Planckian metals and Deconfined quantum criticality

CIFAR Quantum Materials at CCQ
Flatiron Institute, New York, February 11, 2020

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PHYSICS
HARVARD

1. Resonant SYK models
and Planckian metals

2. Deconfined quantum criticality of
random *t*-*J* models

The complex SYK model

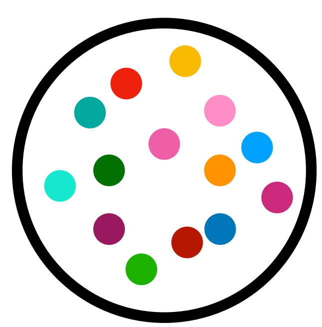
$$H = \frac{1}{(2N)^{3/2}} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta} \, c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta + e \sum_\alpha c_\alpha^\dagger c_\alpha$$

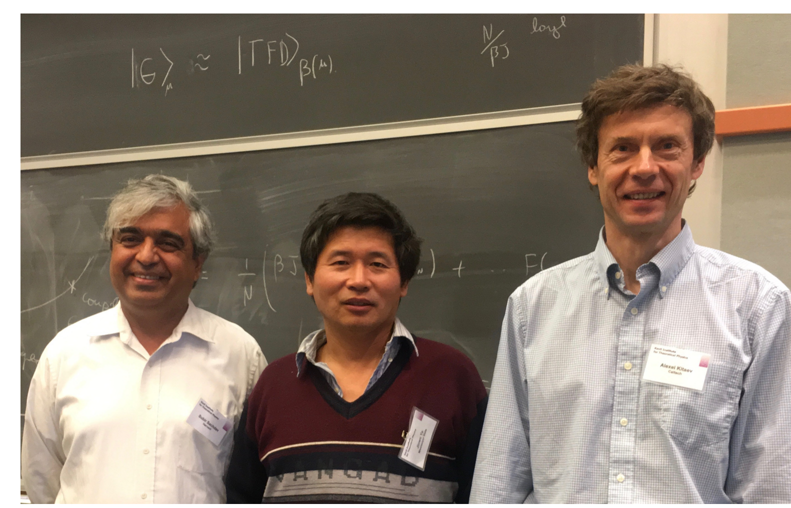
$$c_\alpha c_\beta + c_\beta c_\alpha = 0 \quad , \quad c_\alpha c_\beta^\dagger + c_\beta^\dagger c_\alpha = \delta_{\alpha\beta}$$

$$\mathcal{Q} = \frac{1}{N} \sum_\alpha c_\alpha^\dagger c_\alpha$$

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

with $\overline{U_{\alpha\beta;\gamma\delta}} = 0$ and $\overline{|U_{\alpha\beta;\gamma\delta}|^2} = U^2$





S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

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Random interactions

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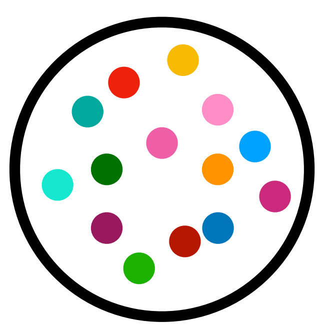

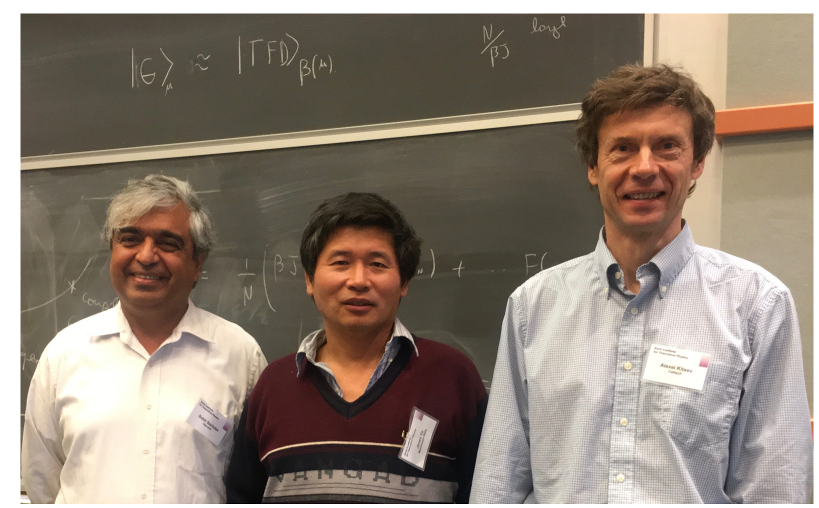

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Random interactions

Flat band

$U_{\alpha\beta;\gamma\delta}$ are independent random variables

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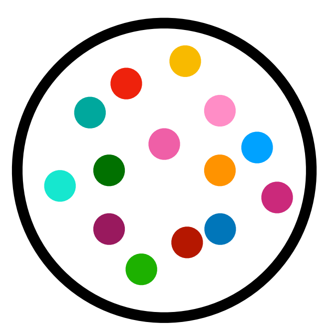

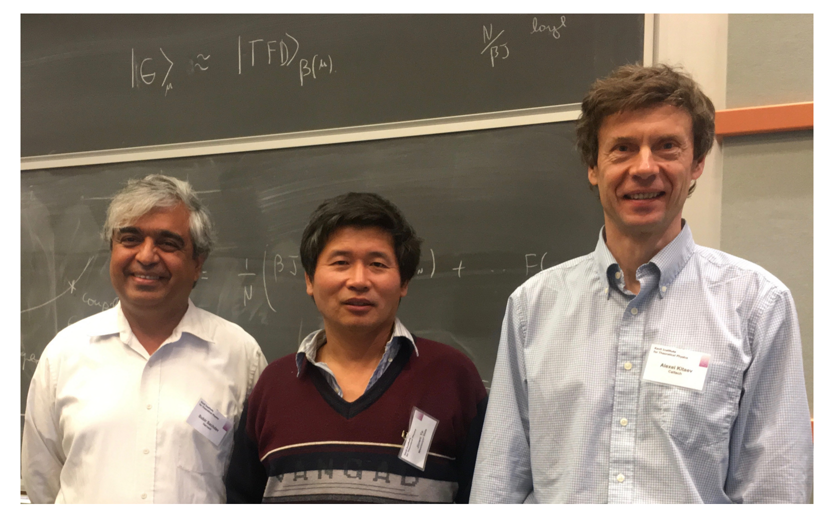

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The complex SYK model

There is a one-parameter family
of critical solutions with varying e/U,
yielding different $0 < \mathcal{Q} < 1$.

For long (imaginary) times $\tau > 0$

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle = A e^{-2\pi \mathcal{E} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)} \right)^{1/2}$$

In a Fermi liquid,

$$\left\langle c_i(\tau) c_i^\dagger(0) \right\rangle \sim \frac{T}{\sin(\pi T \tau)}$$

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A. Georges and O. Parcollet
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Determines the particle-hole asymmetry, and $\mathcal{E} = \mathbb{C} e/U$, with $\mathbb{C} = 0.41$ from a numerical solution.

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Generalized SYK models

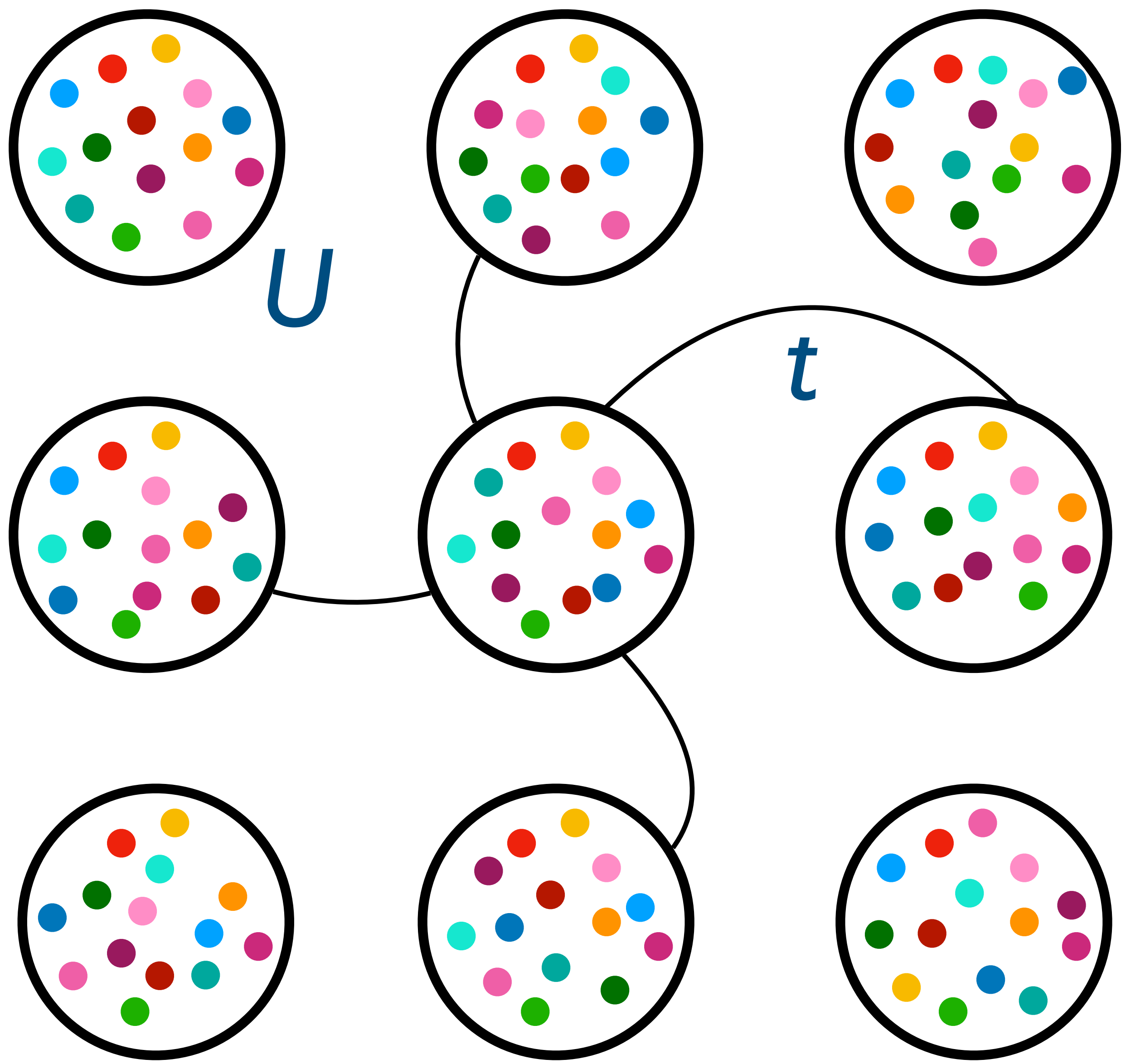
$$H = \frac{1}{(2N)^{3/2}} \sum_i \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{i,\alpha\beta;\gamma\delta}\, c^\dagger_{i\alpha} c^\dagger_{i\beta} c_{i\gamma} c_{i\delta} - t \sum_{\langle ij\rangle} \sum_{\alpha} c^\dagger_{i\alpha} c_{j\alpha}$$

Choose $U \gg t$ on-site, and independent between sites; yields 'incoherent metal' with no Fermi surface for $t^2/U \ll k_B T \ll U$ with

$$G(\mathbf{k},\omega) = G_{\rm SYK}(e, \hbar\omega/(k_B T))$$

independent of $\mathbf{k}$.
There is linear-in-T resistivity but only with bad metal behavior with $\rho > h/e^2$, and co-efficient dependent upon U:

$$\rho \sim \frac{h}{e^2}\frac{k_B T}{t^2/U}$$



Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017); Pengfei Zhang, PRB **96**, 205138 (2017); Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018); Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

See also Antoine Georges and Olivier Parcollet PRB **59**, 5341 (1999); Yingfei Gu, Xiao-Liang Qi, D. Stanford, JHEP (2017) 125

Generalized SYK models

$$H = \frac{1}{(2N)^{3/2}} \sum_{k_a} \sum_{\alpha,\beta,\gamma,\delta=1}^{N} U_{\alpha\beta;\gamma\delta}(k_a)\, c^\dagger_{k_1\alpha} c^\dagger_{k_2\beta} c_{k_3\gamma} c_{k_4\delta}$$

$$+ \sum_{k\alpha} e_k c^\dagger_{k\alpha} c_{k\alpha}$$

Dispersive band. e_k defines m^* and Fermi surface

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

e_k has a bandwidth W.

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Dispersive band. e_k defines m^* and Fermi surface

Random interactions uncorrelated in space

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

e_k has a bandwidth W.

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2\Big[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\Big]$$

Generalized SYK models

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Dispersive band. e_k defines m^* and Fermi surface

Random interactions with spatial correlations

$U_{\alpha\beta;\gamma\delta}(k_a)$ is a random function of $\alpha\beta\gamma\delta$

e_k has a bandwidth W.

We examine a model with weaker $W \lesssim U$, but impose a **resonance condition**.

This leads to a solution which obeys the Planckian ansatz as $T \to 0$.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

$$\overline{U(k_1,k_2,k_3,k_4)U^*(k_5,k_6,k_7,k_8)} = U^2\left[\delta(k_1+k_2-k_3-k_4-k_5-k_6+k_7+k_8)\right] \times \left[\delta(e_{k_1}+e_{k_2}-e_{k_3}-e_{k_4}) + \delta(e_{k_5}+e_{k_6}-e_{k_7}-e_{k_8})\right]$$

Green's function of a Planckian metal

Has a 'remnant' Fermi surface at $e_k = 0$, where the spectral function is particle-hole symmetric.

$$G(\mathbf{k}, \omega) \quad = \quad G_{\rm SYK}\left(\frac{e_k}{U}, \frac{\hbar\omega}{k_B T}\right)$$

$$G(\mathbf{k}, \tau) \quad \sim \quad e^{-(e_k/U) 2\pi \mathbb{C} T \tau} \times \left(\frac{T/U}{\sin(\pi T \tau)}\right)^{1/2}$$

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$$G(\mathbf{k},\tau) \sim e^{-(e_k/U)2\pi\mathbb{C}T\tau} \times \left(\frac{T/U}{\sin(\pi T\tau)}\right)^{1/2}$$

Resistivity of a Planckian metal as $T \to 0$

$$\boxed{\rho = \frac{m^*}{ne^2}\, 2.71\mathbb{C}\, \frac{k_B T}{\hbar}}$$

Note that all explicit dependence on U has cancelled out!

Choosing $\mathbb{C} = 0.41$ as in the SYK model, we have the prefactor $2.71\mathbb{C} = 1.11$.

A. A. Patel and S. Sachdev, PRL **123**, 066601 (2019)

Generalized SYK models

- Spin correlations have the 'marginal' form: $\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim 1/|\tau|$.
- No pseudogap phase.
- Resonance condition should have its origin in $T = 0$ quantum criticality.
- No 'Mottness': on-site Hubbard U is missing.

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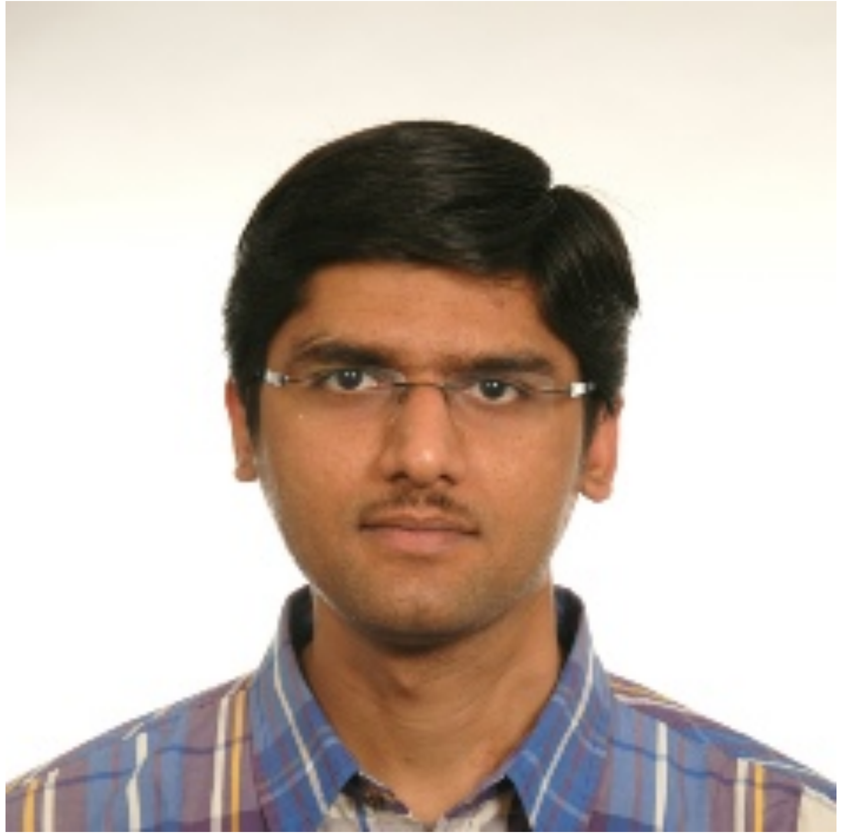
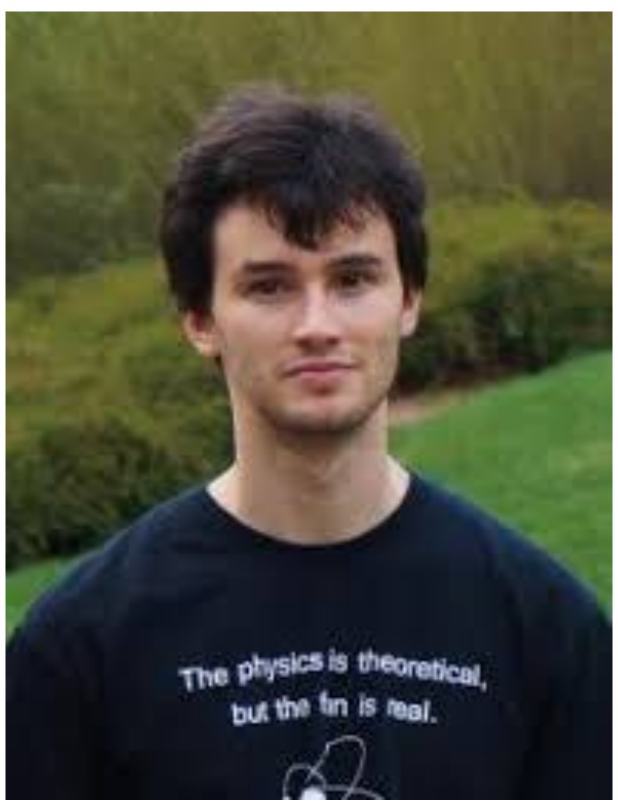


Darshan Joshi

Chenyuan Li

arXiv:1912.08822



Grigory Tarnopolsky

Antoine Georges

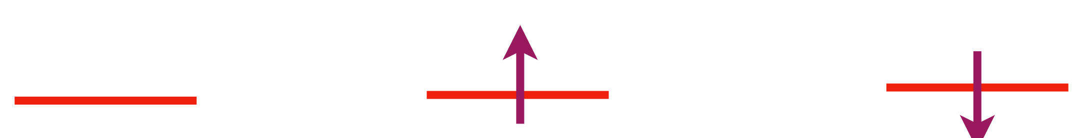
t-J model

$$H = \sum_{i,j=1}^{N} t_{ij}\, c^{\dagger}_{i\alpha} c_{j\alpha} + \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c^{\dagger}_{j\beta}\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c^{\dagger}_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c^{\dagger}_{i\alpha} c_{i\alpha} = 1 - p$$



$|0\rangle$ $\quad c^{\dagger}_{\uparrow}|0\rangle$ $\quad c^{\dagger}_{\downarrow}|0\rangle$

