The onset of spin density wave order in metals

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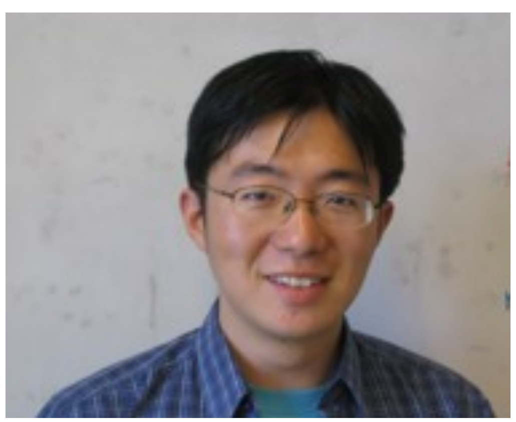
Max Metlitski, Harvard



Cenke Xu

Harvard → UCSB

Yang Qi

Harvard → Tsinghua

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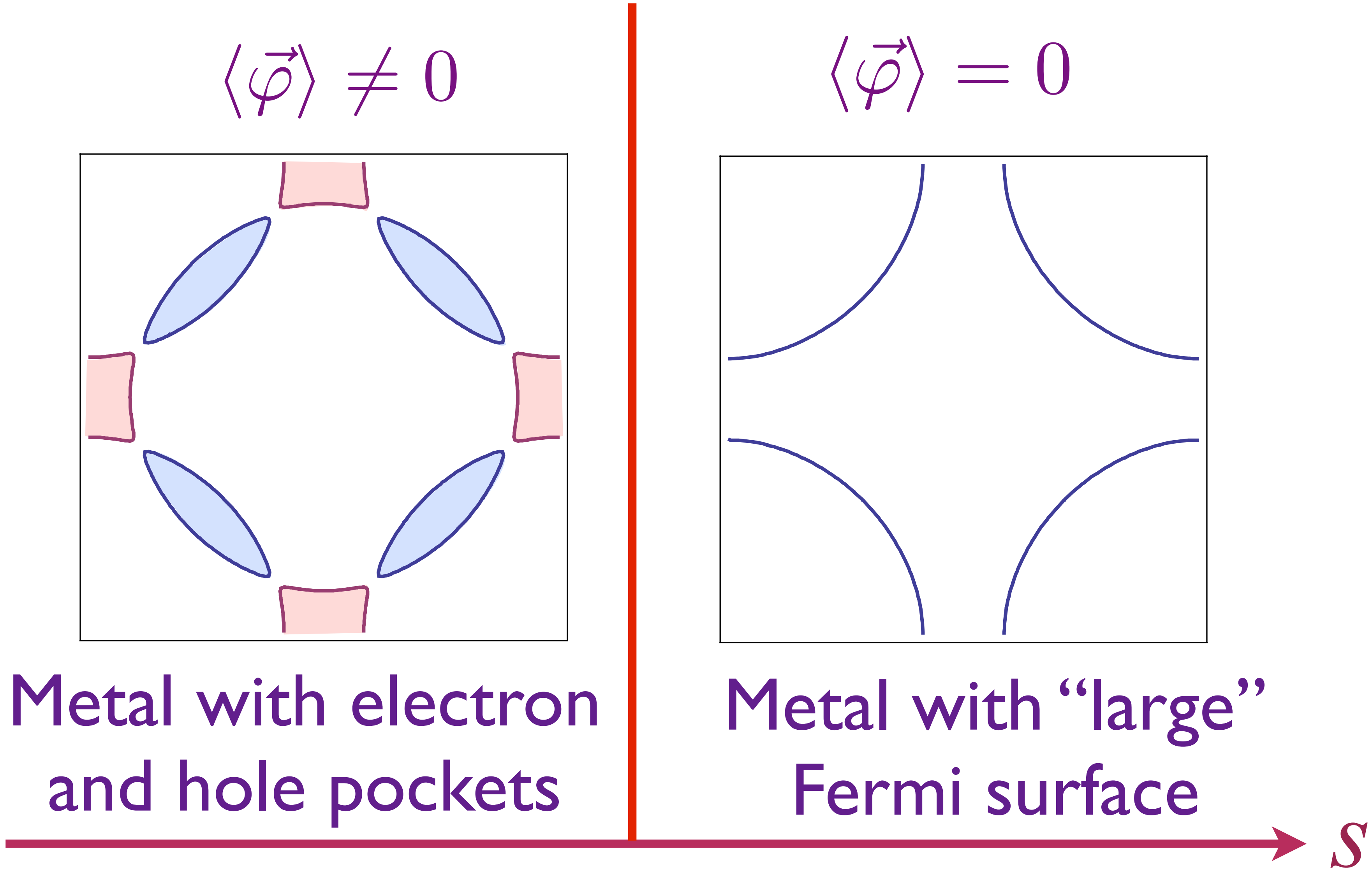
Quantum criticality of the onset of antiferromagnetism in a metal



Outline

1. Formulation of general theory

   Global phase diagram of a SU(2) gauge theory

2. Field theory for a direct transition between two Fermi liquids

   From a large Fermi surface to Fermi pockets

3. Instabilities to other orders

   Unconventional pairing, pseudospin symmetry, and bond order

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A convenient starting point: the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$

To explore the full range of phases at strong coupling, it is useful to replace the SDW order parameter $\vec{\varphi}$ by a fixed length field $\vec{n}$, with $\vec{n}^2 = 1$:

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}c_\alpha \mathcal{D}\vec{n}\,\delta\left(\vec{n}^2 - 1\right)\exp\left(-\mathcal{S}\right) \\
\mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha}\left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}\right) c_{\mathbf{k}\alpha} \\
&\quad - \lambda \int d\tau \sum_i c^\dagger_{i\alpha}\vec{n}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\
&+ \int d\tau d^2r \frac{1}{2g}\left[\left(\boldsymbol{\nabla}_r \vec{n}\right)^2 + \frac{1}{c^2}\left(\partial_\tau \vec{n}\right)^2\right]
\end{aligned}
$$

Now g is the tuning parameter across the quantum phase transition. This allows discussion of exotic phases in which there is local antiferromagnetic order (and so a local gap in the fermion spectrum), but no global order. Such phases require suppression of '*hedgehog*' tunneling events in $\vec{n}$.

Write $\vec{n} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$, and transform fermions to a "rotating reference frame", quantizing spins in the direction of the local antiferromagnetic order:

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

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$\mathrm{SU}(2)_{\mathrm{spin}}$

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$\mathrm{U}(1)_{\mathrm{charge}}$

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$U \times U^{-1}$

$\mathrm{SU(2)}_{\mathrm{s;gauge}}$

S. Sachdev, M. A. Metlitski, Y. Qi, and S. Sachdev *Phys. Rev. B* **80**, 155129 (2009)

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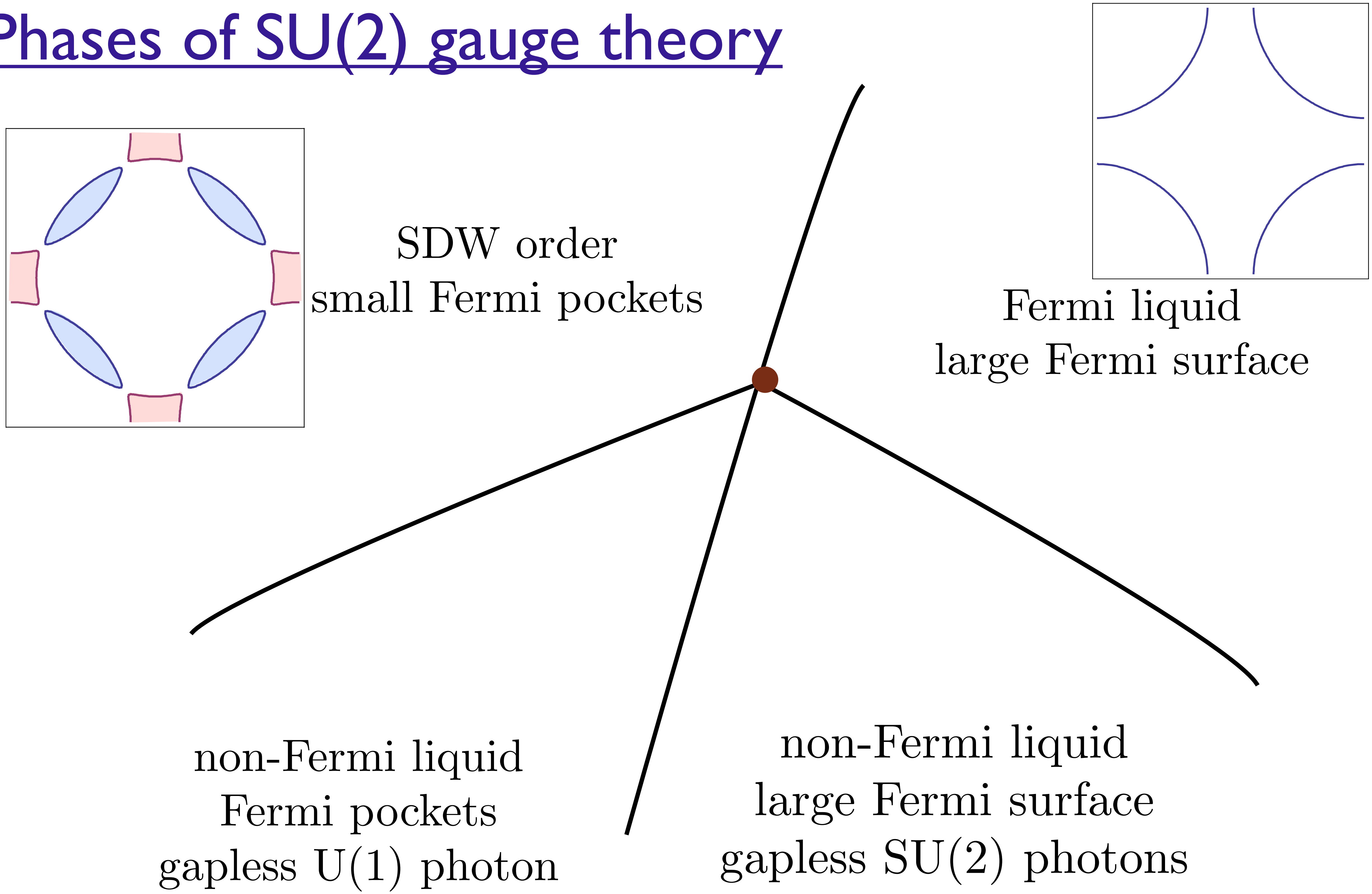
The spin-fermion (or Hubbard) model can be written *exactly* as a lattice gauge theory with a

$$\mathrm{SU(2)}_{s;g} \times \mathrm{SU(2)}_{\mathrm{spin}} \times \mathrm{U(1)}_{\mathrm{charge}}$$

invariance.
The $\mathrm{SU(2)}_{s;g}$ is a gauge invariance,
while $\mathrm{SU(2)}_{\mathrm{spin}} \times \mathrm{U(1)}_{\mathrm{charge}}$ is a global symmetry

Phases of SU(2) gauge theory



S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

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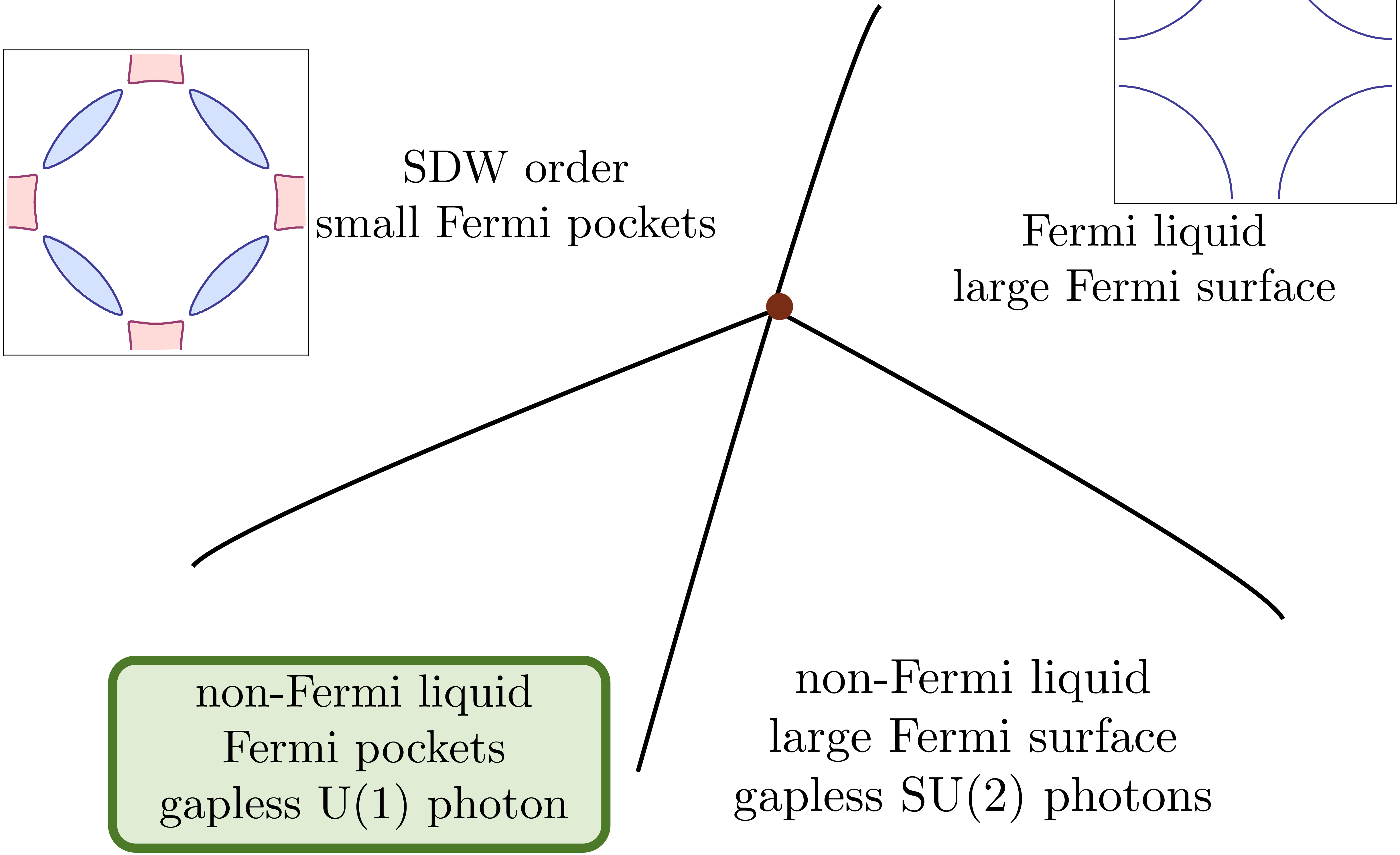
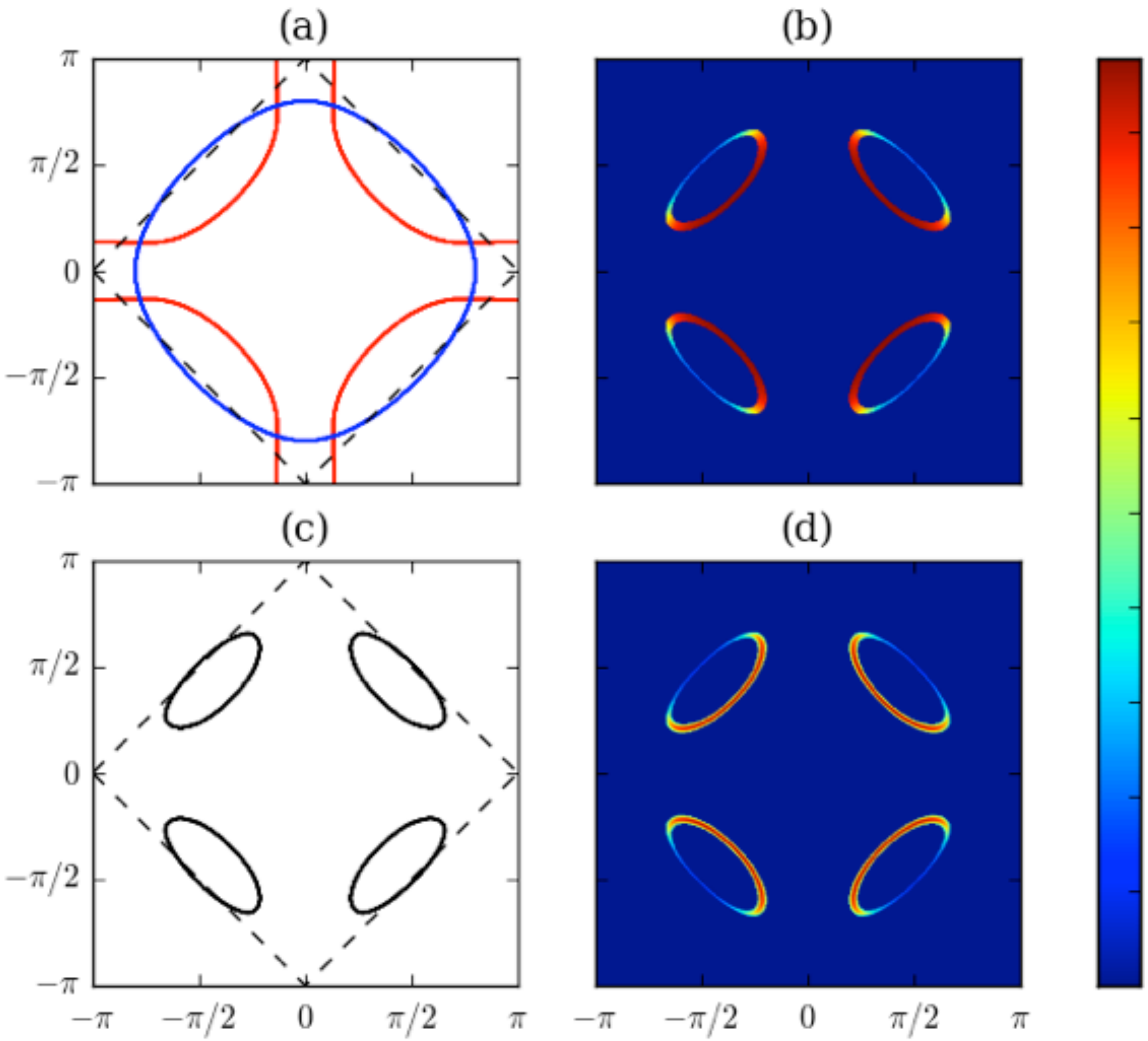


