De Sitter Lecture Series in Theoretical Physics 2009
University of Groningen

Quantum phase transitions: from antiferromagnets and superconductors to black holes

Talk online: sachdev.physics.harvard.edu

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Outline

1. Introduction to quantum phase transitions:
   quantum spin systems and relativistic field theories

2. Quantum phase transitions in *d*-wave
   superconductors and metals

3. The AdS/CFT correspondence:
   quantum criticality at strong coupling

4. The cuprate high temperature superconductors:
   competing orders and quantum criticality

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4. The cuprate high temperature superconductors:
   competing orders and quantum criticality

Victor Galitski, Maryland

Ribhu Kaul, Harvard → Kentucky

Max Metlitski, Harvard

Eun Gook Moon, Harvard

Yang Qi, Harvard

Cenke Xu, Harvard → Santa Barbara

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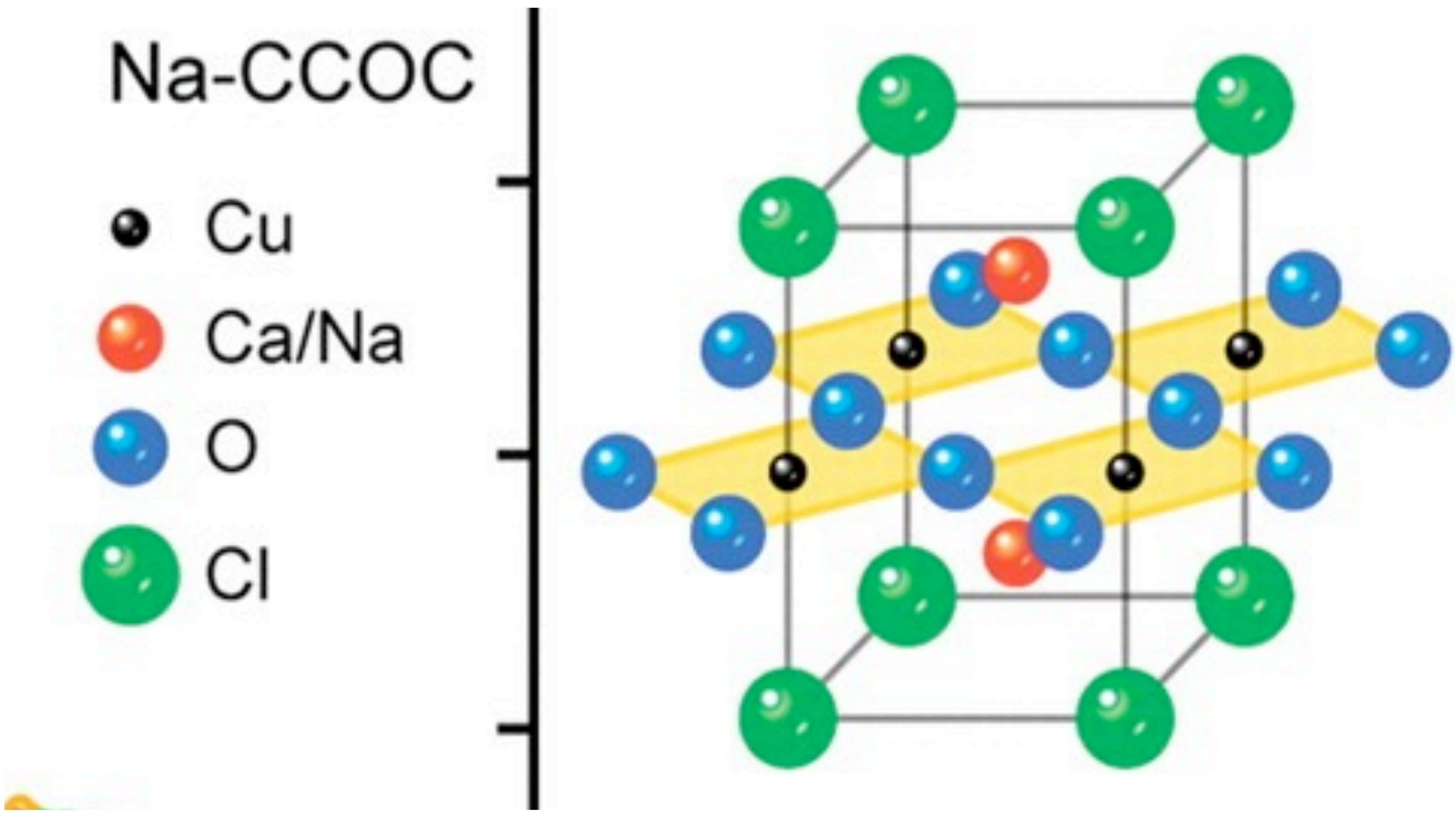
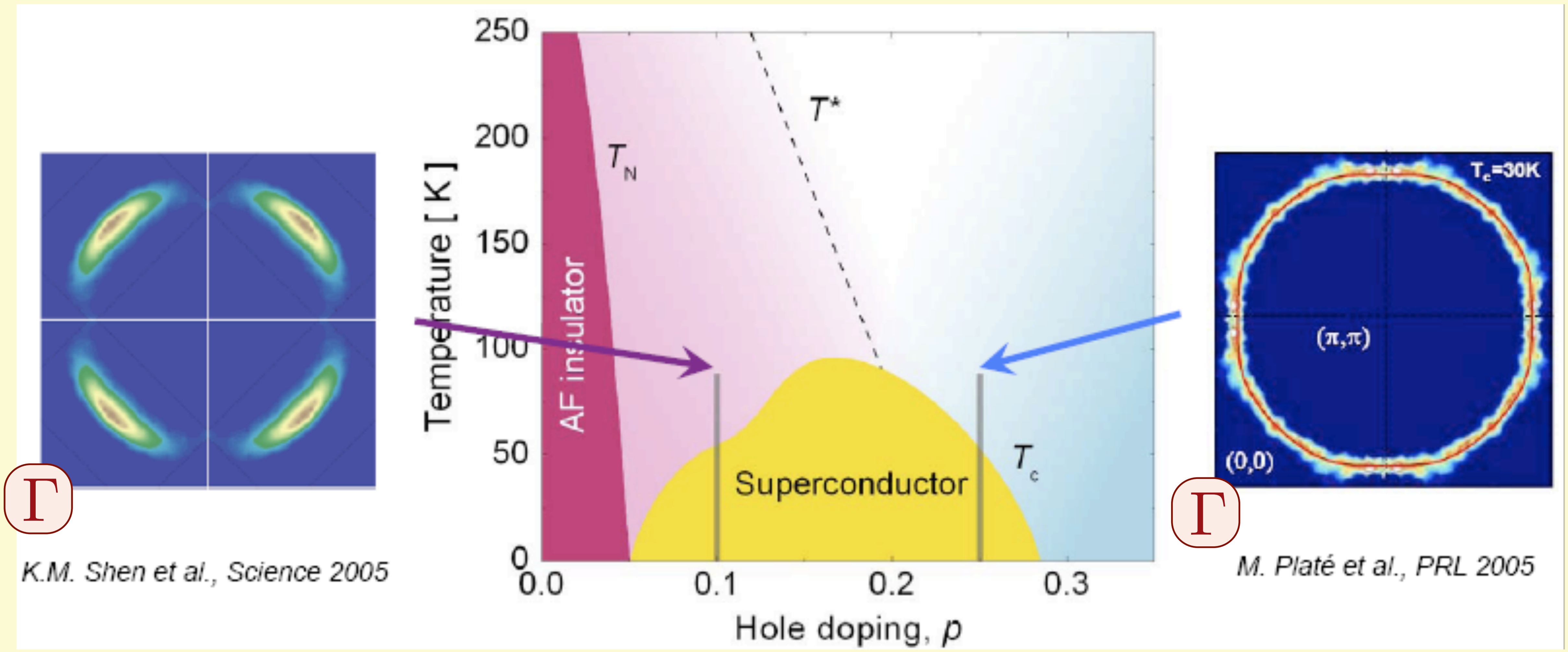
The cuprate superconductors



Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

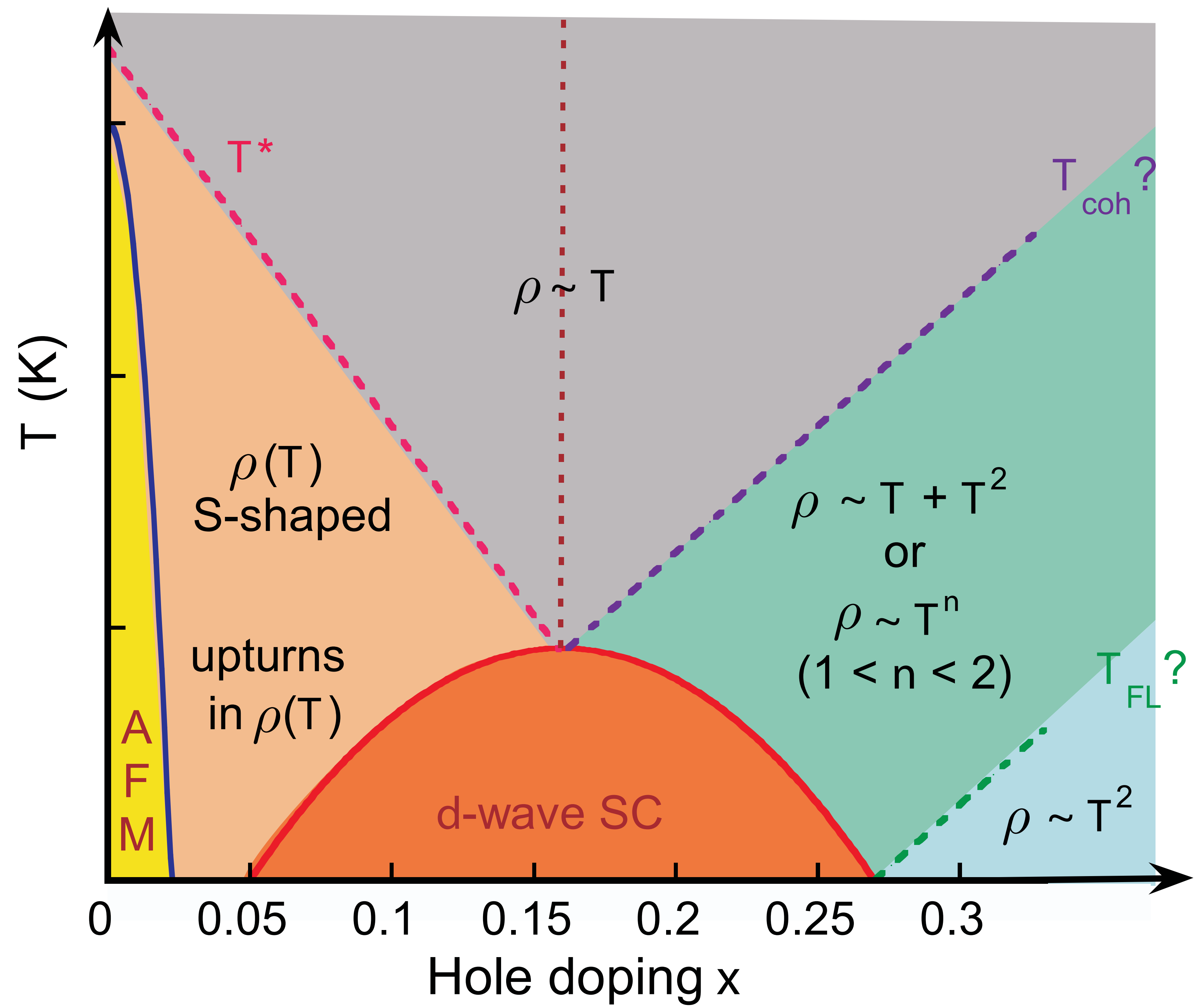
Large hole Fermi surface

Antiferro-
magnetism

d-wave
supercon-
ductivity

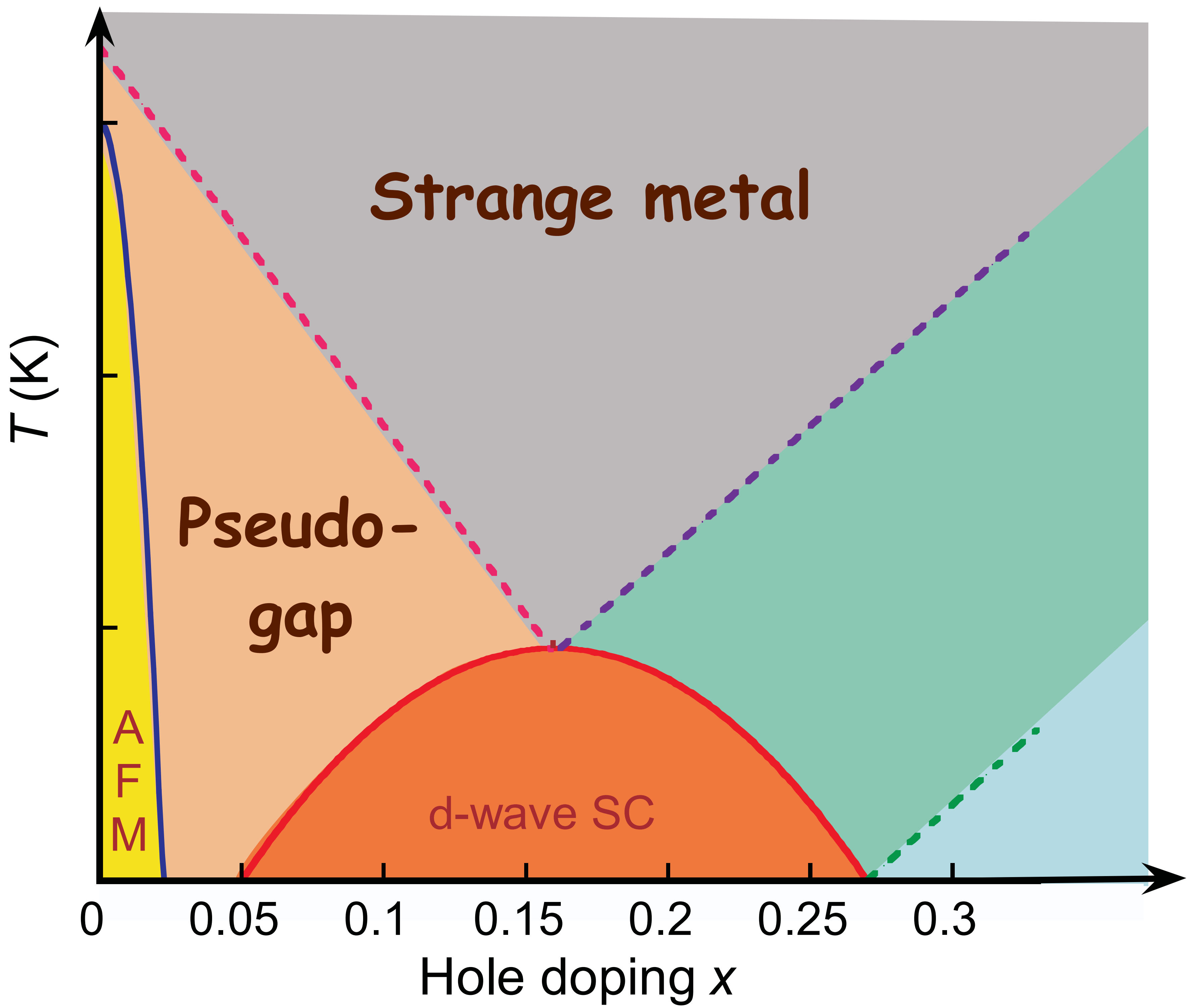
Fermi
surface

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

Crossovers in transport properties of hole-doped cuprates



Antiferro-
magnetism

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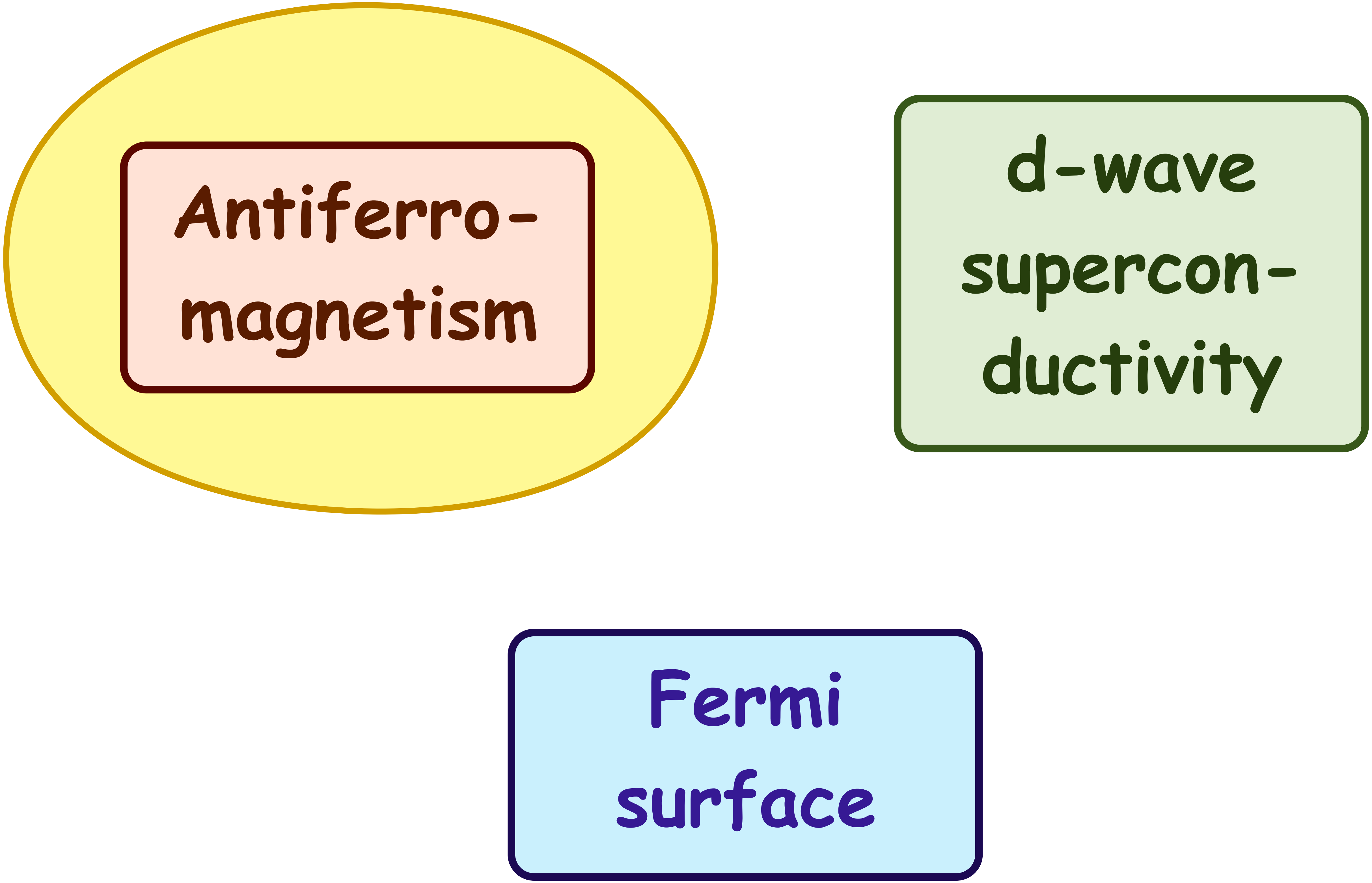
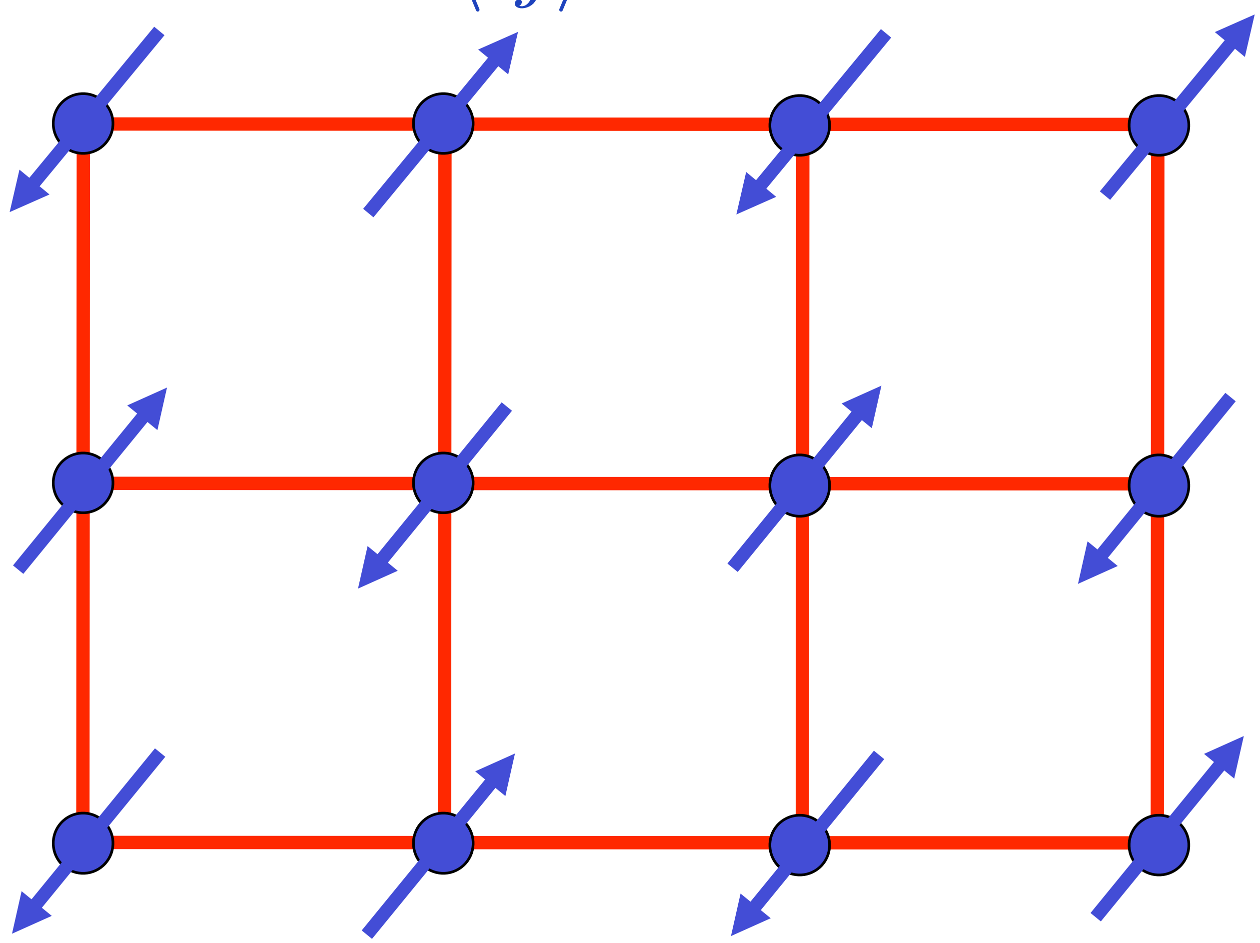

Antiferro-magnetism
d-wave supercon-ductivity
Fermi surface

Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



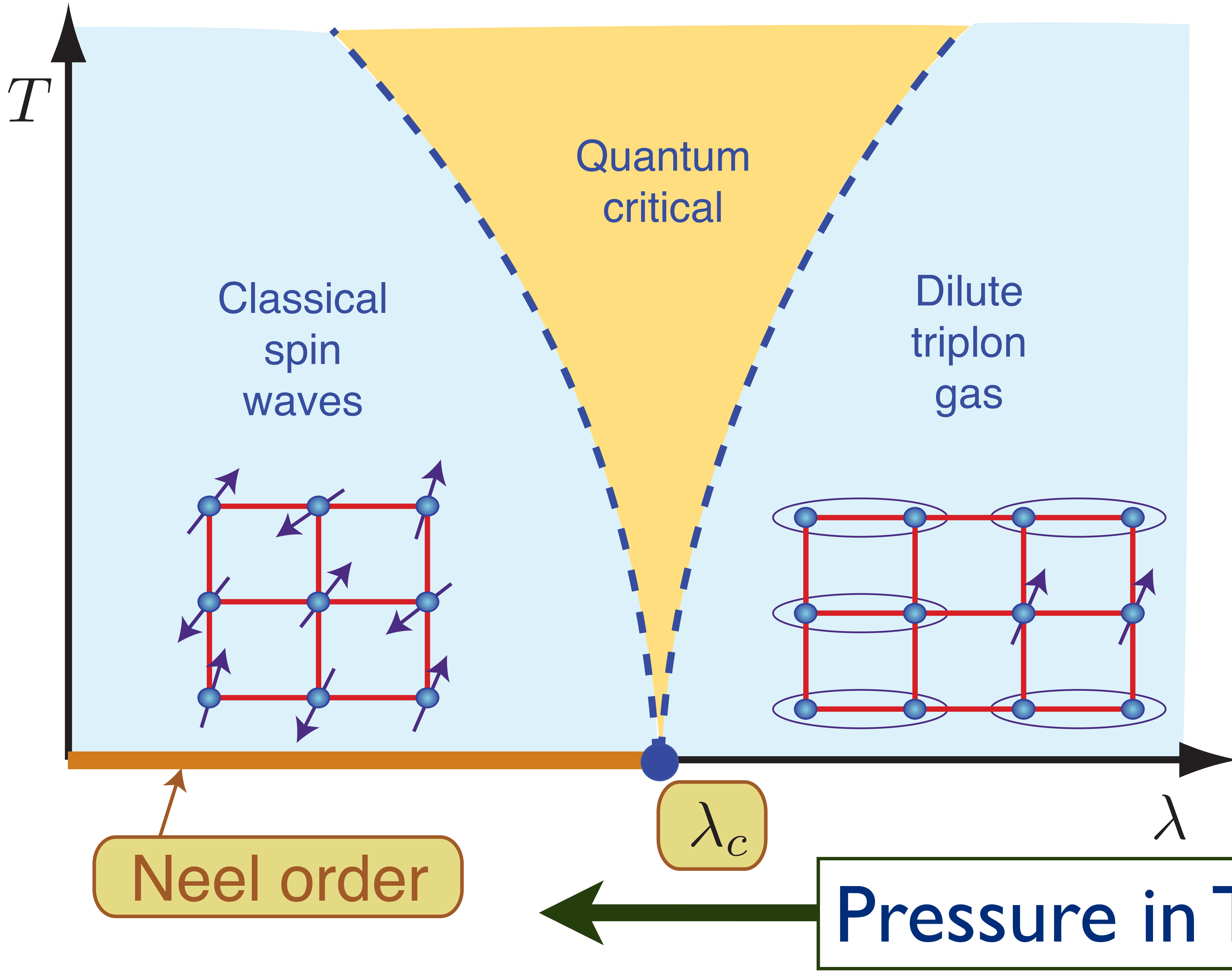
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

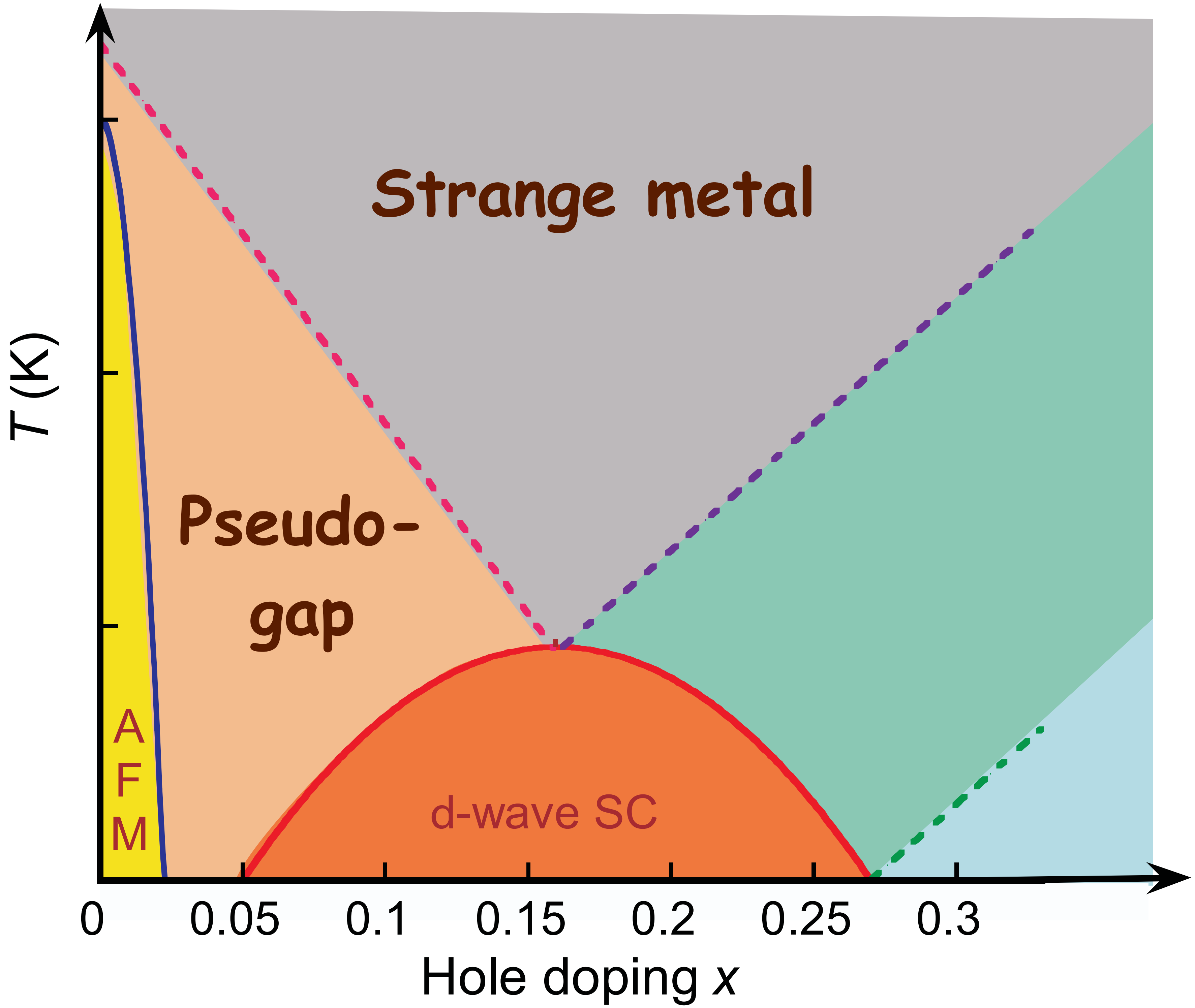
Canonical quantum critical phase diagram of coupled-dimer antiferromagnet



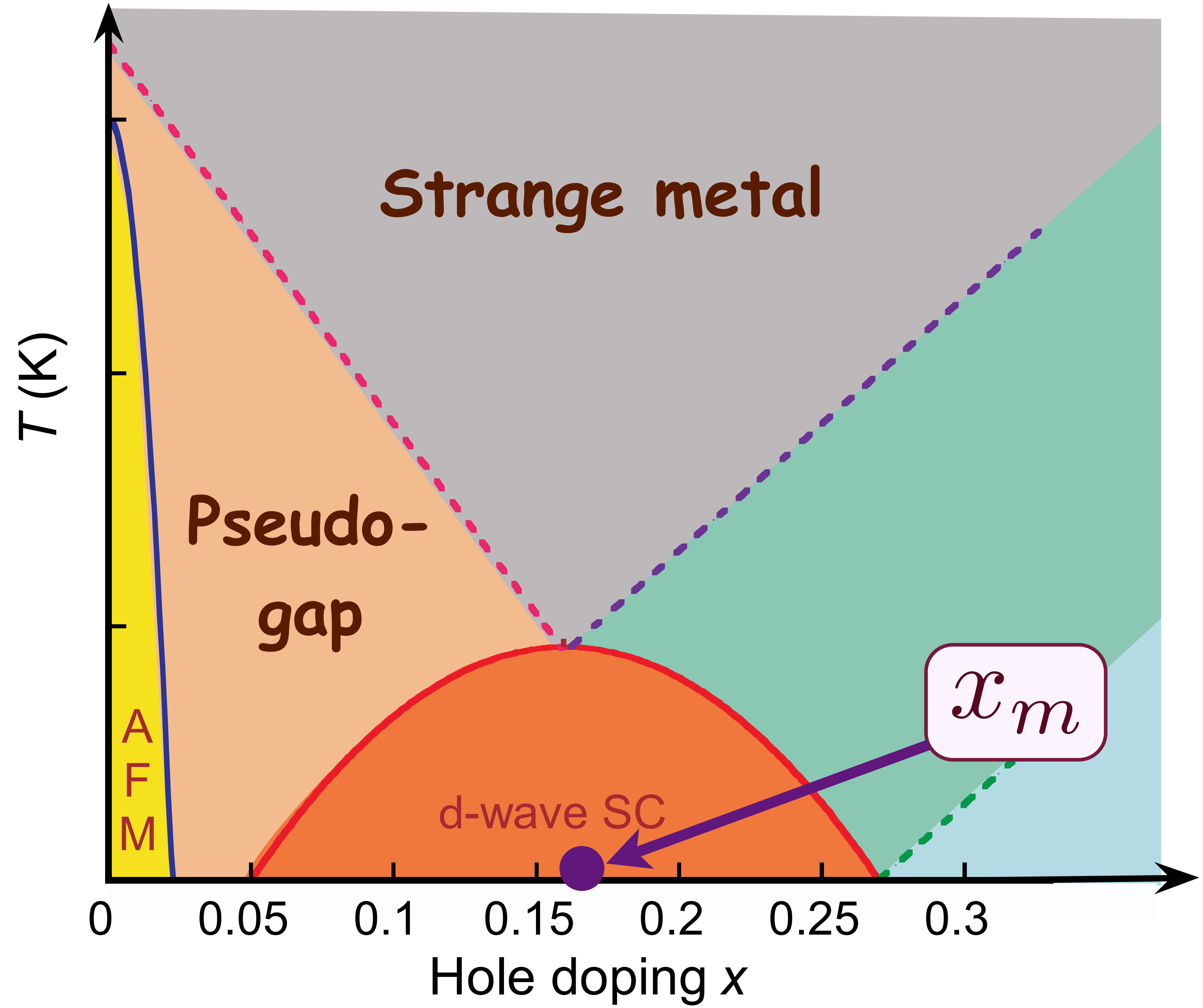
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

Christian Ruegg et al. , *Phys. Rev. Lett.* **100**, 205701 (2008)

Crossovers in transport properties of hole-doped cuprates



Crossovers in transport properties of hole-doped cuprates



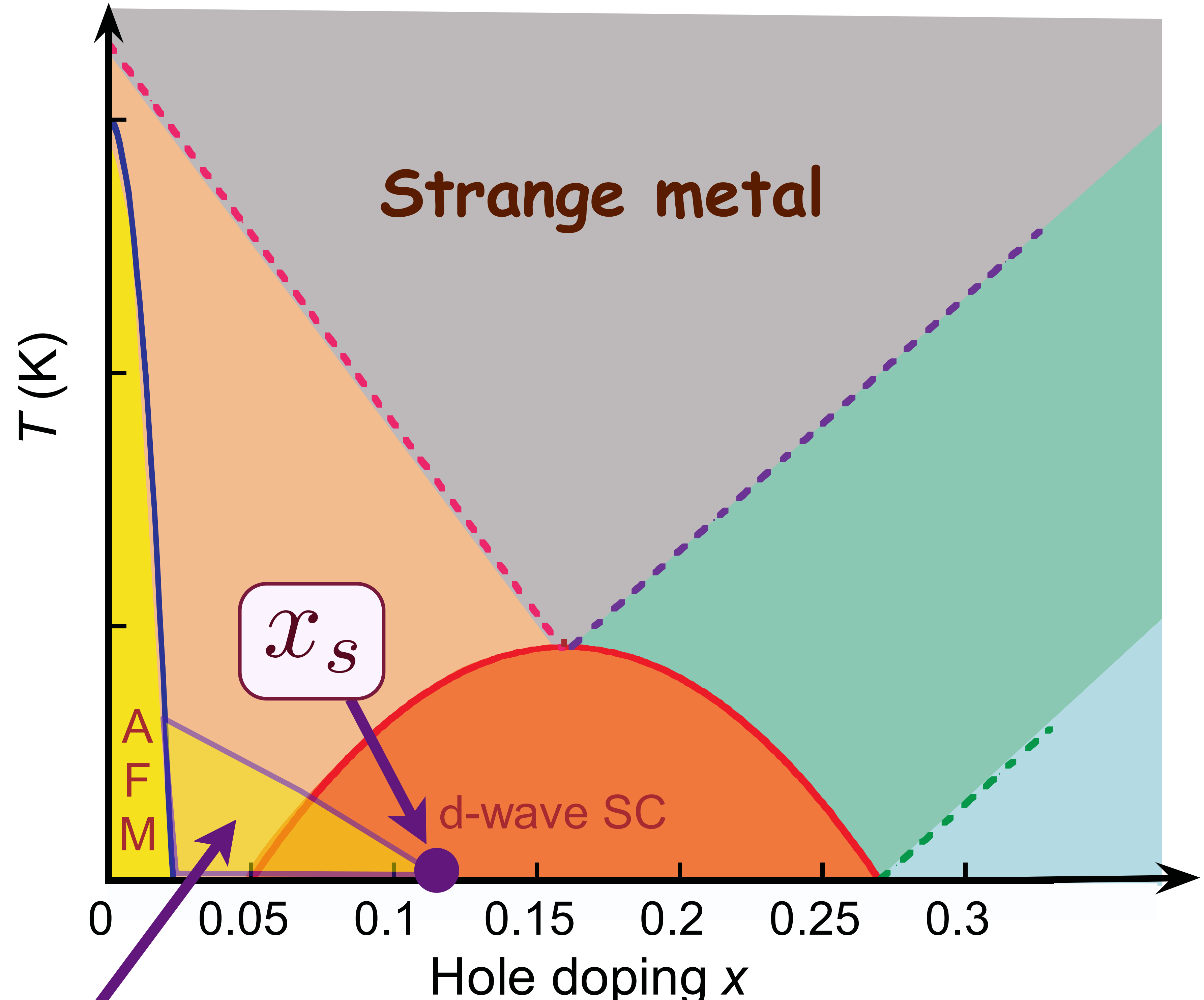
S. Sachdev and J. Ye, *Phys. Rev. Lett.* **69**, 2411 (1992).

