Quantum theory of vortices and quasiparticles in *d*-wave superconductors

Quantum theory of vortices and quasiparticles in *d*-wave superconductors

Physical Review B **73**, 134511 (2006),
Physical Review B **74**, 144516 (2006),
Annals of Physics **321**, 1528 (2006)

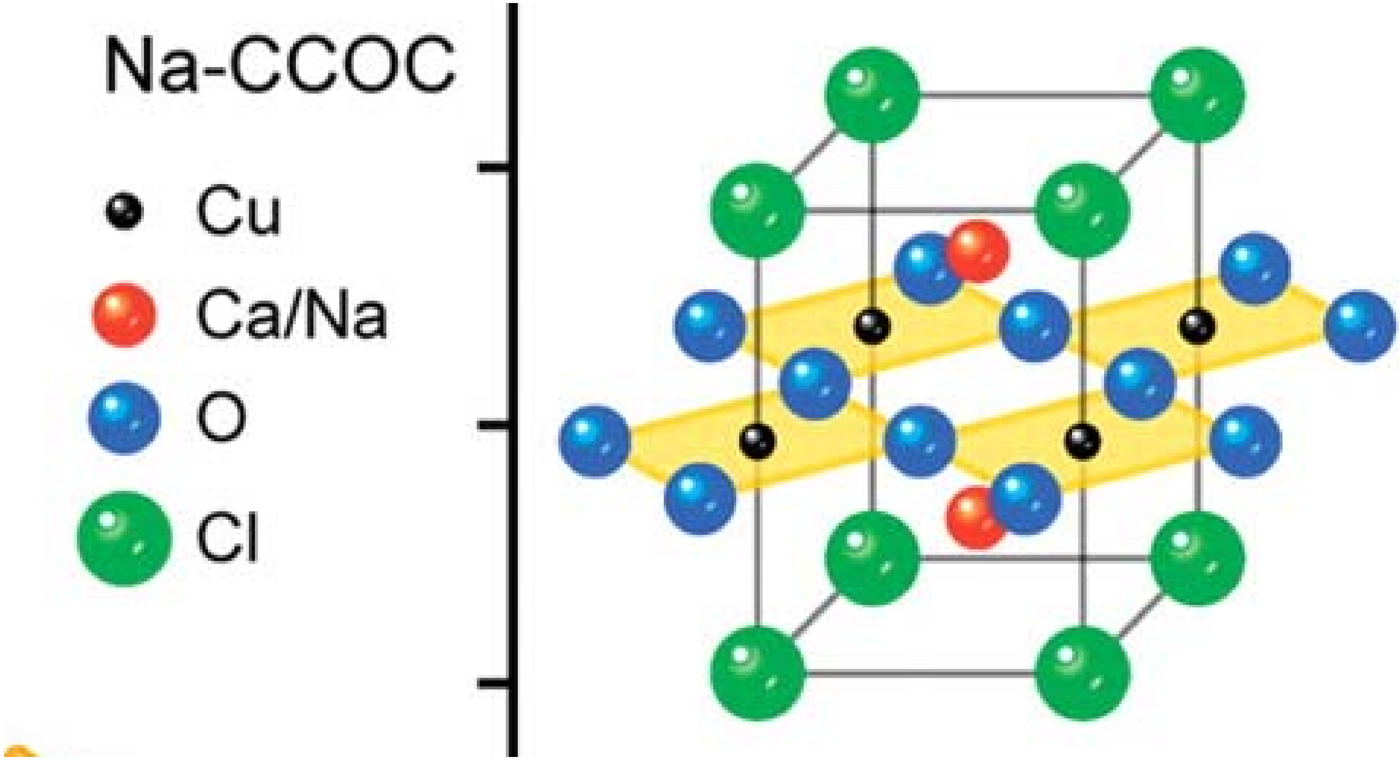
Predrag Nikolic

Subir Sachdev

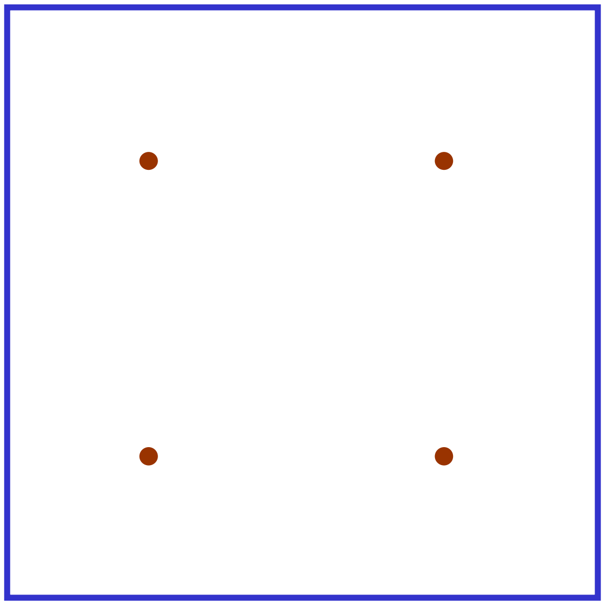
Harvard University

Talks online at http://sachdev.physics.harvard.edu

BCS theory for electronic quasiparticles in a *d*-wave superconductor



BCS theory for electronic quasiparticles in a *d*-wave superconductor

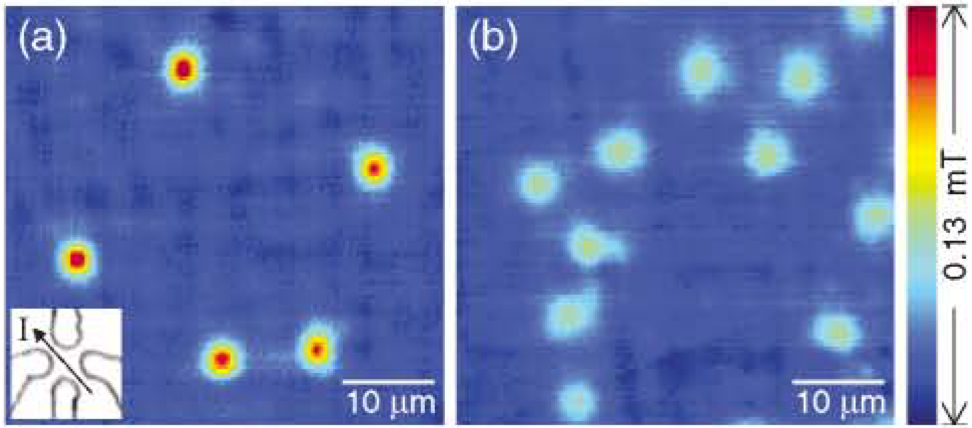


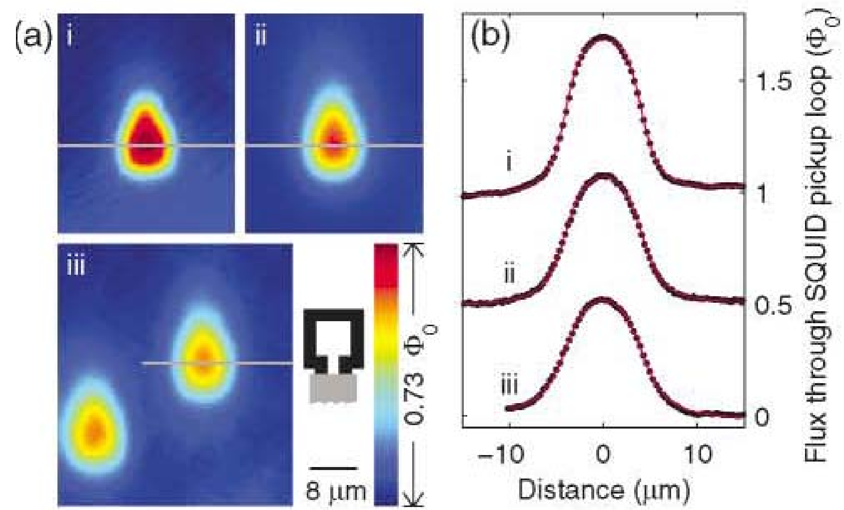
Dispersion of $S = 1/2$ fermionic Bogoliubov quasiparticles:

$$\varepsilon_{\mathbf{k}} = \left((-2t\cos k_x - 2t\cos k_y - \mu)^2 + \Delta_0^2(\cos k_x - \cos k_y)^2 \right)^{1/2}$$

Low energy fermionic excitations near the nodes can be described as 4 Dirac fermions

Quantized fluxoids in $YBa_2Cu_3O_{6+y}$





J. C. Wynn, D. A. Bonn, B.W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler, *Phys. Rev. Lett*. **87**, 197002 (2001).

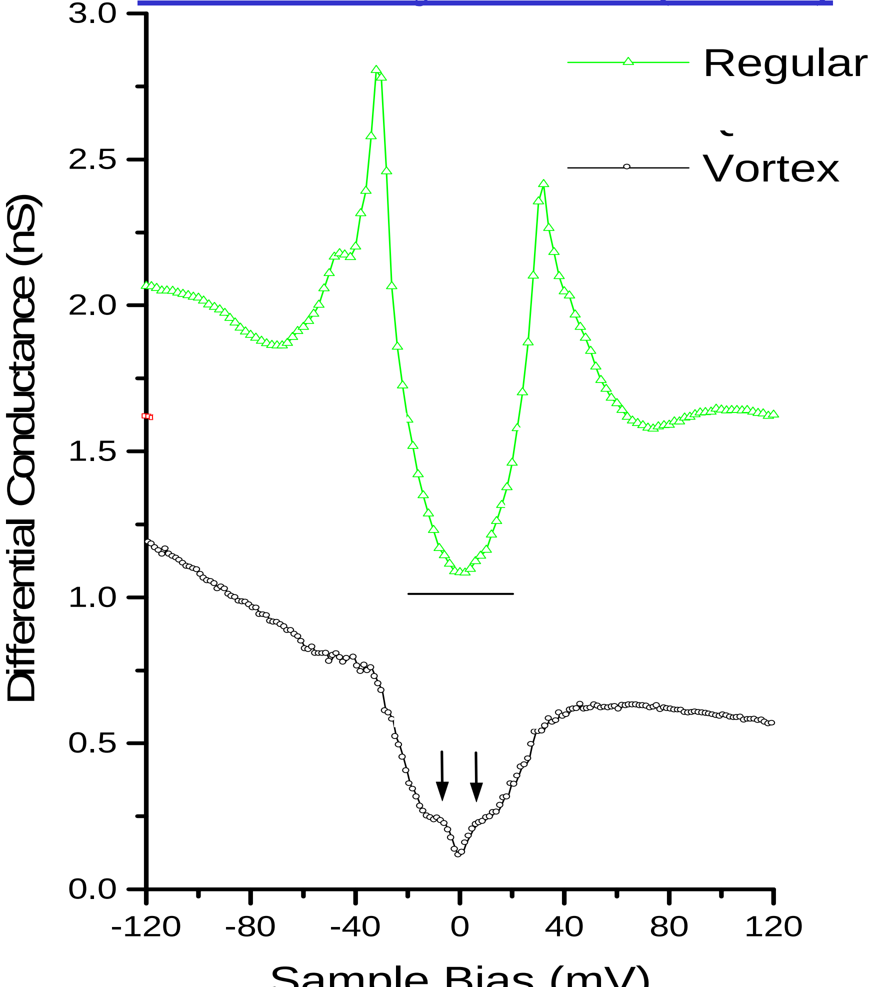
In superconductors, vortices carry quantized magnetic flux:

$$\int \mathbf{B} \cdot d\mathbf{S} = n\frac{hc}{2e}$$

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

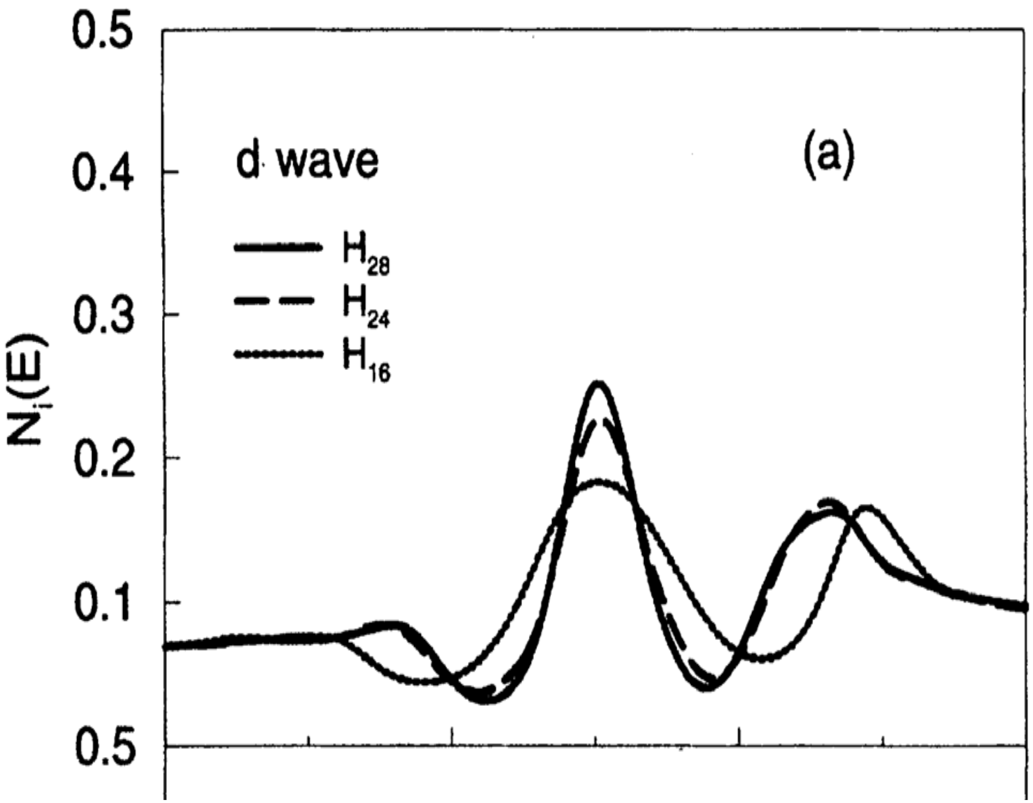
Local density of states (LDOS)



LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

I. Maggio-Aprile *et al.* *Phys. Rev. Lett.* **75**, 2754 (1995).
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

BCS theory for local density of states (LDOS) at the center of a vortex in a *d*-wave superconductor



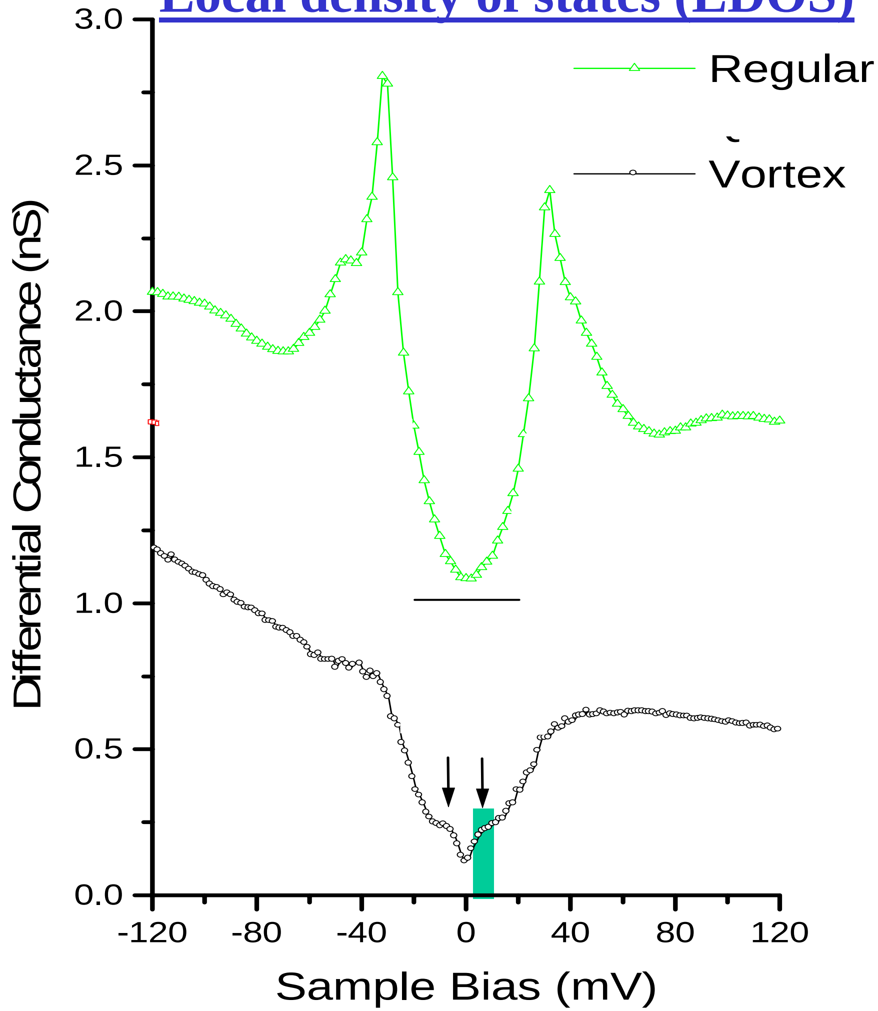
Prominent feature: large peak at zero bias

Y. Wang and A. H. MacDonald, *Phys. Rev.* B **52**, 3876 (1995).
M. Ichioka, N. Hayashi, N. Enomoto, and K. Machida, *Phys. Rev.* B **53**, 15316 (1996).

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states (LDOS)

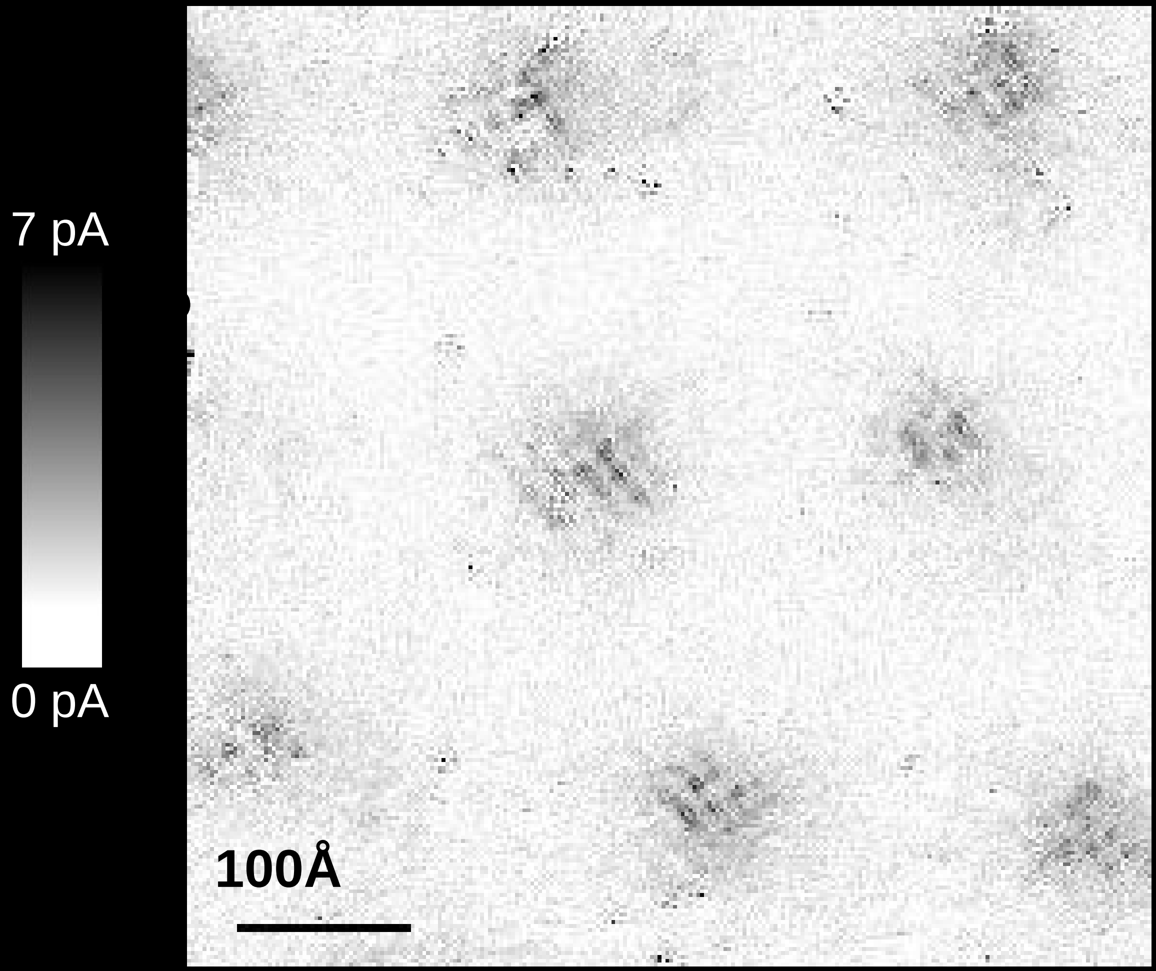


I. Maggio-Aprile *et al.* *Phys. Rev. Lett.* **75**, 2754 (1995).
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

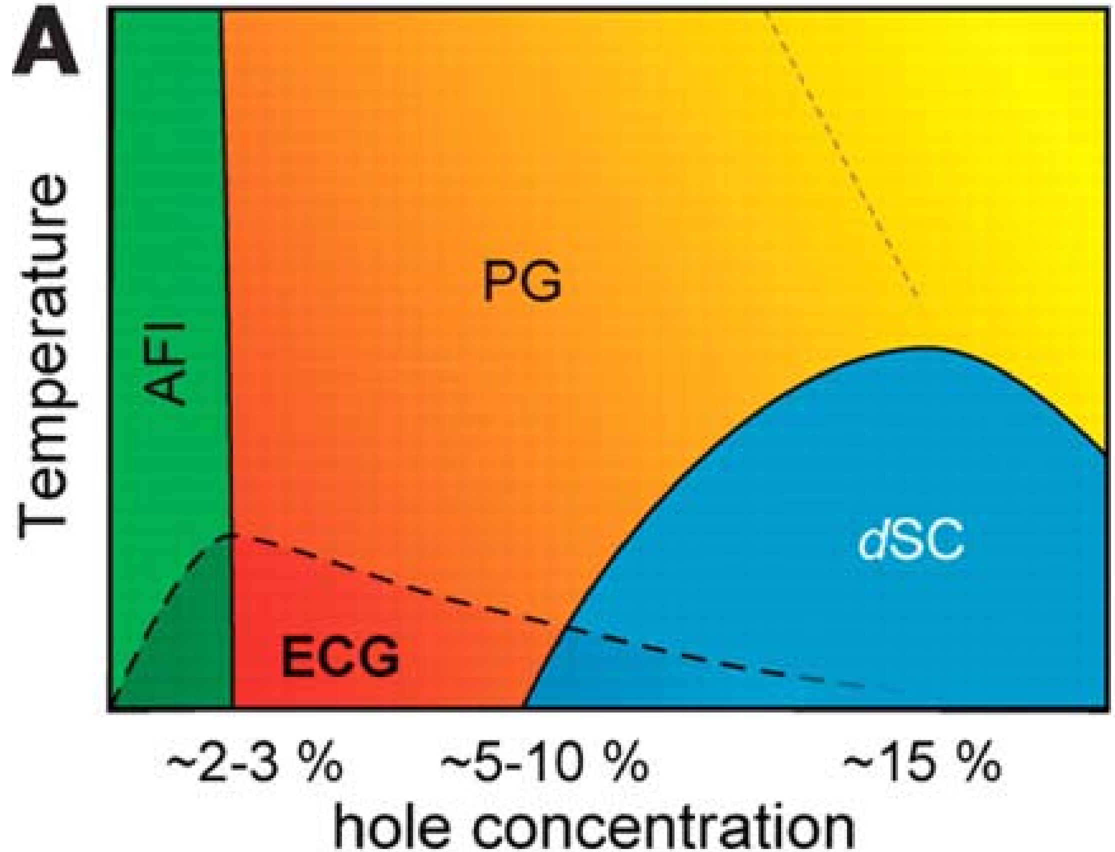
Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



