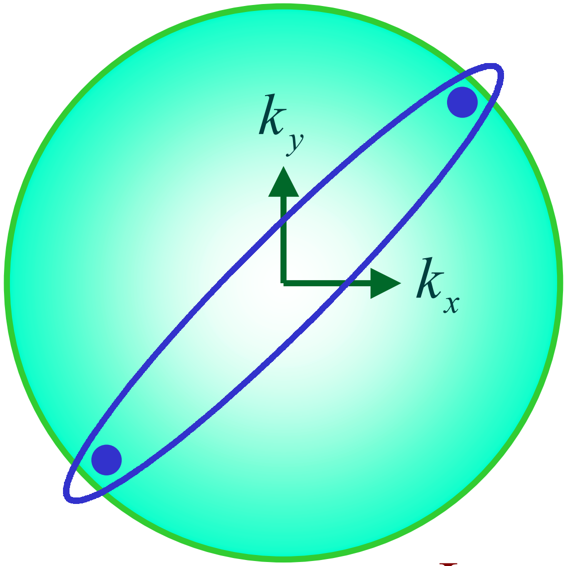


Hole-doped cuprates are BCS superconductors with

$\langle c^\dagger_{\boldsymbol{k}\uparrow} c^\dagger_{\boldsymbol{-k}\downarrow} \rangle \equiv \Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y \right)$ d-wave pairing

$\langle \vec{S} \rangle = 0$ spin-singlet

Low energy excitations:

Superflow: $\Delta_0 \to \Delta_0 e^{i\theta}$

$S = 1/2$ fermionic quasiparticles: $E_{\boldsymbol{k}} = \sqrt{\varepsilon_{\boldsymbol{k}}^2 + \Delta_{\boldsymbol{k}}^2}$

BCS theory also predicts that the Fermi surface, with gapless quasiparticles, will reveal itself when $\Delta_0 \to 0,$ either locally or globally at low temperatures. Δ_0 can be suppressed by a strong magnetic field, and near vortices, impurities and interfaces.

Superconductivity in a doped Mott insulator

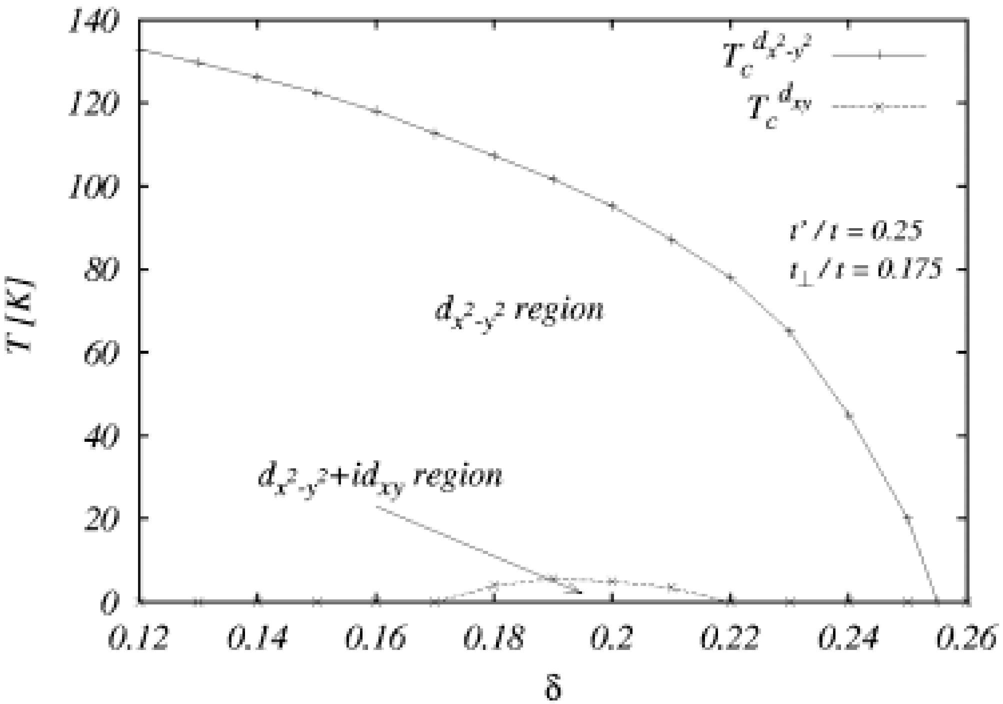
Hypothesis: cuprate superconductors have low energy excitations associated with additional order parameters