A. J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T

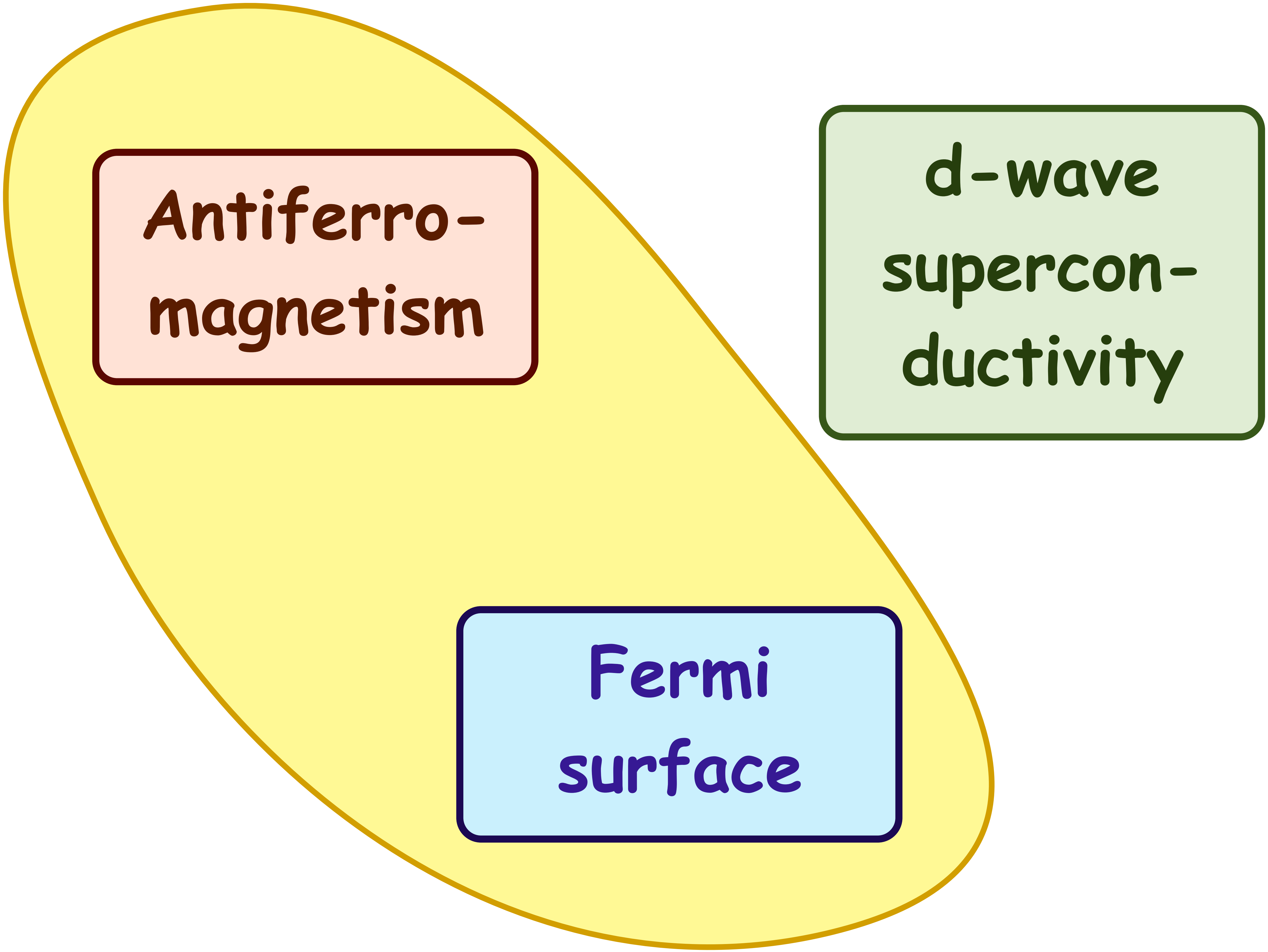


Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

Antiferro-
magnetism

d-wave
supercon-
ductivity

Fermi
surface


Antiferro-
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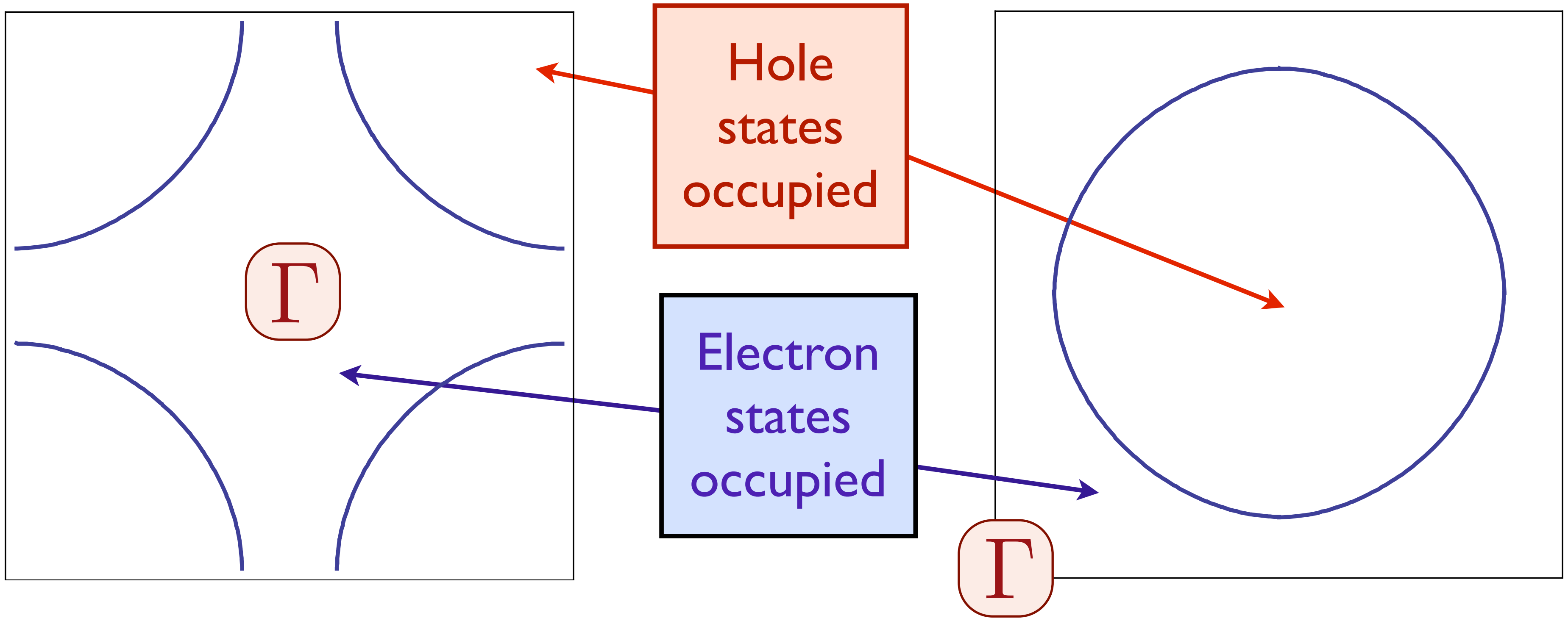
"Large" Fermi surfaces in cuprates



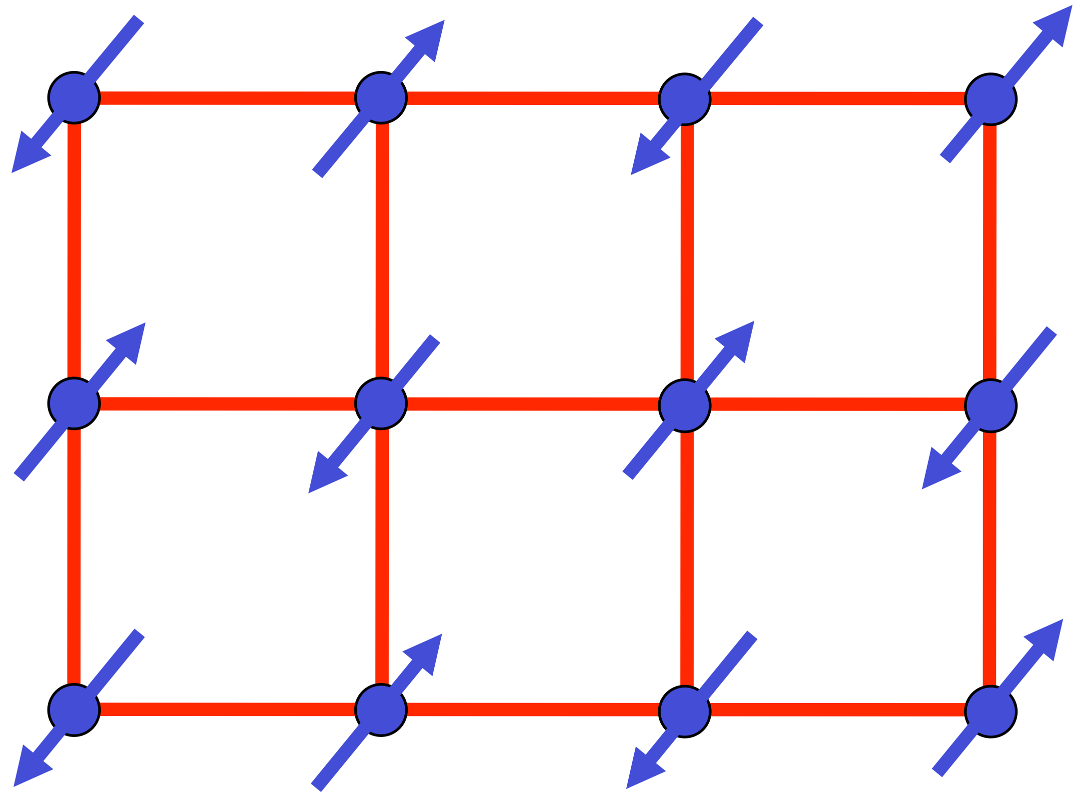
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

$$\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$$

Spin density wave theory

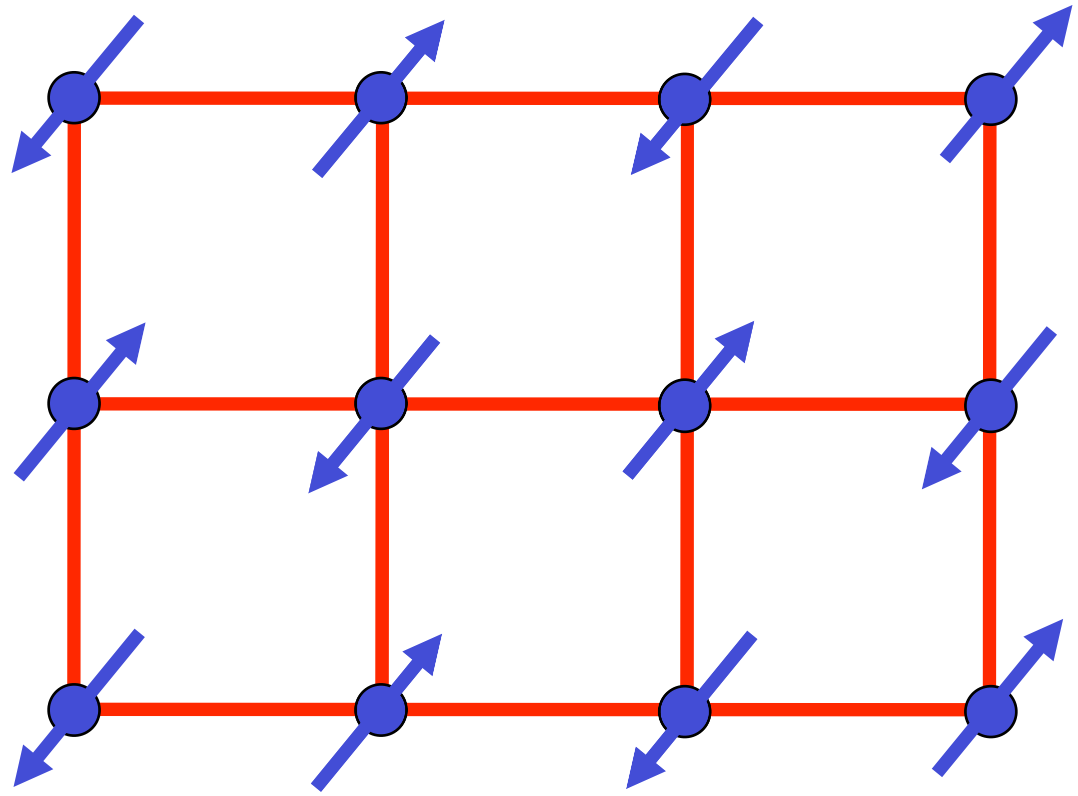


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the spin density wave (SDW) order parameter, and $\mathbf{K}$ is the ordering wavevector. For simplicity, we consider $\mathbf{K} = (\pi, \pi)$.

Spin density wave theory



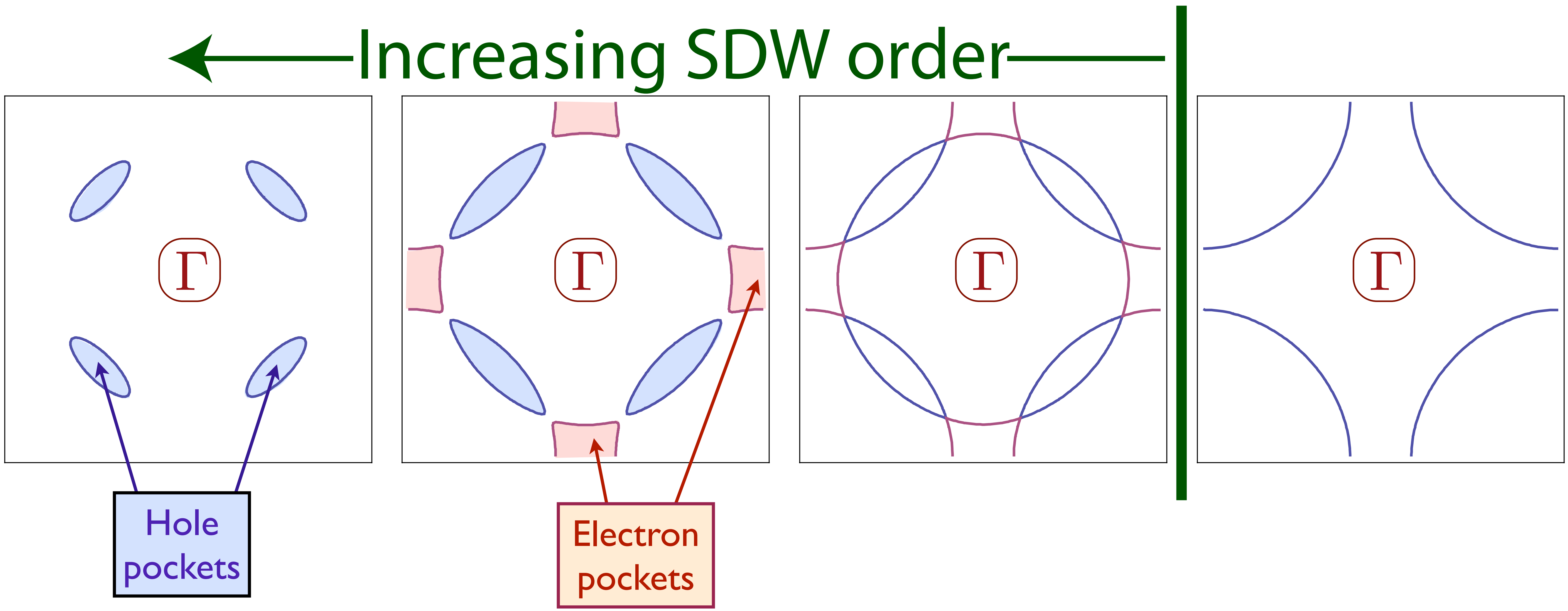
Spin density wave Hamiltonian

$$H_{\rm sdw} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

Diagonalize $H_0 + H_{\rm sdw}$ for $\vec{\varphi} = (0, 0, \varphi)$

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

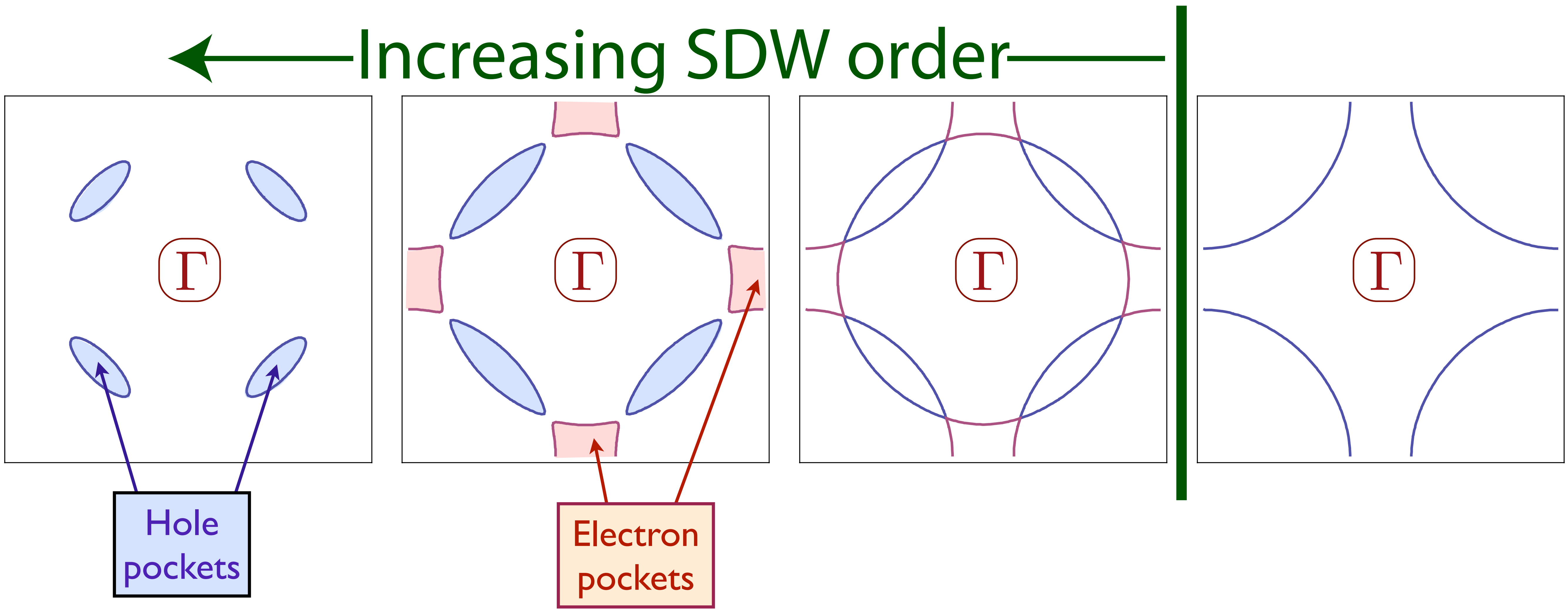
Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



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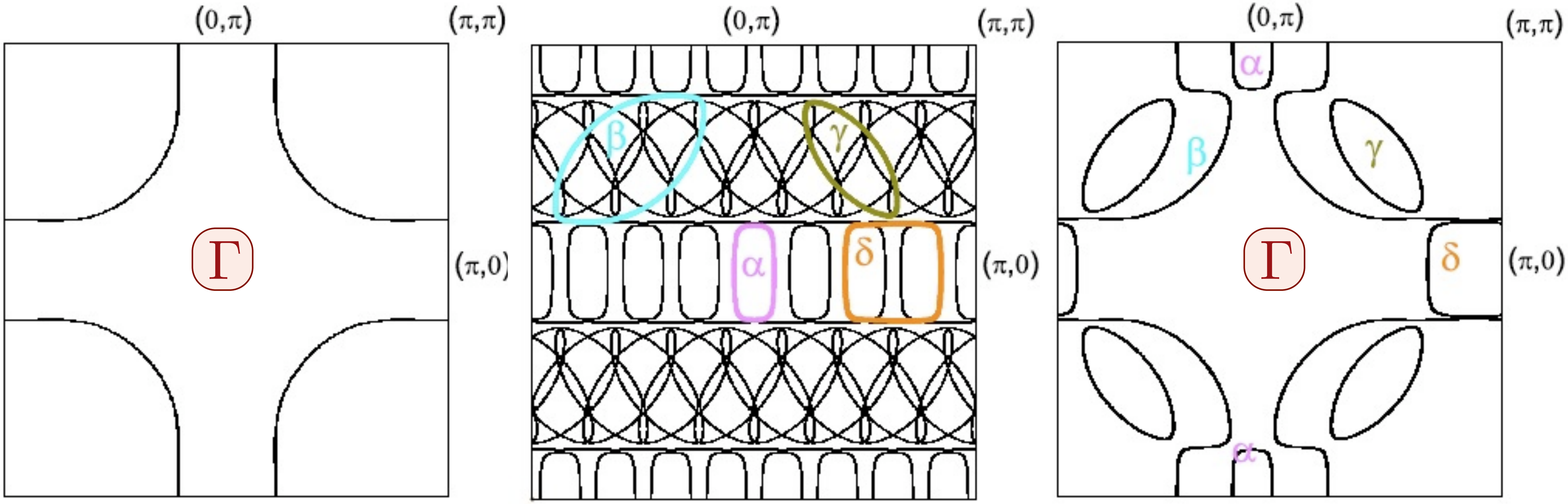
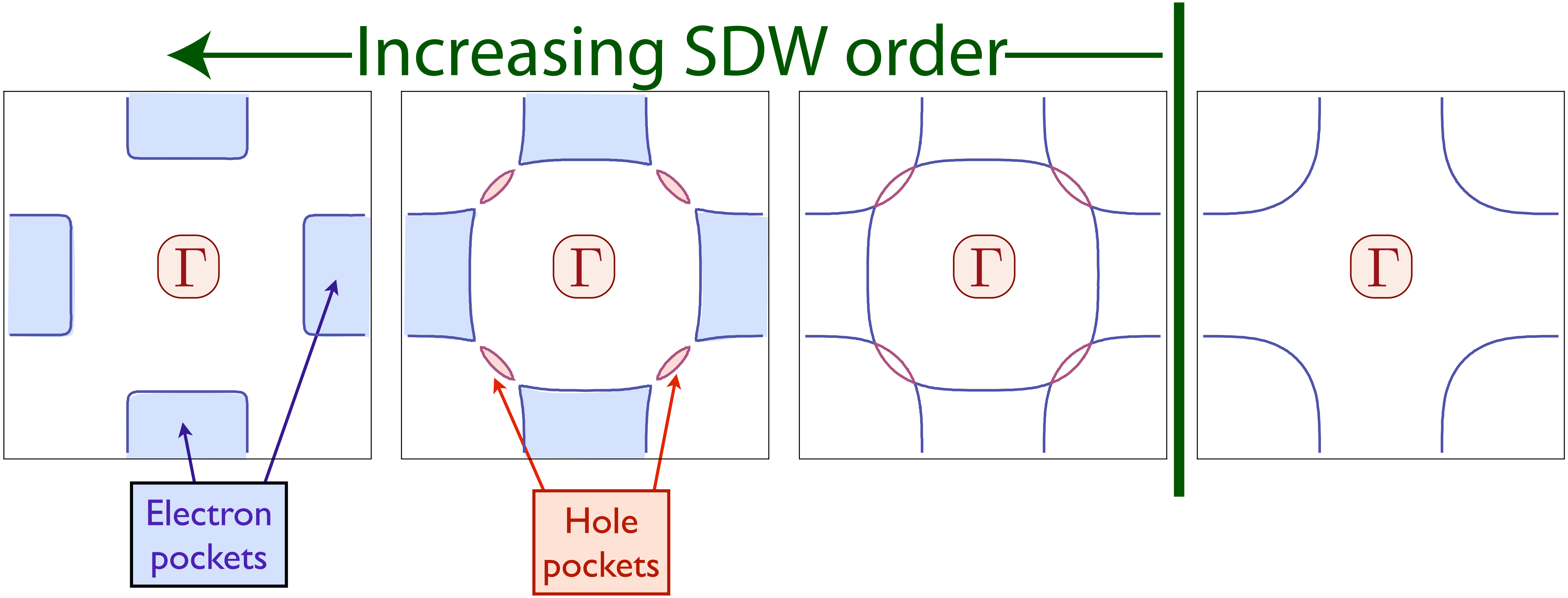
Spin density wave theory in hole-doped cuprates



Incommensurate order in $YBa_2Cu_3O_{6+x}$

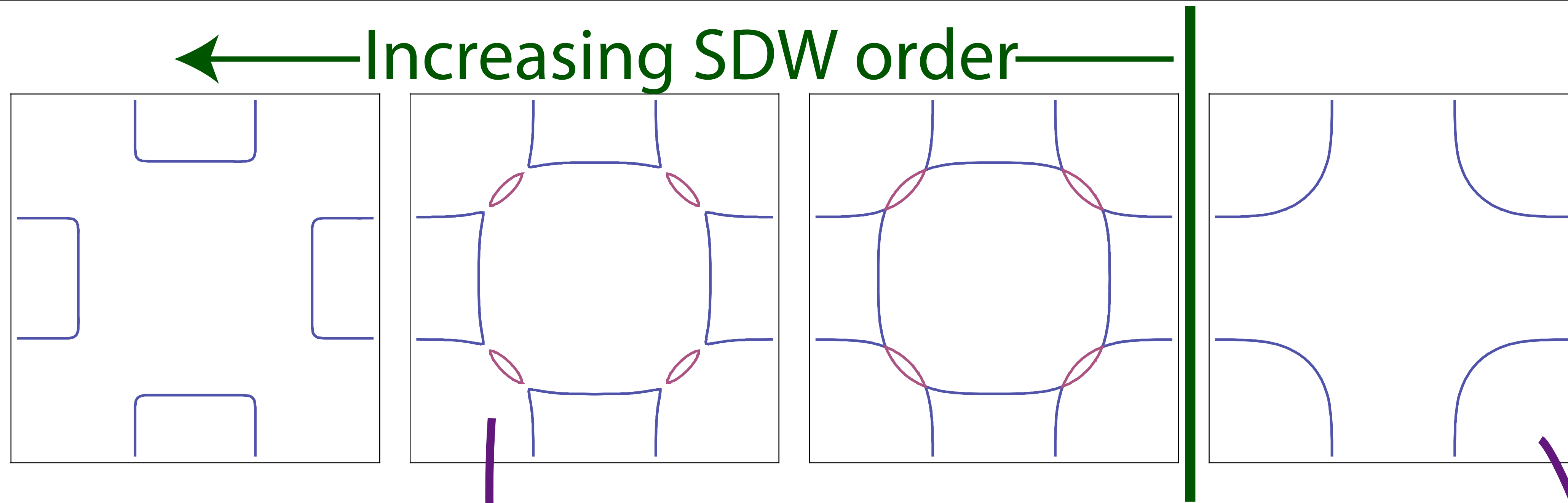
A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Electron-doped cuprates



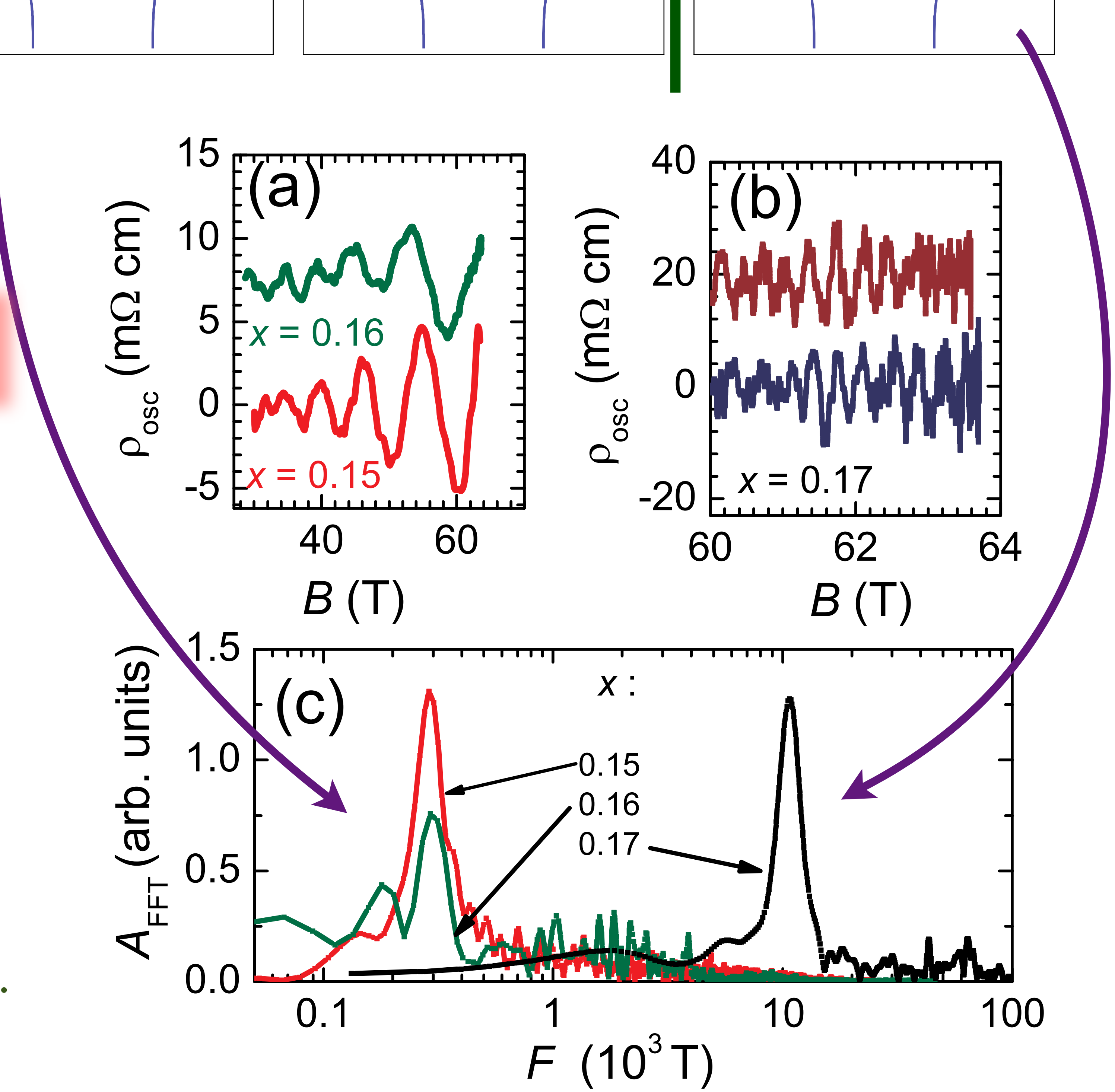
Large Fermi surface breaks up into electron and hole pockets

D. Senechal and A.-M. S. Tremblay, *Physical Review Letters* **92**, 126401 (2004)
J. Lin, and A. J. Millis, *Physical Review B* **72**, 214506 (2005).



Quantum oscillations

$\mathrm{Nd_{2-x}Ce_xCuO_4}$



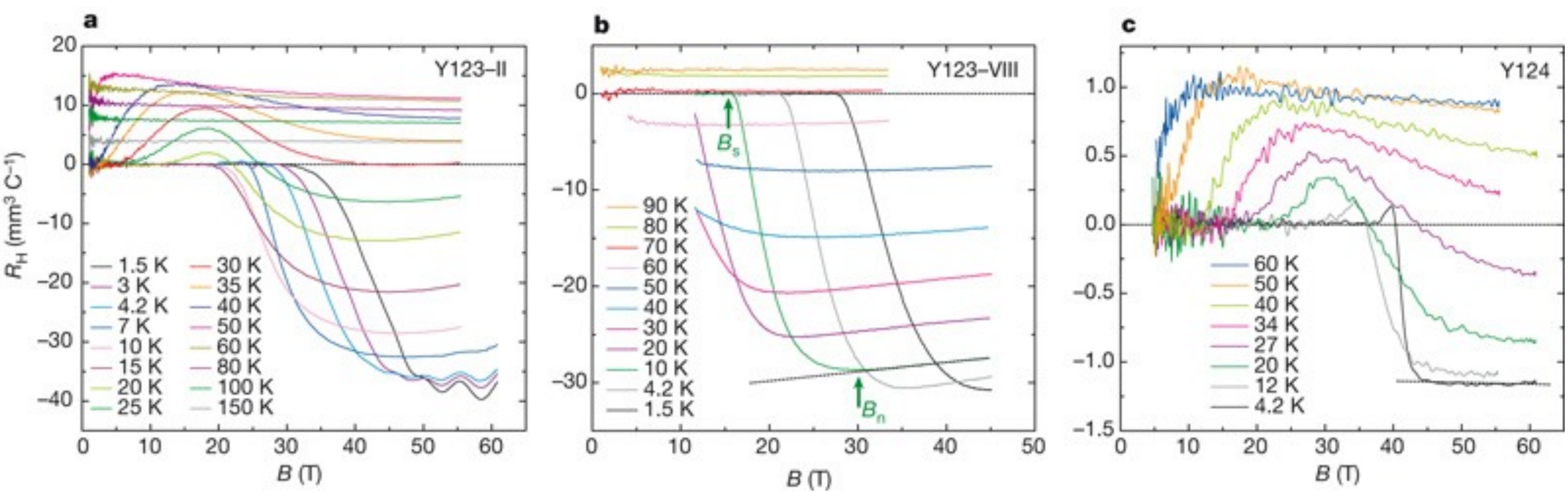
T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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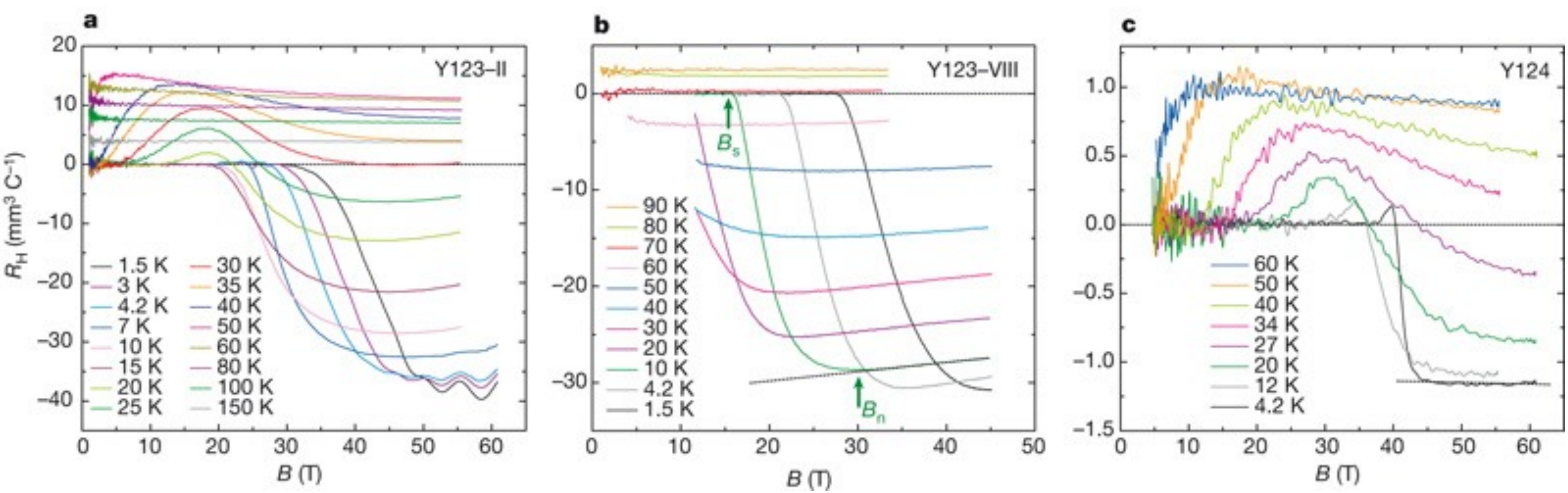
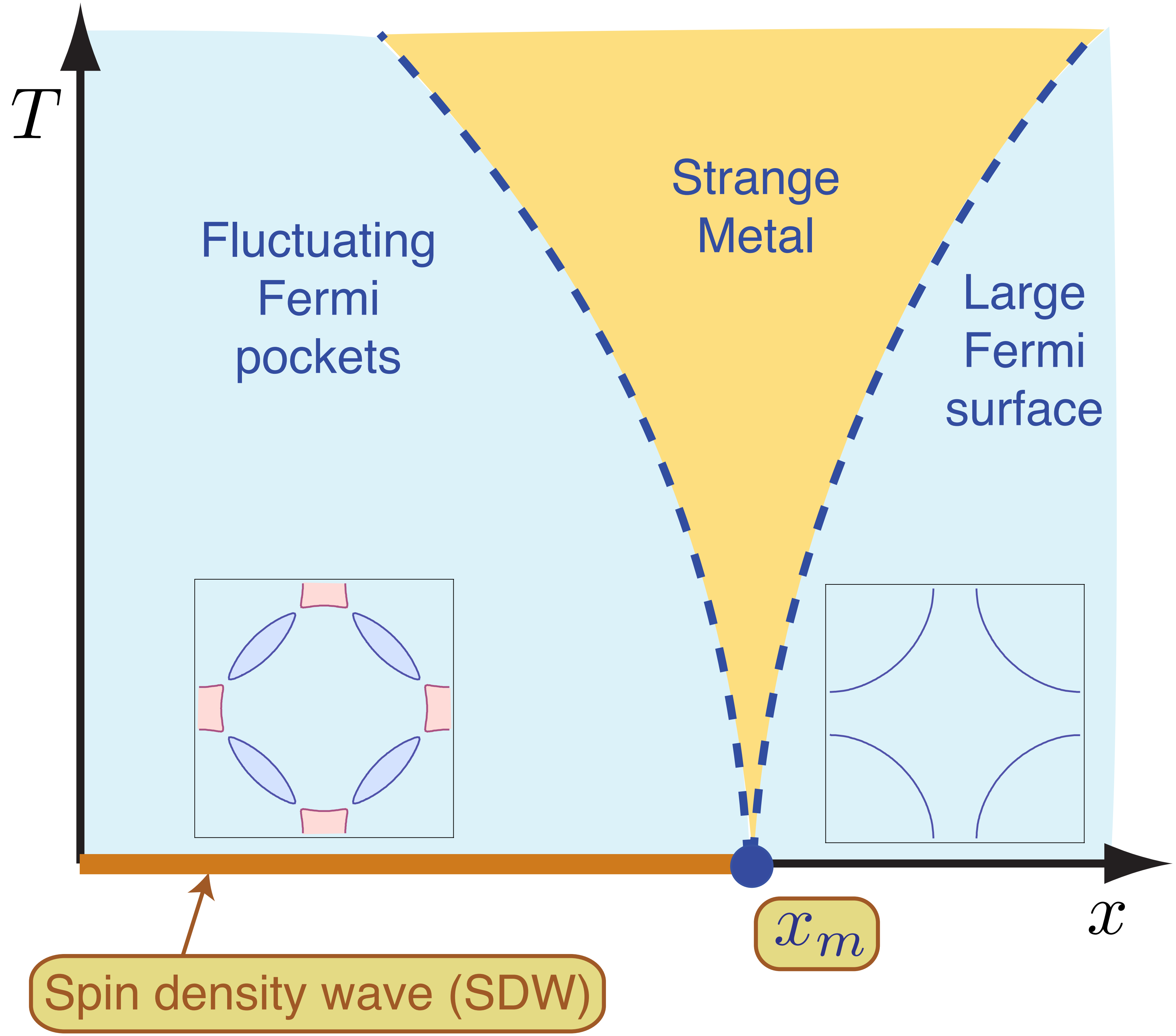