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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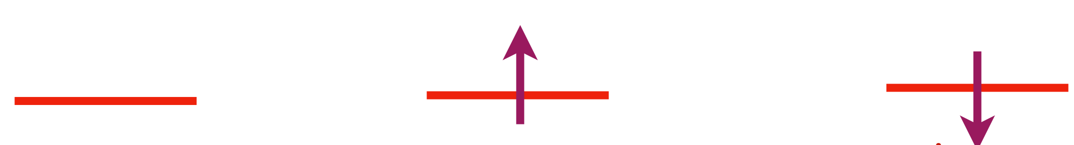
$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$

$$t_{ij} \text{ random}, \ \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$



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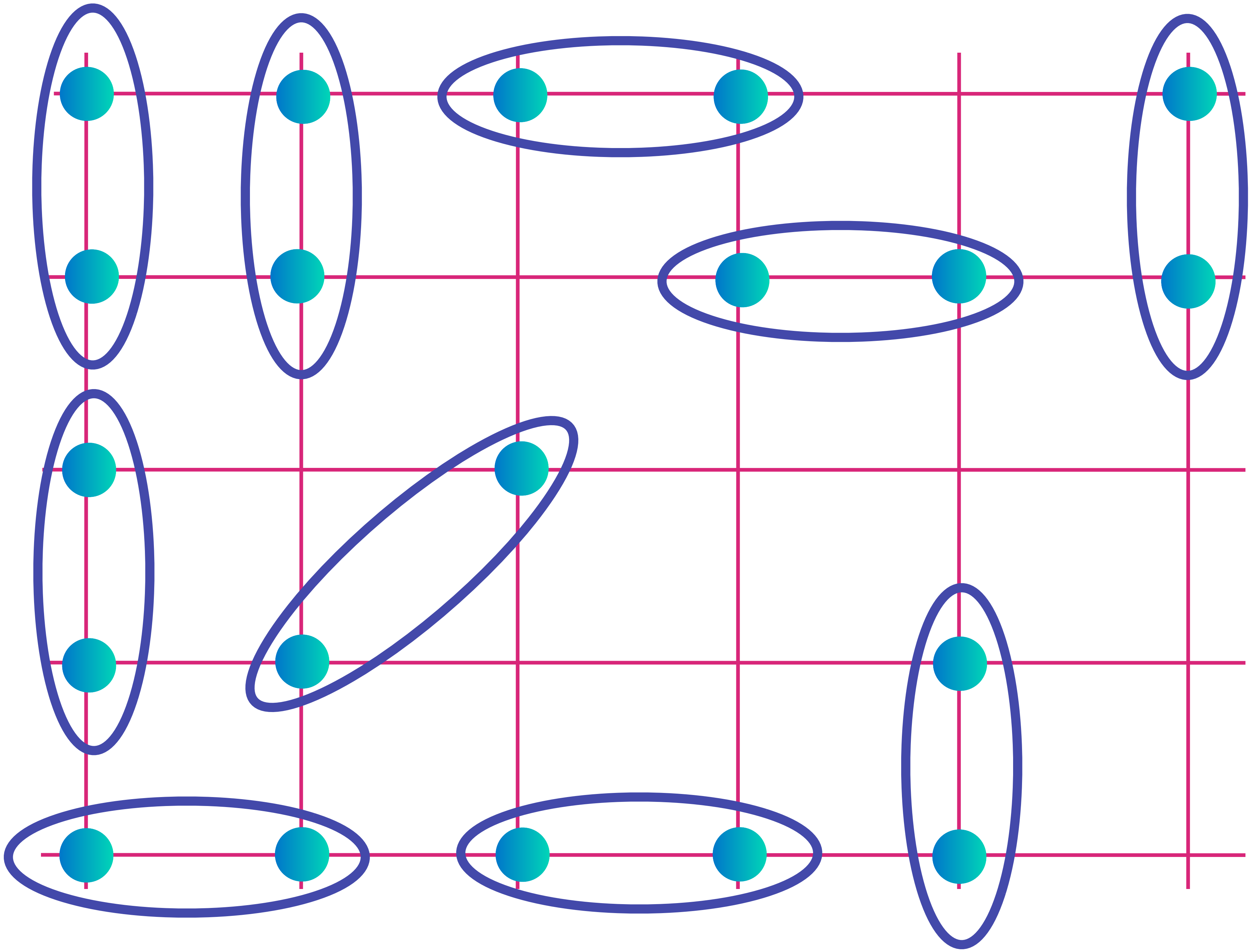
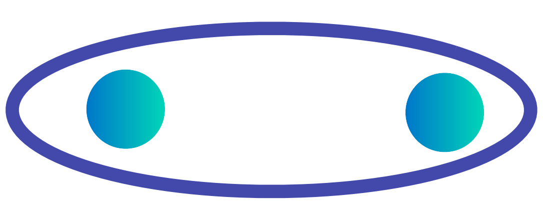
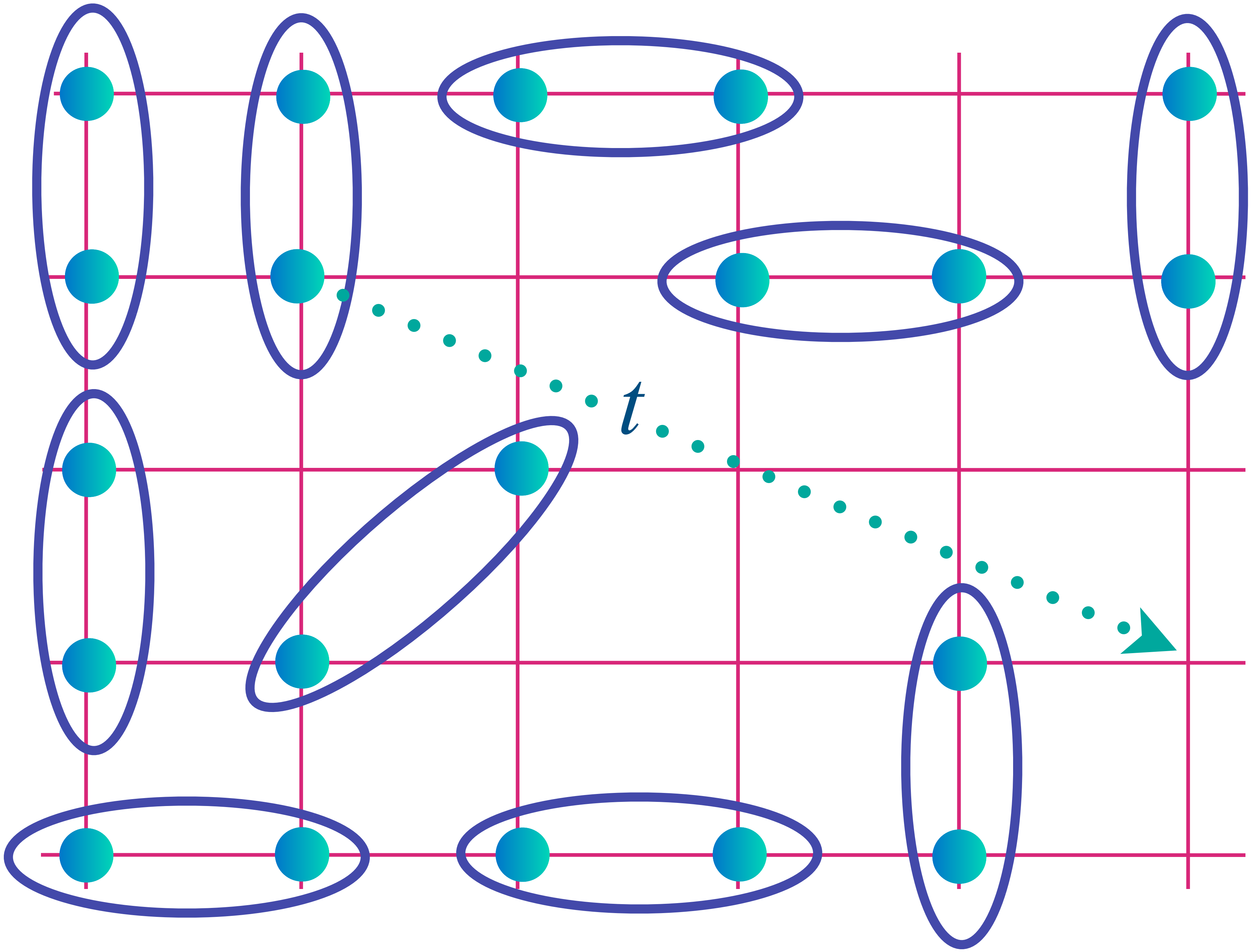


Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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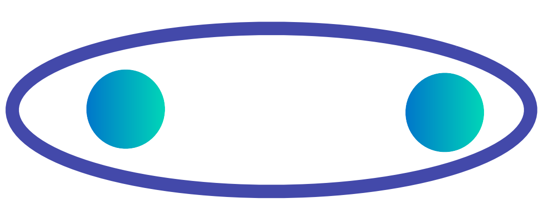

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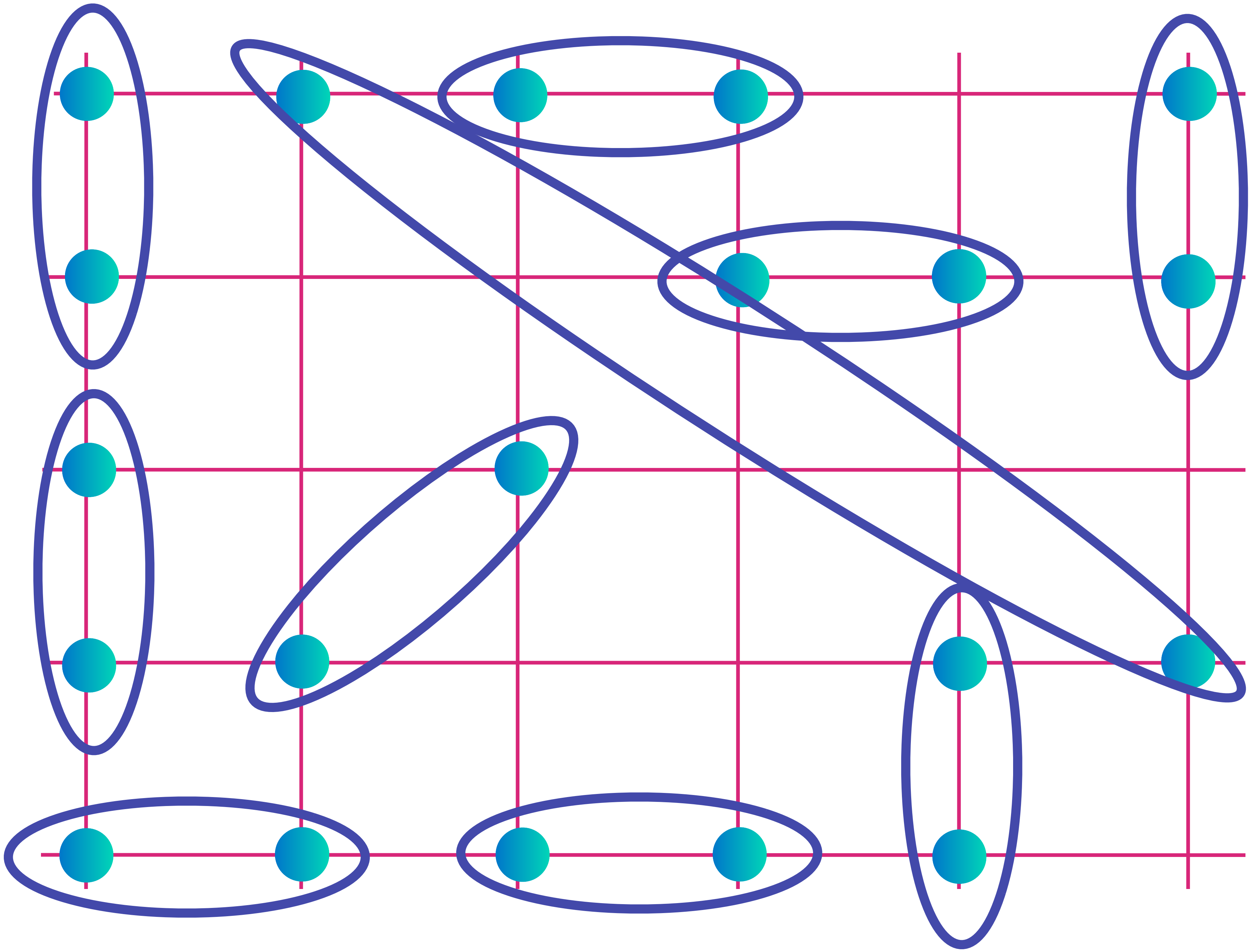

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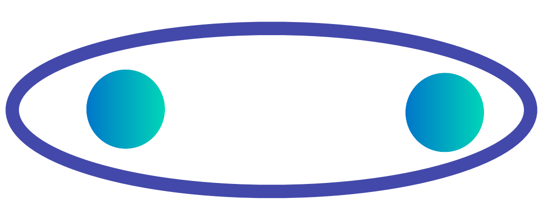

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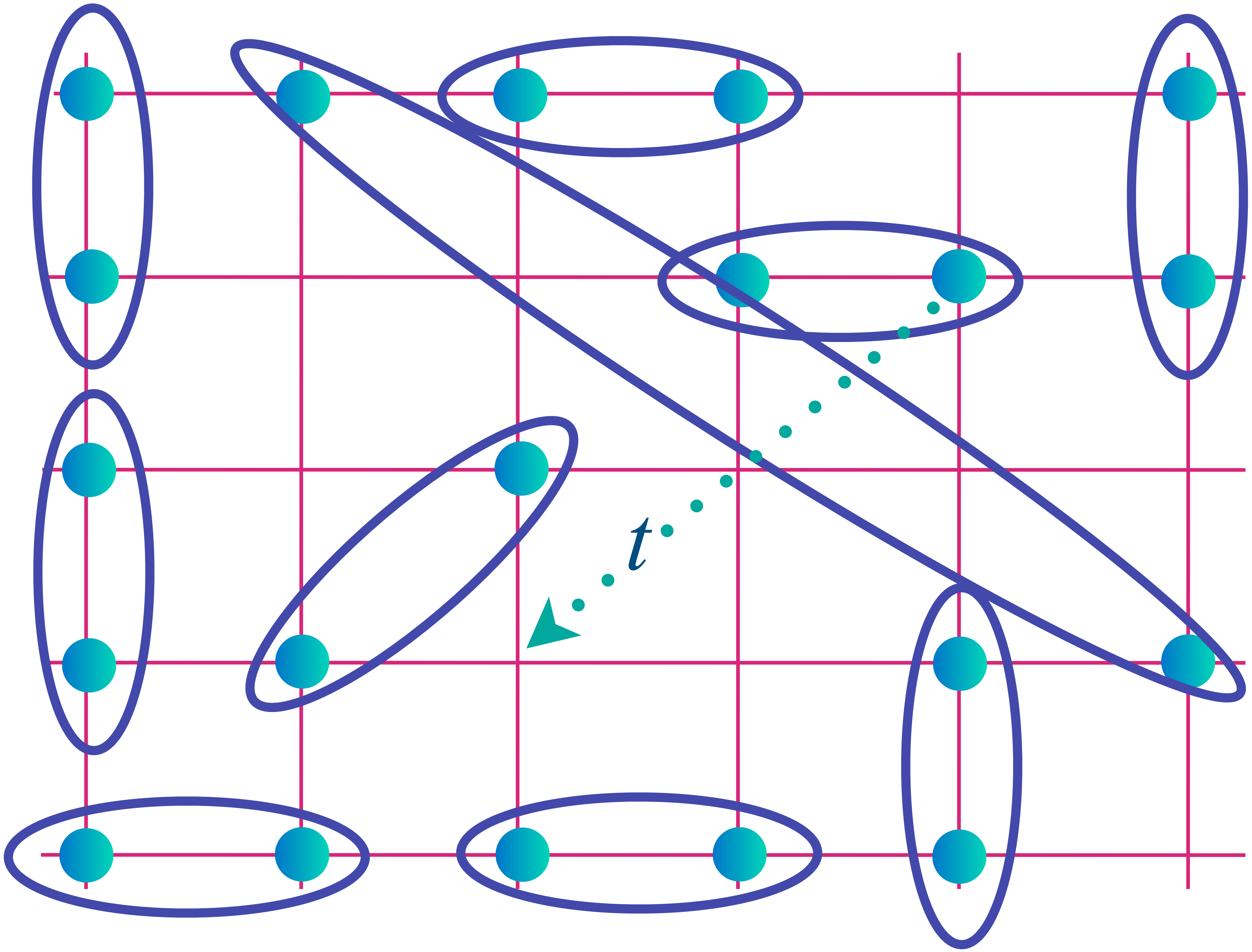

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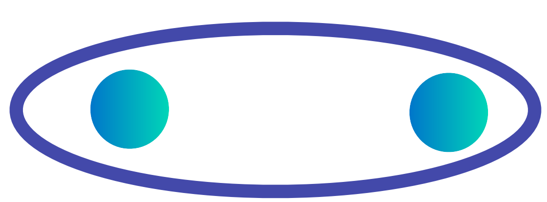

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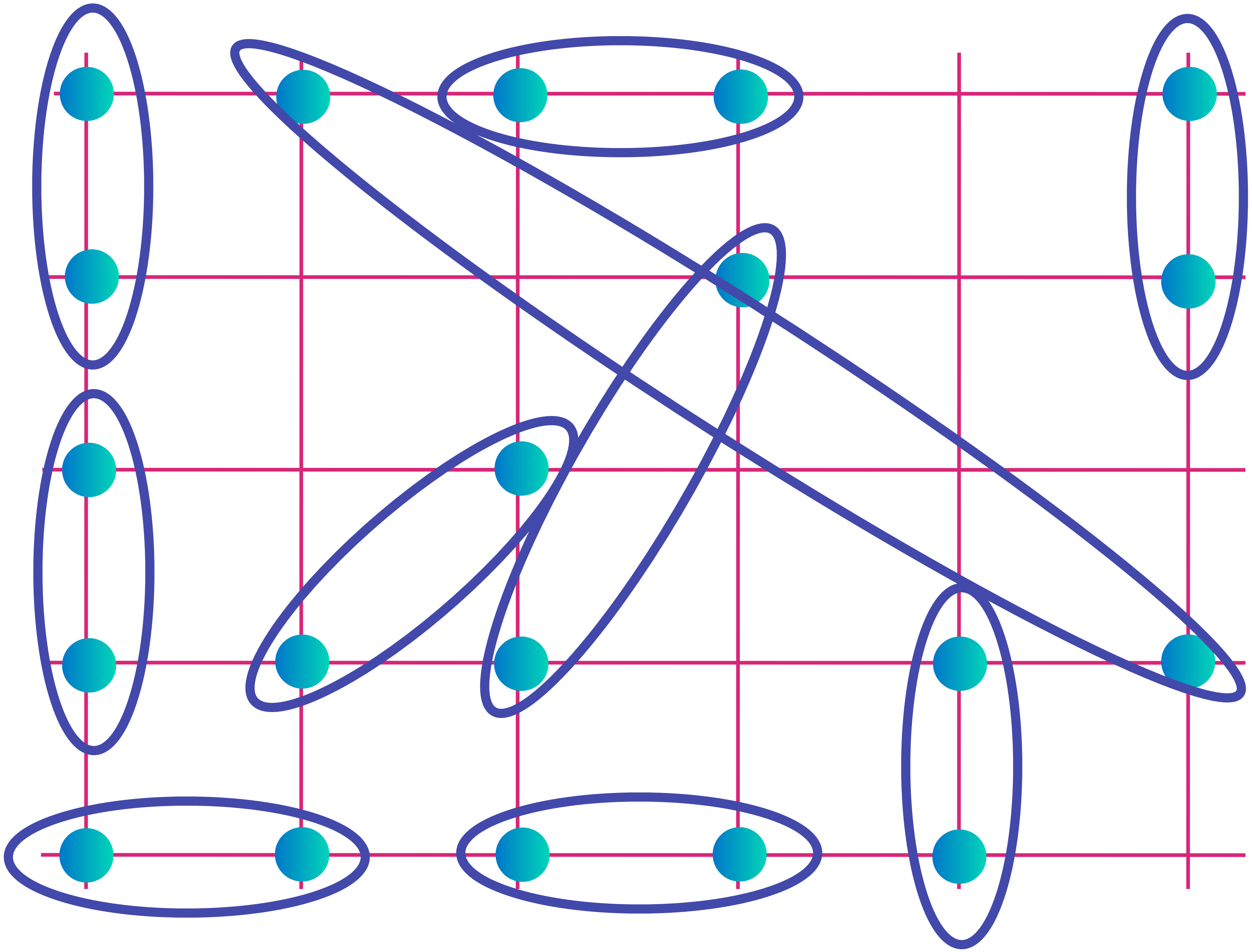

