The 34th Jerusalem School in Theoretical Physics

NEW HORIZONS IN QUANTUM MATTER

27.12, 2016 — 5.1, 2017

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Modern quantum materials realize a remarkably rich set of electronic phases. This school will explore the many new concepts and methods which have been developed in recent years, moving beyond the traditional paradigms of Fermi liquid theory and spontaneous symmetry breaking. In particular, long-range quantum entanglement appears in topological and quantum-critical states, and the school will discuss new techniques required to describe their observable properties.

For more details:
www.as.huji.ac.il/horizons-in-quantum

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Stanford University

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Harvard University

Nati Seiberg
IAS, Princeton

Senthil Todadri
MIT

The SYK model of non-Fermi liquids and black holes

Applications of Gauge-Gravity Duality 2016
Chalmers University of Technology
Gothenburg, Sweden, October 4, 2016

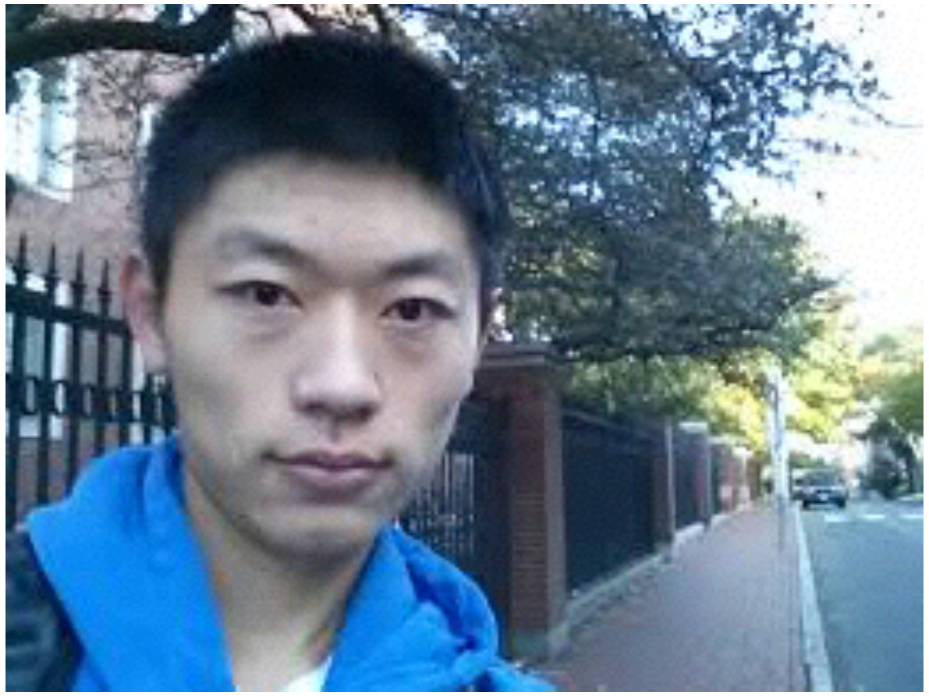
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

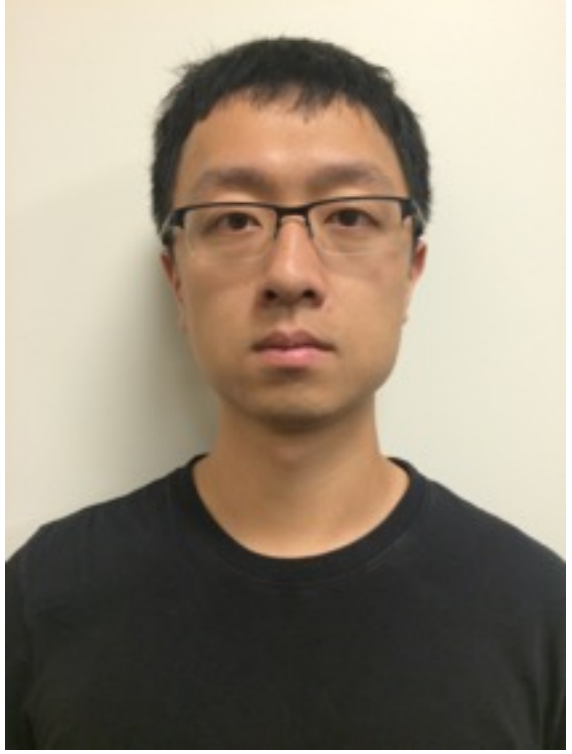
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS HARVARD

Richard Davison, Harvard



Wenbo Fu, Harvard



Yingfei Gu, Stanford

Quantum matter without quasiparticles

- Quasiparticles are long-lived excitations which can be combined to yield the complete low-energy many-body spectrum

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Quantum matter without quasiparticles

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- How do we rule out quasiparticle excitations? Examine the time it takes to reach local thermal equilibrium. Equilibration takes a long time while quasiparticles collide (in Fermi liquids, $\tau \sim 1/T^2$; in gapped systems, $\tau \sim e^{\Delta/T}$). Systems *without* quasiparticles saturate a (conjectured) lower bound on the local-equilibration/de-phasing/transition-to-quantum-chaos time

$$\tau_\varphi \quad \geq \quad C \frac{\hbar}{k_B T}$$

where C is a T-independent constant.

