Quantum magnetism and criticality

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Theory of the Nernst effect near the superfliud-insulator transition

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Outline

1. Quantum "disordering" magnetic order

   Collinear order and confinement

2. Z_2 spin liquids

   Noncollinear order and fractionalization

3. U(1) spin liquids

   Valence bond solid (VBS) order

4. Doped spin liquids

   Superconductors with topological order

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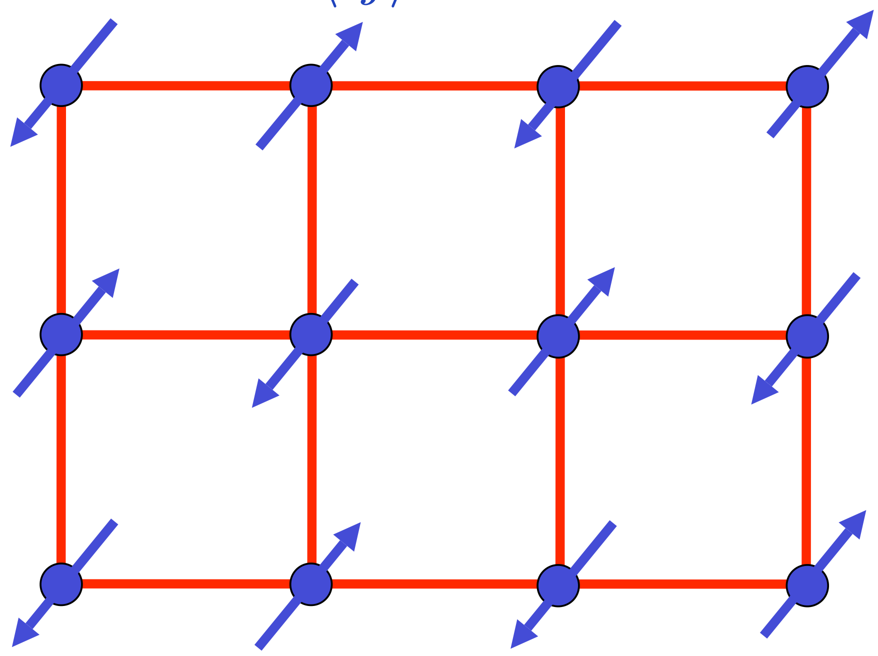
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



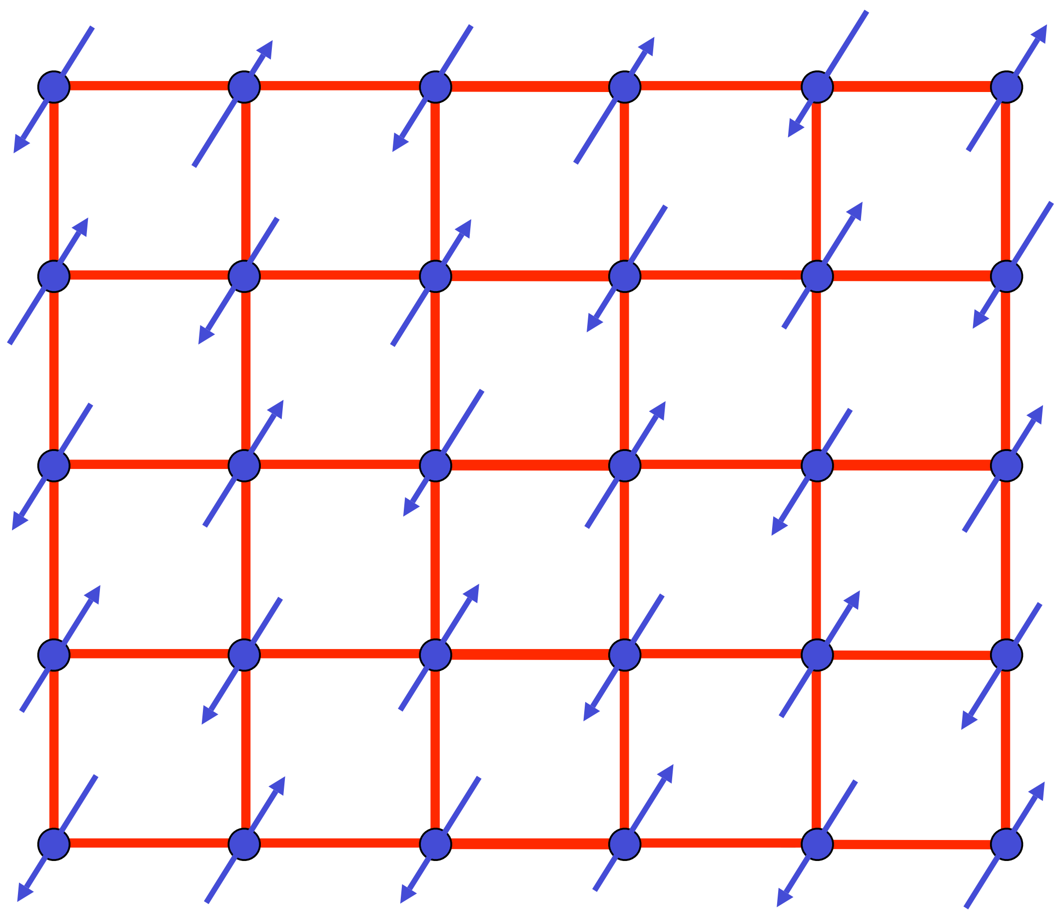
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

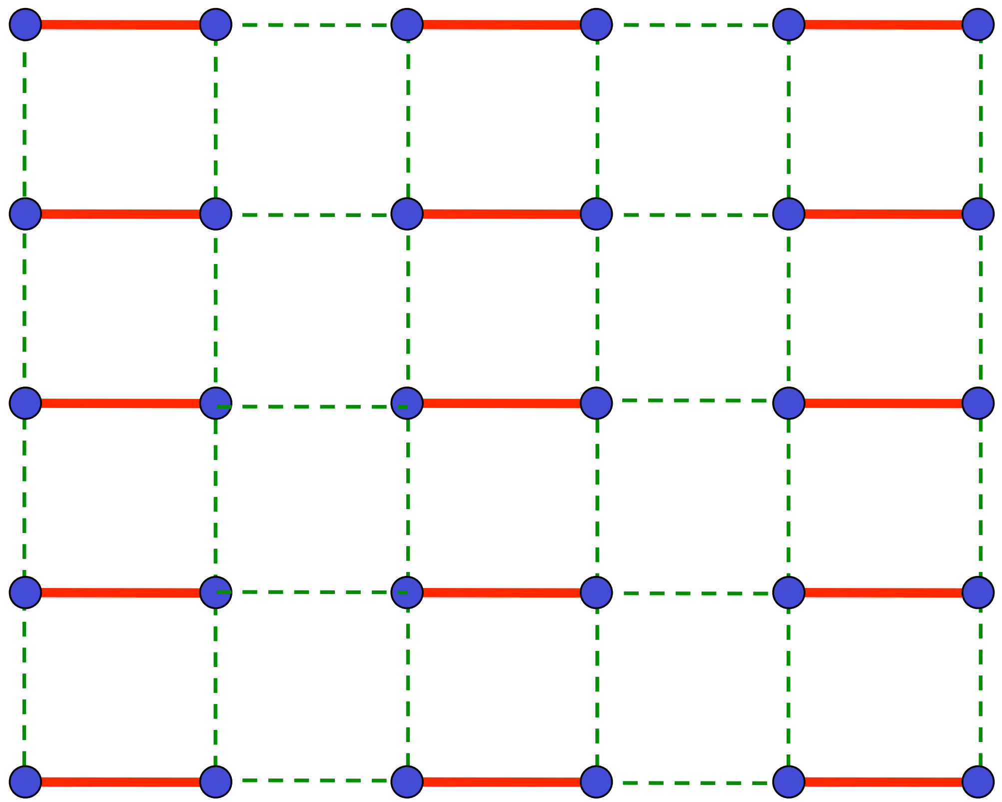
$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Antiferromagnetic (Neel) order in the insulator

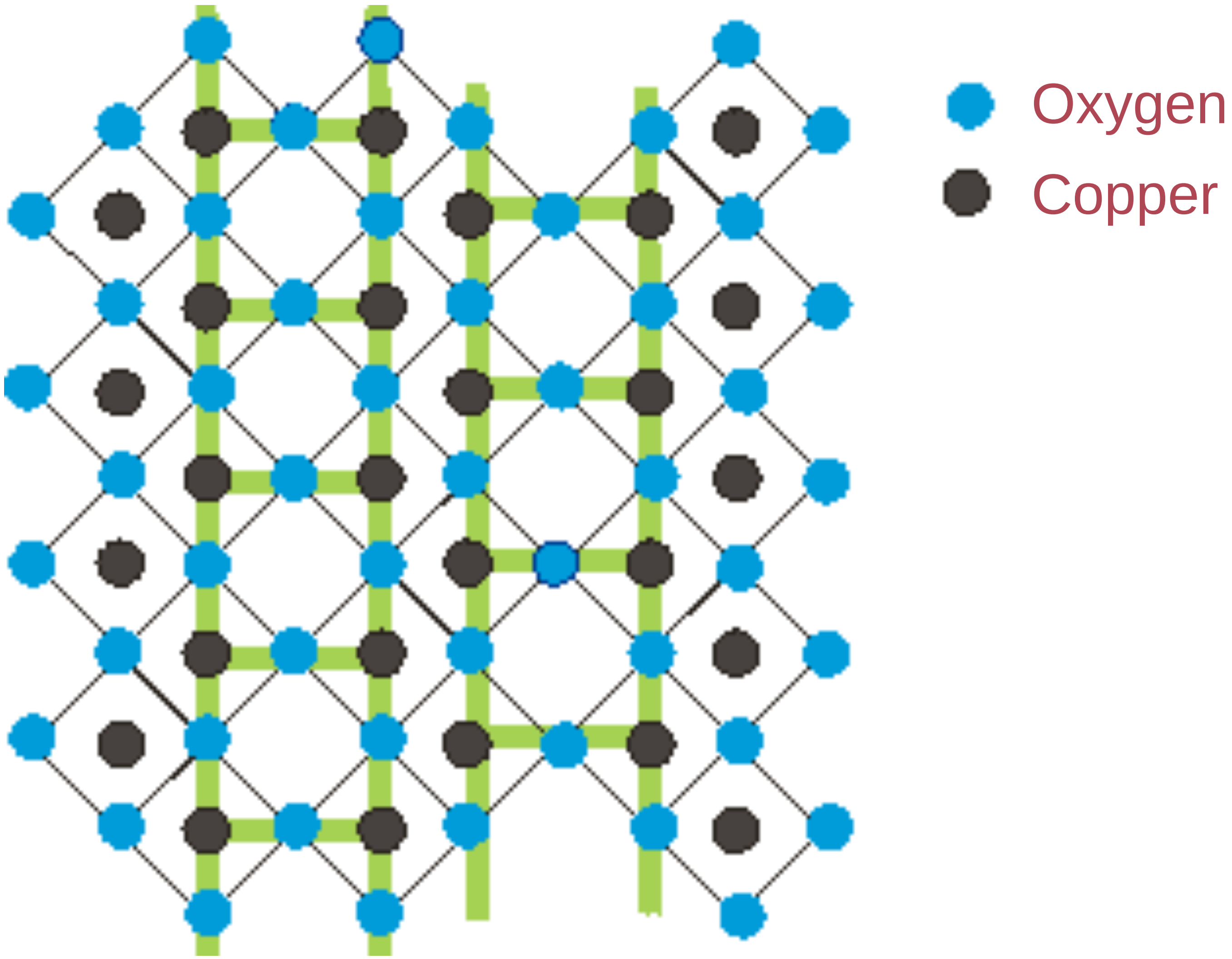


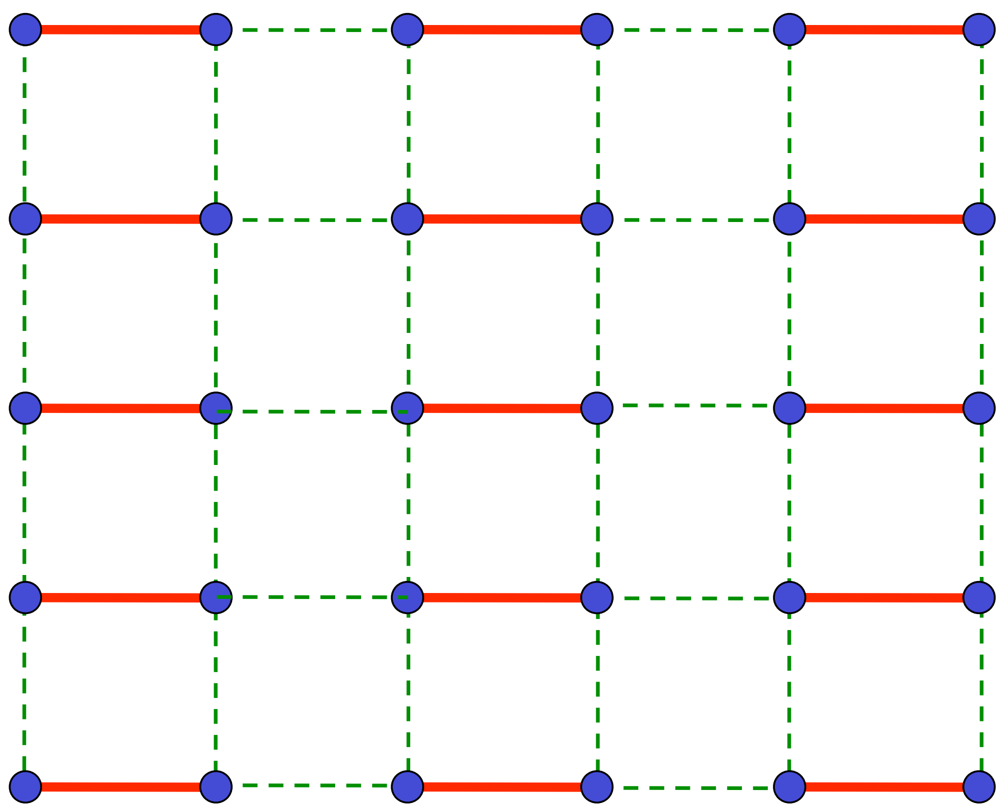
No entanglement of spins

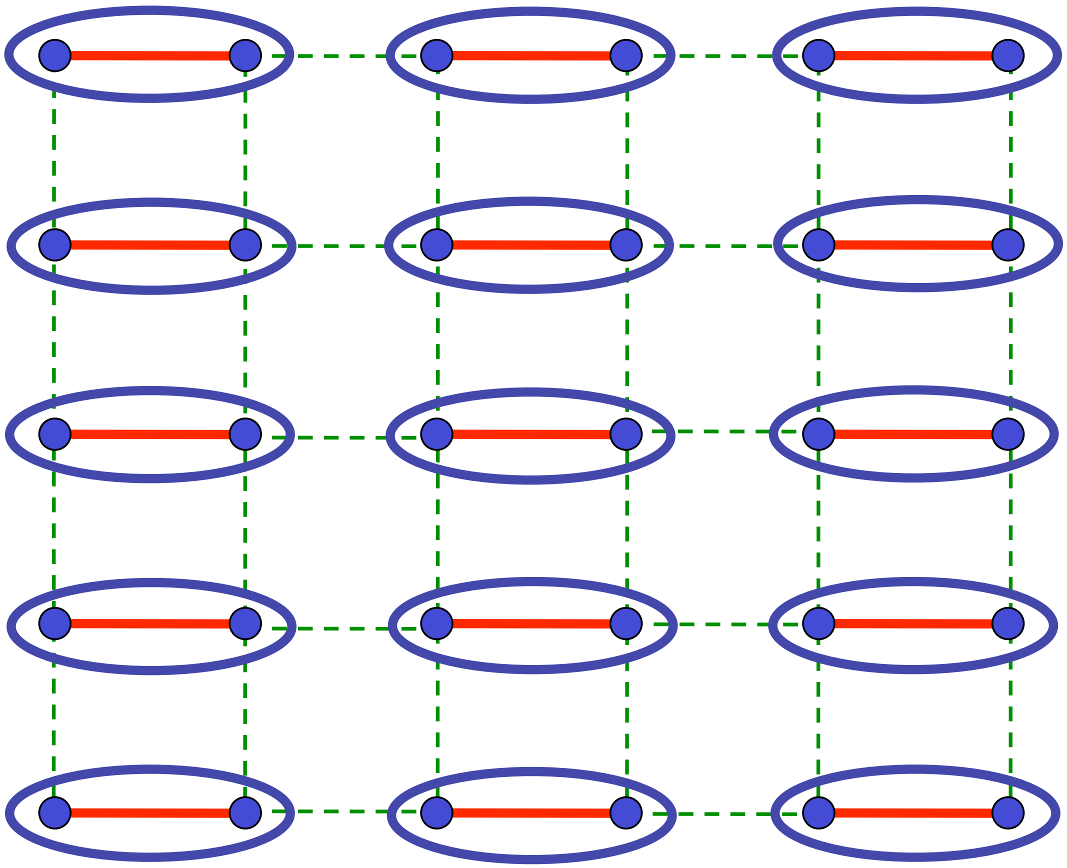


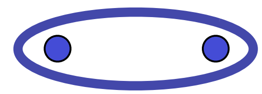
Weaken some bonds to induce spin entanglement in a new quantum phase

$SrCu_2O_3$





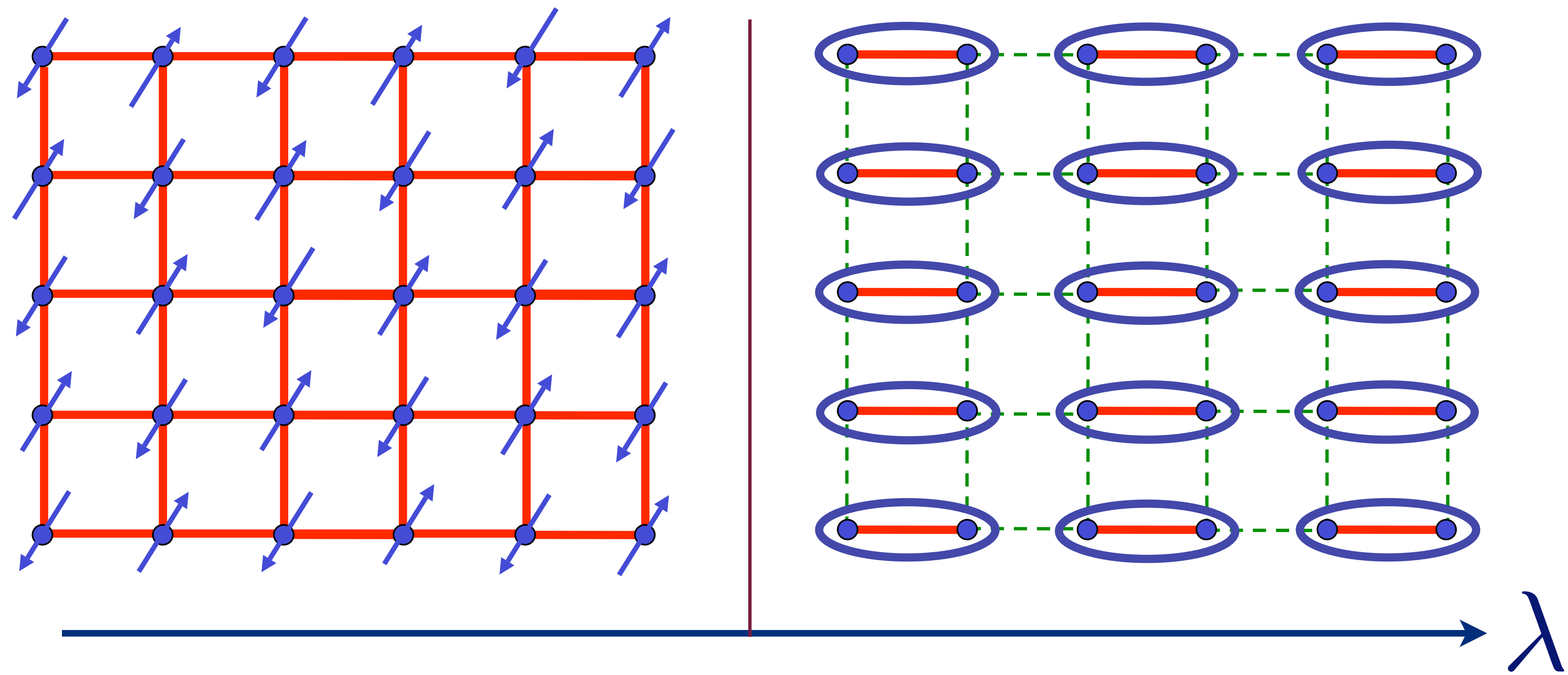


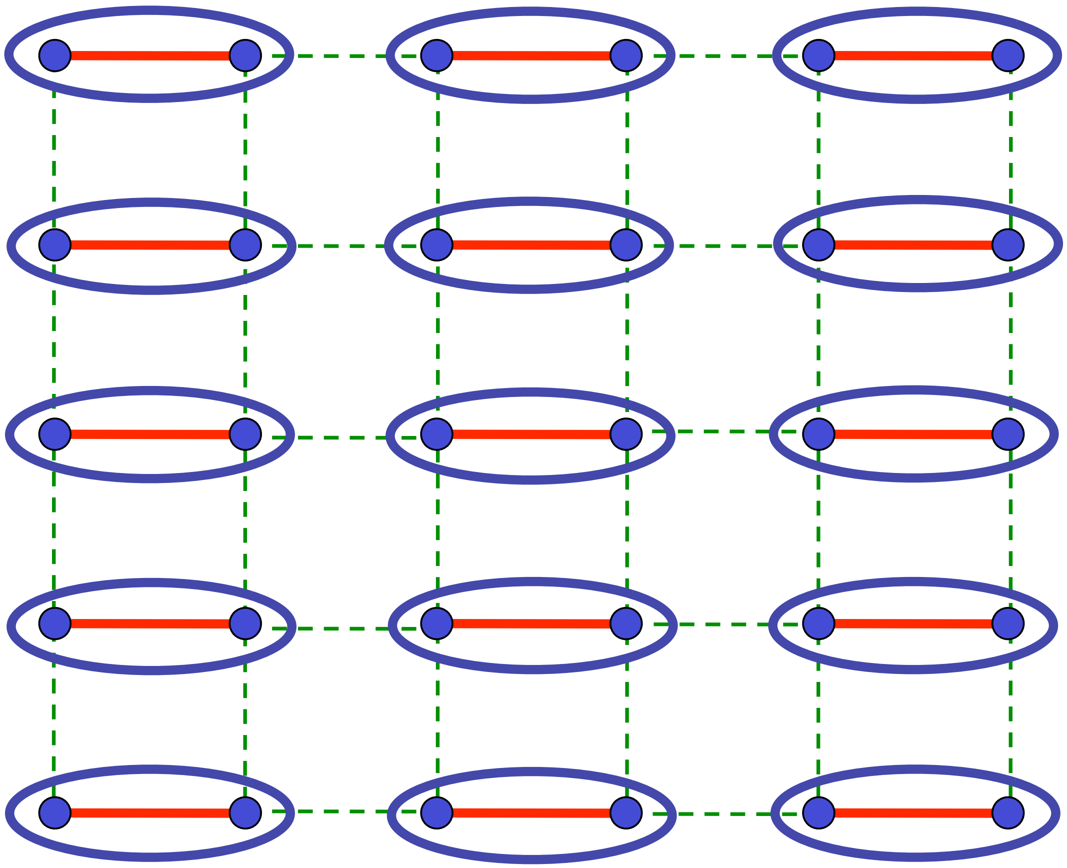


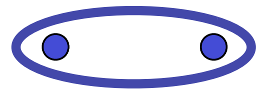
$$= \frac{1}{\sqrt{2}}\Big(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \Big)$$

Ground state is a product of pairs
of entangled spins.

Phase diagram as a function of the ratio of exchange interactions, λ







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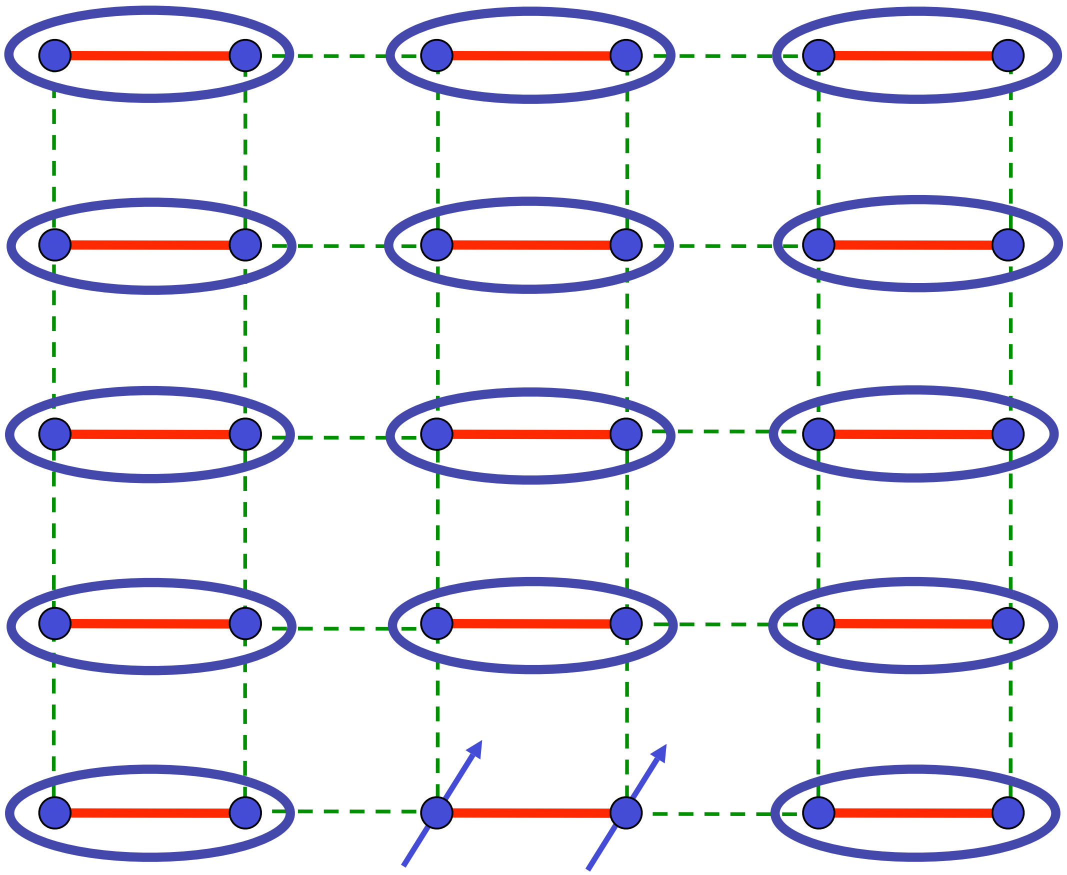

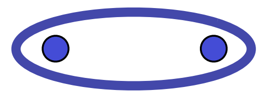

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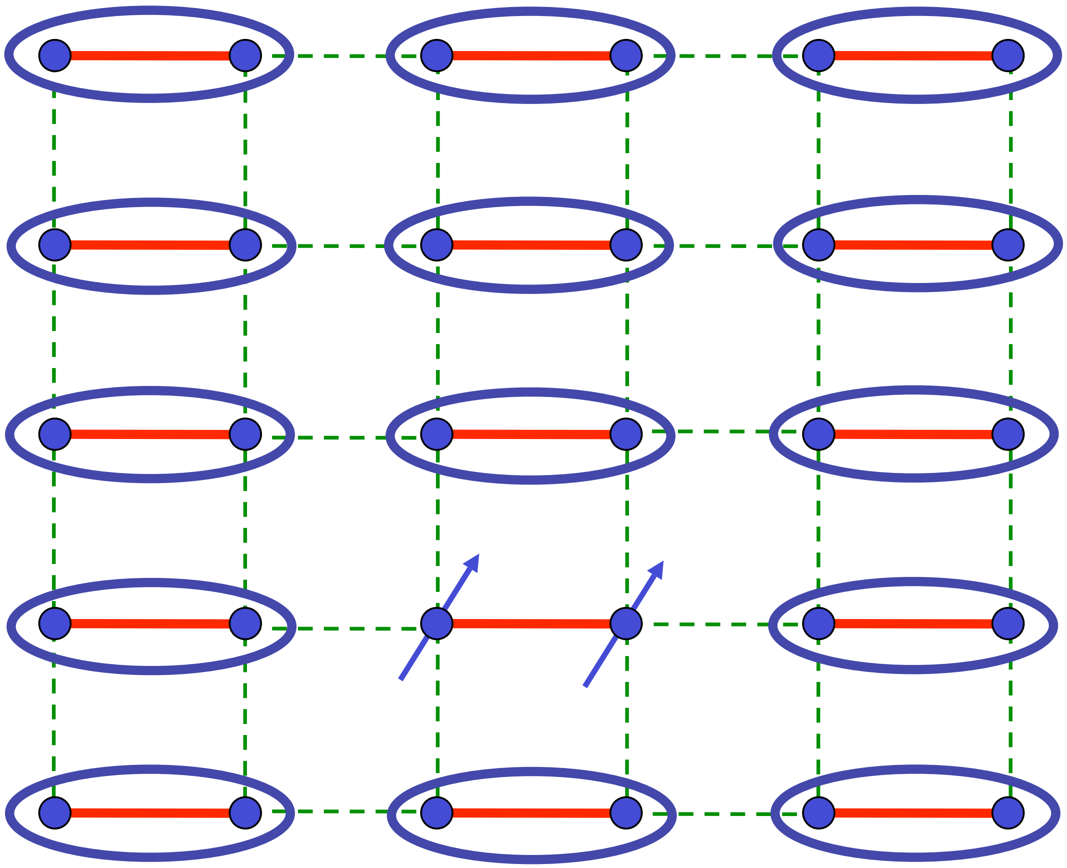
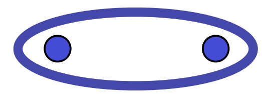
Excitation: S=1 ***triplon***





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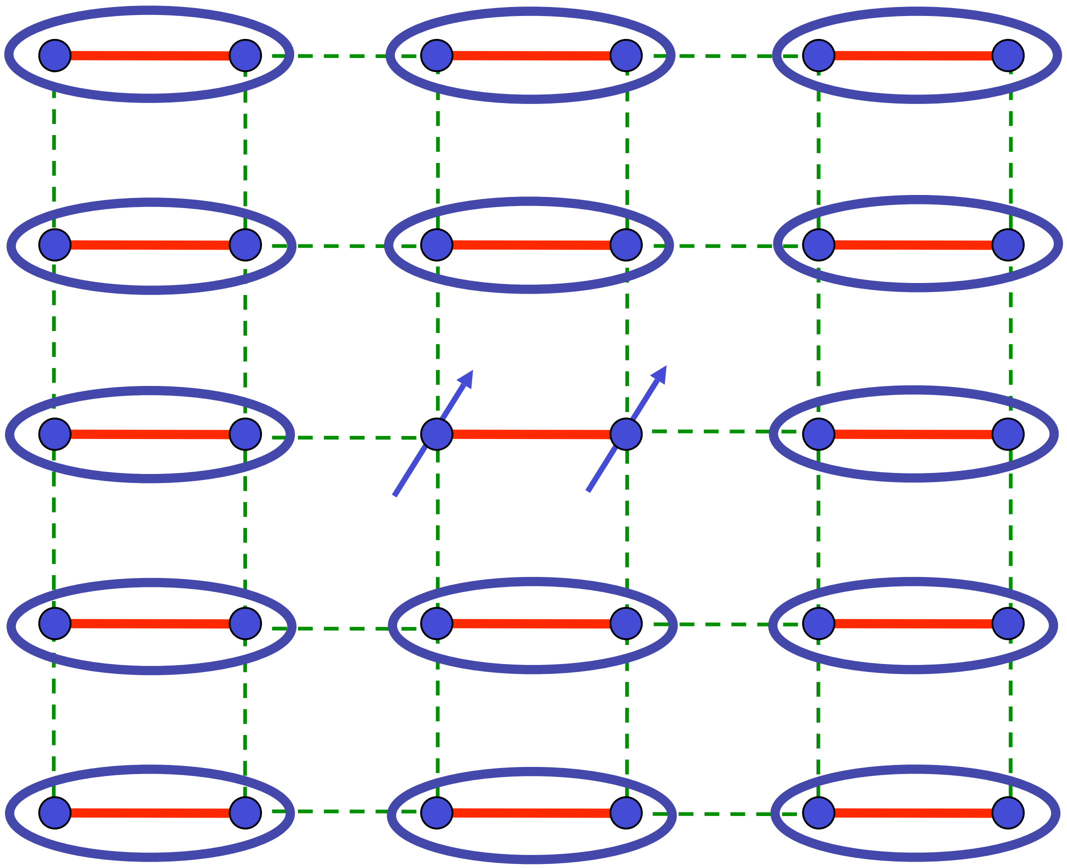

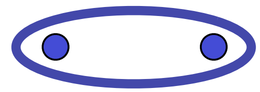

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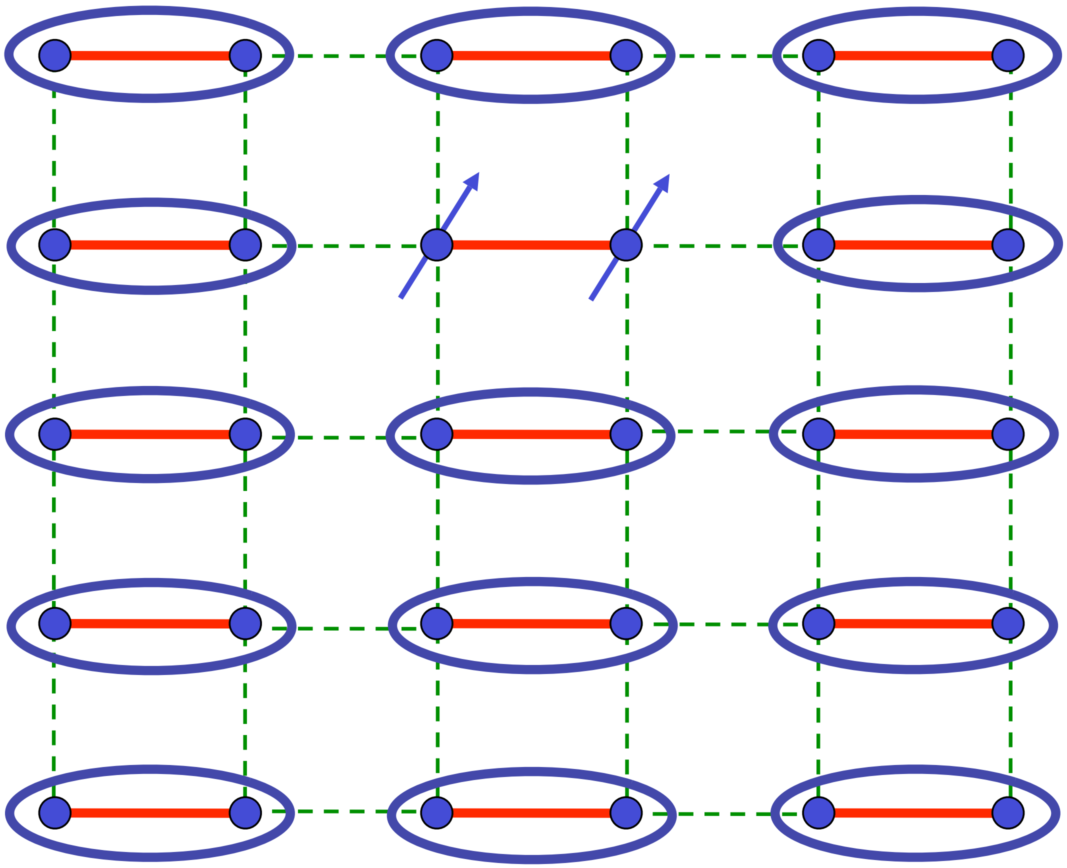

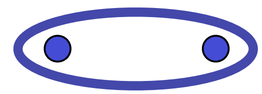

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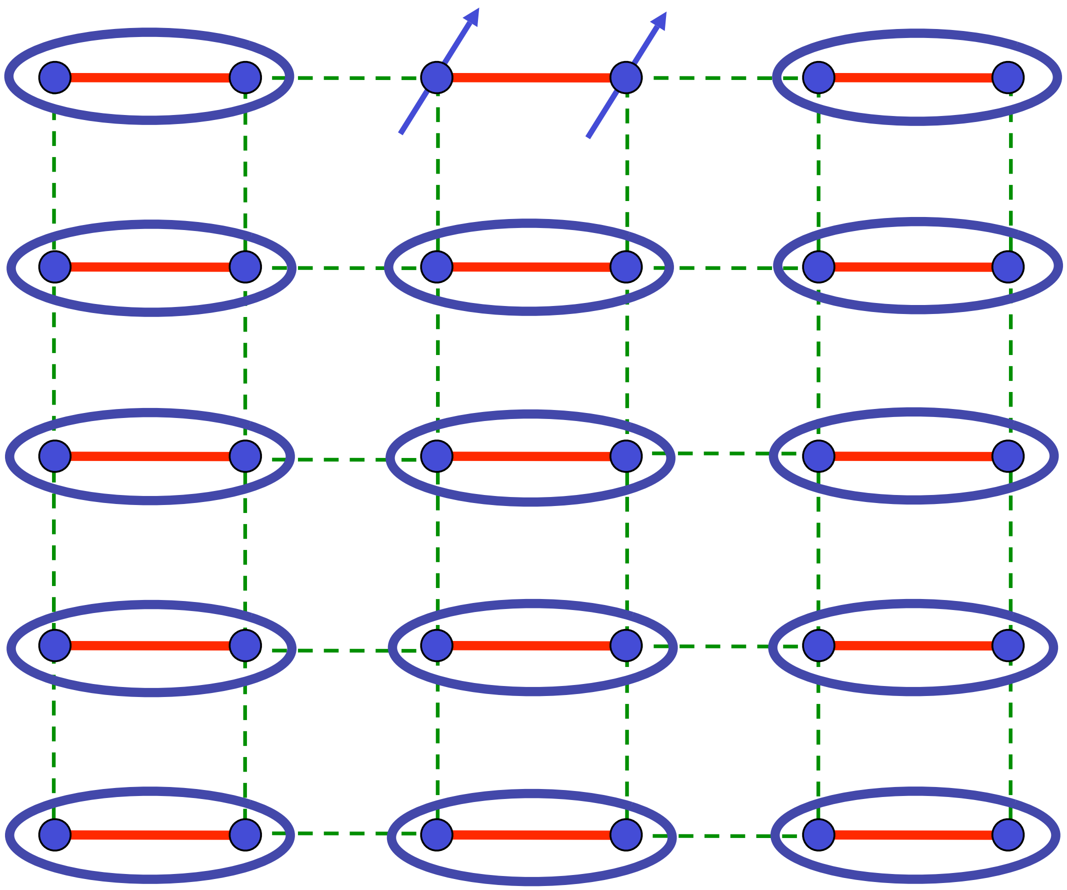

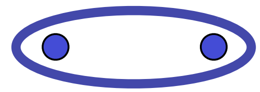

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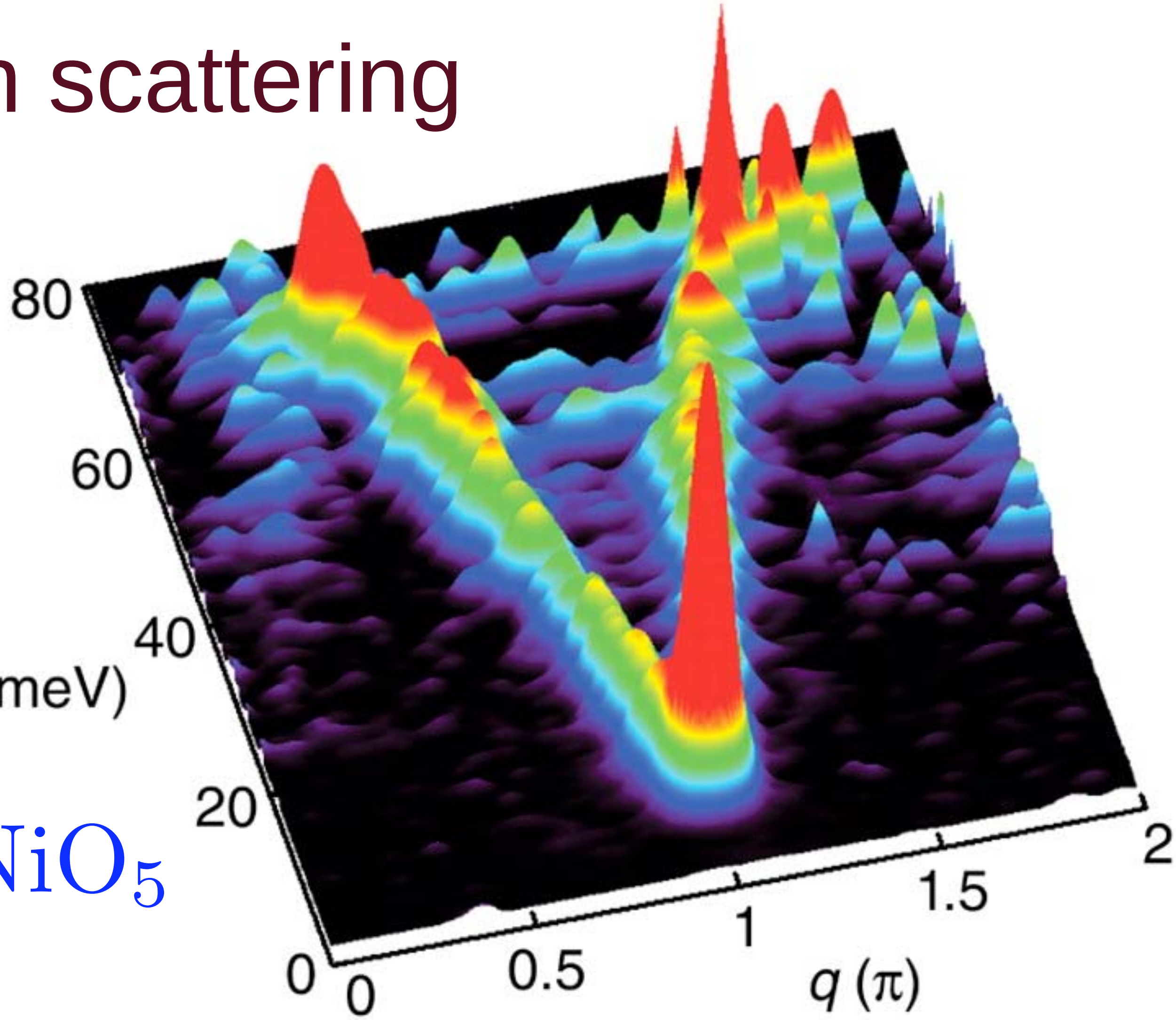
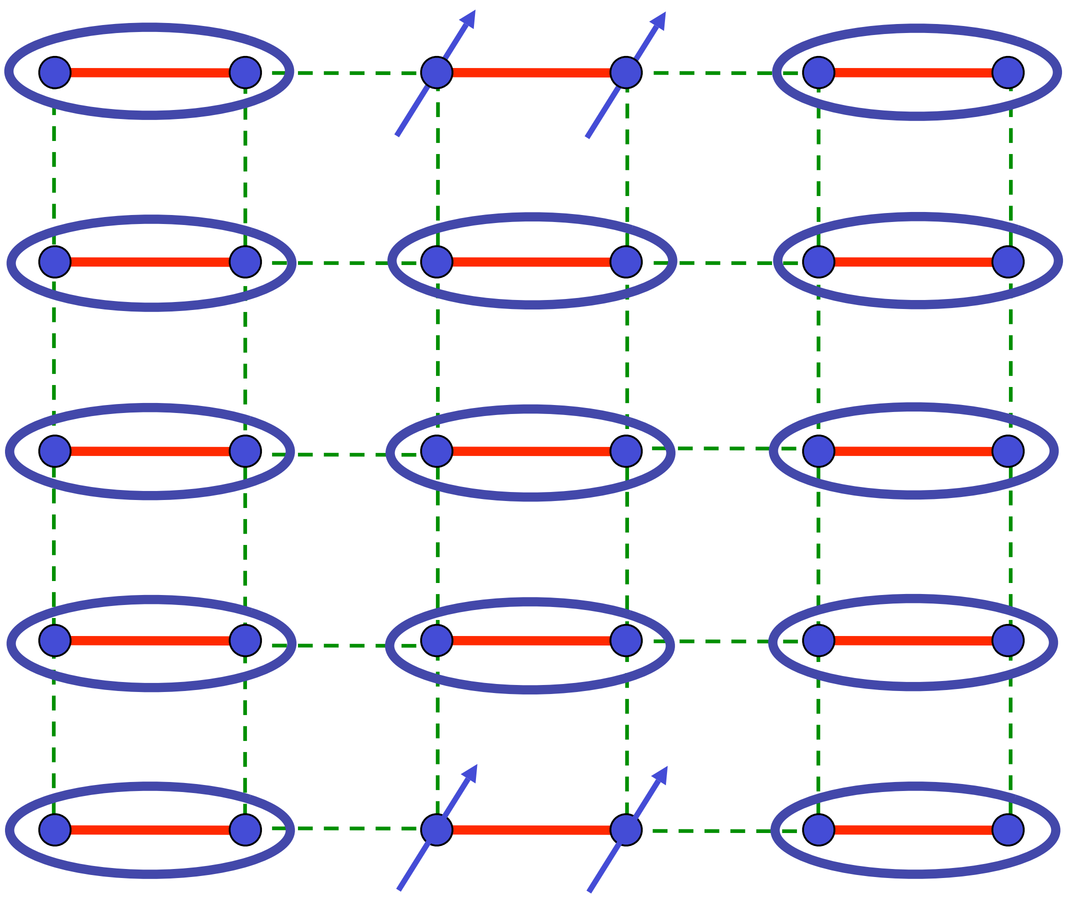
Neutron scattering

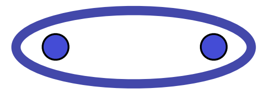


Y_2BaNiO_5

G. Xu, C. Broholm, Yeong-Ah Soh, G. Aeppli, J. F. DiTusa, Y. Chen, M. Kenzelmann, C. D. Frost, T. Ito, K. Oka, and H. Takagi, Science **317**, 1049 (2007).