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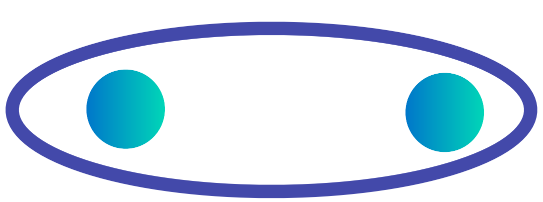
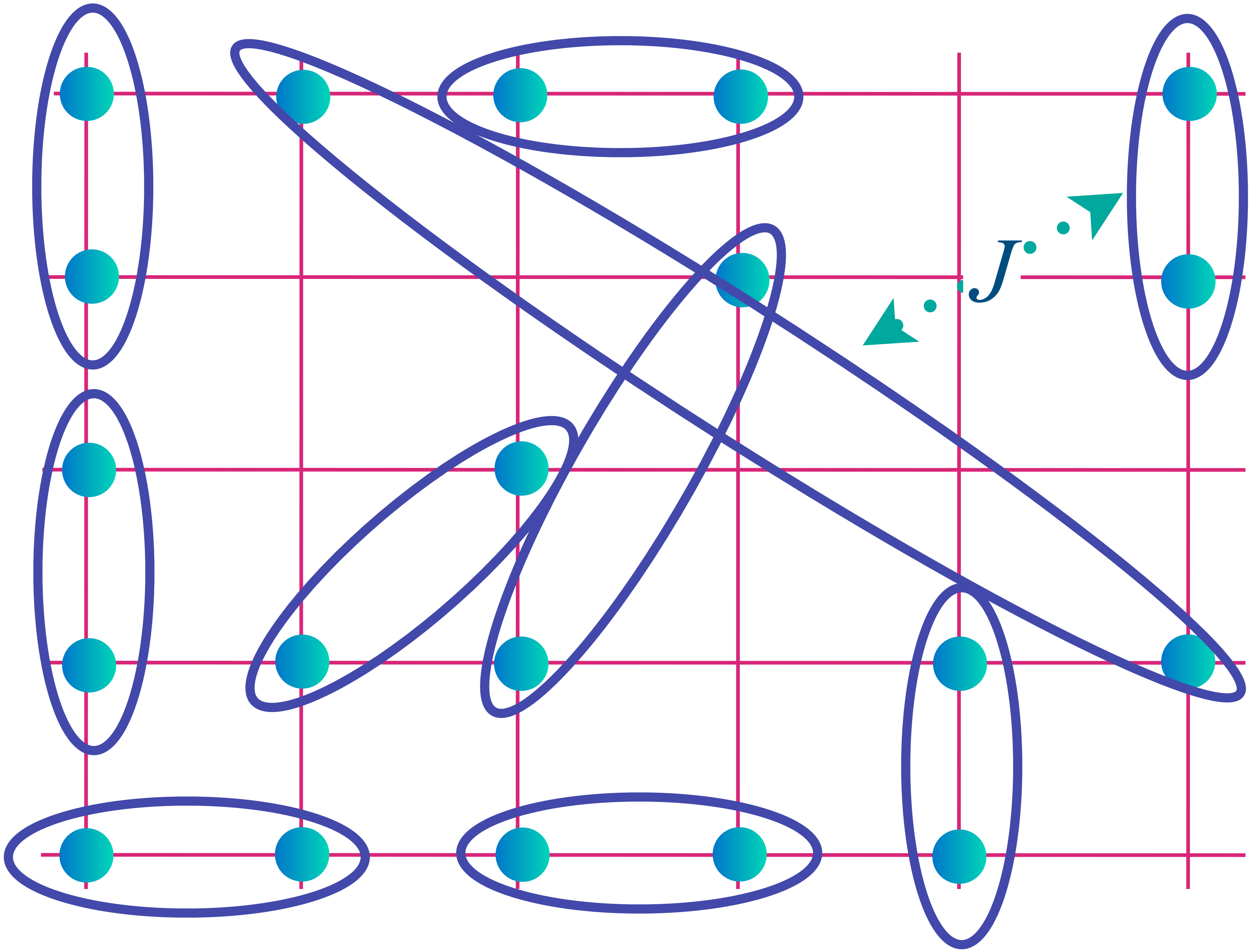
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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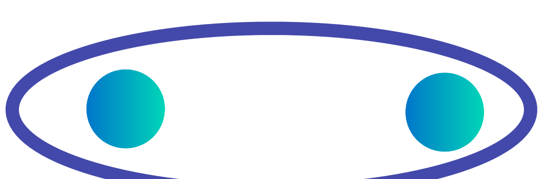

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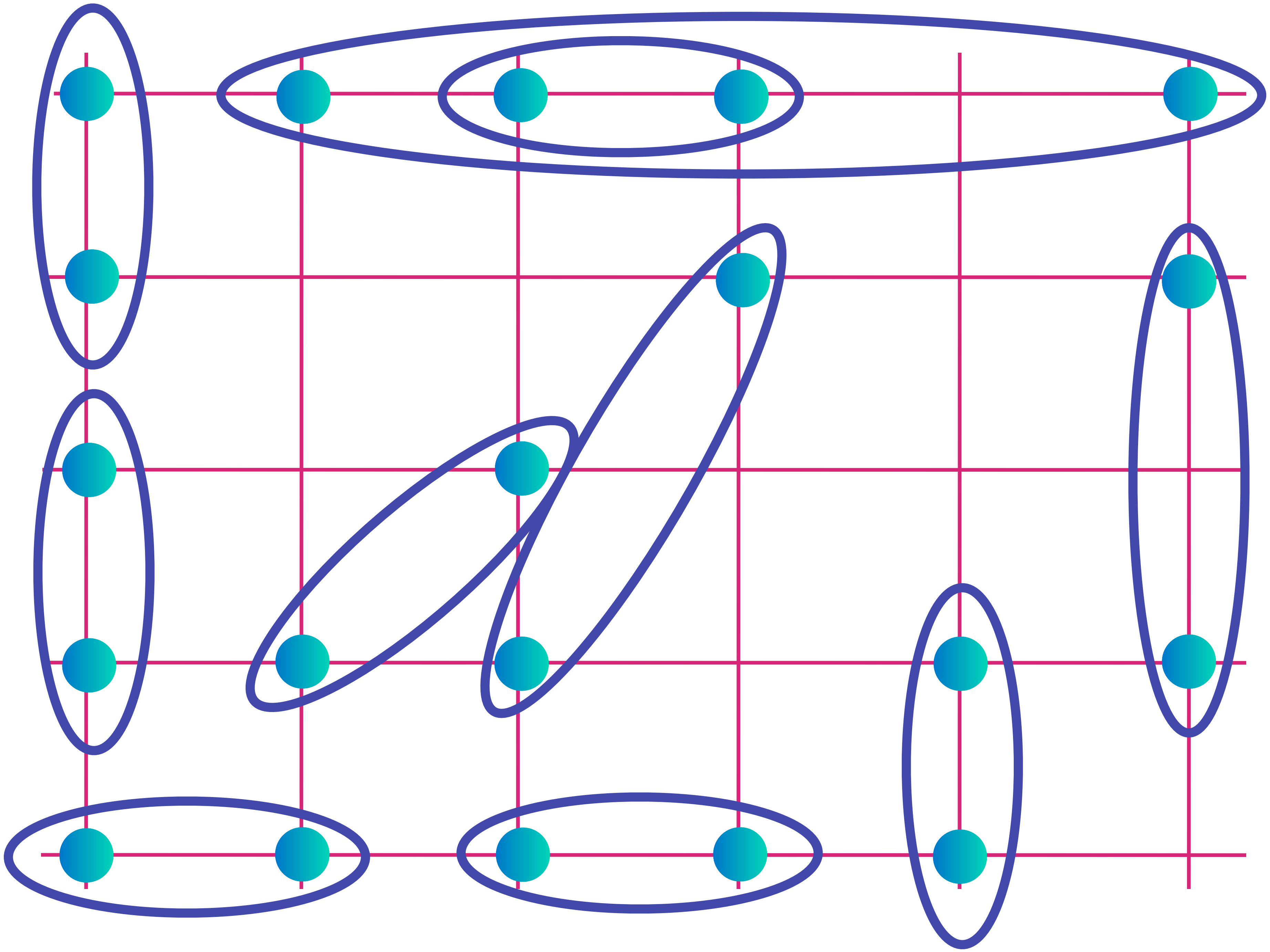

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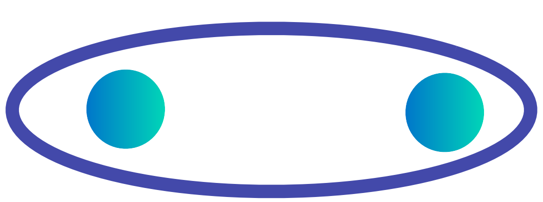

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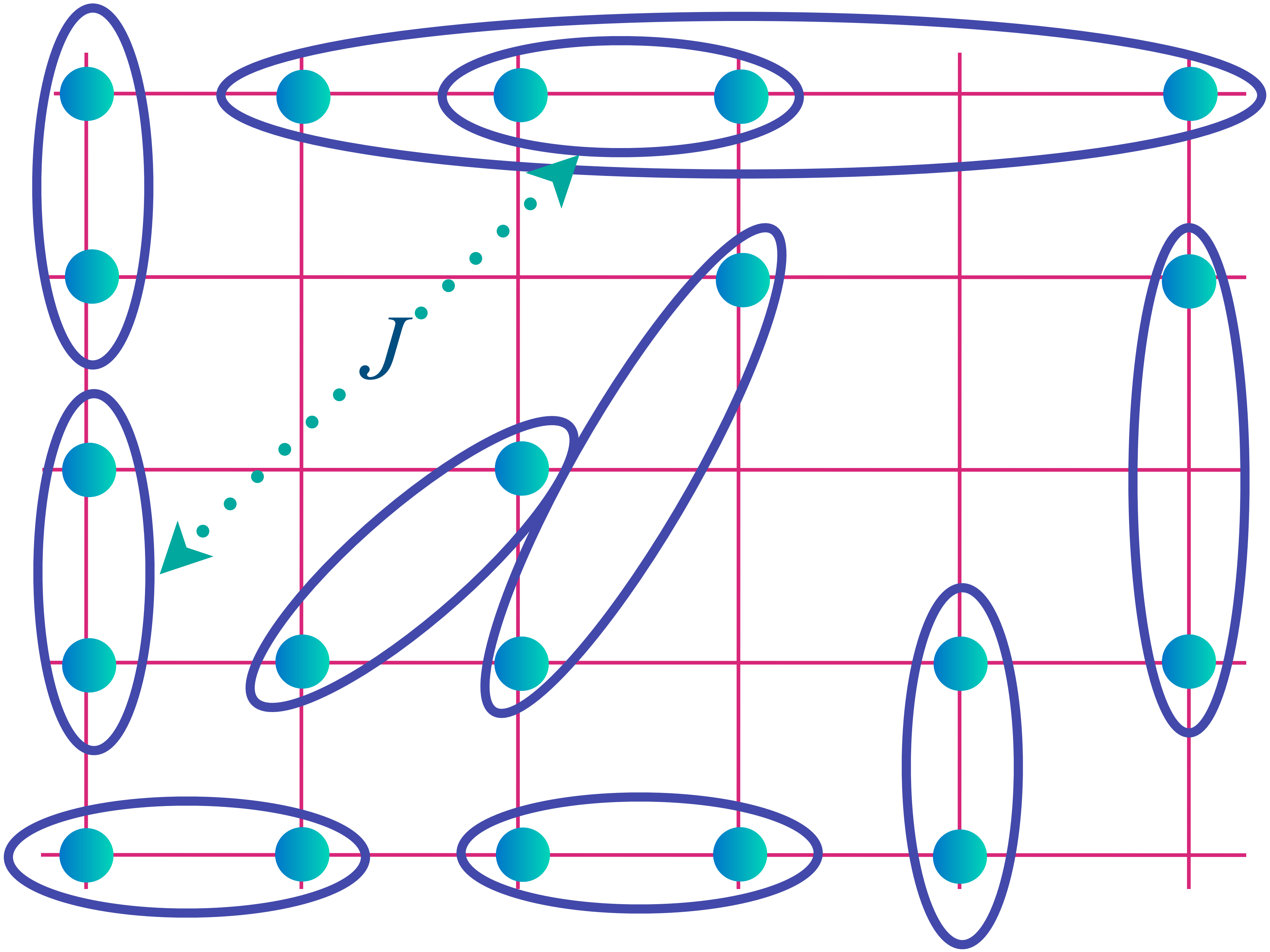

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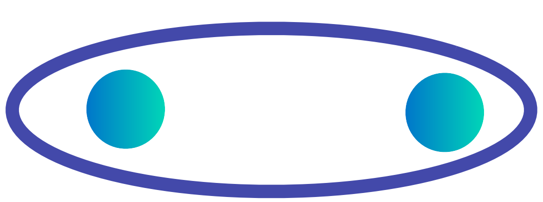
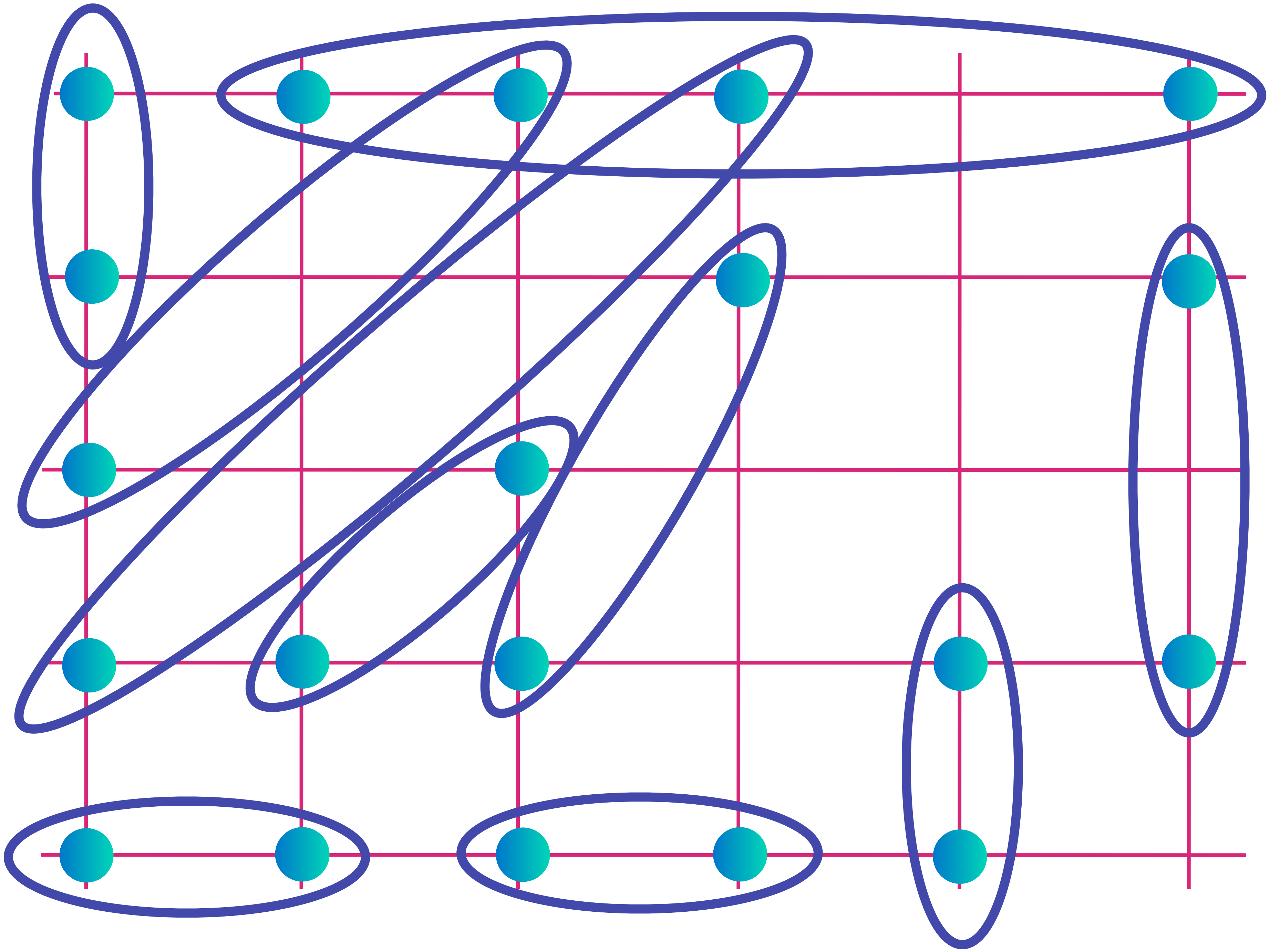
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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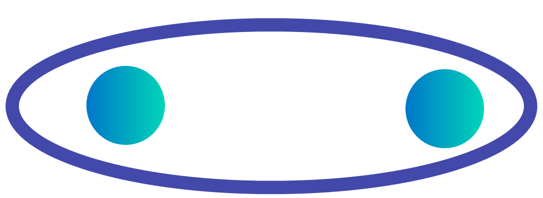

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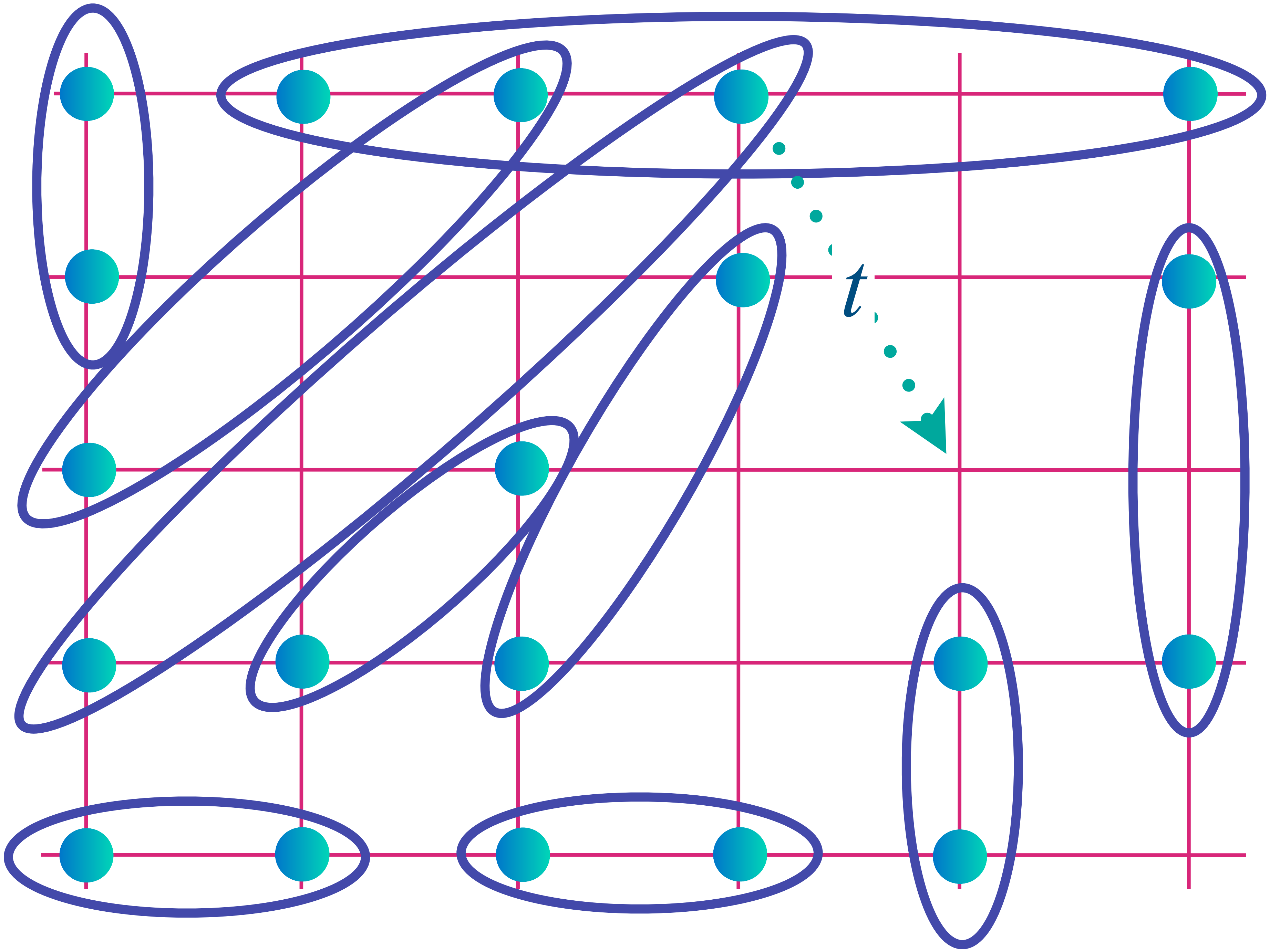