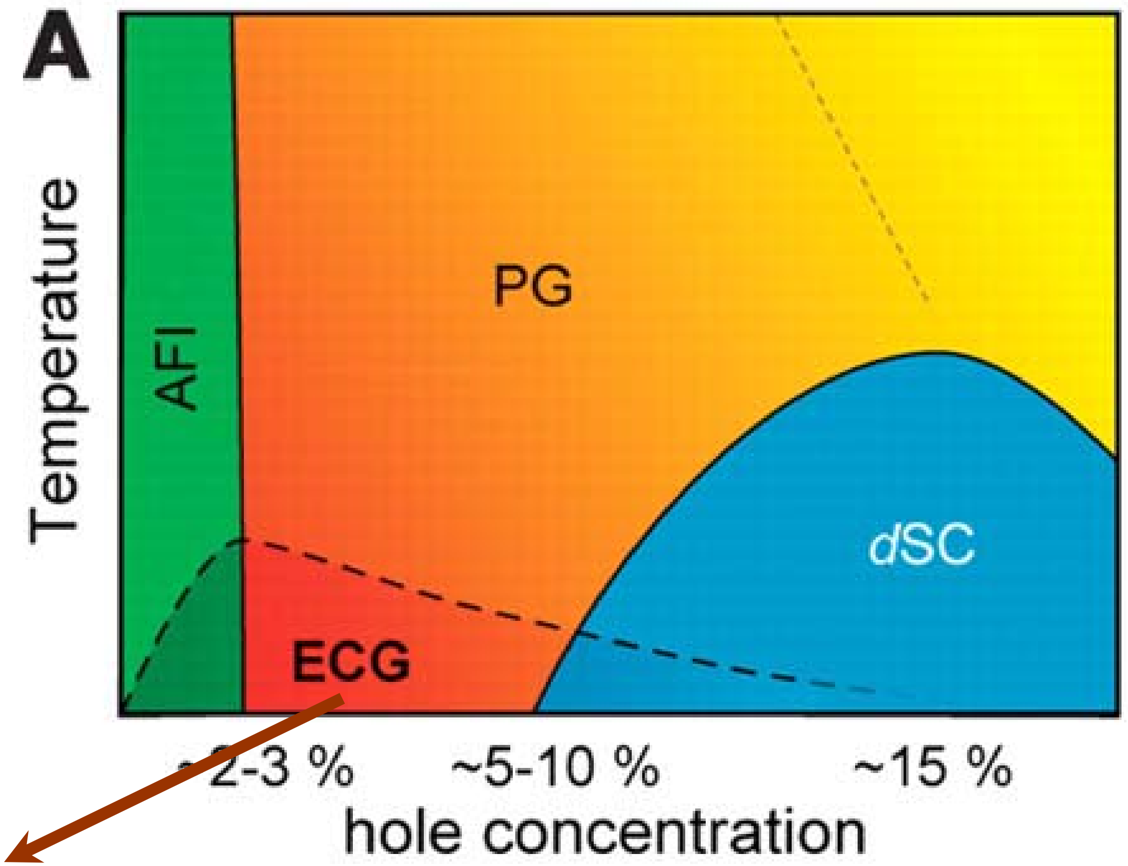
Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman et al., *Science* 295, 466 (2002).
G. Levy et al., Phys. Rev. Lett. 95, 257005 (2005).

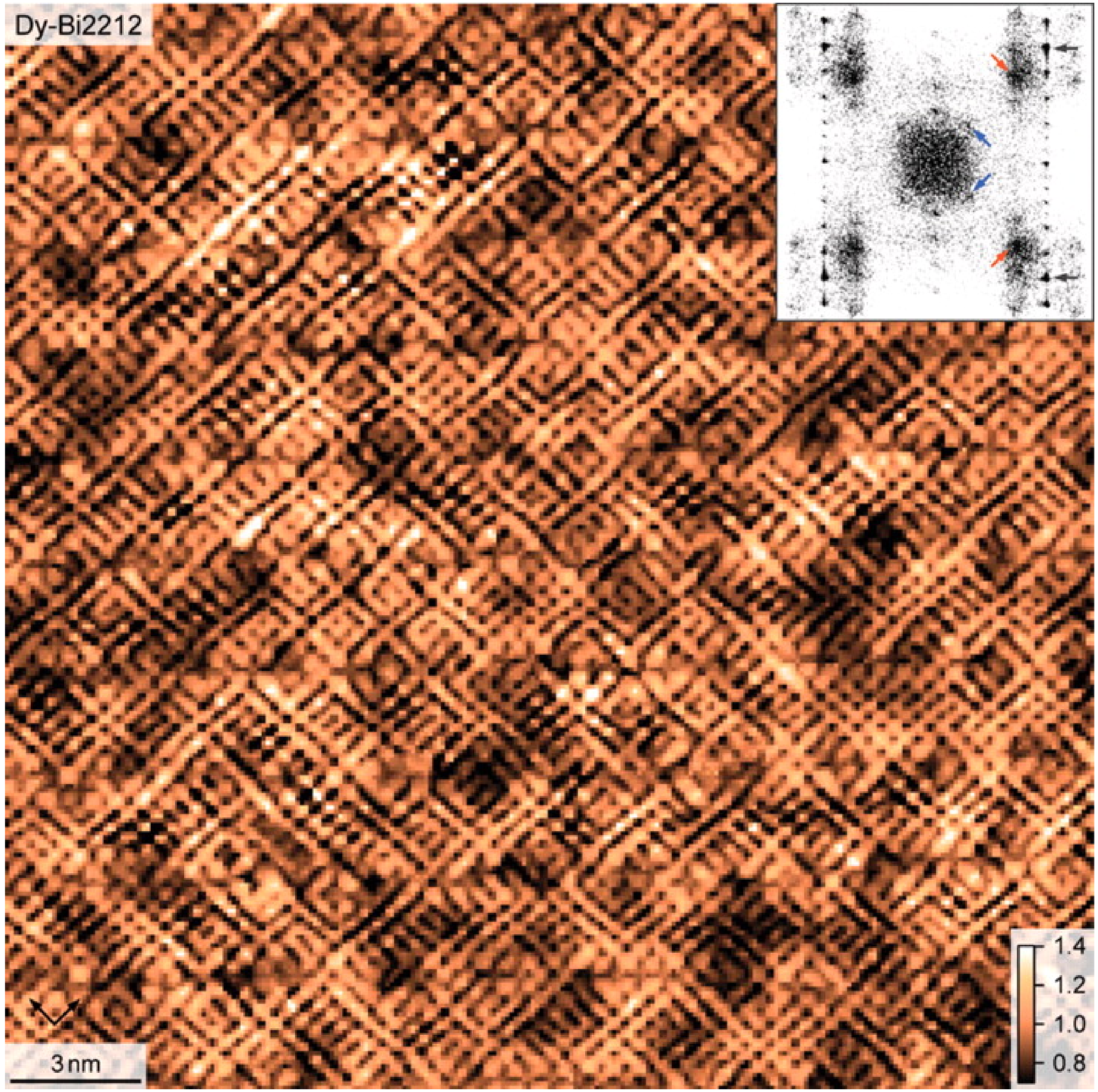
Prediction of periodic LDOS modulations near vortices:
K. Park and S. Sachdev, Phys. Rev. B 64, 184510 (2001).



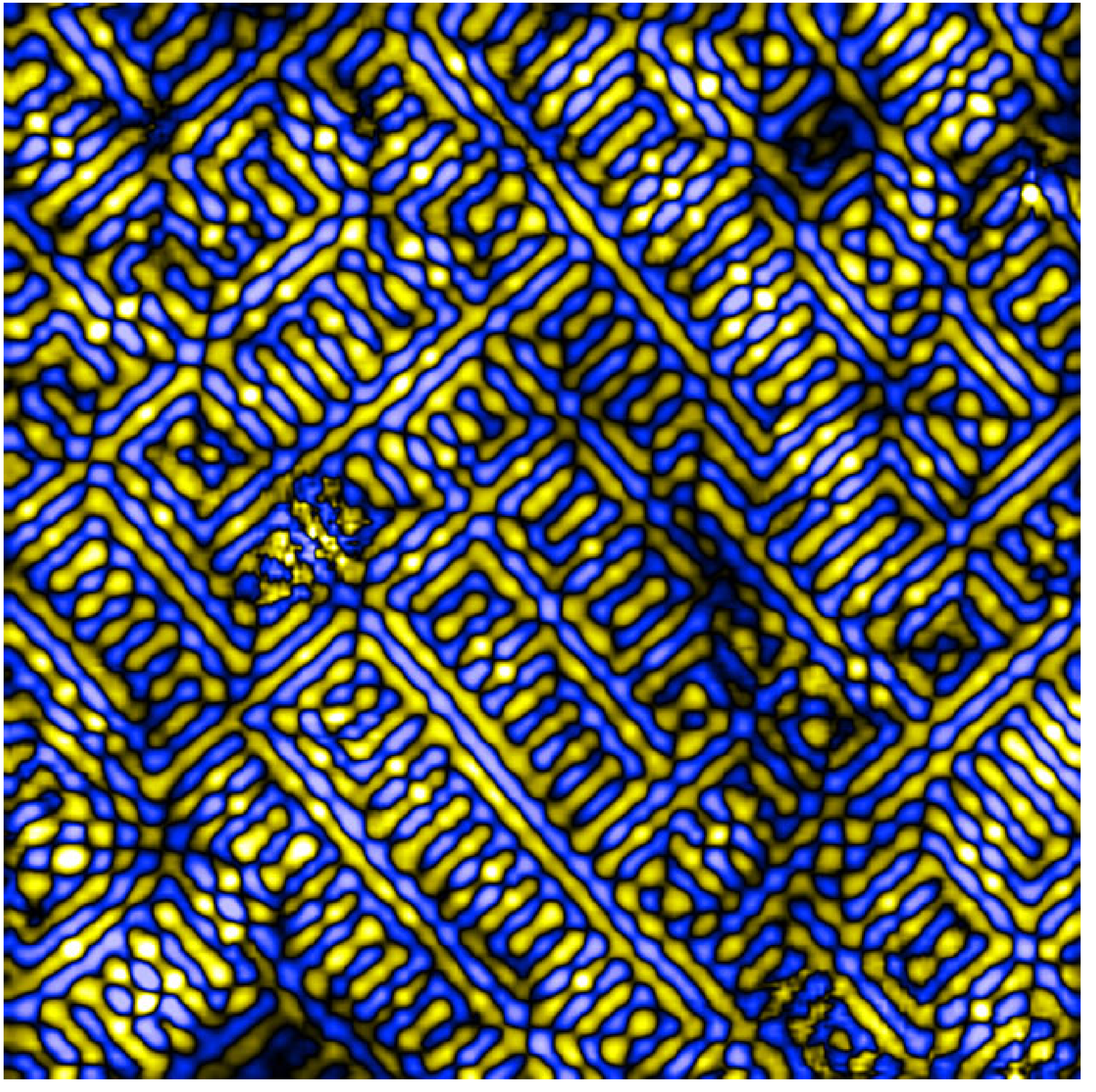
A
Temperature
AFI
PG
ECG
dSC
~2-3 %
~5-10 %
~15 %
hole concentration



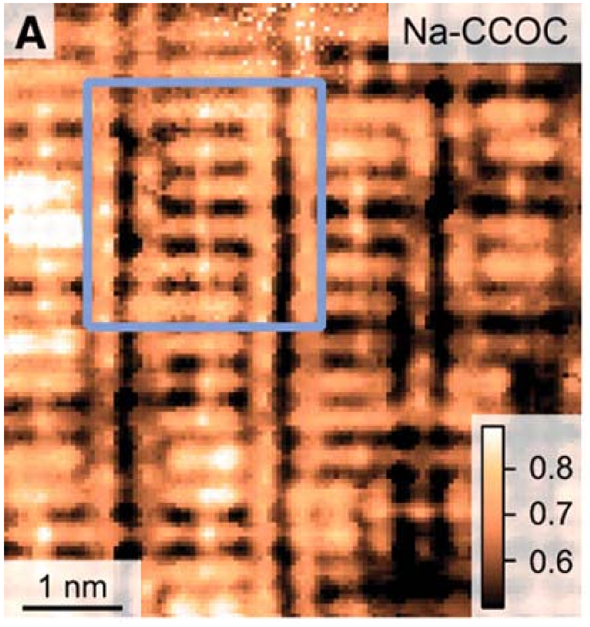
STM in zero field



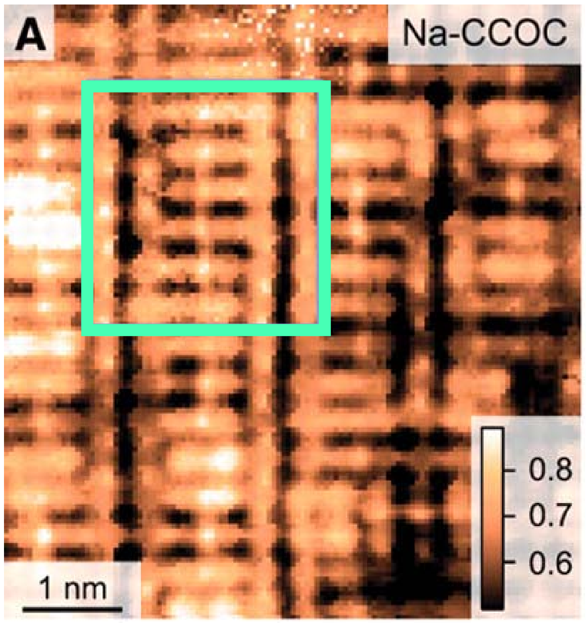
Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