Collision of triplons





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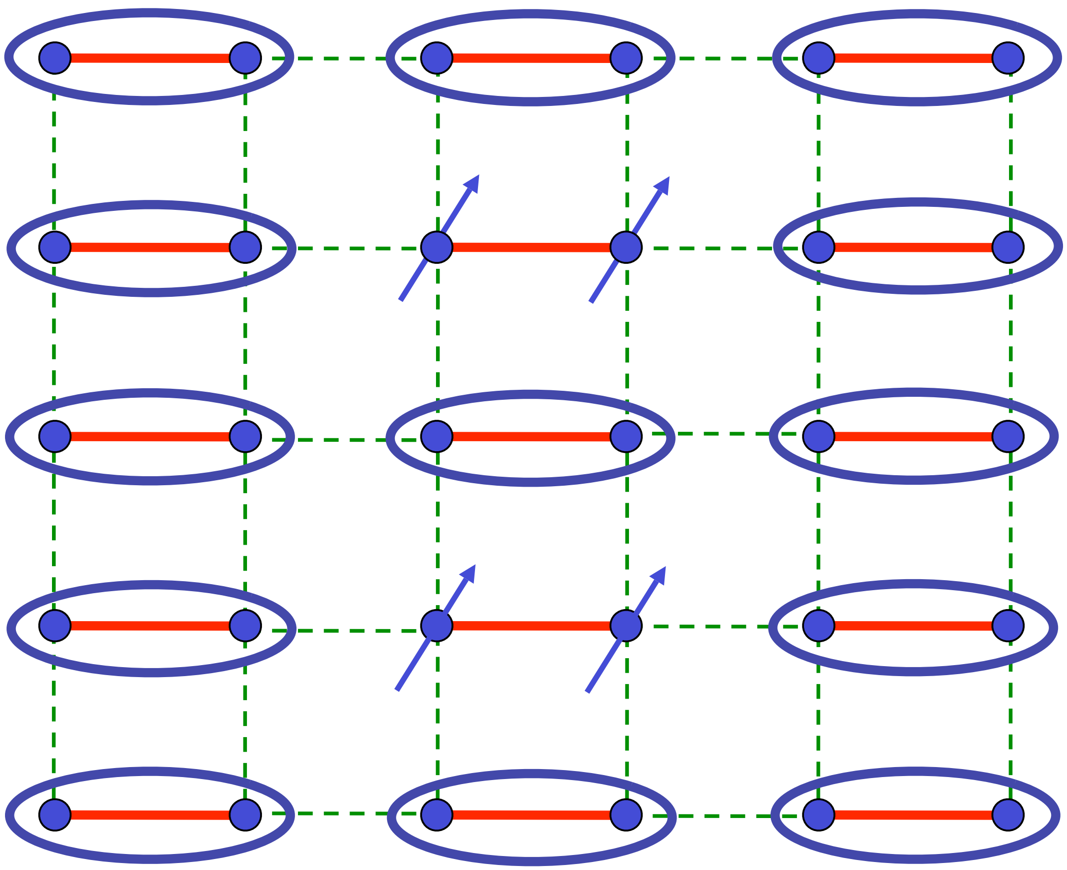

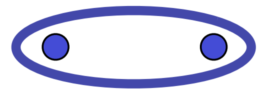

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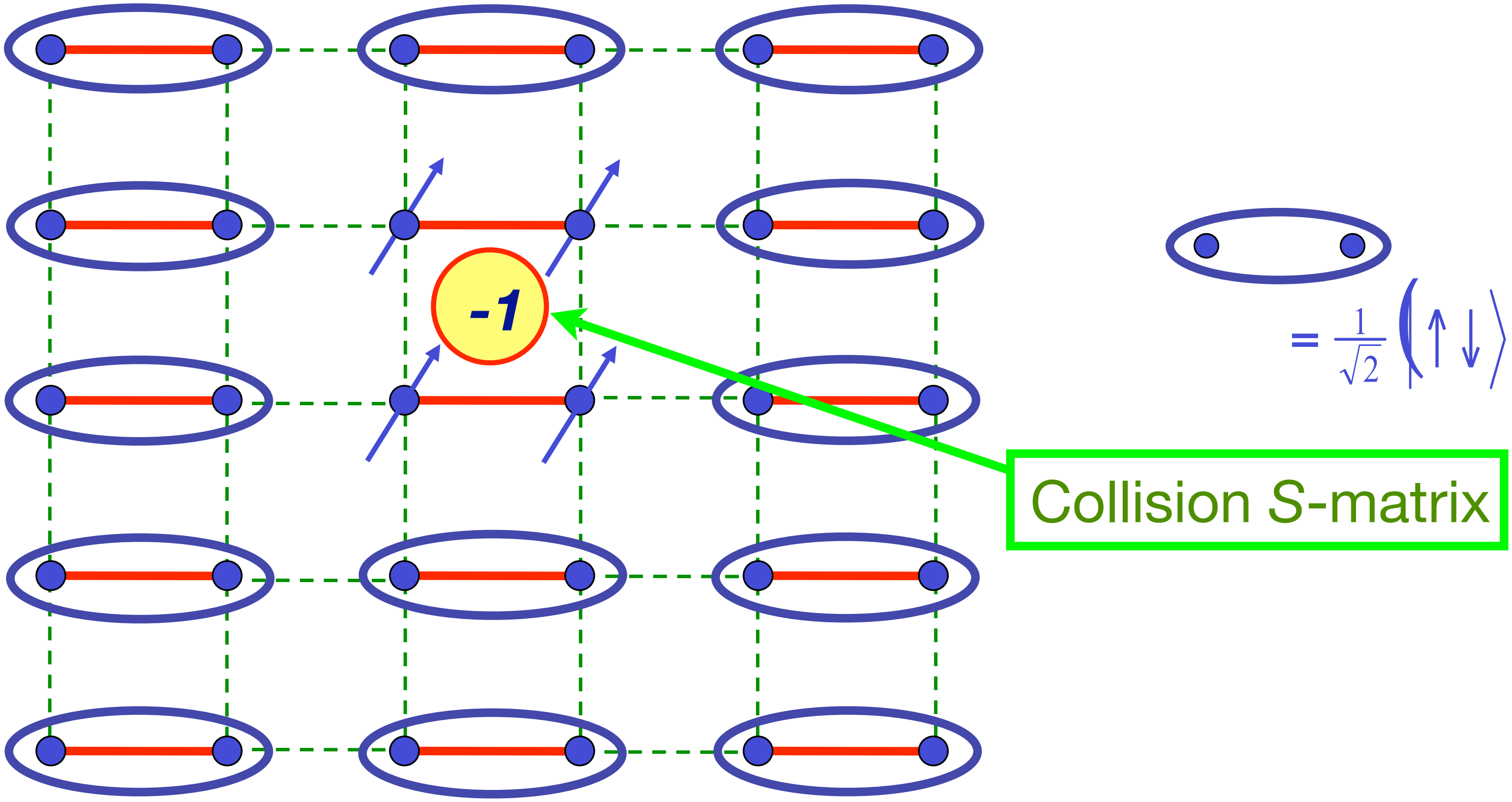

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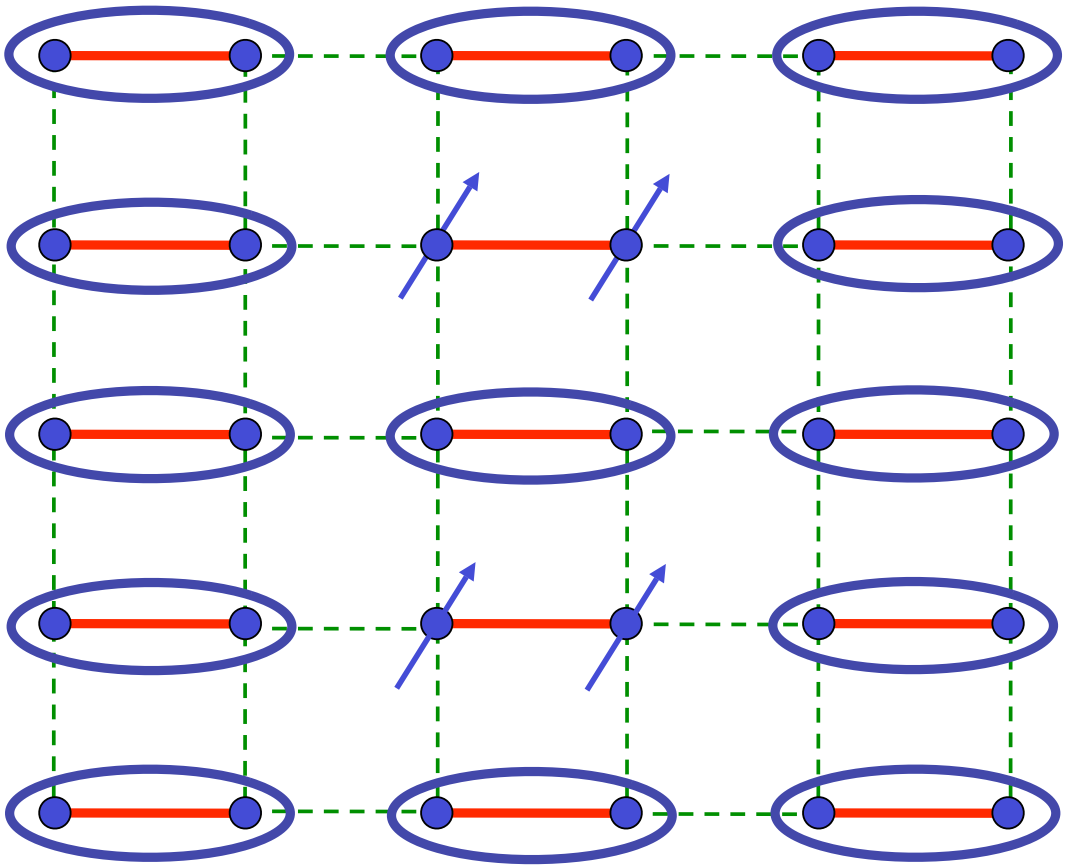

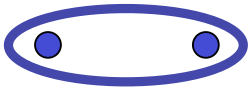

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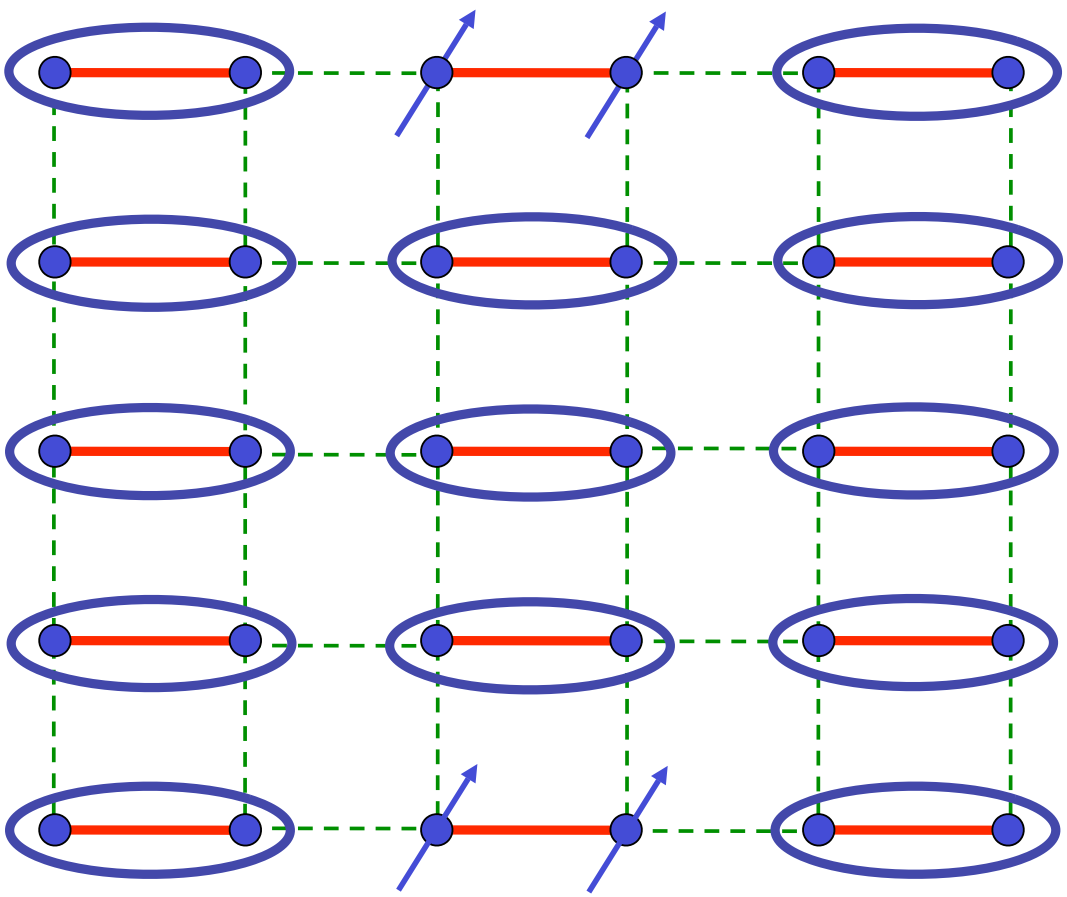

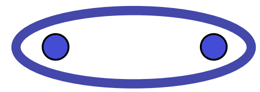

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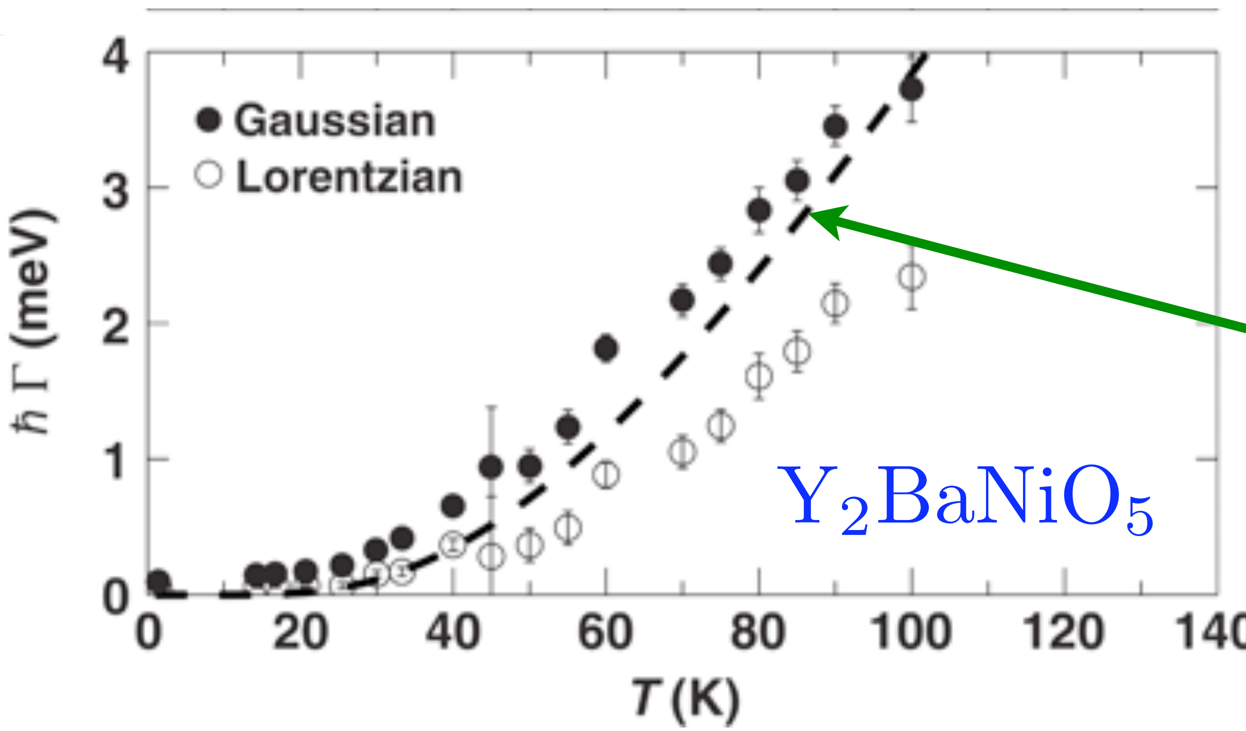
Neutron scattering linewidth



Parameter free prediction by K. Damle and S. Sachdev, Phys. Rev. B **57**, 8307 (1998) from multiple collisions with universal, quantum *S*-matrices

G. Xu, C. Broholm, Yeong-Ah Soh, G. Aeppli, J. F. DiTusa, Y. Chen, M. Kenzelmann, C. D. Frost, T. Ito, K. Oka, and H. Takagi, Science **317**, 1049 (2007).

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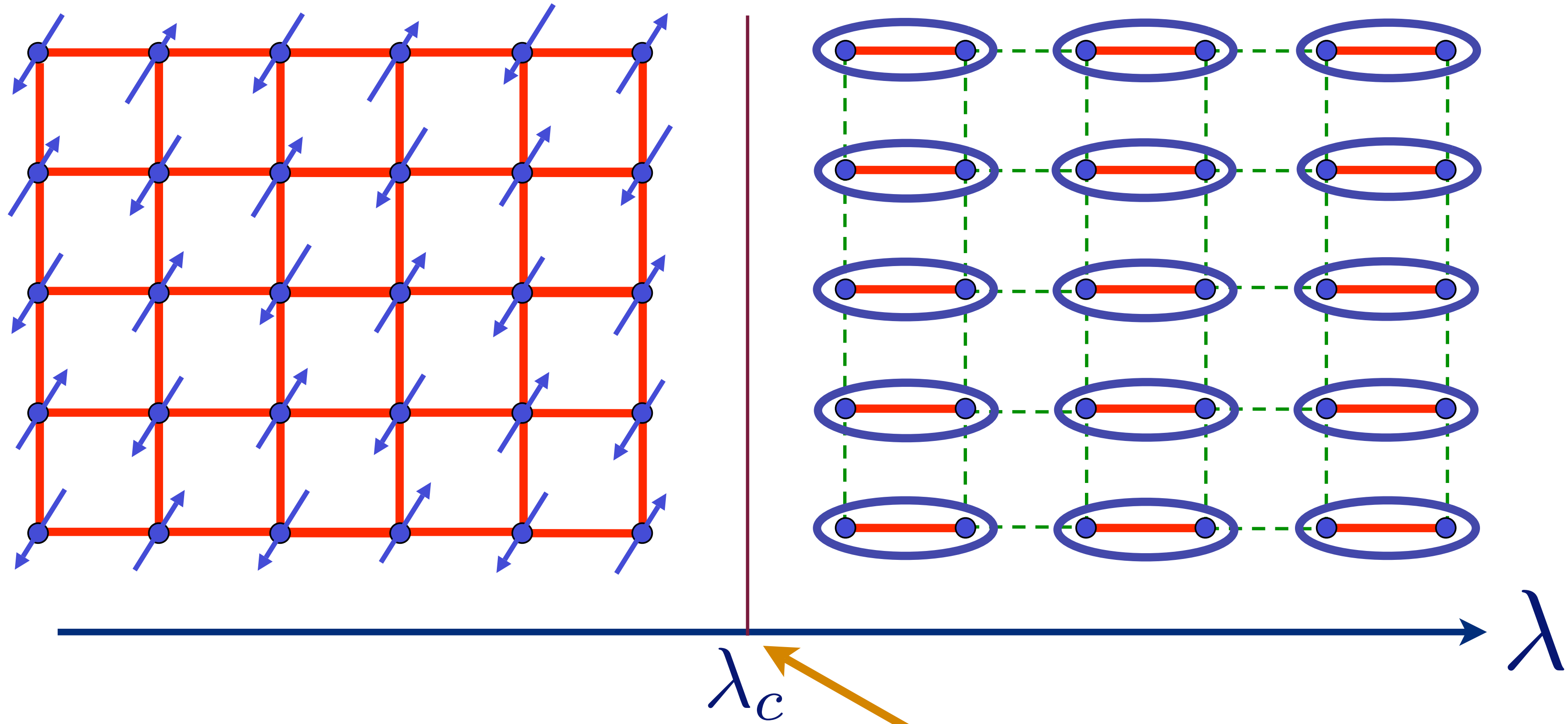
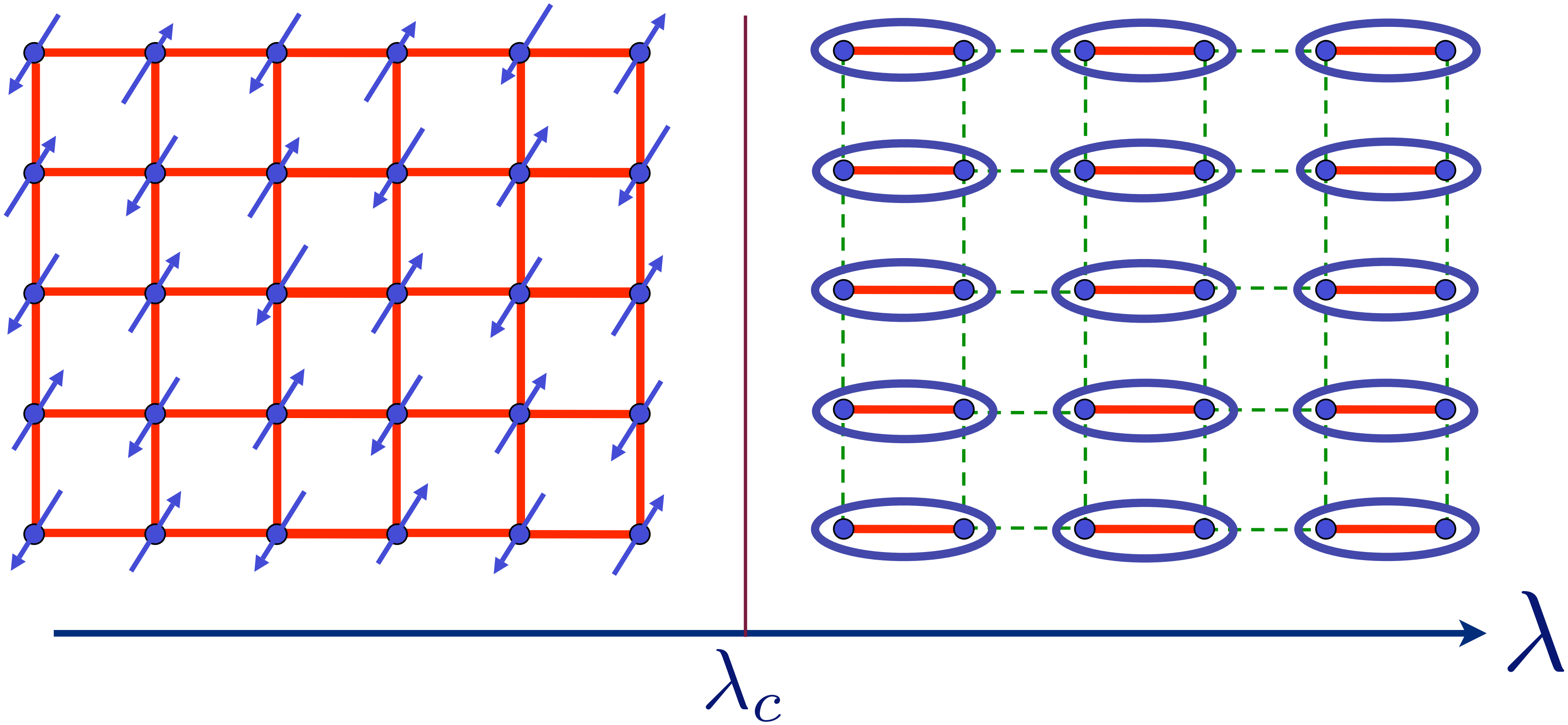


Quantum critical point with non-local entanglement in spin wavefunction

Phase diagram as a function of the ratio of exchange interactions, λ



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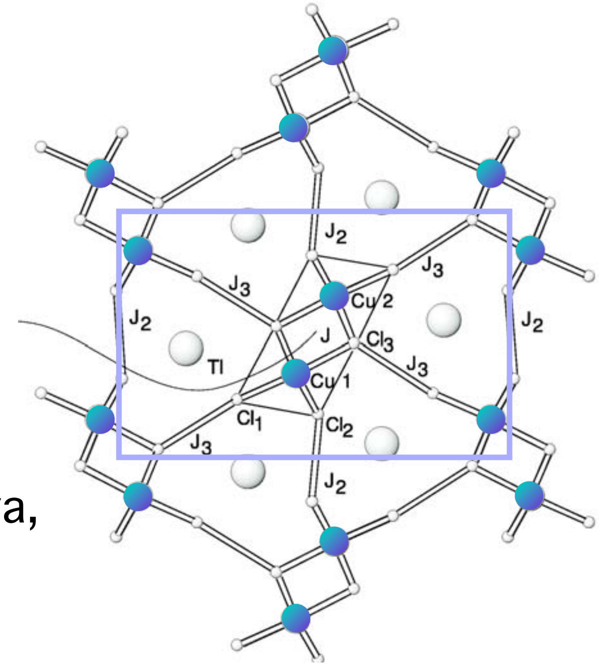
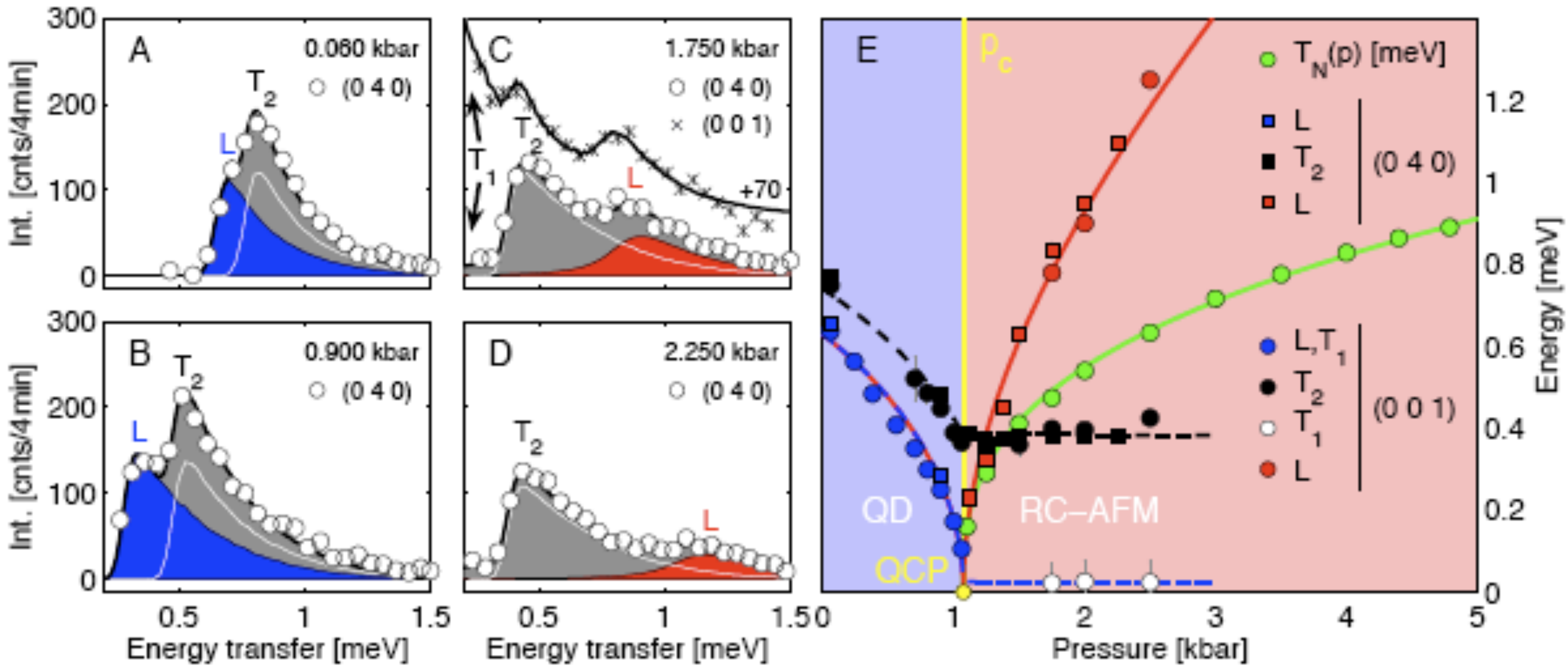
Landau-Ginzburg-Wilson Theory

Observation of longitudinal mode in $TlCuCl_3$

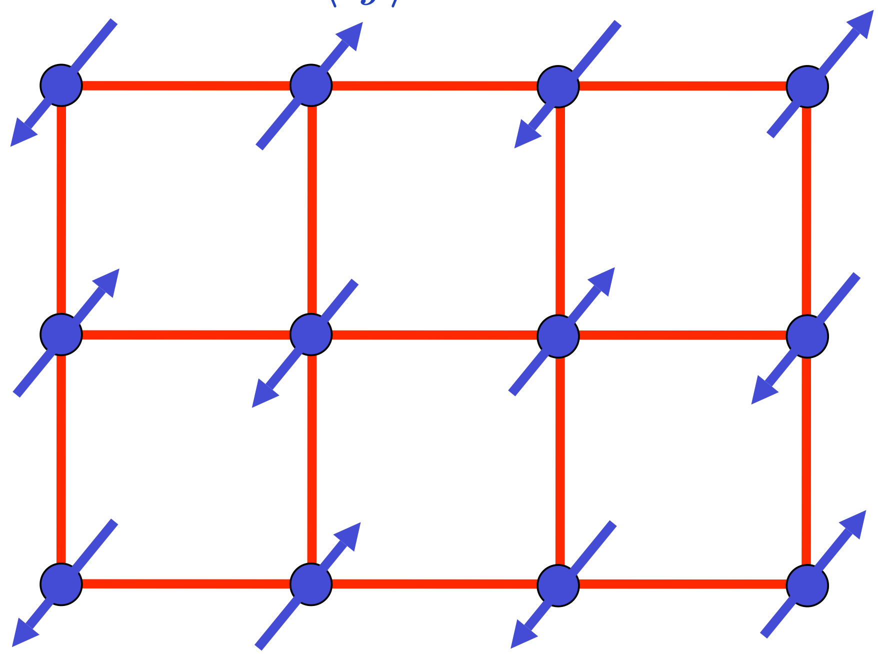
Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm





Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



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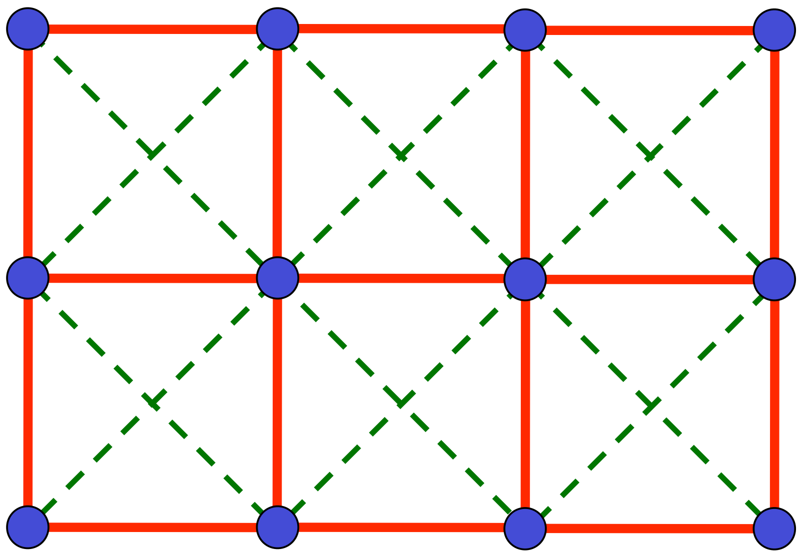

Destroy Neel order by perturbations which preserve full square lattice symmetry *e.g.* second-neighbor or ring exchange.

What is the state with $\langle \vec{\varphi} \rangle = 0$?

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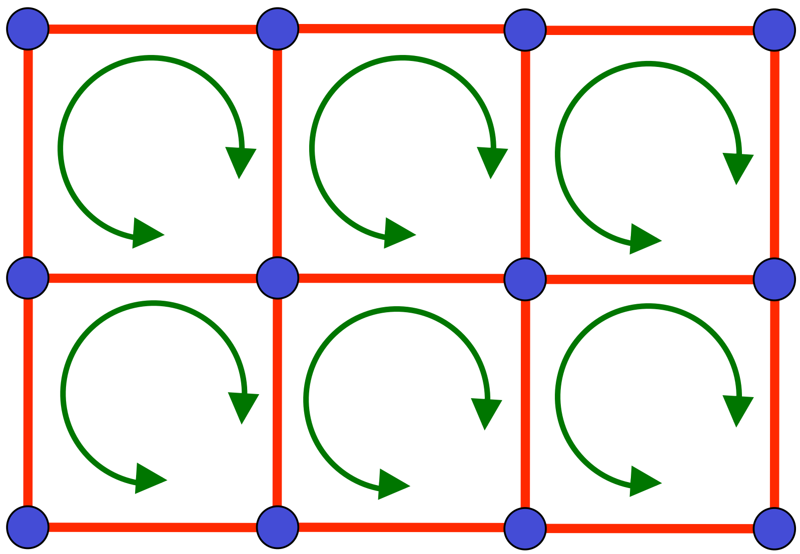

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LGW theory for quantum criticality

Landau-Ginzburg-Wilson theory: write down an effective action for the antiferromagnetic order parameter $\vec{\varphi}$ by expanding in powers of $\vec{\varphi}$ and its spatial and temporal derivatives, while preserving all symmetries of the microscopic Hamiltonian

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S. Chakravarty, B.I. Halperin, and D.R. Nelson, *Phys. Rev.* B **39**, 2344 (1989)

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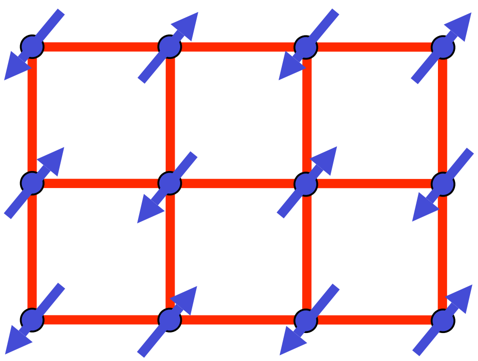
A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)



$\langle \vec{\varphi} \rangle \neq 0$

Néel state

State with no broken symmetries. Fluctuations of $\vec{\varphi}$ about $\vec{\varphi} = 0$ realize a *stable* $S = 1$ quasiparticle with energy $\varepsilon_k = \sqrt{s + c^2 k^2}$

$\langle \vec{\varphi} \rangle = 0$

s_c $\quad$ s

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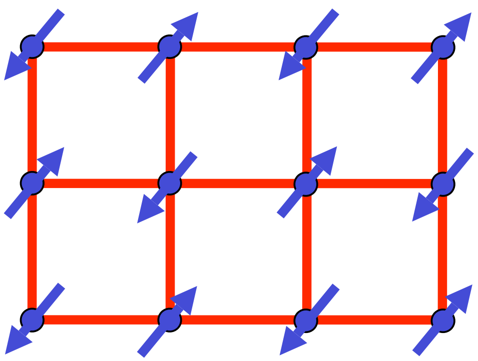

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Néel state

However, $S = 1/2$ antiferromagnets on the square lattice have **no such state**.

$\langle \vec{\varphi} \rangle = 0$

s_c $\qquad$ s

There is no state with a gapped, stable S=1 quasiparticle and no broken symmetries

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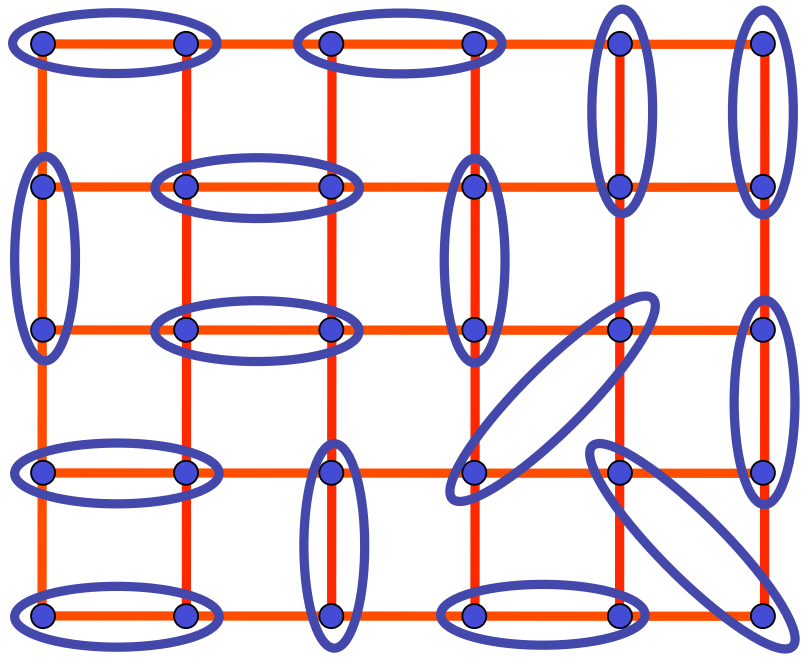

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"Liquid" of valence bonds has fractionalized *S=1/2* excitations

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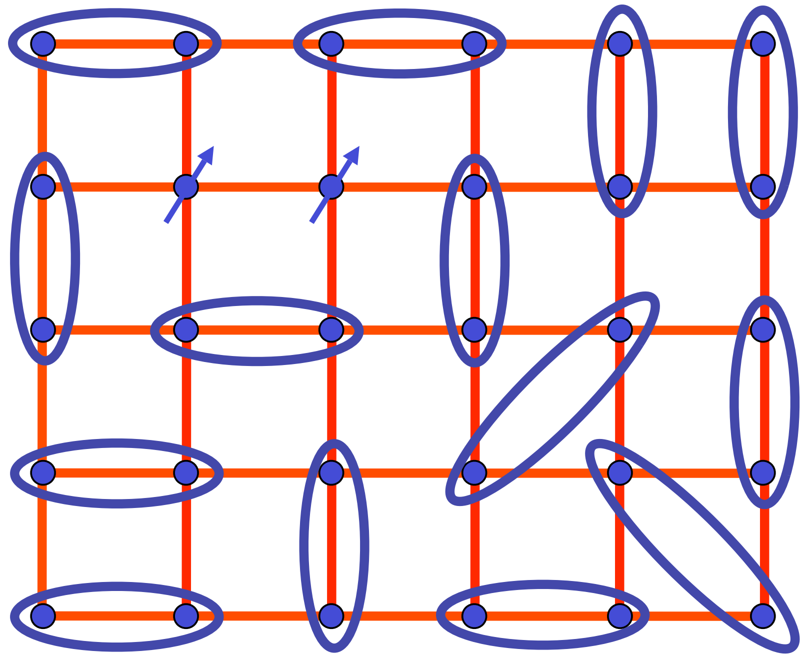

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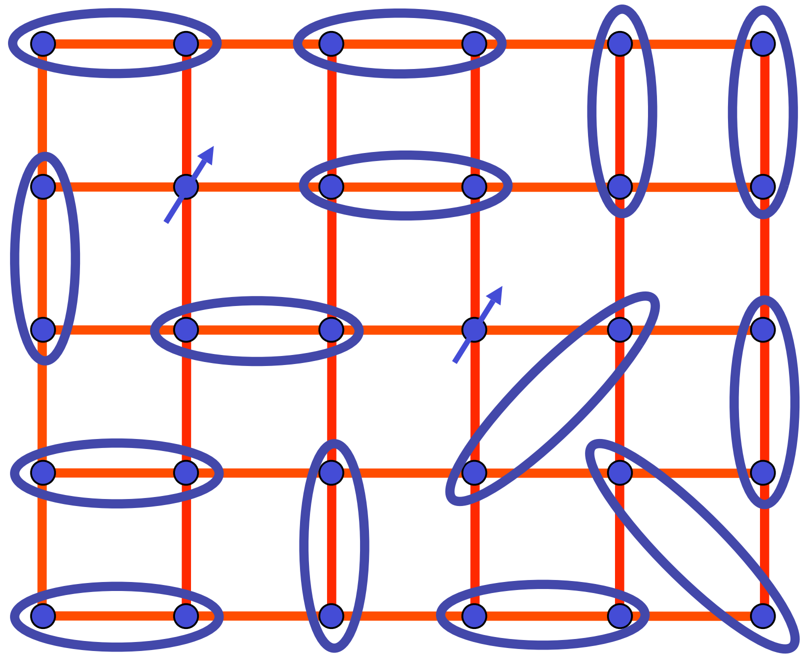

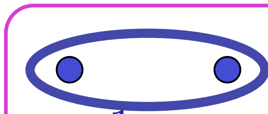

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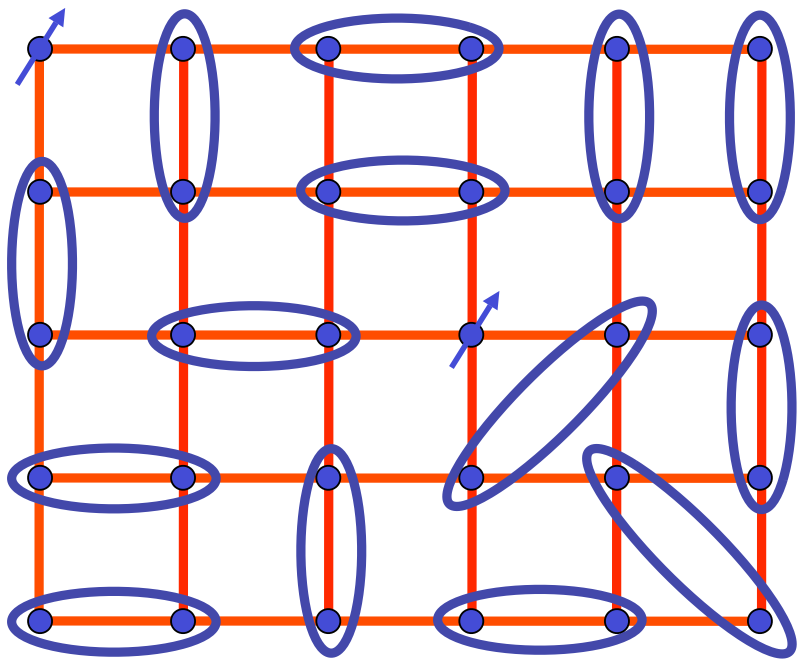


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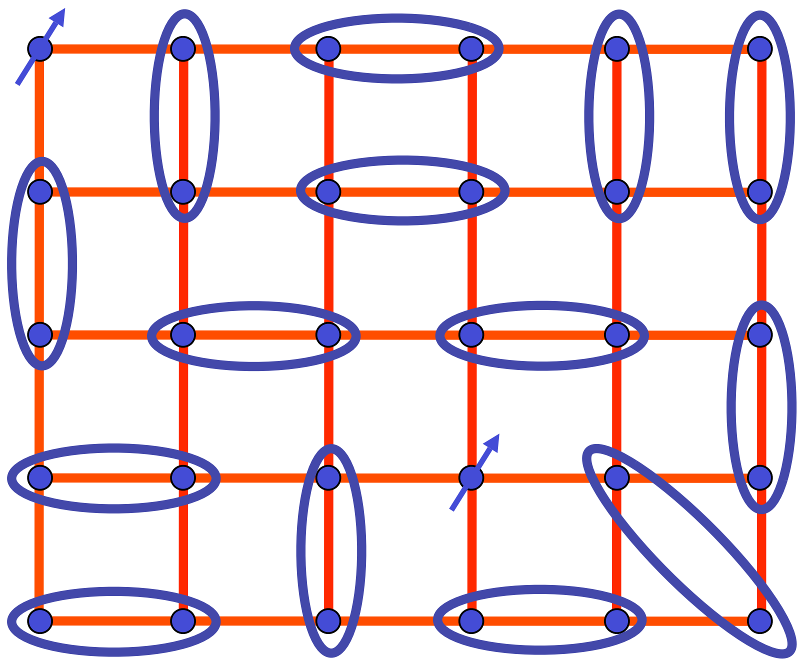

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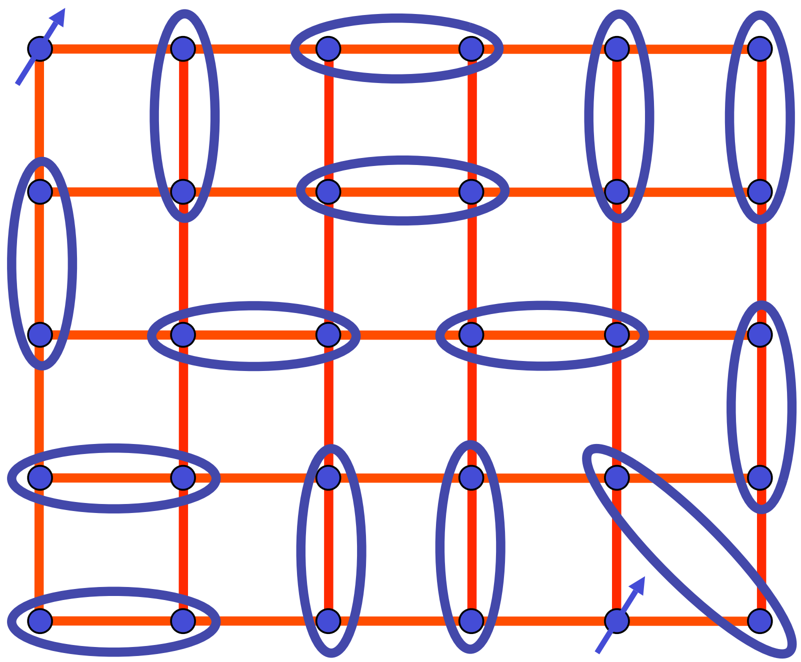

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"Liquid" of valence bonds has fractionalized *S=1/2* excitations

Possible theory for fractionalization and topological order

Decompose the Néel order parameter into *spinors*

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

where $\vec{\sigma}$ are Pauli matrices, and z_α are complex spinors which carry spin $S = 1/2$.

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Key question: Can the z_α become the needed $S = 1/2$ excitations of a fractionalized phase ?

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Decompose the Néel order parameter into *spinors*

$$\vec{\varphi} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

where $\vec{\sigma}$ are Pauli matrices, and z_α are complex spinors which carry spin $S = 1/2$.

Key question: Can the z_α become the needed $S = 1/2$ excitations of a fractionalized phase ?

Effective theory for spinons must be invariant under the U(1) gauge transformation

$$z_\alpha \rightarrow e^{i\theta} z_\alpha$$

Possible theory for fractionalization and topological order

Naive expectation: Low energy spinon theory for "quantum dis-ordering" a Néel state is

$$\mathcal{S}_z = \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s \left|z_\alpha\right|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \Big]$$

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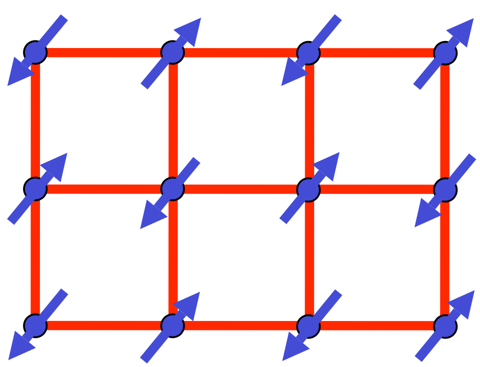


$\langle z_\alpha \rangle \neq 0$

Néel state

Spin liquid state with stable $S = 1/2$ z_α spinons, and a gapless U(1) photon A_μ representing the topological order.

