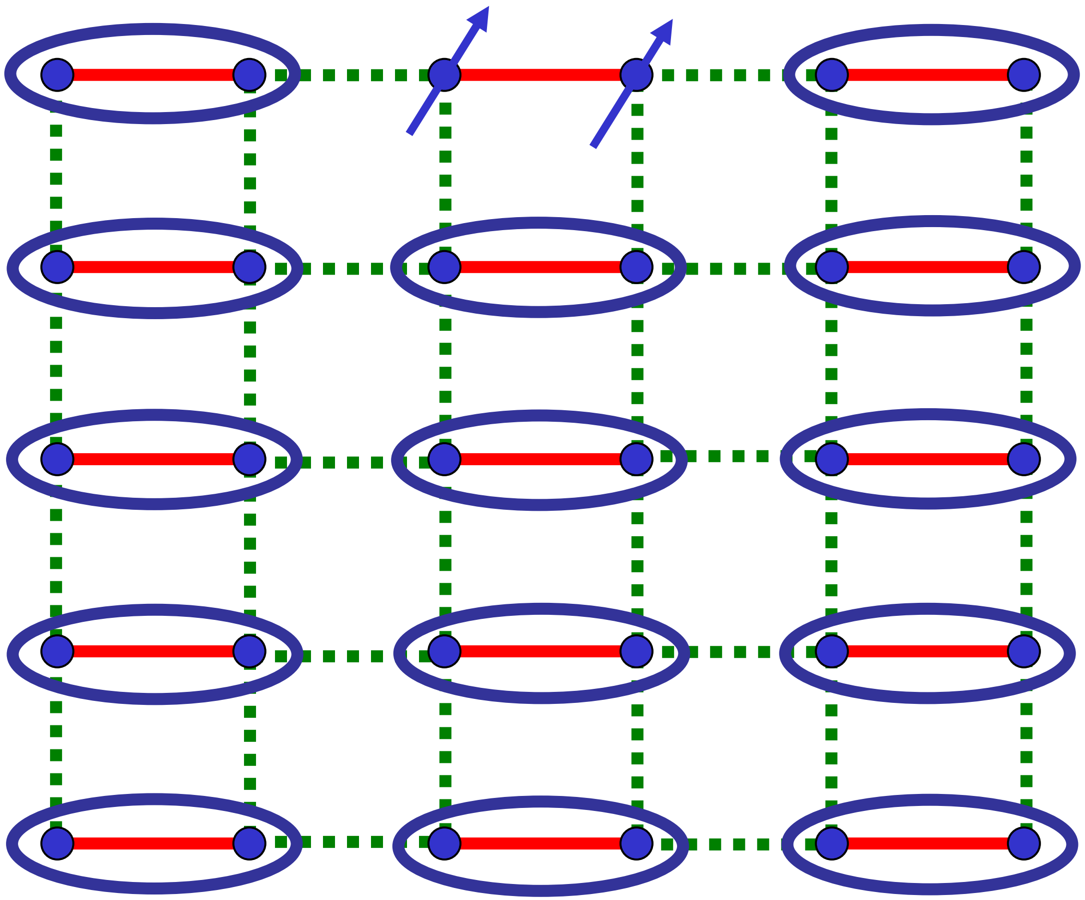

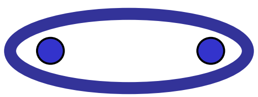 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

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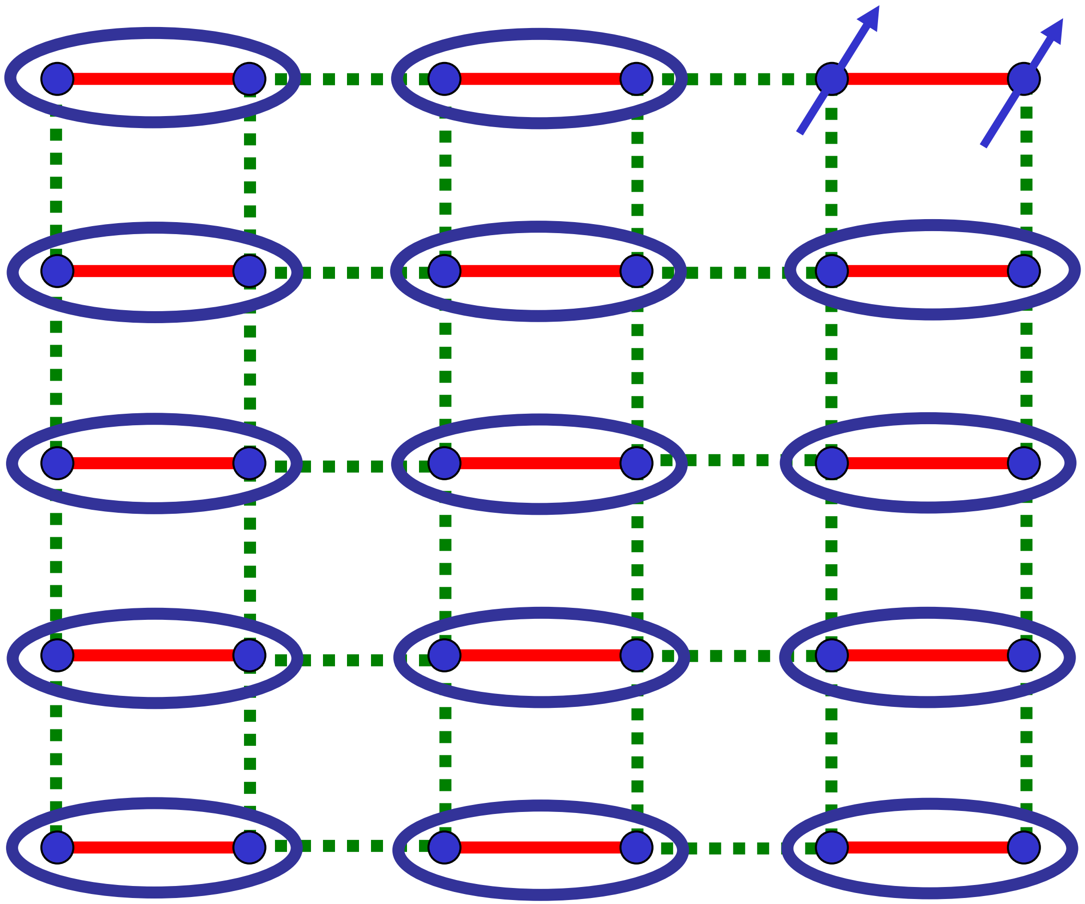
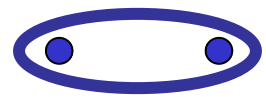


 $= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$

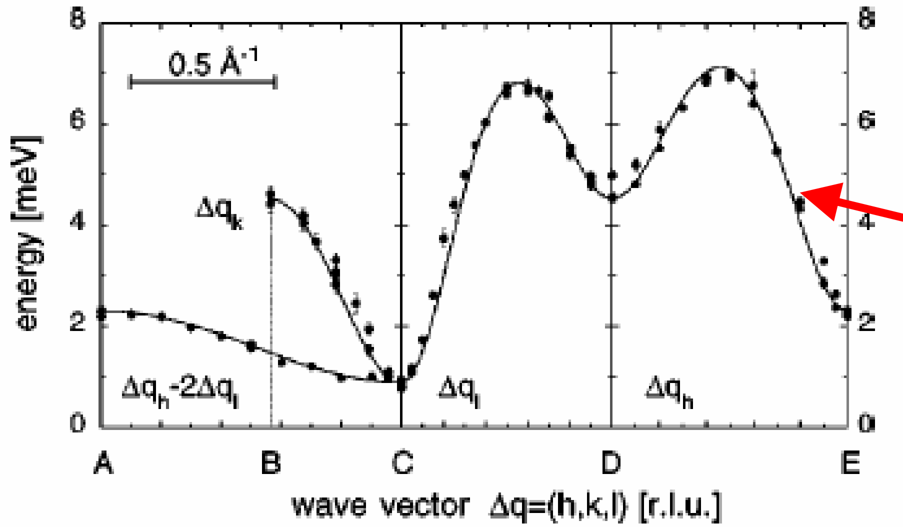
Excitation:
S=1 quasipartcle

Energy dispersion away from antiferromagnetic wavevector

$$\varepsilon_p = \Delta + \frac{c_x^2 p_x^2 + c_y^2 p_y^2}{2\Delta}$$

$\Delta \rightarrow$ spin gap

$TlCuCl_3$



N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer and H. Mutka, *Phys. Rev.* B 63 172414 (2001).

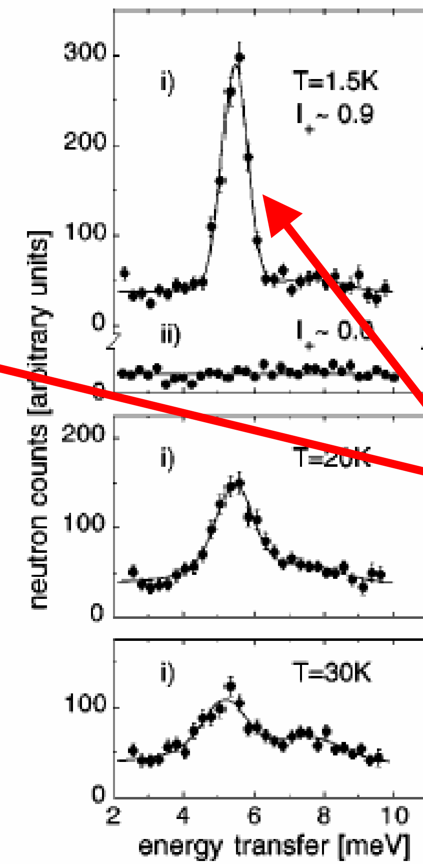
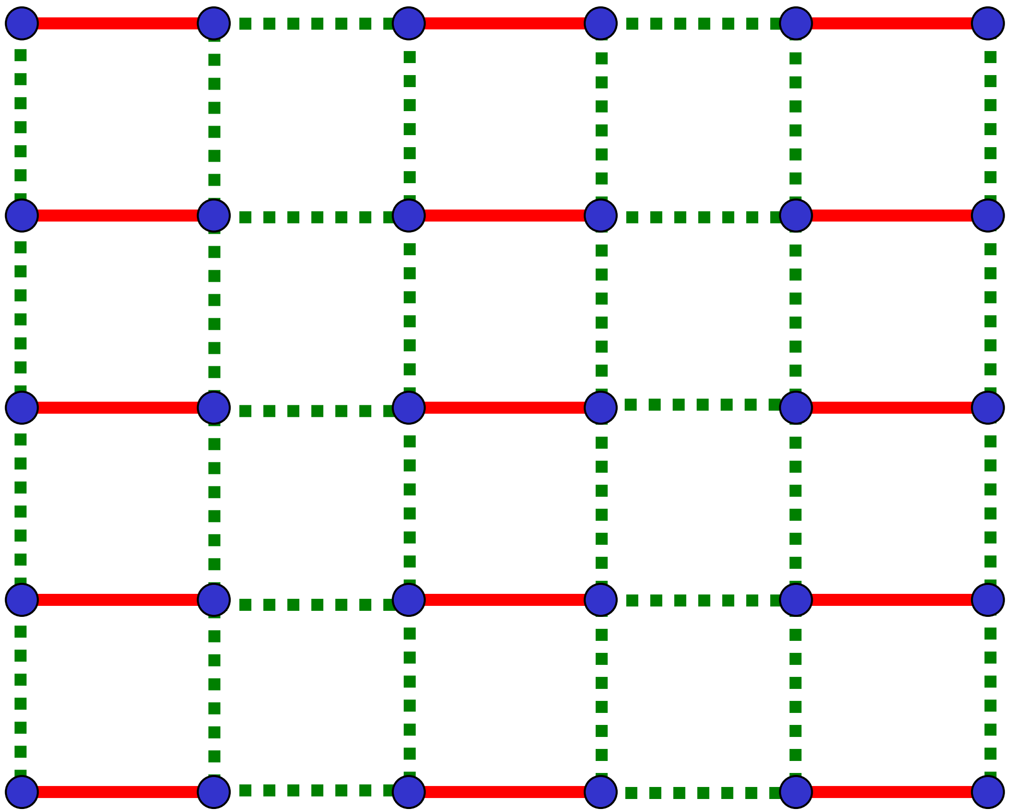


FIG. 1. Measured neutron profiles in the a^*c^* plane of $TlCuCl_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

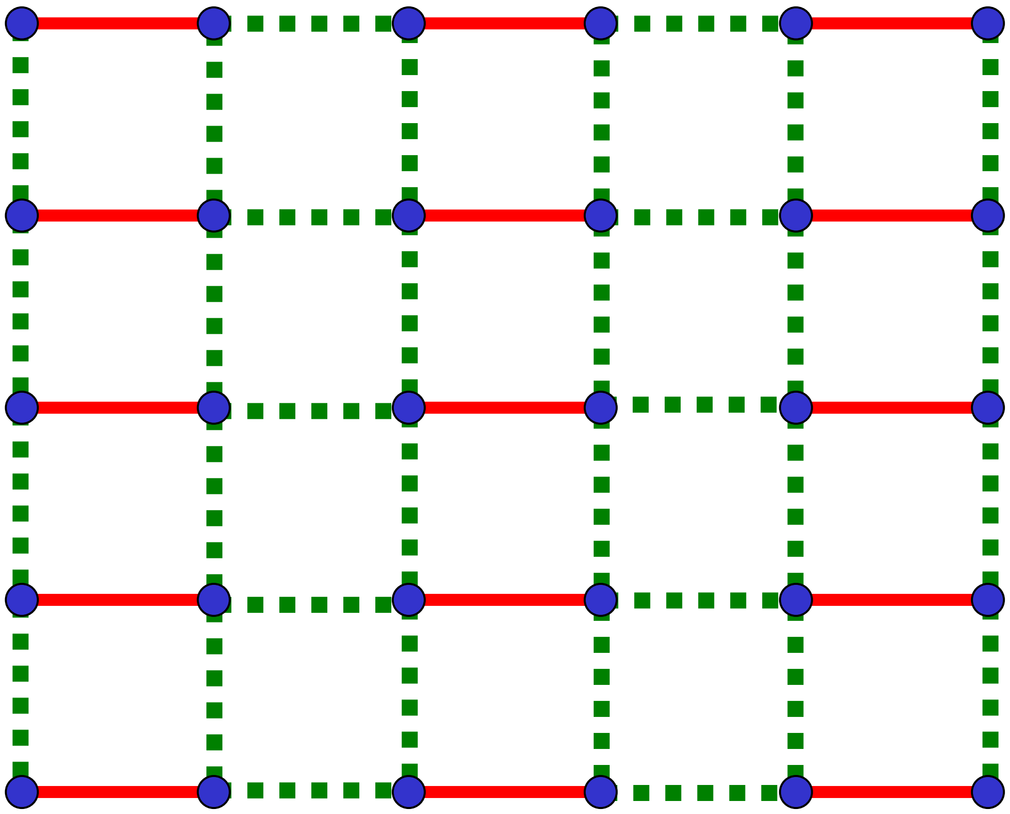
S=1 quasi-particle

Coupled Dimer Antiferromagnet



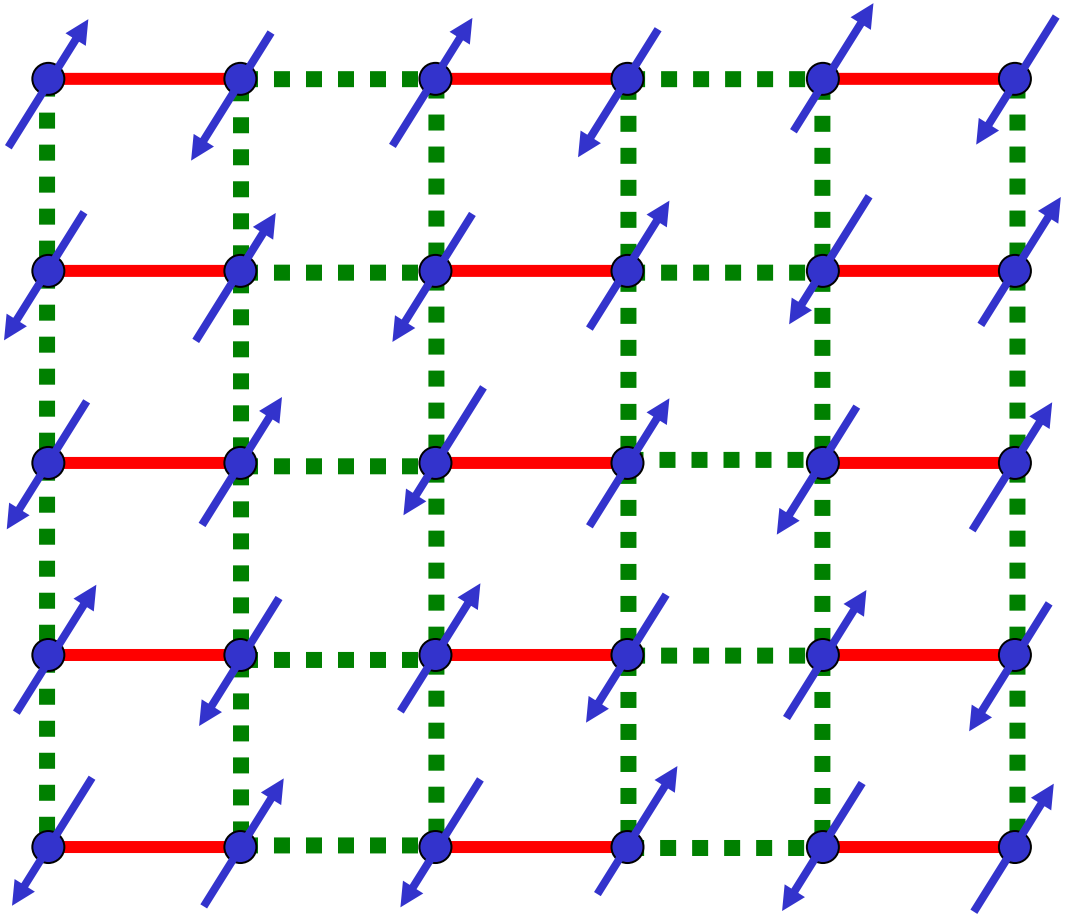
λ close to 1

Weakly dimerized square lattice



λ close to 1

Weakly dimerized square lattice



Excitations:
2 spin waves (***magnons***)

$$\varepsilon_p = \sqrt{c_x^2 p_x^2 + c_y^2 p_y^2}$$

Ground state has long-range spin density wave (Néel) order at wavevector ***K***= (π,π)

$$\langle \vec{\varphi} \rangle \neq 0$$

spin density wave order parameter: $\vec{\varphi} = \eta_i \frac{\vec{S}_i}{S}$; $\eta_i = \pm 1$ on two sublattices