S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Gapless U(1) photon phase with "topological" order

No long-range antiferromagnetism, but hedgehogs suppressed (spacetime analog of monopole-free phase in pyrochlores)



Total area of 4 pockets = $(2\pi^2)\times$ (hole density x).

Leading approximation for Green's function is similar (but *not* identical) to the YRZ phenomenological form

Y. Qi and S. Sachdev, *Physical Review B* **81**, 115129 (2010)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Physical Review B* **75** , 235122 (2007)
R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008)

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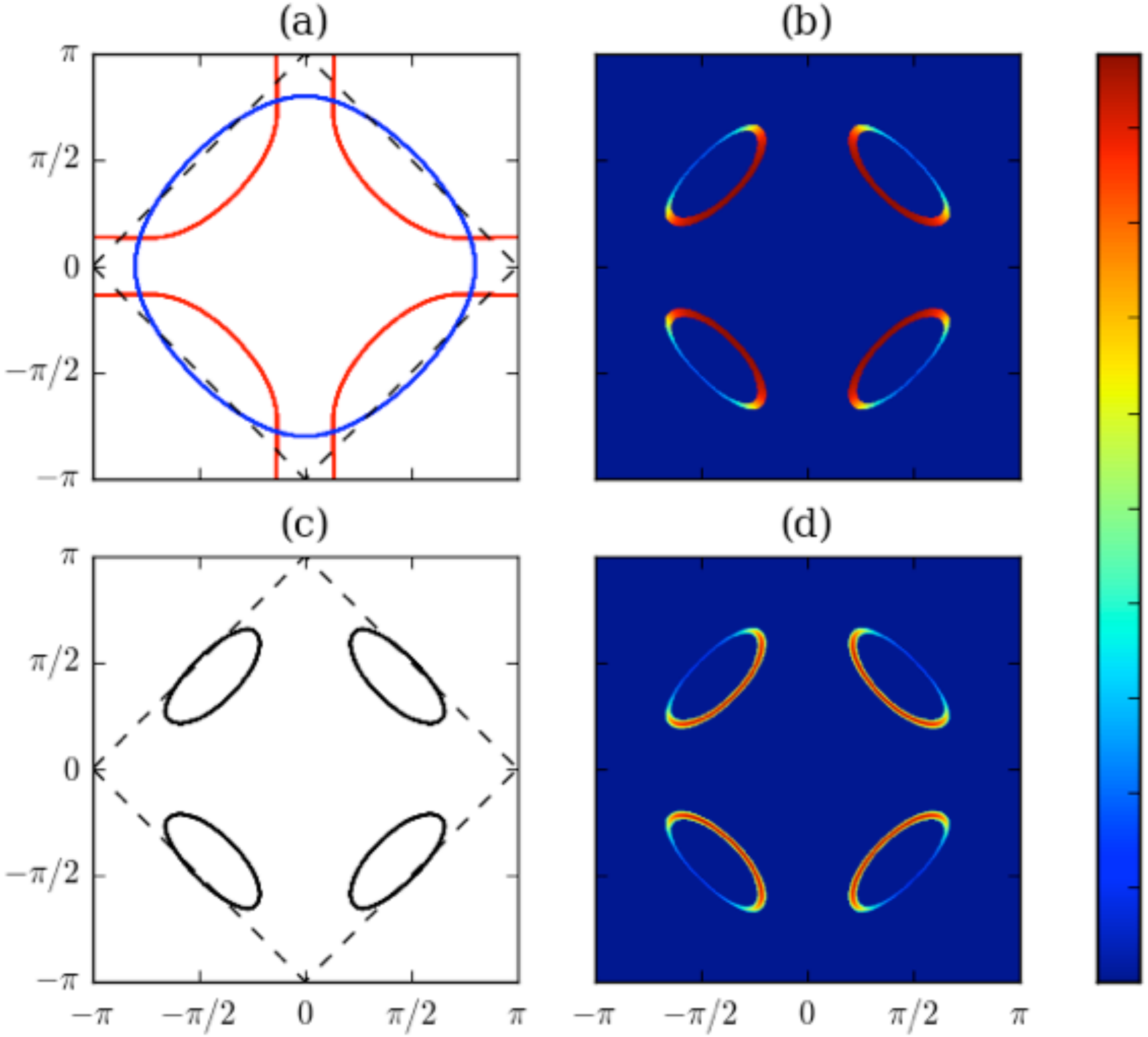

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Phase is a *fractionalized Fermi liquid*, previously proposed for Kondo lattice models (and possibly found in $YbRh_2(Si_{0.95}Ge_{0.05})_2$, J. Custers, P. Gegenwart, C. Geibel, F. Steglich, P. Coleman, and S. Paschen, Phys. Rev. Lett. **104**, 186402 (2010).)

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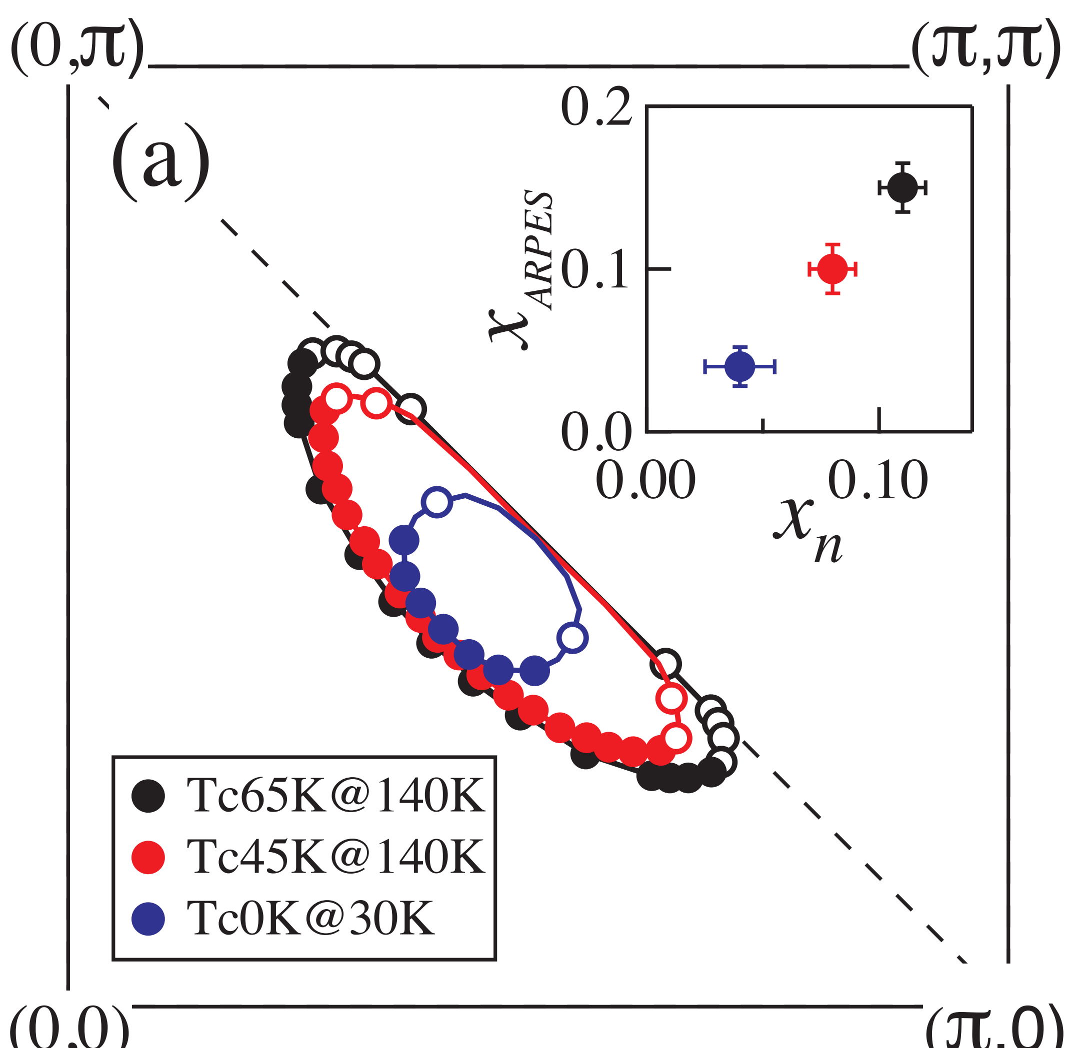
On the Reconstructed Fermi Surface in the Underdoped Cuprates

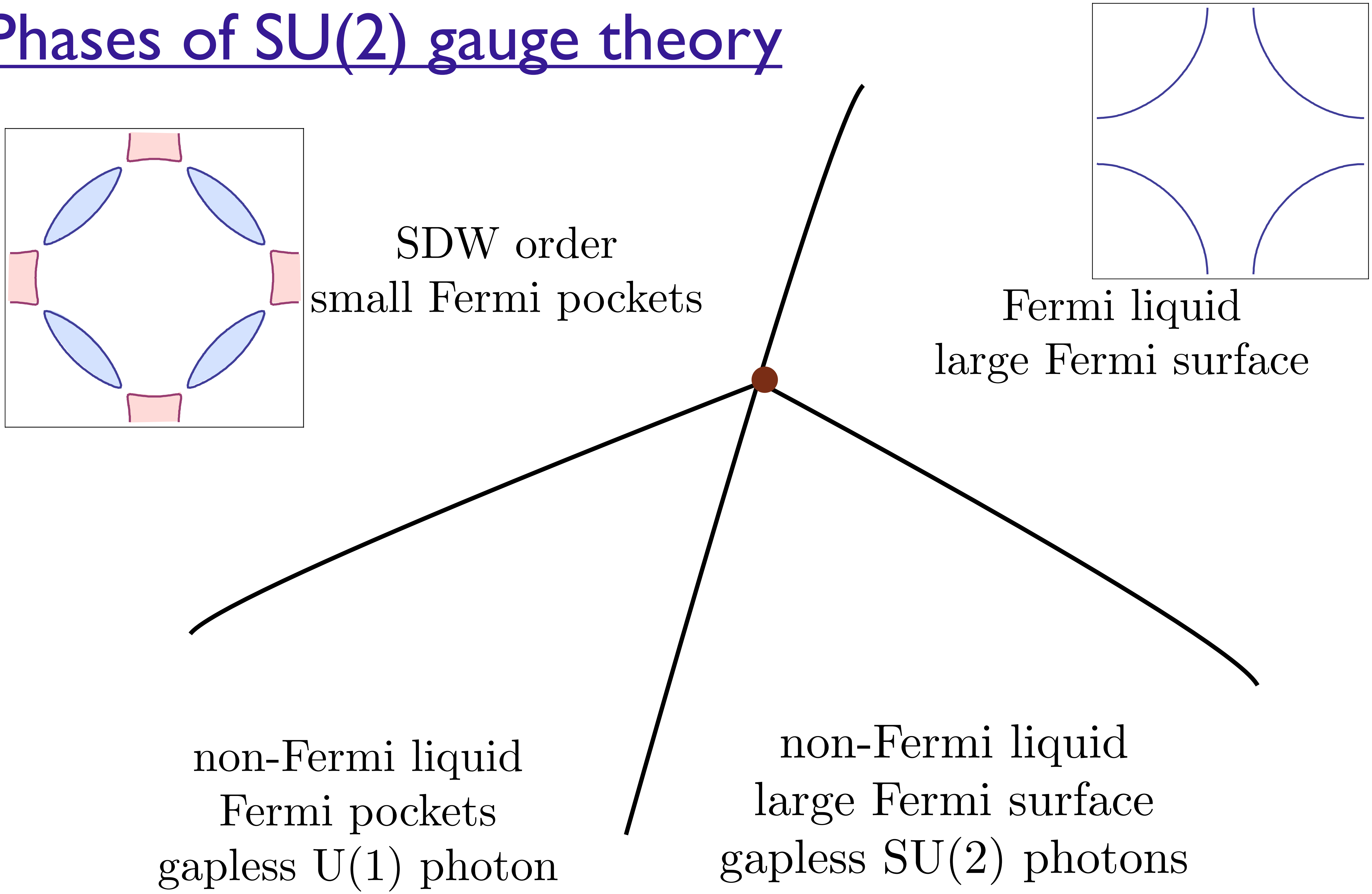
H.-B. Yang,[1] J. D. Ramaeu,[1] Z.-H. Pan,[1] G.D. Gu,[1] P. D. Johnson,[1] R. H. Claus,[2] D. G. Hinks,[2] and T. E. Kidd[3]