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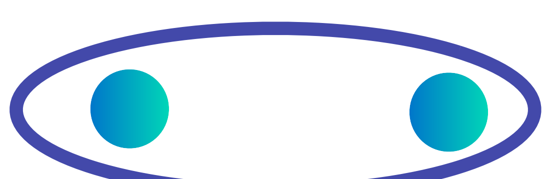
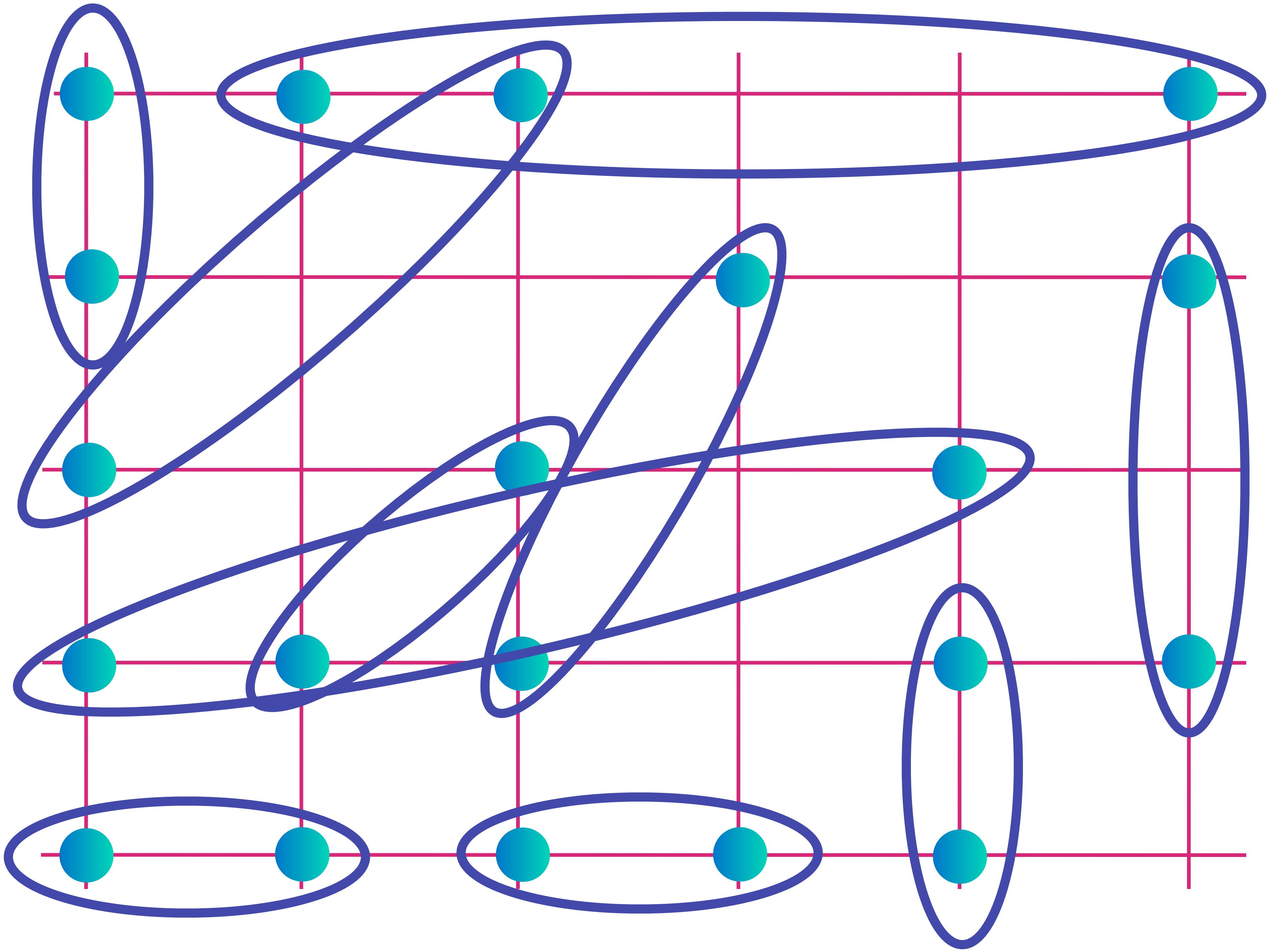
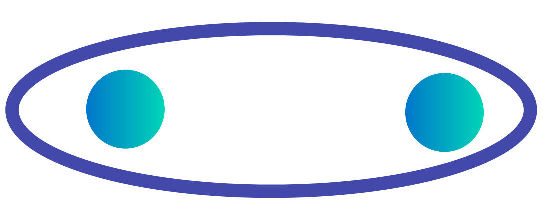
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

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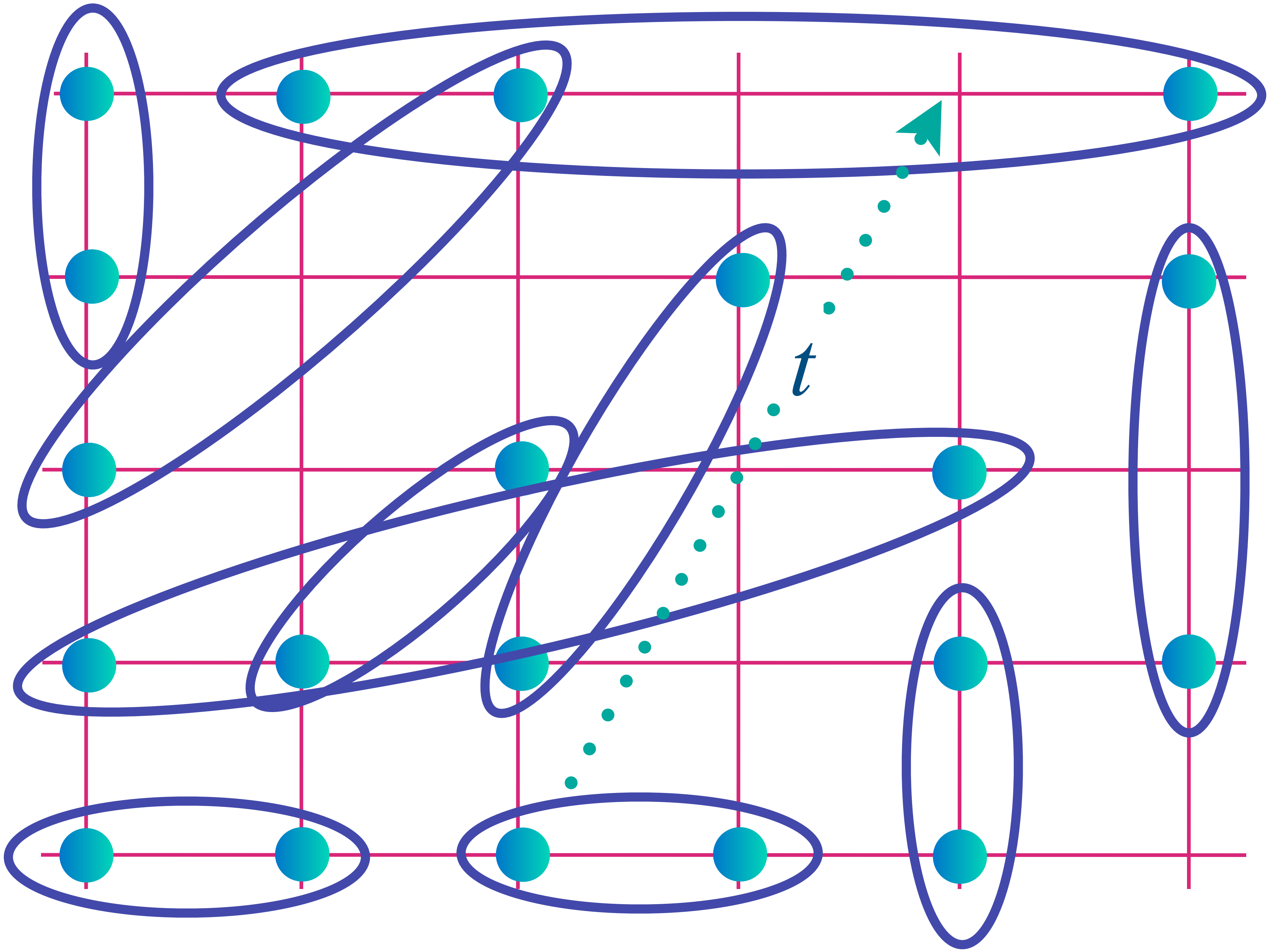

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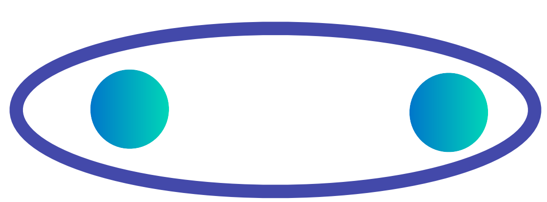
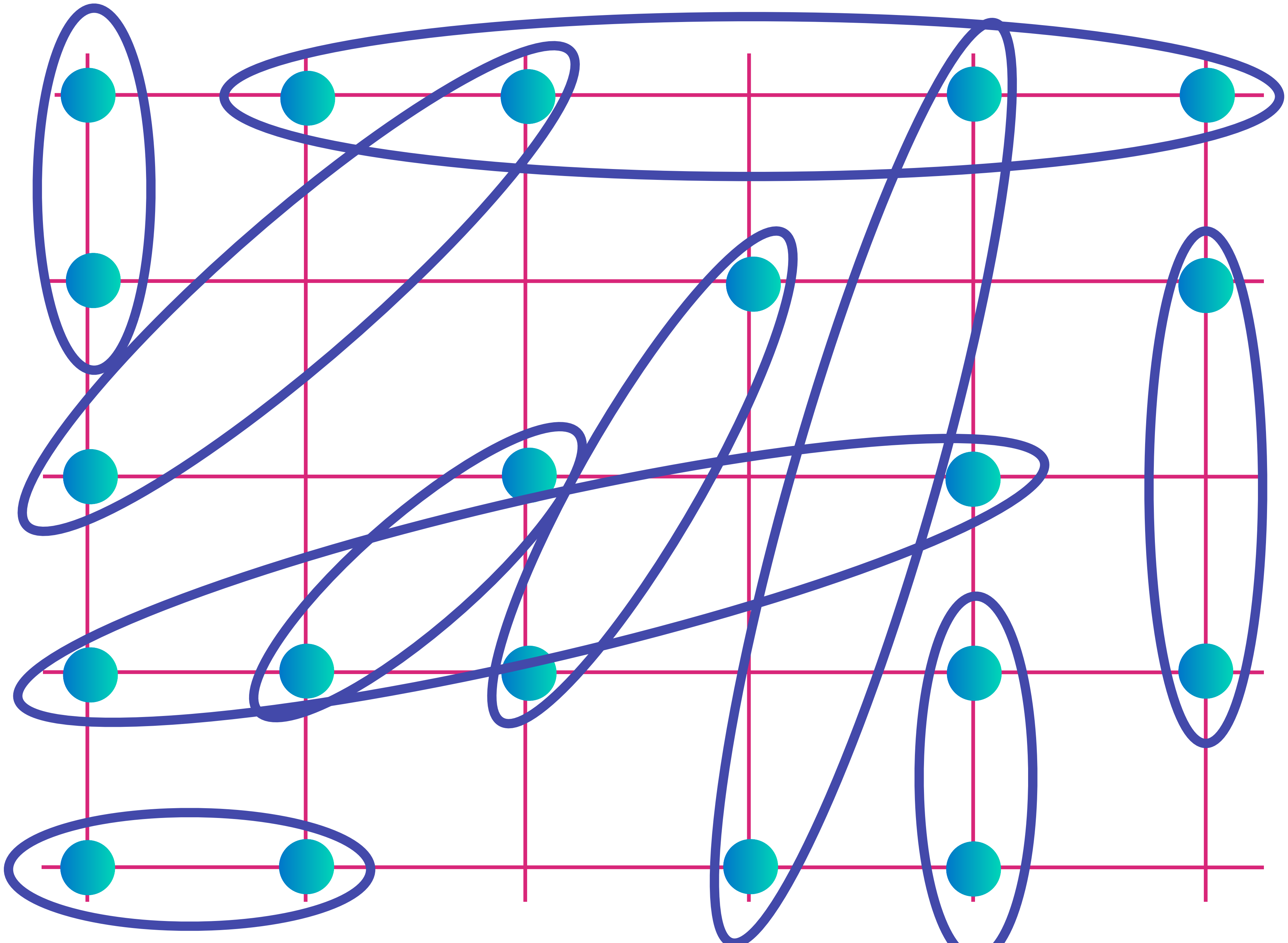
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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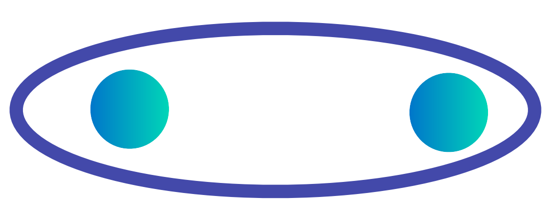
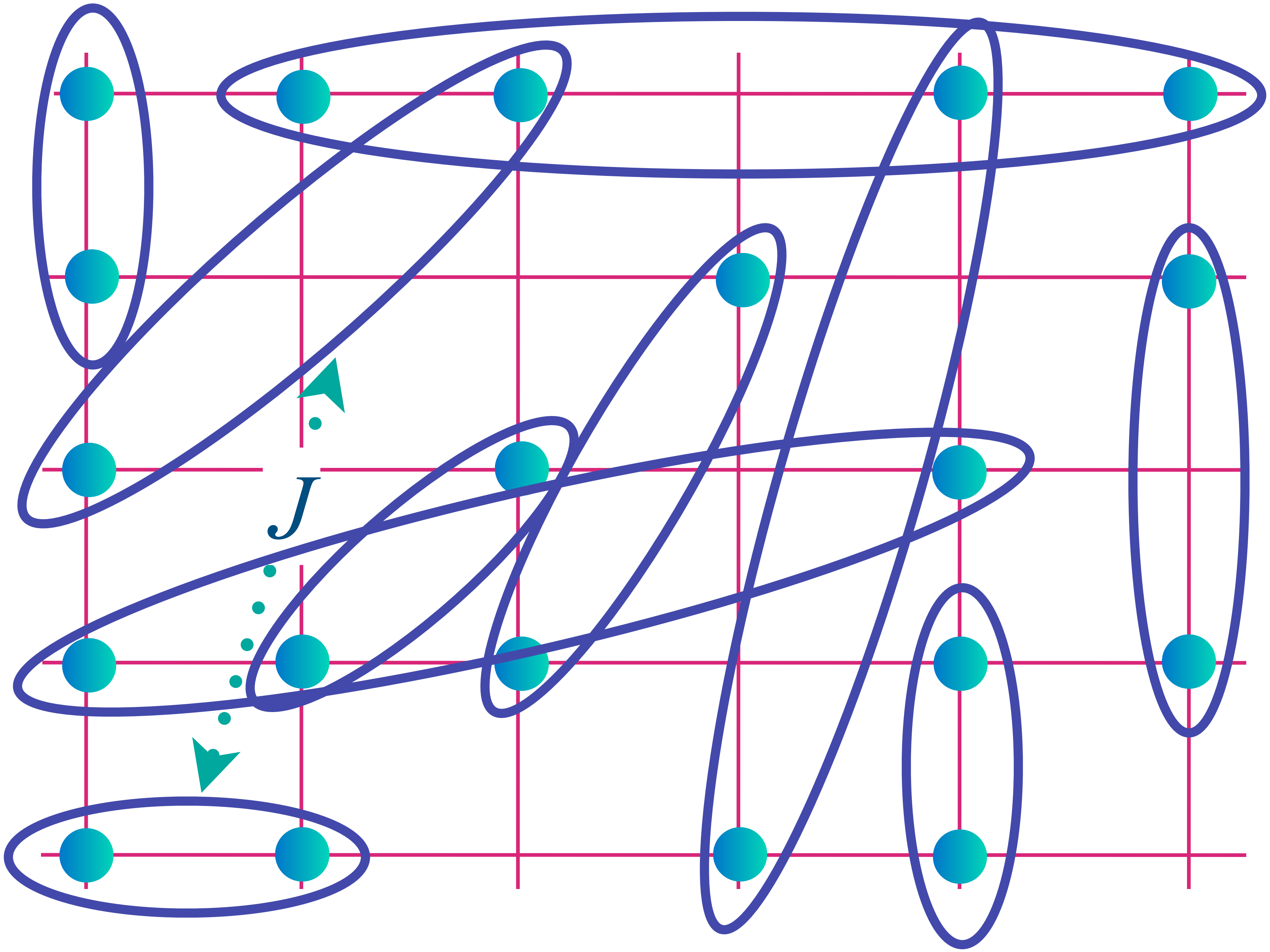
 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