$TlCuCl_3$

Neutron Diffraction Study of the Pressure-Induced Magnetic Ordering in the Spin Gap System $TlCuCl_3$

Akira OOSAWA*, Masashi FUJISAWA[1], Toyotaka OSAKABE, Kazuhisa KAKURAI and Hidekazu TANAKA[2]

Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195
[1]*Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*
[2]*Research Center for Low Temperature Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152-8551*

(Received February 3, 2003)

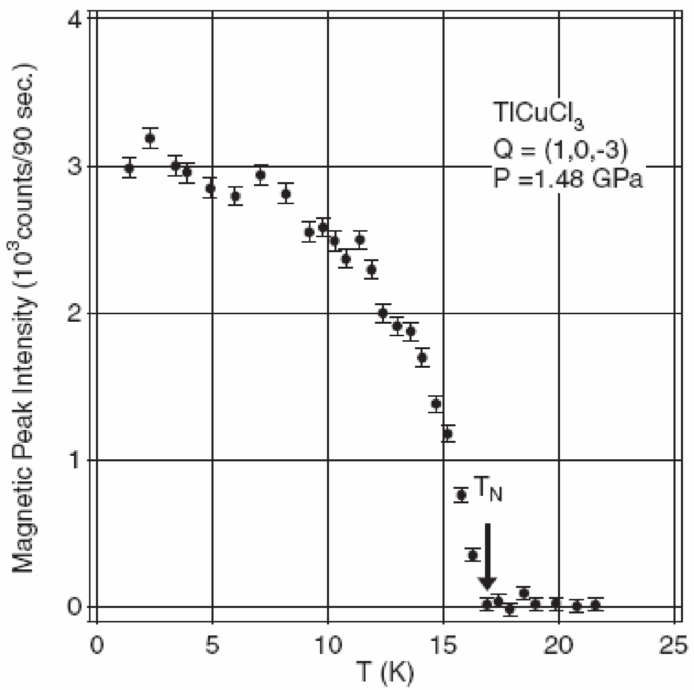


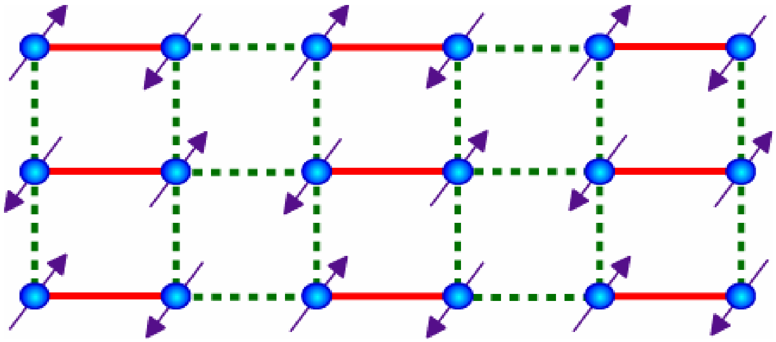
Fig. 3. Temperature dependence of the magnetic Bragg peak intensity for $Q = (1, 0, -3)$ reflection measured at $P = 1.48$ GPa in $TlCuCl_3$.

J. Phys. Soc. Jpn **72**, 1026 (2003)

T=0

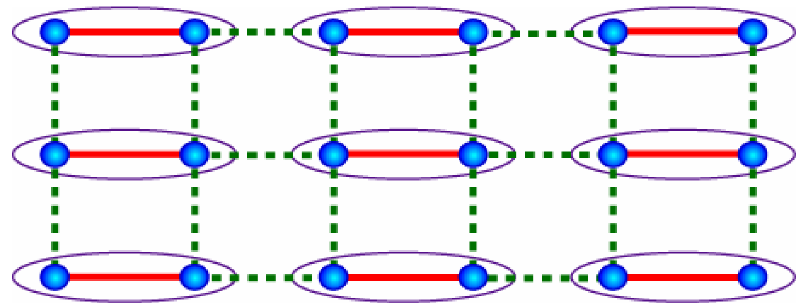
$\lambda_c = 0.52337(3)$

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002)



Néel state

$\langle \vec{\varphi} \rangle \neq 0$



Quantum paramagnet

$\langle \vec{\varphi} \rangle = 0$

λ 1 λ_c

Pressure in $TlCuCl_3$

The method of bond operators (S. Sachdev and R.N. Bhatt, *Phys. Rev.* B **41**, 9323 (1990)) provides a quantitative description of spin excitations in $TlCuCl_3$ across the quantum phase transition (M. Matsumoto, B. Normand, T.M. Rice, and M. Sigrist, *Phys. Rev. Lett.* **89**, 077203 (2002))

LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + c^2 \left(\partial_\tau \vec{\varphi}\right)^2 + \left(\lambda_c - \lambda\right) \vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

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S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

For $\lambda < \lambda_c$, oscillations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ constitute the *triplon* excitation

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

Outline

I. Magnetic quantum phase transitions in “dimerized” Mott insulators:
   Landau-Ginzburg-Wilson (LGW) theory

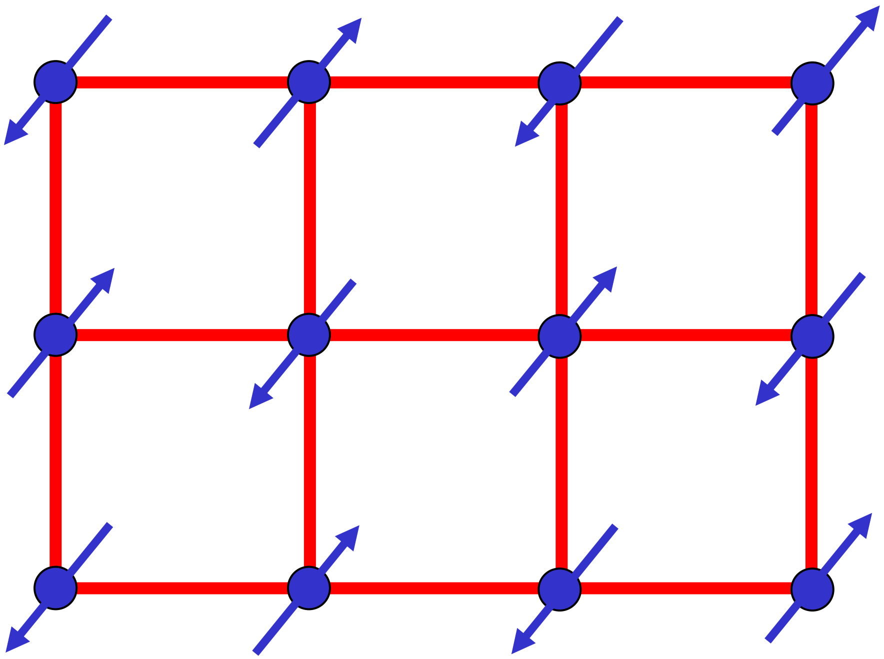
II. Magnetic quantum phase transitions of Mott insulators on the square lattice
   A. Breakdown of LGW theory
   B. Berry phases
   C. Spinor formulation and deconfined criticality

II. Magnetic quantum phase transitions of Mott insulators on the square lattice:

A. Breakdown of LGW theory

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{ spin operator with } S=1/2$$



Ground state has long-range Néel order

Order parameter $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$$\langle \vec{\varphi} \rangle \neq 0$$

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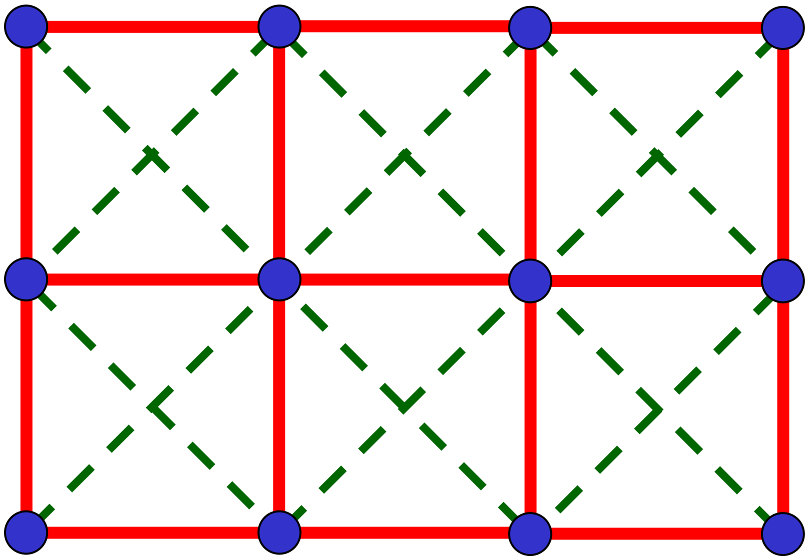

Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What is the state with $\langle \vec{\varphi} \rangle = 0$?

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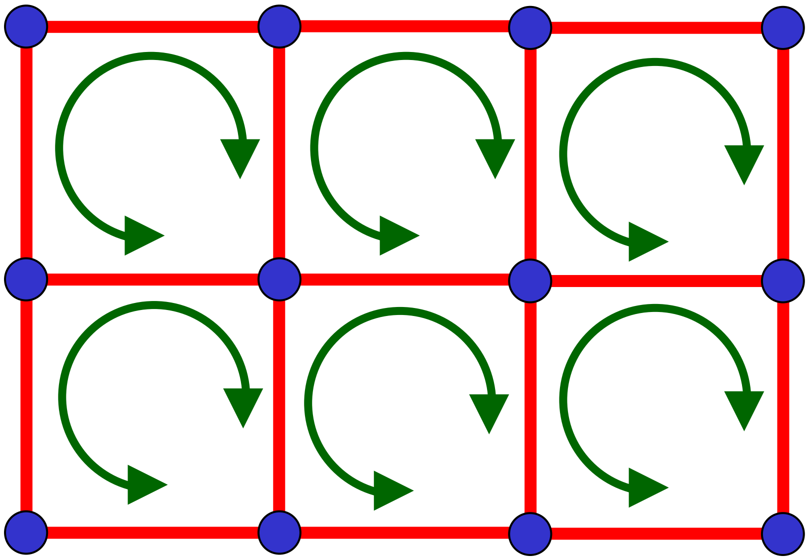

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$$S_\varphi = \int d^2x d\tau \left[\frac{1}{2}\left(\left(\nabla_x \vec{\varphi}\right)^2 + c^2 \left(\partial_\tau \vec{\varphi}\right)^2 + r\, \vec{\varphi}^2 \right) + \frac{u}{4!}\left(\vec{\varphi}^2\right)^2 \right]$$

The ground state for $r > 0$ has no broken symmetry and a gapped S=1 quasiparticle excitation $\left(\text{oscillations of } \vec{\varphi} \text{ about } \vec{\varphi} = 0\right)$

Problem: there is no state with a gapped, stable *S=1* quasiparticle and no broken symmetries

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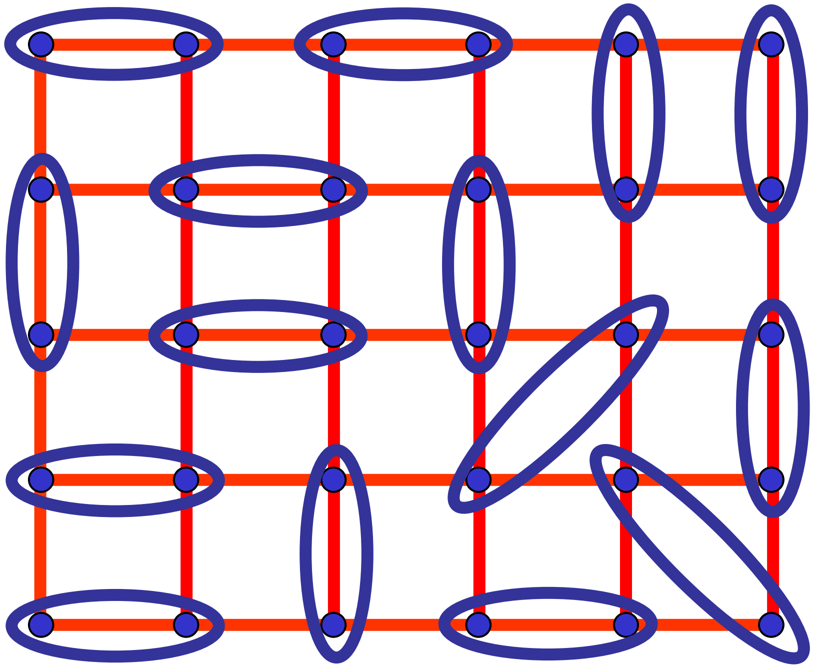

"Liquid" of valence bonds has fractionalized *S=1/2* excitations

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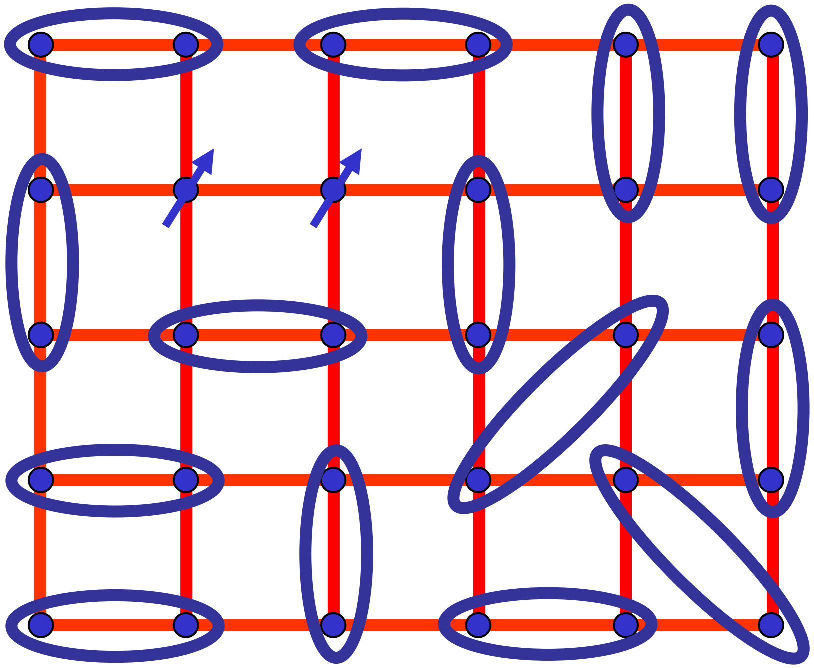

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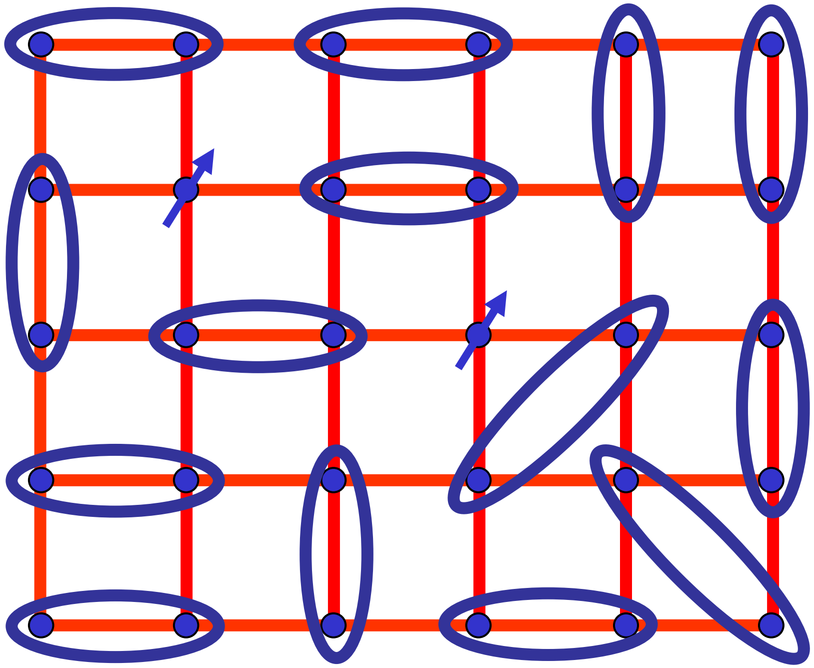

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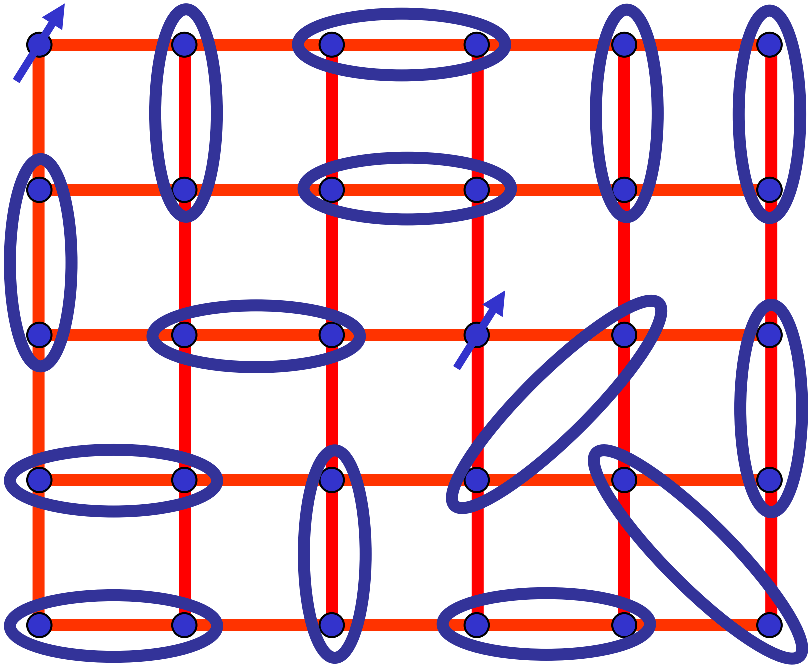

