De Sitter Lecture Series in Theoretical Physics 2009
University of Groningen

Quantum phase transitions: from antiferromagnets and superconductors to black holes

Talk online: sachdev.physics.harvard.edu

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Outline

1. Introduction to quantum phase transitions:
   quantum spin systems and relativistic field theories

2. Quantum phase transitions in *d*-wave
   superconductors and metals

3. The AdS/CFT correspondence:
   quantum criticality at strong coupling

4. The cuprate high temperature superconductors:
   competing orders and quantum criticality

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3. The AdS/CFT correspondence: quantum criticality at strong coupling

A. Quantum critical transport of CFTs: the collisionless-to-hydrodynamic crossover

B. Quantum matter at non-zero density: hydrodynamic thermoelectric transport

C. Quantum matter at non-zero density: Fermi surfaces, Green's functions and quantum oscillations

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The Superfluid-Insulator transition

Boson Hubbard model

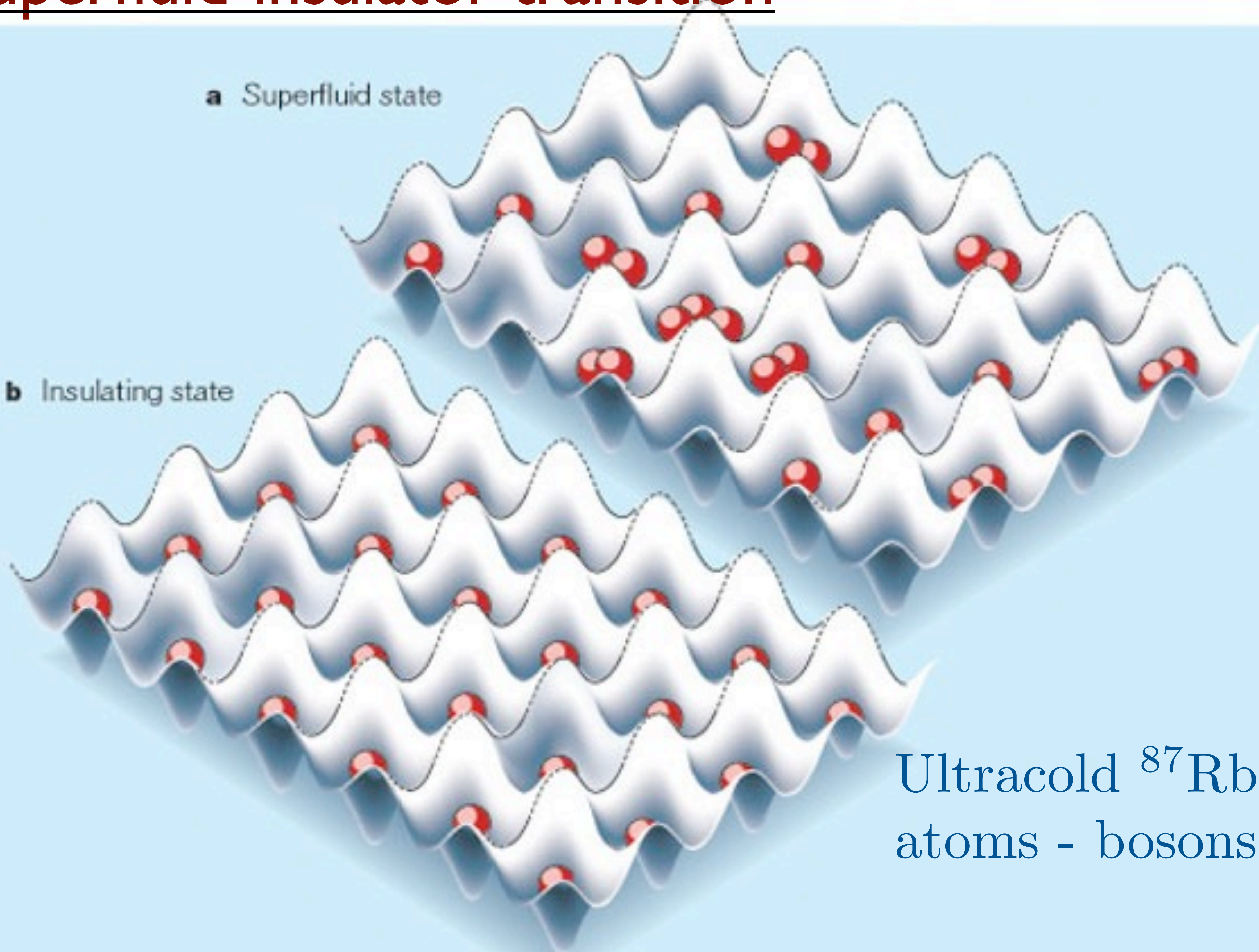
Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

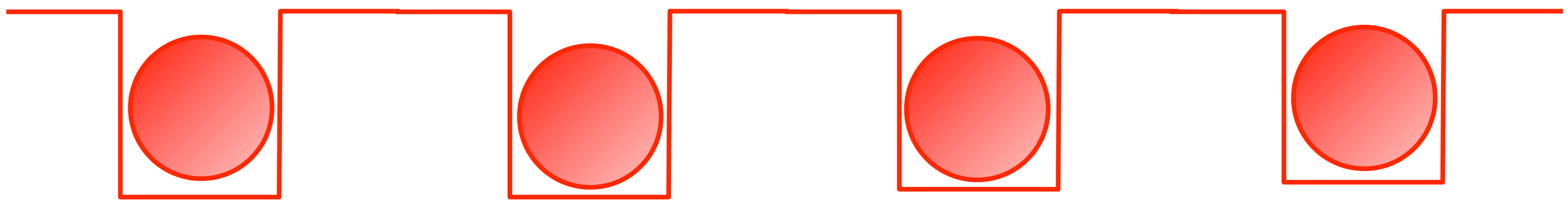
M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

Superfluid-insulator transition



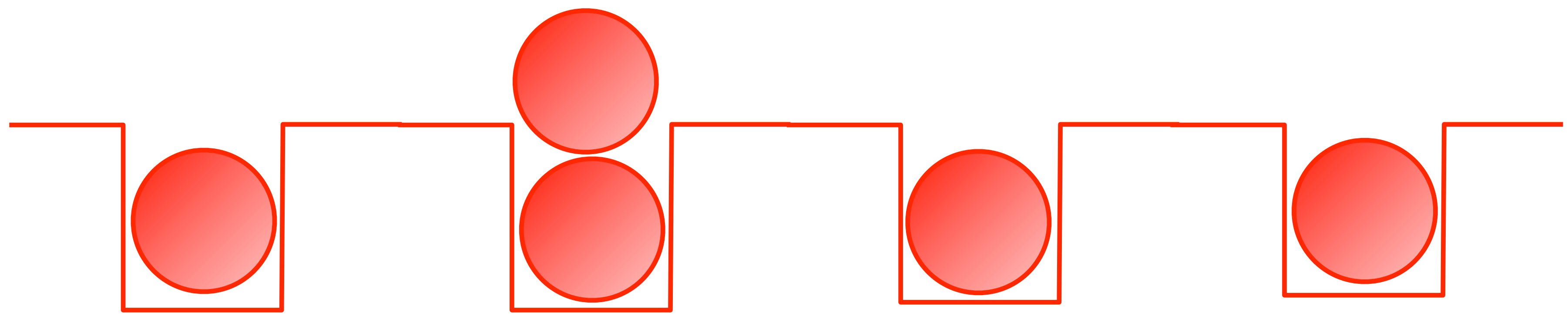
Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



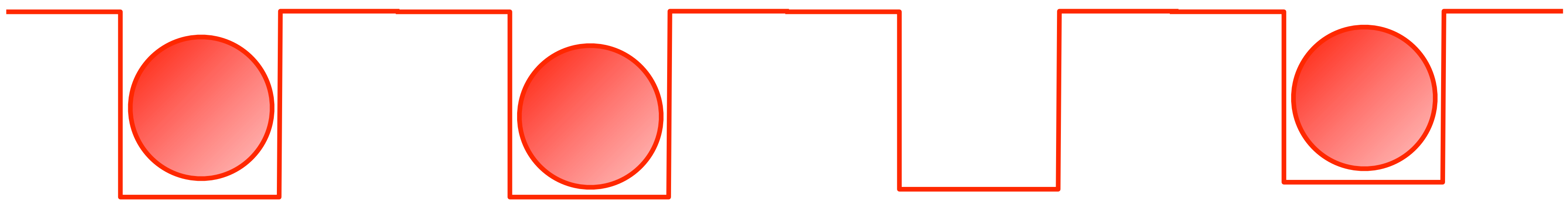
Insulator (the vacuum) at large U

Excitations:



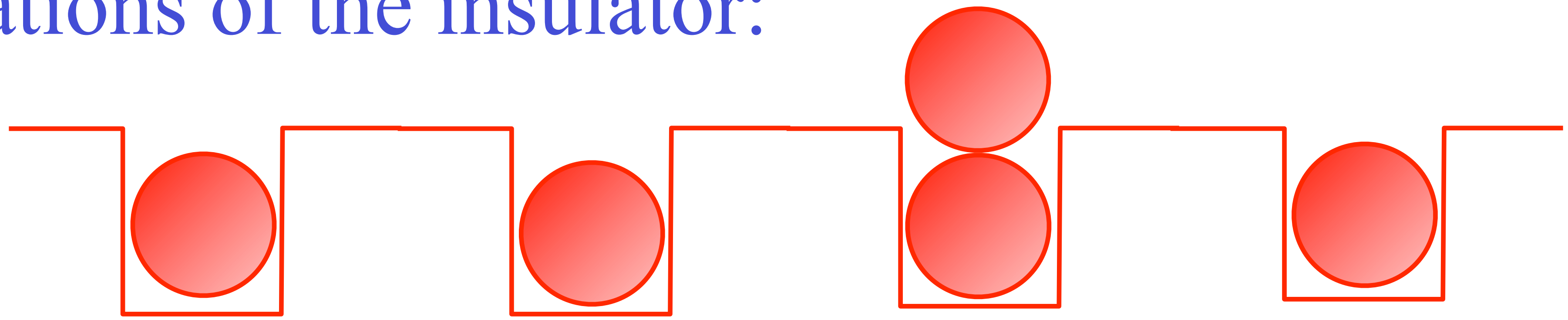
Particles $\sim \psi^\dagger$

Excitations:



Holes ~ ψ

Excitations of the insulator:



Particles ~ $\psi^\dagger$

Holes ~ ψ

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

.

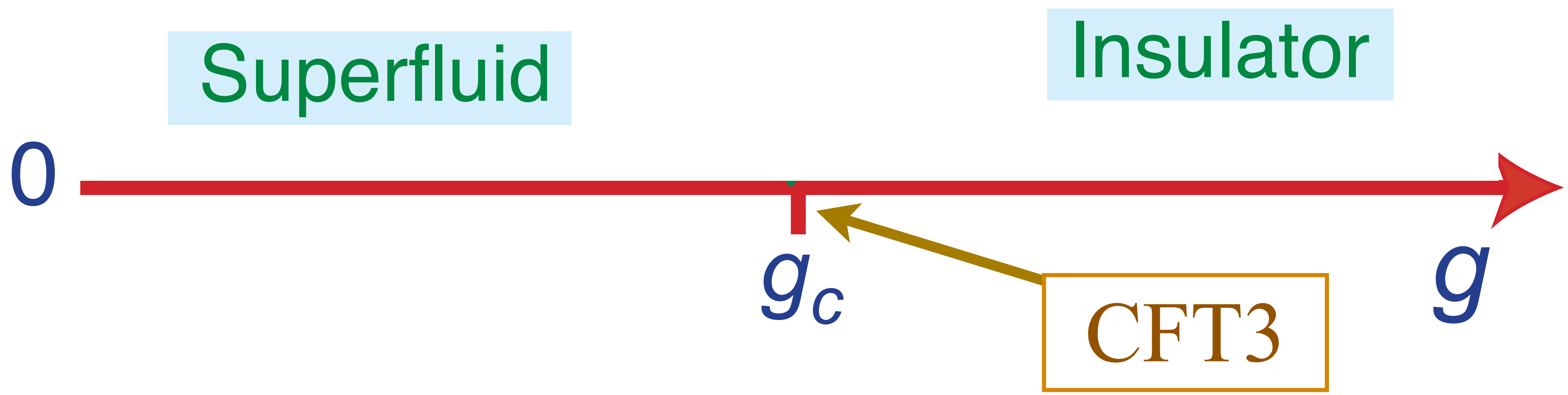
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

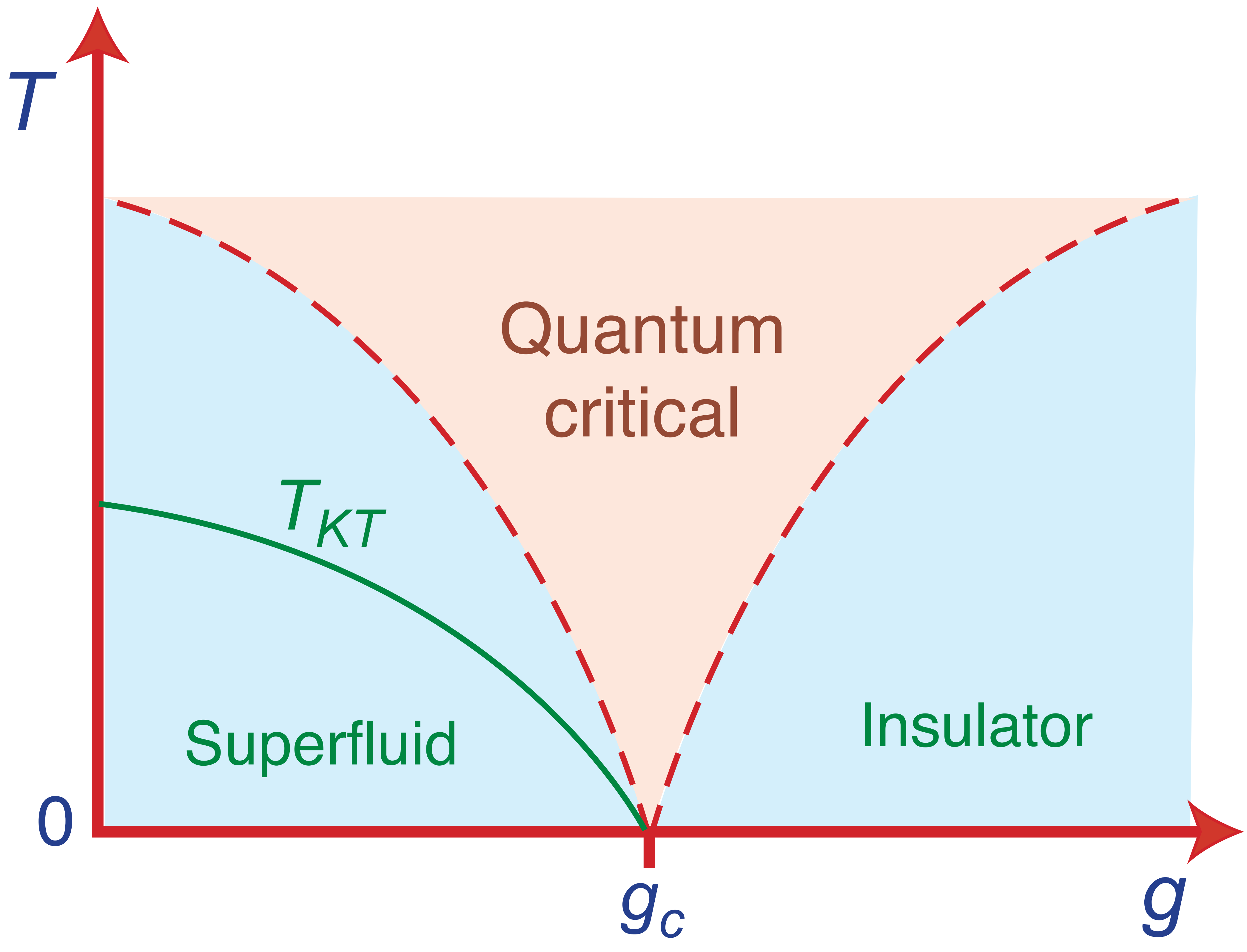
Insulator $\Leftrightarrow$ $\langle \psi \rangle = 0$

Superfluid $\Leftrightarrow$ $\langle \psi \rangle \neq 0$

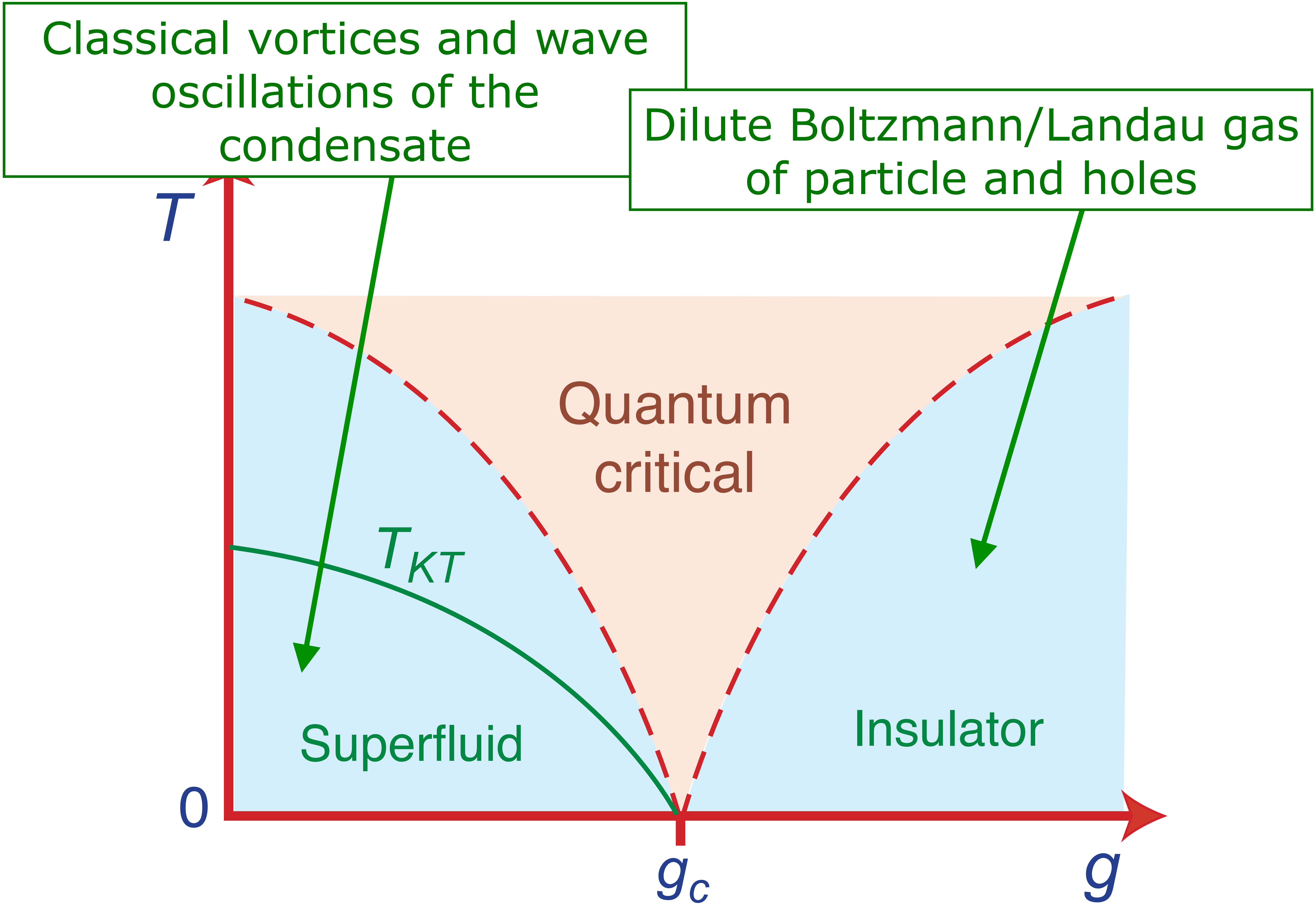
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

