PHYSICAL REVIEW B VOLUME 38, NUMBER 1 1 JULY 1988

# Functional integral theories of low-dimensional quantum Heisenberg models

Daniel P. Arovas
*The James Franck Institute, The University of Chicago, 5640 South Ellis Avenue, Chicago, Illinois 60637*

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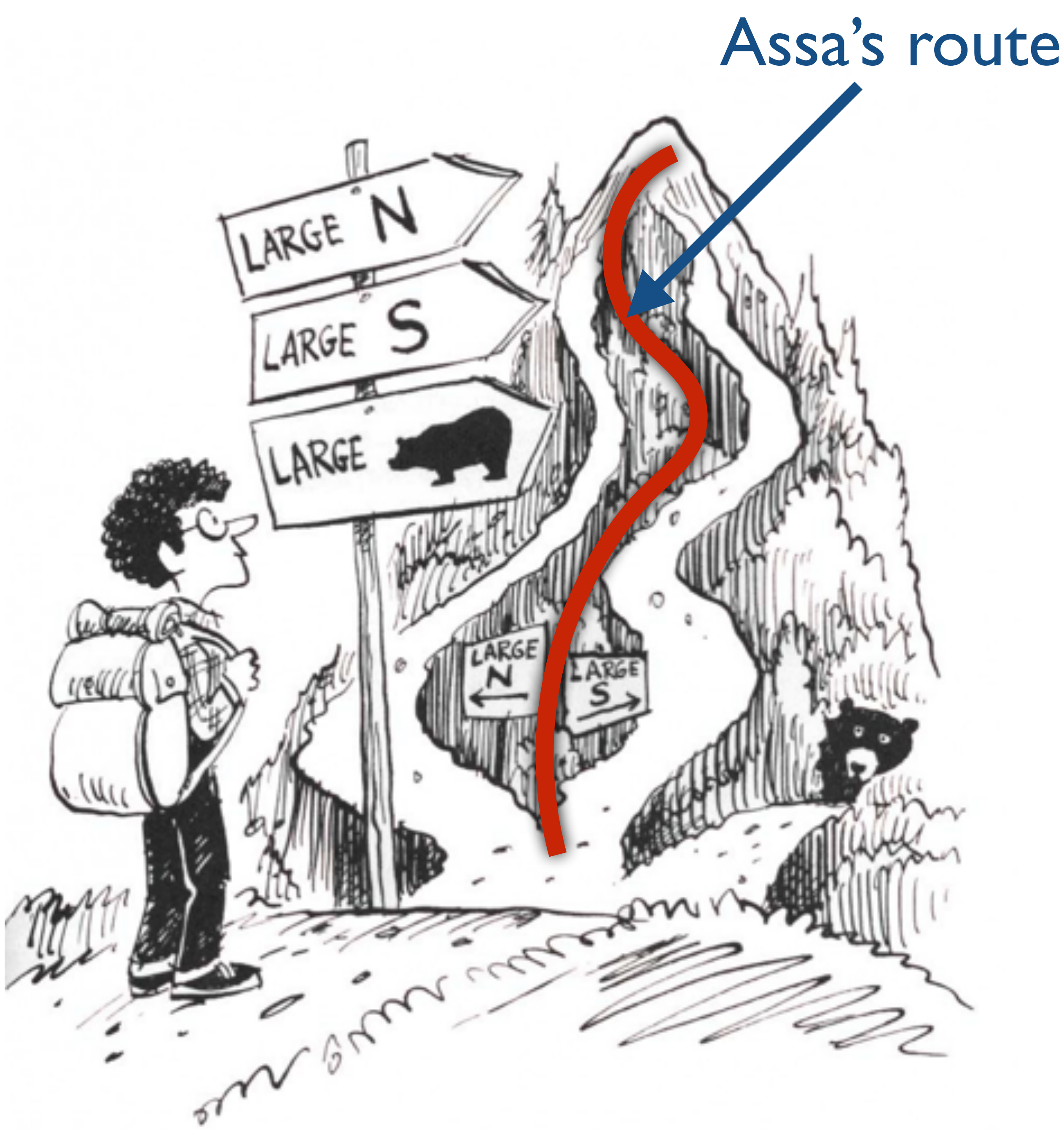

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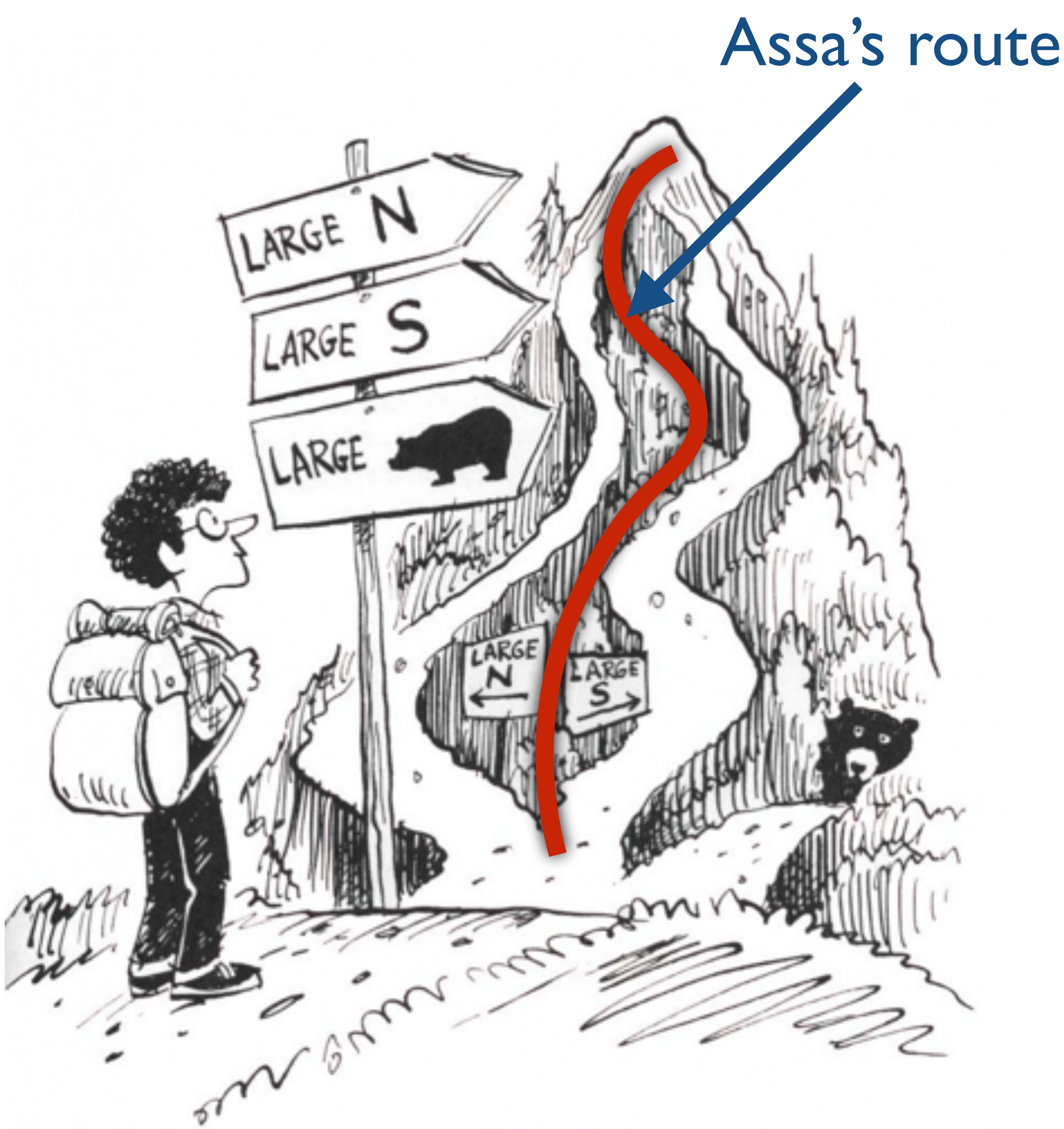


Fluctuations on a bipartite lattice:

- U(1) gauge theories and monopoles
- Monopole Berry phases
- Valence bond solids
- Deconfined criticality
- Monopole scaling dimensions
- ..........

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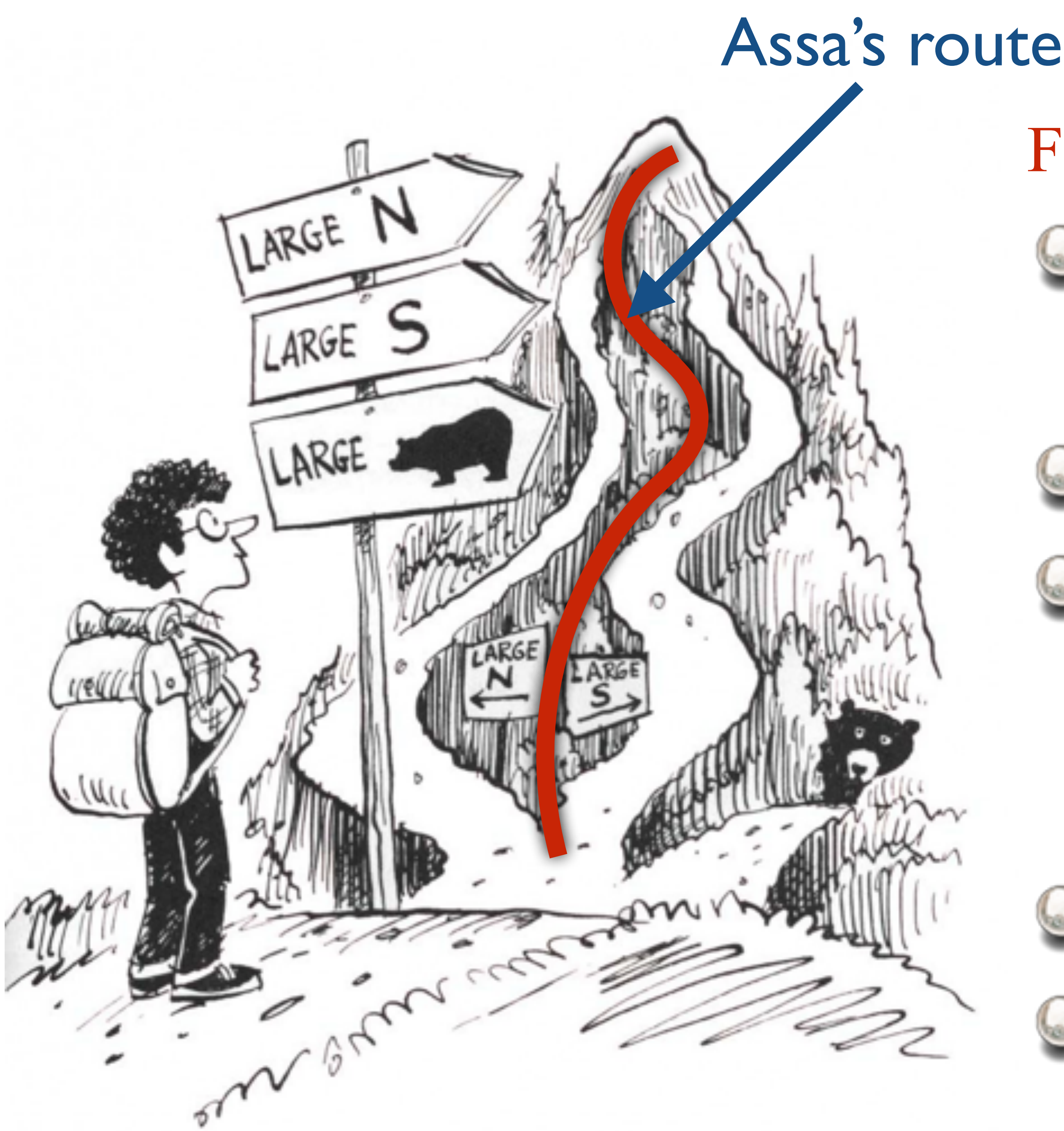



Fluctuations on a non-bipartite lattice:

- $Z_2$ spin liquid: maps to "odd" $Z_2$ gauge theory
- Visons
- First quantum state with anyons ("topological order") and time-reversal symmetry
- Toric code and protected quantum memory
- …………..

# Quantum matter without quasiparticles: random fermion models, black holes, and graphene

Interacting Electrons and Quantum Magnetism
Technion, Haifa
June 21, 2016

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS HARVARD

*Quantum matter without quasiparticles:*

*1. Ground states disconnected from independent electron states: many-particle entanglement*

*2. No quasiparticles*

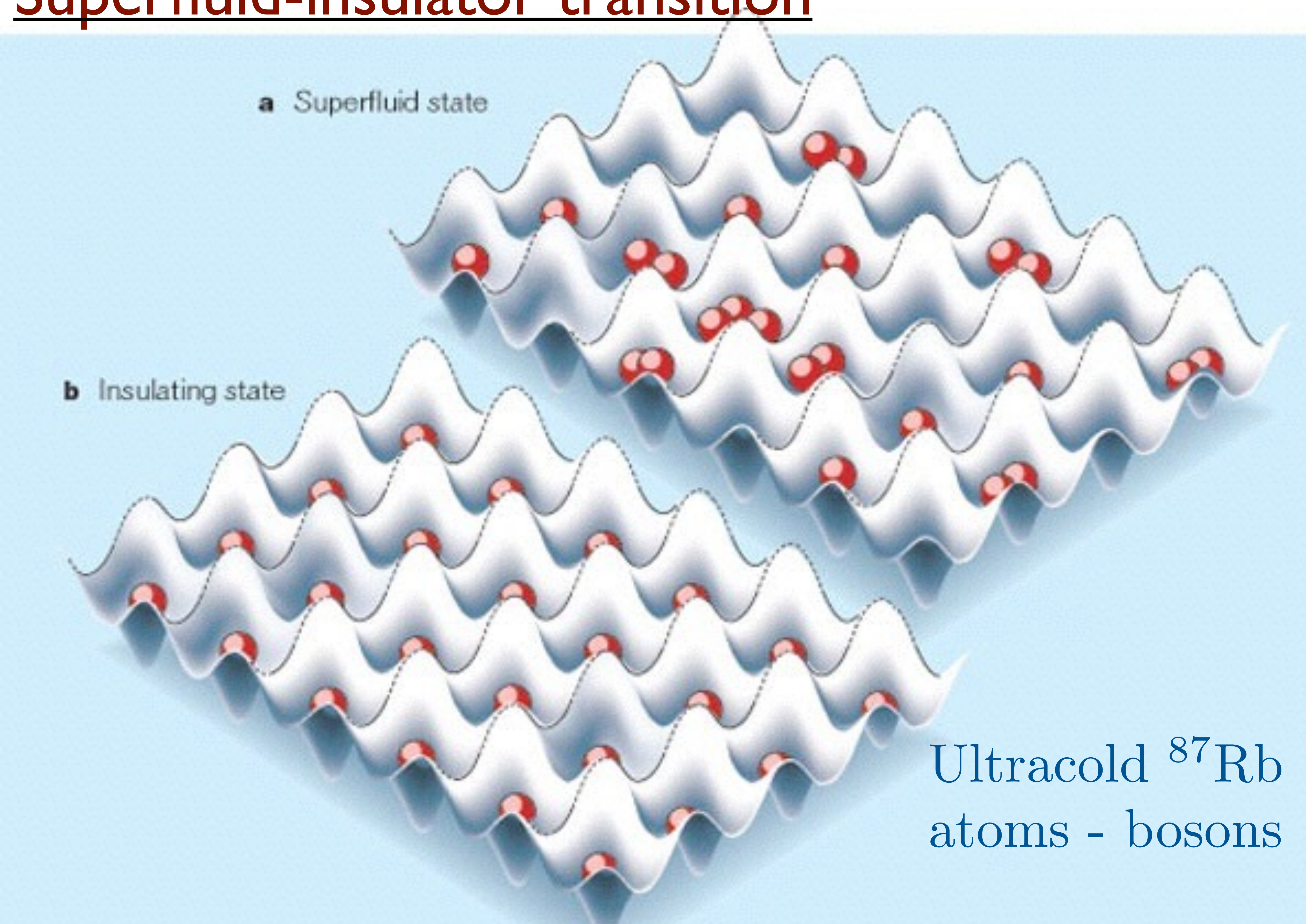
- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
- Graphene

*Note: Most states with long-range entanglement, like the fractional quantum Hall states, do have quasiparticles*

# *Quantum matter without quasiparticles:*

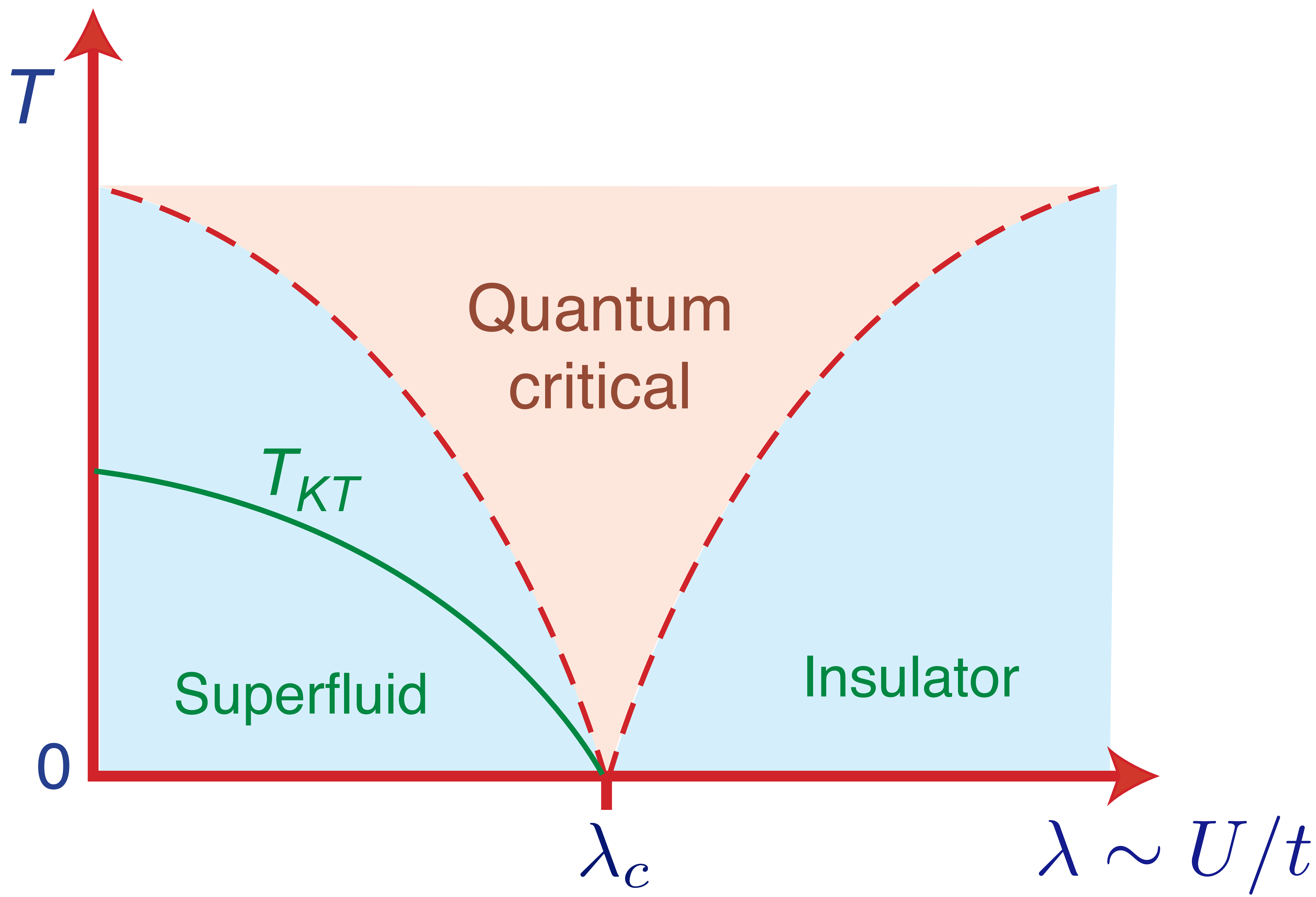
- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
- Graphene