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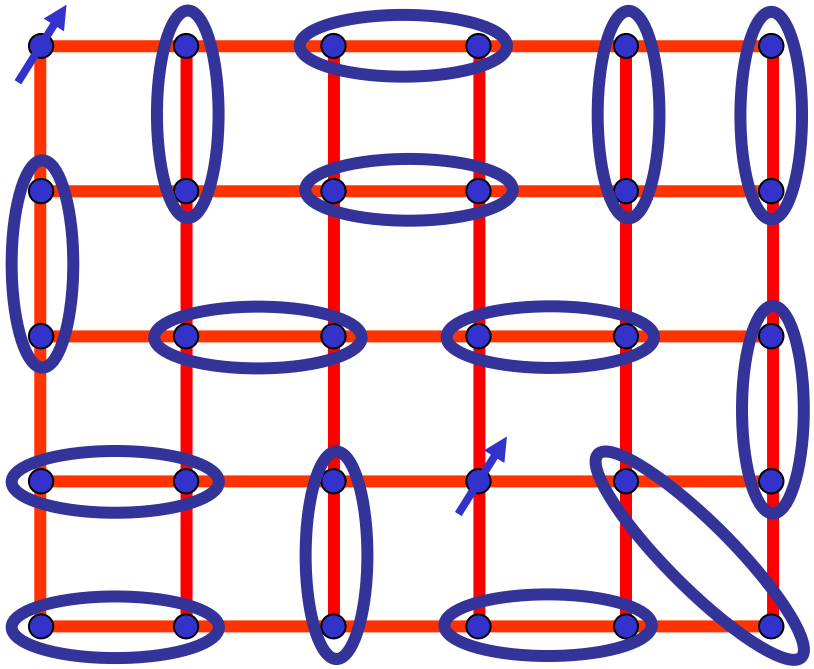

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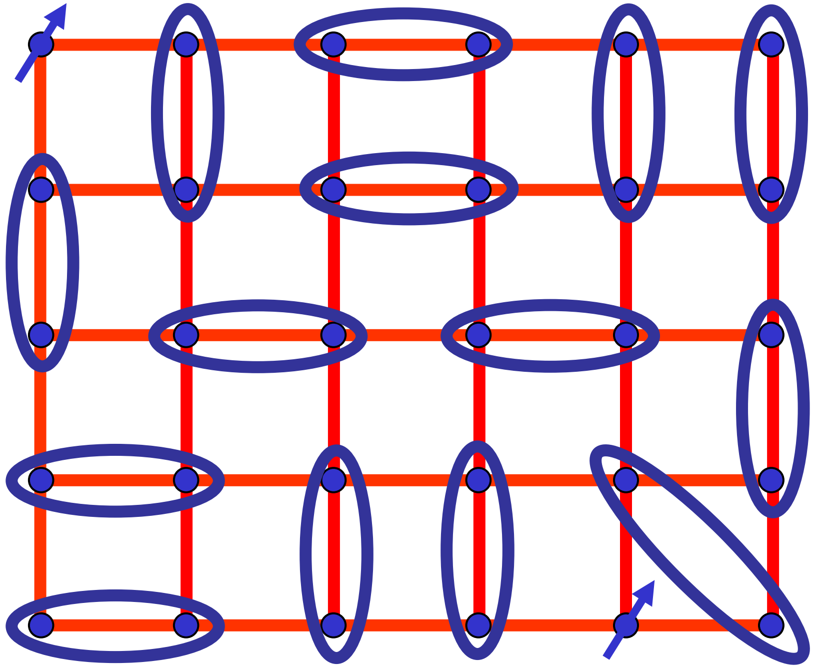

"Liquid" of valence bonds has fractionalized *S=1/2* excitations

Large scale Quantum Monte Carlo studies

Easy-plane model:

$$\mathcal{H}_{\rm XY} = 2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

A.W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, *Phys. Rev. Lett.* **89**, 247201 (2002); A.W. Sandvik and R.G. Melko, cond-mat/0604451.

SU(2)-invariant model:

$$\mathcal{H}_{\rm SU(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4} \right)$$

A.W. Sandvik, cond-mat/0611343

Easy-plane model

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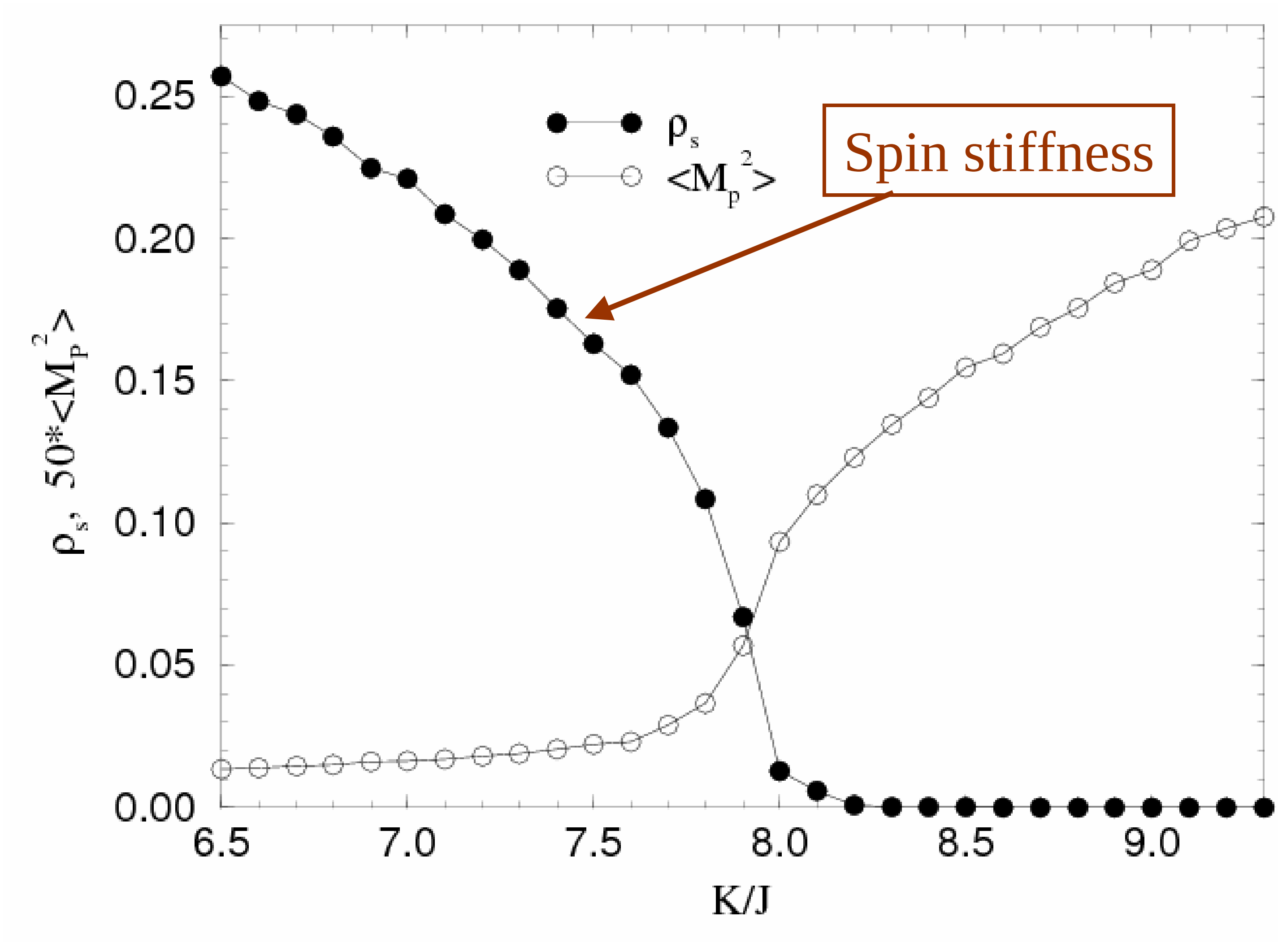

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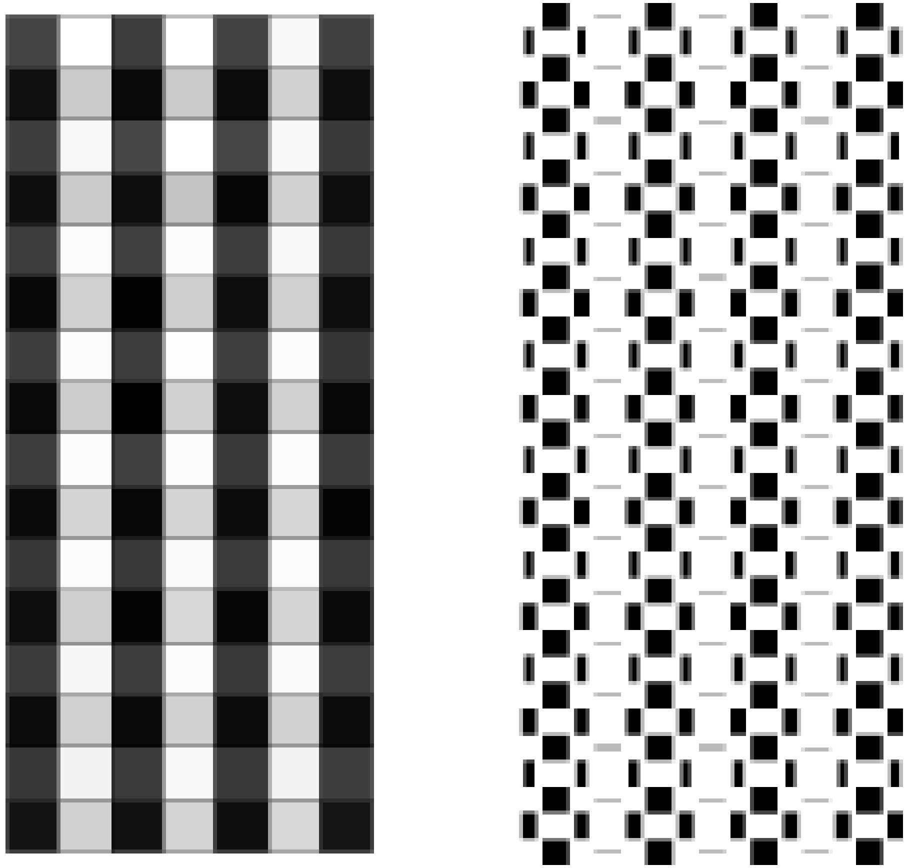
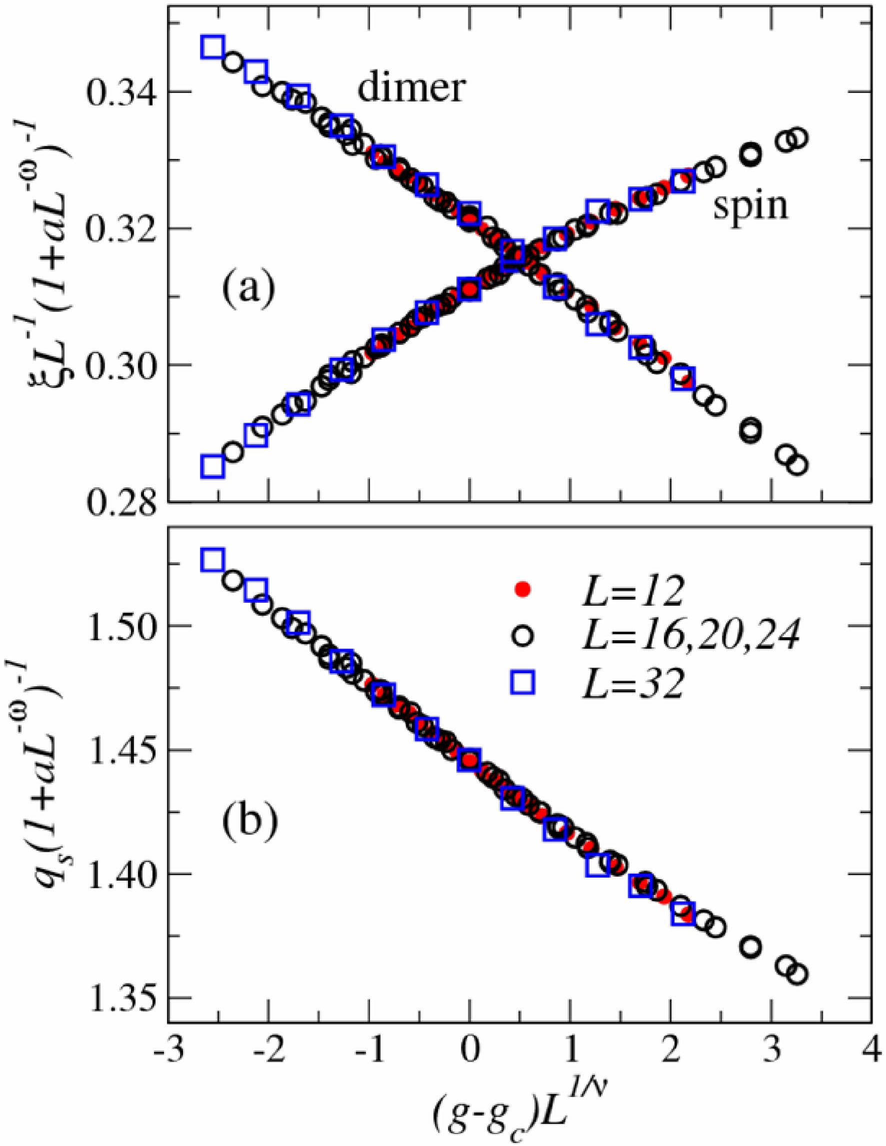


Valence bond solid (VBS) order in expectation values of plaquette and exchange terms

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

SU(2) invariant model

$$\mathcal{H}_{\rm SU(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$

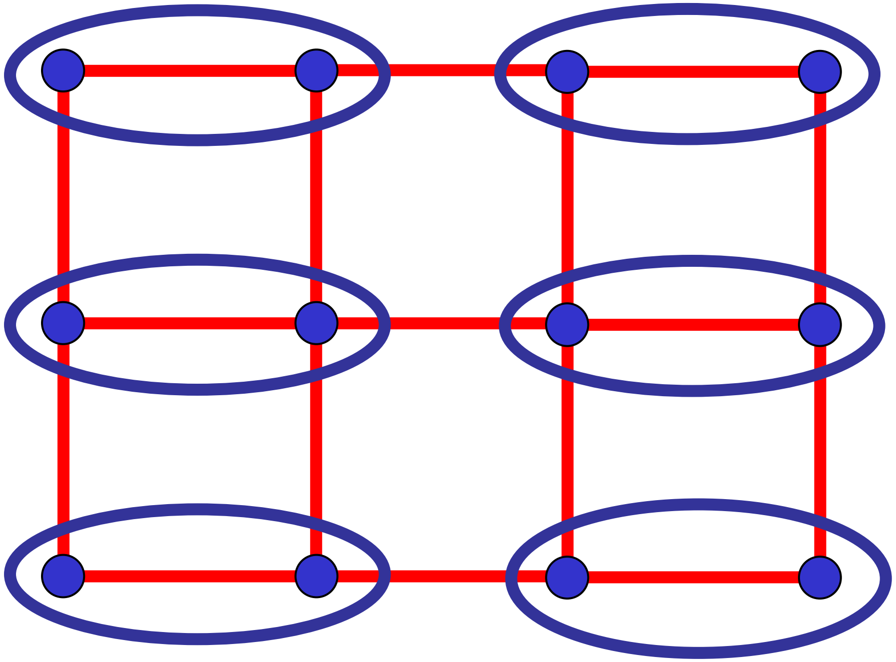


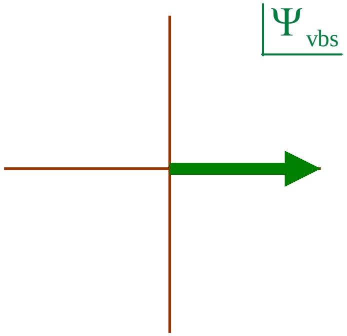
Strong evidence for a continuous "deconfined" quantum critical point

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

A.W. Sandvik, cond-mat/0611343

Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$





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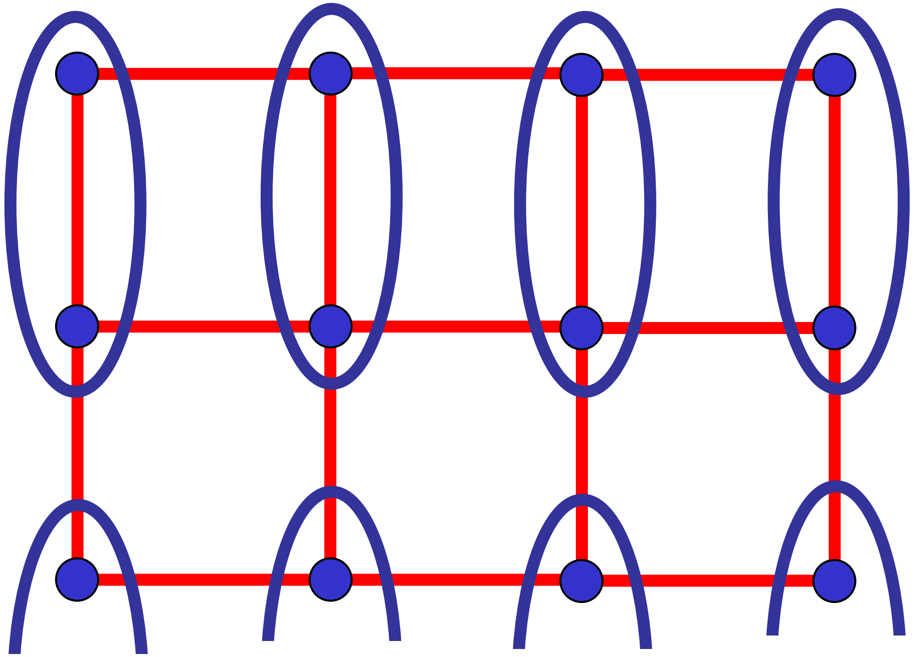