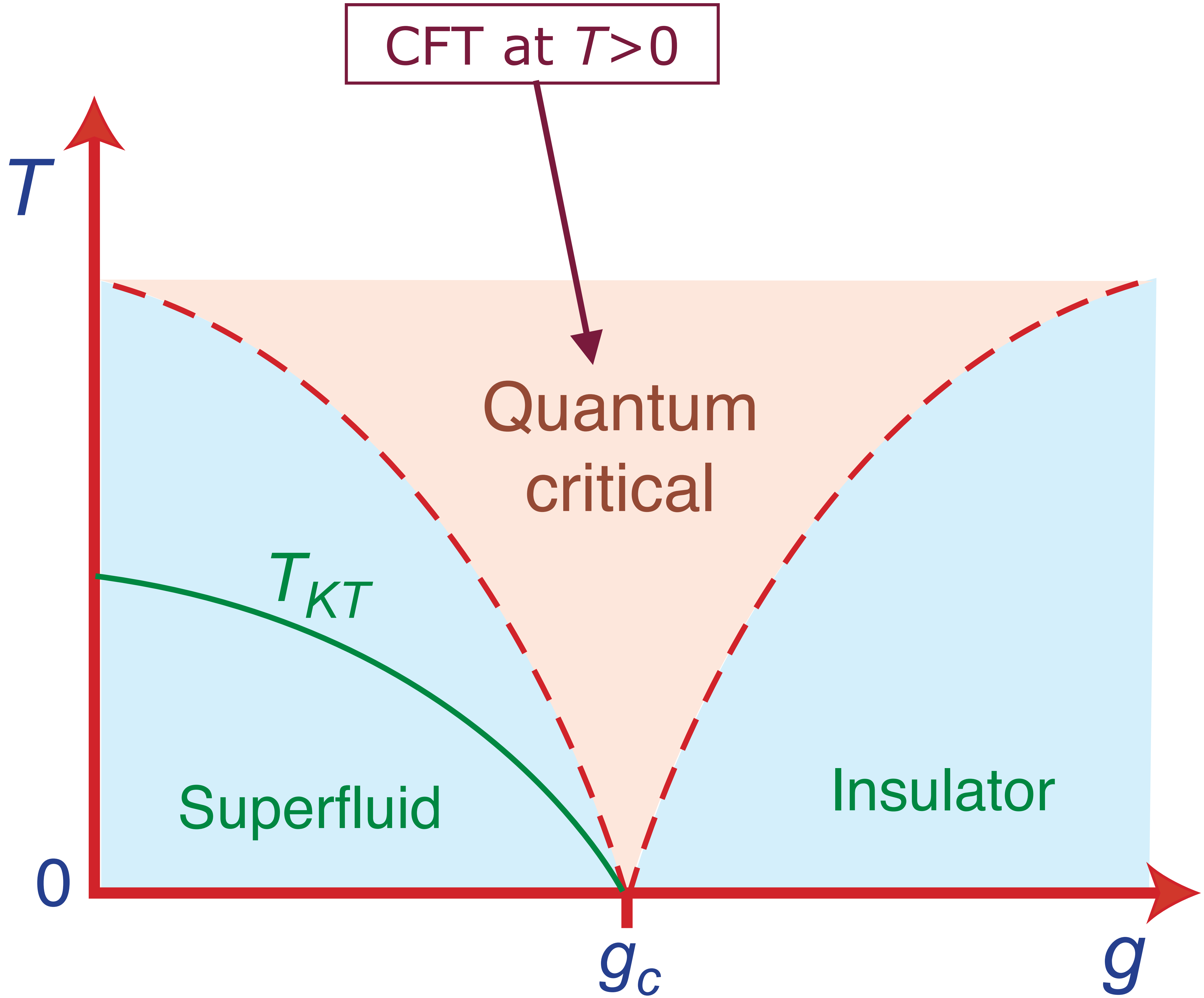
$$\langle \psi \rangle \neq 0 \qquad\qquad \langle \psi \rangle = 0$$





T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


Classical vortices and wave oscillations of the condensate
Dilute Boltzmann/Landau gas of particle and holes
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\begin{aligned}\sigma_{\text{Superconductor}}(T \to 0) &= \infty \\ \sigma_{\text{Insulator}}(T \to 0) &= 0 \\ \sigma_{\text{Quantum critical point}}(T \to 0) &\approx \frac{4e^2}{h}\end{aligned}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

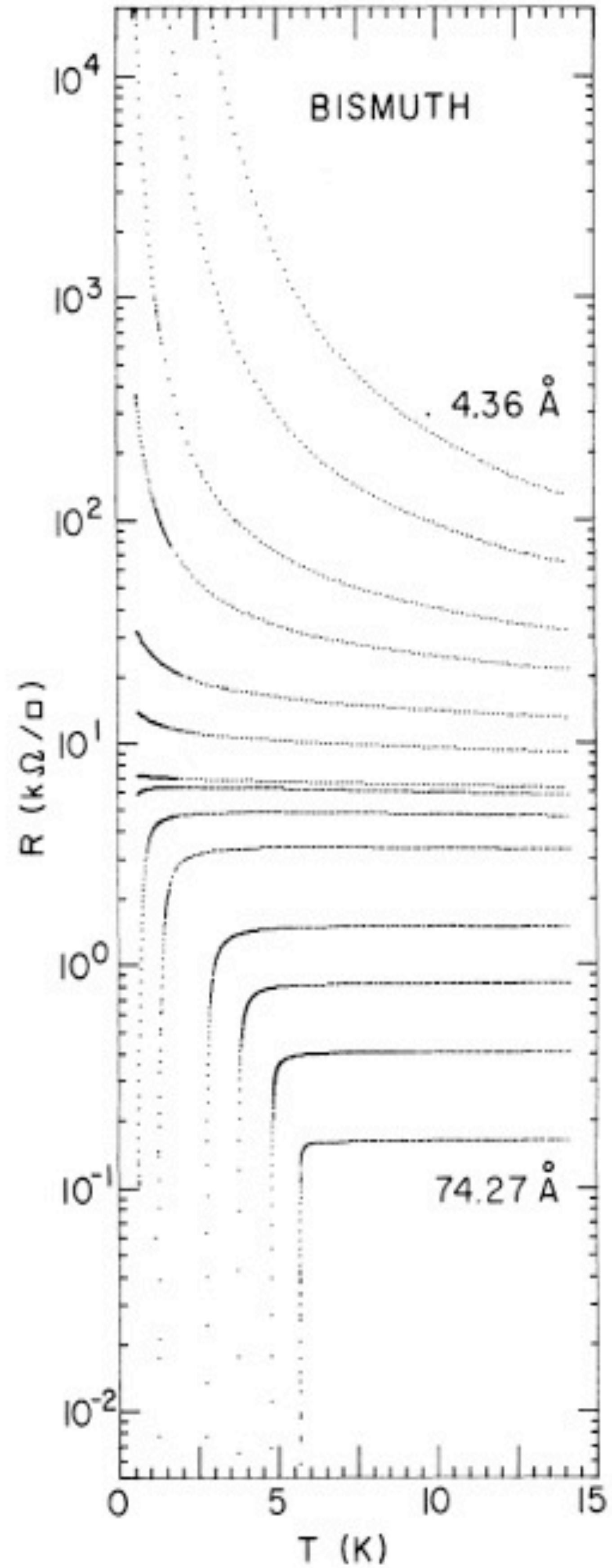


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Quantum critical transport

Quantum *"perfect fluid"* with shortest possible relaxation time, τ_R

$$\boxed{\tau_R \gtrsim \frac{\hbar}{k_B T}}$$

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Electrical conductivity

$$\boxed{\sigma = \frac{4e^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]}$$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

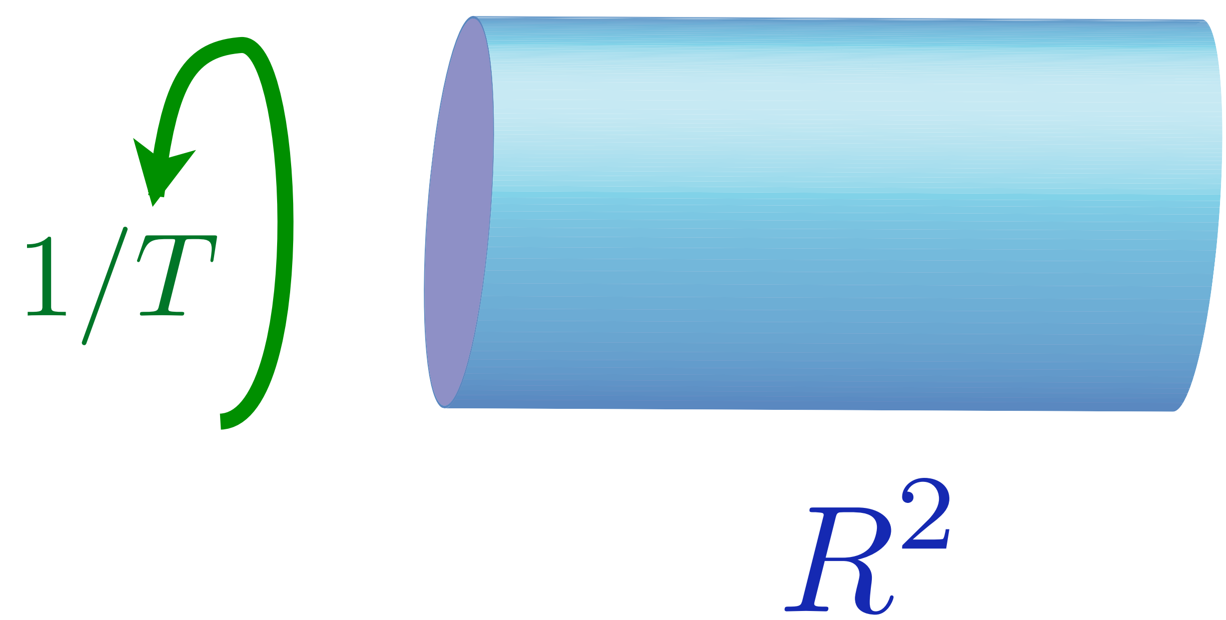
$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times \left[\text{Universal constant } \mathcal{O}(1)\ \right]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Euclidean field theory:

Compute current correlations on
$R^2 \times S^1$ with circumference $1/T$



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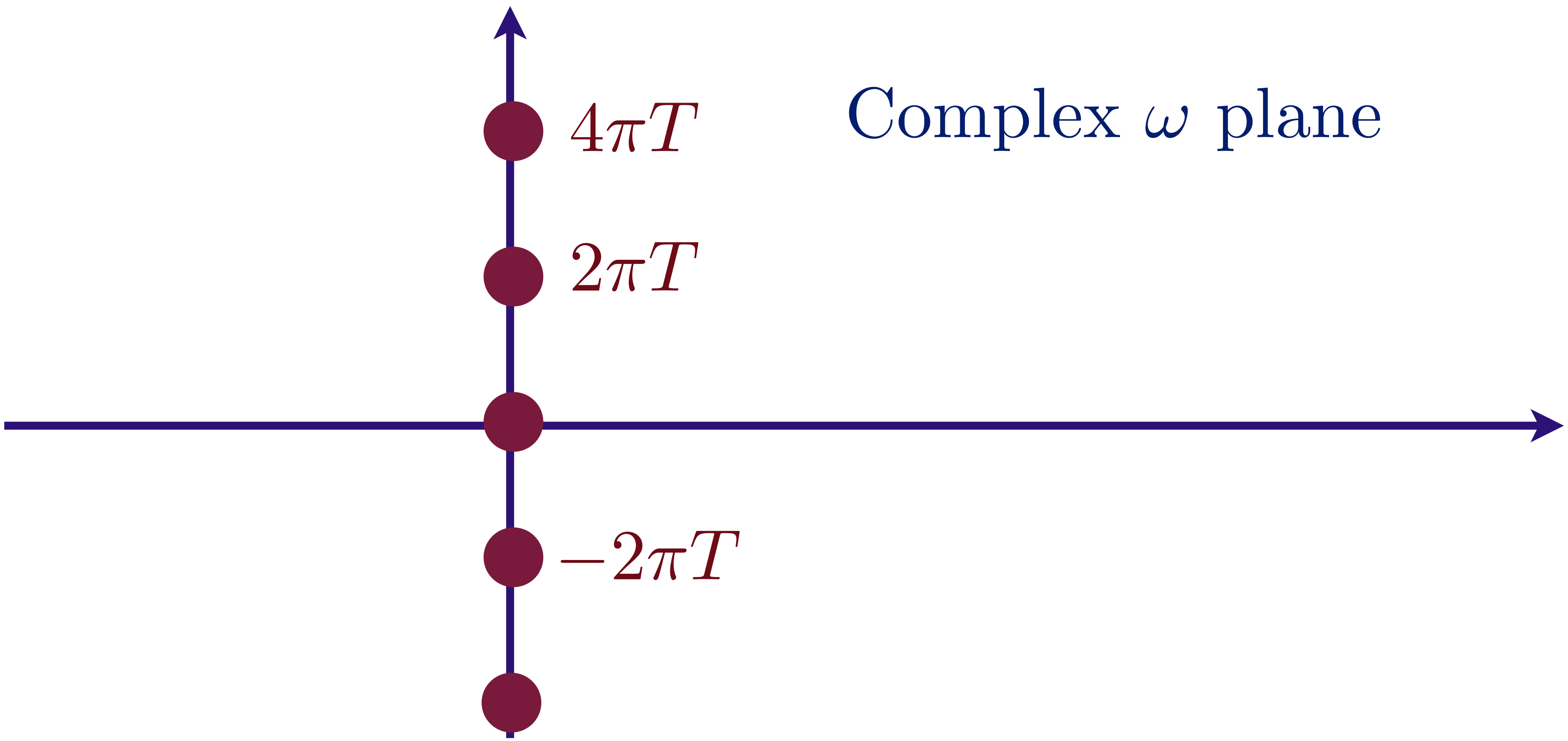

Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer) or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Quantum critical transport

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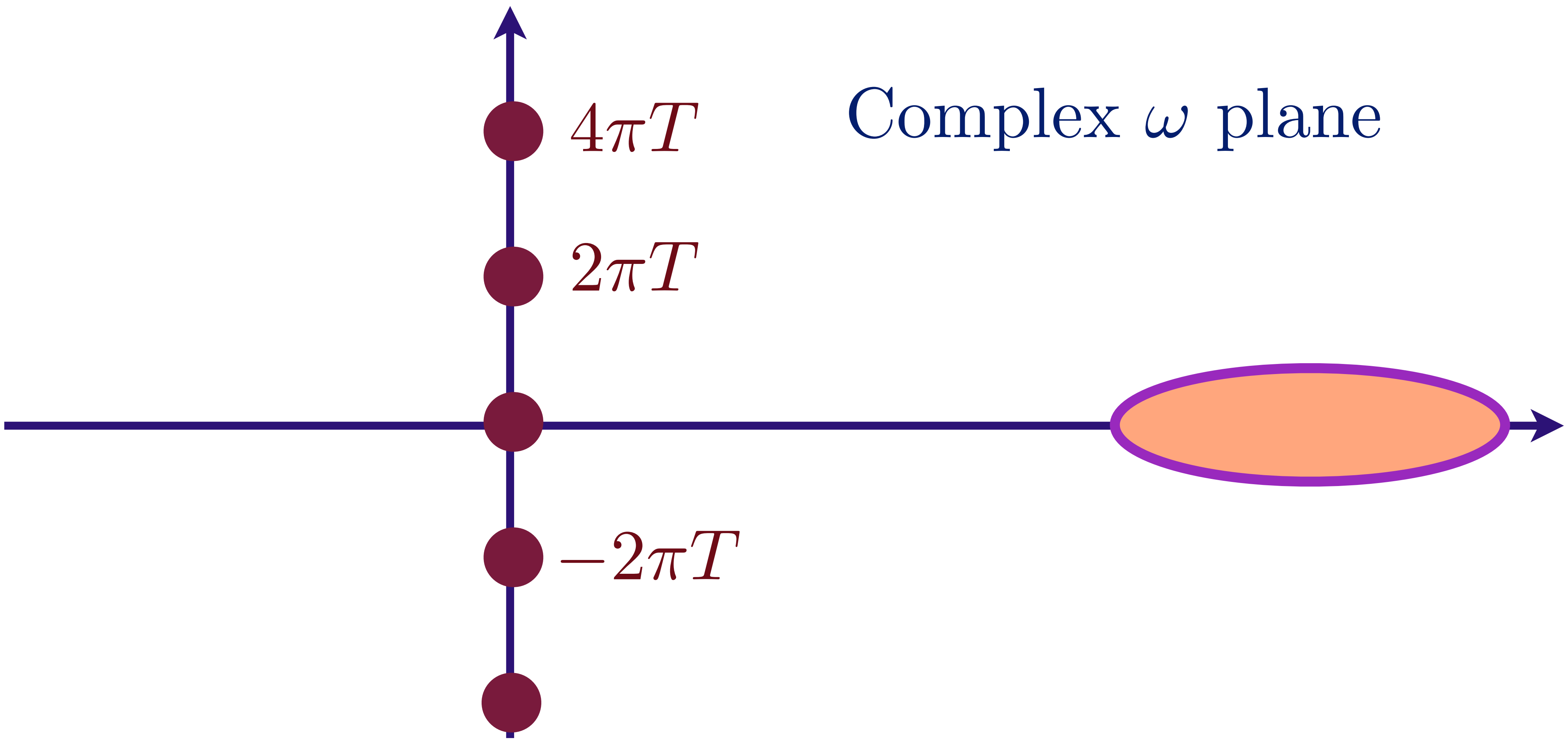

Direct $1/N$ or ϵ expansions for correlators at the Euclidean frequencies $\omega_n = 2\pi n T i$ (n integer) or in the conformal "collisionless" regime, $\hbar\omega \gg k_B T$.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k,\omega)$

For *all* CFT2s, at $\hbar\omega \gg k_B T$

$$\chi(k,\omega) \quad = \quad \frac{4e^2}{h} K \frac{vk^2}{v^2k^2 - \omega^2} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} \frac{Kv}{-i\omega}$$

where K is a universal number characterizing the CFT2 (the level number), and v is the velocity of "light".

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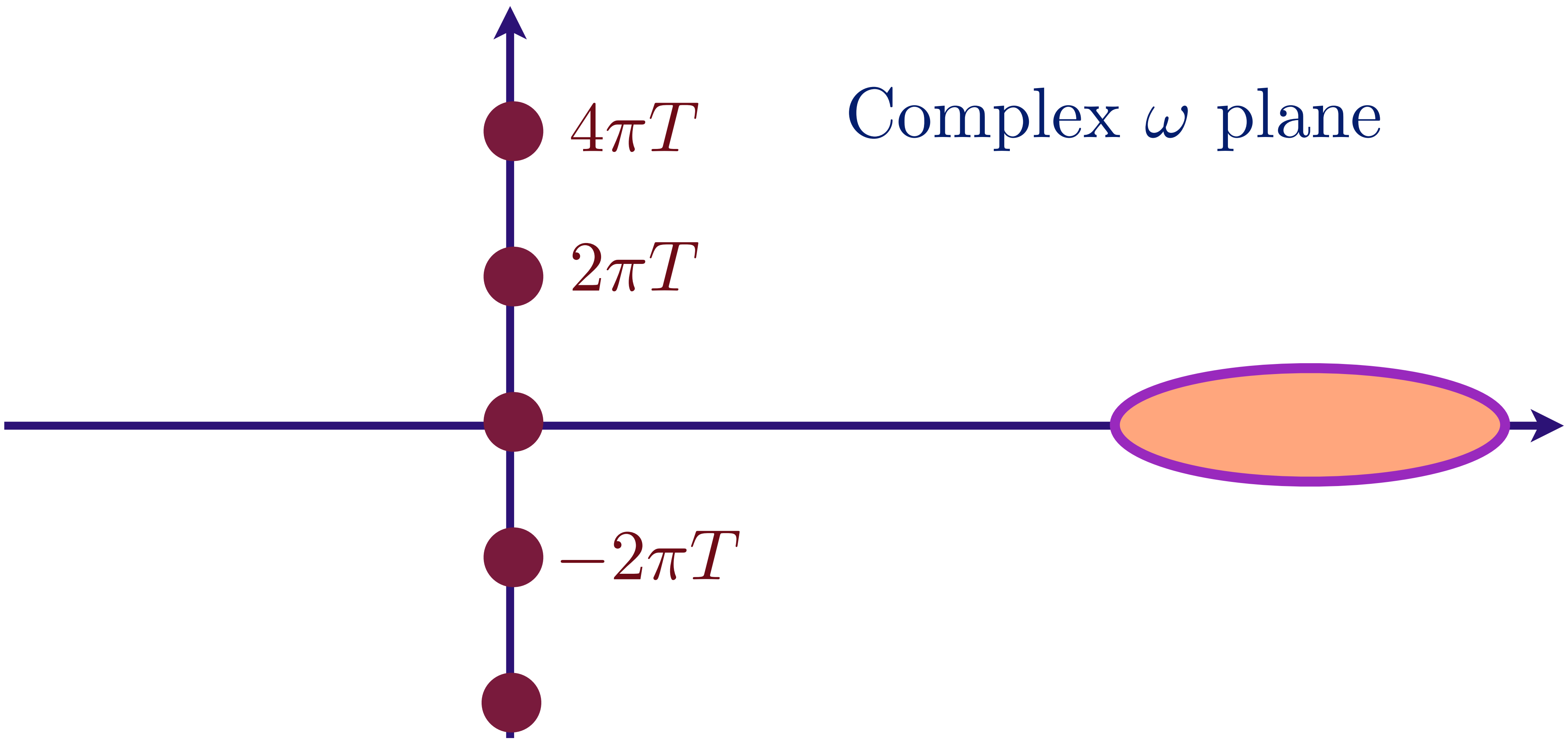

