Universal conductance of nanowires near the superconductor-metal quantum transition

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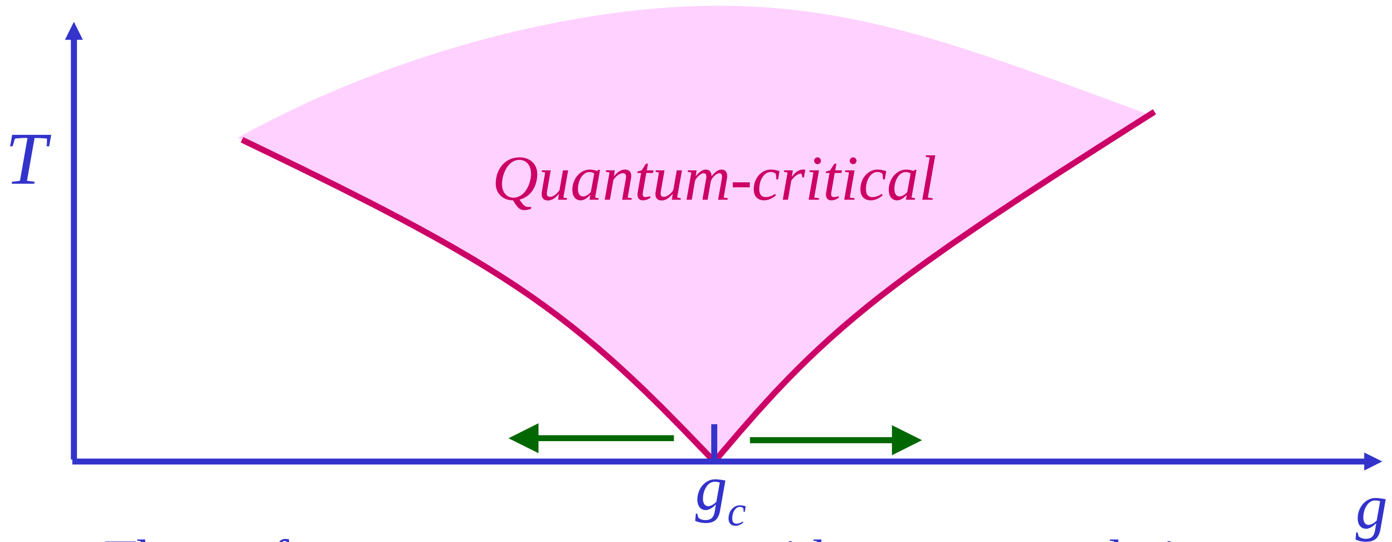
Talk online at http://sachdev.physics.harvard.edu

Why study quantum phase transitions ?



• Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.

• Critical point is a novel state of matter without quasiparticle excitations

• Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

Important property of ground state at $g=g_c$:
temporal and spatial scale invariance;
characteristic energy scale at other values of g: $\Delta \sim \left| g - g_c \right|^{z\nu}$

Outline

I. Quantum Ising Chain

II. Landau-Ginzburg-Wilson theory
   Mean field theory and the evolution of the excitation spectrum.

III. Superfluid-insulator transition
   Boson Hubbard model at integer filling.

IV. Superconductor-metal transition in nanowires
   Universal conductance and sensitivity to leads

I. Quantum Ising Chain

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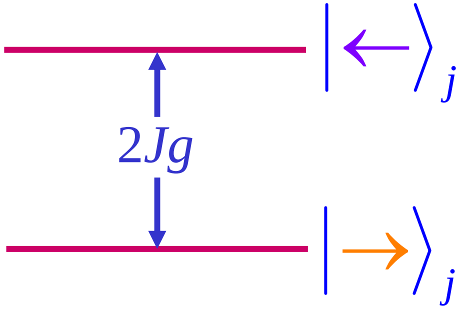
Degrees of freedom: $j = 1 \ldots N$ qubits, N "large"

$$|\uparrow\rangle_j\,,\; |\downarrow\rangle_j$$

or $$|\rightarrow\rangle_j = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_j + |\downarrow\rangle_j\right)\,,\; |\leftarrow\rangle_j = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_j - |\downarrow\rangle_j\right)$$

Hamiltonian of decoupled qubits:

$$H_0 = -Jg\sum_j \sigma_j^x$$



Coupling between qubits:

$$H_1 = -J \sum_j \sigma_j^z \sigma_{j+1}^z$$

$$\Big(\left|\rightarrow\right\rangle_j \left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_j \left\langle\rightarrow\right| \Big) \Big(\left|\rightarrow\right\rangle_{j+1} \left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_{j+1} \left\langle\rightarrow\right| \Big)$$

Prefers neighboring qubits

are *either* $\left|\uparrow\right\rangle_j \left|\uparrow\right\rangle_{j+1}$ *or* $\left|\downarrow\right\rangle_j \left|\downarrow\right\rangle_{j+1}$

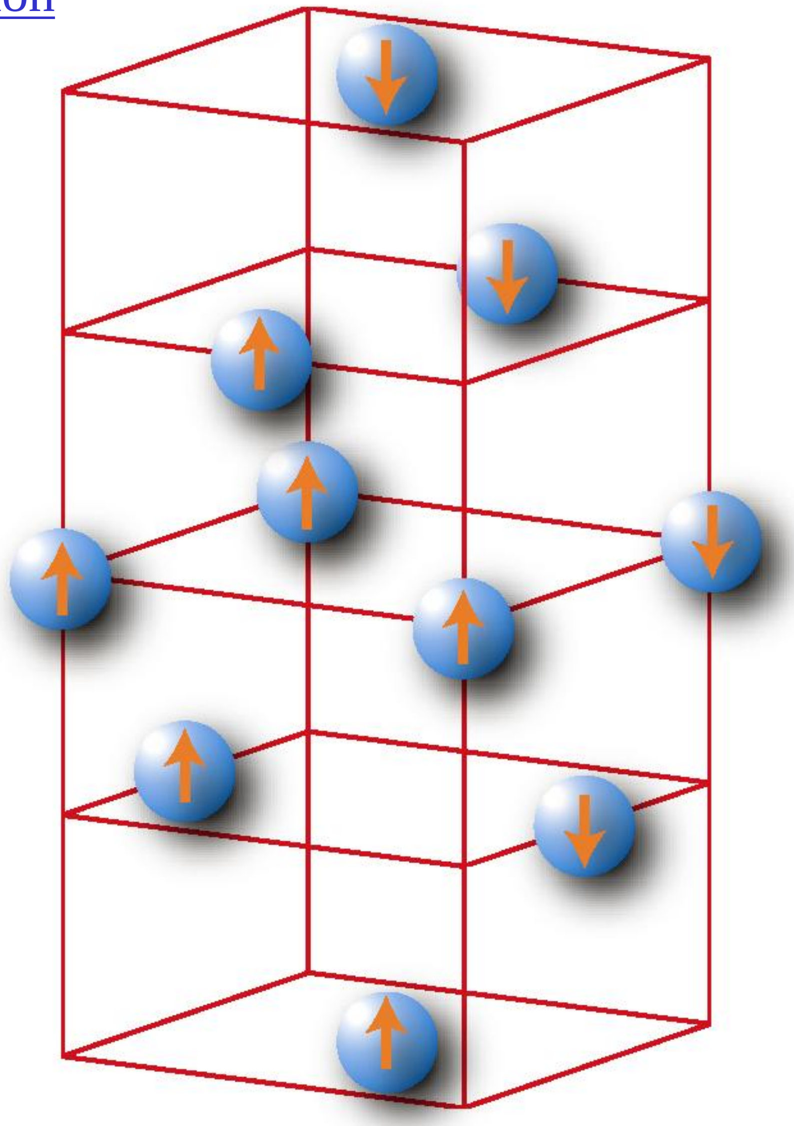
(not entangled)

Full Hamiltonian

$$H = H_0 + H_1 = -J \sum_j \left(g\sigma_j^x + \sigma_j^z \sigma_{j+1}^z \right)$$

leads to entangled states at *g* of order unity

Experimental realization



$LiHoF_4$

Weakly-coupled qubits $(g \gg 1)$

Ground state:

$$\left|G\right\rangle = \left|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\right\rangle$$

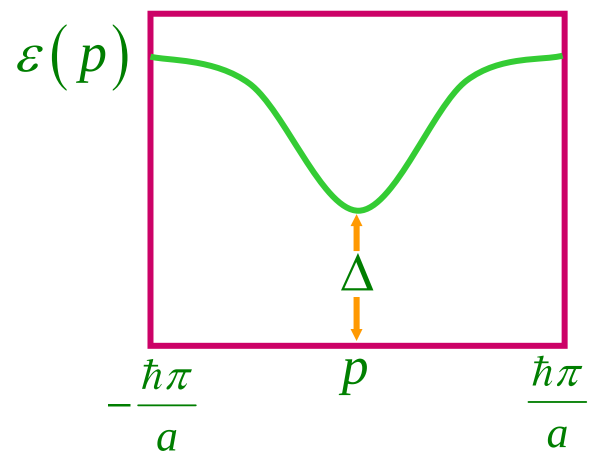
$$-\frac{1}{2g}\left|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\leftarrow\leftarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\right\rangle - \cdots$$

Lowest excited states:

$$\left|\ell_j\right\rangle = \left|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\leftarrow_j\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\right\rangle + \cdots$$

Coupling between qubits creates "flipped-spin" *quasiparticle* states at momentum p

$$\left|p\right\rangle = \sum_j e^{ipx_j/\hbar}\left|\ell_j\right\rangle$$



Excitation energy $\varepsilon(p) = \Delta + 4J\sin^2\left(\frac{pa}{2\hbar}\right) + O\left(g^{-1}\right)$