S. Sachdev, *Quantum Phase Transitions* (1999)
K. Damle and Sachdev, PRB **56**, 8714 (1997)

Quantum matter without quasiparticles

- Shortest possible local-equilibration/de-phasing/ transition-to-quantum-chaos with

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S. Sachdev, *Quantum Phase Transitions* (1999)
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$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

P. Kovtun, D.T. Son, and A.O. Starinets, PRL **94**, 111601 (2005)

$$\frac{D}{v_b^2} \geq \tilde{C} \frac{\hbar}{k_B T}$$

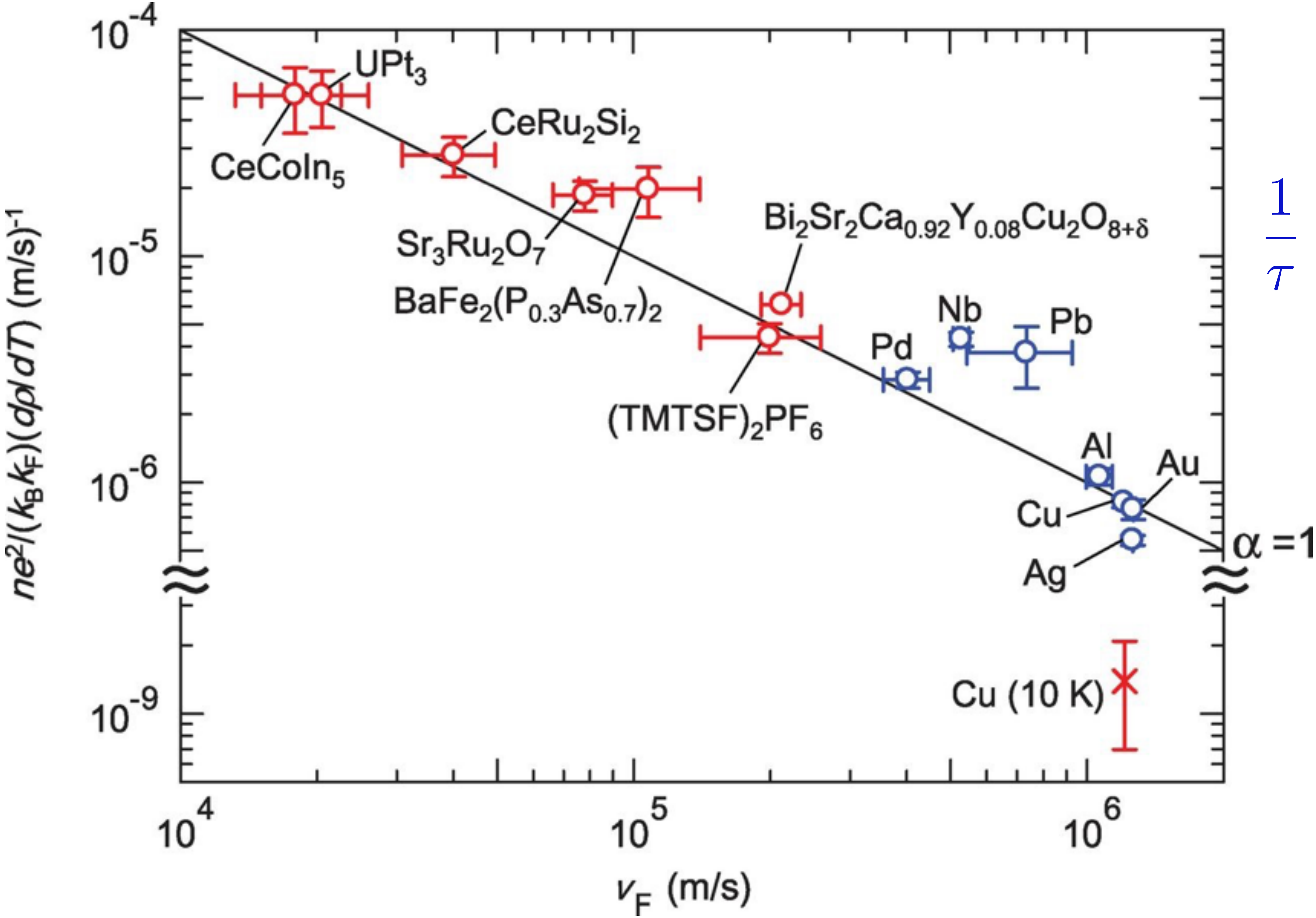
Saturation requires fixed point with <u>disorder *and* interactions</u>

S.A. Hartnoll, Nature Physics **11**, 54 (2015)
M. Blake, PRL **117**, 091601 (2016)

$$\tau_L \geq \frac{1}{2\pi} \frac{\hbar}{k_B T}$$

J. Maldacena, S. H. Shenker and D. Stanford, JHEP 08 (2016)106

In Fermi liquids, $\tau \sim 1/T^2$;
in gapped systems, $\tau \sim e^{\Delta/T}$.



