Building strange metals from SYK models

Subir Sachdev June 25, 2018 Correlated Electron Systems Gordon Research Conference Mount Holyoke College







Wenbo Fu Harvard



Yingfei Gu Harvard



Grisha Tarnopolsky Harvard

arXiv:1804.04130







arXiv:1712.05026

and **To appear**

Daniel Arovas UCSD

John McGreevy UCSD

Aavishkar Patel Harvard Quasiparticles are ubiquitous:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

What are quasiparticles ?

• Quasiparticles are additive excitations: The low-lying excitations of the many-body system can be identified as a set $\{n_{\alpha}\}$ of quasiparticles with energy ε_{α}

$$E = \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.



• Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$au_{\rm eq} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad {\rm as} \ T \to 0,$$

where E_F is the Fermi energy.

. Solvable model without quasiparticles SYK model of a `quantum island'

2. Lattice models of SYK islands Theories of strange metals

 SYK U(1) gauge theory
Solvable model with finite density of fermions, emergent gauge fields, and disorder

The Sachdev-Ye-Kitaev (SYK) model



Pick a set of random positions



Place electrons randomly on some sites















This describes both a strange metal and a black hole!

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell} c_i^{\dagger} c_j^{\dagger} c_k c_\ell - \mu \sum_i c_i^{\dagger} c_i$$
$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^{\dagger} + c_j^{\dagger} c_i = \delta_{ij}$$
$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^{\dagger} c_i$$

 $U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $|\overline{U_{ij;k\ell}}|^2 = U^2$ $N \to \infty$ yields critical strange metal.



S. Sachdev and J.Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$



S. Sachdev and J.Ye, Phys. Rev. Lett. 70, 3339 (1993)

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i(\tau) + \mu - \Sigma(i\omega)}, \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots$$
, $G(z) = \frac{A}{\sqrt{z}}$

where $A = e^{-i\pi/4} (\pi/U^2)^{1/4}$ at half-filling. The ground state is a non-Fermi liquid, with a continuously variable density Q.

S. Sachdev and J.Ye, Phys. Rev. Lett. 70, 3339 (1993)

Many-body level spacing \sim $2^{-N} = e^{-N \ln 2}$ There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size N = 12. The $T \rightarrow 0$ state has an entropy $S_{GPS} = Ns_0$ with

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848...$$

< $\ln 2$

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$ where G is Catalan's constant, for the half-filled case Q = 1/2.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB 94, 035135 (2016)

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size N = 12. The $T \to 0$ state Many-body has an entropy $S_{GPS} = Ns_0$ level spacing \sim with $2^{-N} = e^{-N \ln 2}$ $s_0 = \frac{G}{\pi} + \frac{\ln(2)}{\Lambda} = 0.464848\dots$ No quasiparticles ! $E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha}$ Non-quasiparticle $+\sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$ excitations with spacing $\sim e^{-Ns_0}$ PRB 63, 134406 (2001)

W. Fu and S. Sachdev, PRB 94, 035135 (2016)

(No quasiparticles)

• Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_BT} \quad , \quad {\rm as} \ T \to 0. \qquad \begin{array}{l} \mbox{A. Georges and O. Parcollet} \\ \mbox{PRB 59, 5341 (1999)} \\ \mbox{A. Eberlein, V. Kasper, S. Sachdev, and} \\ \mbox{J. Steinberg, PRB 96, 205123 (2017)} \end{array}$$

No quasiparticles

• Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad {\rm as} \ T \to 0. \qquad \begin{array}{c} \mbox{A. Georges and O. Parcollet} \\ \mbox{PRB 59, 5341 (1999)} \\ \mbox{A. Eberlein, V. Kasper, S. Sachdev, and} \\ \mbox{J. Steinberg, PRB 96, 205123 (2017)} \end{array}$$

• Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad {\rm as} \ T \to 0. \quad {\rm S. \, Sachdev, \, Quantum \, Phase \, Transitions, \, Cambridge \, (1999)}$$

Absence of quasiparticles \Leftrightarrow Fastest possible thermalization

SYK models and black holes

- Black holes have an entropy proportional to their surface area, and a temperature, $T_H = \hbar c^3 / (8\pi G M k_B)$.
- Black holes relax to thermal equilibrium in a 'Planckian' time $\sim \hbar/(k_B T_H) = 8\pi G M/c^3$.
- Black holes in d+1 spatial dimensions are similar to a quantum system without quasiparticles in d spatial dimensions.