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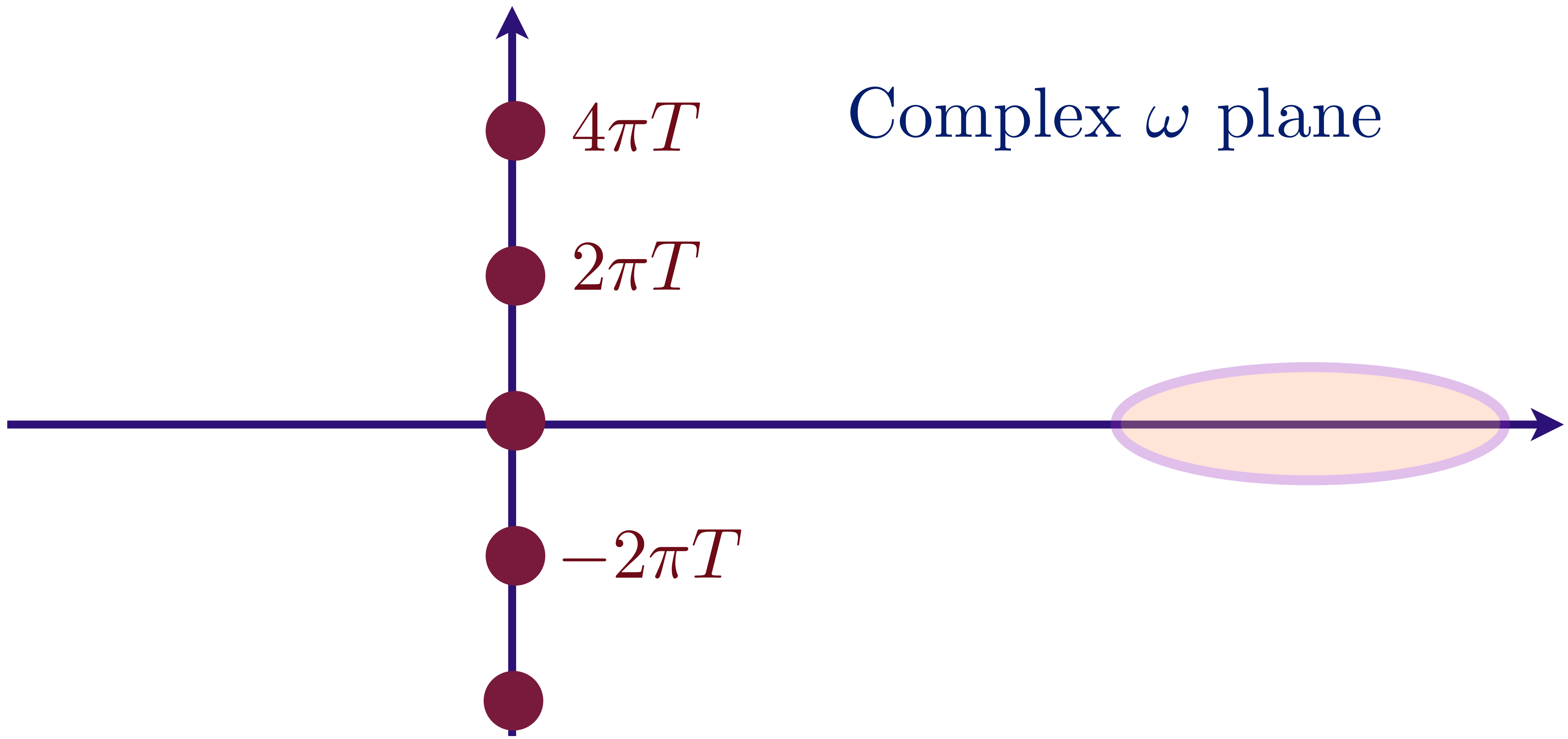

Strong coupling problem:
Correlators at $\hbar\omega \ll k_B T$, along the real axis, in the collision-dominated hydrodynamic regime.

Quantum critical transport

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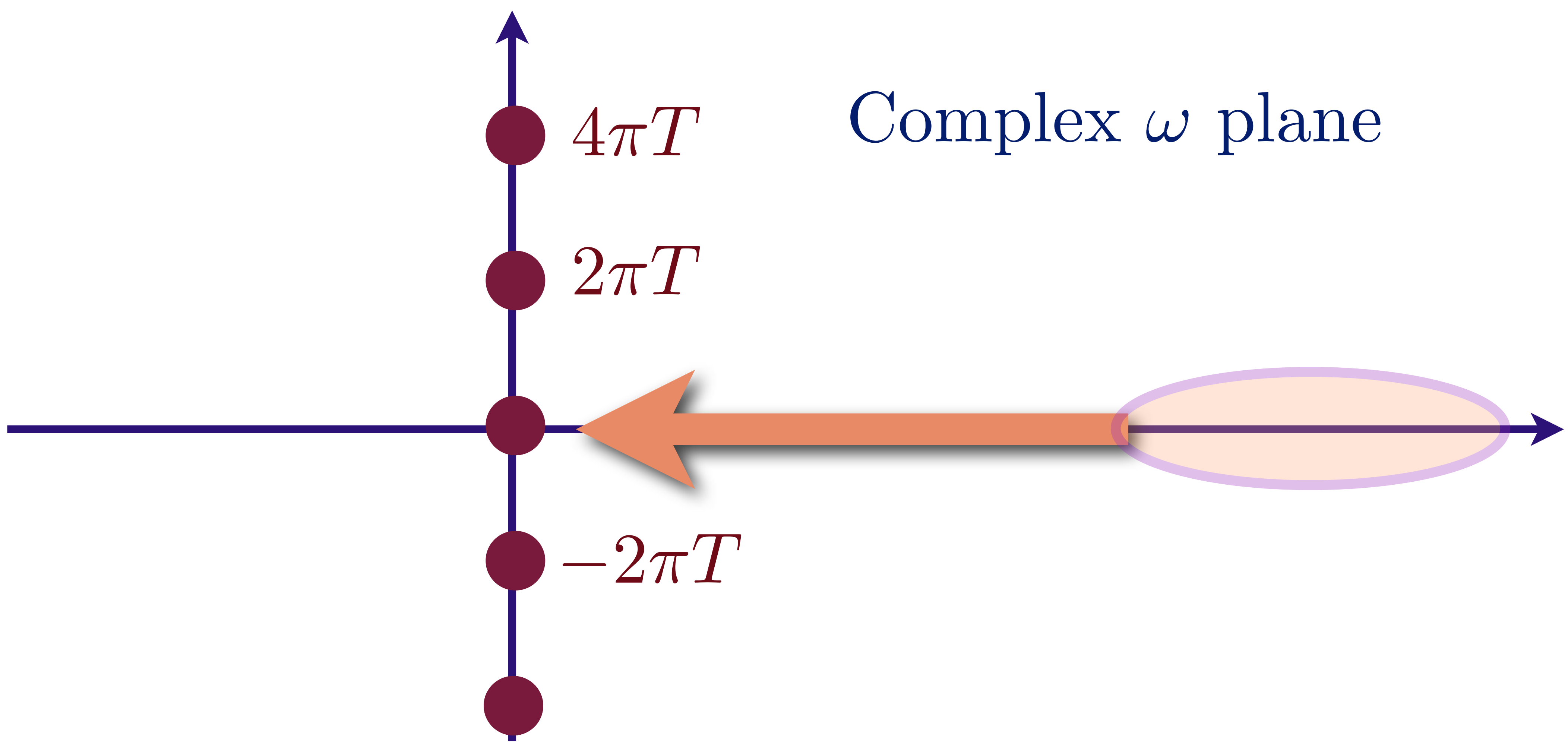

Strong coupling problem:
Correlators at $\hbar\omega \ll k_B T$, along the real axis, in the collision-dominated hydrodynamic regime.

CFT correlator of $U(1)$ current J_μ in 1+1 dimensions

Charge density correlation at $T = 0$:

$$\langle J_R(x,\tau) J_R(0) \rangle \sim \frac{1}{(\tau + ix)^2}$$

$$\langle J_t(k,\omega) J_t(-k,-\omega) \rangle \sim \frac{k^2}{k^2 - \omega^2}$$

CFT correlator of $\mathrm{U}(1)$ current J_μ in 1+1 dimensions

Charge density correlation at $T \geq 0$:

$$\left\langle J_R(x,\tau) J_R(0) \right\rangle \sim \frac{\pi^2 T^2}{\sin^2\left(\pi T(\tau + ix)\right)}$$

$$\left\langle J_t(k, i\omega_n) J_t(-k, -i\omega_n) \right\rangle \sim \frac{k^2}{k^2 + \omega_n^2}$$

Conformal mapping of plane to cylinder with circumference *1/T*

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Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT2s, at all $\hbar\omega / k_B T$

$$\chi(k, \omega) = \frac{4e^2}{h} K \frac{v k^2}{v^2 k^2 - \omega^2} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} \frac{K v}{-i\omega}$$

where K is a universal number characterizing the CFT2 (the level number), and v is the velocity of "light".
This follows from the conformal mapping of the plane to the cylinder, which relates correlators at $T = 0$ to those at $T > 0$.

No hydrodynamics in CFT2s.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k\to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

However, for *all* CFT3s, at $\underline{\hbar\omega \ll k_B T}$, we have the Einstein relation

$$\chi(k,\omega) = 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T > 0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\[2ex] \dfrac{4e^2}{h} \Theta_1 \Theta_2 \equiv \sigma_Q & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

The AdS/CFT correspondence

Field theories in D spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

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where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $D+1$ dimensions.

At the RG fixed point, $\beta(g) = 0$, the D dimensional field theory is invariant under the scale transformation

$$x^\mu \to x^\mu / b \quad , \quad u \to b\, u$$

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This is an invariance of the *metric* of the theory in $D+1$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 dx^\mu dx_\mu + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $z = L^2/u$

$$ds^2 = L^2 \frac{dx^\mu dx_\mu + dz^2}{z^2}.$$

This is the space AdS$_{D+1}$, and L is the AdS radius.

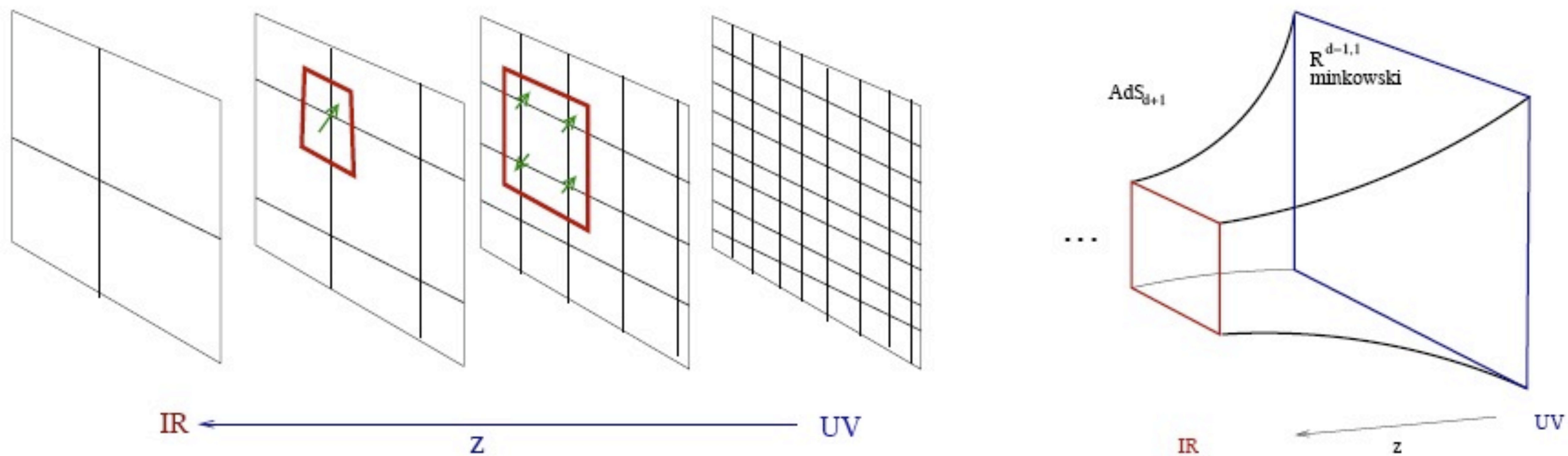

Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory. The left figure indicates a series of block spin transformations labelled by a parameter z. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

J. McGreevy, arXiv0909.0518

Bonus: AdS_{D+1} is a solution of Einstein's equations with a negative cosmological constant, and is a symmetric space; the full group of symmetries of the metric is $\mathrm{SO}(D+1,1)$ (in Euclidean signature)

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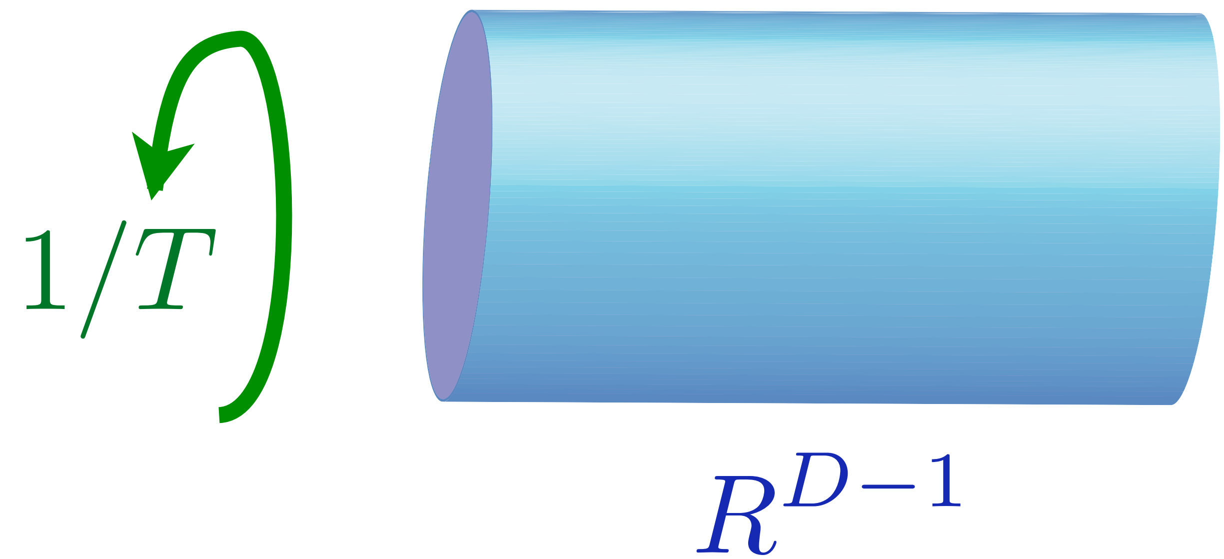
$\mathrm{SO}(D+1,1)$ is the group of conformal transformations in D dimensions, and relativistic field theories at the RG fixed point are conformally invariant.

At $T > 0$, the Euclidean field theory is on the cylinder $R^{D-1} \times S^1$, where the time co-ordinate is periodic under $\tau \rightarrow \tau + 1/T$.



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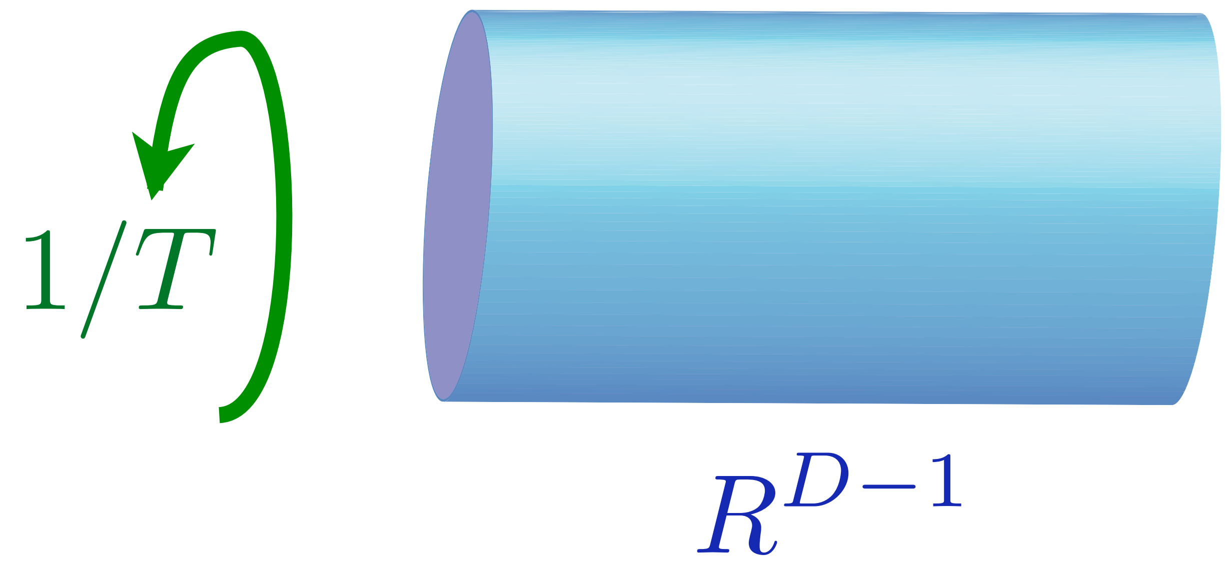


Solving Einstein's equations with a negative cosmological constant we have the solution

$$ds^2 = \frac{L^2}{z^2}\left(f(z)d\tau^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right) \quad ; \quad f(z) = 1 - \left(\frac{z}{z_H}\right)^D$$

This is a AdS-Schwarzschild black hole with a horizon at $z = z_H$. This space is periodic in τ with period $1/T$ for

$$T = \frac{d}{4\pi z_H}$$

SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- Has a single dimensionful coupling constant, e_0, which flows to a strong-coupling fixed point $e_0 = e_0^*$ in the infrared.
- The CFT3 describing this fixed point resembles "critical spin liquid" theories.
- This CFT3 is the low energy limit of string theory on an M2 brane. The AdS/CFT correspondence provides a dual description using 11-dimensional supergravity on $\mathrm{AdS}_4 \times S_7$.
- The CFT3 has a global SO(8) R symmetry, and correlators of the SO(8) charge density can be computed exactly in the large N limit, even at $T > 0$.

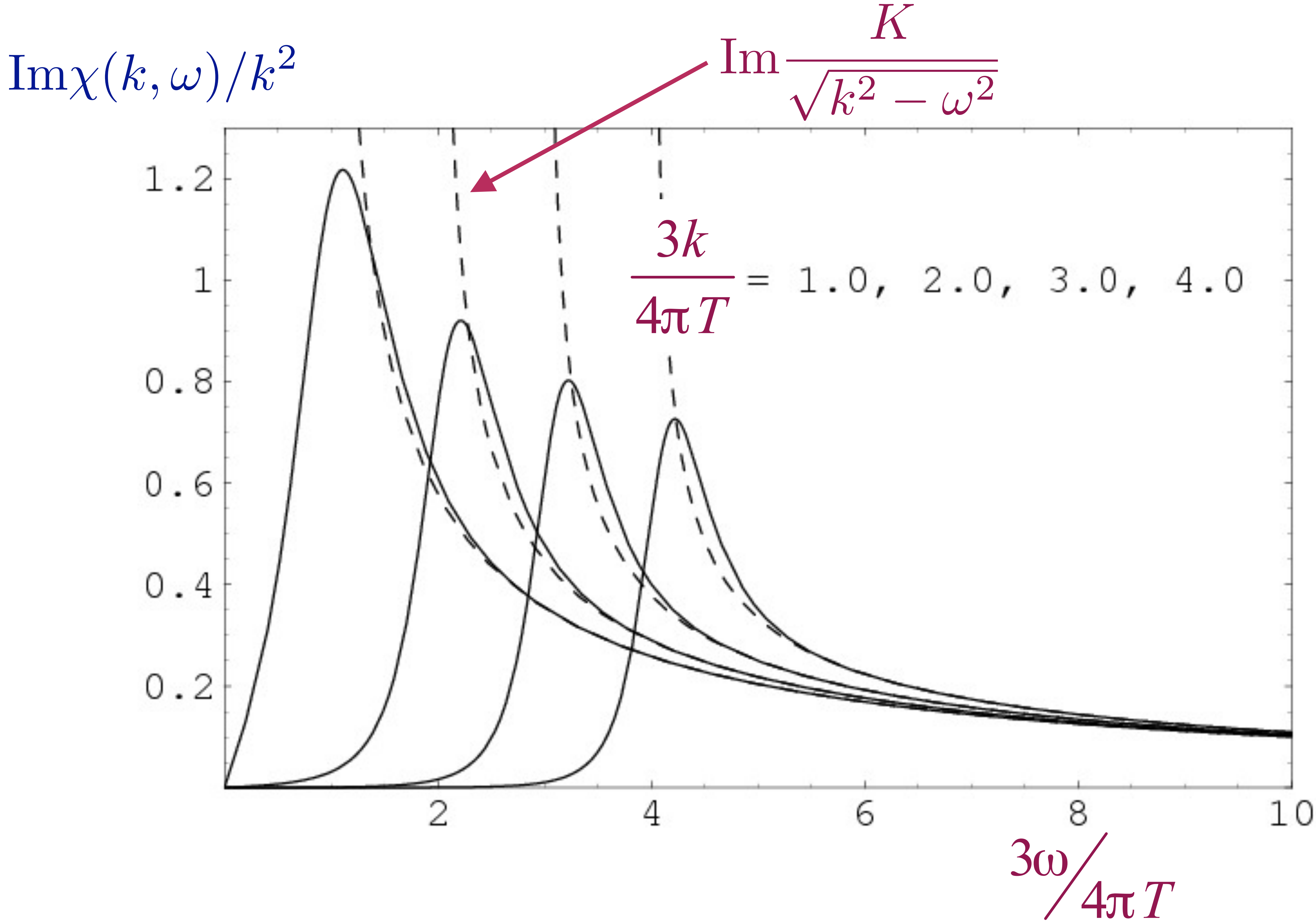
SU(N) SYM3 with $\mathcal{N} = 8$ supersymmetry

- The SO(8) charge correlators of the CFT3 are given by the usual AdS/CFT prescription applied to the following gauge theory on AdS4:

$$\mathcal{S} = -\frac{1}{4g_{4D}^2} \int d^4x \sqrt{-g} g^{MA} g^{NB} F^a_{MN} F^a_{AB}$$

  where $a = 1 \ldots 28$ labels the generators of SO(8). Note that in large N theory, this looks like 28 copies of an Abelian gauge theory.