The Fermi surface topologies of underdoped samples the high-T_C superconductor Bi2212 have been measured with angle resolved photoemission. By examining thermally excited states above the Fermi level, we show that the Fermi surfaces in the pseudogap phase of underdoped samples are actually composed of fully enclosed hole pockets. The spectral weight of these pockets is vanishingly small at the anti-ferromagnetic zone boundary, which creates the illusion of Fermi "arcs" in standard photoemission measurements. The area of the pockets as measured in this study is consistent with the doping level, and hence carrier density, of the samples measured. Furthermore, the shape and area of the pockets is well reproduced by a phenomenological model of the pseudogap phase as a spin liquid.



arXiv:1008.3121

Phases of SU(2) gauge theory



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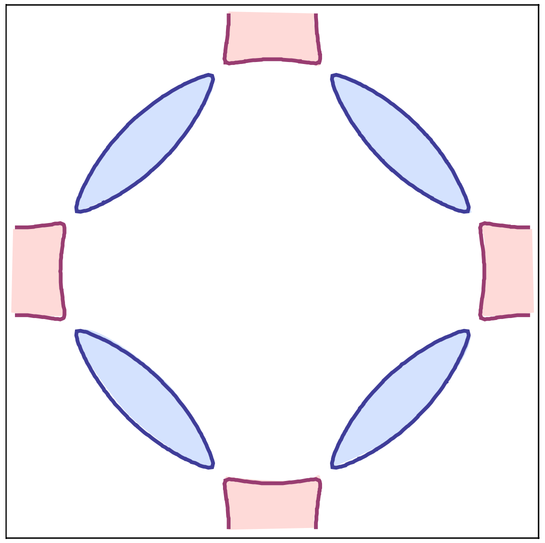
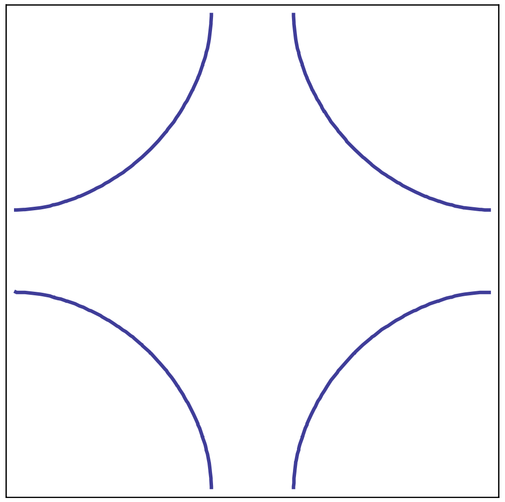
Phases of SU(2) gauge theory



SDW order
small Fermi pockets

Original spin-fermion model: remainder of talk



Fermi liquid
large Fermi surface

non-Fermi liquid
Fermi pockets
gapless U(1) photon

non-Fermi liquid
large Fermi surface
gapless SU(2) photons

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

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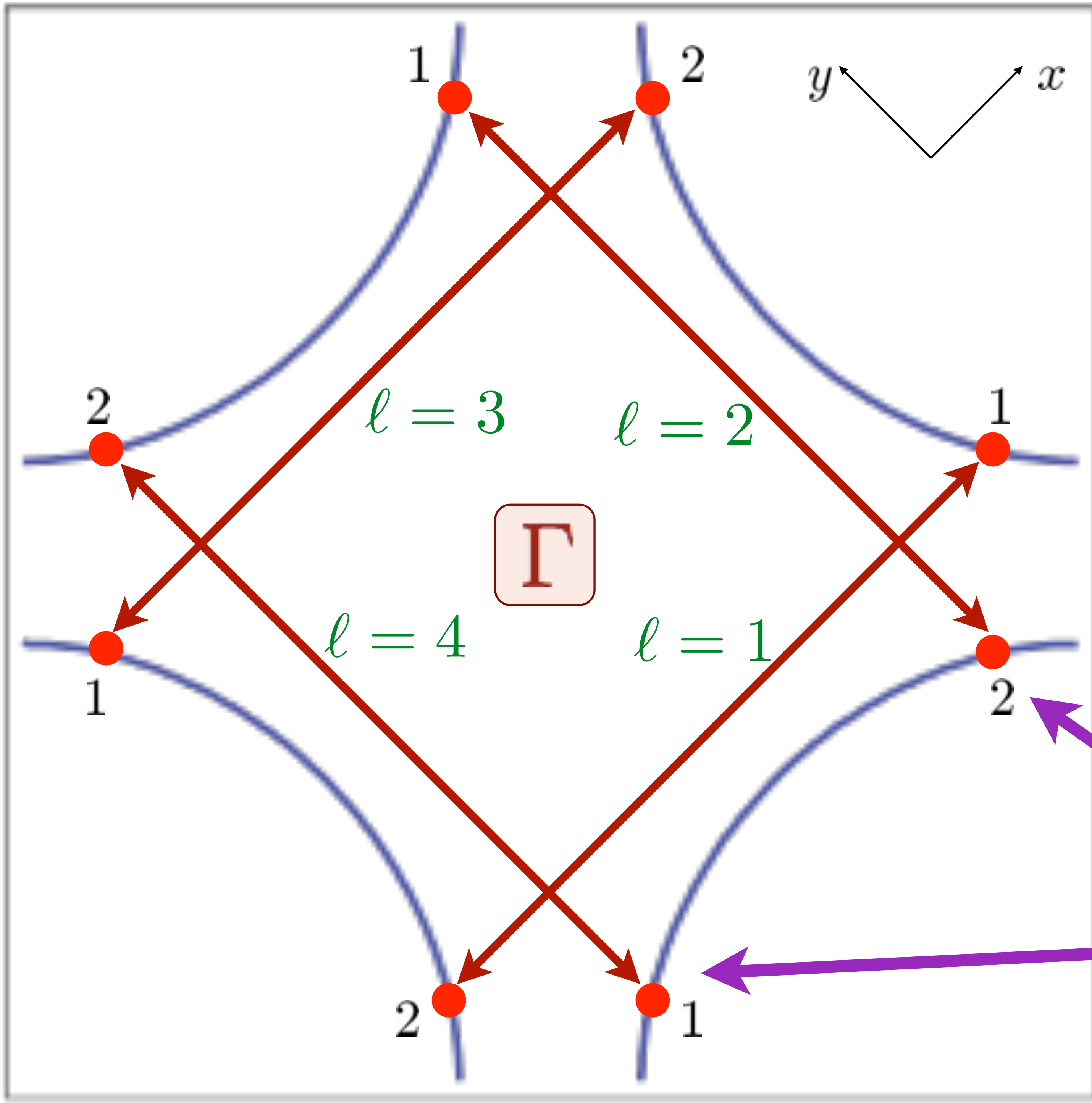
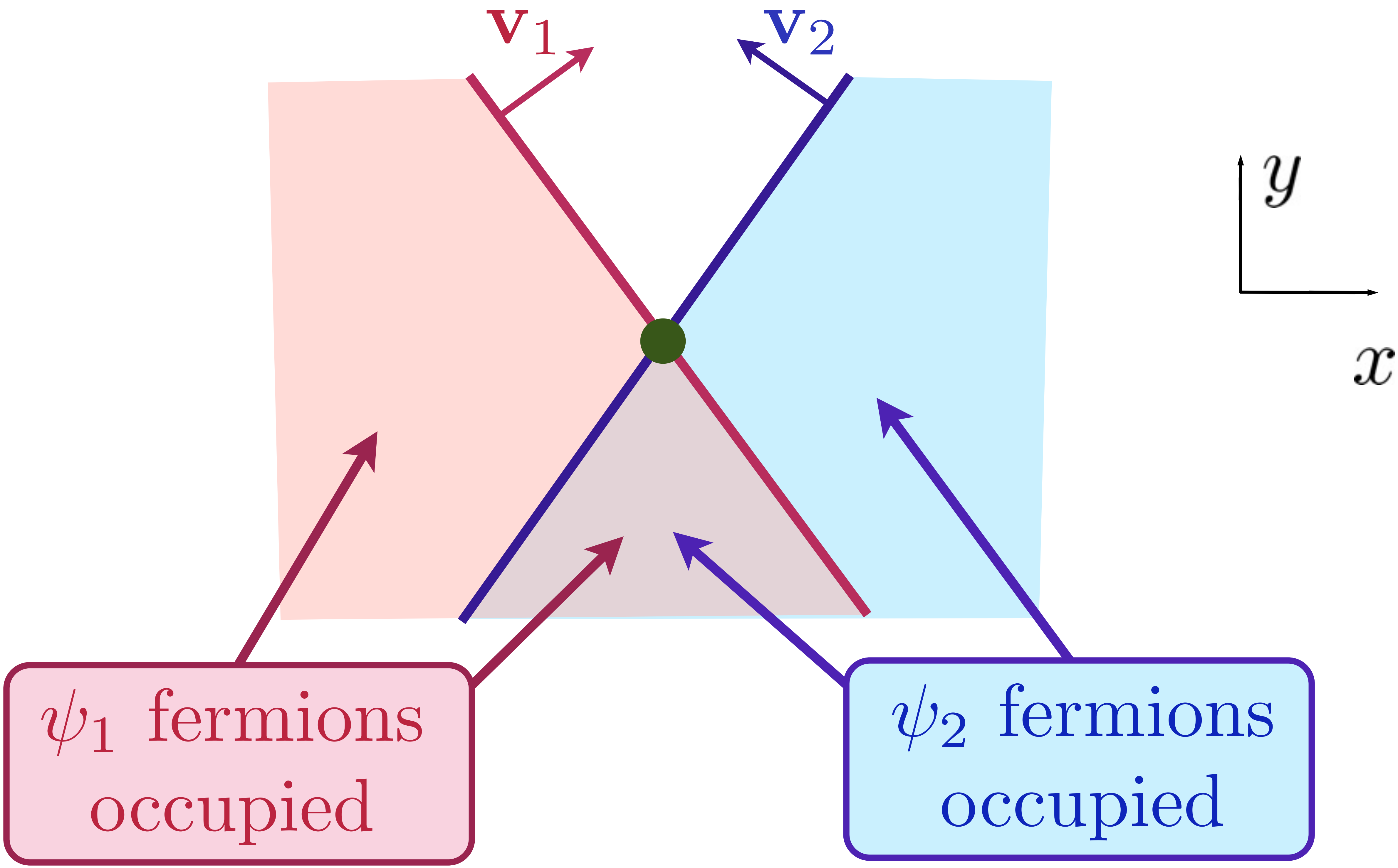


Low energy fermions at hot spots $\mathbf{k} = \mathbf{k}_\ell$:
$\psi^\ell_{1\alpha}$, $\psi^\ell_{2\alpha}$
$\ell = 1, \ldots, 4.$
with $c_{\mathbf{k}+\mathbf{k}_\ell} = \psi^\ell(\mathbf{k})$

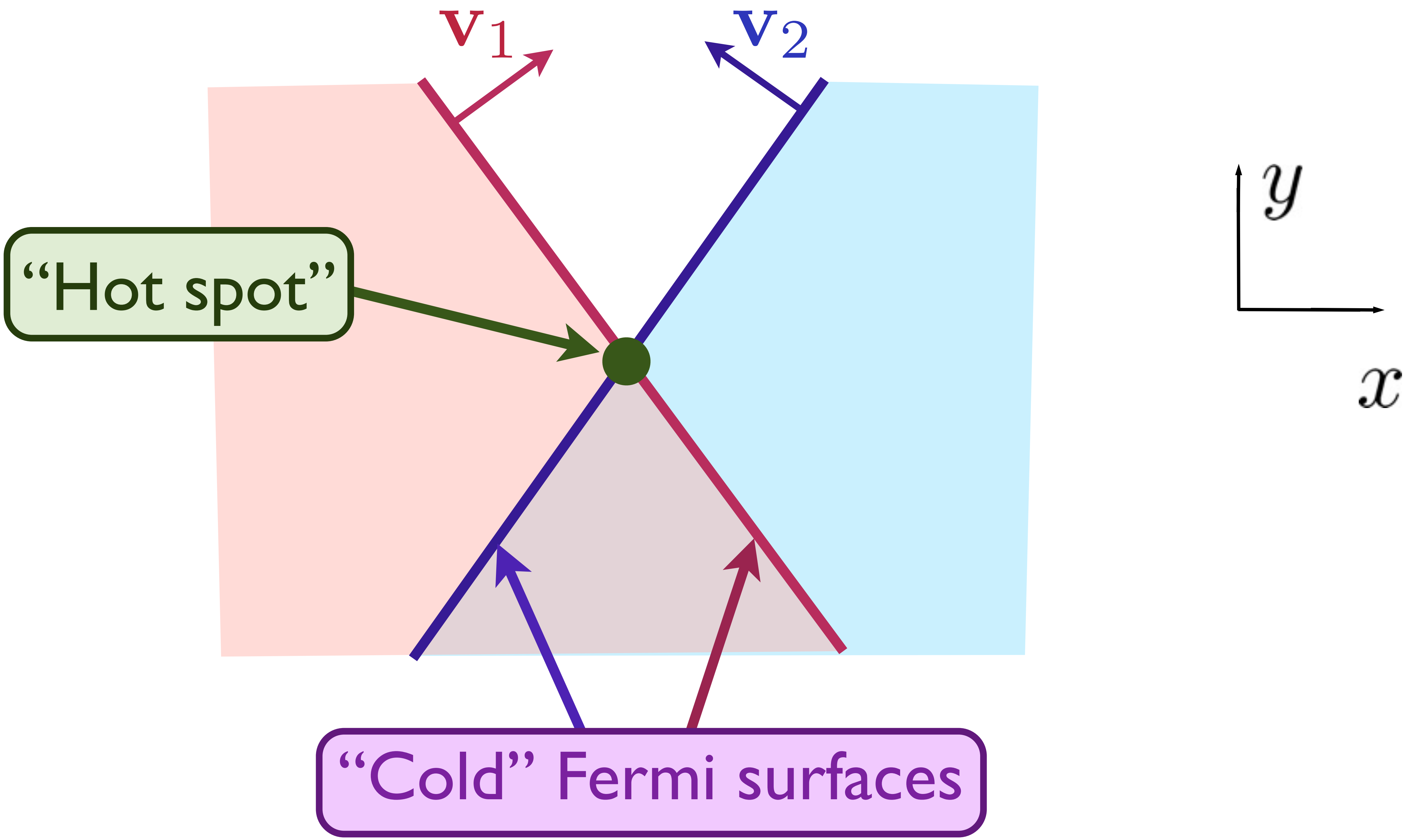
$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

$$\mathbf{v}^{\ell=1}_1 = (v_x, v_y),\ \mathbf{v}^{\ell=1}_2 = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$