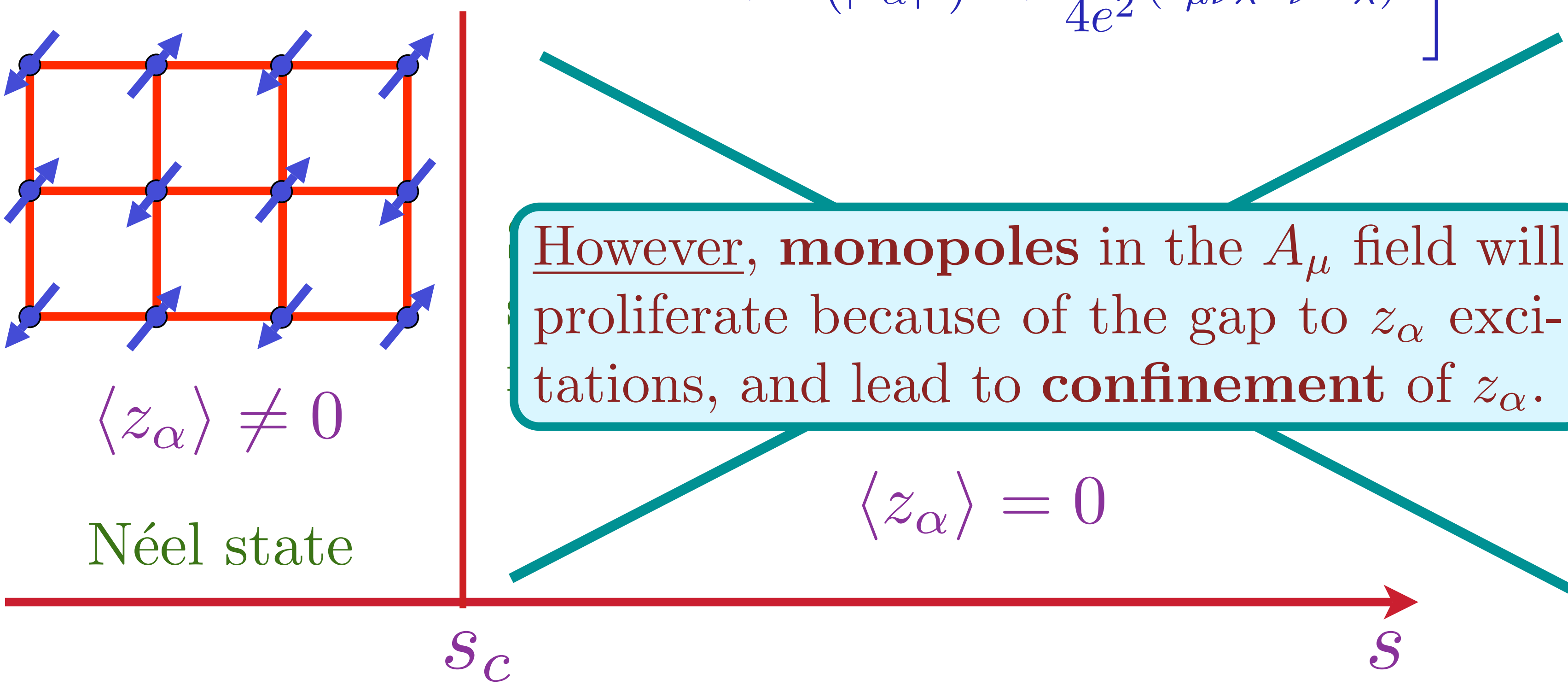
$\langle z_\alpha \rangle = 0$

s_c s

Possible theory for fractionalization and topological order

Naive expectation: Low energy spinon theory for "quantum disordering" a Néel state is

$$\mathcal{S}_z = \int d^2x d\tau \Big[c^2 \left|(\nabla_x - iA_x) z_\alpha\right|^2 + \left|(\partial_\tau - iA_\tau) z_\alpha\right|^2 + s \left|z_\alpha\right|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}\left(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda\right)^2 \Big]$$



However, **monopoles** in the A_μ field will proliferate because of the gap to z_α excitations, and lead to **confinement** of z_α.

Outline

1. Quantum "disordering" magnetic order
   Collinear order and confinement

2. Z_2 spin liquids
   Noncollinear order and fractionalization

3. U(1) spin liquids
   Valence bond solid (VBS) order

4. Doped spin liquids
   Superconductors with topological order

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Z_2 gauge theory for fractionalization and topological order

- *Discrete* gauge theories do have deconfined phases in 2+1 dimensions.

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- *Discrete* gauge theories do have deconfined phases in 2+1 dimensions.
- Find a collective excitation Φ with the gauge transformation

$$\Phi \to e^{2i\theta}\Phi$$

- Higgs state with $\langle\Phi\rangle \neq 0$ is described by the fractionalized phase of a Z_2 gauge theory in the which the spinons z_α carry Z_2 gauge charges (E. Fradkin and S. Shenker, Phys. Rev. D **19**, 3682 (1979)).

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- *What is Φ in the antiferromagnet* ? Its physical interpretation becomes clear from its allowed coupling to the spinons:

$$\mathcal{S}_{z,\Phi} = \int d^2r d\tau \left[\lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.}\right]$$

From this coupling it follows that the states with $\langle\Phi\rangle \neq 0$ have **coplanar spin correlations**.

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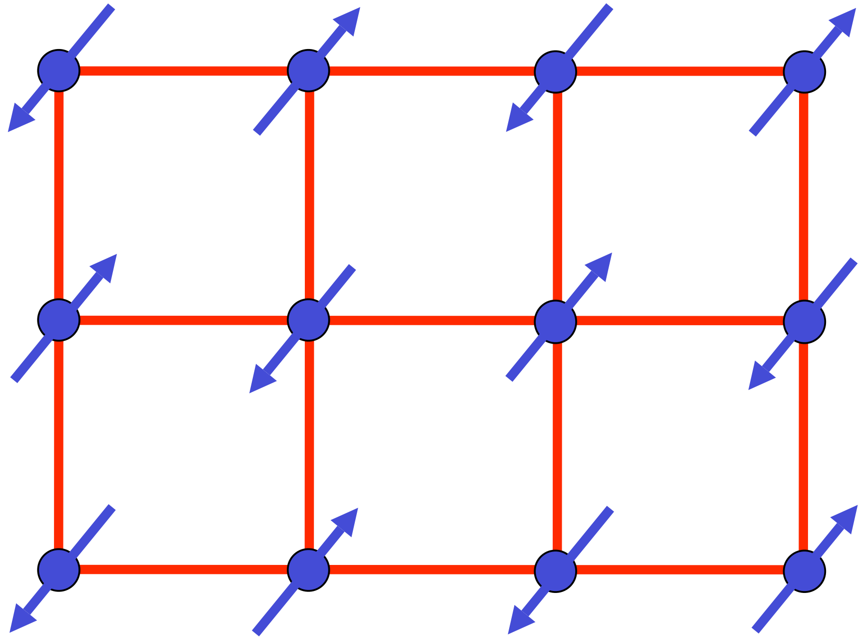
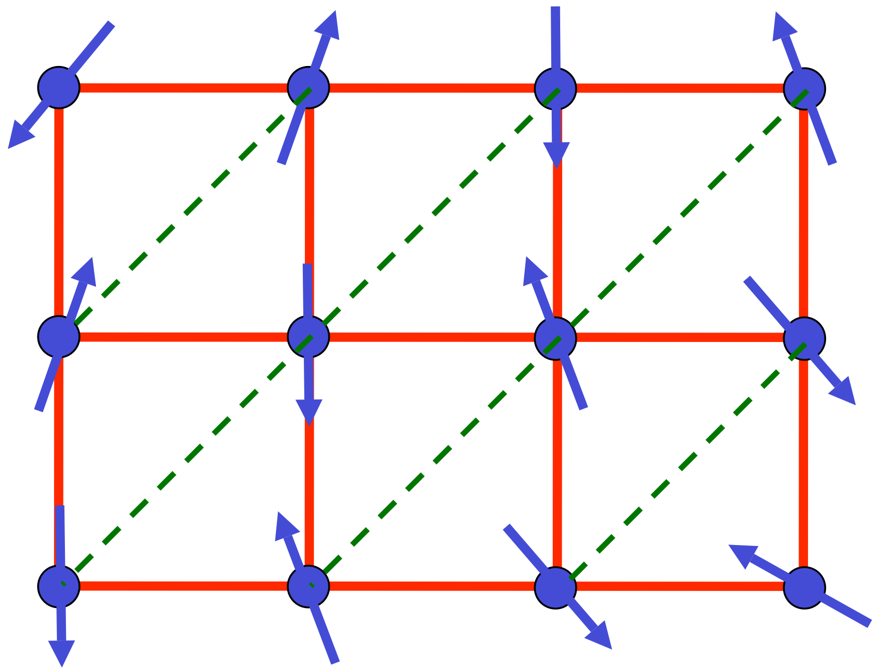


Collinear magnetic order with $\langle \Phi \rangle = 0$.

A spin density wave:

$$\langle \vec{S}_i \rangle \propto (\cos(\mathbf{K} \cdot \mathbf{r}_i), \sin(\mathbf{K} \cdot \mathbf{r}_i), 0)$$

$$\mathbf{K} = (\pi, \pi).$$



Coplanar magnetic order with $\langle\Phi\rangle \neq 0$.

A spin density wave:

$$\langle\vec{S}_i\rangle \propto (\cos(\mathbf{K}\cdot\mathbf{r}_i), \sin(\mathbf{K}\cdot\mathbf{r}_i), 0)$$

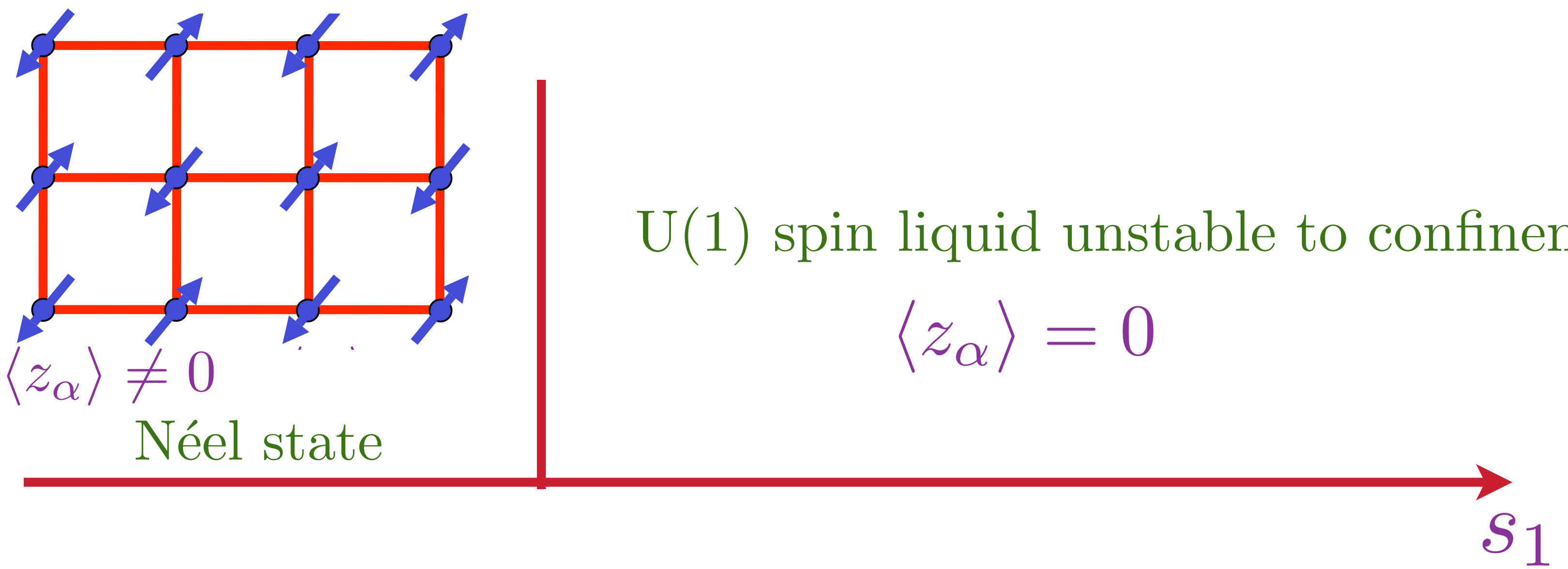
with

$$\mathbf{K} = (\pi + \langle\Phi\rangle, \pi + \langle\Phi\rangle).$$

Experimental realization: $CsCuCl_3$

Phase diagram of gauge theory of spinons

$$\mathcal{S}_z = \int d^2x d\tau \left[|(\partial_\mu - iA_\mu) z_\alpha|^2 + s_1 |z_\alpha|^2 + u \left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2} (\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right.$$



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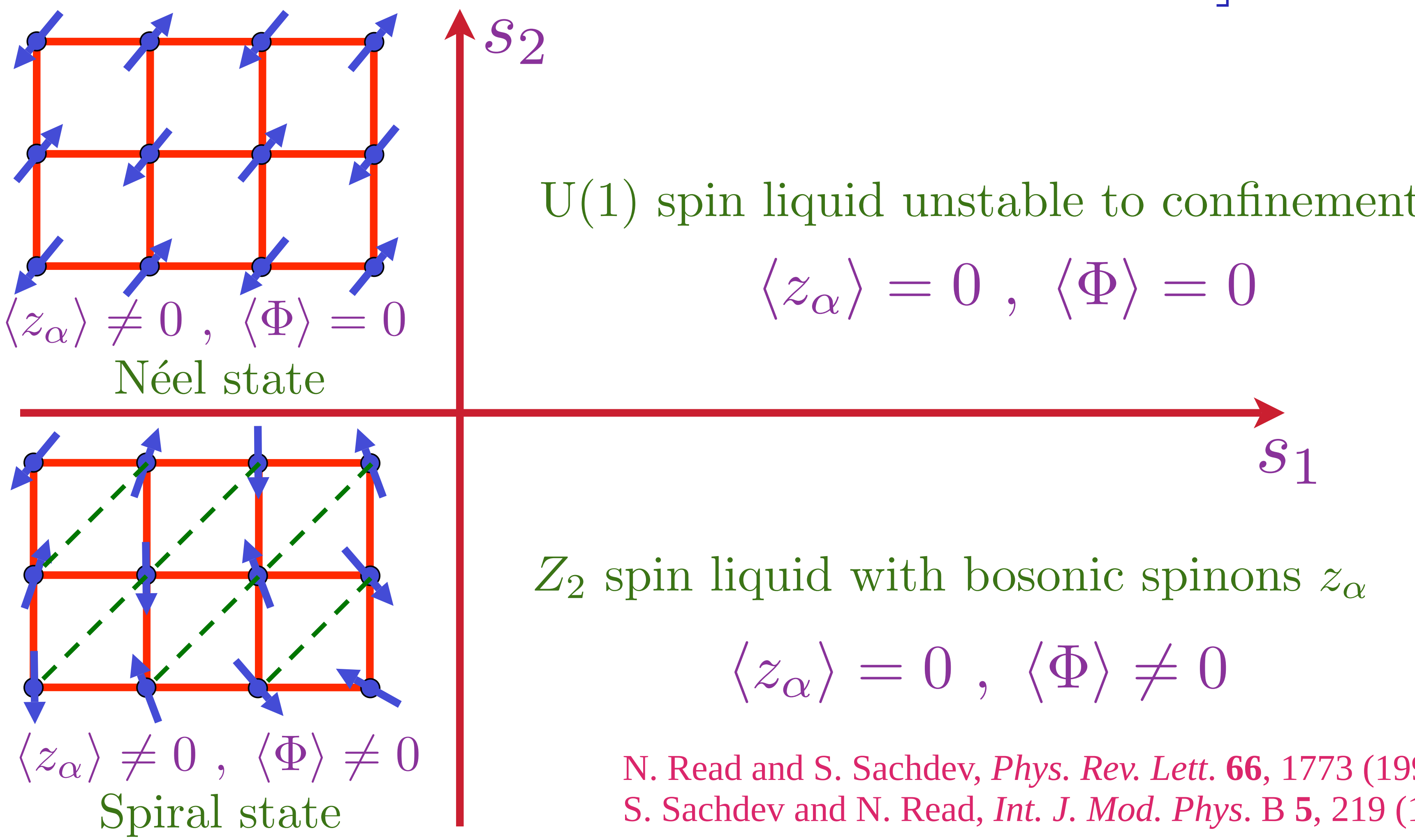

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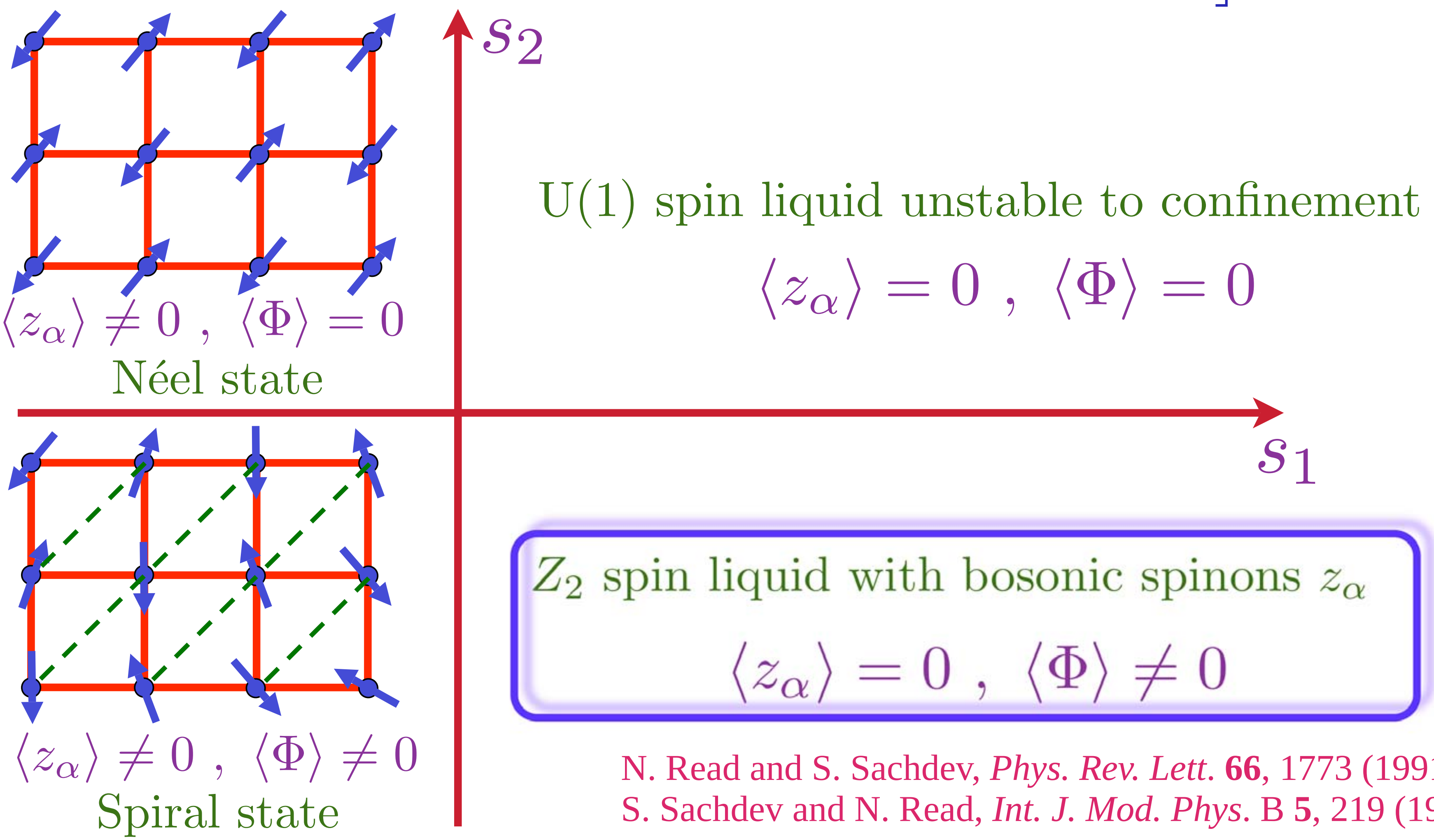

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Characteristics of Z_2 spin liquid

- Two classes of gapped excitations:
  - Bosonic spinons z_α which carry Z_2 gauge charge
  - Z_2 vortex associated with $2\pi n$ winding in phase of Φ. This vortex appears as a π flux-tubes to spinons

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- Same states (without spinons) and Z_2 gauge theories found to describe liquid phases of quantum dimer models (R. Moessner and S. L. Sondhi, Phys. Rev. Lett. **86**, 1881 (2001).

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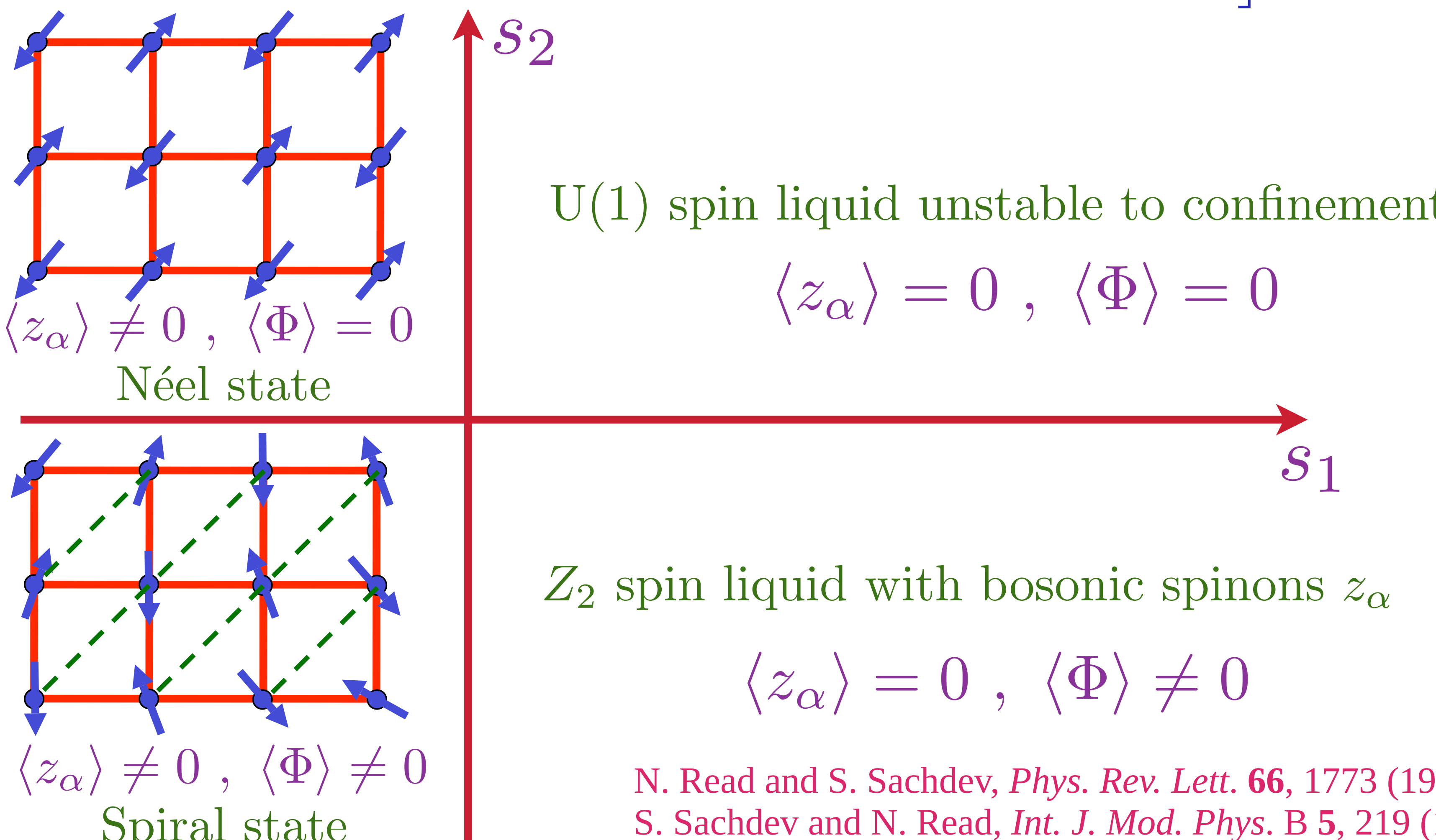

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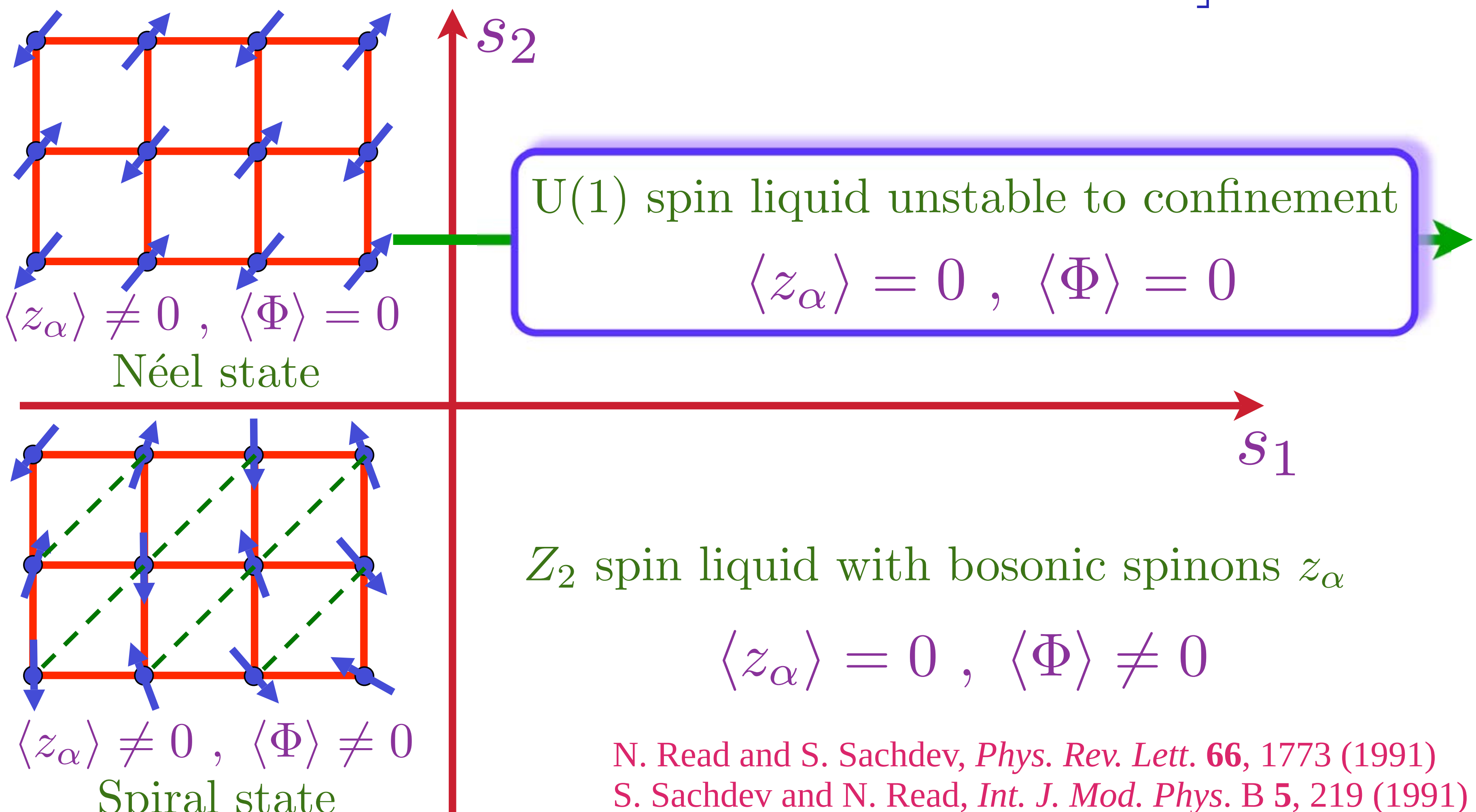

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Quantum theory for destruction of Neel order

Partition function on cubic lattice in spacetime

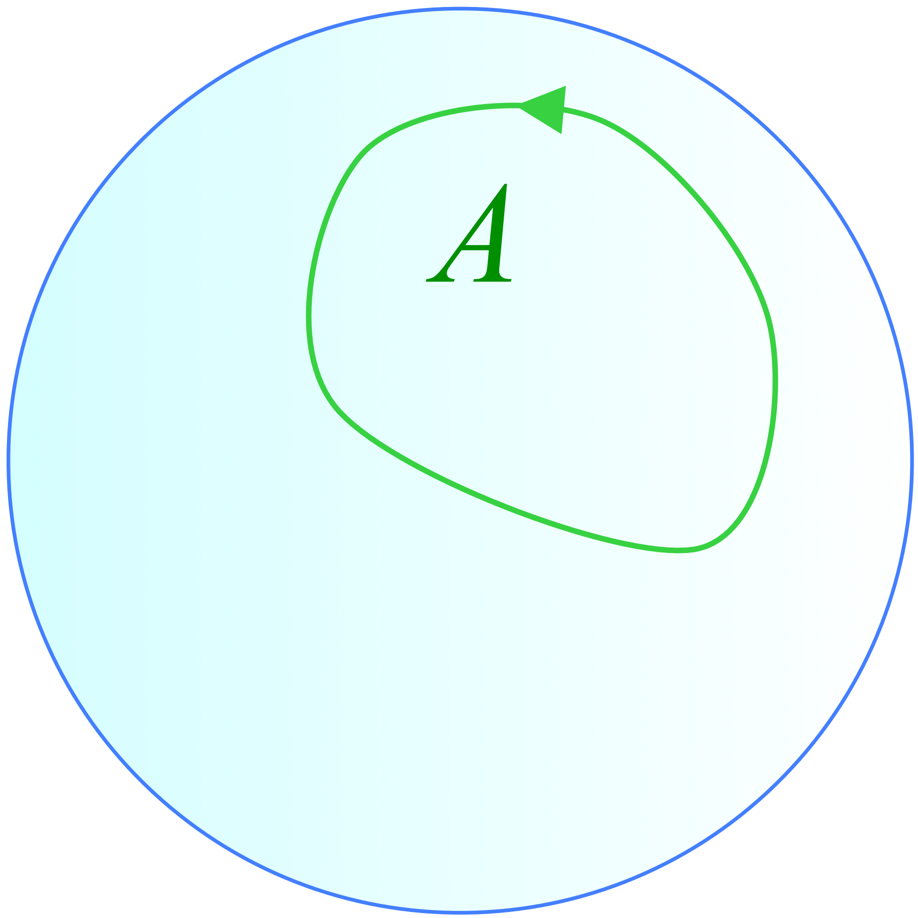
$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu}\right)$$

LGW theory: weights in partition function are those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Missing ingredient: Spin Berry Phases



$$e^{iA/2}$$

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Quantum theory for destruction of Neel order

Coherent state path integral on cubic lattice in spacetime

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\mathrm{Berry}}\right)$$

Modulus of weights in partition function: those of a classical ferromagnet at a "temperature" g

Small $g \Rightarrow$ ground state has Neel order with $\langle \vec{\varphi} \rangle \neq 0$

Large $g \Rightarrow$ paramagnetic ground state with $\langle \vec{\varphi} \rangle = 0$

Berry phases lead to large cancellations between different time histories

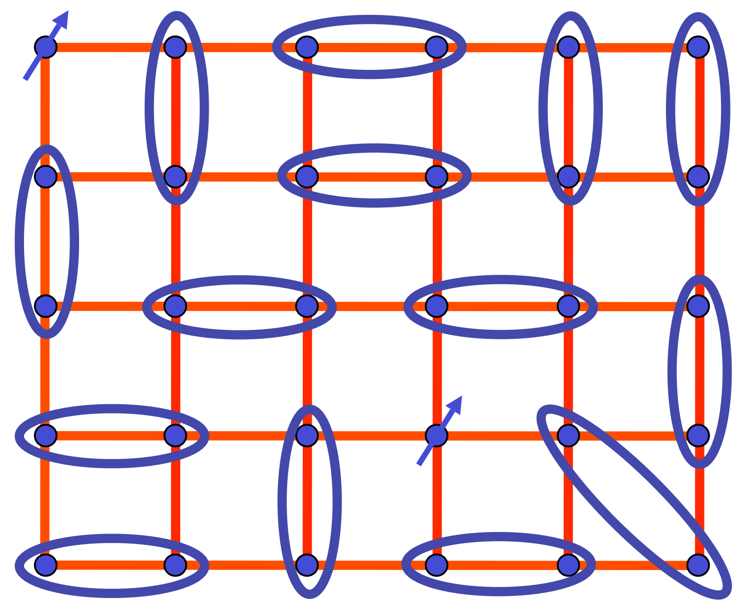
Quantum theory for destruction of Neel order

Partition function on cubic lattice

$$\mathcal{Z} = \prod_a \int d\vec{\varphi}_a \delta\left(\vec{\varphi}_a^2 - 1\right) \exp\left(\frac{1}{g}\sum_{a,\mu} \vec{\varphi}_a \cdot \vec{\varphi}_{a+\mu} + i\mathcal{S}_{\text{Berry}}\right)$$

Rewrite partition function in terms of spinors $z_{a\alpha}$, with $\alpha = \uparrow, \downarrow$ and

$$\vec{\varphi}_a = z_{a\alpha}^* \vec{\sigma}_{\alpha\beta} z_{a\beta}$$



S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

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Partition function expressed as a gauge theory of spinor degrees of freedom

$$\mathcal{Z} = \prod_a \int dz_{a\alpha} dA_{a\mu} \delta\left(\sum_\alpha |z_{a\alpha}|^2 - 1\right)$$
$$\times \exp\left(\frac{1}{g}\sum_{a,\mu} z^*_{a\alpha} e^{iA_{a\mu}} z_{a+\mu,\alpha} + i\sum_a \eta_a A_{a\tau}\right)$$

S. Sachdev and K. Park, *Annals of Physics*, **298**, 58 (2002)

Large g effective action for the $A_{a\mu}$ after integrating $z_{\alpha\mu}$

$$\mathcal{Z} = \prod_{a,\mu} \int dA_{a\mu} \exp\left(\frac{1}{2e^2} \sum_{\square} \cos\left(\Delta_\mu A_{a\nu} - \Delta_\nu A_{a\mu}\right) + i \sum_a \eta_a A_{a\tau} \right)$$

with $e^2 \sim g^2$

This is compact QED in 3 spacetime dimensions with static charges ± 1 on two sublattices.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
S. Sachdev and R. Jalabert, *Mod. Phys. Lett.* B **4**, 1043 (1990).
K. Park and S. Sachdev, *Phys. Rev.* B **65**, 220405 (2002).

Duality mapping:
The low energy continuum theory is

$$\int d^2r d\tau \left[\frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2\right]$$

Decouple this to

$$\int d^2r d\tau \left[\frac{e^2}{2}J_\mu^2 + iJ_\mu\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda\right]$$

Integrate over A_μ to obtain constraint $\epsilon_{\mu\nu\lambda}\partial_\nu J_\lambda = 0$. Solve this constraint by $J_\mu = \partial_\mu \chi$ to obtain the **dual theory**

$$\int d^2r d\tau \left[\frac{e^2}{2}(\partial_\mu \chi)^2\right]$$

This theory has a global shift symmetry $\chi \rightarrow \chi +$ constant. This symmetry is spontaneously broken, and the massless χ particle (*i.e.* the photon) is the Goldstone boson of this shift symmetry.

Consequences of Berry phases:

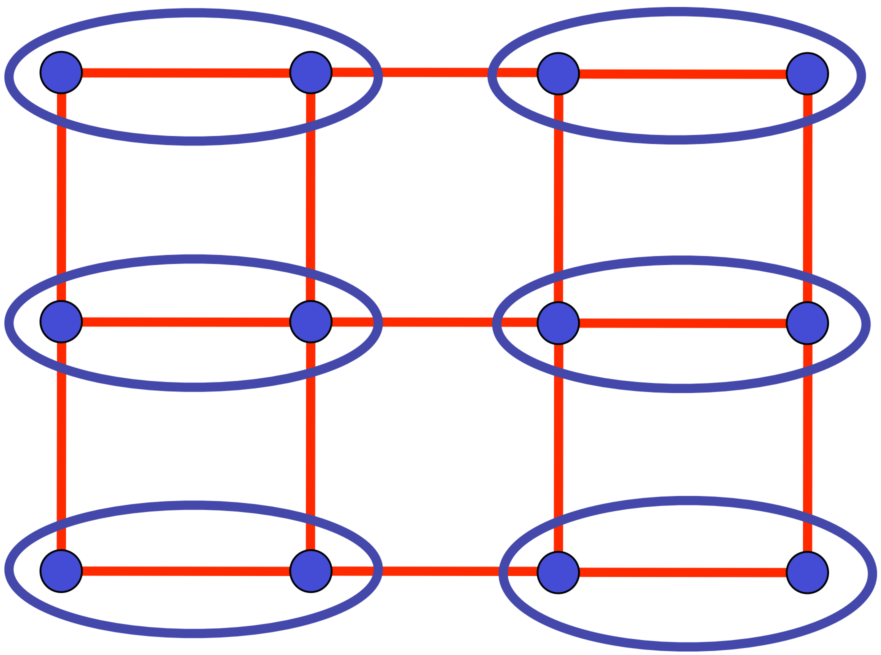
- The continuous shift symmetry is an enlargement of the Z_4 spatial rotation symmetry of the square lattice. So this spatial rotation symmetry is spontaneously broken in the free photon phase.

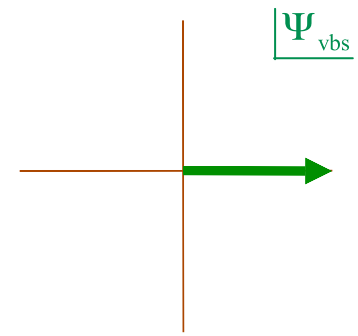
- The monopole operator

$$V = \exp\left(i\frac{2\pi\chi}{e_0^2}\right)$$

  is equivalent to the valence bond solid (VBS) operator $\Psi_{\rm vbs}$, and $\langle V\rangle \sim \langle \Psi_{\rm vbs}\rangle \neq 0$

Characterization of VBS state with $\langle\vec{\varphi}\rangle = 0$





Such a state breaks the symmetry of rotations by $n\pi / 2$ about lattice sites, and has $\langle\Psi_{vbs}\rangle \neq 0$, where Ψ_{vbs} is the ***VBS order parameter***

$$\Psi_{\rm vbs}(i) = \sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j e^{i\,\arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

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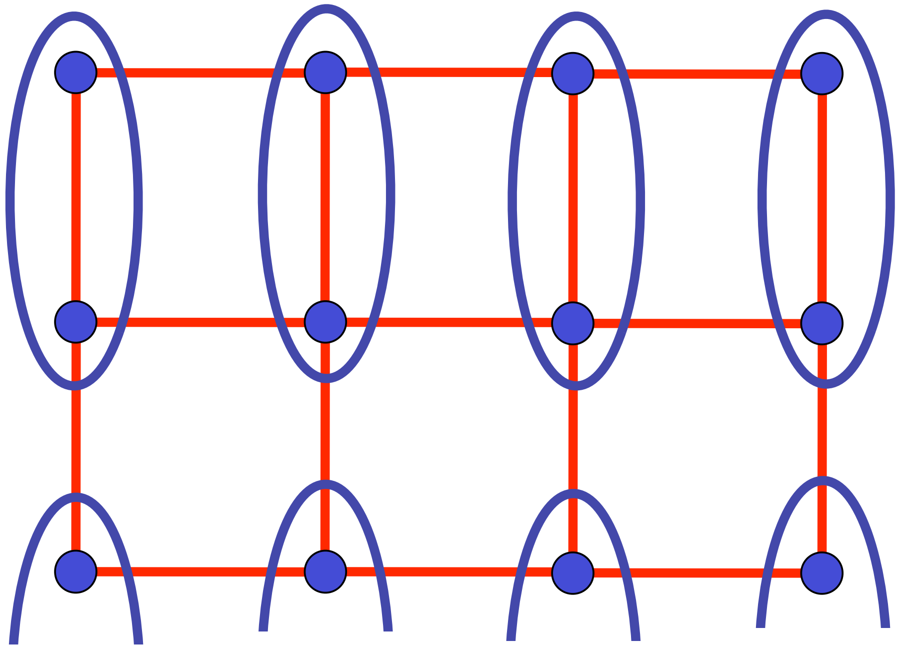

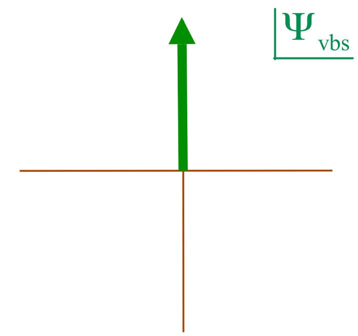

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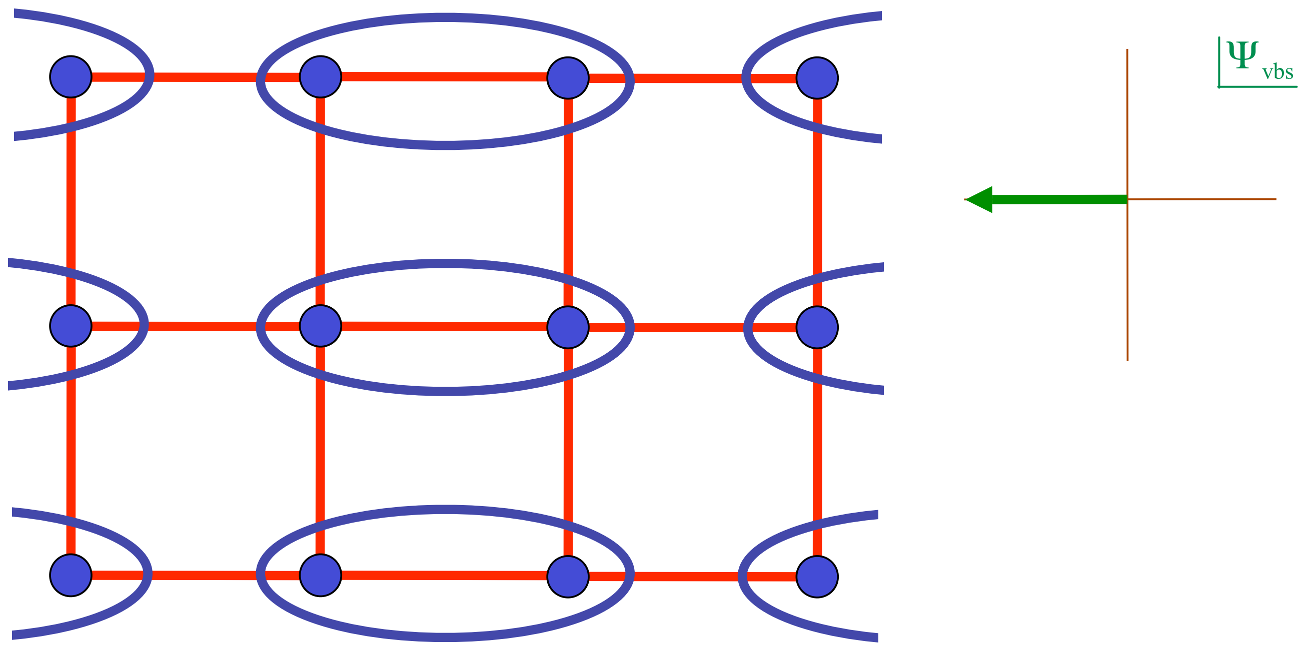

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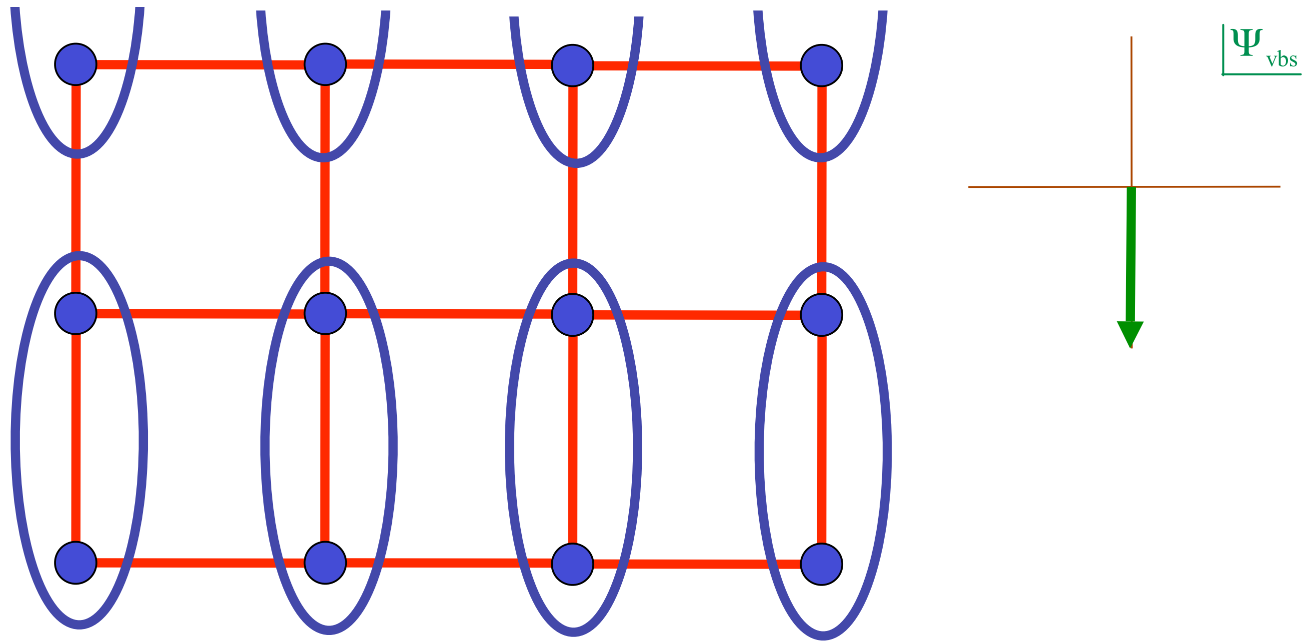

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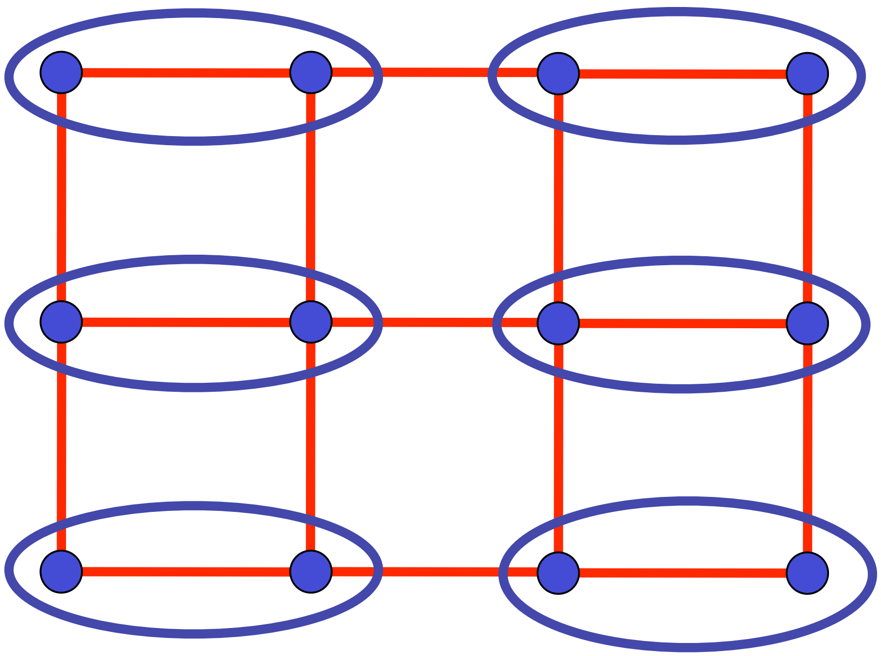

