Quantum matter without quasiparticles: strange metals and black holes

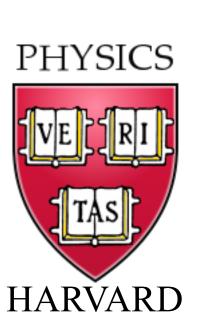
Dartmouth College October 20, 2017

Subir Sachdev

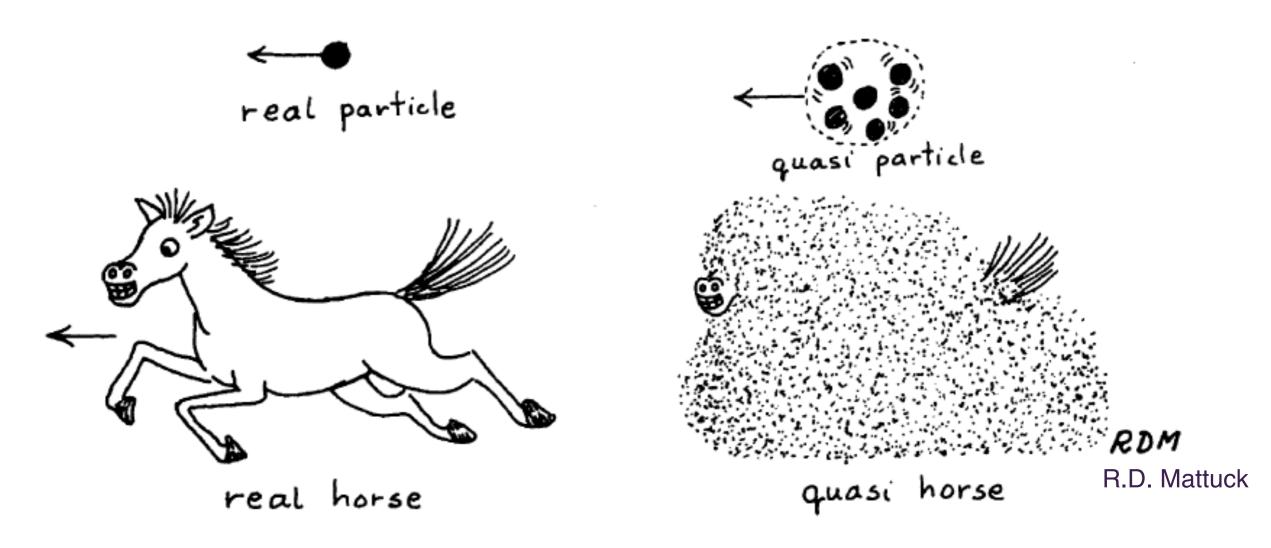
Talk online: sachdev.physics.harvard.edu

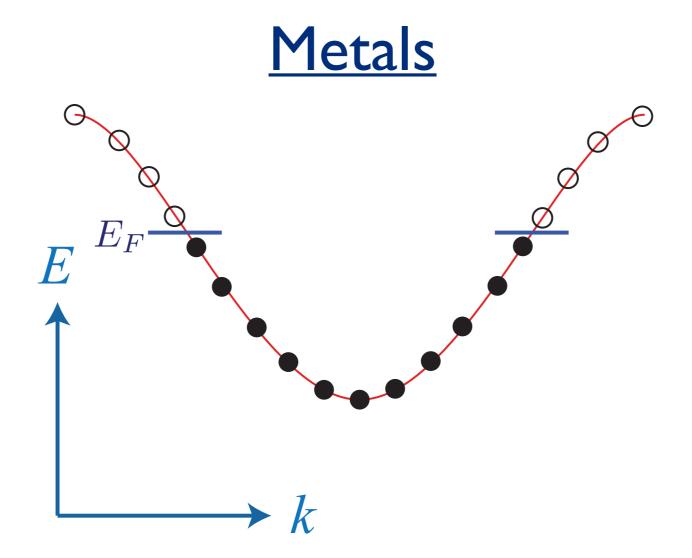


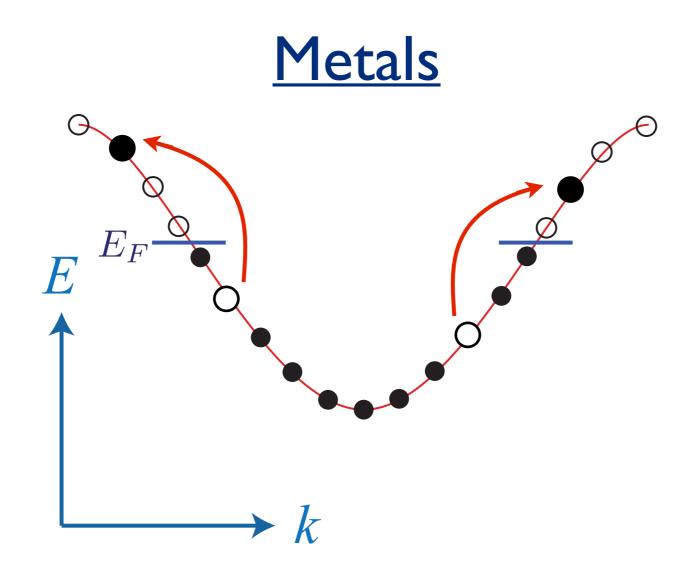




A quasiparticle is an "excited lump" in the manyelectron state which responds just like an ordinary particle.



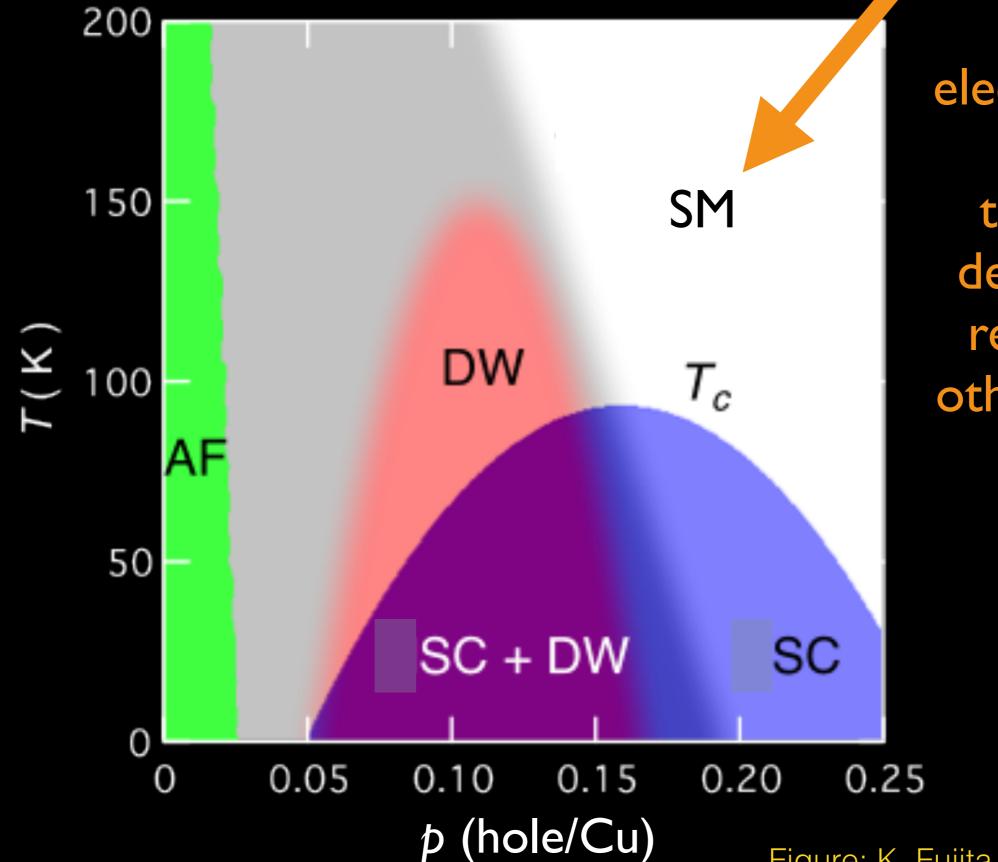




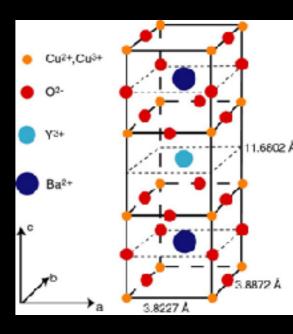
The quasiparticle idea is the key reason for the many successes of quantum condensed matter physics:

- Fermi liquid theory of metals, insulators, semiconductors
- Theory of superconductivity (pairing of quasiparticles)
- Theory of disordered metals and insulators (diffusion and localization of quasiparticles)
- Theory of metals in one dimension (collective modes as quasiparticles)
- Theory of the fractional quantum Hall effect (quasiparticles which are `fractions' of an electron)

Strange metal



Entangled electrons lead to "strange" temperature dependence of resistivity and other properties





"Strange",





or "Incoherent",

metal has a resistivity, ρ , which obeys

$$\rho \sim T$$
,

and

$$\rho \gg h/e^2$$

(in two dimensions), where h/e^2 is the quantum unit of resistance.



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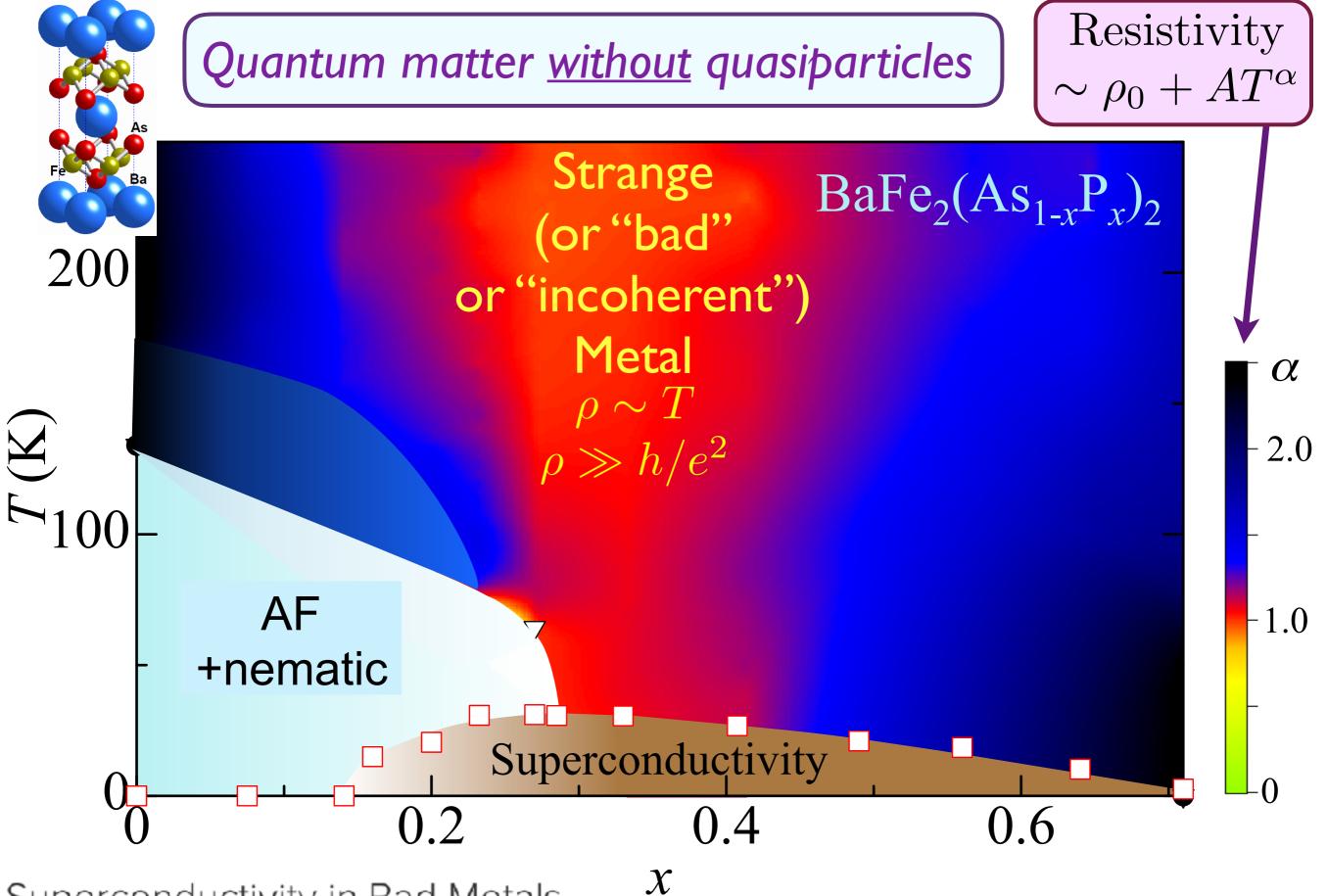
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Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson Phys. Rev. Lett. **74**, 3253 – Published 17 April 1995 S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *PRB* 81, 184519 (2010)

• Quasiparticles are additive excitations:

The low-lying excitations of the many-body system can be identified as a set $\{n_{\alpha}\}$ of quasiparticles with energy ε_{α}

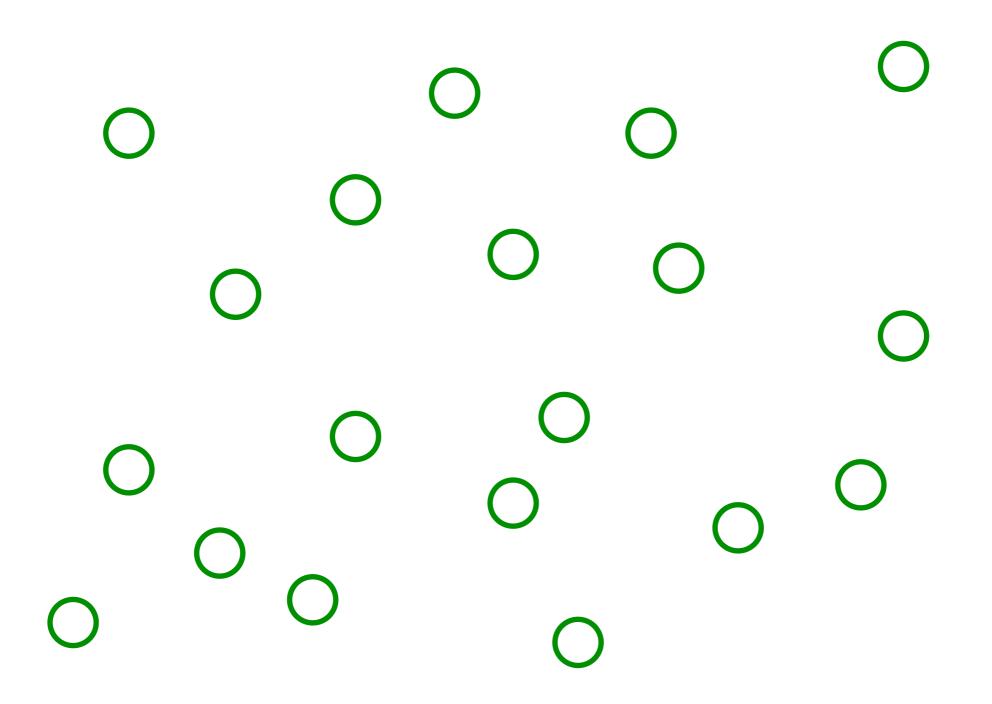
$$E = \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