# Superfluid-insulator transition

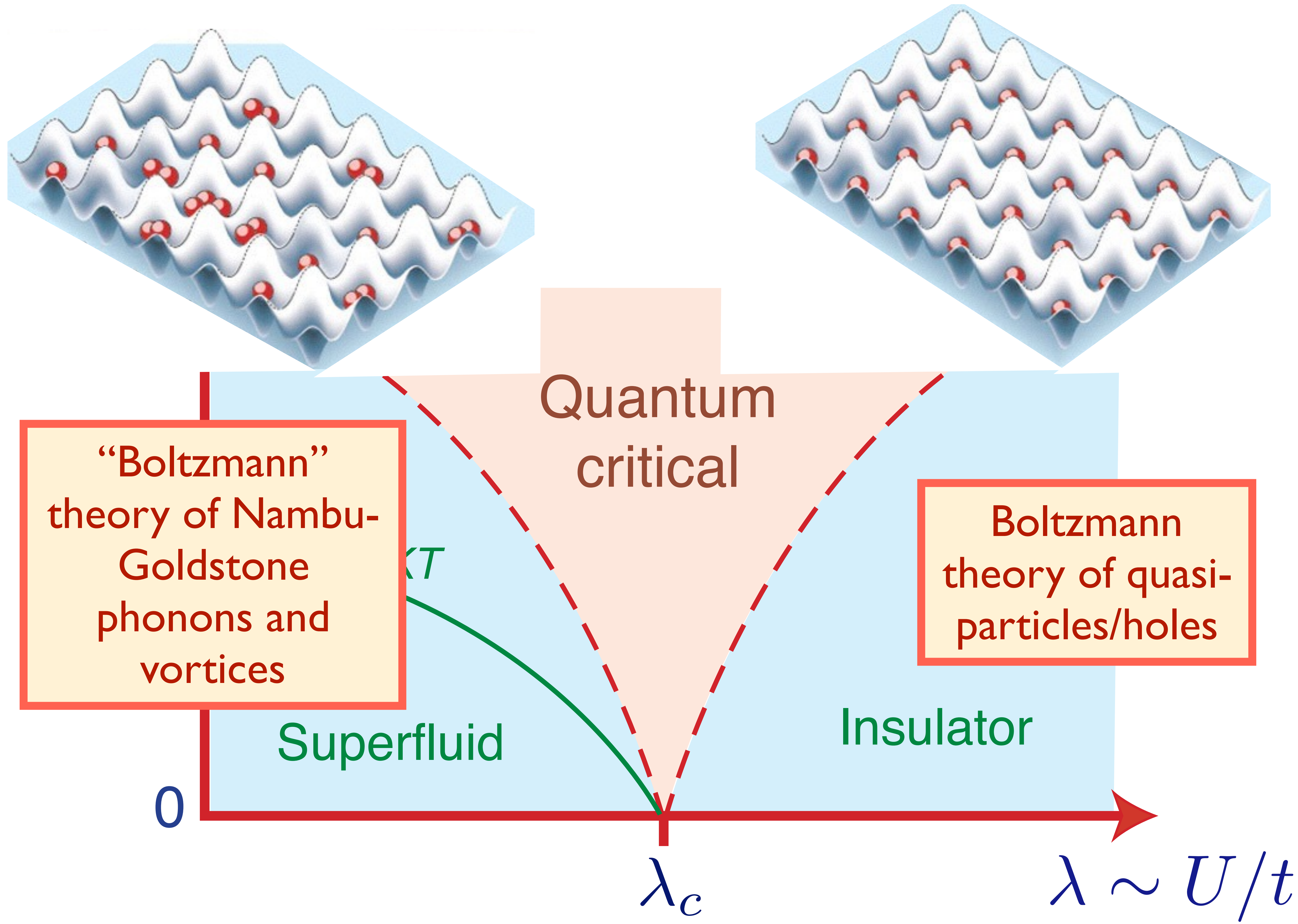



Ultracold $^{87}$Rb atoms - bosons

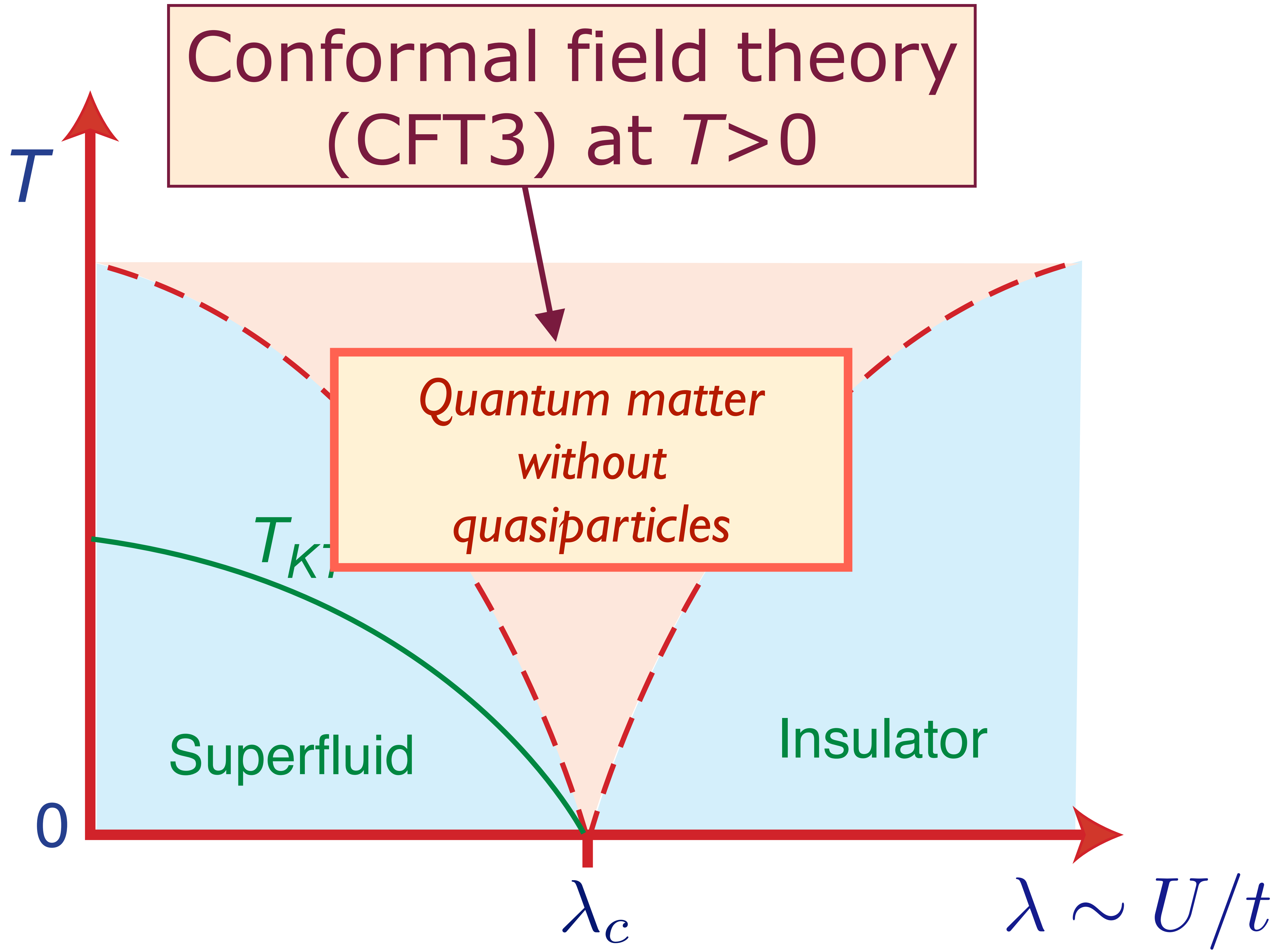
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).
Xibo Zhang, Chen-Lung Hung, Shih-Kuang Tung, and Cheng Chin, *Science* **335**, 1070 (2012)



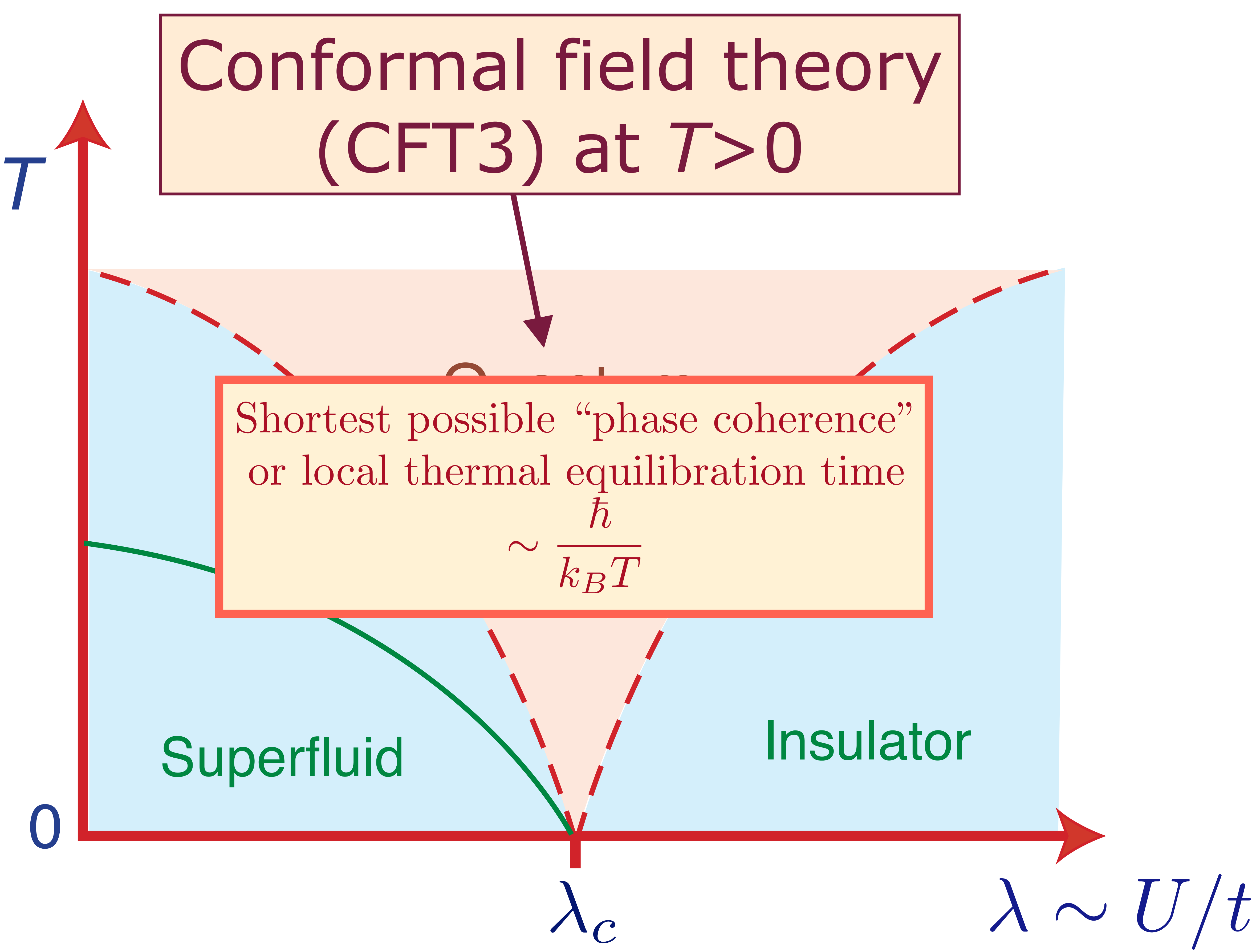
T
Quantum
critical
$T_{KT}$
Superfluid
Insulator
0
$\lambda_c$
$\lambda \sim U/t$



Quantum
critical
"Boltzmann"
theory of Nambu-
Goldstone
phonons and
vortices
KT
Boltzmann
theory of quasi-
particles/holes
Superfluid
Insulator
0
$\lambda_c$
$\lambda \sim U/t$



Conformal field theory (CFT3) at $T>0$
Quantum matter without quasiparticles
$T$
$T_{KT}$
Superfluid
Insulator
0
$\lambda_c$
$\lambda \sim U/t$




A.V. Chubukov, S. Sachdev, and J. Ye, PRB **49**, 11919 (1994); K. Damle and S. Sachdev, PRB **56**, 8714 (1997);
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

## Local thermal equilibration or phase coherence time, $\tau_\varphi$:

- As $T \to 0$, there is an *lower bound* on $\tau_\varphi$ in all many-body quantum systems of order $\hbar/(k_B T)$,

$$\tau_\varphi > C \frac{\hbar}{k_B T},$$

  and the lower bound is realized by systems *without* quasiparticles.

- In systems *with* quasiparticles, $\tau_\varphi$ is parametrically larger at low $T$;
  *e.g.* in Fermi liquids $\tau_\varphi \sim 1/T^2$,
  and in gapped insulators $\tau_\varphi \sim e^{\Delta/(k_B T)}$ where $\Delta$ is the energy gap.

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

## A bound on quantum chaos:

- The time over which a many-body quantum system becomes chaotic is given by $\tau_L = 1/\lambda_L$, where $\lambda_L$ is the "Lyapunov exponent" determining memory of initial conditions (the "butterfly effect"):

$$D(t) = \langle W(t)V(0)W(t)V(0)\rangle \sim c_0 - \epsilon\, c_1 e^{\lambda_L t},$$

where we make a (system-dependent) choice to arrange $\epsilon \ll 1$. As $T \to 0$, this Lyapunov time is argued to obey the lower bound

$$\boxed{\tau_L \geq \frac{1}{2\pi}\frac{\hbar}{k_B T}}$$

- Theories holographically dual to Einstein gravity have the shortest possible $\tau_L = \hbar/(2\pi k_B T)$

A.I. Larkin and Y. N. Ovchinnikov, JETP **28**, 6 (1969)
S. H. Shenker and D. Stanford, arXiv:1306.0622
J. Maldacena, S. H. Shenker and D. Stanford, arXiv:1503.01409

## A bound on quantum chaos:

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- Theories holographically dual to Einstein gravity have the shortest possible $\tau_L = \hbar/(2\pi k_B T)$

Quantum matter without quasiparticles
$\approx$ fastest possible many-body quantum chaos

## *Quantum matter without quasiparticles:*

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
- Graphene

# Infinite-range model with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

$t_{ij}$ are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

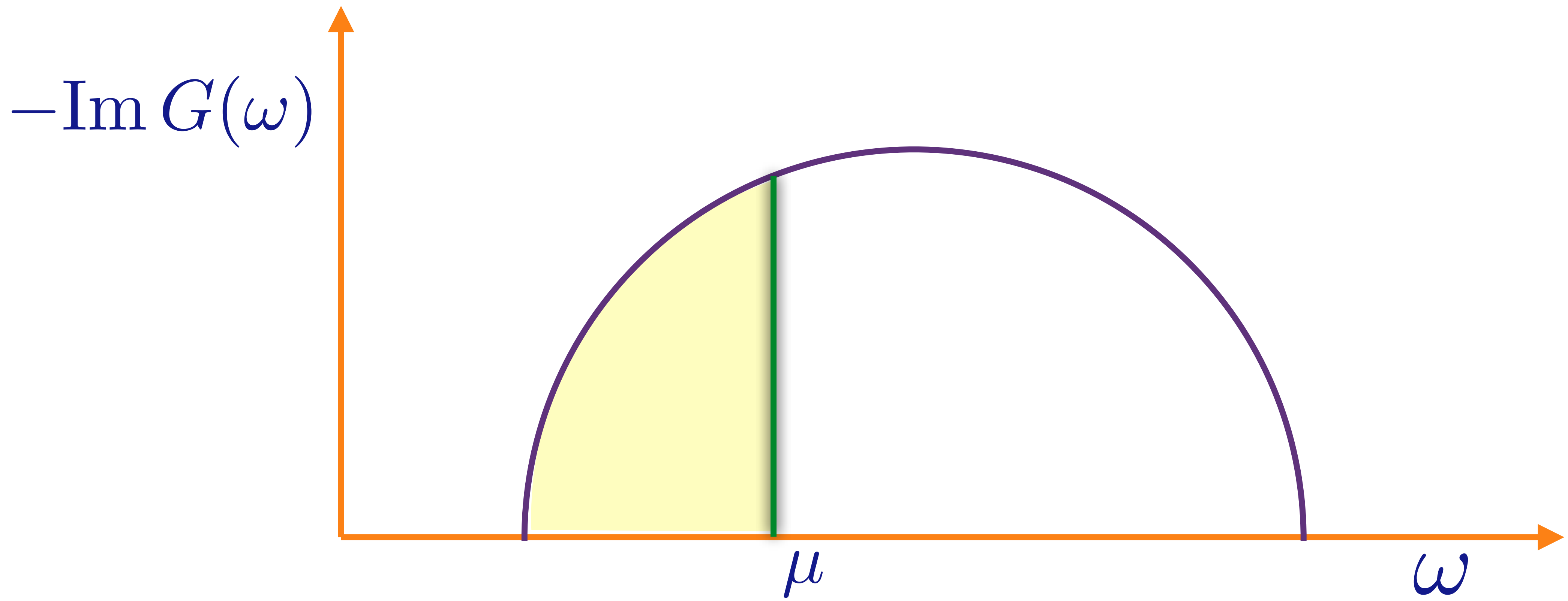
Fermions occupying the eigenstates of a *N* x *N* random matrix

# Infinite-range model with quasiparticles

Feynman graph expansion in $t_{ij..}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = t^2 G(\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$G(\omega)$ can be determined by solving a quadratic equation.