$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$



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Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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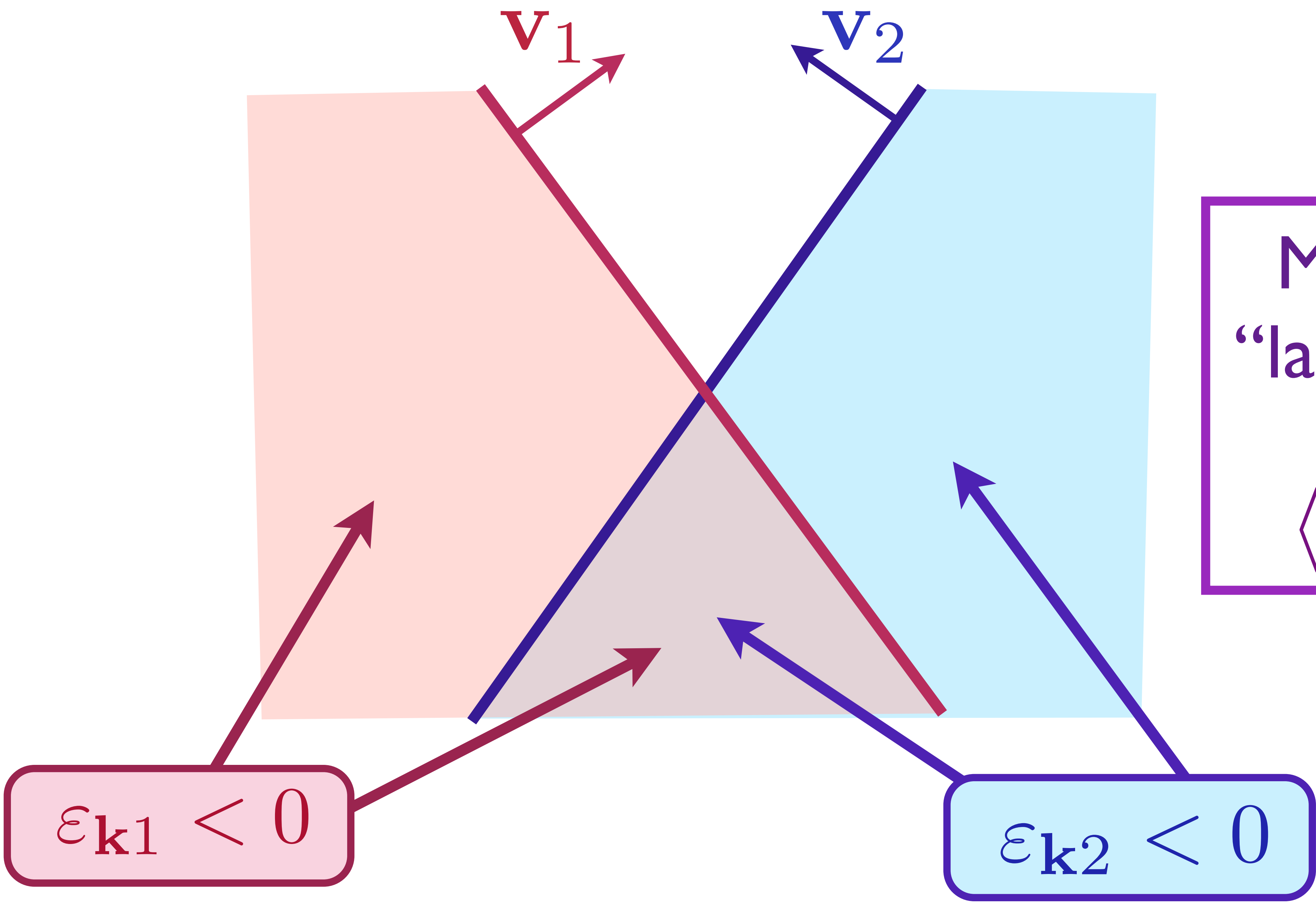
"Yukawa" coupling: $\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

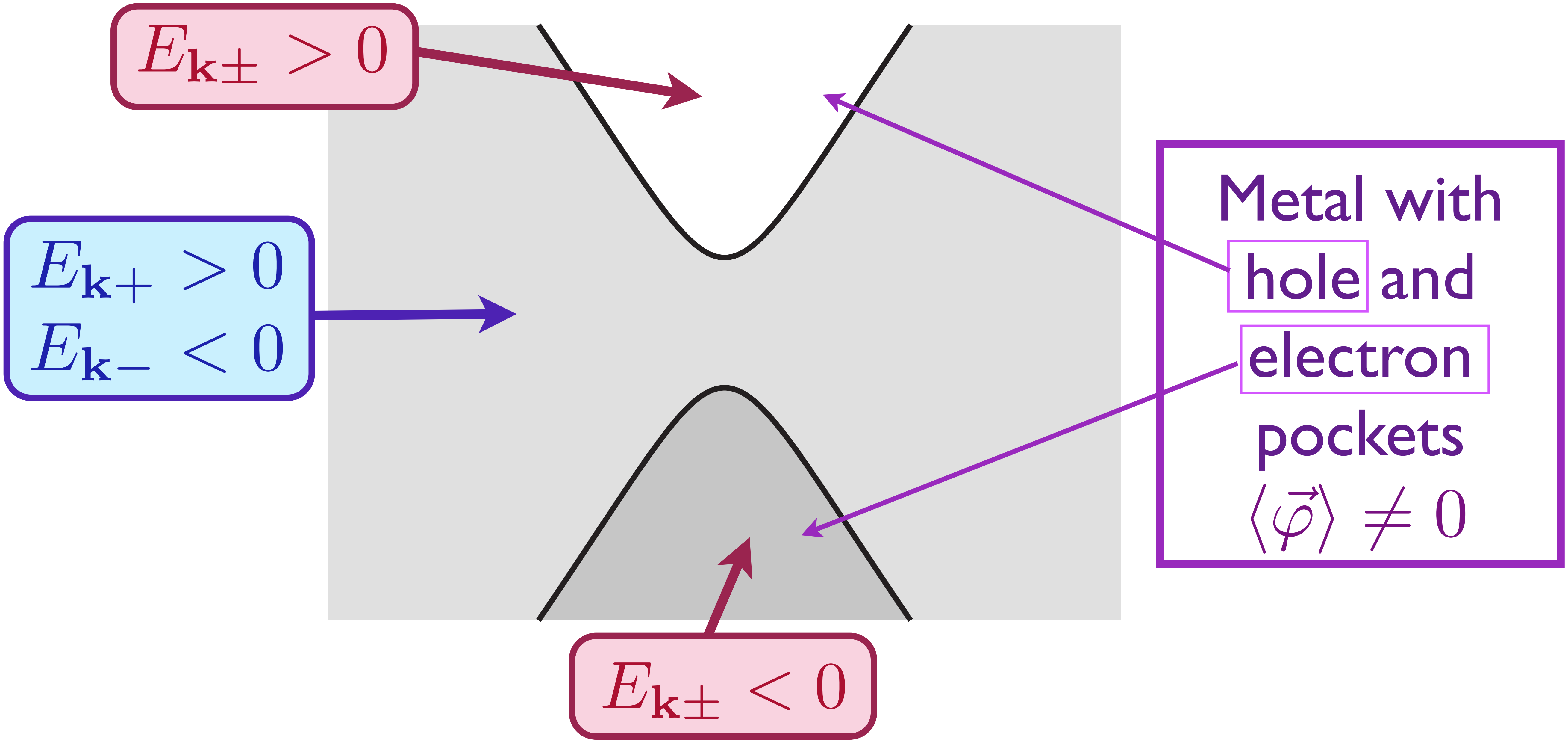
$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$



Metal with "large" Fermi surface
$\langle\vec{\varphi}\rangle = 0$

Fermion dispersions: $\varepsilon_{\mathbf{k}1} = \mathbf{v}_1 \cdot \mathbf{k}$ and $\varepsilon_{\mathbf{k}2} = \mathbf{v}_2 \cdot \mathbf{k}$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$
$$-\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$$



Fermion dispersions:

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}1} + \varepsilon_{\mathbf{k}2}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}1} - \varepsilon_{\mathbf{k}2}}{2}\right)^2 + \lambda^2|\vec{\varphi}|^2}$$

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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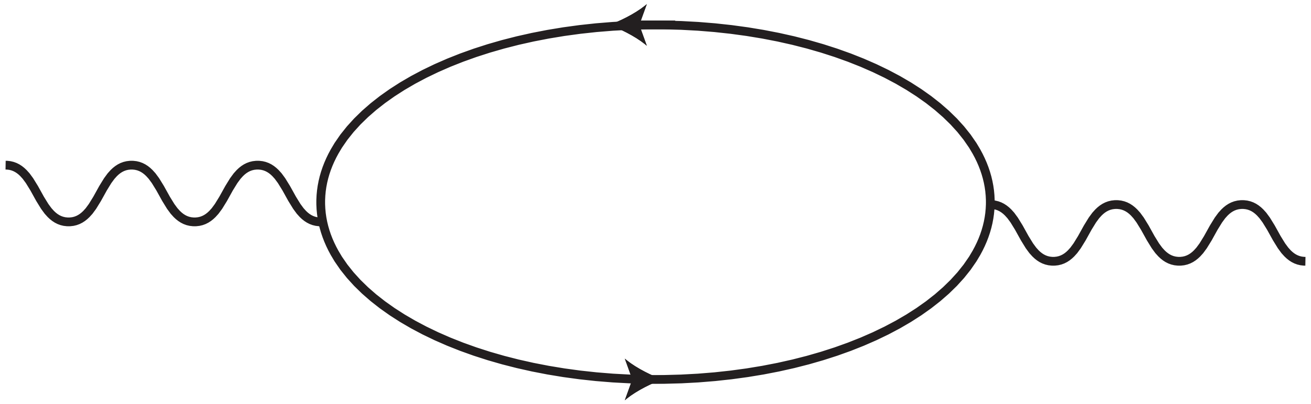
Hertz theory

Integrate out fermions and obtain an effective action for the boson field $\vec{\varphi}$ alone. Because the fermions are gapless, this is potentially dangerous, and will lead to non-local terms in the $\vec{\varphi}$ effective action. Hertz focused on only the simplest such non-local term. However, there are an infinite number of non-local terms at higher order, and these lead to a breakdown of the Hertz theory in $d = 2$.

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

Hertz action.

Upon integrating the fermions out, the leading term in the $\vec{\varphi}$ effective action is $-\Pi(q,\omega_n)|\vec{\varphi}(q,\omega_n)|^2$, where $\Pi(q,\omega_n)$ is the fermion polarizability. This is given by a simple fermion loop diagram



which evaluates to

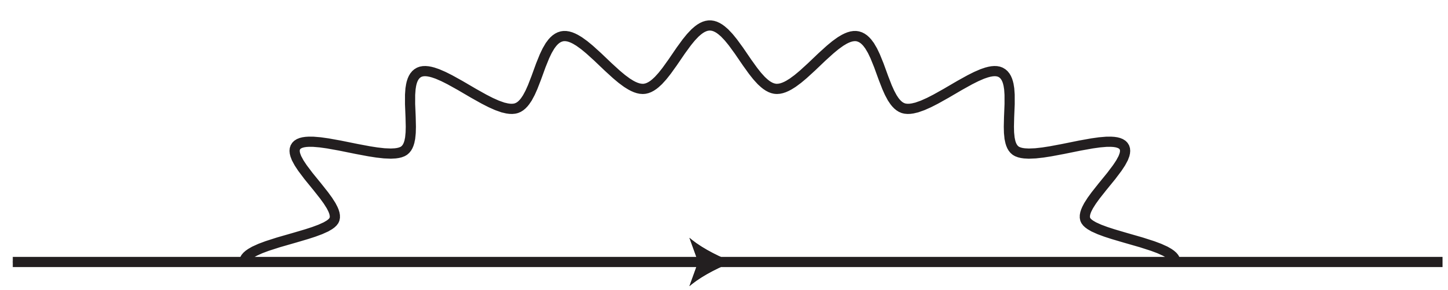
$$\Pi(q,\omega_n) = -\frac{|\omega_n|\Lambda^{d-2}}{4\pi|\mathbf{v}_1 \times \mathbf{v}_2|}. \quad (1)$$

We have dropped a frequency-independent, cutoff-dependent constant which can absorbed into a redefinition of s. Notice also that the factor of ζ has cancelled.
Inserting this fermion polarizability in the effective action for $\vec{\varphi}$, we

obtain the Hertz action for the SDW transition:

$$\mathcal{S}_H = \int \frac{d^d k}{(2\pi)^d} T \sum_{\omega_n} \frac{1}{2} \left[k^2 + \gamma|\omega_n| + s\right] |\vec{\varphi}(k, \omega_n)|^2 + \frac{u}{4} \int d^d x d\tau \left(\vec{\varphi}^2(x, \tau)\right)^2 . \quad (2)$$

Let us, for now, assume the validity of the Hertz Gaussian action, and compute the leading correction to the electronic Green's function. This is given by the following Feynman graph for the electron self energy, Σ. At zero momentum for the ψ_1 fermion we have



$$\Sigma_1(0, \omega_n) = \lambda^2 \int \frac{d^d q}{(2\pi)^d} \int \frac{d\epsilon_n}{2\pi} \frac{1}{[q^2 + \gamma|\epsilon_n|]\,[-i\zeta(\epsilon_n + \omega_n) + \mathbf{v}_2 \cdot \mathbf{q}]} . \quad (3)$$

Evaluation of the integrals shows that

$$\Sigma_1(0,\omega_n) \sim |\omega_n|^{(d-1)/2} \tag{4}$$

The most important case is $d = 2$, where we have

$$\Sigma_1(0,\omega_n) = i\,\frac{\lambda^2}{\pi|v_2|\sqrt{\gamma}}\text{sgn}(\omega_n)\sqrt{|\omega_n|} \quad , \quad d = 2. \tag{5}$$

Strong coupling physics in $d = 2$

The theory so far has the boson propagator

$$\sim \frac{1}{q^2 + \gamma|\omega|}$$

which scales with dynamic exponent $z_b = 2$, and now a fermion propagator

$$\sim \frac{1}{-i\zeta\omega + c_1|\omega|^{(d-1)/2} + \mathbf{v}\cdot\mathbf{q}}.$$

First note that for $d < 3$, the bare $-i\zeta\omega$ term is less important than the contribution from the self energy at low frequencies. This indicates that ζ is *irrelevant* in the critical theory, and we can set $\zeta \to 0$. Fortunately, all the loop diagrams evaluated so far are independent of ζ.

Setting $\zeta = 0$, we see that the fermion propagator scales with dynamic exponent $z_f = 2/(d-1)$. For $d > 2$, $z_f < z_b$, and so at small momenta the boson fluctuations have lower energy than the fermion fluctuations. Thus it seems reasonable to assume that the fermion fluctuations are not as singular, and we can focus on an effective theory of the SDW order parameter $\vec{\varphi}$ alone. In other words, the Hertz assumptions appear valid for $d > 2$.

However, in $d = 2$, we have $z_f = z_b = 2$. Thus fermionic and bosonic fluctuations are equally important, and it is not appropriate to integrate the fermions out at an initial stage. We have to return to the original theory of coupled bosons and fermions. This turns out to be strongly coupled, and exhibits complex critical behavior.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$$

Perform RG on both fermions and $\vec{\varphi}$, using *local* field theory above.