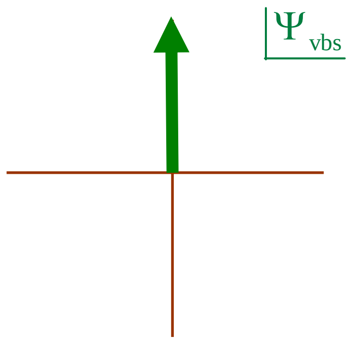

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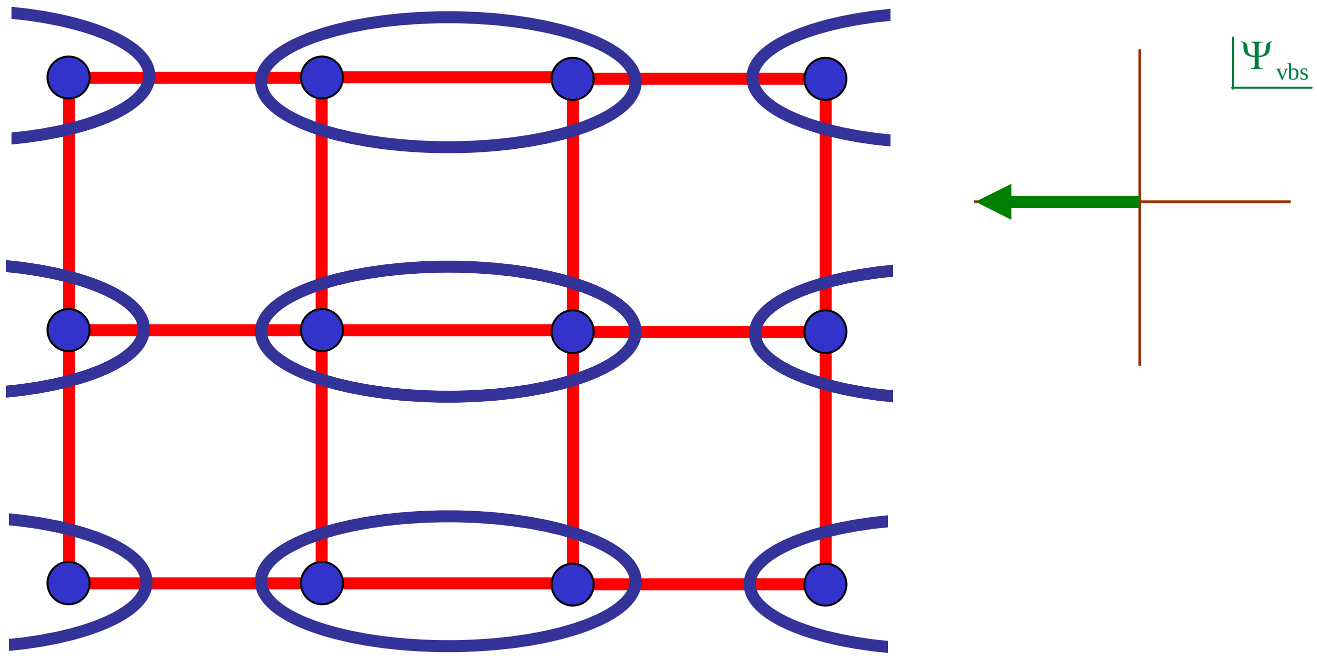

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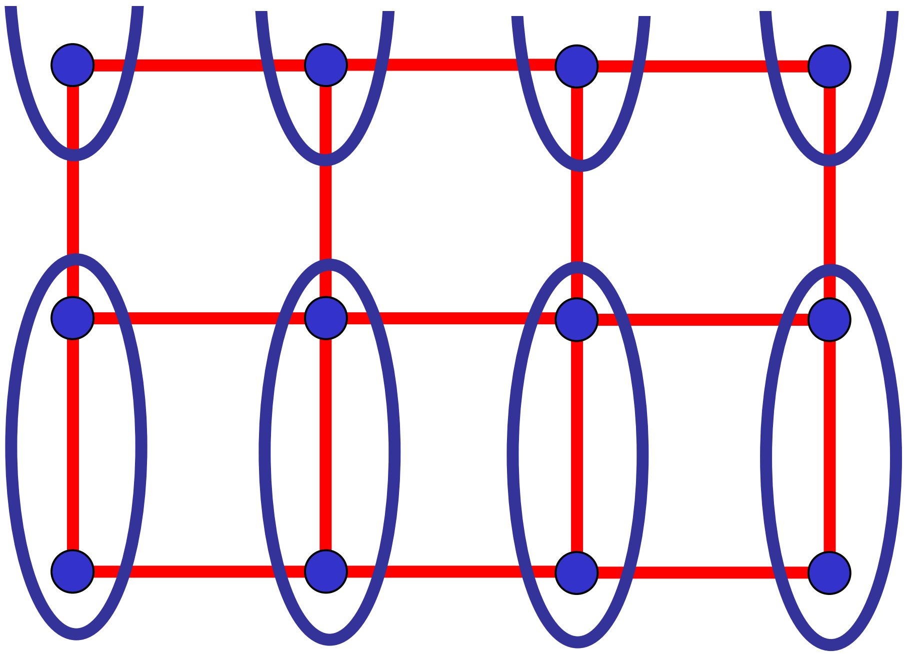

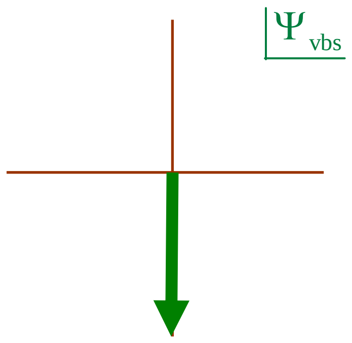

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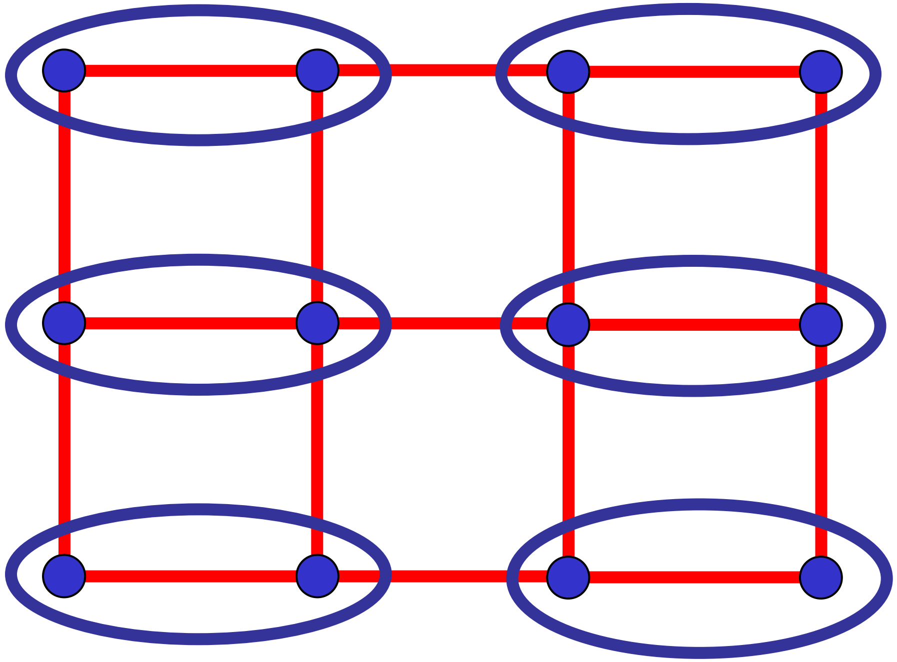

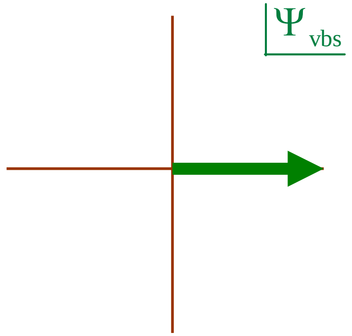

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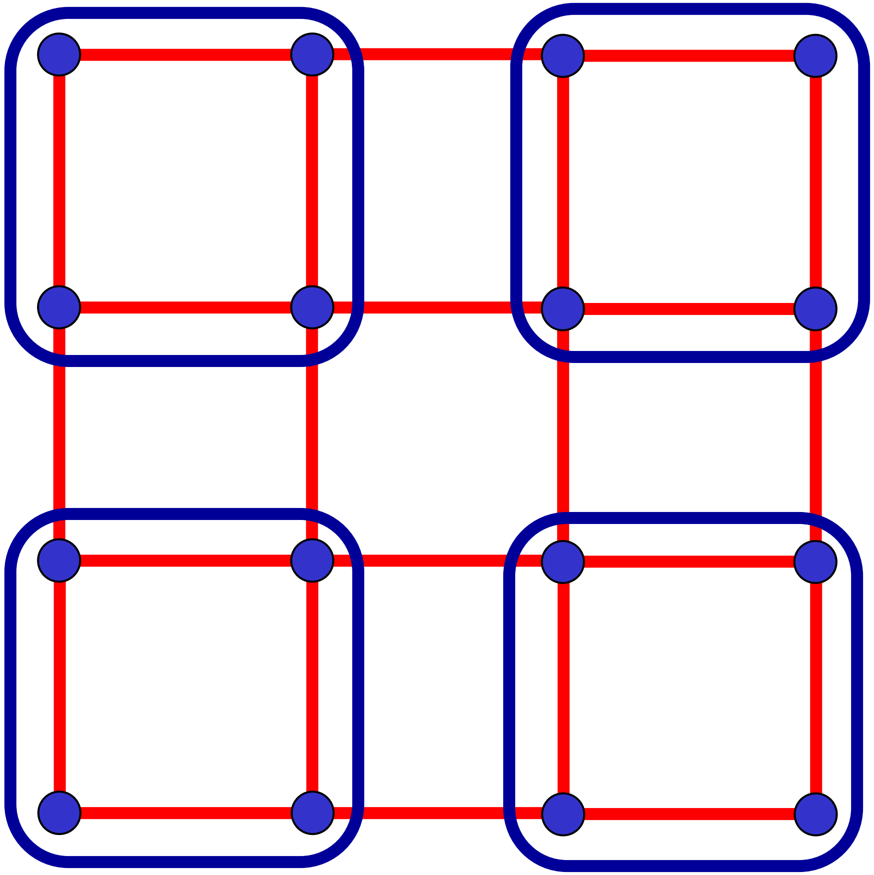

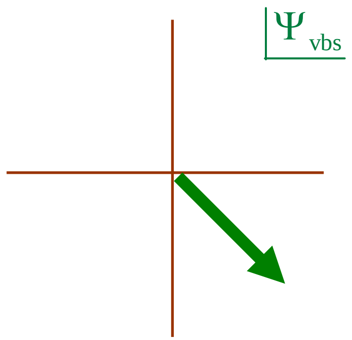

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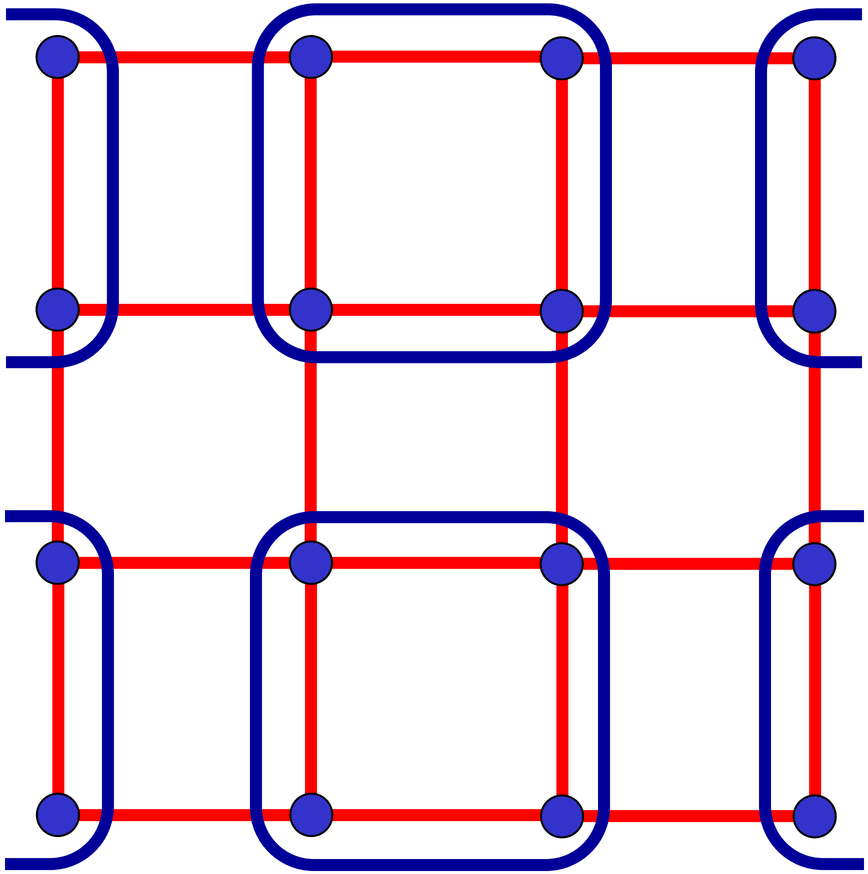

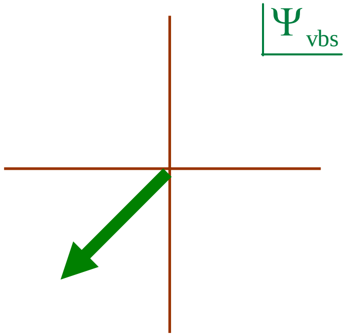

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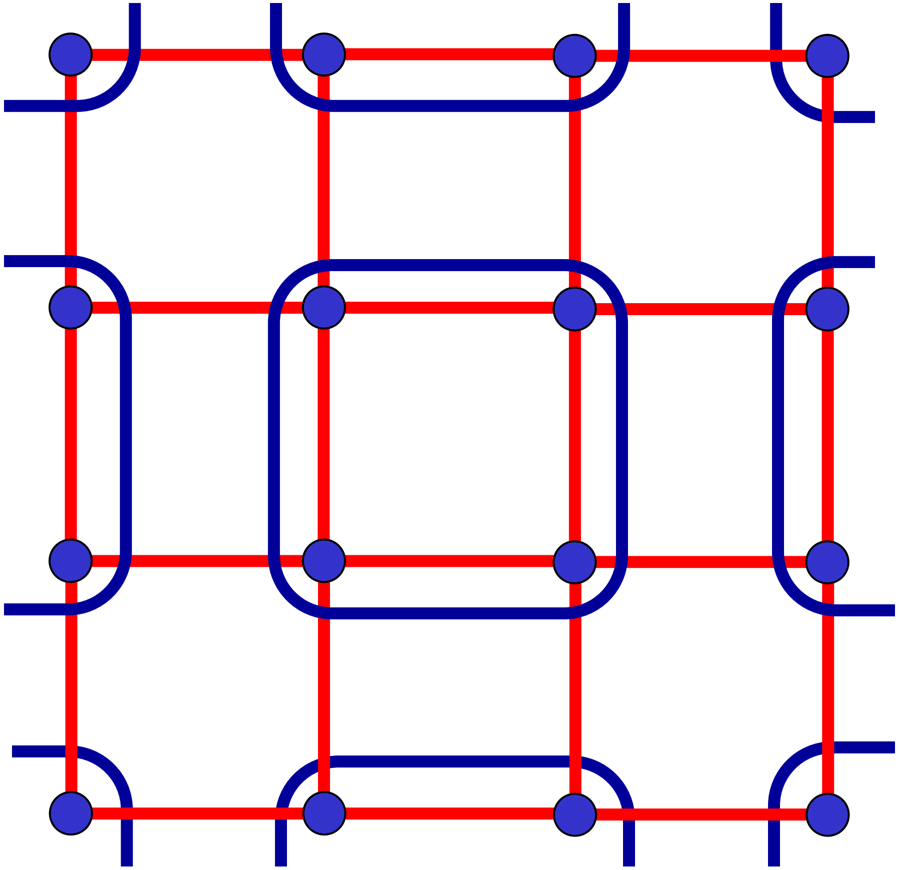