Fermi liquid state: Two-body interactions lead to a scattering time of quasiparticle excitations from in (random) single-particle eigen-states which diverges as $\sim T^{-2}$ at the Fermi level.

# Infinite-range (SY) model without quasiparticles

$$H = \frac{1}{(NM)^{1/2}} \sum_{i,j=1}^{N} \sum_{\alpha,\beta=1}^{M} J_{ij} c_{i\alpha}^{\dagger} c_{i\beta} c_{j\beta}^{\dagger} c_{j\alpha}$$

$$c_{i\alpha} c_{j\beta} + c_{j\beta} c_{i\alpha} = 0 \quad , \quad c_{i\alpha} c_{j\beta}^{\dagger} + c_{j\beta}^{\dagger} c_{i\alpha} = \delta_{ij} \delta_{\alpha\beta}$$

$$\frac{1}{M} \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} = \mathcal{Q}$$

$J_{ij}$ are independent random variables with $\overline{J_{ij}} = 0$ and $\overline{J_{ij}^2} = J^2$

$N \to \infty$ at $M = 2$ yields spin-glass ground state.

$N \to \infty$ and then $M \to \infty$ yields critical strange metal

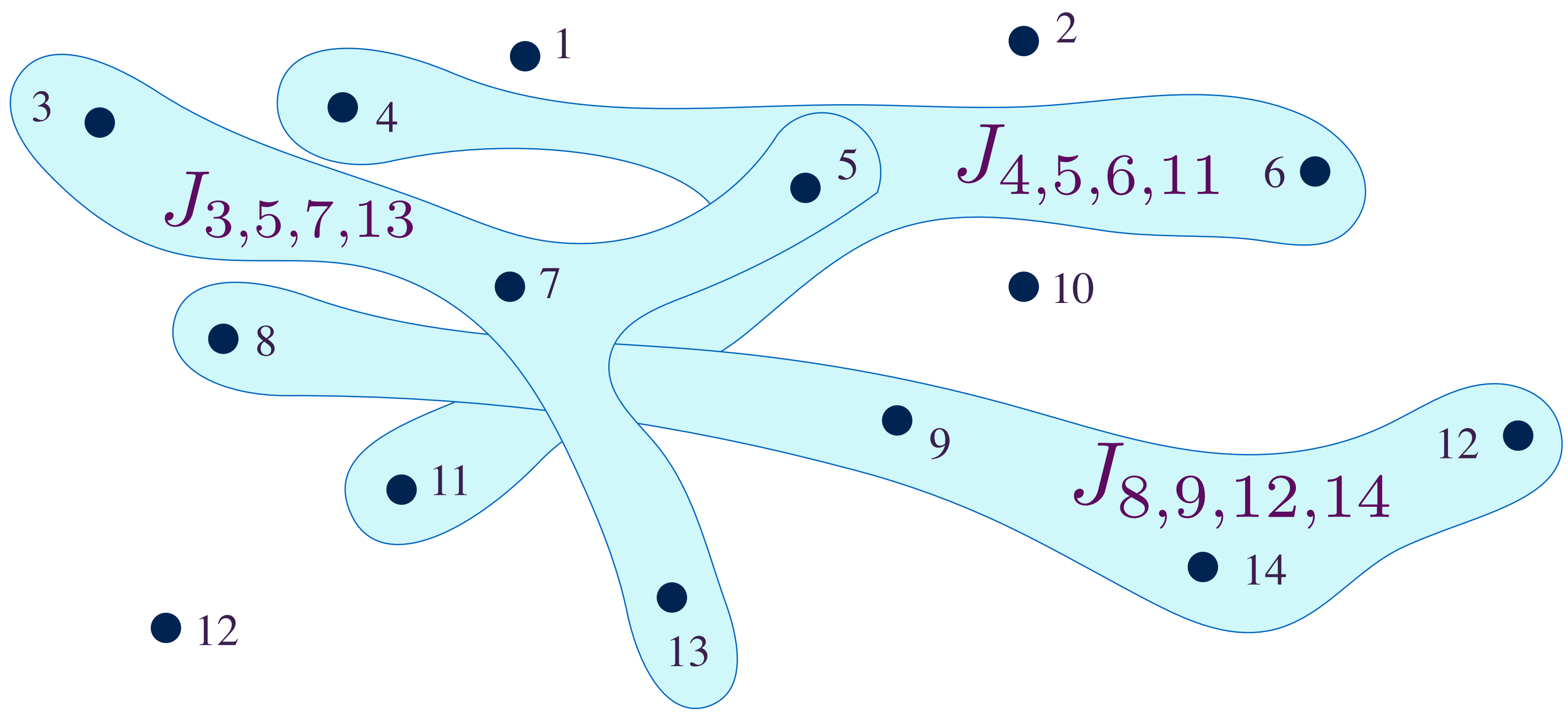
S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

# SYK model without quasiparticles

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$




$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$

$N \to \infty$ yields <u>same</u> critical strange metal; simpler to study numerically

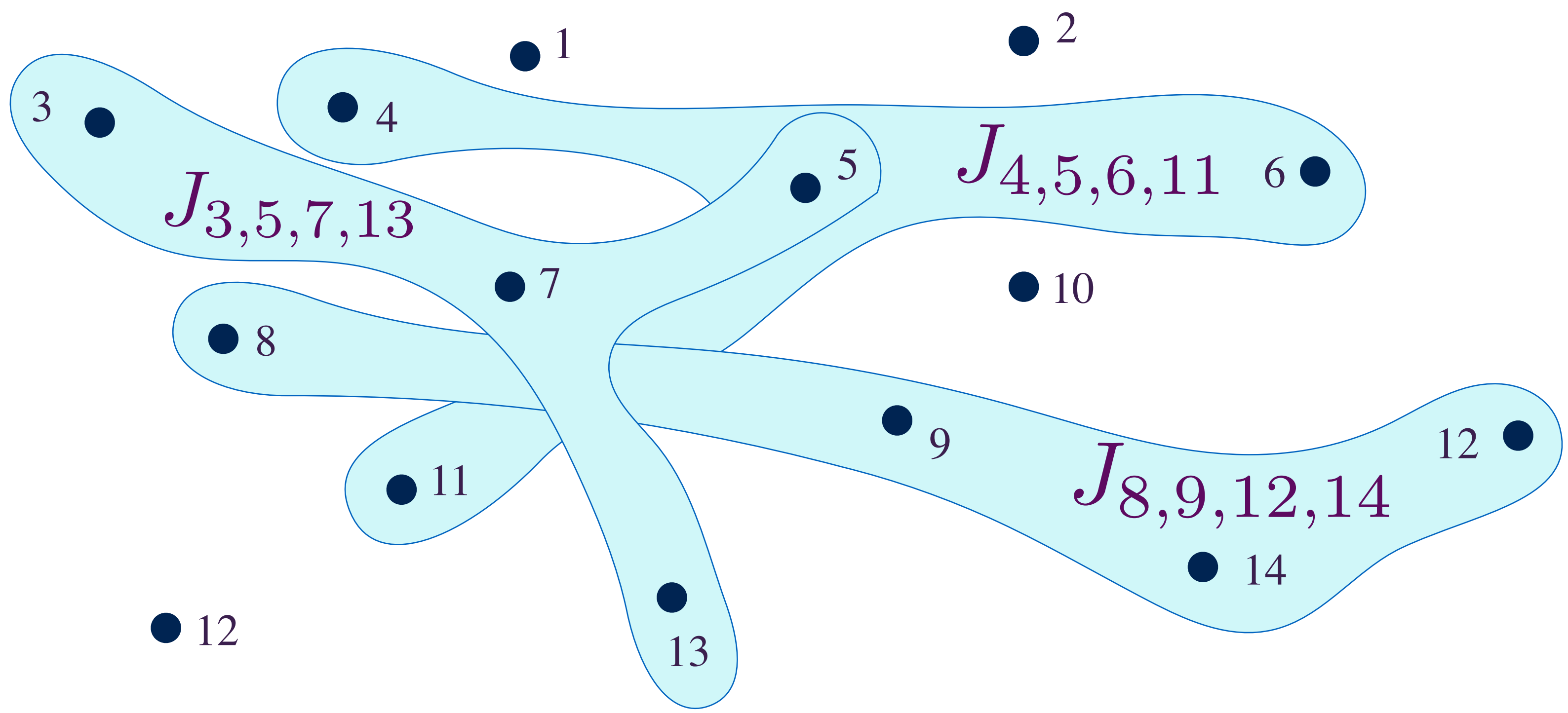
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# SYK model without quasiparticles

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$




A fermion can move only by entangling with another fermion: the Hamiltonian has "nothing but entanglement".

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# SYK model without quasiparticles

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large $N$ limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

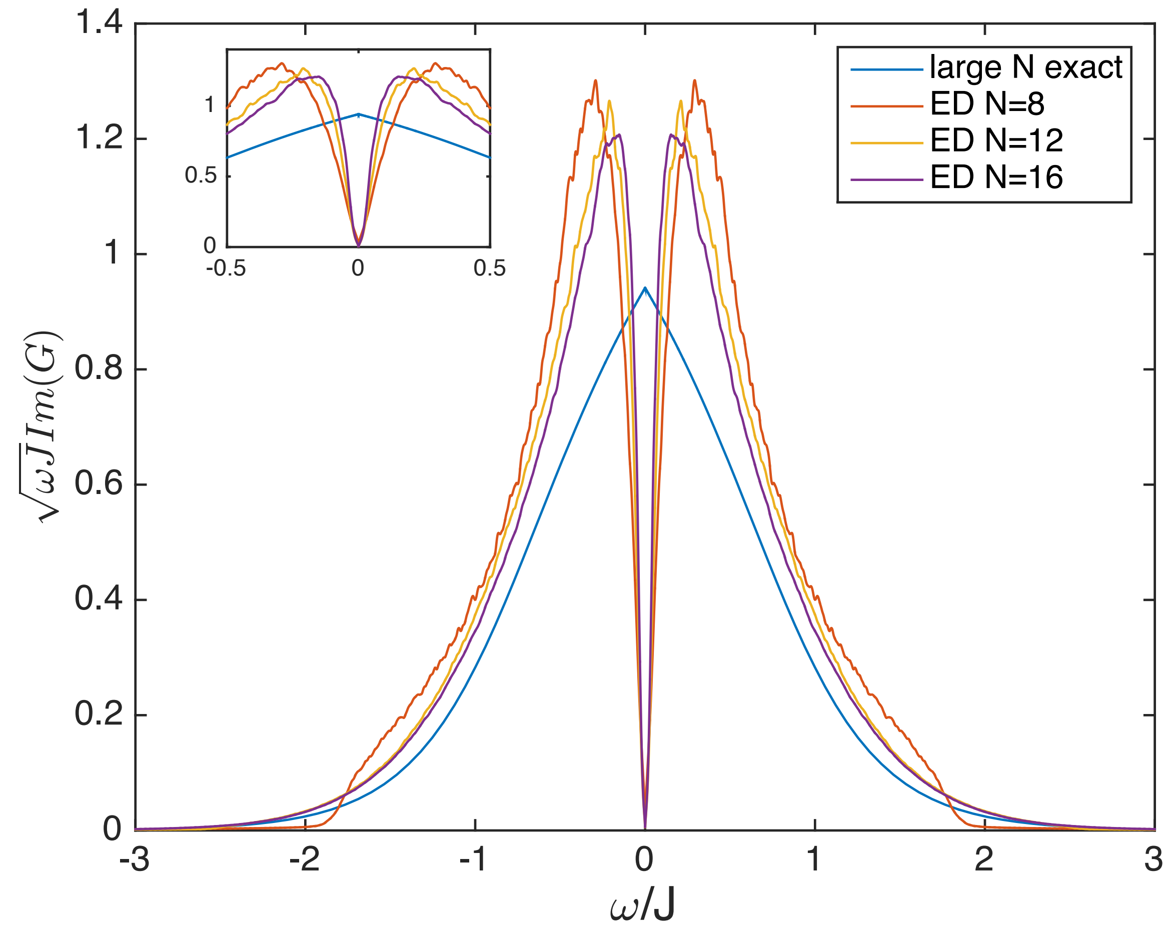
Low frequency analysis shows that the solutions must be gapless and obey

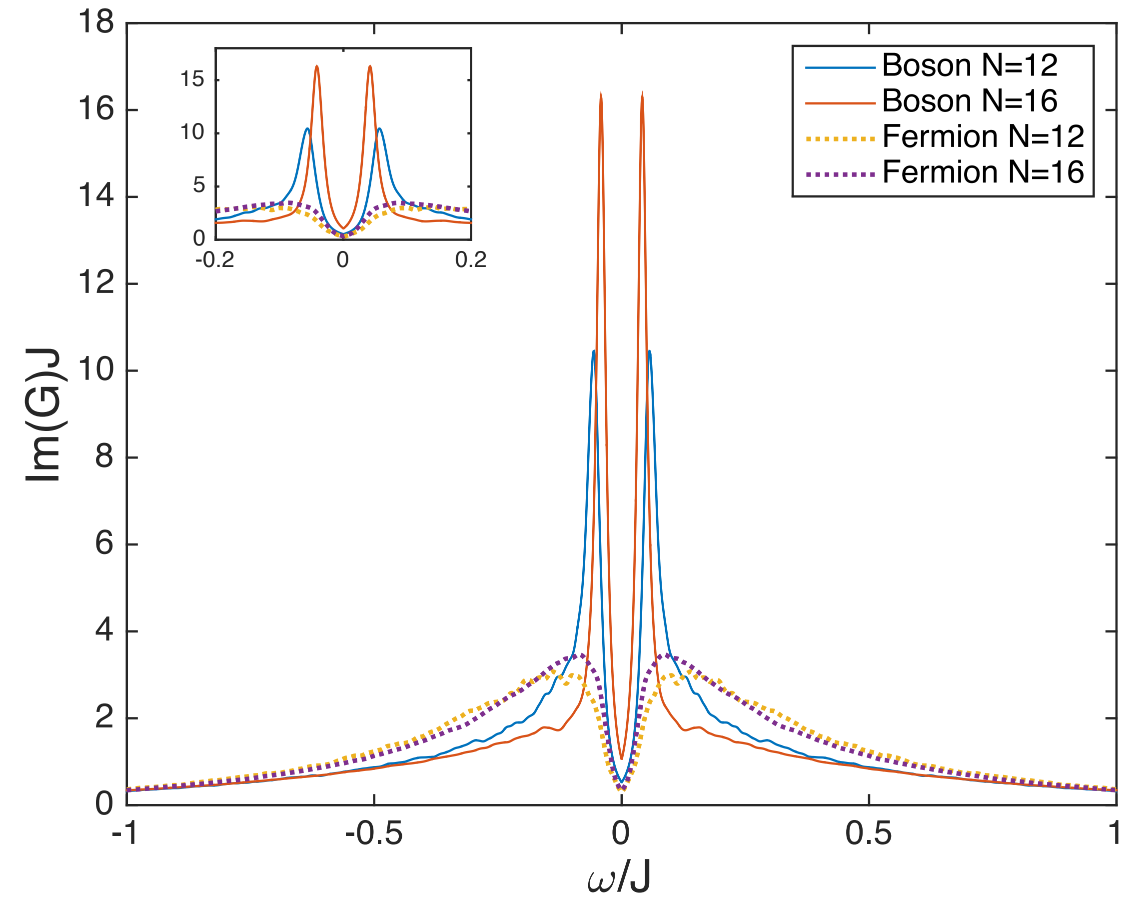
$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \ldots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex $A$. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

# SYK model without quasiparticles




Large $N$ solution of equations for $G$ and $\Sigma$ agree well with exact diagonalization of the finite N Hamiltonian.

However, exact diagonalization of the same model with hard-core bosons indicates the presence of spin-glass order in the ground state.

W. Fu and S. Sachdev, arXiv: 1603.05246

# SYK model without quasiparticles

Local fermion density of states

$$\rho(\omega) = -\text{Im}\, G(\omega) \sim \begin{cases} \omega^{-1/2} & , \omega > 0 \\ e^{-2\pi\mathcal{E}} \, |\omega|^{-1/2}, & \omega < 0. \end{cases}$$

$\mathcal{E}$ encodes the particle-hole asymmetry

While $\mathcal{E}$ determines the *low* energy spectrum, it is determined by the *total* fermion density $\mathcal{Q}$:

$$\mathcal{Q} = \frac{1}{4}(3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi}\tan^{-1}\left(e^{2\pi\mathcal{E}}\right).$$

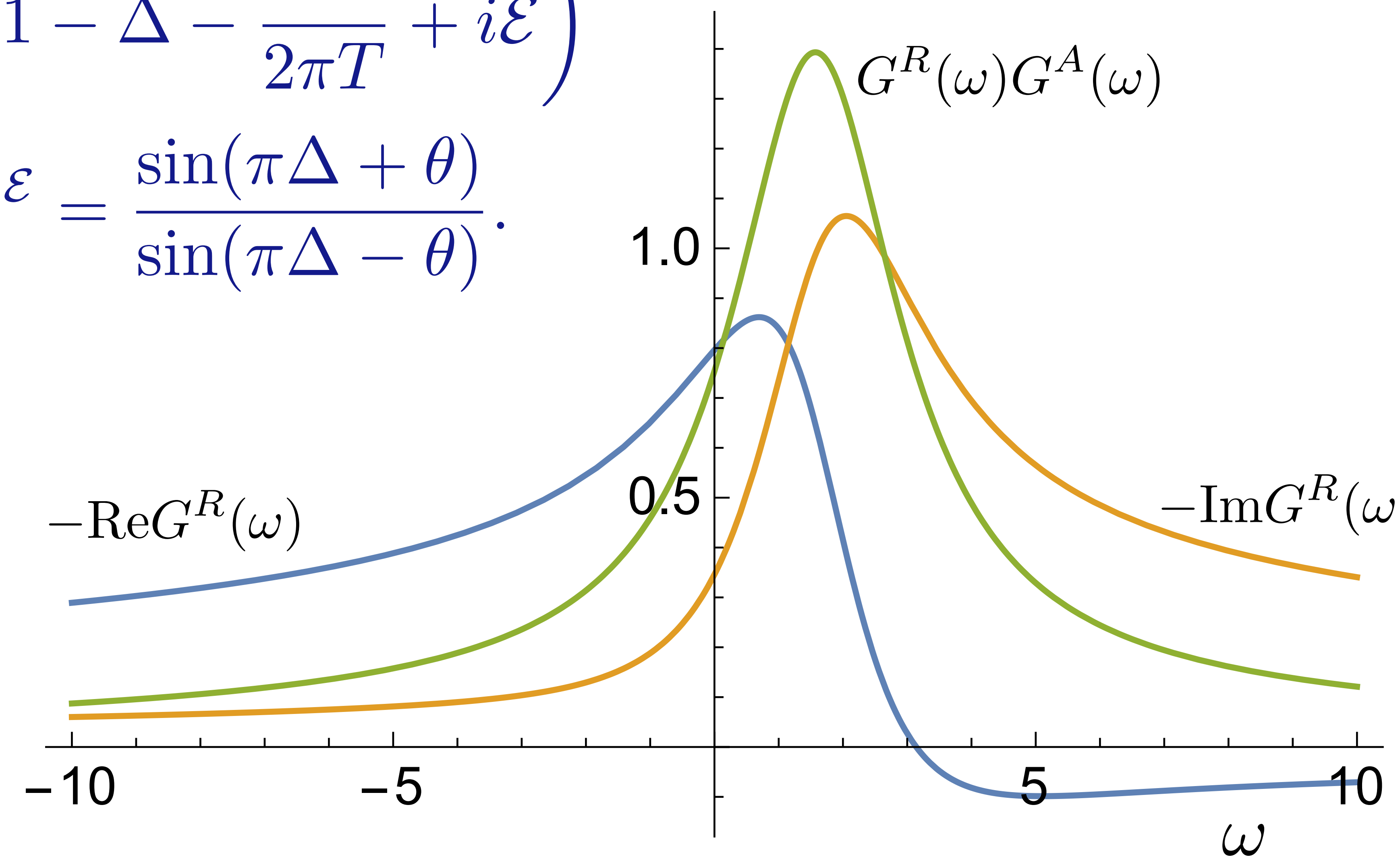

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

# SYK model without quasiparticles

At non-zero temperature, $T$, the Green's function also fully determined by $\mathcal{E}$.

$$G^R(\omega) = \frac{-iCe^{-i\theta}}{(2\pi T)^{1-2\Delta}} \frac{\Gamma\left(\Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}{\Gamma\left(1 - \Delta - \dfrac{i\omega}{2\pi T} + i\mathcal{E}\right)}$$

where $\Delta = 1/4$ and $\quad e^{2\pi\mathcal{E}} = \dfrac{\sin(\pi\Delta + \theta)}{\sin(\pi\Delta - \theta)}$.