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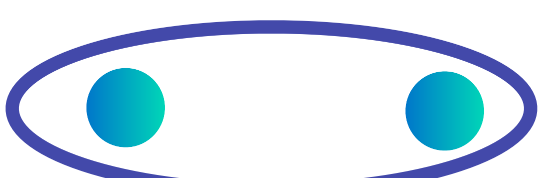
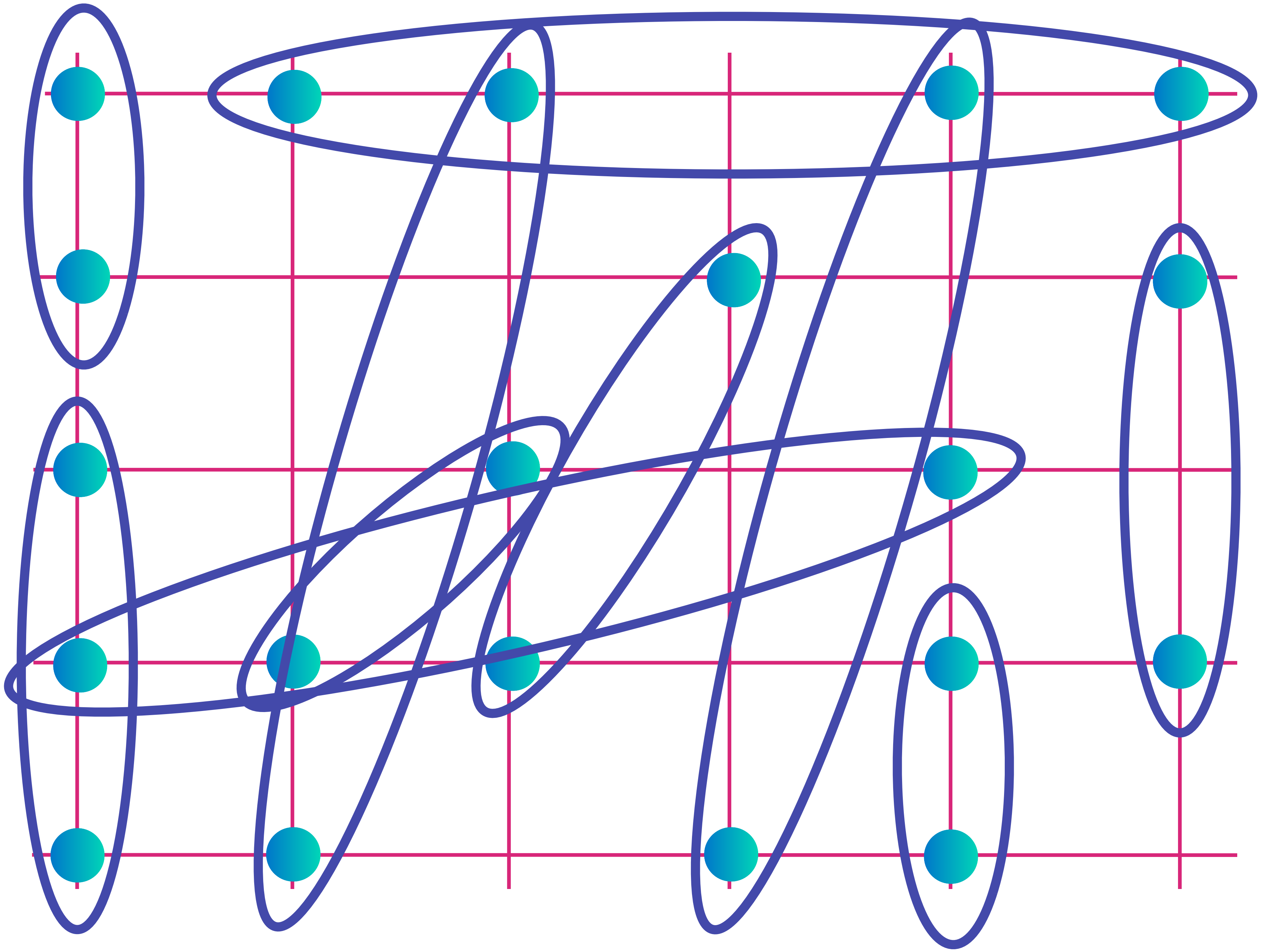
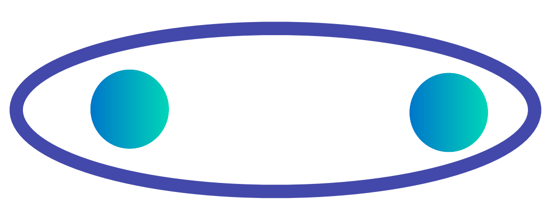


$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



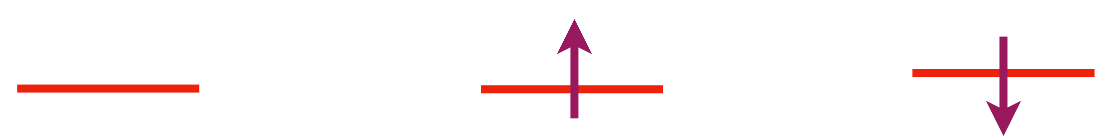
Allow electron motion and bond exchange between ANY pair of sites, all with a random amplitude

 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$$b^\dagger |v\rangle \qquad f_\uparrow^\dagger |v\rangle \qquad f_\downarrow^\dagger |v\rangle$$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad b \to b e^{i\phi}\,, \quad f_\alpha \to f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

<u>*t-J* model</u>

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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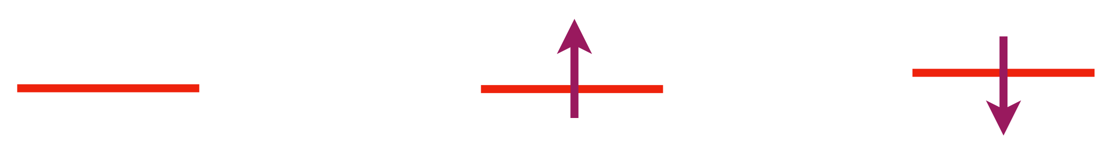


$$\mathfrak{f}^{\dagger} |v\rangle \qquad \mathfrak{b}_{\uparrow}^{\dagger} |v\rangle \qquad \mathfrak{b}_{\downarrow}^{\dagger} |v\rangle$$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^{\dagger} \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^{\dagger} \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^{\dagger} \mathfrak{b}_\alpha + \mathfrak{f}^{\dagger}\mathfrak{f} &= 1 \end{aligned}$$

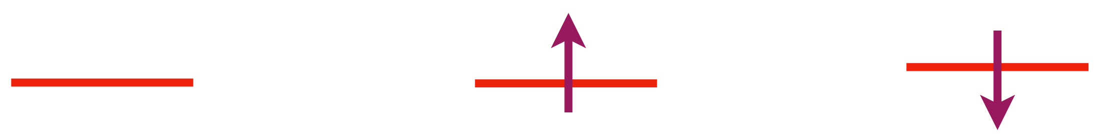
U(1) gauge invariance, $\quad \mathfrak{f} \rightarrow \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \rightarrow \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$\mathfrak{f}^{\dagger}\,|v\rangle$ $\quad$ $\mathfrak{b}_{\uparrow}^{\dagger}\,|v\rangle$ $\quad$ $\mathfrak{b}_{\downarrow}^{\dagger}\,|v\rangle$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^{\dagger} \\ \vec{S} &= \frac{1}{2}\mathfrak{b}_\alpha^{\dagger} \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^{\dagger}\mathfrak{b}_\alpha + \mathfrak{f}^{\dagger}\mathfrak{f} &= 1 \end{aligned}$$

$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$

U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Insulating J model

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} = 1$$

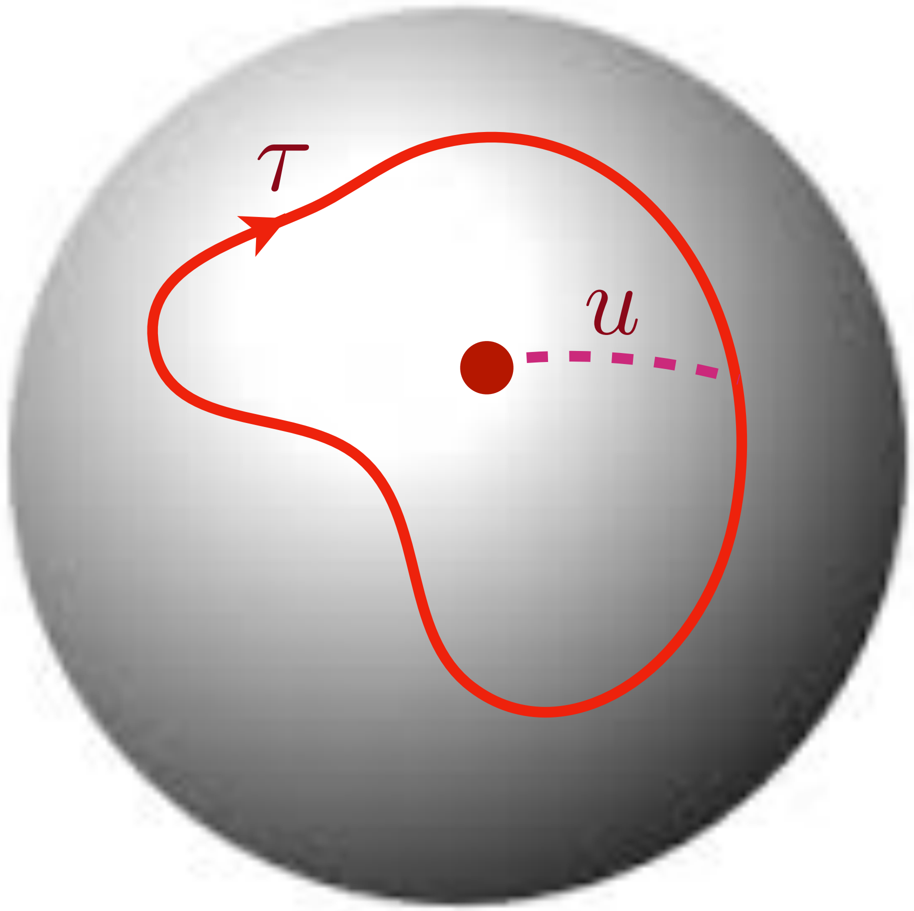
$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J_{ij}^2} = J^2$$

Insulating J model

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J}$$

$$\mathcal{S}_B = \frac{i}{2}\int_0^1 du \int d\tau \vec{S}\cdot\left(\frac{\partial\vec{S}}{\partial\tau}\times\frac{\partial\vec{S}}{\partial u}\right)$$

$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J}$$

$$\mathcal{S}_B = \frac{i}{2}\int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right)$$

$$\mathcal{S}_J = -\frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') \, .$$

From this action we compute

$$\overline{Q}(\tau - \tau') = \frac{1}{3}\left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \overline{Q}(\tau).$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

t-J model

$$\mathcal{Z} = \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}}$$

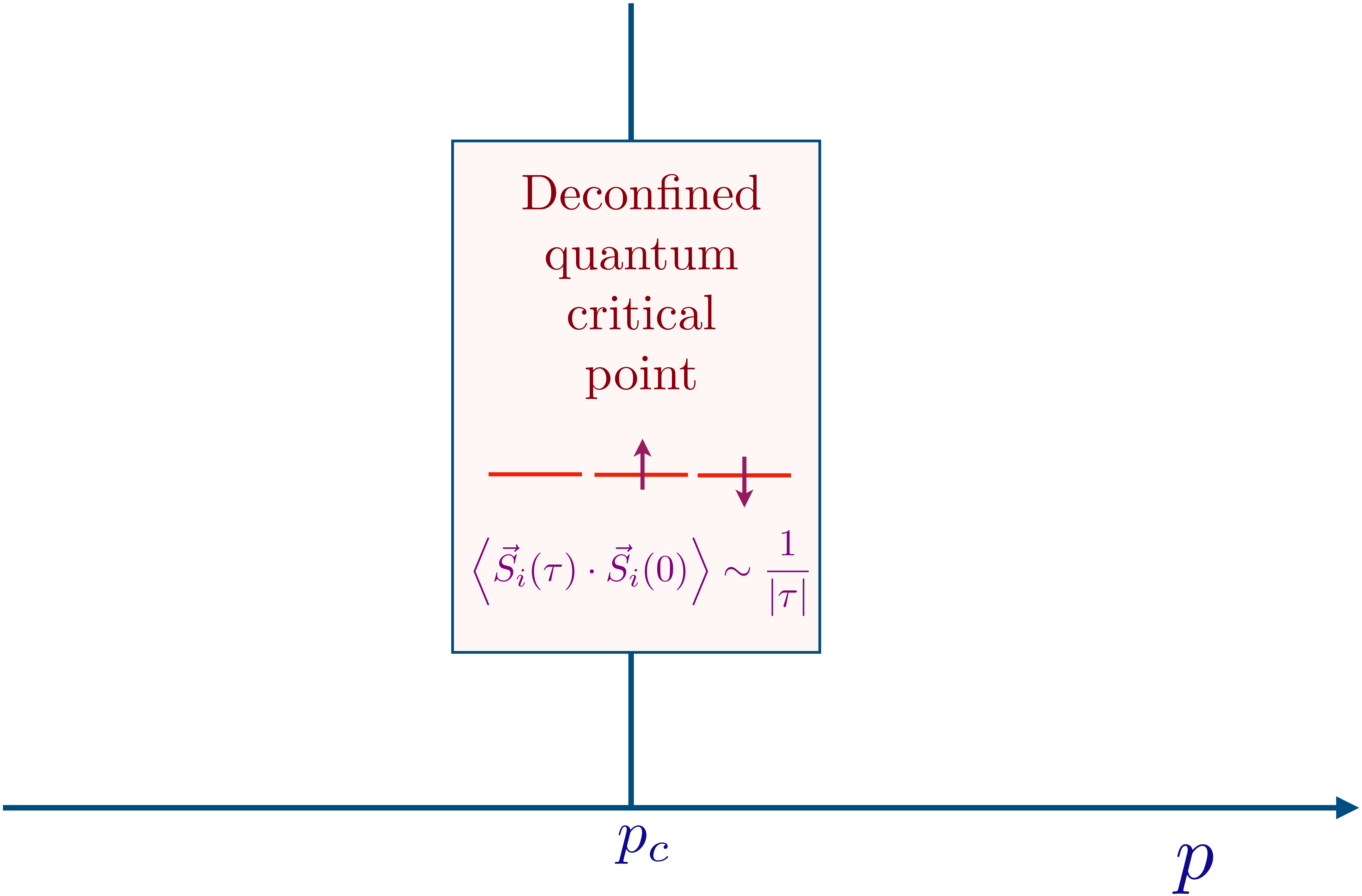
$$\mathcal{S}_B = i \int_0^1 du \int d\tau \, \mathrm{Tr}\left(\mathcal{P}\partial_\tau \mathcal{P} \partial_u \mathcal{P}\right)$$

$$\mathcal{S}_{tJ} = \int d\tau d\tau' \, \mathrm{Tr}\left(\mathcal{P}(\tau)\mathcal{Q}(\tau - \tau')\mathcal{P}(\tau')\right) + \int d\tau \mathrm{Tr}\left(s_0 \, \mathcal{P}(\tau)\right) \, .$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a 'Zeeman superfield' s_0.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



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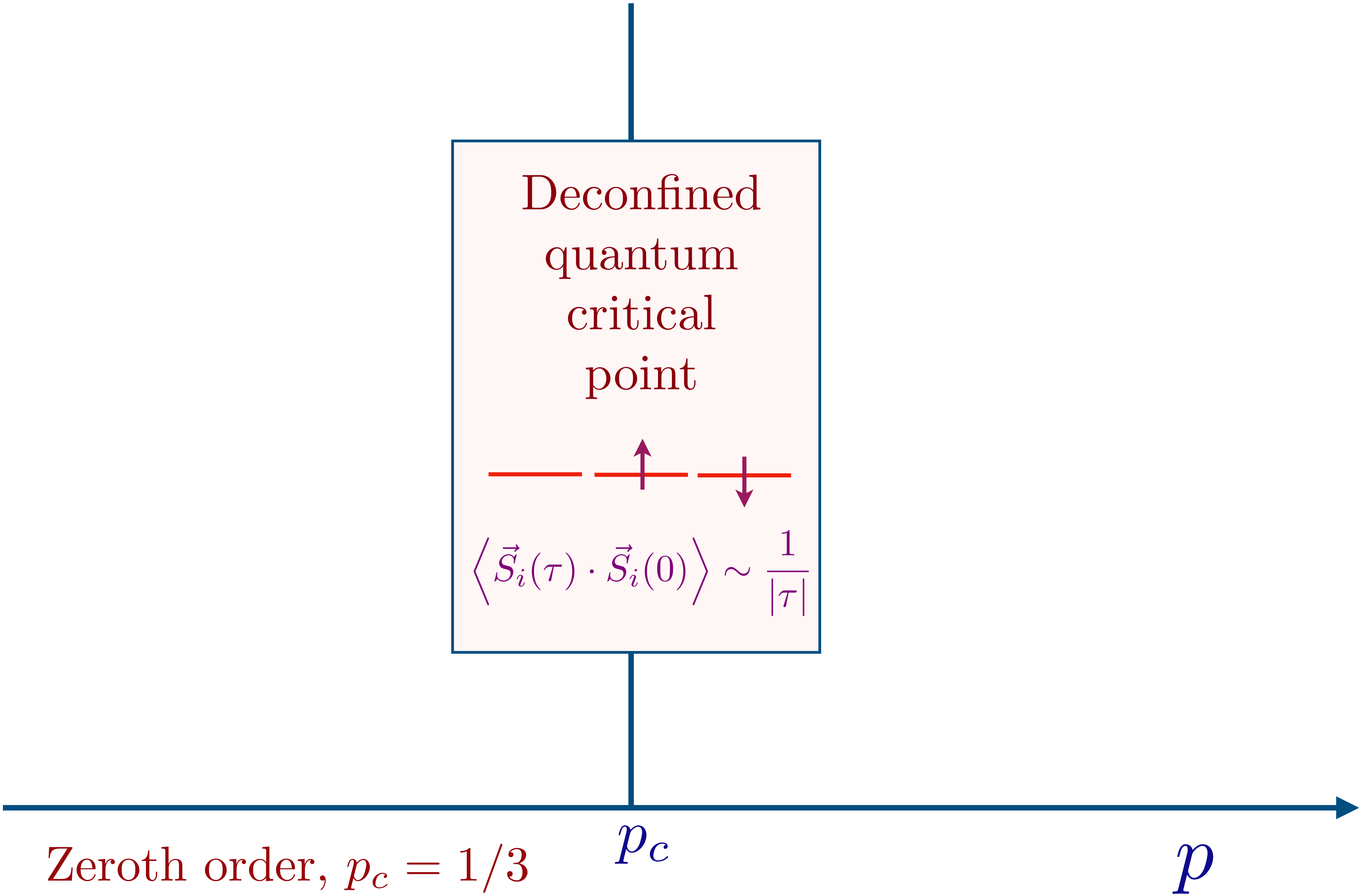