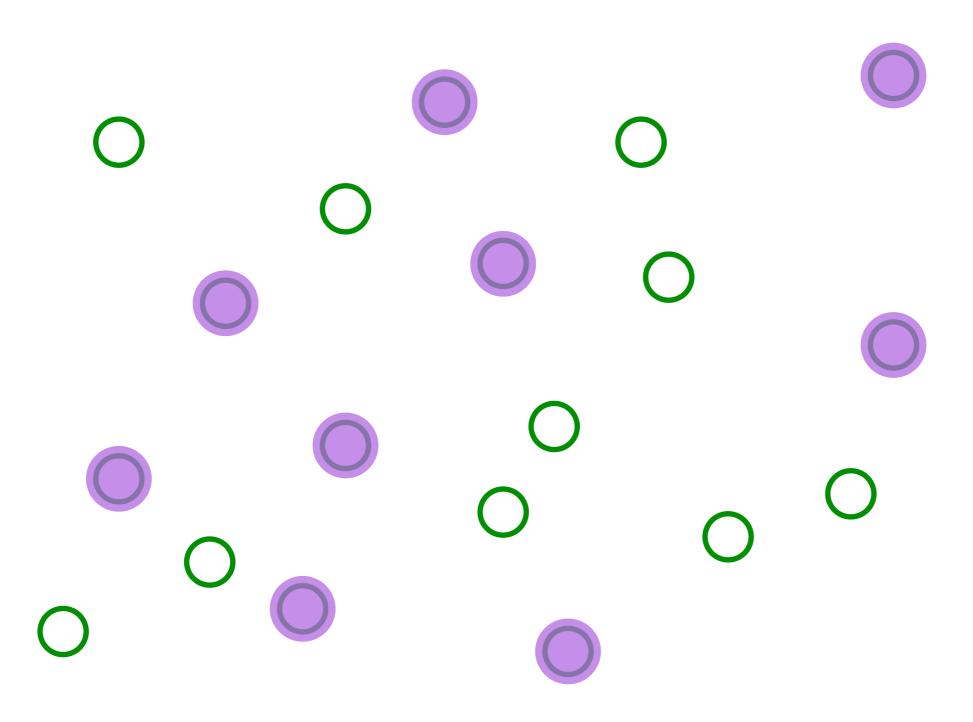
• Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$au_{
m eq} \sim rac{\hbar E_F}{(k_B T)^2} \quad , \quad {
m as} \ T o 0,$$

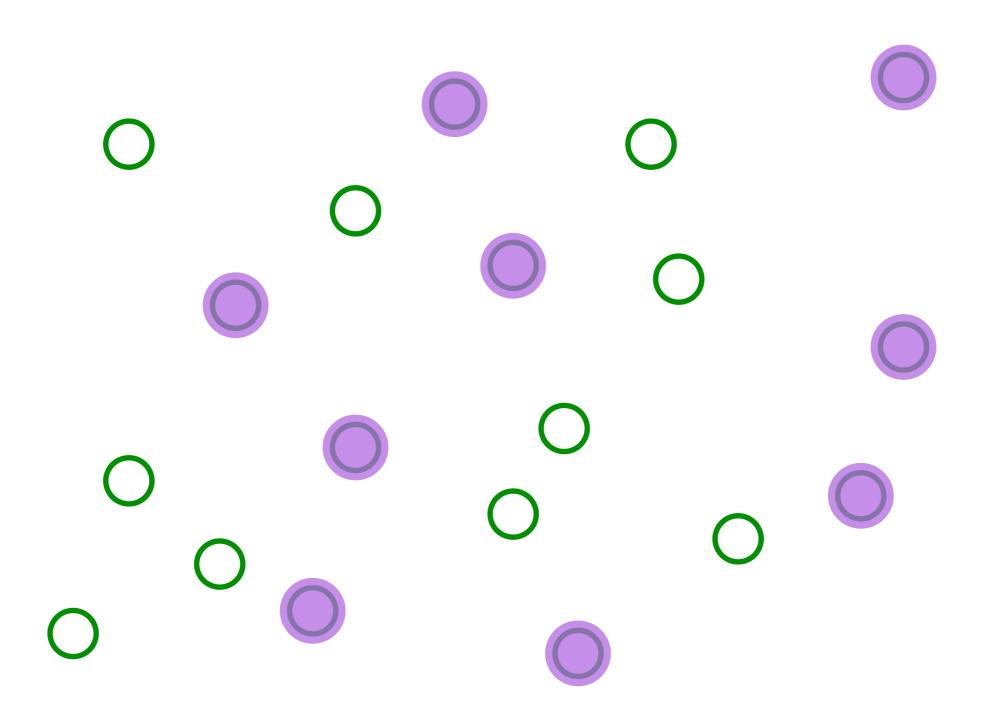
where E_F is the Fermi energy.