The new order parameters can
- have long range order co-existing with superconductivity
- appear with long-range correlations when Δ_0 is suppressed globally
- appear with short-range correlations when Δ_0 is suppressed locally
- characterize states which are "nearby" in parameter space, which could be either insulators or superconductors

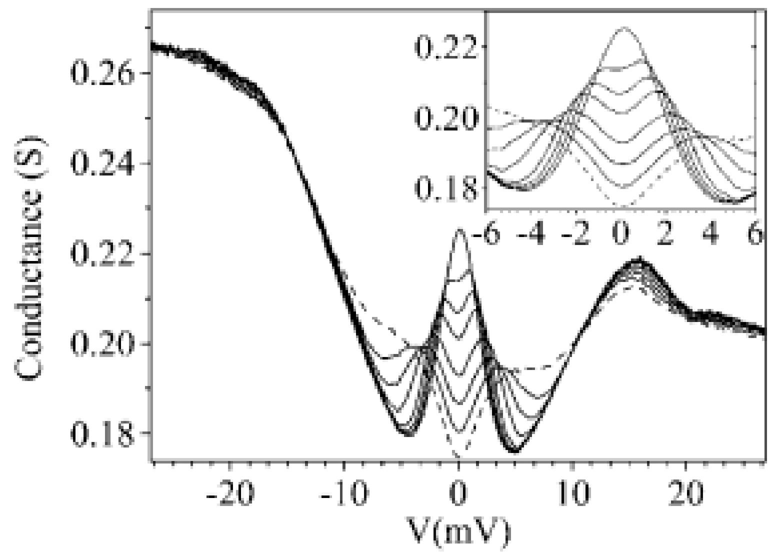
Candidates

I. Broken time-reversal symmetry

$$d + id' \text{ pairing: } \Delta_{\boldsymbol{k}} = \Delta_0 \left(\cos k_x - \cos k_y \right) + i\Delta_1 \sin k_x \sin k_y$$



G. Sangiovanni, M. Capone, S. Caprara, C. Castellani, C. Di Castro, M. Grilli, cond-mat/0111107

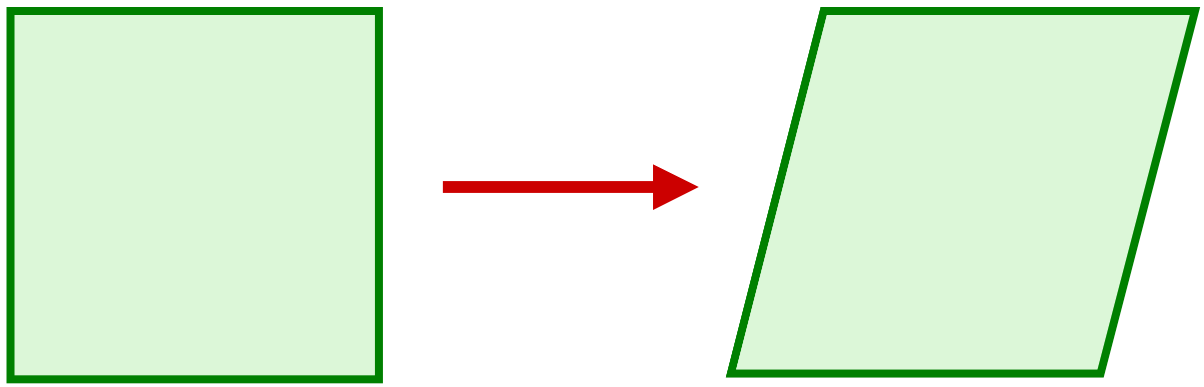


Some experimental evidence in tunneling measurements on YBCO by Y. Dagan and G. Deutscher, *Phys. Rev. Lett.* **87**, 177004 (2001).