S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

# SYK model without quasiparticles

The entropy per site, $\mathcal{S}$, has a non-zero limit as $T \to 0$. This is *not* due to an extensive degeneracy, but due to an energy level spacing $\sim e^{-aN}$ in the entire many-body spectrum all the way down to the ground state. The $T \to 0$ limit of $\mathcal{S}$ obeys

$$\left(\frac{\partial \mathcal{S}}{\partial \mathcal{Q}}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_{\mathcal{Q}} = 2\pi\mathcal{E}$$

Note that $\mathcal{S}$ and $\mathcal{E}$ involve low-lying states, while $\mathcal{Q}$ depends upon *all* states, and details of the UV structure

$$\mathcal{S}(\mathcal{Q}, T \to 0) = 2\pi \int_{-\infty}^{f^{-1}(\mathcal{Q})} dx\, x f'(x) \quad , \quad f(x) = \frac{(3 - \tanh(2\pi x))}{4} - \frac{\tan^{-1}(e^{2\pi x})}{\pi}$$


O. Parcollet, A. Georges, G. Kotliar, and A. Sengupta Phys. Rev. B **58**, 3794 (1998)
A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)

# Infinite-range (SYK) model without quasiparticles

After integrating the fermions, the partition function can be written as a path integral with an action $S$ analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

$$S = \ln \det \left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right] + \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left[G(\tau_2, \tau_1) + (J^2/2) G^2(\tau_2, \tau_1) G^2(\tau_1, \tau_2)\right]$$

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, PRX **5**, 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[f'(\sigma_1) f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

# Infinite-range (SYK) model without quasiparticles

Let us write the large $N$ saddle point solutions of $S$ as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

## Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for $\Sigma$) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.


J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

# Infinite-range (SYK) model without quasiparticles

However the effective action must vanish for SL(2,R) transformations because $G_s, \Sigma_s$ are invariant under it. In this manner we obtain the effective action as a Schwarzian

$$N S_{\text{eff}} = -\frac{N\gamma}{4\pi^2} \int d\tau \, \{f, \tau\} \quad , \quad \{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2} \left( \frac{f''}{f'} \right)^2 ,$$

where the co-efficient $\gamma$ determines the specific heat, $\mathcal{C}$

$$\mathcal{C} = T \frac{\partial \mathcal{S}}{\partial T} = N \gamma \, T$$

The Schwarzian describes fluctuations of the energy operator with scaling dimension $h = 2$.

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

# Infinite-range (SYK) model without quasiparticles

The Schwarzian effective action implies that the SYK model *saturates* the lower bound on the Lyapunov time

$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

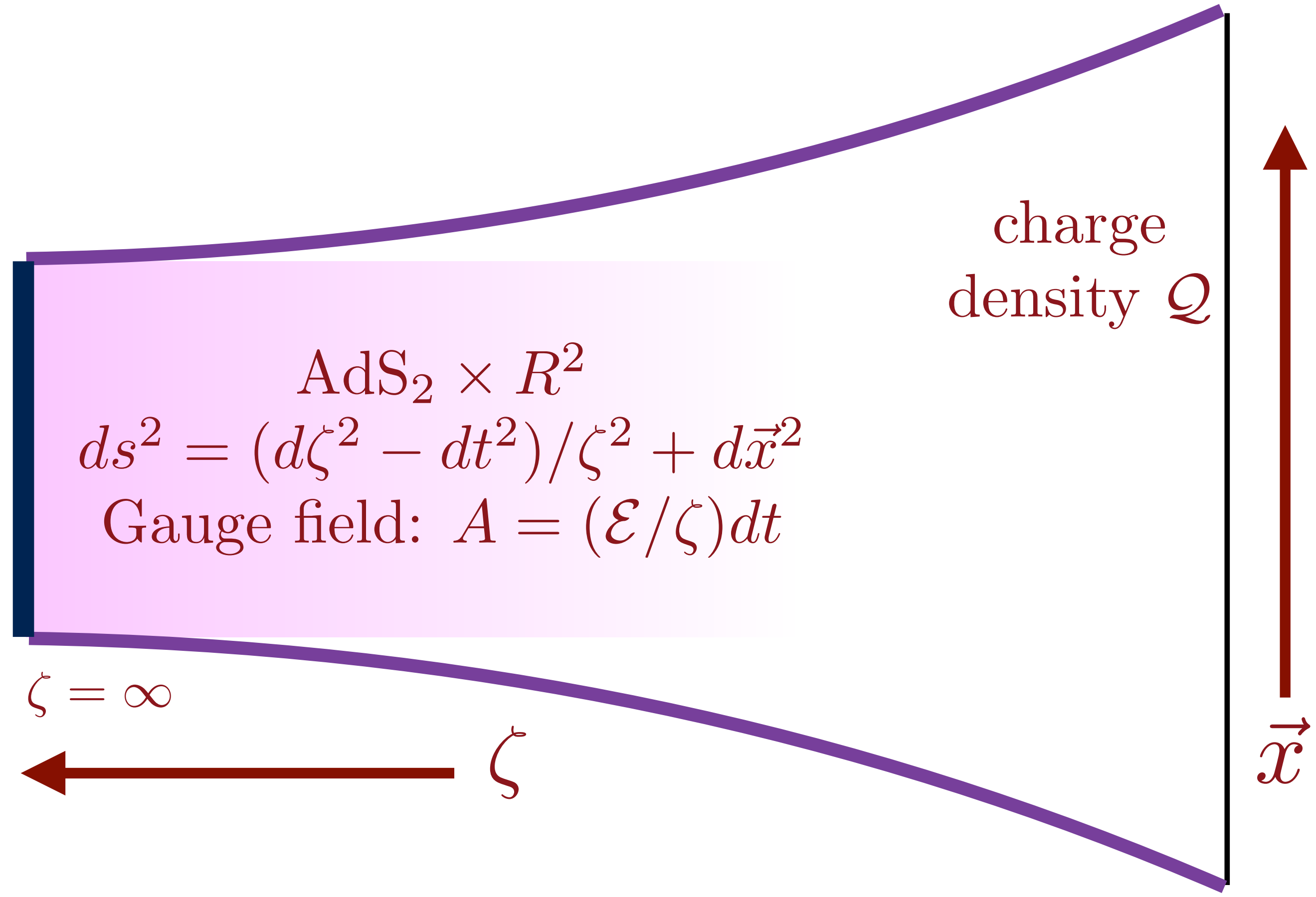
Apart from the energy operator associated with the Schwarzian, there are an infinite number of other scalar operators with irrational scaling dimensions given by the roots of

$$\tan\left(\frac{\pi(2h-1)}{4}\right) = \frac{1-2h}{3}$$

$$\Rightarrow \quad h = 3.77354\ldots,\ 5.67946\ldots,\ 7.63197\ldots,\ 9.60396\ldots,\ldots$$

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

# SYK and $AdS_2$




PHYSICAL REVIEW LETTERS **105, 151602 (2010)**

## Holographic Metals and the Fractionalized Fermi Liquid

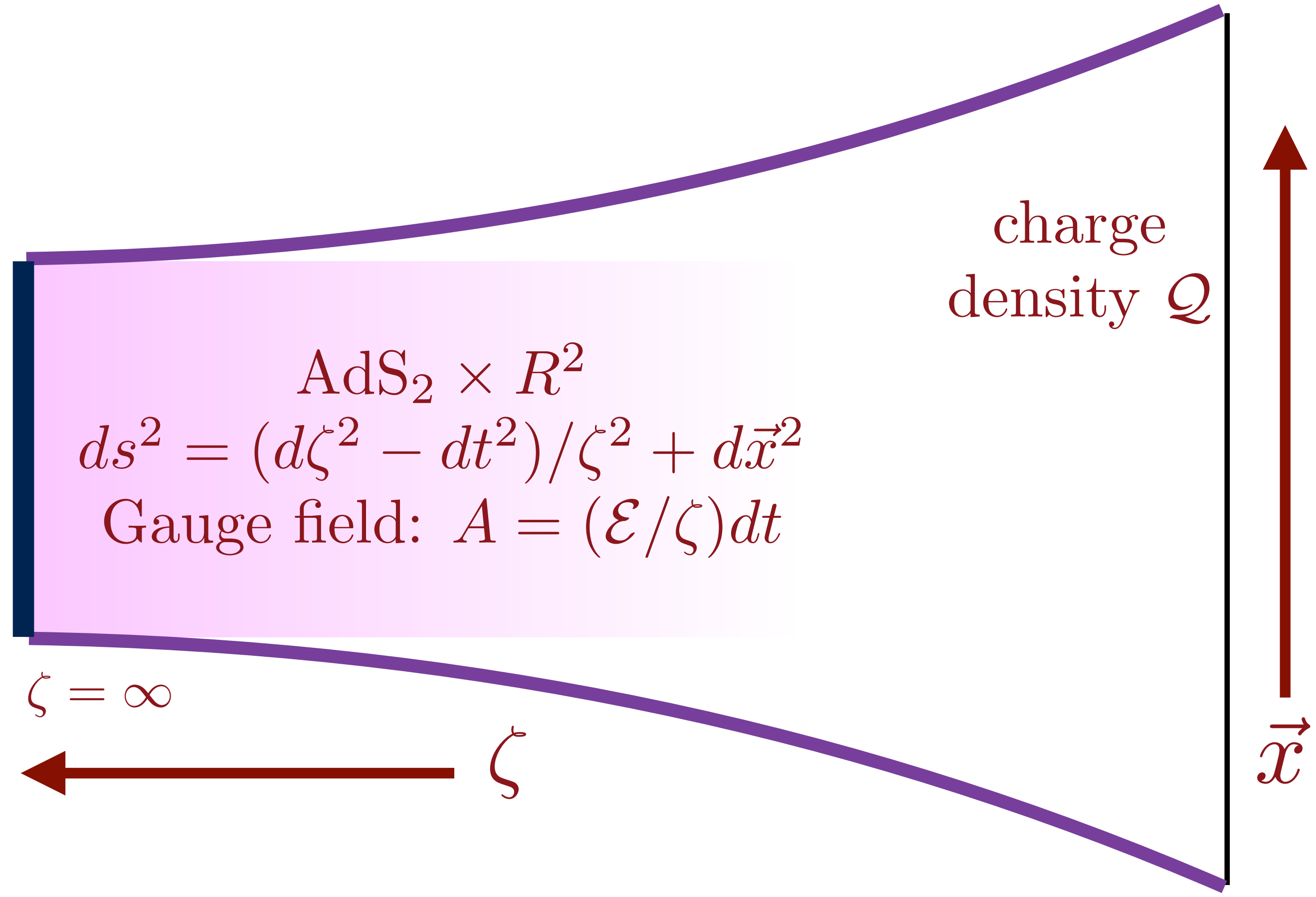
Subir Sachdev

*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

# SYK and AdS$_2$



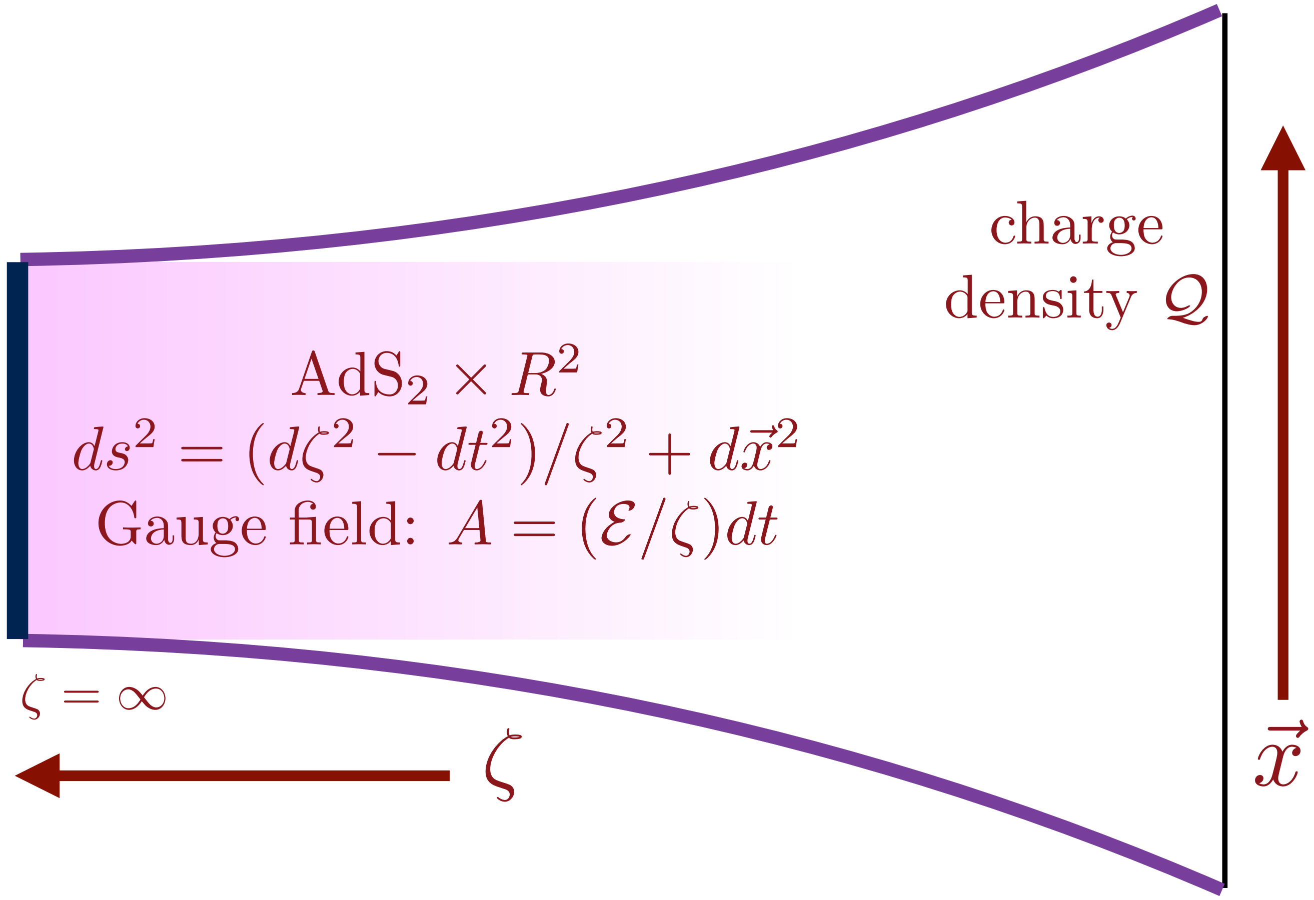

- The non-zero $T \to 0$ entropy density, $\mathcal{S}$, matches the Bekenstein-Hawking-Wald entropy density of extremal AdS$_2$ horizons, and the dependence of the fermion Green's function on $\omega$, $T$, and $\mathcal{E}$, matches that of a Dirac fermion in AdS$_2$ (as computed by T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD **83**, 125002 (2011)).

S. Sachdev, PRL **105**, 151602 (2010)

- More recently, it was noted that the relation $(\partial\mathcal{S}/\partial\mathcal{Q})_T = 2\pi\mathcal{E}$ also matches between SYK and gravity, where $\mathcal{E}$, the electric field on the horizon, also determines the spectral asymmetry of the Dirac fermion.