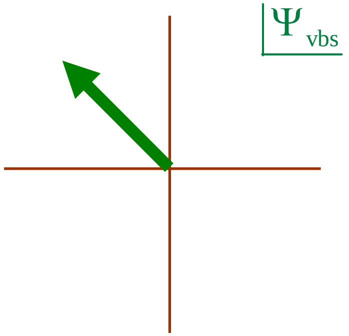

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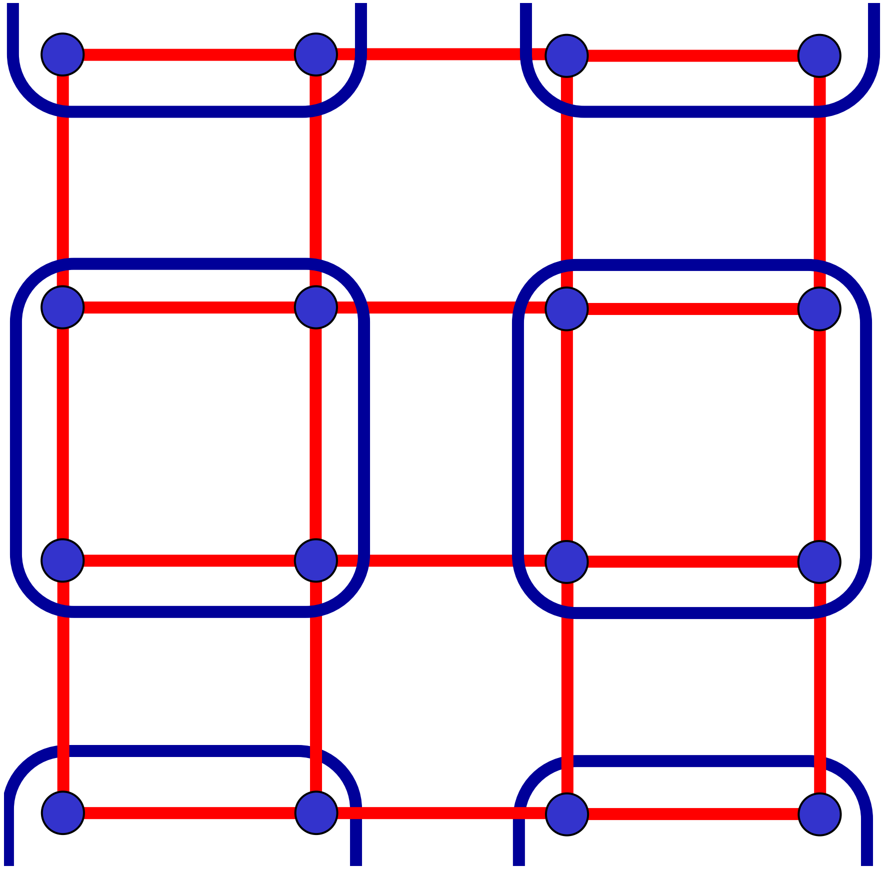

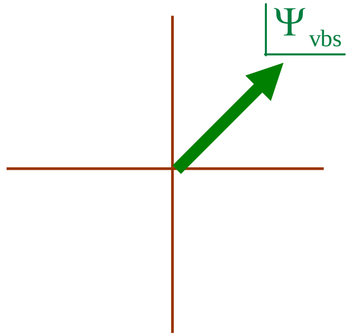

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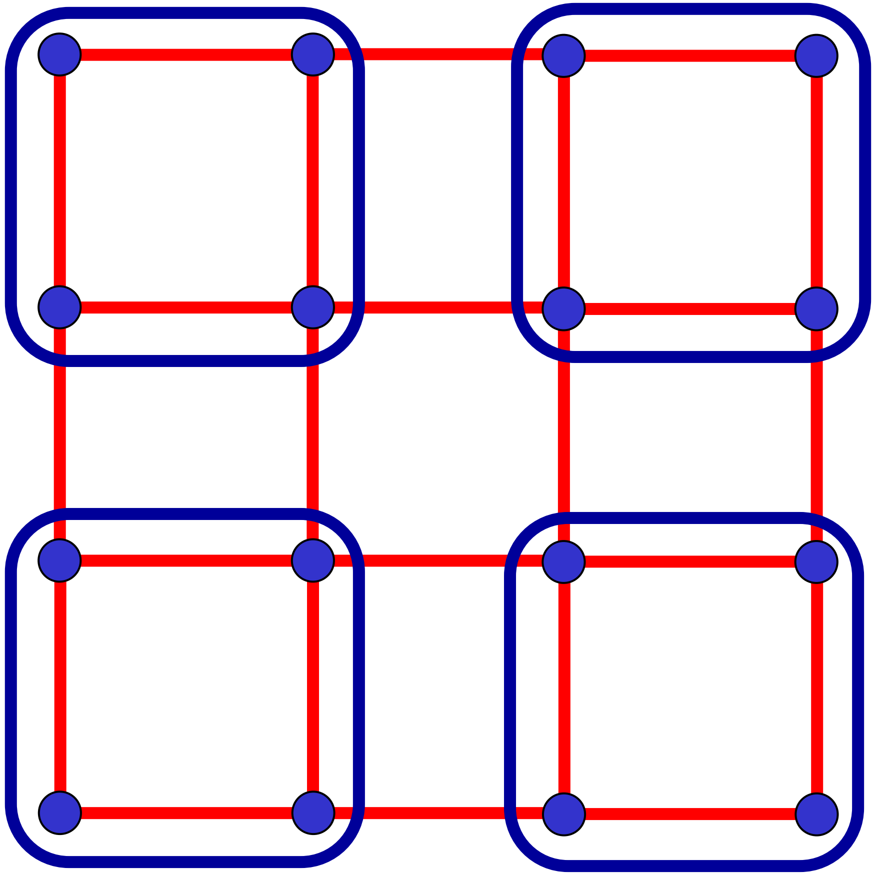

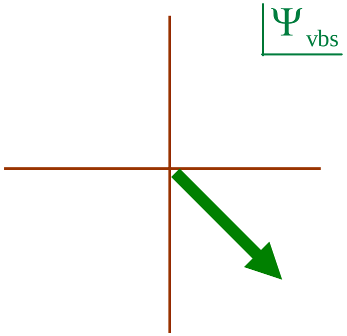

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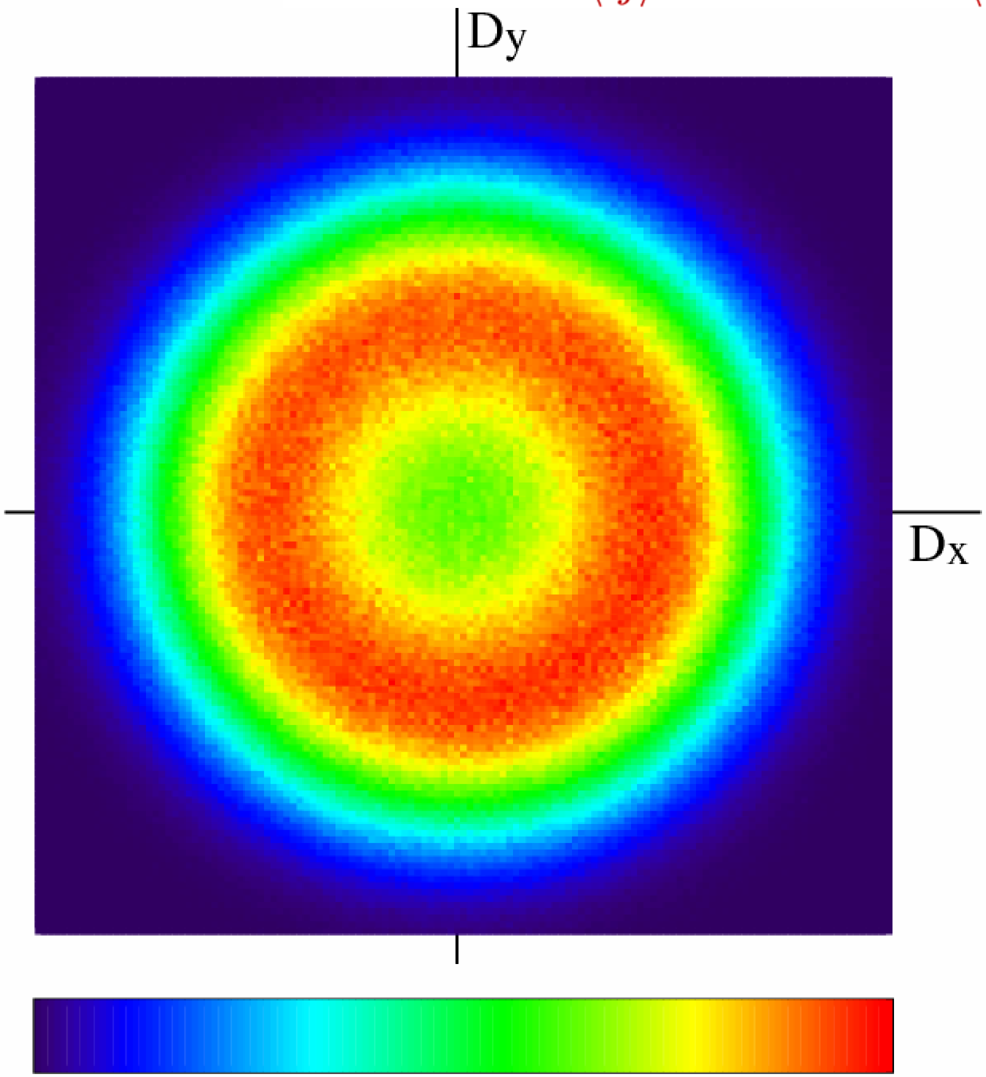
SU(2) invariant model

$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} \right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \frac{1}{4} \right)$$



A.W. Sandvik, cond-mat/0611343

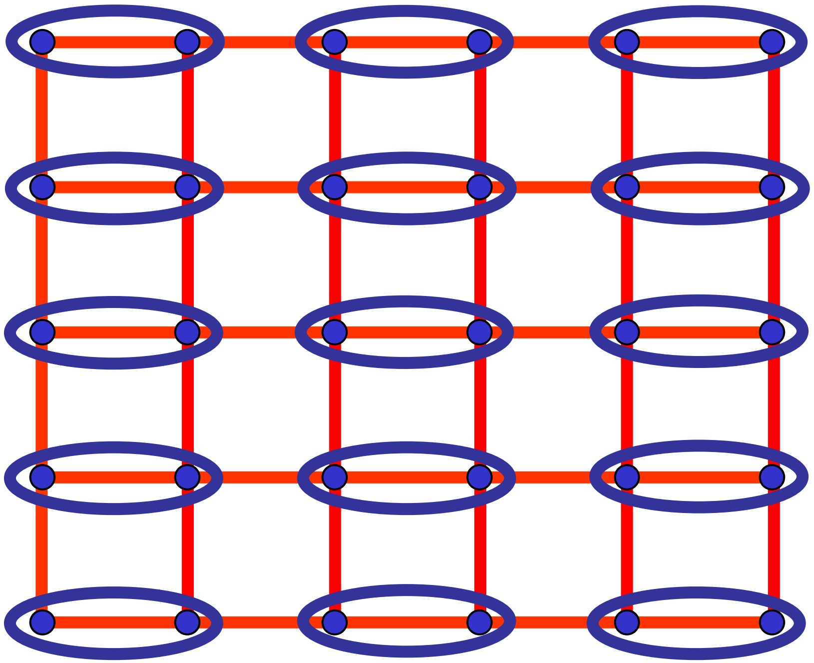
Probability distribution of VBS order Ψ at quantum critical point

Emergent circular symmetry is a consequence of a gapless photon excition

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

The VBS state does have a stable *S=1* quasiparticle excitation

$$\left\langle \Psi_{\text{vbs}} \right\rangle \neq 0, \ \left\langle \vec{\varphi} \right\rangle = 0$$



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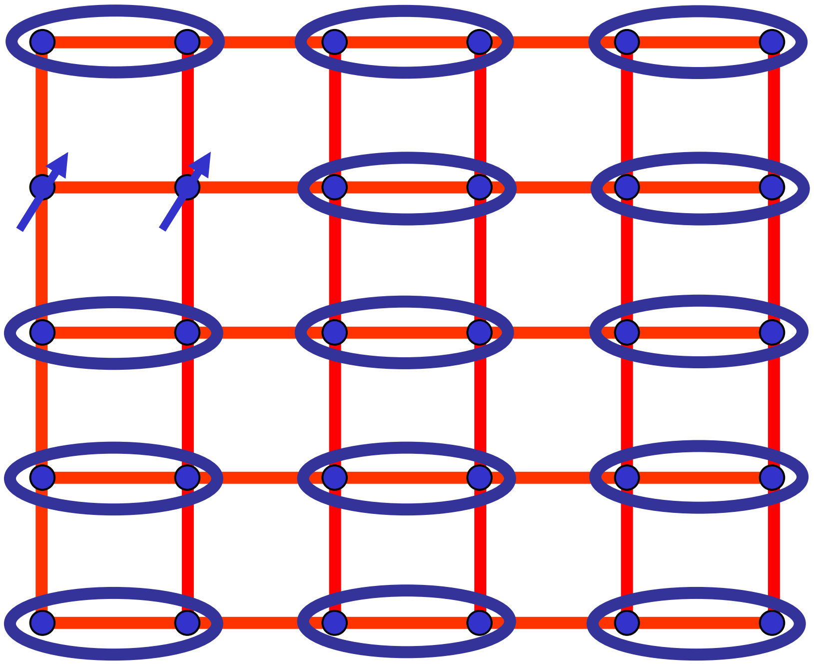

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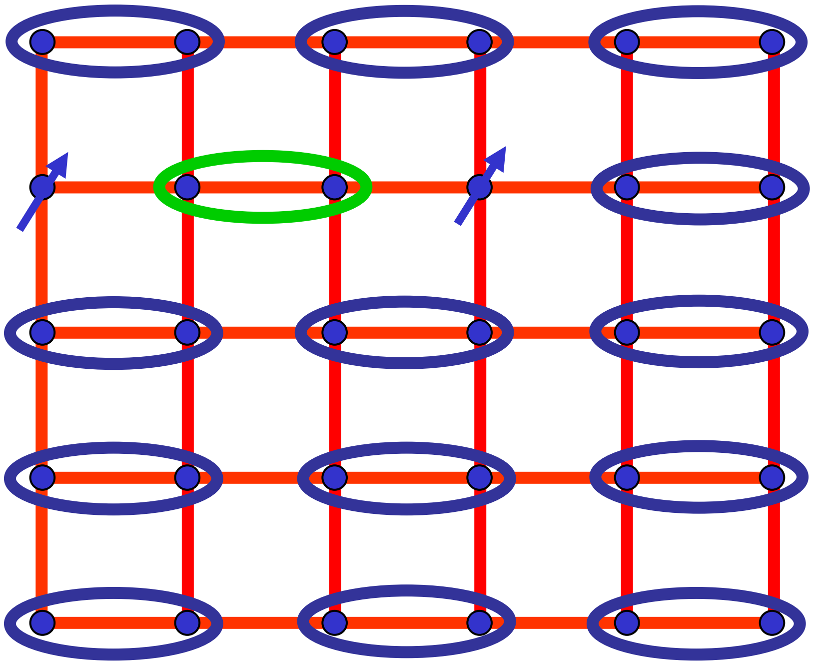

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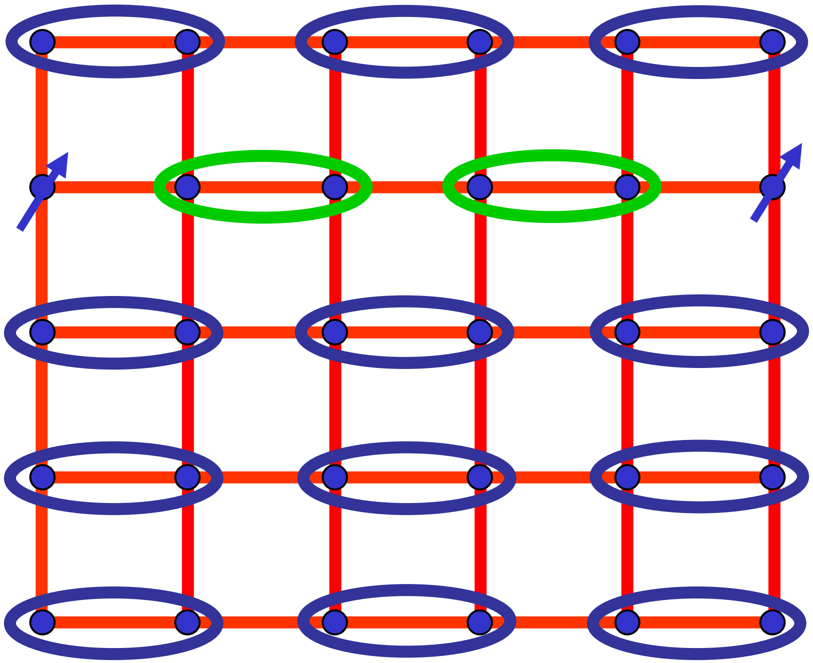

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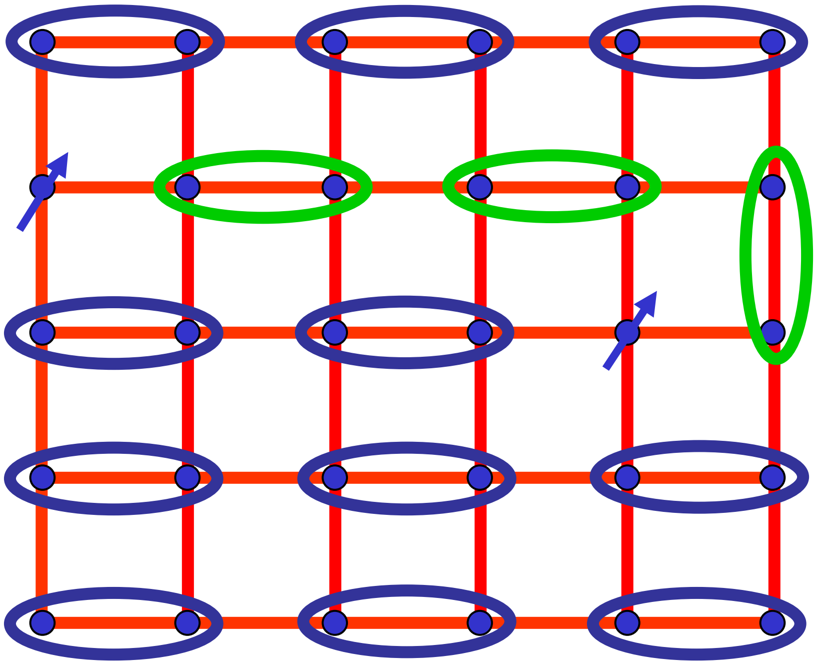