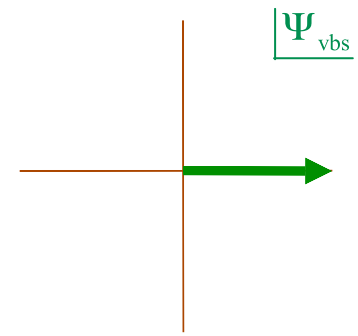

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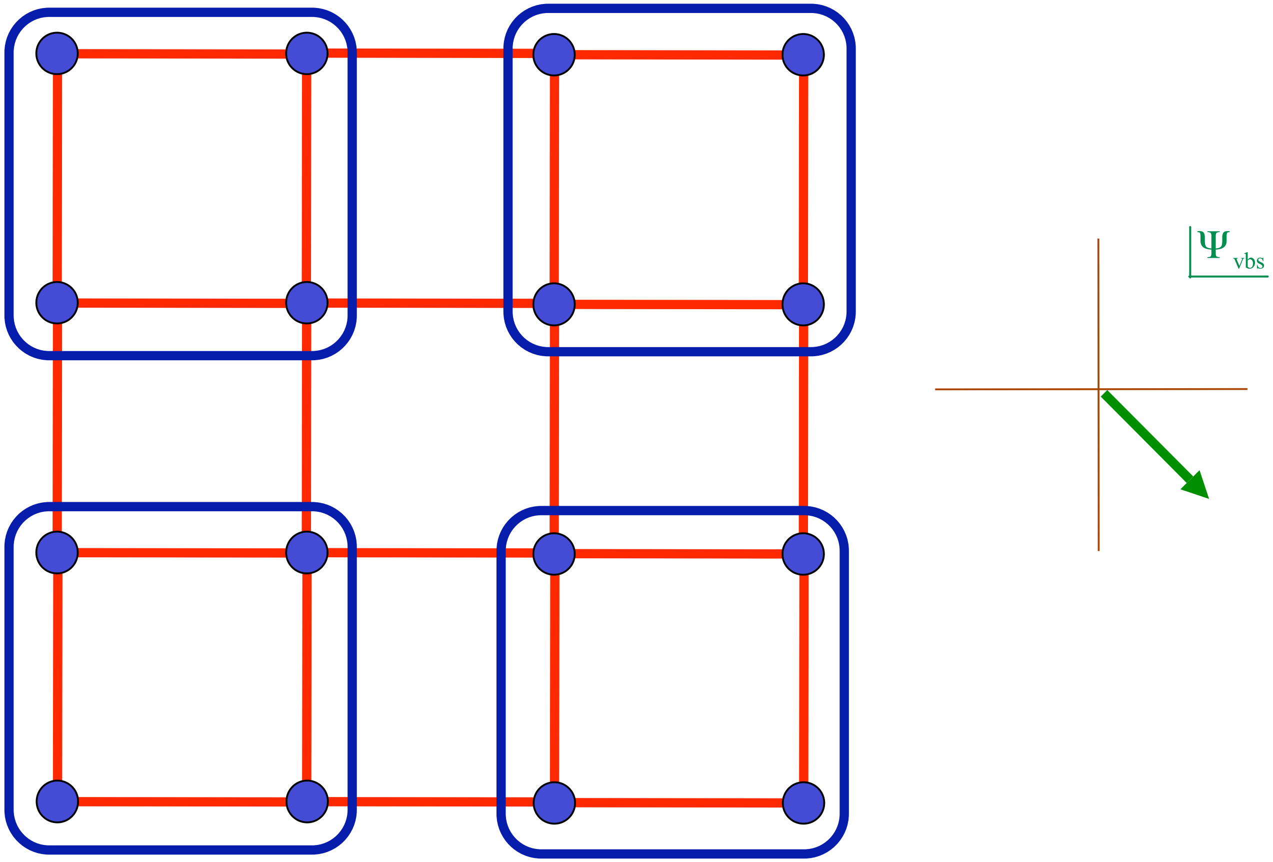
$$\Psi_{\text{vbs}}(i) = \sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j e^{i\,\arctan(\mathbf{r}_j - \mathbf{r}_i)}$$

Characterization of VBS state with $\langle \vec{\varphi} \rangle = 0$



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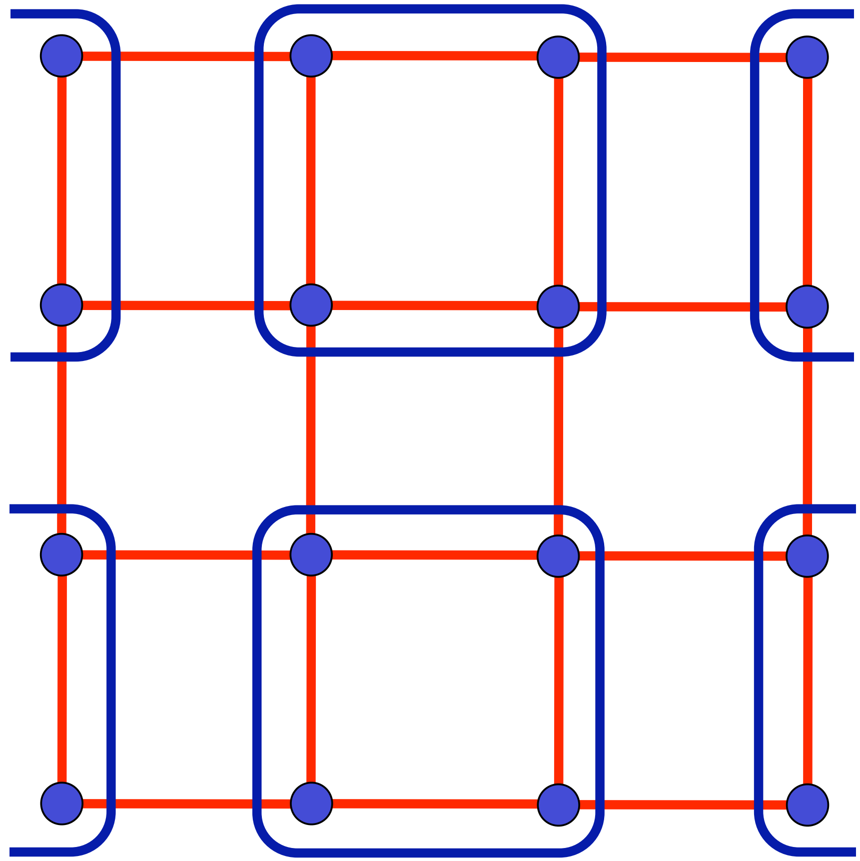

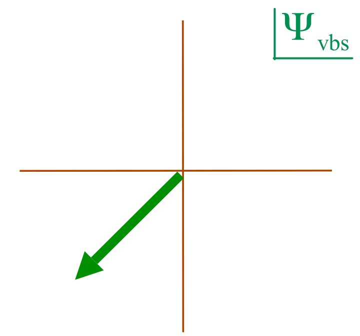

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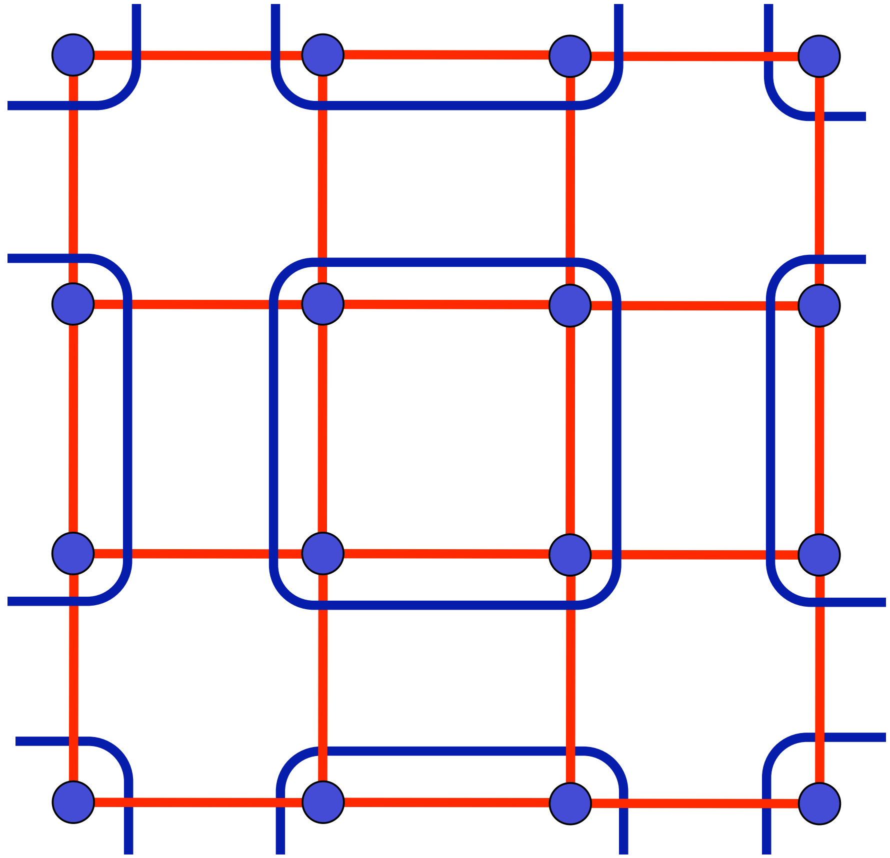

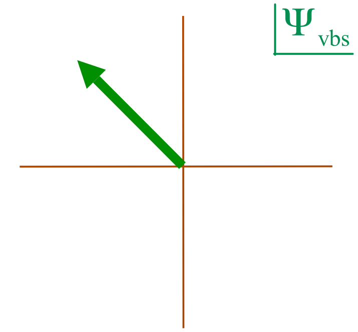

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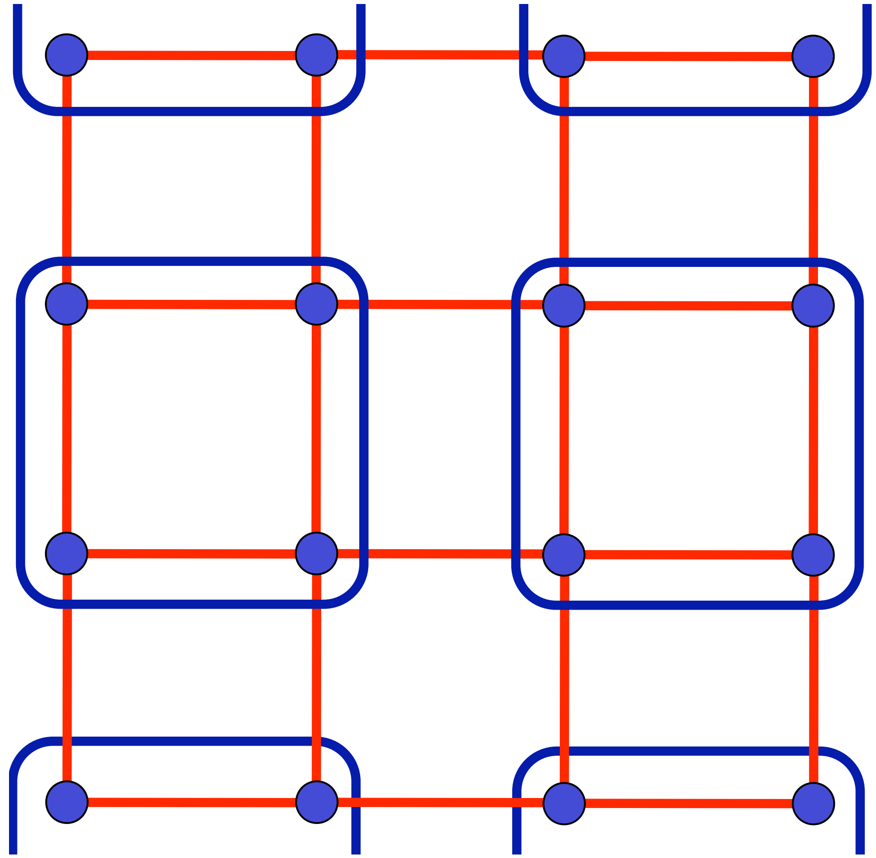

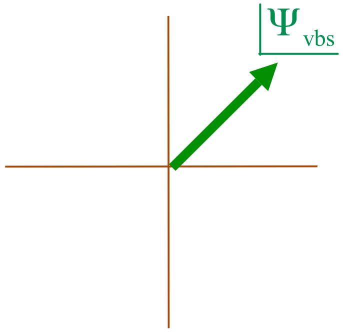

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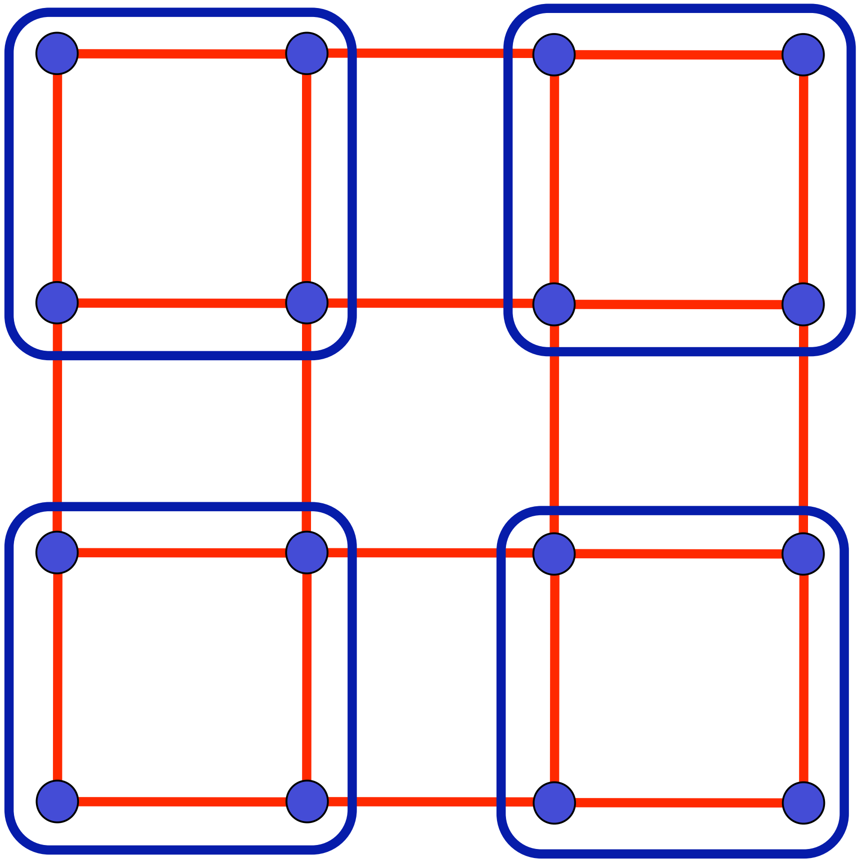

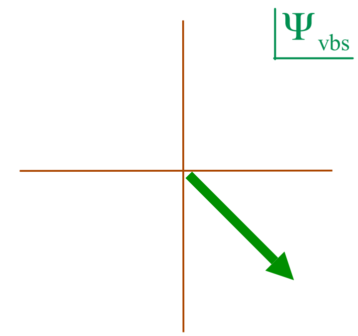

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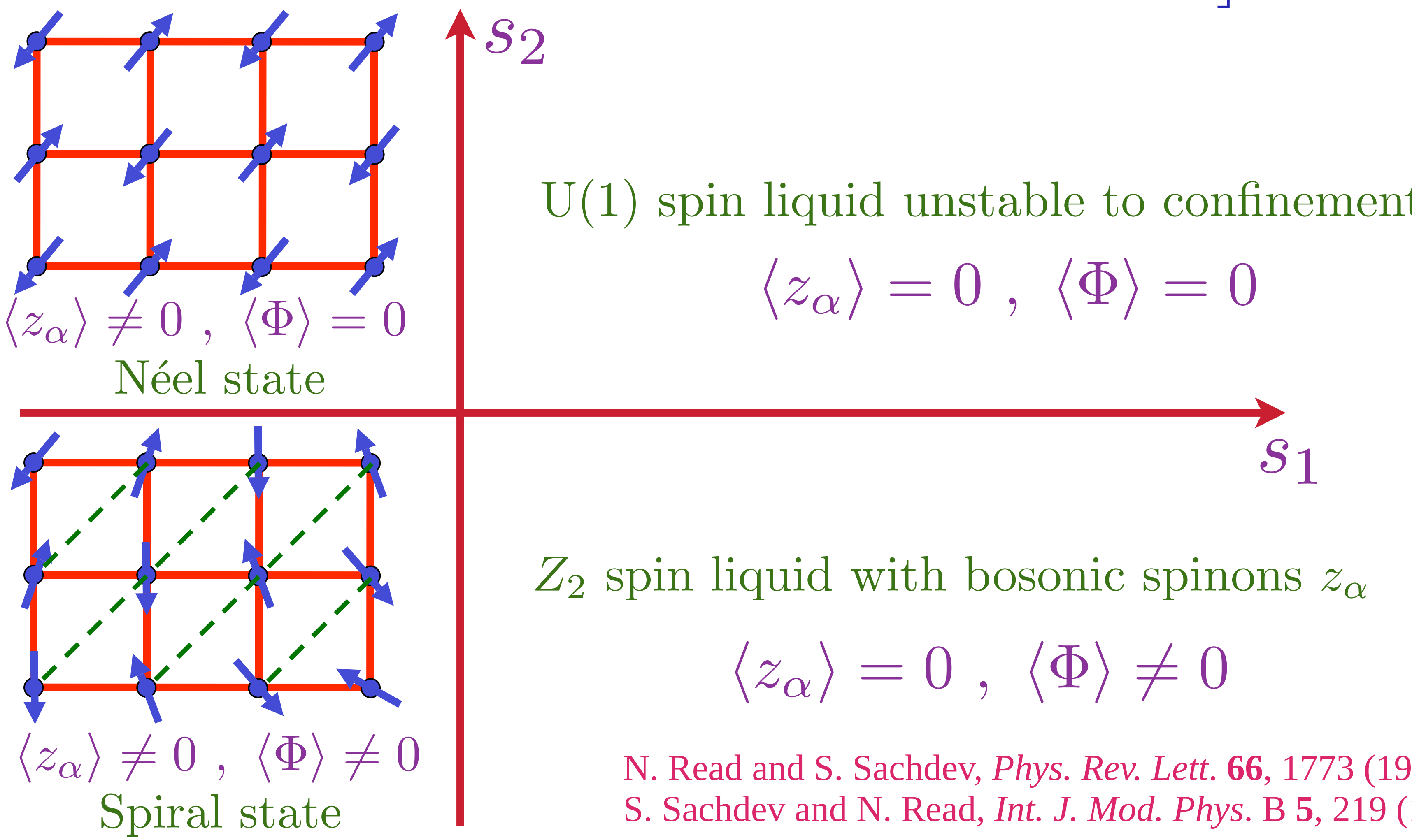
Phase diagram of gauge theory of spinons

$$\mathcal{S}_{z,\Phi} = \int d^2x d\tau \left[\left|(\partial_\mu - iA_\mu) z_\alpha\right|^2 + s_1 \left|z_\alpha\right|^2 + u\left(|z_\alpha|^2\right)^2 + \frac{1}{4e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right.$$

$$\left. + |(\partial_\mu - 2iA_\mu)\Phi|^2 + s_2|\Phi|^2 + \tilde{u}|\Phi|^4 + \lambda \Phi^* \epsilon_{\alpha\beta} z_\alpha \partial_x z_\beta + \text{c.c.} \right]$$



N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
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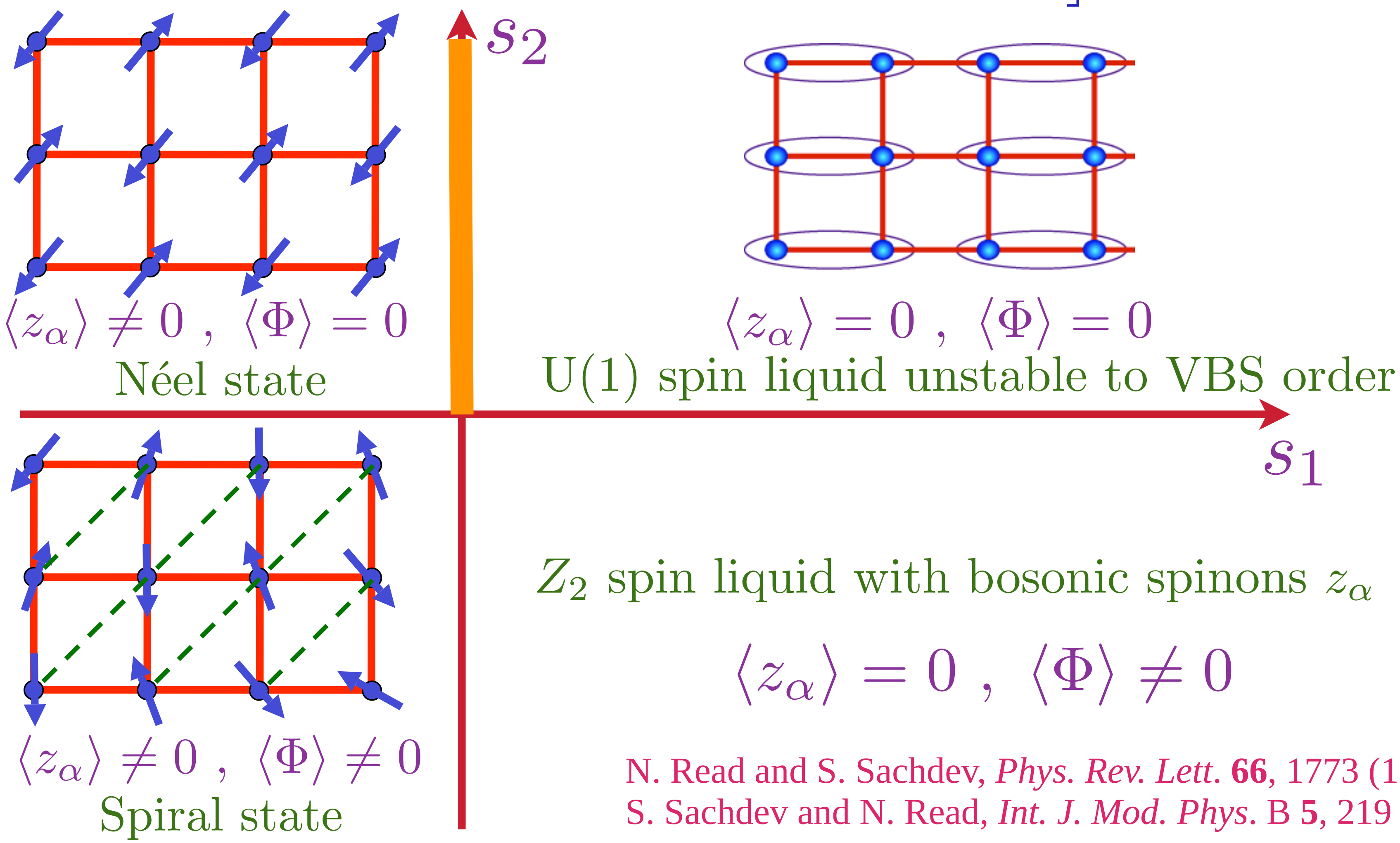

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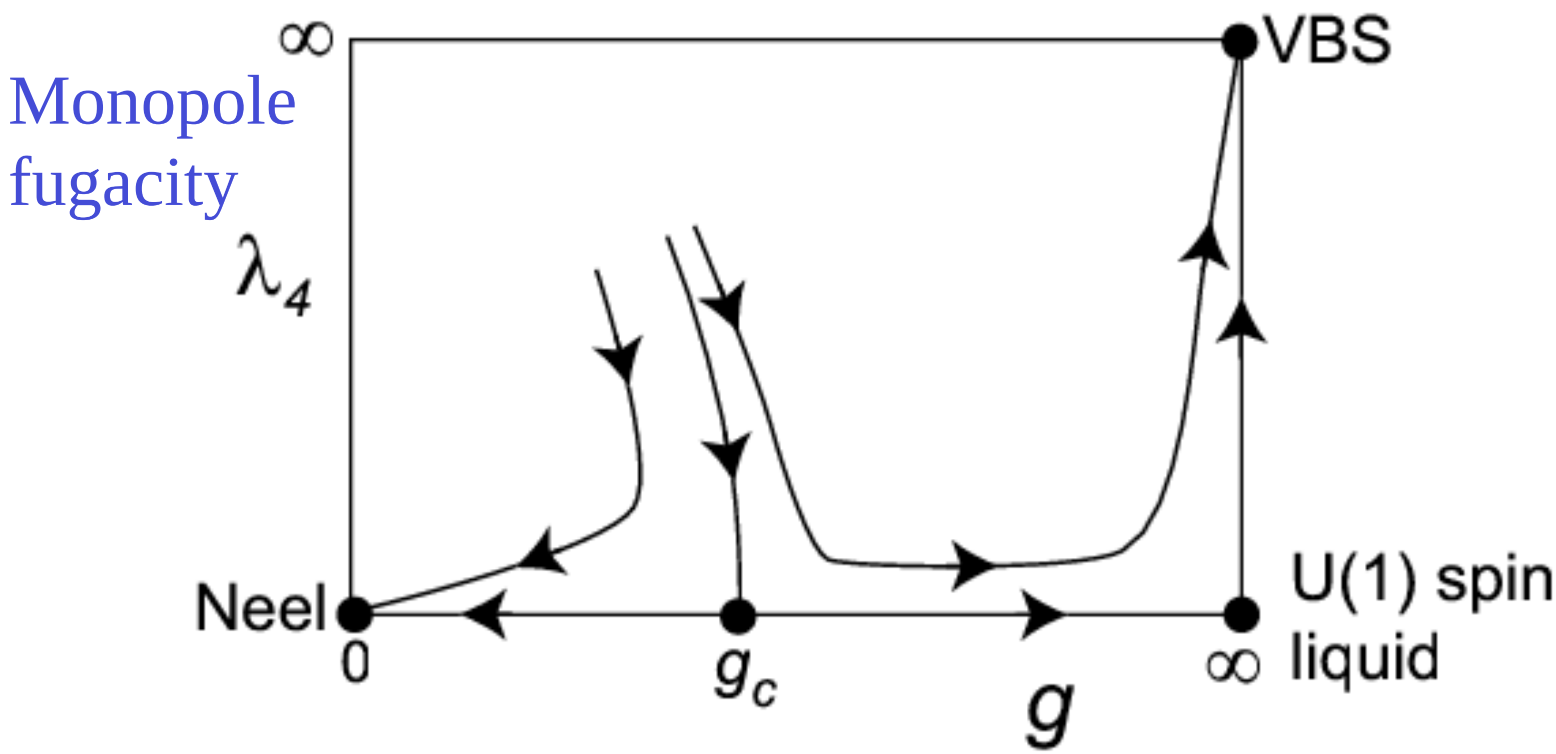

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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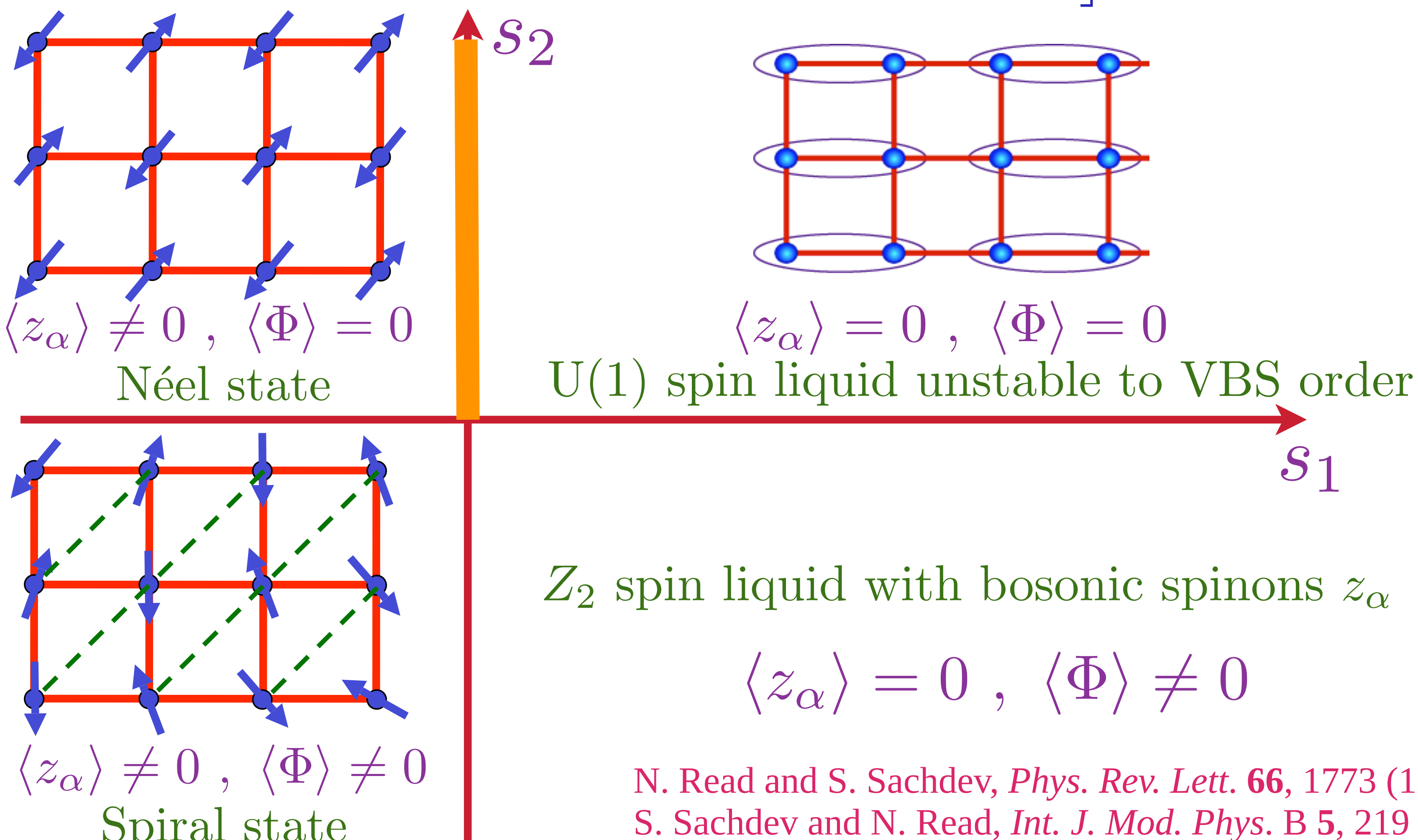

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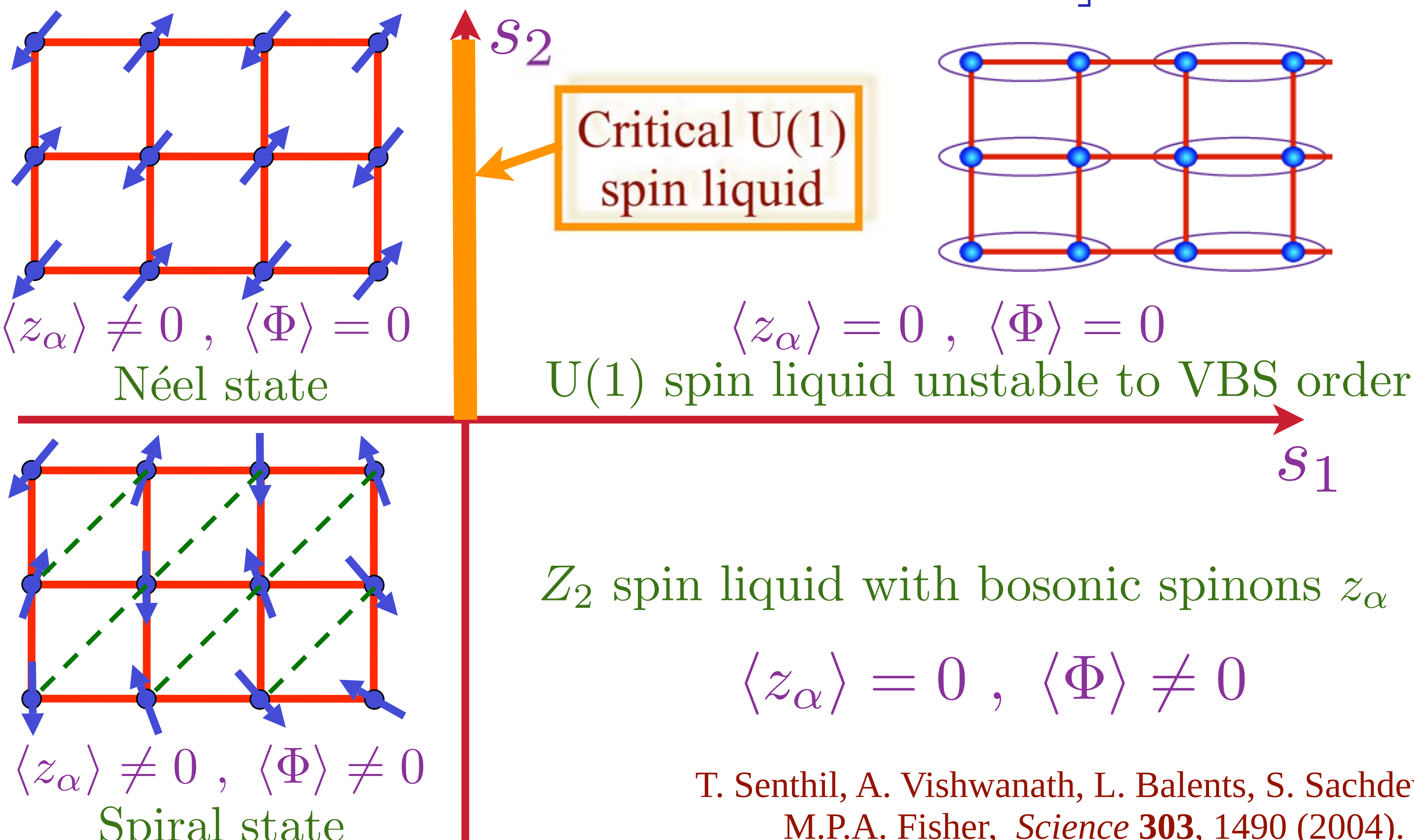
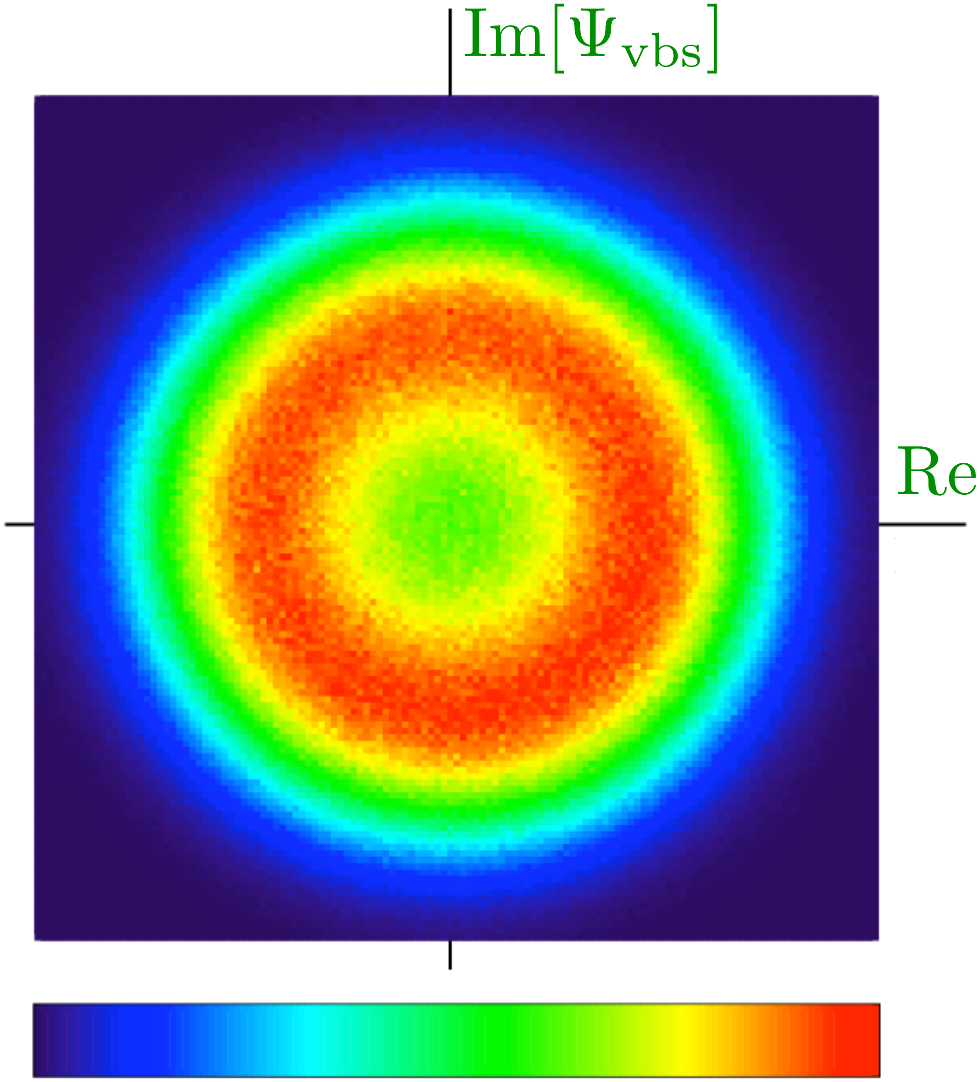


T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

$$\mathcal{H}_{\mathrm{SU(2)}} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right)\left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$



Probability distribution of VBS order Ψ_{vbs} at quantum critical point

Emergent circular symmetry is evidence for U(1) photon and topological order

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Quantum magnetism and criticality

Ribhu Kaul, Yong-Baek Kim, Alexei Kolezhuk,
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Theory of the Nernst effect near the superfliud-insulator transition

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Outline

1. Quantum "disordering" magnetic order

   Collinear order and confinement

2. Z_2 spin liquids

   Noncollinear order and fractionalization

3. U(1) spin liquids

   Valence bond solid (VBS) order

4. Doped spin liquids

   Superconductors with topological order

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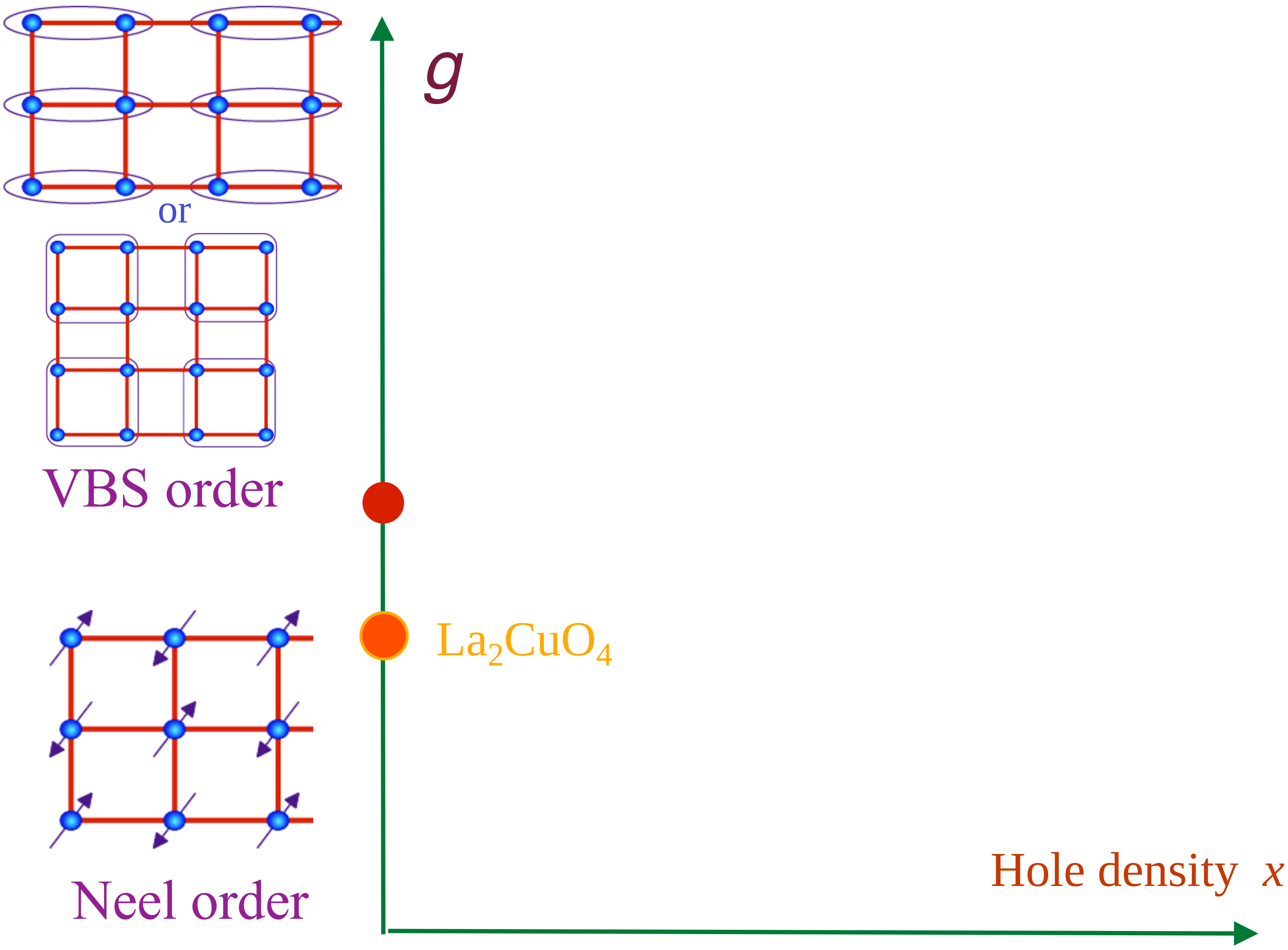
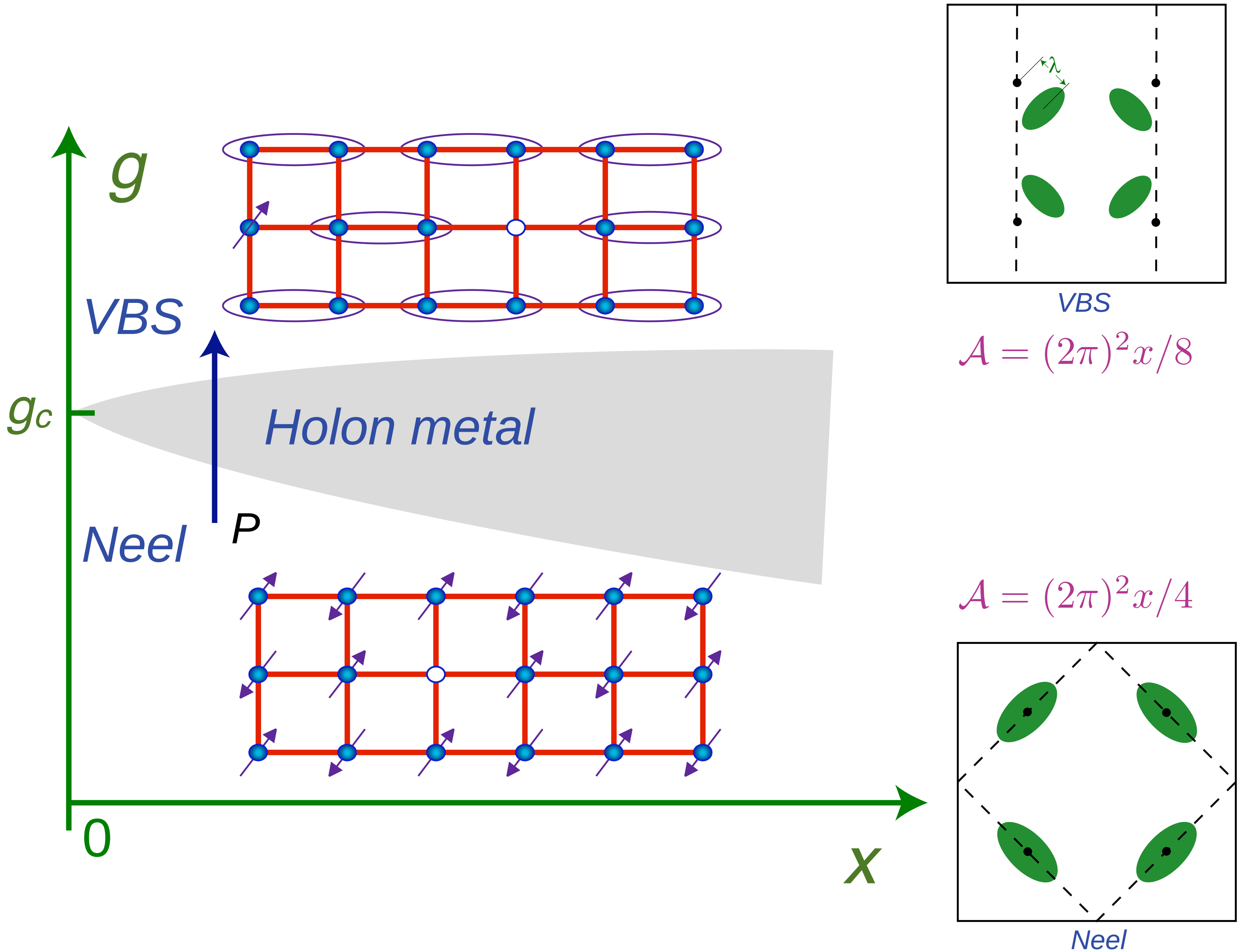
Hole dynamics in an antiferromagnet across a deconfined quantum critical point,
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,
Physical Review B **75** , 235122 (2007)

Algebraic charge liquids and the underdoped cuprates,
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil,
arXiv:0706.2187, Nature Physics, in press.

Phase diagram of doped antiferromagnets



Phase diagram of lightly doped antiferromagnet



Pictorial explanation of factor of 2:

- In the Néel phase, sublattice index is identical to spin index. So for each valley and momentum, degeneracy of the hole state is 2.

- In the VBS state, the sublattice index and the spin index are distinct. So for each valley and momentum, degeneracy of the hole state is 4.

• Begin with the representation of the quantum antiferromagnet as the lattice CP^1 model:

$$\mathcal{S}_z = -\frac{1}{g}\sum_{a,\mu} z_{a\alpha}^* e^{iA_{a\mu}} z_{a+\mu,\alpha} + i\sum_a \eta_a A_{a\tau}$$

• Write the electron operator at site r, $c_\alpha(r)$ in terms of **fermionic holon** operators $f_\pm$

$$c_\alpha(r) = \begin{cases} f_+^\dagger(r) z_{r\alpha} & \text{for } r \text{ on sublattice A} \\ \varepsilon_{\alpha\beta} f_-^\dagger(r) z_{r\beta}^* & \text{for } r \text{ on sublattice B} \end{cases}$$

Note that the holons f_s have charge s under the U(1) gauge field A_μ.