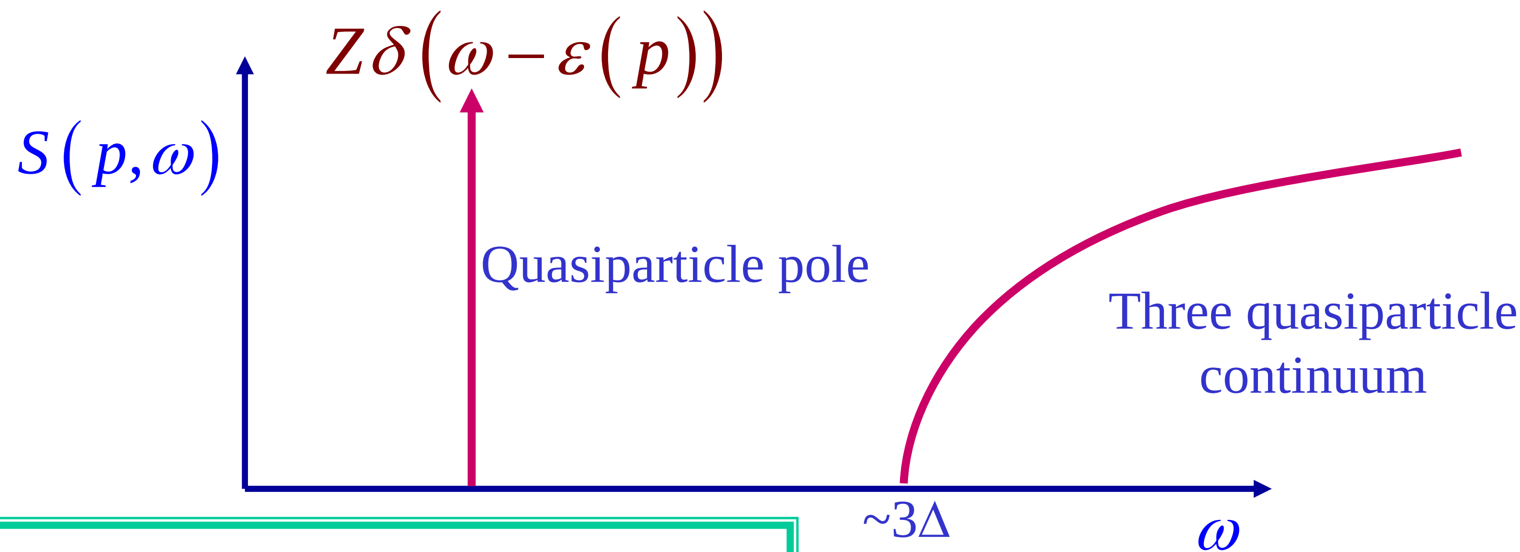
Excitation gap $\Delta = 2gJ - 2J + O\left(g^{-1}\right)$

Entire spectrum can be constructed out of multi-quasiparticle states

Dynamic Structure Factor $S(p,\omega)$:

Weakly-coupled qubits $(g \gg 1)$

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



Structure holds to all orders in $1/g$

At $T>0$, collisions between quasiparticles broaden pole to a Lorentzian of width $1/\tau_\varphi$ where the ***phase coherence time*** τ_φ is given by

$$\frac{1}{\tau_\varphi} = \frac{2k_BT}{\pi\hbar} e^{-\Delta/k_BT}$$

S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Strongly-coupled qubits $\left(g \ll 1\right)$

Ground states:

$$\left|G\uparrow\right\rangle = \left|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\right\rangle$$

$$-\frac{g}{2}\left|\cdots\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\right\rangle - \cdots$$

Ferromagnetic moment

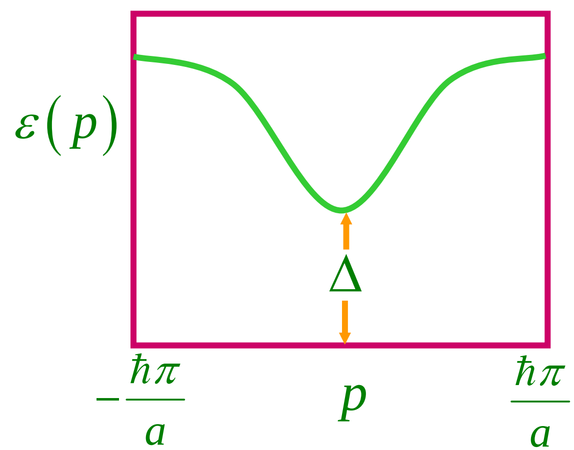
$$N_0 = \left\langle G\left|\sigma^z\right|G\right\rangle \neq 0$$

Second state $\left|G\downarrow\right\rangle$ obtained by $\uparrow \Leftrightarrow \downarrow$

$\left|G\downarrow\right\rangle$ and $\left|G\uparrow\right\rangle$ mix only at order g^N

Lowest excited states: domain walls

$$\left|d_j\right\rangle = \left|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow_j\downarrow\downarrow\downarrow\downarrow\downarrow\cdots\right\rangle + \cdots$$



Coupling between qubits creates new "domain-wall" *quasiparticle* states at momentum p

$$\left|p\right\rangle = \sum_j e^{ipx_j/\hbar}\left|d_j\right\rangle$$

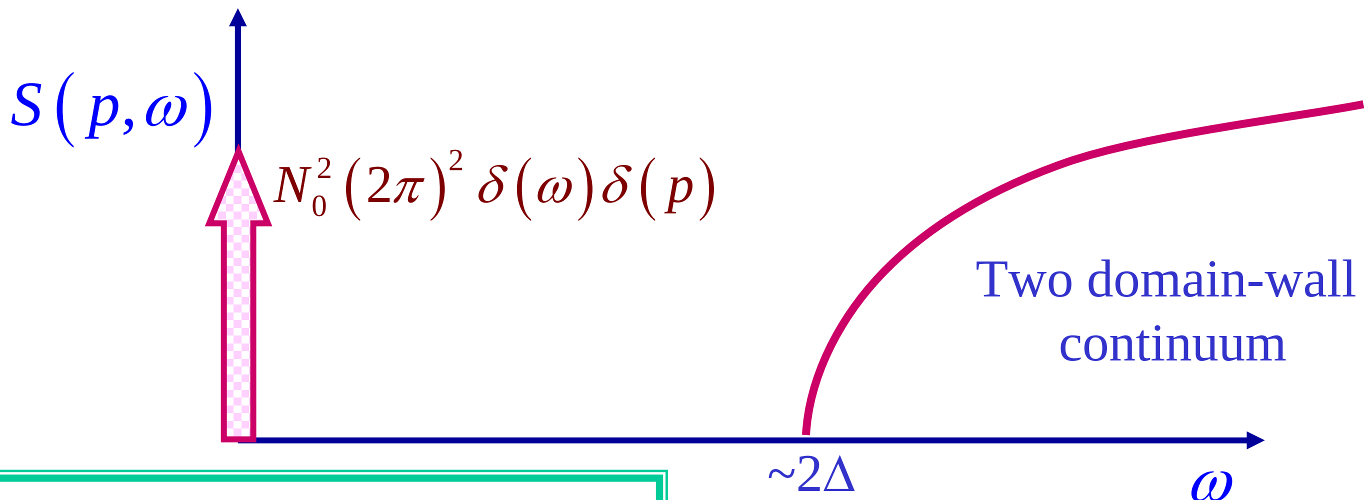
Excitation energy $\varepsilon\left(p\right) = \Delta + 4Jg\sin^2\left(\frac{pa}{2\hbar}\right) + O\left(g^2\right)$

Excitation gap $\Delta = 2J - 2gJ + O\left(g^2\right)$

Dynamic Structure Factor $S(p,\omega)$:

Strongly-coupled qubits $(g \ll 1)$

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



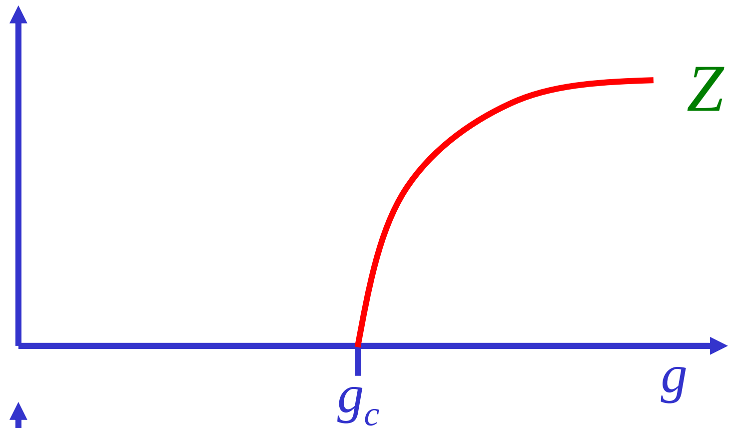
Structure holds to all orders in g

At $T > 0$, motion of domain walls leads to a finite ***phase coherence time*** τ_φ, and broadens coherent peak to a width $1/\tau_\varphi$ where $$\frac{1}{\tau_\varphi} = \frac{2k_BT}{\pi\hbar} e^{-\Delta/k_BT}$$

S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Entangled states at g of order unity