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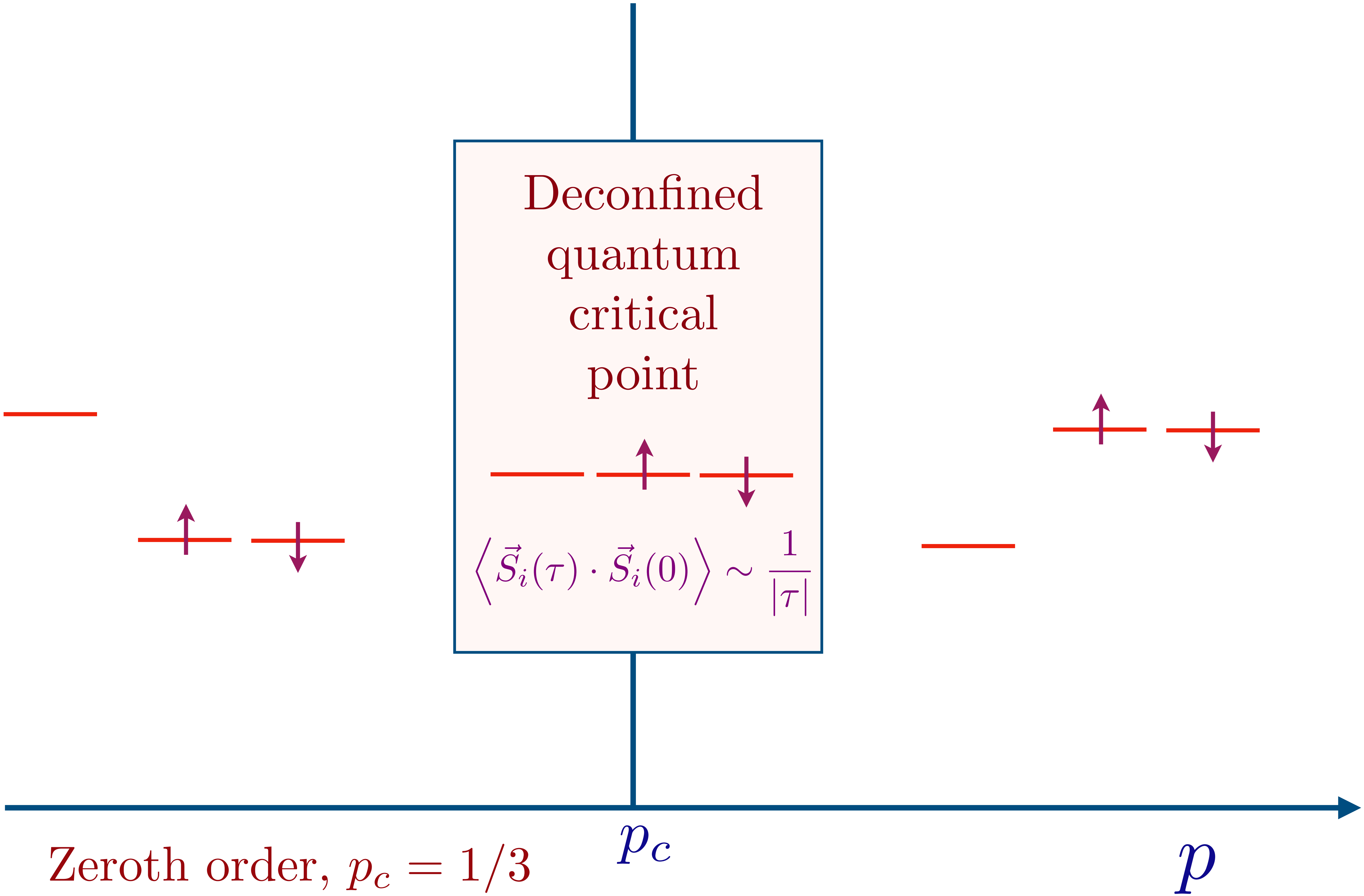
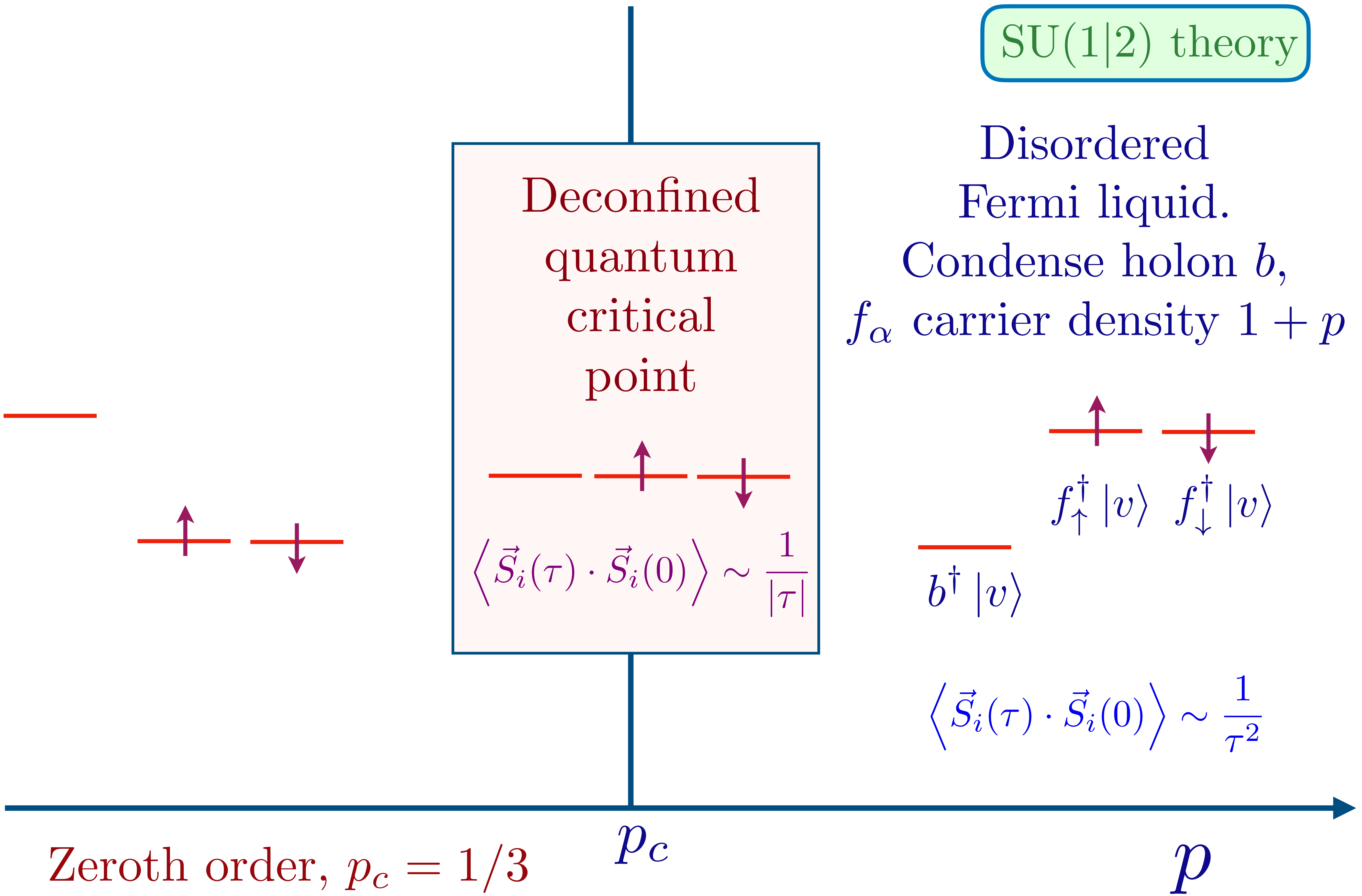


Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

t-J model phase diagram

SU(2|1) theory

SU(1|2) theory

Metallic spin glass. Condense spinon $\mathfrak{b}_\alpha$, $\mathfrak{f}$ carrier density p

Deconfined quantum critical point

Disordered Fermi liquid. Condense holon b, f_α carrier density $1+p$

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

p_c

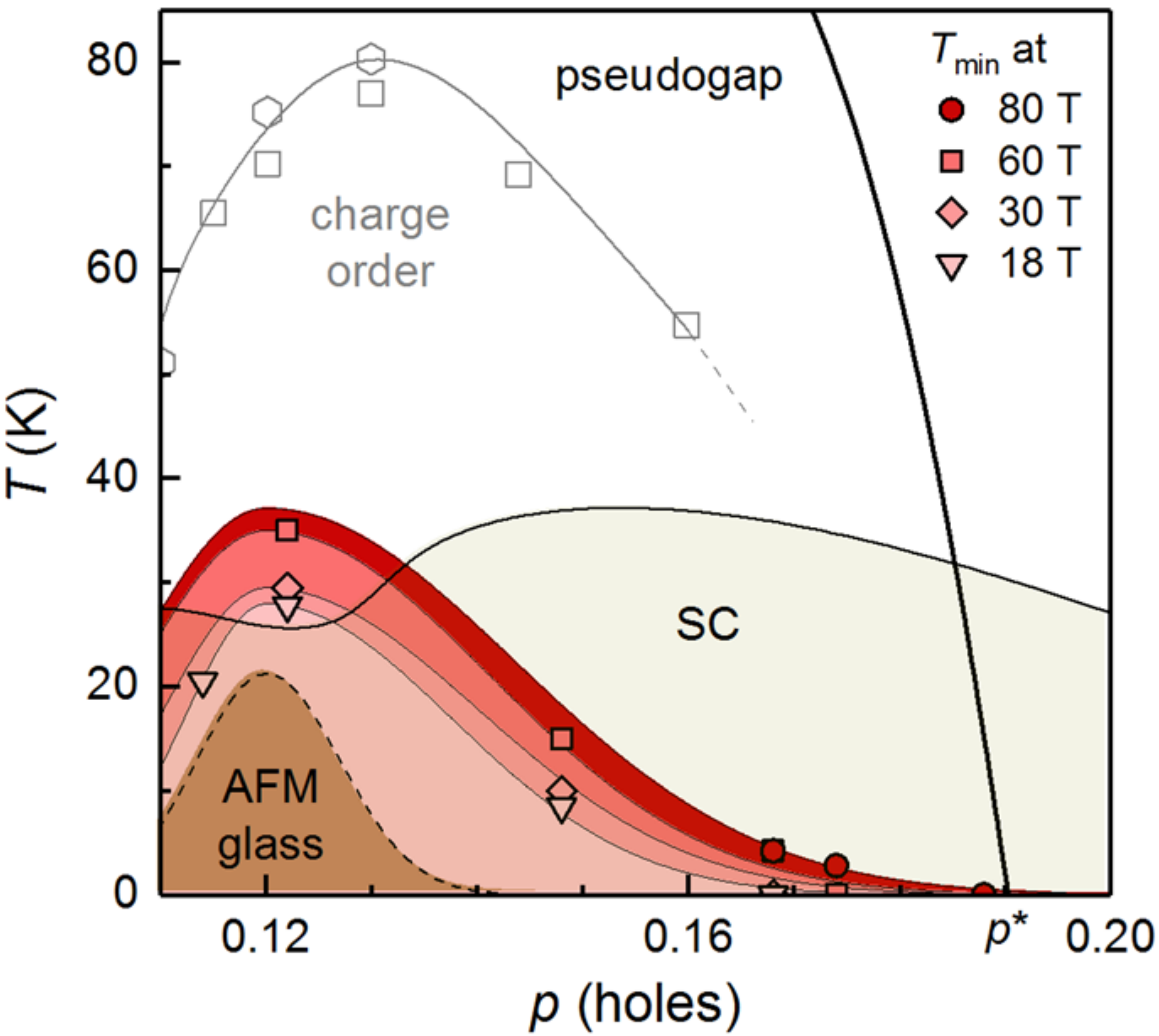
p

Zeroth order, $p_c = 1/3$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

Hidden magnetism at the pseudogap critical point of a high temperature superconductor

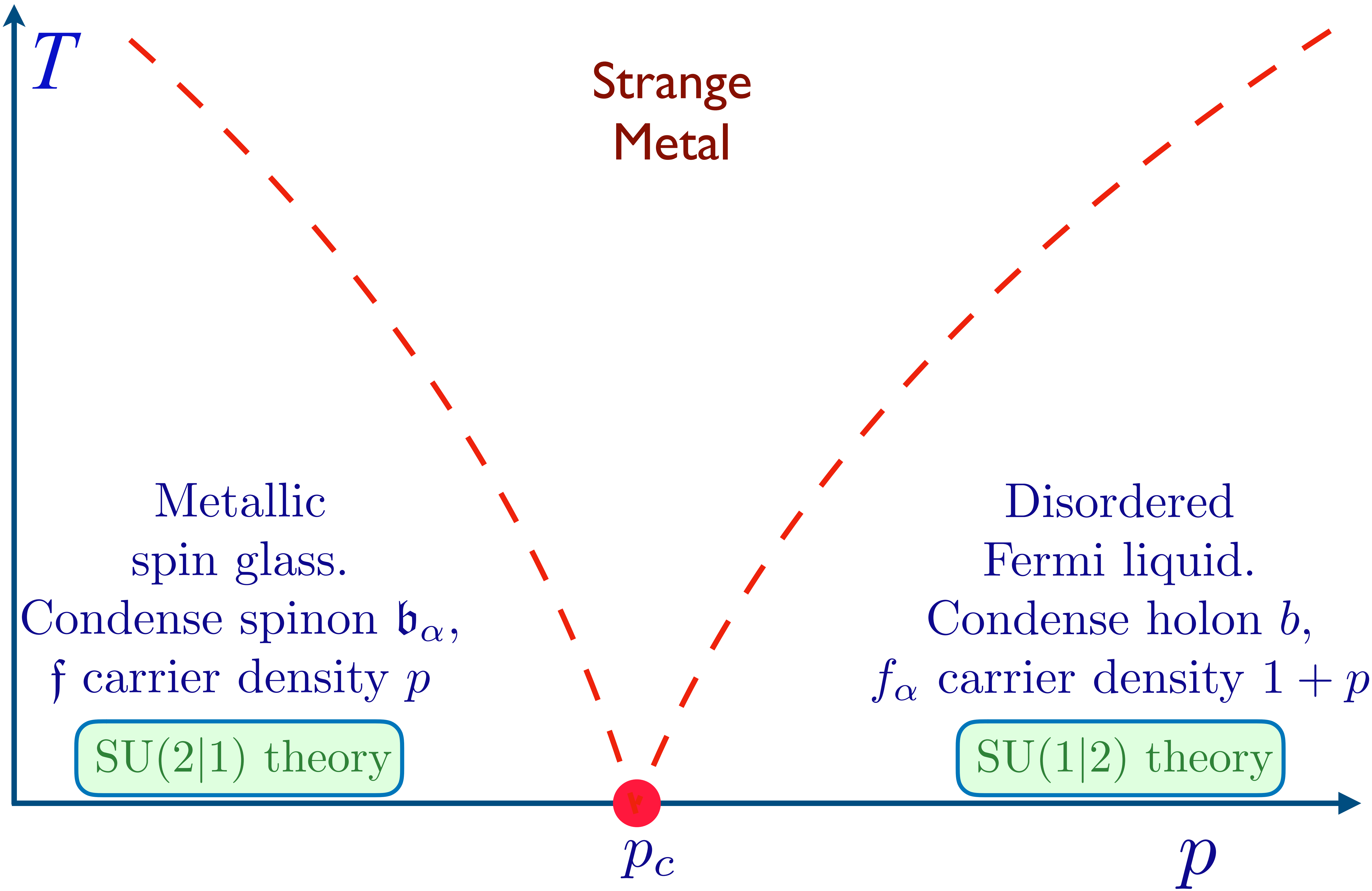
Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