SYK models and black holes

PHYSICAL REVIEW LETTERS **105**, 151602 (2010)

Ś

Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA (Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.



I. Solvable model without quasiparticles SYK model of a `quantum island'

2. Lattice models of SYK islands Theories of strange metals

 SYK U(1) gauge theory
Solvable model with finite density of fermions, emergent gauge fields, and disorder

SYK quantum islands of electrons with random hopping between them.



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017) See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands Can also use non-random *t*, and the same *U* on all "islands".



Low 'coherence' scale











Mobile electrons (c) interacting with SYK quantum islands (f) with random exchange interactions.



Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX 8, 021049 (2018)

Mobile electrons (c) interacting with SYK quantum islands (f) with <u>non-random</u> exchange interactions.



Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, arXiv:1801.06178 (see poster by Debanjan Chowdhury)









Physical Review B 81, 184519 (2010)

Linear-in-B magnetoresistance with B/T scaling



I. M. Hayes, R. D. McDonald, N. P. Breznay, T. Helm, P. J. W. Moll, M. Wartenbe, A. Shekhter, and J. G. Analytis, Nature Physics 12, 916 (2016) See talk by James Analytis



Solvable model without quasiparticles
SYK model of a `quantum island'

2. Lattice models of SYK islands Theories of strange metals

 SYK U(1) gauge theory Solvable model with finite density of fermions, emergent gauge fields, and disorder

SCIENCE VOL 323 30 JANUARY 2009 **Anomalous Criticality in the Electrical Resistivity of La_{2-x}Sr_xCuO₄**

603

R. A. Cooper,¹ Y. Wang,¹ B. Vignolle,² O. J. Lipscombe,¹ S. M. Hayden,¹ Y. Tanabe,³ T. Adachi,³ Y. Koike,³ M. Nohara,⁴* H. Takagi,⁴ Cyril Proust,² N. E. Hussey¹†



Universal *T*-linear resistivity and Planckian limit in overdoped cuprates

arXiv:1805.02512

- A. Legros^{1,2}, S. Benhabib³, W. Tabis^{3,4}, F. Laliberté¹, M. Dion¹, M. Lizaire¹,
- B. Vignolle³, D. Vignolles³, H. Raffy⁵, Z. Z. Li⁵, P. Auban-Senzier⁵,
- N. Doiron-Leyraud¹, P. Fournier^{1,6}, D. Colson², L. Taillefer^{1,6}, and C. Proust^{3,6}

From the resistivity, they determined the value of the number α defined by

$$\rho(T) = \rho_0 + \alpha \, \frac{h}{2e^2} \left(\frac{T}{T_F}\right)$$

where $T_F = (\pi \hbar^2 / k_B)(n/m^*)$ and m^* is determined from the specific heat. This expression is obtained from the Drude form $\rho = m^*/(ne^2\tau)$ and $\hbar/\tau = \alpha k_B T$.