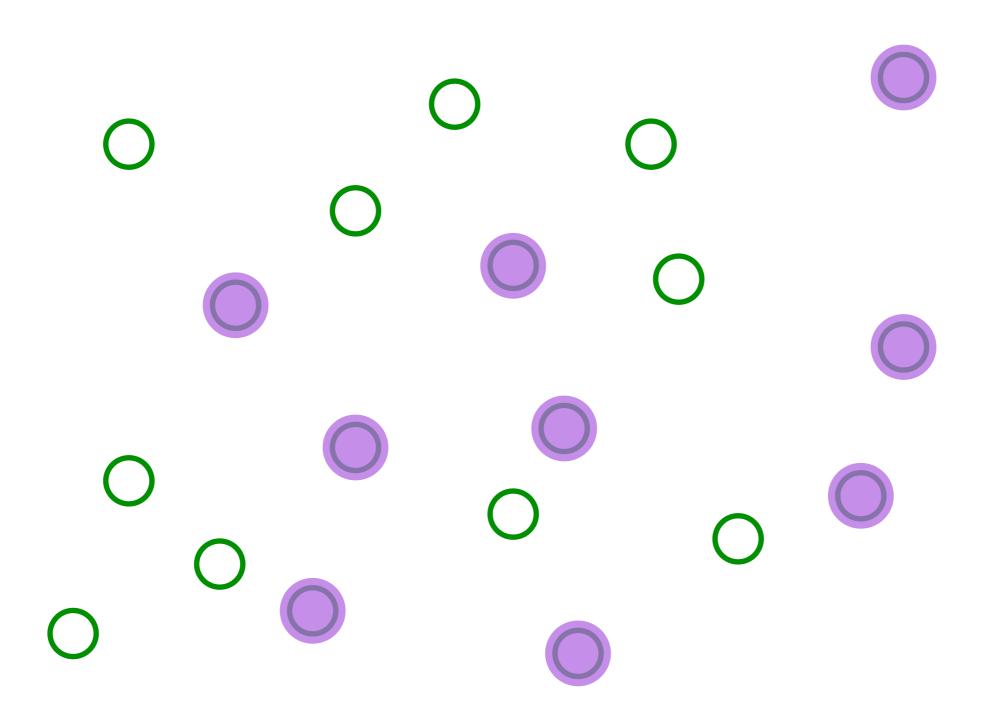


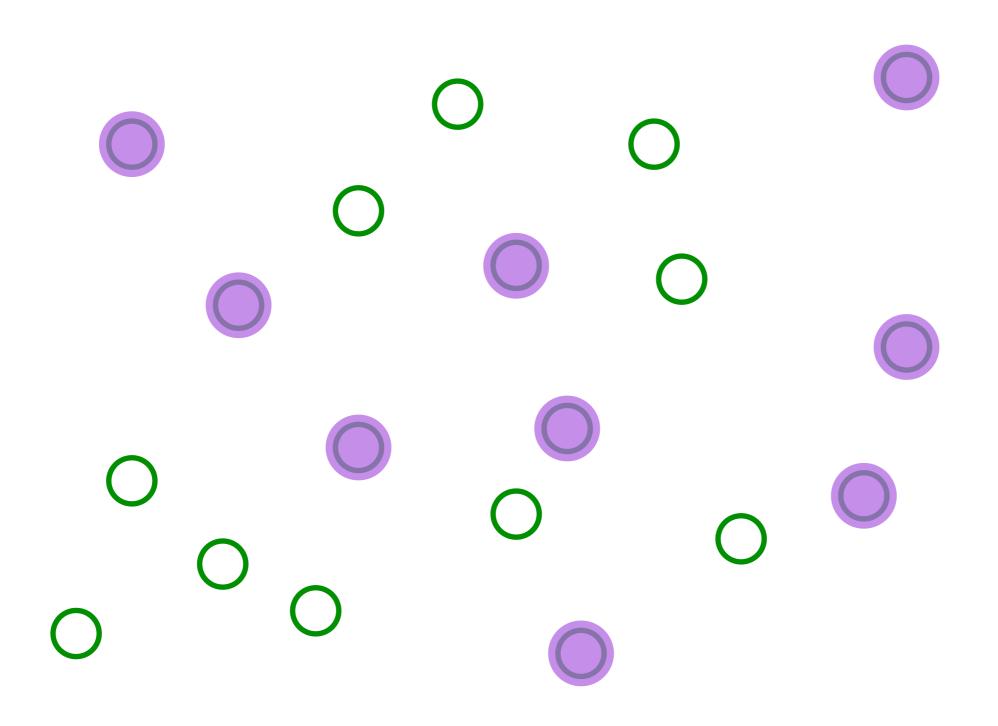
Pick a set of random positions

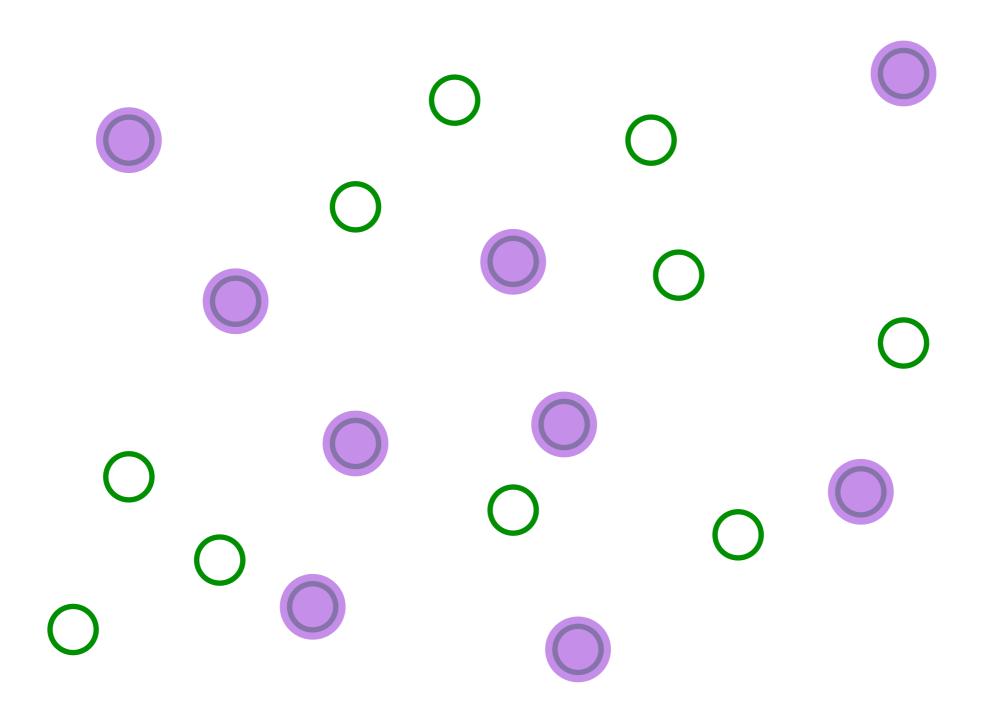


Place electrons randomly on some sites









$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^{\dagger} c_j + \dots$$

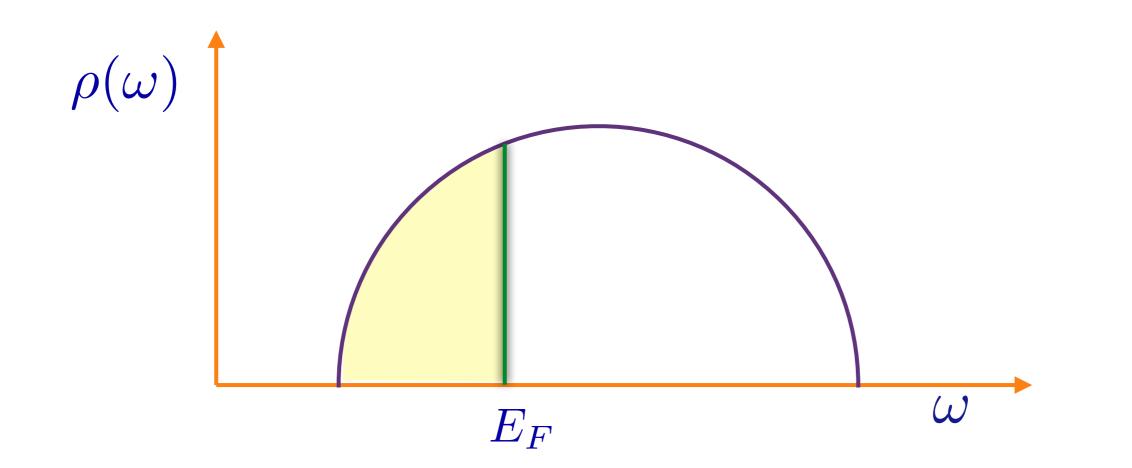
$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^{\dagger} + c_j^{\dagger} c_i = \delta_{ij}$$

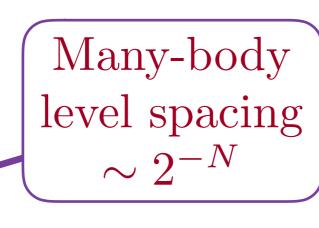
$$\frac{1}{N} \sum_i c_i^{\dagger} c_i = \mathcal{Q}$$

 t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $|t_{ij}|^2 = t^2$

Fermions occupying the eigenstates of a $N \times N$ random matrix

Let ε_{α} be the eigenvalues of the matrix t_{ij}/\sqrt{N} . The fermions will occupy the lowest $N\mathcal{Q}$ eigenvalues, upto the Fermi energy E_F . The density of states is $\rho(\omega) = (1/N) \sum_{\alpha} \delta(\omega - \varepsilon_{\alpha})$.