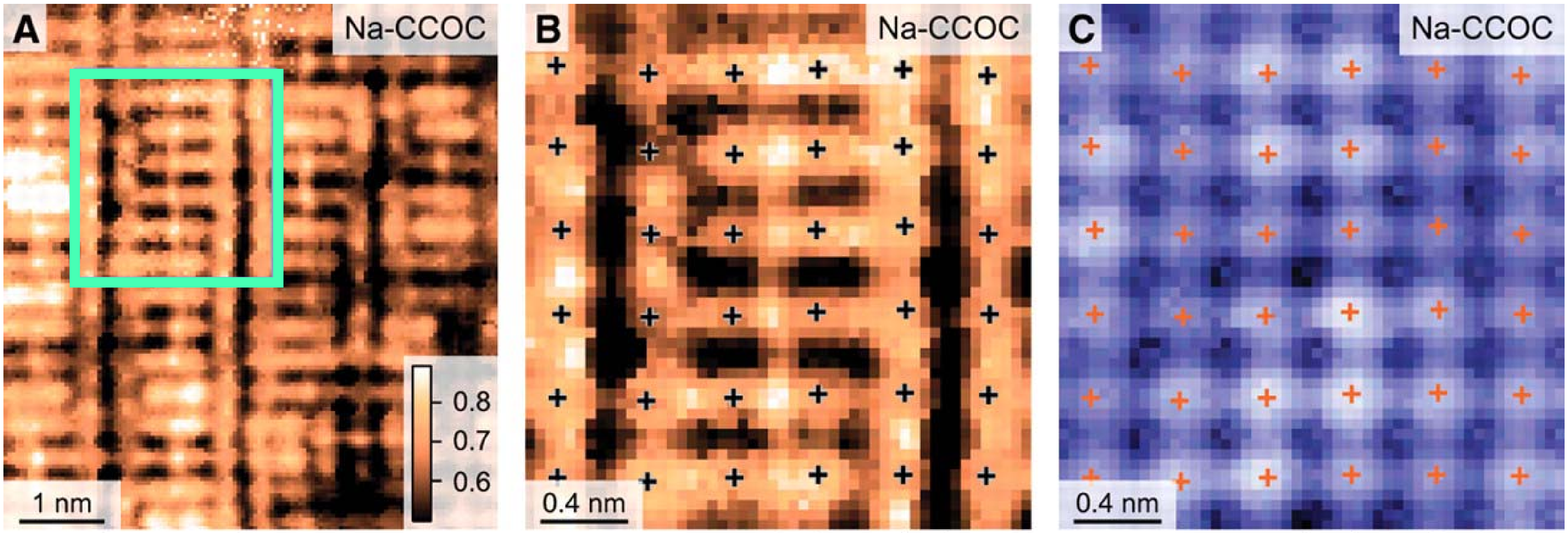
Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



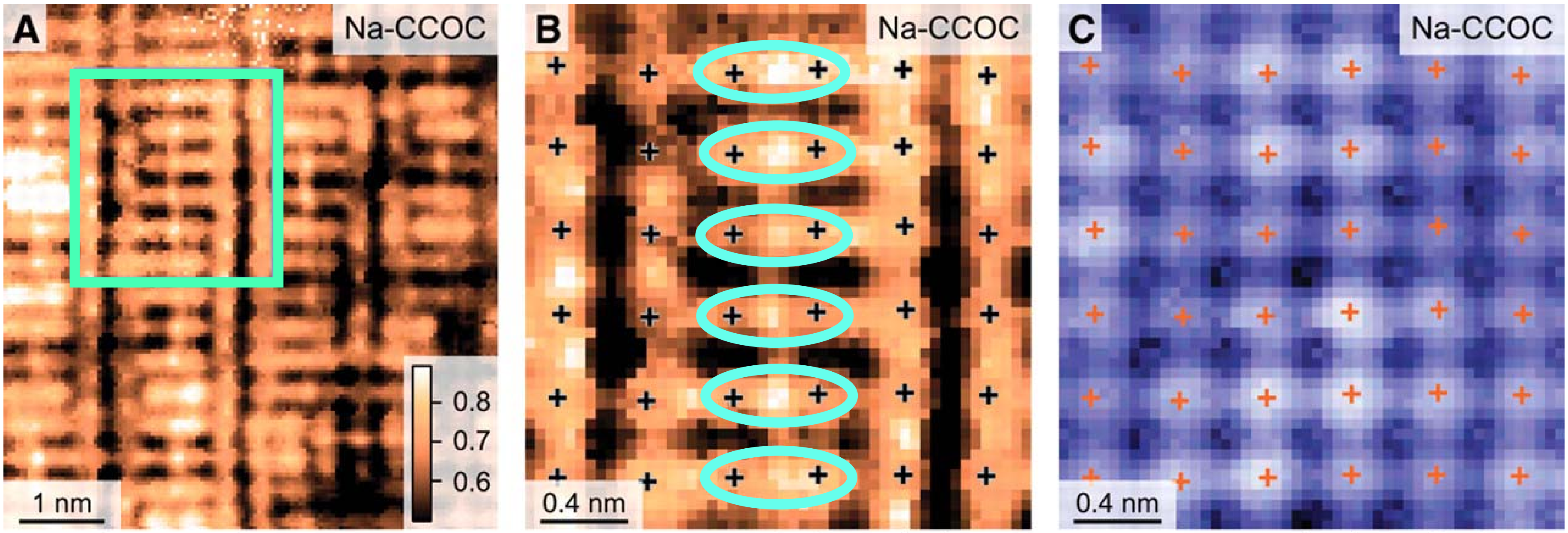
Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



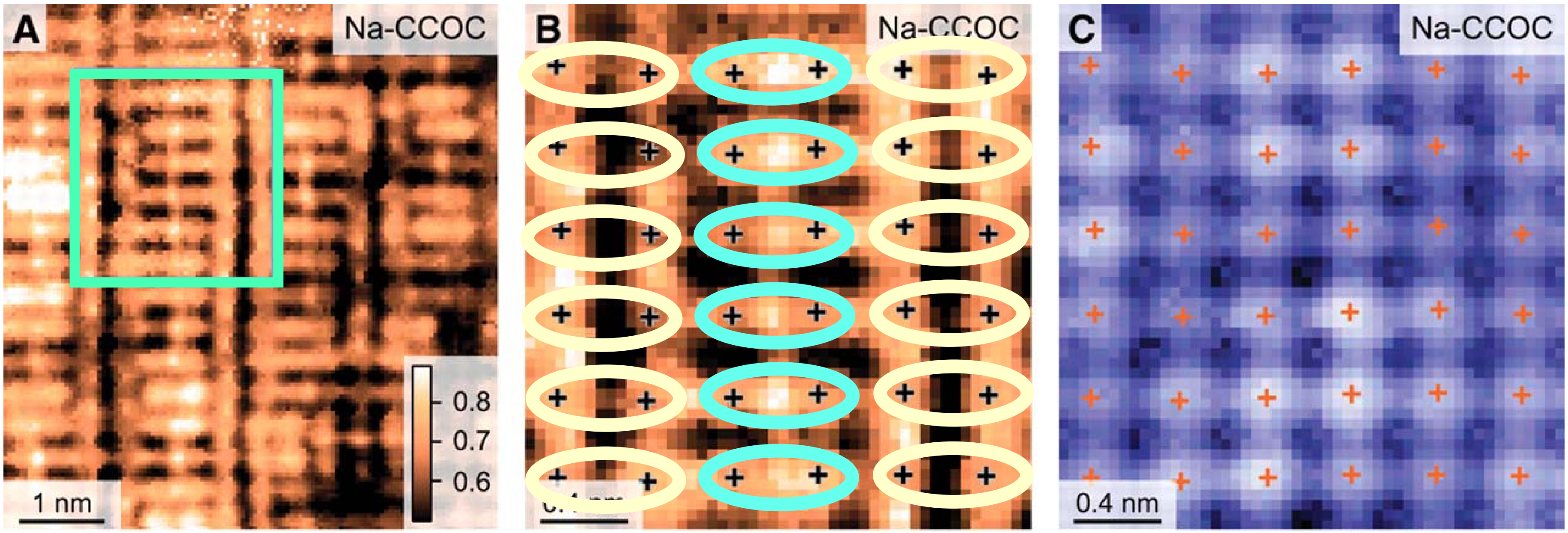
Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)



“Glassy” Valence Bond Solid (VBS)

Y. Kohsaka, C. Taylor, K. Fujita, A. Schmidt, C. Lupien, T. Hanaguri, M. Azuma, M. Takano, H. Eisaki, H. Takagi, S. Uchida, and J. C. Davis, *Science* **315**, 1380 (2007)

Outline

1. Our model

2. Influence of electronic quasiparticles on vortex motion

3. Influence of vortex quantum zero-point motion on electronic quasiparticles

4. Aharonov-Bohm phases in vortex quantum fluctuations and VBS modulations in LDOS

I. The model

Degrees of freedom

- We consider a *point vortex* (with vanishing core radius) whose (first-quantized) position is $\mathbf{r}_v(\tau)$. The τ dependence represents the zero-point quantum motion of this vortex.

- The Bogoliubov quasiparticles are represented at low energies by the (second-quantized) Dirac field $\Psi(\mathbf{r}, \tau)$.

- The vortex motion is also influenced by the background density of Cooper pairs, via the Magnus force. We will ignore this for now, and consider its important effects later.

A single vortex in a d-wave superconductor.
Effective low energy action

After the Franz-Tesanovic gauge transformation, this vortex appears as a π flux tube to the fermionic quasiparticles. The complete low energy theory for the vortex and the fermionic "Dirac" quasiparticles is then

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\Psi(\mathbf{r},\tau)\mathcal{D}\mathbf{r}_v(\tau)\exp\left(-\mathcal{S}\right) \\
\mathcal{S} &= \int d^2r d\tau \overline{\Psi}\gamma^\mu(\partial_\mu - i a_\mu)\Psi \\
&+ \text{ additional terms from the "Doppler shift"}
\end{aligned}
$$

where

$$\vec{\nabla} \times \vec{a} = \pi\delta(\mathbf{r} - \mathbf{r}_v(\tau))$$

A single vortex in a d-wave superconductor. Effective low energy action

After the Franz-Tesanovic gauge transformation, this vortex appears as a π flux tube to the fermionic quasiparticles. The complete low energy theory for the vortex and the fermionic "Dirac" quasiparticles is then

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\Psi(\mathbf{r},\tau)\mathcal{D}\mathbf{r}_v(\tau)\exp\left(-\mathcal{S}\right) \\
\mathcal{S} &= \int d^2 r d\tau \overline{\Psi}\gamma^{\mu}(\partial_\mu - i a_\mu)\Psi \\
&+ \quad \text{additional terms from the “Doppler shift”}
\end{aligned}
$$

where

$$\vec{\nabla} \times \vec{a} = \pi\delta(\mathbf{r} - \mathbf{r}_v(\tau))$$

Note: The action is has no coupling constants, and much can be deduced simply by a $z = 1$ scaling analysis.