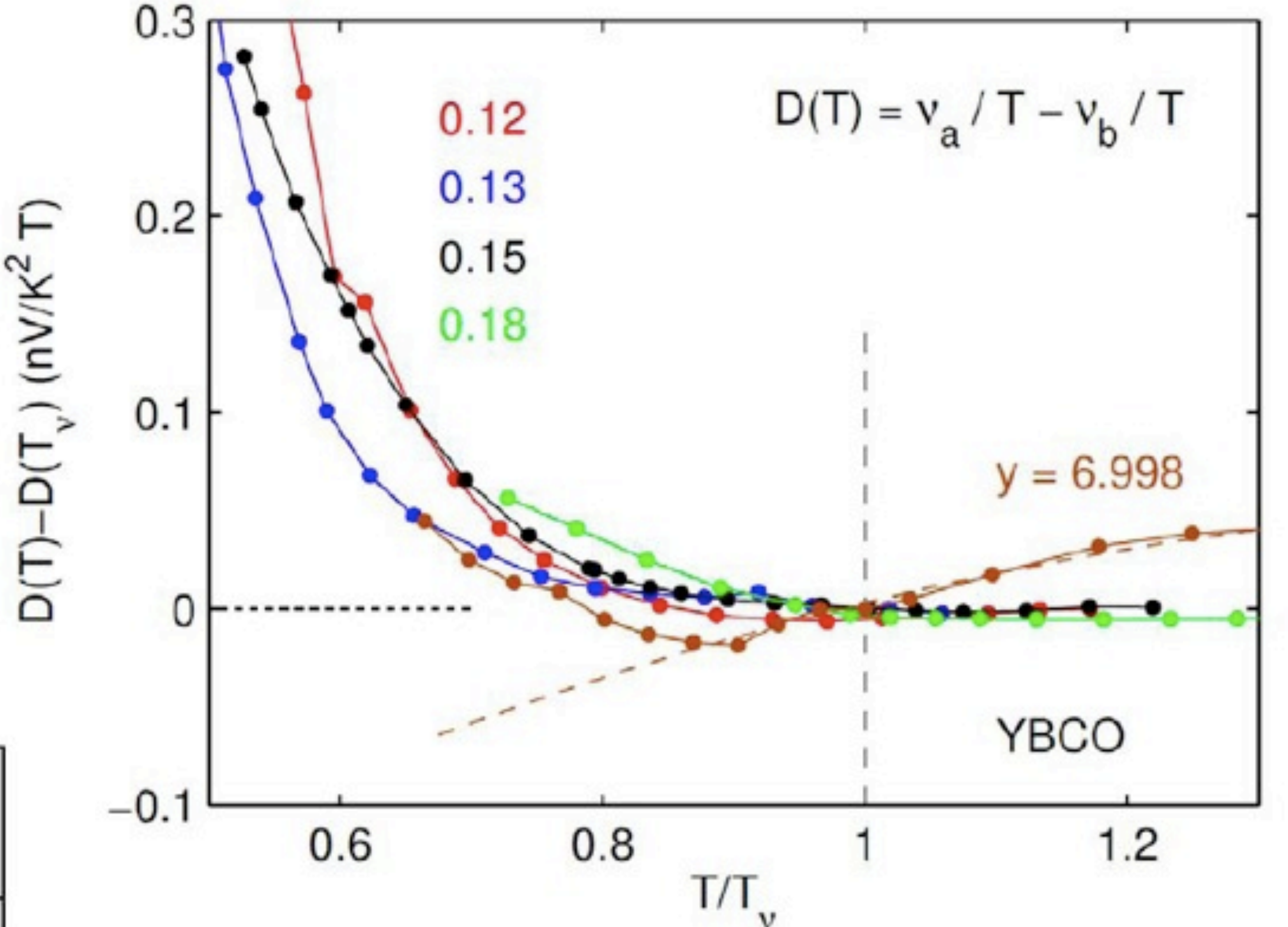
Theory of quantum criticality in the cuprates

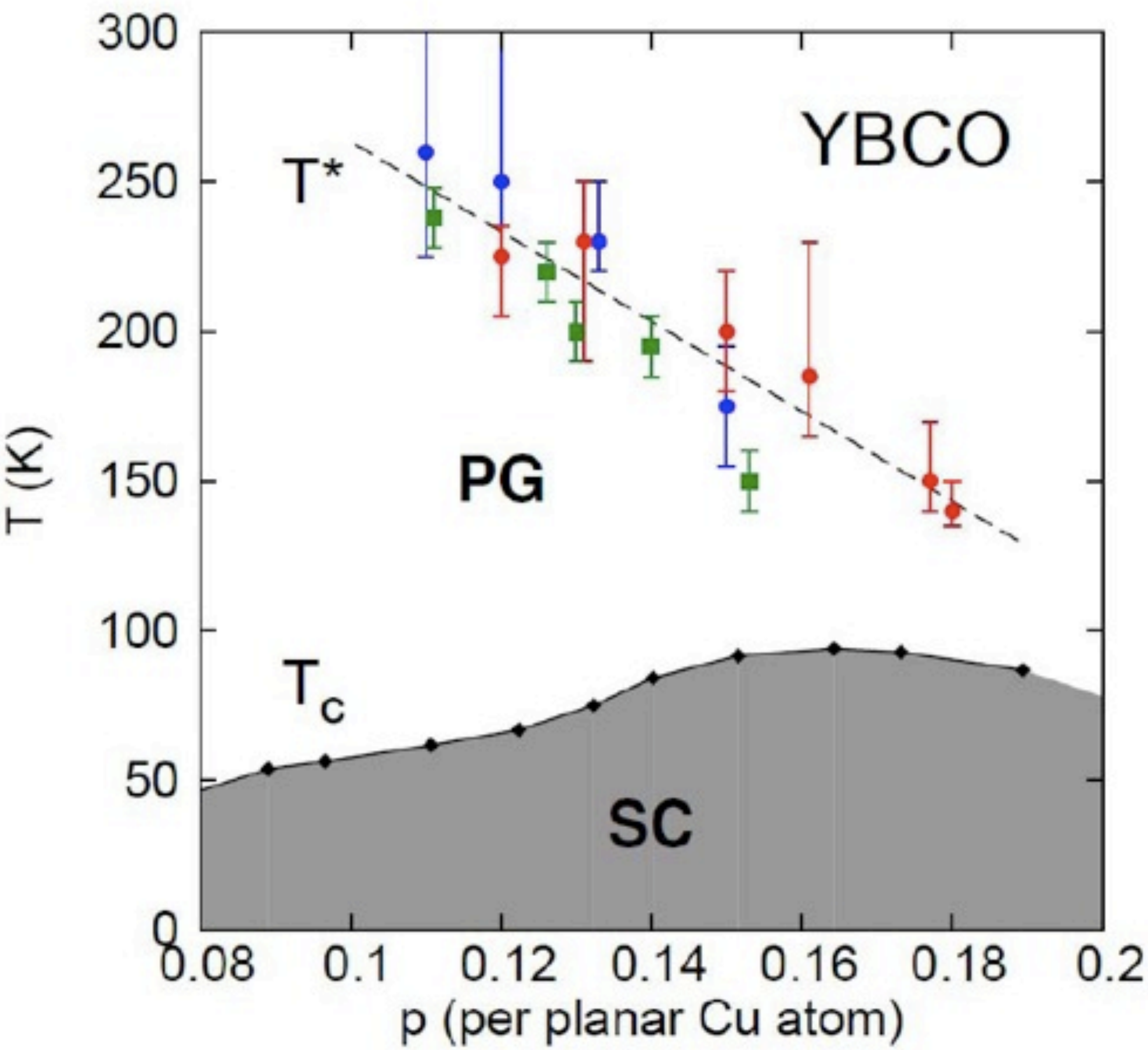


Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

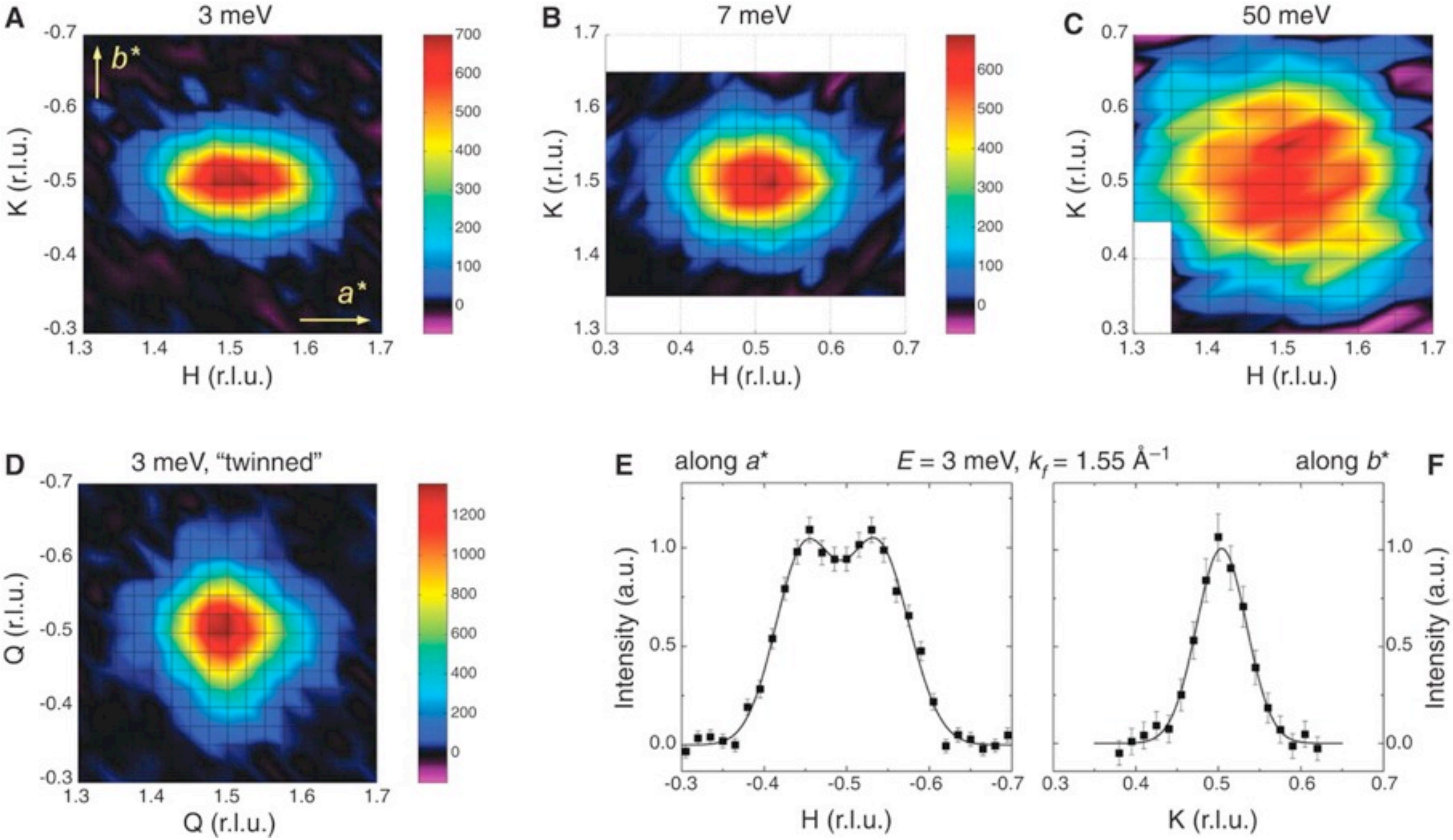
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D. A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430





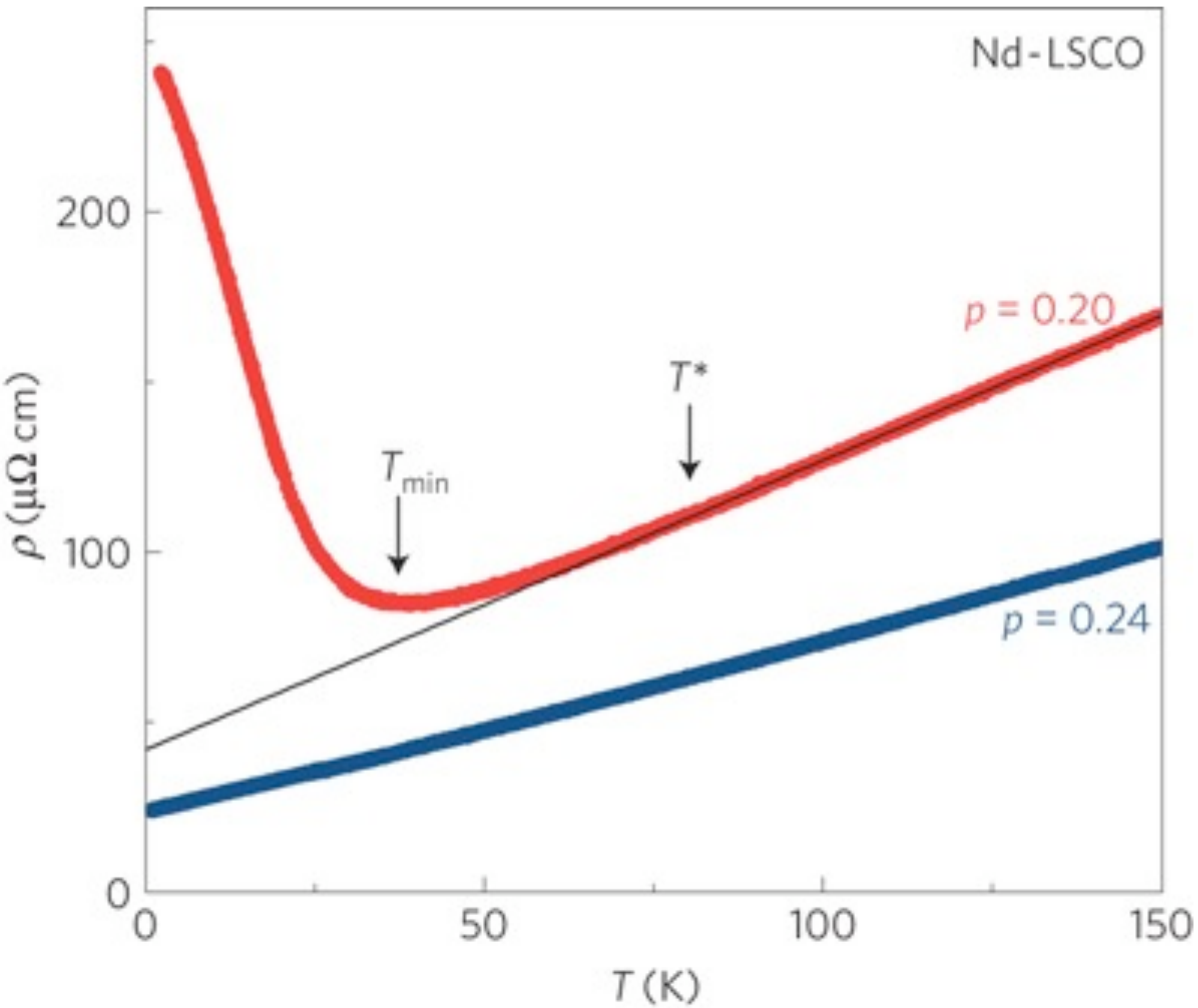
S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).



Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c

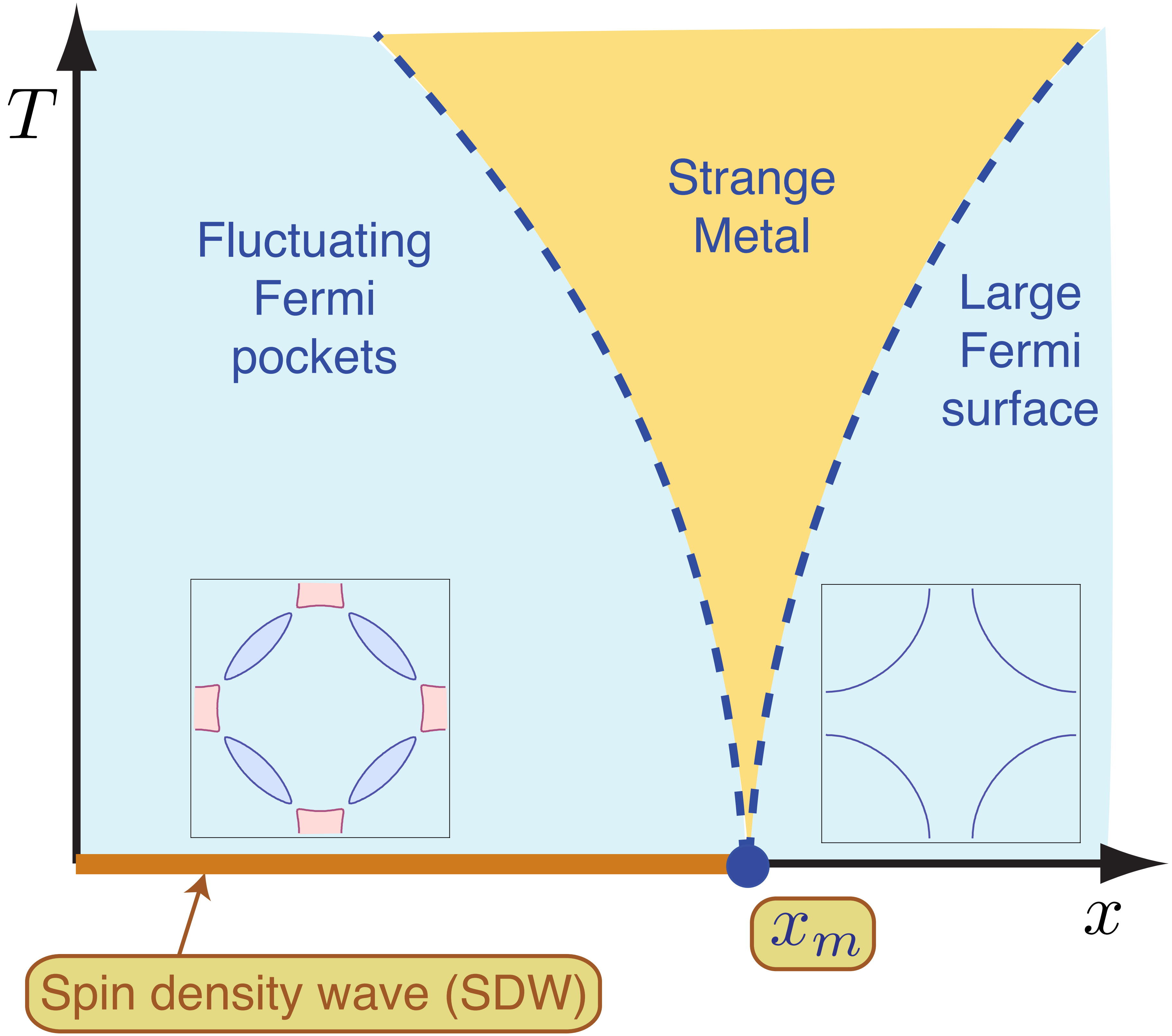


Magnetic field of upto 35 T used to suppress superconductivity

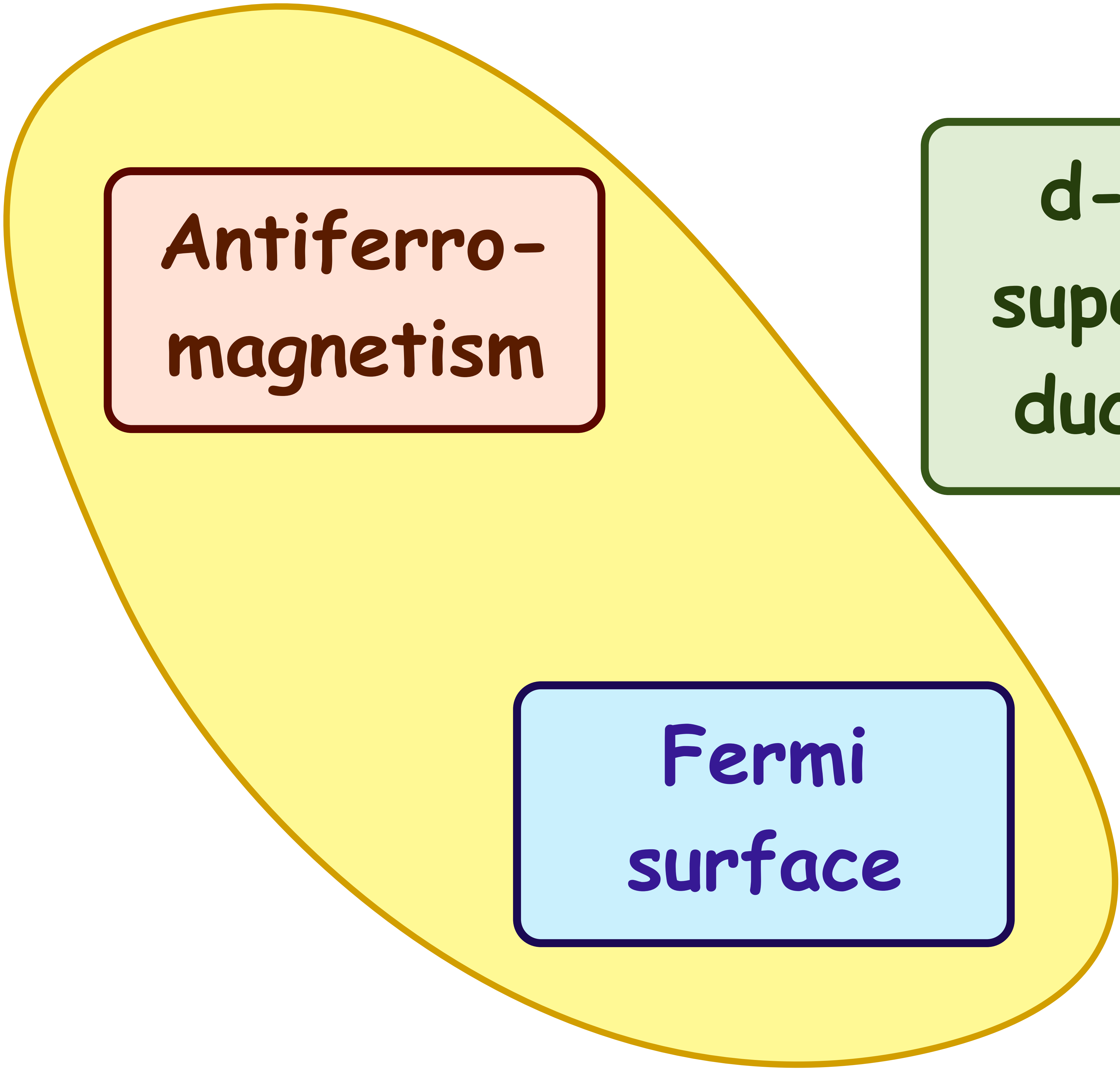
Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-T_c superconductor

R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$


Antiferro-
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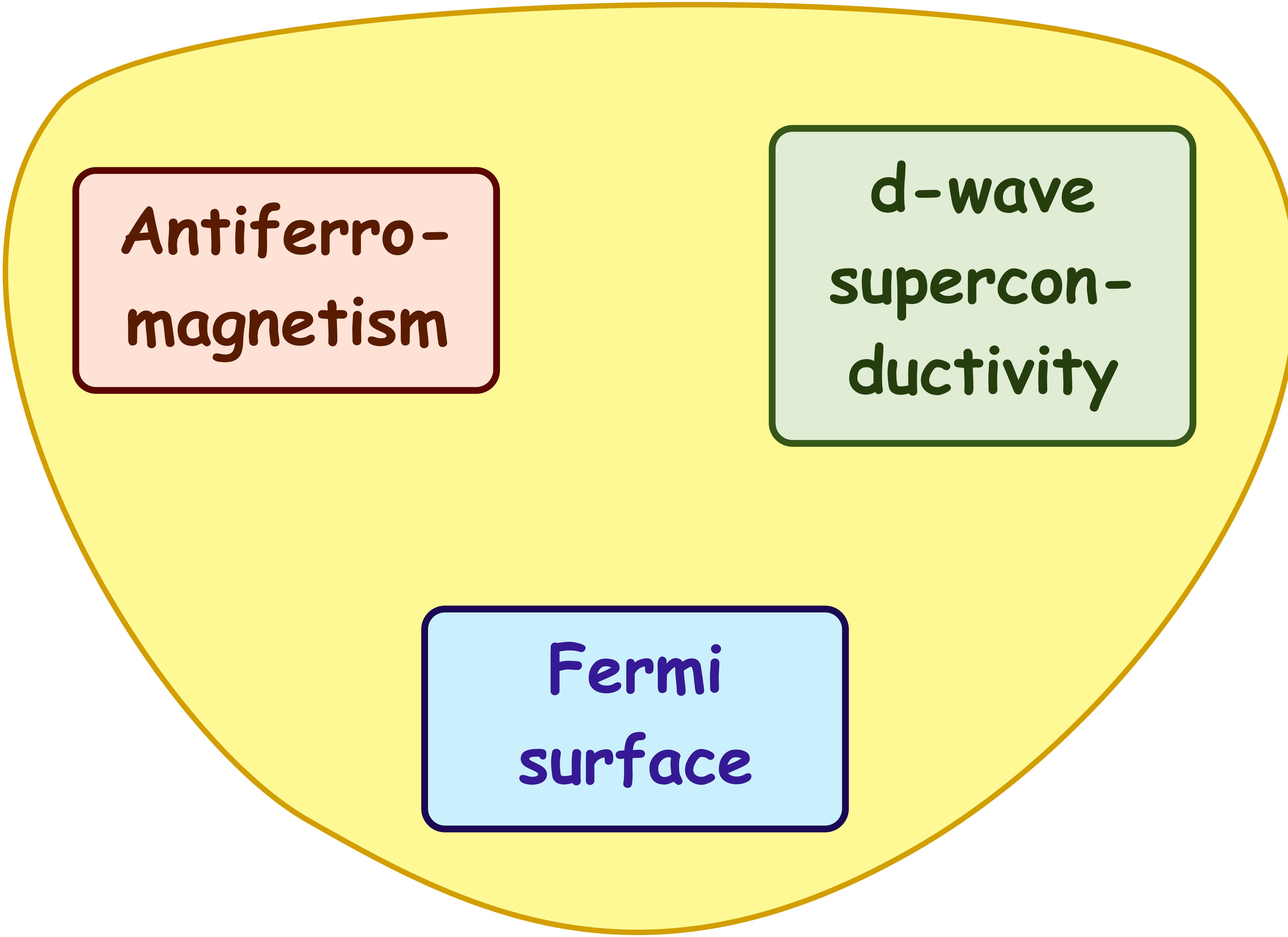
d-wave
supercon-
ductivity
Fermi
surface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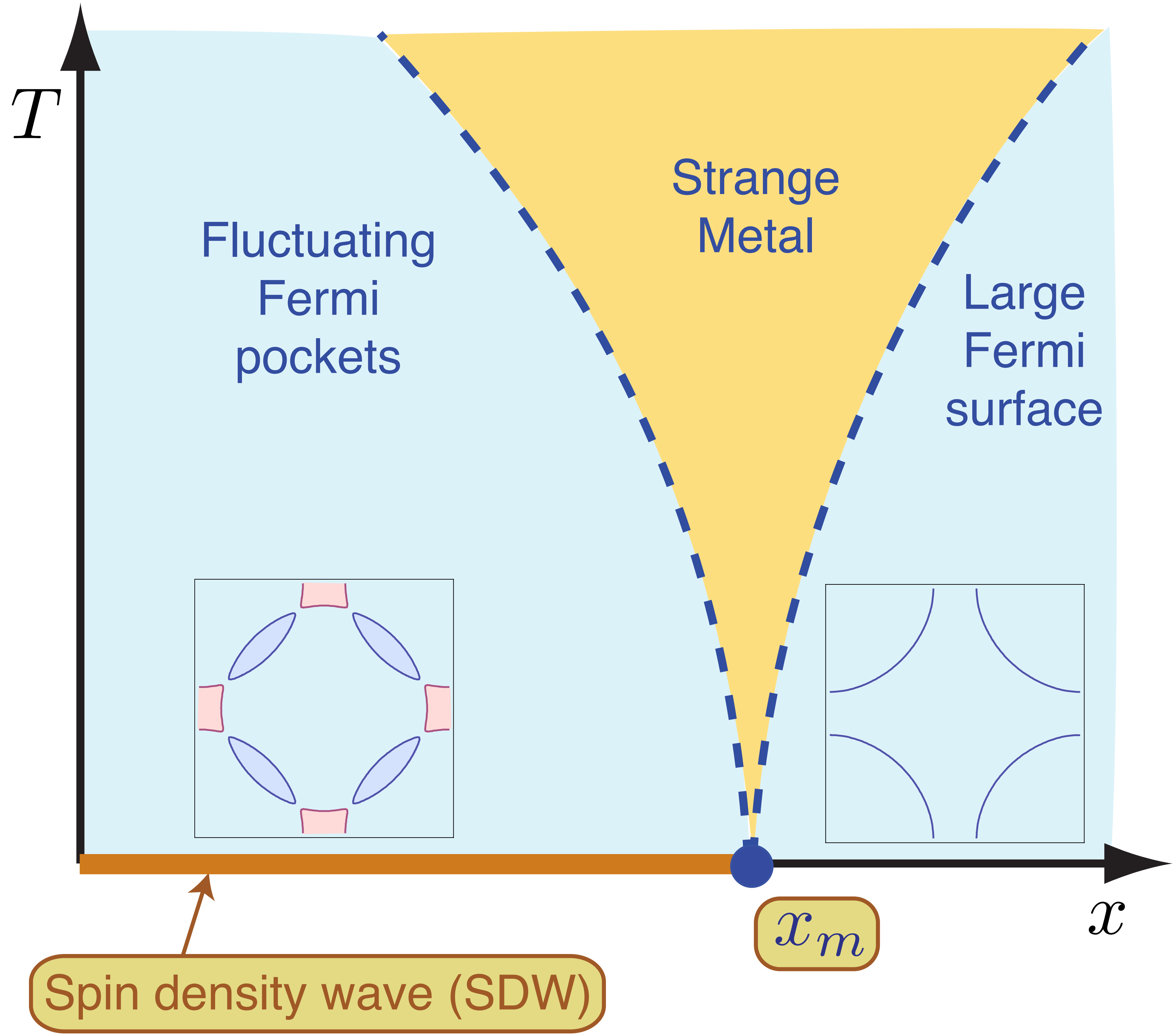

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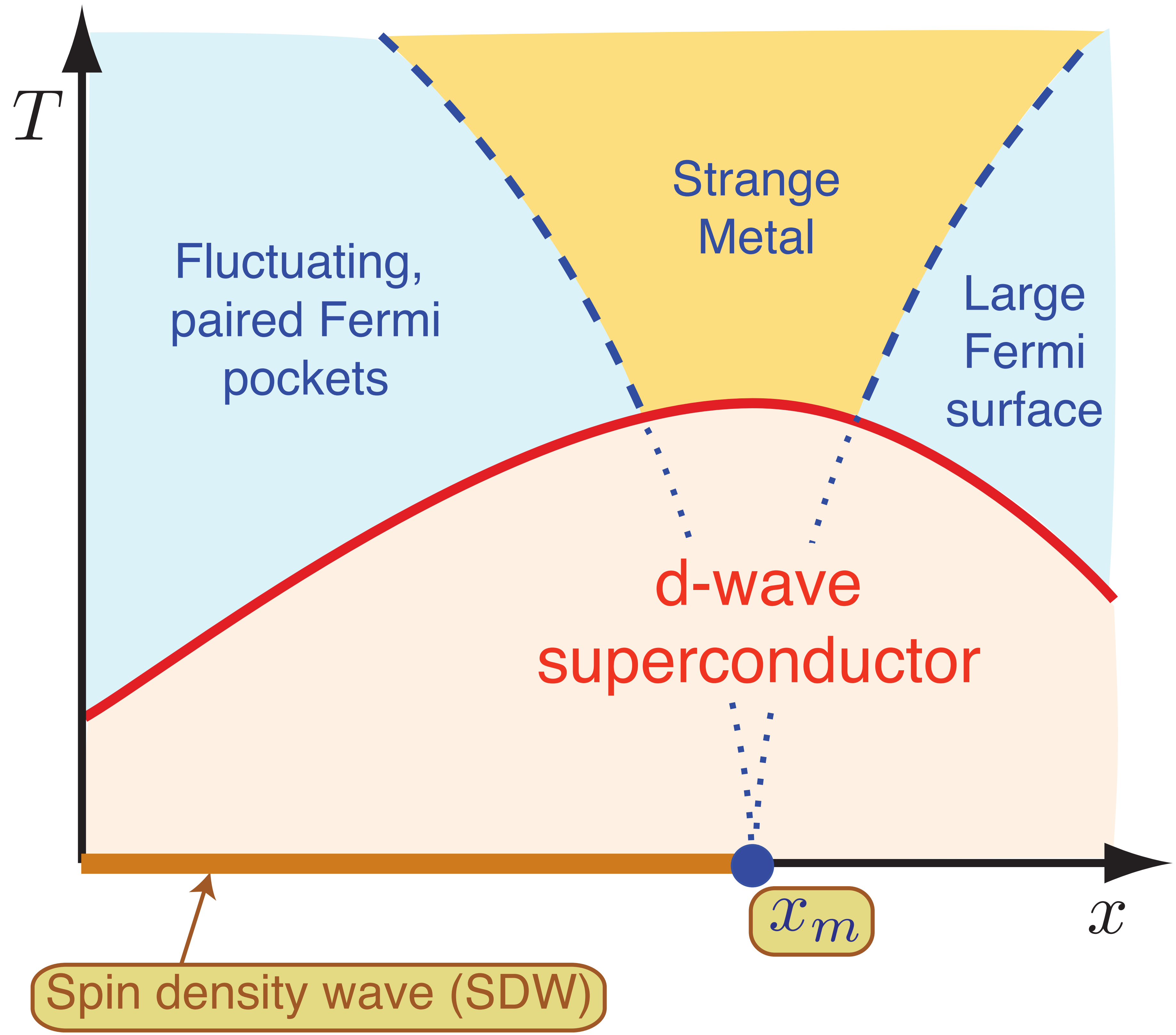
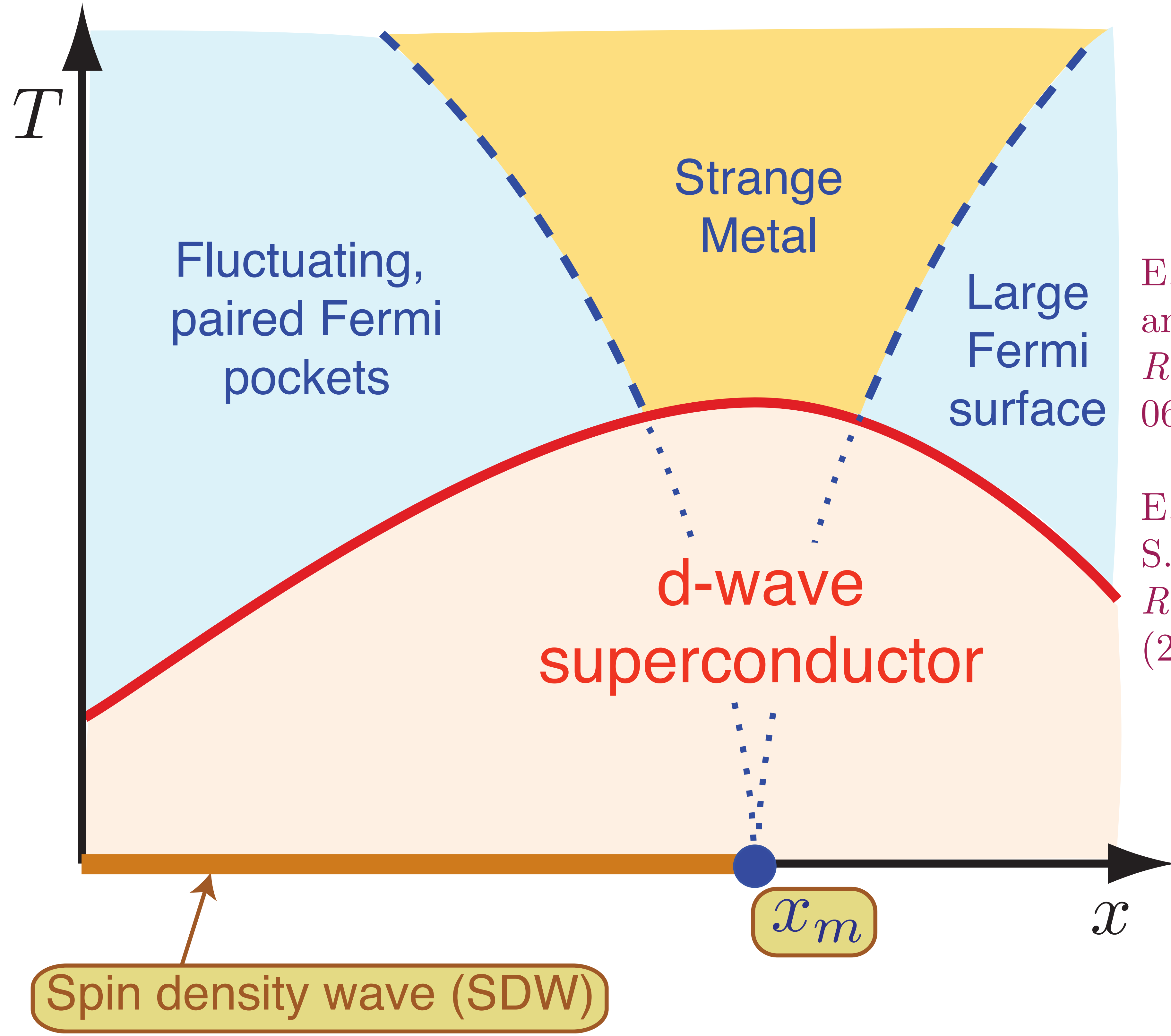


Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



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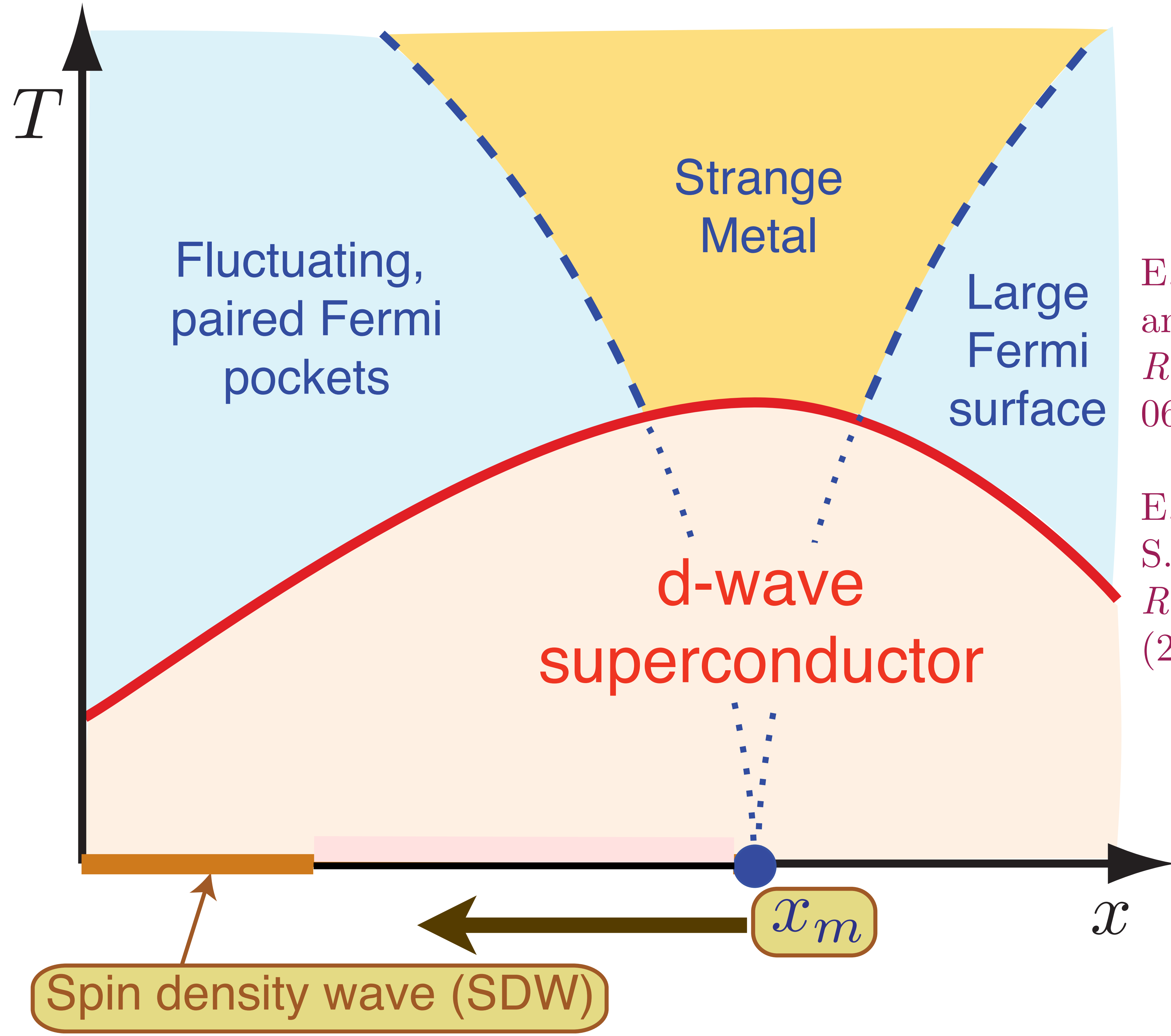
E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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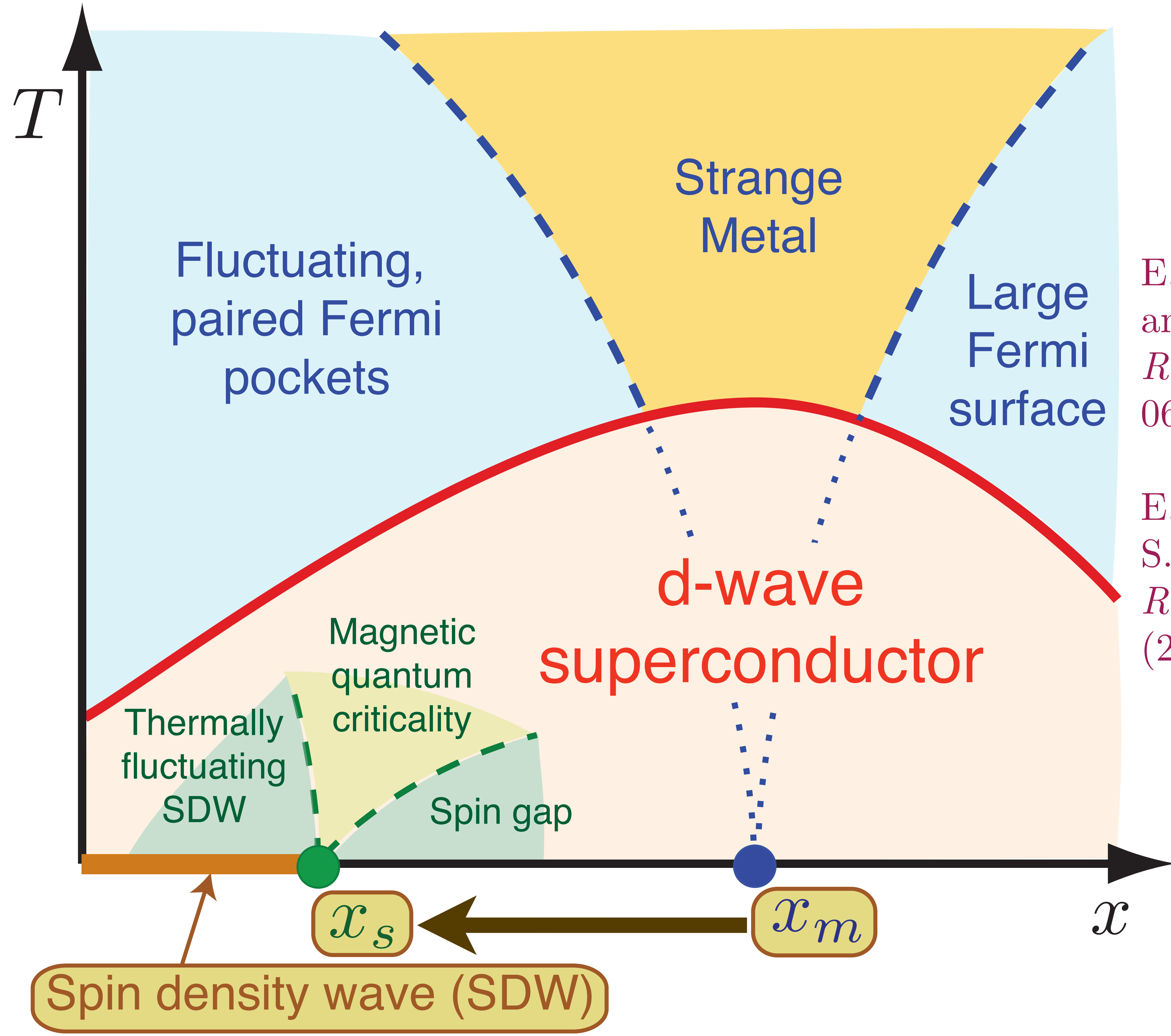

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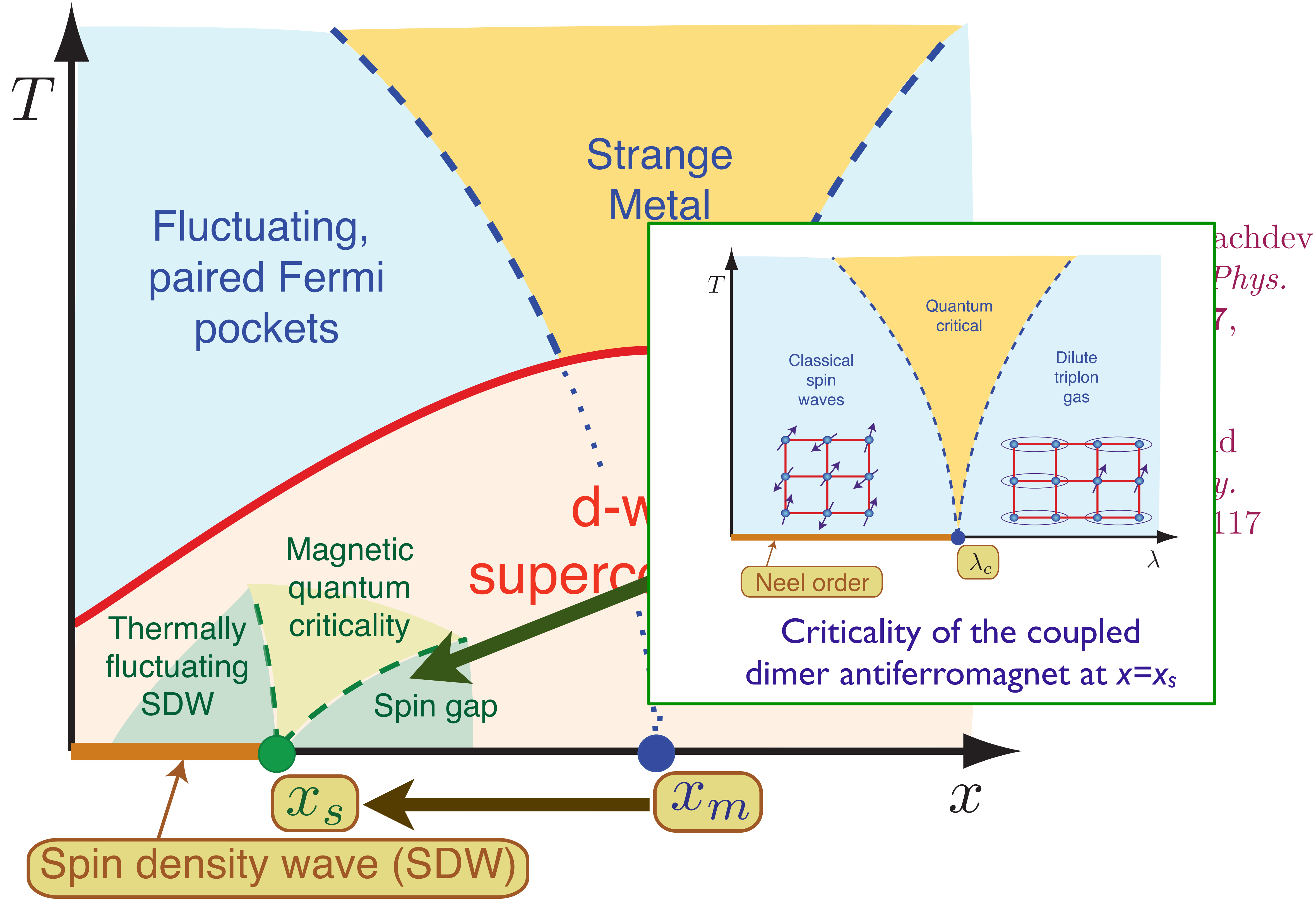

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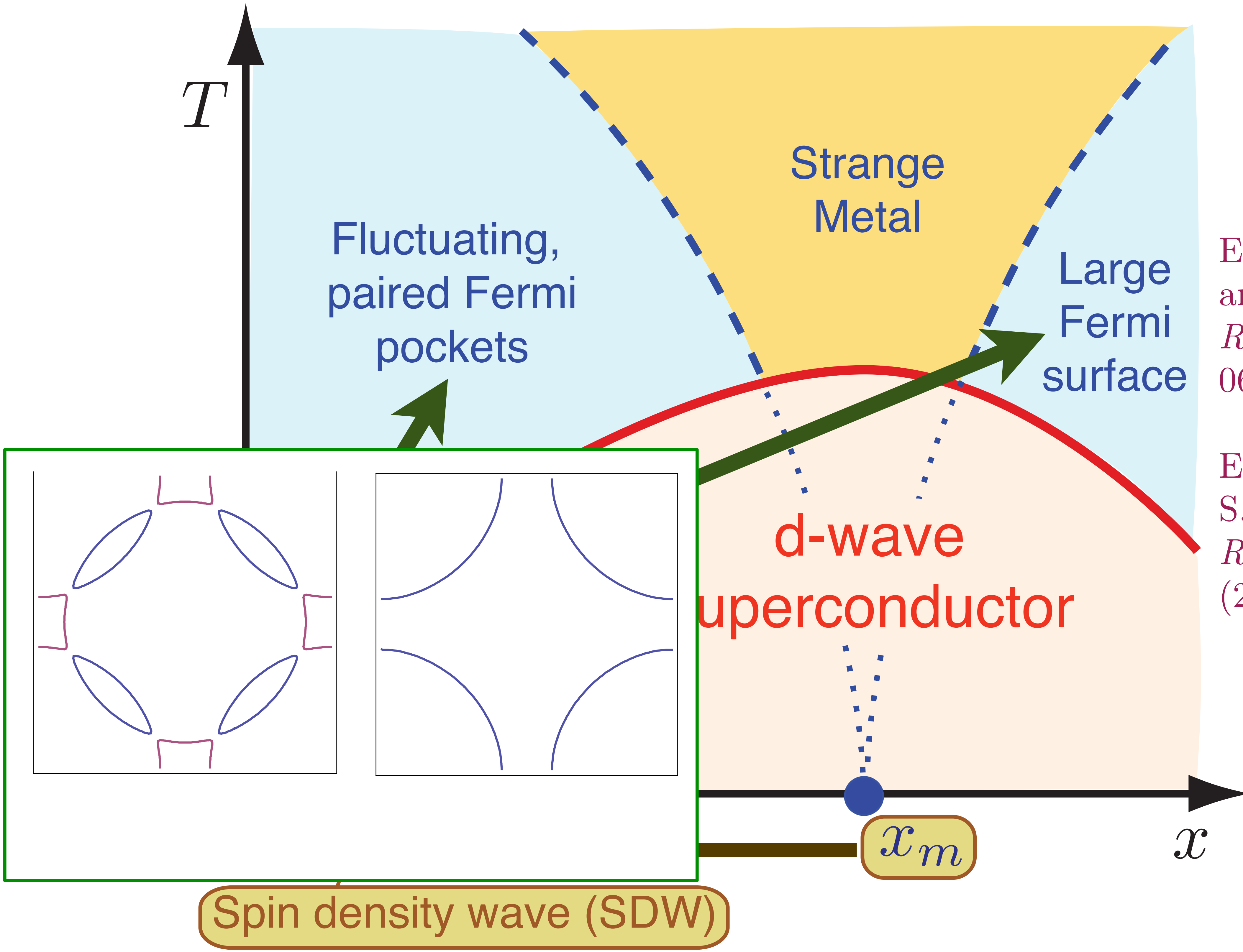

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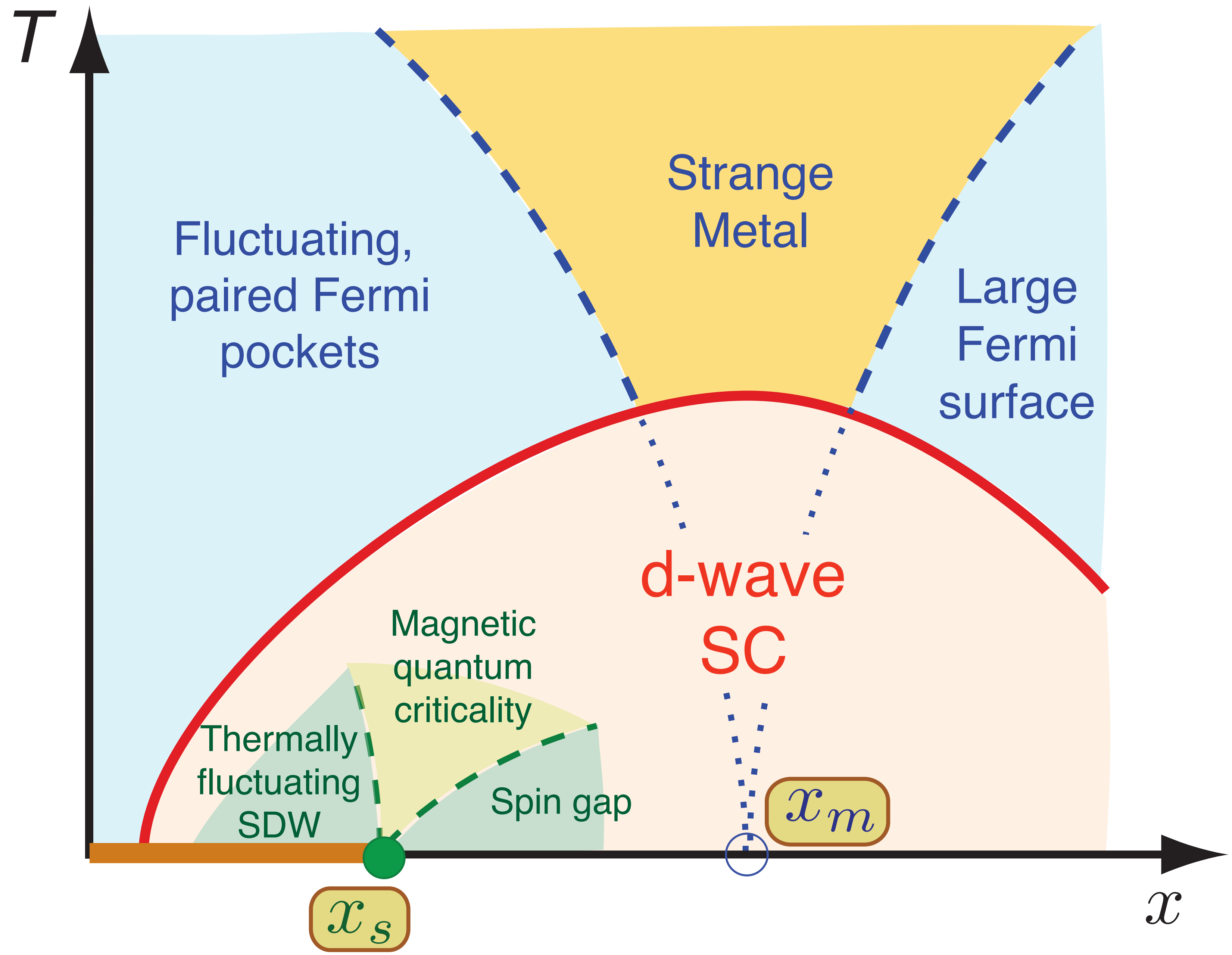
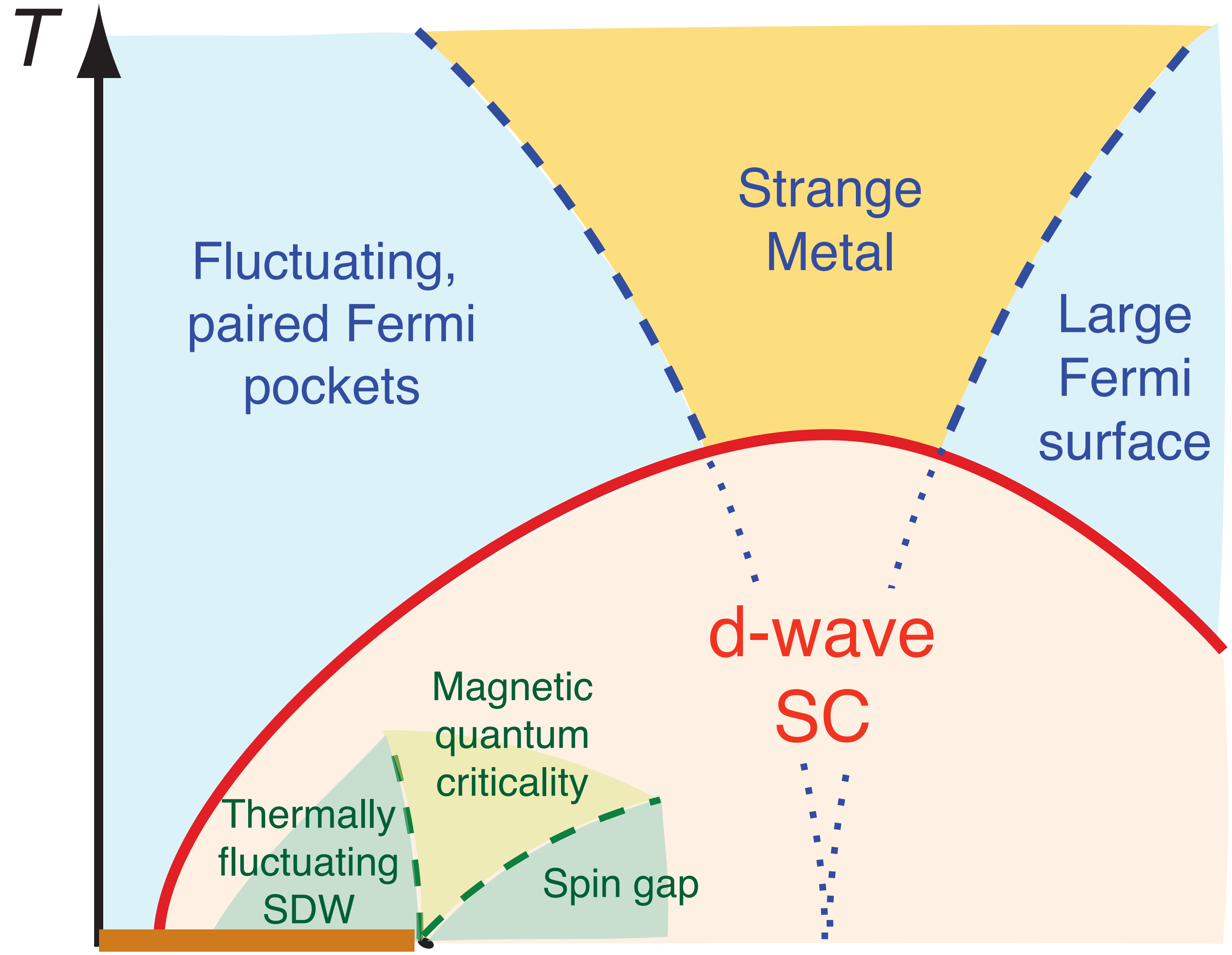

T
Fluctuating,
paired Fermi
pockets
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x_s
x



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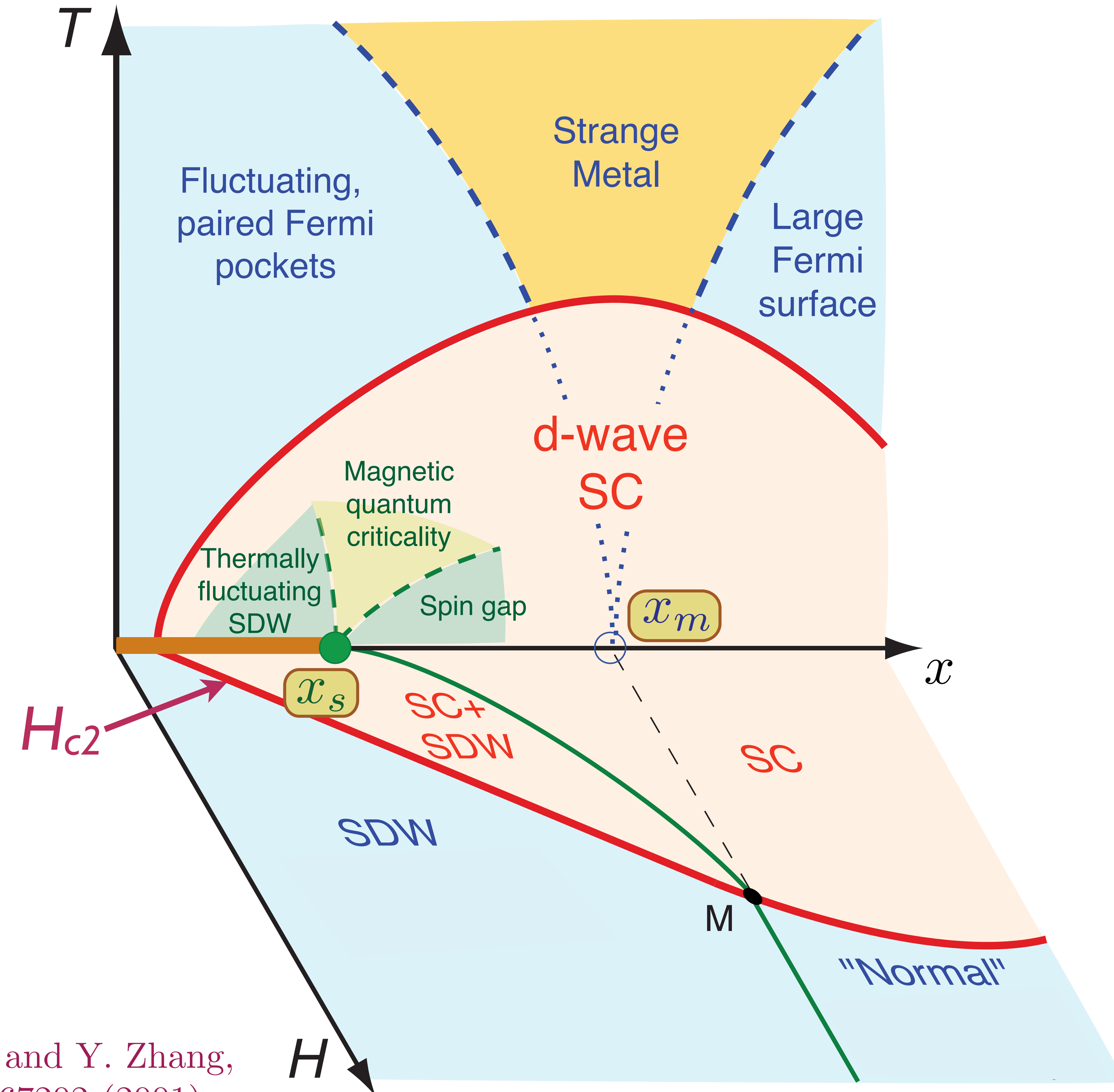

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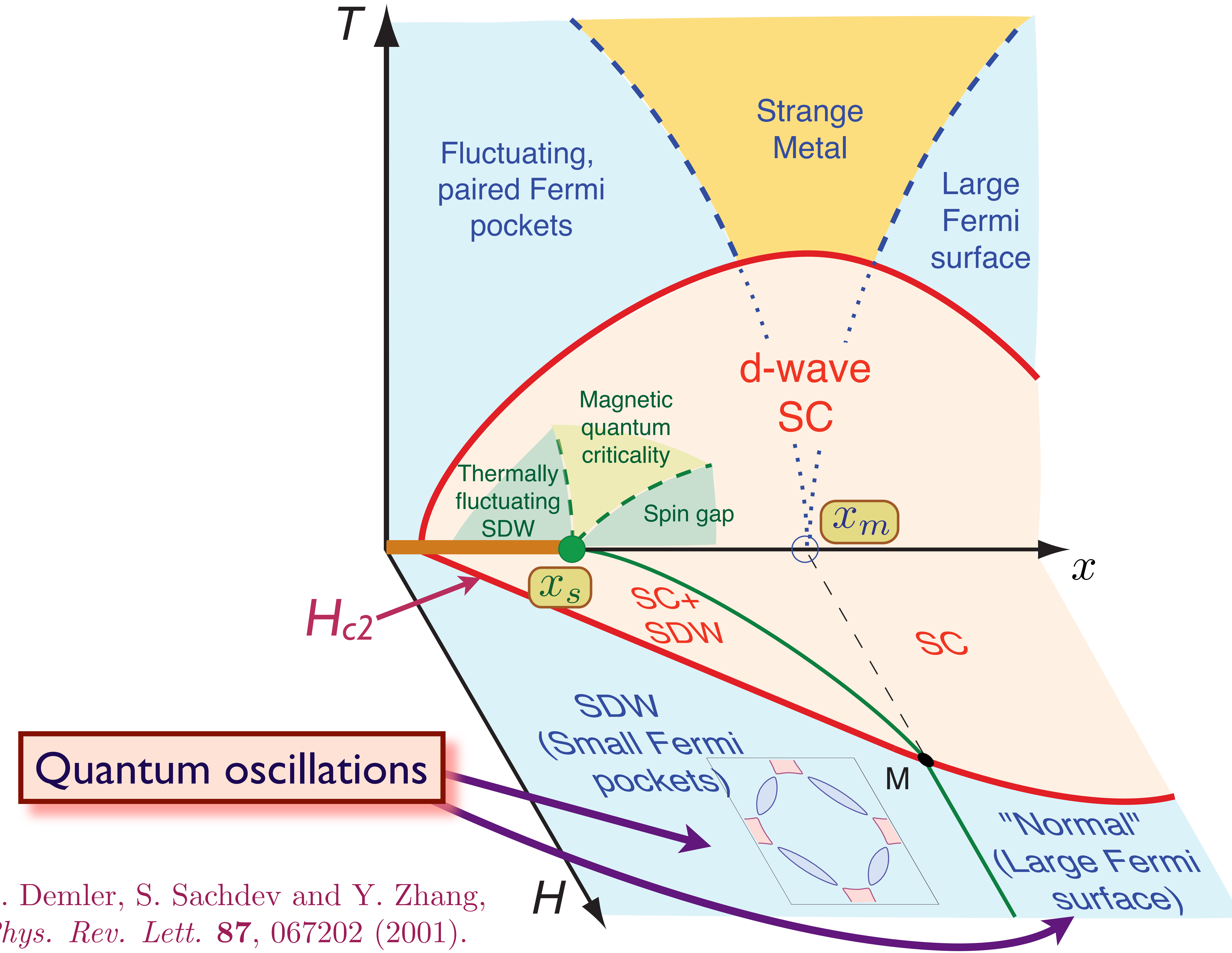

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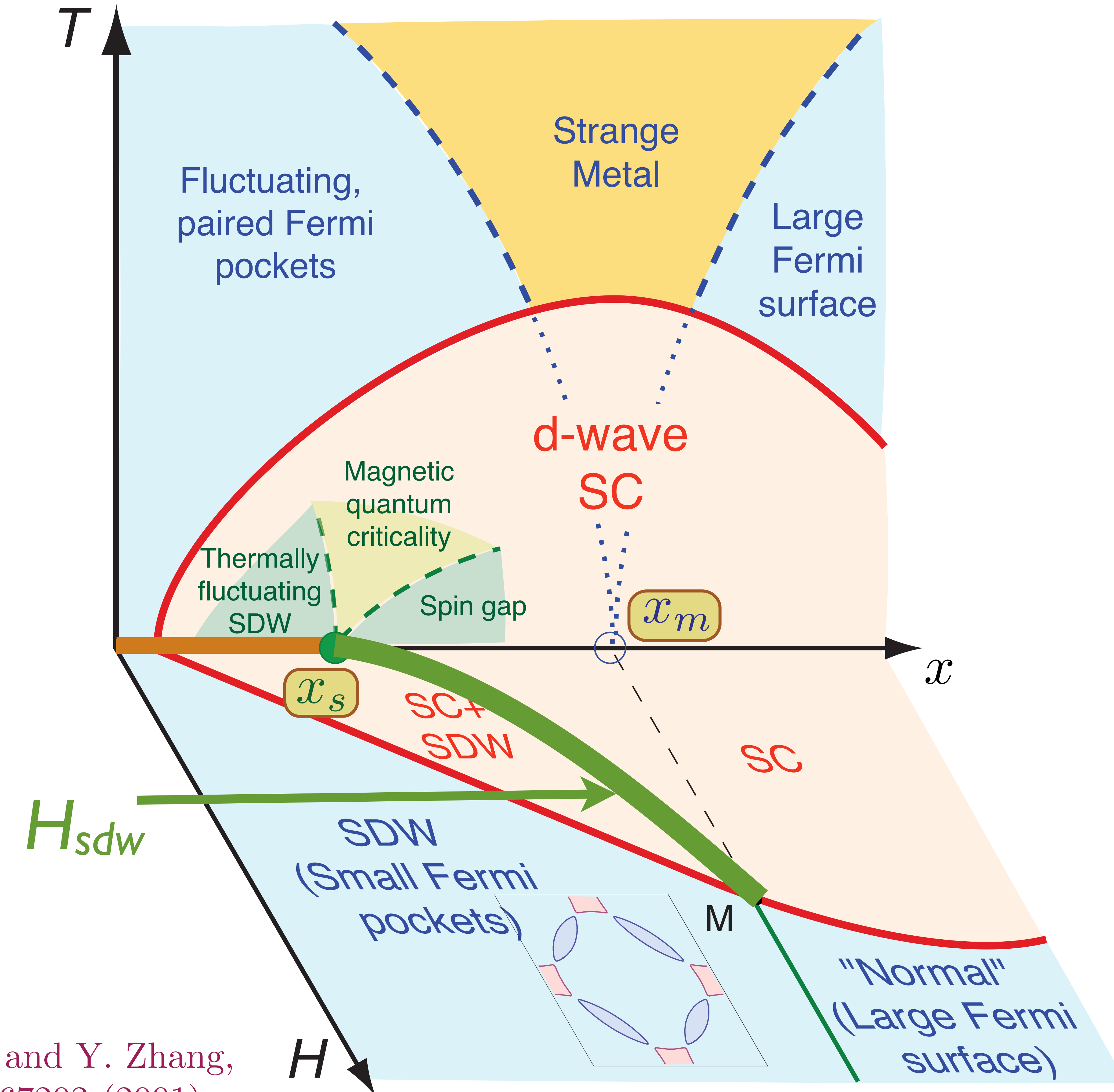

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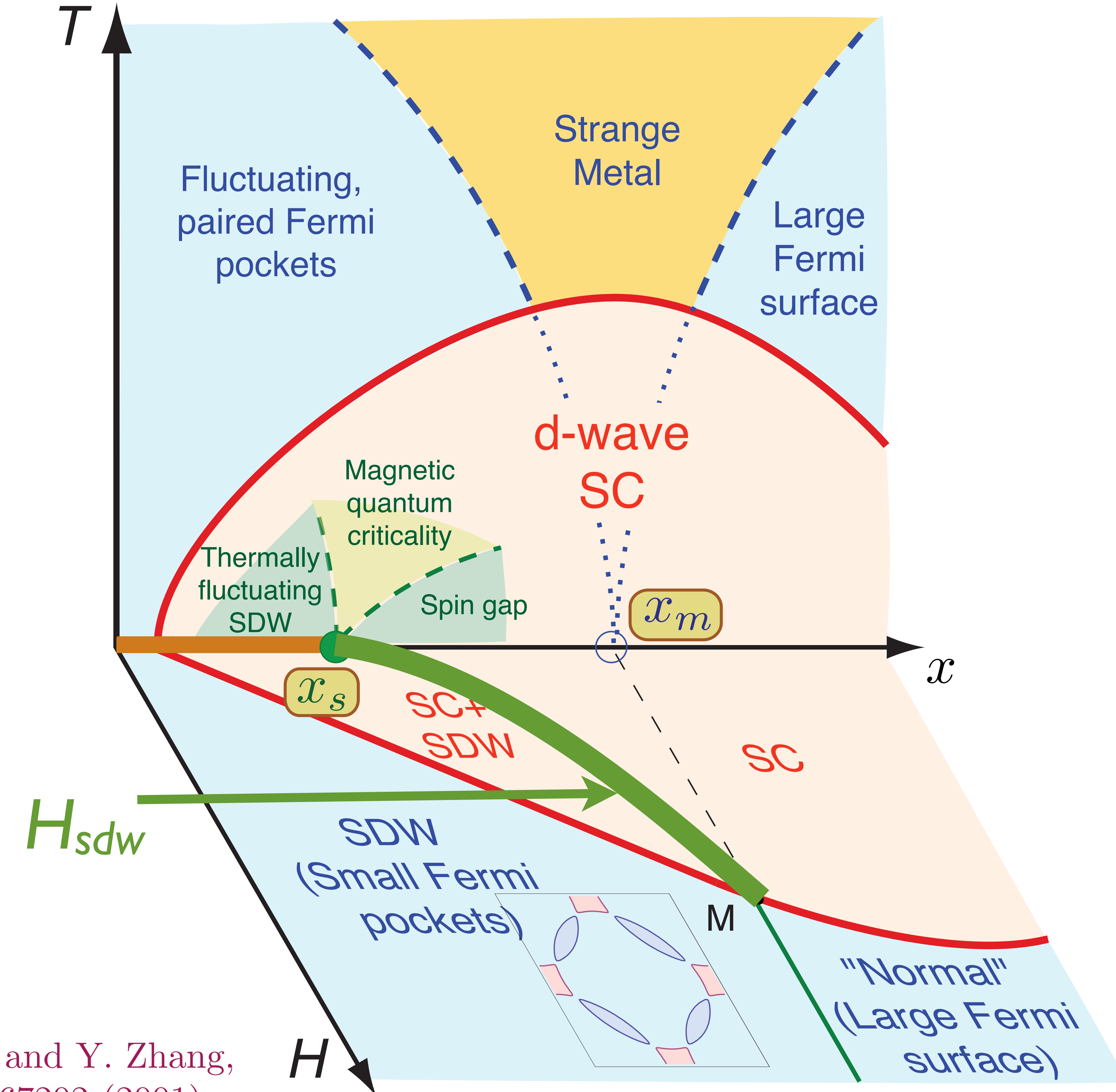