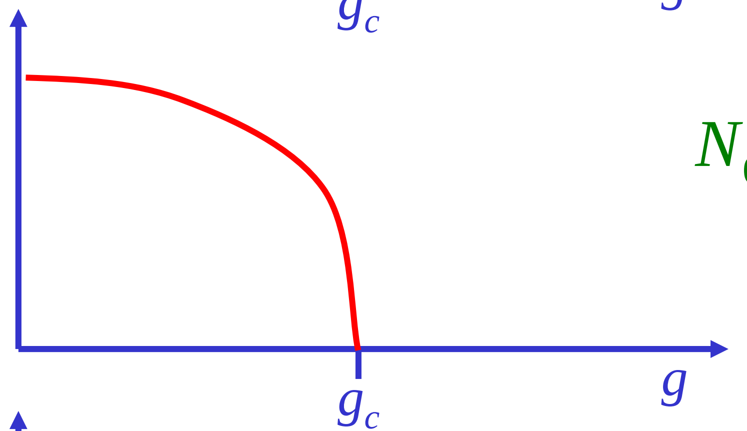
"Flipped-spin" Quasiparticle weight Z



$$Z \sim (g - g_c)^{1/4}$$

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

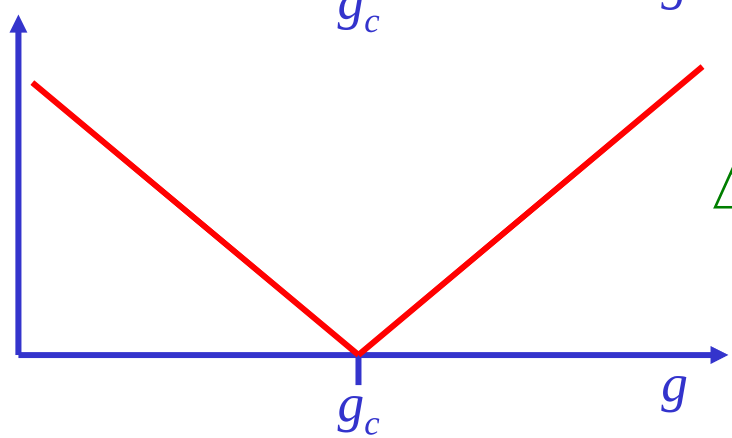
Ferromagnetic moment N_0



$$N_0 \sim (g_c - g)^{1/8}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

Excitation energy gap Δ

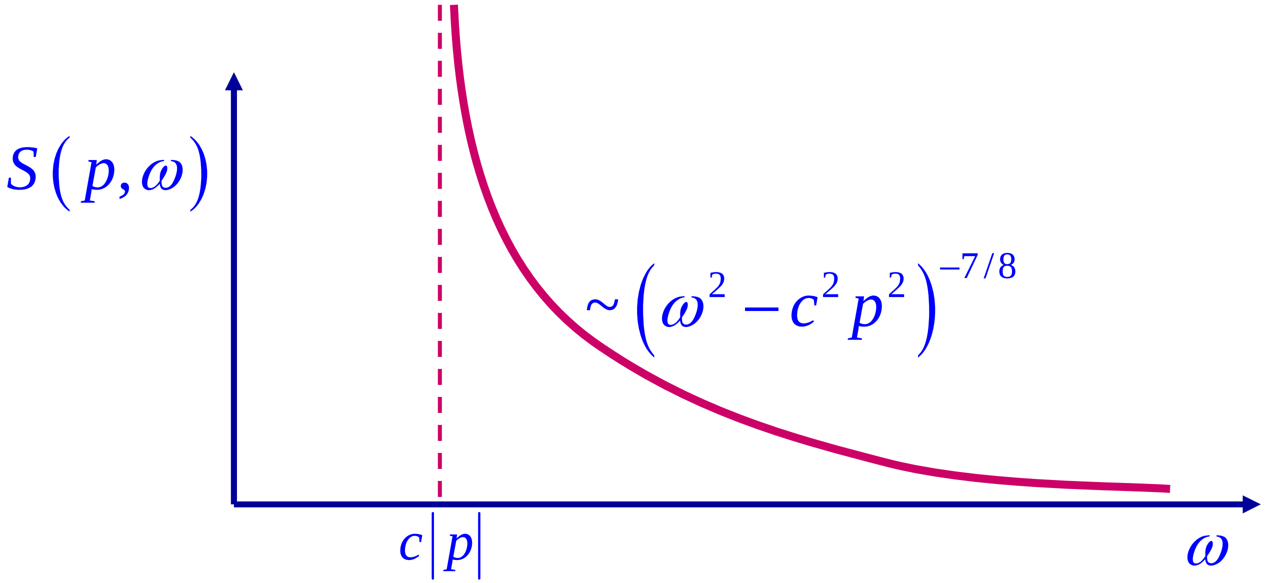


$$\Delta \sim |g - g_c|$$

Dynamic Structure Factor $S(p,\omega)$:

Critical coupling $(g = g_c)$

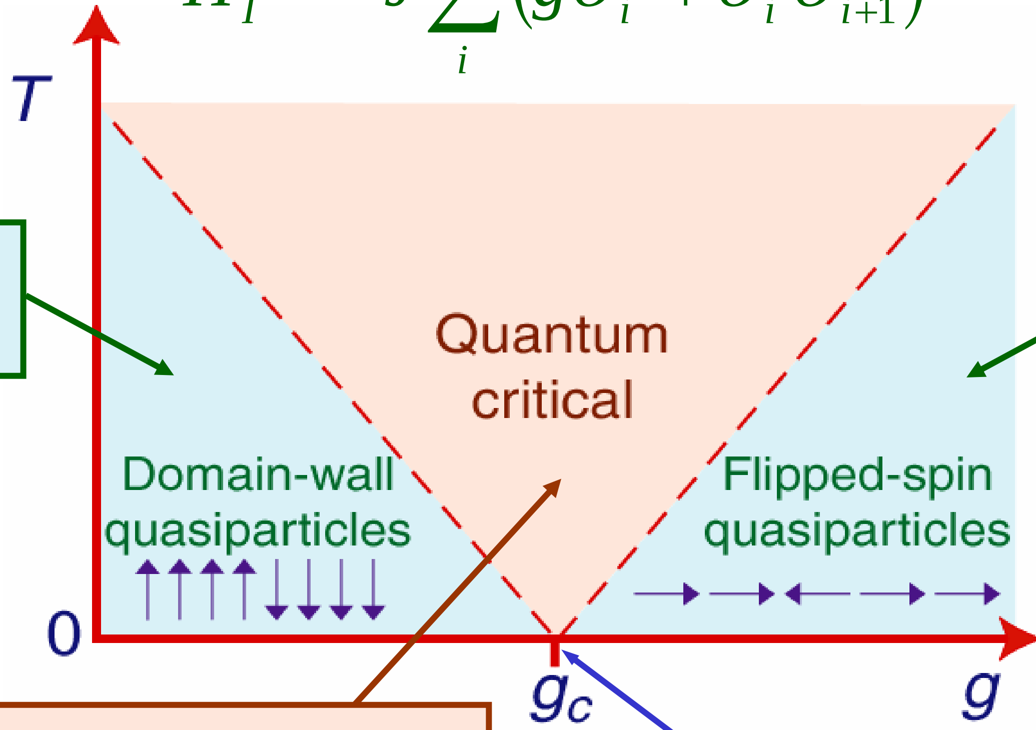
Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



No quasiparticles --- dissipative critical continuum

$$H_I = -J \sum_i \left(g\sigma_i^x + \sigma_i^z \sigma_{i+1}^z \right)$$

Quasiclassical dynamics



Quasiclassical dynamics

$$\chi(\omega) = \frac{i}{\hbar} \sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4} \left(1 - i\omega/\Gamma_R + \ldots\right)}$$

$$\Gamma_R = \left(2 \tan \frac{\pi}{16} \right) \frac{k_B T}{\hbar}$$

$$\left\langle \sigma_j^z \sigma_k^z \right\rangle \sim \frac{1}{|j-k|^{1/4}}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

S. Sachdev and J. Ye, Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young, Phys. Rev. Lett. **78**, 2220 (1997).

Outline

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II. Landau-Ginzburg-Wilson theory

Mean field theory and the evolution of the excitation spectrum

- Identify order parameter $\phi(x,\tau) \sim \sigma_j^z$
- Symmetries:

$$\begin{aligned} \text{Spin inversion:} &\quad \phi \to -\phi \\ \text{Time reversal} &\quad \tau \to -\tau \\ \text{Spatial inversion} &\quad x \to -x \end{aligned}$$

- Write down most general Lagrangian consistent with symmetries

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\phi(x,\tau) \exp\left(-\int d^d x \int d\tau \mathcal{L}[\phi]\right) \\ \mathcal{L}[\phi] &= \frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla_x \phi)^2 + \frac{r}{2}\phi^2 + \frac{u}{4}\phi^4 + \ldots \end{aligned}$$

- Identify phases at $r \gg 0$ and $r \ll 0$ with the paramagnet and the ferromagnet respectively.

Quantum field theory formally resembles the classical statistical mechanics of an Ising model in $d+1$ dimensions. Theory of second-order classical phase transitions implies that at the critical point the susceptibility depends on the $d+1$ dimensional momentum k as

$$\chi(k) \sim \frac{1}{k^{2-\eta}}$$

After analytic continuation, and using the "Lorentz invariance" of the critical theory, the quantum critical point therefore has the following dynamic susceptibility at $T = 0$.

$$\chi(p, \omega) \sim \frac{1}{(c^2 p^2 - \omega^2)^{1-\eta/2}}$$

At $T > 0$, we have to consider a classical statistical mechanics problem in finite geometry with a 'temporal' direction of extent $L_\tau = \hbar/(k_B T)$. *Finite size scaling* now implies that the susceptibility at the critical point obeys

$$\chi(k) \sim L_\tau^{2-\eta} F\left(k L_\tau\right)$$

After analytic continuation, the quantum system has the dynamic response (note: can no longer use "Lorentz invariance")

$$\chi''(0, \omega) \sim \frac{1}{T^{2-\eta}} \Phi\left(\frac{\hbar\omega}{k_B T}\right)$$

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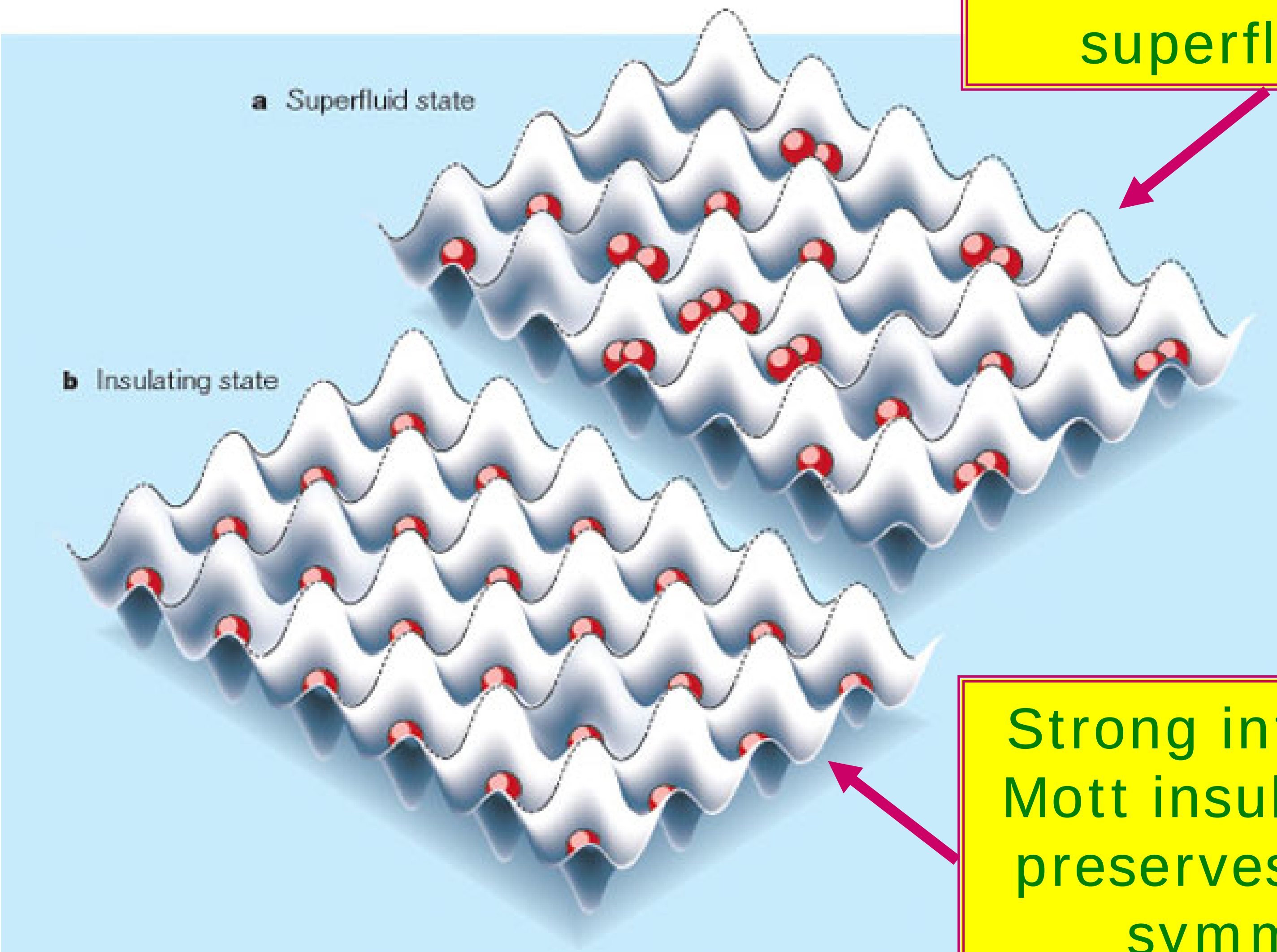
III. Superfluid-insulator transition