See also M. Covington, M. Aprili, E. Paraoanu, L. H. Greene, F. Xu, J. Zhu, and C. A. Mirkin *Phys. Rev. Lett.* **79**, 277 (1997)

Photoemission measurements of
A. Kaminski, S. Rosenkranz, H. M. Fretwel, J. C. Campuzano, Z. Li, H. Raffy, W. G. Cullen, H. You, C. G. Olson, C. M. Varma, and H. Ho, *Nature* **416**, 610 (2002).

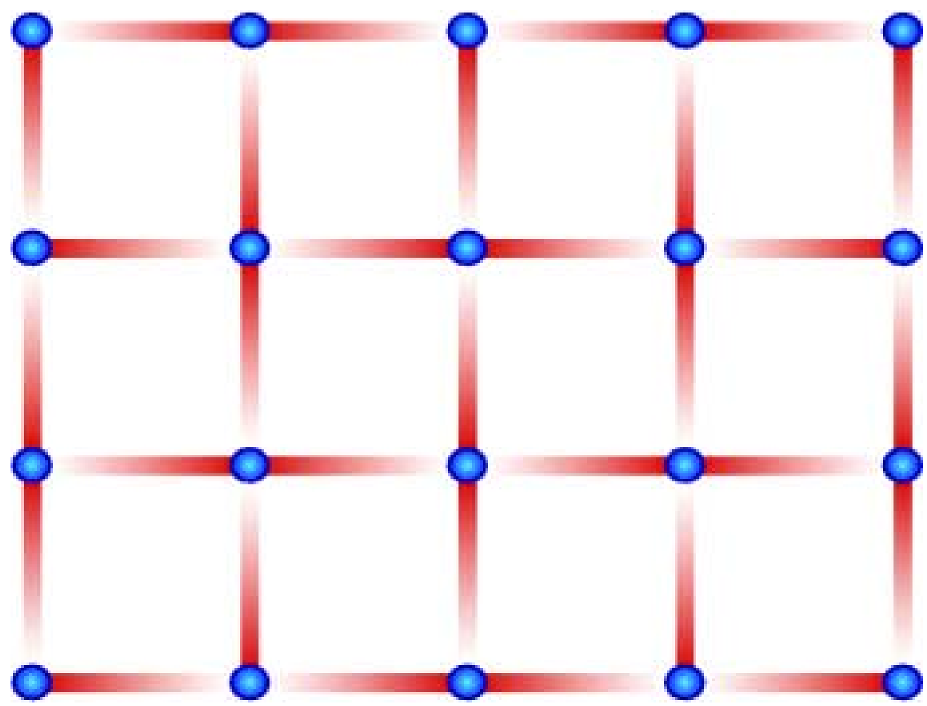
Breaking of time-reversal symmetry or parity ?



II. Staggered-flux state

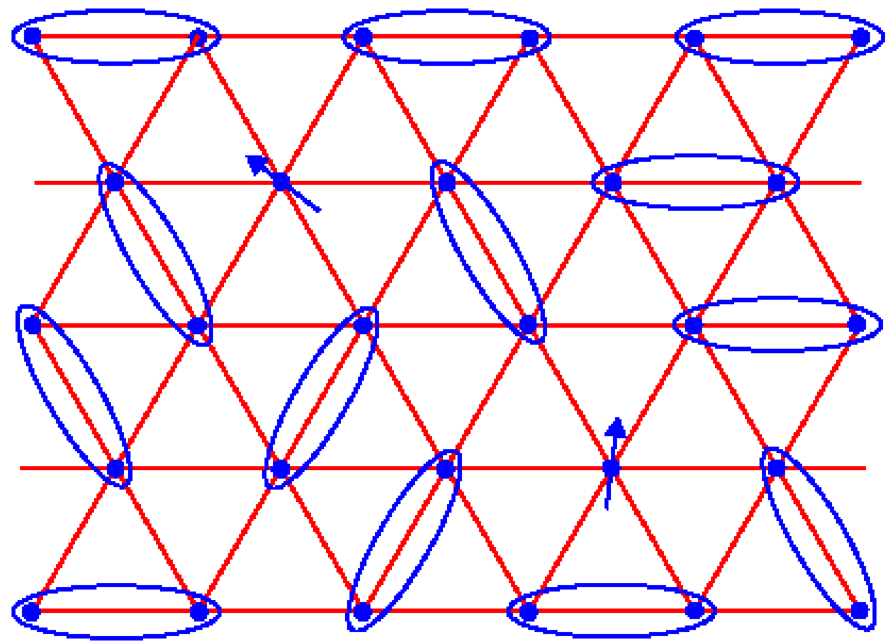
Broken time-reversal symmetry +

translational symmetry breaking at momentum (π,π)