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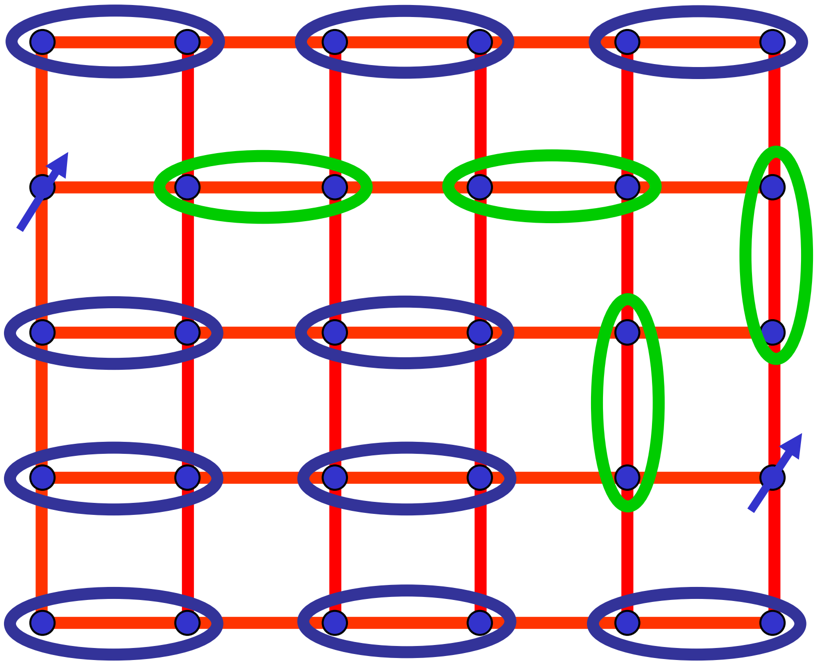


LGW theory of multiple order parameters

$$F = F_{\text{vbs}}\left[\Psi_{\text{vbs}}\right] + F_{\varphi}\left[\vec{\varphi}\right] + F_{\text{int}}$$

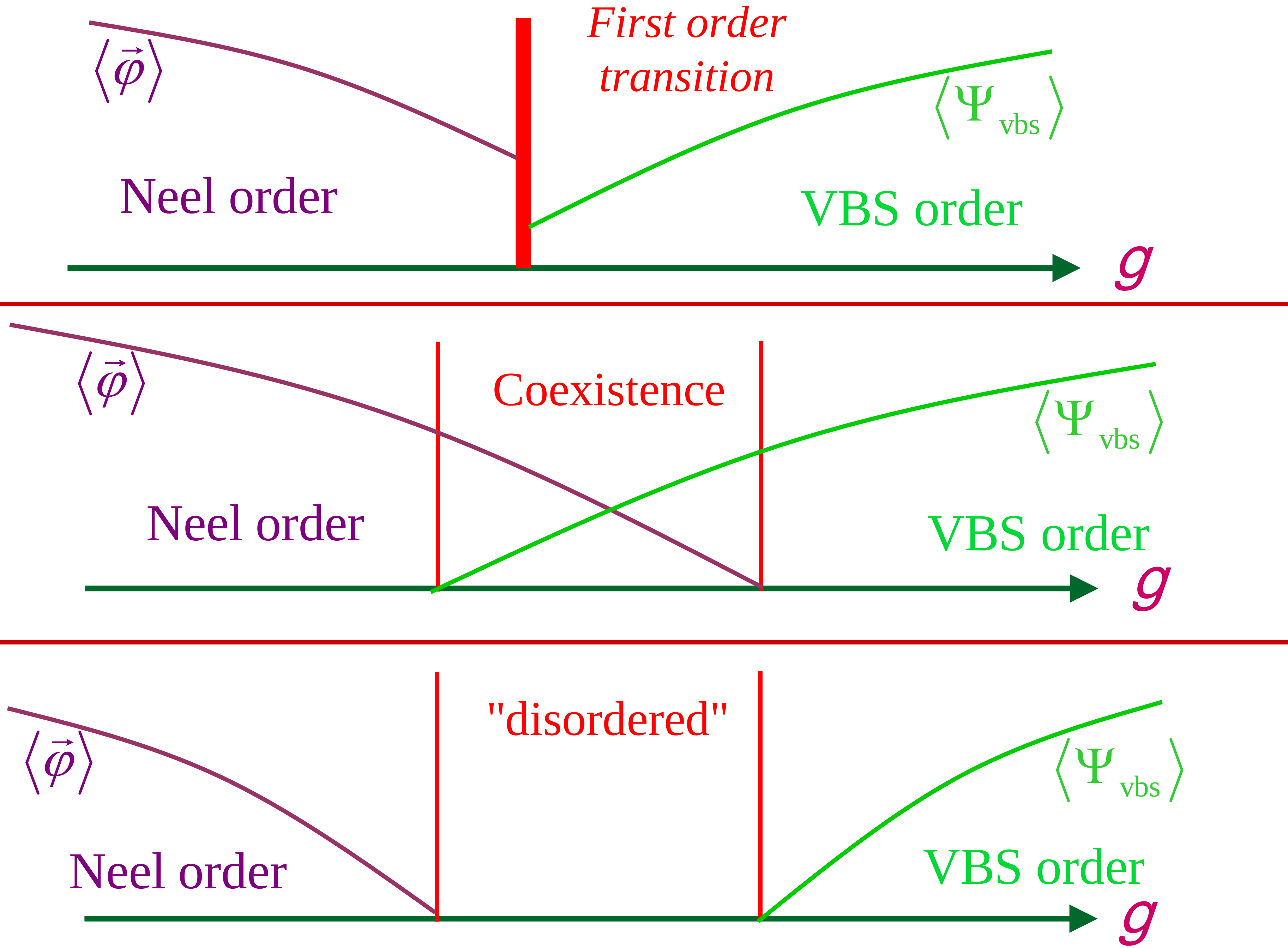
$$F_{\text{vbs}}\left[\Psi_{\text{vbs}}\right] = r_1 \left|\Psi_{\text{vbs}}\right|^2 + u_1 \left|\Psi_{\text{vbs}}\right|^4 + \cdots$$

$$F_{\varphi}\left[\vec{\varphi}\right] = r_2 \left|\vec{\varphi}\right|^2 + u_2 \left|\vec{\varphi}\right|^4 + \cdots$$

$$F_{\text{int}} = v \left|\Psi_{\text{vbs}}\right|^2 \left|\vec{\varphi}\right|^2 + \cdots$$

Distinct symmetries of order parameters permit couplings only between their energy densities

LGW theory of multiple order parameters



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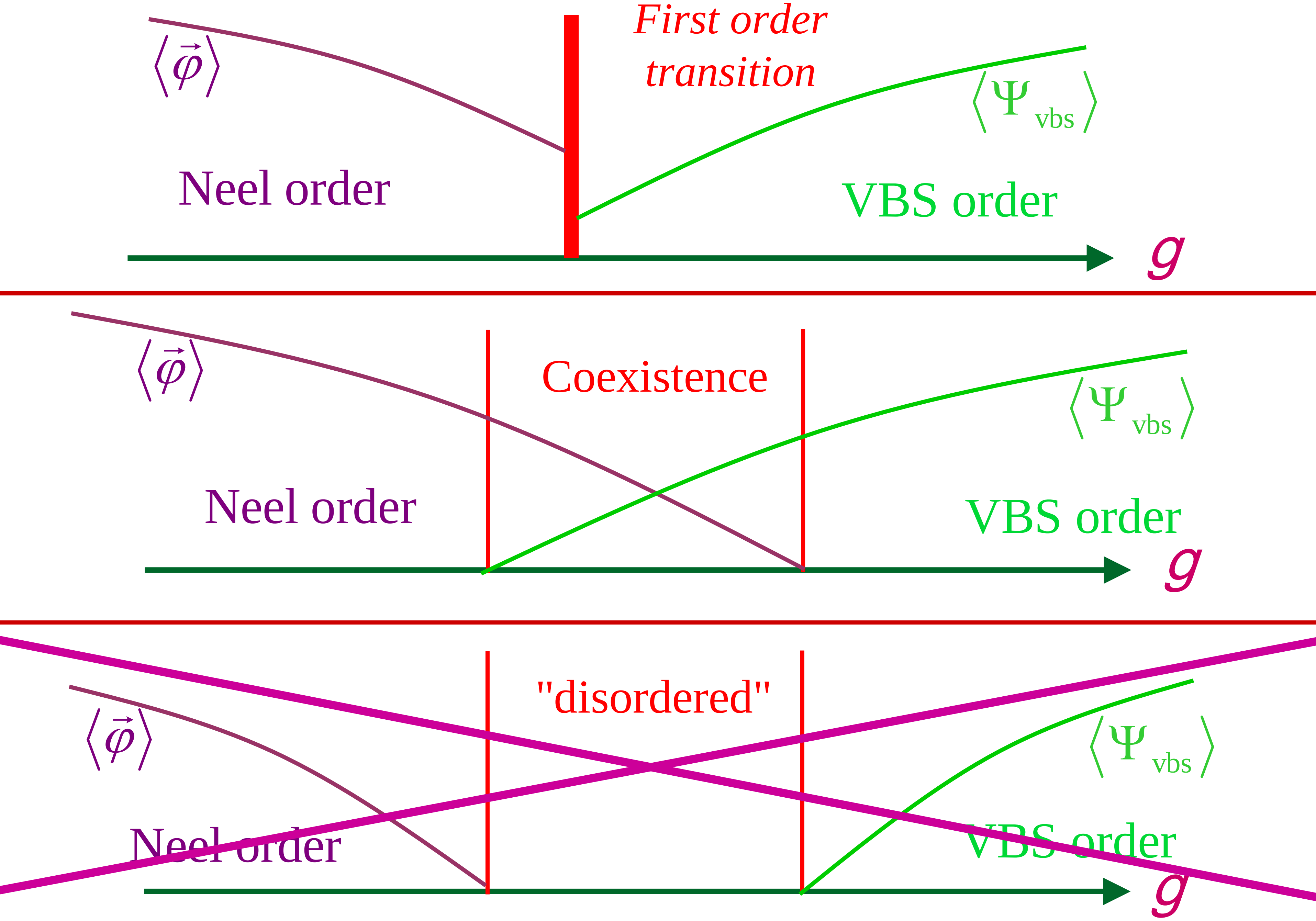


Outline

I. Magnetic quantum phase transitions in "dimerized" Mott insulators:
Landau-Ginzburg-Wilson (LGW) theory

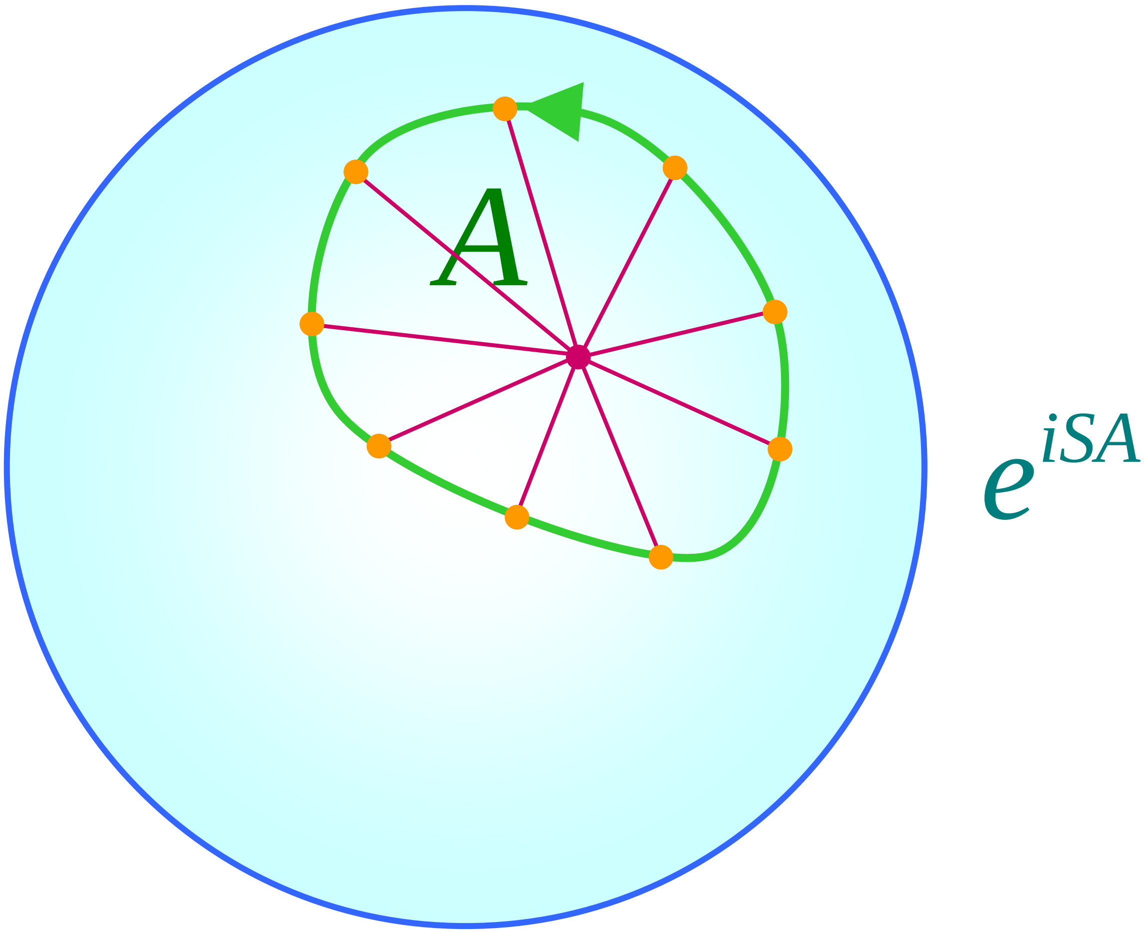
II. Magnetic quantum phase transitions of Mott insulators on the square lattice
A. Breakdown of LGW theory
B. Berry phases
C. Spinor formulation and deconfined criticality

II. Magnetic quantum phase transitions of Mott insulators on the square lattice:

B. Berry phases

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



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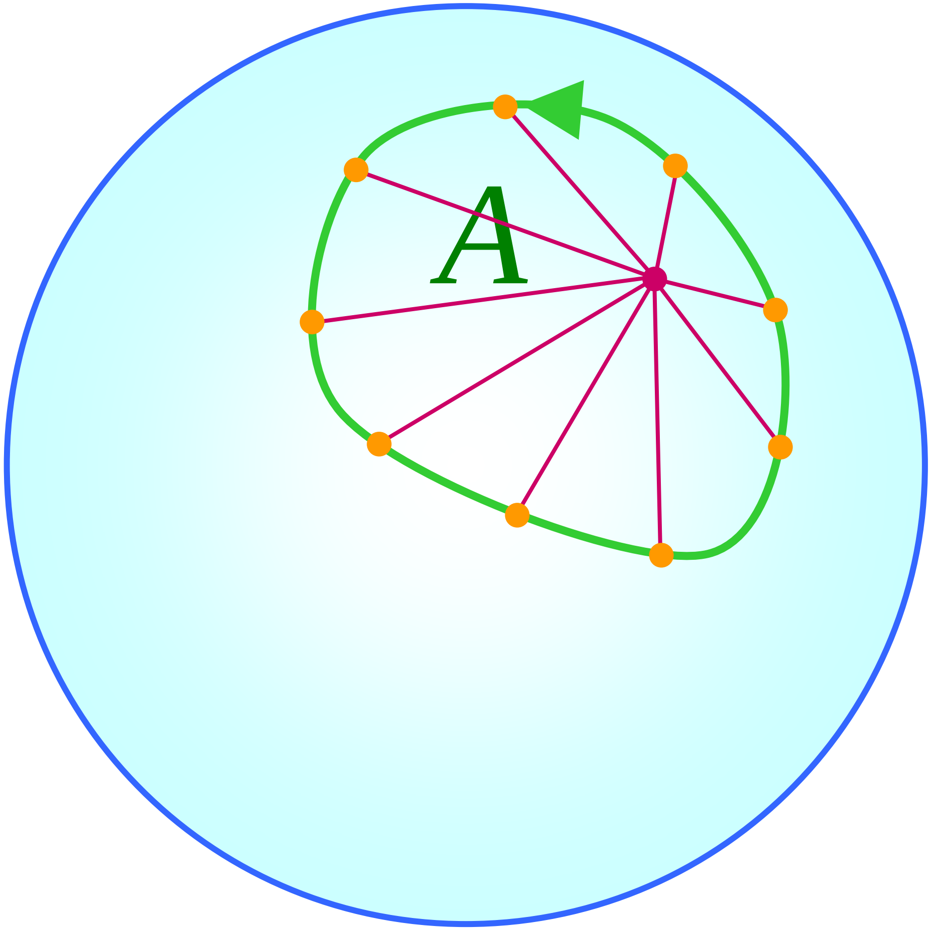
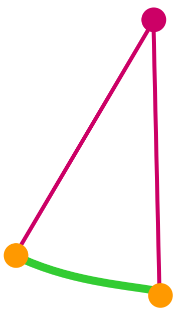


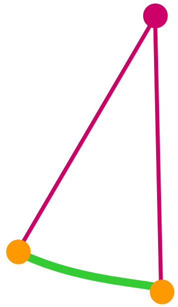
$$e^{iSA}$$

Quantum theory for destruction of Neel order



Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points a

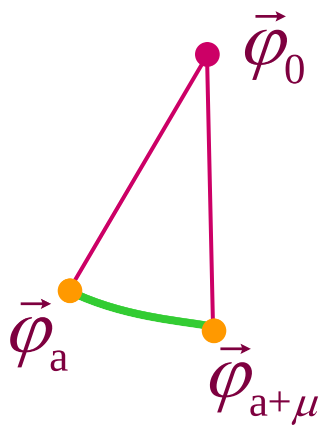


Quantum theory for destruction of Neel order

Discretize imaginary time: path integral is over fields on the sites of a cubic lattice of points *a*

Recall $\vec{\varphi}_{\rm a} = 2\eta_a \vec{S}_a \rightarrow \vec{\varphi}_{\rm a} = (0,0,1)$ in classical Neel state;

$\eta_{\rm a} \rightarrow \pm 1$ on two square sublattices ;



$(\mu = x, y, \tau)$

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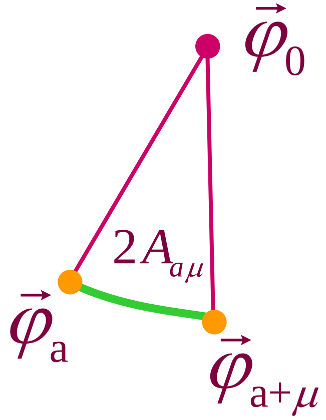
$A_{a\mu} \rightarrow$ *half* oriented area of spherical triangle

formed by $\vec{\varphi}_a$, $\vec{\varphi}_{a+\mu}$, and an arbitrary reference point $\vec{\varphi}_0$