Boson Hubbard model at integer filling

Bosons at density $f = 1$

Weak interactions: superfluidity



Strong interactions: Mott insulator which preserves all lattice symmetries

LGW theory: continuous quantum transitions between these states

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

I. The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

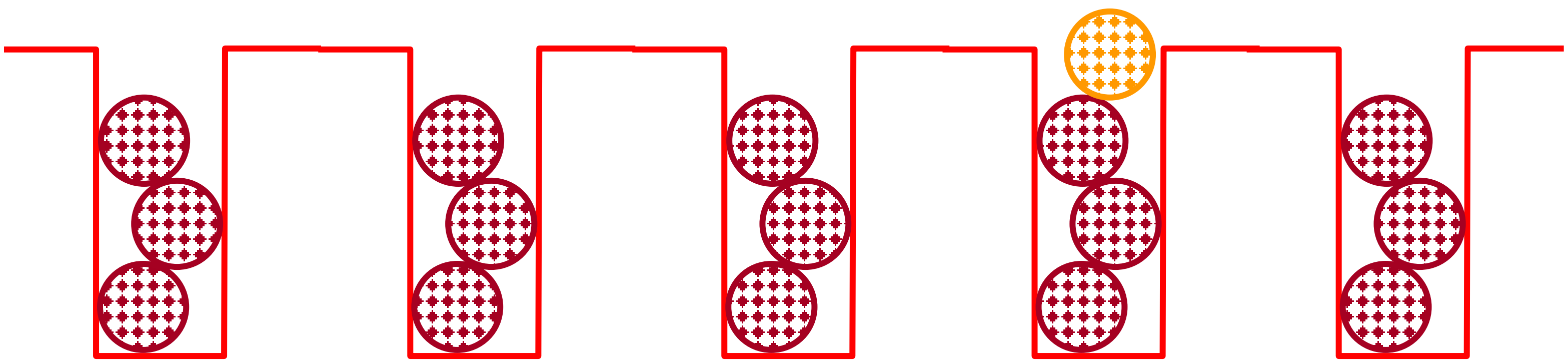
M.PA. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher *Phys. Rev. B* **40**, 546 (1989).

For small *U/t*, ground state is a superfluid BEC with superfluid density $\approx$ density of bosons

What is the ground state for large *U/t* ?

Typically, the ground state remains a superfluid, but with

superfluid density $\ll$ density of bosons

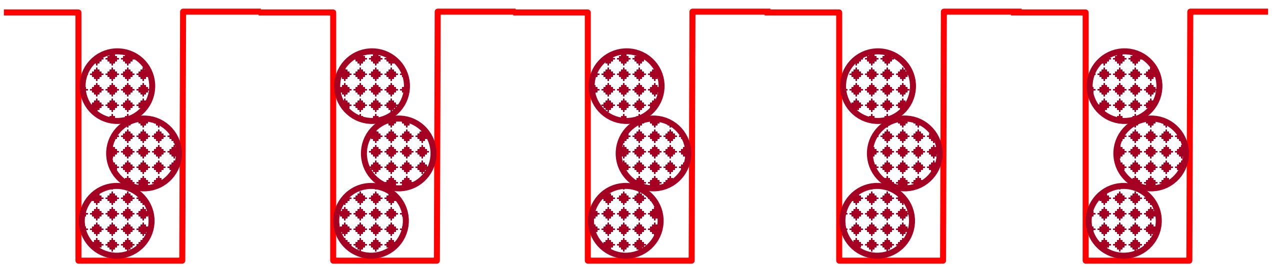


The superfluid density evolves smoothly from large values at small *U/t*, to small values at large *U/t*, and there is no quantum phase transition at any intermediate value of *U/t*.

(In systems with Galilean invariance and at zero temperature, superfluid density=density of bosons always, independent of the strength of the interactions)

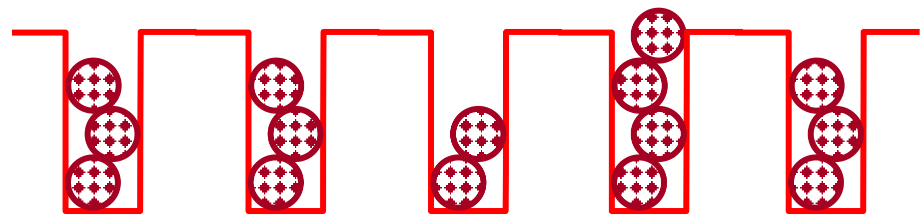
What is the ground state for large *U/t* ?

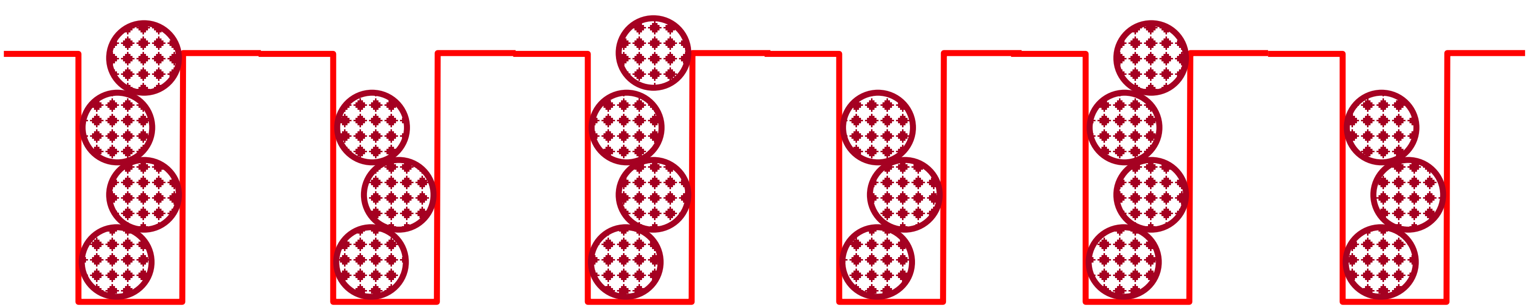
Incompressible, insulating ground states, with zero superfluid density, appear at special commensurate densities



$$\langle n_j \rangle = 3$$

$$-\frac{t}{U}$$

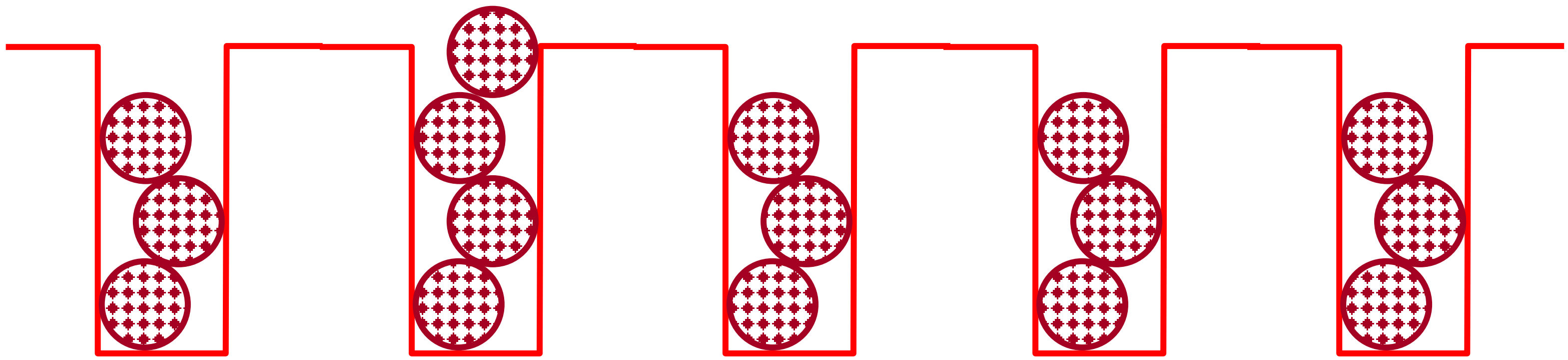




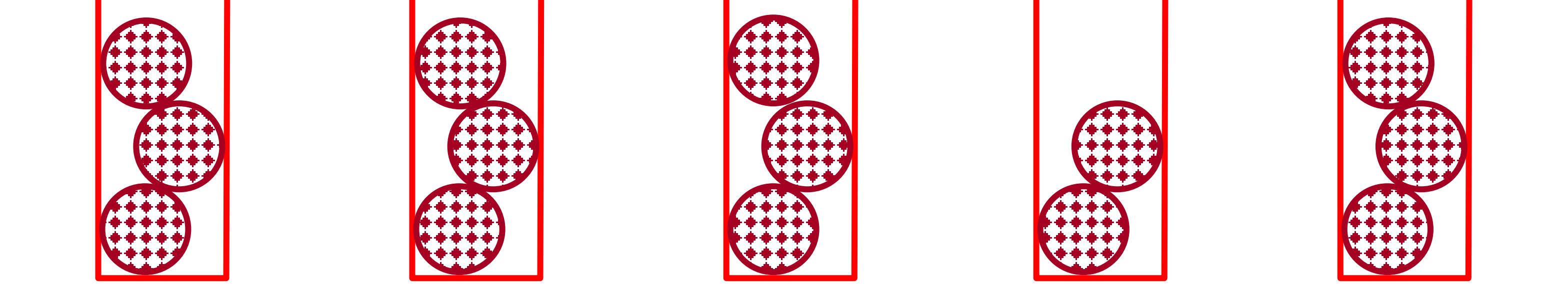
$$\langle n_j \rangle = 7/2$$

Ground state has “density wave” order, which spontaneously breaks lattice symmetries

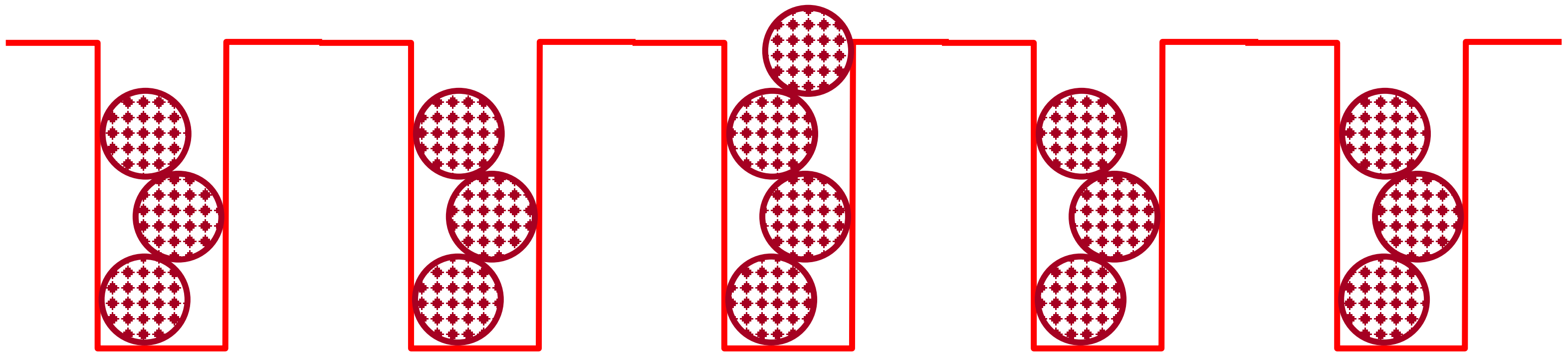
Excitations of the insulator: infinitely long-lived, finite energy *quasiparticles* and *quasiholes*