Evidence in some neutron scattering observations of YBCO by H. A. Mook, Pengcheng Dai, and F. Dogan, Phys. Rev. B **64**, 012502 (2001), but not confirmed by higher precision studies of C. Stock , W.J.L. Buyers , Z. Tun , R. Liang , D. Peets , D. Bonn, W.N. Hardy, and L. Taillefer, cond-mat/0203076

III. Fractionalized spin liquid



Leads to:

(a) Stable hc/e vortices

S. Sachdev, Phys. Rev. B **45**, 389 (1992).

(b) Senthil flux memory effect

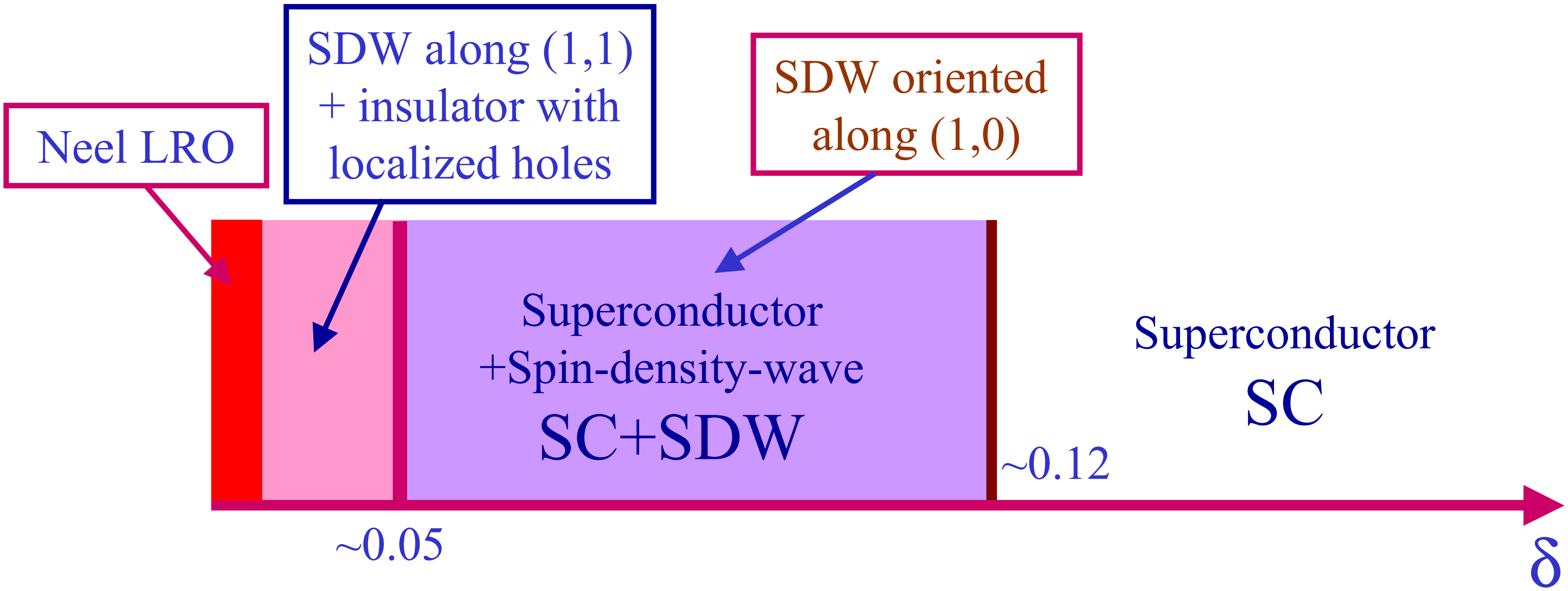
T. Senthil and M.P.A. Fisher, Phys. Rev. Lett. **86**, 292 (2001)

Not observed

J. C. Wynn, D. A. Bonn, B. W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler *Phys. Rev. Lett*. **8**7, 197002 (2001).
D. A. Bonn, J. C. Wynn, B. W. Gardner, Yu-Ju Lin, Ruixing Liang, W. N. Hardy, J. R. Kirtley, and K. A. Moler *Nature*, **414**, 887 (2001).