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

Electronic spectrum in pseudogap metal is well described by the Higgs phase of a SU(2) gauge theory

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero, PRX **8**, 021048 (2018)

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, PNAS **II5**, E3665 (2018)

See talk by Antoine Georges and poster by Mathias Scheurer





Electronic spectrum in pseudogap metal is well described by the Higgs phase of a SU(2) gauge theory

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero, PRX **8**, 021048 (2018)

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, PNAS **II5**, E3665 (2018)

See talk by Antoine Georges and poster by Mathias Scheurer

Optimal doping critical point is associated with vanishing of the Higgs condensate. Overdoped regime is described by (a large Fermi surface of) electrically-charged fermions coupled to an emergent SU(2) gauge field in the presence of disorder

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, PRB 80, 155129 (2009)

D. Chowdhury and S. Sachdev, PRB 91, 115123 (2015)



Fermions with random hopping coupled to a fluctuating U(1) gauge field

 $\Sigma(i\omega_n) = t^2 G(i\omega_n) + t^2 T \sum \frac{G(i\omega_n + i\Omega_m) - G(i\omega_n)}{\Pi(i\Omega_m) - \Pi(i\Omega_m = 0)},$



Aavishkar Patel (see poster)

$$\Pi(i\Omega_m) = 2t^2 T \frac{M}{N} \sum_{\omega_n} G(i\omega_n) G(i\omega_n + i\Omega_m), \quad G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}.$$





where \mathcal{E} is a parameter universally related to the filling fraction $(\mathcal{E} = 0 \text{ at half-filling})$. The exponent x is the solution to

$$\frac{(1/x-2)(\cosh(2\pi\mathcal{E})-\cos(\pi x))}{\tan(\pi x)\sin(\pi x)} = \frac{2M}{N}.$$



Disordered strange metal as $T \rightarrow 0$ with <u>all</u> electrons contributing to transport.

No quasiparticles

• Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad {\rm as} \ T \to 0. \qquad \begin{array}{c} \mbox{A. Georges and O. Parcollet} \\ \mbox{PRB 59, 5341 (1999)} \\ \mbox{A. Eberlein, V. Kasper, S. Sachdev, and} \\ \mbox{J. Steinberg, PRB 96, 205123 (2017)} \end{array}$$

• Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad {\rm as} \ T \to 0. \quad {\rm S. \, Sachdev, \, Quantum \, Phase \, Transitions, \, Cambridge \, (1999)}$$

Absence of quasiparticles \Leftrightarrow Fastest possible thermalization

• Solvable model without quasiparticles: SYK model of a 'quantum island'

- Solvable model without quasiparticles: SYK model of a 'quantum island'
- Lattice models of SYK islands: Bad metal behavior with $\rho \sim (T/E_c)(h/e^2)$ for $T > E_c$, and Fermi liquid behavior for $T < E_c$.

- Solvable model without quasiparticles: SYK model of a 'quantum island'
- Lattice models of SYK islands: Bad metal behavior with $\rho \sim (T/E_c)(h/e^2)$ for $T > E_c$, and Fermi liquid behavior for $T < E_c$.
- SYK-Kondo lattice models: Bad metal behavior with $\rho \sim (T/T_0)(h/e^2)$ for $T > T_0$, and marginal Fermi liquid (MFL) behavior for $T < T_0$ with $\rho \sim (T/T_0)(h/e^2)$. MFL regime has <u>small</u> Fermi surface, and magnetoresistance B/T scaling (with mesoscopic disorder).

- Solvable model without quasiparticles: SYK model of a 'quantum island'
- Lattice models of SYK islands: Bad metal behavior with $\rho \sim (T/E_c)(h/e^2)$ for $T > E_c$, and Fermi liquid behavior for $T < E_c$.
- SYK-Kondo lattice models: Bad metal behavior with $\rho \sim (T/T_0)(h/e^2)$ for $T > T_0$, and marginal Fermi liquid (MFL) behavior for $T < T_0$ with $\rho \sim (T/T_0)(h/e^2)$. MFL regime has <u>small</u> Fermi surface, and magnetoresistance B/T scaling (with mesoscopic disorder).
- SYK U(1) gauge theory: solvable model with finite density of fermions, emergent gauge fields, and disorder. Strange metal behavior with $\rho \sim (T/t)^{2x}(h/e^2)$ as $T \rightarrow 0$, with <u>all</u> electrons mobile.