S. Sachdev, PRX **5**, 041025 (2015)

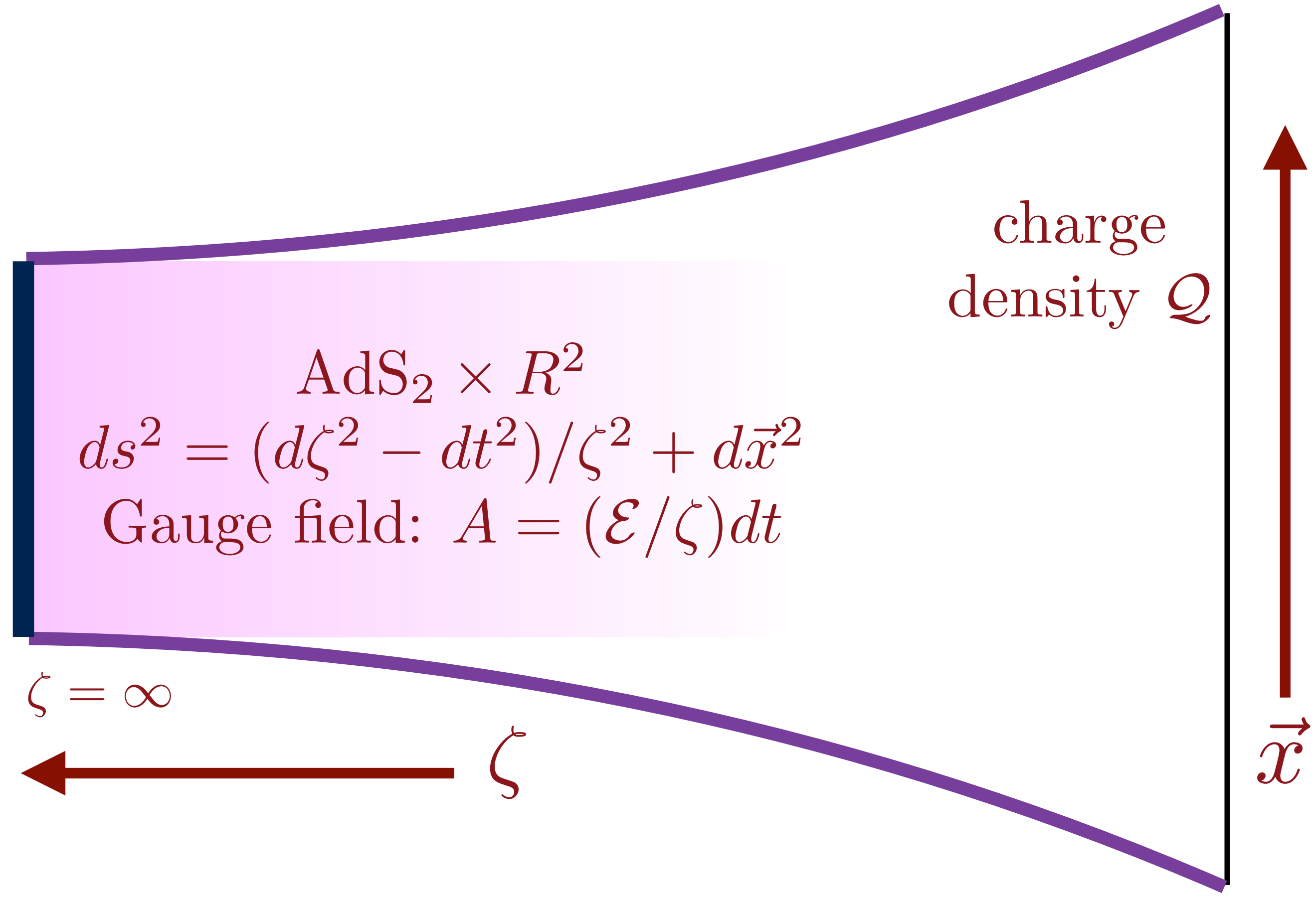
# SYK and $AdS_2$




The same Schwarzian effective action describes low energy fluctuations on the boundary theory of gravity theories with $\mathrm{AdS}_2$ near-horizon geometries (including the AdS-Reissner-Nordstrom solution of Einstein-Maxwell theory in 4 space-time dimensions). And the co-efficient of the Schwarzian, $N\gamma/4\pi^2$, determines the specific heat $\mathcal{C} = N\gamma T$.

A. Kitaev, unpublished; A. Almheiri and J. Polchinski, JHEP **1511** (2015) 014; J. Polchinski and V. Rosenhaus, arXiv: 1601.06768; J. Maldacena and D. Stanford, arXiv:1604.07818; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438; A. Almheiri and B. Kang, arXiv: 1606.04108

# SYK and $AdS_2$




The Schwarzian effective action implies that both the SYK model and the $\text{AdS}_2$ theories *saturate* the lower bound on the Lyapunov time

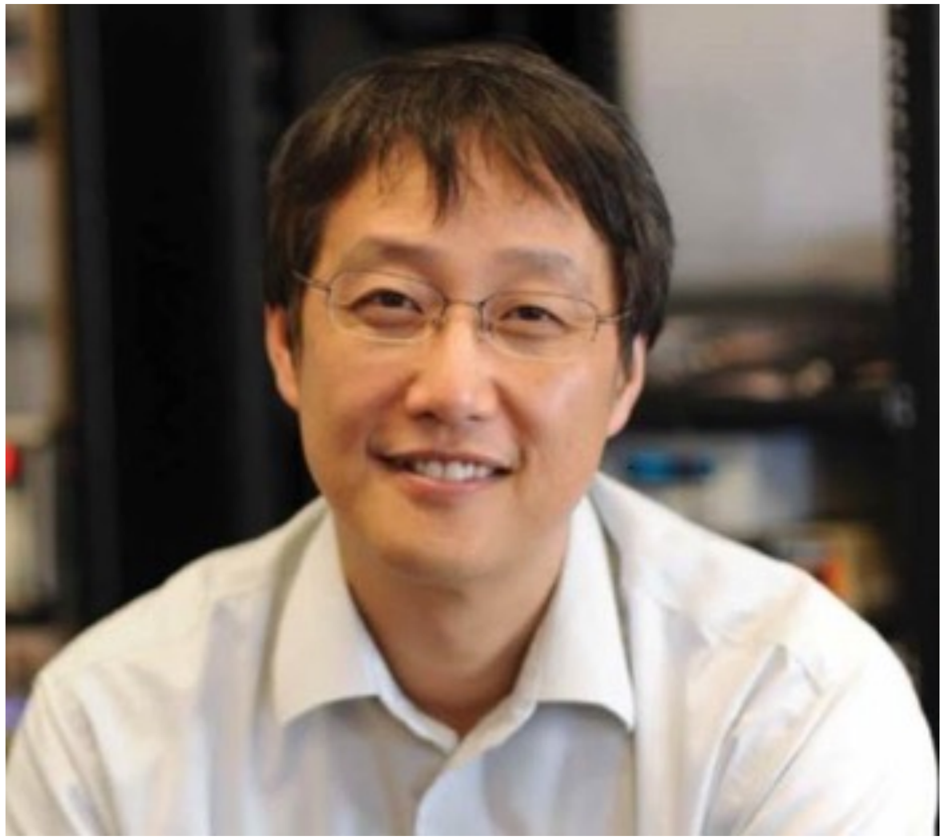
$$\tau_L = \frac{1}{2\pi} \frac{\hbar}{k_B T}.$$

This is additional evidence for an $\text{AdS}_2$ dual of the SYK model.


A. Kitaev, unpublished; A. Almheiri and J. Polchinski, JHEP **1511** (2015) 014; J. Polchinski and V. Rosenhaus, arXiv: 1601.06768; J. Maldacena and D. Stanford, arXiv:1604.07818; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438; A. Almheiri and B. Kang, arXiv: 1606.04108

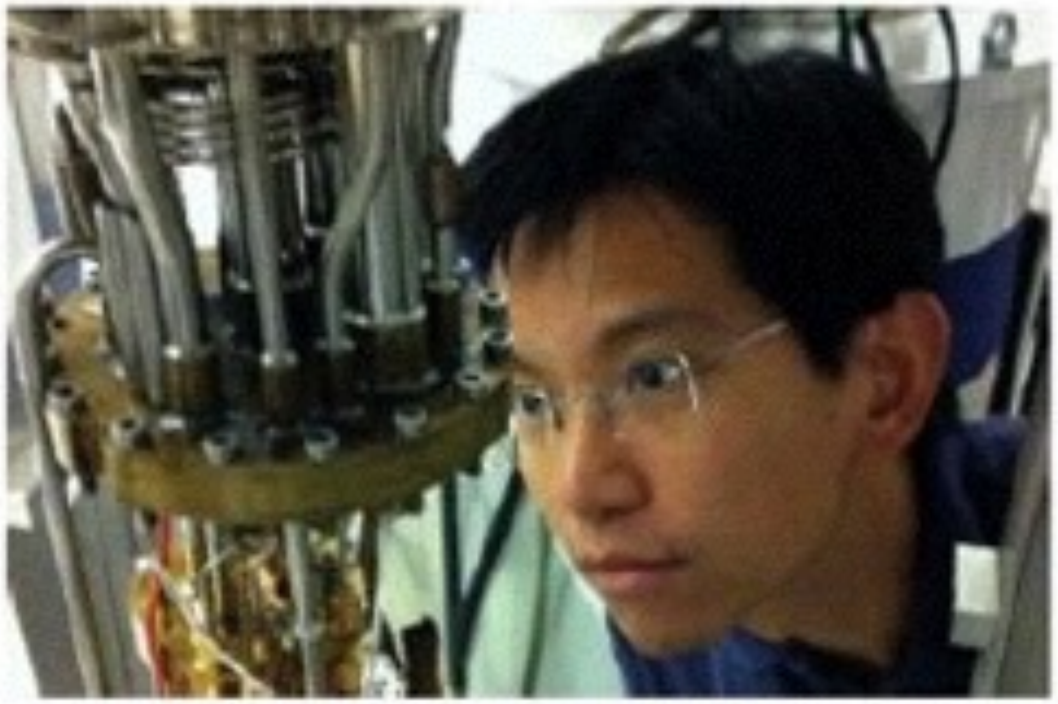
# *Quantum matter without quasiparticles:*

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Solvable random fermion Sachdev-Ye-Kitaev (SYK) model
- Charged black hole horizons in anti-de Sitter space
- Graphene