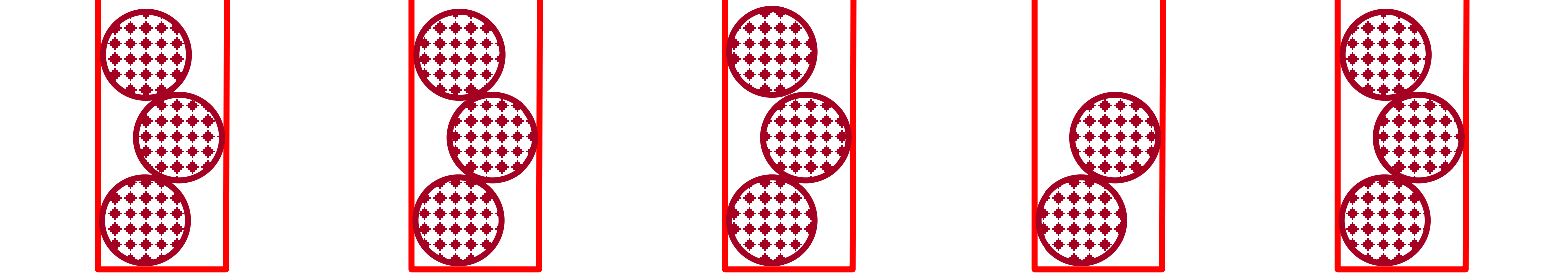
Energy of quasi-particles/holes: $\varepsilon_{p,h}(p) = \Delta_{p,h} + \frac{p^2}{2m^*_{p,h}}$



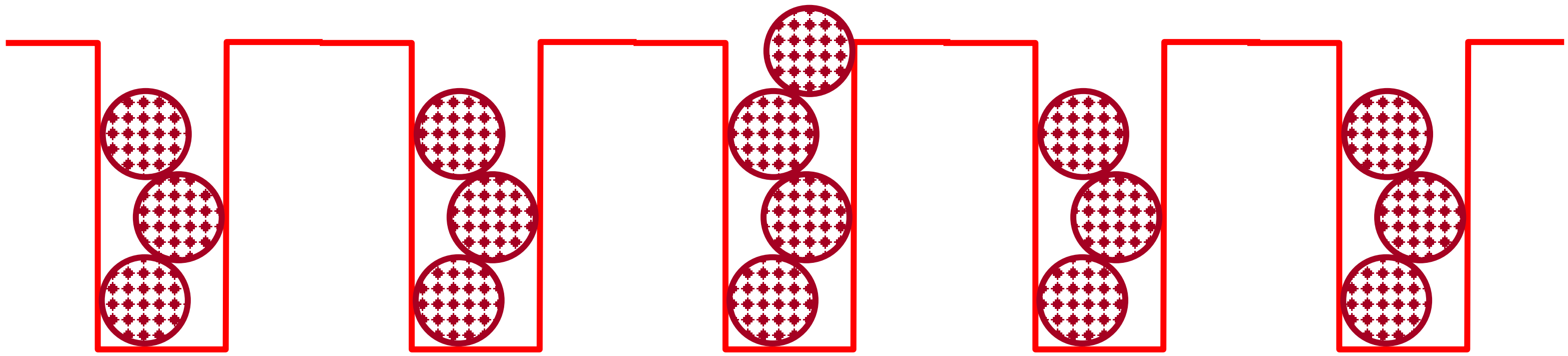
Excitations of the insulator: infinitely long-lived, finite energy *quasiparticles* and *quasiholes*



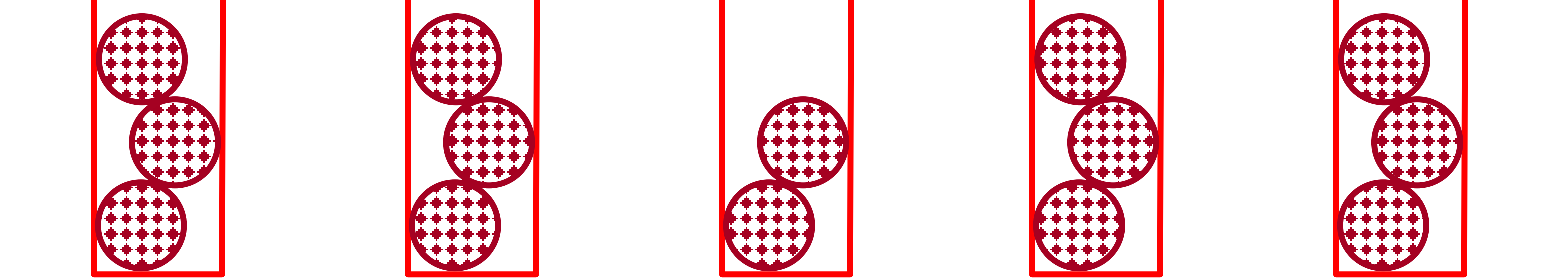
Energy of quasi-particles/holes: $\varepsilon_{p,h}(p) = \Delta_{p,h} + \frac{p^2}{2m^*_{p,h}}$



Excitations of the insulator: infinitely long-lived, finite energy *quasiparticles* and *quasiholes*



Energy of quasi-particles/holes: $\varepsilon_{p,h}(p) = \Delta_{p,h} + \frac{p^2}{2m^*_{p,h}}$



LGW theory of the superfluid insulator transition

- Identify order parameter $\Psi(x,\tau) \sim b_j^\dagger$
- Symmetries:

$$\begin{aligned} \text{Gauge invariance:} \quad & \Psi \rightarrow \Psi e^{i\theta} \\ \text{Time reversal} \quad & \tau \rightarrow -\tau \;\; ; \;\; \Psi \rightarrow \Psi^* \\ \text{Spatial inversion} \quad & x \rightarrow -x \end{aligned}$$

- Write down most general Lagrangian consistent with symmetries

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\Psi(x,\tau) \exp\left(- \int d^d x \int d\tau \mathcal{L}\left[\Psi\right] \right) \\ \mathcal{L}\left[\Psi\right] &= K\Psi^* \frac{\partial \Psi}{\partial \tau} + \left|\partial_\tau \Psi\right|^2 + c^2 \left|\nabla_x \Psi\right|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \ldots \end{aligned}$$

- Identify phases at $r \gg 0$ and $r \ll 0$ with the insulator and the superfluid respectively.
- For $K \neq 0$, the particle and hole excitations have different energies.

- Gauge-invariance of the underlying boson Hamiltonian shows that

$$K = -\frac{\partial r}{\partial \mu}$$

- In mean-field theory, the ground state energy, E, across the superfluid-insulator transition has the non-analytic term

$$E = E_0 - \frac{r^2}{2u}\theta(-r)$$

(Beyond mean-field theory, the non-analytic term is $E \sim r^{(d+z)\nu}$).

- Because the density of bosons $= -\partial E/\partial \mu$, this implies a change in the boson density across the transition *unless* $\partial r/\partial \mu = 0$

- A superfluid-insulator transition at fixed boson density must have.

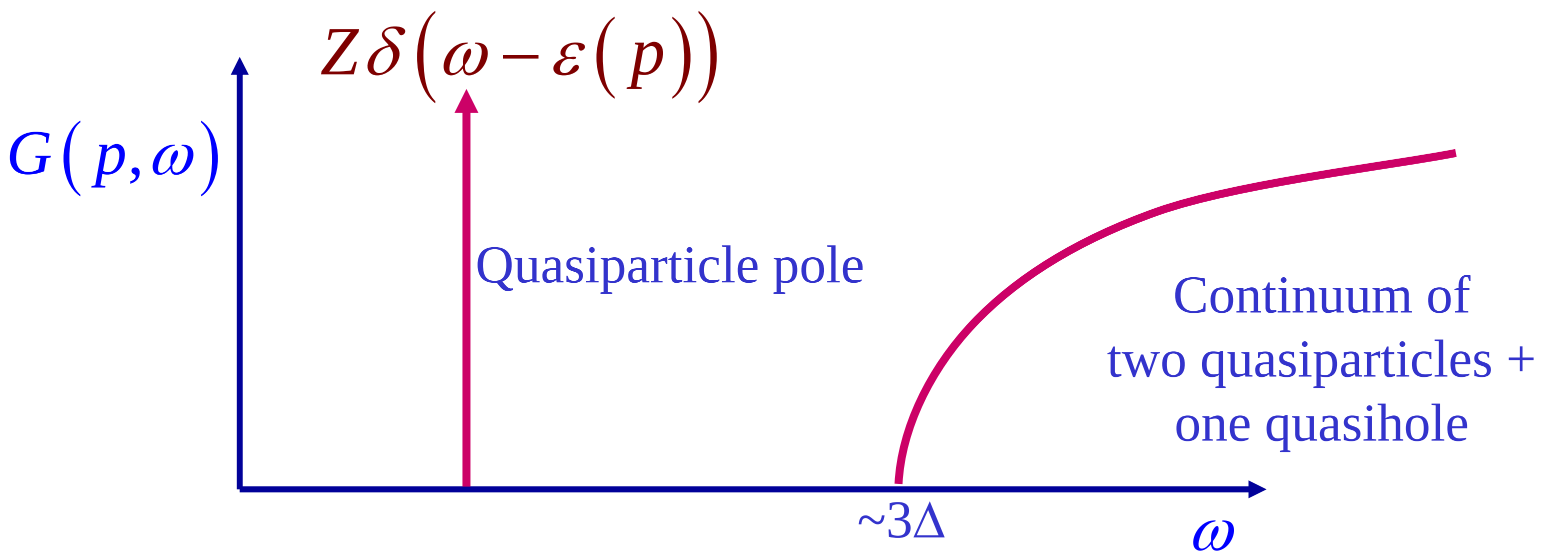
$$K = 0$$

Boson Green's function $G(p,\omega)$:

Insulating ground state

Cross-section to add a boson

while transferring energy $\hbar\omega$ and momentum p



Similar result for quasi-hole excitations obtained by removing a boson

Entangled states at $g \equiv t/U$ of order unity

Quasiparticle weight Z

$$Z \sim \left(g_c - g\right)^{\eta\nu}$$

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

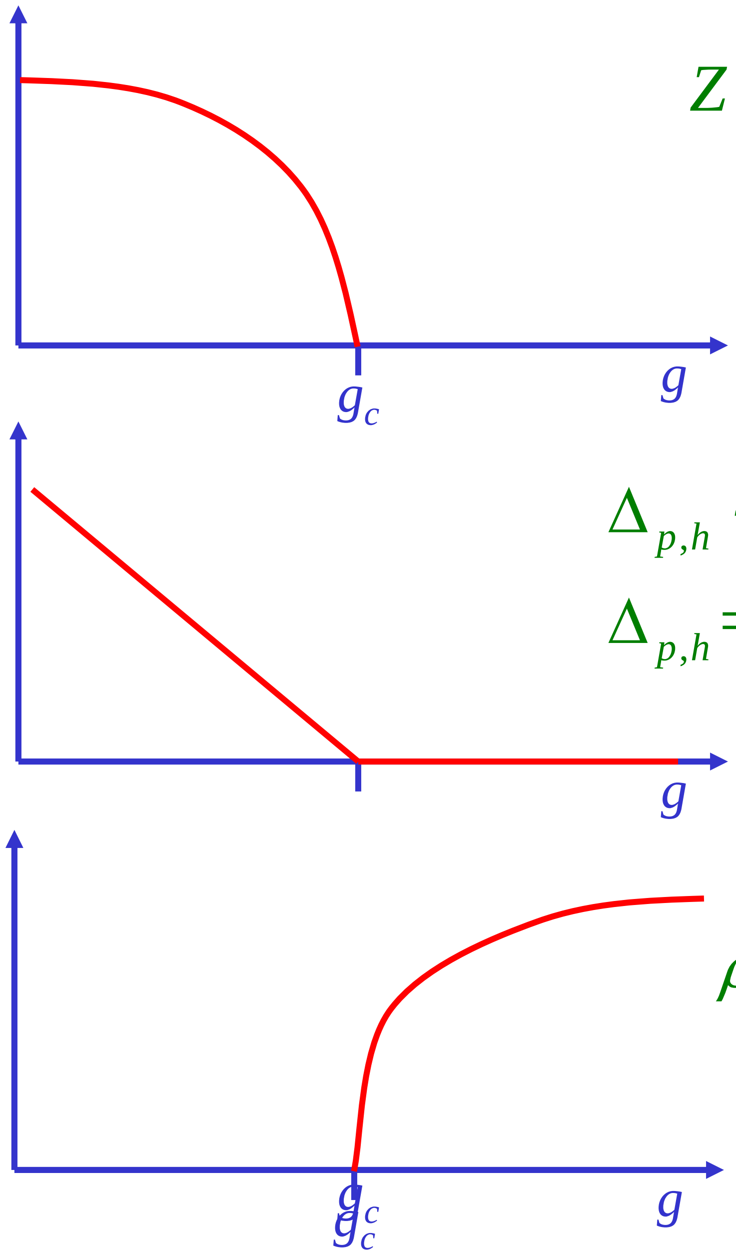
Excitation energy gap Δ

$$\Delta_{p,h} \sim \left(g_c - g\right)^{\nu} \text{ for } g < g_c$$

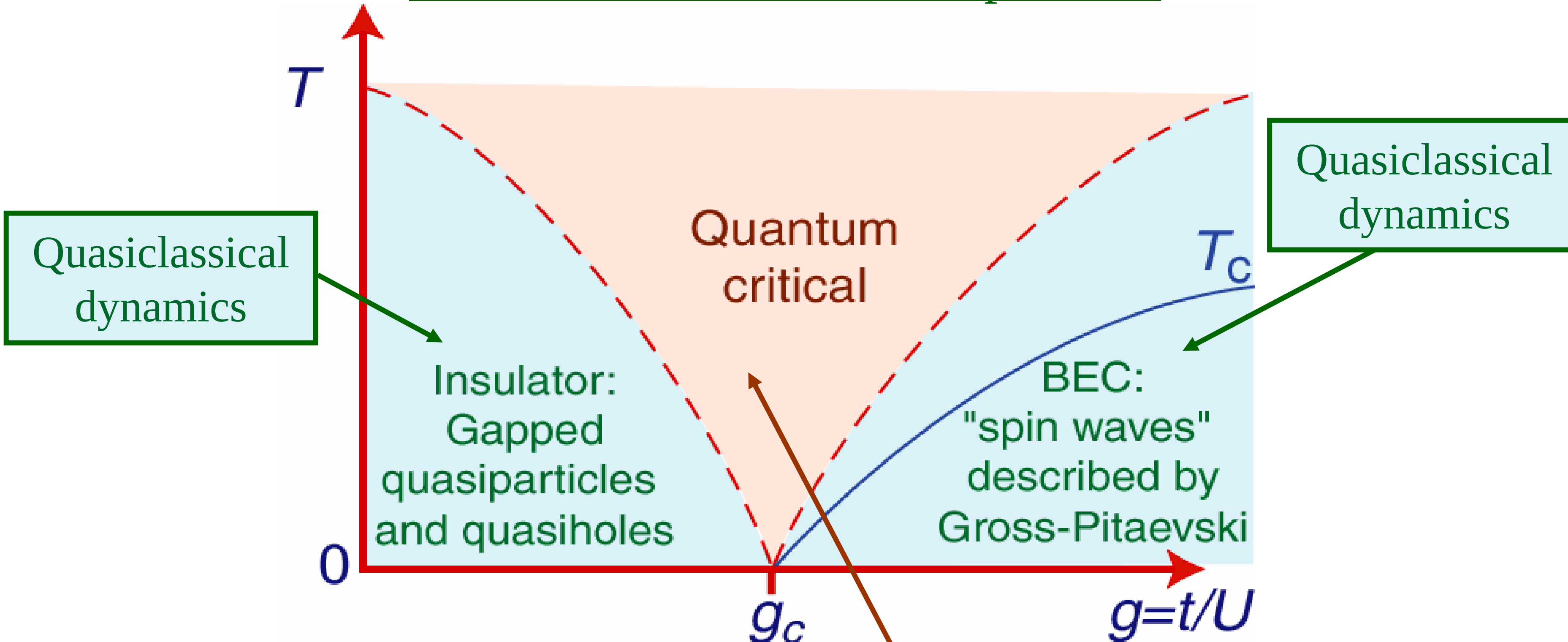
$$\Delta_{p,h} = 0 \text{ for } g > g_c$$

Superfluid density ρ_s

$$\rho_s \sim \left(g - g_c\right)^{(d+z-2)\nu}$$



Crossovers at nonzero temperature



Relaxational dynamics ("Bose molasses") with phase coherence/relaxation time τ_φ given by

$$\frac{1}{\tau_\varphi} = \left(\text{Universal number}\right)\frac{k_B T}{\hbar} \quad \left(1\mu\text{K} = 20.9\text{kHz}\right)$$

S. Sachdev and J. Ye, *Phys. Rev. Lett*. **69**, 2411 (1992).
K. Damle and S. Sachdev *Phys. Rev*. B **56**, 8714 (1997).

Conductivity (in d=2) = $\frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right)$ $\quad \Sigma \rightarrow$ universal function

M.P.A. Fisher, G. Girvin, and G. Grinstein, *Phys. Rev. Lett*. **64**, 587 (1990).
K. Damle and S. Sachdev *Phys. Rev*. B **56**, 8714 (1997).

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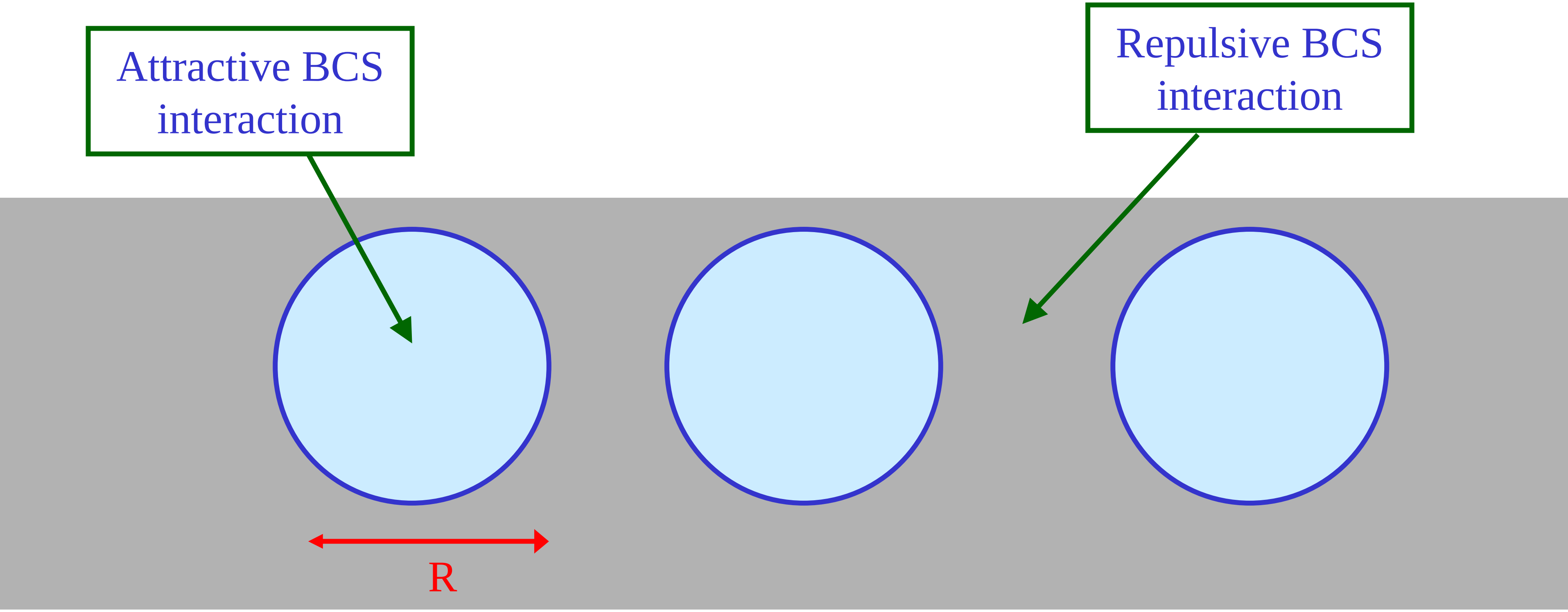
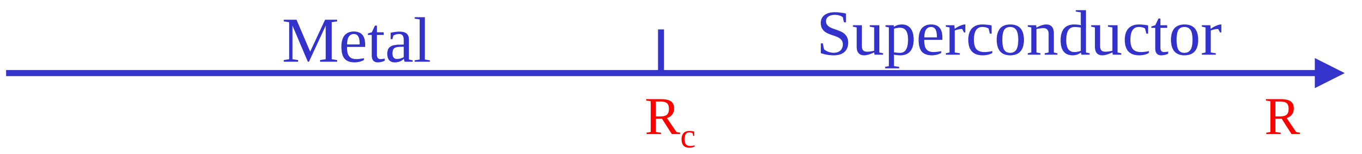
IV. Superconductor-metal transition in nanowires