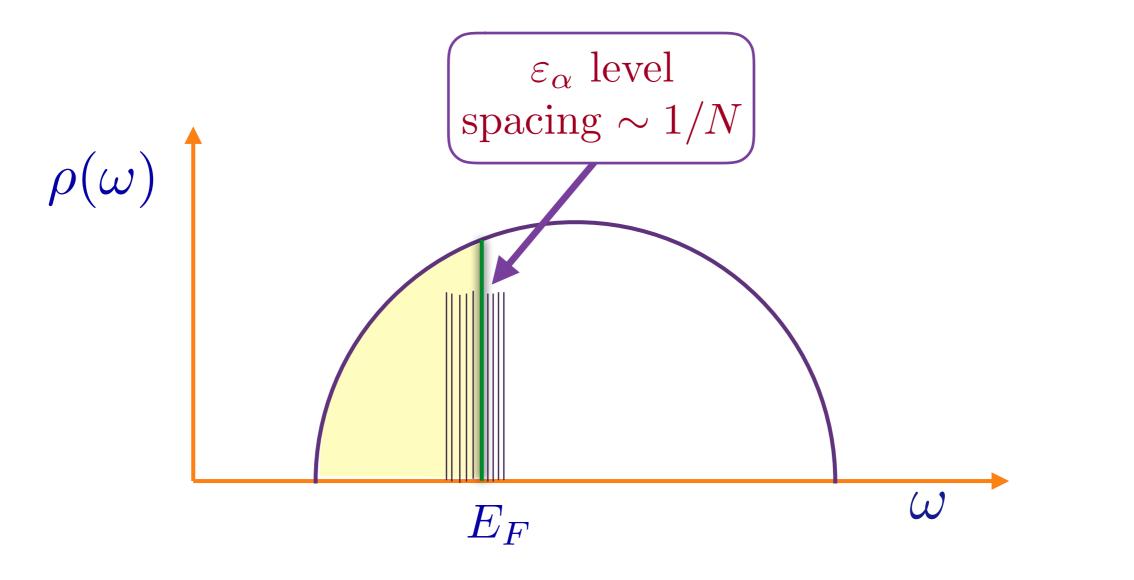
Quasiparticle excitations with spacing $\sim 1/N$

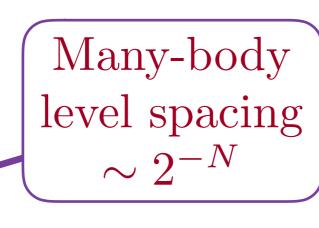
There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_{\alpha} \varepsilon_{\alpha},$$

where $n_{\alpha} = 0, 1$. Shown are all values of E for a single cluster of size N = 12. The ε_{α} have a level spacing $\sim 1/N$.

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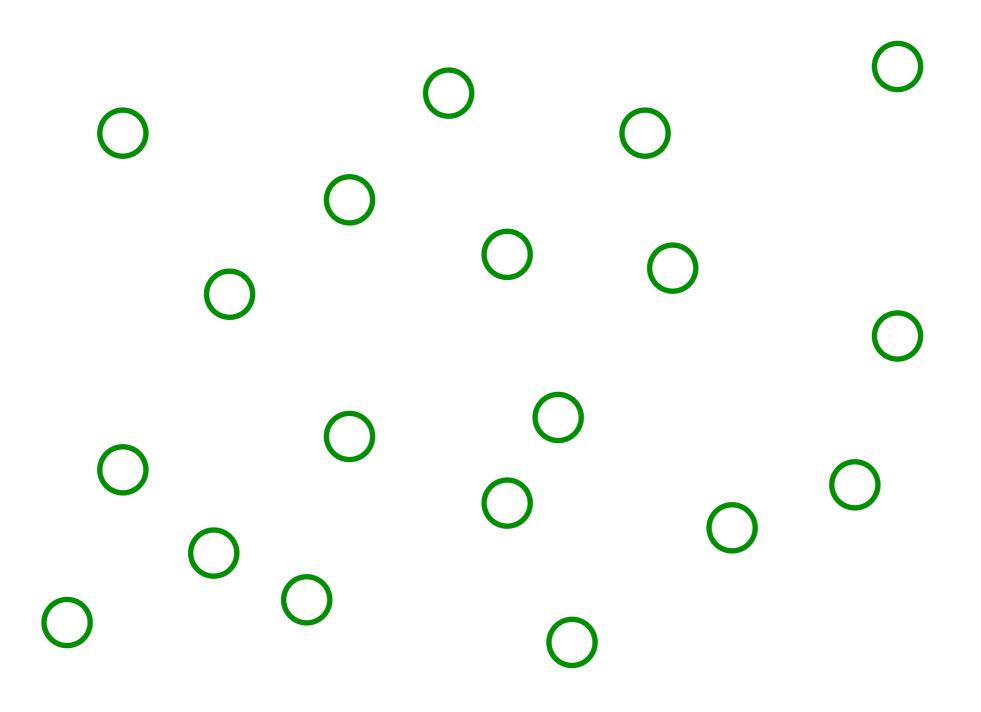
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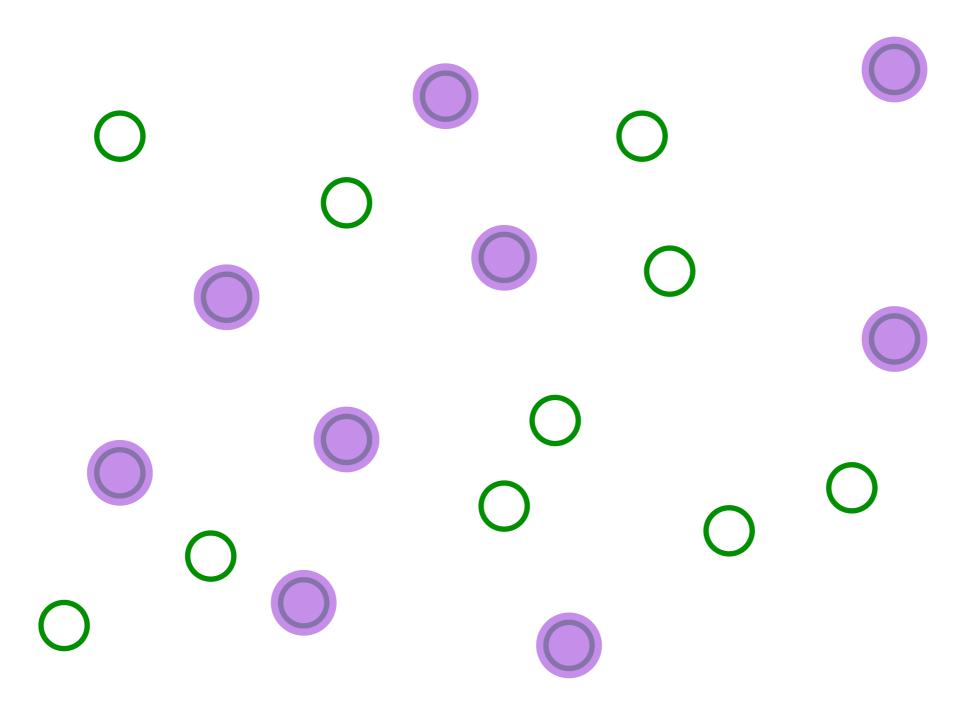
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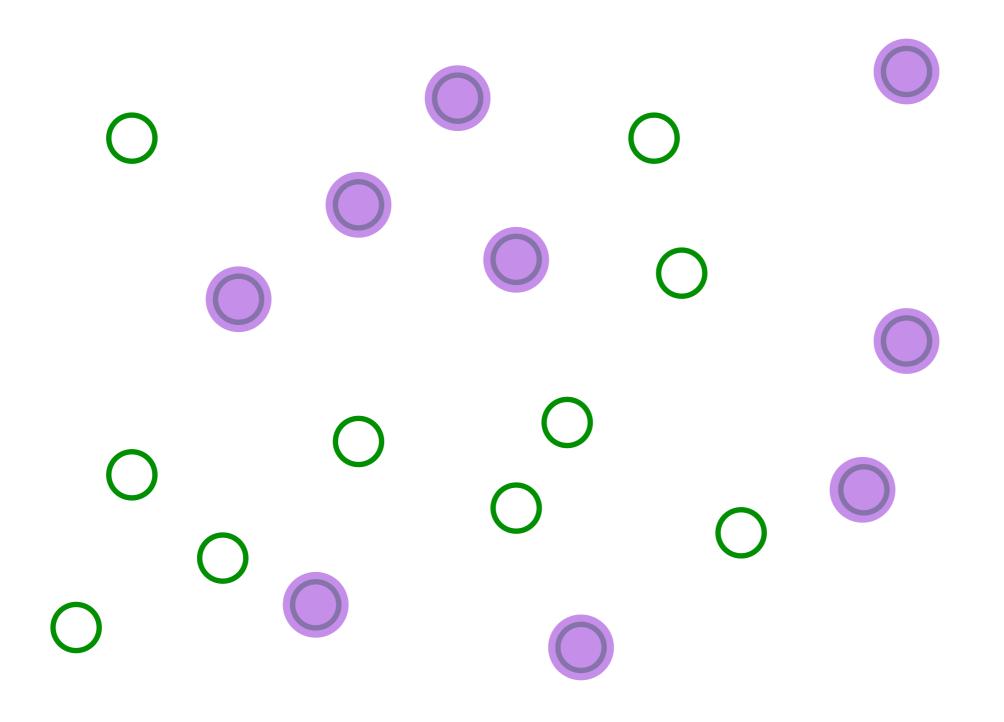
The Sachdev-Ye-Kitaev (SYK) model