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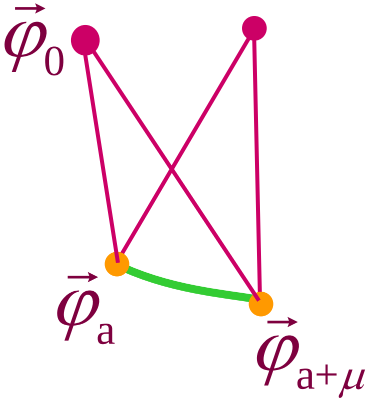

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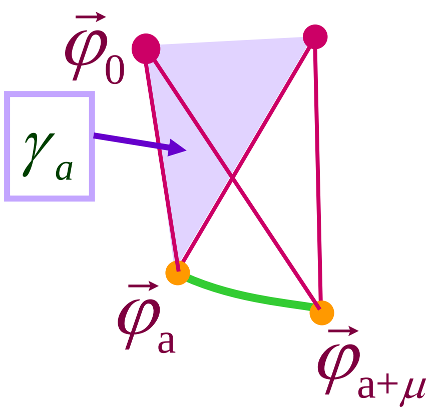

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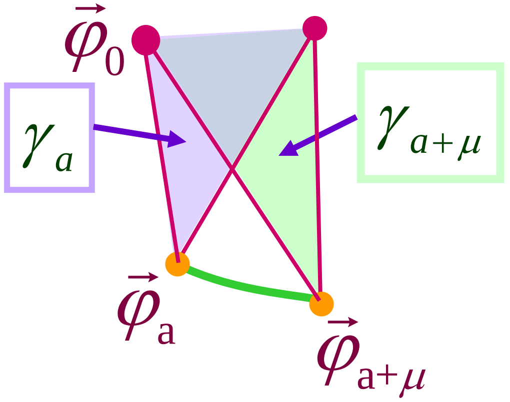

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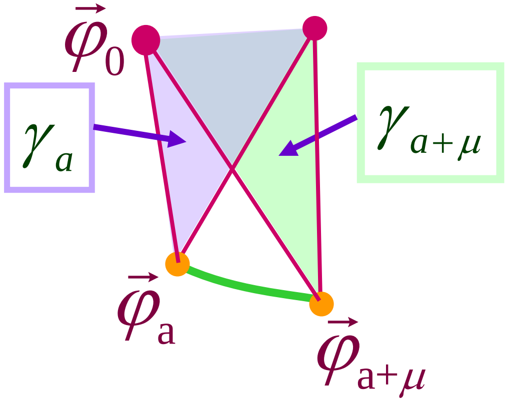
$$2A_{a\mu} \rightarrow 2A_{a\mu} - \gamma_{a+\mu} + \gamma_a$$

Change in choice of $\vec{\varphi}_0$ is like a "gauge transformation"



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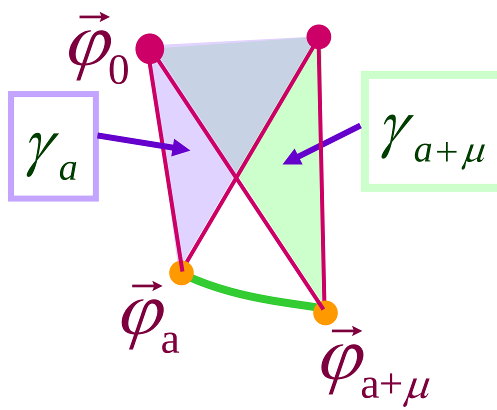
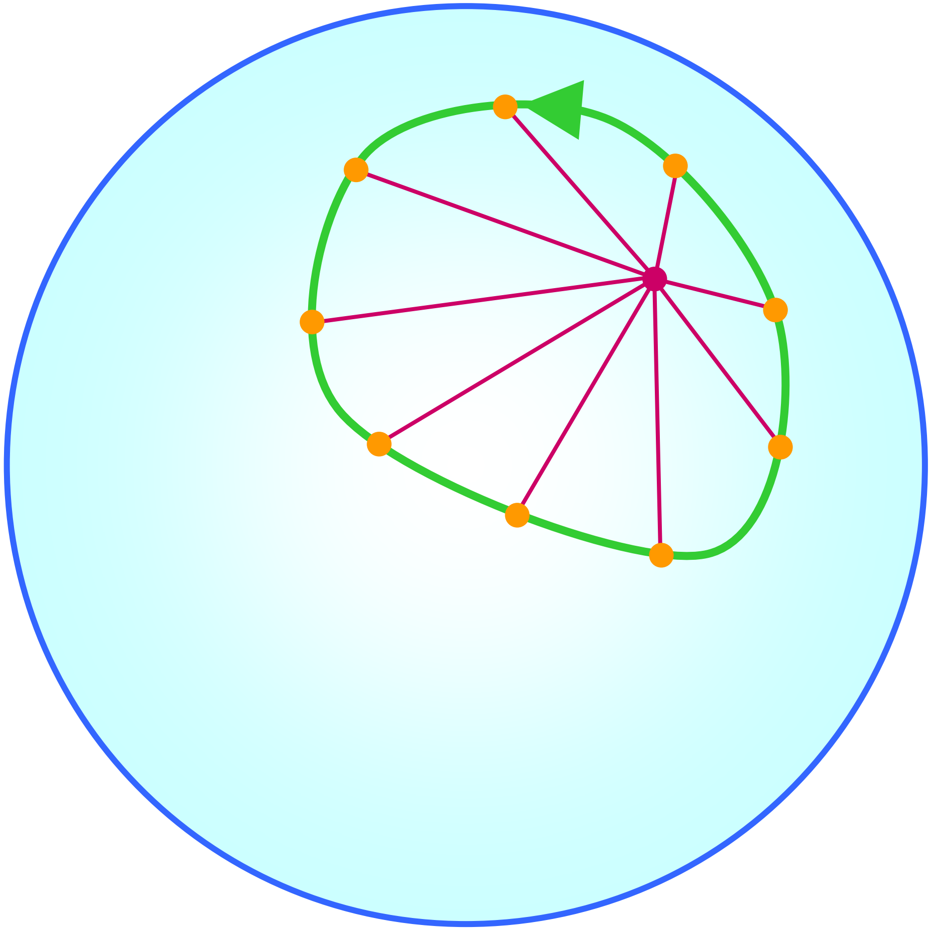


The area of the triangle is uncertain modulo 4π, and the action has to be invariant under $A_{a\mu} \rightarrow A_{a\mu} + 2\pi$

Quantum theory for destruction of Neel order

Ingredient missing from LGW theory: Spin Berry Phases



$$\exp\left(i \sum_a \eta_a A_{a\tau} \right)$$

Sum of Berry phases of all spins on the square lattice.

Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} \right)$$

LGW theory: weights in partition function are those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\left\langle \vec{\varphi} \right\rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\left\langle \vec{\varphi} \right\rangle = 0$

Quantum theory for destruction of Neel order

Partition function on cubic lattice

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Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\left\langle \vec{\varphi} \right\rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\left\langle \vec{\varphi} \right\rangle = 0$

Berry phases lead to large cancellations between different time histories $\rightarrow$ need an effective action for $A_{a\mu}$ at large g

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

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C. Spinor formulation and deconfined criticality

II. Magnetic quantum phase transitions of Mott insulators on the square lattice:

C. Spinor formulation and deconfined criticality

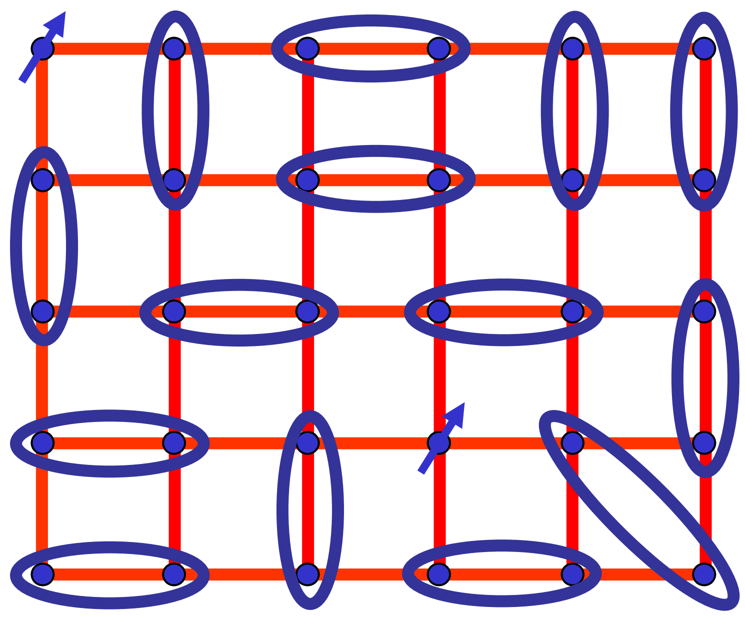
Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$Z = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i \sum_a \eta_a A_{a\tau} \right)$$

Rewrite partition function in terms of spinors $z_{a\alpha}$, with $\alpha = \uparrow, \downarrow$ and

$$\vec{\varphi}_a = z_{a\alpha}^* \vec{\sigma}_{\alpha\beta} z_{a\beta}$$



S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

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Identity from spherical trigonometry

$$\text{Arg}\left[z_{a\alpha}^* z_{a+\mu,\alpha} \right] = A_{a\mu}$$

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Quantum theory for destruction of Neel order

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Partition function expressed as a gauge theory of spinor degrees of freedom

$$Z \approx \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\left| z_{a\alpha} \right|^2 - 1 \right)$$

$$\times \exp\left(\frac{1}{g} \sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau} \right)$$

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Large g effective action for the $A_{a\mu}$ after integrating $z_{\alpha\mu}$

$$Z = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right) - i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

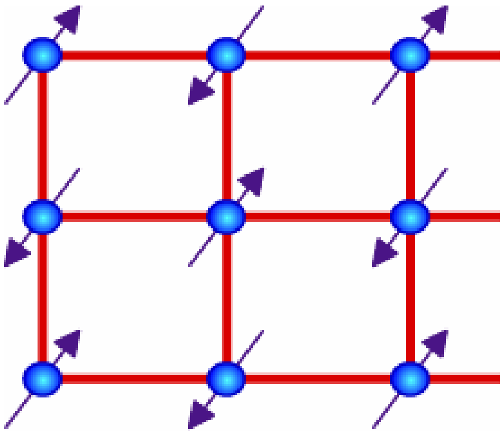
This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

This theory can be reliably analyzed by a duality mapping.

The gauge theory is in a *confining* phase, and there is VBS order in the ground state. (Proliferation of monopoles in the presence of Berry phases).

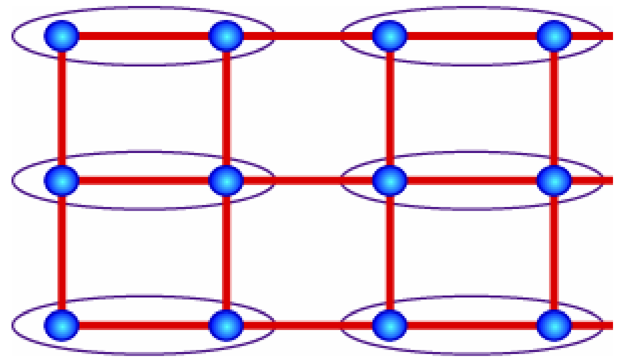
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

$$Z \approx \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\left|z_{a\alpha}\right|^2 - 1\right) \exp\left(\frac{1}{g} \sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i \sum_a \eta_a A_{a\tau} \right)$$



Neel order

$$\langle \vec{\varphi} \rangle \neq 0$$



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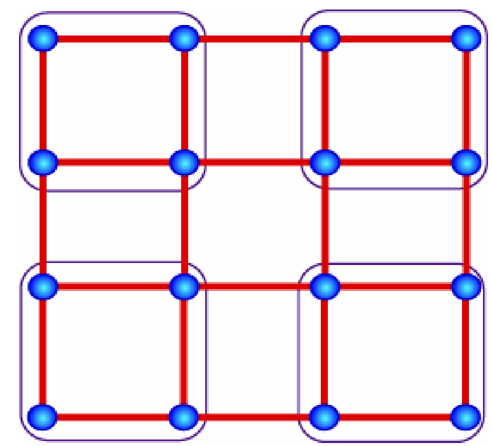


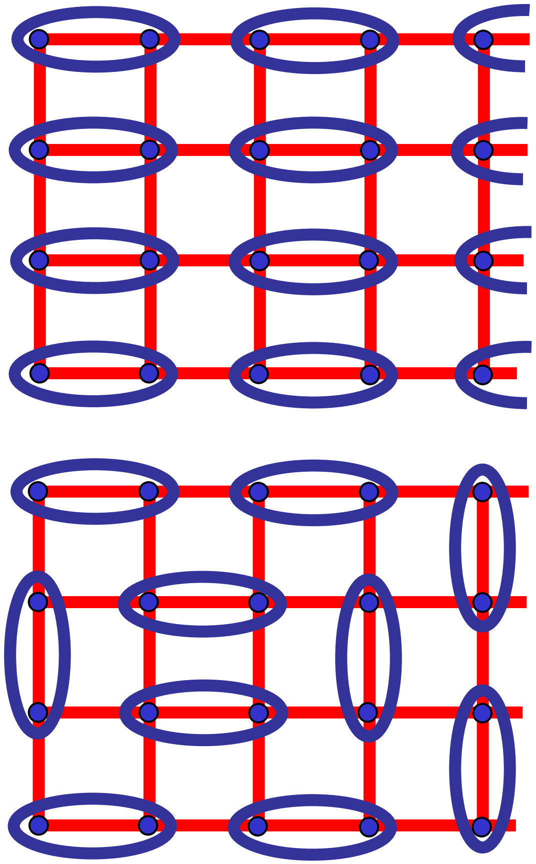
VBS order

$$\langle \Psi_{\text{vbs}} \rangle \neq 0$$

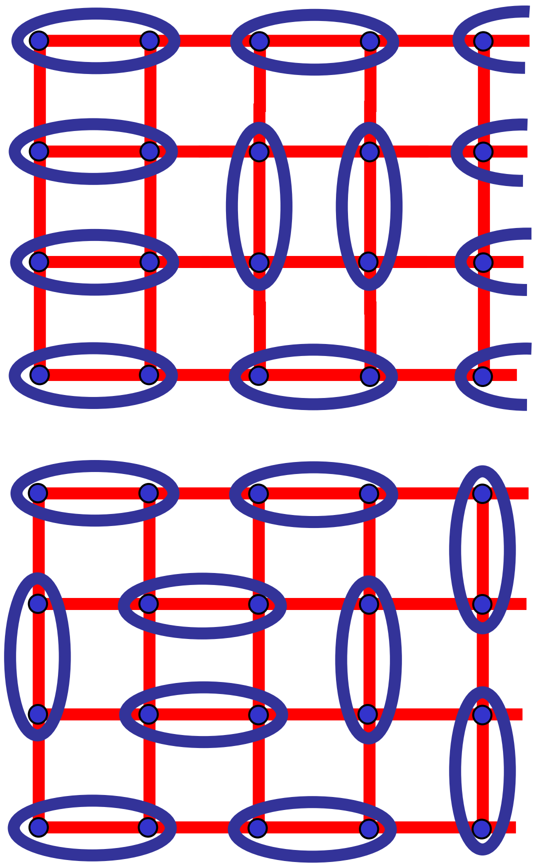
Not present in LGW theory of $\vec{\varphi}$ order

0 $\longrightarrow$ g

Ordering by quantum fluctuations



Ordering by quantum fluctuations



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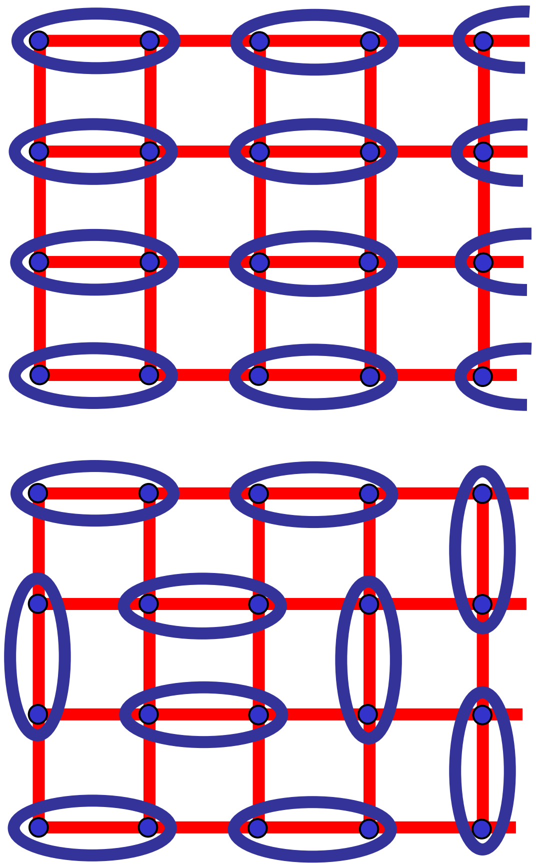