$$\frac{1}{\tau} = \alpha \frac{k_B T}{\hbar}$$

J. A. N. Bruin, H. Sakai, R. S. Perry, A. P. Mackenzie, *Science*. **339,** 804 (2013)

Theories of non-Fermi liquids

- Sachdev-Ye-Kitaev (SYK) model

- Coupled SYK models:
  diffusive metals without
  quasiparticles

- Holographic Einstein-Maxwell-axion
  theory with momentum dissipation

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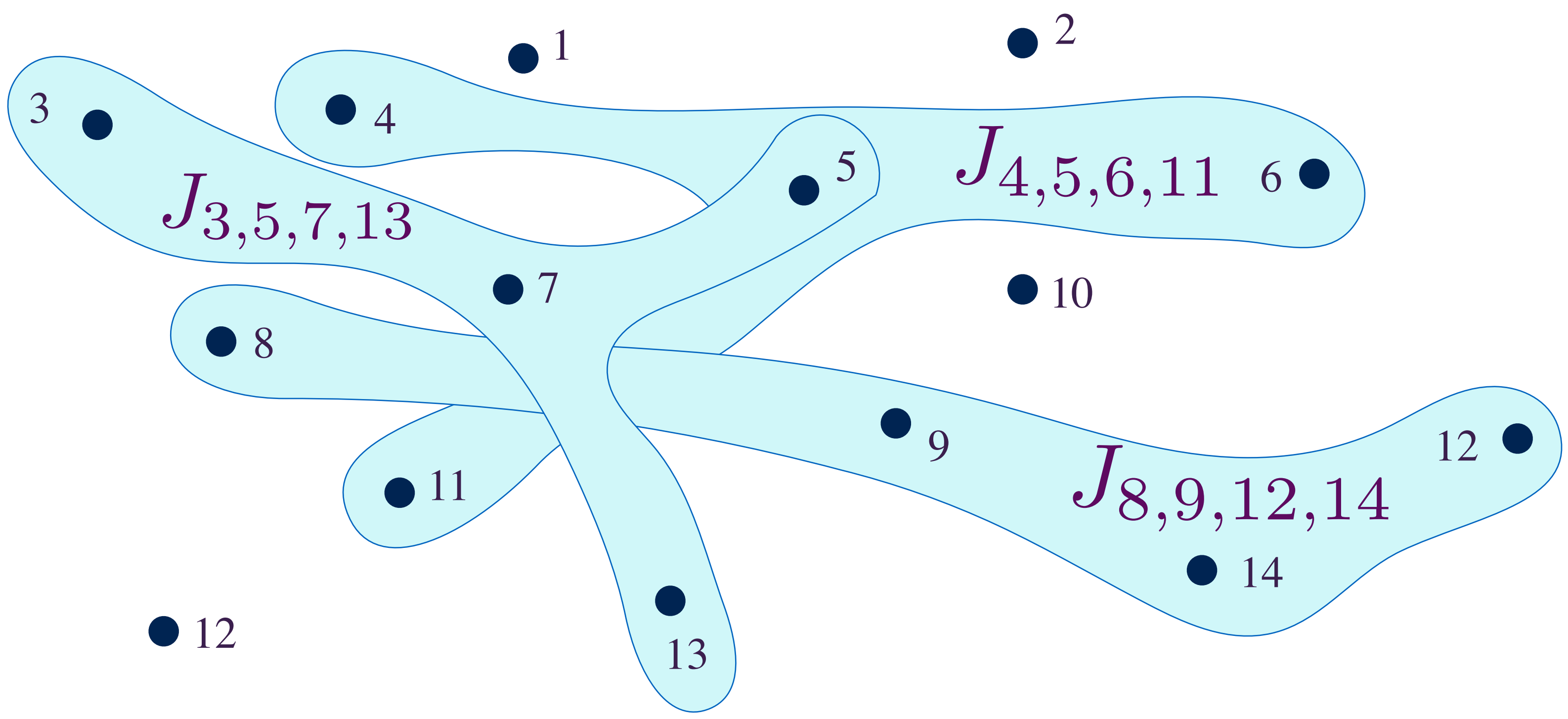
SYK model

$$H_{\mathrm{SYK}} = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

A fermion can move only by entangling with another fermion: the Hamiltonian has "nothing but entanglement".



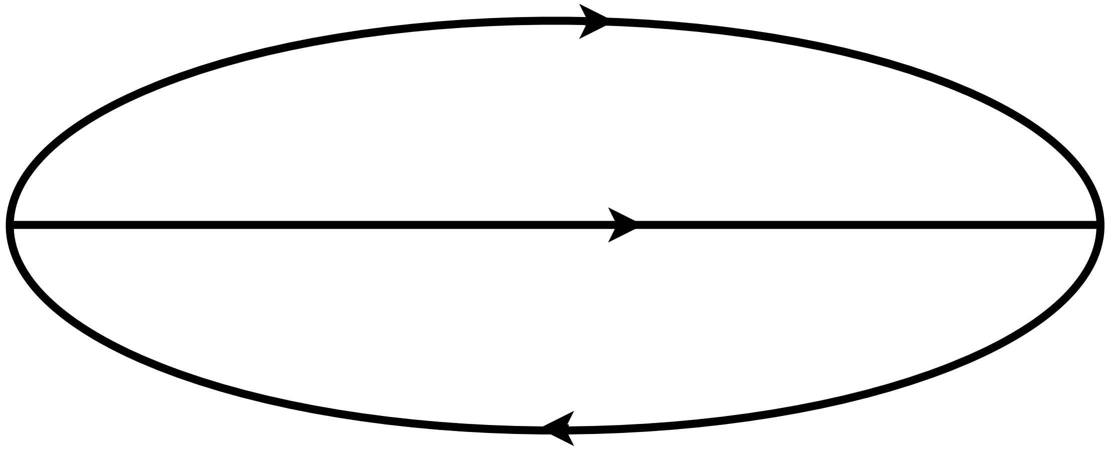
Cold atom realization:
I. Danshita, M. Hanada, and M. Tezuka, arXiv:1606.02454

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

SYK model

Feynman graph expansion in $J_{ij..}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$

$\Sigma =$ 

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{1}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A}{\sqrt{z}}$$

for some complex A. The ground state is a non-Fermi liquid, with a continuously variable density $\mathcal{Q}$.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

SYK model

- $T = 0$ Green's function $G \sim 1/\sqrt{\tau}$

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  $G \sim 1/(\sin(\pi T \tau))^{1/2}$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)