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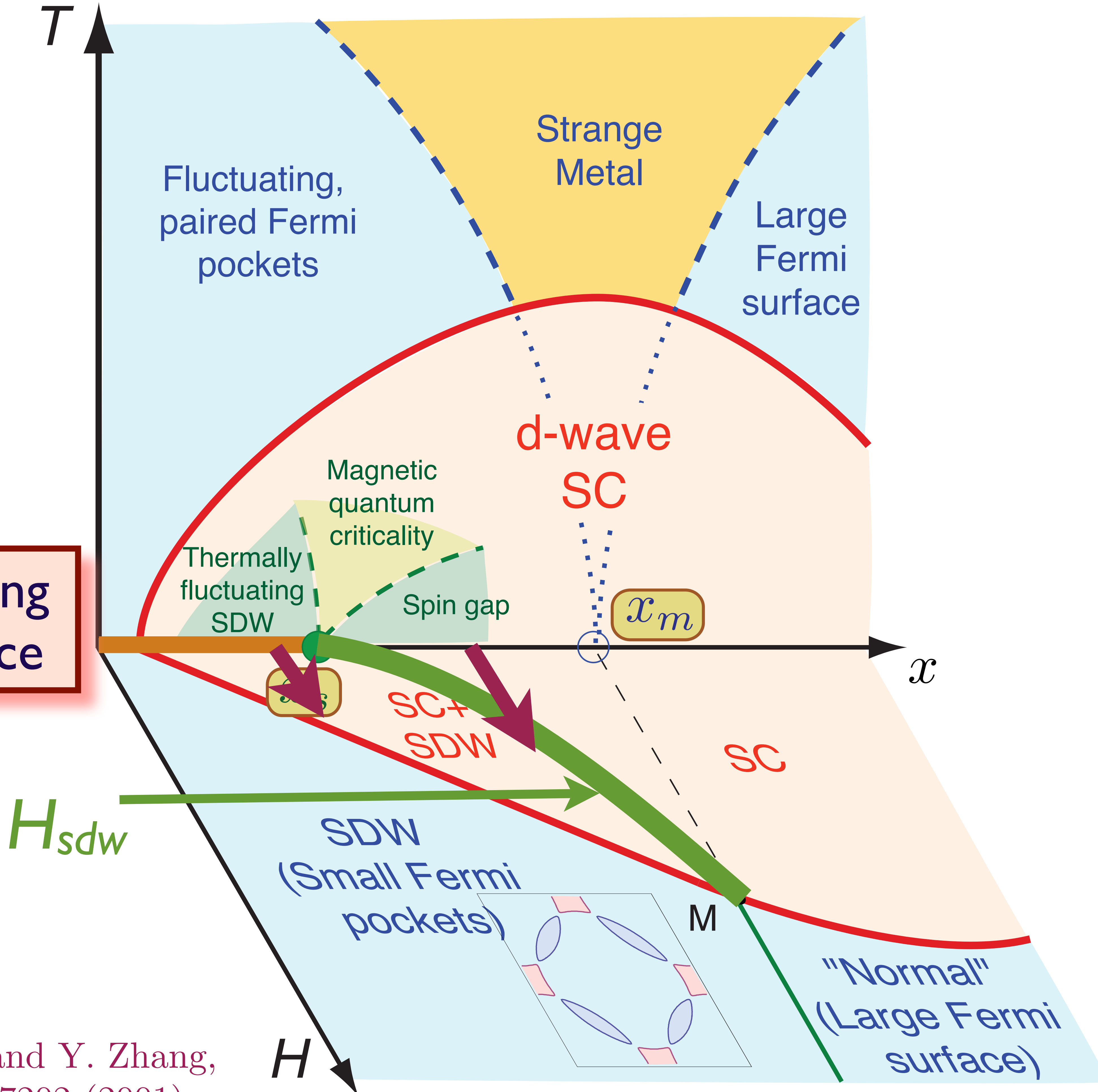
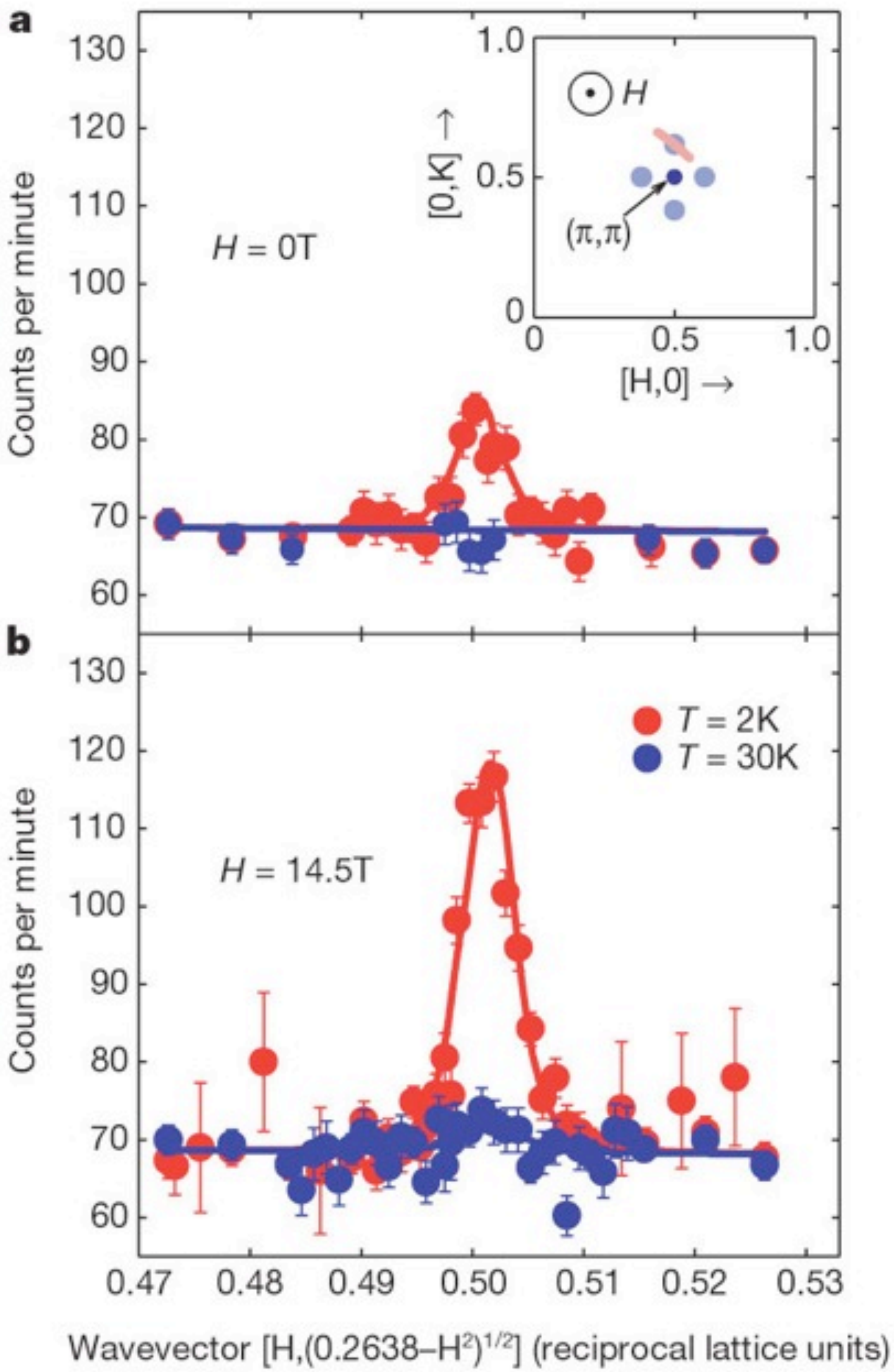
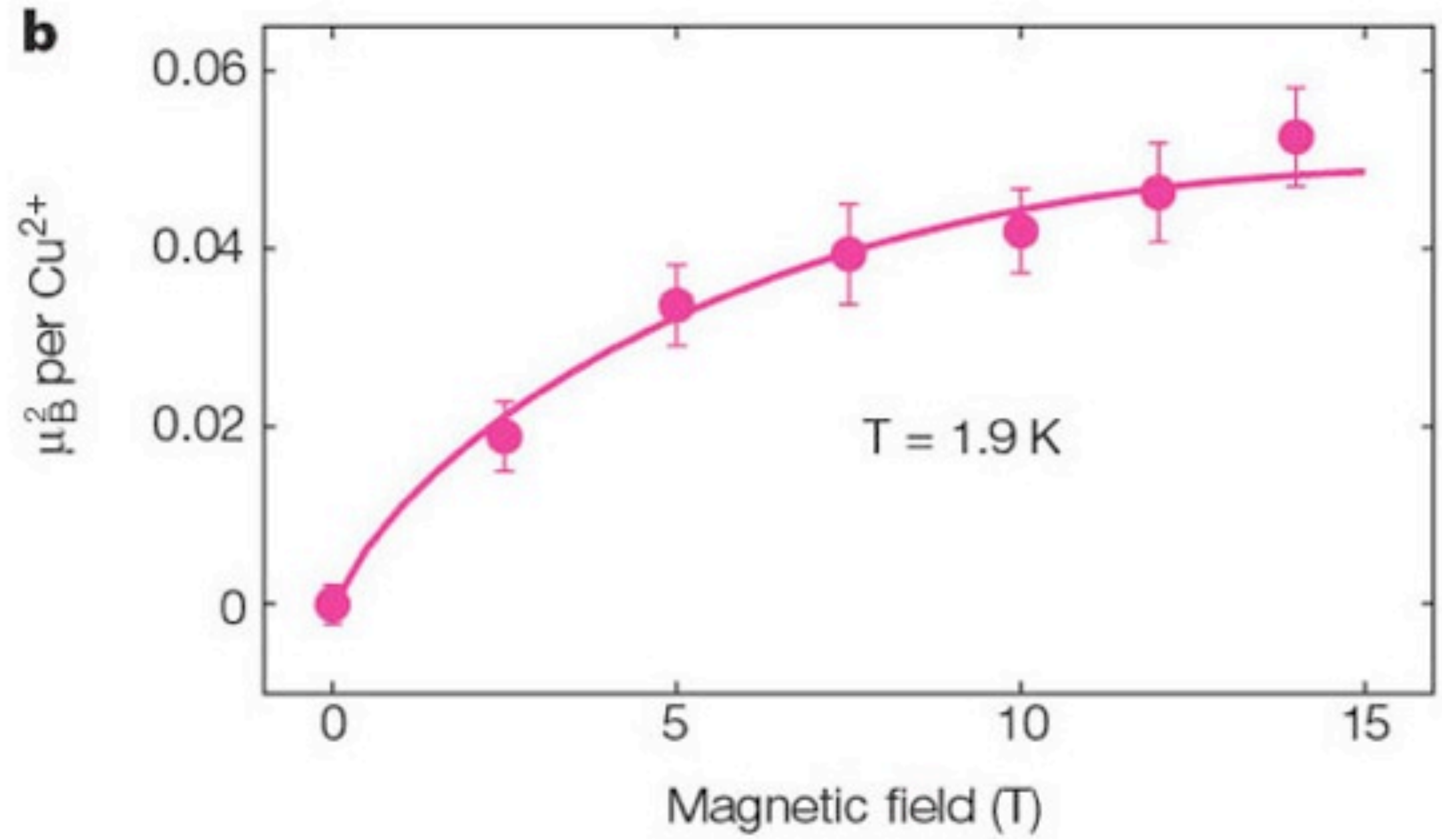
Neutron scattering & muon resonance



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).





B. Lake, H. M. Rønnow, N. B. Christensen, G. Aeppli, K. Lefmann, D. F. McMorrow, P. Vorderwisch, P. Smeibidl, N. Mangkorntong, T. Sasagawa, M. Nohara, H. Takagi, and T. E. Mason, Nature ***415***, 299 (2002)

B. Lake, G. Aeppli, K. N. Clausen, D. F. McMorrow, K. Lefmann, N. E. Hussey, N. Mangkorntong, M. Nohara, H. Takagi, T. E. Mason, and A. Schröder *Science* **291**, 1759 (2001).

PHYSICAL REVIEW B **71**, 220508(R) (2005)

Field-induced transition between magnetically disordered and ordered phases in underdoped $La_{2-x}Sr_xCuO_4$

B. Khaykovich,[1] S. Wakimoto,[2] R. J. Birgeneau,[3] M. A. Kastner,[1] Y. S. Lee,[1] P. Smeibidl,[4] P. Vorderwisch,[4] and K. Yamada[5]

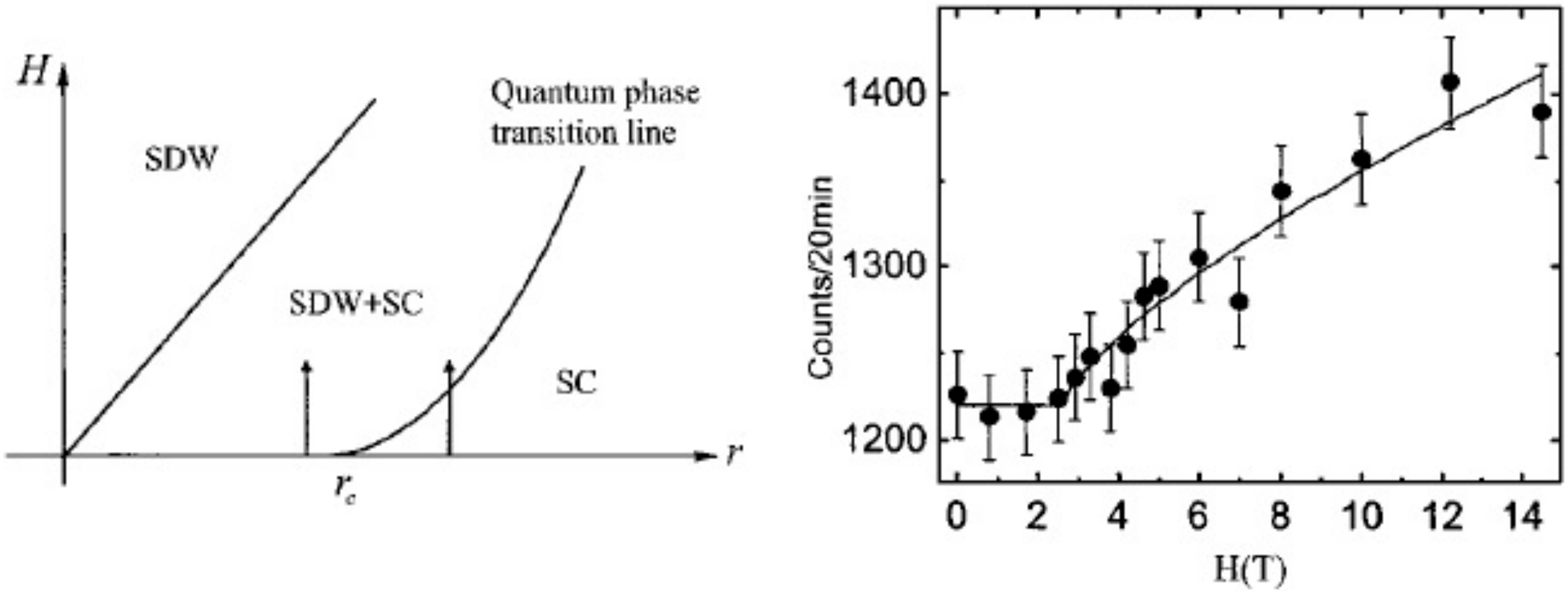
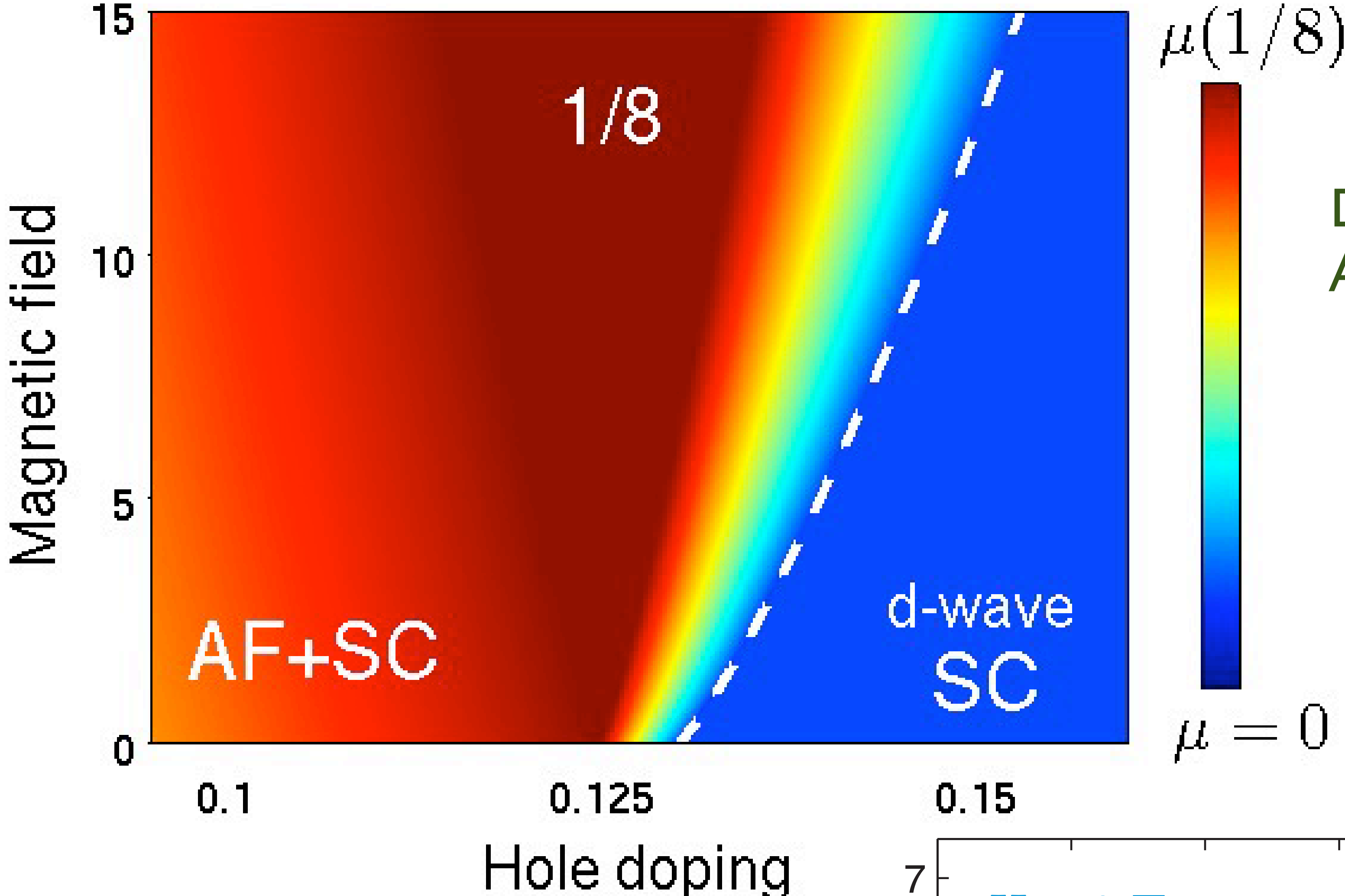


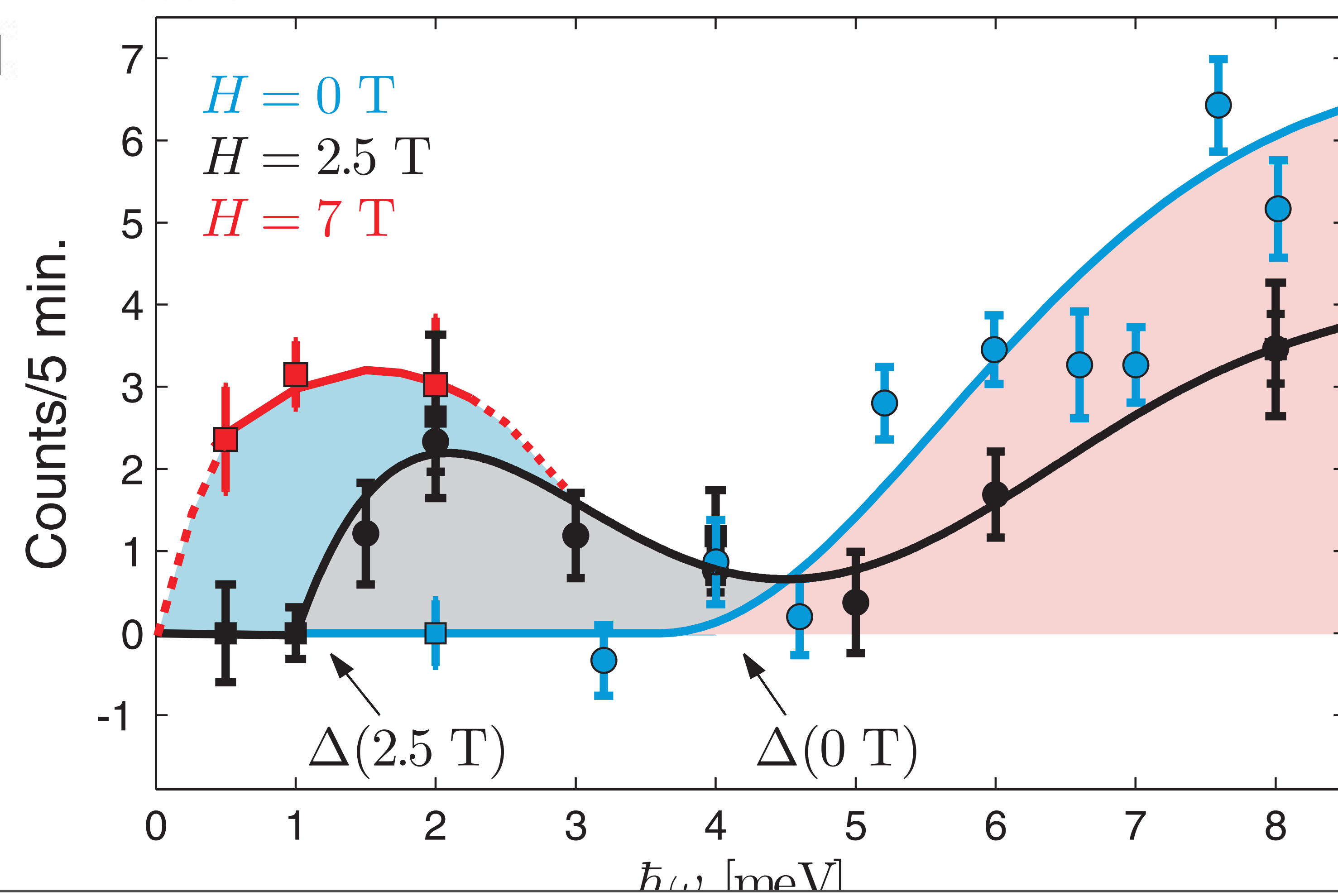
FIG. 1. (a) A fragment of the theoretical phase diagram, adopted from Refs. 4 and 20. The vertical axis is the magnetic field and the horizontal axis is the coupling strength between superconductivity and magnetic order. (b) Field dependence of the magnetic Bragg peak corresponding to the incommensurate SDW peak at $Q=(1.125,0.125,0)$. Every point is measured after field cooling at $T=1.5$ K. The data are fitted to $I=I_0+A|H-H_c|^{2\beta}$ above H_c as explained in the text. Spectrometer configuration: 45-60-Be—S—Be-60-open; cold Be filters were used before and after the sample to eliminate contamination from high-energy neutrons; $E=4$ meV.

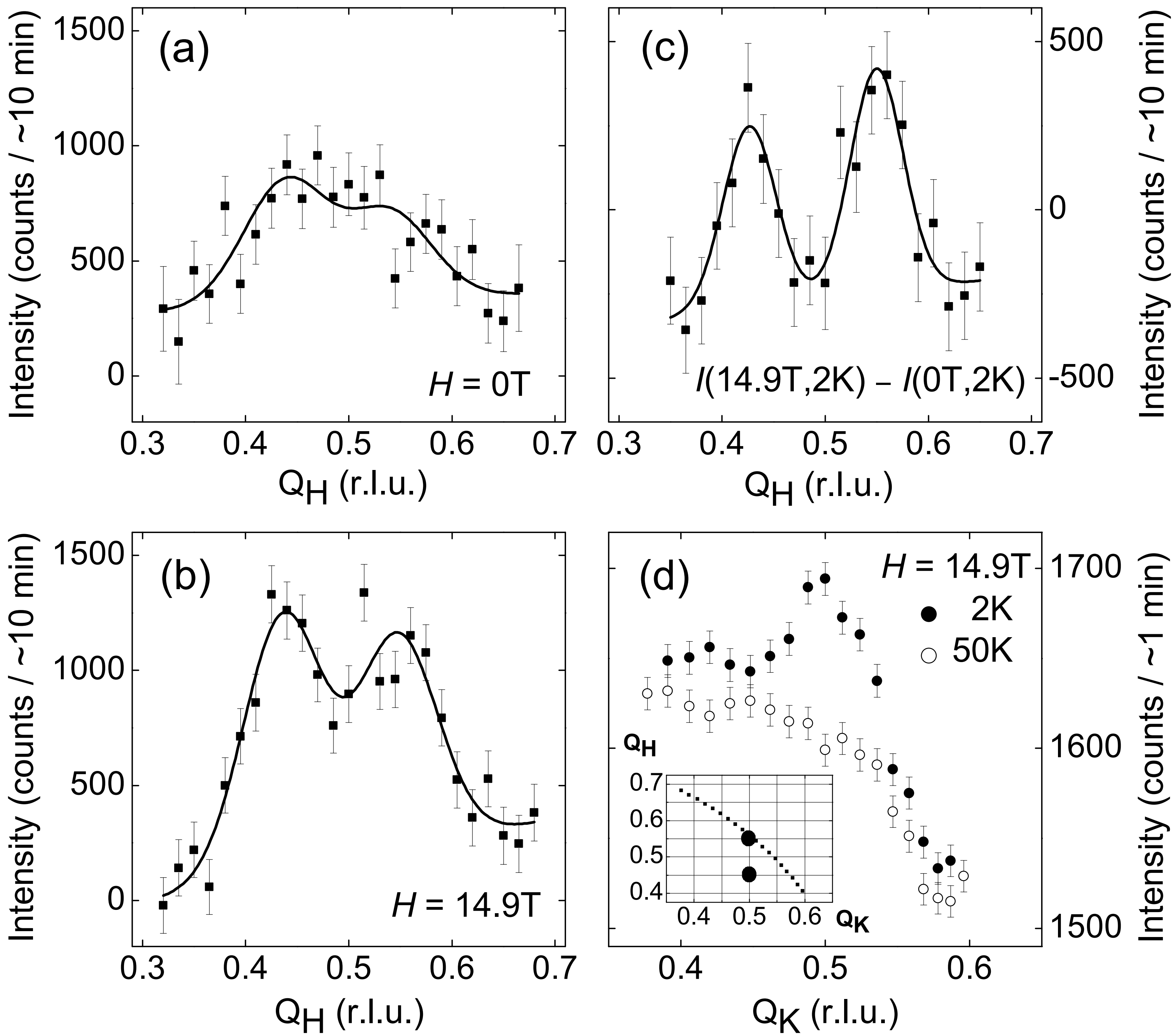
Thursday, November 19, 2009

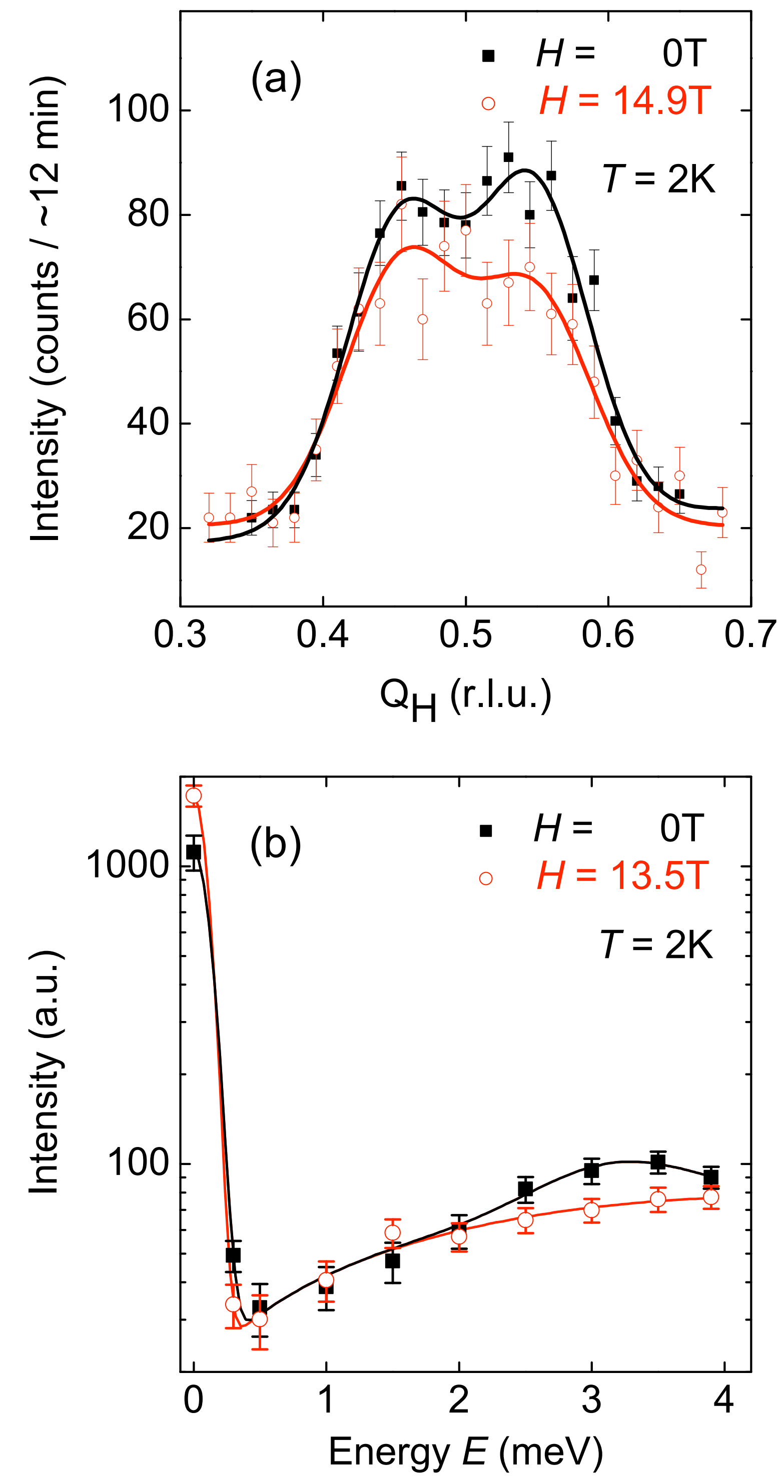


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).

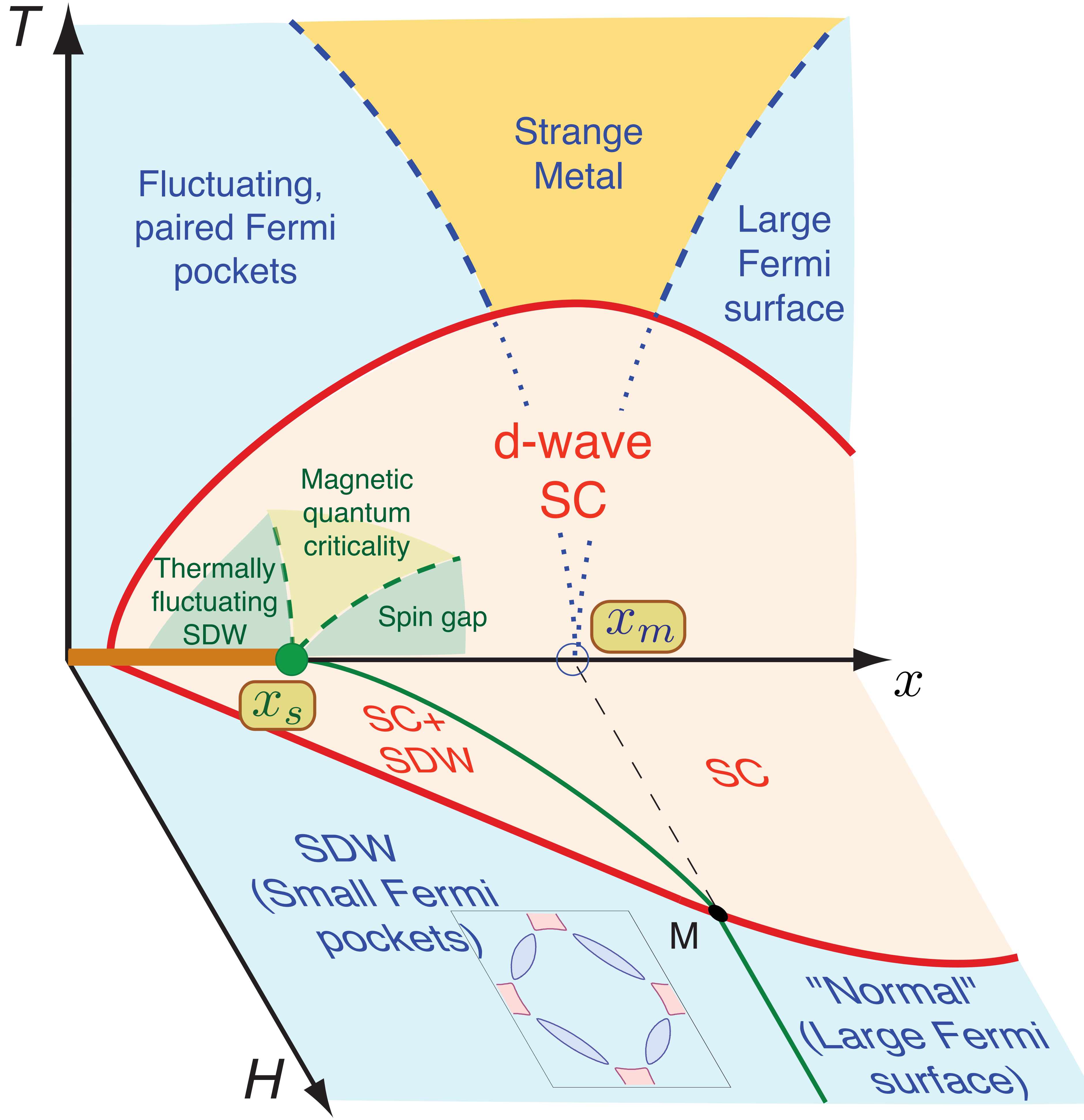
J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).