• Choose the dispersion, $\epsilon(\vec{k})$ of the $f_\pm$ in momentum space so that its minima are at $(\pm\pi/2, \pm\pi/2)$. *To avoid double-counting, these dispersions must be restricted to be within the diamond Brillouin zone.*

$$\mathcal{S}_f = \int d\tau \sum_{s=\pm} \int_\diamond \frac{d^2k}{4\pi^2} f_s^\dagger(\vec{k}) \left(\partial_\tau - isA_\tau + \epsilon(\vec{k} - s\vec{A})\right) f_s(\vec{k})$$

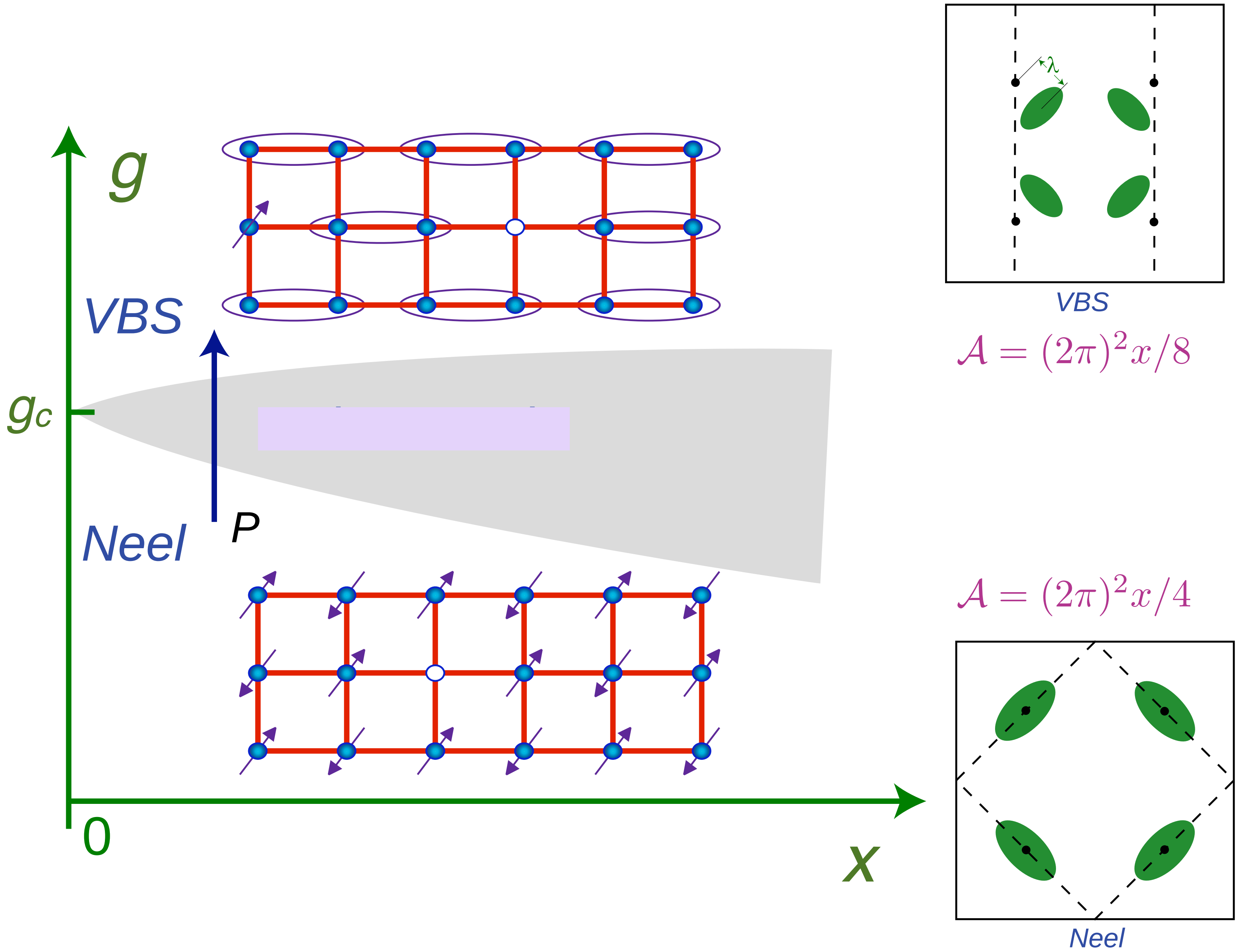
• Include the hopping between opposite sublattices (Shraiman-Siggia term):

$$\begin{aligned} \mathcal{S}_t &= -t \sum_{\langle rr' \rangle} c_\alpha^\dagger(r) c_\alpha(r') + \text{h.c.} \\ &= -t \sum_{\langle rr' \rangle} (f_+^\dagger(r) z_{r\alpha})^\dagger \varepsilon_{\alpha\beta} f_-^\dagger(r') z_{r\beta}^* \end{aligned}$$

• Complete theory for doped antiferromagnet:

$$\mathcal{S} = \mathcal{S}_z + \mathcal{S}_f + \mathcal{S}_t$$

Phase diagram of lightly doped antiferromagnet



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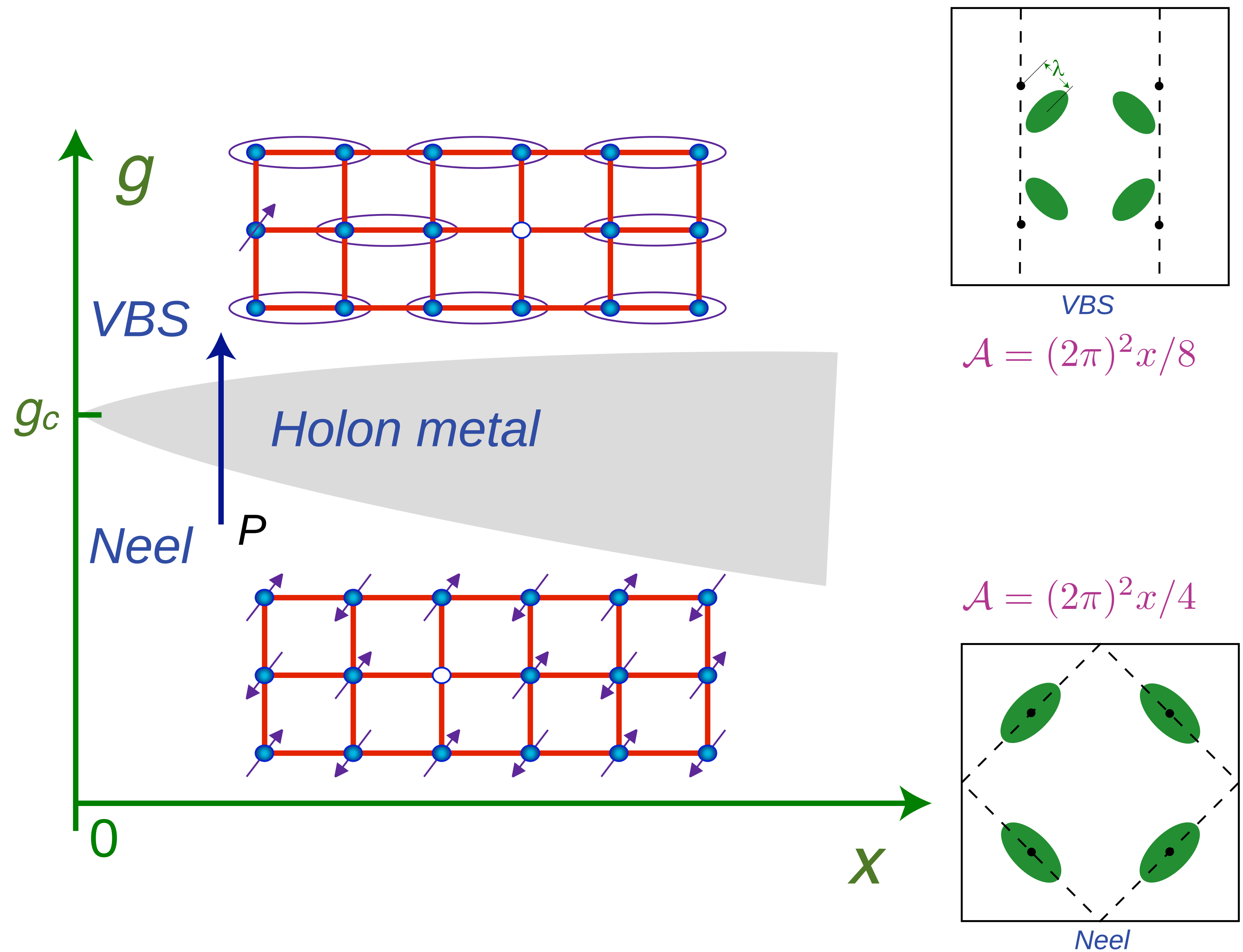
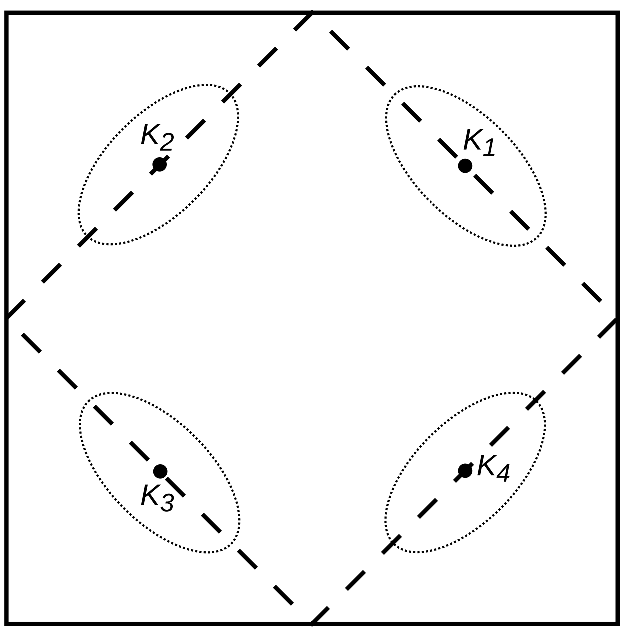


A new non-Fermi liquid phase:
The holon metal
An algebraic *charge* liquid.

- Ignore compactness in A_μ and Berry phase term.
- Neutral spinons z_α are gapped.
- Charge e fermions f_s form Fermi surfaces and carry charges $s = \pm 1$ under the U(1) gauge field A_μ.
- Quasi-long range order in a variety of VBS and pairing correlations.

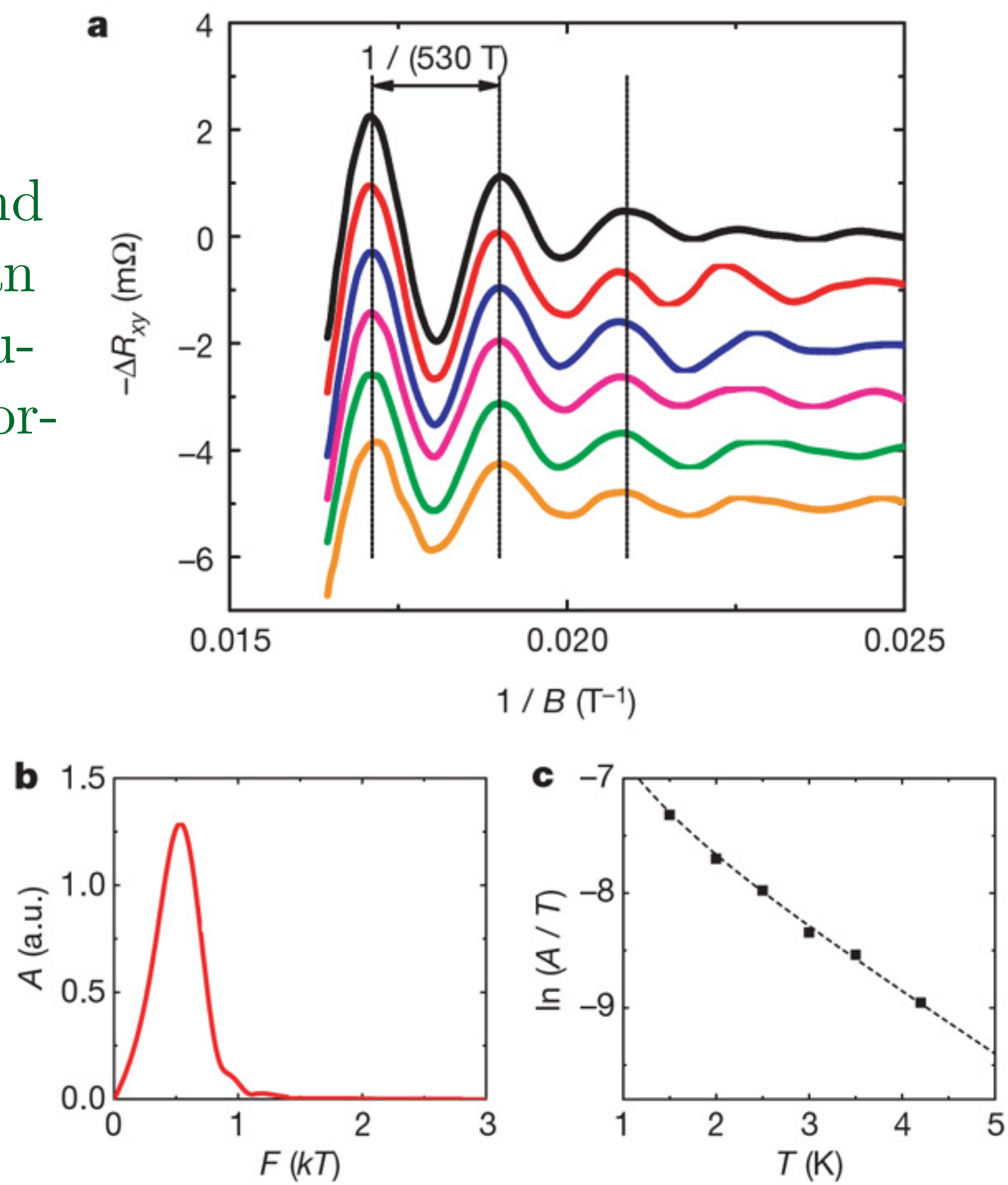


Area of each Fermi pocket,

$$\mathcal{A} = (2\pi)^2 x/4.$$

The Fermi pocket will show sharp magnetoresistance oscillations, but it is invisible to photoemission.

Quantum oscillations and the Fermi surface in an underdoped high T_c superconductor (ortho-II ordered $YBa_2Cu_3O_{6.5}$).



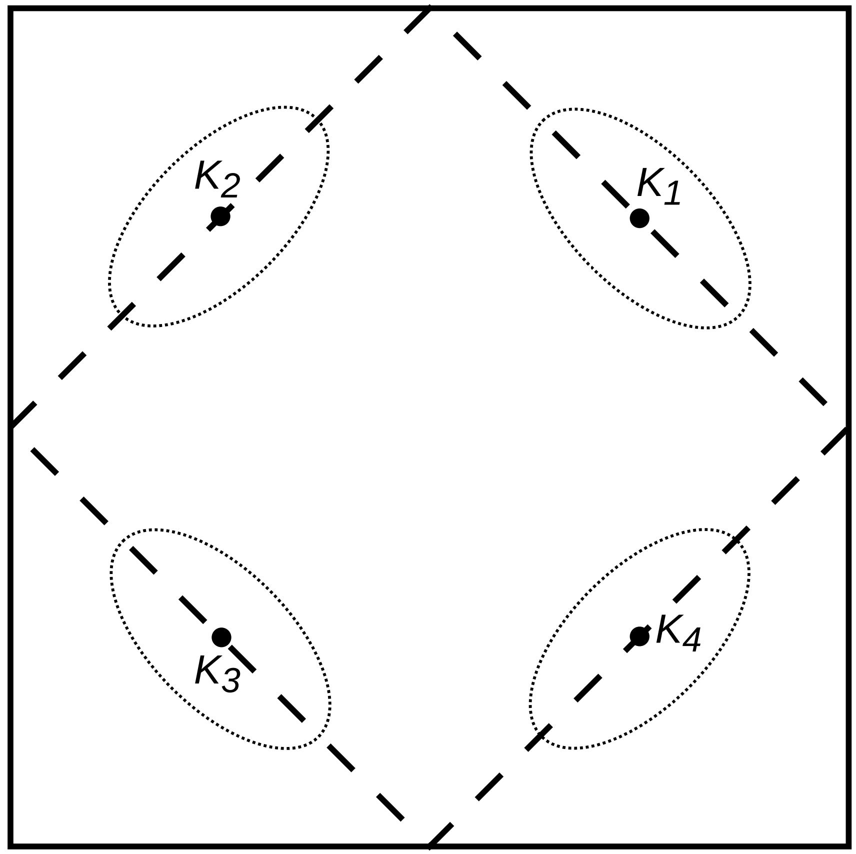
N. Doiron-Leyraud, C. Proust, D. LeBoeuf, J. Levallois, J.-B. Bonnemaison, R. Liang, D. A. Bonn, W. N. Hardy, and L. Taillefer, *Nature* **447**, 565 (2007)

Holon pairing leading to *d*-wave superconductivity

- First consider holon pairing in the Neel state, where holon=hole.

- This was studied in V. V. Flambaum, M. Yu. Kuchiev, and O. P. Sushkov, *Physica* C **227**, 267 (1994); V. I. Belincher *et al.*, *Phys. Rev.* B **51**, 6076 (1995). They found *p*-wave pairing of holons, induced by spin-wave exchange from the sublattice mixing term $\mathcal{S}_t$. This corresponds to *d*-wave pairing of physical electrons

Holon pairing leading to *d*-wave superconductivity



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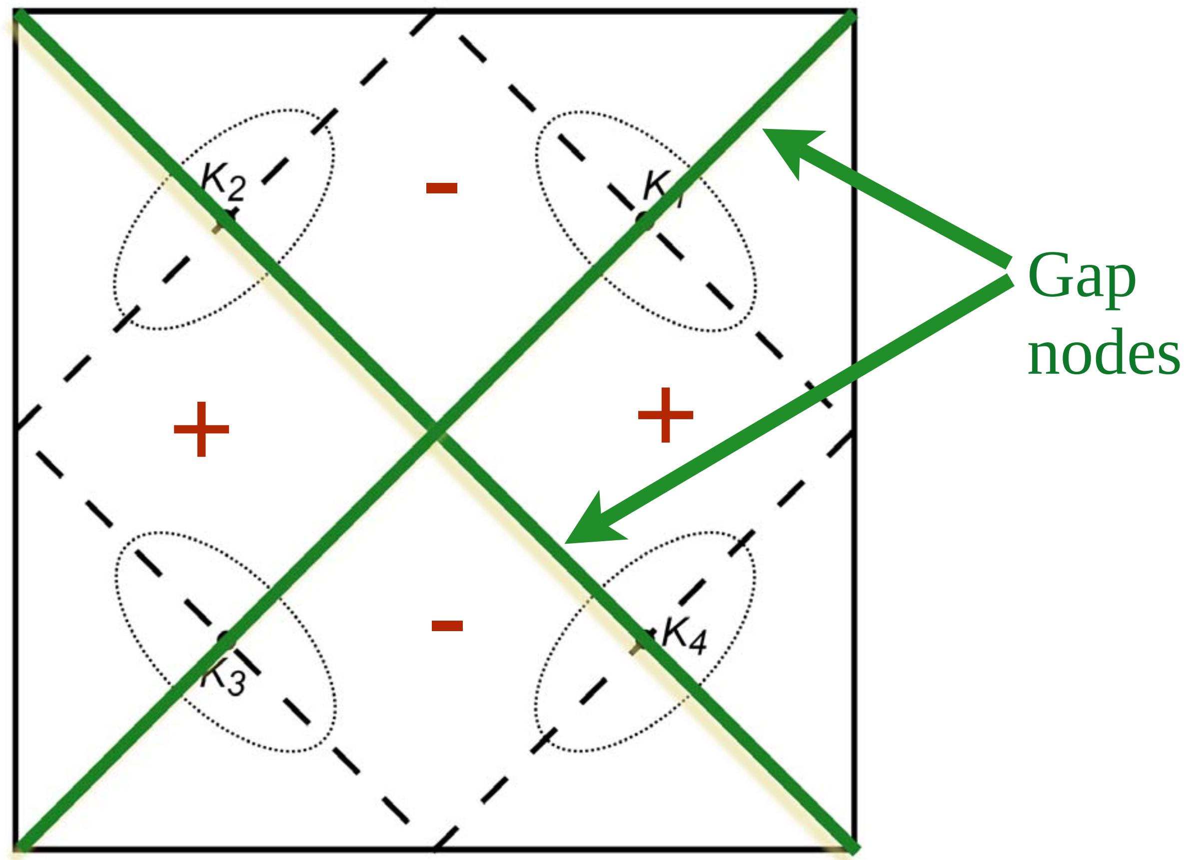
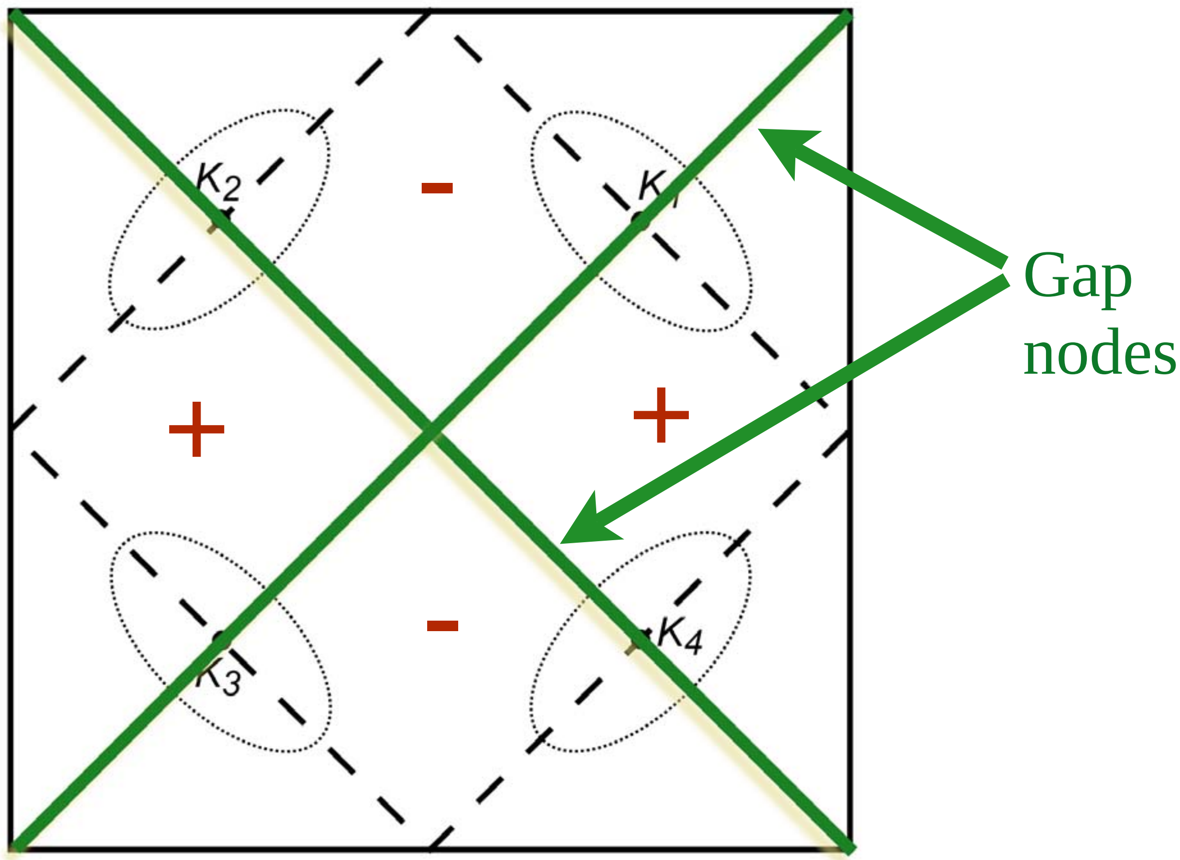


Holon pairing leading to d-wave superconductivity



We assume the same pairing holds across a transition involving loss of long-range Néel order. The resulting phase is another algebraic charge liquid - the *holon superconductor*. This superconductor has gapped spinons with no electrical charge, and spinless, nodal Bogoliubov-Dirac quasiparticles. The superconductivity does **not** gap the U(1) gauge field A_μ, because the Cooper pairs are gauge neutral.

Low energy theory of holon superconductor

4 two-component Dirac quasiparticles coupled to a U(1) gauge field

$$\mathcal{S}_{\text{holon superconductor}} = \int d\tau d^2 r \Bigg[\frac{1}{2e_0^2} \left(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda\right)^2 + \sum_{i=1}^{4} \psi_i^\dagger \left(D_\tau - i v_F(\partial_x - iA_x)\tau^x - i v_F(\partial_y - iA_y)\tau^y\right) \psi_i \Bigg]$$

Low energy theory of holon superconductor

External vector potential $\vec{A}$ couples as

$$\mathcal{H}_A = \vec{j} \cdot \vec{A}$$

where

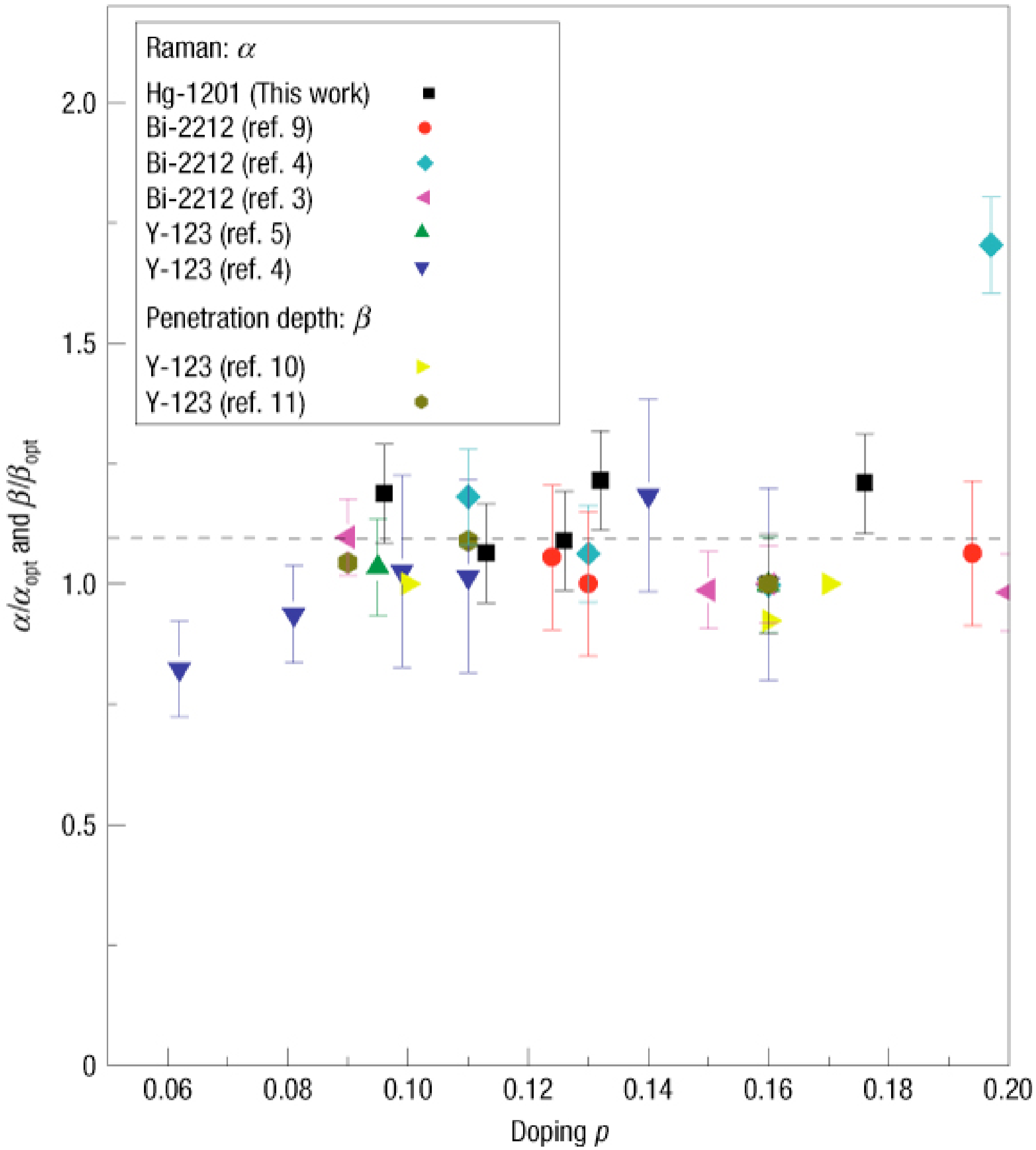
$$j_x = v_F \left(\psi_3^\dagger \psi_3 - \psi_1^\dagger \psi_1 \right) \quad , \quad j_x = v_F \left(\psi_4^\dagger \psi_4 - \psi_2^\dagger \psi_2 \right)$$

are conserve charges of $\mathcal{S}_{\text{holon superconductor}}$.
Fundamental property: The superfluid density, ρ_s, has the following x and T dependence:

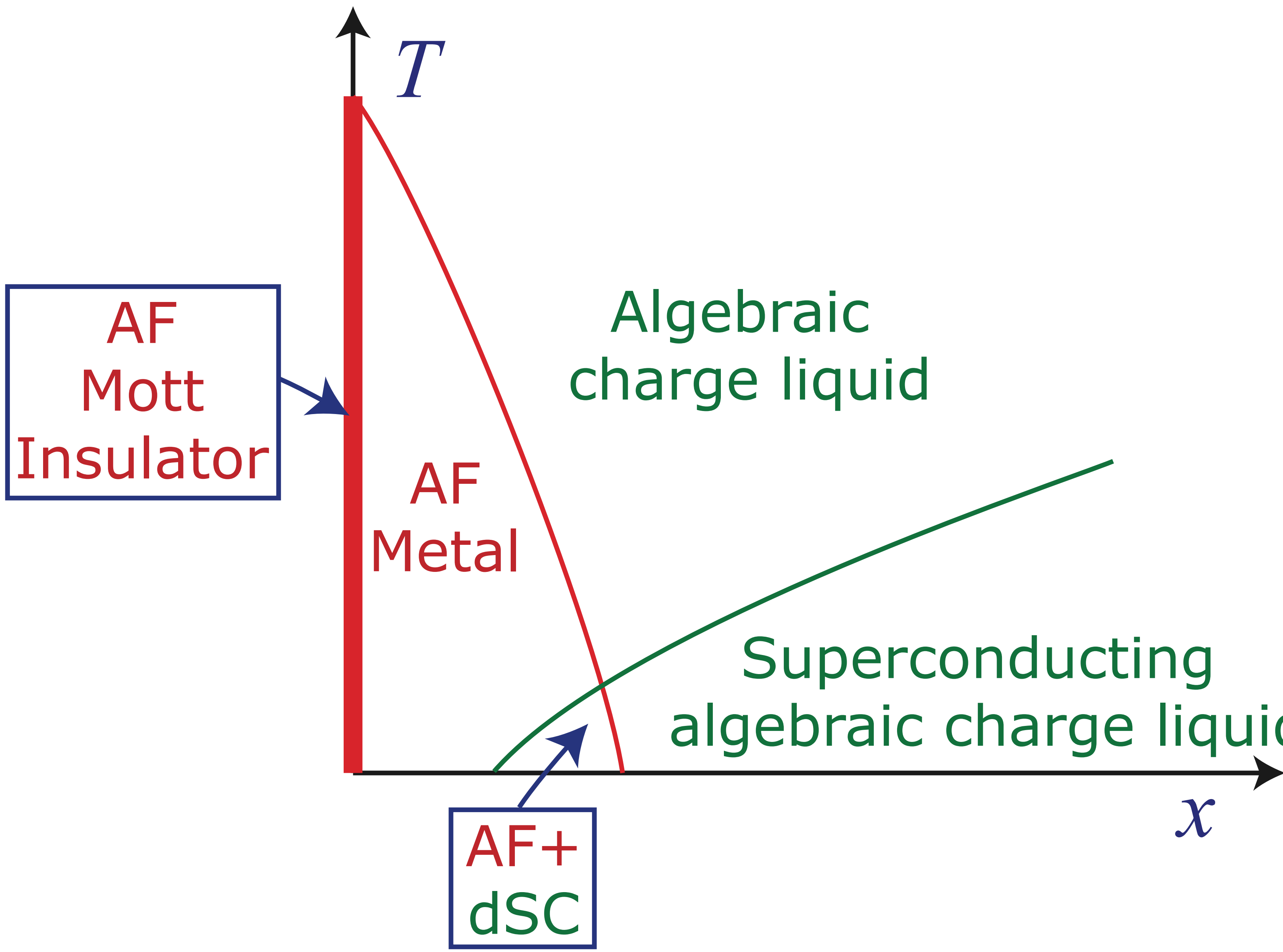
$$\rho_s(x, T) = cx - \mathcal{R} k_B T$$

where c is a non-universal constant and $\mathcal{R}$ is a universal constant obtained in a $1/N$ expansion ($N = 4$ is the number of Dirac fermions):

$$\mathcal{R} = 0.4412 + \frac{0.307}{N} + \ldots$$



Raman: α
Hg-1201 (This work)
Bi-2212 (ref. 9)
Bi-2212 (ref. 4)
Bi-2212 (ref. 3)
Y-123 (ref. 5)
Y-123 (ref. 4)
Penetration depth: β
Y-123 (ref. 10)
Y-123 (ref. 11)
α/α_{opt} and β/β_{opt}
2.0
1.5
1.0
0.5
0
0.06
0.08
0.10
0.12
0.14
0.16
0.18
0.20
Doping p


T
AF
Mott
Insulator
AF
Metal
Algebraic
charge liquid
Superconducting
algebraic charge liquid
AF+
dSC
x

Conclusions

1. Theory for Z_2 and U(1) spin liquids in quantum antiferromagnets, and evidence for their realization in model spin systems.

2. *Algebraic charge liquids* appear naturally upon adding fermionic carriers to spin liquids with bosonic spinons. These are conducting states with topological order.

3. The holon metal/superconductor, obtained by doping a Neel-ordered insulator, matches several observed characteristics of the underdoped cuprates.

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   Integer and fractional filling

2. Quantum-critical transport
   Collisionless-t0-hydrodynamic crossover of CFT3s

3. SYM3 with $\mathcal{N} = 8$ supersymmetry

4. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

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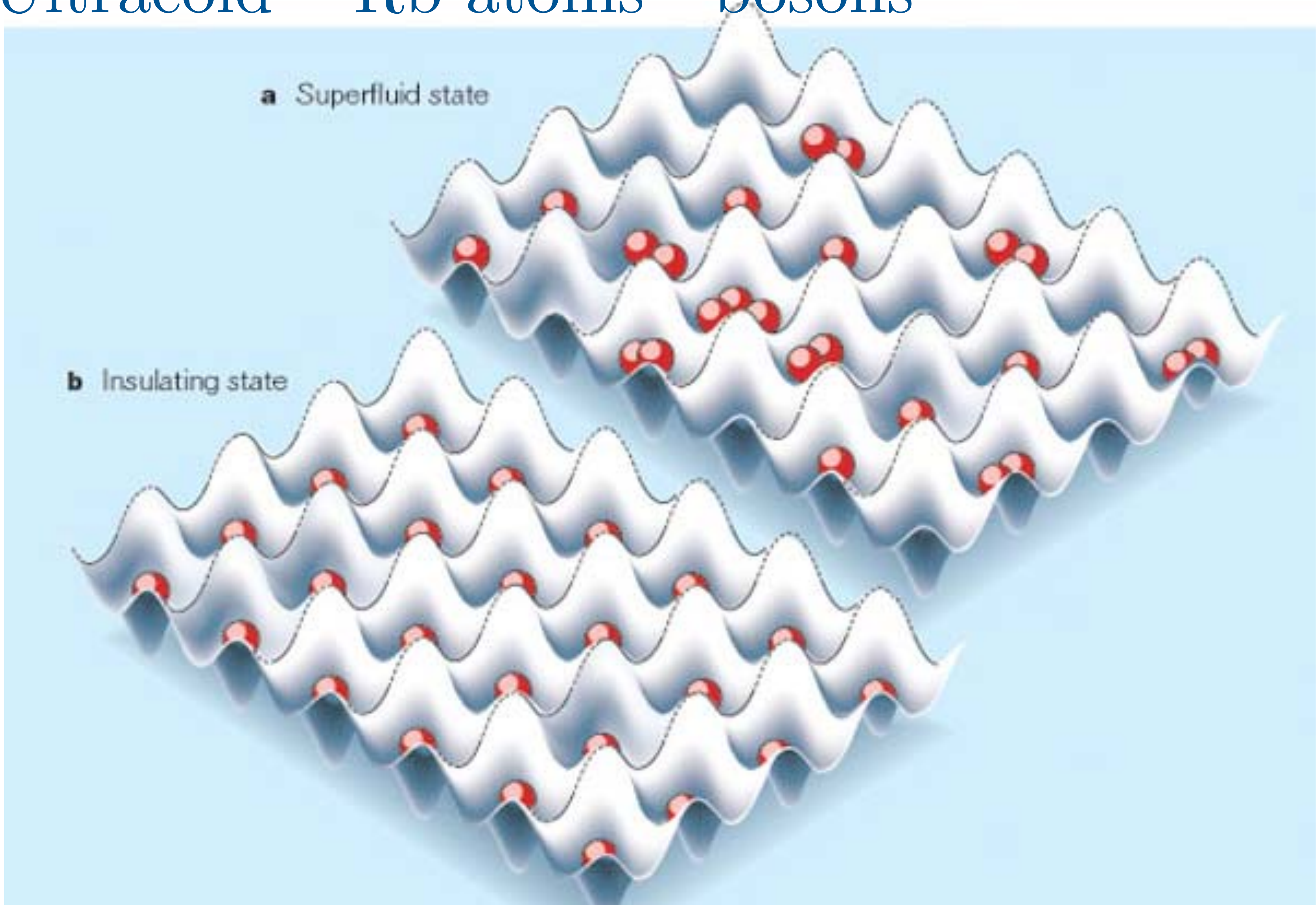
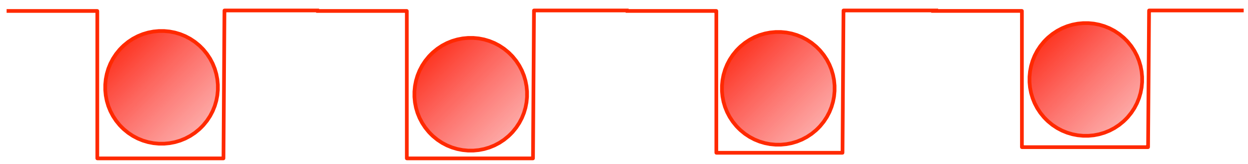
Ultracold ^{87}Rb atoms - bosons

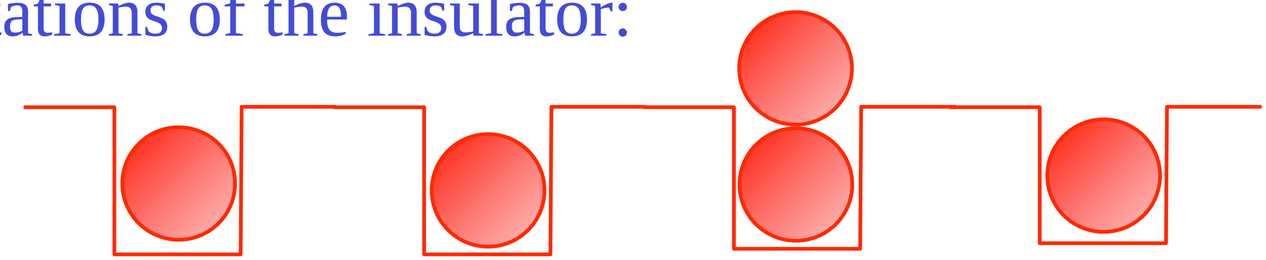


M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

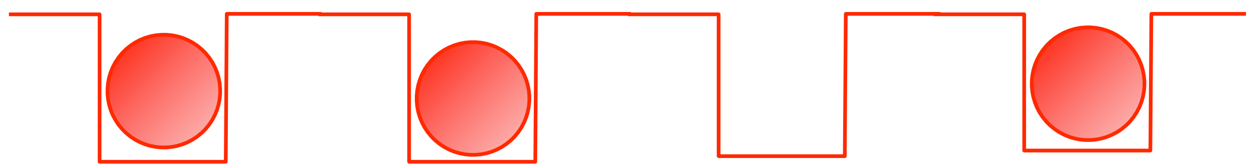
The insulator:



Excitations of the insulator:

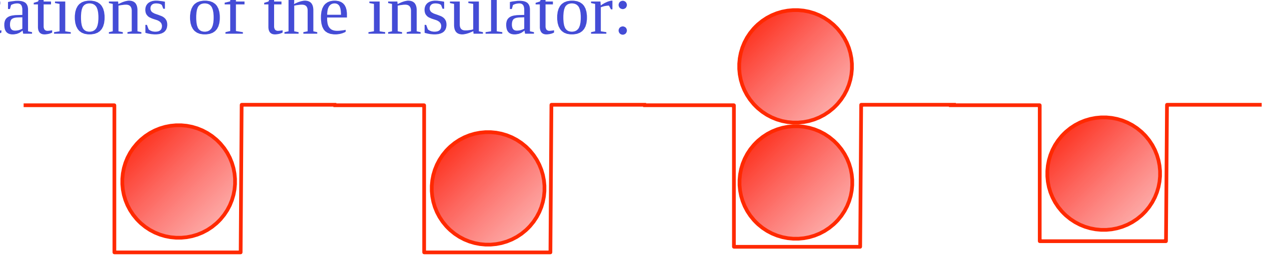


Particles $\sim \psi^{\dagger}$

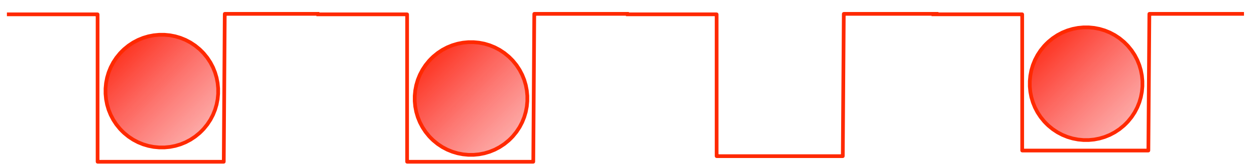


Holes $\sim \psi$

Excitations of the insulator:



Particles ~ $\psi^\dagger$



Holes ~ ψ

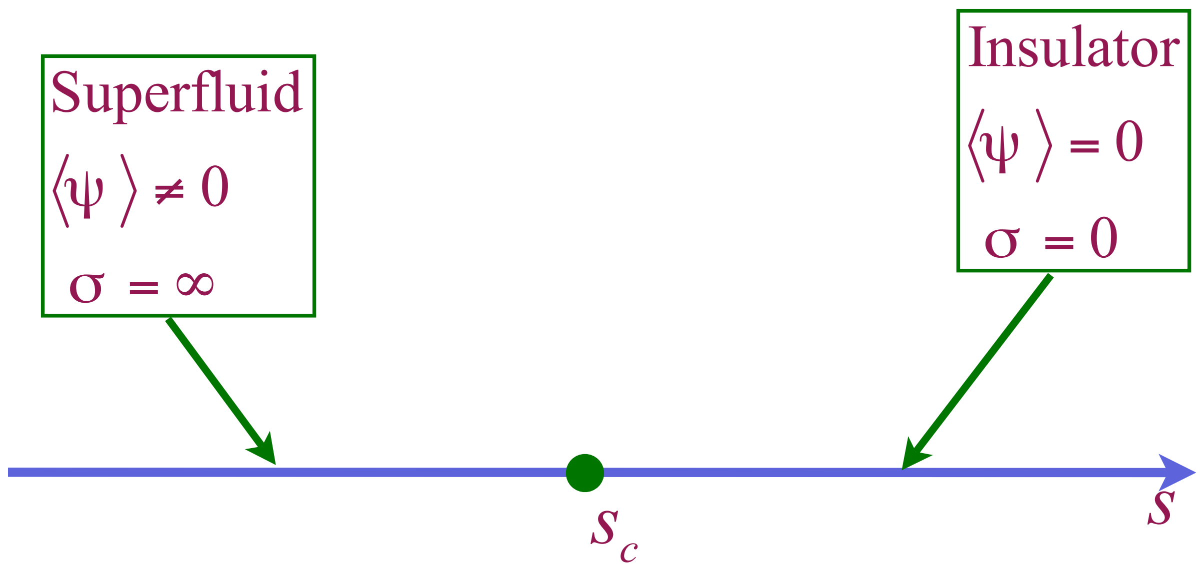
Density of particles = density of holes ⇒

. "relativistic" field theory for ψ:

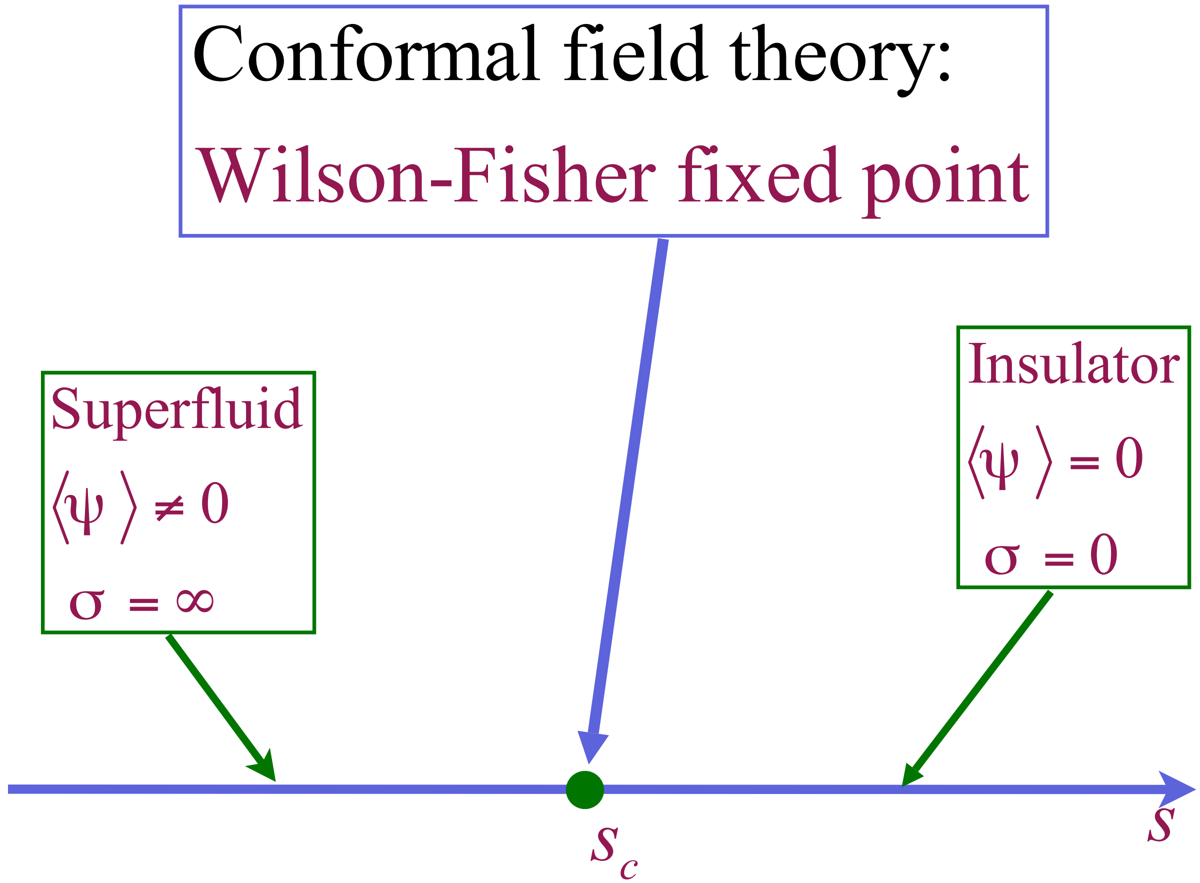
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla}\psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \quad \Leftrightarrow \quad \langle \psi \rangle = 0$$

$$\text{Superfluid} \quad \Leftrightarrow \quad \langle \psi \rangle \neq 0$$



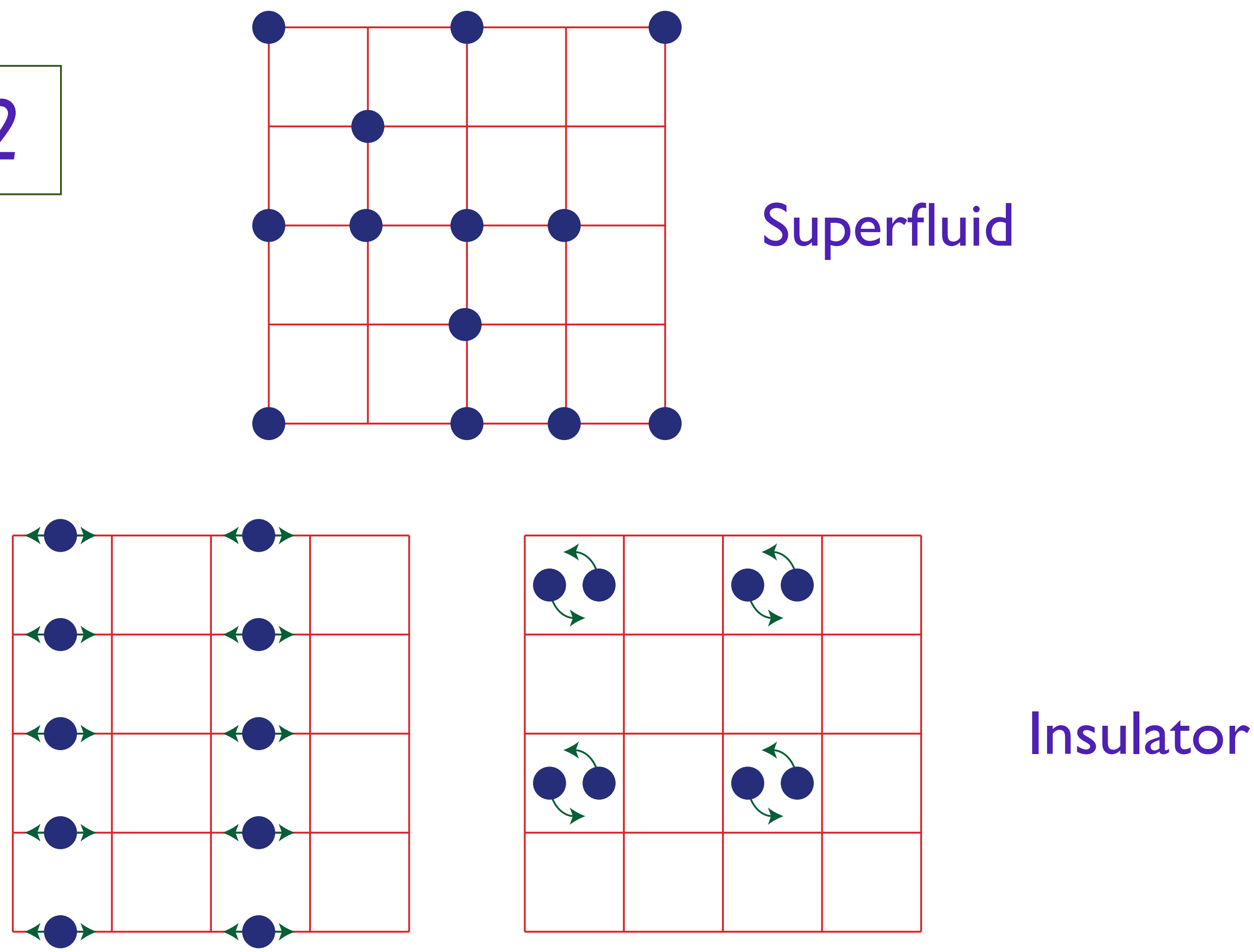
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla}\psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$



$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

Superfluid-insulator transition at fractional filling f

$f = 1/2$



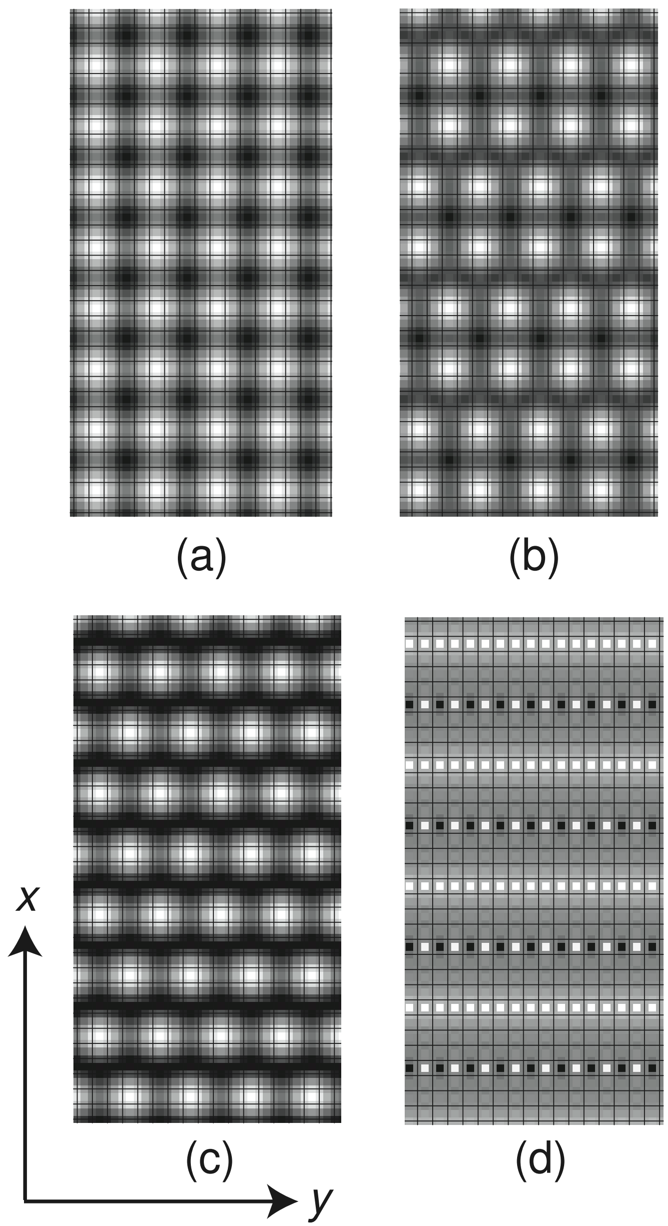
Possible continuous superfluid-insulator transition is described by a more complex CFT

L. Balents, L. Bartosch, A. Burkov, S. Sachdev, and K. Sengupta, Physical Review B **71**, 144508 (2005)

Superfluid-insulator transition at fractional filling f

$f = 1/16$

Possible continuous superfluid-insulator transition is described by a more complex CFT

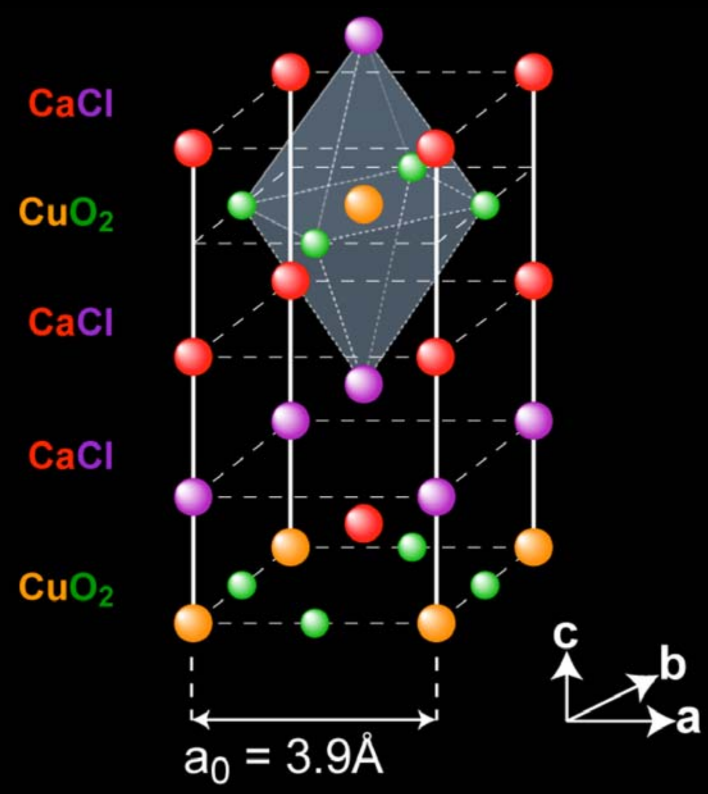


Insulator

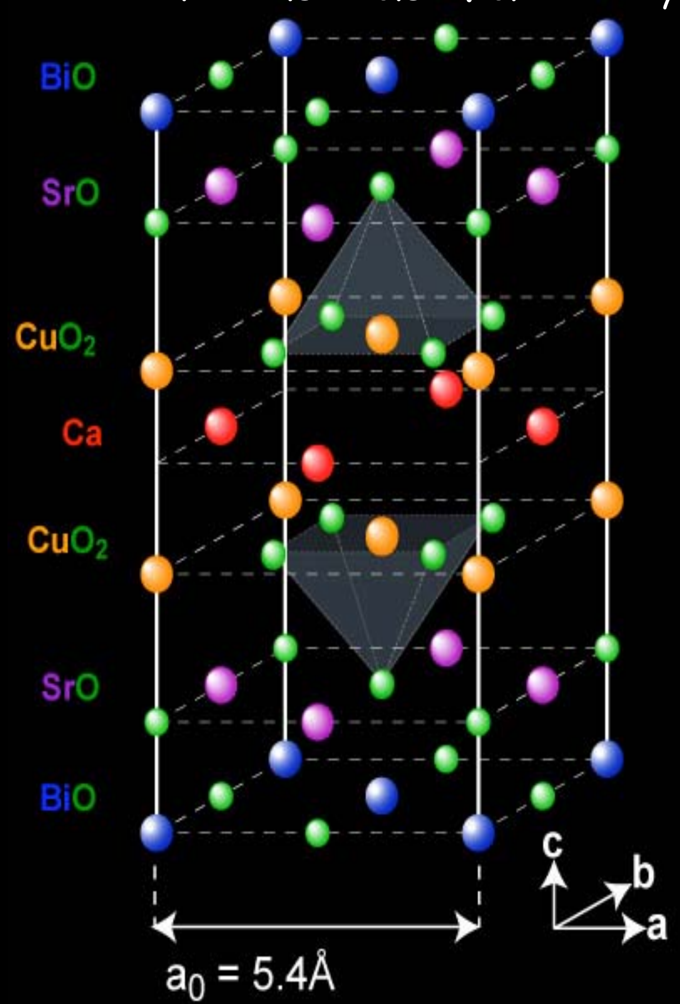
L. Balents, L. Bartosch, A. Burkov, S. Sachdev, and K. Sengupta, Physical Review B **71**, 144508 (2005)

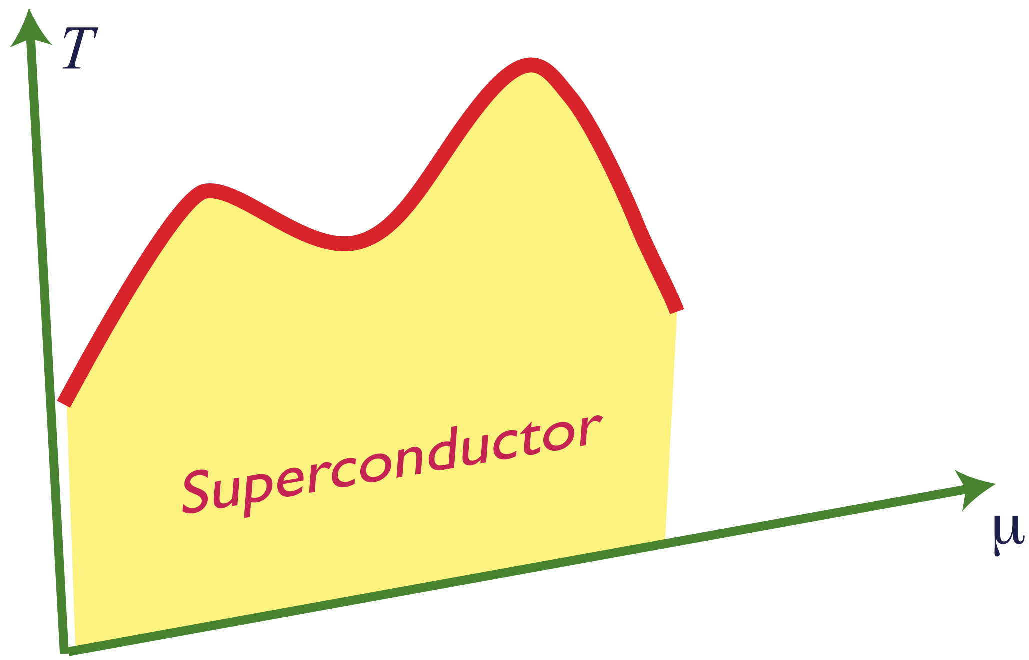
Dope the antiferomagnets with charge carriers of density x by applying a chemical potential μ

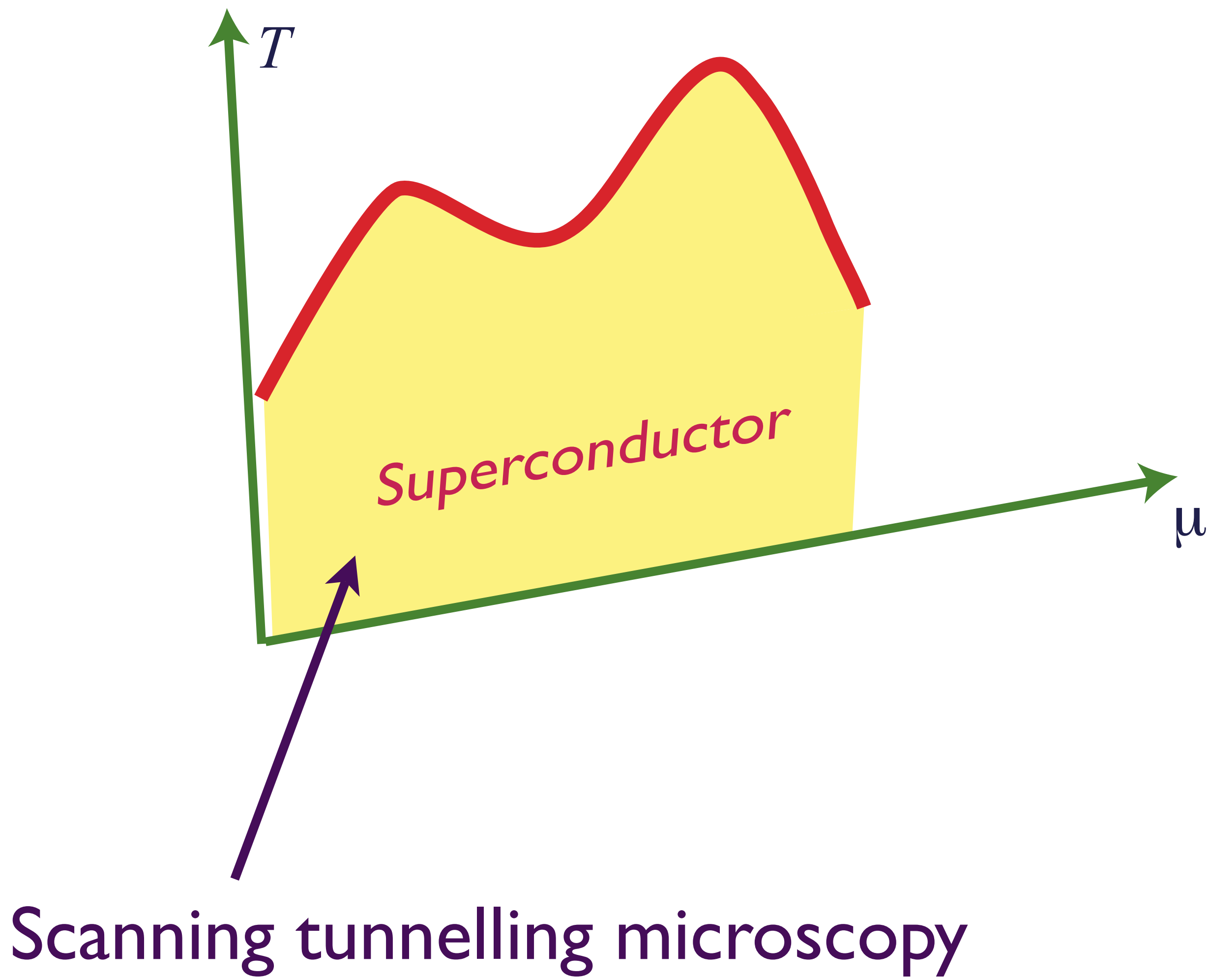
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

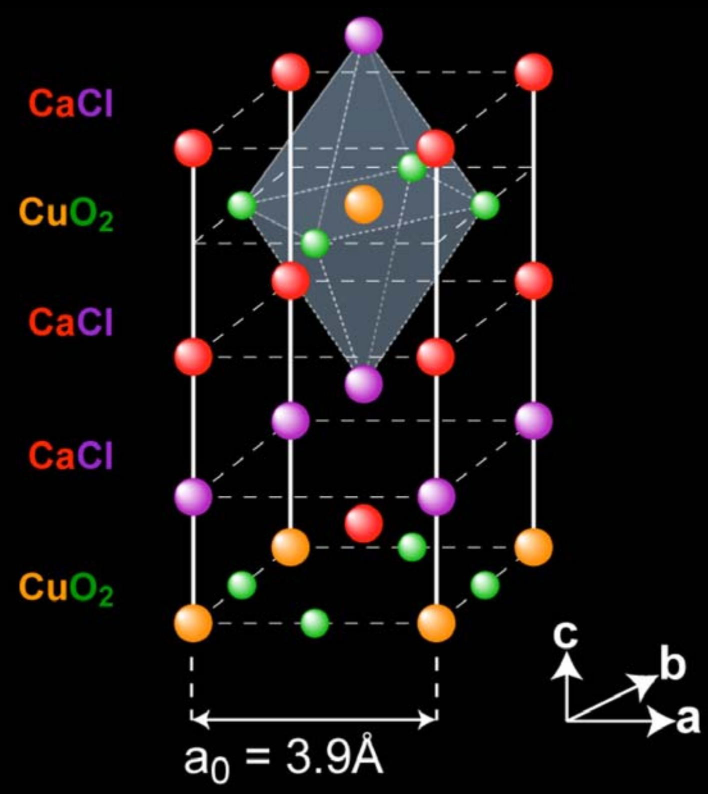



T
Superconductor
μ


T
Superconductor
μ
Scanning tunnelling microscopy

STM studies of the underdoped superconductor

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



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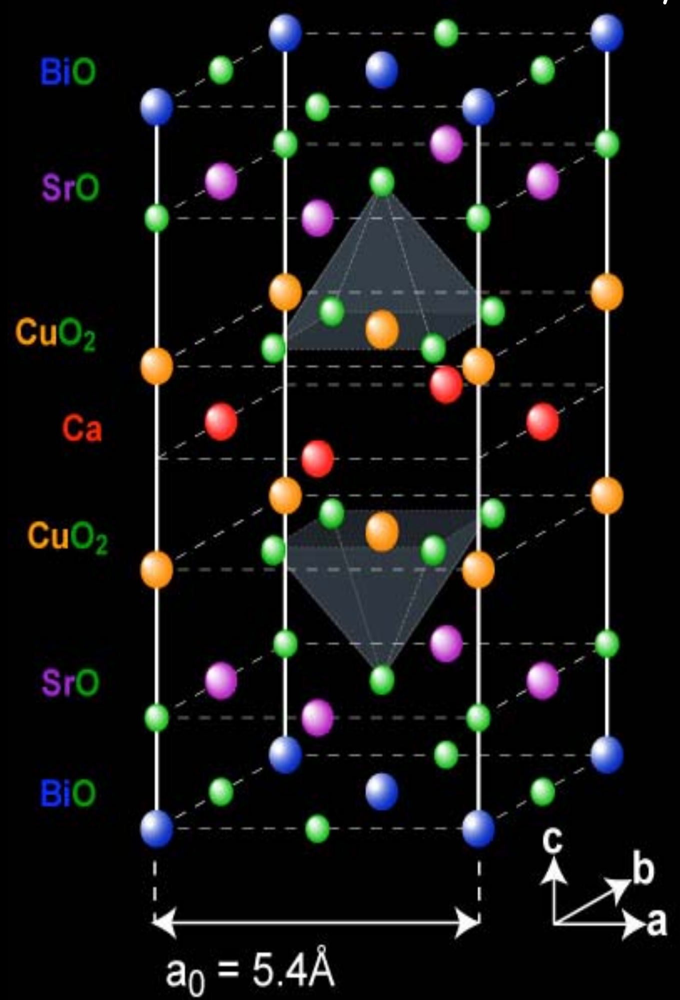
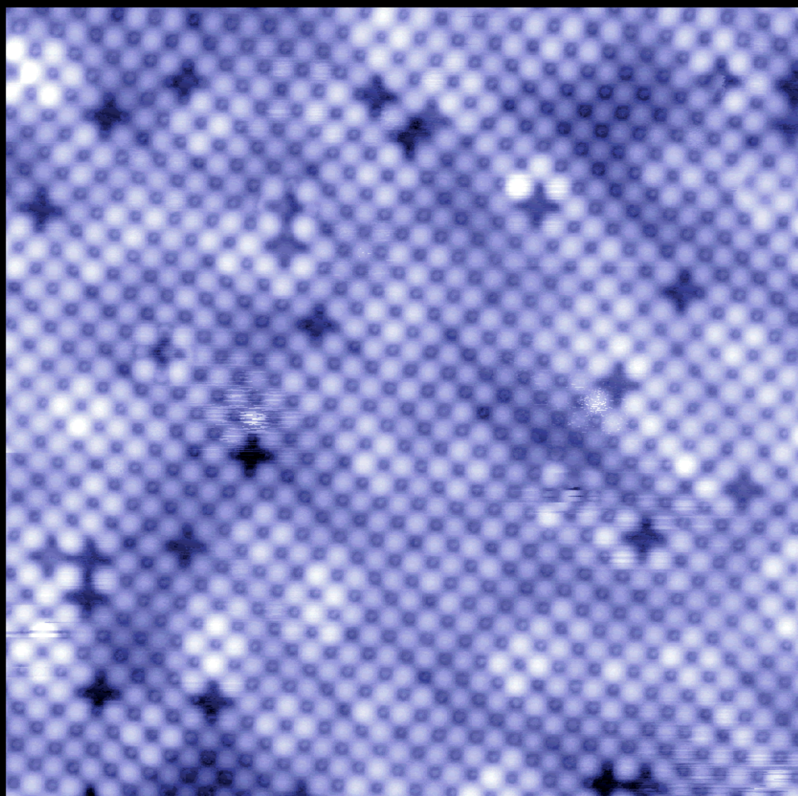
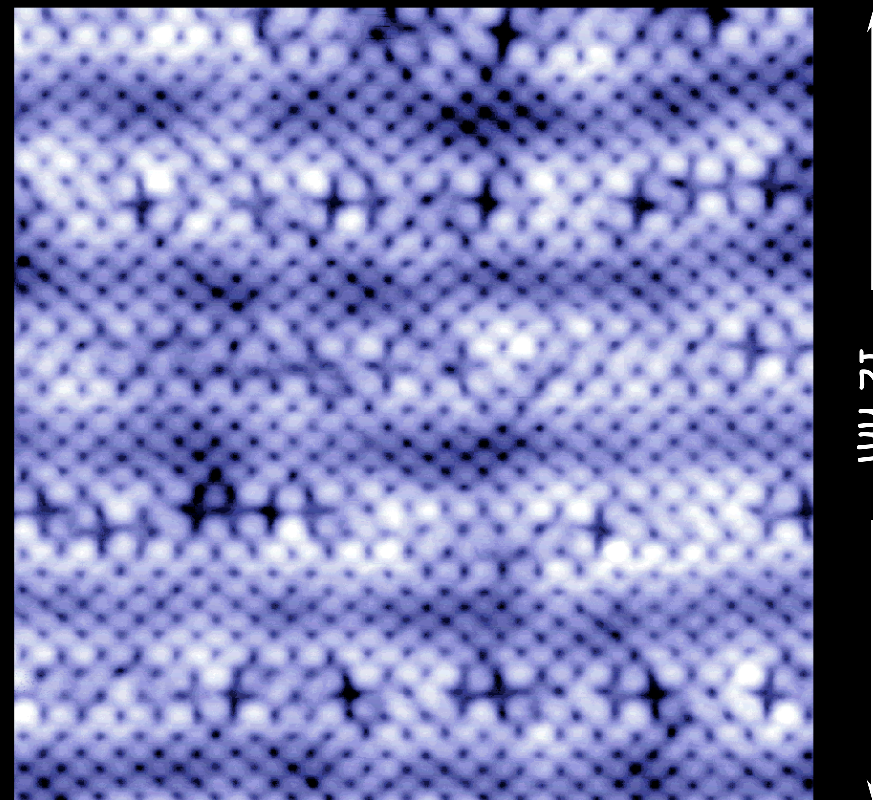


Topograph

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$

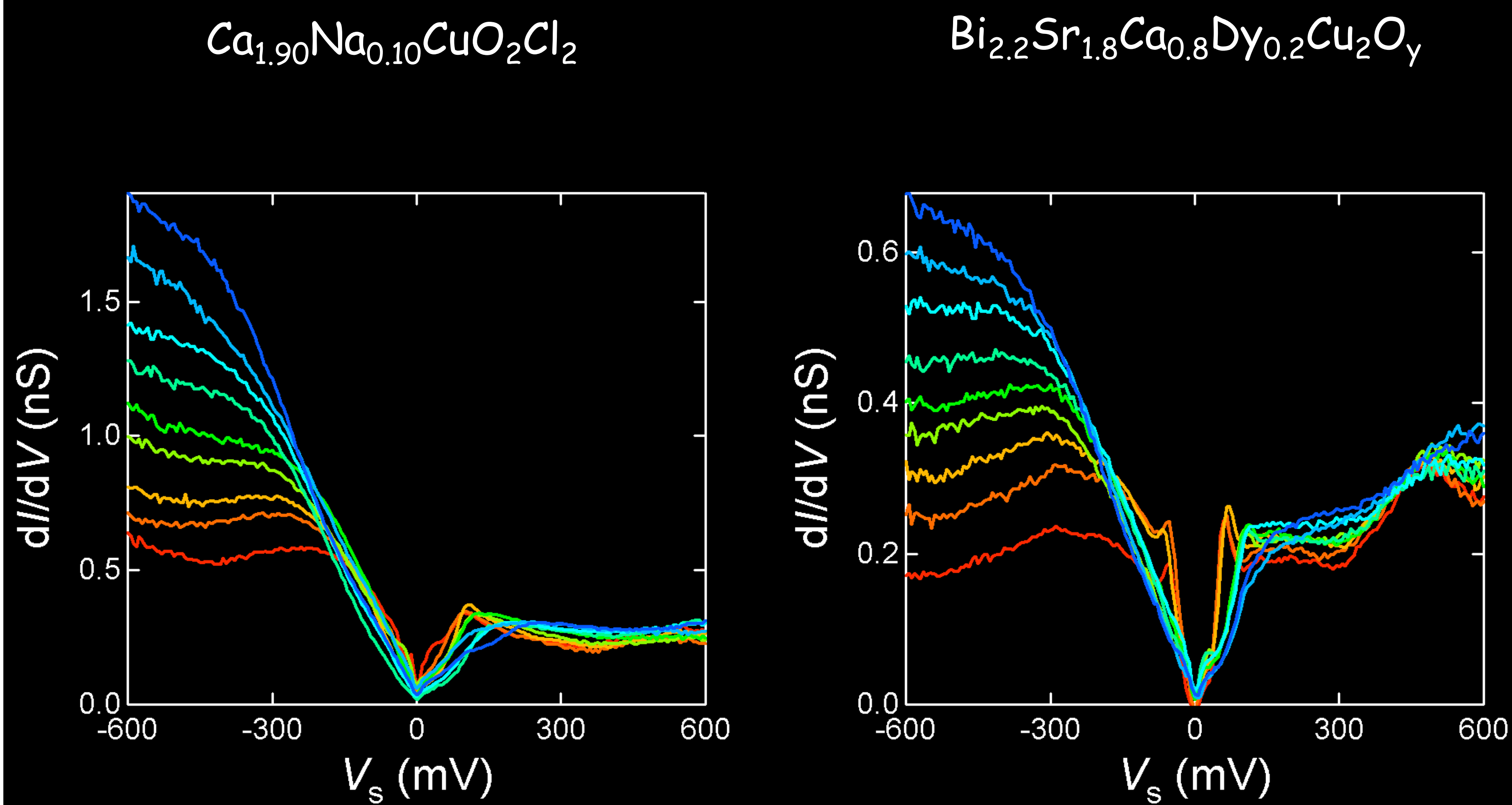


$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Y. Kohsaka et al. *Science* 315, 1380 (2007)

dI/dV Spectra

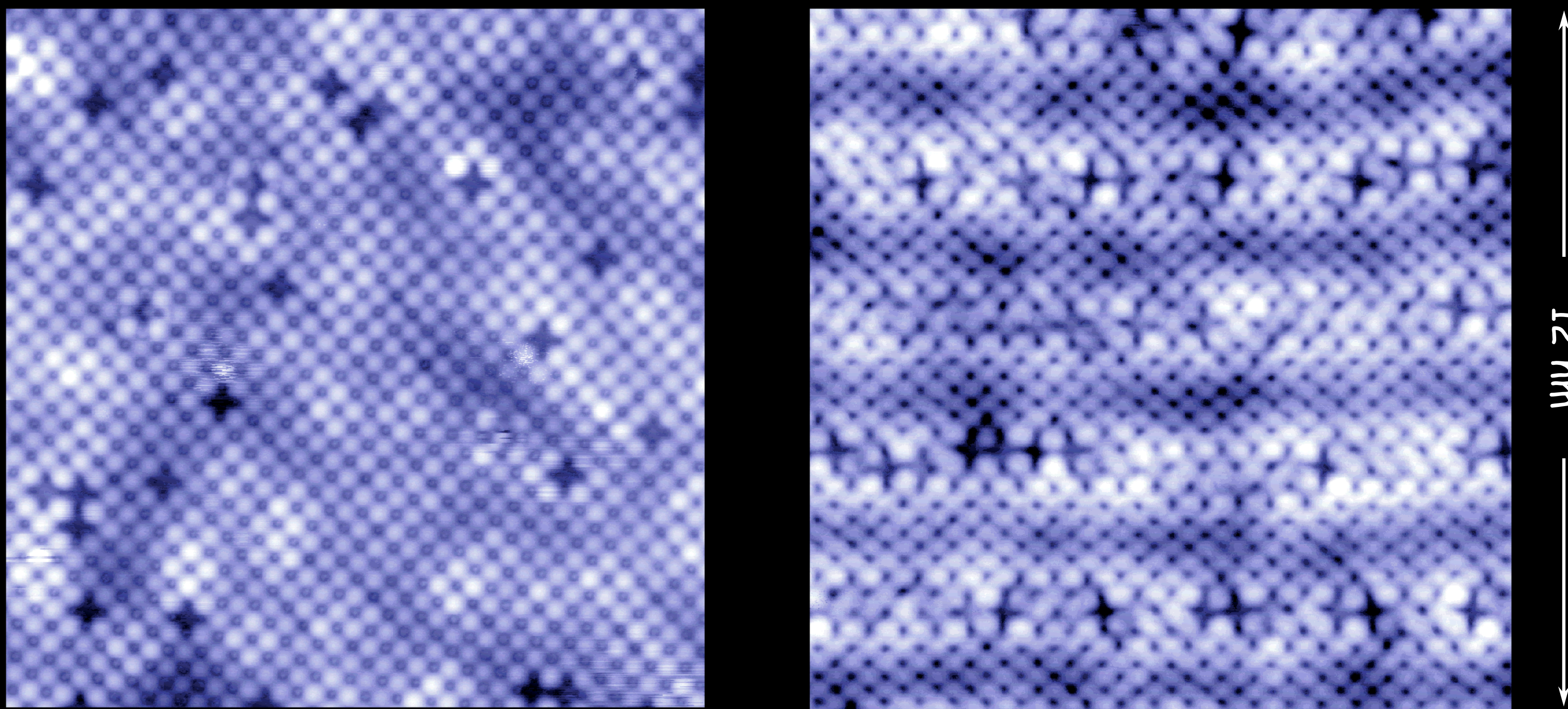


Intense Tunneling-Asymmetry (TA)
variation are highly similar

Y. Kohsaka et al. Science 315, 1380 (2007)

Topograph

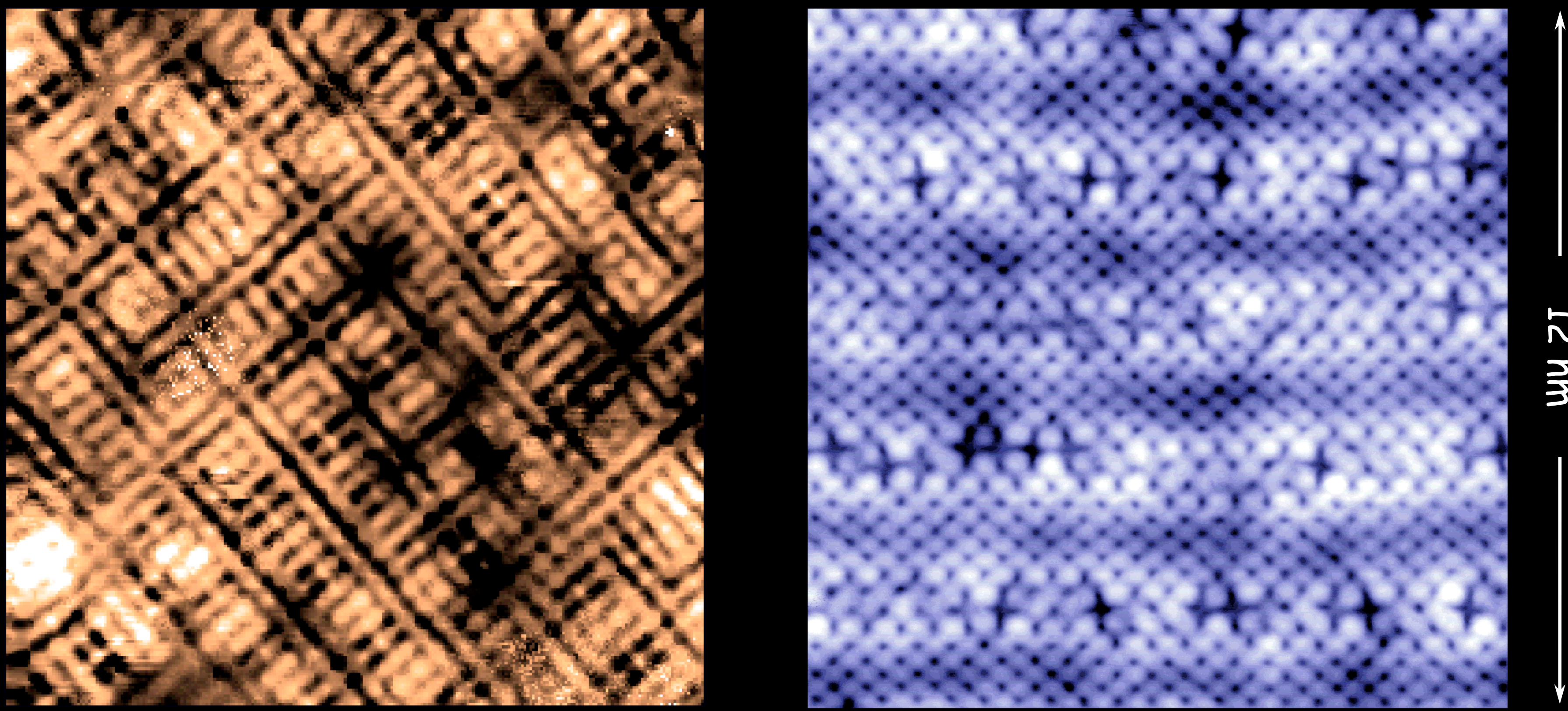
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$ $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Y. Kohsaka et al. Science 315, 1380 (2007)

Tunneling Asymmetry (TA)-map at E=150meV

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$ $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Y. Kohsaka et al. Science 315, 1380 (2007)

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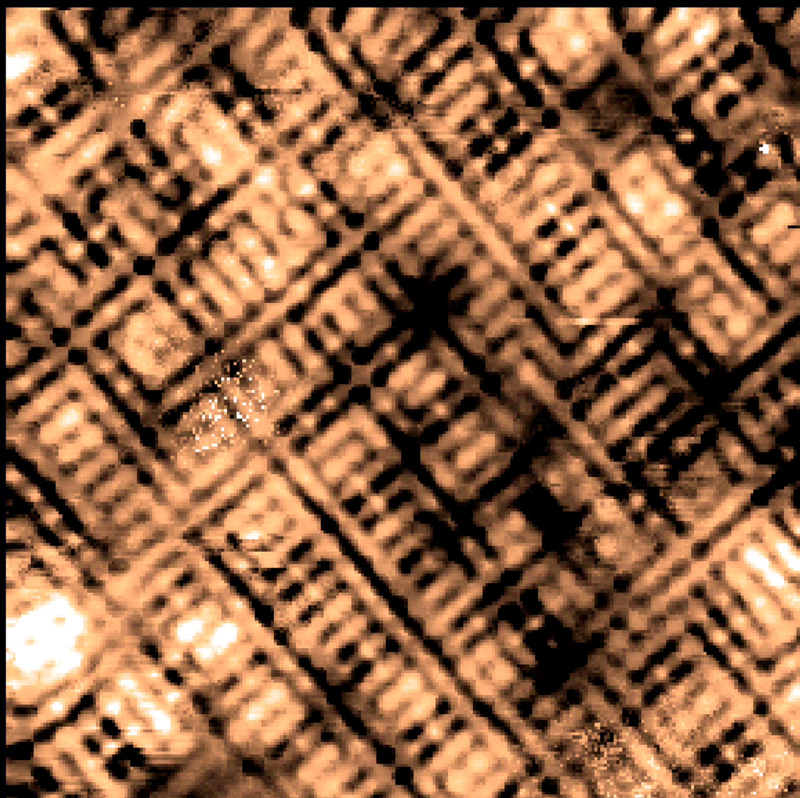
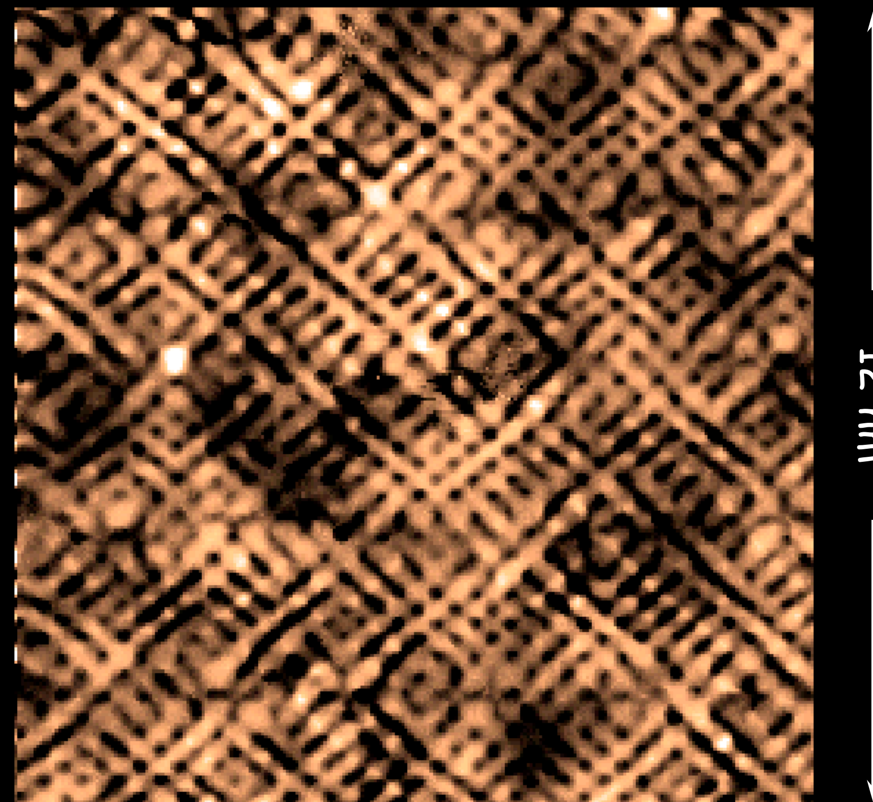


$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Y. Kohsaka et al. Science 315, 1380 (2007)

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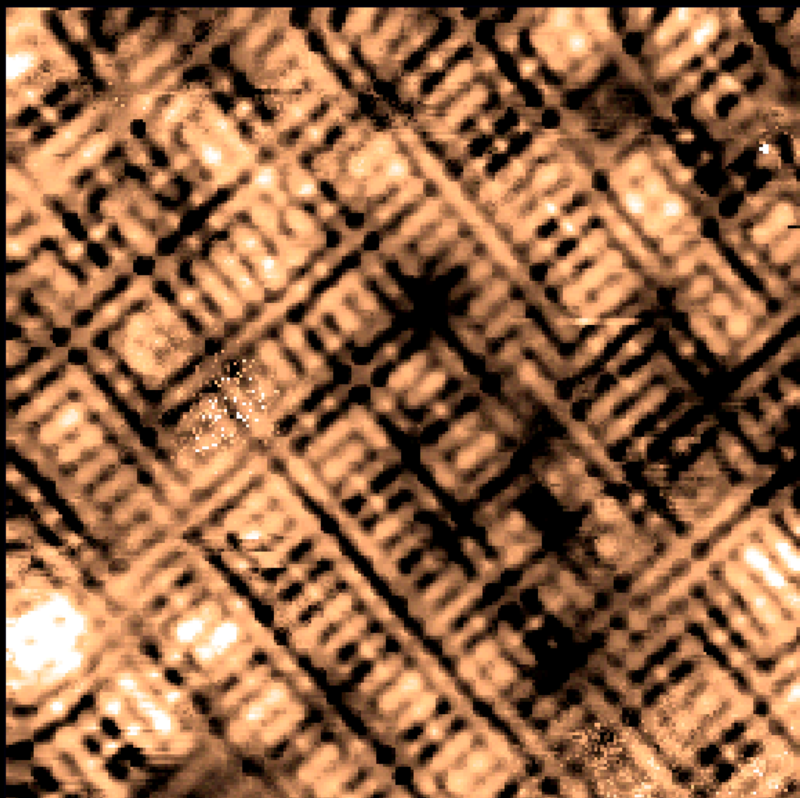
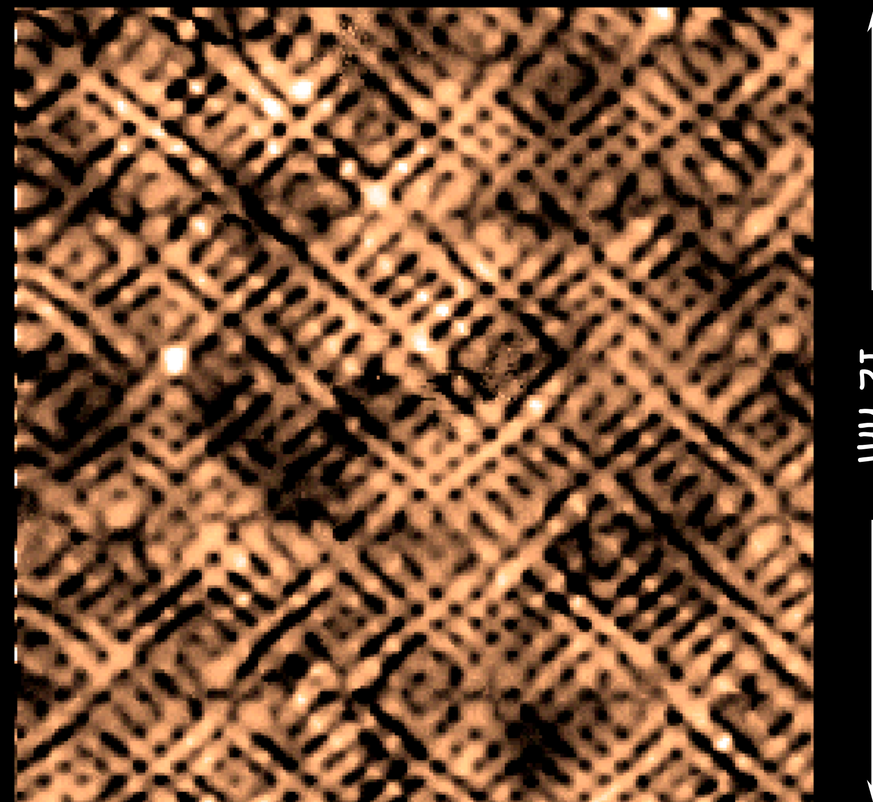
$Ca_{1.90}Na_{0.10}CuO_2Cl_2$