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- $T = 0$ Green's function $G \sim 1/\sqrt{\tau}$
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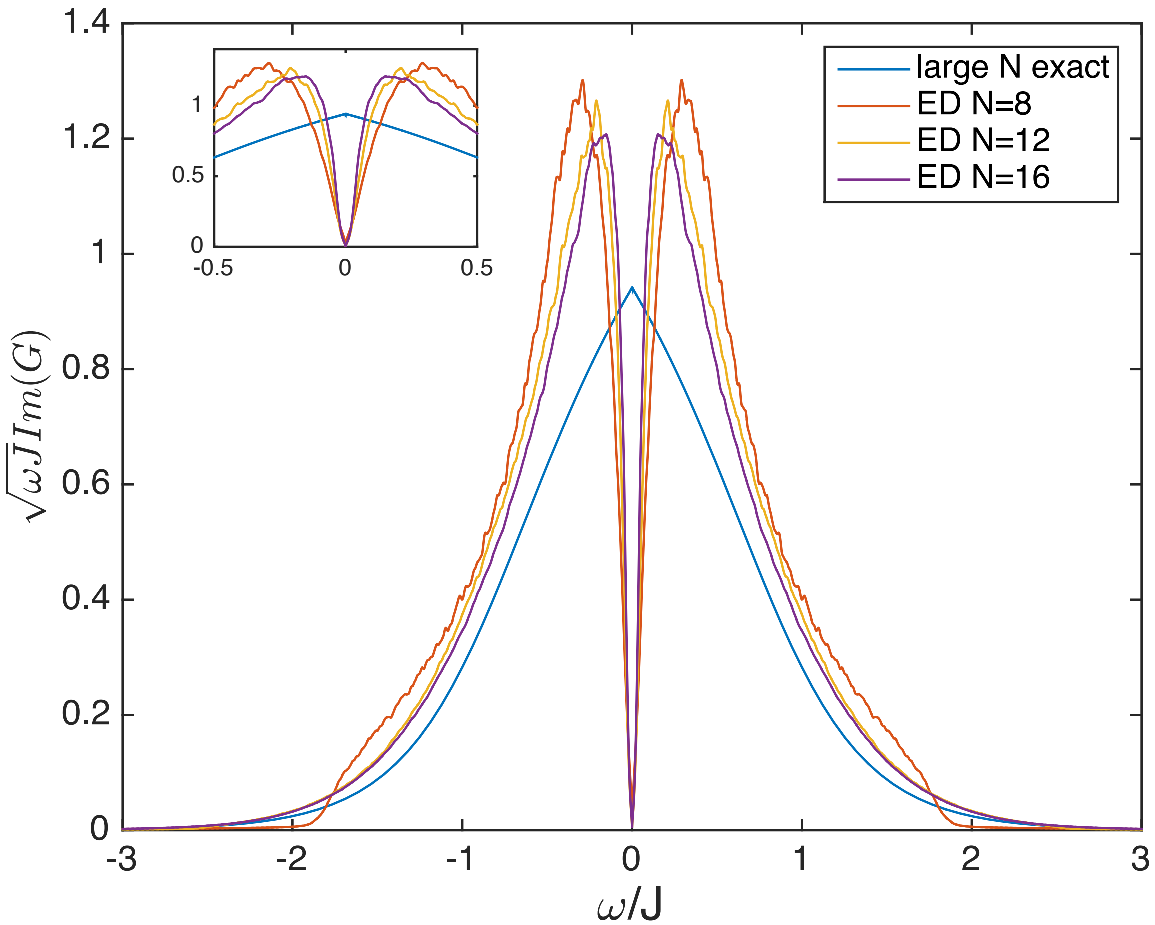
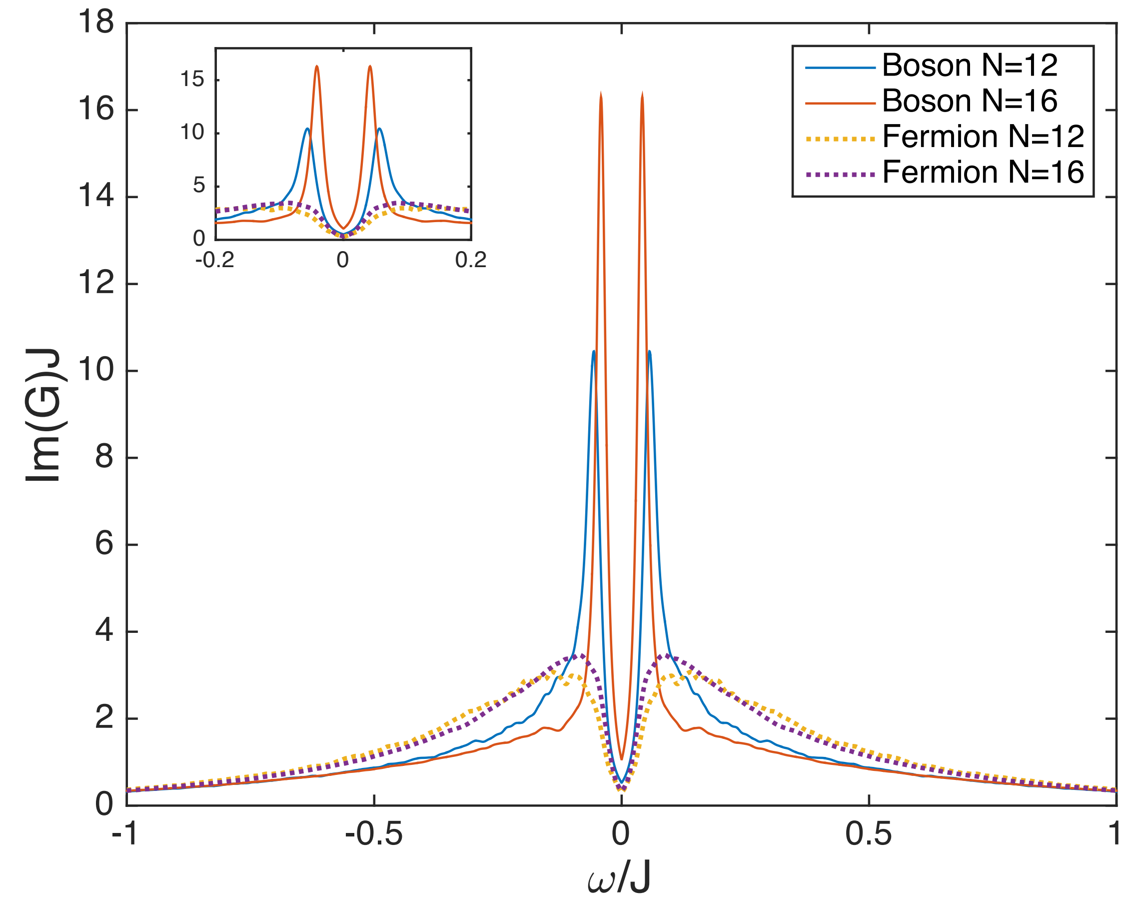
A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

SYK model





Large N solution of equations for G and Σ agree well with exact diagonalization of the finite N Hamiltonian $\Rightarrow$ no spin-glass order

However, exact diagonalization of the same model with hard-core bosons indicates the presence of spin-glass order in the ground state.