IV. Spin and "charge" density waves ("stripes")

Phase diagram of LSCO at zero temperature measured by neutron scattering



B. Keimer *et al*. Phys. Rev. B **46**, 14034 (1992).

S. Wakimoto, G. Shirane *et al*., Phys. Rev. B **60**, R769 (1999).

G. Aeppli, T.E. Mason, S.M. Hayden, H.A. Mook, J. Kulda, Science **278**, 1432 (1997).

Y. S. Lee, R. J. Birgeneau, M. A. Kastner *et al.,* Phys. Rev. B **60**, 3643 (1999).

J. E. Sonier *et al*., cond-mat/0108479.

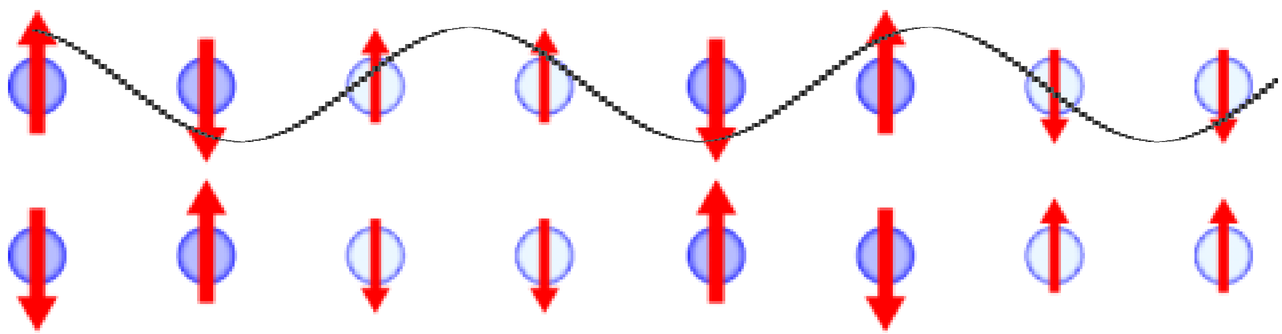
C. Panagopoulos, B. D. Rainford, J. L. Tallon, T. Xiang, J. R. Cooper, and C. A. Scott, preprint.

Spin density wave order parameter for general ordering wavevector

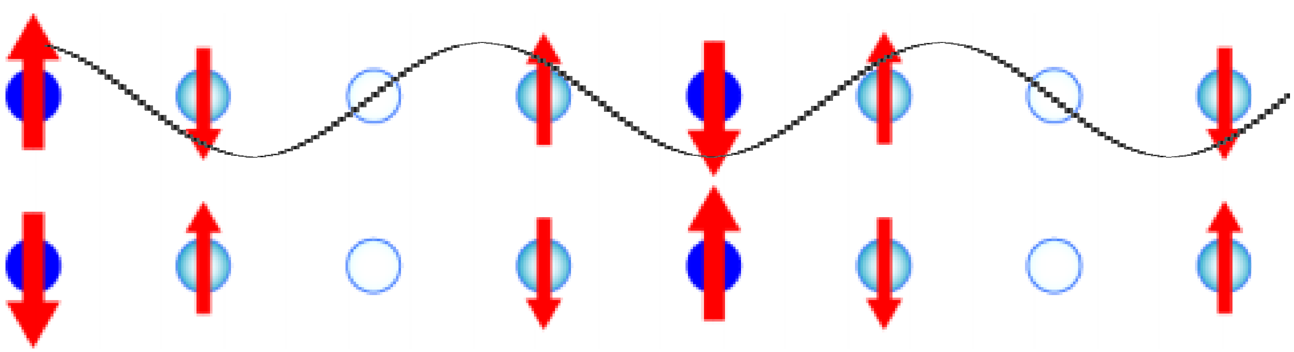
$$S_\alpha(\boldsymbol{r}) = \Phi_\alpha(\boldsymbol{r}) e^{i\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