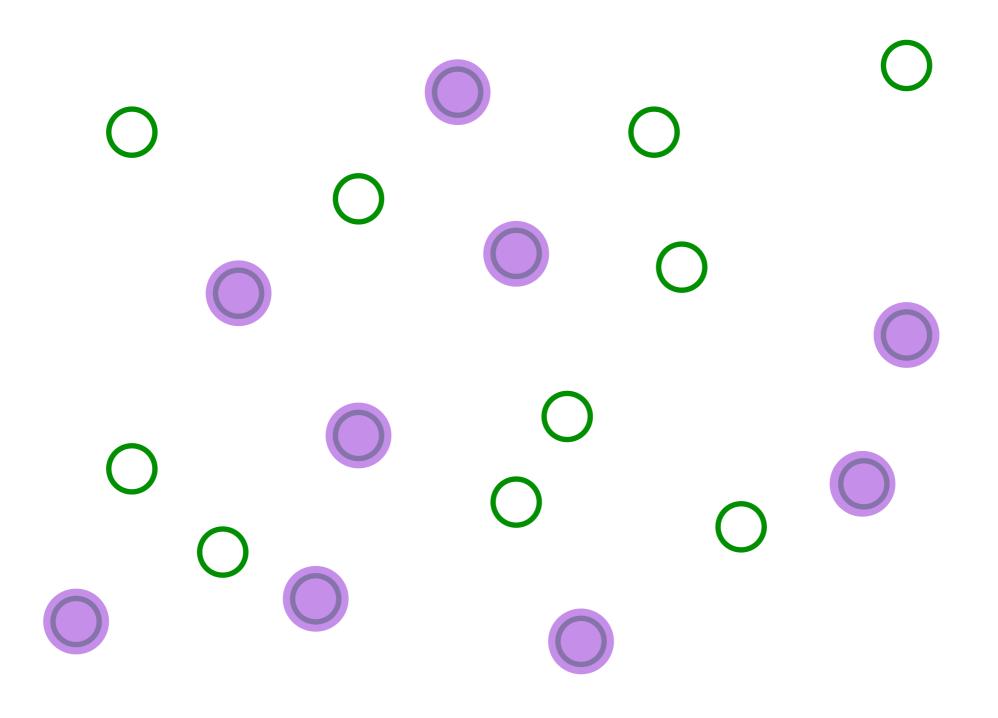


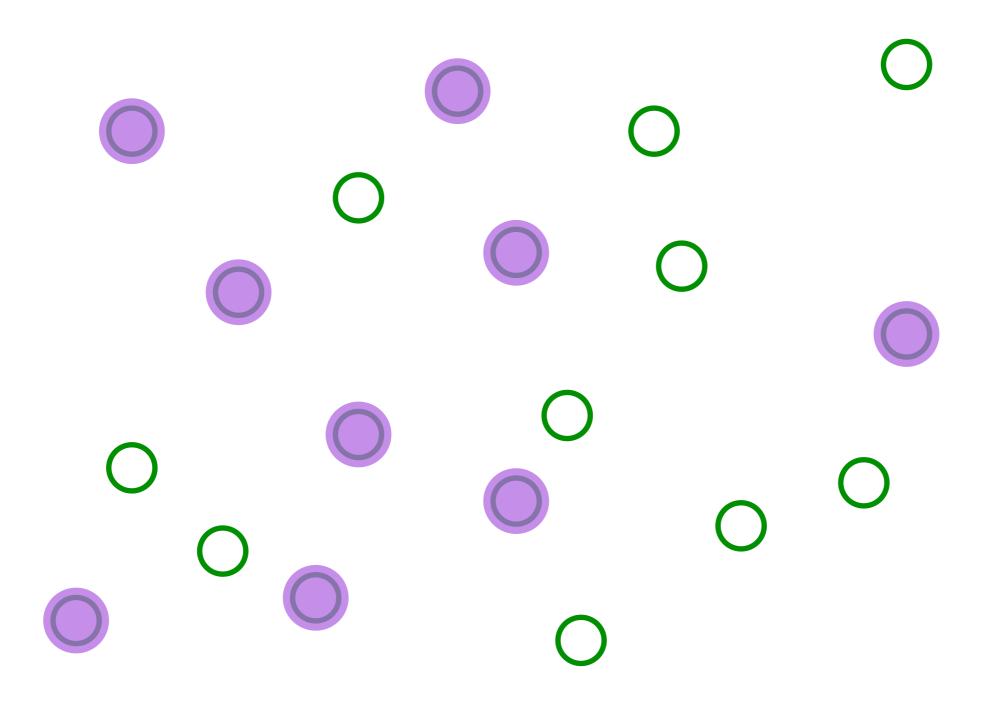
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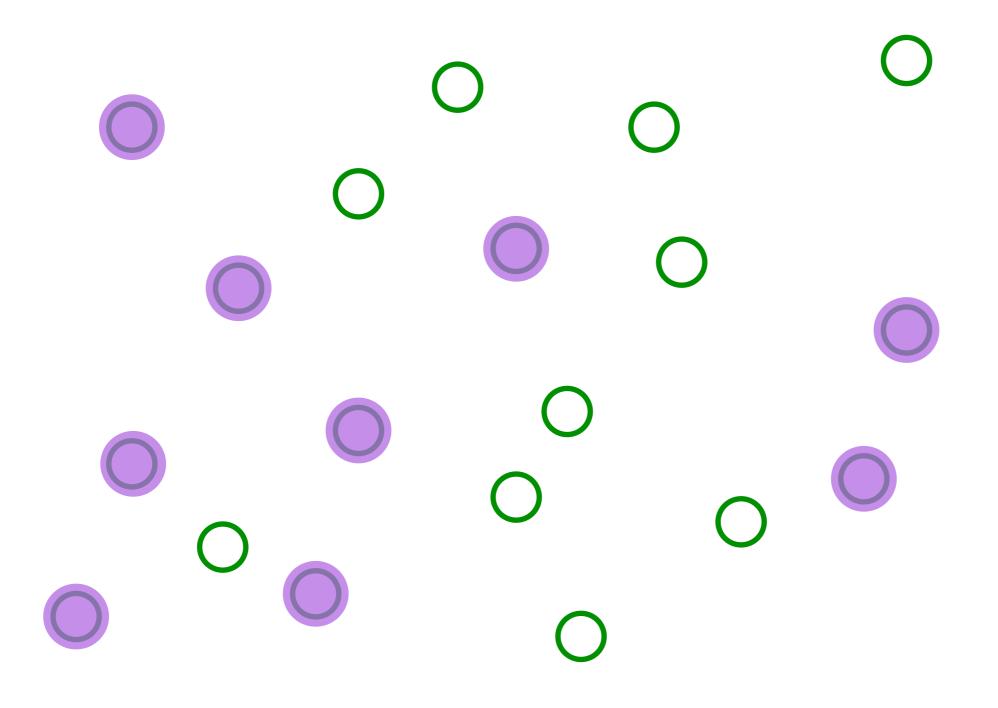