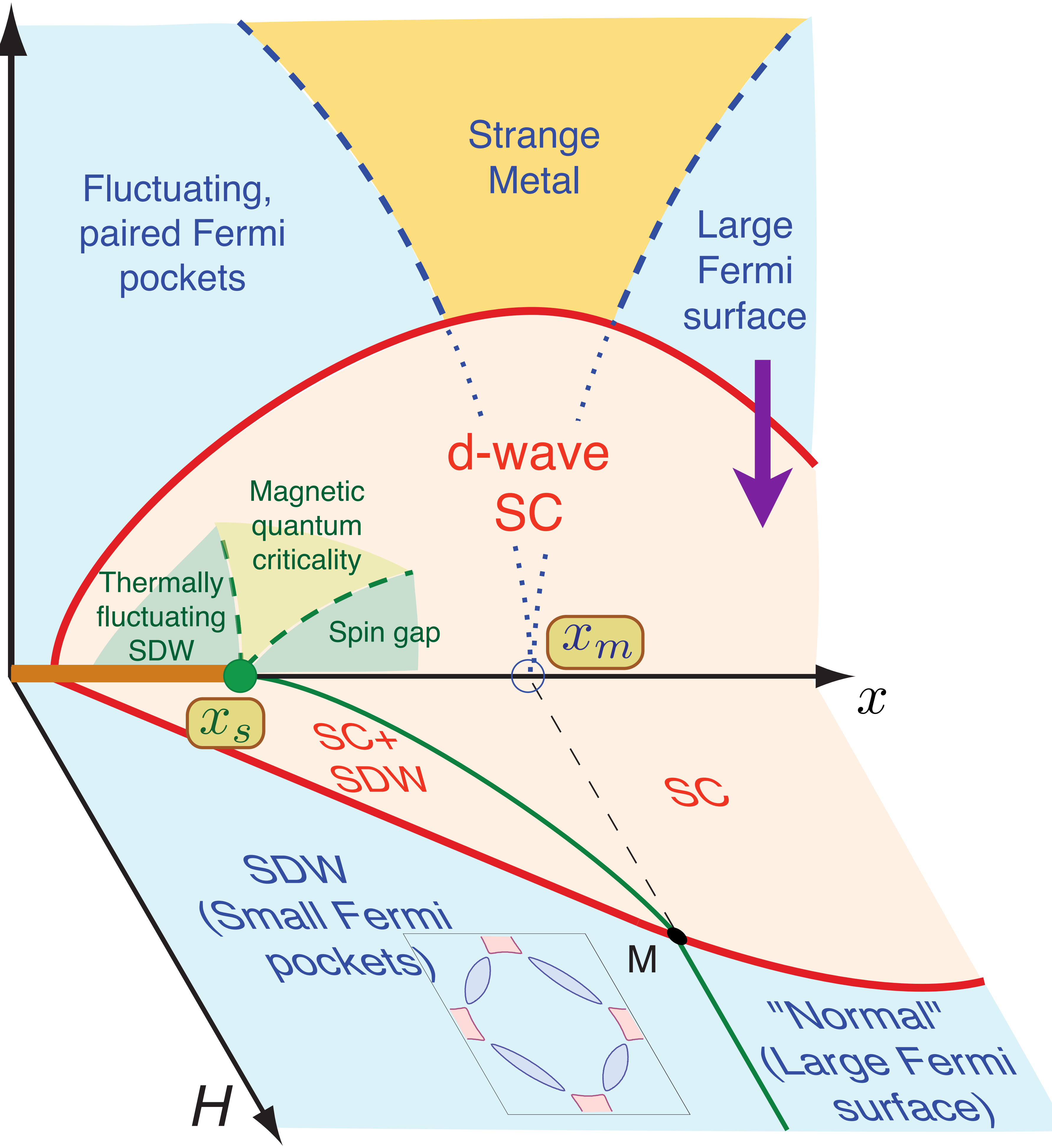


D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)

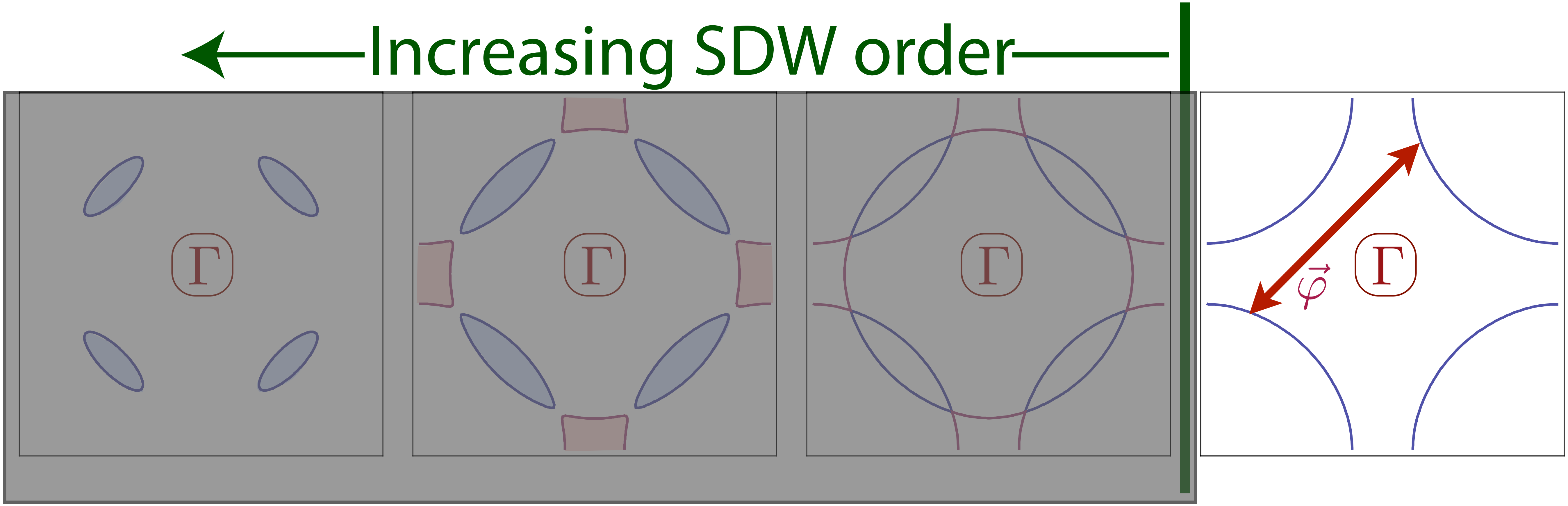


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x
x_s
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Theory of the onset of d-wave superconductivity from a large Fermi surface



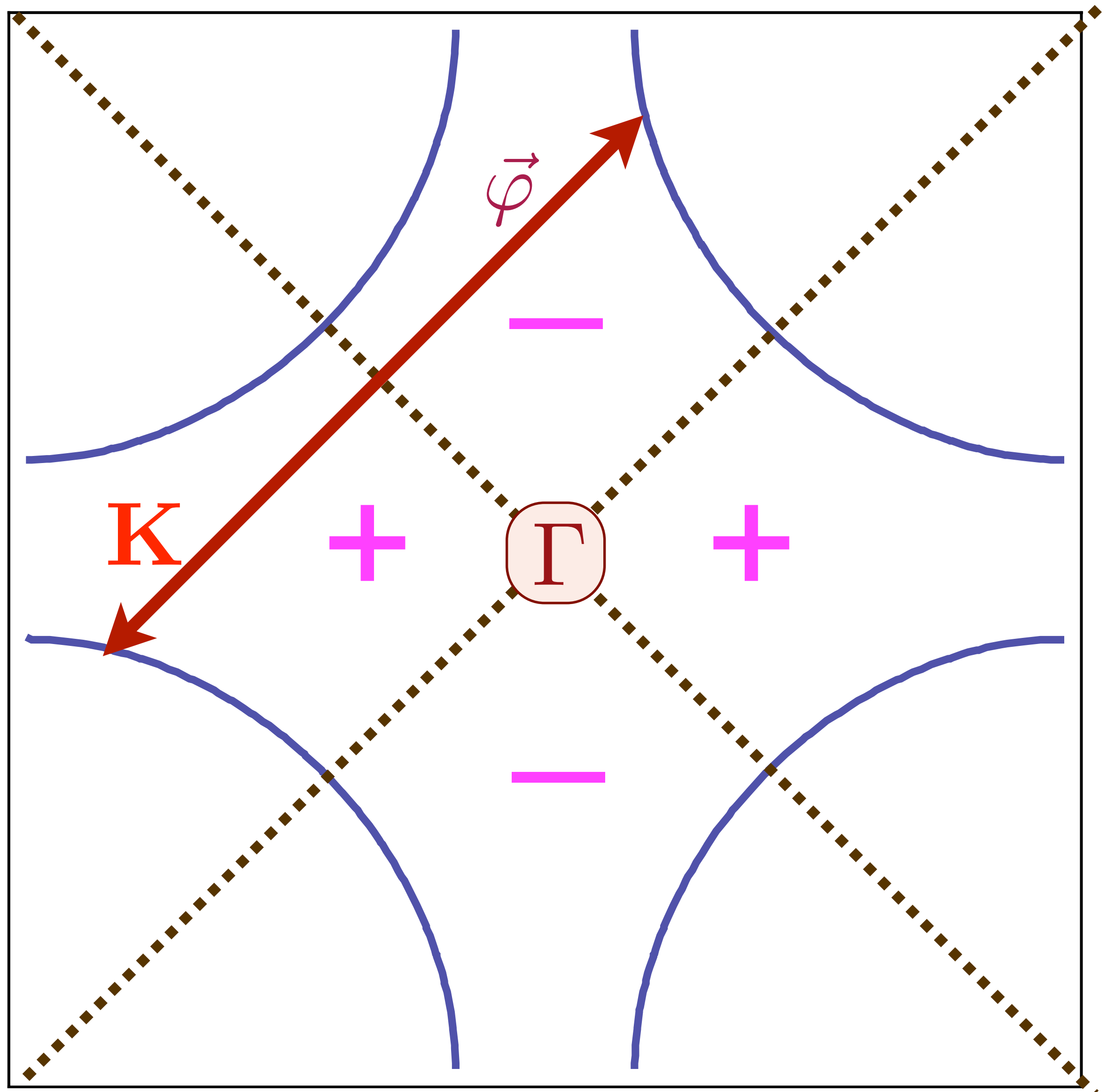
Spin-fluctuation exchange theory of d-wave superconductivity in the cuprates



Fermions at the *large* Fermi surface exchange fluctuations of the SDW order parameter $\vec{\varphi}$.

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

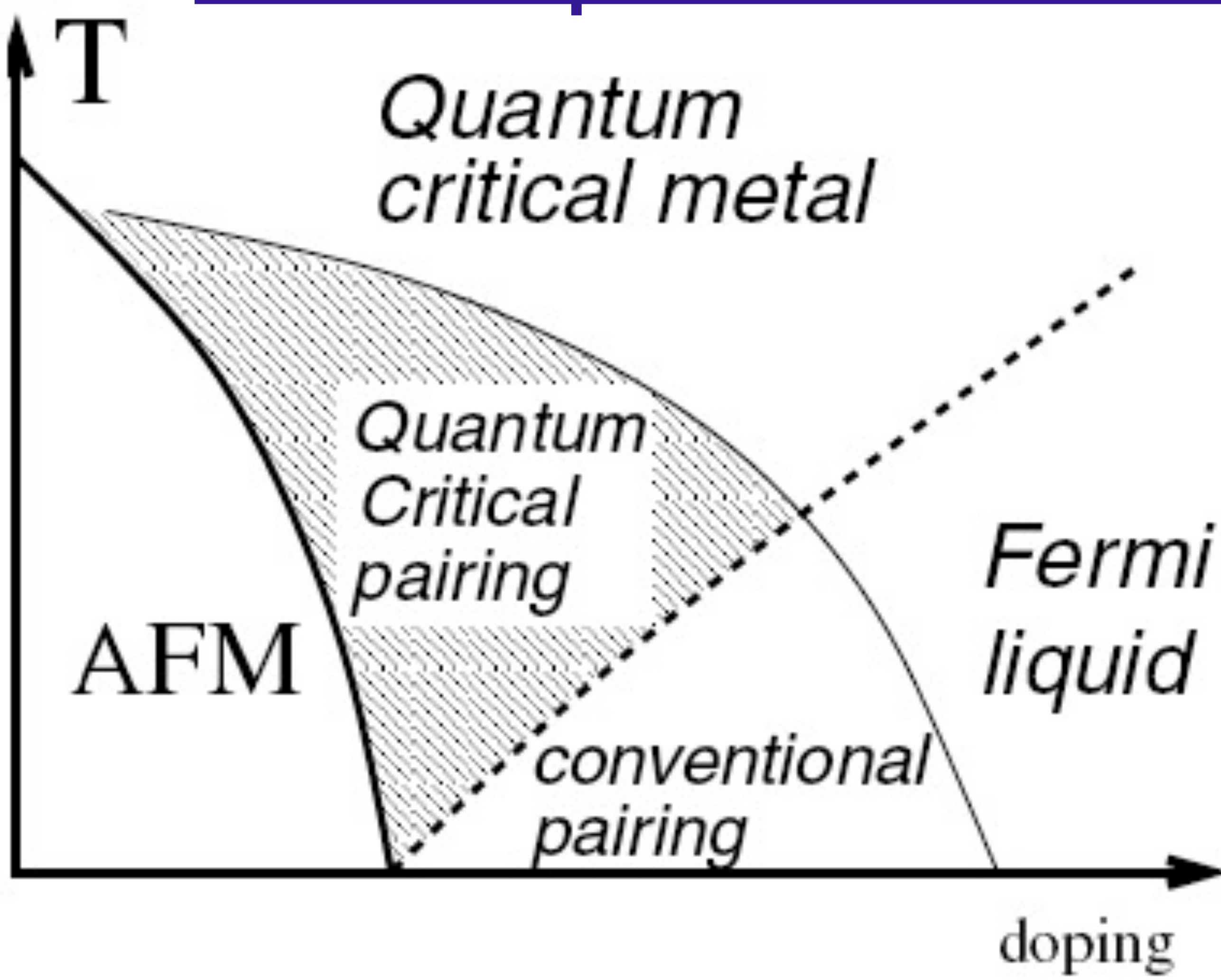
d-wave pairing of the large Fermi surface



$$\langle c_{\mathbf{k}\uparrow} c_{-\mathbf{k}\downarrow} \rangle \propto \Delta_{\mathbf{k}} = \Delta_0(\cos(k_x) - \cos(k_y))$$

D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)

Approaching the onset of antiferromagnetism in the spin-fluctuation theory

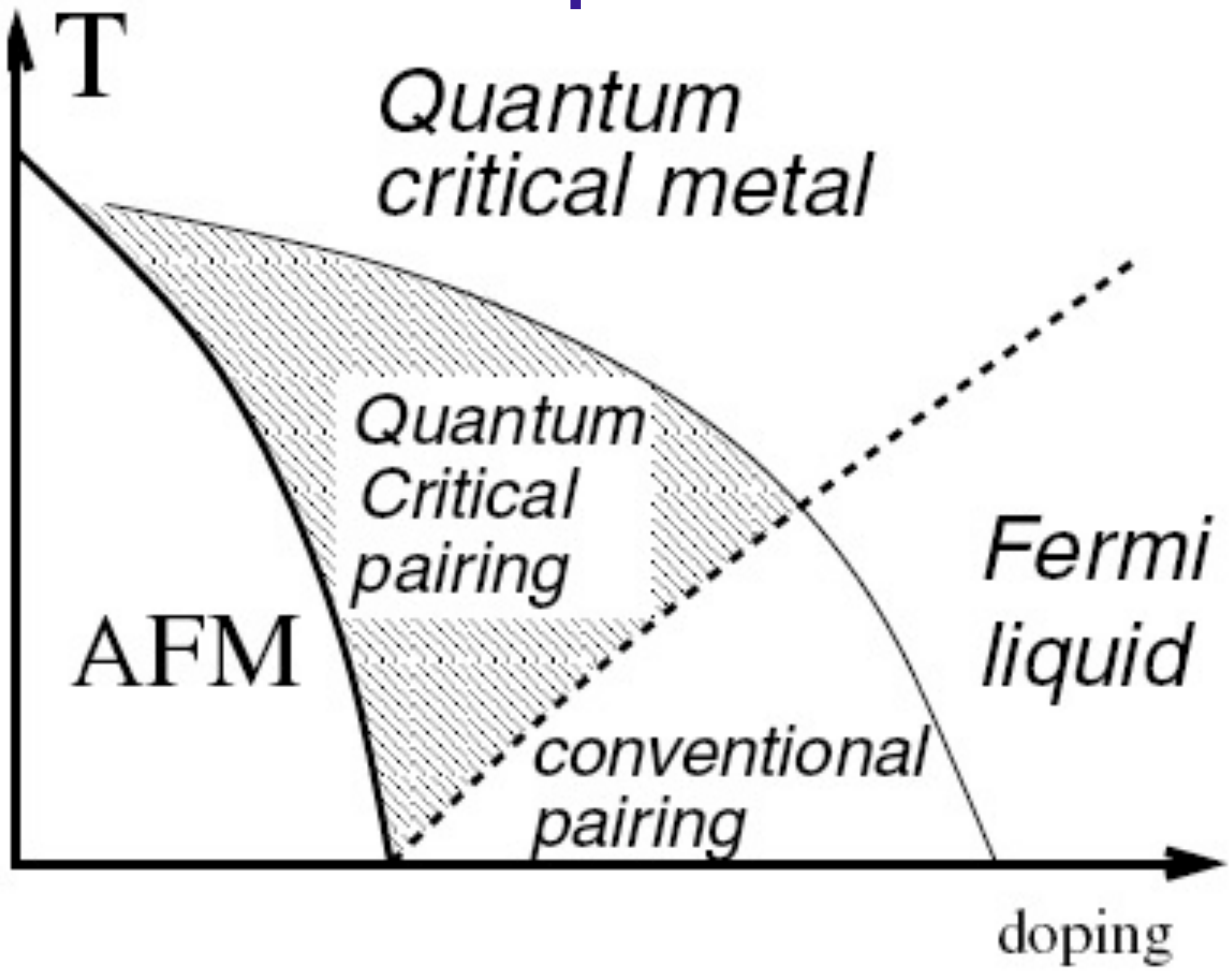


:

- T_c *increases* upon approaching the SDW transition.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).

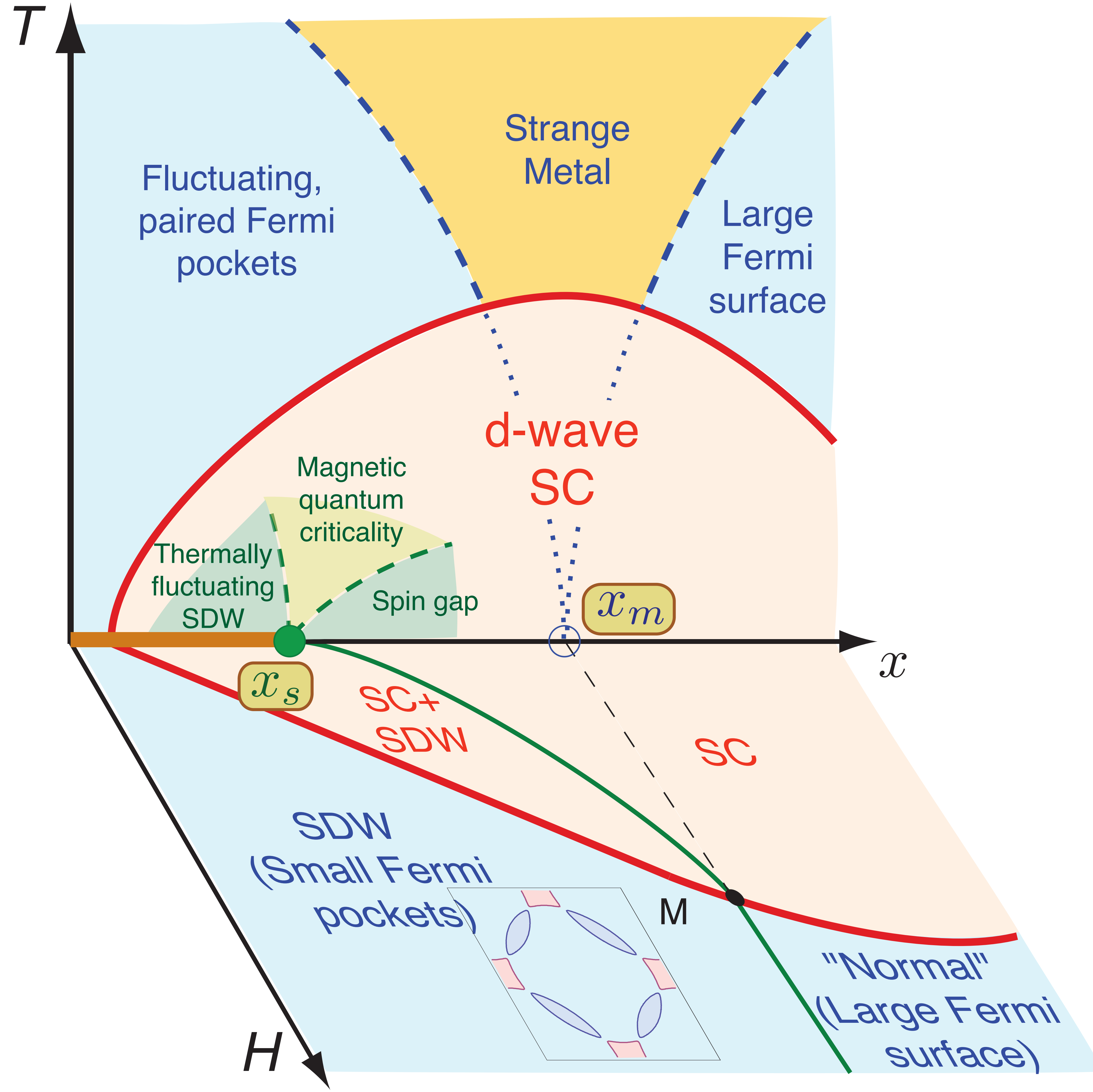
Approaching the onset of antiferromagnetism in the spin-fluctuation theory



:

- T_c *increases* upon approaching the SDW transition.
- Pairing from SDW fluctuations: SDW and SC orders do not compete, but attract each other.

Ar. Abanov, A.V. Chubukov and J. Schmalian, *Advances in Physics* **52**, 119 (2003).


T
Fluctuating,
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pockets
Strange
Metal
Large
Fermi
surface
d-wave
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Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x
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SC+
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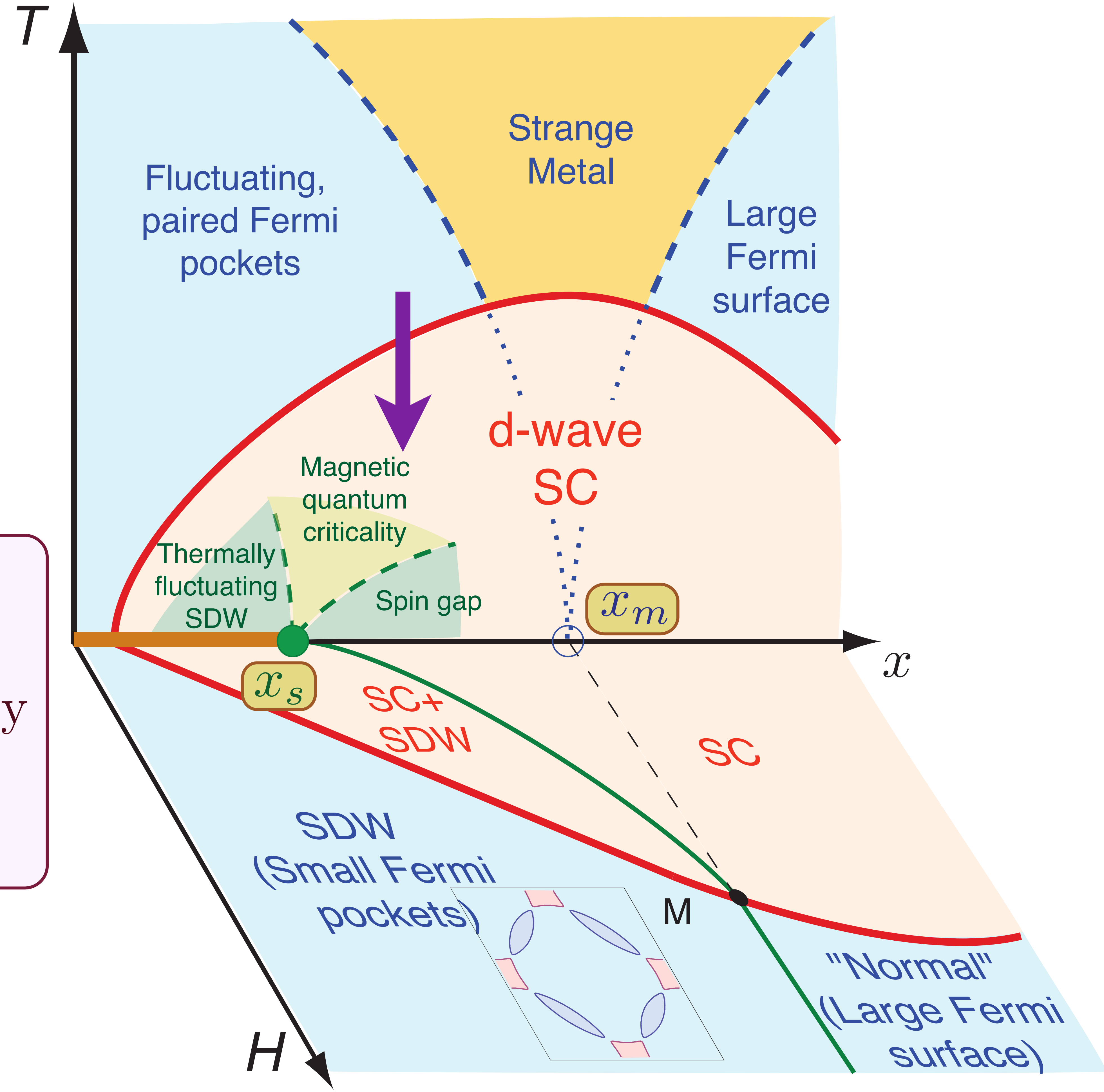
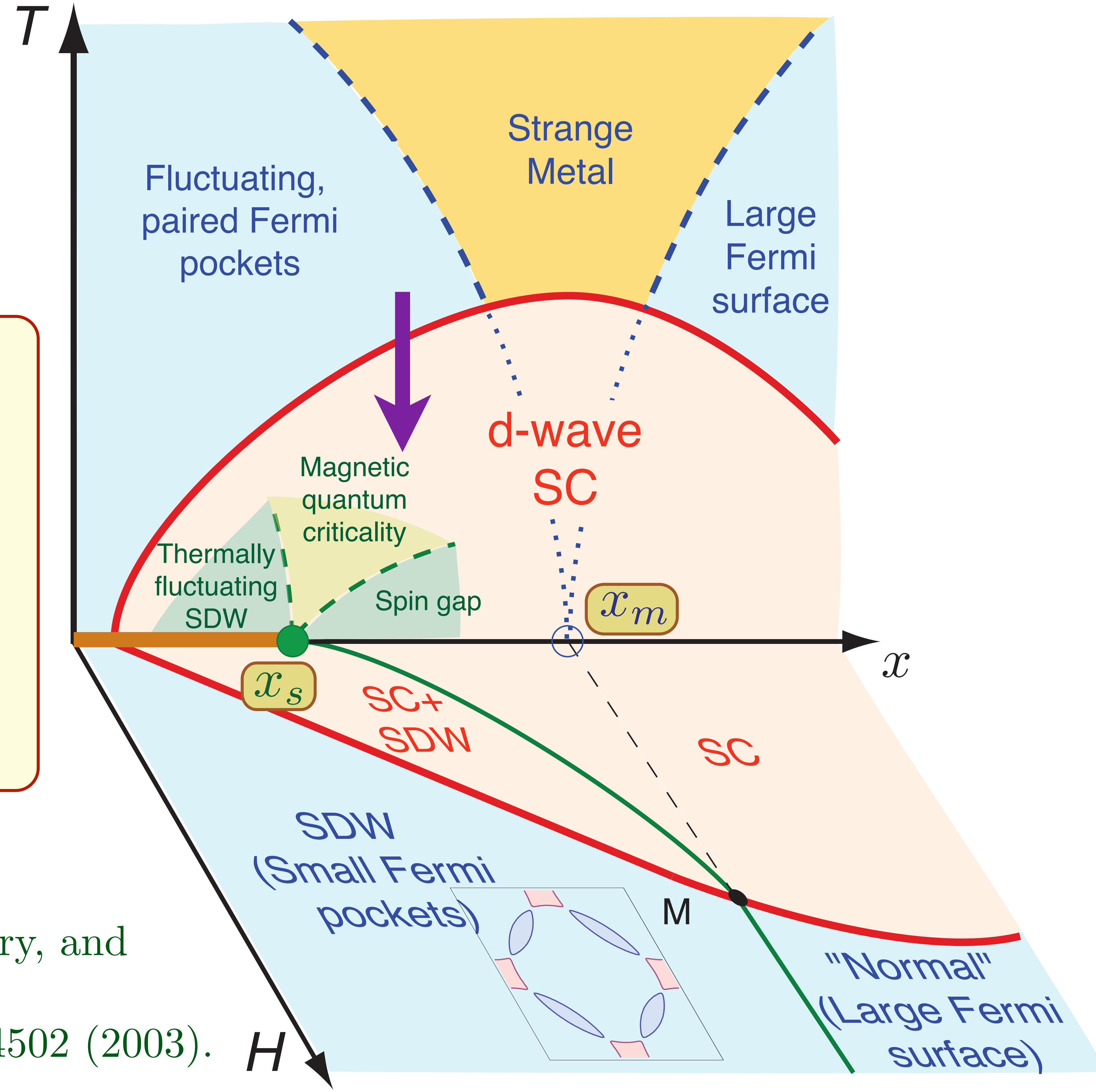
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

Theory of the onset of d-wave superconductivity from small Fermi pockets

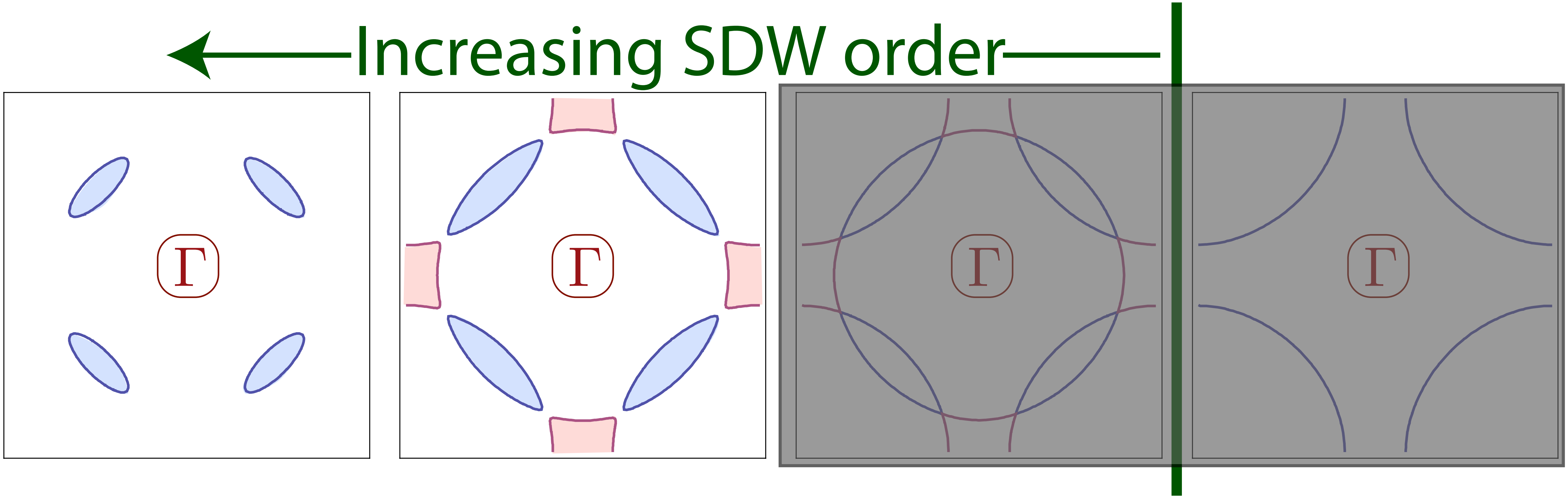


Physics of competition: *d*-wave SC and SDW “eat up’ same pieces of the large Fermi surface.



B. Kyung, J.-S. Landry, and A.-M. S. Tremblay, Phys. Rev. B **68**, 174502 (2003).

Theory of underdoped cuprates



Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order $\hat{\vec{\varphi}}$

$$\begin{pmatrix} c_\uparrow \\ c_\downarrow \end{pmatrix} = R \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \quad ; \quad R^\dagger \hat{\vec{\varphi}} \cdot \vec{\sigma} R = \sigma^z \quad ; \quad R^\dagger R = 1$$

H. J. Schulz, *Physical Review Letters* **65**, 2462 (1990)
B. I. Shraiman and E. D. Siggia, *Physical Review Letters* **61**, 467 (1988).
J. R. Schrieffer, *Journal of Superconductivity* **17**, 539 (2004)

Theory of underdoped cuprates

$$\text{With} \quad R = \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix} \quad \text{or} \quad \hat{\vec{\varphi}} = z_\alpha^* \vec{\sigma}_{\alpha\beta} z_\beta$$

the theory is invariant under

$$z_\alpha \to e^{i\theta} z_\alpha \;\; ; \;\; \psi_+ \to e^{-i\theta} \psi_+ \;\; ; \;\; \psi_- \to e^{i\theta} \psi_{.-}$$

We obtain a U(1) gauge theory of

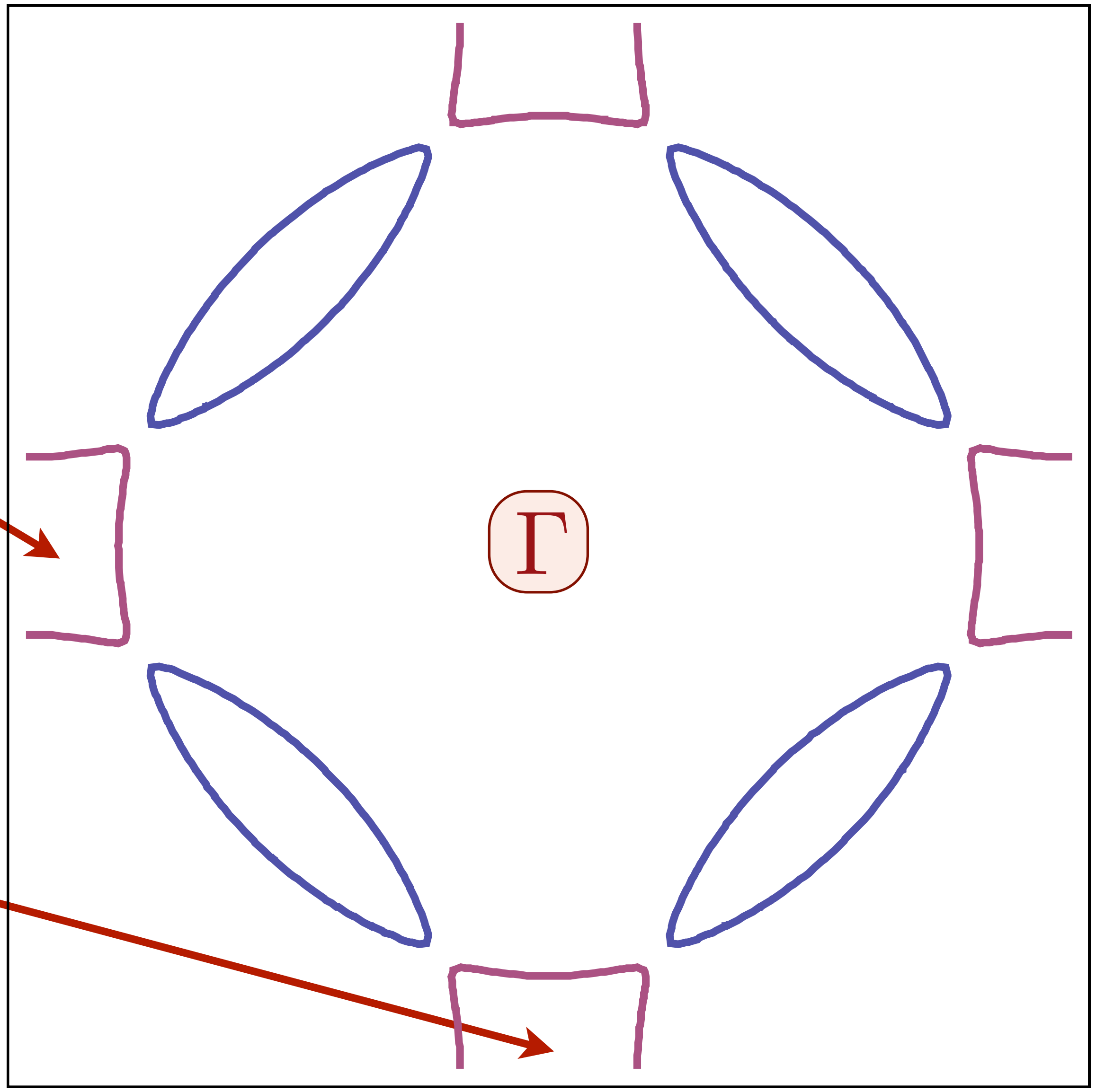
- bosonic neutral spinons z_α;
- spinless, charged fermions $\psi_\pm$;
- an emergent U(1) gauge field A_μ.

X.-G. Wen, *Phys. Rev. B* **39**, 7223 (1989).
P. A. Lee, *Phys. Rev. Lett.* **63**, 680 (1989).
R. Shankar, *Phys. Rev. Lett.* **63**, 203 (1989).
L. B. Ioffe and P. B. Wiegmann, *Phys. Rev. Lett.* **65**, 653 (1990).
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, *Phys. Rev. B* **75** , 235122 (2007).

Pairing across T_c

Electron $c_{2\alpha}$,
spinless fermion $g_\pm$

Electron $c_{1\alpha}$,
spinless fermion $g_\pm$



Focus on pairing near $(\pi, 0)$, $(0, \pi)$, where $\psi_\pm \equiv g_\pm$,
and the electron operators are

$$\begin{pmatrix} c_{1\uparrow} \\ c_{1\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ g_- \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{2\uparrow} \\ c_{2\downarrow} \end{pmatrix} = \mathcal{R}_z \begin{pmatrix} g_+ \\ -g_- \end{pmatrix}$$

$$\mathcal{R}_z \equiv \begin{pmatrix} z_\uparrow & -z_\downarrow^* \\ z_\downarrow & z_\uparrow^* \end{pmatrix}.$$

R. K. Kaul, M. Metlitksi, S. Sachdev,
and Cenke Xu,
Phys. Rev. B **78**, 045110 (2008).

Pairing across T_c

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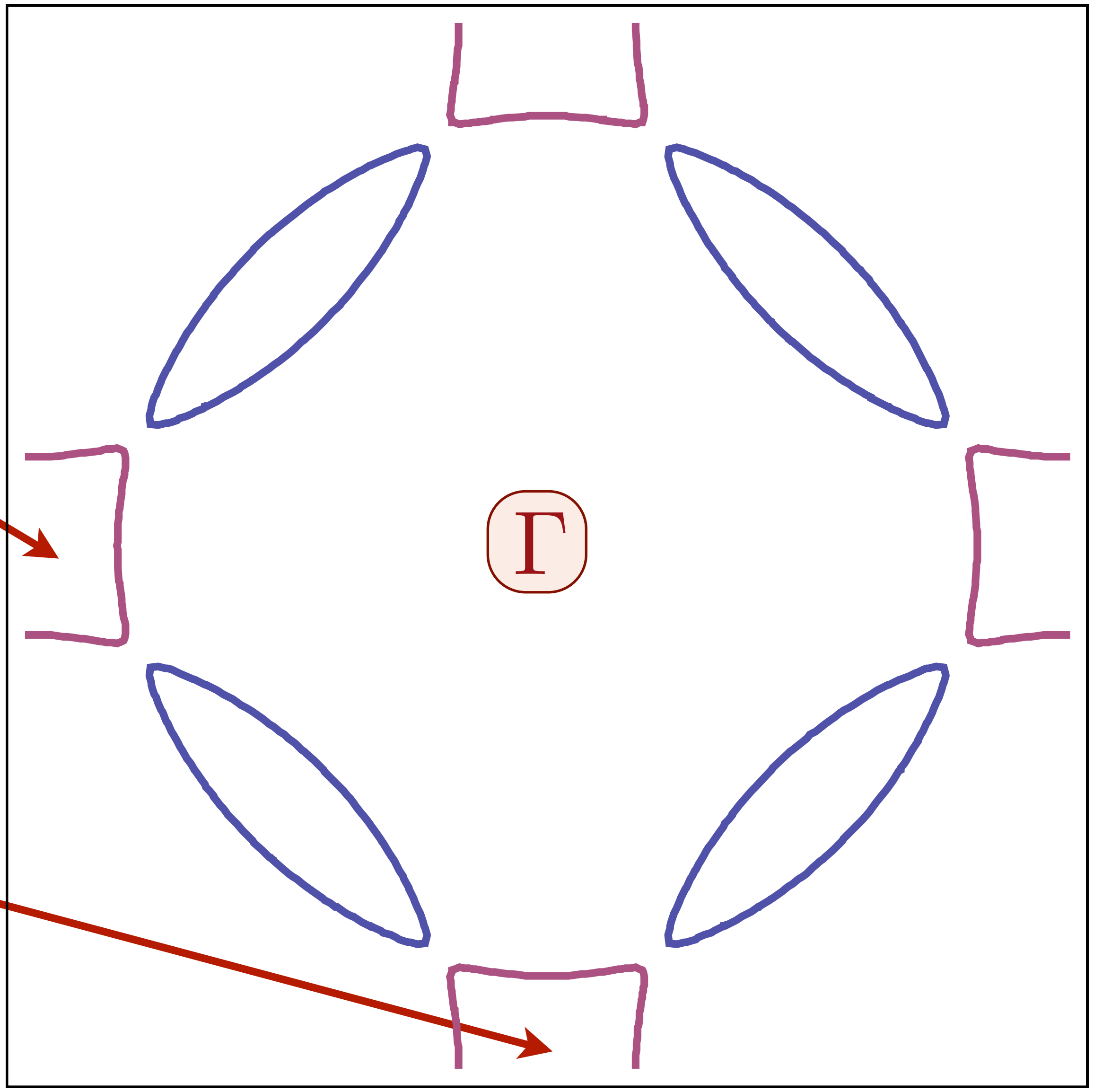

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R. K. Kaul, M. Metlitksi, S. Sachdev,
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Phys. Rev. B **78**, 045110 (2008).