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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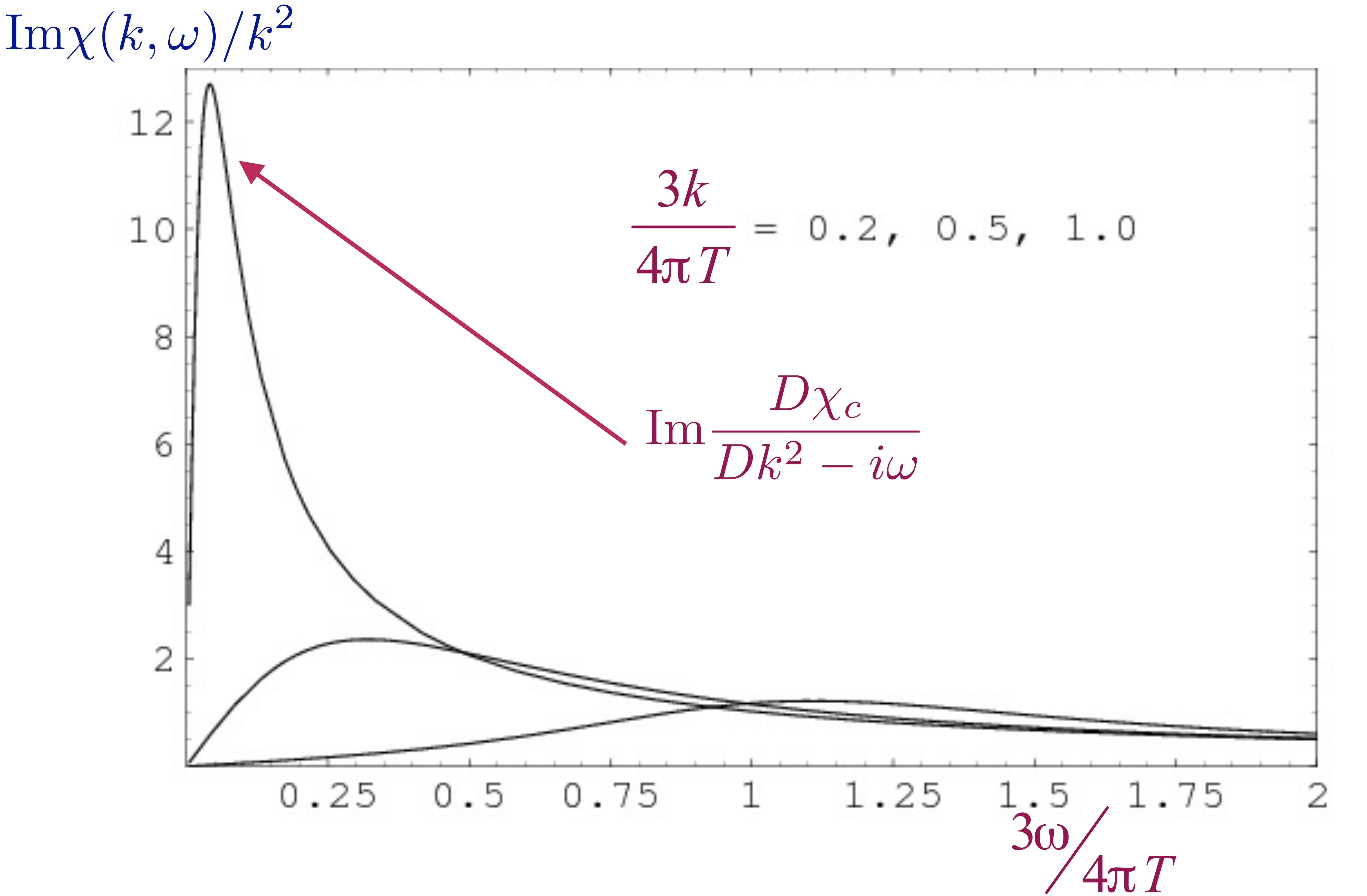

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Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(hv)^2}\Theta_1$$

$$D = \frac{hv^2}{k_B T}\Theta_2$$

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h}K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h}\Theta_1\Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$\boxed{\begin{aligned} K &= \frac{\sqrt{2}N^{3/2}}{3} \\ \Theta_1 &= \frac{8\pi^2\sqrt{2}N^{3/2}}{9} \\ \Theta_2 &= \frac{3}{8\pi^2} \end{aligned}}$$

C. Herzog, JHEP **0212**, 026 (2002)

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Electromagnetic self-duality

- Unexpected result, $K = \Theta_1 \Theta_2$.
- This is traced to a *four*-dimensional electromagnetic self-duality of the theory on AdS_4. In the large N limit, the SO(8) currents decouple into 28 U(1) currents with a Maxwell action for the U(1) gauge fields on AdS_4.
- This special property is not expected for generic CFT3s.

Conformal field theory:

Wilson-Fisher fixed point

Superfluid

$\langle \psi \rangle \neq 0$

$\sigma = \infty$

Insulator

$\langle \psi \rangle = 0$

$\sigma = 0$

Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981)

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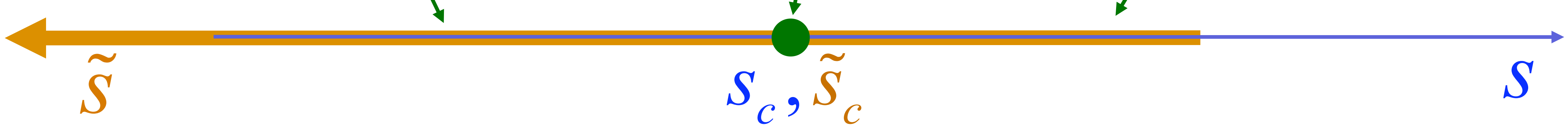

Using the boson quasiparticle excitations of the insulator $\sim \psi$

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

<u>is dual to</u>

Using the vortex quasiparticle excitations of the superfluid $\sim \varphi$

$$\mathcal{S}_{\rm dual} = \int d^3x \left[|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + \frac{\tilde{u}}{2}|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981)

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Electromagnetic self-duality

- Unexpected result, $K = \Theta_1 \Theta_2$.

- This is traced to a *four*-dimensional electromagnetic self-duality of the theory on AdS_4. In the large N limit, the SO(8) currents decouple into 28 U(1) currents with a Maxwell action for the U(1) gauge fields on AdS_4.

- This special property is not expected for generic CFT3s.

- Although there is no boson-vortex *self*-duality at the Wilson-Fisher fixed point, the applicability of AdS/CFT suggests that the conductivity may be close to its self-dual value, $\sigma \approx 4e^2/h$.

Outline

3. The AdS/CFT correspondence:
quantum criticality at strong coupling

A. Quantum critical transport of CFTs: the collisionless-to-hydrodynamic crossover

B. Quantum matter at non-zero density: hydrodynamic thermoelectric transport

C. Quantum matter at non-zero density: Fermi surfaces, Green's functions and quantum oscillations

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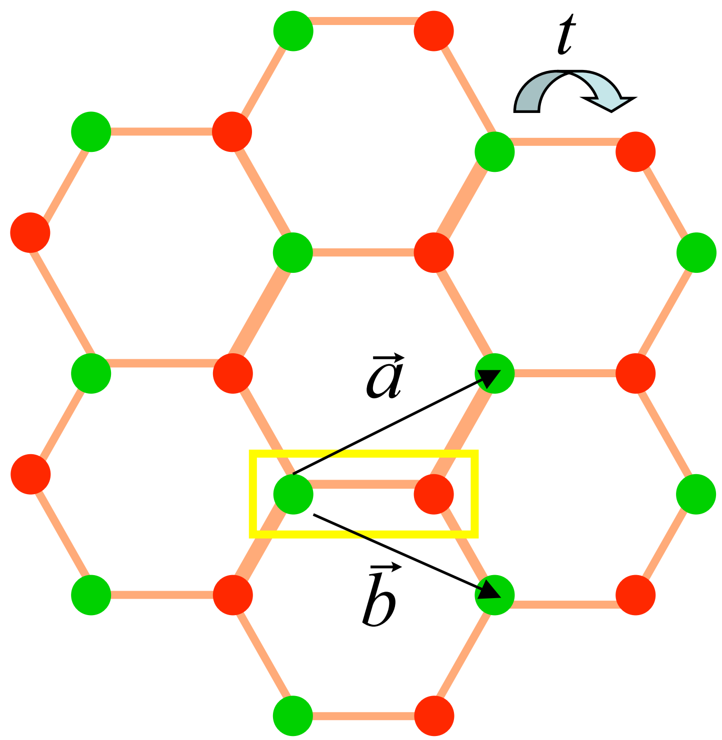
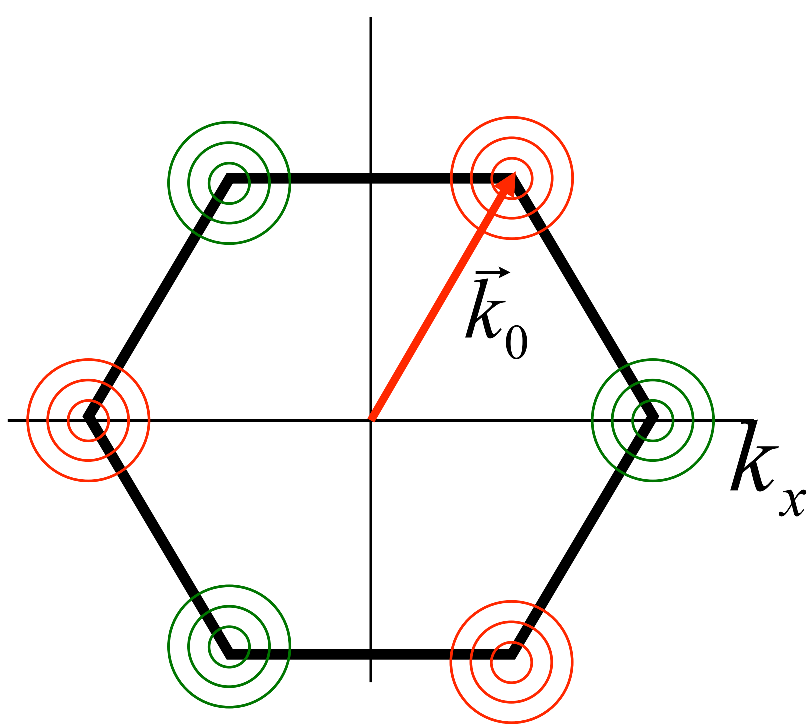
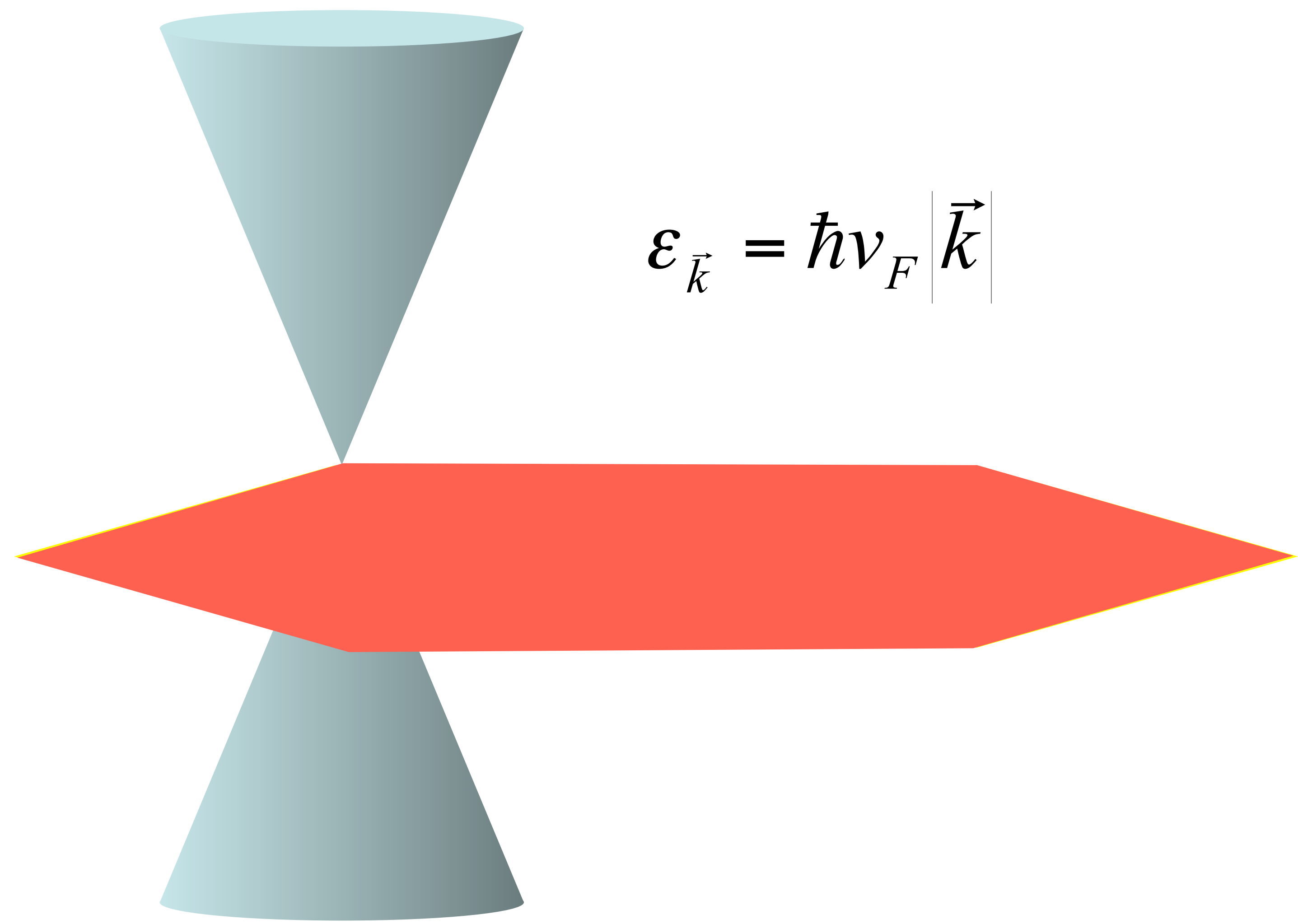
Graphene