$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

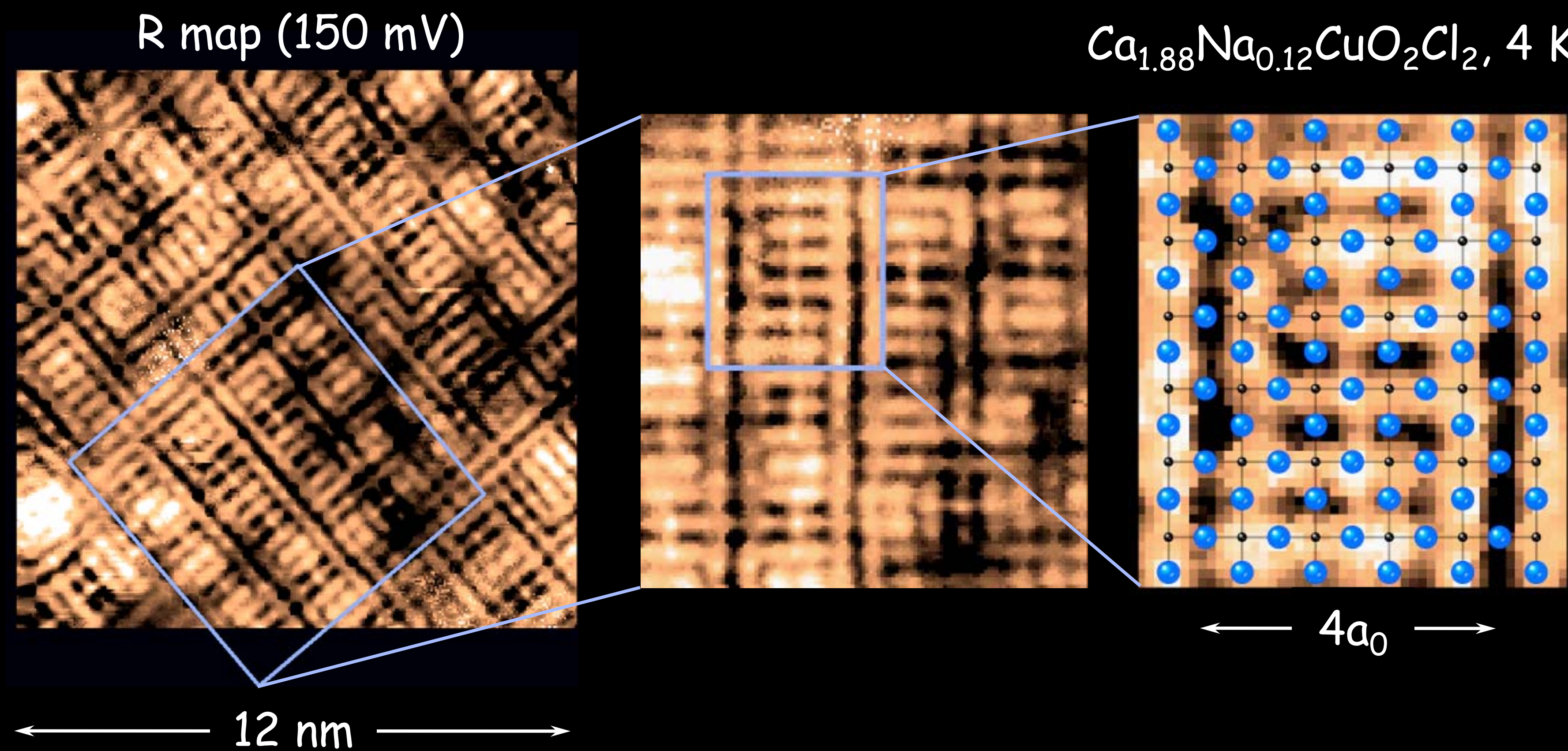


Indistinguishable bond-centered TA contrast

with disperse $4a_0$-wide nanodomains

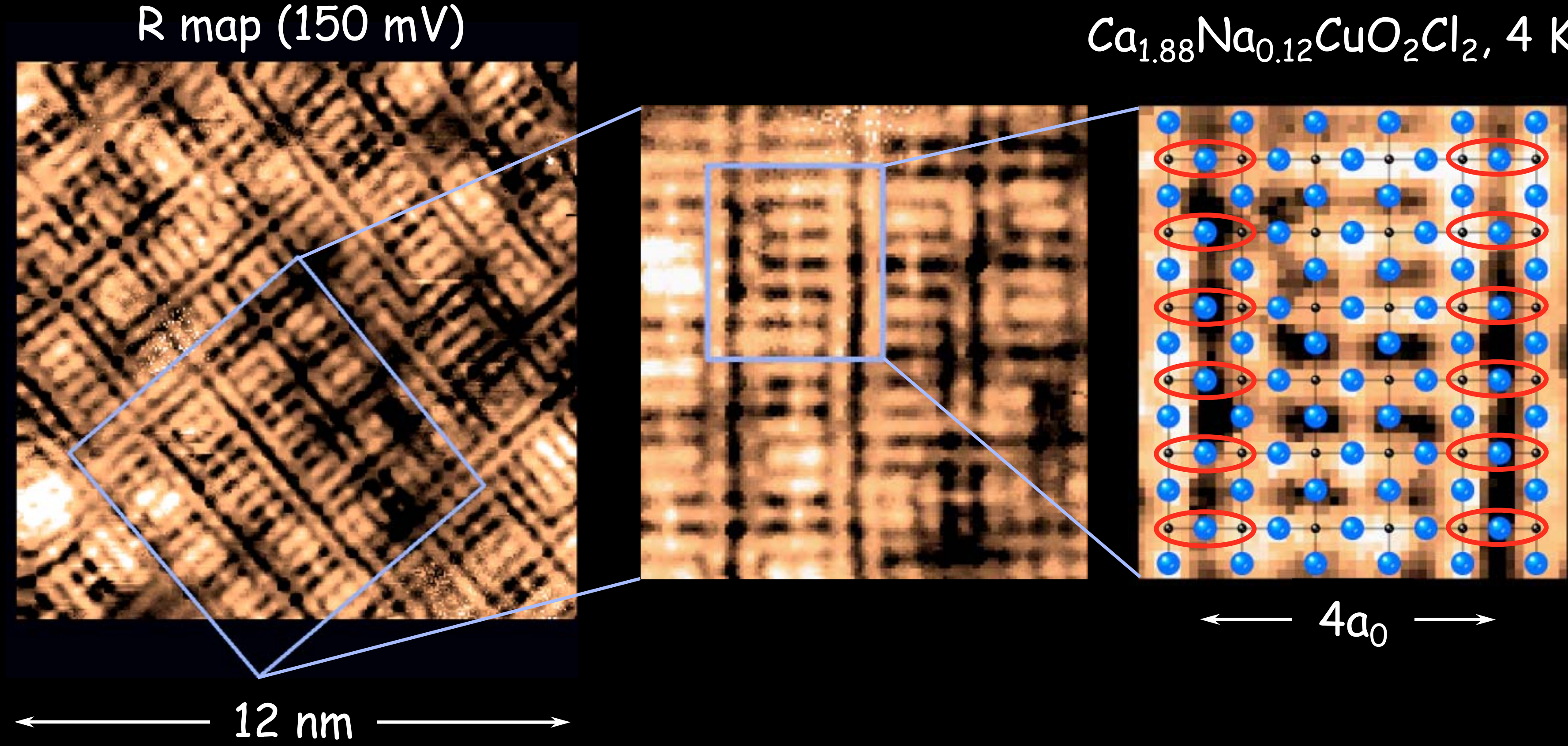
Y. Kohsaka et al. Science 315, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



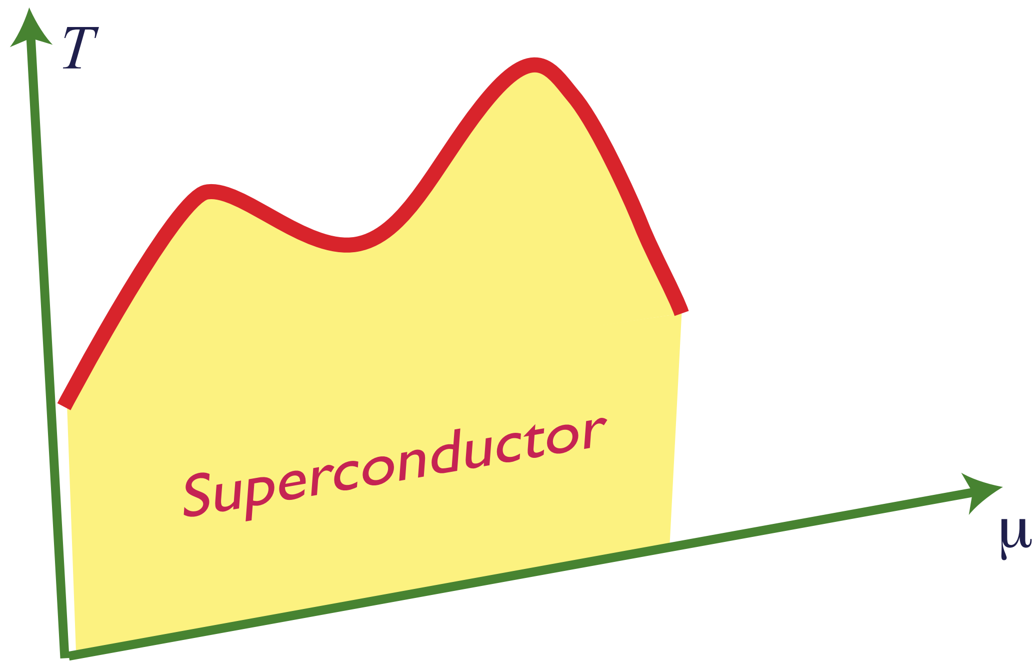
Y. Kohsaka et al. Science 315, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

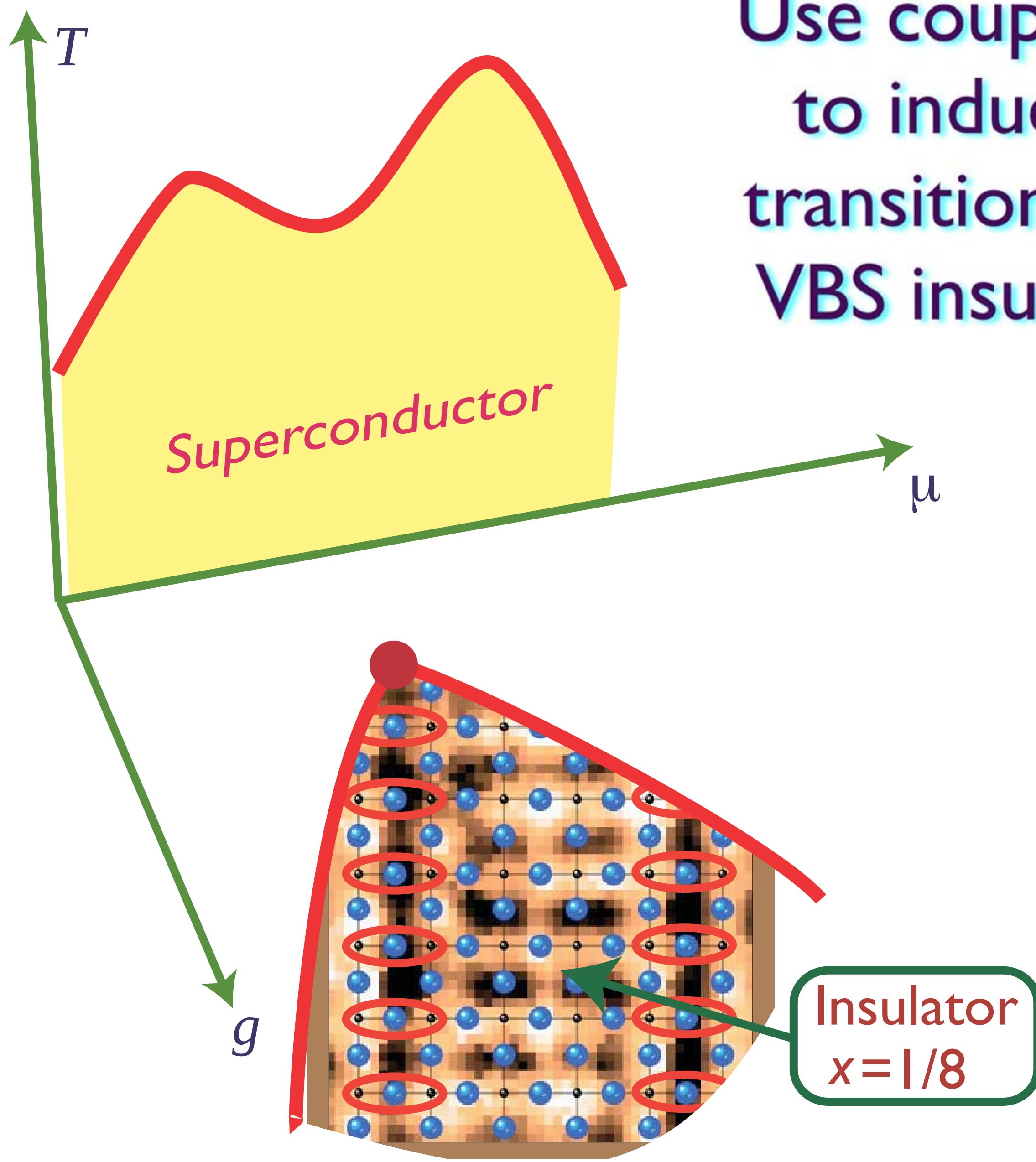


Evidence for VBS order - a valence bond supersolid

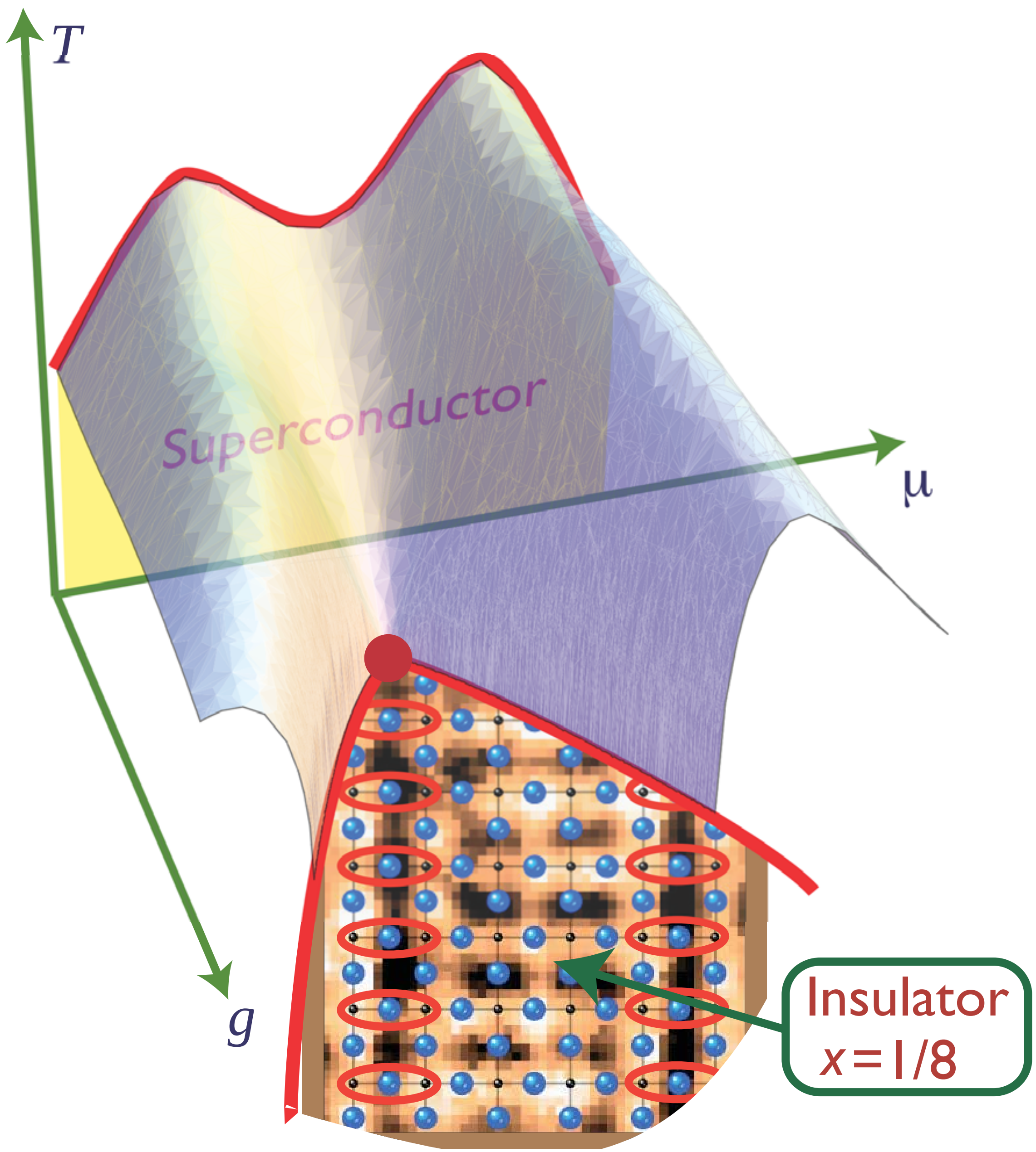
S. Sachdev and N. Read, Int. J. Mod. Phys. B 5, 219 (1991).


T
Superconductor
μ

Use coupling g to induce a transition to a VBS insulator



Proposed generalized phase diagram



Outline

1. Superfluid-insulator transition

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   Collisionless-t0-hydrodynamic crossover of CFT3s

3. SYM3 with $\mathcal{N} = 8$ supersymmetry

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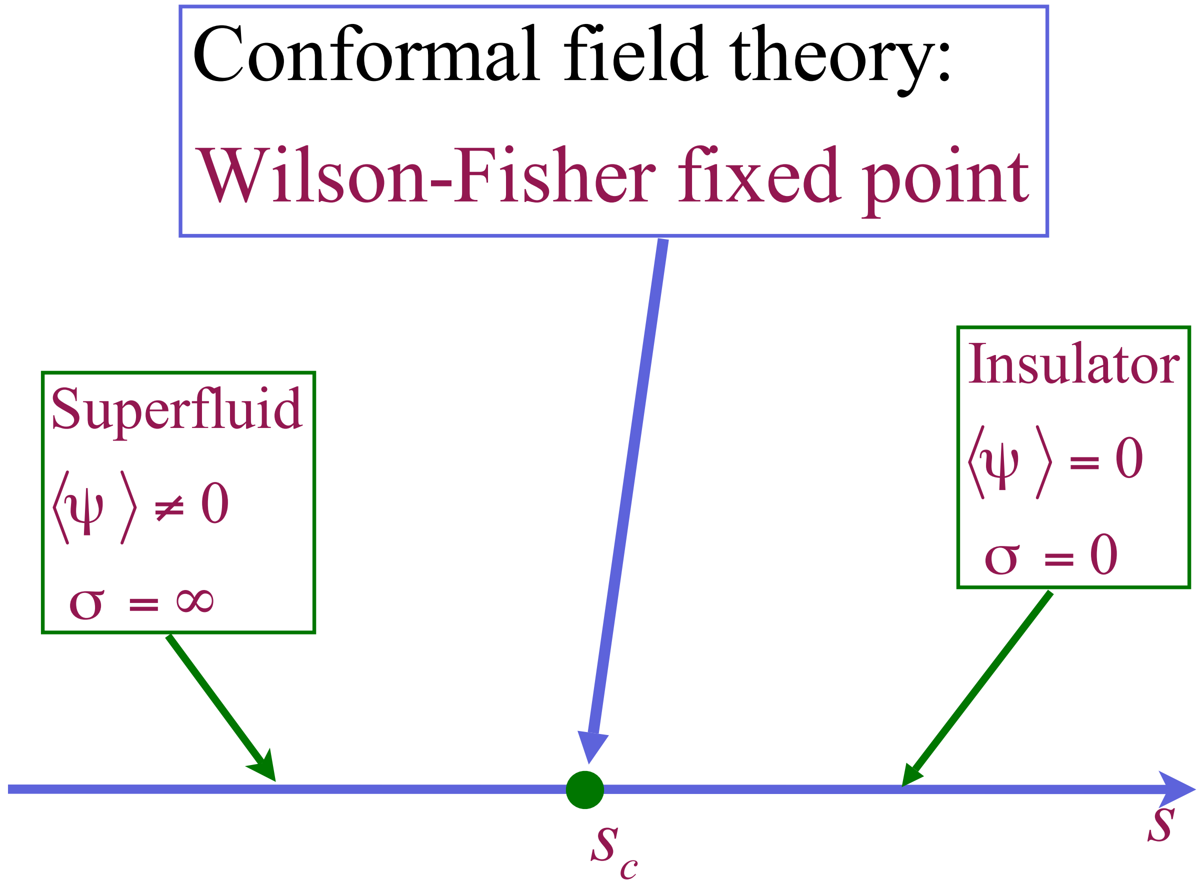

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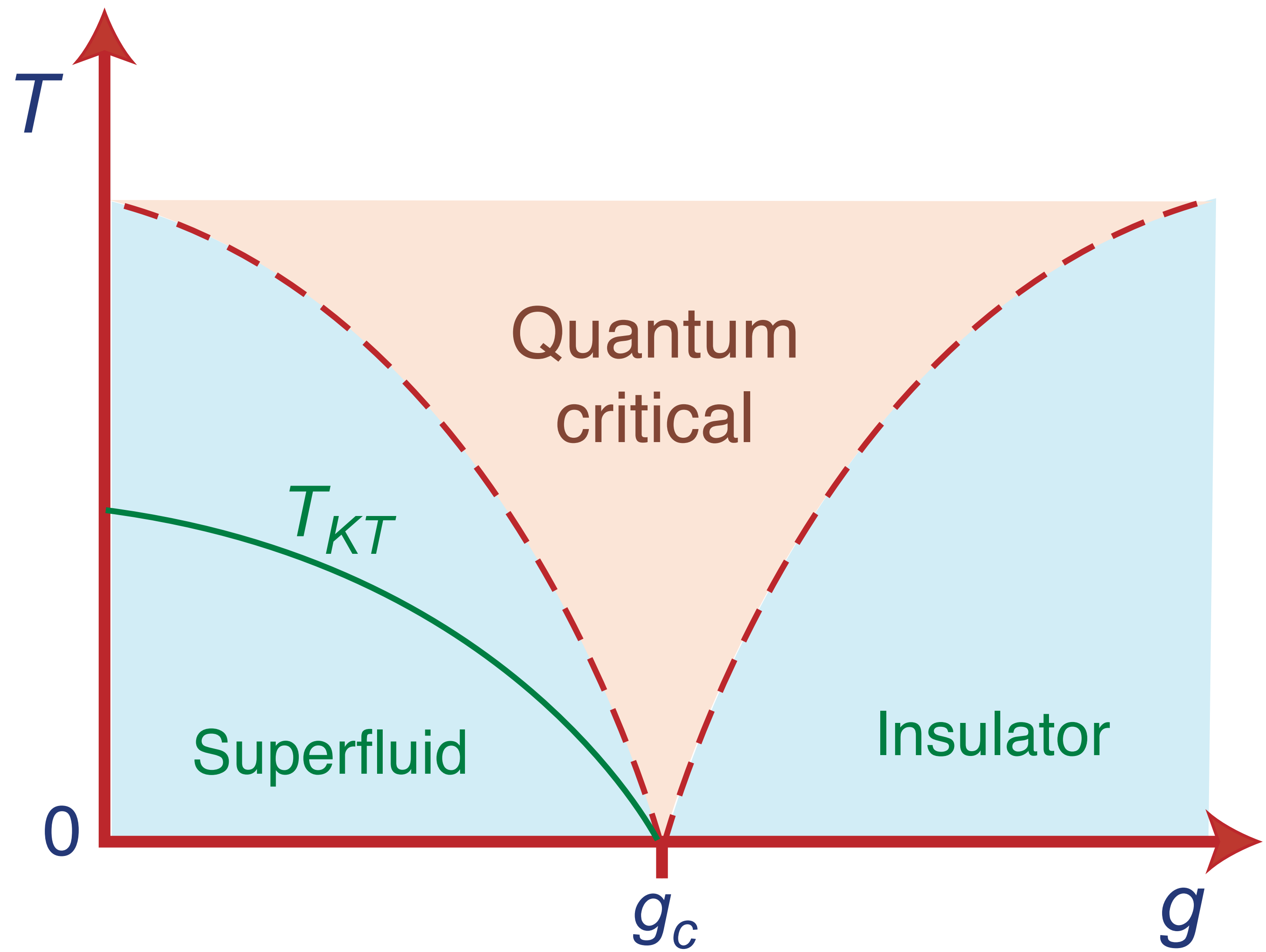
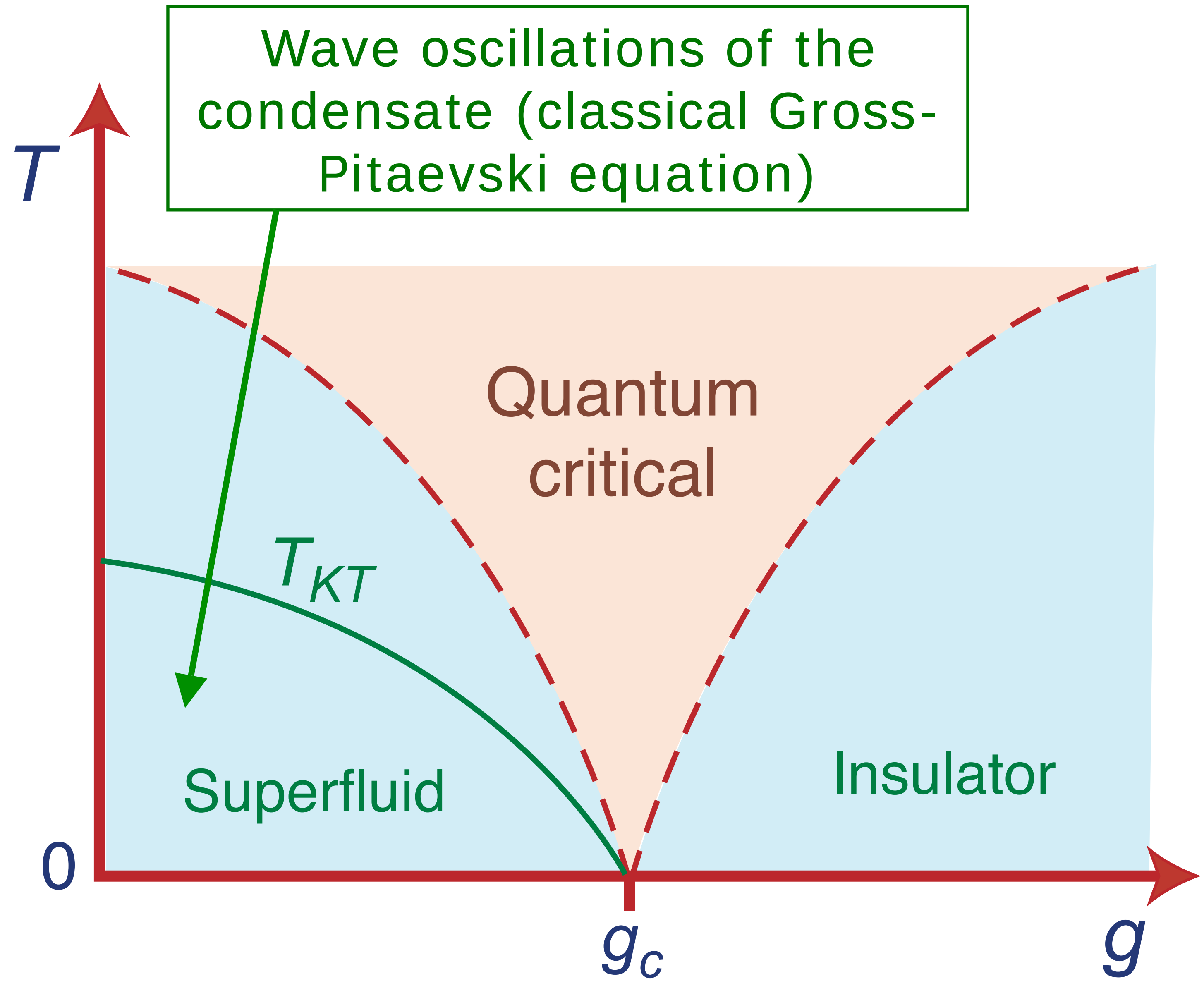
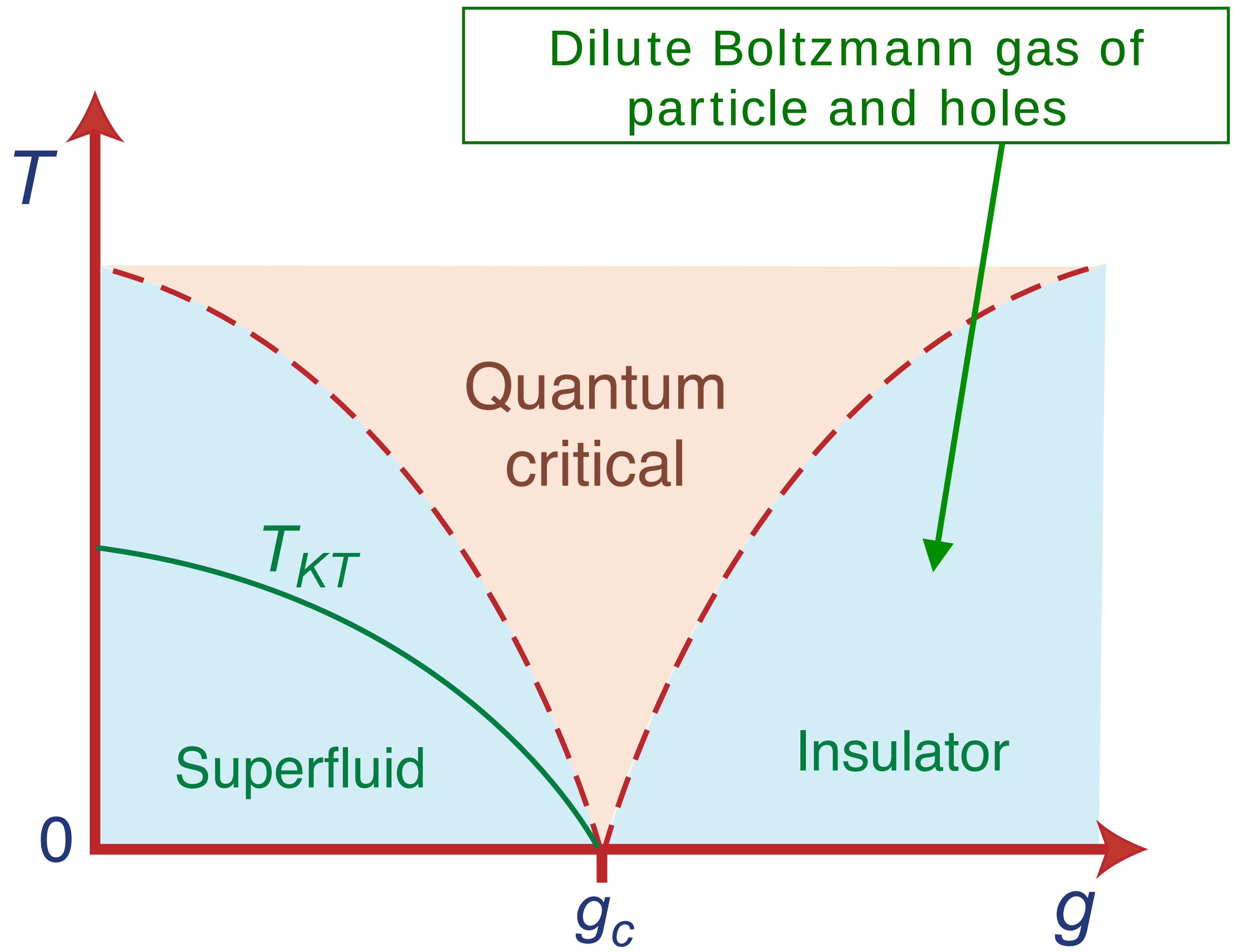
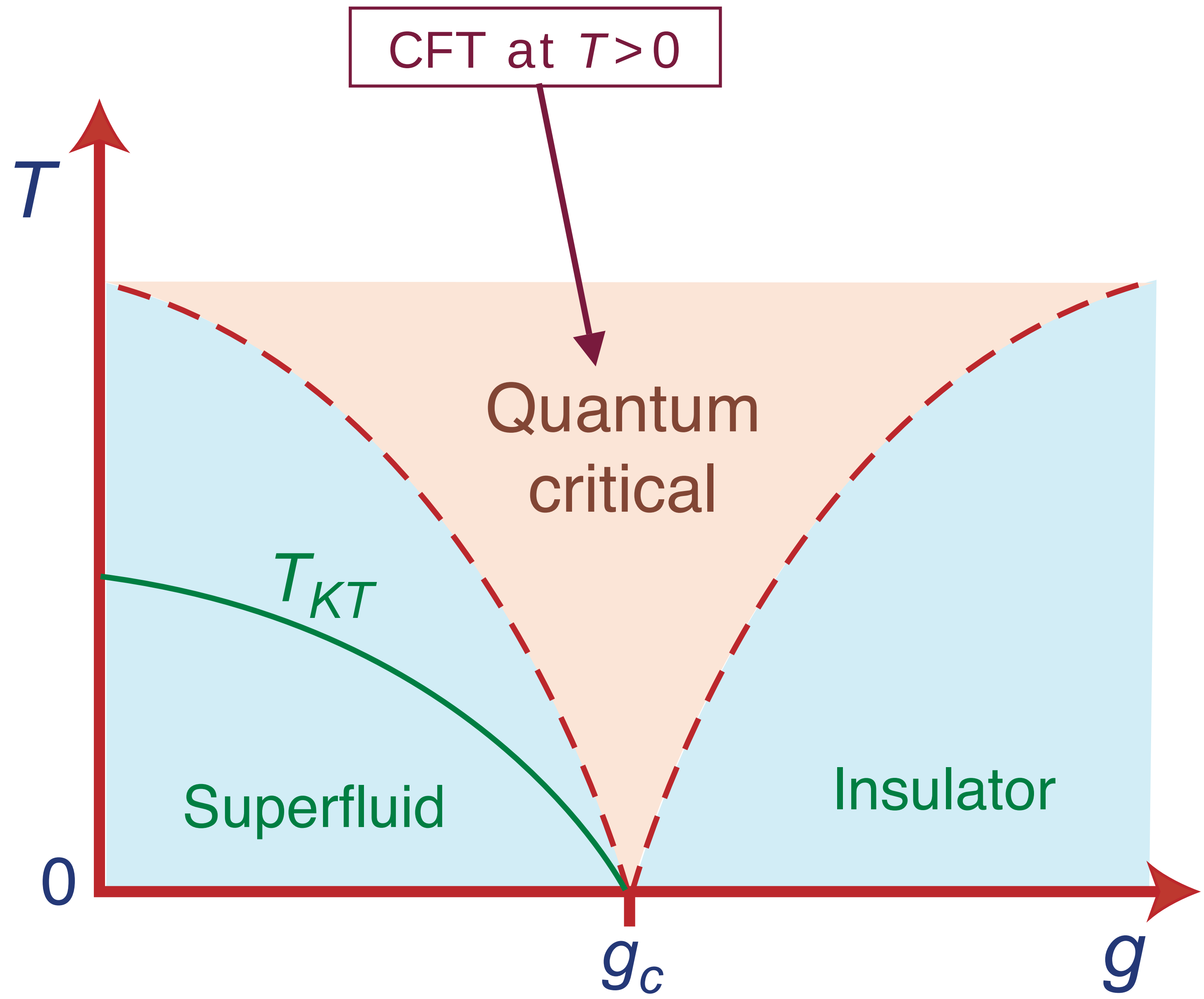


T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g



Wave oscillations of the condensate (classical Gross-Pitaevski equation)
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g


Dilute Boltzmann gas of particle and holes
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$
$$\sigma_{\text{Insulator}}(T \to 0) = 0$$
$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

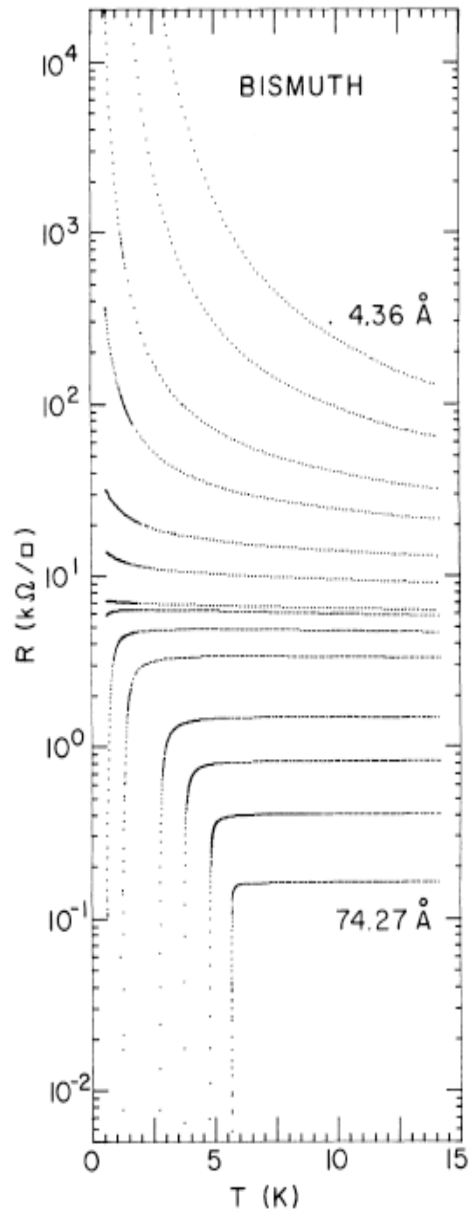


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Density correlations in CFTs at T >0

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k,\omega)$

For *all* CFT2s, at all $\hbar\omega / k_B T$

$$\chi(k,\omega) \;=\; \frac{4e^2}{h} K \frac{vk^2}{v^2k^2 - \omega^2} \;\; ; \;\; \sigma(\omega) = \frac{4e^2}{h} \frac{Kv}{-i\omega}$$

where K is a universal number characterizing the CFT2 (the level number), and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k,\omega) \;=\; \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2k^2-\omega^2}} \;\; ; \;\; \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

However, for *all* CFT3s, at $\underline{\hbar\omega \ll k_B T}$, we have the Einstein relation

$$\chi(k, \omega) = 4e^2 \chi_c \frac{Dk^2}{Dk^2 - i\omega} \; ; \; \sigma(\omega) = 4e^2 D \chi_c = \frac{4e^2}{h} \Theta_1 \Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2} \Theta_1 \; ; \; D = \frac{hv^2}{k_B T} \Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T > 0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h} \Theta_1 \Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

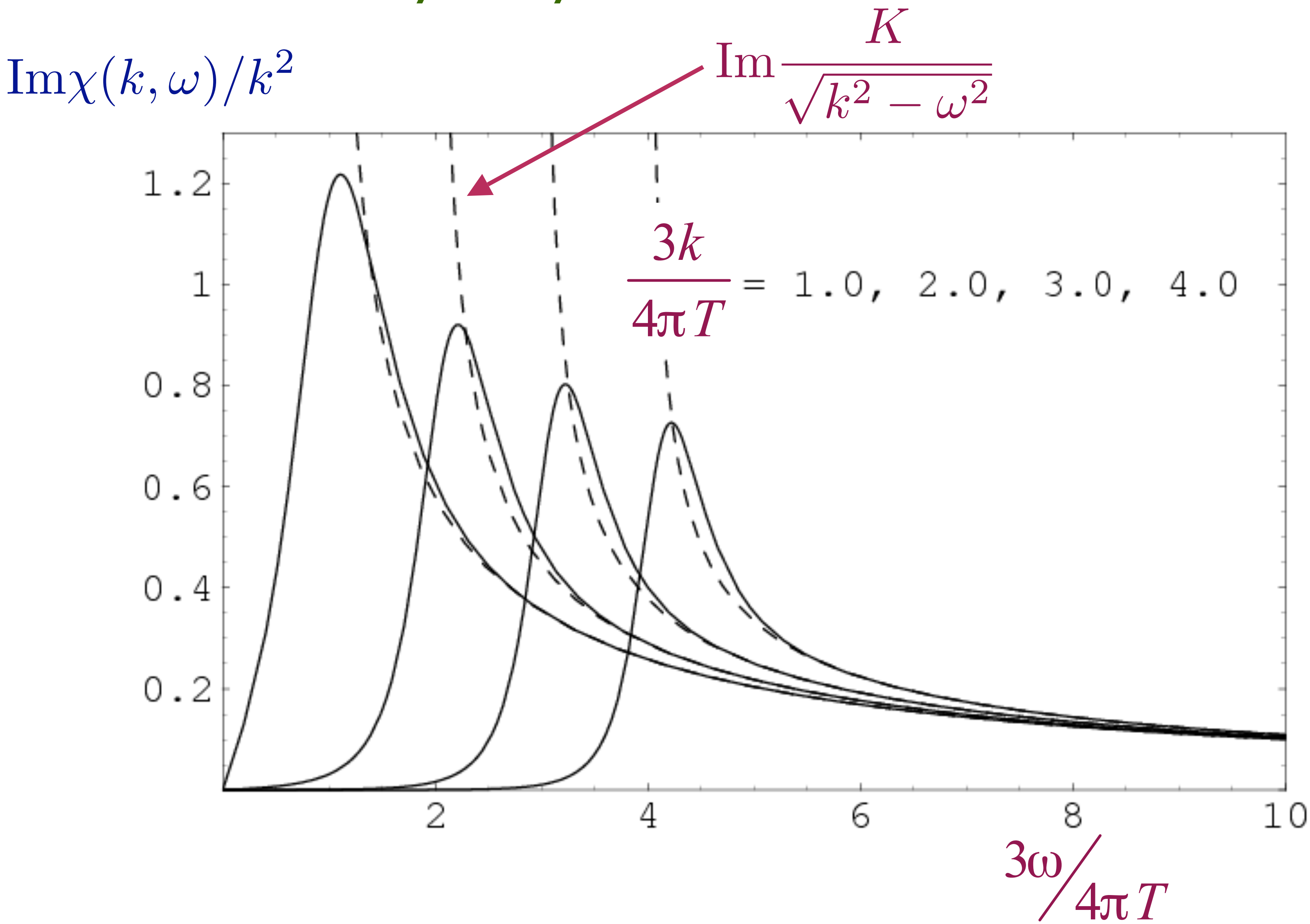
- Has a single dimensionful coupling constant, g, which flows to a strong-coupling fixed point $g = g^*$ in the infrared.

- The CFT3 describing this fixed point resembles "critical spin liquid" theories.

- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on $\text{AdS}_4 \times S_7$.

- The CFT3 has a global SO(8) R symmetry, and correlators of the SO(8) charge density can be computed exactly in the large N limit, even at $T > 0$.

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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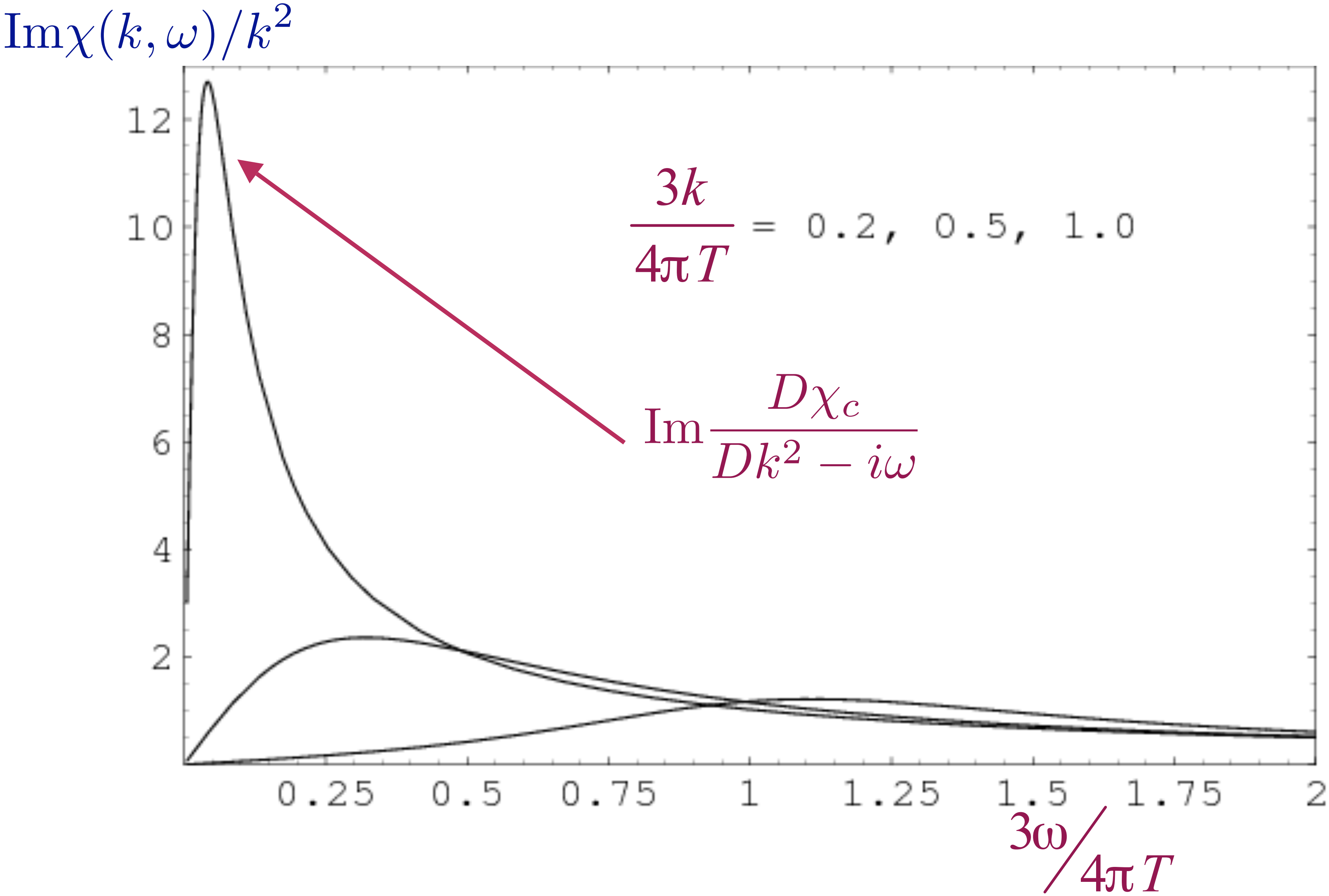

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(hv)^2}\Theta_1$$

$$D = \frac{hv^2}{k_B T}\Theta_2$$

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h}K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h}\Theta_1\Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$K = \frac{\sqrt{2}N^{3/2}}{3}$$

$$\Theta_1 = \frac{8\pi^2\sqrt{2}N^{3/2}}{9}$$

$$\Theta_2 = \frac{3}{8\pi^2}$$

C. Herzog, JHEP **0212**, 026 (2002)
P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Electromagnetic self-duality

- Unexpected result, $K = \Theta_1\Theta_2$.
- This is traced to a *four*-dimensional electromagnetic self-duality of the theory on AdS_4. In the large N limit, the SO(8) currents decouple into 28 U(1) currents with a Maxwell action for the U(1) gauge fields on AdS_4.
- This special property is not expected for generic CFT3s.
- Open question: Does $K = \Theta_1\Theta_2$ hold beyond the $N \to \infty$ limit ? In other words, does this "self-duality" survive in the full M theory.