Fluctuating pocket theory for electrons near $(0, \pi)$ and $(\pi, 0)$

Attractive gauge forces lead to simple s-wave pairing of the $g_\pm$

$$\langle g_+ g_- \rangle = \Delta$$

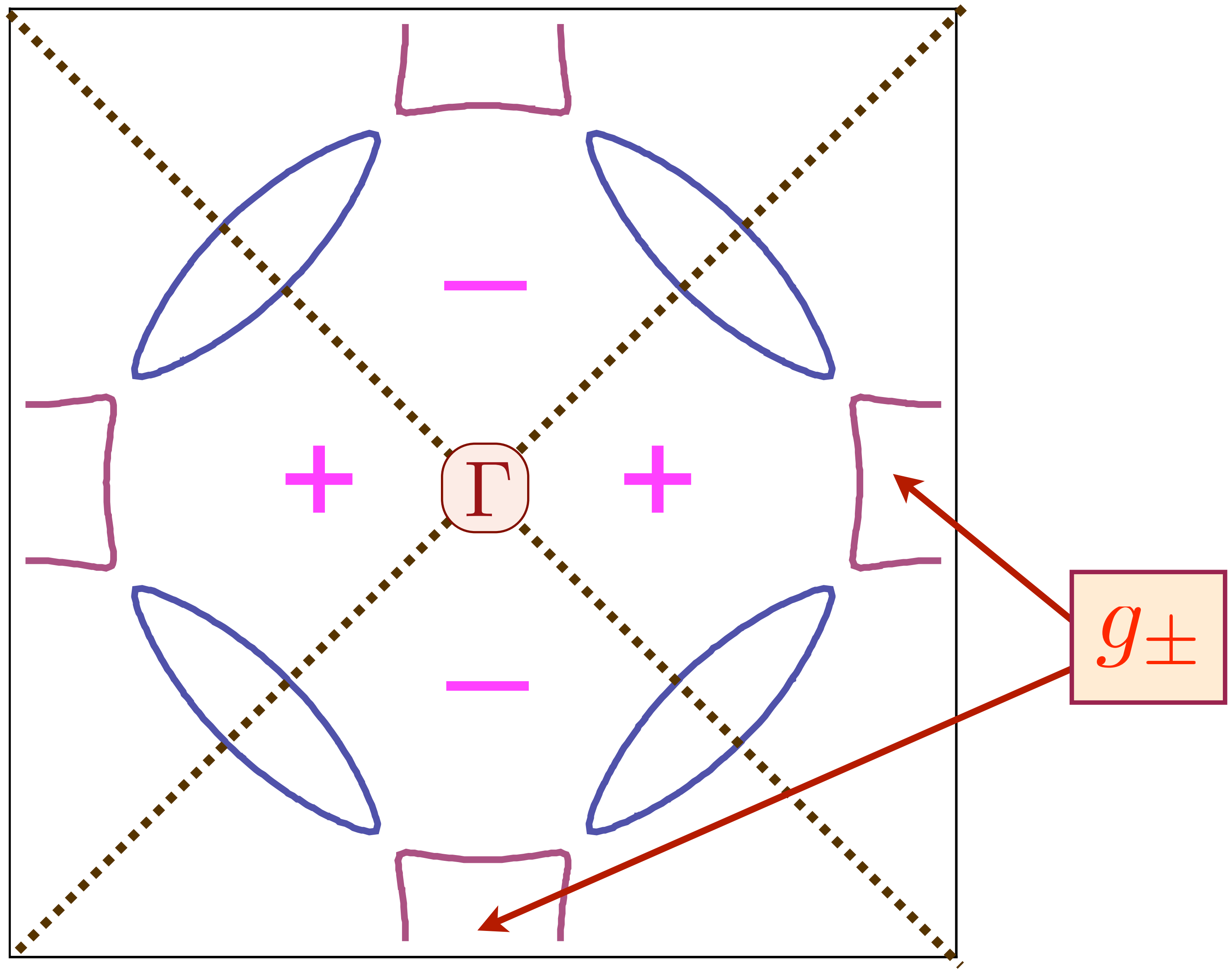
For the physical electron operators, this pairing implies

$$\begin{aligned} \langle c_{1\uparrow} c_{1\downarrow} \rangle &= \Delta \left\langle |z_\alpha|^2 \right\rangle \\ \langle c_{2\uparrow} c_{2\downarrow} \rangle &= -\Delta \left\langle |z_\alpha|^2 \right\rangle \end{aligned}$$

$i.e.$ d-wave pairing !

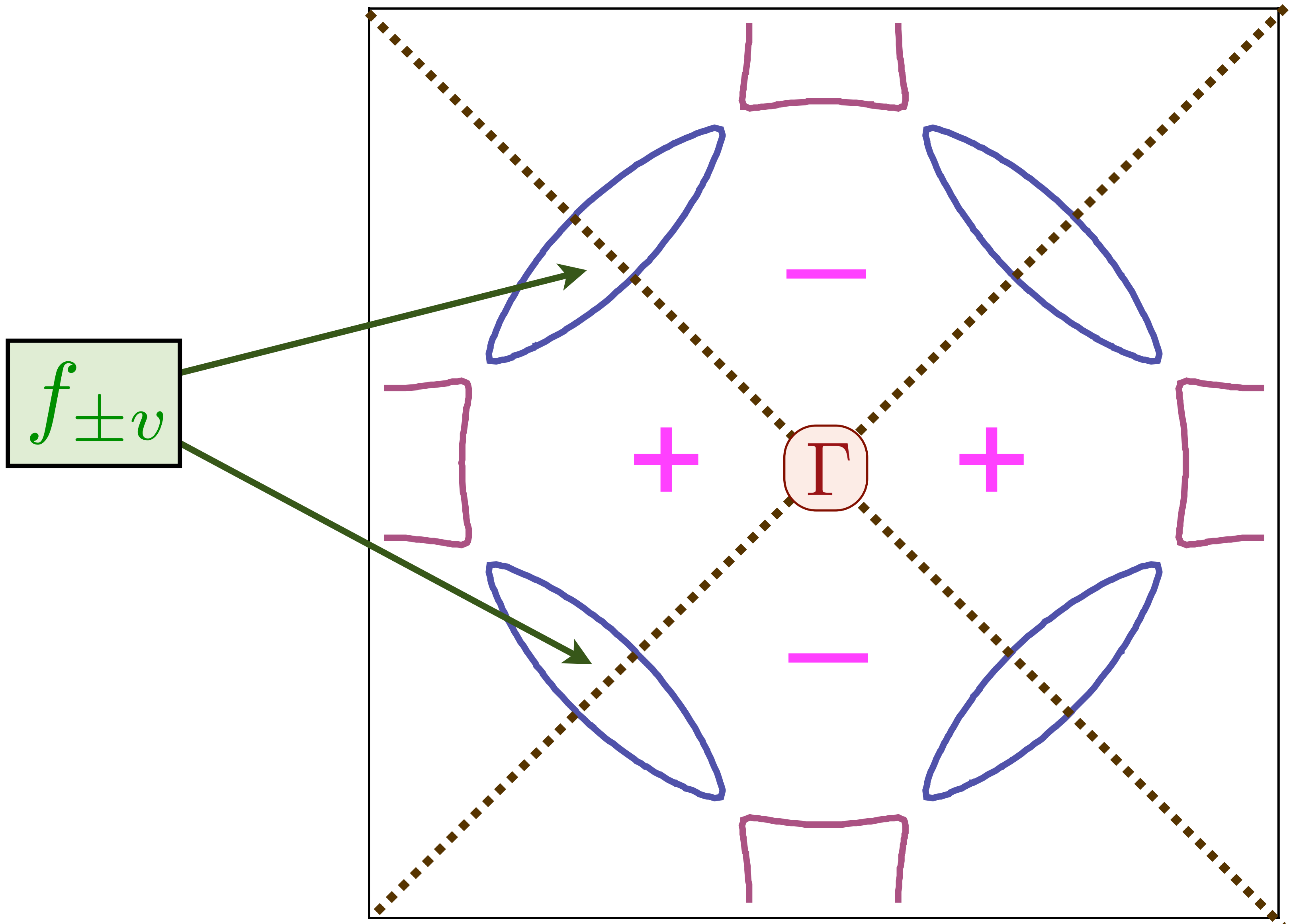
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

Strong pairing of the $g_\pm$ electron pockets



$$\langle g_+ g_- \rangle = \Delta$$

Weak pairing of the $f_\pm$ hole pockets



$$\langle f_{+1}(\mathbf{k}) f_{-1}(-\mathbf{k}) \rangle \sim (k_x - k_y) J \langle g_+ g_- \rangle;$$
$$\langle f_{+2}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle \sim (k_x + k_y) J \langle g_+ g_- \rangle;$$
$$\langle f_{+1}(\mathbf{k}) f_{-2}(-\mathbf{k}) \rangle = 0,$$

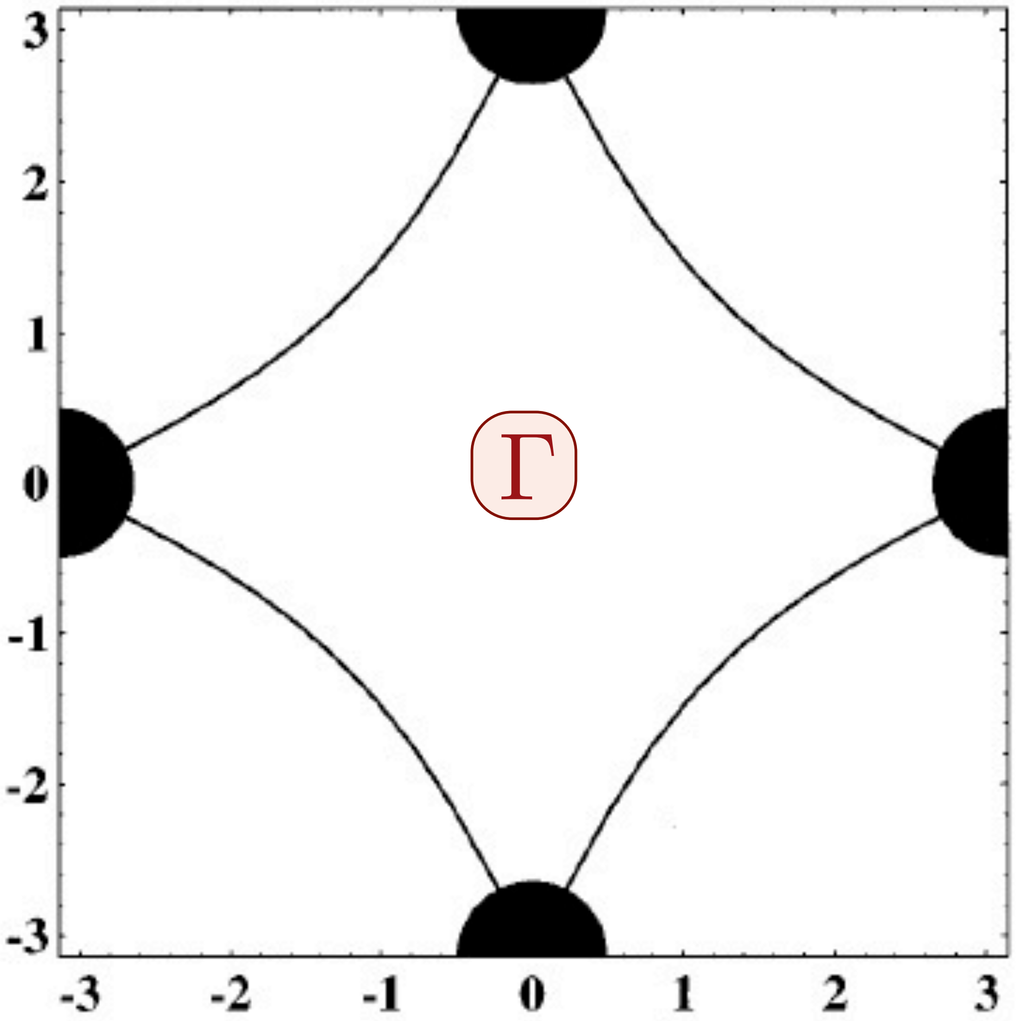


FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping $t' = -0.3t$; it is similar to the Fermi line observed in the underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.

$$H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q}{}' V_{p,q}(b_q^\dagger c_{p\uparrow} c_{q-p\downarrow} + \text{H.c.})$$

$$+ \sum_p \xi_p c_{p,\sigma}^\dagger c_{p,\sigma};$$

$$V_{p,q} = Va^2(p_x^2 - p_y^2)$$

-2e bosons at antinodes,
+e fermion "arcs" at nodes,
and proximity "Josephson" coupling

V. B. Geshkenbein, L. B. Ioffe, and A. I. Larkin, Phys. Rev. B **55**, 3173 (1997).

Similar features in our theory

V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

Features of superconductivity

- d-wave superconductivity.
- Nodal-anti-nodal dichotomy: strong pairing near $(\pi, 0)$, $(0, \pi)$, and weak pairing near zone diagonals.

V. Galitski and S. Sachdev,
Physical Review B **79**, 134512 (2009).

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- T_c decreases as spin correlation increases (competing order effect).
- Shift in quantum critical point of SDW ordering: gauge fluctuations are stronger in the superconductor.

Eun Gook Moon and S. Sachdev,
Physical Review B **80**, 035117 (2009).

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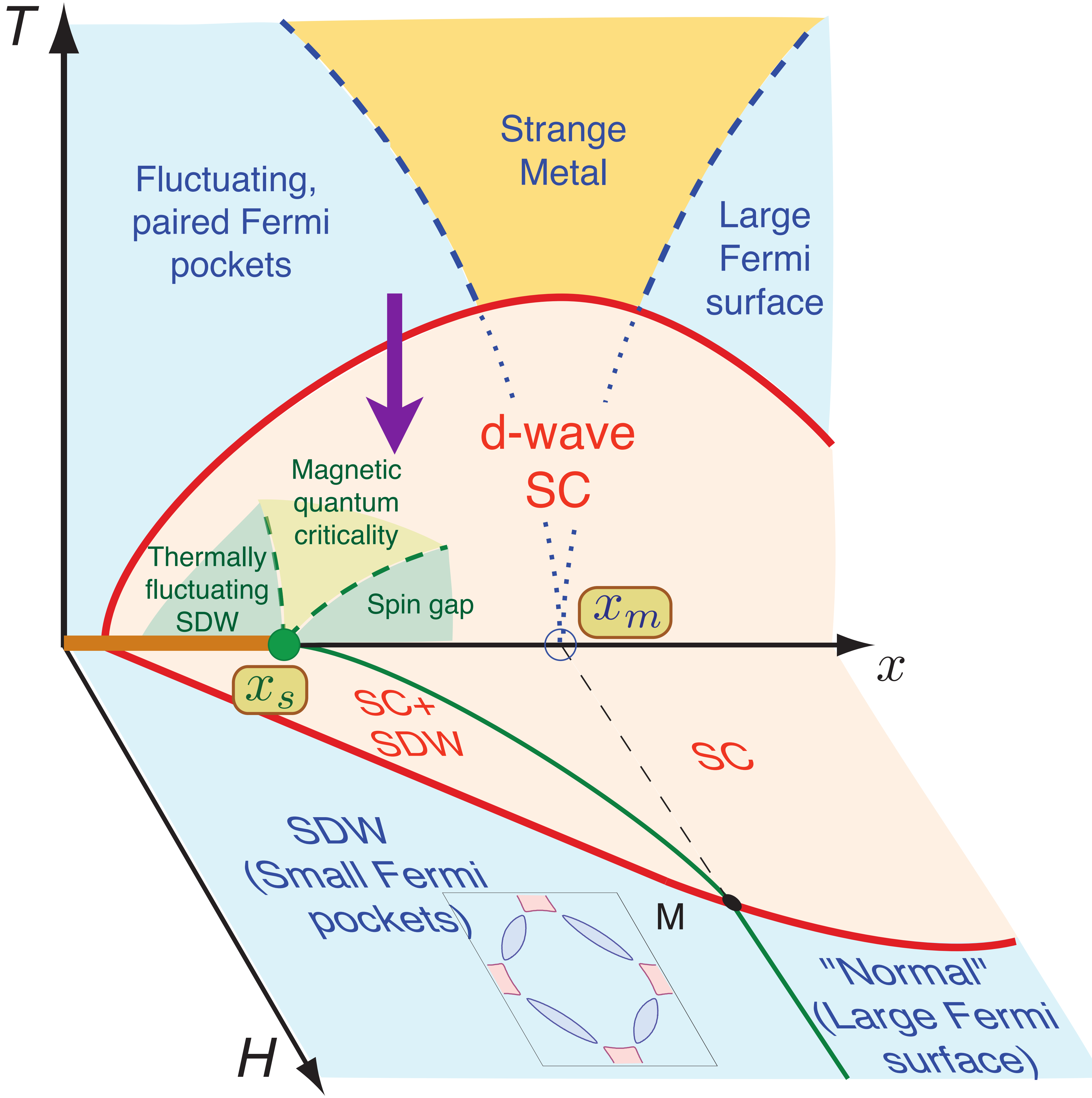
- After onset of superconductivity, monopoles condense and lead to confinement and **nematic** and/or **valence bond solid** (VBS) order.

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).



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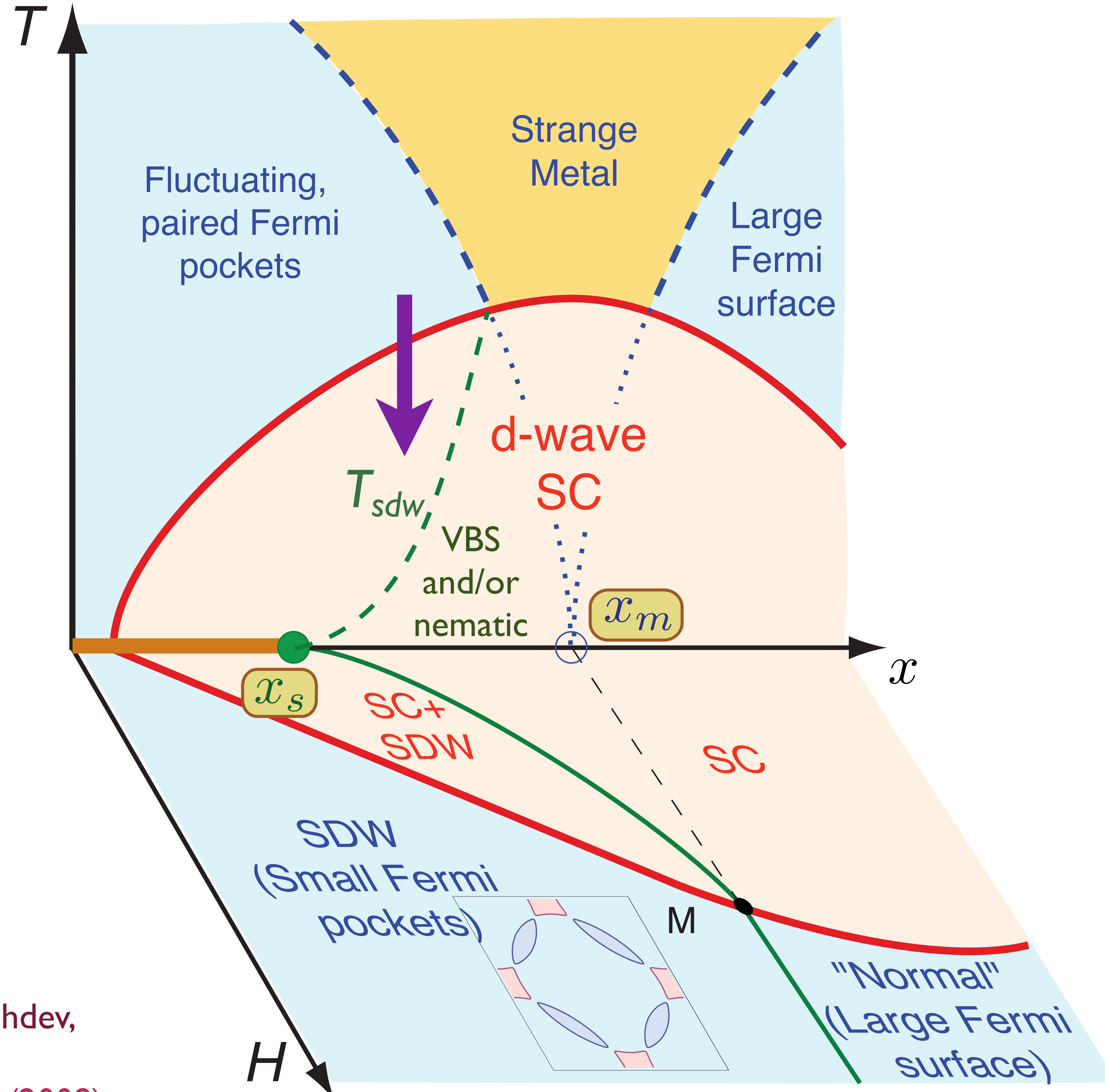

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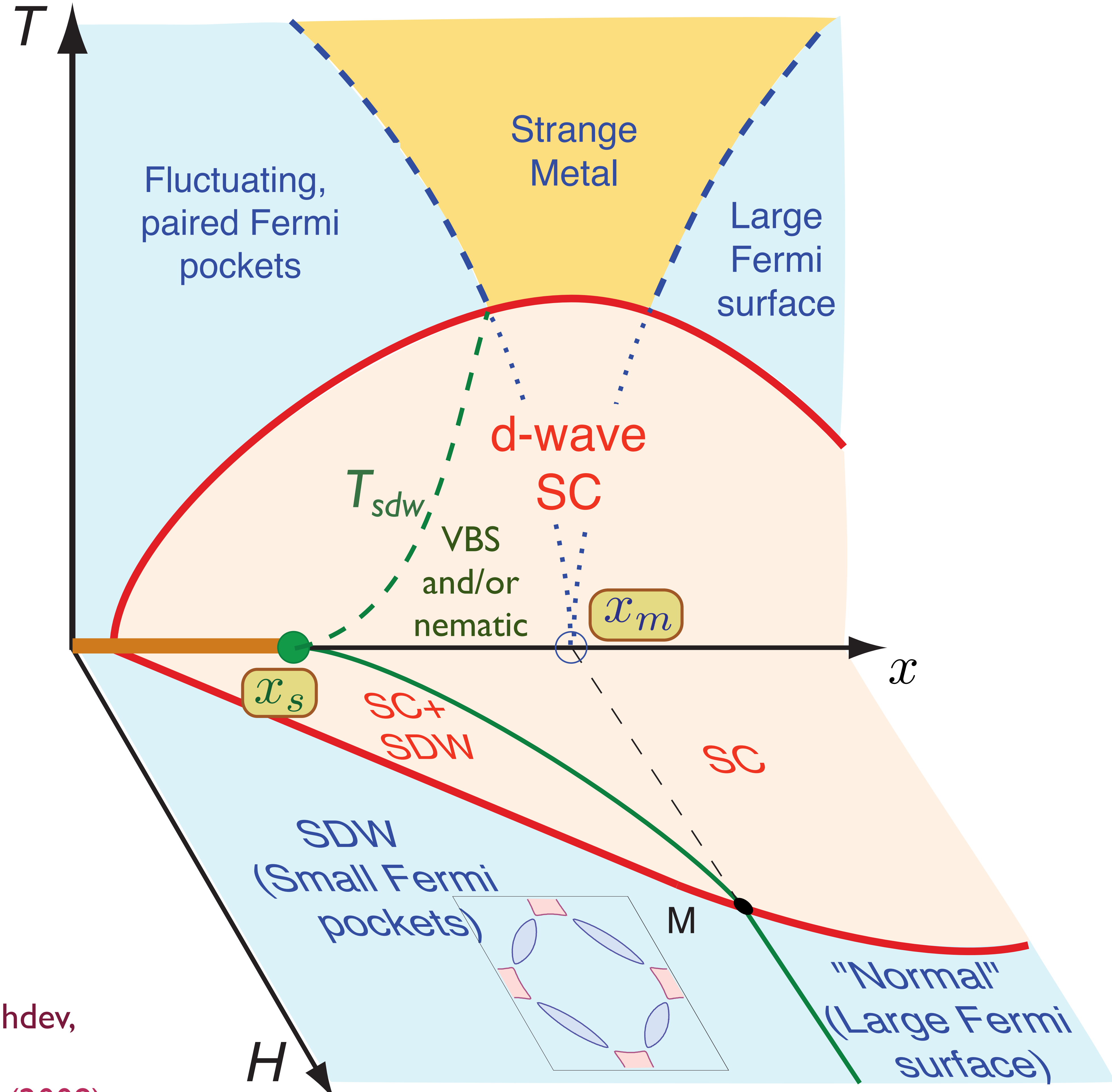
Physical Review B **78**, 045110 (2008).



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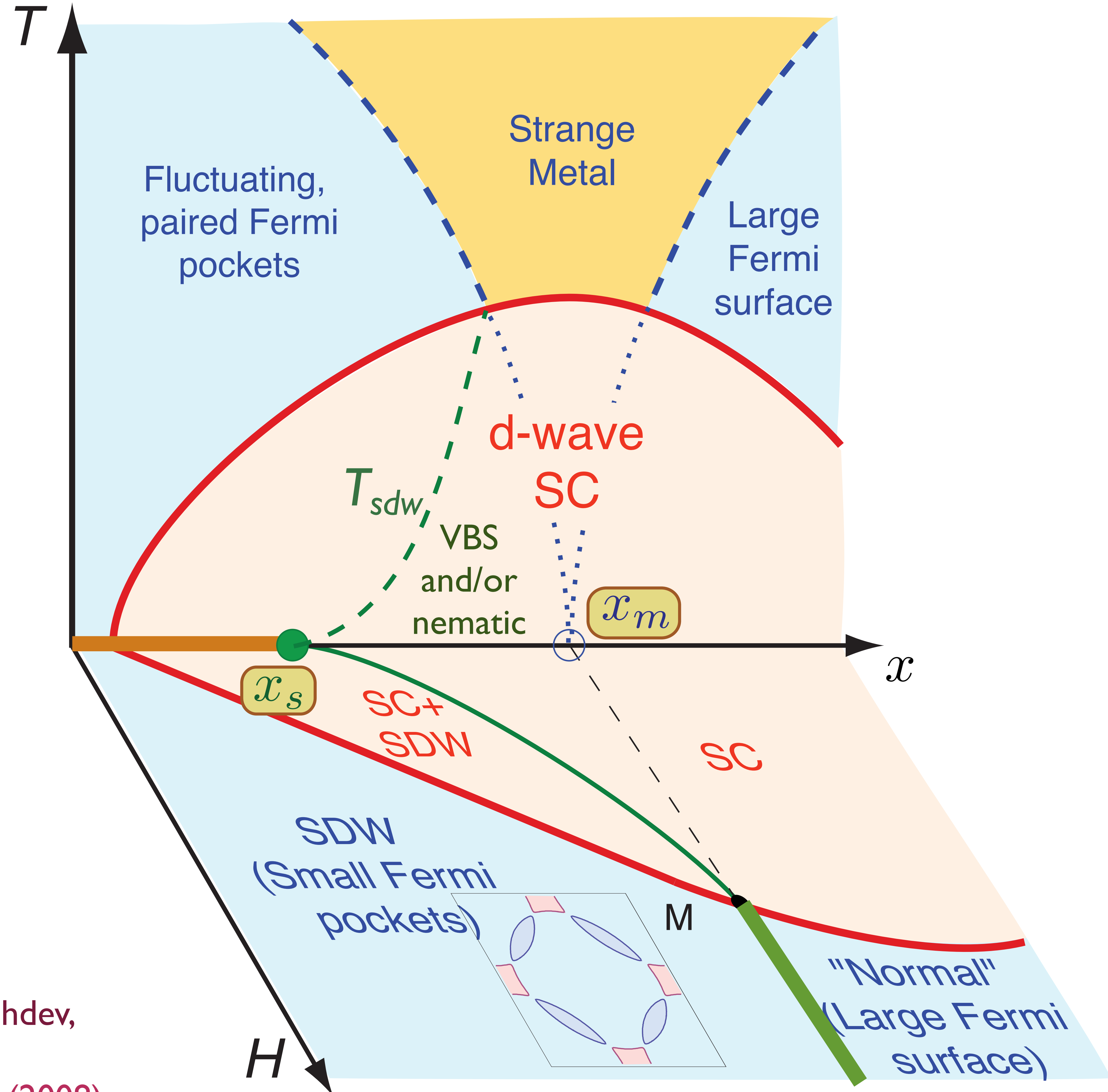
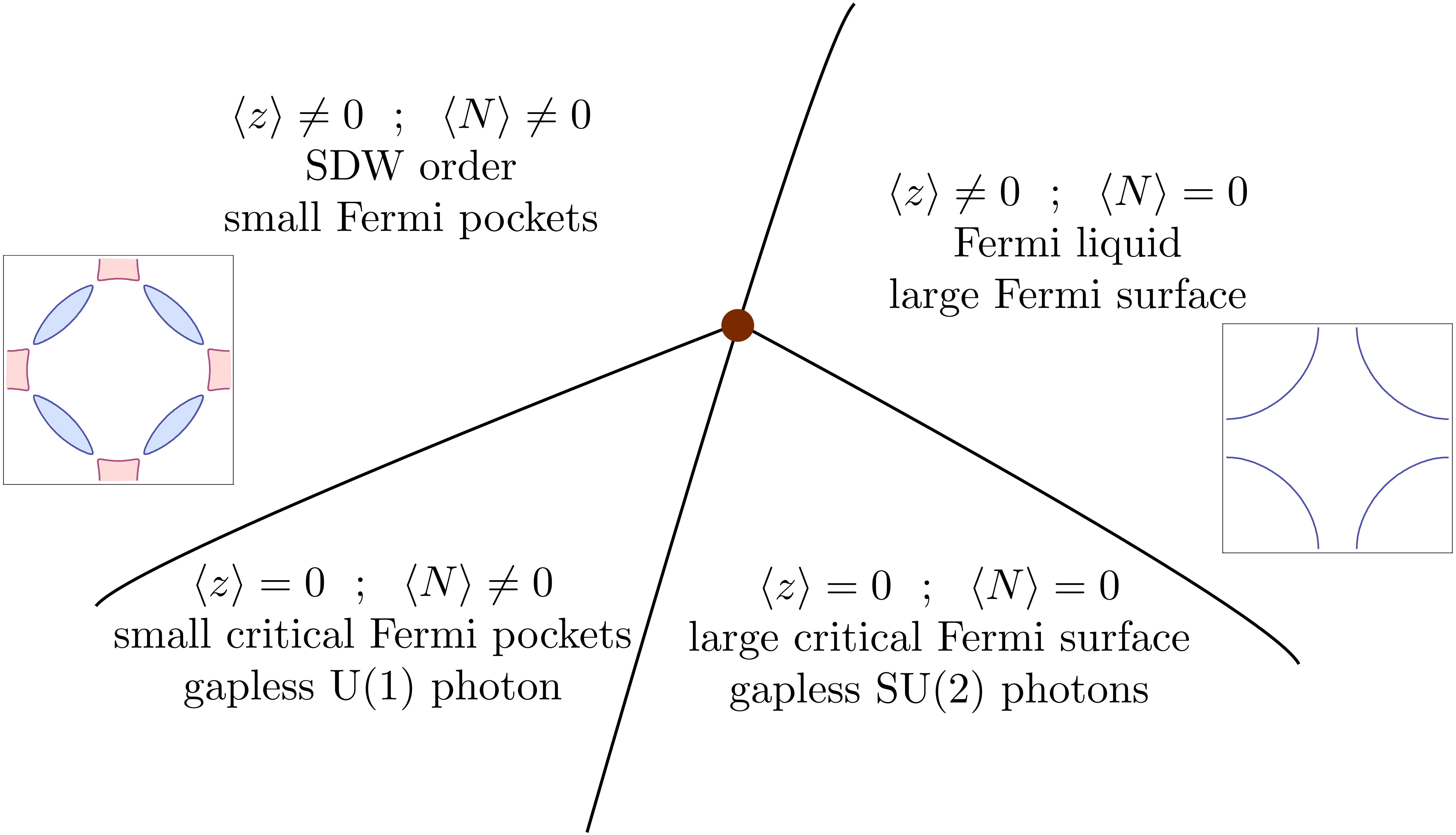


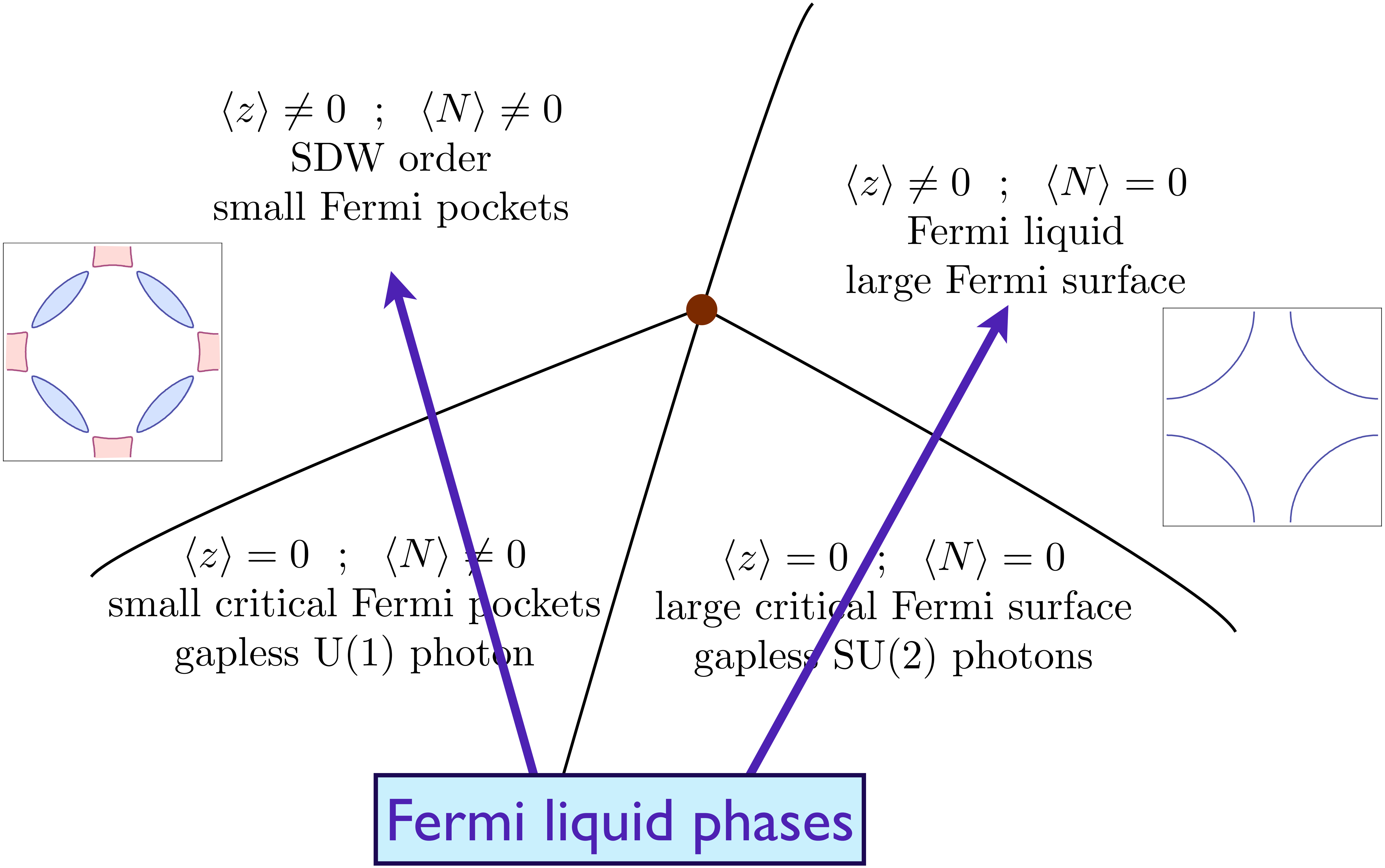
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

Quantum phase transitions in metal



S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Quantum phase transitions in metal