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

SYK model

After integrating the fermions, the partition function can be written as a path integral with an action S analogous to a Luttinger-Ward functional

$$Z = \int \mathcal{D}G(\tau_1, \tau_2)\mathcal{D}\Sigma(\tau_1, \tau_2) \exp(-NS)$$

A. Georges, O. Parcollet, and S. Sachdev, Phys. Rev. B **63**, 134406 (2001)

$$S = \ln \det \left[\delta(\tau_1 - \tau_2)(\partial_{\tau_1} + \mu) - \Sigma(\tau_1, \tau_2)\right] + \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2) \left[G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

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$$+ \int d\tau_1 d\tau_2 \Sigma(\tau_1, \tau_2)\left[G(\tau_2, \tau_1) + (J^2/2)G^2(\tau_2, \tau_1)G^2(\tau_1, \tau_2)\right]$$

A. Georges, O. Parcollet, and S. Sachdev,
Phys. Rev. B **63**, 134406 (2001)

At frequencies $\ll J$, the time derivative in the determinant is less important, and without it the path integral is invariant under the reparametrization and gauge transformations

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Kitaev, unpublished
S. Sachdev, PRX **5**, 041025 (2015)

$$\tau = f(\sigma)$$

$$G(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-1/4} \frac{g(\sigma_1)}{g(\sigma_2)} G(\sigma_1, \sigma_2)$$

$$\Sigma(\tau_1, \tau_2) = \left[f'(\sigma_1)f'(\sigma_2)\right]^{-3/4} \frac{g(\sigma_1)}{g(\sigma_2)} \Sigma(\sigma_1, \sigma_2)$$

where $f(\sigma)$ and $g(\sigma)$ are arbitrary functions.

SYK model

Let us write the large N saddle point solutions of S as

$$G_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-1/2} \quad , \quad \Sigma_s(\tau_1 - \tau_2) \sim (\tau_1 - \tau_2)^{-3/2}.$$

These are not invariant under the reparametrization symmetry but are invariant only under a SL(2,R) subgroup under which

$$f(\tau) = \frac{a\tau + b}{c\tau + d} \quad , \quad ad - bc = 1.$$

So the (approximate) reparametrization symmetry is spontaneously broken.

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

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Th ... in-
var ...

So ... en.

Connections of SYK to gravity and AdS_2 horizons

- Reparameterization and gauge invariance are the 'symmetries' of the Einstein-Maxwell theory of gravity and electromagnetism
- SL(2,R) is the isometry group of AdS_2.

J. Maldacena and D. Stanford, arXiv:1604.07818
See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

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So the (approximate) reparametrization symmetry is spontaneously broken.

Reparametrization zero mode

Expand about the saddle point by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1) f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))$$

(and similarly for Σ) and obtain an effective action for $f(\tau)$. This action does not vanish because of the time derivative in the determinant which is not reparameterization invariant.

J. Maldacena and D. Stanford, arXiv:1604.07818

See also A. Kitaev, unpublished, and J. Polchinski and V. Rosenhaus, arXiv:1601.06768

SYK model

With $g(\tau) = e^{-i\phi(\tau)}$, the action for $\phi(\tau)$ and $f(\tau) = \dfrac{1}{\pi T}\tan(\pi T(\tau + \epsilon(\tau))$ fluctuations is

$$S_{\phi,f} = \frac{K}{2}\int_0^{1/T} d\tau (\partial_\tau \phi + i(2\pi \mathcal{E} T)\partial_\tau \epsilon)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \, \{f, \tau\},$$

where $\{f, \tau\}$ is the Schwarzian:

$$\{f, \tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 .$$

The couplings are given by thermodynamics (Ω is the grand potential)

$$K = -\left(\frac{\partial^2 \Omega}{\partial \mu^2}\right)_T \quad , \quad \gamma + 4\pi^2 \mathcal{E}^2 K = -\left(\frac{\partial^2 \Omega}{\partial T^2}\right)_\mu$$

$$2\pi \mathcal{E} = \frac{\partial S_0}{\partial \mathcal{Q}}$$

Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished

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where $\{f,\tau\}$ is the Schwarzian:

$$\{f,\tau\} \equiv \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 .$$

The correlators of the density fluctuations, $\mathcal{Q}(\tau)$, and the energy fluctuations $\delta E - \mu\delta\mathcal{Q}(\tau)$ are time independent and given by

$$\left(\begin{array}{cc} \langle \delta\mathcal{Q}(\tau)\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle / T \\ \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))(\delta E(0) - \mu\delta\mathcal{Q}(0))\rangle / T \end{array} \right) = T\chi_s$$

where χ_s is the static susceptibility matrix given by

$$\chi_s \equiv \left(\begin{array}{cc} -(\partial^2\Omega/\partial\mu^2)_T & -\partial^2\Omega/(\partial T\partial\mu) \\ -T\partial^2\Omega/(\partial T\partial\mu) & -T(\partial^2\Omega/\partial T^2)_\mu \end{array} \right).$$

Wenbo Fu, Yingfei Gu, S. Sachdev, unpublished

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Coupled SYK models

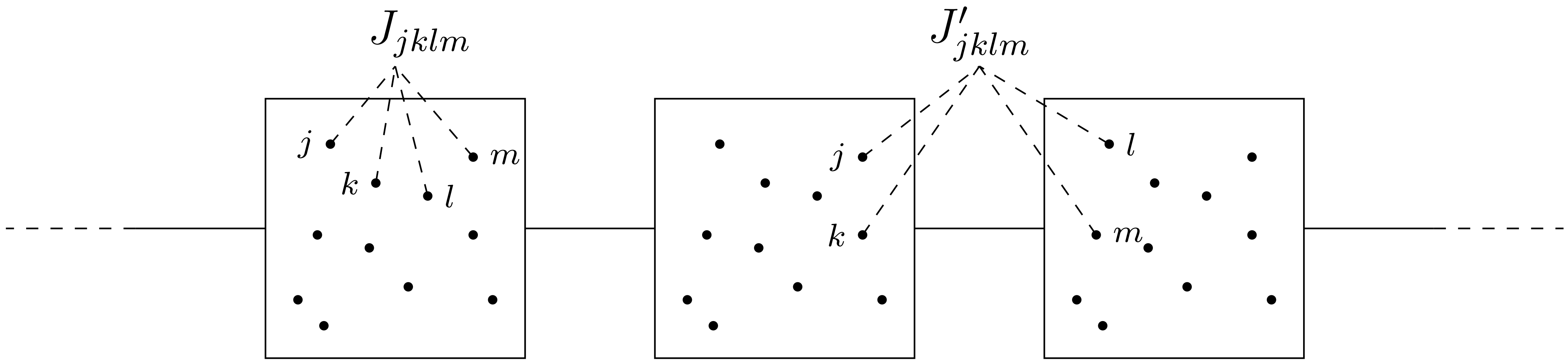

Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.

Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832

SYK model

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Coupled SYK models

$$\left(\begin{array}{cc} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} / T \\ \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; E - \mu \mathcal{Q} \rangle_{k,\omega} / T \end{array} \right) = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \left(\begin{array}{cc} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{array} \right) \chi_s^{-1}.$$

Coupled SYK models

$$\begin{pmatrix} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} /T \\ \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; E - \mu \mathcal{Q} \rangle_{k,\omega} /T \end{pmatrix} = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

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The coupled SYK models realize a diffusive, metal with no quasiparticle excitations. (a "strange metal")

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- $T = 0$ Green's function $G \sim 1/\sqrt{\tau}$
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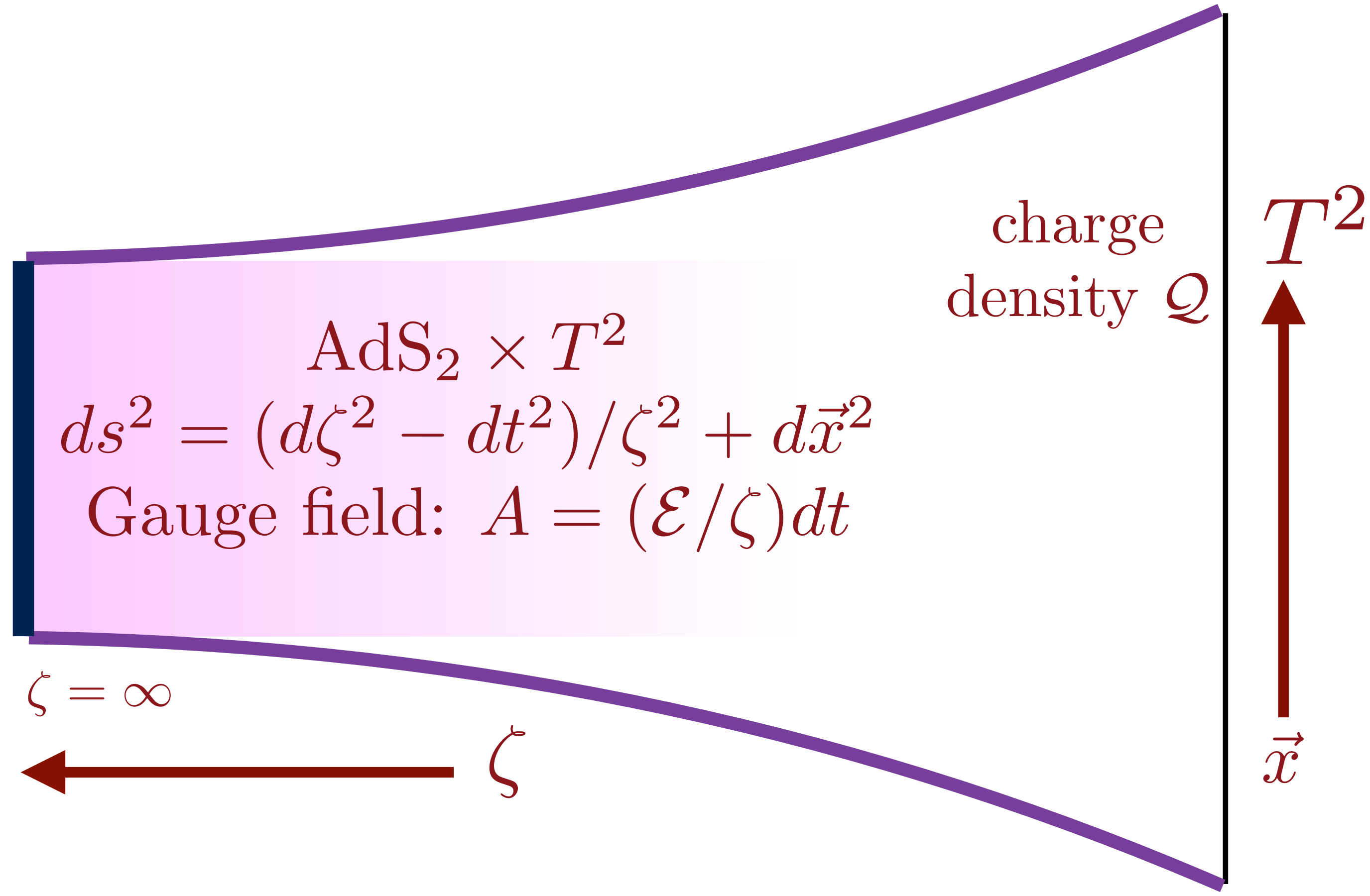
- Non-zero entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$
- These features indicate that the SYK model is dual to the low energy limit of a quantum gravity theory of black holes with AdS_2 near-horizon geometry. The Bekenstein-Hawking entropy is $N S_0$.

S. Sachdev, PRL **105**, 151602 (2010)

SYK and AdS_2



PHYSICAL REVIEW LETTERS 105, 151602 (2010)

Holographic Metals and the Fractionalized Fermi Liquid

Subir Sachdev

Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 23 June 2010; published 4 October 2010)

We show that there is a close correspondence between the physical properties of holographic metals near charged black holes in anti–de Sitter (AdS) space, and the fractionalized Fermi liquid phase of the lattice Anderson model. The latter phase has a "small" Fermi surface of conduction electrons, along with a spin liquid of local moments. This correspondence implies that certain mean-field gapless spin liquids are states of matter at nonzero density realizing the near-horizon, $AdS_2 \times R^2$ physics of Reissner-Nordström black holes.

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- Non-zero entropy as $T \to 0$, $S(T \to 0) = N S_0 + \ldots$
- These features indicate that the SYK model is dual to the low energy limit of a quantum gravity theory of black holes with AdS_2 near-horizon geometry. The Bekenstein-Hawking entropy is $N S_0$.
- There is a scalar zero mode associated with the breaking of reparameterization invariance down to SL(2,R). The same pattern of symmetries is present in gravity theories on AdS_2.