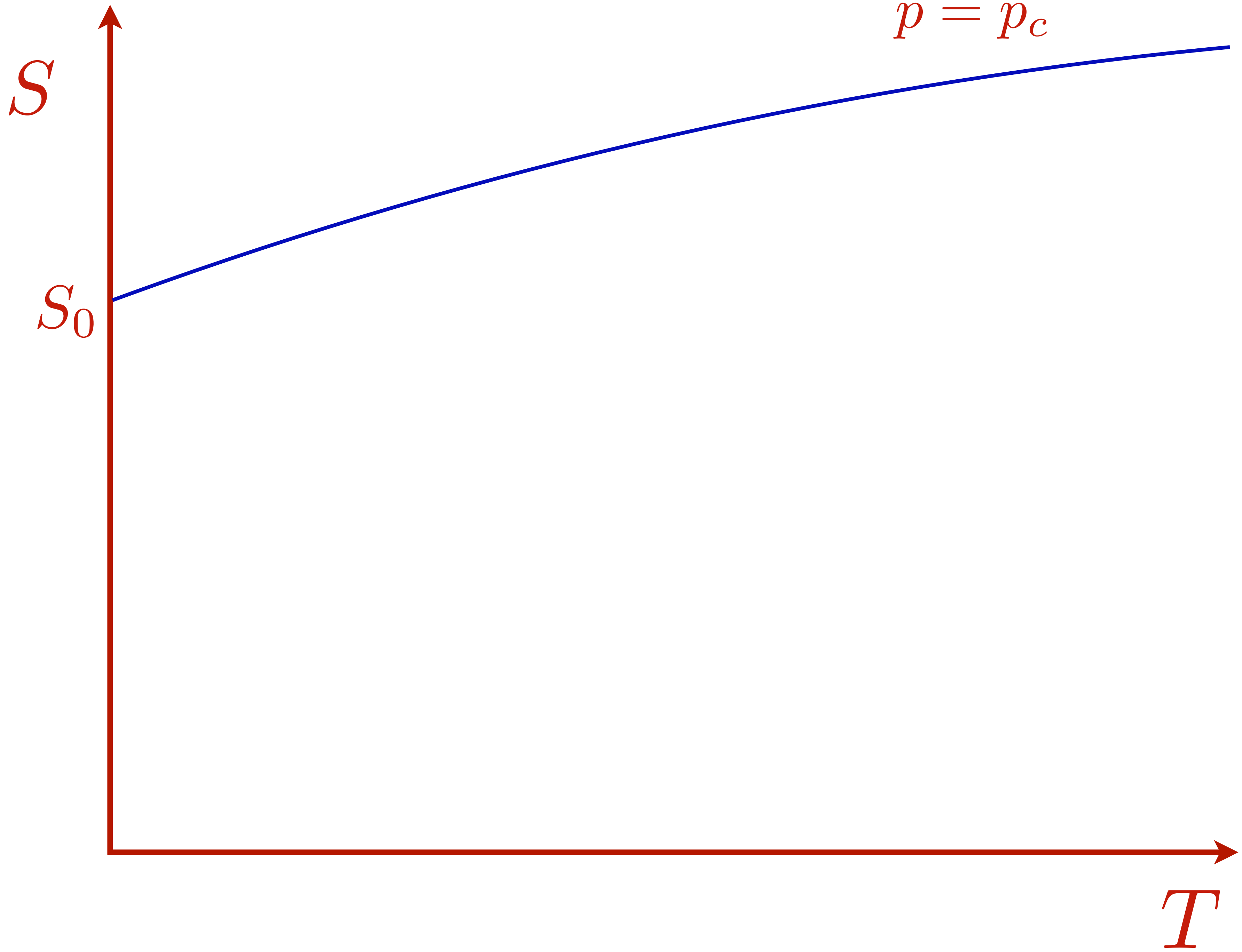
arXiv:1909.10258

Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

t-J model phase diagram

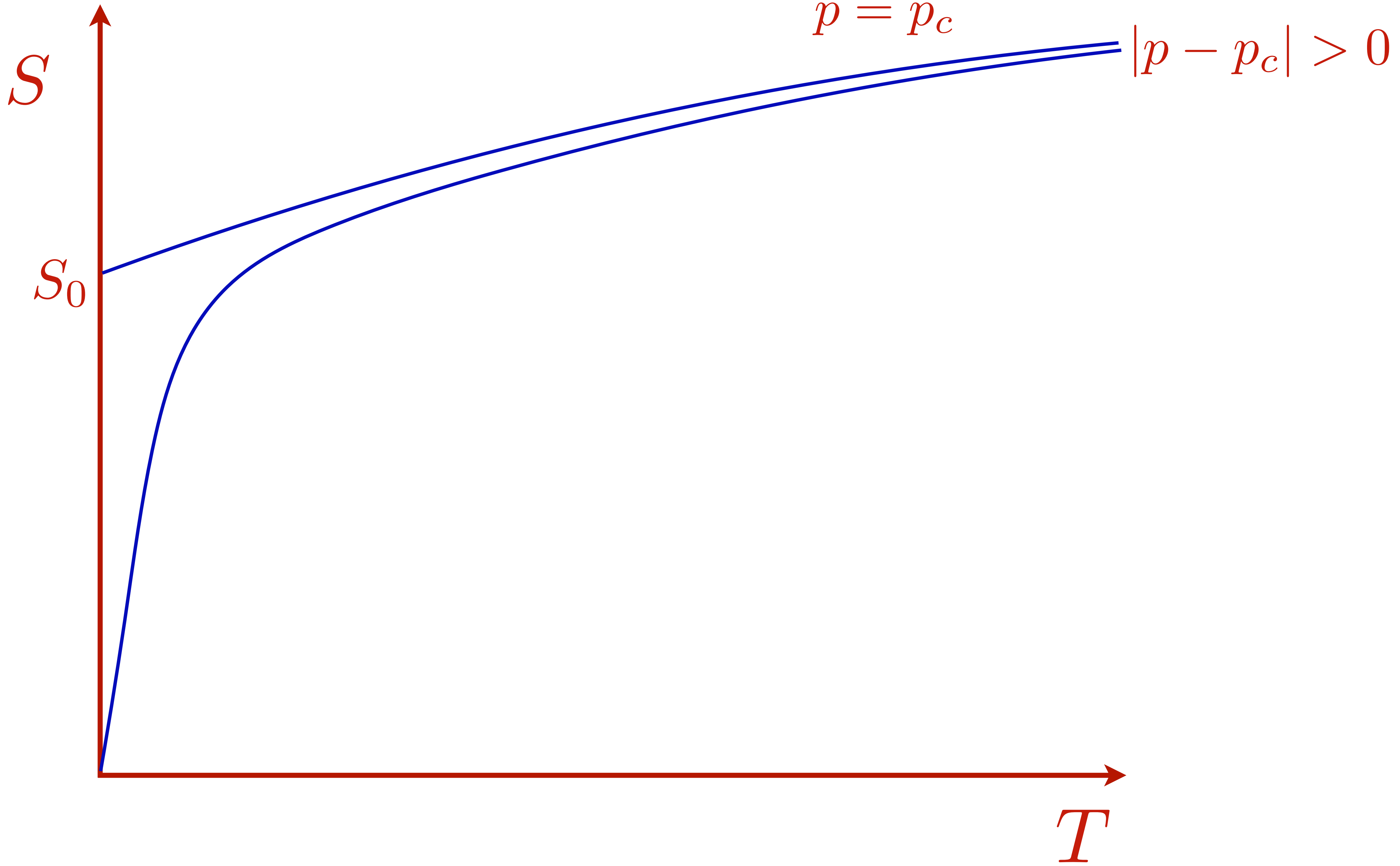


t-J model entropy



$$\frac{C}{T} = \frac{dS}{dT}$$

t-J model entropy



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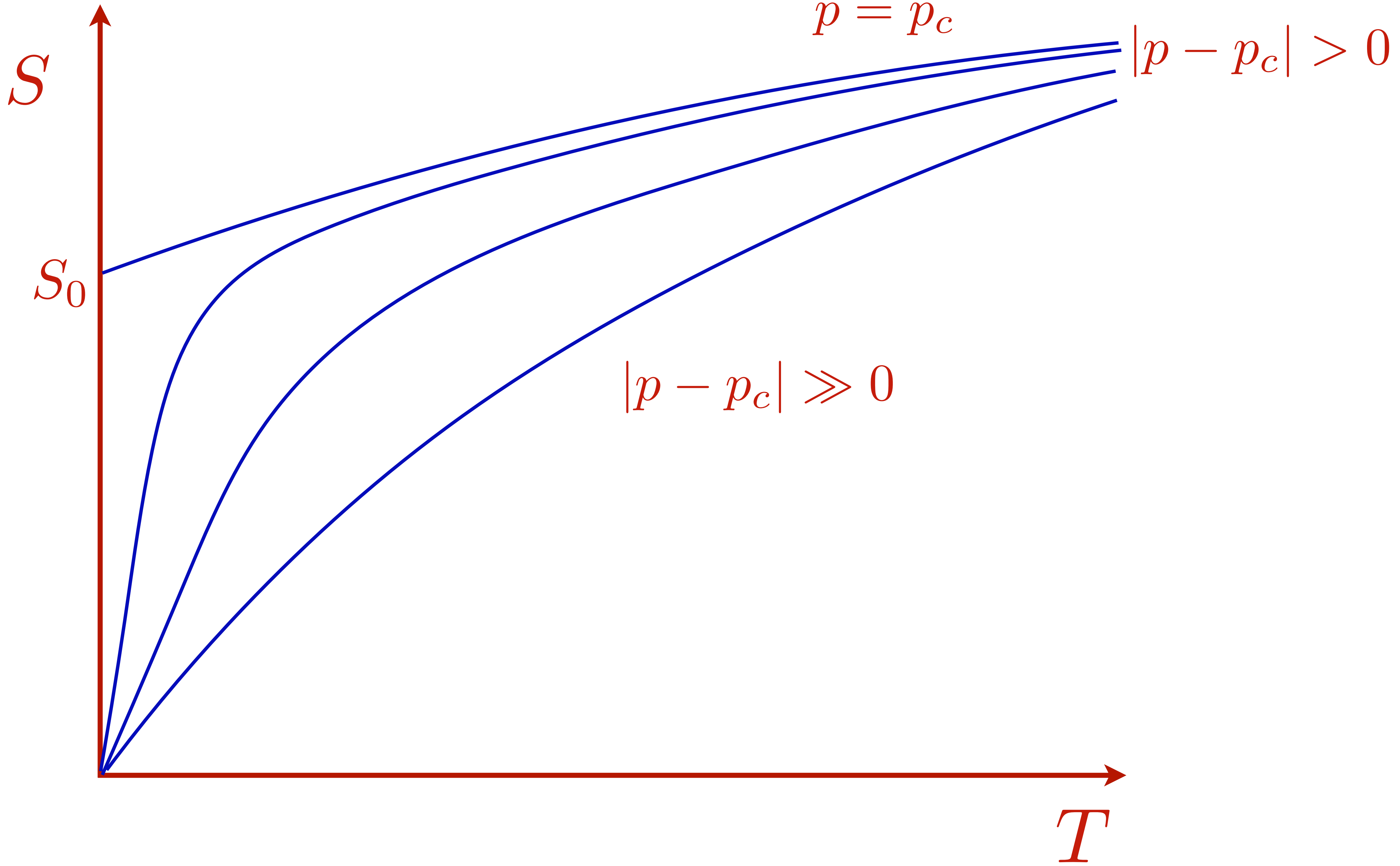
t-J model entropy



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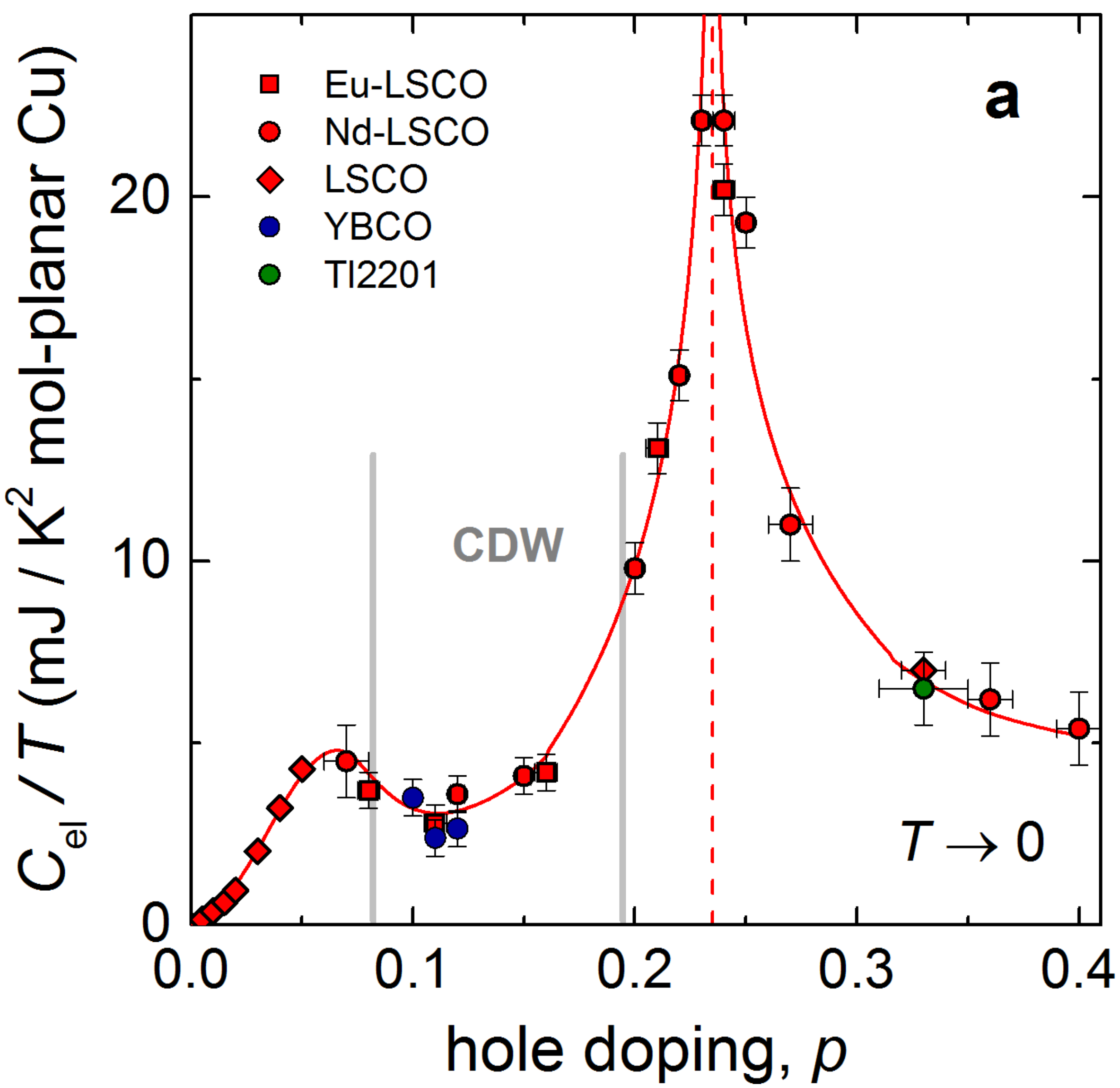
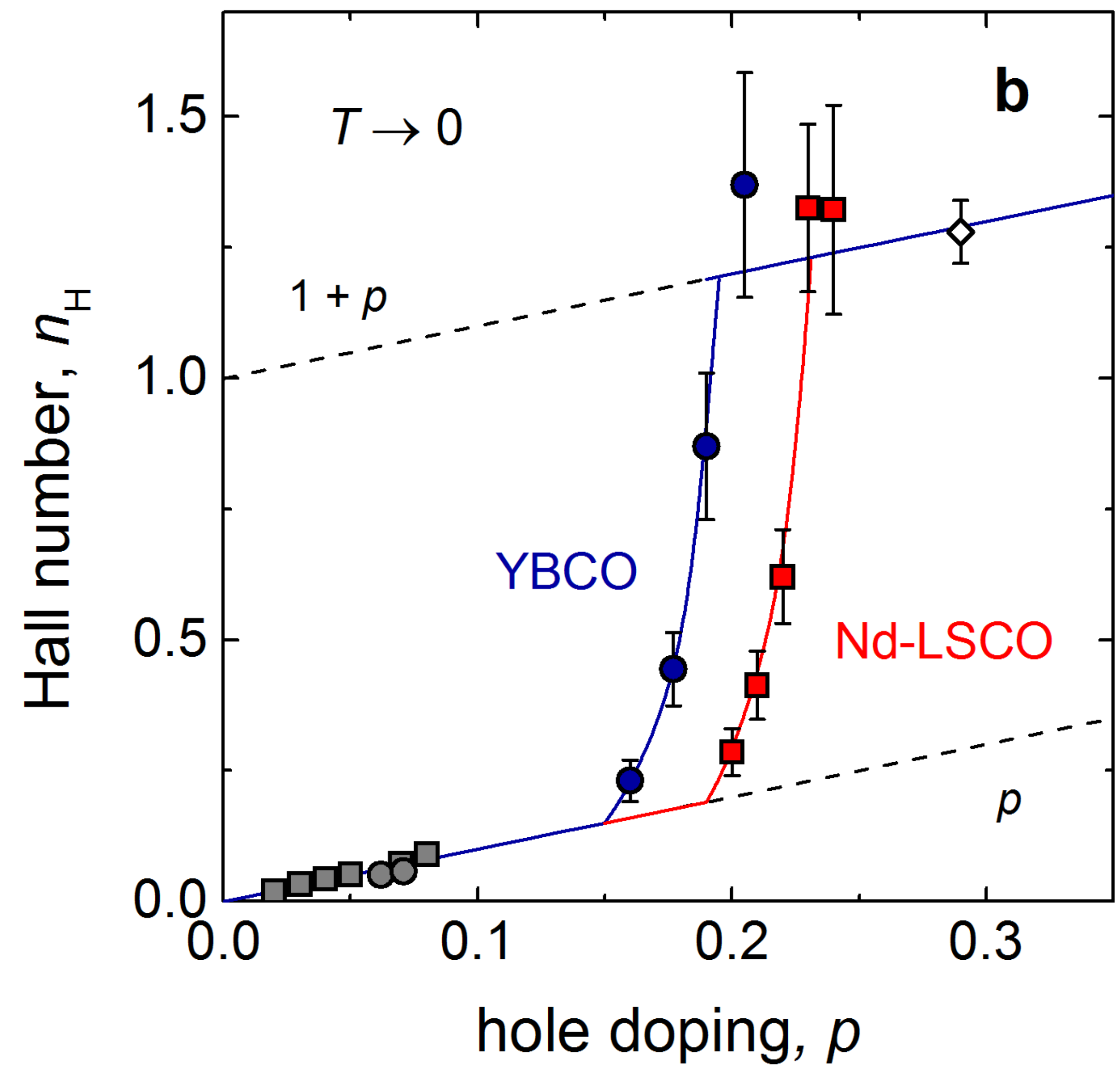
Hole doped cuprates

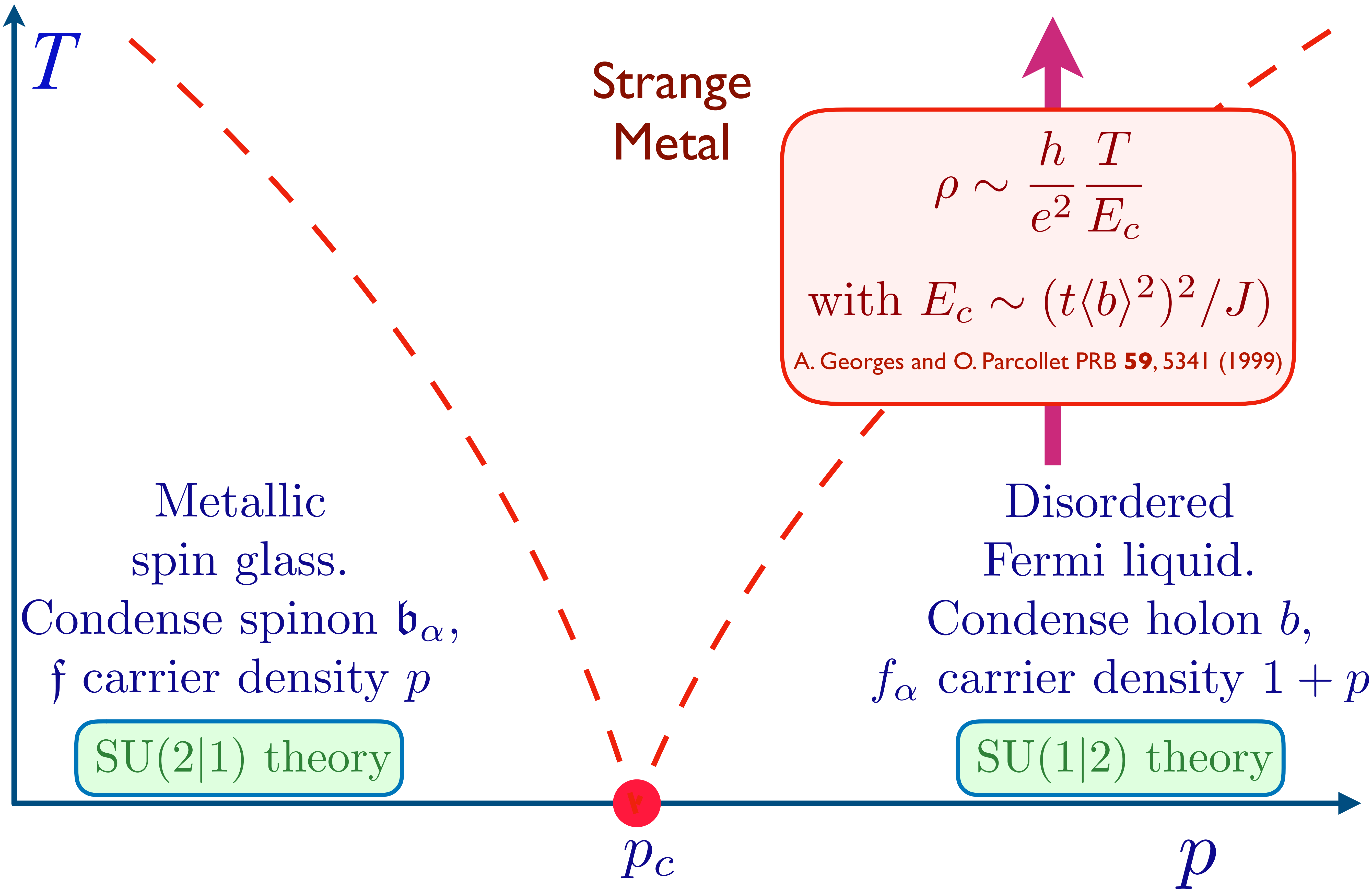
The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507