Outline

1. Our model

2. Influence of electronic quasiparticles on vortex motion

3. Influence of vortex quantum zero-point motion on electronic quasiparticles

4. Aharonov-Bohm phases in vortex quantum fluctuations and VBS modulations in LDOS

II. Influence of electronic quasiparticles on vortex motion

Integrate out the nodal quasiparticles and expand the resulting action in powers of $d\mathbf{r}_v/d\tau$. This gives a result of the form

$$\mathcal{S}\left[\mathbf{r}_v(\tau)\right] = \int \frac{d\omega}{2\pi} \left|\mathbf{r}_v(\omega)\right|^2 K(\omega)$$

Translational invariance implies $K(0) = 0$. The scaling dimension of $K(\omega)$ is 3, and this allows us to deduce its functional form.

Integrate out the nodal quasiparticles and expand the resulting action in powers of $d\mathbf{r}_v/d\tau$. We obtained:

$$\mathcal{S}\left[\mathbf{r}_v(\tau)\right] = \int \frac{d\omega}{2\pi} \left|\mathbf{r}_v(\omega)\right|^2 \left[\frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega|\right] + \ldots$$

Integrate out the nodal quasiparticles and expand the resulting action in powers of $d\mathbf{r}_v/d\tau$. We obtained:

$$\mathcal{S}\left[\mathbf{r}_v(\tau)\right] = \int \frac{d\omega}{2\pi} \left|\mathbf{r}_v(\omega)\right|^2 \left[\frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega|\right] + \ldots$$

A *finite* effective mass $m_v \sim \Lambda / v_F^2$

where $\Lambda \sim \Delta$ is a high energy cutoff. By power-counting, there are no infra-red singulatities to this order, and hence only an analytic dependence on ω is possible.

Integrate out the nodal quasiparticles and expand the resulting action in powers of $d\mathbf{r}_v/d\tau$. We obtained:

$$\mathcal{S}\left[\mathbf{r}_v(\tau)\right] = \int \frac{d\omega}{2\pi} \left|\mathbf{r}_v(\omega)\right|^2 \left[\frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega|\right] + \ldots$$

A *finite* effective mass $m_v \sim \Lambda / v_F^2$

where $\Lambda \sim \Delta$ is a high energy cutoff. By power-counting, there are no infra-red singulatities to this order, and hence only an analytic dependence on ω is possible.

Disagrees with N. B. Kopnin, and V. M. Vinokur, Phys. Rev. Lett **81**, 3952 (1998), who obtained a divergent mass $m_v \sim 1 / \sqrt{H}$ in an applied field H

Integrate out the nodal quasiparticles and expand the resulting action in powers of $d\mathbf{r}_v/d\tau$. We obtained:

$$\mathcal{S}\left[\mathbf{r}_v(\tau)\right] = \int \frac{d\omega}{2\pi} \left|\mathbf{r}_v(\omega)\right|^2 \left[\frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega|\right] + \ldots$$

↓

sub-Ohmic damping with

$$C_1 = v_F^{-2} \times \left(\text{Universal function of } v_\Delta / v_F \right)$$

Integrate out the nodal quasiparticles and expand the resulting action in powers of $d\mathbf{r}_v/d\tau$. We obtained:

$$\mathcal{S}\left[\mathbf{r}_v(\tau)\right] = \int \frac{d\omega}{2\pi} \left|\mathbf{r}_v(\omega)\right|^2 \left[\frac{m_v \omega^2}{2} + C_1 |\omega|^3 + C_2 T^2 |\omega|\right] + \ldots$$

Bardeen-Stephen viscous drag with

$$C_2 = v_F^{-2} \times \left(\text{Universal function of } {v_\Delta}/{v_F} \right)$$

Integrate out the nodal quasiparticles and expand the resulting action in powers of $d\mathbf{r}_v/d\tau$. We obtained:

$$\mathcal{S}\left[\mathbf{r}_v(\tau)\right] = \int \frac{d\omega}{2\pi} \left|\mathbf{r}_v(\omega)\right|^2 \left[\frac{m_v \omega^2}{2} + C_1|\omega|^3 + C_2 T^2 |\omega|\right] + \ldots$$

Bardeen-Stephen viscous drag with

$$C_2 = v_F^{-2} \times \left(\text{Universal function of } {}^{v_\Delta}\!/\!{}_{v_F} \right)$$

Negligible damping of vortex from nodal quasiparticles at T=0; can expect significant quantum zero-point motion. Damping increases as T^2 at higher T

Outline

1. Our model
2. Influence of electronic quasiparticles on vortex motion
3. Influence of vortex quantum zero-point motion on electronic quasiparticles
4. Aharonov-Bohm phases in vortex quantum fluctuations and VBS modulations in LDOS

III. Influence of vortex quantum zero-point motion on electronic quasiparticles

A single vortex in a d-wave superconductor. Effective low energy action for electronic quasipaticles

Add a harmonic pinning potential to the vortex, and ignore damping of vortex motion

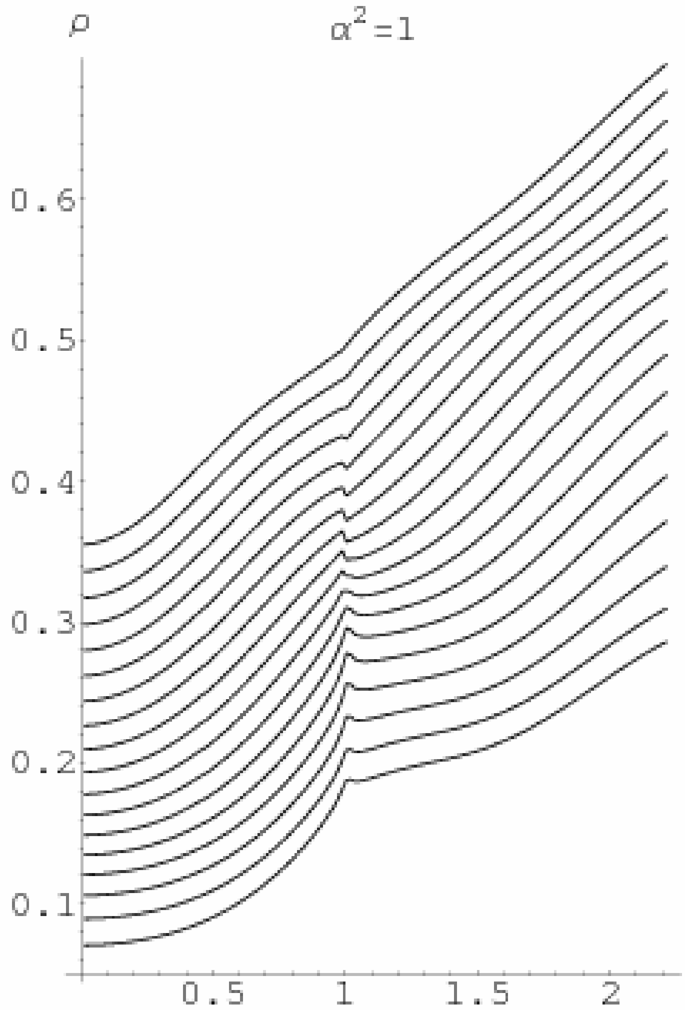
$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\Psi(\mathbf{r},\tau)\mathcal{D}\mathbf{r}_v(\tau)\exp\left(-\mathcal{S}\right) \\
\mathcal{S} &= \int d^2 r d\tau \overline{\Psi}\gamma^{\mu}(\partial_\mu - i a_\mu)\Psi \\
&+ \frac{1}{2}m_v\left(\frac{d\mathbf{r}_v}{d\tau}\right)^2 + \frac{1}{2}m_v\omega_v^2\mathbf{r}_v^2
\end{aligned}
$$

where

$$\vec{\nabla}\times\vec{a} = \pi\delta(\mathbf{r}-\mathbf{r}_v(\tau))$$

Now integrate out $\mathbf{r}_v$ and determine change in electronic LDOS.

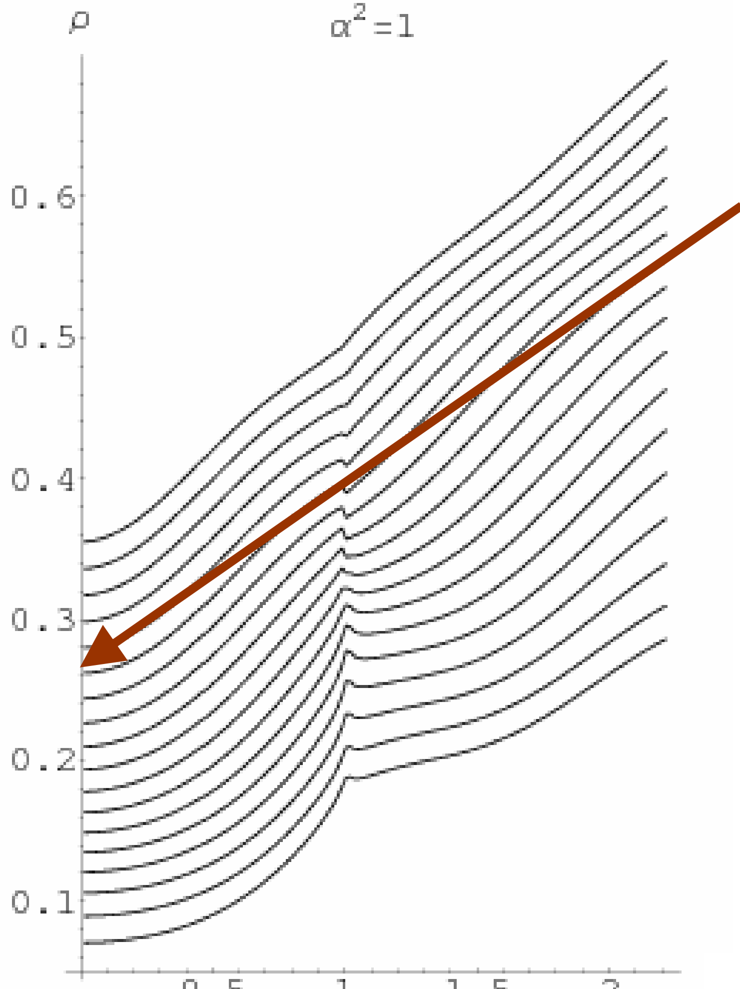
Influence of the quantum oscillating vortex on the LDOS



ω / ω_v

$$\alpha^2 = \frac{m v_F^2}{\omega_v} = 1$$

Influence of the quantum oscillating vortex on the LDOS



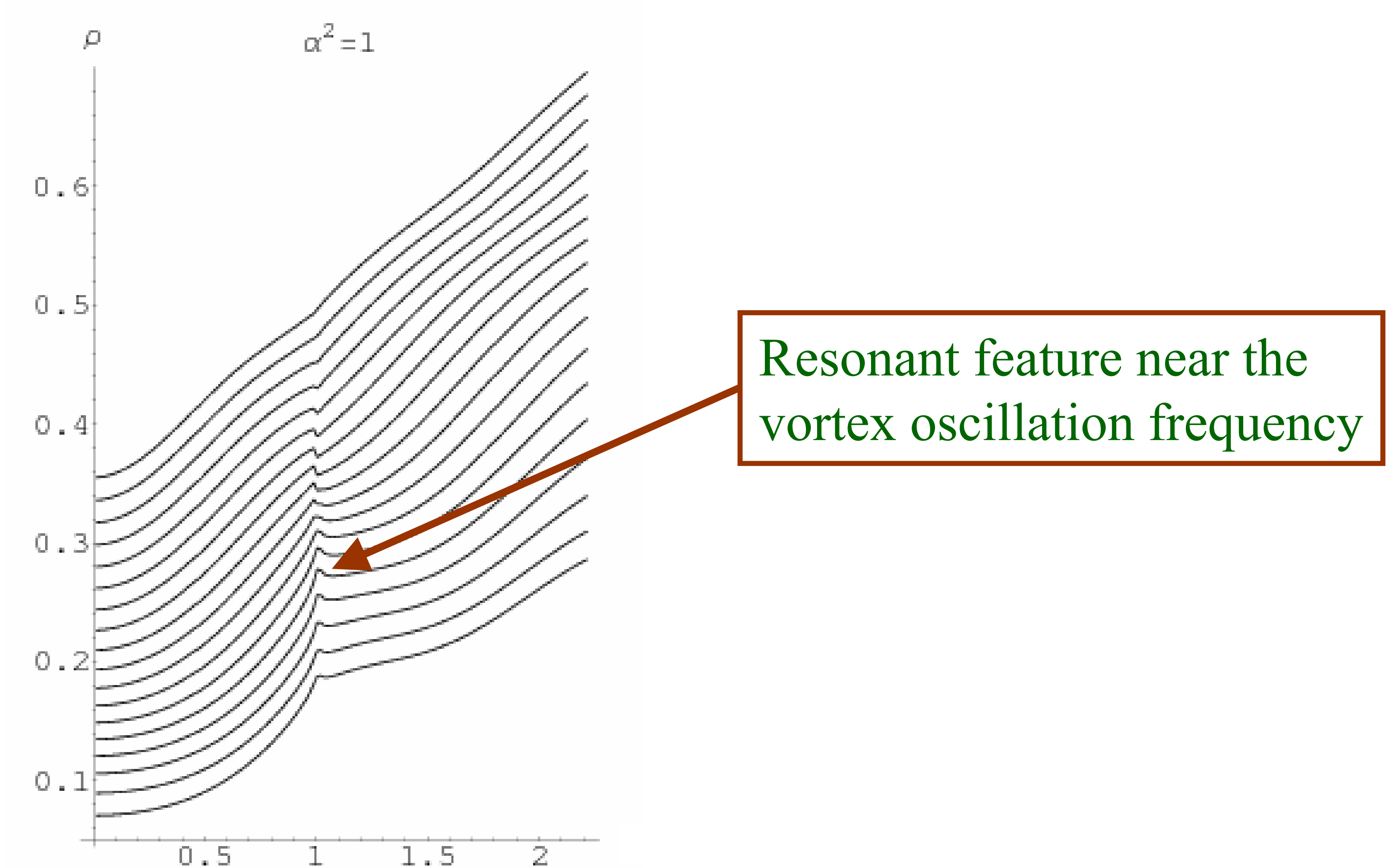
ω / ω_v

$$\alpha^2 = \frac{mv_F^2}{\omega_v} = 1$$

No zero bias peak.