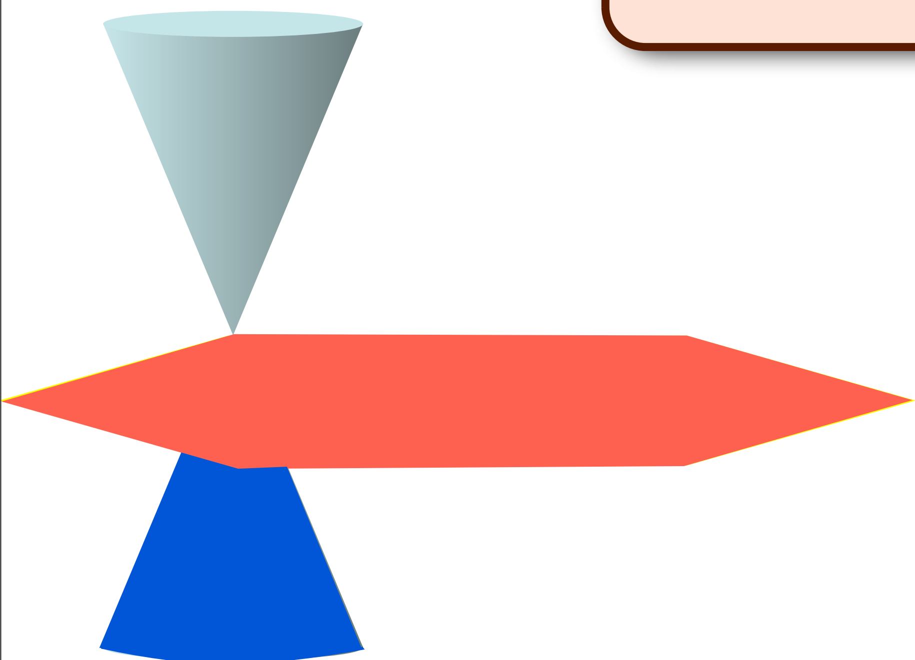




$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

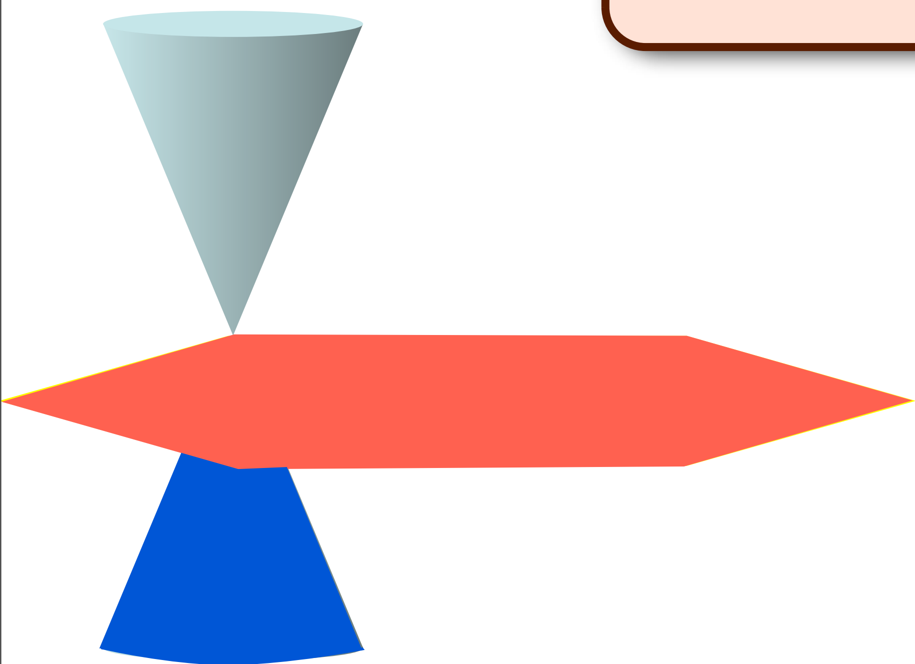
Conical Dirac dispersion

Conformal field theory
in 2+1 dimensions at $T = 0$



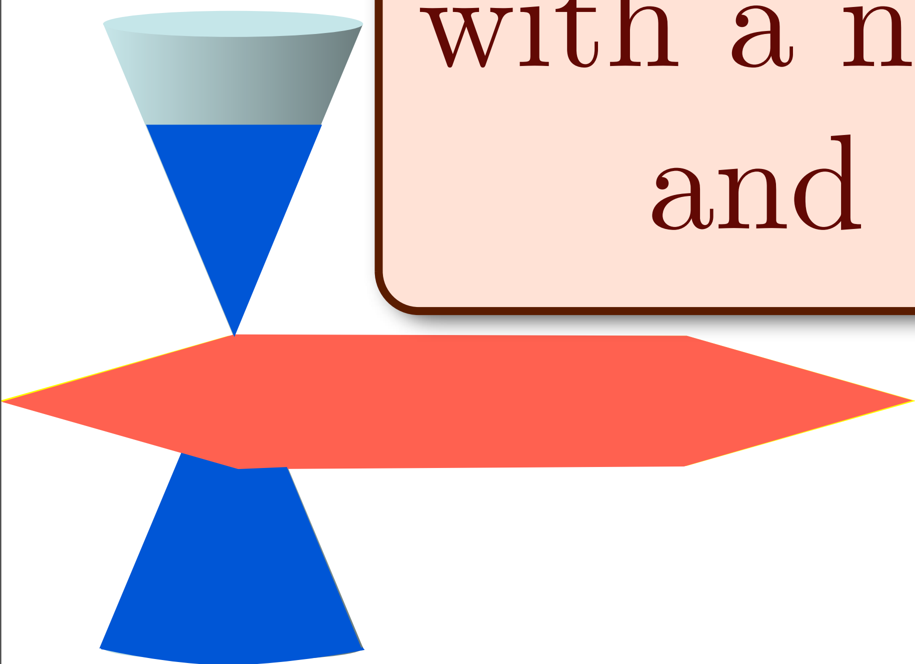
Einstein gravity
on AdS_4

Conformal field theory
in 2+1 dimensions at $T > 0$

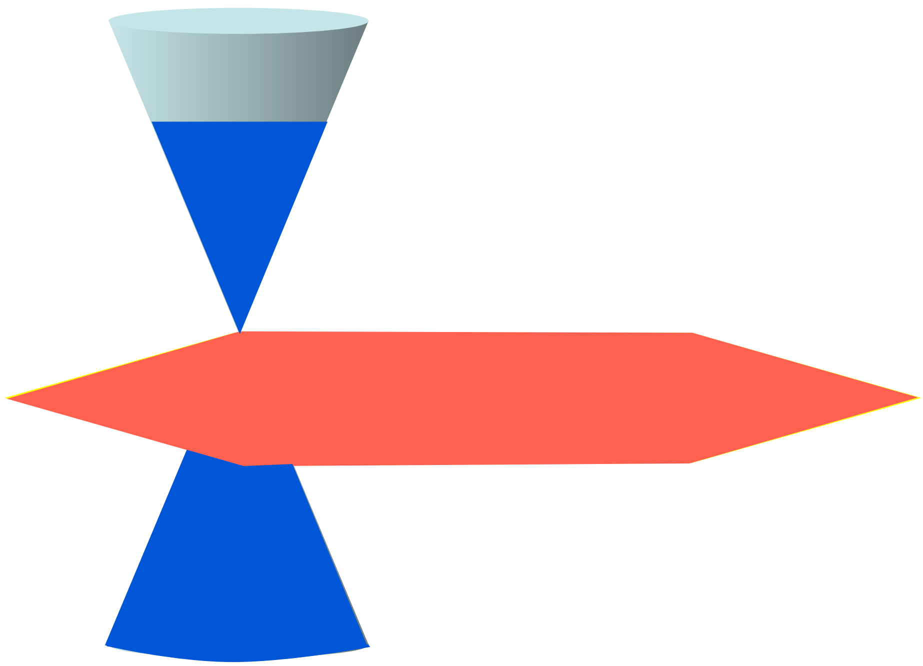


Einstein gravity on AdS_4
with a Schwarzschild
black hole

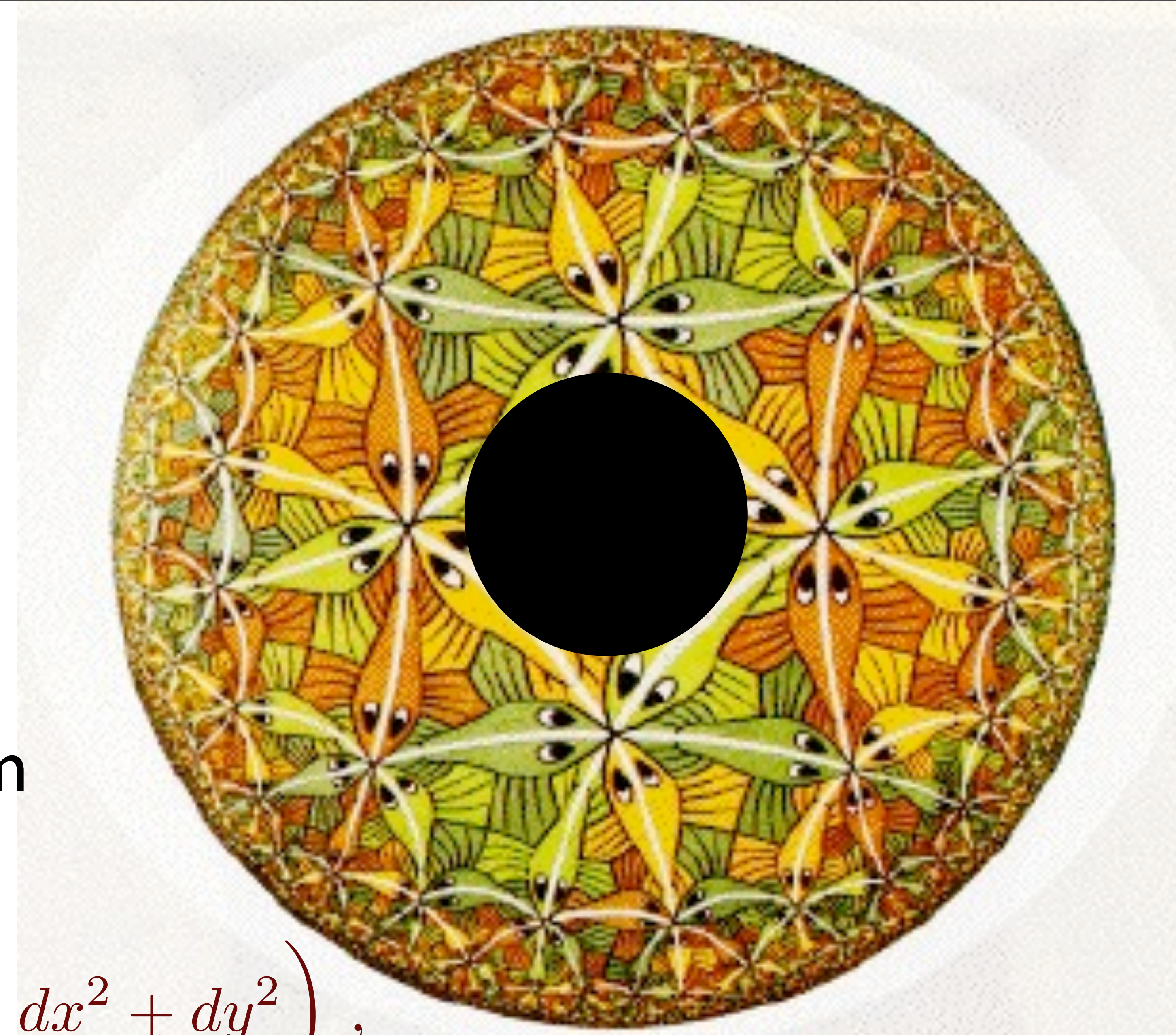
Conformal field theory
in 2+1 dimensions at $T > 0$,
with a non-zero chemical potential, μ
and applied magnetic field, B



Einstein gravity on AdS_4
with a Reissner-Nordstrom
black hole carrying electric
and magnetic charges



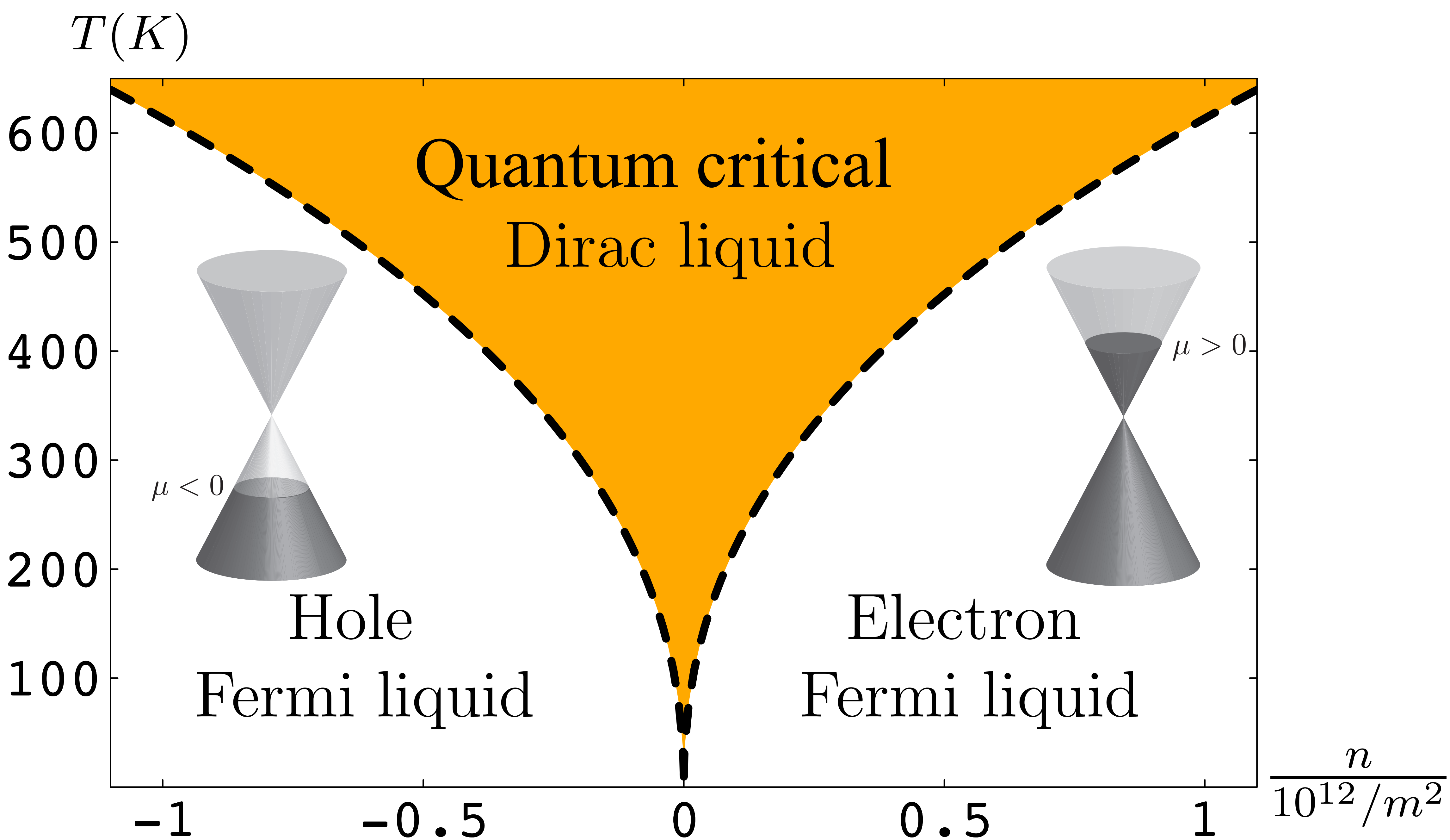
AdS$_4$-Reissner-Nordstrom black hole



$$
\begin{aligned}
ds^2 &= \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right), \\
f(r) &= 1 - \left(1 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^3 + \frac{(r_+^2\mu^2 + r_+^4 B^2)}{\gamma^2}\left(\frac{r}{r_+}\right)^4, \\
A &= i\mu\left[1 - \frac{r}{r_+}\right] d\tau + Bx\,dy\,. \\
T &= \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).
\end{aligned}
$$

Hydrodynamic theory

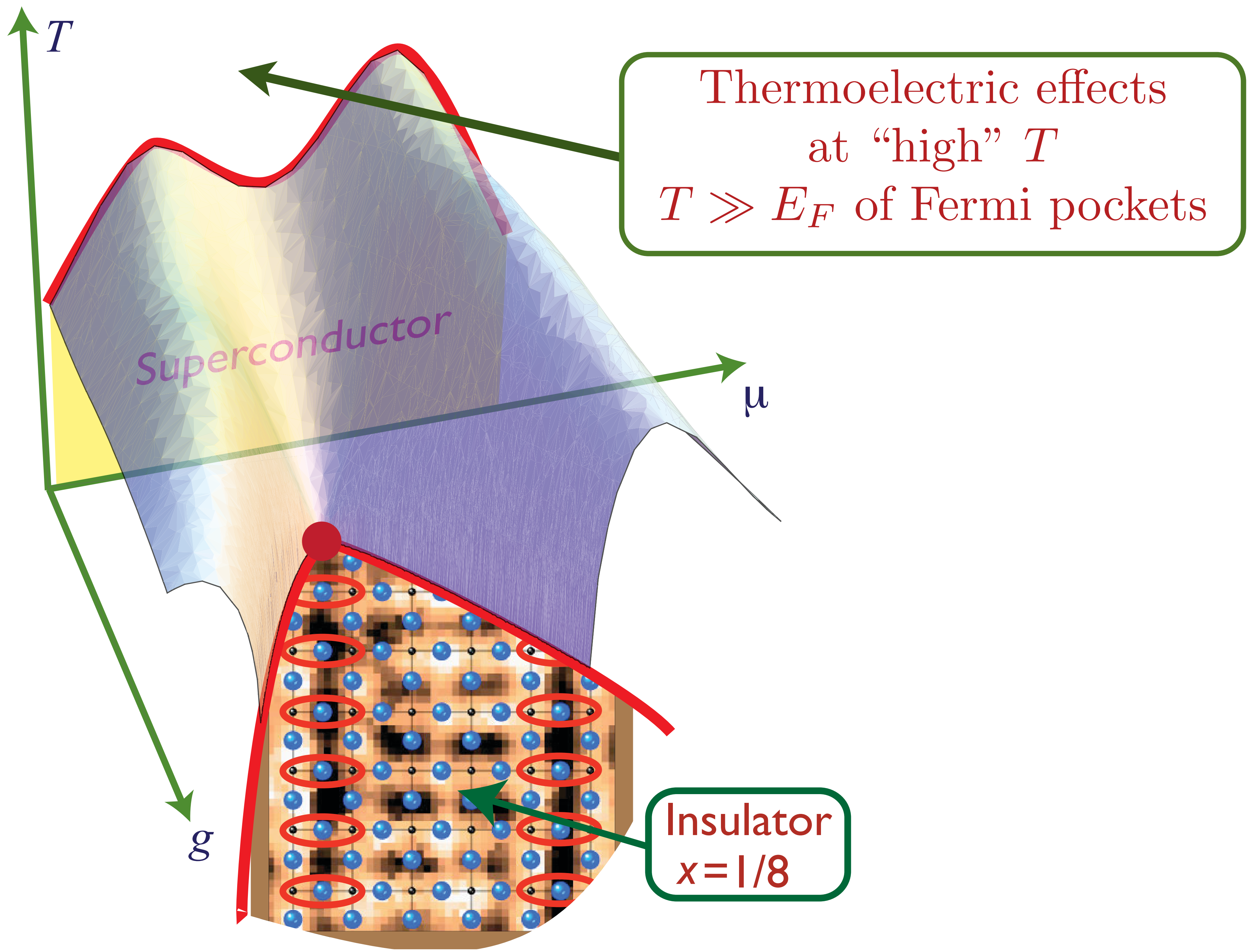
- Promising applications to graphene.



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Hydrodynamic theory

- Assume proximity to a superfluid-insulator transition at $x = 1/8$ in cuprates



S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

The **same** results were later obtained from the equations of generalized relativistic magnetohydrodynamics.

So the results apply to experiments on graphene, the cuprates, *and* to the dynamics of black holes.

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

As a simple example, in zero magnetic field, we can write the electrical conductivity as

$$\sigma = \sigma_Q + \frac{e^{*2}\rho^2 v^2}{\varepsilon + P}\pi\delta(\omega)$$

where σ_Q is the universal conductivity of the CFT, ρ is the charge density, ε is the energy density and P is the pressure.

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

The same quantities also determine a "Wiedemann-Franz"-like relation for thermal conductivity, κ at $B = 0$

$$\kappa = \sigma_Q \left(\frac{k_B^2 T}{e^{*2}} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2 .$$

At $B \neq 0$ and $\rho = 0$ we have a "Wiedemann-Franz" relation for "vortices"

$$\kappa = \frac{1}{\sigma_Q} k_B^2 T \left(\frac{v(\varepsilon + P)}{k_B T B} \right)^2 .$$

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

A second example: In an applied magnetic field B, the dynamic transport co-efficients exhibit a **hydrodynamic cyclotron resonance** at a frequency ω_c

$$\omega_c = \frac{e^* B \rho v^2}{c(\varepsilon + P)}$$

and damping constant γ

$$\gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}.$$

The same constants determine the **quasinormal frequency** of the Reissner-Nordstrom black hole.

Magnetohydrodynamics of quantum criticality

We used the AdS/CFT connection to derive many new relations between thermoelectric transport co-efficients in the quantum critical regime.

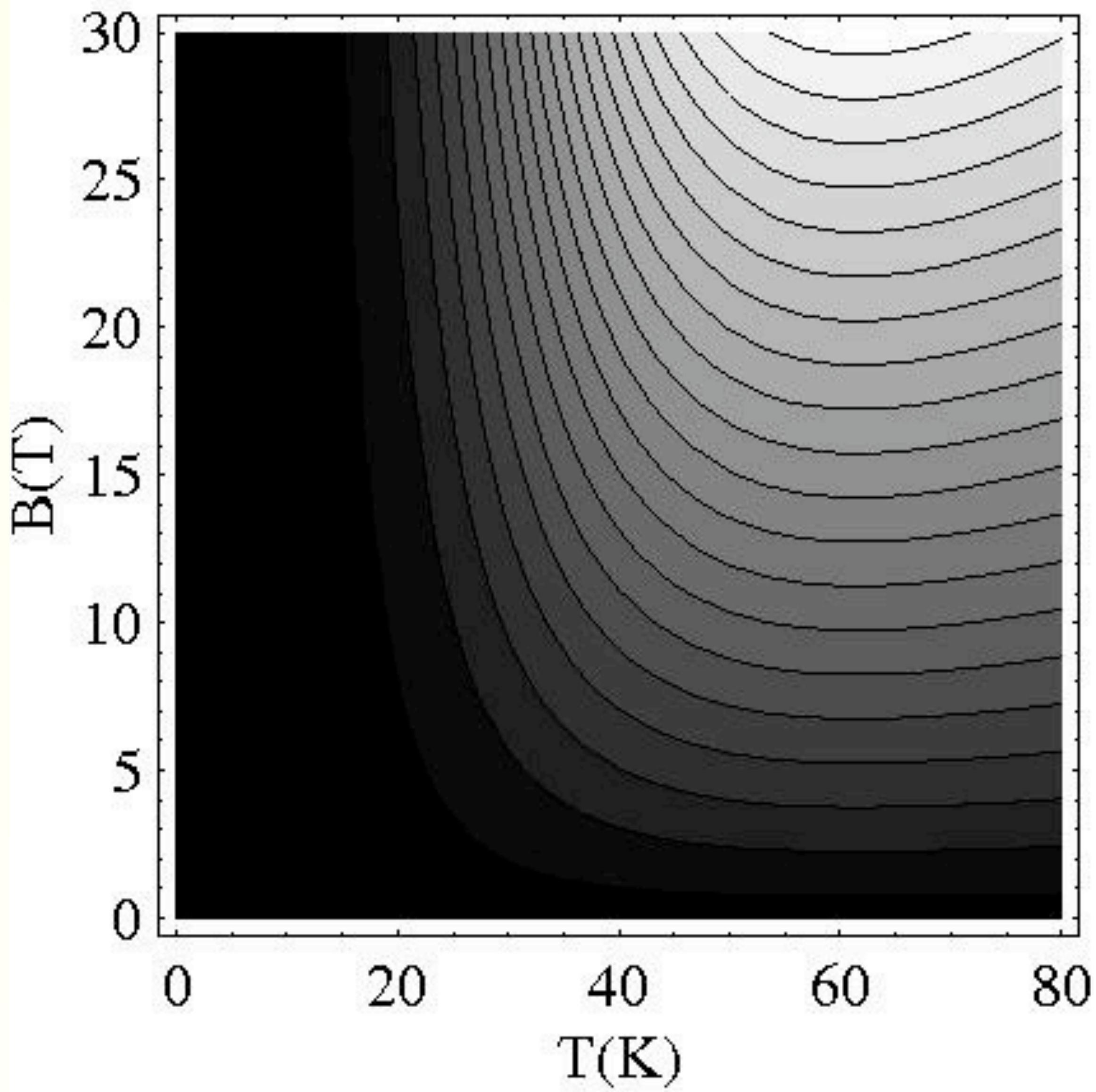
We also obtain the Nernst signal (transverse thermoelectric response)

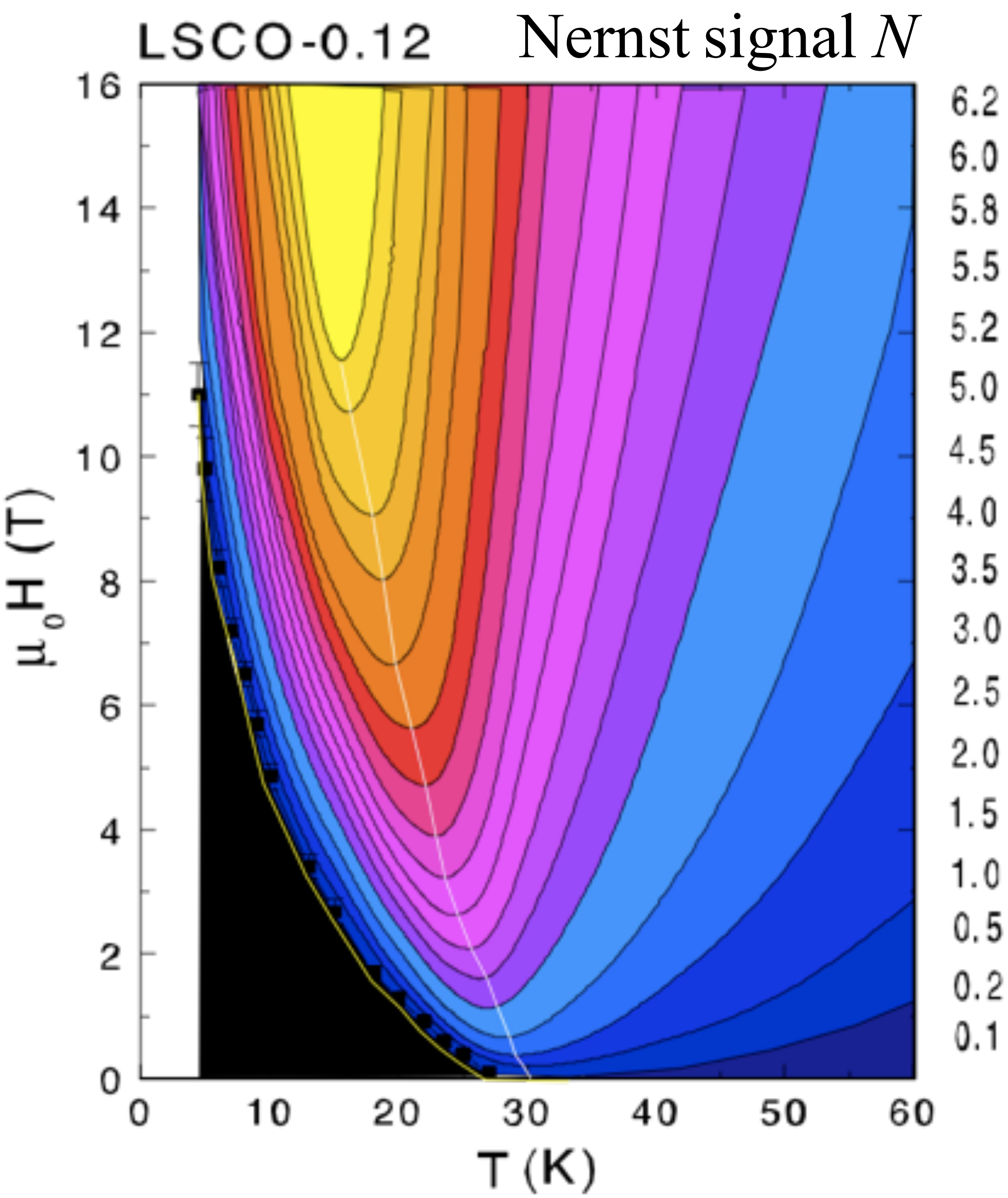
$$e_N = \left(\frac{k_B}{e^*}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

where $\tau_{\rm imp}$ is the momentum relaxation time due to impurities or umklapp scattering.