t-J model phase diagram



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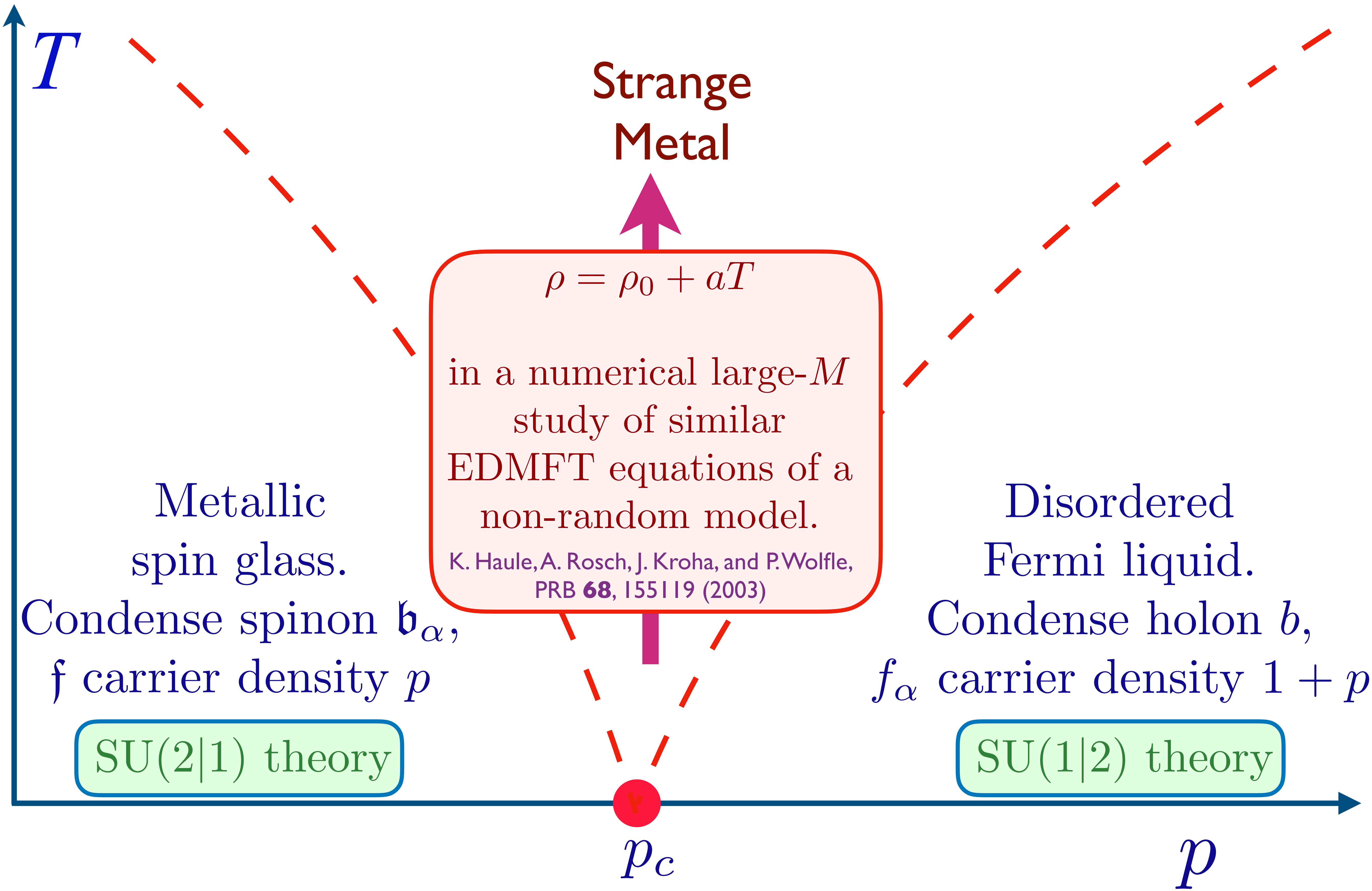
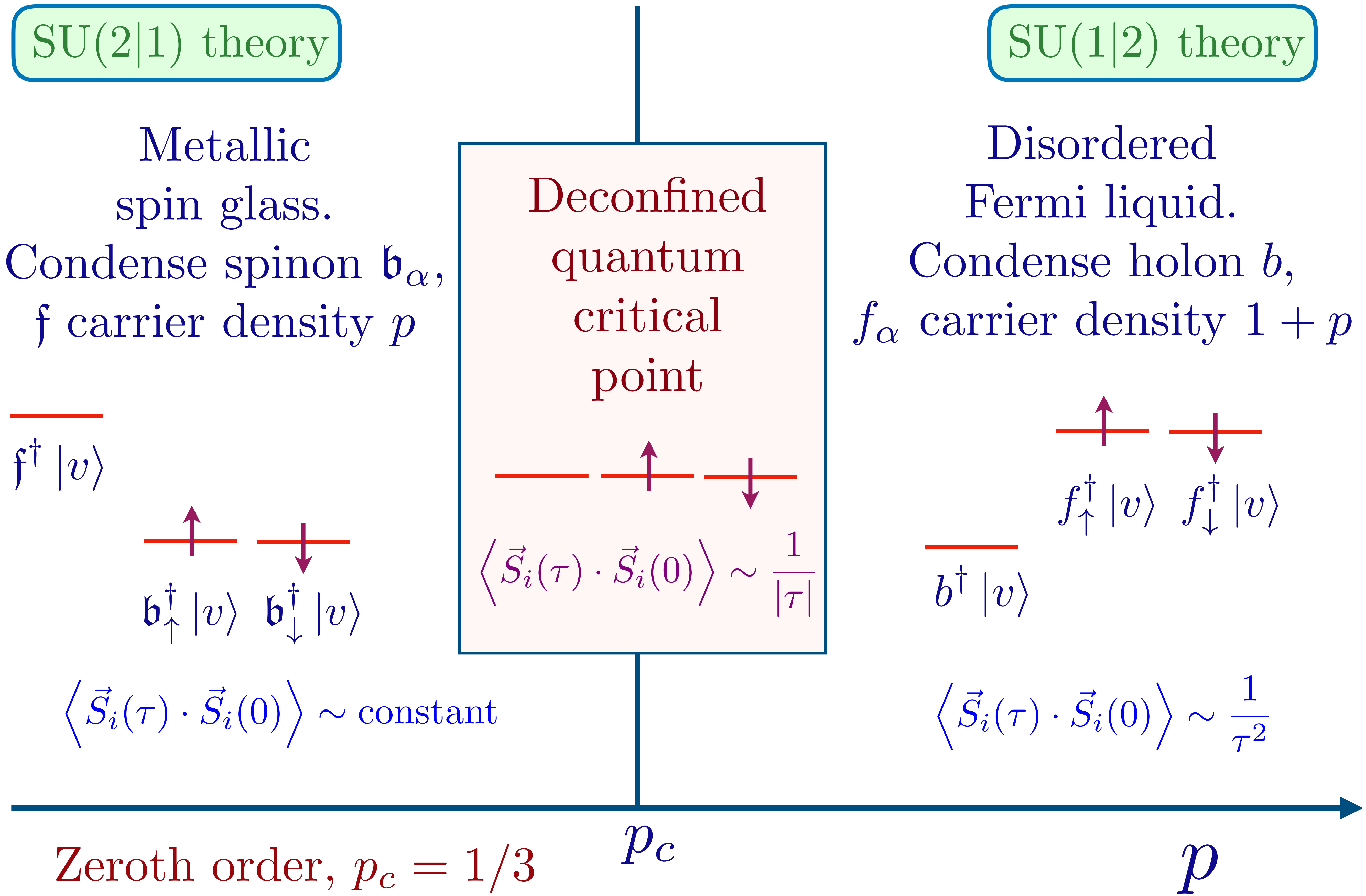


t-J model phase diagram



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

- t-J models with random and all-to-all interactions exhibit a deconfined critical point at a non-zero doping $p = p_c$, flanked by conventional confining phases with carrier densities p and $1 + p$.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

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- Underlying reason for quantum criticality: statistical transmutation from bosonic to fermionic spins. Degeneracy of 3 states of t-J model yields zeroth order prediction, $p_c = 1/3$.

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- Closely related saddle-point equations were obtained in an "extended dynamical mean-field theory (EDMFT)" of non-random lattice t-J models.

- The structure of the DQCP is similar to the SYK models: both have local spin correlations which decay as $\sim 1/|\tau|$ in imaginary time τ.

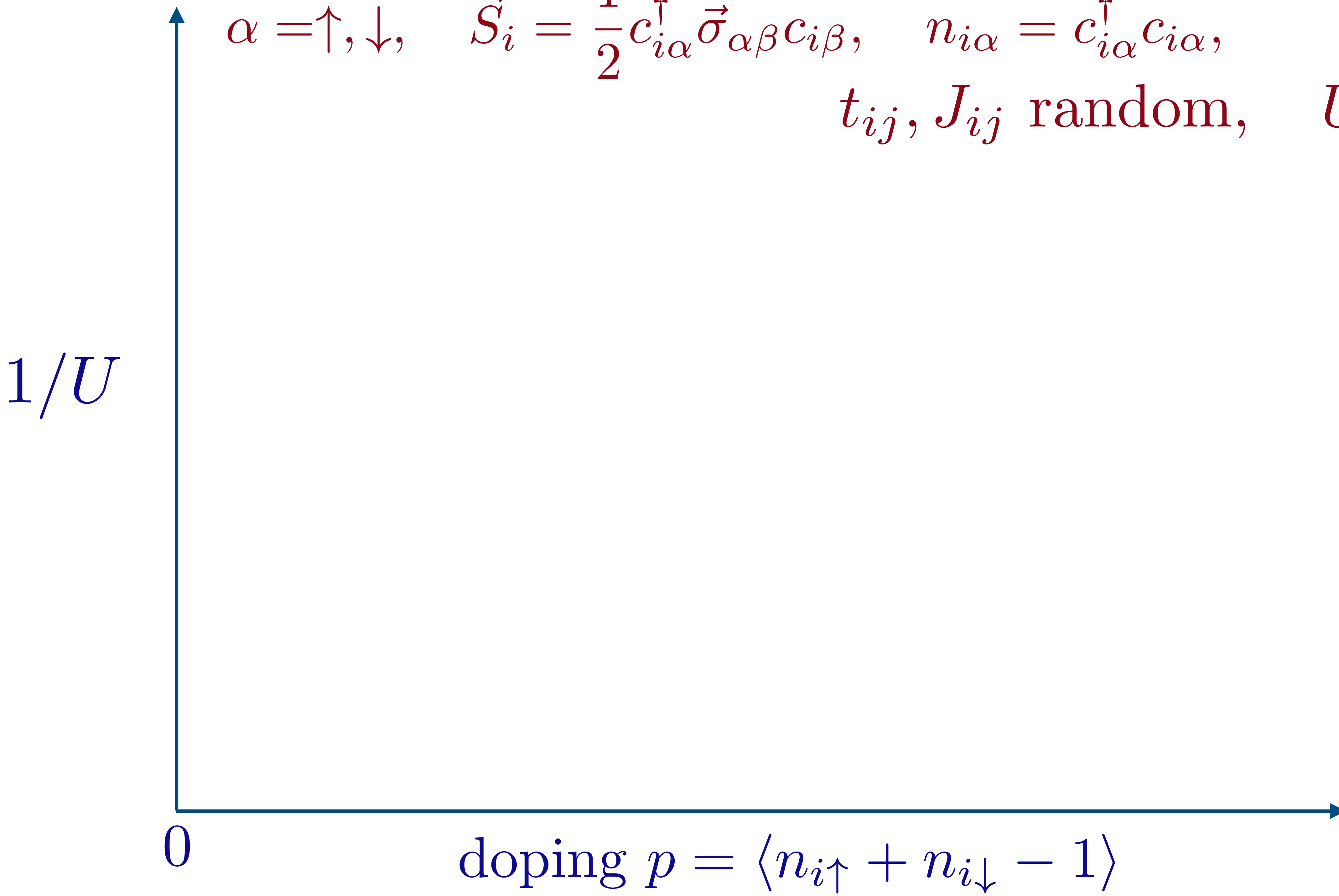
D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, arXiv:1912.08822

Random *t-J-U* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^{\dagger} c_{i\alpha},$$

$$t_{ij}, J_{ij} \text{ random}, \quad U > 0$$



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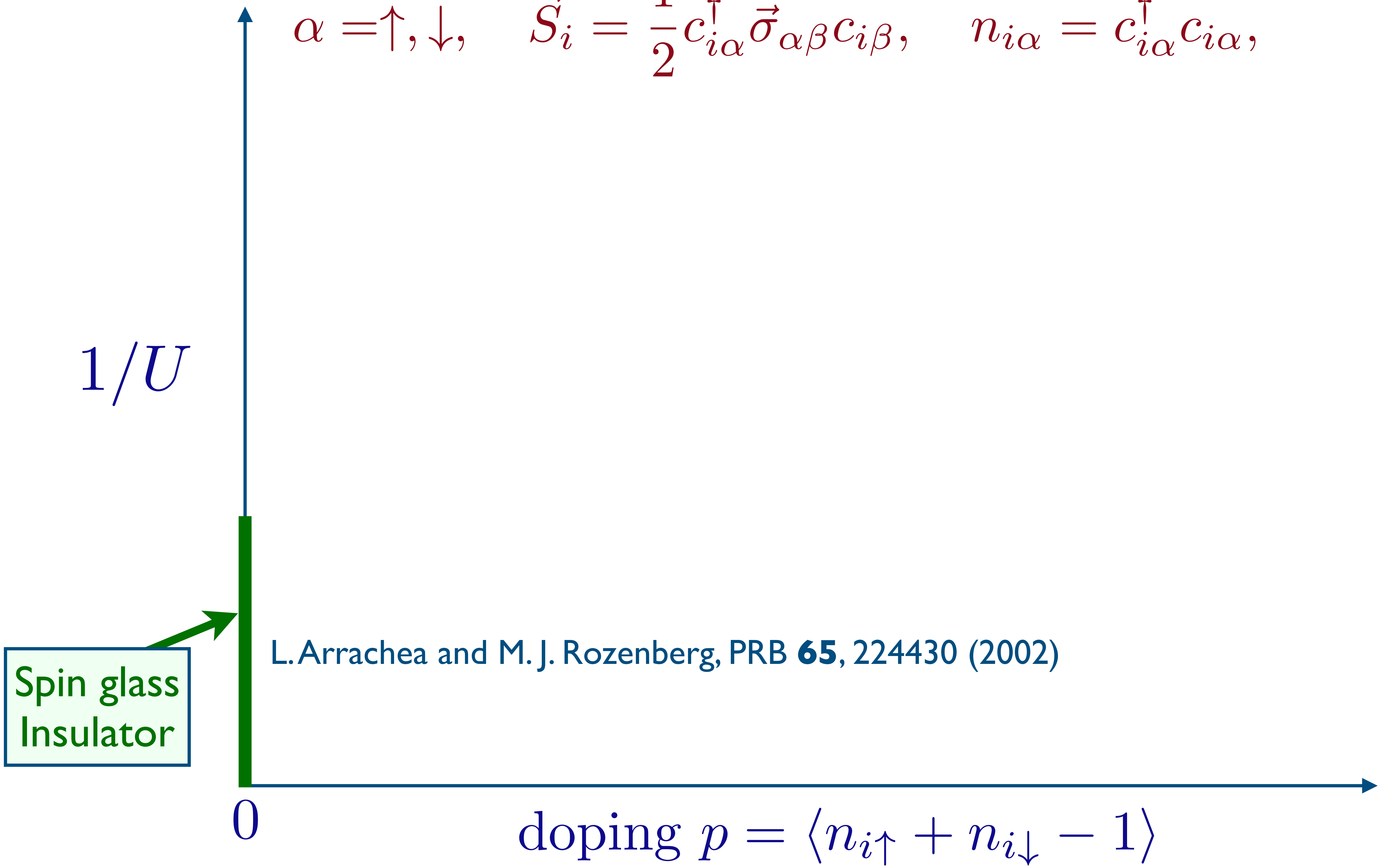
$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c_{i\alpha}^\dagger c_{i\alpha},$$



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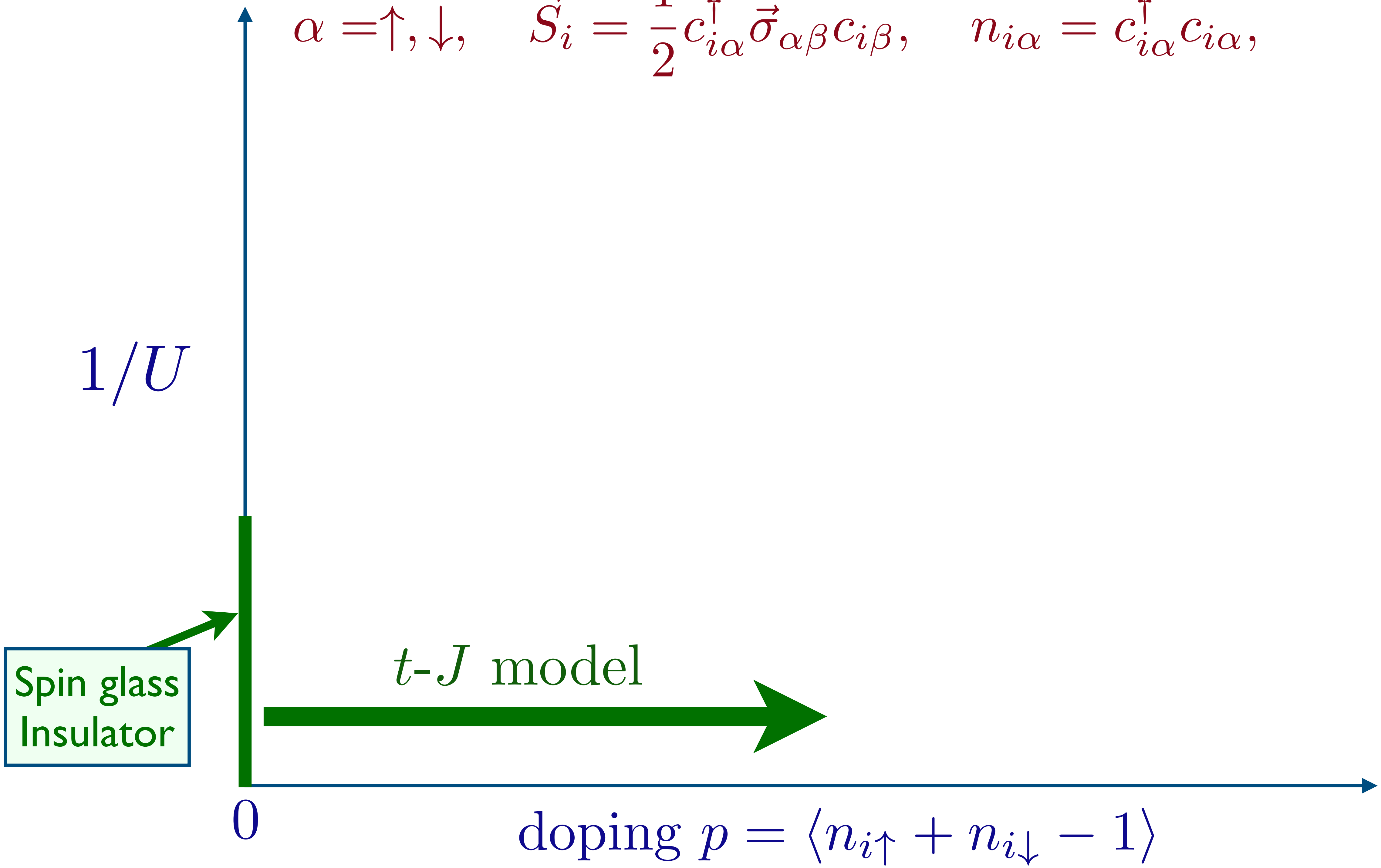

Random *t-J-U* model

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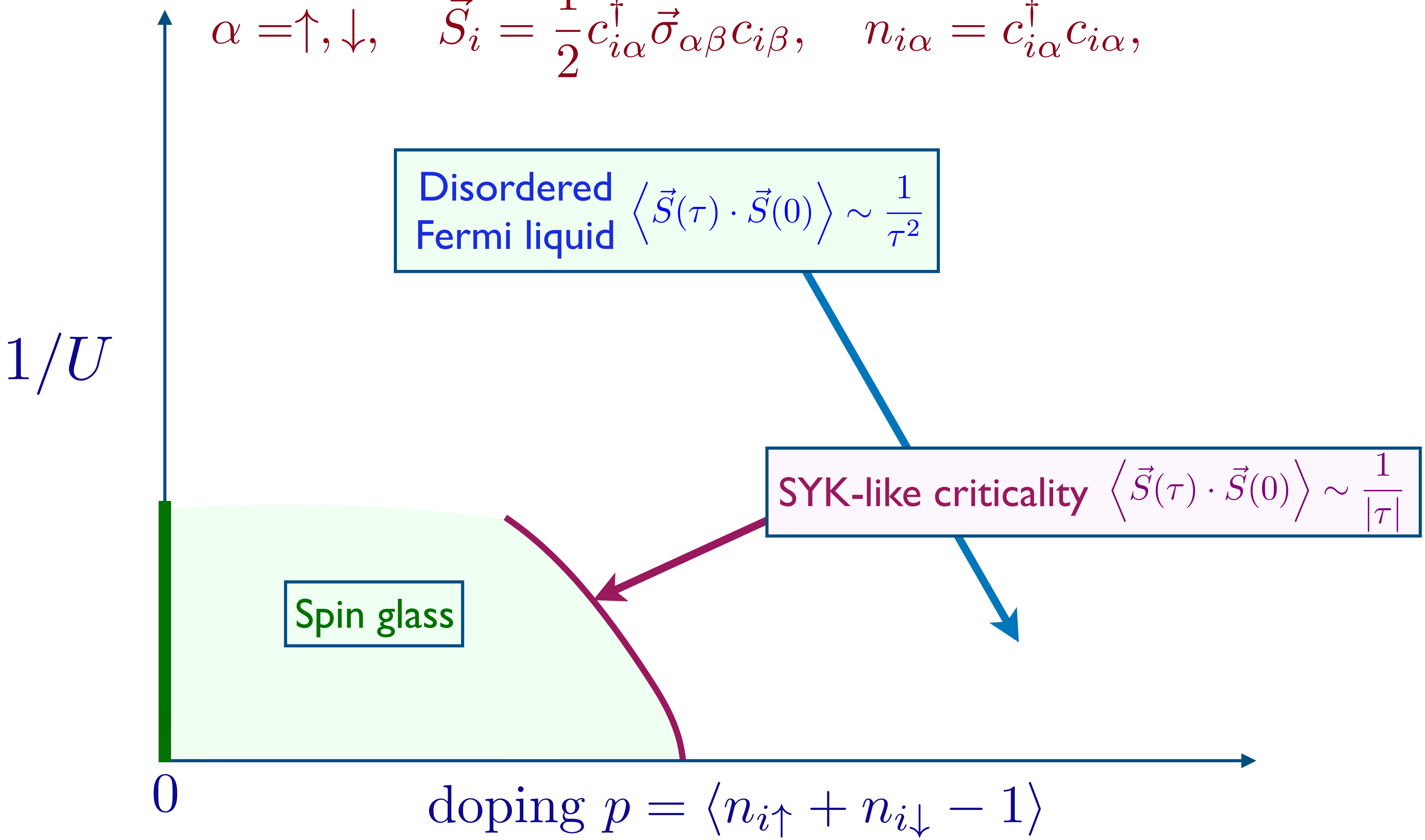

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