$\Phi_\alpha(\boldsymbol{r})$ is a ***complex*** field and $\boldsymbol{K} = (3\pi/4, \pi)$

Spin density wave is ***longitudinal*** (and not spiral): $\Phi_\alpha = e^{i\theta} n_\alpha$



Bond-centered



Site-centered

A longitudinal spin density wave necessarily has an accompanying modulation in the site charge densities, exchange and pairing energy per link *etc*. at half the wavelength of the SDW

"Charge" order: periodic modulation in local observables invariant under spin rotations and time-reversal

$$\text{Order parmeter} \sim \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r})$$

$$\delta\rho(\boldsymbol{r}) \propto S_{\alpha}^{2}(\boldsymbol{r}) = \sum_{\alpha} \Phi_{\alpha}^{2}(\boldsymbol{r}) e^{i2\boldsymbol{K}\cdot\boldsymbol{r}} + \text{c.c.}$$

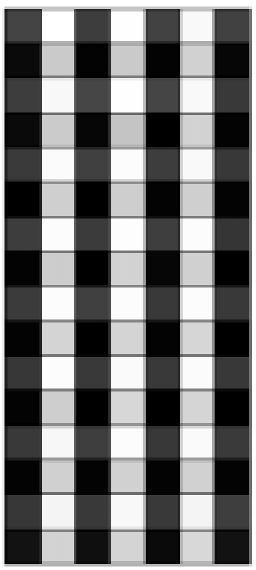
J. Zaanen and O. Gunnarsson, *Phys. Rev.* B **40**, 7391 (1989).
H. Schulz, *J. de Physique* **50**, 2833 (1989).
O. Zachar, S. A. Kivelson, and V. J. Emery, *Phys. Rev.* B **57**, 1422 (1998).

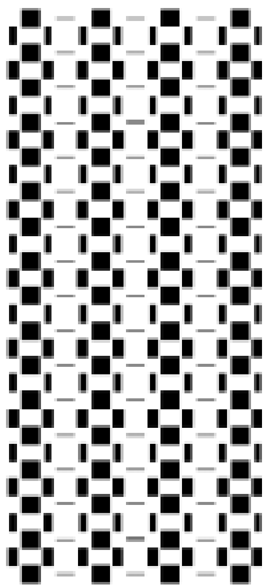
Spin Peierls order

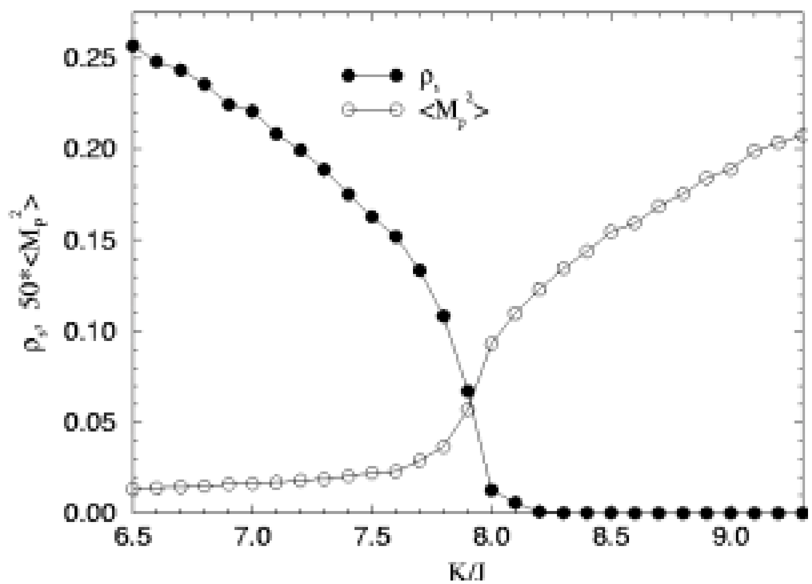
Bond-centered stripe order in a frustrated *S=1/2* XY magnet

A. W. Sandvik, S. Daul, R. R. P. Singh, and D. J. Scalapino, cond-mat/0205270

First large scale numerical study of the destruction of Neel order in a *S=1/2* antiferromagnet with full square lattice symmetry