LSCO Experiments

Theory N





B and T dependencies are in semi-quantitative agreement with observations on cuprates, with reasonable values for only 2 adjustable parameters, $\tau_{\rm imp}$ and v.

Y. Wang, L. Li, and N. P. Ong, *Phys. Rev.* B **73**, 024510 (2006).
S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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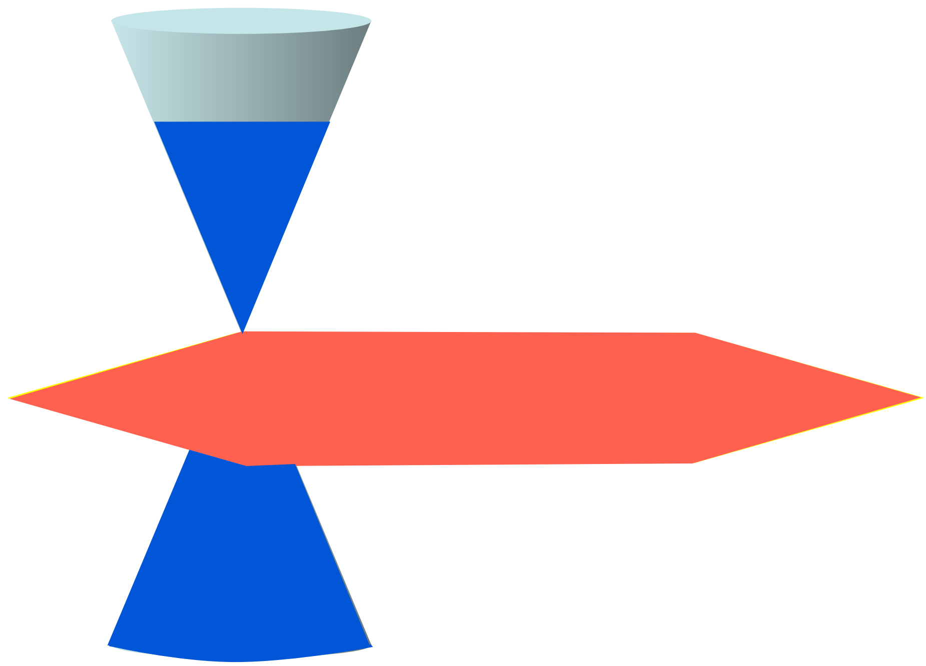
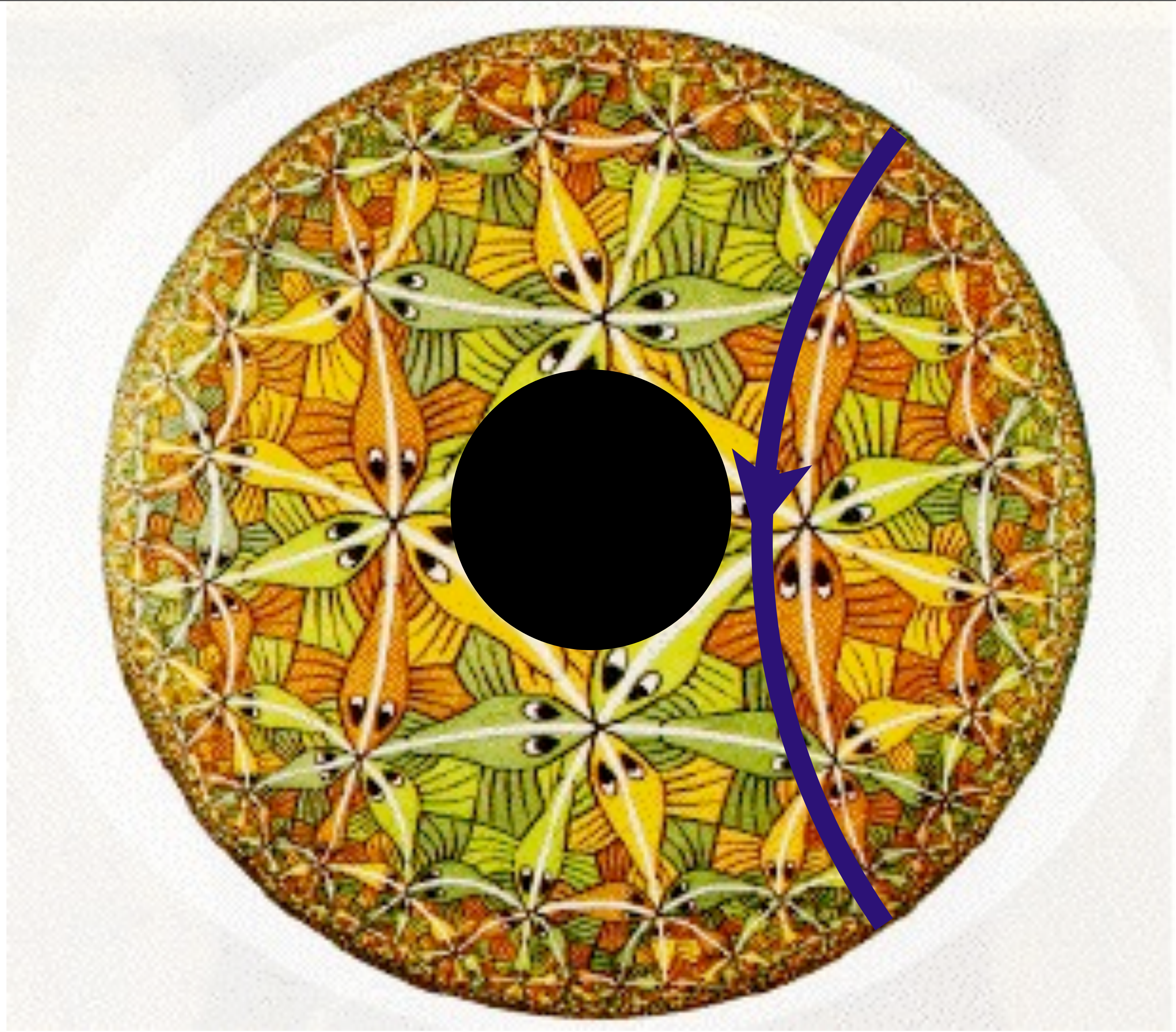
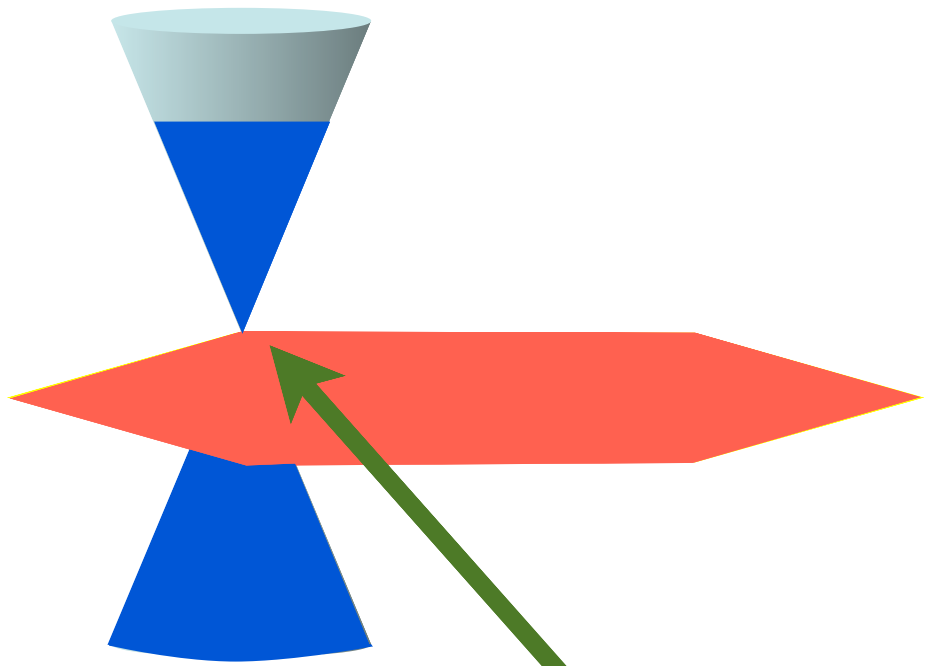
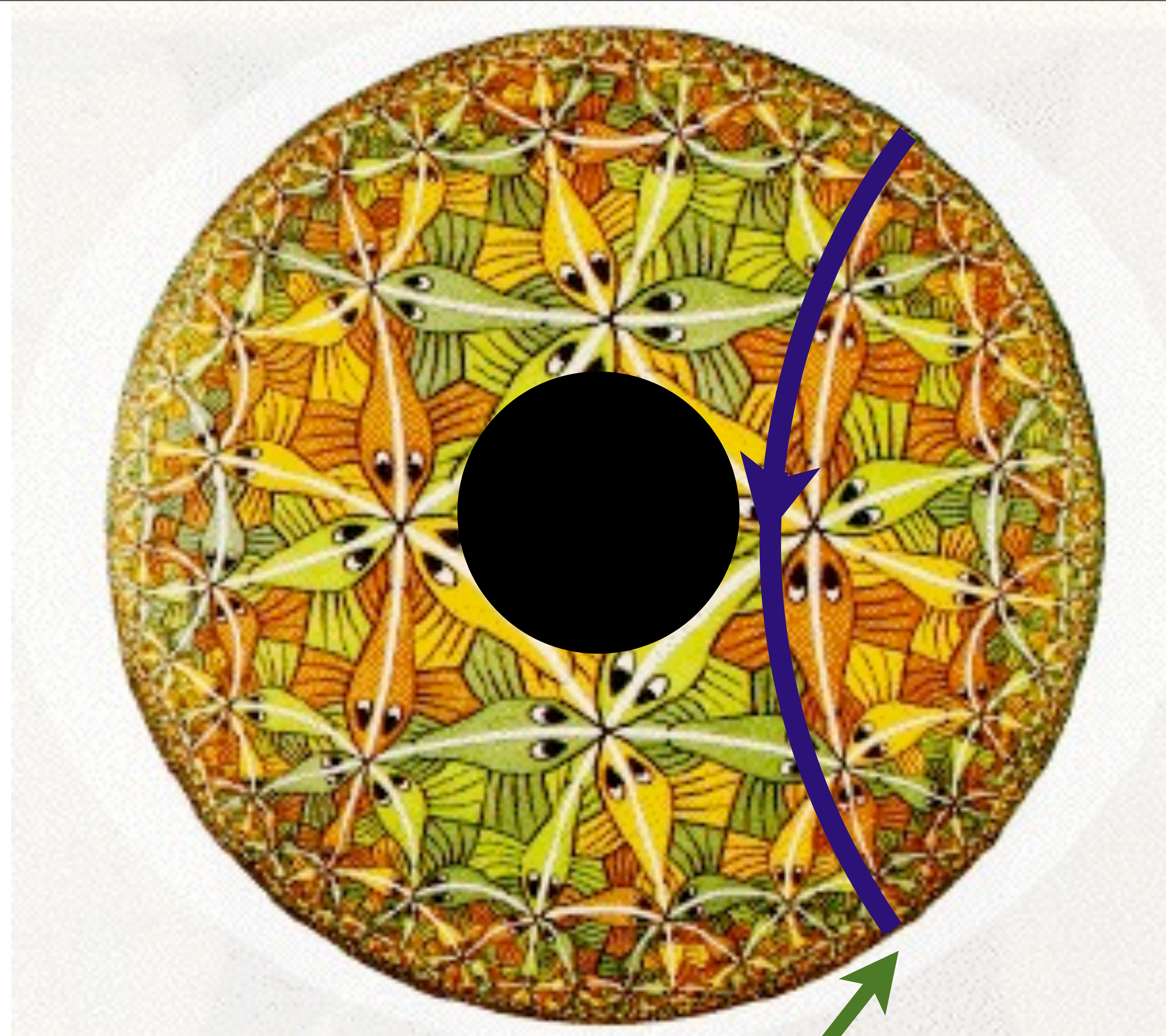




Examine free energy and Green's function of a probe particle

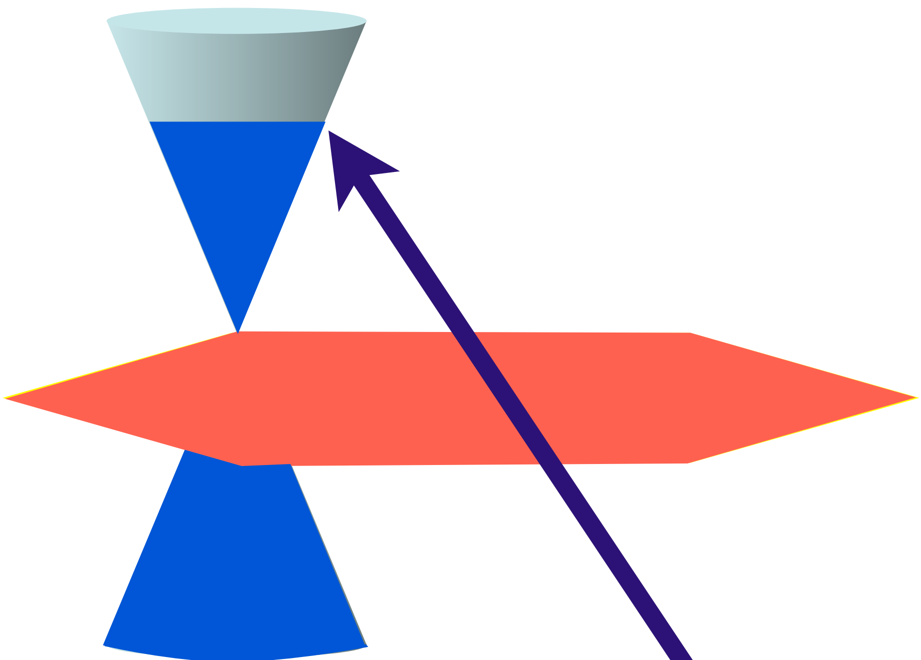
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

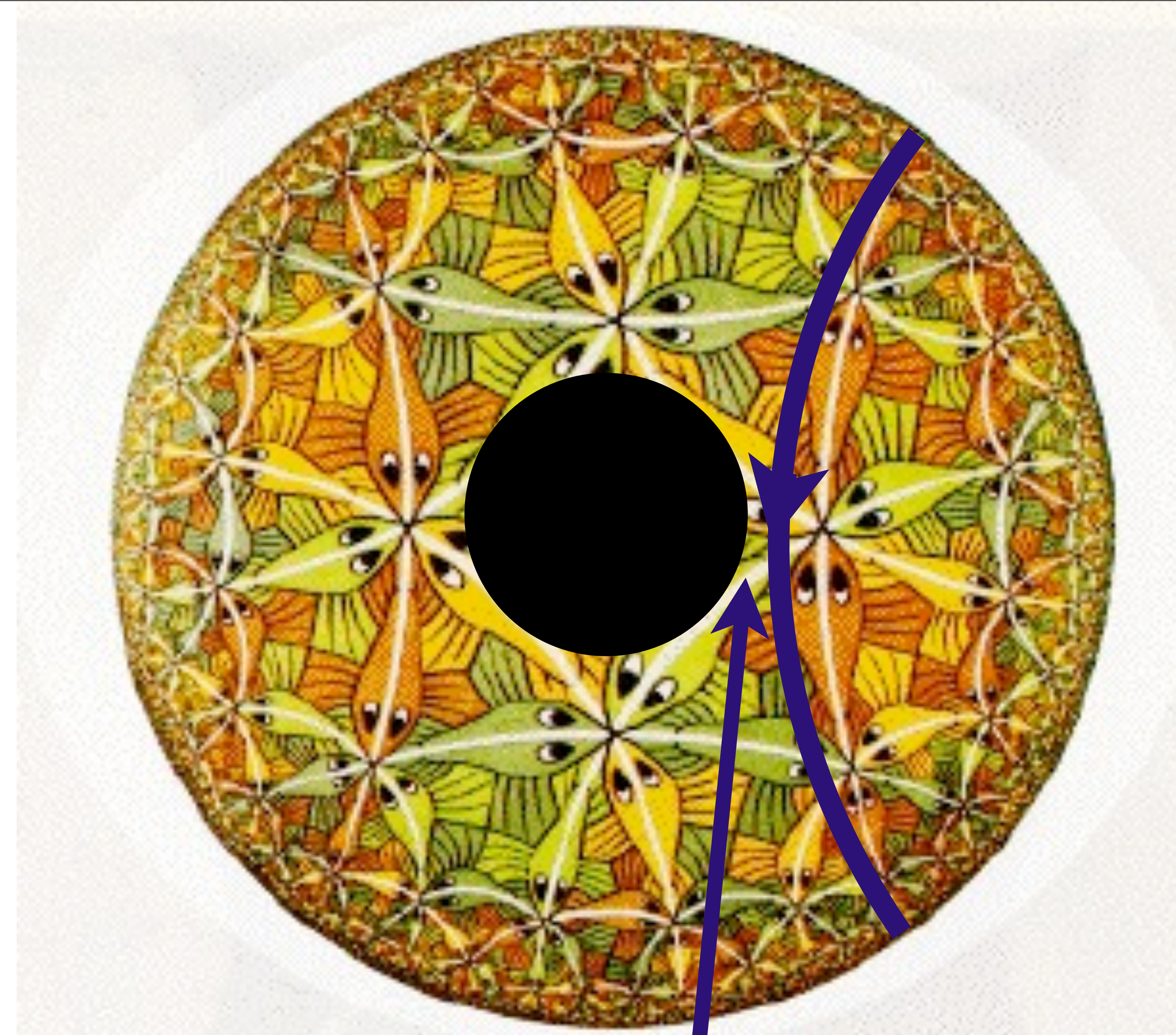




Short time behavior depends upon conformal AdS_4 geometry near boundary

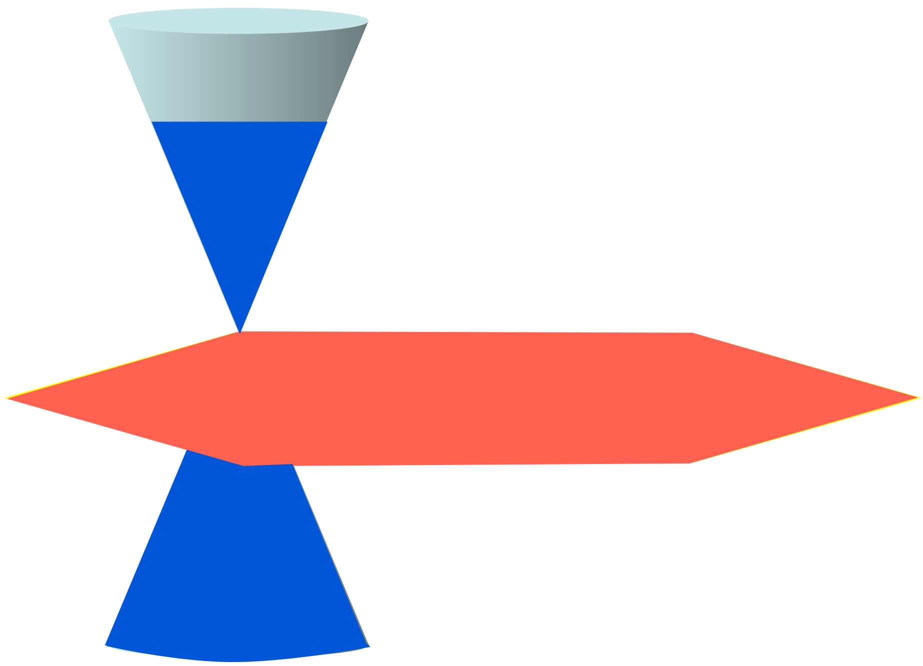
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