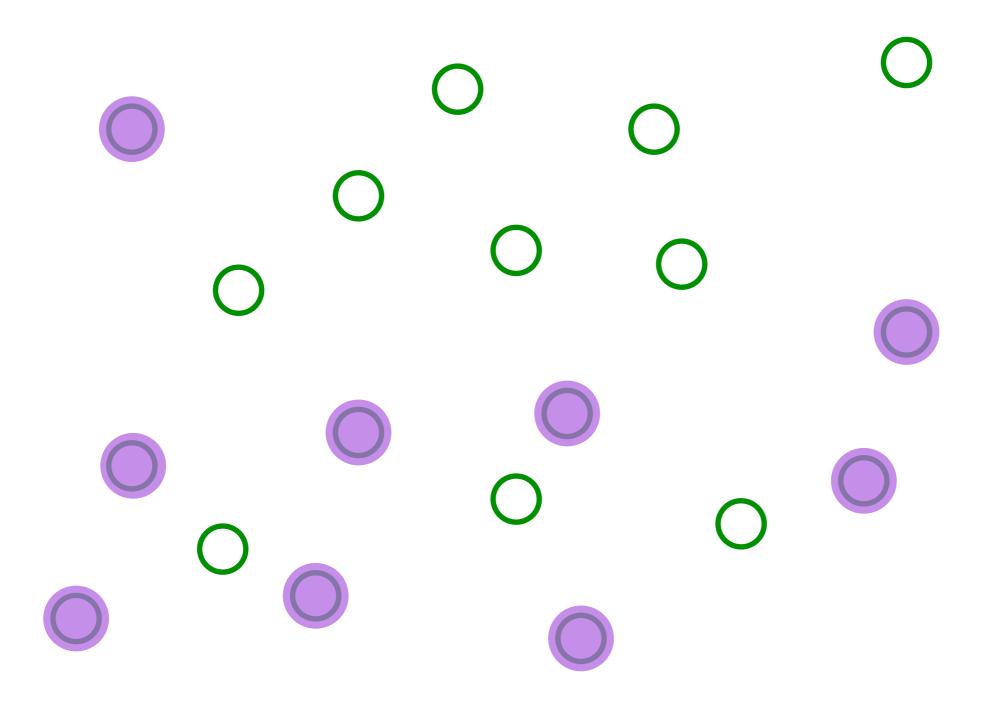
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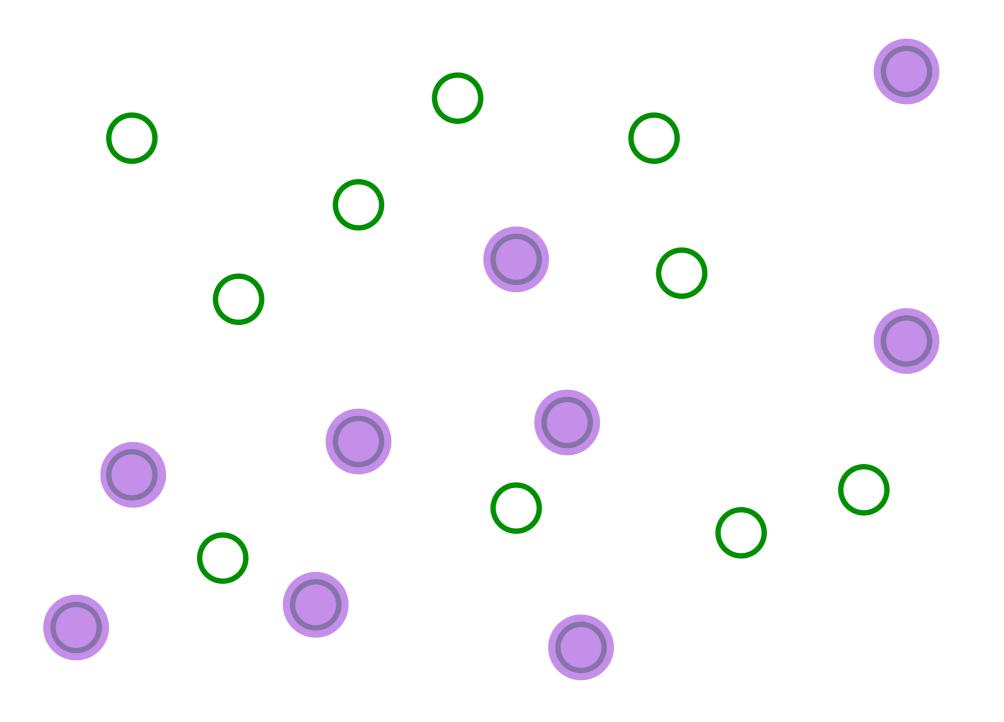


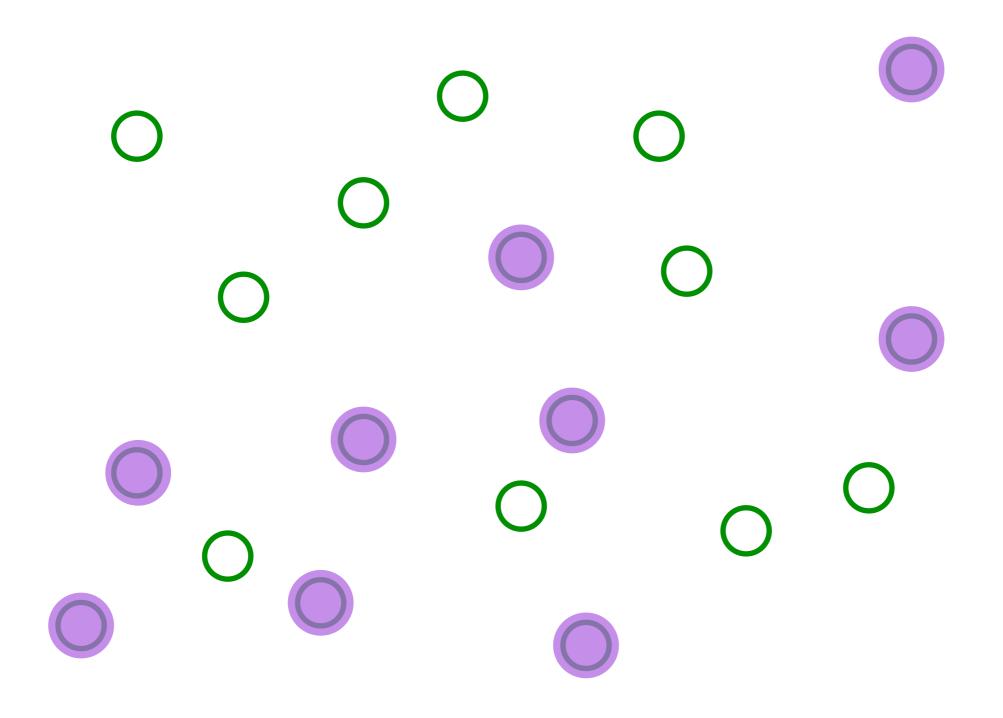










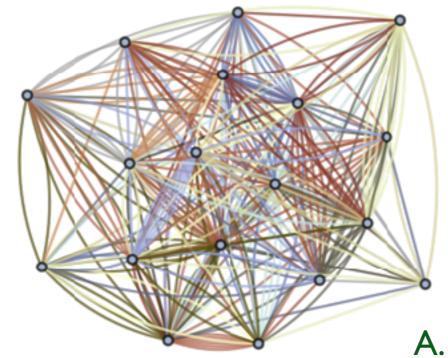


This describes both a strange metal and a black hole!