Absent because of small core size.

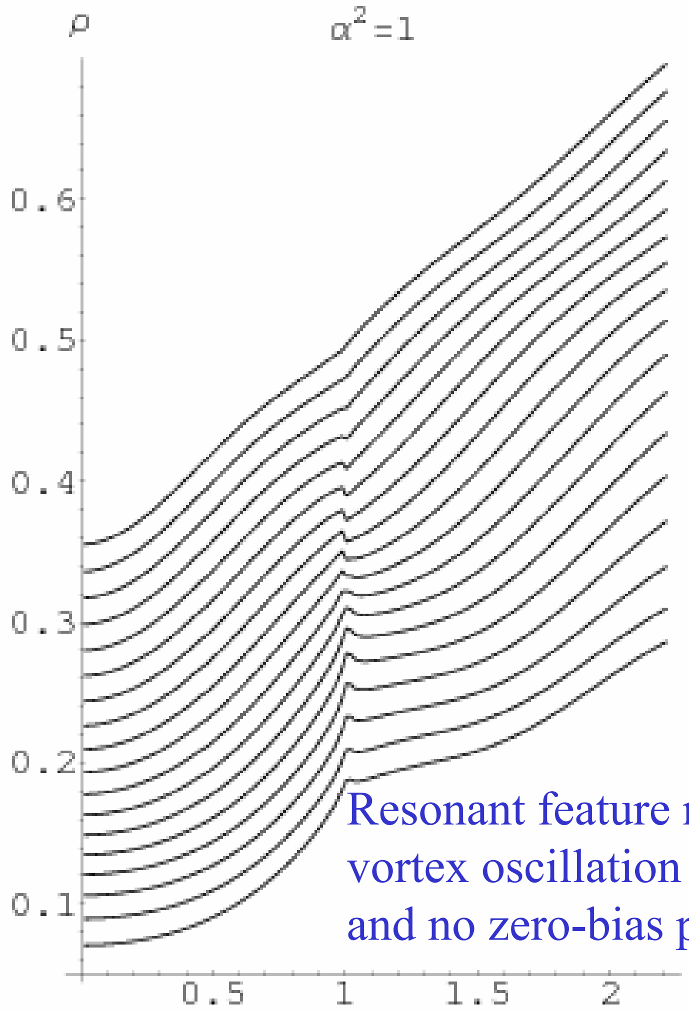
Influence of the quantum oscillating vortex on the LDOS



ω / ω_v

$$\alpha^2 = \frac{m v_F^2}{\omega_v} = 1$$

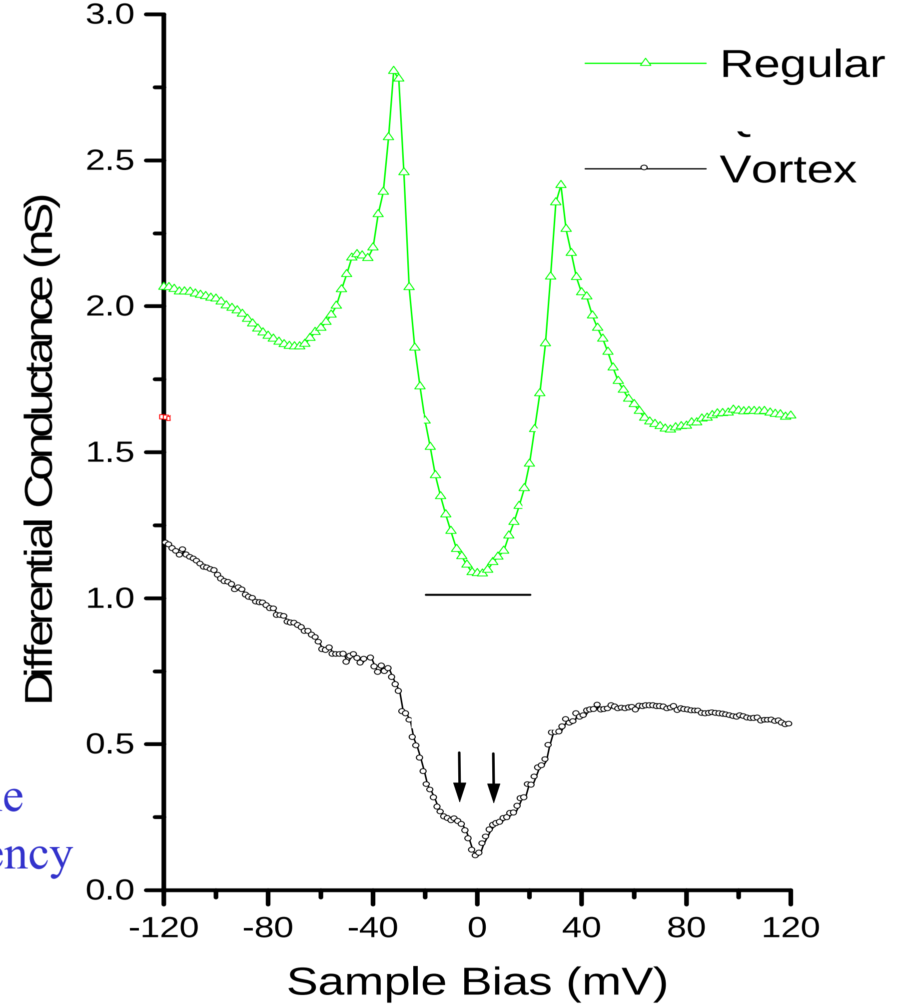
Influence of the quantum oscillating vortex on the LDOS



Resonant feature near the vortex oscillation frequency and no zero-bias peak

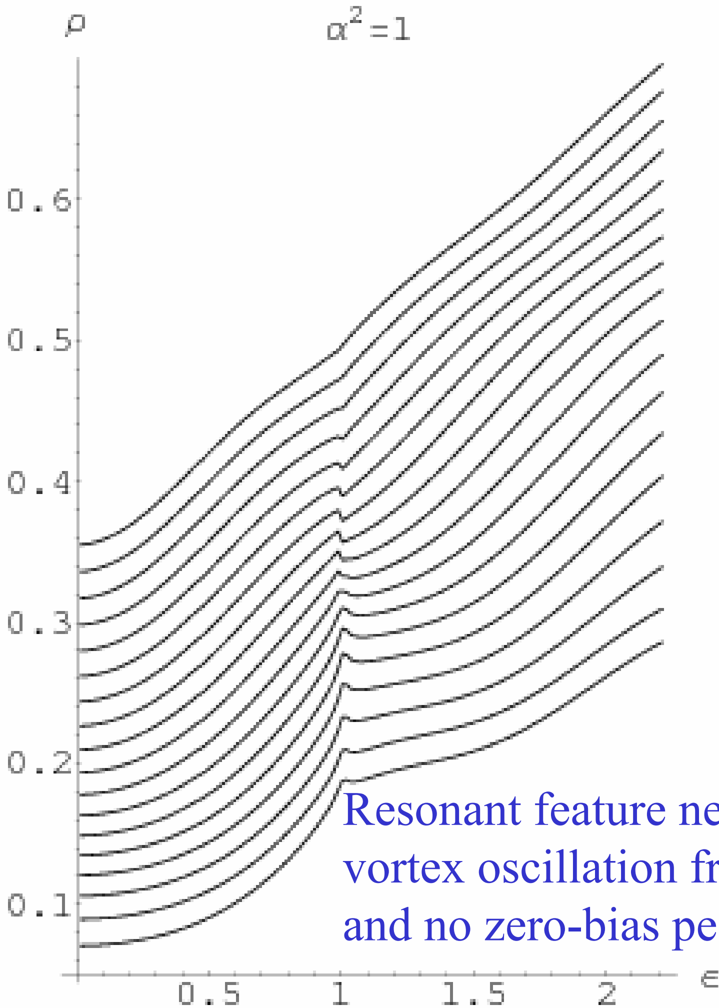
ω / ω_v

$$\alpha^2 = \frac{mv_F^2}{\omega_v} = 1$$

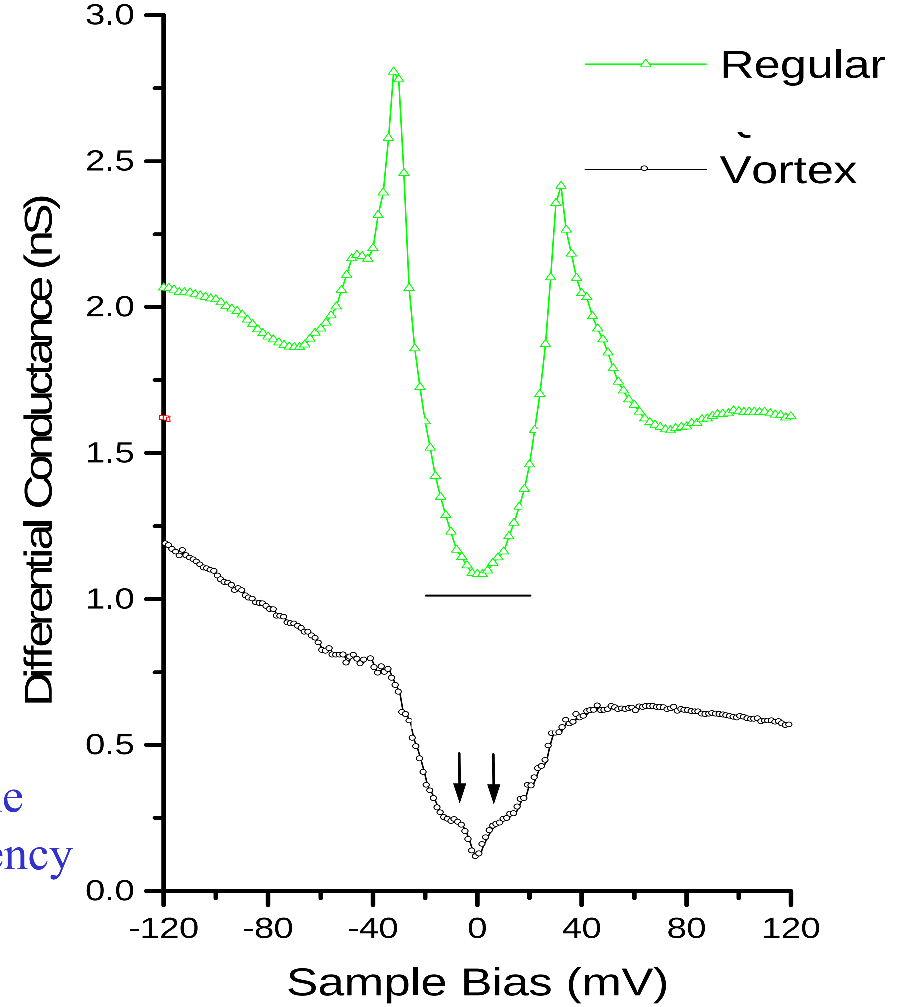


I. Maggio-Aprile *et al.* *Phys. Rev. Lett.* **75**, 2754 (1995)
S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

Influence of the quantum oscillating vortex on the LDOS



Resonant feature near the vortex oscillation frequency and no zero-bias peak



I. Maggio-Aprile *et al.* *Phys. Rev. Lett.* **75**, 2754 (1995)

S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

Is there an independent way to determine m_v and ω_v ?