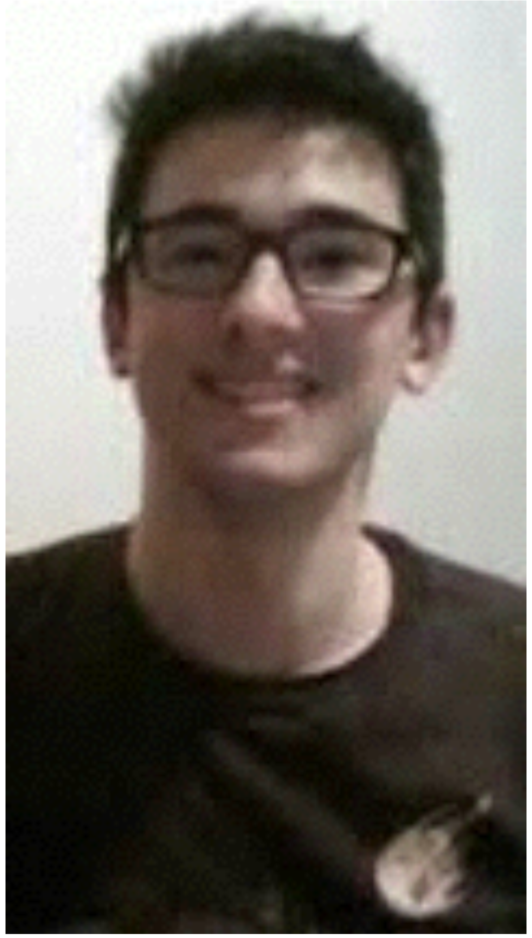


Philip Kim

Jesse Crossno

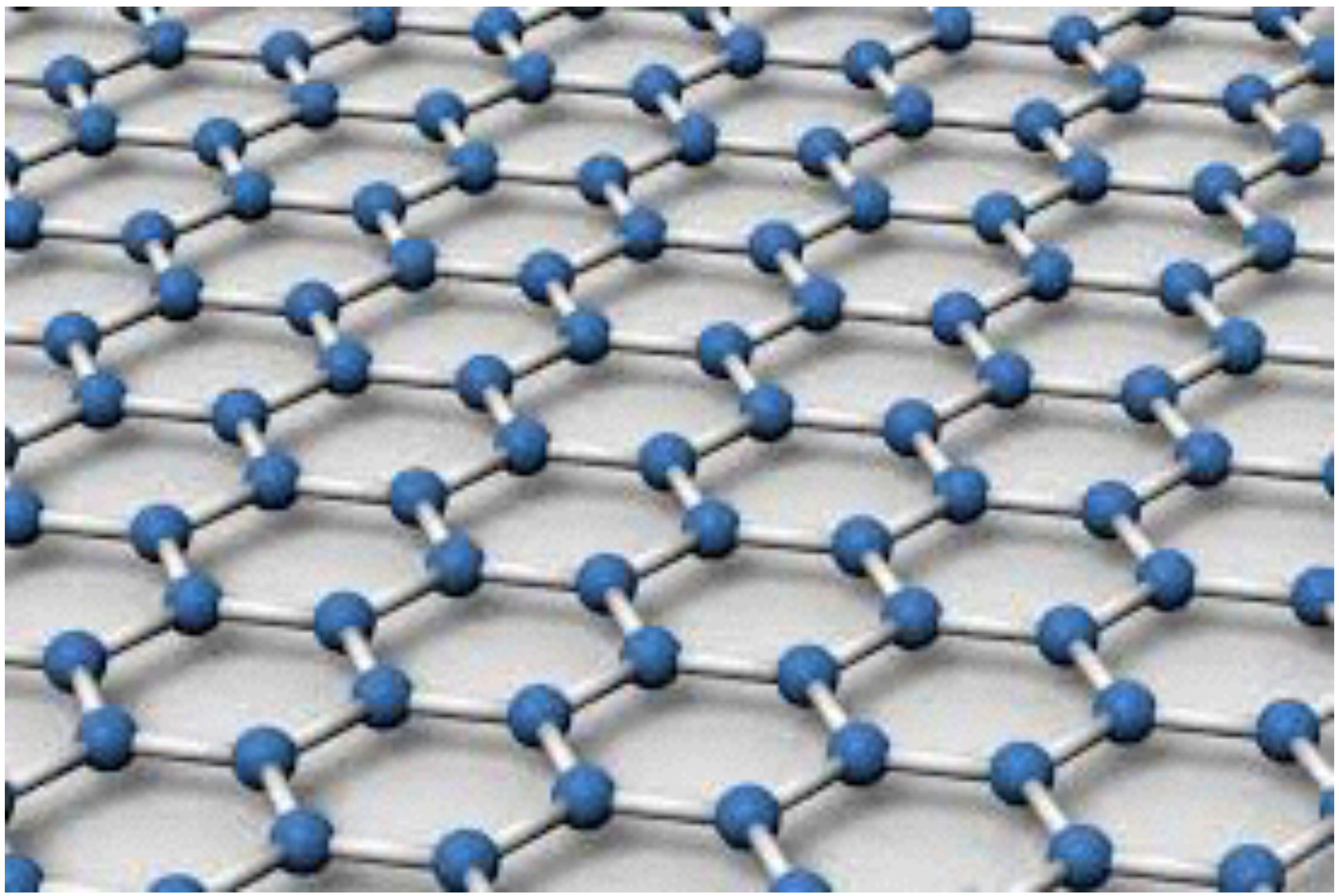


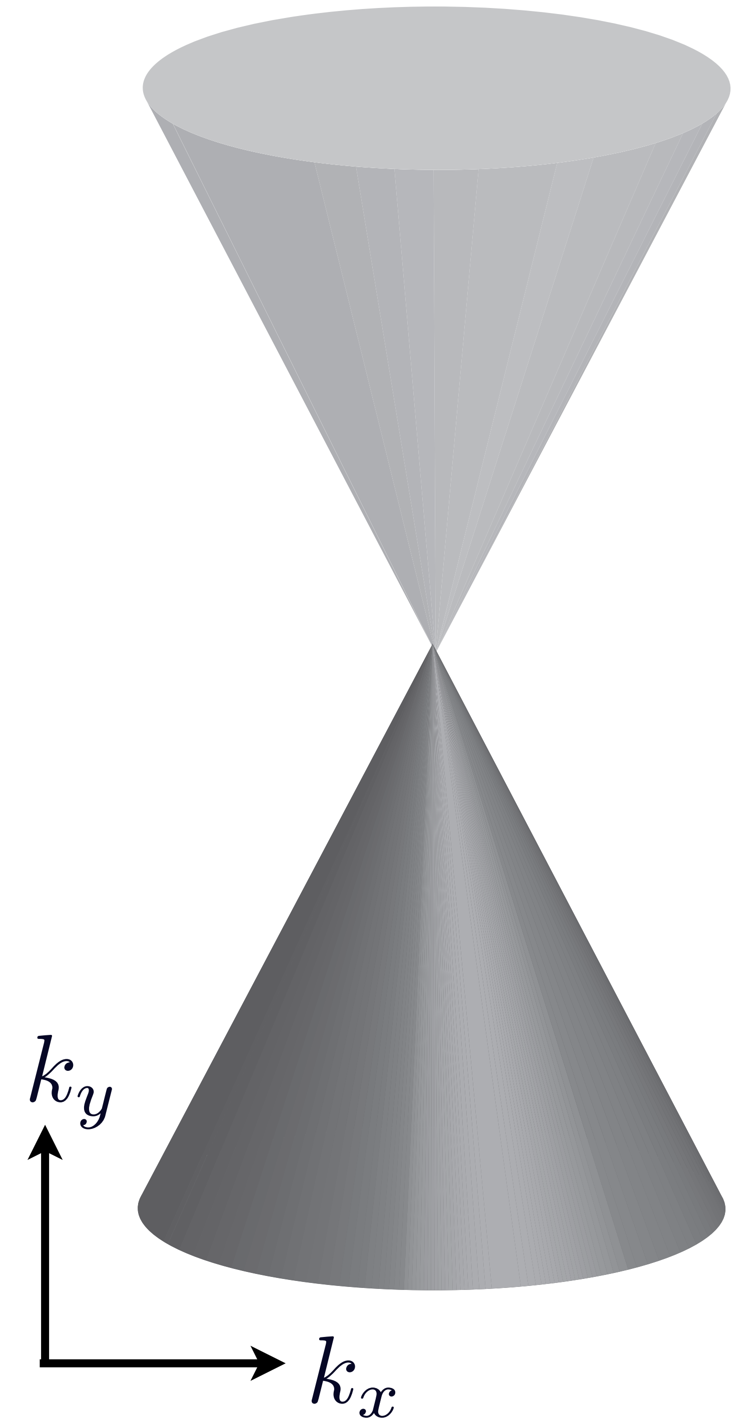
Kin Chung Fong



Andrew Lucas

# Graphene

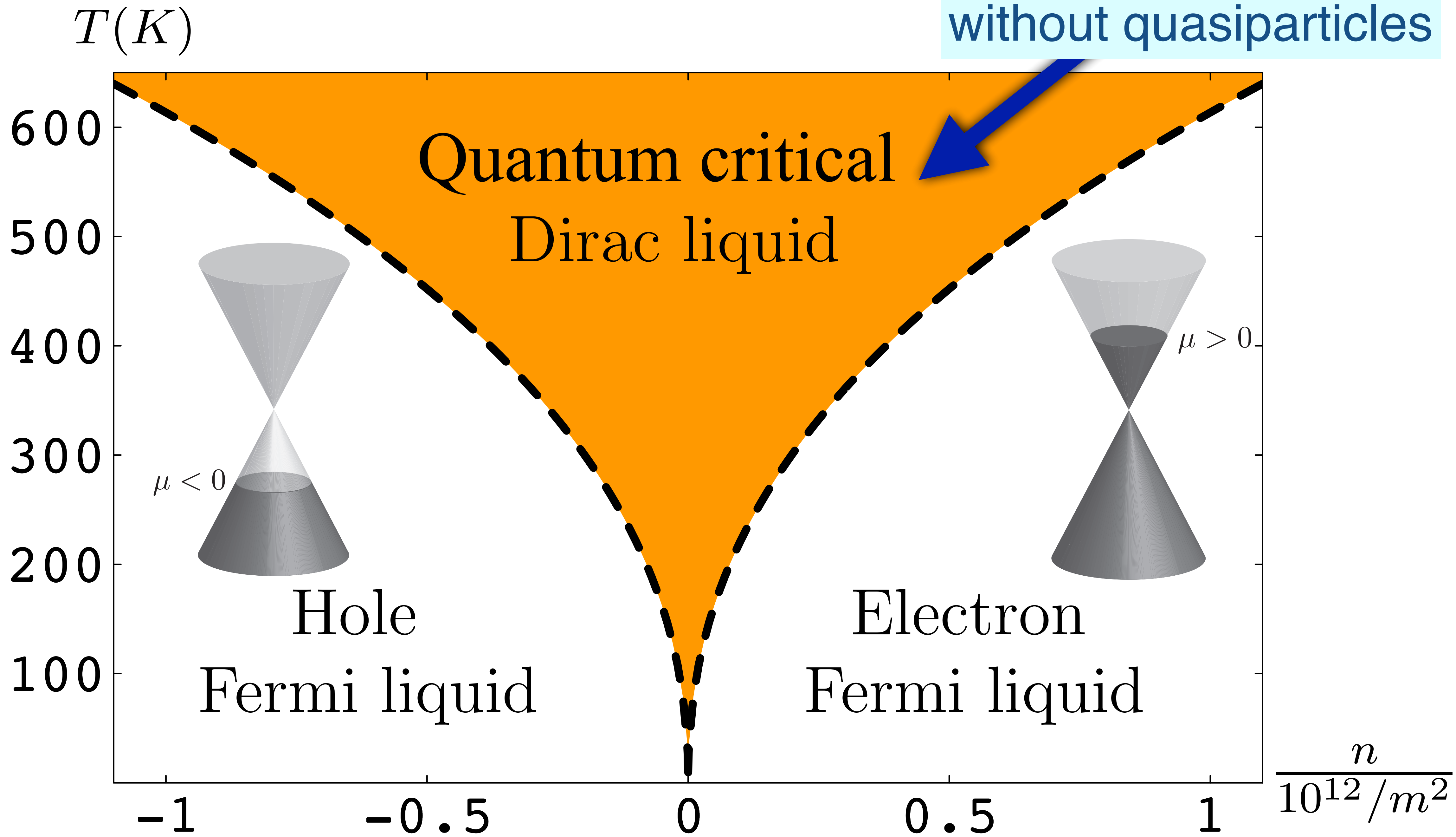




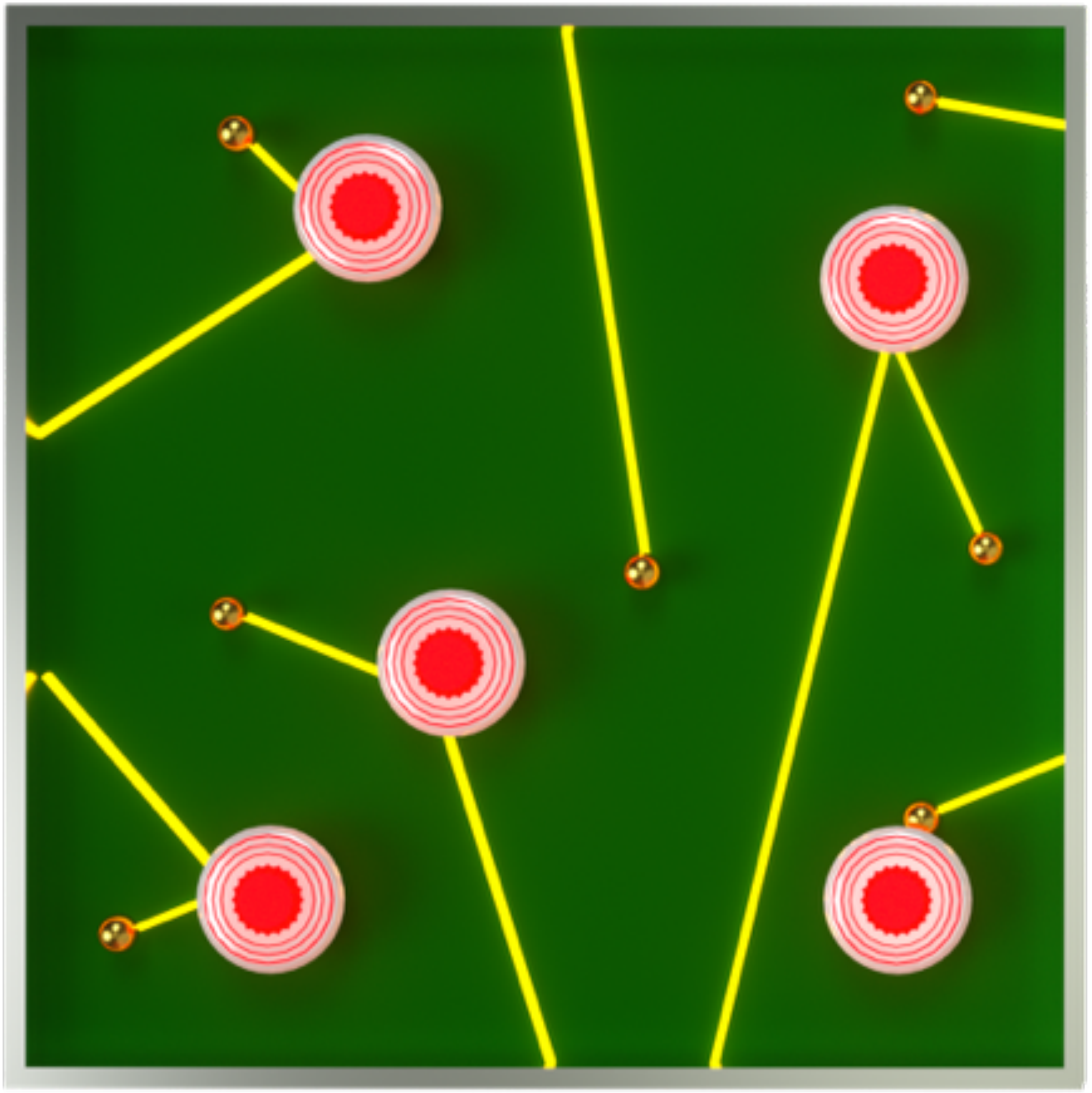

Same "Hubbard" model as for ultracold atoms, but for electrons on the honeycomb lattice

# Graphene

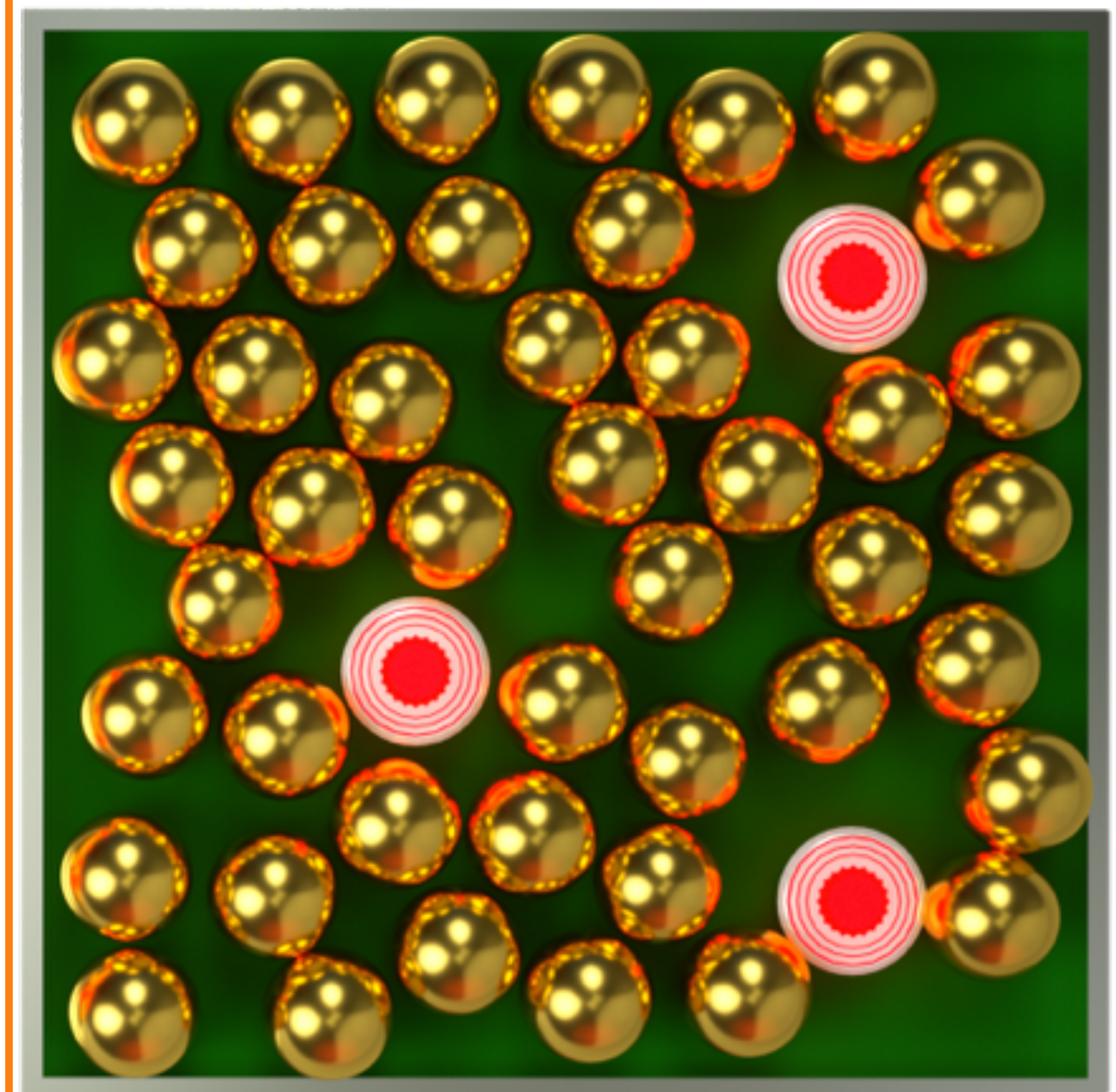
Predicted "strange metal" without quasiparticles




M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

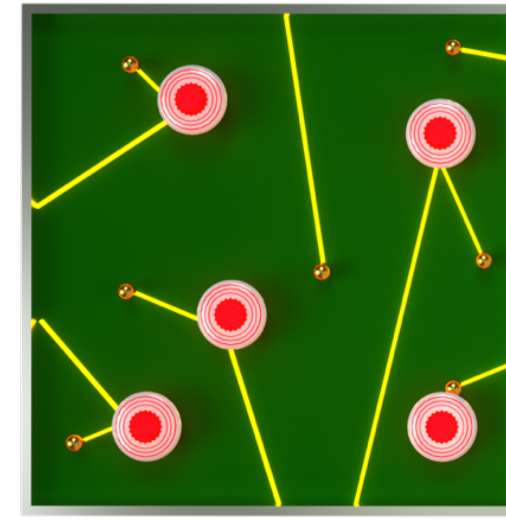


Fermi liquids: quasiparticles moving ballistically between impurity (red circles) scattering events



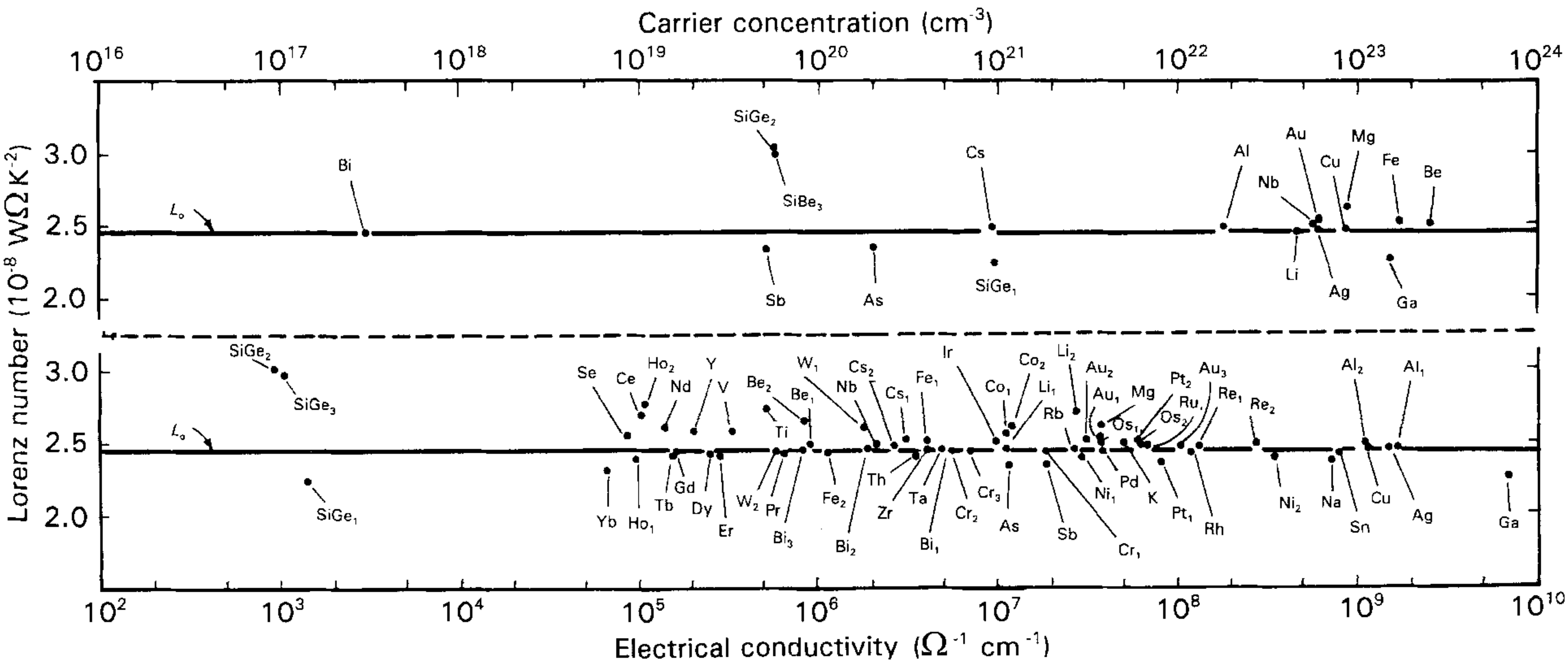
Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron “liquid” then “flows” around impurities

# Thermal and electrical conductivity with quasiparticles



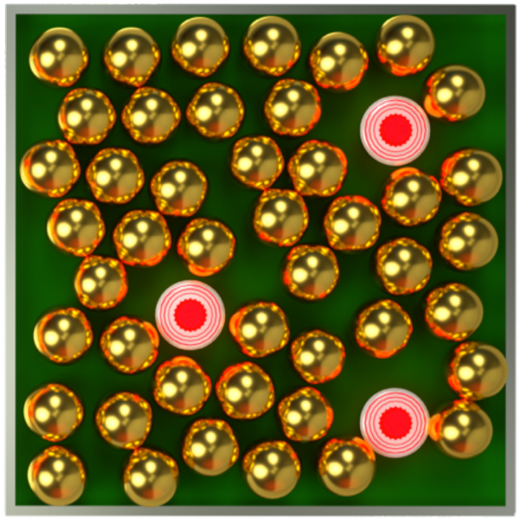
- Wiedemann-Franz law in a Fermi liquid:

$$L_0 = \frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8} \, \frac{\mathrm{W} \cdot \Omega}{\mathrm{K}^2}.$$




G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

# Transport in Strange Metals



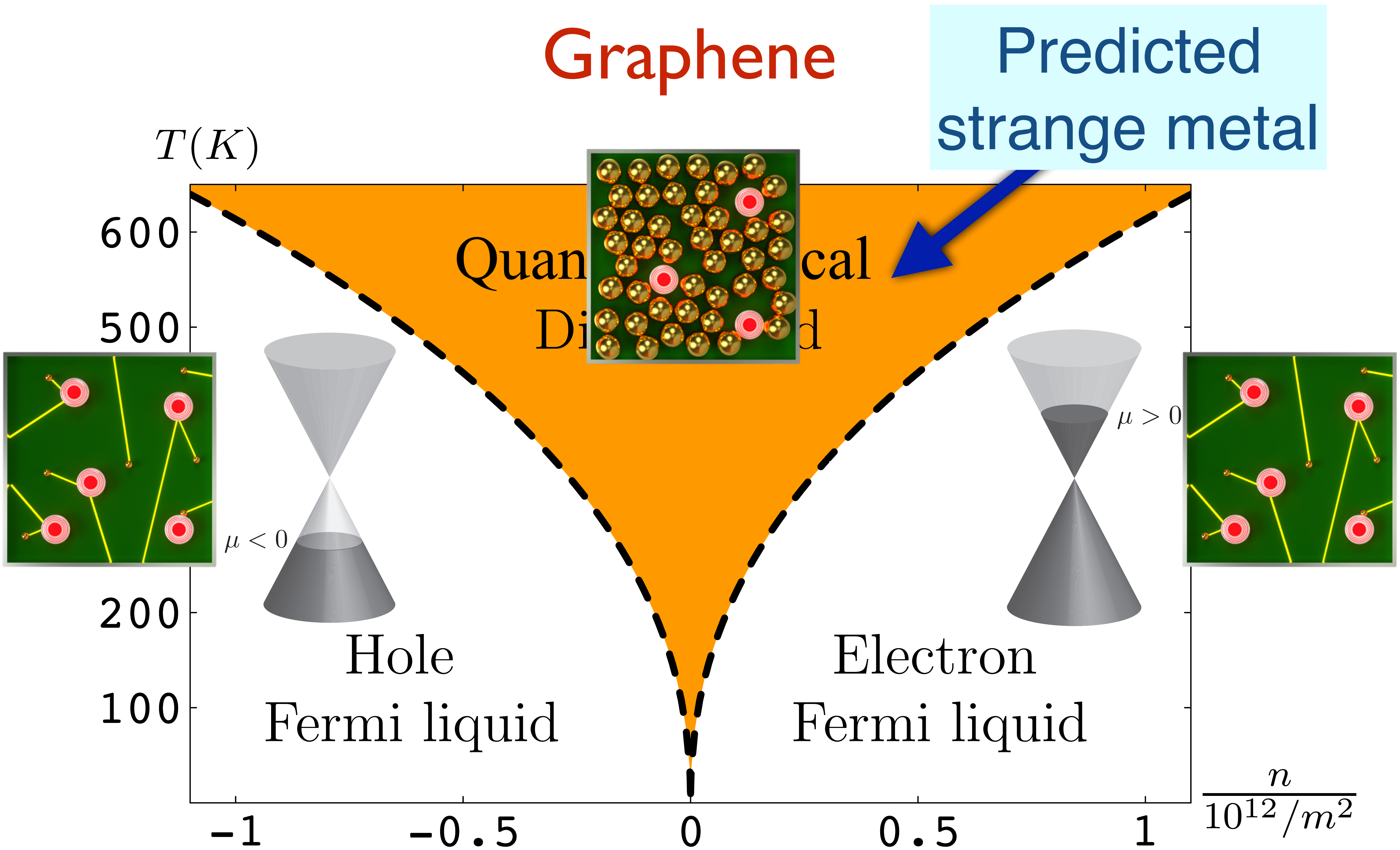
For a strange metal
with a "relativistic" Hamiltonian,
hydrodynamic, holographic,
and memory function methods yield
Lorentz ratio $L = \kappa/(T\sigma)$

$$= \frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H}\sigma_Q)\right)^2}$$