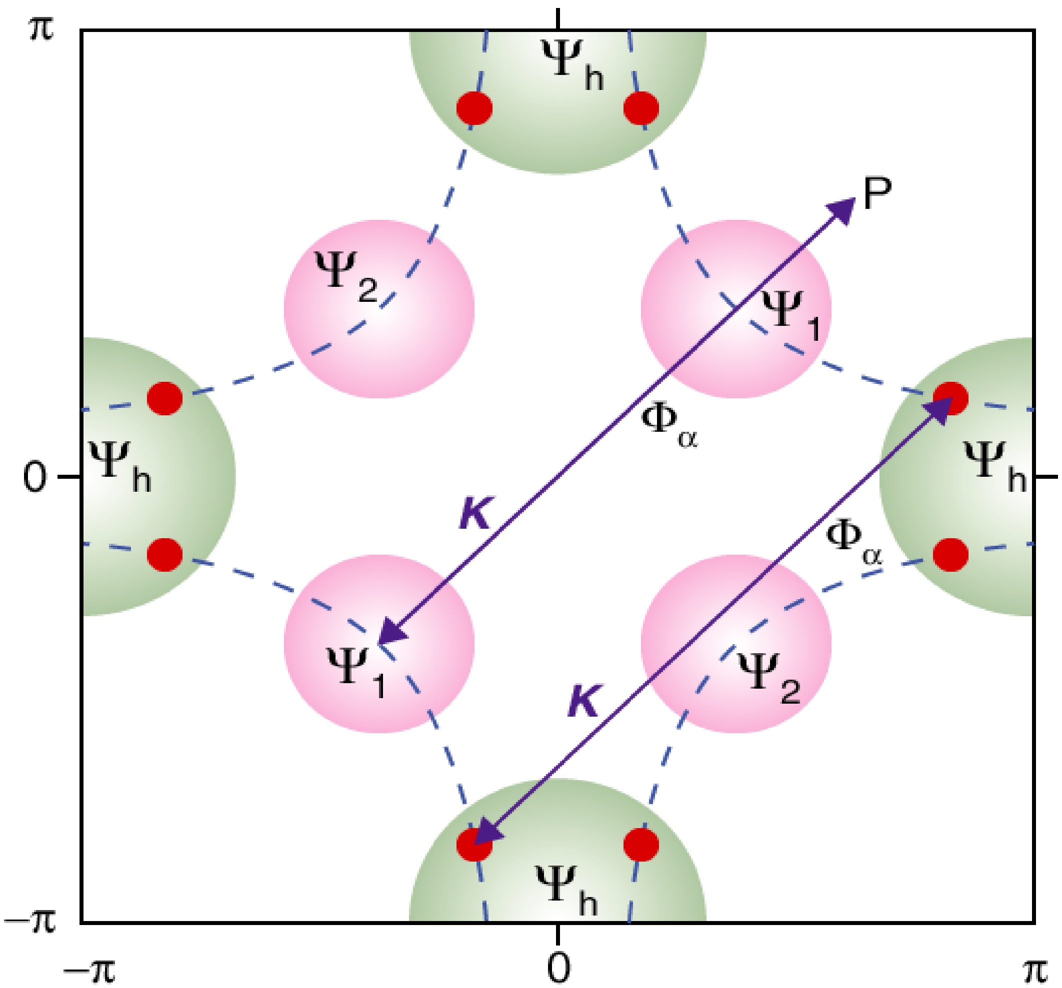




$$H = -2J \sum_{\langle ij \rangle} \left(S_i^x S_j^x + S_i^y S_j^y \right) - K \sum_{\langle ijkl \rangle \subset \Box} \left(S_i^+ S_j^- S_k^+ S_l^- + S_i^- S_j^+ S_k^- S_l^+ \right)$$

Leads to even period bond-centered charge order upon doping

Coupling between S=1/2 fermionic quasiparticles and collective mode of spin density wave order



Strong constraints on the mixing of quasiparticles and the Φ_α collective mode by momentum and energy conservation: nodal quasiparticles can be essentially decoupled from nonzero ***K*** collective modes

Quantitative connection between spectra of quasiparticles (STM) and collective mode (neutron scattering) ?