<u>Outline</u>

1. Our model

2. Influence of electronic quasiparticles on vortex motion

3. Influence of vortex quantum zero-point motion on electronic quasiparticles

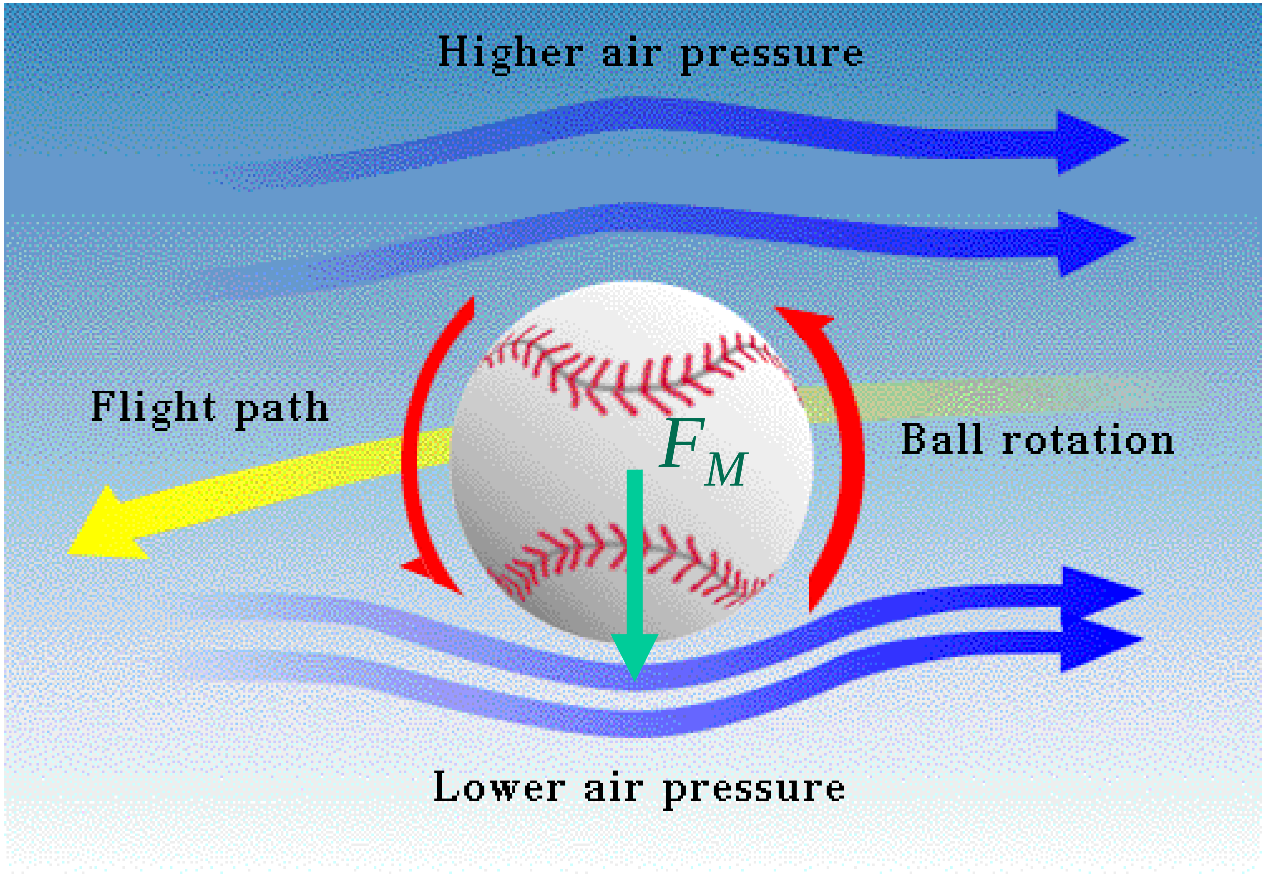
4. Aharonov-Bohm phases in vortex quantum fluctuations and VBS modulations in LDOS

IV. Aharonov-Bohm phases in vortex motion and VBS modulations in LDOS

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
C. Lannert, M.P.A. Fisher, and T. Senthil, *Phys. Rev.* B **63**, 134510 (2001).
S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002).
T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, *Phys. Rev.* B **70**, 144407 (2004).
L. Balents, L. Bartosch, A. Burkov, S. Sachdev, and K. Sengupta, *Phys. Rev.* B **71**, 144508 (2005).

See also Z. Tesanovic, *Phys. Rev. Lett.* **93**, 217004 (2004); A. Melikyan and Z. Tesanovic, *Phys. Rev.* B **71**, 214511 (2005).

In ordinary fluids, vortices experience the Magnus Force



$$F_M = (\text{mass density of air}) \bullet (\text{velocity of ball}) \bullet (\text{circulation})$$

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s\cdot d\mathbf{r}\right)\\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\end{aligned}$$

where ρ = number density of bosons

$\mathbf{v}_s$ = local velocity of superfluid

$\mathbf{r}_v$ = position of vortex

For a vortex in a superfluid, this is

$$\begin{aligned}\mathbf{F}_M &= (m\rho)\left(\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\right)\left(\oint \mathbf{v}_s\cdot d\mathbf{r}\right)\\ &= nh\rho\left(\mathbf{v}_s - \frac{d\mathbf{r}_v}{dt}\right)\times\hat{\mathbf{z}}\\ &= n\left(\mathbf{E} + \frac{d\mathbf{r}_v}{dt}\times\mathbf{B}\right)\end{aligned}$$

where $\mathbf{E} = \rho\mathbf{v}_s \times \hat{\mathbf{z}}$ and $\mathbf{B} = -h\rho\hat{\mathbf{z}}$

Dual picture:
The vortex is a quantum particle with dual "electric" charge *n*, moving in a dual "magnetic" field of strength = $h\times$(number density of Bose particles)

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett.* **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev.* B **39**, 2756 (1989)

Influence of the periodic potential on vortex motion

Let the Hamiltonian of a single vortex be $\mathcal{H}_v$.

In general, this is a very complicated object, but we can obtain all needed information by symmetry considerations.

The Hamiltonian $\mathcal{H}_v$ should commute with T_x, the operator which translates the square lattice by one site in the x direction (and similarly for T_y):

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

However, T_x and T_y do not commute with each other.

Under translation along a distance $\mathbf{s}$, a vortex picks up a a Aharanov-Bohm phase factor $\exp\left(i \int_0^{\mathbf{s}} d\mathbf{r} \cdot \mathbf{A}\right)$.

Consequently

$$\boxed{T_x T_y = \exp\left(i\phi\right) T_y T_x}$$

where ϕ is the dual "flux" through a unit cell, This "flux" has the value

$$\boxed{\phi = 2\pi f}$$

where f is the filling fraction of bosons (Cooper pairs). We will consider the case of rational filling fraction $f = p/q$, where p, q are relatively prime integers.

Bosons on the square lattice at filling fraction f=p/q

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

$$T_x T_y = \exp\left(2\pi i p/q\right) T_y T_x$$

Bosons on the square lattice at filling fraction f=p/q

$$[T_x, \mathcal{H}_v] = 0$$

$$[T_y, \mathcal{H}_v] = 0$$

$$T_x T_y = \exp\left(2\pi i p/q\right) T_y T_x$$

Theorem:
The ground state of $\mathcal{H}_v$ is at least q-fold degenerate. We can choose a basis, $|m\rangle$ $(m = 0 \ldots (q-1))$, for the ground states such that

$$\begin{aligned} T_x|m\rangle &= |m+1\rangle \\ T_y|m\rangle &= e^{2\pi i m p/q}|m\rangle \end{aligned}$$

Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any impurity breaks translational invariance, and so chooses a prefered orientation in vortex "flavor space". This chooses some linear combination among the ground states: $|G\rangle = \sum_{m=0}^{q-1} c_m |m\rangle$

Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any impurity breaks translational invariance, and so chooses a prefered orientation in vortex "flavor space". This chooses some linear combination among the ground states: $|G\rangle = \sum_{m=0}^{q-1} c_m |m\rangle$

- In terms of these states, we can also introduce the operators ρ_{mn} defined by

$$\rho_{mn} = e^{i\pi mnp/q} \sum_{\ell=0}^{q-1} e^{2\pi i \ell m} |\ell\rangle\langle \ell + n|$$

  which transform T_x, T_y like the Fourier components of a density $\rho_{\mathbf{Q}}$ at the wavevectors $\mathbf{Q} = 2\pi f(m,n)$:

$$T_x : \rho_{\mathbf{Q}} \to e^{i\mathbf{Q}\cdot\hat{x}} \rho_{\mathbf{Q}} \qquad T_y : \rho_{\mathbf{Q}} \to e^{i\mathbf{Q}\cdot\hat{y}} \rho_{\mathbf{Q}}$$

Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any impurity breaks translational invariance, and so chooses a prefered orientation in vortex "flavor space". This chooses some linear combination among the ground states: $|G\rangle = \sum_{m=0}^{q-1} c_m |m\rangle$
- In terms of these states, we can also introduce the operators ρ_{mn} defined by

$$\rho_{mn} = e^{i\pi mnp/q} \sum_{\ell=0}^{q-1} e^{2\pi i \ell m} |\ell\rangle\langle \ell + n|$$

which transform T_x, T_y like the Fourier components of a density $\rho_{\mathbf{Q}}$ at the wavevectors $\mathbf{Q} = 2\pi f(m,n)$:

$$T_x : \rho_{\mathbf{Q}} \to e^{i\mathbf{Q}\cdot\hat{x}} \rho_{\mathbf{Q}} \qquad T_y : \rho_{\mathbf{Q}} \to e^{i\mathbf{Q}\cdot\hat{y}} \rho_{\mathbf{Q}}$$

- It is not possible to choose the c_m such that $\langle G|\rho_{\mathbf{Q}}|G\rangle = 0$ for all $\mathbf{Q} \neq 0$.

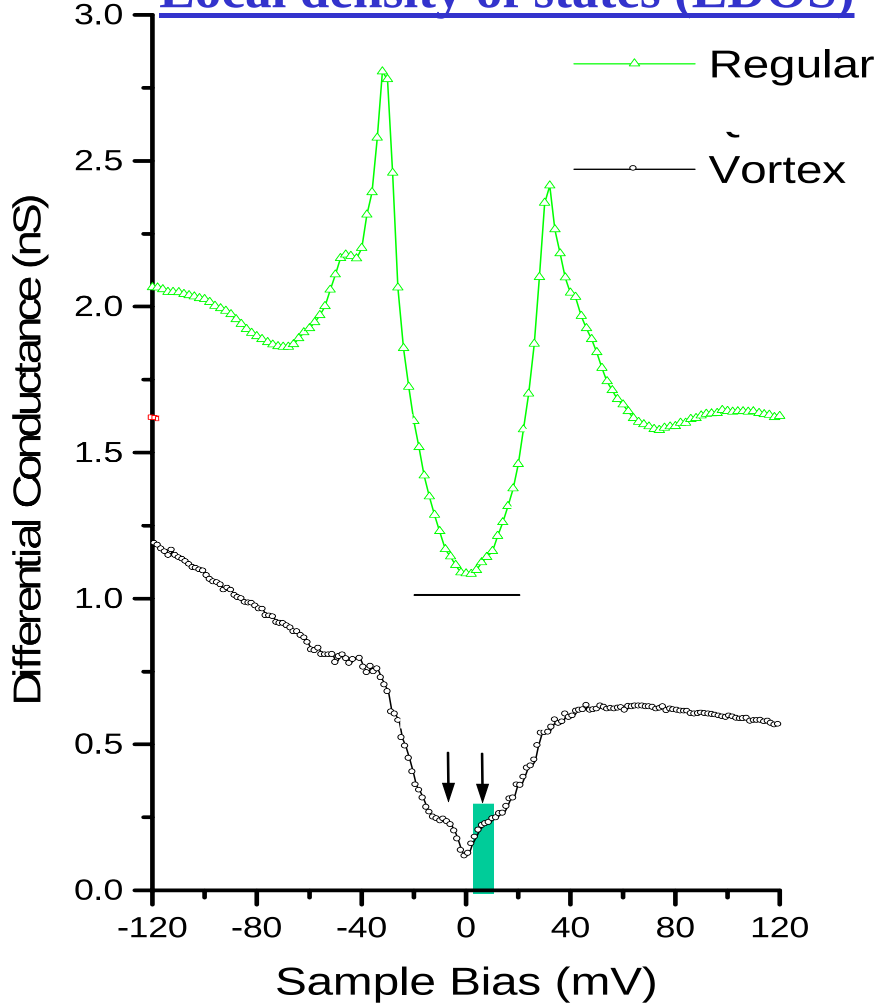
Properties of a quantum-fluctuating vortex weakly pinned by an impurity.