M. A. Metlitski and S. Sachdev, *Physical Review B* **82**, 075127 (2010)

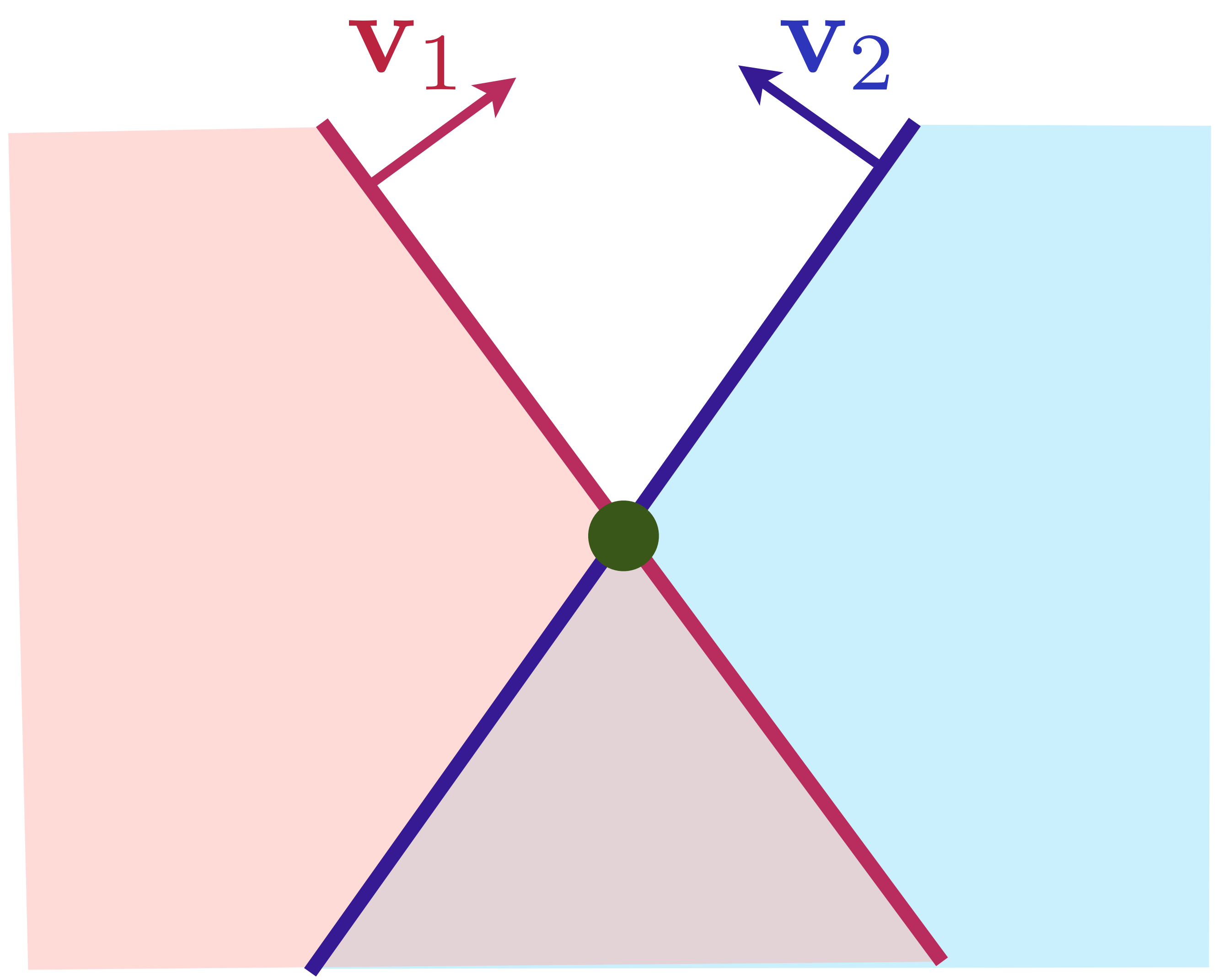
In principle, the RG analysis can be organized an expansion in $1/N$, where N is the number of hot-spots. Apart from the field scale renormalizations, and the dynamic exponent z, the only coupling constants are the velocity ratio $\alpha = v_y/v_x$, and the boson quartic coupling u. We assume s has been tuned to reach the critical point, and the scaling limit has $\zeta \to 0$ (characteristic of all non-Fermi liquid fixed points) so that the boson-fermion coupling λ can be absorbed into the fermion field scale.

At two-loop order, the $1/N$ expansion is well-behaved, and we can determine consistent RG flow equations. However, at higher loops we find corrections to the renormalizations which require summation of all planar graphs even at the leading order in $1/N$, and the $1/N$ expansion appears to be organized as a genus expansion of random surfaces. But even this genus expansion breaks down in the renormalization of u. In the following, we just describe the two loop results.

The main RG improved two-loop results are:

- The position of the Fermi surface renormalizes to

$$p_y = -\frac{12}{\pi N} p_x \log(1/|p_x|)$$



Bare Fermi surface

The main RG improved two-loop results are:

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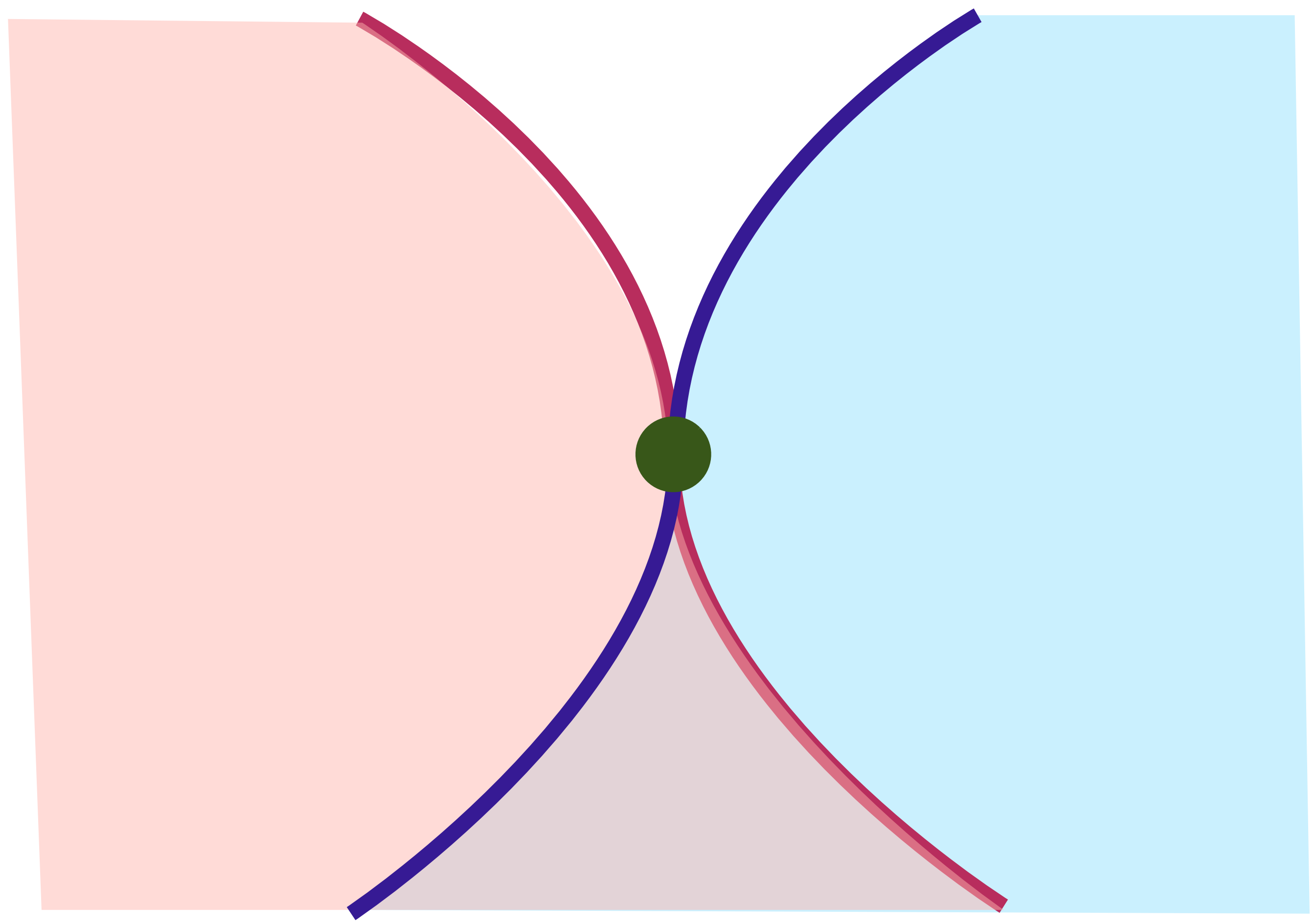
$$p_y = -\frac{12}{\pi N} p_x \log(1/|p_x|)$$



Dressed Fermi surface

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

The main RG improved two-loop results are:

- The position of the Fermi surface renormalizes to

$$p_y = -\frac{12}{\pi N} p_x \log(1/|p_x|)$$

- The fermion self-energy at the hot spot is,

$$\Sigma(\omega, \vec{p} = 0) \sim -i \exp\left(-\frac{3}{\pi^2 N^3} \log^3 \frac{1}{|\omega|}\right) |\omega|^{1/2} \mathrm{sgn}(\omega),$$

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- Along the Fermi surface away from the hot spot, the quasiparticle residue and Fermi velocity behave as,

$$v_F \sim \exp\left(\frac{48}{\pi^2 N^3} \log^3 \frac{1}{p_{\|}}\right) p_{\|}, \quad \mathcal{Z} \sim \left(\log \frac{1}{p_{\|}}\right)^{-1/2} p_{\|}$$

- The characteristic frequency of the bosonic spectrum is

$$\omega \sim \vec{q}^2 \exp\left(\frac{48}{\pi^2 N^3} \log^3 \frac{1}{|\vec{q}|}\right),$$

and the the bosonic propagator obeys

$$D^{-1}(\omega, \vec{q}=0) \quad \sim \quad |\omega|^{1-\frac{1}{N}} \exp\left(\frac{6}{\pi^2 N^4} \log^3 \frac{1}{|\omega|}\right) \left(\log \frac{1}{|\omega|}\right)^{-1/3}$$

$$D^{-1}(\omega=0, \vec{q}) \quad \sim \quad |\vec{q}|^2 \exp\left(\frac{48}{\pi^2 N^3} \log^3 \frac{1}{|\vec{q}|}\right)$$

Outline

1. Formulation of general theory

   Global phase diagram of a SU(2) gauge theory

2. Field theory for a direct transition between two Fermi liquids

   From a large Fermi surface to Fermi pockets

3. Instabilities to other orders

   Unconventional pairing, pseudospin symmetry, and bond order

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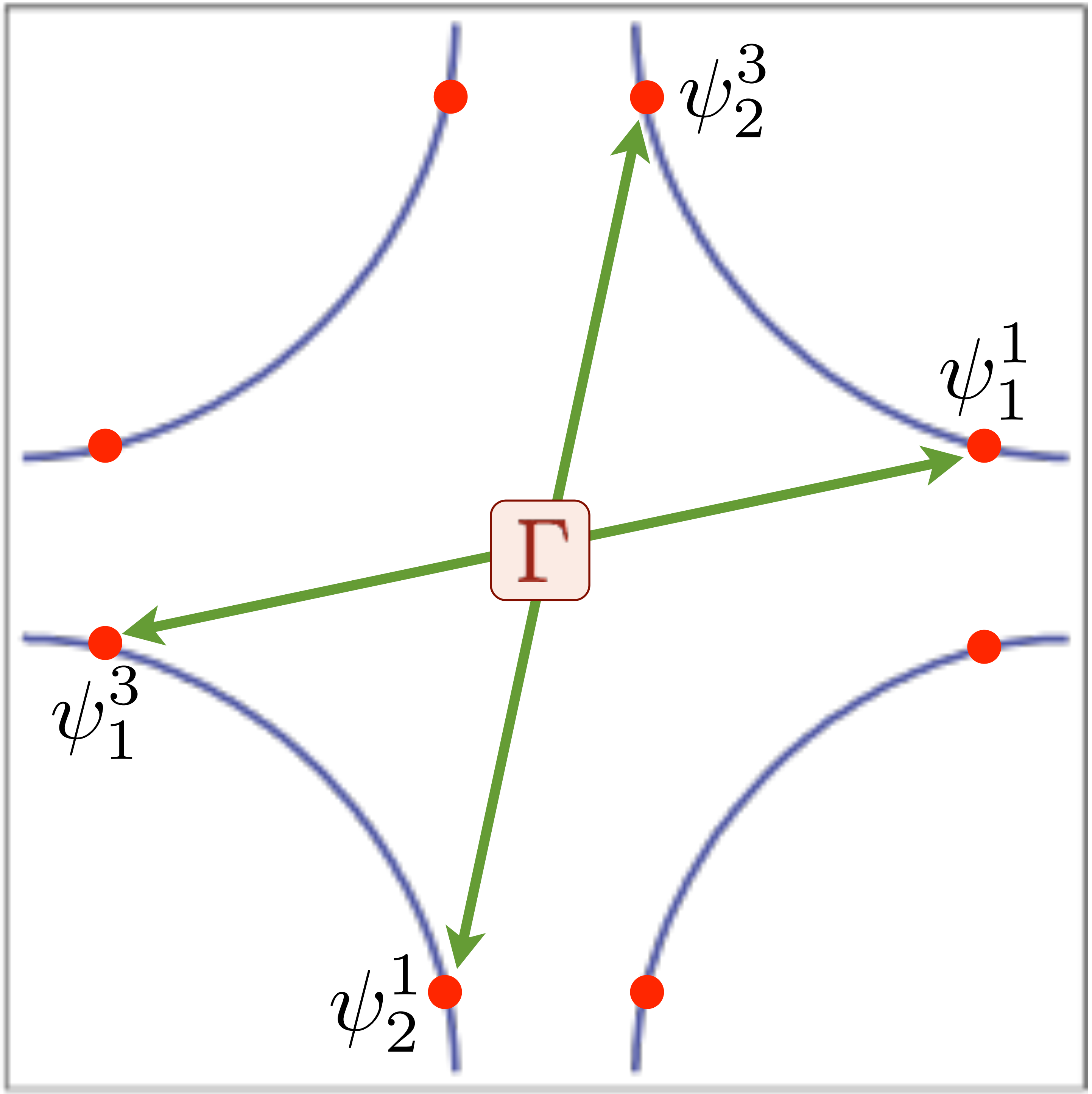
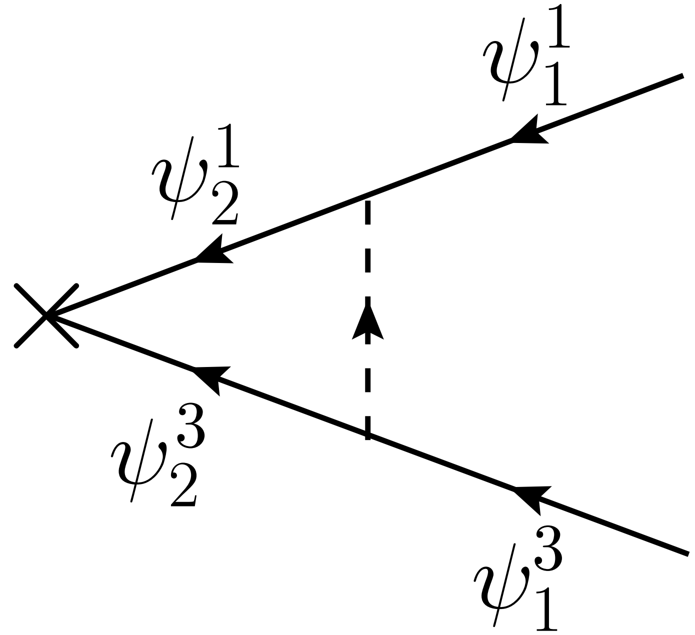
d-wave pairing in the theory of hotspots



Hot spots have strong instability to d-wave pairing near SDW critical point. This instability is stronger than the BCS instability of a Fermi liquid.

Pairing order parameter: $\varepsilon^{\alpha\beta}\left(\psi^3_{1\alpha}\psi^1_{1\beta} - \psi^3_{2\alpha}\psi^1_{2\beta}\right)$



At leading order, the pairing vertex is enhanced by the factor

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2\left(\frac{1}{\omega}\right).$$

Note that this is not suppressed by a factor of $1/N$. It is not clear how to improve this using the RG. However, we can note that the coupling α is of order unity, and so the pairing is enhanced as the frequency crosses the Fermi energy.