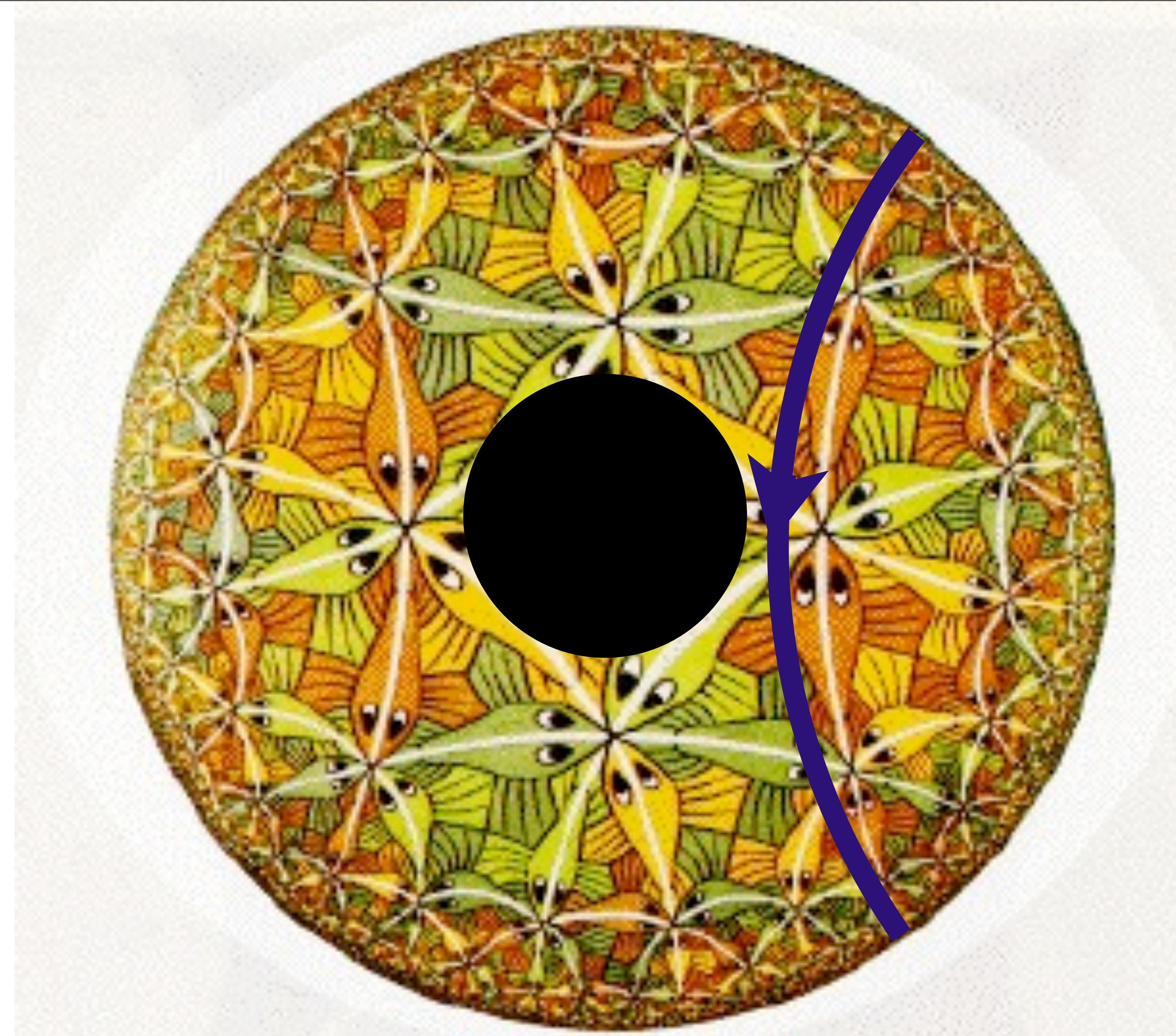




Long time behavior depends upon
near-horizon geometry of black hole

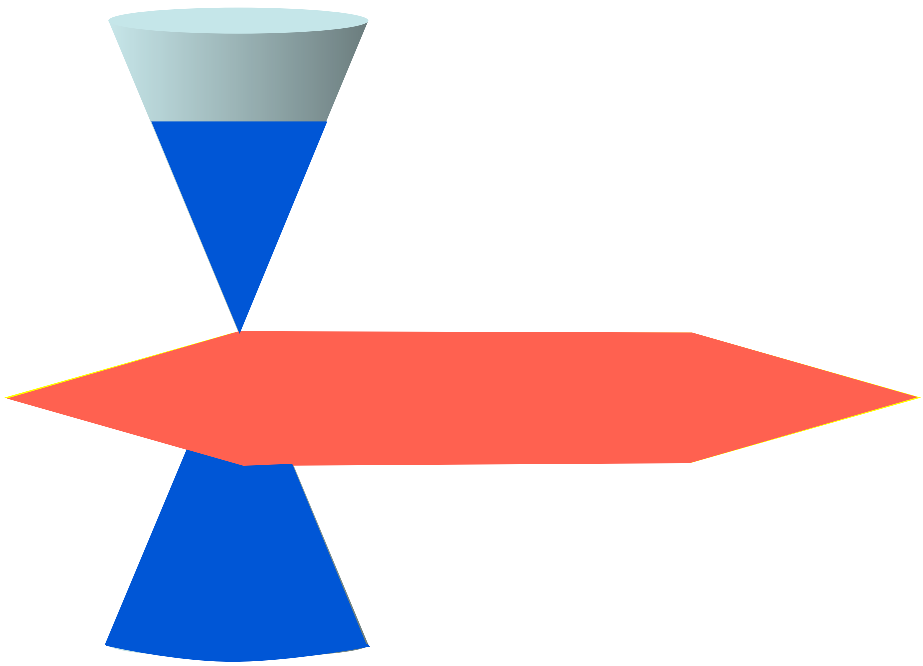
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

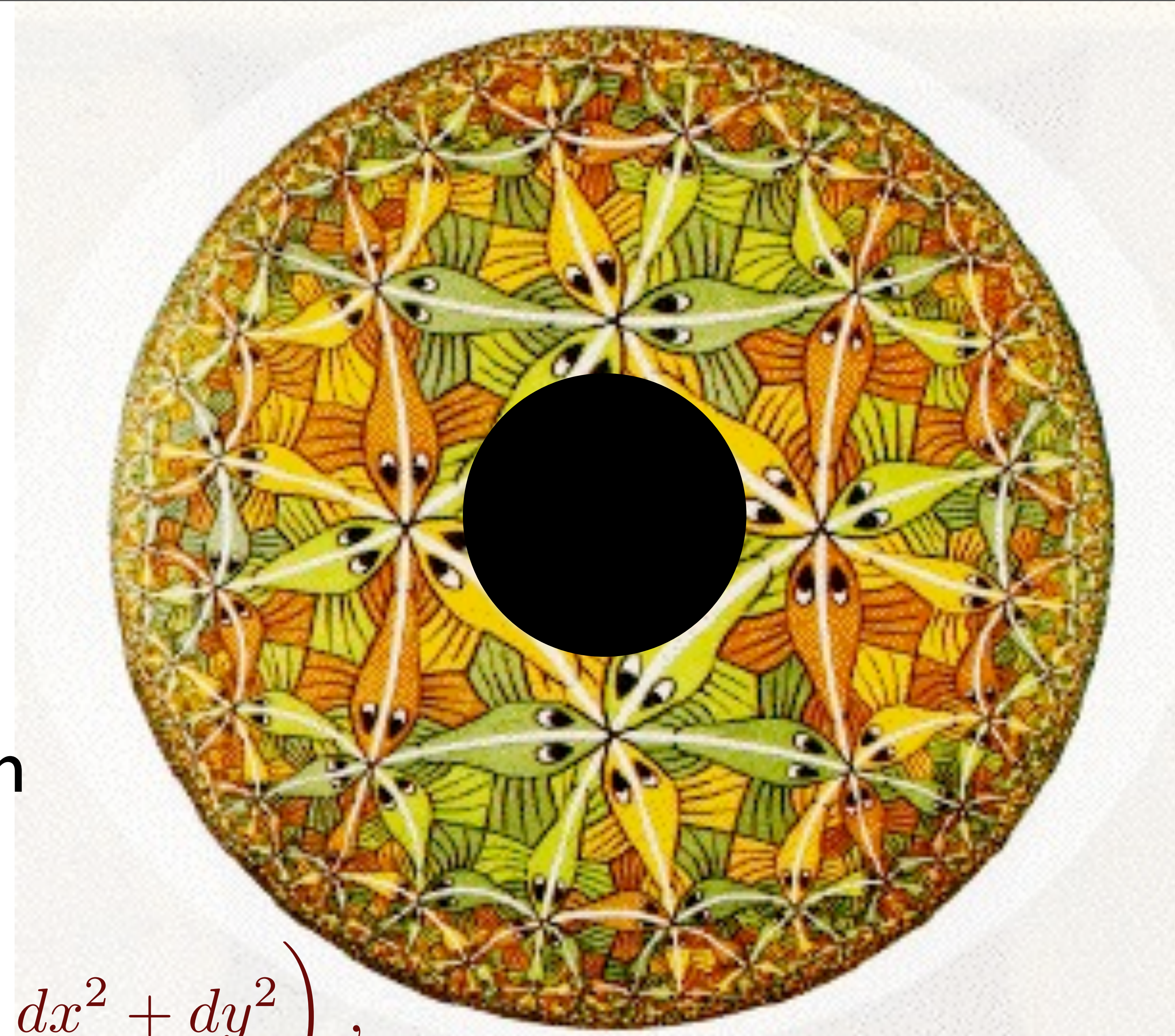




Radial direction of gravity theory is measure of energy scale in CFT

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694
F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788





AdS$_4$-Reissner-Nordstrom black hole

$$
\begin{aligned}
ds^2 &= \frac{L^2}{r^2}\left(f(r)d\tau^2 + \frac{dr^2}{f(r)} + dx^2 + dy^2\right), \\
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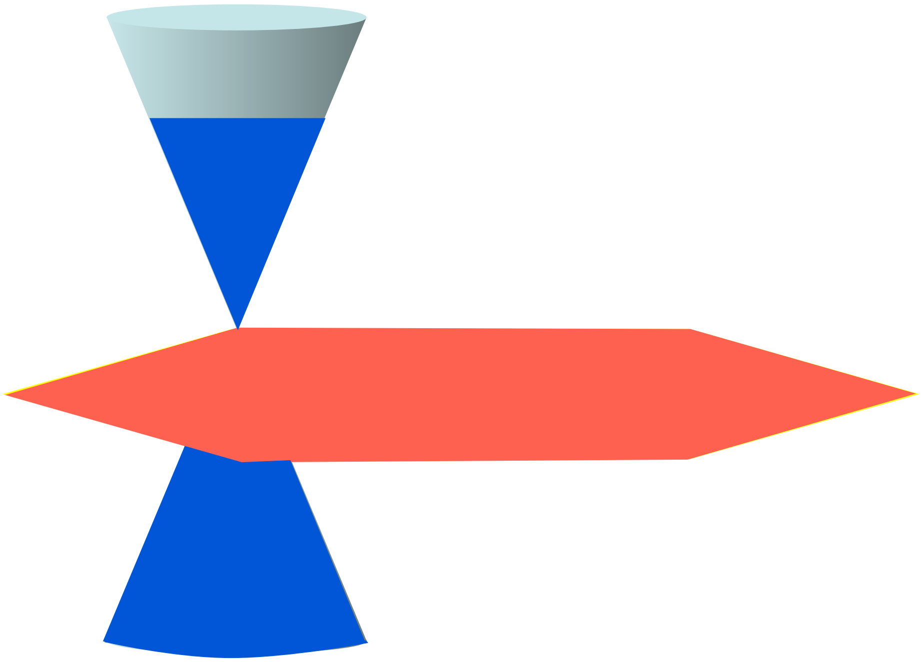
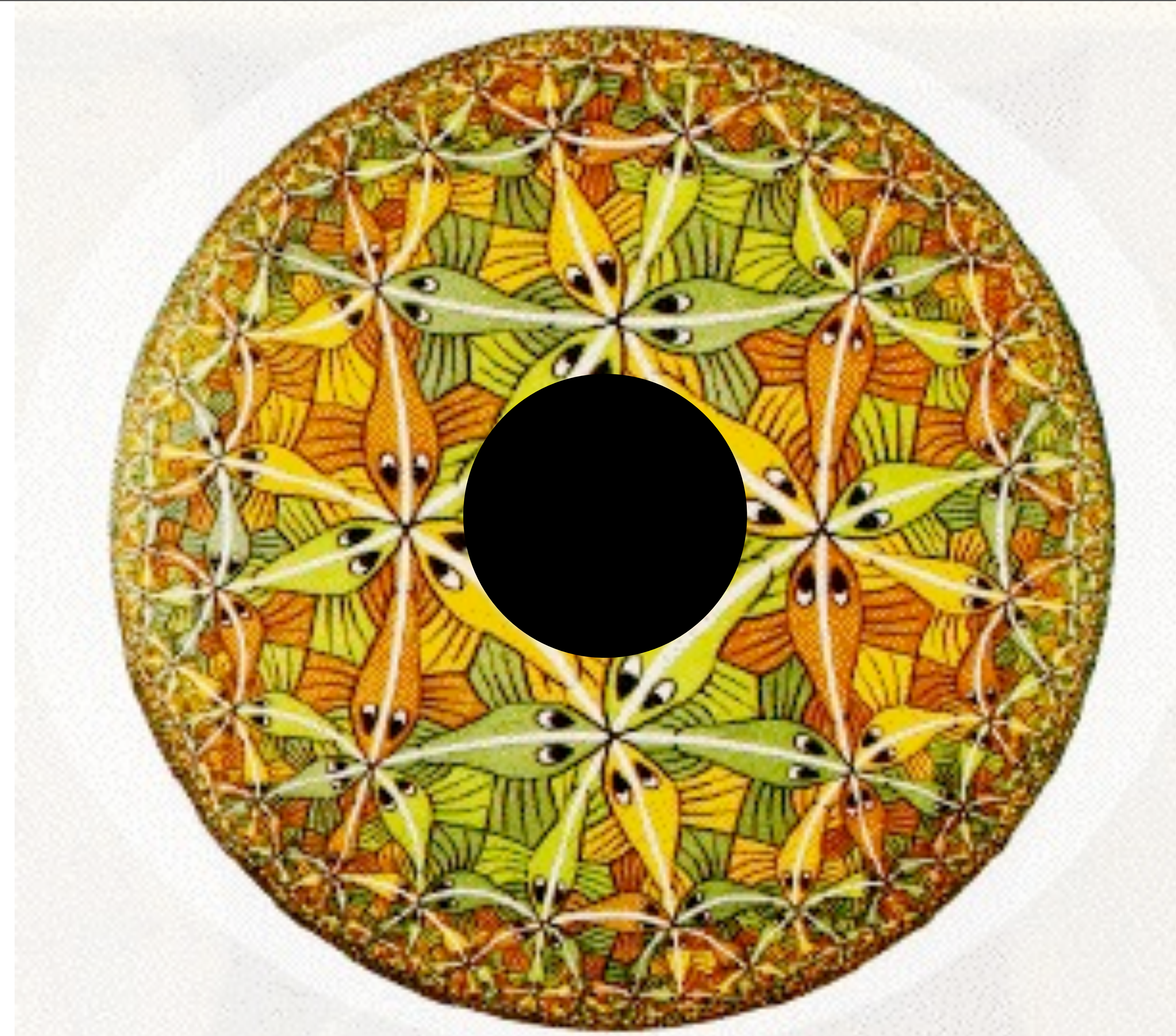
A &= i\mu\left[1 - \frac{r}{r_+}\right] d\tau + Bx\, dy\,. \\
T &= \frac{1}{4\pi r_+}\left(3 - \frac{r_+^2\mu^2}{\gamma^2} - \frac{r_+^4 B^2}{\gamma^2}\right).
\end{aligned}
$$

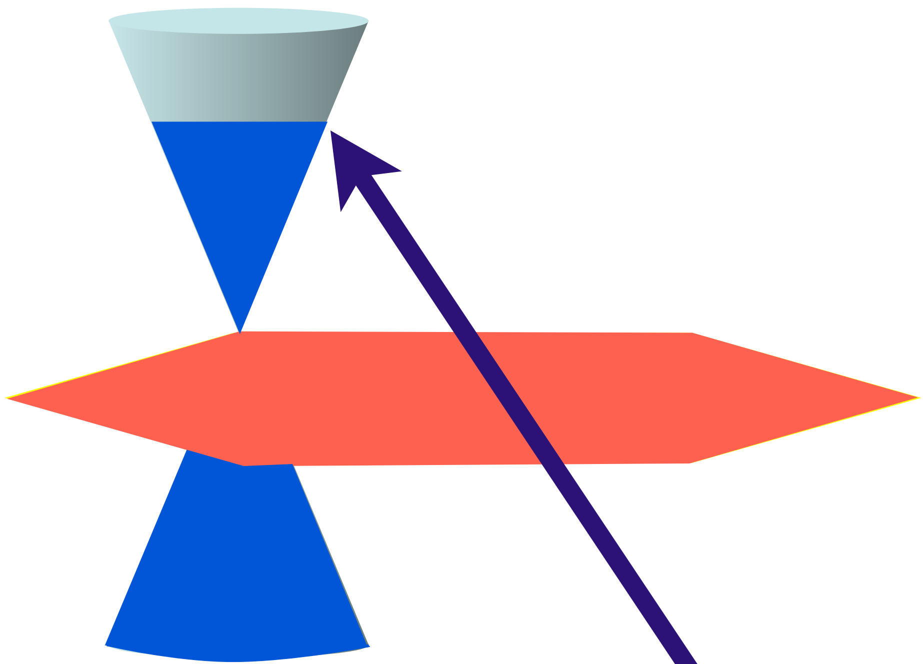


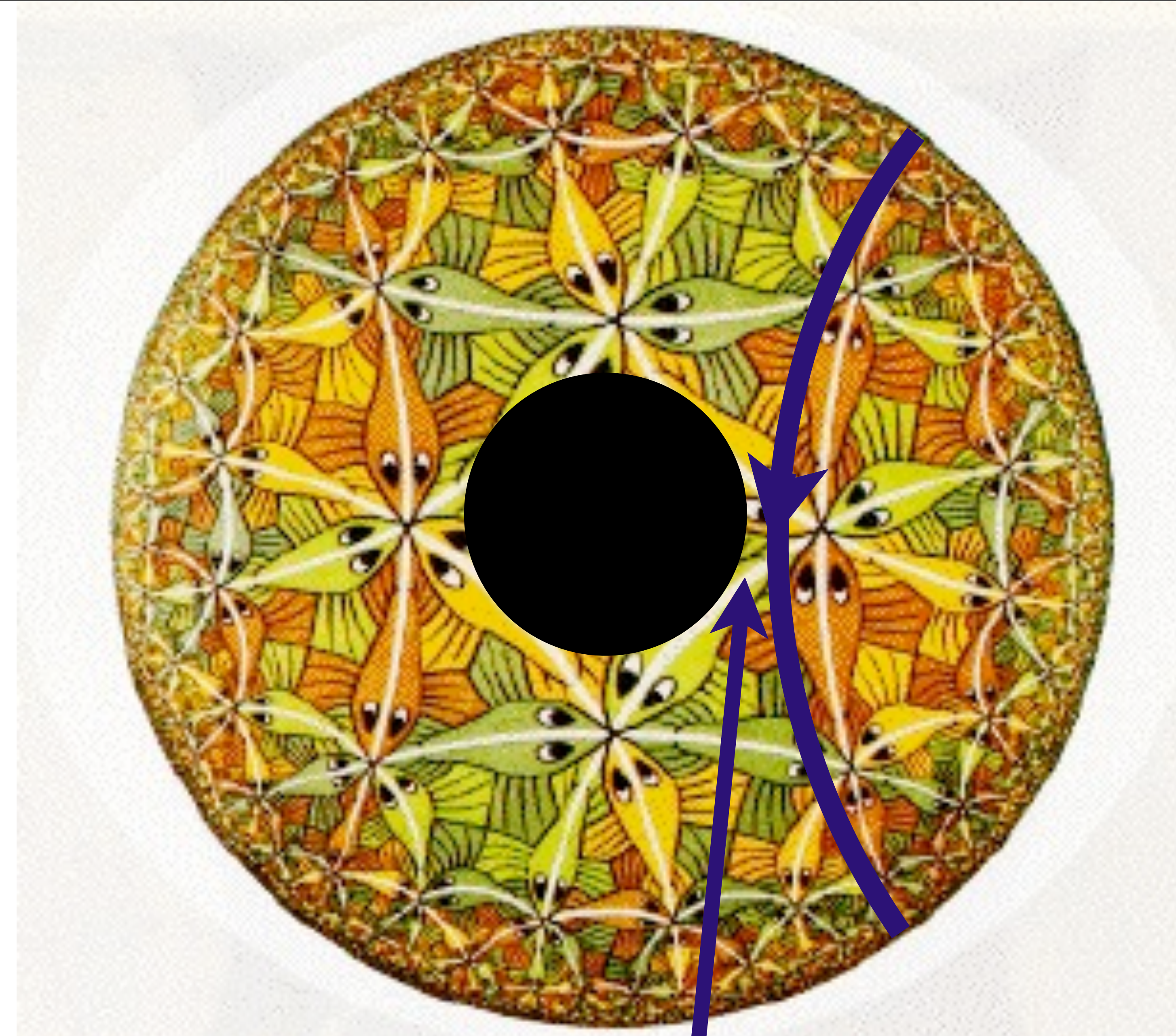
AdS$_2$ x R^2 near-horizon geometry



$$r - r_+ \quad \sim \quad \frac{1}{\zeta}$$

$$ds^2 \quad = \quad \frac{R^2}{\zeta^2}\left(-d\tau^2 + d\zeta^2\right) + \frac{r_+^2}{R^2}\left(dx^2 + dy^2\right)$$