A. Kitaev, KITP talk, 2015

SYK model

- The dependence of S_0 on the density $\mathcal{Q}$ matches the behavior of the Wald-Bekenstein-Hawking entropy of AdS_2 horizons in a large class of gravity theories.

S. Sachdev PRX **5**, 041025 (2015)

SYK model

- The dependence of S_0 on the density $\mathcal{Q}$ matches the behavior of the Wald-Bekenstein-Hawking entropy of AdS_2 horizons in a large class of gravity theories.

- The scalar zero mode leads to a linear-in-T specific heat

$$S(T \to 0) = NS_0 + N\gamma T + \ldots.$$

An identical scalar zero model is also present in the low energy limit of theories of quantum gravity on AdS_2.

J. Maldacena and D. Stanford, arXiv:1604.07818

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An identical scalar zero model is also present in the low energy limit of theories of quantum gravity on AdS_2.

- The Lyapunov time to quantum chaos saturates the lower bound both in the SYK model and in quantum gravity.

$$\tau_L = \frac{1}{2\pi}\frac{\hbar}{k_B T}$$

A. Kitaev, KITP talk, 2015

J. Maldacena and D. Stanford, arXiv:1604.07818

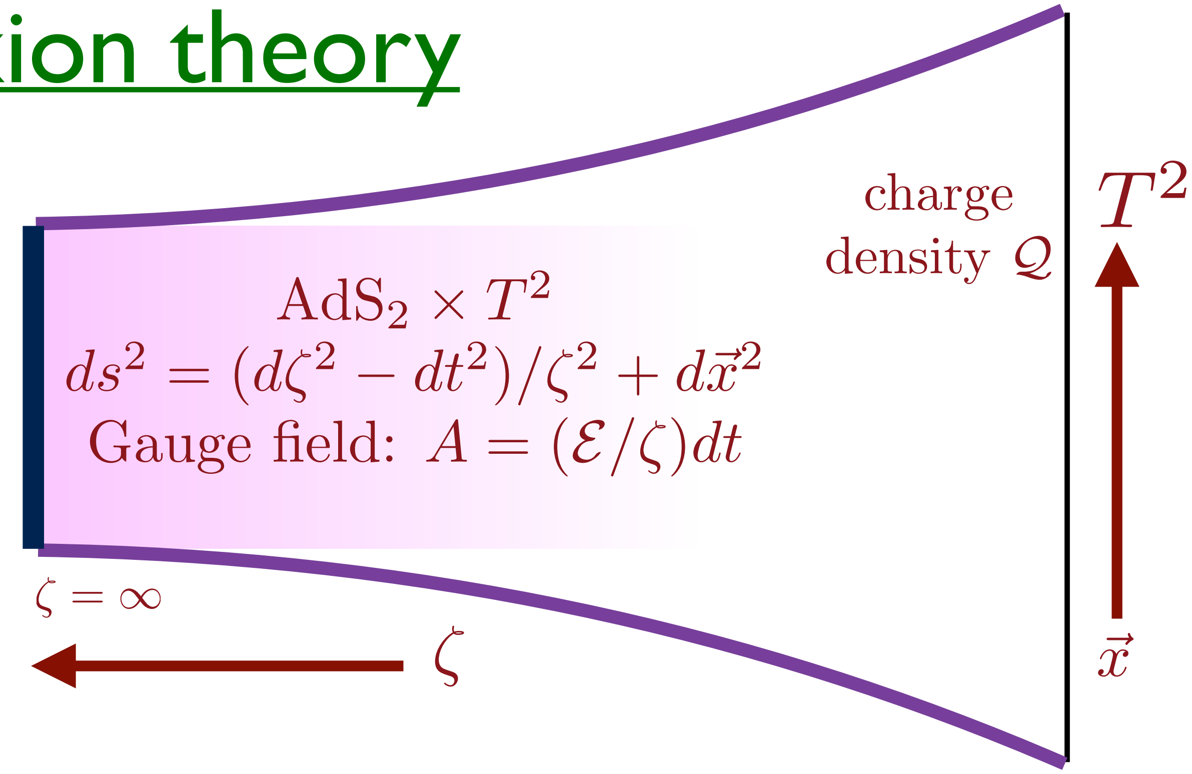
It would be nice to have a solvable model of holography.

theory	bulk dual	anom. dim.	chaos	solvable in $1/N$	black hole
SYM	Einstein grav.	large	maximal	no	yes
$O(N)$	Vasiliev	$1/N$	$1/N$	yes	no
SYK	"$\ell_s \sim \ell_{AdS}$"	$O(1)$	maximal	yes	yes