We also note that in the two-loop RG, the coupling $\alpha = v_y/v_x$ has a flow towards weak coupling

$$\frac{d\alpha}{d\ell} = -\frac{12}{\pi N}\frac{\alpha^2}{(1+\alpha^2)}$$

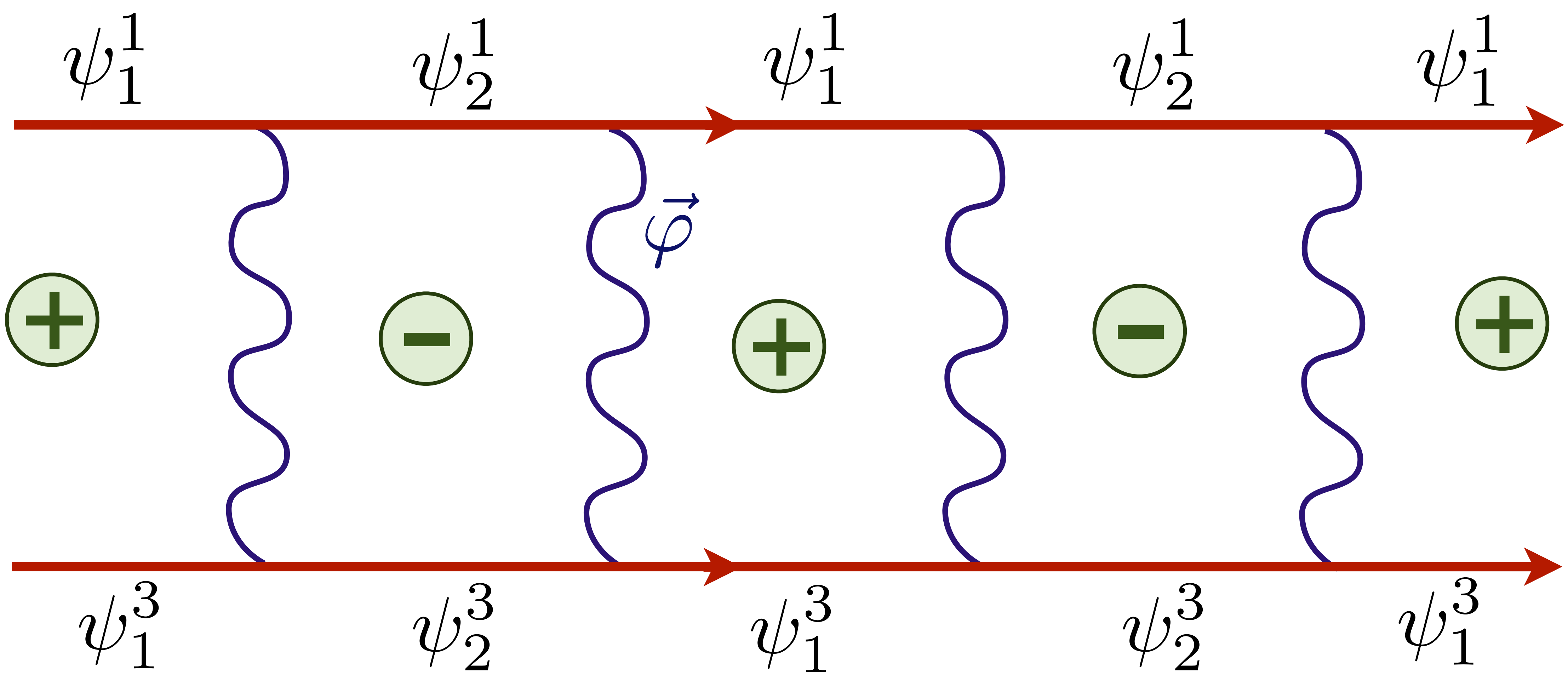
but it not appropriate to simply insert the integrated value from this flow into the pairing enhancement.

Emergent Pseudospin symmetry

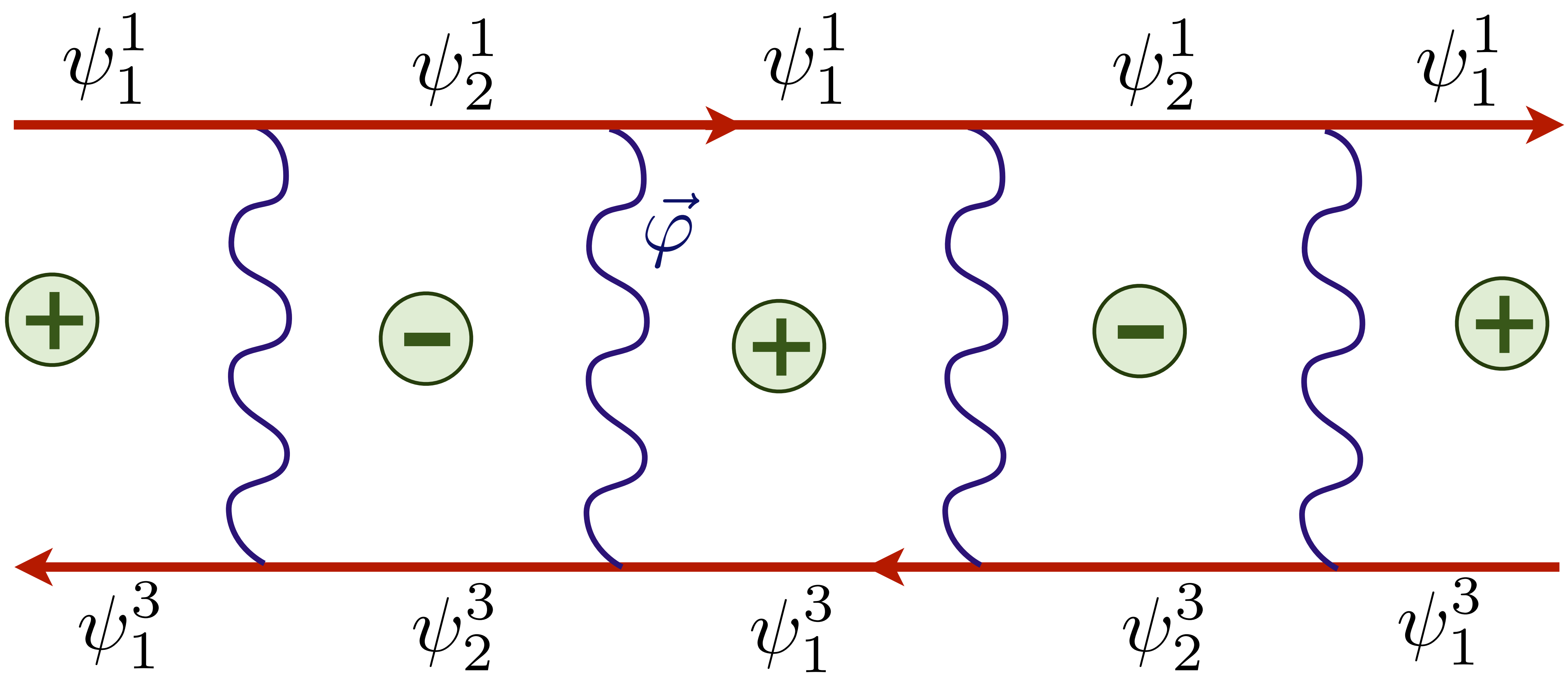
Continuum theory of hotspots in invariant under:

$$\left(\begin{array}{c} \psi^{\ell}_{\uparrow} \\ \psi^{\ell \dagger}_{\downarrow} \end{array} \right) \rightarrow U^{\ell} \left(\begin{array}{c} \psi^{\ell}_{\uparrow} \\ \psi^{\ell \dagger}_{\downarrow} \end{array} \right)$$

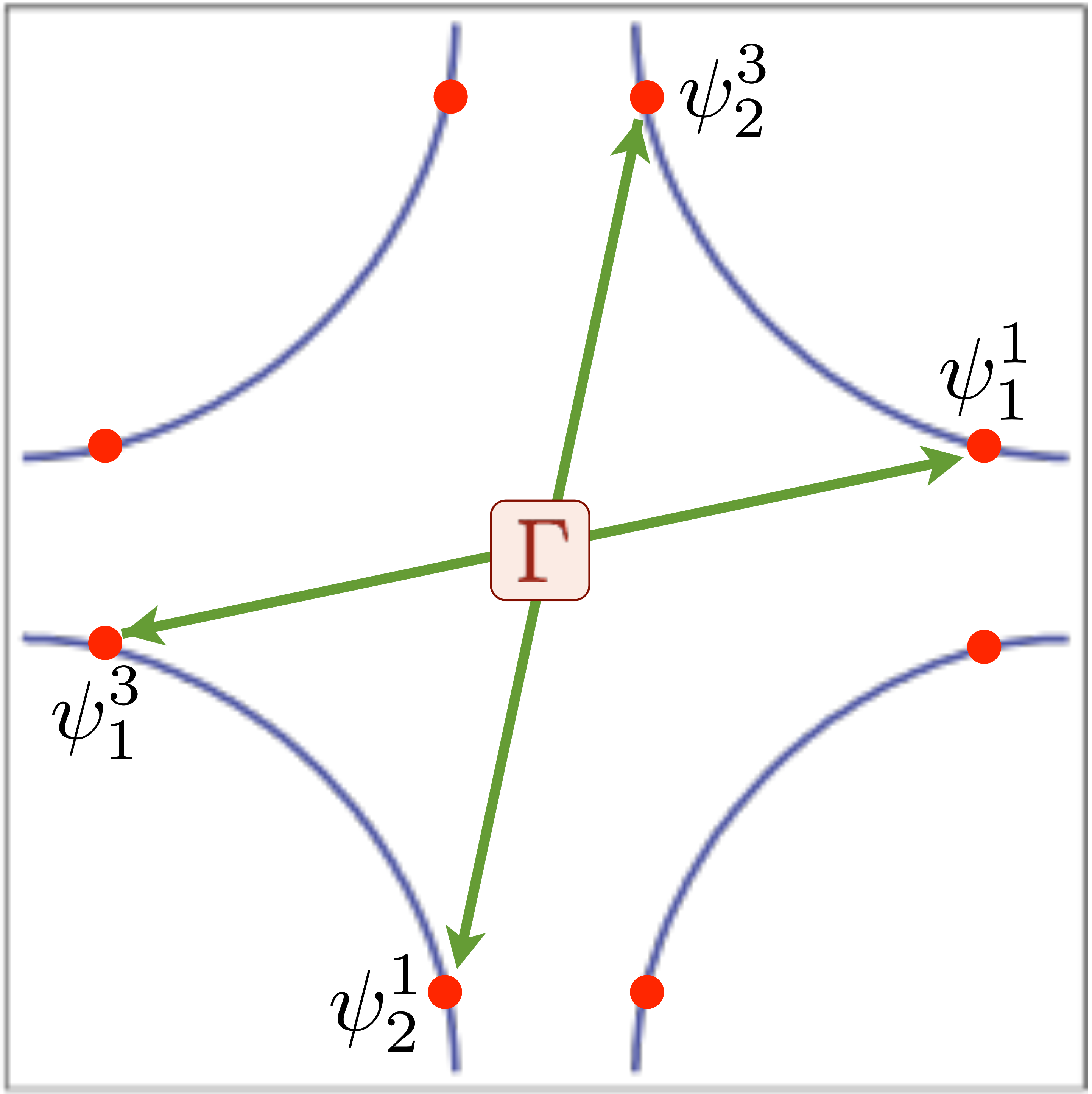
where U^{ℓ} are arbitrary SU(2) matrices which can be *different* on different hotspots ℓ.



d-wave Cooper pairing instability in particle-particle channel



Bond density wave (with local Ising-nematic order) instability in particle-hole channel



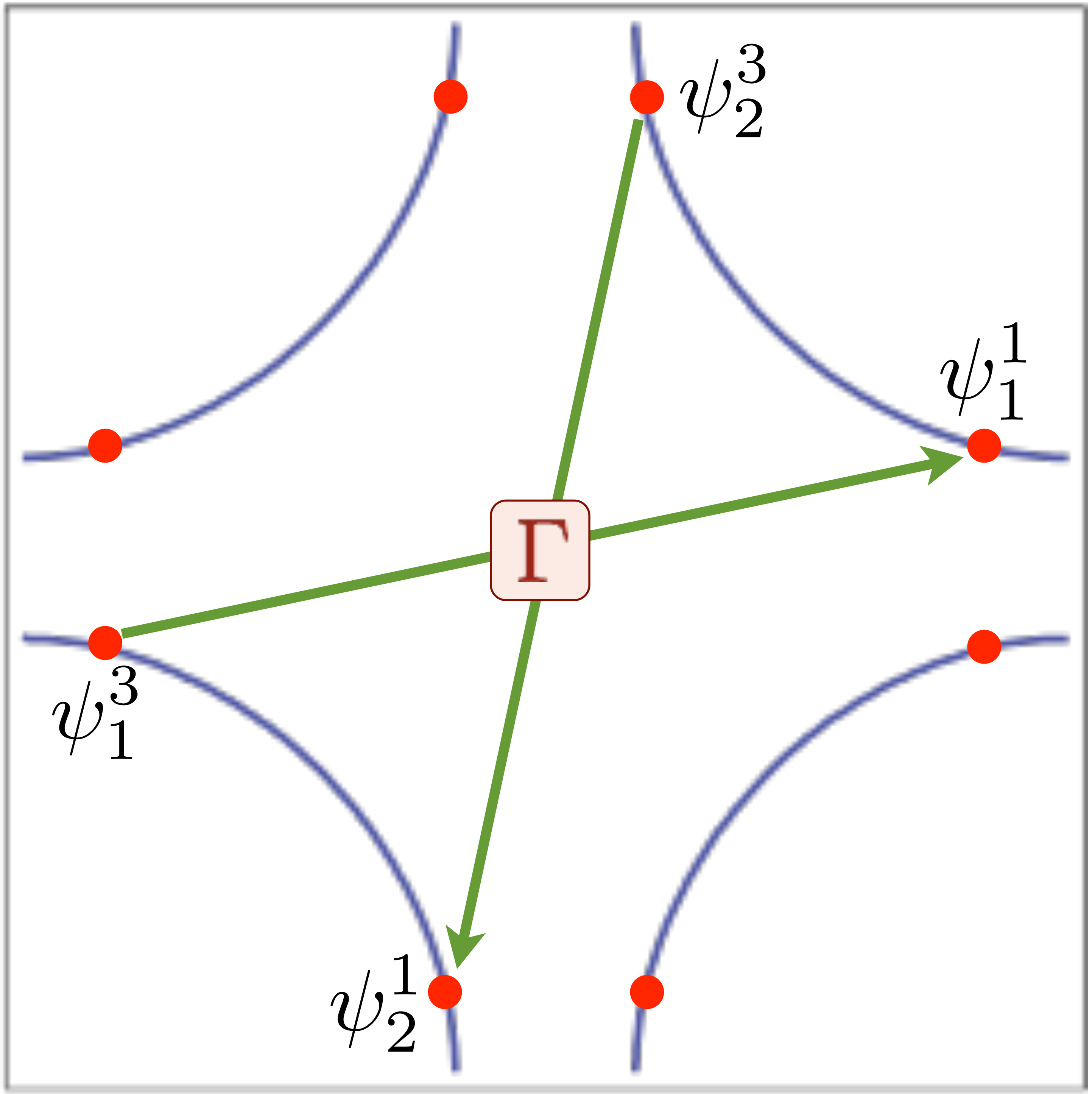
Recall d-wave pairing instability in the particle-particle channel