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Dope the antiferomagnets with charge carriers of density *x* by applying a chemical potential μ

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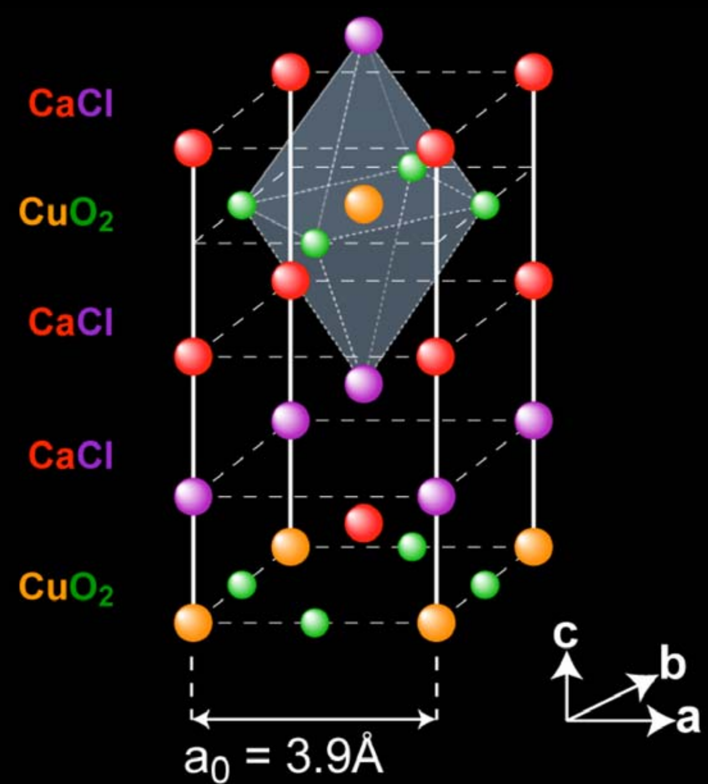
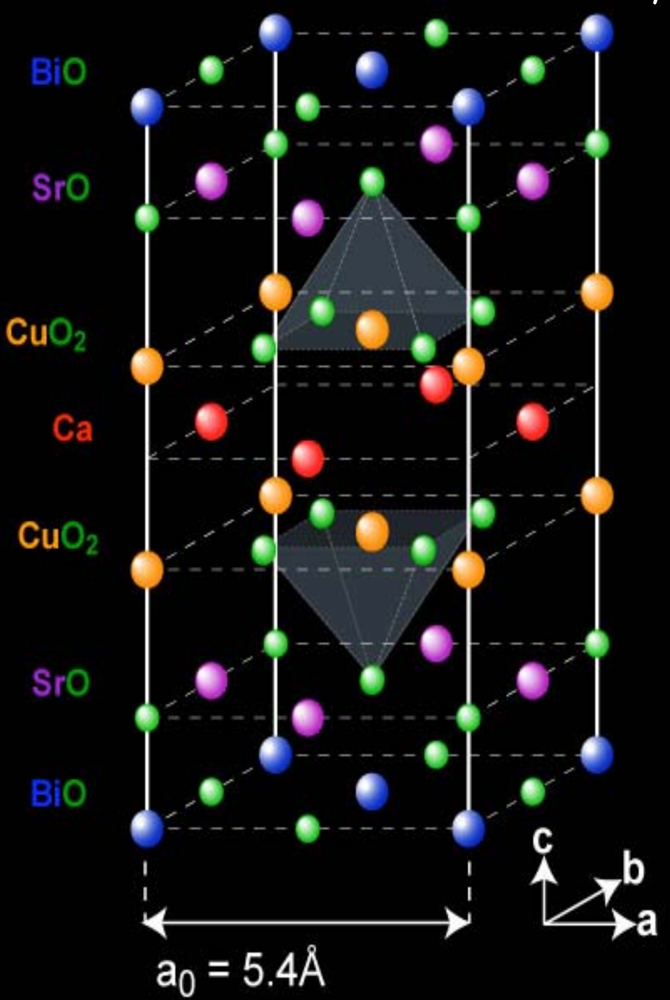
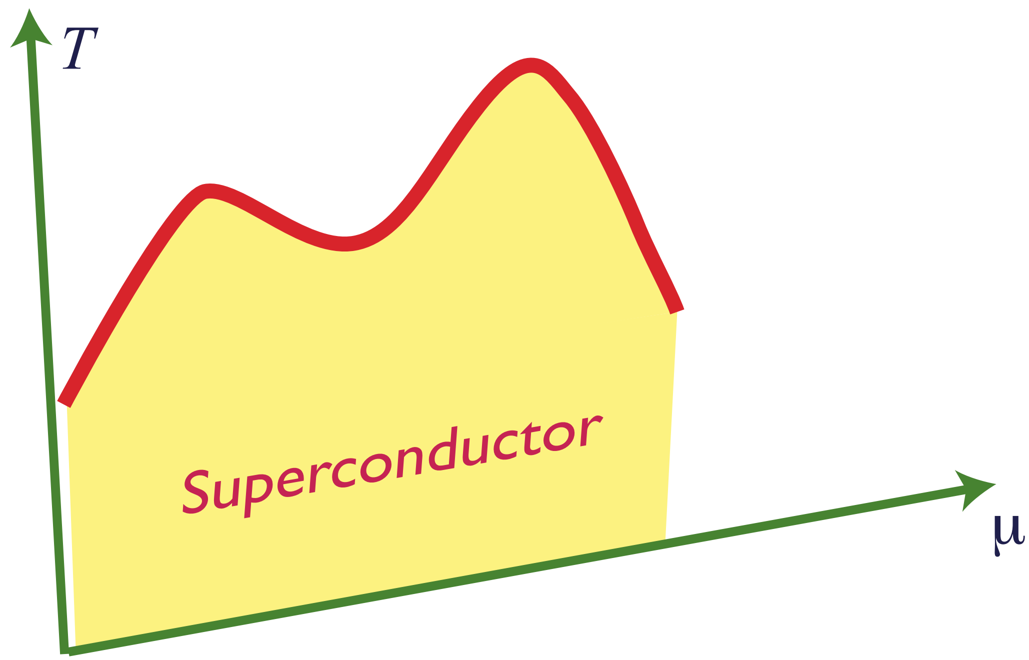
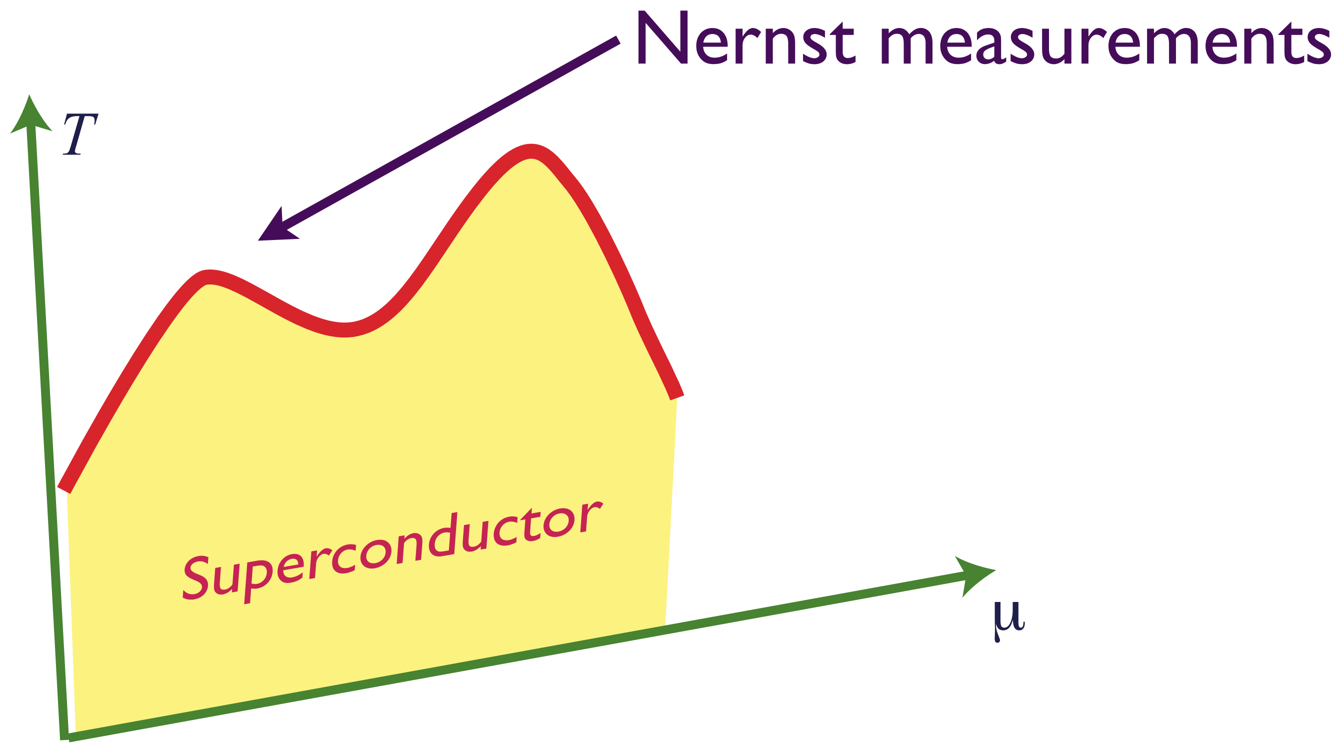


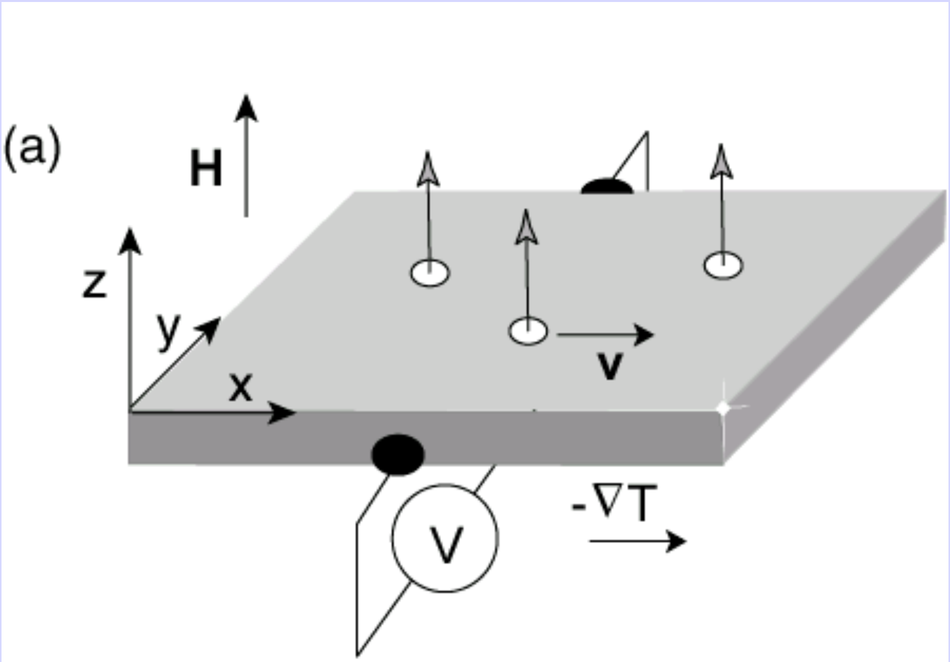
$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

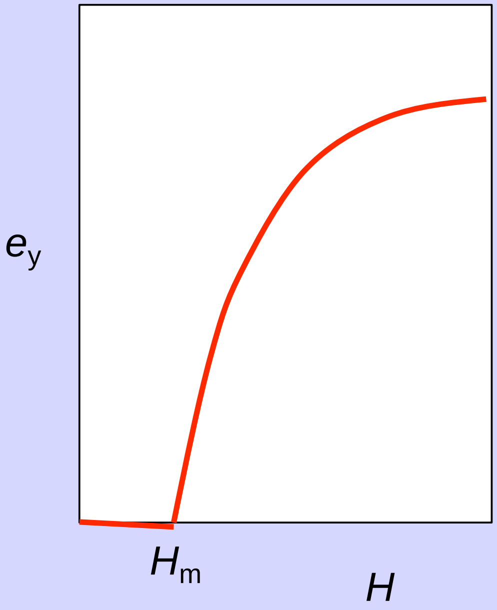



T
Superconductor
μ

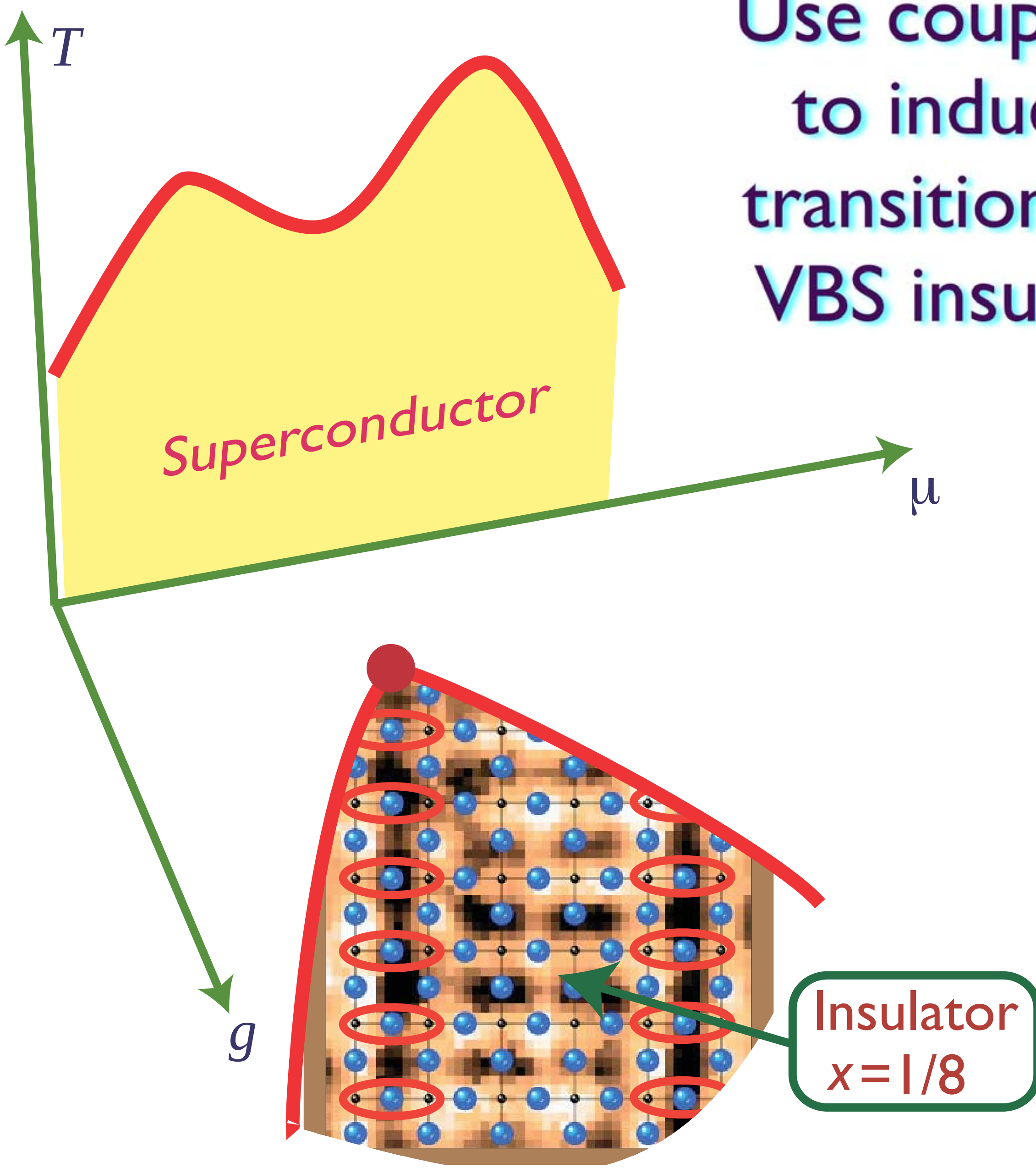

Nernst measurements
T
Superconductor
μ

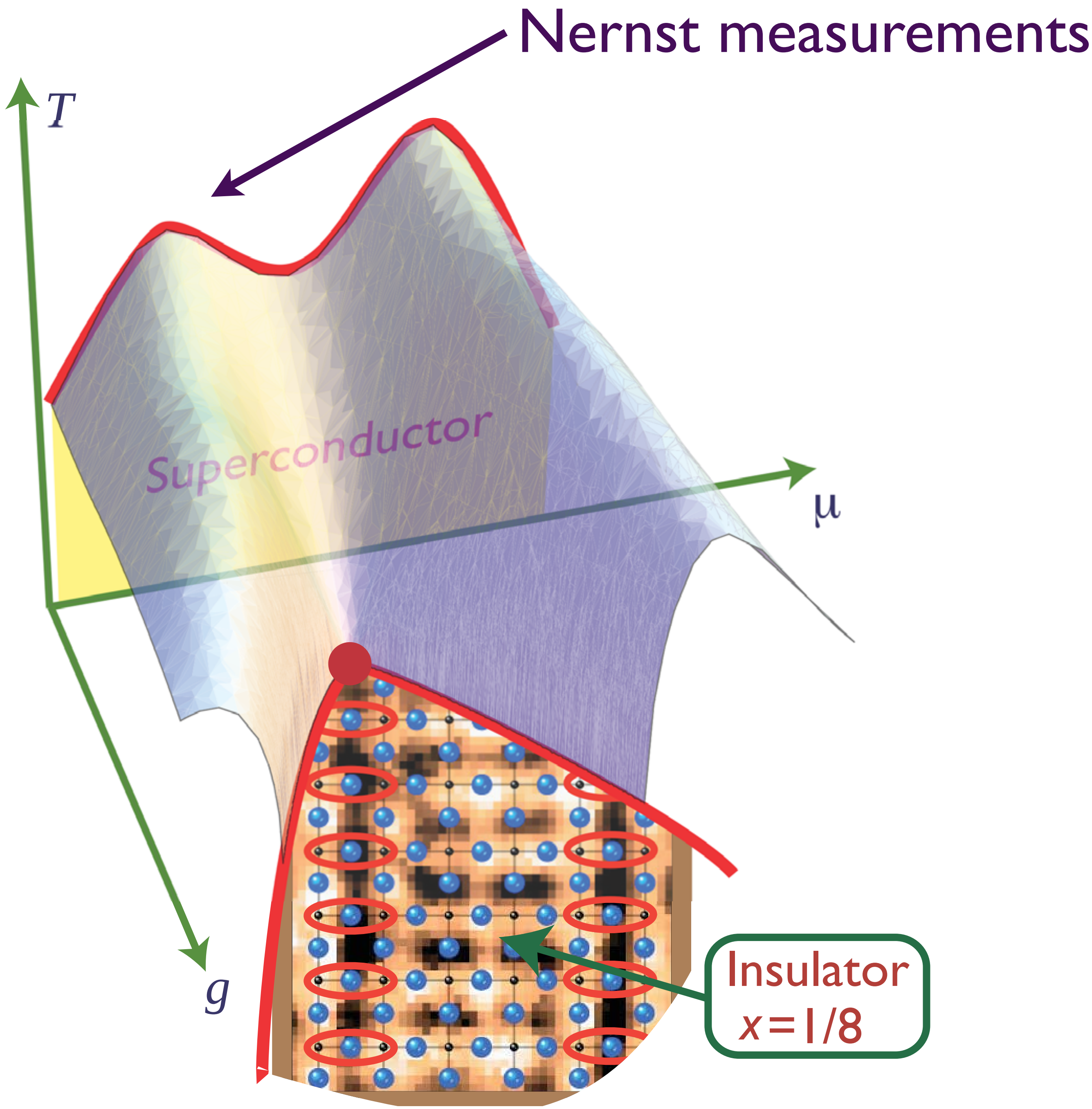
Nernst experiment





Use coupling g to induce a transition to a VBS insulator



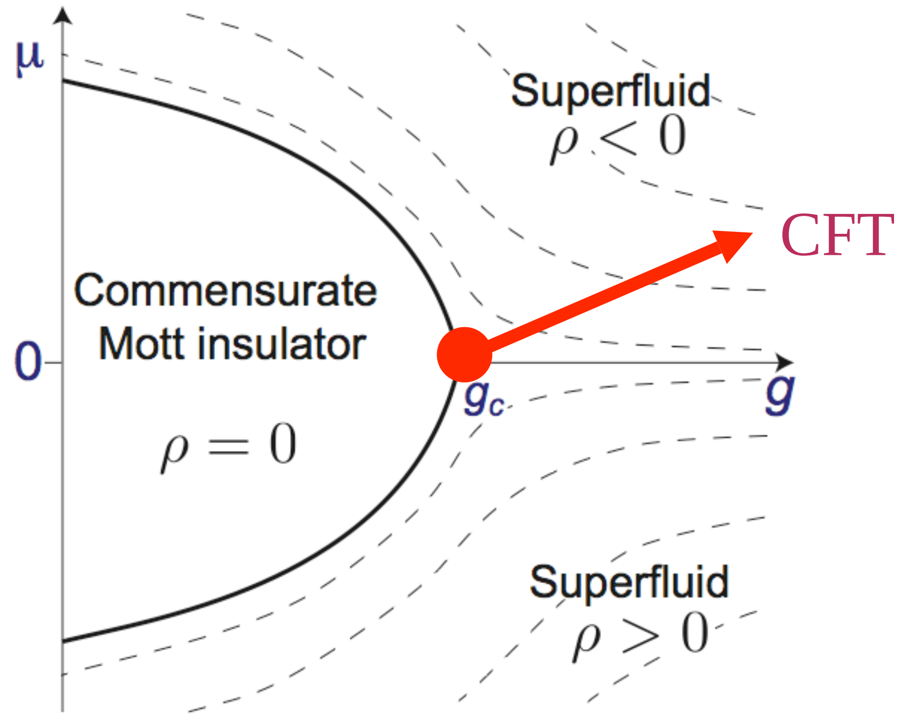

Nernst measurements
T
Superconductor
μ
g
Insulator
x=1/8

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field *B*

In the gravity dual theory, these perturbations correspond to electric and magnetic charges on the black hole



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left|(\vec{\nabla} - i\vec{A})\psi\right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

In the hydrodynamic regime, $\hbar\omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.
The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^{μ}, the current,
- ρ, the local number density,
- ε, the local energy density,
- P, the local pressure,
- u^{μ}, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \end{aligned}$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu
\end{aligned}
$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda \right) + \mu \frac{\partial_\mu T}{T} \right]
\end{aligned}$$

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

For experimental applications, we must move away from the ideal CFT

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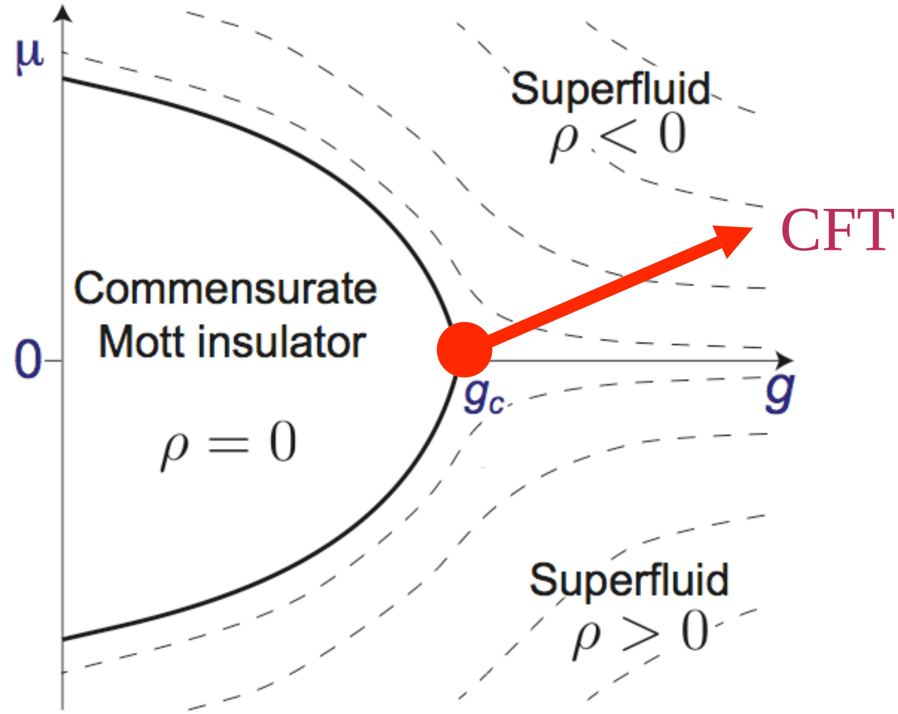

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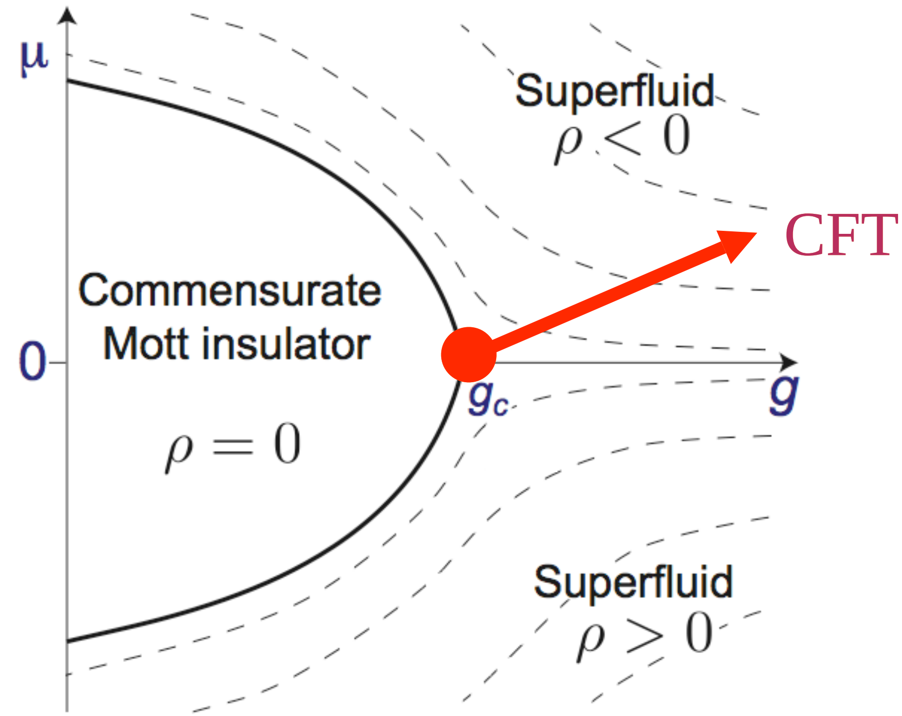
$$\nabla \times \vec{A} = B$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field B

- An impurity scattering rate $1/\tau_{imp}$ (its T dependence follows from scaling arguments)



e.g.

$$\mathcal{S} = \int d^2 r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left| (\vec{\nabla} - i\vec{A})\psi \right|^2 - g|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\rm imp}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} \left(\delta^\mu_\nu + u^\mu u_\nu\right) T^{\nu\gamma} u_\gamma \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q \left(g^{\mu\nu} + u^\mu u^\nu\right) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda\right) + \mu \frac{\partial_\mu T}{T} \right]
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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\rm imp})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\rm imp})}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2} \right] .$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\text{imp}})} \quad \text{as } B \to 0$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Hall conductivity

$$\begin{aligned} \sigma_{xy} &= -\frac{2e\rho c}{B}\left[\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\rm imp}}{(\omega + i\gamma + i/\tau_{\rm imp})^2 - \omega_c^2}\right] \\ &= B\left[\sigma_Q \frac{4e\rho v^2}{(\varepsilon + P)(1/\tau_{\rm imp} - i\omega)} + \frac{8e^3\rho^3 v^4}{(\varepsilon + P)^2(1/\tau_{\rm imp} - i\omega)^2}\right] \\ & \qquad\qquad \text{as } B \to 0 \end{aligned}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right) \left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\rm imp})}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right] \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\rm imp})}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right] \end{aligned}$$

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$$\kappa_{xx} = \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \quad \boxed{\rightarrow 1 \text{ as } B \rightarrow 0}$$

$$= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]$$

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S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\text{imp}}}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right]$$

$$\frac{k_B}{2e} = 43.086 \mu\text{V}/\text{K}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Transverse thermoelectric co-efficient

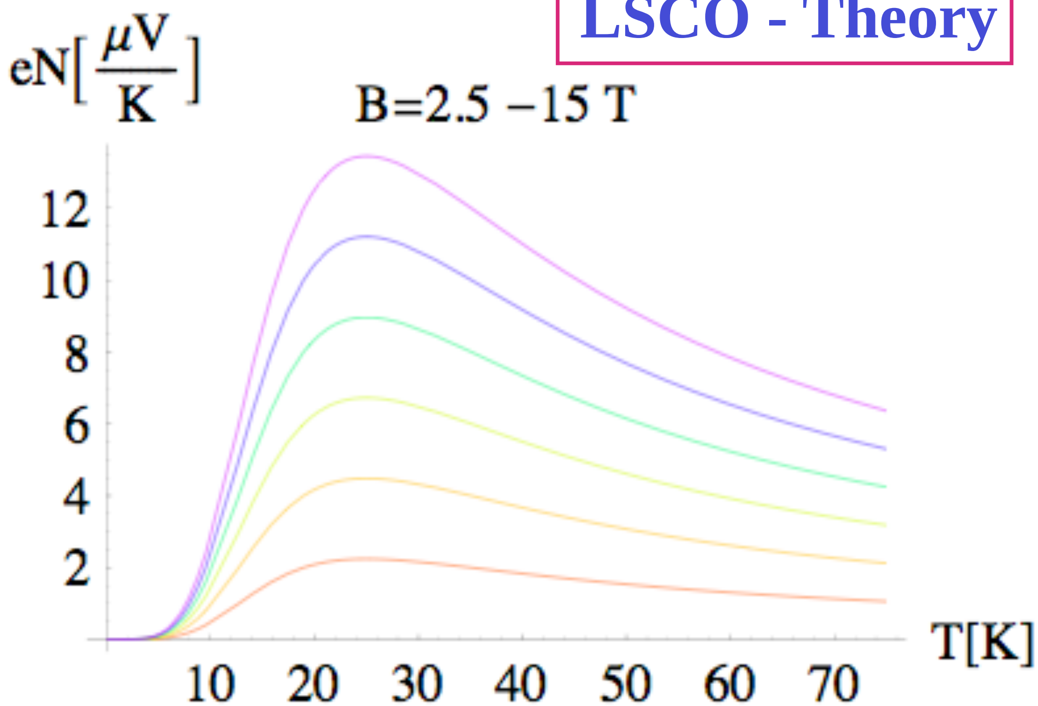
$$\left(\frac{h}{2ek_B}\right)\alpha_{xy} = \Phi_s \overline{B}\,(k_B T)^2 \left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2 \frac{\overline{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P}(k_B T)^3\,\hbar/2\pi\tau_{\rm imp}}{\Phi_{\varepsilon+P}^2(k_B T)^6 + \overline{B}^2\overline{\rho}^2(2\pi\tau_{\rm imp}/\hbar)^2},$$

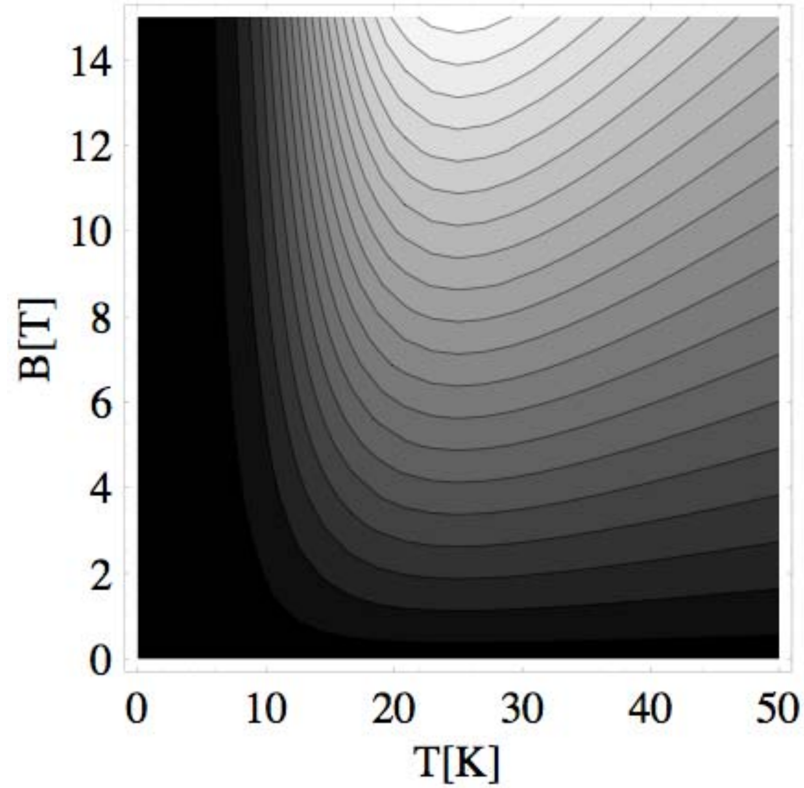
where

$$B = \overline{B}\phi_0/(\hbar v)^2 \quad ; \quad \rho = \overline{\rho}/(\hbar v)^2.$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

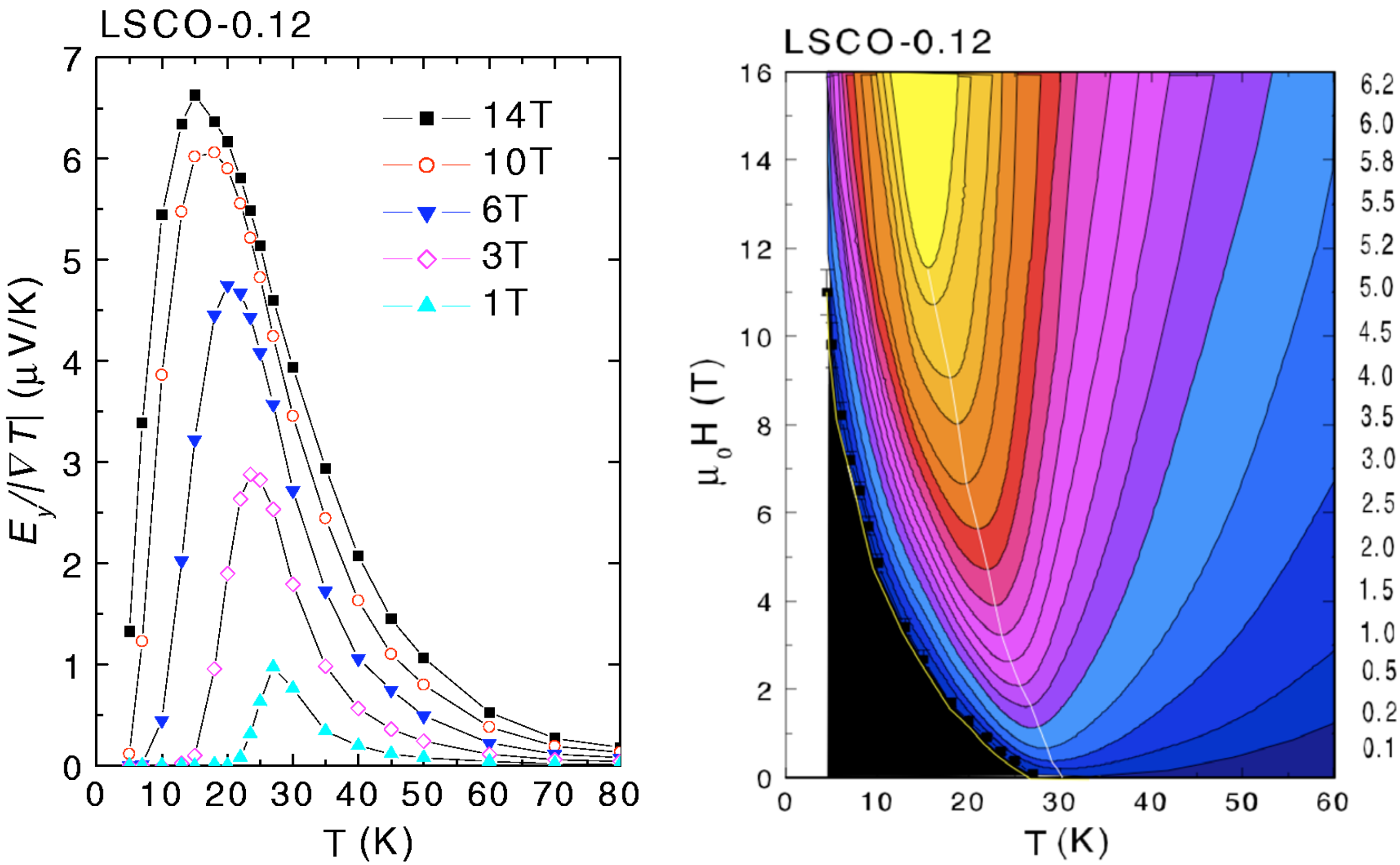
LSCO - Theory





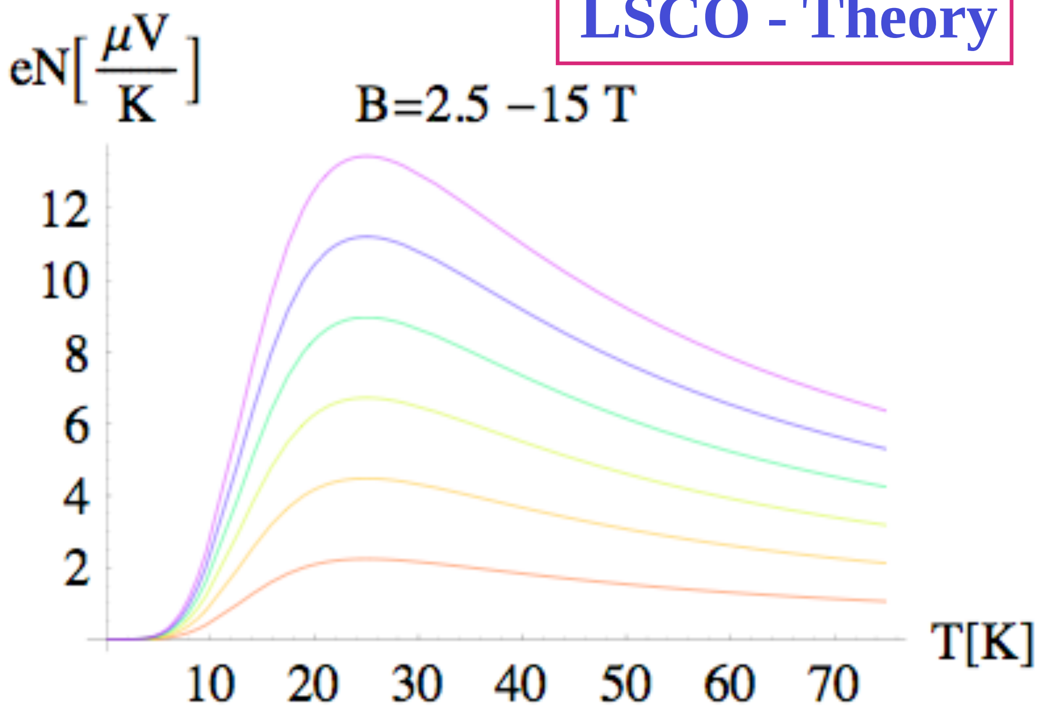
S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

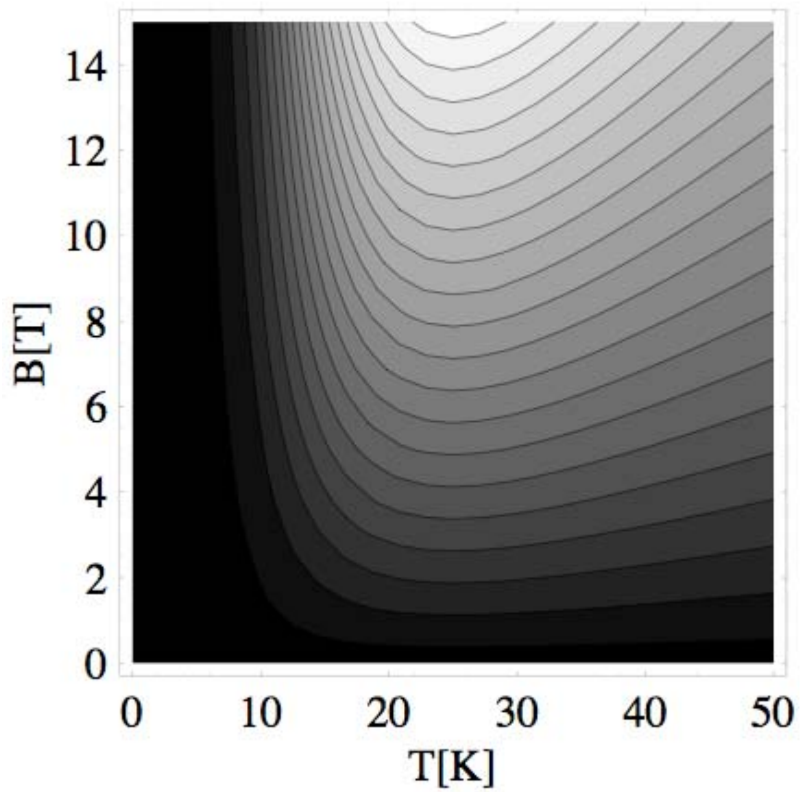
LSCO - Experiments



N. P. Ong *et al.*

LSCO - Theory





Only input parameters

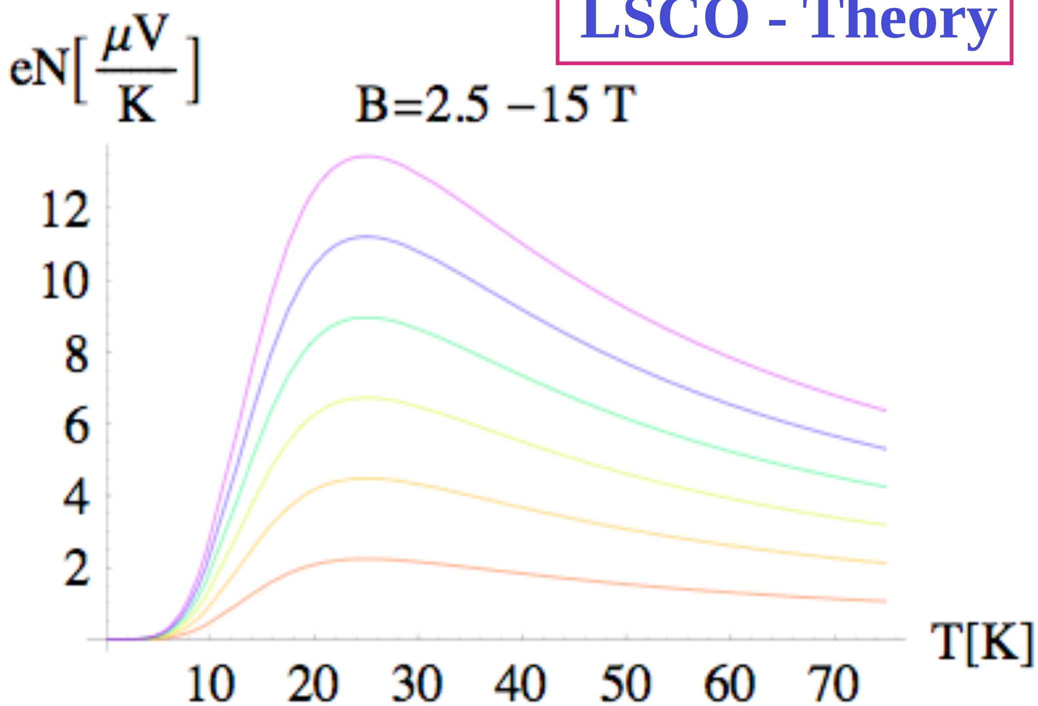
$$\hbar v = 47 \text{ meV Å}$$
$$\tau_{\text{imp}} \approx 10^{-12} \text{ s}$$

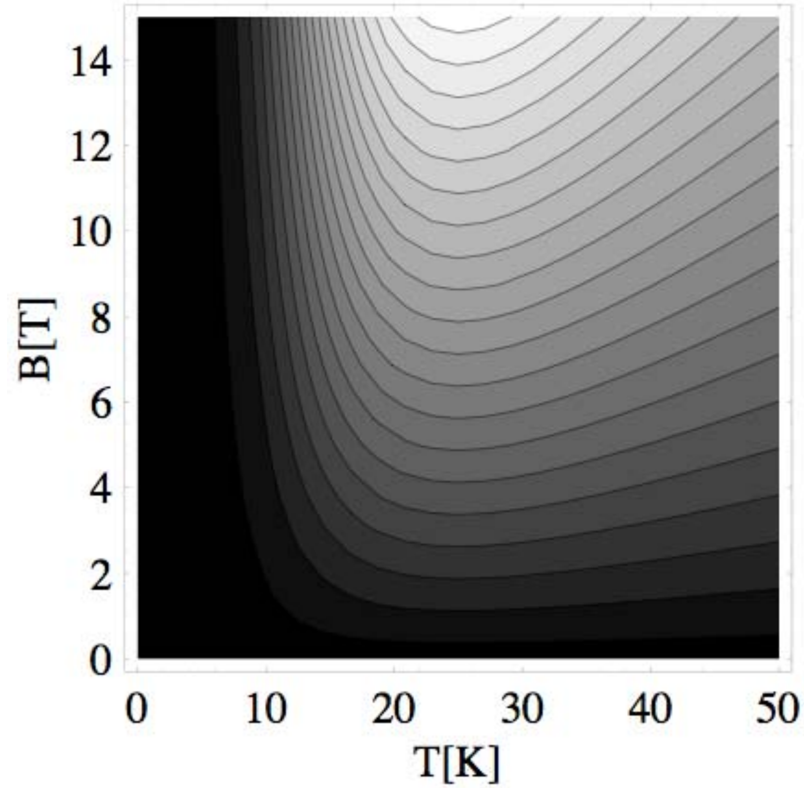
Output

$$\omega_c = 6.2\text{GHz} \cdot \frac{B}{1\text{T}} \left(\frac{35\text{K}}{T}\right)^3$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

LSCO - Theory





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Similar to velocity estimates by
A.V. Balatsky and Z-X. Shen, *Science* **284**, 1137 (1999).

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential μ
- A magnetic field B

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in *precise* accord with *all* hydrodynamic results presented earlier

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

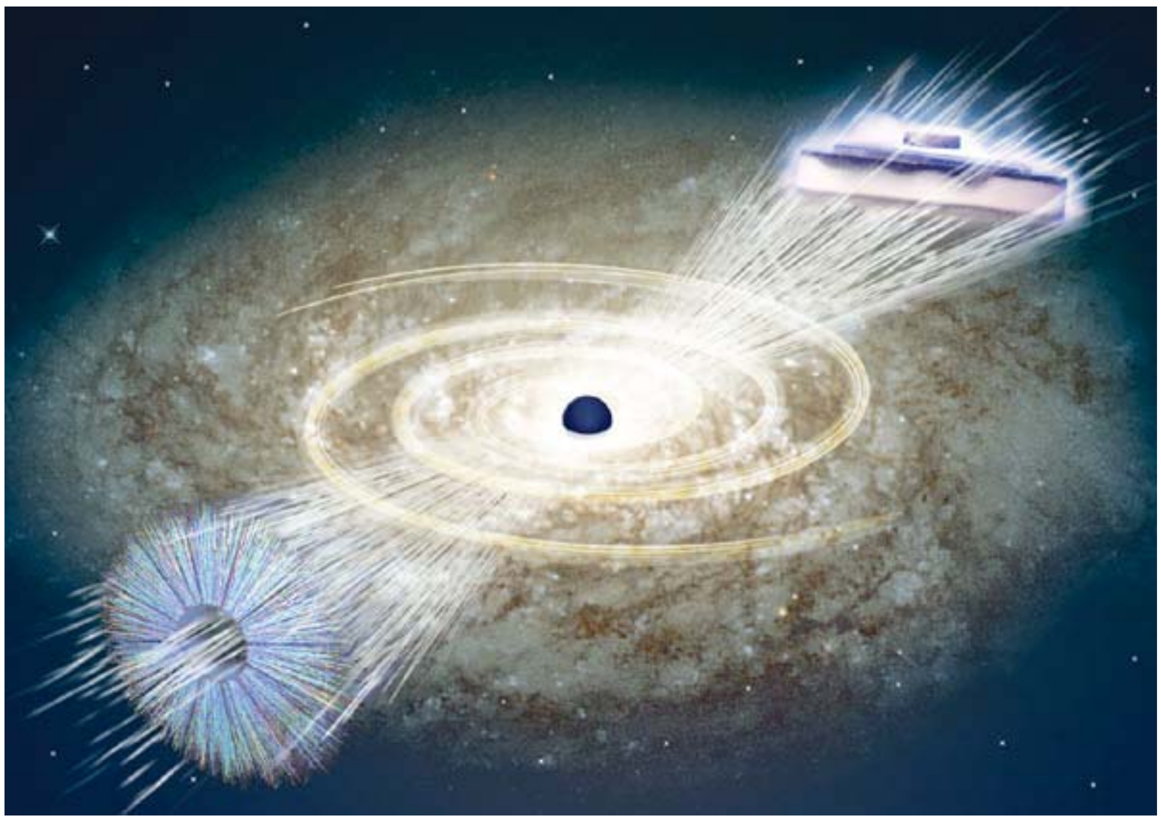
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007



Conclusions

- Condensed matter systems realize several interesting CFT3s.
- Collisionless-to-hydrodynamic crossover in CFT3s at T>0.
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.