- Any pinned vortex exhibits modulations in VBS-like observables at the wavevectors Q over the region in which the vortex executes its quantum zero-point motion.

STM around vortices induced by a magnetic field in the superconducting state

J. E. Hoffman, E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* **295**, 466 (2002).

Local density of states (LDOS)



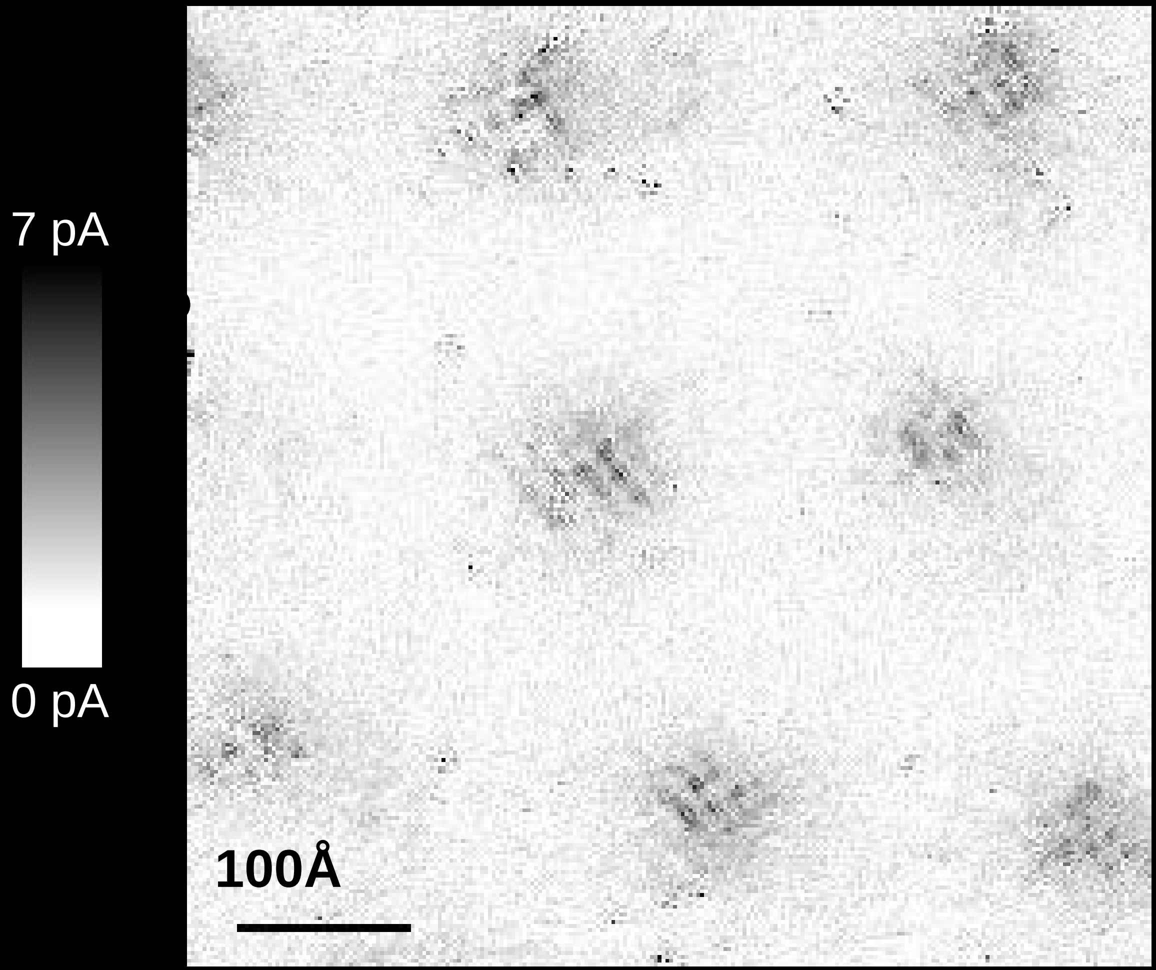
1Å spatial resolution image of integrated LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$

(1meV to 12 meV) at B=5 Tesla.

I. Maggio-Aprile *et al. Phys. Rev. Lett.* **75**, 2754 (1995).

S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



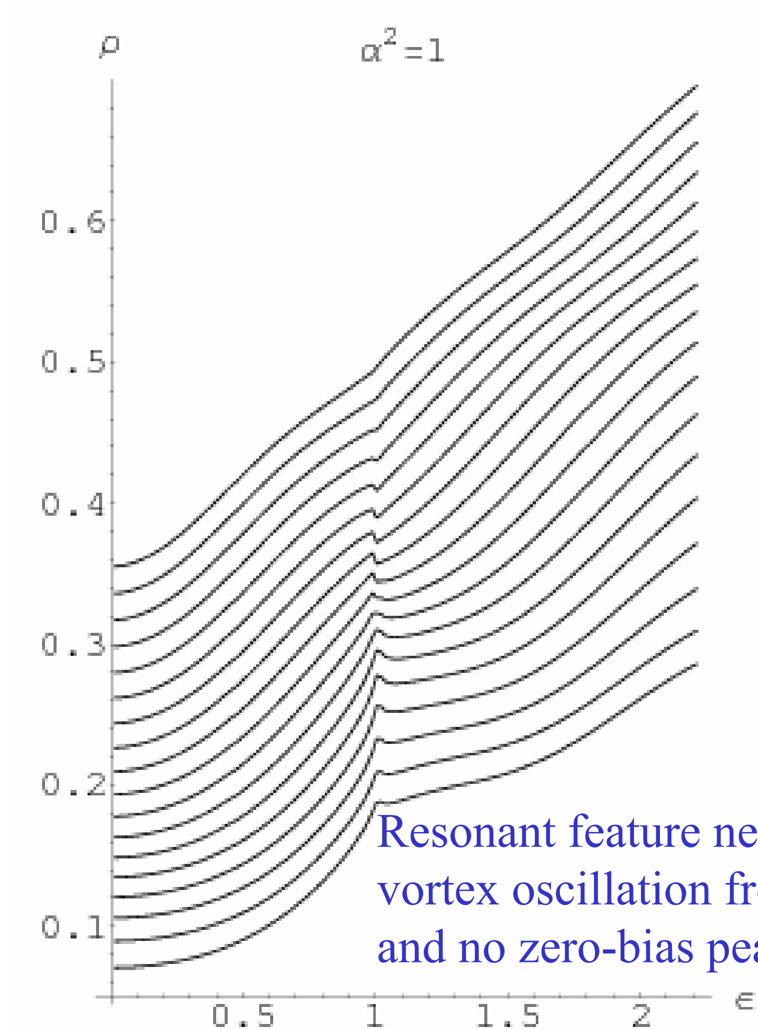
Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

J. Hoffman et al., *Science* 295, 466 (2002).
G. Levy et al., Phys. Rev. Lett. 95, 257005 (2005).

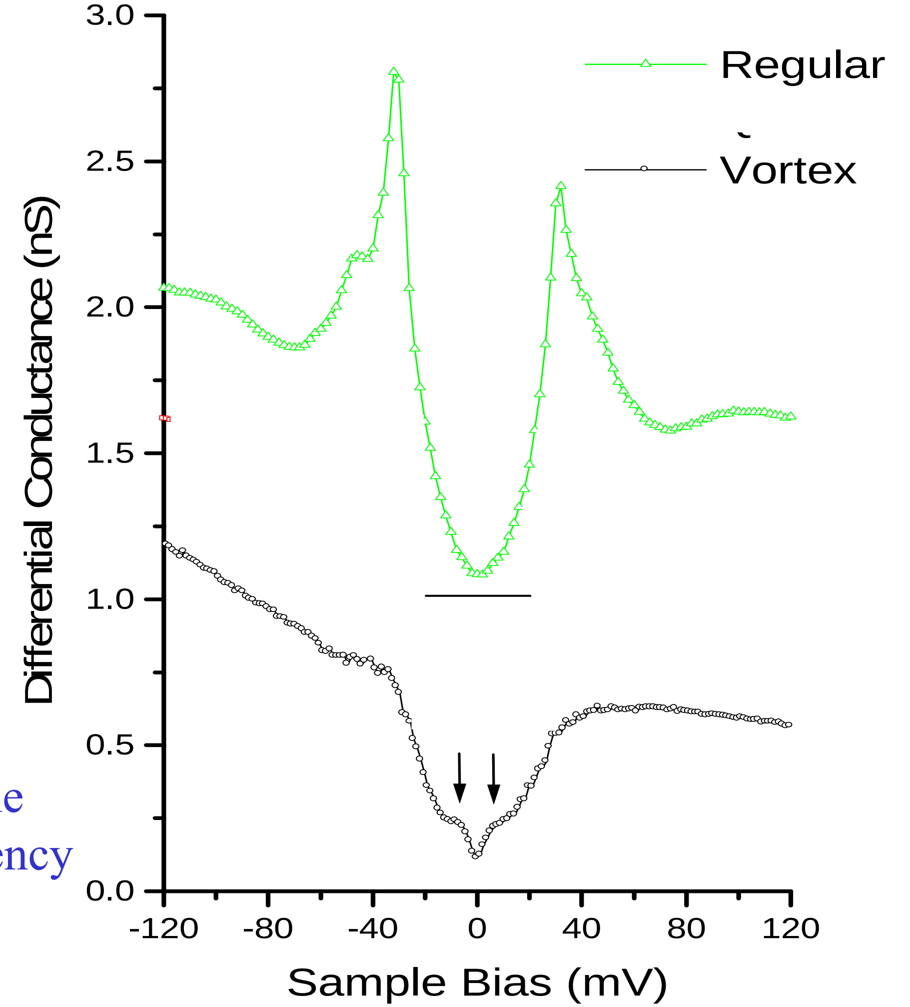
Prediction of periodic LDOS modulations near vortices:
K. Park and S. Sachdev, Phys. Rev. B 64, 184510 (2001).

Using as <u>input</u> (i) the size of the "checkerboard halo" in STM as a measure of the zero-point motion radius of the vortex, and (ii) the forces between the vortices as determined from an estimate of the superfluid stiffness, we obtain as <u>output</u> an estimate of $m_v \approx 2 - 9 m_e$ and the vortex oscillation frequency $\omega_v \approx 2 - 7$ meV.

Influence of the quantum oscillating vortex on the LDOS



Resonant feature near the vortex oscillation frequency and no zero-bias peak



I. Maggio-Aprile *et al.* *Phys. Rev. Lett.* **75**, 2754 (1995)

S.H. Pan *et al*. *Phys. Rev. Lett*. **85**, 1536 (2000).

Independent estimate of ω_v gives a consistency check.

Deconfined quantum criticality

What happens when the vortex quantum fluctuation length-scale becomes large ?

Deconfined quantum criticality

What happens when the vortex quantum fluctuation length-scale becomes large ?

Landau-forbidden quantum phase transition between a superfluid and an insulator with VBS order.

Conclusions

- Quantum zero point motion of vortices provides a unified explanation for many LDOS features observed in STM experiments.
- Size of LDOS modulation halo allows estimate of the inertial mass of a vortex
- The deduced energy of the LDOS sub-gap peak provides a strong consistency check of our proposal
- Direct detection of vortex zero-point motion may be possible in inelastic neutron or light-scattering experiments