$\mathcal{Q} \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density
$\sigma_Q \rightarrow$ quantum critical conductivity
$\tau_{\rm imp} \rightarrow$ momentum relaxation time from impurities.
Note that for a clean system ($\tau_{\rm imp} \rightarrow \infty$ first),
the Lorentz ratio diverges $L \sim 1/\mathcal{Q}^4$,
as we approach "zero" electron density at the Dirac point.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

# Graphene

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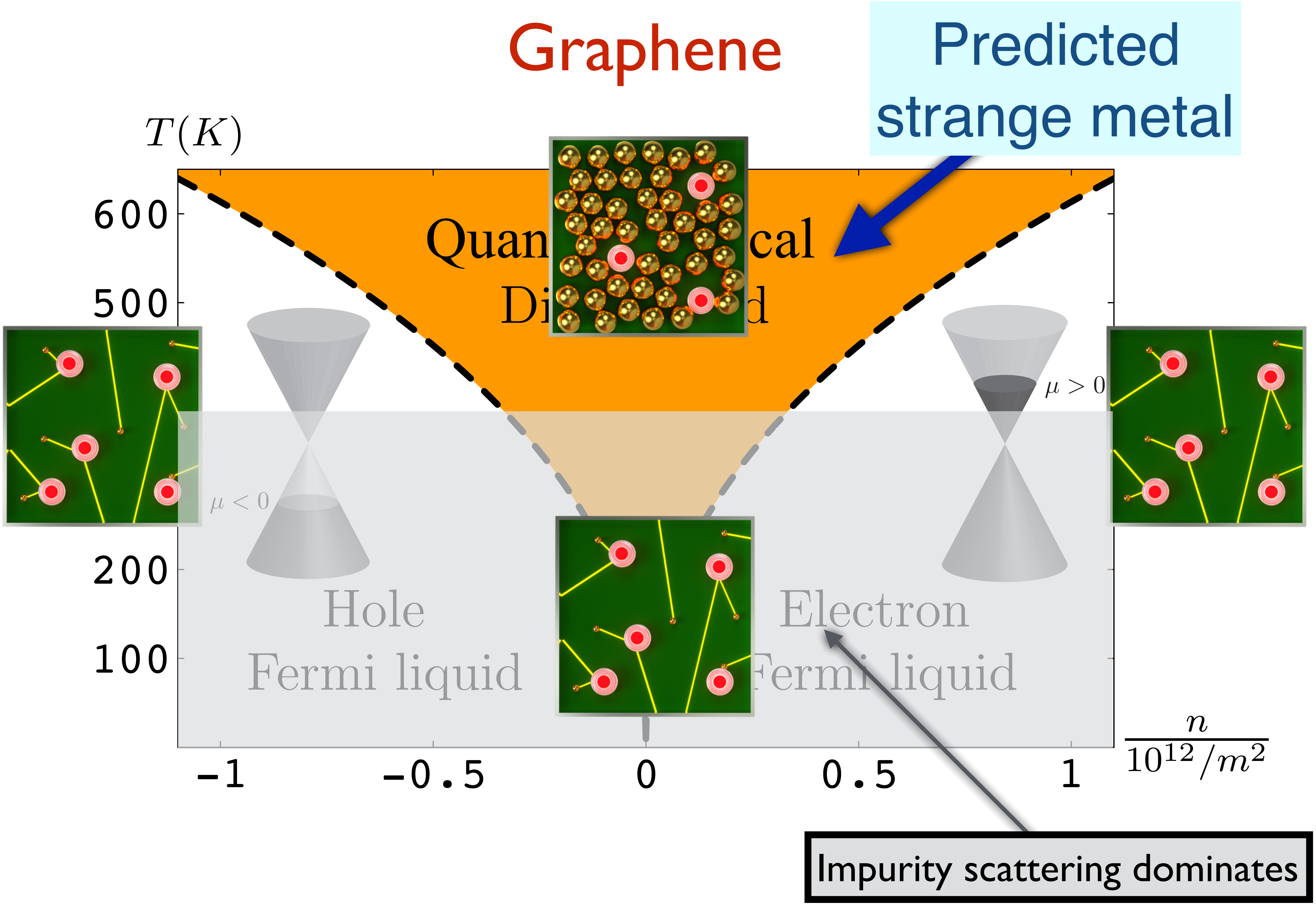


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
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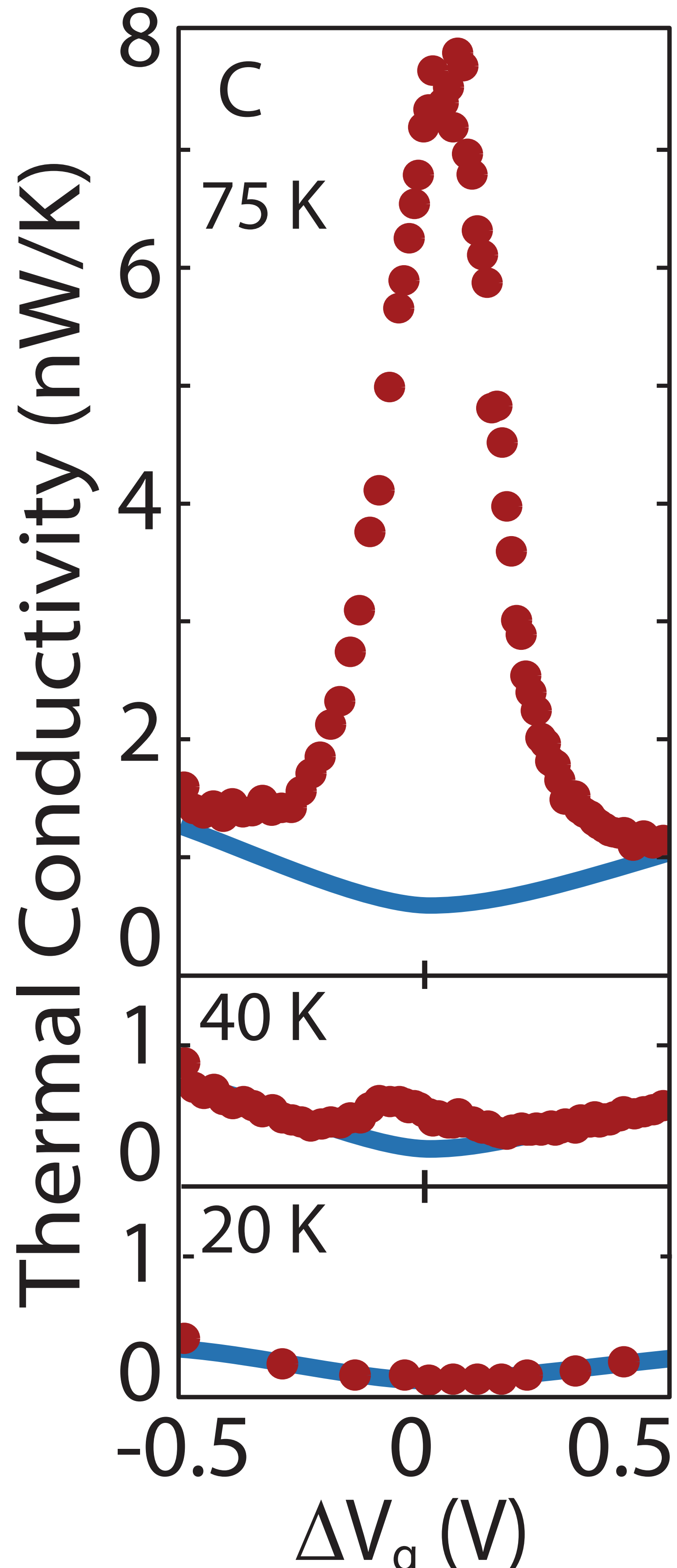
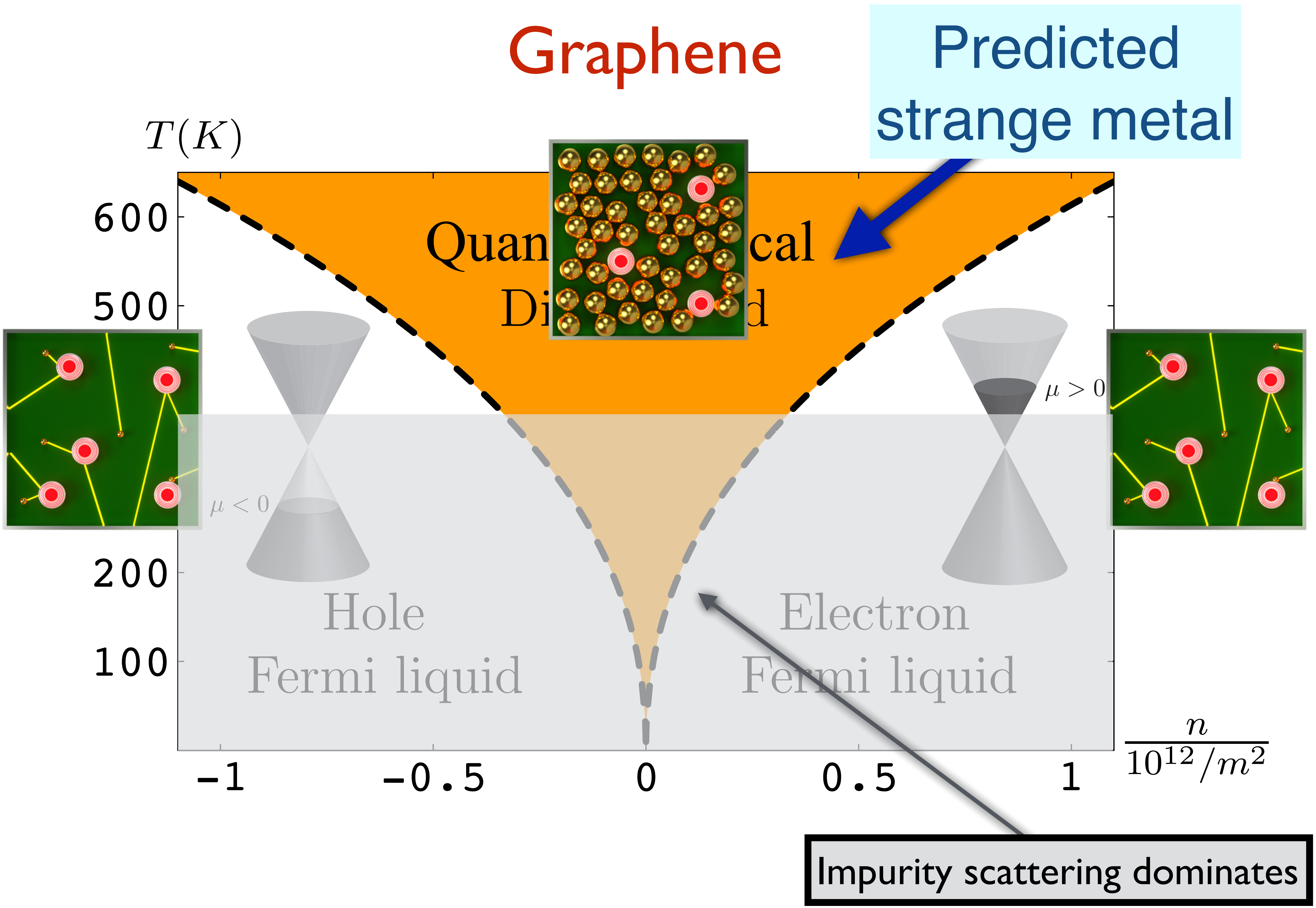
J. Crossno et al., Science **351**, 1058 (2016)




Red dots: data

Blue line: value for $L = L_0$

# Graphene




M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
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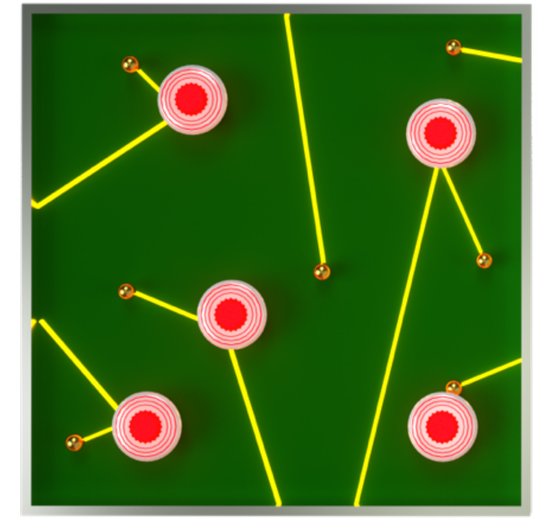
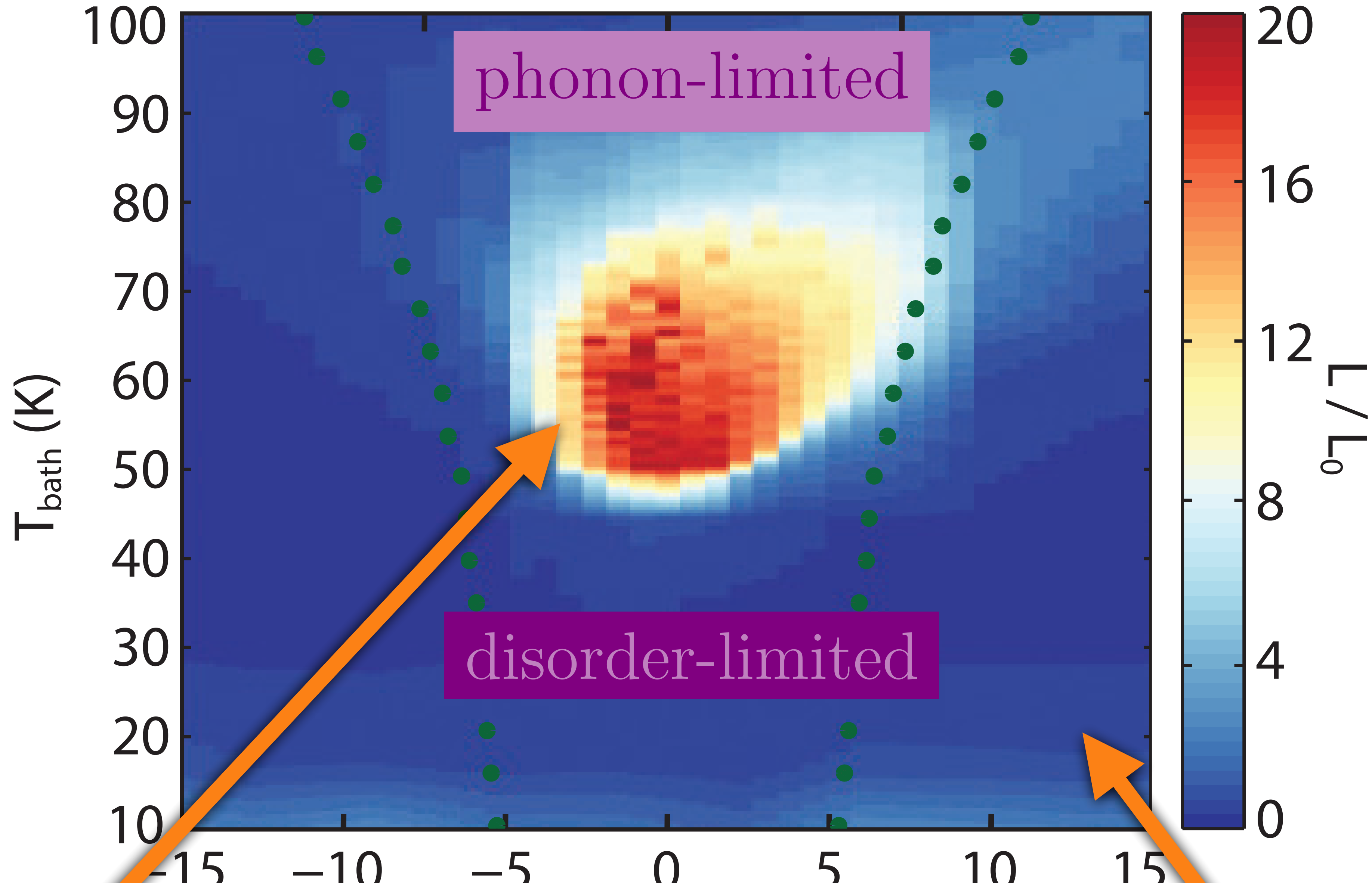
J. Crossno et al., Science **351**, 1058 (2016)