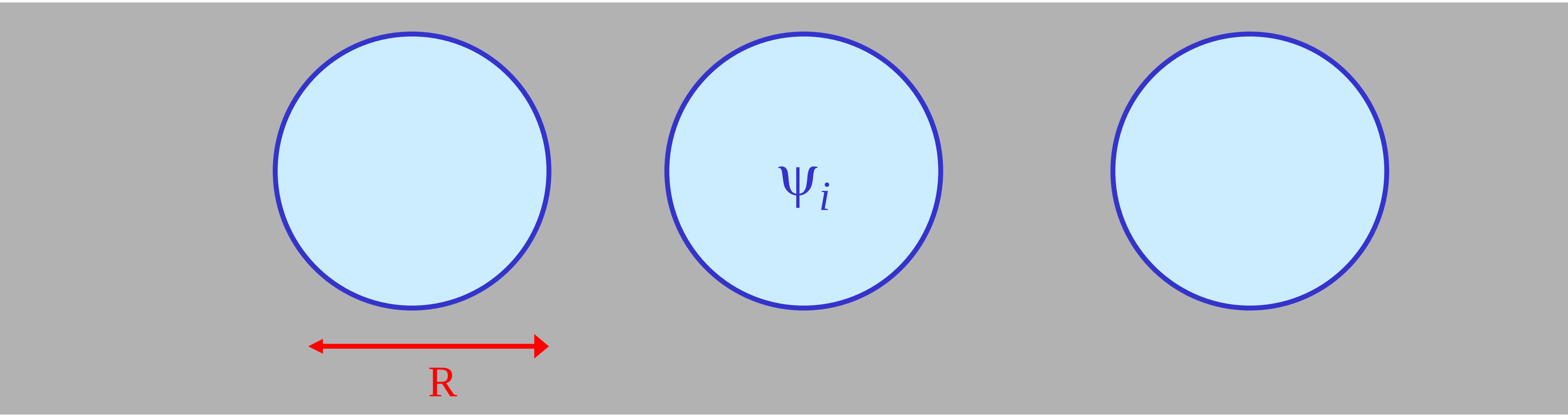
T=0 Superconductor-metal transition





M.V. Feigel'man and A.I. Larkin, *Chem. Phys*. **235**, 107 (1998)
B. Spivak, A. Zyuzin, and M. Hruska, *Phys. Rev*. B **64**, 132502 (2001).

T=0 Superconductor-metal transition



$$S = -\int d\tau \sum_{i,j} J_{ij}\psi_i^*(\tau)\psi_j(\tau) - \int d\tau d\tau' \sum_i \frac{\psi_i^*(\tau)\psi_i(\tau')}{(\tau-\tau')^2}$$

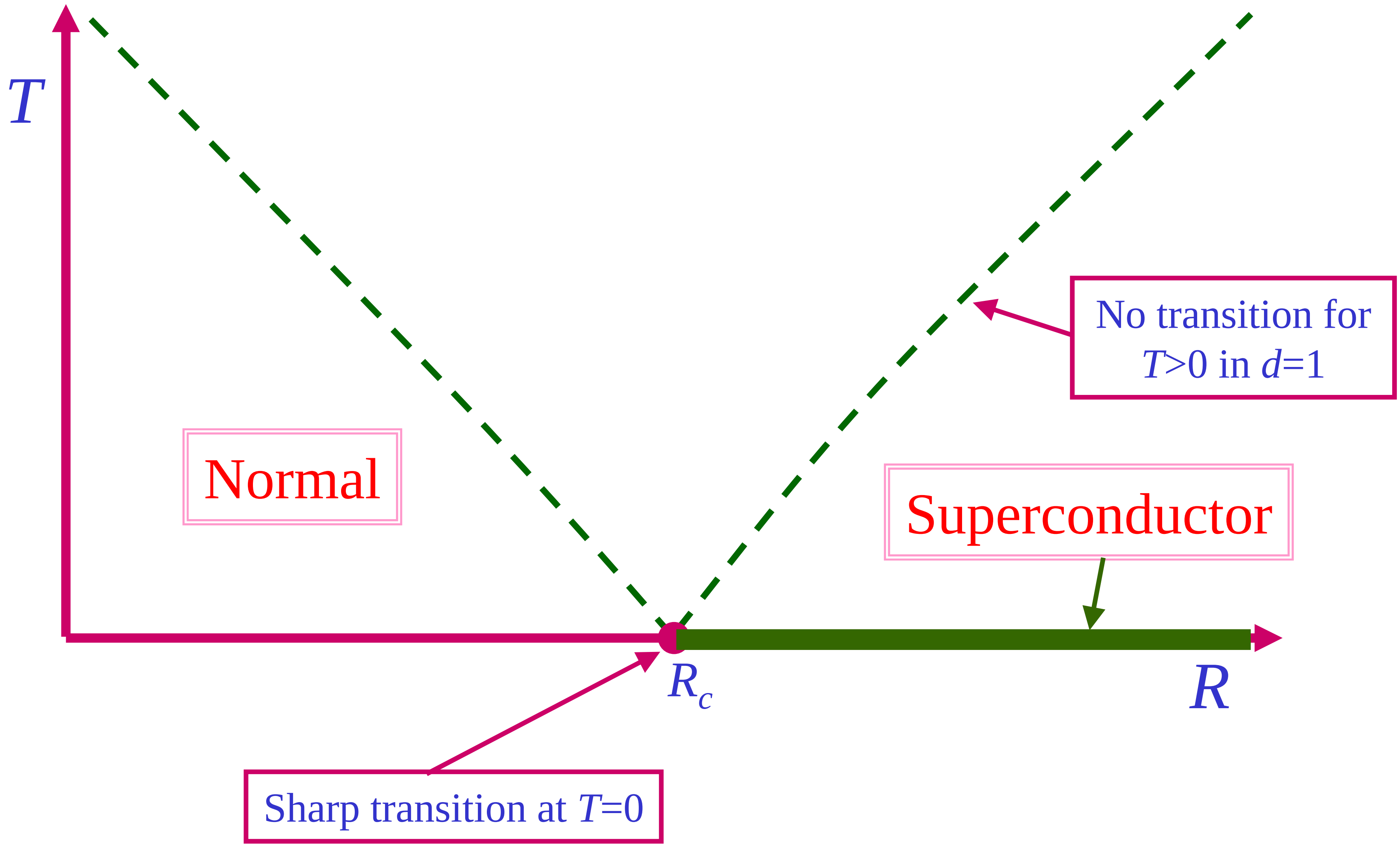
Continuum theory for quantum critical point

$$\mathcal{S}_{\text{bulk}} = \frac{A}{\hbar}\int_0^L dx \left[\int_0^\beta d\tau \left(\delta|\partial_x\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4\right) + \frac{\hbar\gamma}{\beta}\sum_{\omega_n}|\omega_n||\psi(x,\omega_n)|^2\right],$$

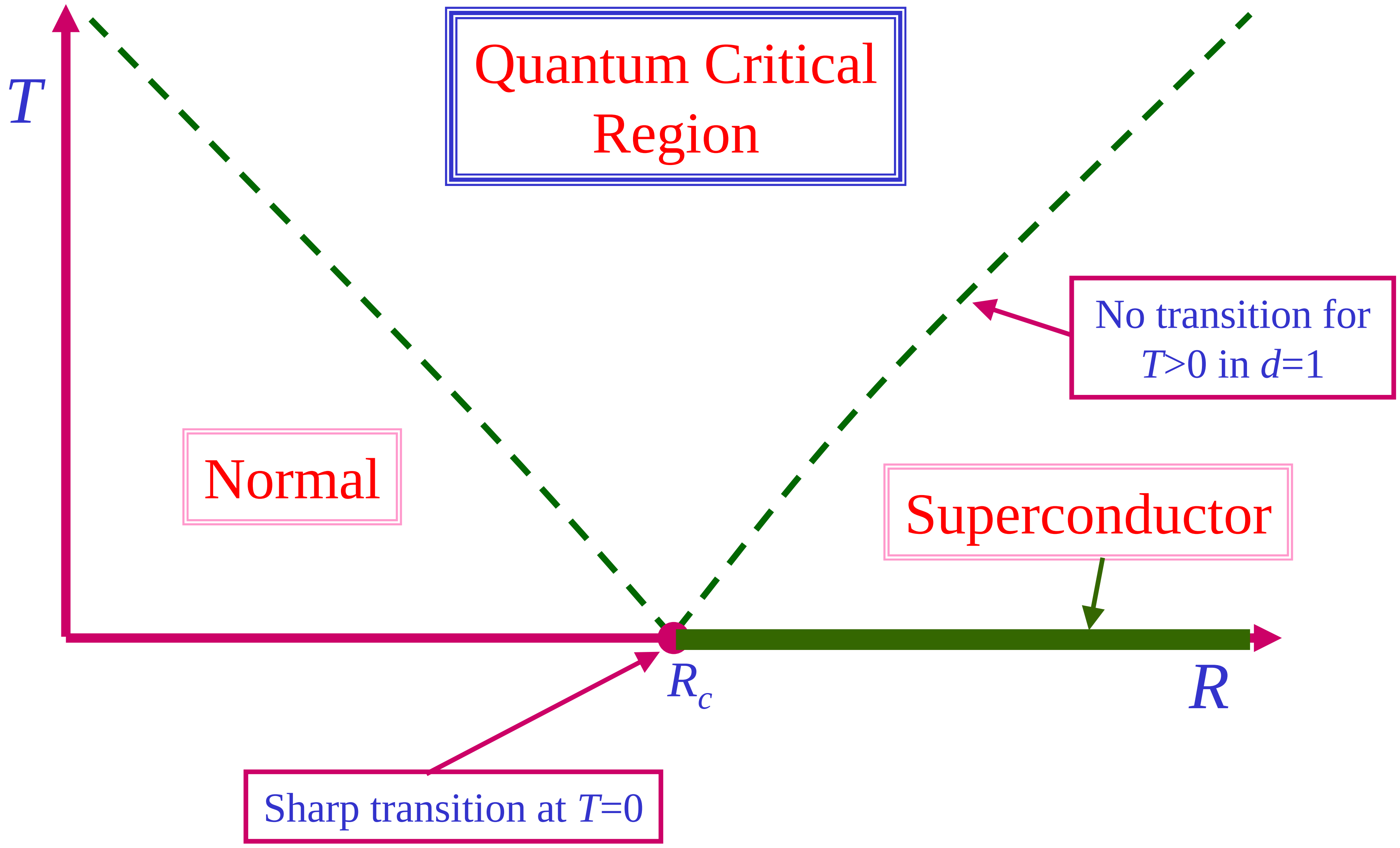
Obeys strong hyperscaling properties in spatial dimensions $d < 2$.
Critical properties can be determined by an expansion in $\epsilon = 2 - d$ in a theory with n-component fields ($n = 2$ here).

$$\begin{aligned} z &= 2 - \eta \\ \eta &= \frac{(n+2)(12-\pi^2)}{4(n+8)^2}\epsilon^2 \\ \nu &= \frac{1}{2} + \frac{(n+2)}{4(n+8)}\epsilon + \frac{(n+2)(n^2+(38-7\pi^2/6)n+132-19\pi^2/3)}{8(n+8)^3}\epsilon^2 \end{aligned}$$

Consequences of hyperscaling



Consequences of hyperscaling



Consequences of hyperscaling

Quantum Critical Region

The conductance g obeys

$$g = \frac{4e^2}{h} \Phi \left(c_1 T L^z, \frac{\hbar\omega}{k_B T} \right)$$

where Φ is a universal function and only constant c_1 is non-universal.