This had the enhancement factor

$$1 + \frac{\alpha}{\pi(1+\alpha^2)} \log^2 \left(\frac{1}{\omega}\right)$$

Pairing order parameter:

$$\varepsilon^{\alpha\beta} \left(\psi_{1\alpha}^3 \psi_{1\beta}^1 - \psi_{2\alpha}^3 \psi_{2\beta}^1\right)$$



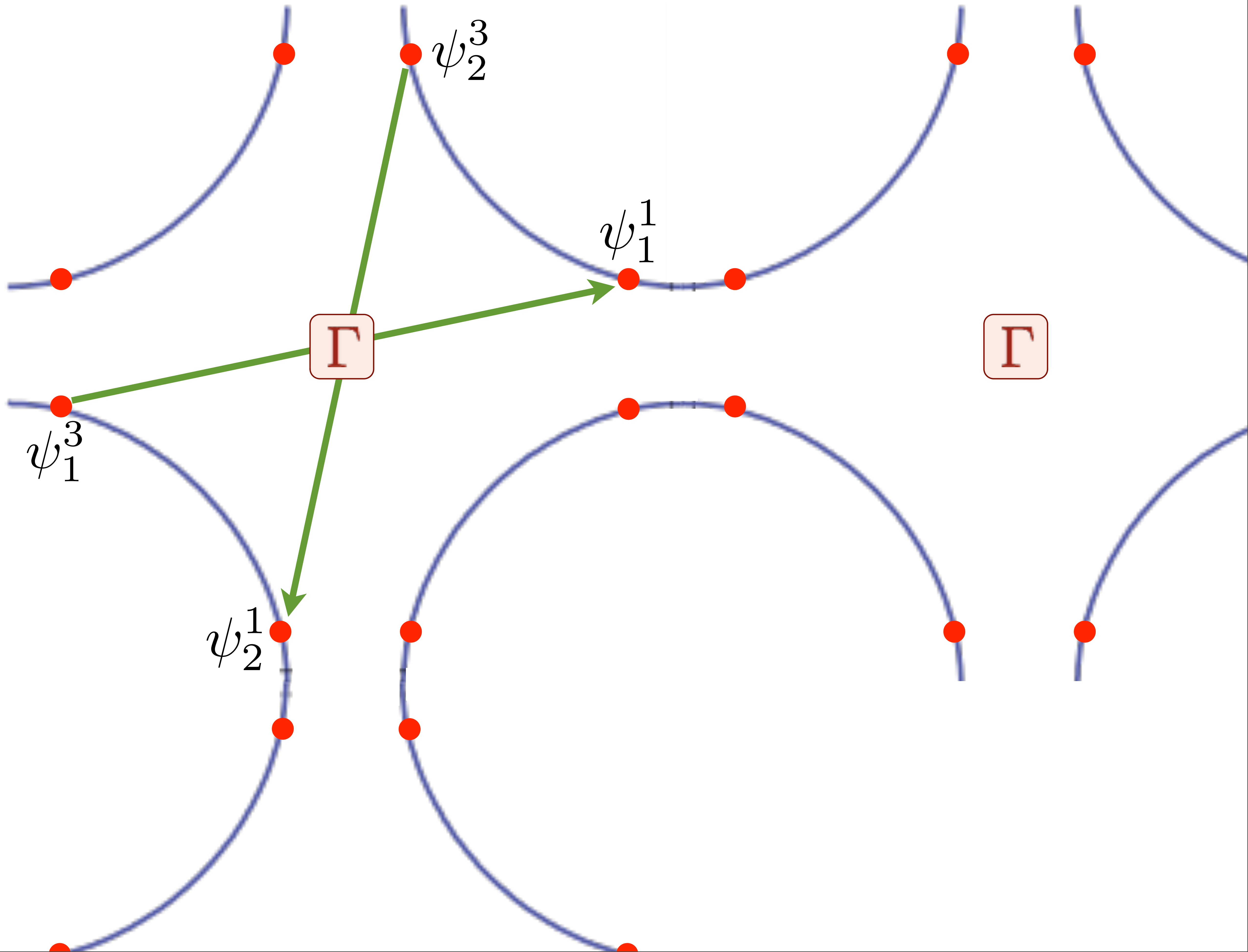
d-wave pairing has a partner instability in the particle-hole channel

This has the enhancement factor

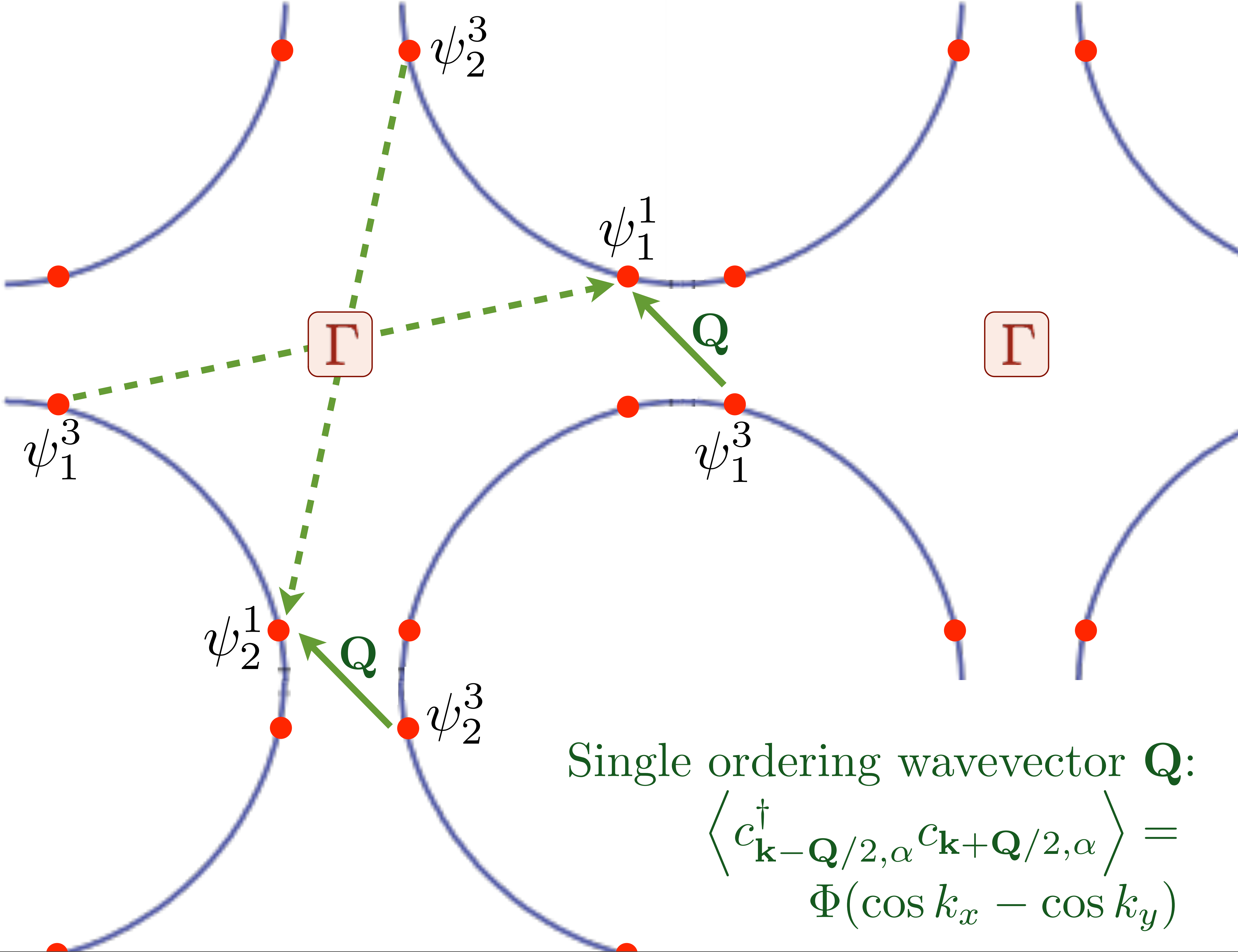
$$1 + \frac{\alpha}{3\pi(1+\alpha^2)} \log^2 \left(\frac{1}{\omega}\right)$$

Density-wave order parameter:

$$\left(\psi^{3\dagger}_{1\alpha}\psi^{1}_{1\alpha} - \psi^{3\dagger}_{2\alpha}\psi^{1}_{2\alpha}\right)$$

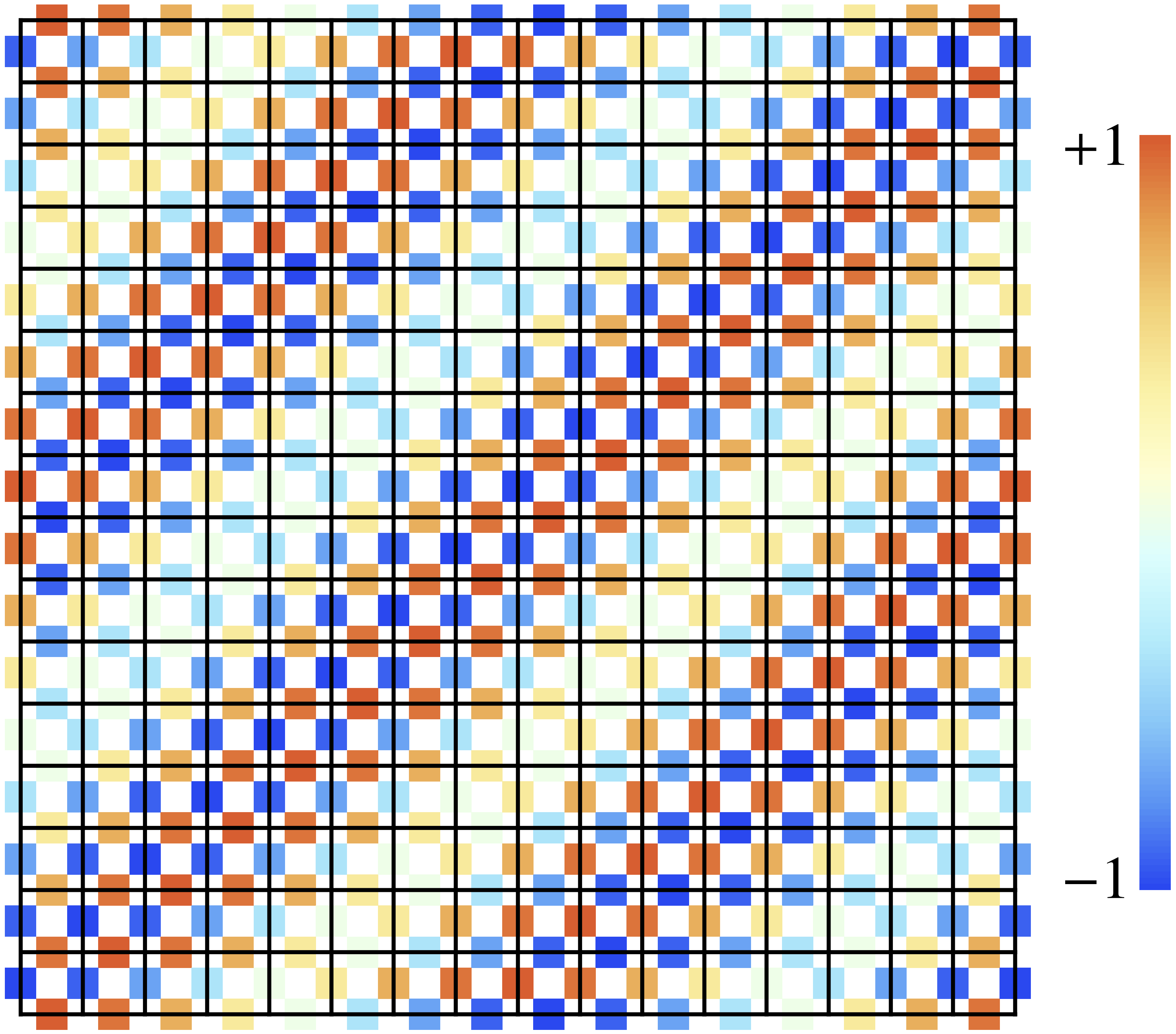


ψ_2^3
ψ_1^1
Γ
Γ
ψ_1^3
ψ_2^1



Single ordering wavevector $\mathbf{Q}$:

$$\left\langle c^{\dagger}_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond: VBS order

No modulations on sites. Modulated bond-density wave with local Ising-nematic ordering:

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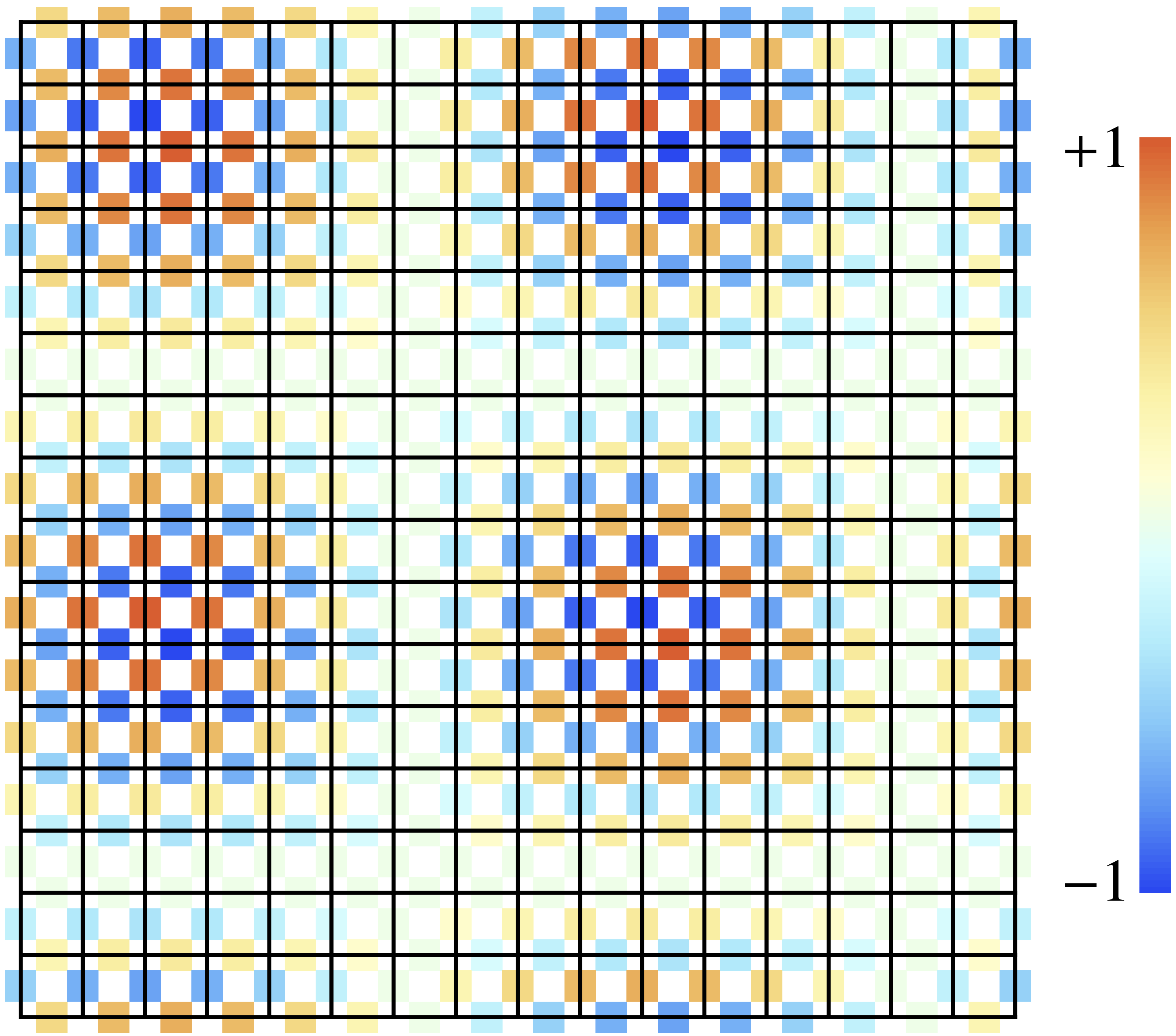