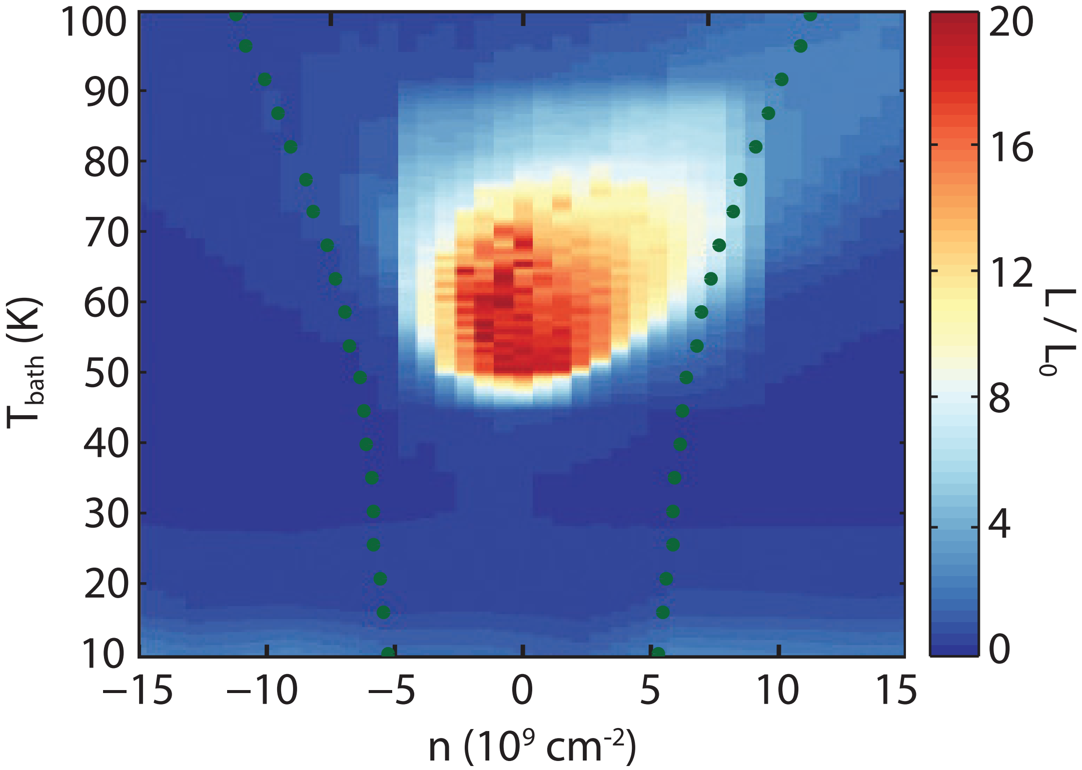
# Strange metal in graphene

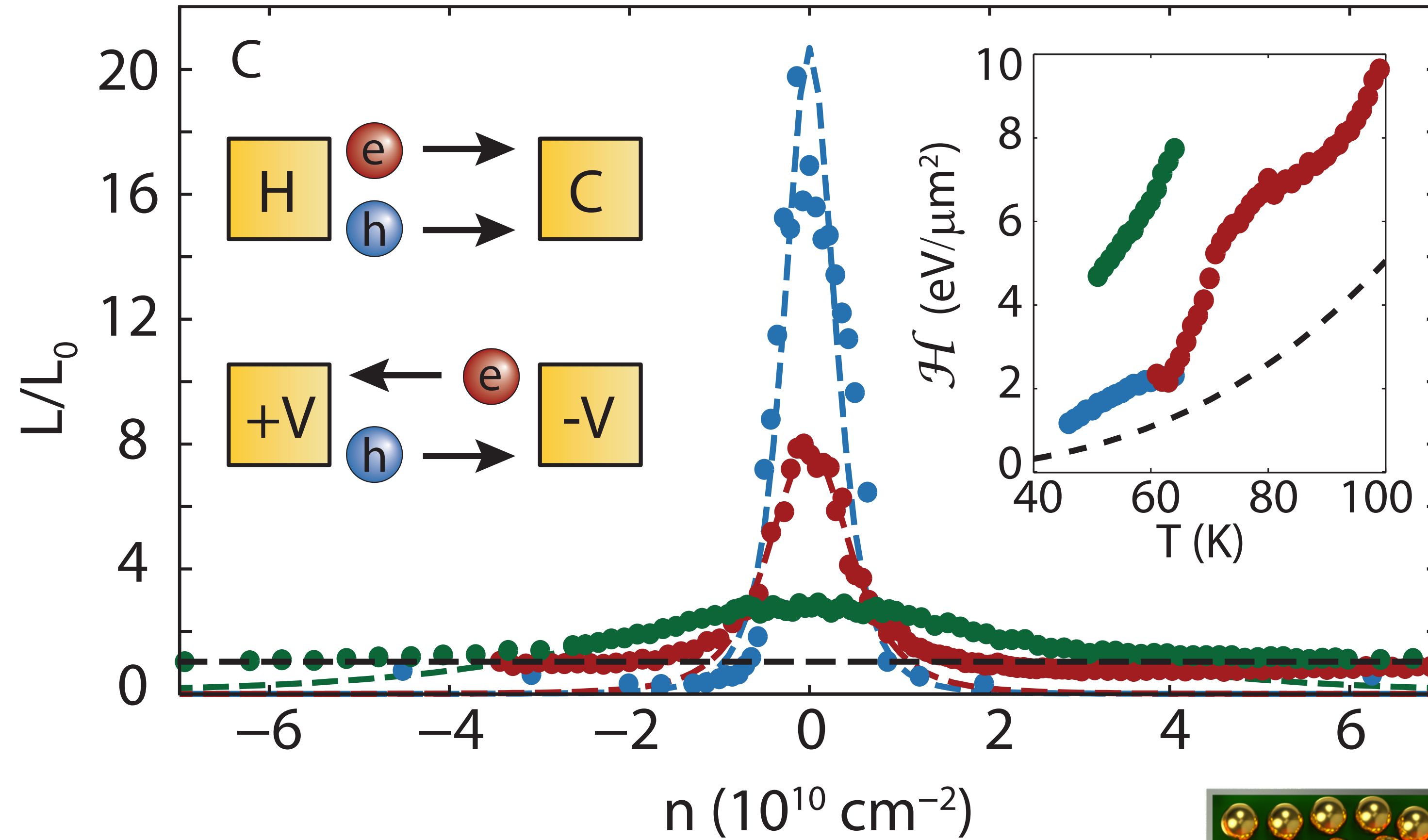


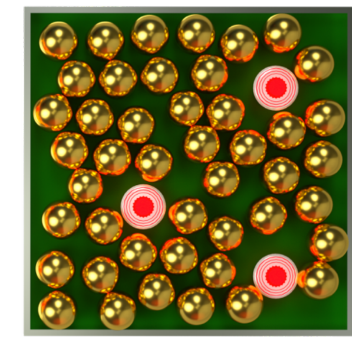


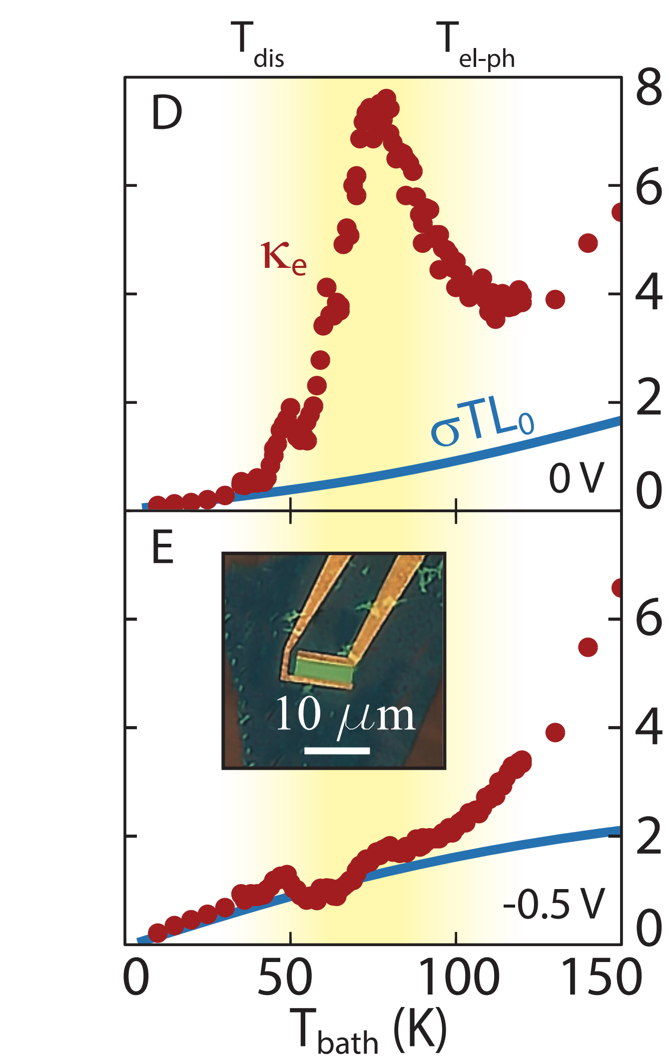

Wiedemann-Franz violated !

Wiedemann-Franz obeyed






Lorentz ratio $L = \kappa/(T\sigma)$

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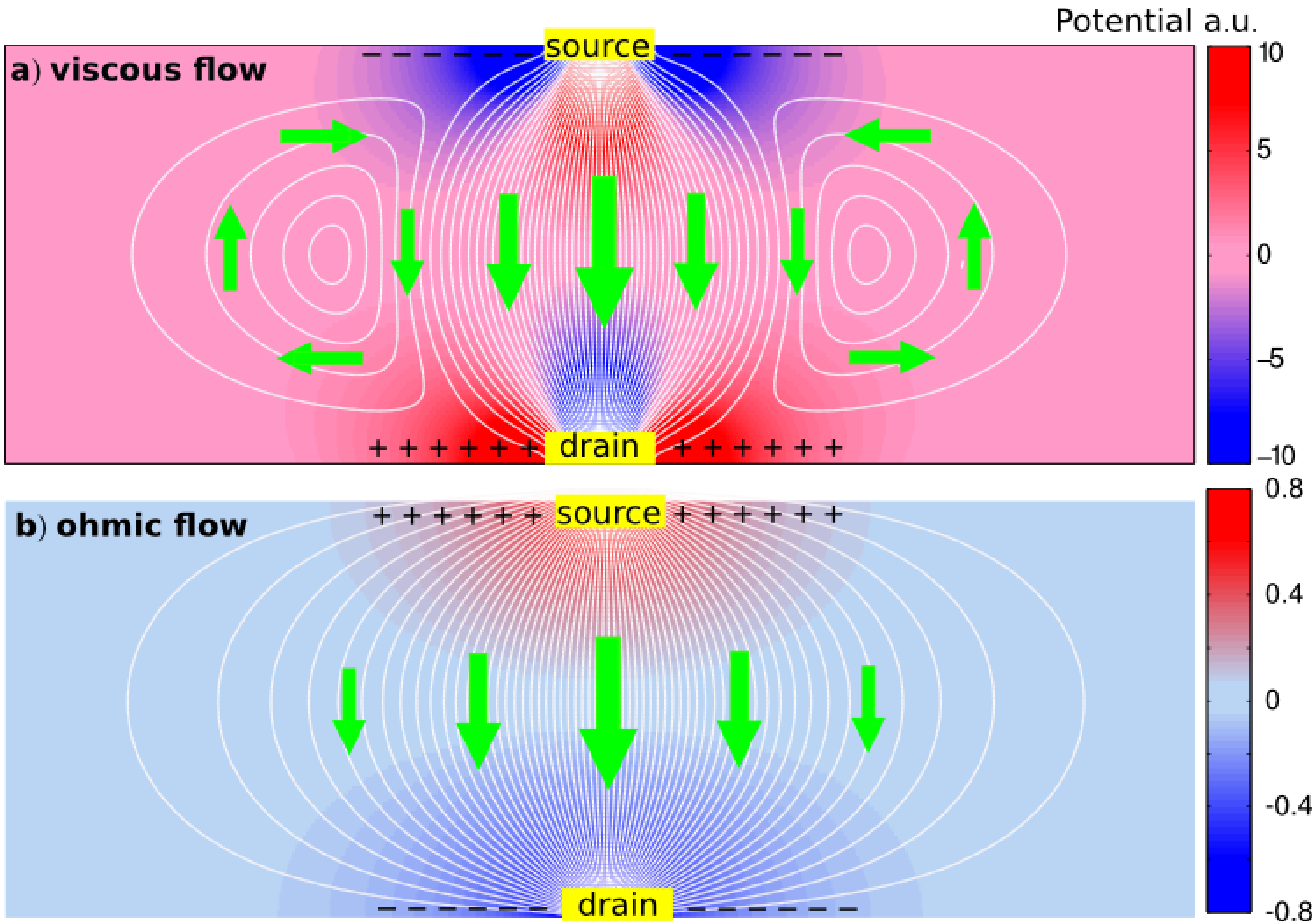
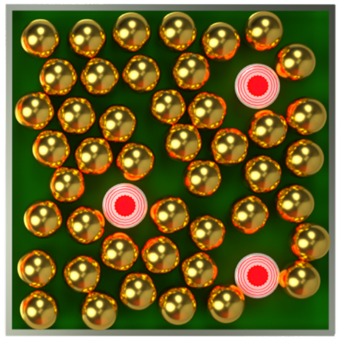
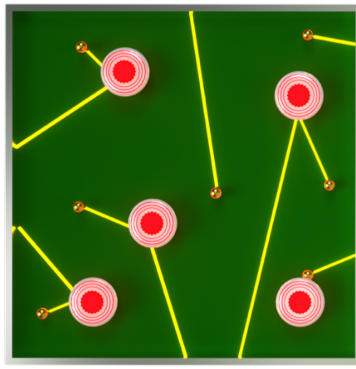
J. Crossno et al., Science **351**, 1058 (2016)

# Strange metal in graphene

**Negative local resistance due to viscous electron backflow in graphene**





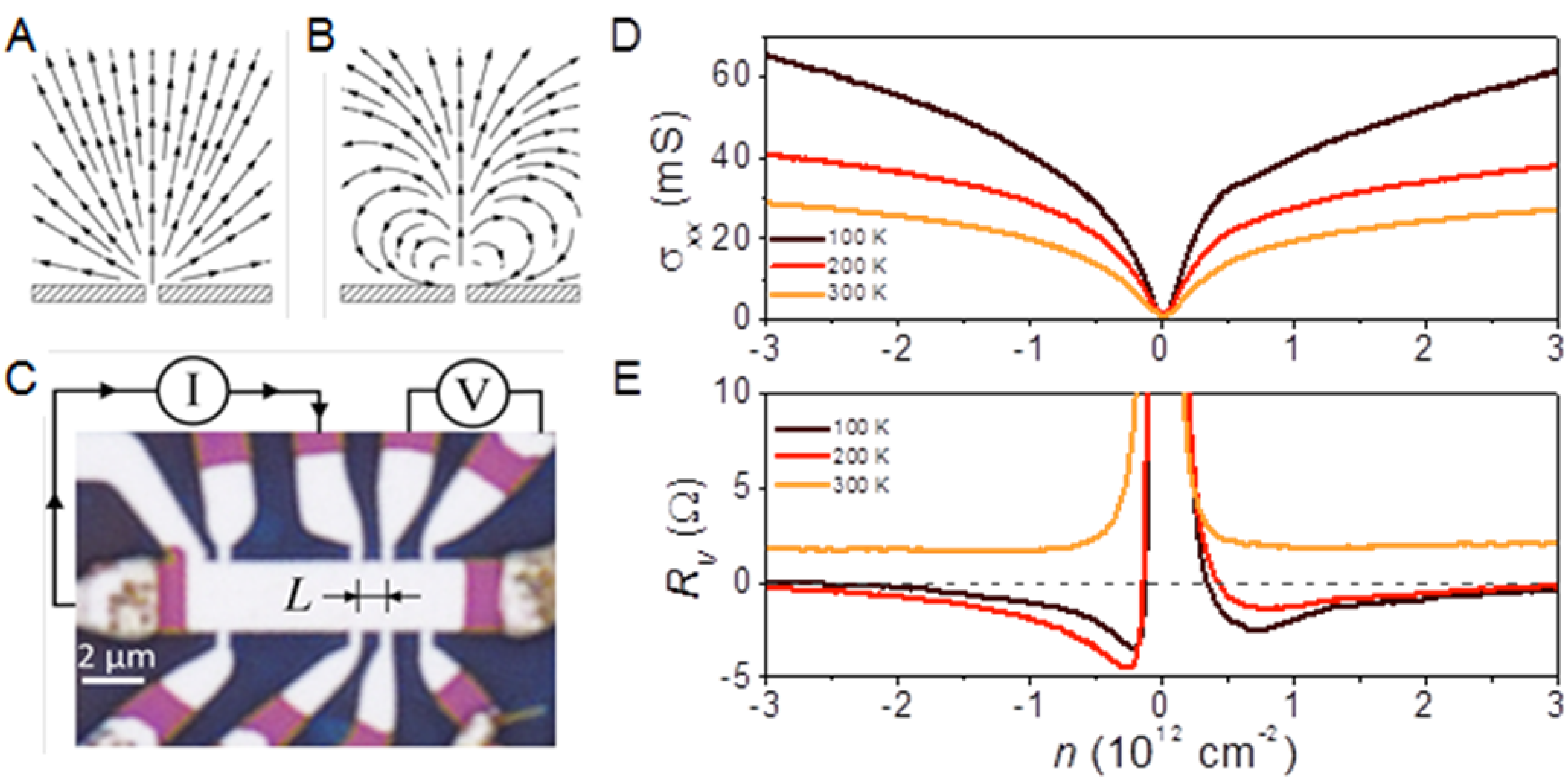
L. Levitov and G. Falkovich, arXiv:1508.00836, *Nature Physics online*

# Strange metal in graphene

Science **351**, 1055 (2016)

## Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Auton[5], E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3,6]



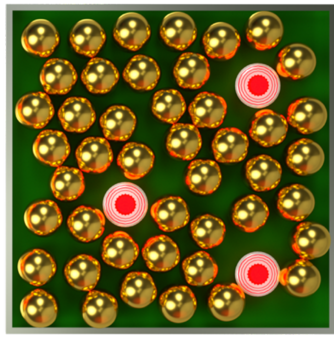



**Figure 1.** Viscous backflow in doped graphene. (**a,b**) Steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero $\nu$ (a) and a viscous Fermi liquid (b). (**c**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**d,e**) Longitudinal conductivity $\sigma_{xx}$ and $R_{\mathrm{V}}$ for this device as a function of $n$ induced by applying gate voltage. $I = 0.3$ μA; $L = 1$ μm. For more detail, see Supplementary Information.

# *Entangled quantum matter without quasiparticles*

- No quasiparticle excitations
- Shortest possible "phase coherence" time, fastest possible local equilibration time, or fastest possible Lyapunov time towards quantum chaos, all of order $\dfrac{\hbar}{k_B T}$
- Realization in solvable SYK model, which saturates the lower bound on the Lyapunov time. Its properties have some similarities to non-rational, large central charge CFT2s.
- Remarkable match between SYK and quantum gravity of black holes with $\mathrm{AdS}_2$ horizons, including a SL(2,R)-invariant Schwarzian effective action for thermal energy fluctuations.
- Experiments on graphene agree well with predictions of a theory of a nearly relativistic quantum liquid without quasiparticles.