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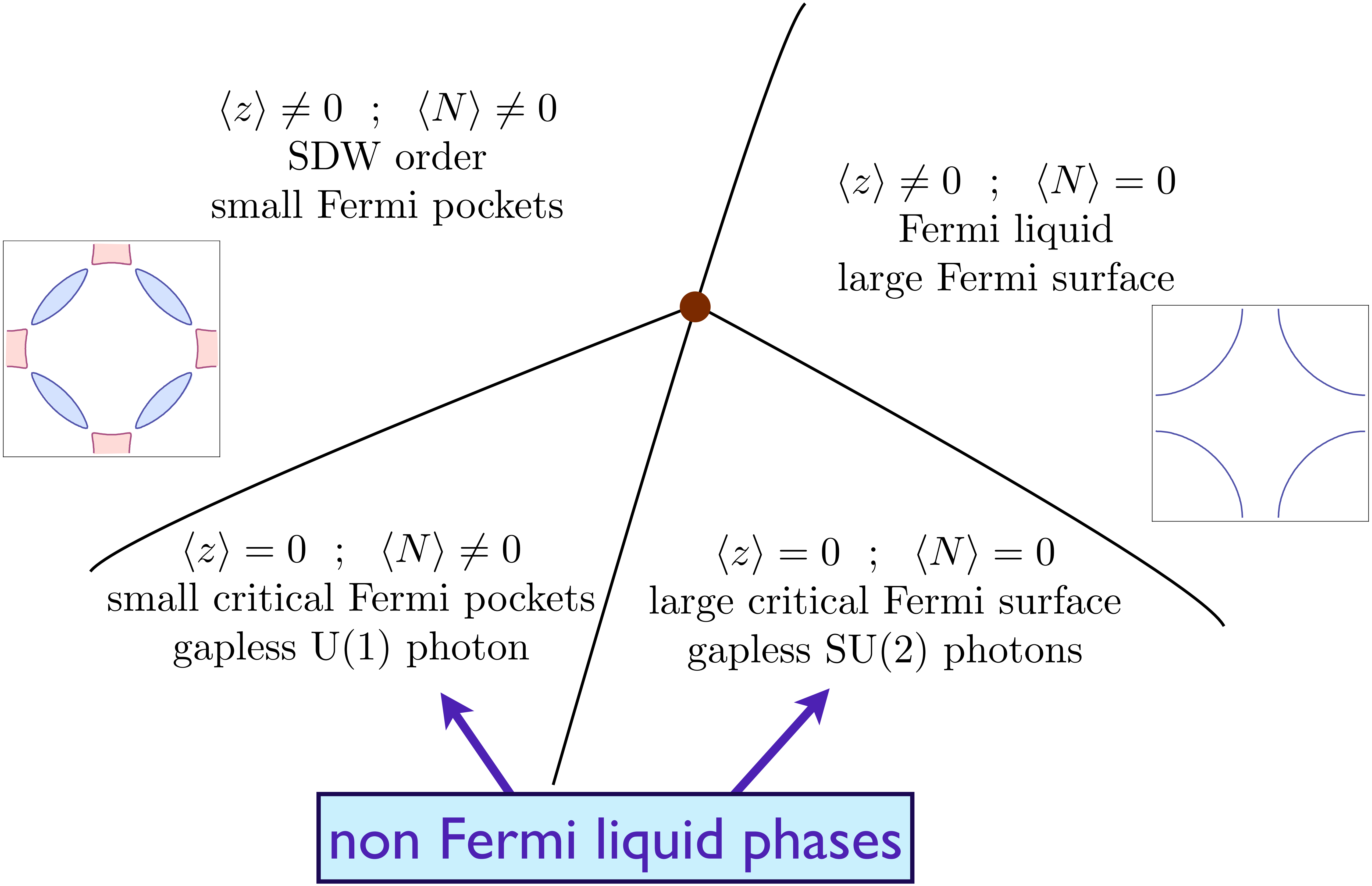

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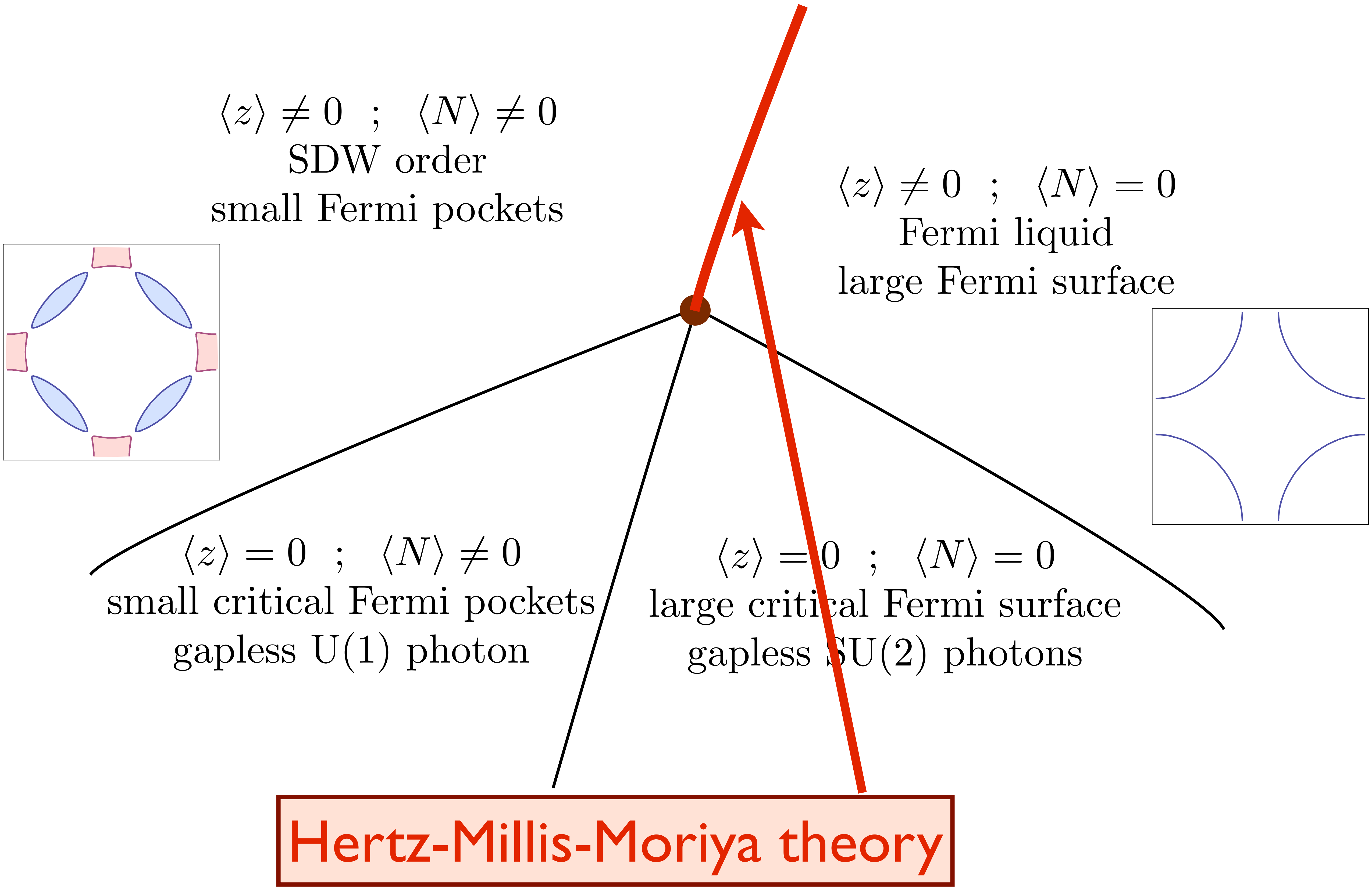


S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Max Metlitski

M. Metlitski and S. Sachdev, *to appear*

Ar. Abanov, A.V. Chubukov, and J. Schmalian,
Advances in Physics **52**, 119 (2003)

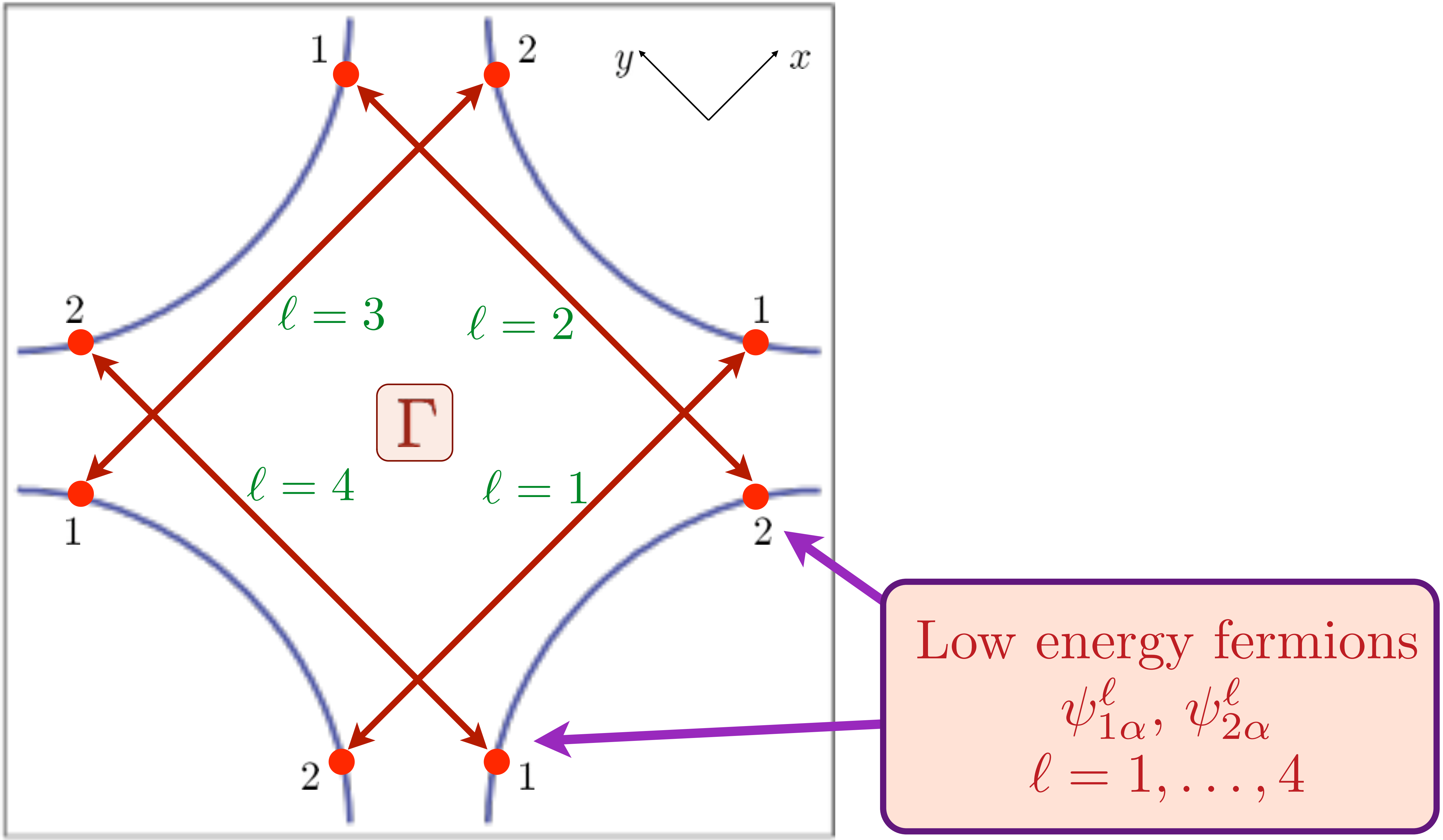
Sung-Sik Lee, arXiv:0905.4532.

Hertz-Millis-Moriya (HMM) theory:
mean field theory
+
Gaussian fluctuations of overdamped paramagnons.

Theory for the onset of spin density wave order in metals is ***strongly*** coupled in two dimensions

Start from the "spin-fermion" model

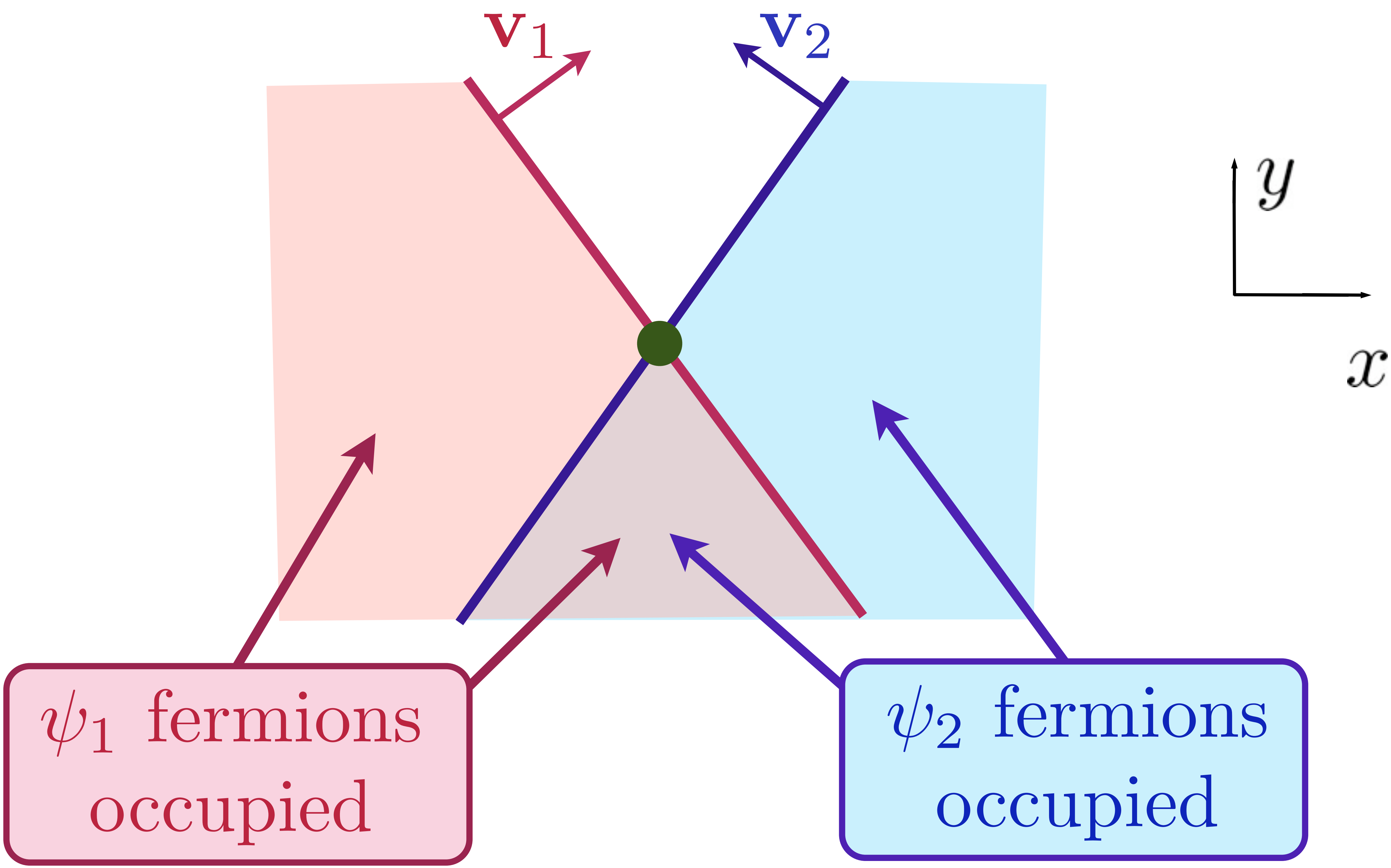
$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right) \\
\mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\
&\quad - \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i} \\
&\quad + \int d\tau d^2 r \left[\left(\partial_r \vec{\varphi}\right)^2 + \frac{1}{c^2} \left(\partial_\tau \vec{\varphi}\right)^2 \right]
\end{aligned}
$$



$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

$$\mathbf{v}^{\ell=1}_1 = (v_x, v_y),\ \mathbf{v}^{\ell=1}_2 = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$



$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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HMM theory

Integrate out fermions and obtain non-local corrections to $\mathcal{L}_\varphi$

$$\mathcal{L}_\varphi = \frac{1}{2}\vec{\varphi}^2\left[\mathbf{q}^2 + \gamma|\omega|\right]/2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

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Exponent $z = 2$ and mean-field criticality (upto logarithms)

But, higher order terms contain an infinite number of marginal couplings ......

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Perform RG on both fermions and $\vec{\varphi}$, using a *local* field theory.

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

$$\frac{1}{N}\frac{1}{\left(q^2+\gamma|\omega|\right)}$$

fermion propagator

$$\frac{1}{\mathbf{v}\cdot\mathbf{q}+i\zeta\omega+i\dfrac{1}{N\sqrt{\gamma}v}\sqrt{\omega}F\left(\dfrac{v^2q^2}{\omega}\right)}$$

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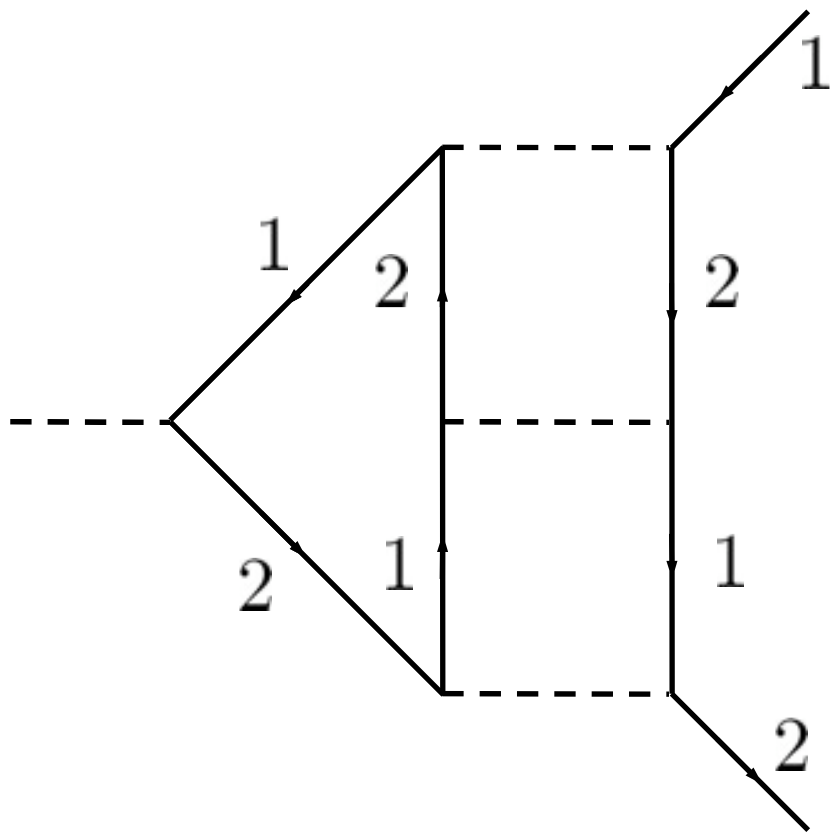
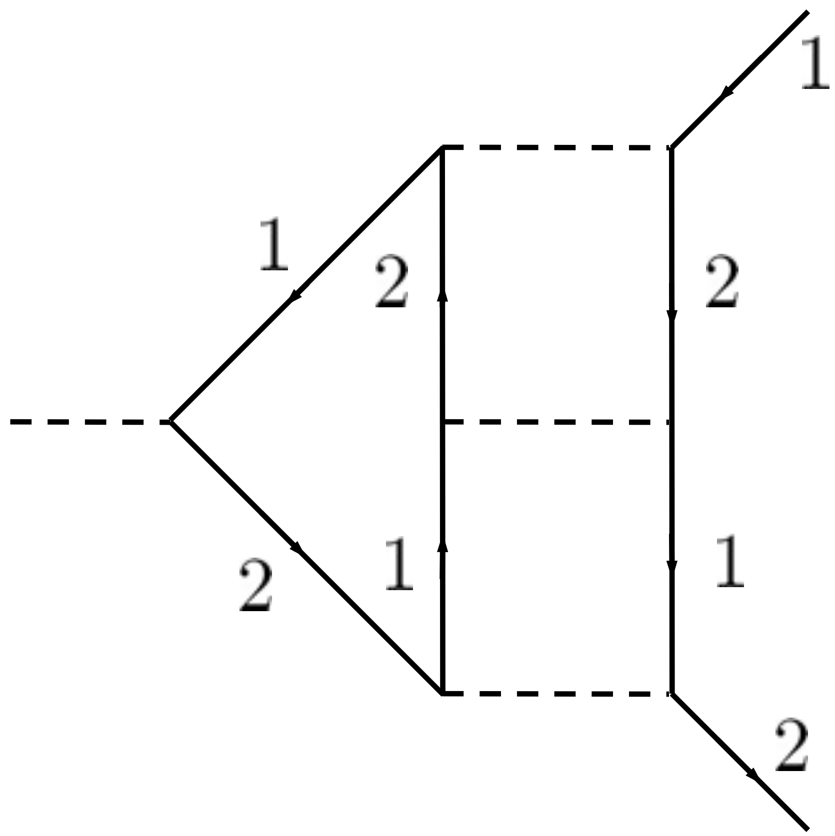
⚠ Dangerous

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \dfrac{1}{N^2} \times \dfrac{1}{\zeta^2} \times \dfrac{1}{\omega}$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



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Actual order $\sim \frac{1}{N^0}$

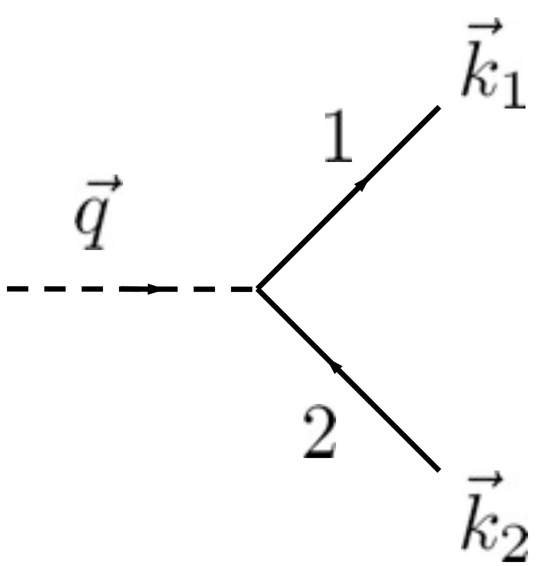
Double line representation

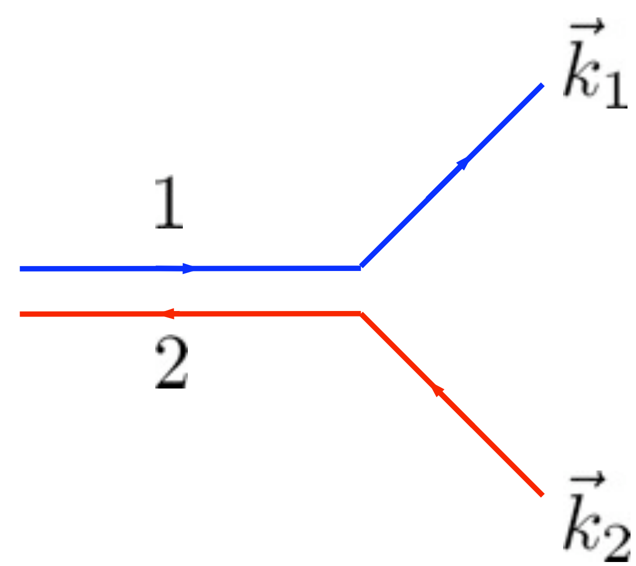
- A way to compute the order of a diagram.
- Extra powers of N come from the Fermi-surface

$$G(\omega,\vec{k}) = \frac{1}{-\Sigma_1(\omega,\vec{k}) - \vec{v}\cdot\vec{k}} \qquad \Sigma_1 \sim \frac{1}{N}$$

- What are the conditions for all propagators to be on the Fermi surface?
- Concentrate on diagrams involving a single pair of hot-spots
- Any bosonic momentum may be (uniquely) written as

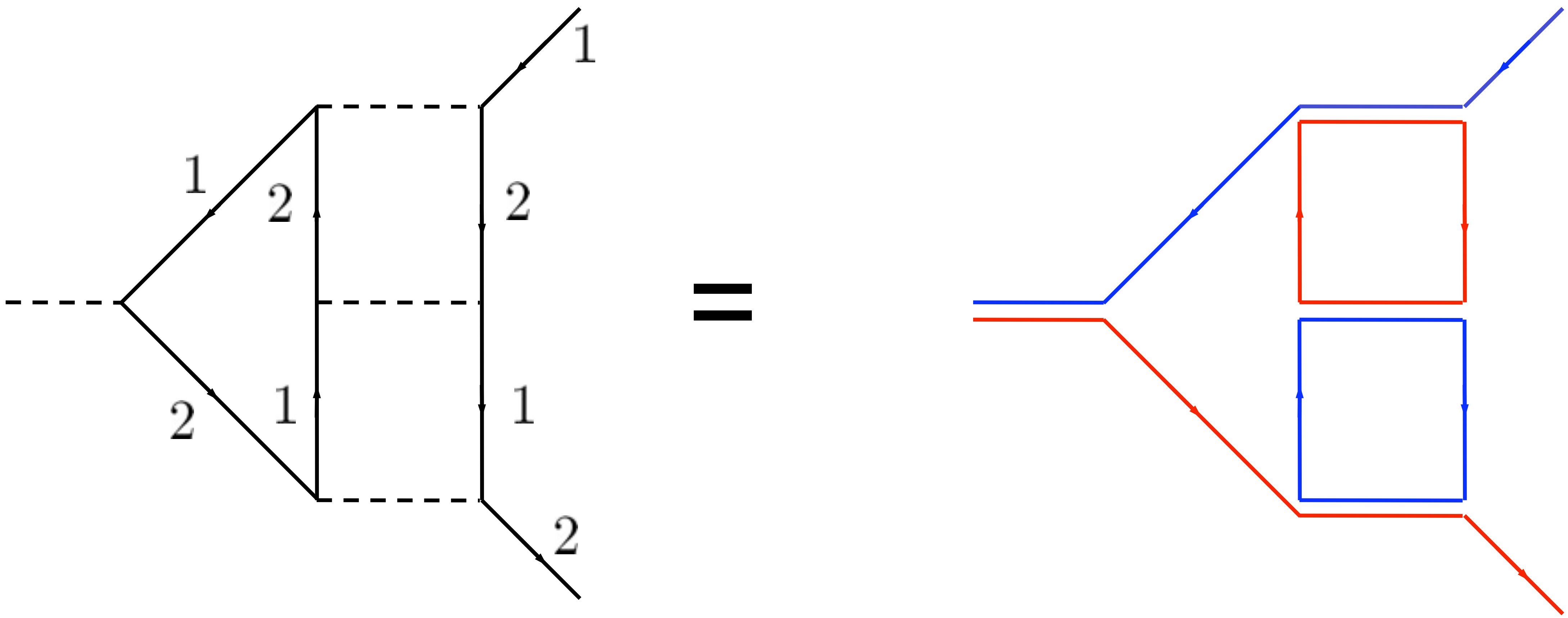
$$\vec{q} = \vec{k}_1 - \vec{k}_2 \qquad \vec{k}_1 \in \mathrm{FS\,of}\ \psi_1 \qquad \vec{k}_2 \in \mathrm{FS\,of}\ \psi_2$$





R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
S. W. Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Phys. Rev. B **72**, 054531 (2005).

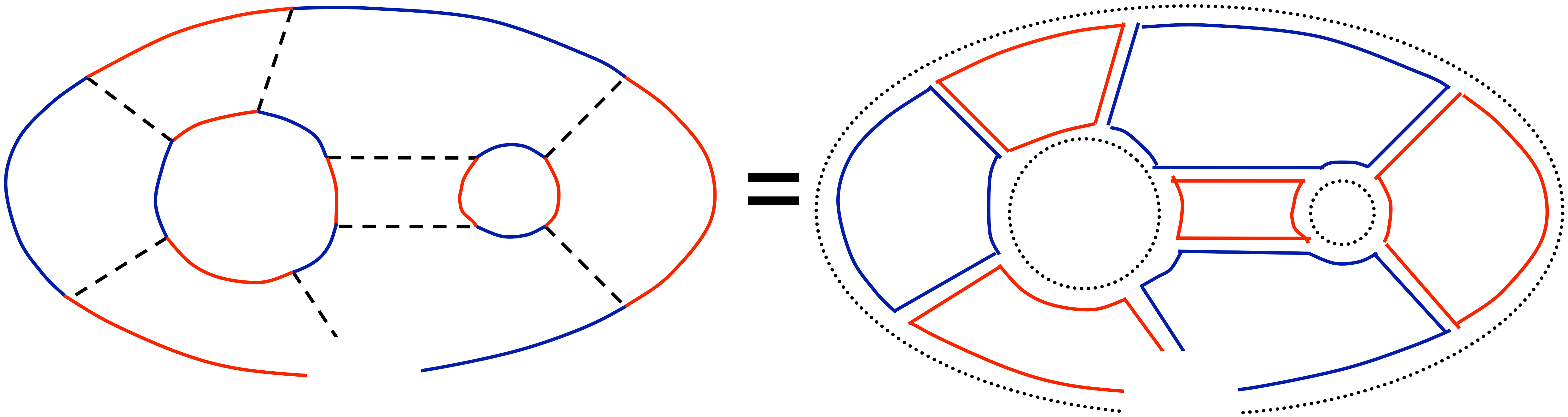
New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Singularities as $\zeta \to 0$ appear when fermions in closed blue and red line loops are exactly on the Fermi surface

$$\text{Actual order} \sim \frac{1}{N^0}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

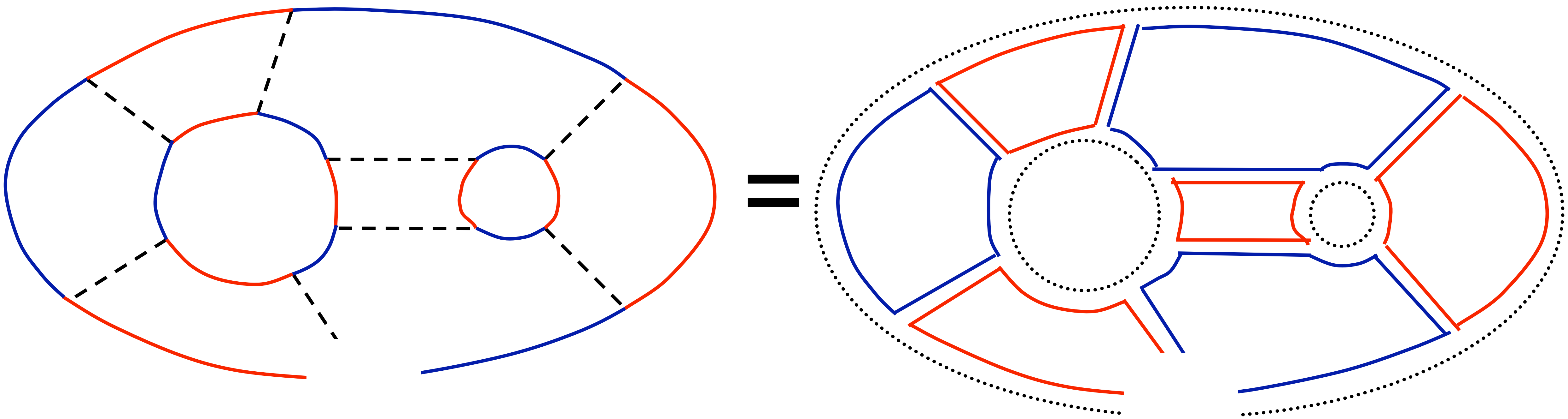


$$\text{Actual order} \sim \frac{1}{N^0}$$

Graph is **planar** after turning fermion propagators also into double lines by drawing additional dotted single line loops for each fermion loop

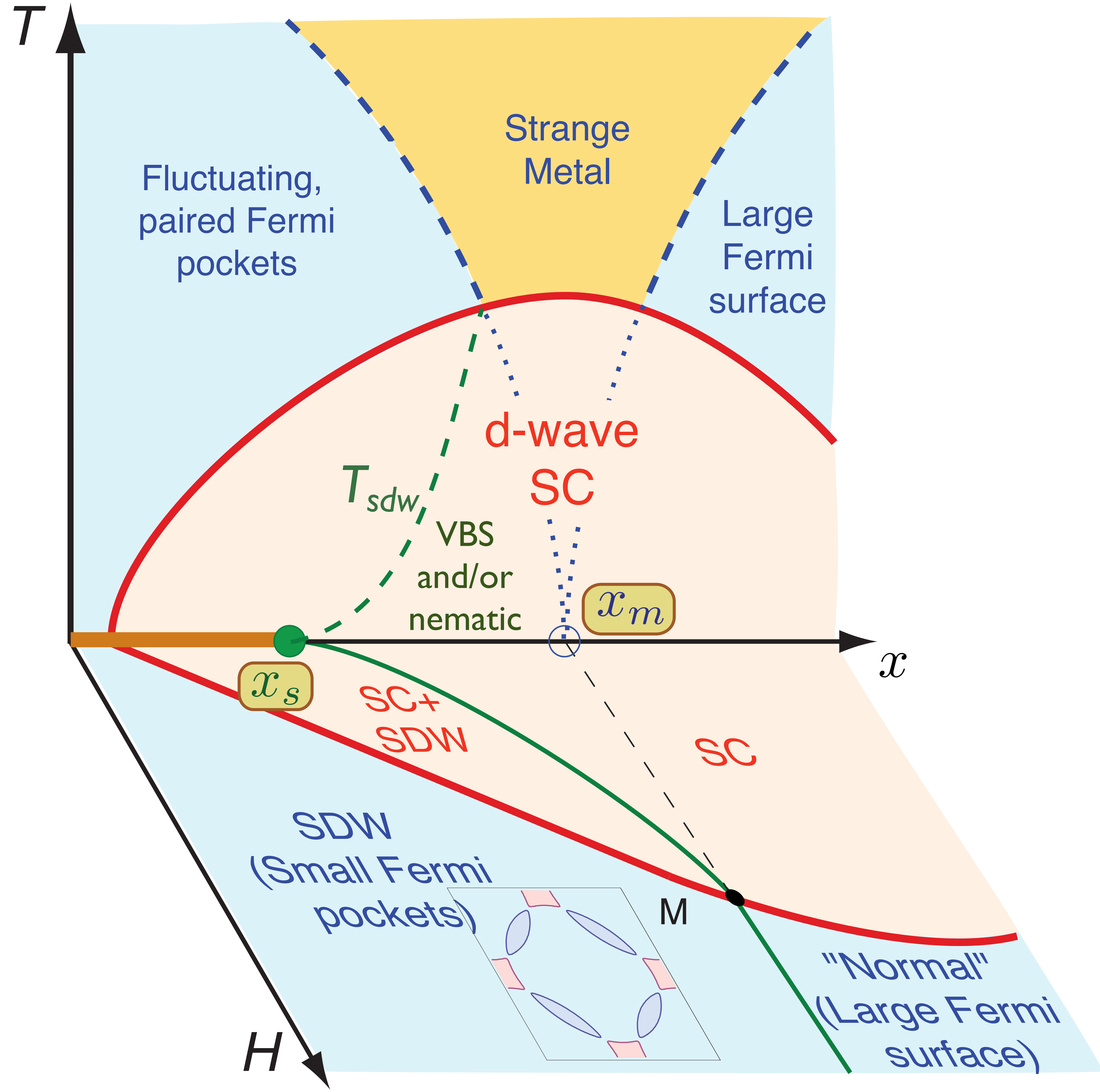
Sung-Sik Lee, arXiv:0905.4532

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

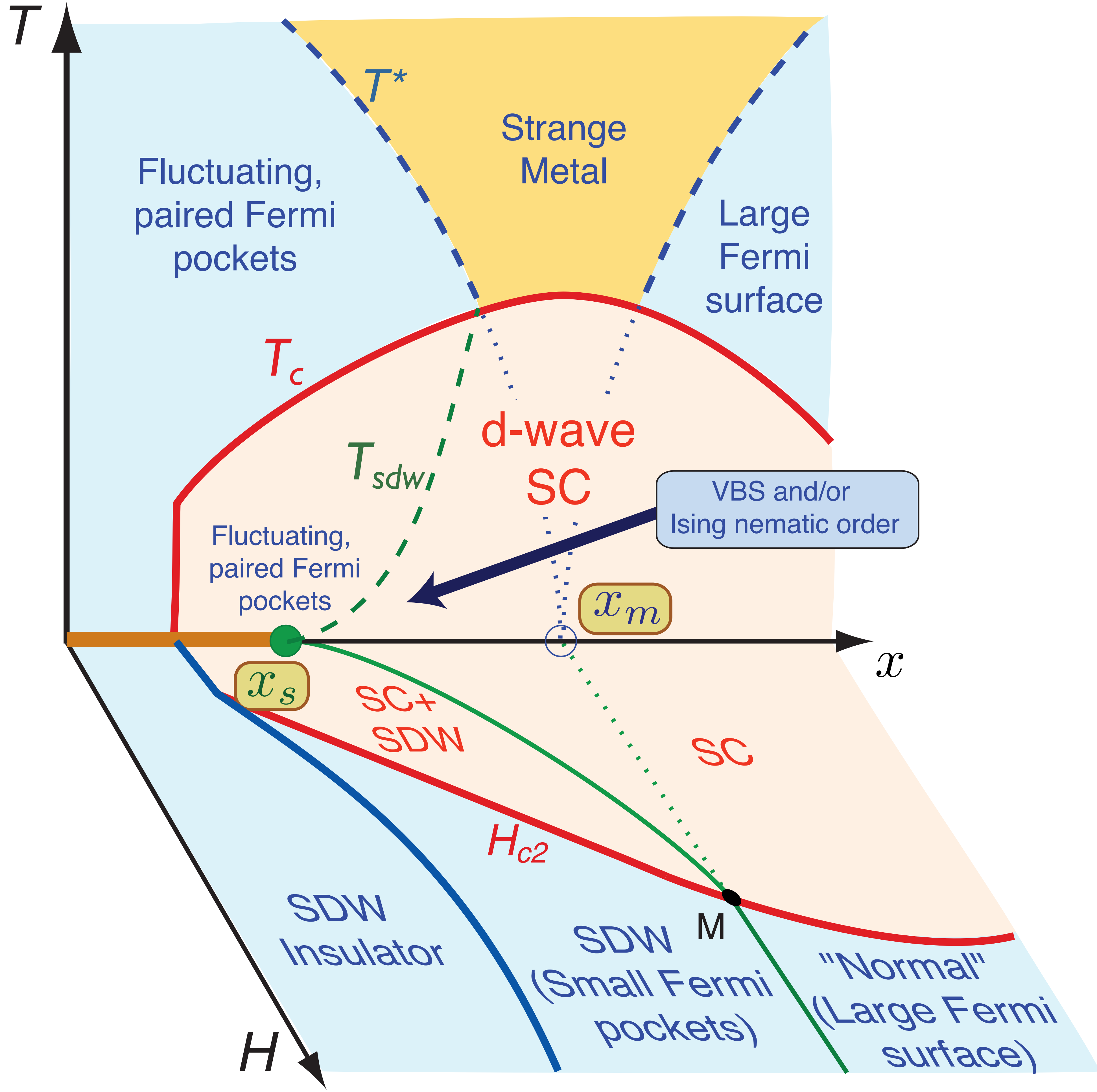


$$\text{Actual order} \sim \frac{1}{N^0}$$

A consistent analysis requires resummation of all planar graphs



Thursday, November 19, 2009


T
T*
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
T_c
T_{sdw}
d-wave
SC
VBS and/or
Ising nematic order
Fluctuating,
paired Fermi
pockets
x_m
x_s
x
SC+
SDW
SC
H_{c2}
M
SDW
Insulator
SDW
(Small Fermi
pockets)
"Normal"
(Large Fermi
surface)
H

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity

Conclusions

Theory for the onset of spin density wave order in metals is *strongly* coupled in two dimensions