For $L > (c_1 T)^{-1/z}$, we have hydrodynamic, "incoherent" transport and $g = \sigma / L$, where σ is the conductivity which is *independent of the leads* and obeys

$$\sigma = \frac{4e^2}{h} \frac{1}{(c_1 T)^{1/z}} \Phi_1 \left(\frac{\hbar\omega}{k_B T} \right)$$

Consequences of hyperscaling

Quantum Critical Region

The conductance g obeys

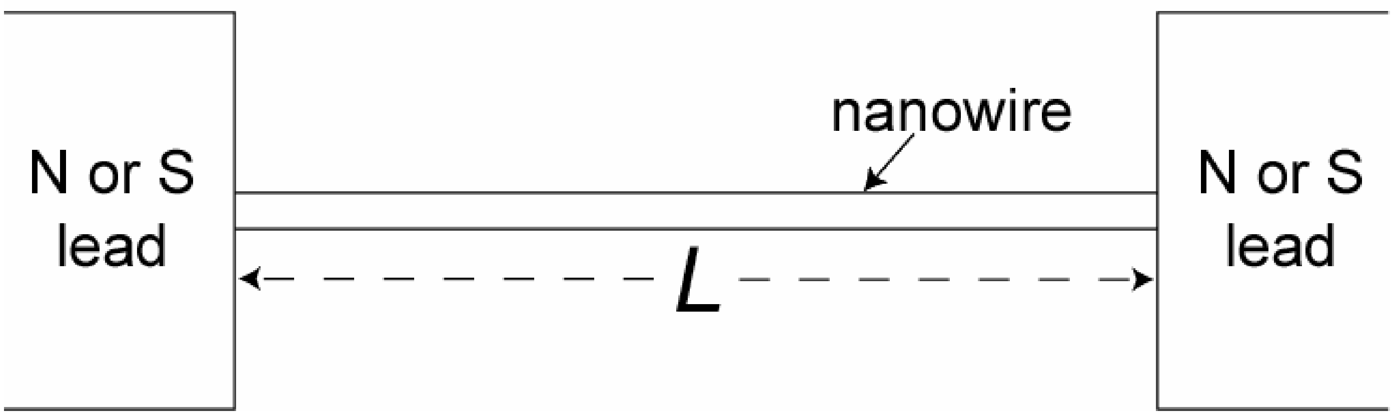
$$g = \frac{4e^2}{h}\Phi\left(c_1 T L^z, \frac{\hbar\omega}{k_B T}\right)$$

where Φ is a universal function and only constant c_1 is non-universal.

For $L < (c_1 T)^{-1/z}$, we have "coherent" transport, and the d.c. conductance is independent of L, but sensitive to the nature of the leads.

$$g = \frac{4e^2}{h}F\left(c_1 \omega L^z\right)$$

Effect of the leads



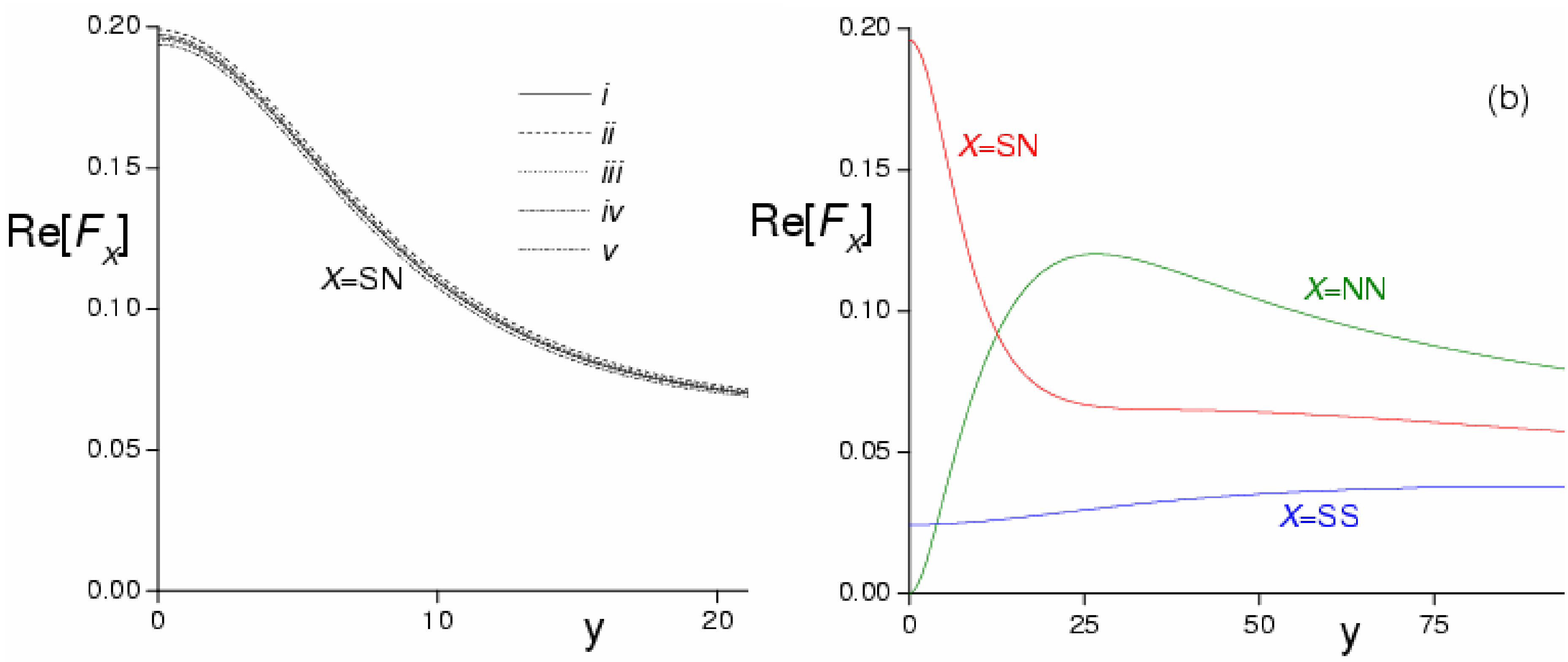
$$\mathcal{S}_{\text{lead}} = \int d\tau \left[-H^* \psi(0,\tau) - H\psi^*(0,\tau) + C|\Psi(0,\tau)|^2 \right]$$

where $H \neq 0$ for a superconducting lead.

Both H and C scale to strong-coupling, and therefore we have Dirichlet boundary conditions ($\Psi = 0$) for a N lead, and Fixed boundary conditions for a S lead

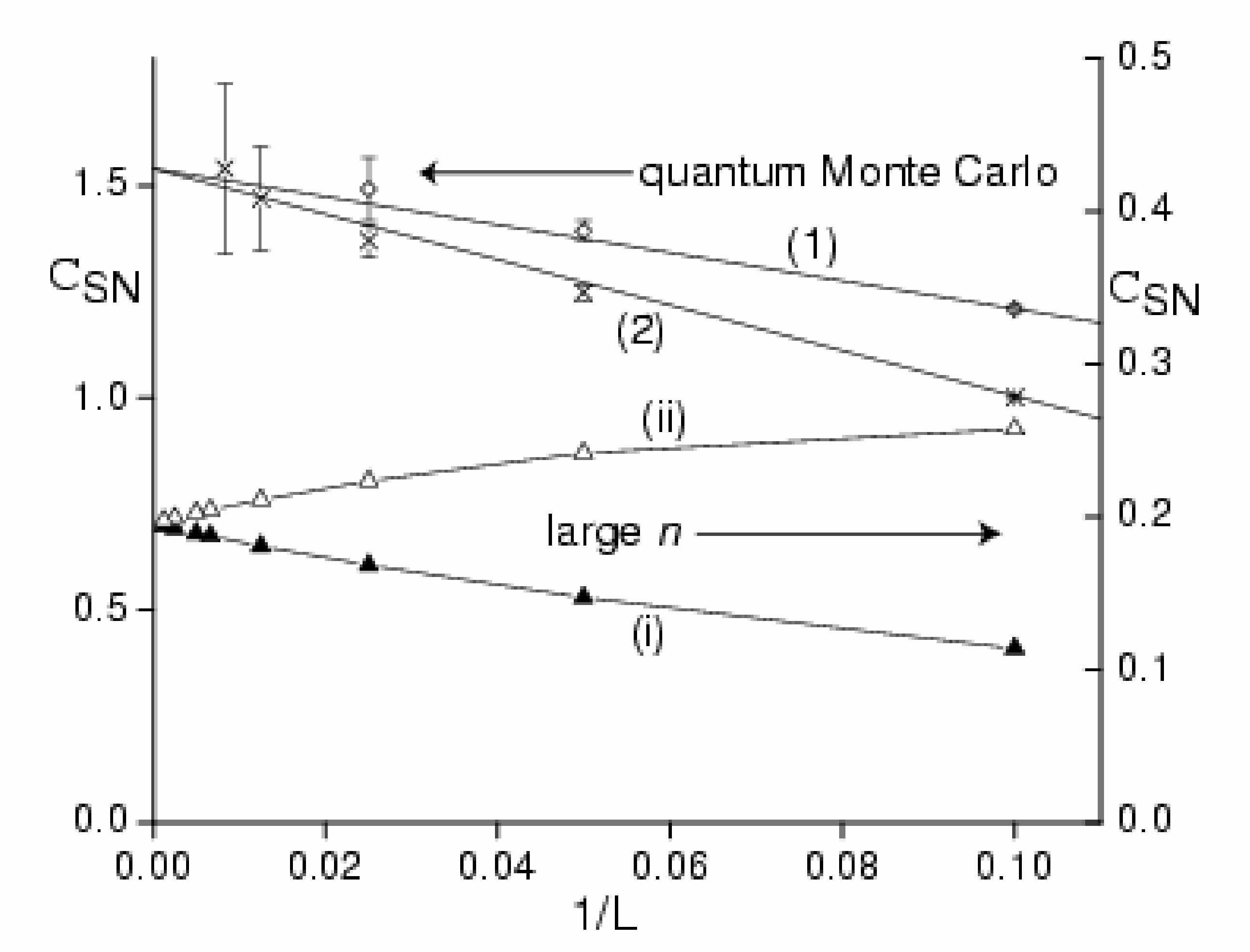
Conductance is *independent* of the specific bare values of H and C.

Large *n* computation of conductance



$$g = \frac{4e^2}{h} F_X(y) \quad ; \quad y = c_1 \omega L^z$$

Quantum Monte Carlo and large n computation of d.c. conductance



$$g = \frac{4e^2}{h} C_{SN}$$

Conclusions

- Universal transport in wires near the superconductor-metal transition
- Theory includes contributions from thermal and quantum phase slips ---- reduces to the classical LAMH theory at high temperatures
- Sensitivity to leads should be a generic feature of the ``coherent'' transport regime of quantum critical points.