column added by SS

Slide by D. Stanford at Strings 2016, Beijing

Einstein-Maxwell-axion theory



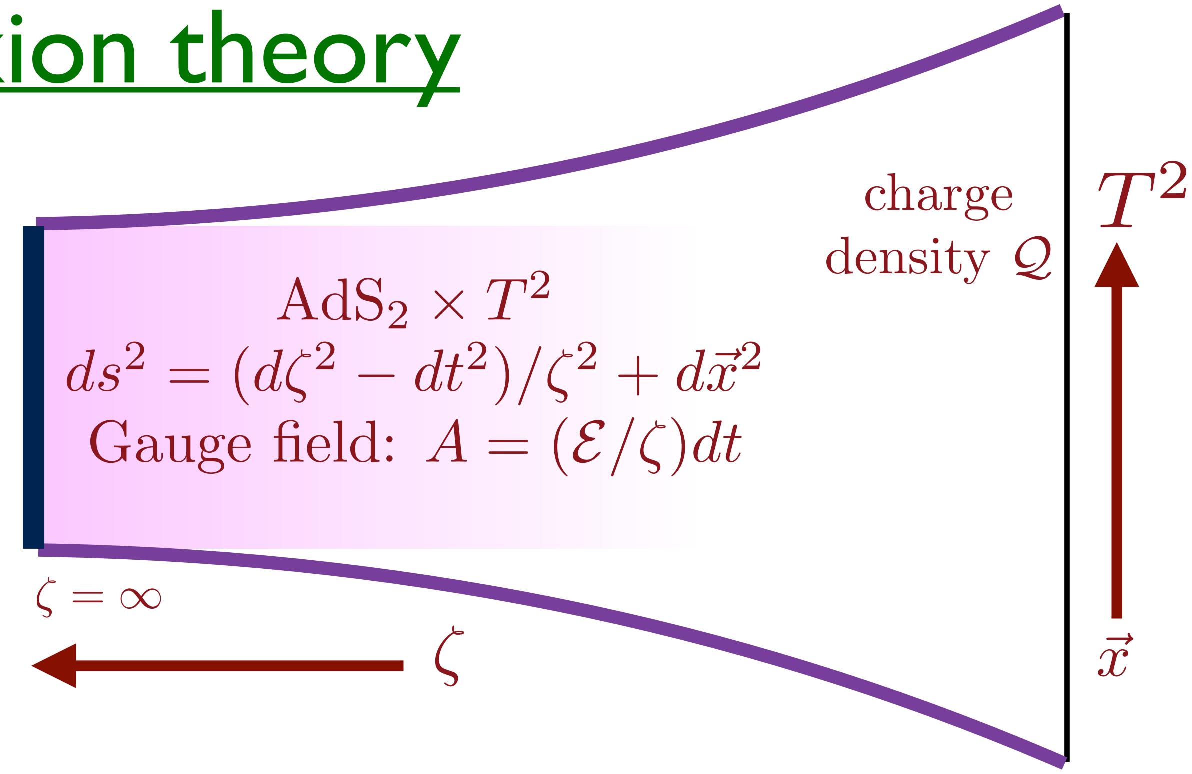
Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054
D. Vegh, arXiv:1301.0537.
R. A. Davison, PRD 88 (2013) 086003.
M. Blake and D. Tong, PRD 88 (2013), 106004.
T. Andrade and B. Withers, JHEP 05 (2014) 101.

$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

- For $\hat{\varphi}_i = 0$, we obtain the Reissner-Nördstrom-AdS charged black hole, with a near-horizon $\text{AdS}_2 \times T^2$ near-horizon geometry.

- For $\hat{\varphi}_i = k x_i$, we obtain a similar solution but with momentum dissipation (a bulk massive graviton).

Einstein-Maxwell-axion theory



Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054
D. Vegh, arXiv:1301.0537.
R. A. Davison, PRD 88 (2013) 086003.
M. Blake and D. Tong, PRD 88 (2013), 106004.
T. Andrade and B. Withers, JHEP 05 (2014) 101.

$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{2}\sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

In the small torus limit, $T \ll 1/R$, where R is the size of the torus, the theory dimensionally reduces to an Einstein-Maxwell-dilaton theory in two dimensions

$$S = \int d^2x \sqrt{-g} \left(e^{\phi}\mathcal{R} + e^{\phi/2}(6/L^2) - m^2 e^{-\phi/2} - \frac{1}{4} e^{3\phi/2} F_{ab} F^{ab} \right),$$

A. Almheiri and J. Polchinski, JHEP **1511** (2015) 014; A. Almheiri and B. Kang, arXiv:1606.04108;
M. Cvetic and I. Papadimitriou, arXiv:1608.07018

Einstein-Maxwell-axion theory

The Einstein-Maxwell-dilaton theory of the small torus limit, $T \ll 1/R$, is equivalent on its boundary to the Schwarzian theory discussed earlier for the SYK model

$$S_{\phi,f} = \frac{K}{2}\int_0^{1/T} d\tau (\partial_\tau \phi)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau \, \{f,\tau\} + i\frac{L}{4\pi^2}\int_0^{1/T} d\tau (\partial_\tau \phi)\{f,\tau\}.$$

So the correlators of the density fluctuations, $\mathcal{Q}(\tau)$, and the energy fluctuations $\delta E - \mu \delta \mathcal{Q}(\tau)$ are time independent and given by

$$\left(\begin{array}{cc} \langle \delta\mathcal{Q}(\tau)\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle /T \\ \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))\delta\mathcal{Q}(0)\rangle & \langle (\delta E(\tau) - \mu\delta\mathcal{Q}(\tau))(\delta E(0) - \mu\delta\mathcal{Q}(0))\rangle /T \end{array} \right) = T\chi_s$$

where χ_s is the static susceptibility matrix given by

$$\chi_s \equiv \left(\begin{array}{cc} -(\partial^2\Omega/\partial\mu^2)_T & -\partial^2\Omega/(\partial T\partial\mu) \\ -T\partial^2\Omega/(\partial T\partial\mu) & -T(\partial^2\Omega/\partial T^2)_\mu \end{array} \right).$$

A. Kitaev, unpublished; J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

Einstein-Maxwell-axion theory

Finally, in the large torus limit, $T \gg 1/R$, we have the behavior of the diffusive metal without quasiparticles found in the coupled SYK models

$$\left(\begin{array}{cc} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} / T \\ \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; E - \mu \mathcal{Q} \rangle_{k,\omega} / T \end{array} \right) = \left[i\omega(-i\omega + Dk^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \left(\begin{array}{cc} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{array} \right) \chi_s^{-1}.$$

Y. Bardoux, M. M. Caldarelli, and C. Charmousis, JHEP 05 (2012) 054
D. Vegh, arXiv:1301.0537.
R. A. Davison, PRD 88 (2013) 086003.
M. Blake and D. Tong, PRD 88 (2013), 106004.
T. Andrade and B. Withers, JHEP 05 (2014) 101.

Non-Fermi liquids

- Shortest possible "phase coherence" time, fastest possible local equilibration time, or fastest possible Lyapunov time towards quantum chaos, all of order $\frac{\hbar}{k_B T}$

- Realization in solvable SYK model, which saturates the lower bound on the Lyapunov time.

- Coupled SYK models realize diffusive metal without quasiparticles.

- Remarkable holographic match to Einstein-Maxwell-axion theories with momentum dissipation via the Schwarzian effective action.