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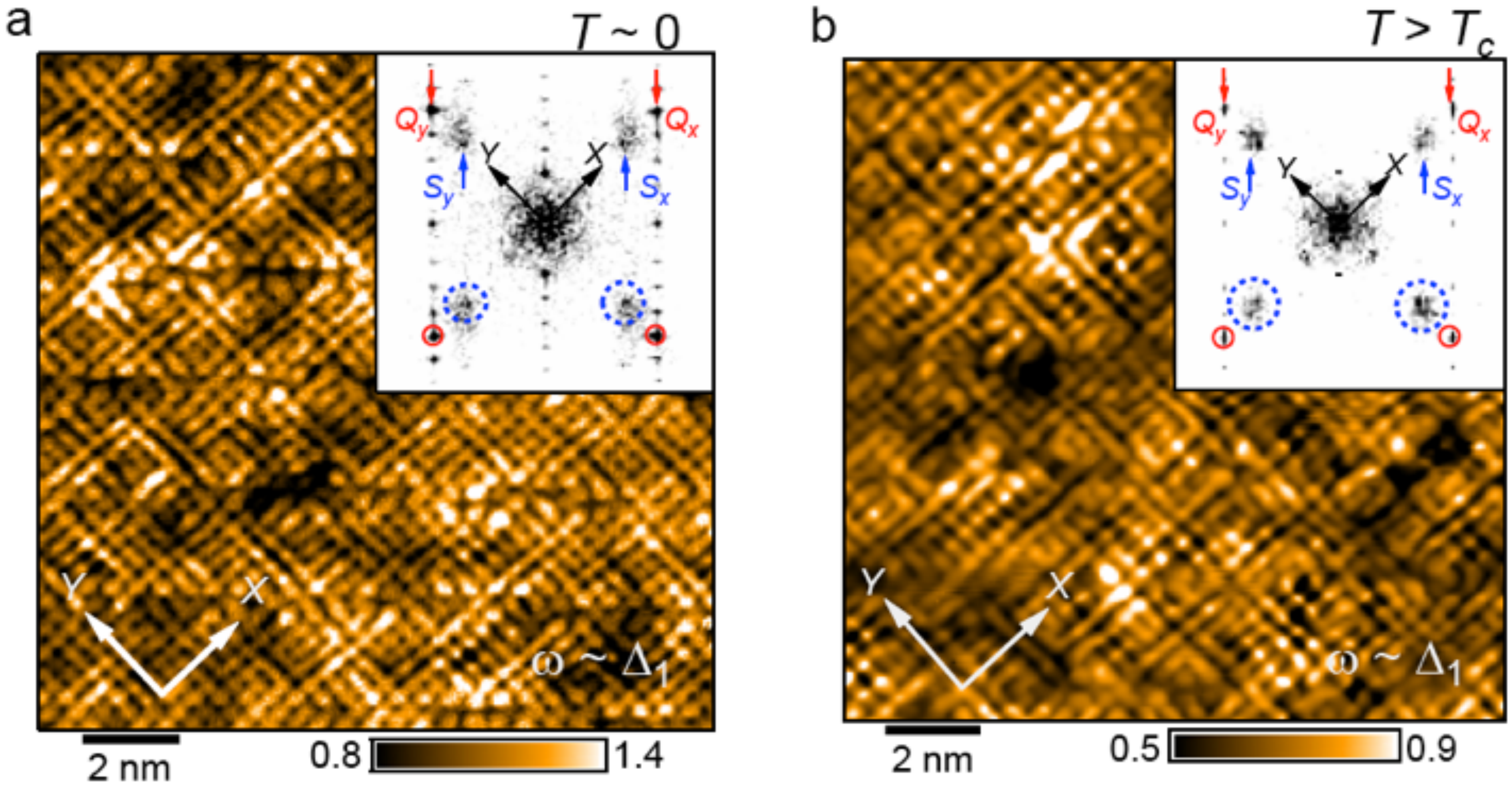
STM measurements of $Z(r)$, the energy asymmetry in density of states in $Bi_2Sr_2CaCu_2O_{8+\delta}$.



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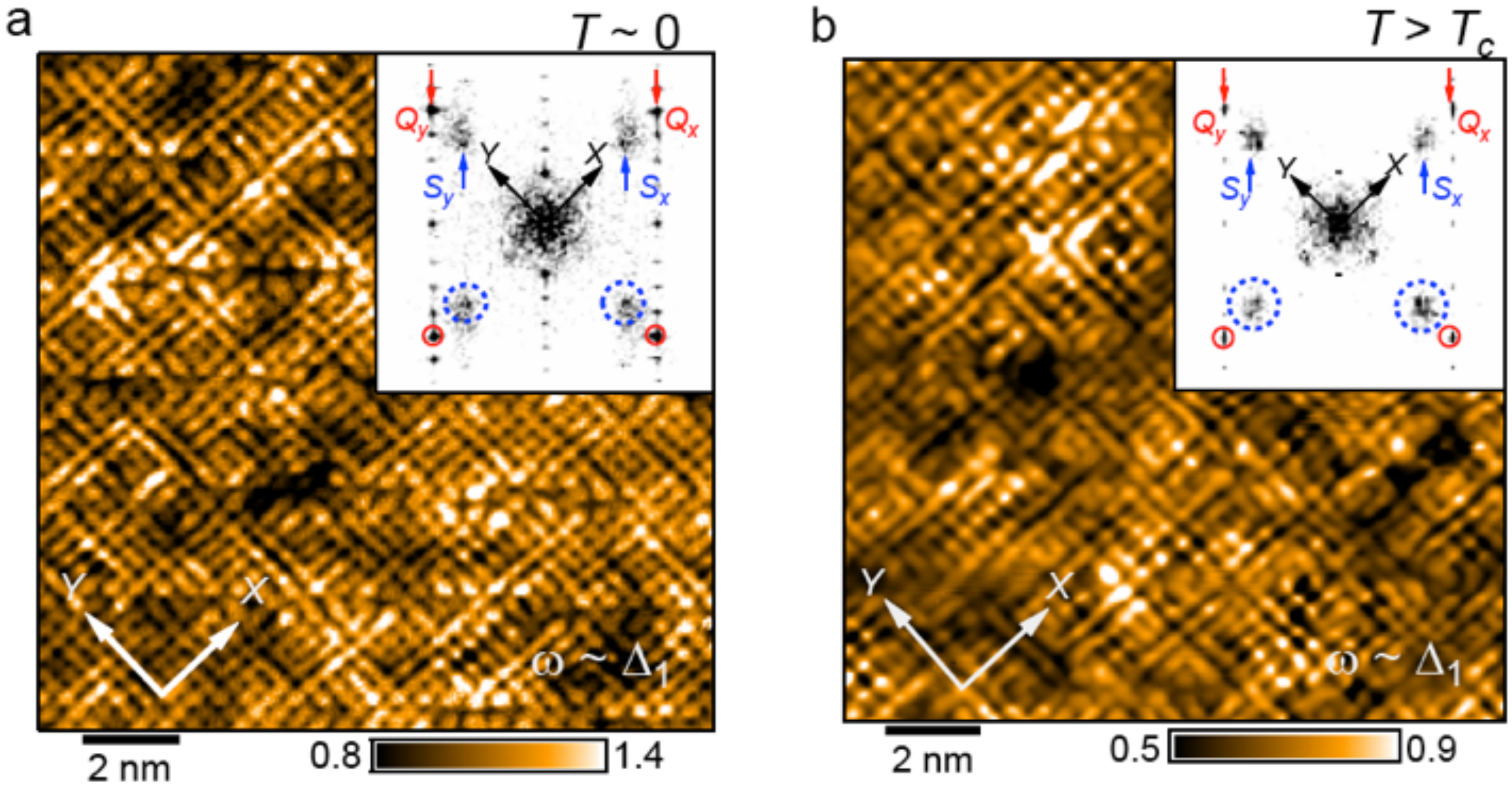

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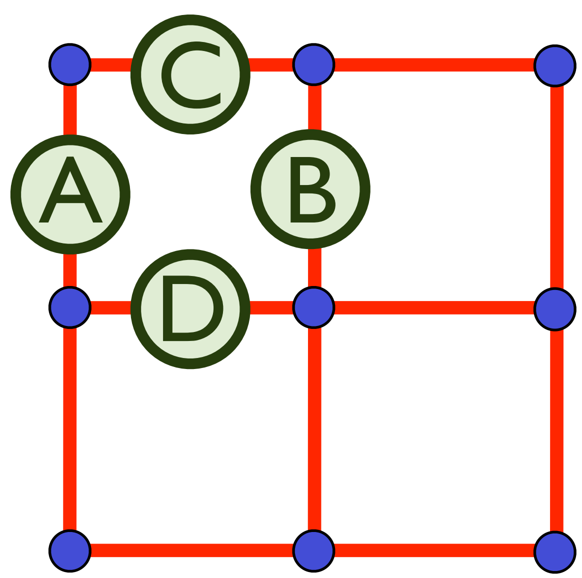

$$O_N = Z_A + Z_B - Z_C - Z_D$$

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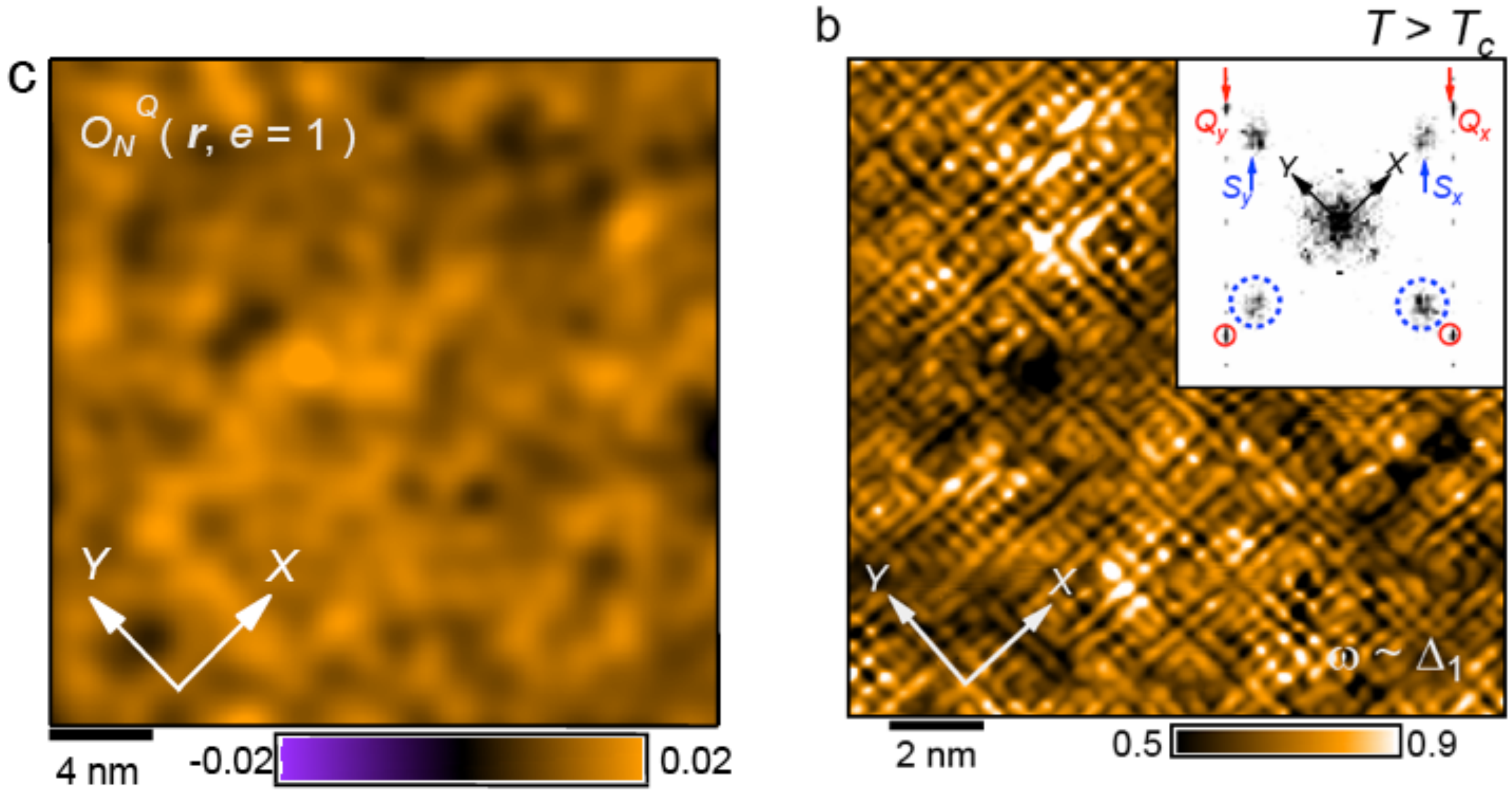

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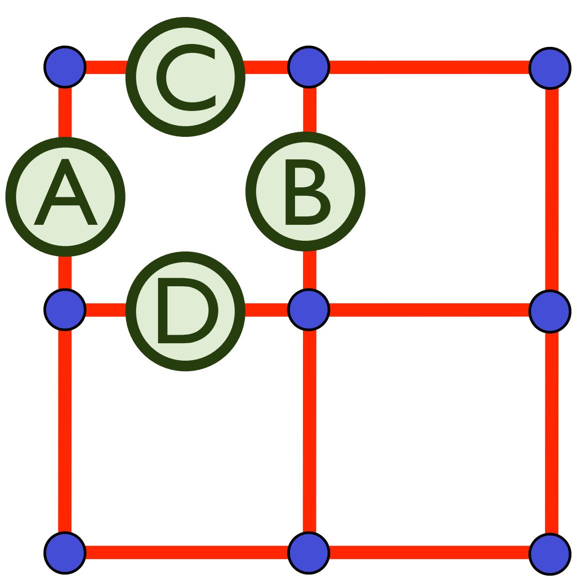

$$O_N = Z_A + Z_B - Z_C - Z_D$$

Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

Conclusions

Presented global phase diagram of a SU(2) gauge theory for spin-density wave (SDW) ordering in metals.

Theory has a phase with no long-range SDW order but with "hedgehogs" suppressed. This phase has pocket Fermi surfaces similar to recent photoemission observations.

Then we discussed the field theory for a direct transition between two Fermi liquids: from a large Fermi surface to Fermi pockets. This theory flows to strong coupling in two spatial dimensions.

We found a strong instability to d-wave pairing near the critical point. The critical theory also had emergent pseudospin symmetries, which implied an additional instability to bond-ordering with a local Ising-nematic character.