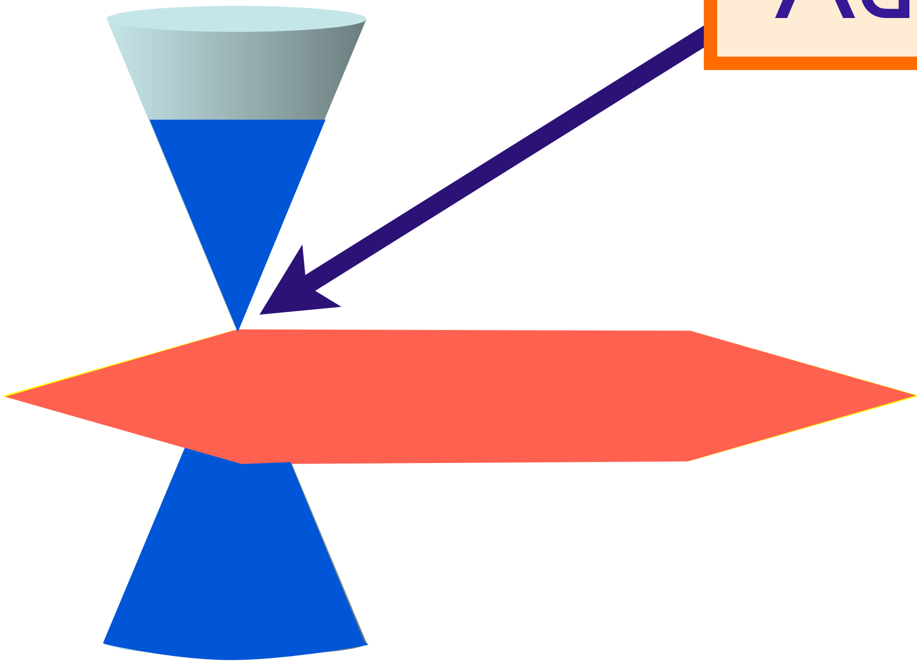


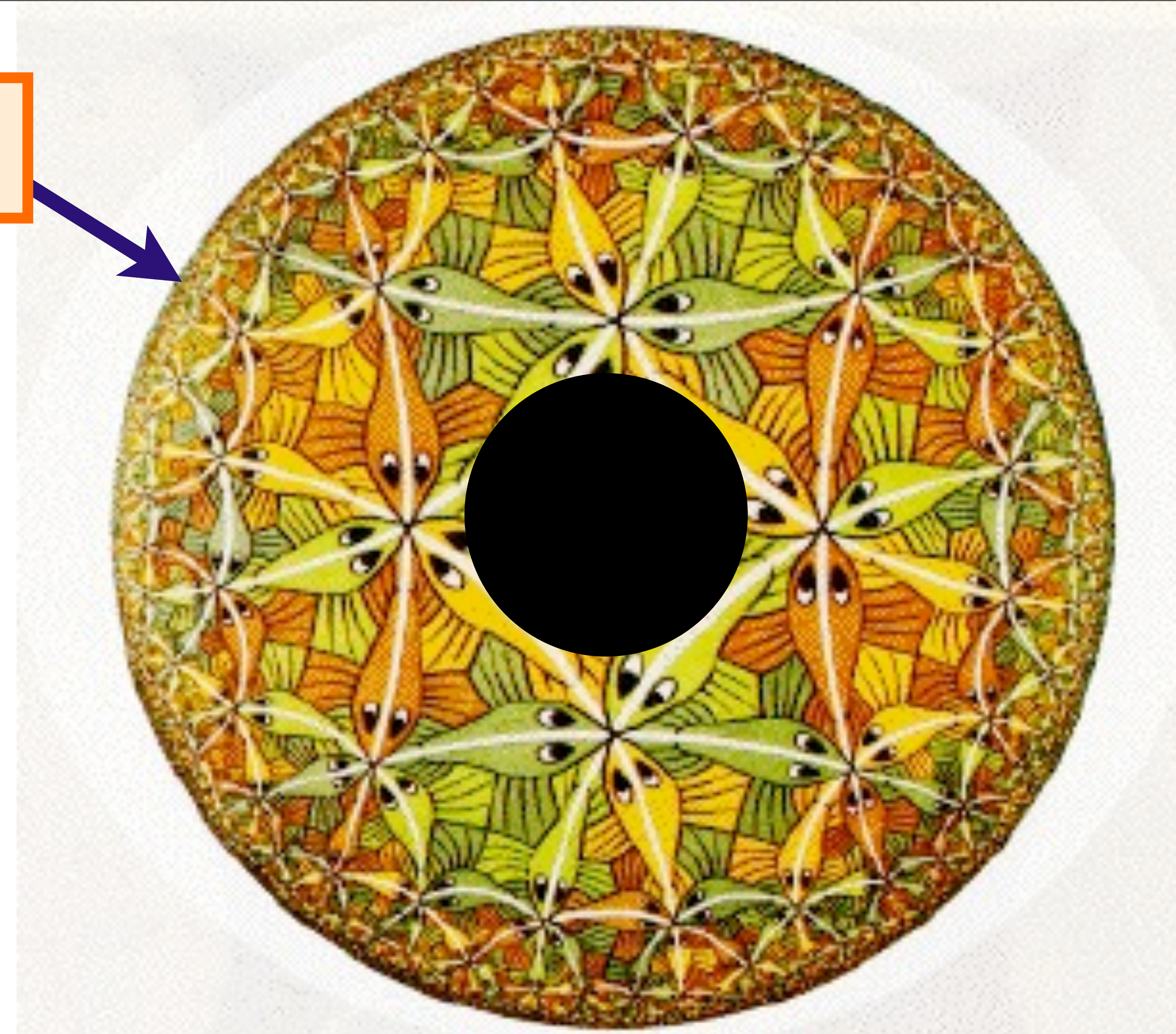


Infrared physics of Fermi surface is linked to the near horizon AdS_2 geometry of Reissner-Nordstrom black hole

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

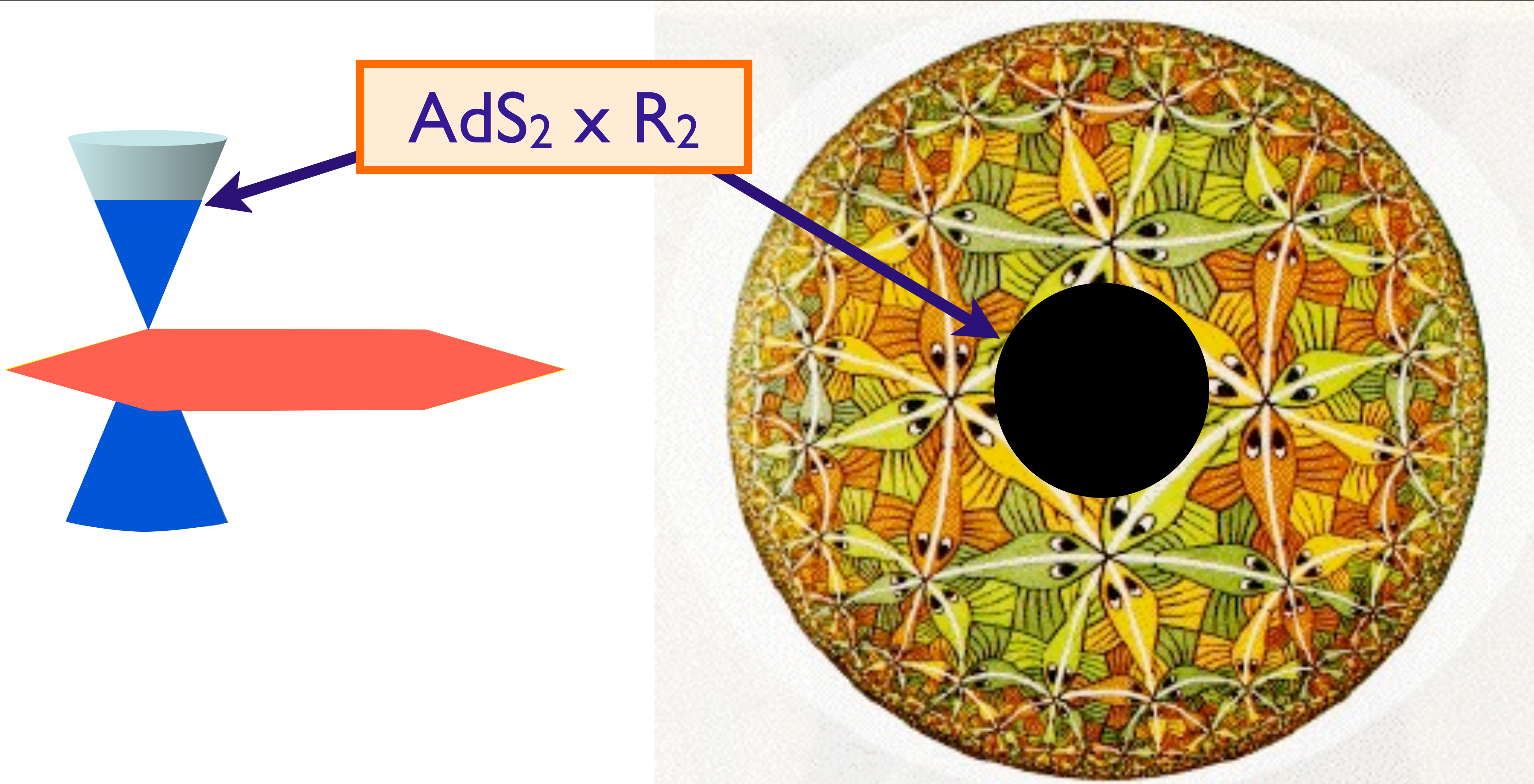
AdS$_4$





Geometric interpretation of RG flow

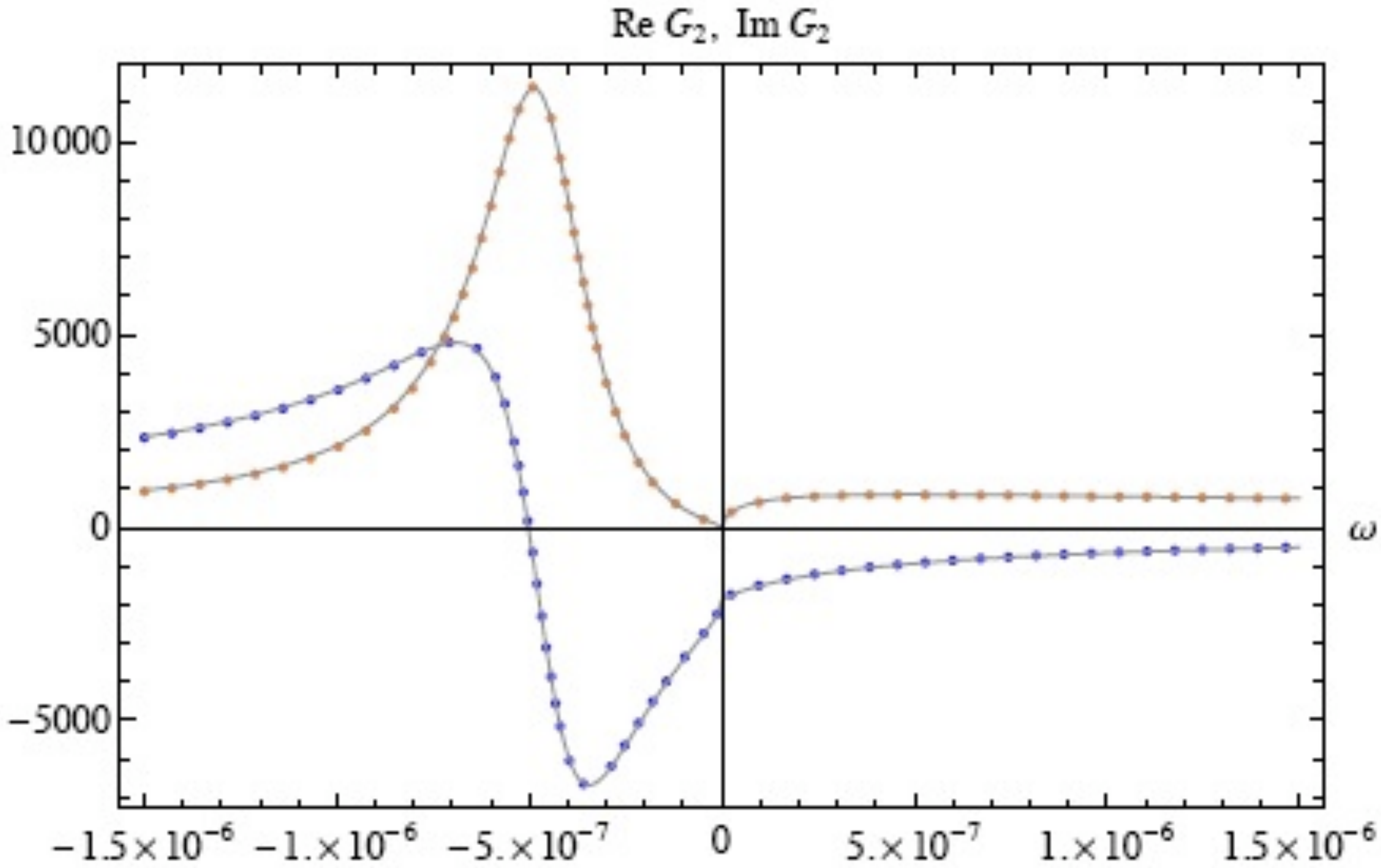
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694



Geometric interpretation of RG flow

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Green's function of a fermion

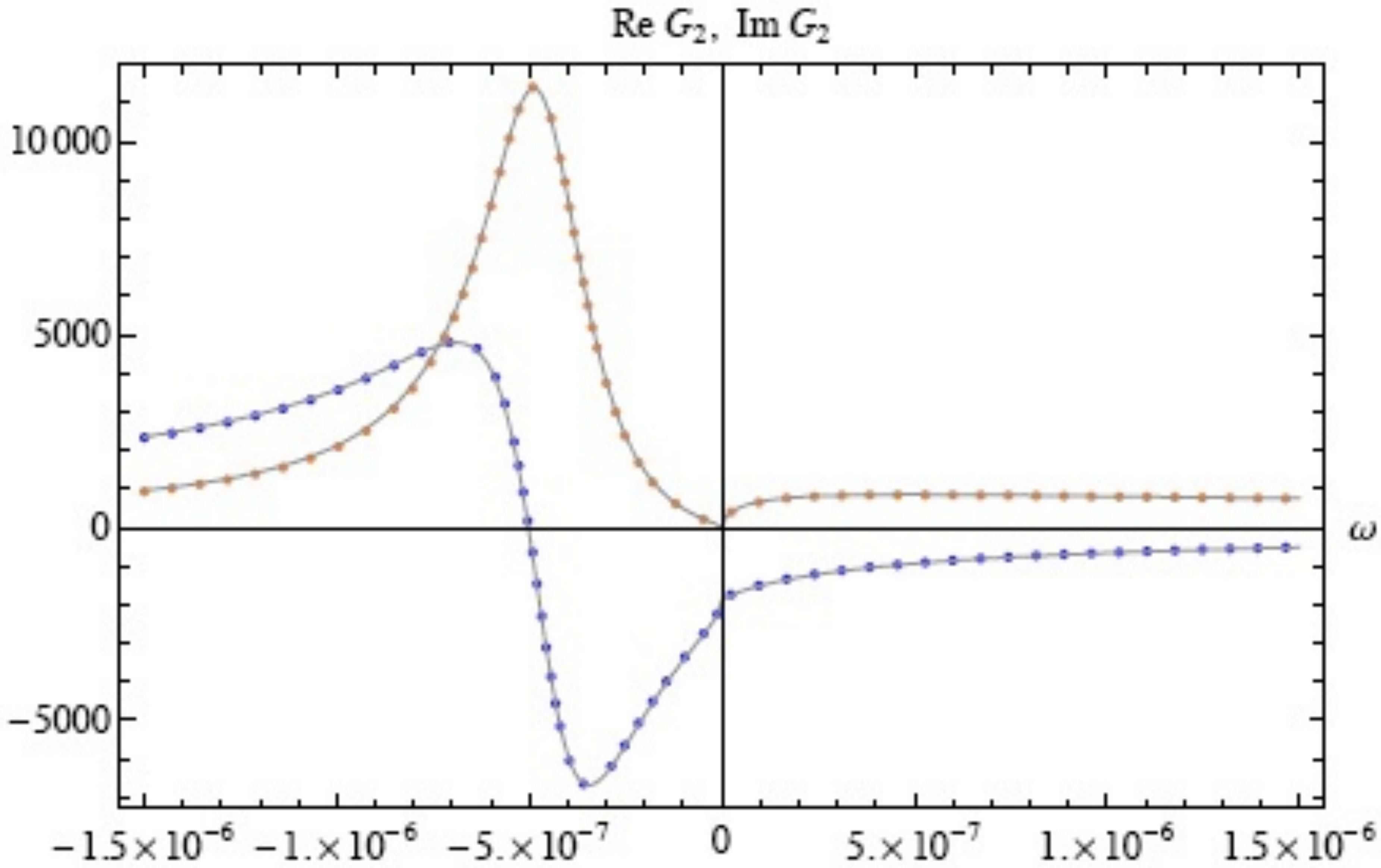


T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

See also M. Cubrovic, J. Zaanen, and K. Schalm, arXiv:0904.1993

Green's function of a fermion



T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

$$G(k,\omega) \approx \frac{1}{\omega - v_F(k-k_F) - i\omega^{\theta(k)}}$$

Similar to non-Fermi liquid theories of Fermi surfaces coupled to gauge fields, and at quantum critical points

Free energy from gravity theory

The free energy is expressed as a sum over the "quasinormal frequencies", z_ℓ, of the black hole. Here ℓ represents any set of quantum numbers:

$$\mathcal{F}_{\text{boson}} = -T \sum_\ell \ln \left(\frac{|z_\ell|}{2\pi T} \left| \Gamma \left(\frac{i z_\ell}{2\pi T} \right) \right|^2 \right)$$

$$\mathcal{F}_{\text{fermion}} = T \sum_\ell \ln \left(\left| \Gamma \left(\frac{i z_\ell}{2\pi T} + \frac{1}{2} \right) \right|^2 \right)$$

Application of this formula shows that the fermions exhibit the dHvA quantum oscillations with expected period (2π/(Fermi surface ares)) in $1/B$, but with an amplitude corrected from the Fermi liquid formula of Lifshitz-Kosevich.

F. Denef, S. Hartnoll, and S. Sachdev, arXiv:0908.1788

Conclusions

General theory of finite temperature dynamics and transport near quantum critical points, with applications to antiferromagnets, graphene, and superconductors

Conclusions

The AdS/CFT offers promise in providing a new understanding of strongly interacting quantum matter at non-zero density