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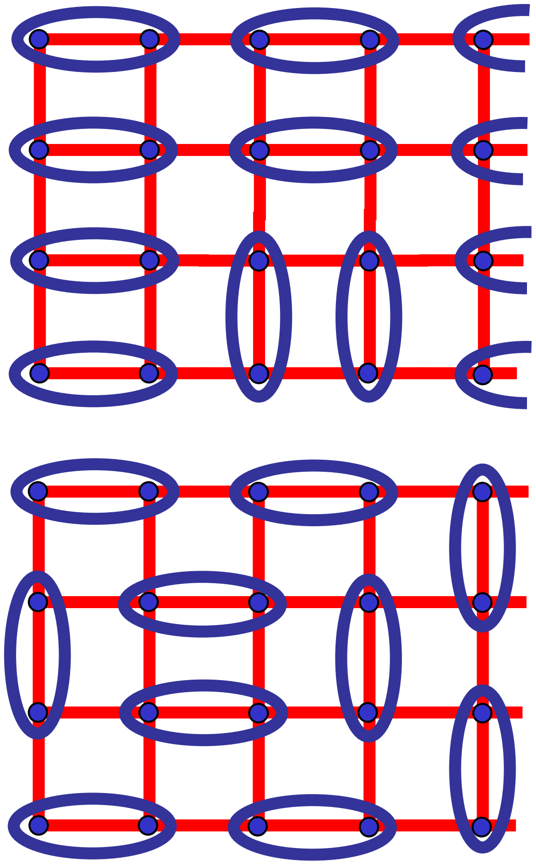

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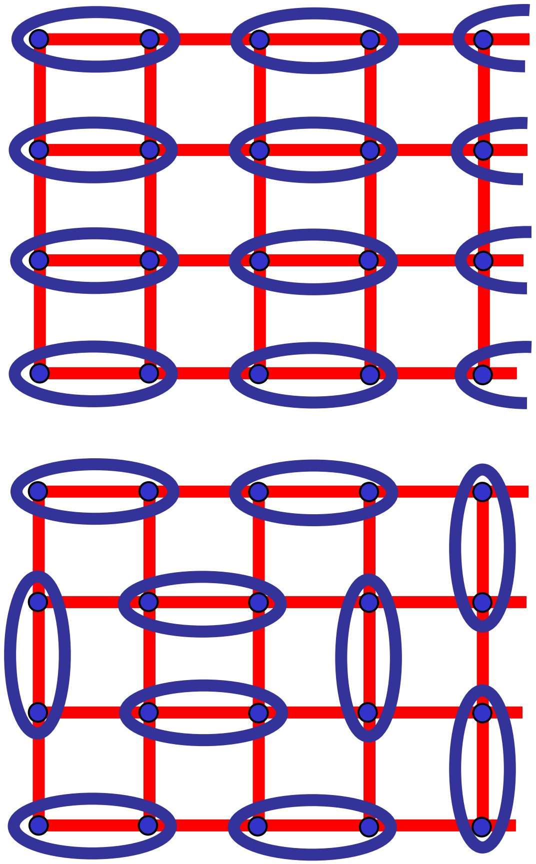

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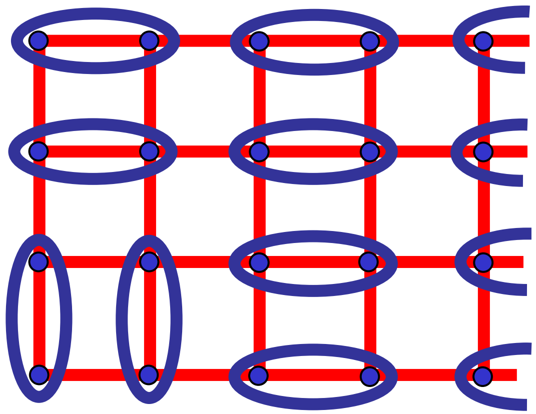

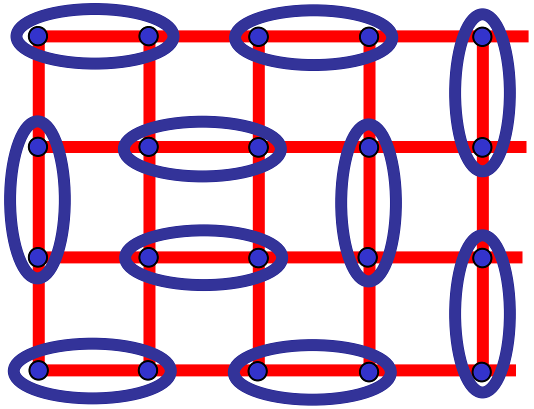

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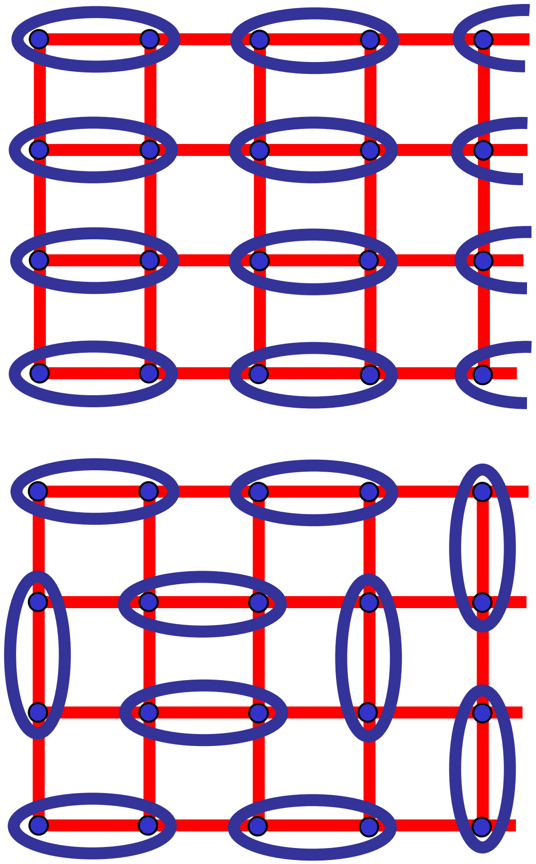

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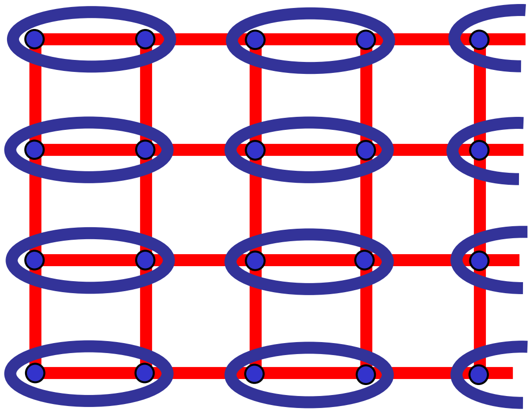

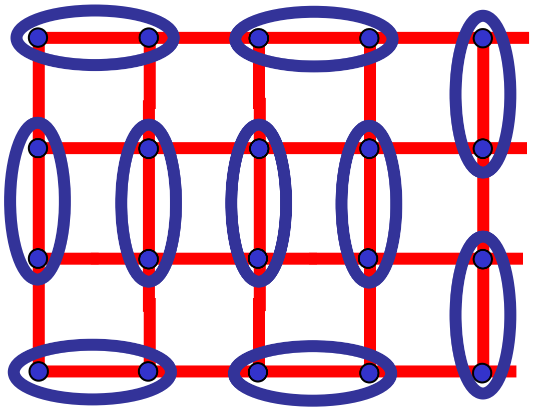

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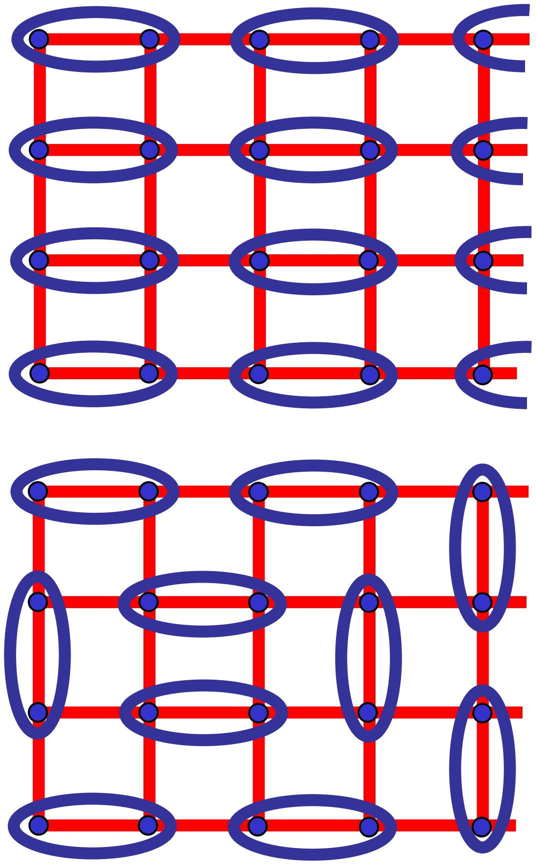
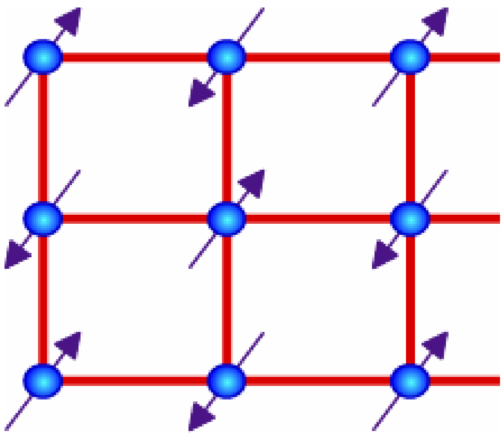


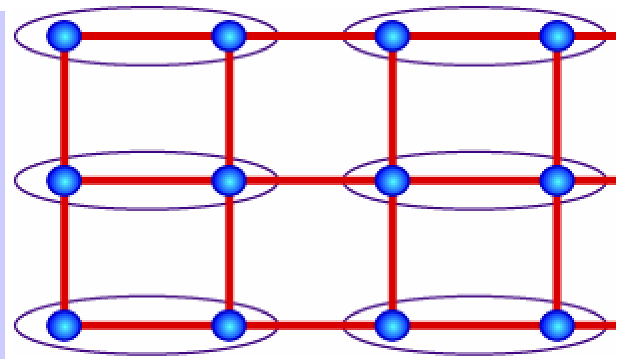
$$Z \approx \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\left|z_{a\alpha}\right|^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i\sum_a \eta_a A_{a\tau}\right)$$



Neel order

$\langle \vec{\varphi} \rangle \neq 0$

?



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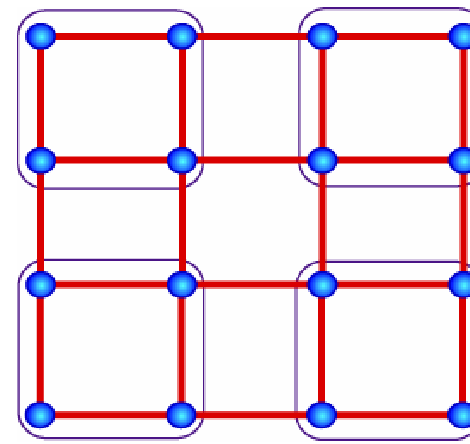


VBS order

$\langle \Psi_{\text{vbs}} \rangle \neq 0$

Not present in LGW theory of $\vec{\varphi}$ order

0 $\longrightarrow$ g

Theory of a second-order quantum phase transition between Neel and VBS phases

At the quantum critical point:

- $A_\mu \rightarrow A_\mu + 2\pi$ periodicity can be ignored

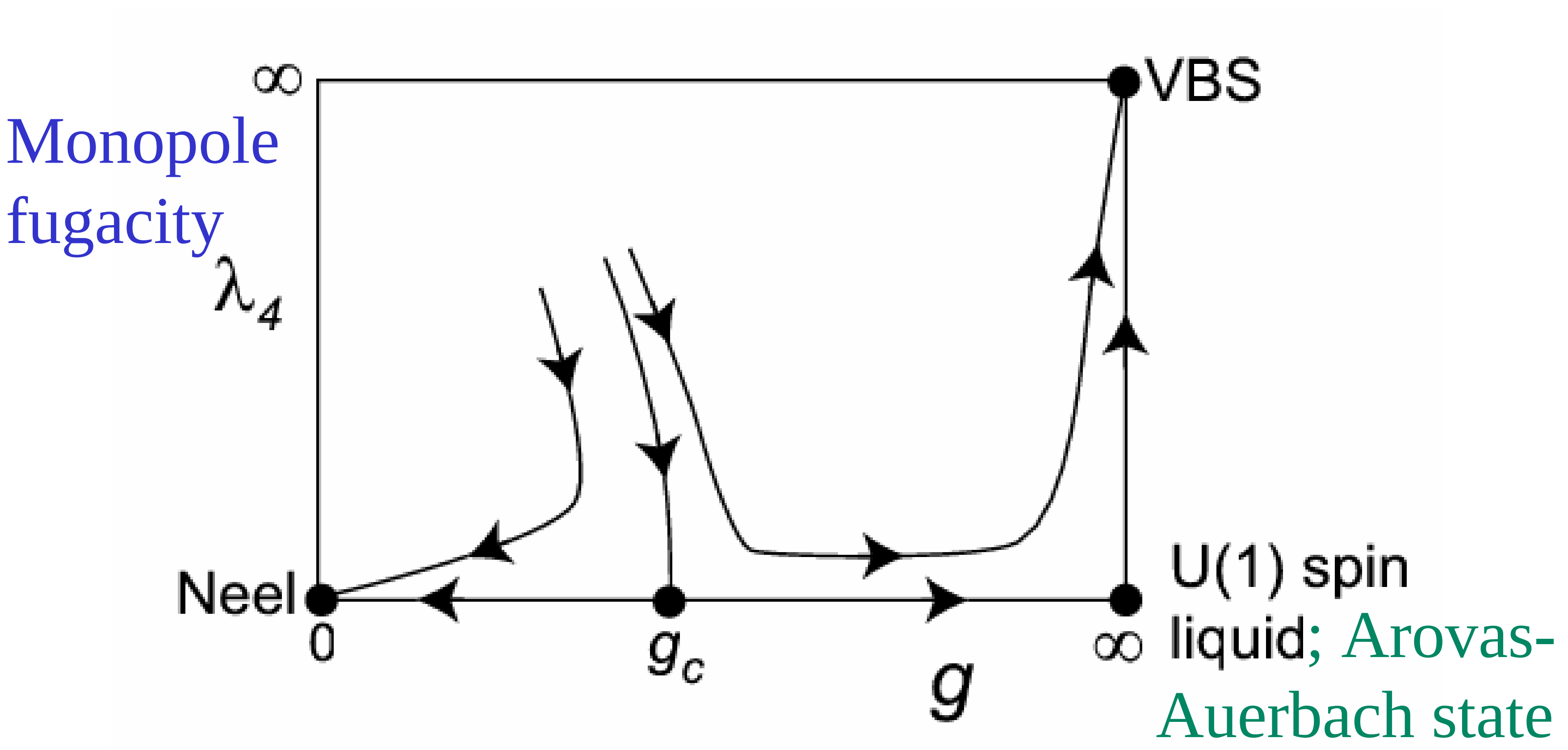
(Monopoles interfere destructively and are dangerously irrelevant).

- $S = 1/2$ *spinons* z_α, with $\vec{\varphi} \sim z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, are globally propagating degrees of freedom.

Second-order critical point described by emergent fractionalized degrees of freedom (A_μ and z_α); Order parameters (φ and $\vec{\Psi}_{\text{vbs}}$) are "composites" and of secondary importance

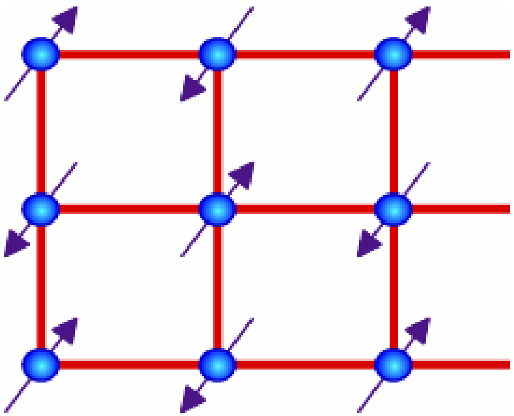
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990); G. Murthy and S. Sachdev, *Nuclear Physics* B **344**, 557 (1990); C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001); S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002); O. Motrunich and A. Vishwanath, *Phys. Rev.* B **70**, 075104 (2004)

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).



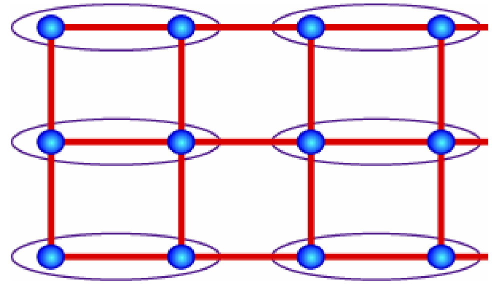
Monopole fugacity
λ_4
∞
VBS
Neel
0
g_c
g
∞
U(1) spin liquid; Arovas-Auerbach state

Phase diagram of S=1/2 square lattice antiferromagnet



Neel order

$$\langle \vec{\varphi} \rangle \sim \langle z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta \rangle \neq 0$$



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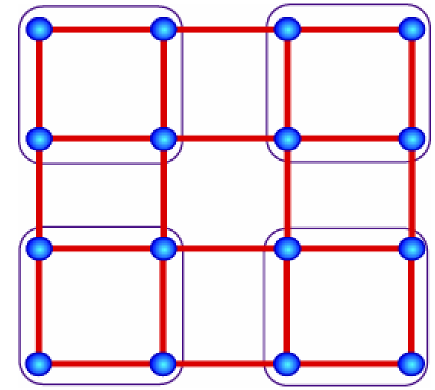

VBS order $\langle \Psi_{\text{vbs}} \rangle \neq 0$

(associated with condensation of monopoles in A_μ),

$S = 1/2$ spinons z_α confined,

$S = 1$ triplon excitations

g

Second-order critical point described by

$$\mathcal{S}_{\text{critical}} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + r\left|z_\alpha\right|^2 + \frac{u}{2}\left(\left|z_\alpha\right|^2\right)^2 + \frac{1}{4e^2}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 \right]$$

at its critical point $r = r_c$, where A_μ is *non-compact*

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Aharanov-Bohm or Berry phases lead to surprising kinematic duality relations between seemingly distinct orders. These phase factors allow for continuous quantum phase transitions in situations where such transitions are forbidden by Landau-Ginzburg-Wilson theory.