(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

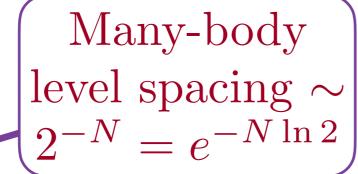
$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell} c_i^{\dagger} c_j^{\dagger} c_k c_{\ell} - \mu \sum_i c_i^{\dagger} c_i$$
$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^{\dagger} + c_j^{\dagger} c_i = \delta_{ij}$$
$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^{\dagger} c_i$$

 $U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$ $N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)



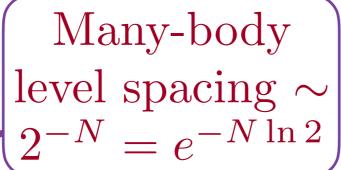
There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size N=12. The $T\to 0$ state has an entropy $S_{GPS}=Ns_0$ with

$$s_0 = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848...$$
< $\ln 2$

where G is Catalan's constant, for the half-filled case Q = 1/2.

Non-quasiparticle excitations with spacing $\sim e^{-Ns_0}$

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)



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No quasiparticles!

$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

PRB 63, 134406 (2001)

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A. Georges, O. Parcollet, and S. Sachdev, PRB 63, 134406 (2001)

D. Stanford and E. Witten, 1703.04612

A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593

D. Bagrets, A. Altland, and A. Kamenev, 1607.00694

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• Low temperature entropy $S = Ns_0 + N\gamma T + \dots$

A. Kitaev, unpublished J. Maldacena and D. Stanford, 1604.07818

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S. Sachdev, Quantum Phase Transitions, Cambridge (1999)

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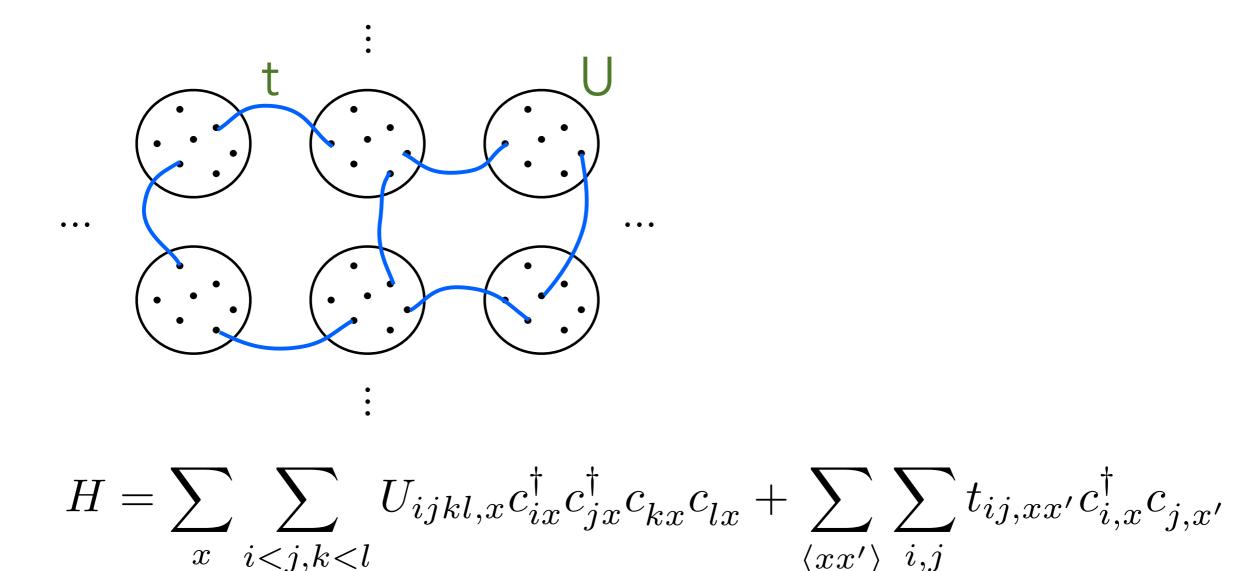
• Systems without quasiparticles are the fastest possible in reaching local equilibrium, and all many-body quantum systems obey, as $T \to 0$

$$au_{\rm eq} > C \frac{\hbar}{k_B T} \,.$$

S. Sachdev, Quantum Phase Transitions, Cambridge (1999)

- In Fermi liquids $\tau_{\rm eq} \sim 1/T^2$, and so the bound is obeyed as $T \to 0$.
- This bound rules out quantum systems with e.g. $\tau_{eq} \sim \hbar/(Jk_BT)^{1/2}$.
- There is no bound in classical mechanics ($\hbar \to 0$). By cranking up frequencies, we can attain equilibrium as quickly as we desire.

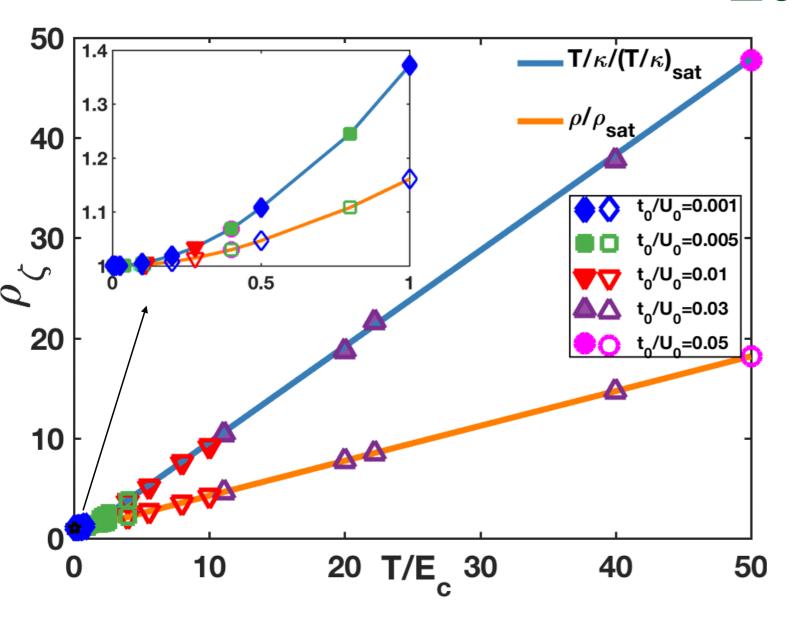
Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models Authors: <u>Xue-Yang Song</u>, <u>Chao-Ming Jian</u>, <u>Leon Balents</u>



$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N.$$

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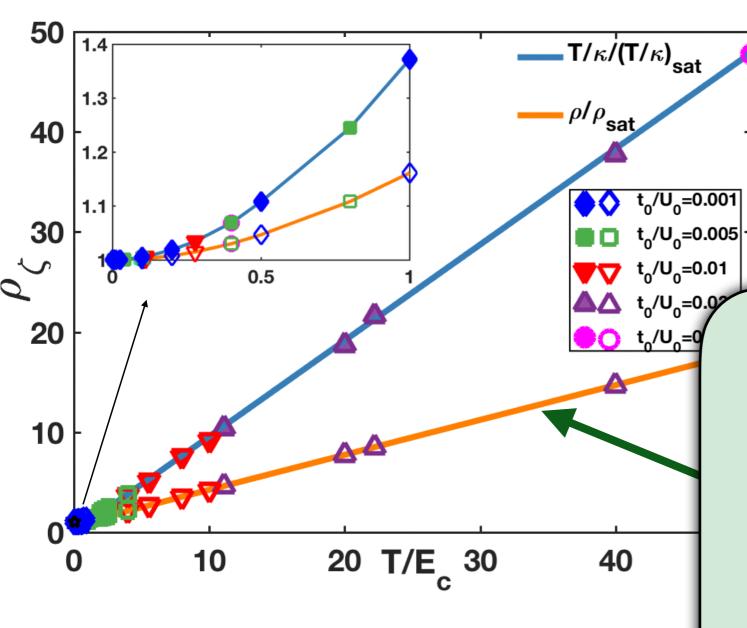
Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

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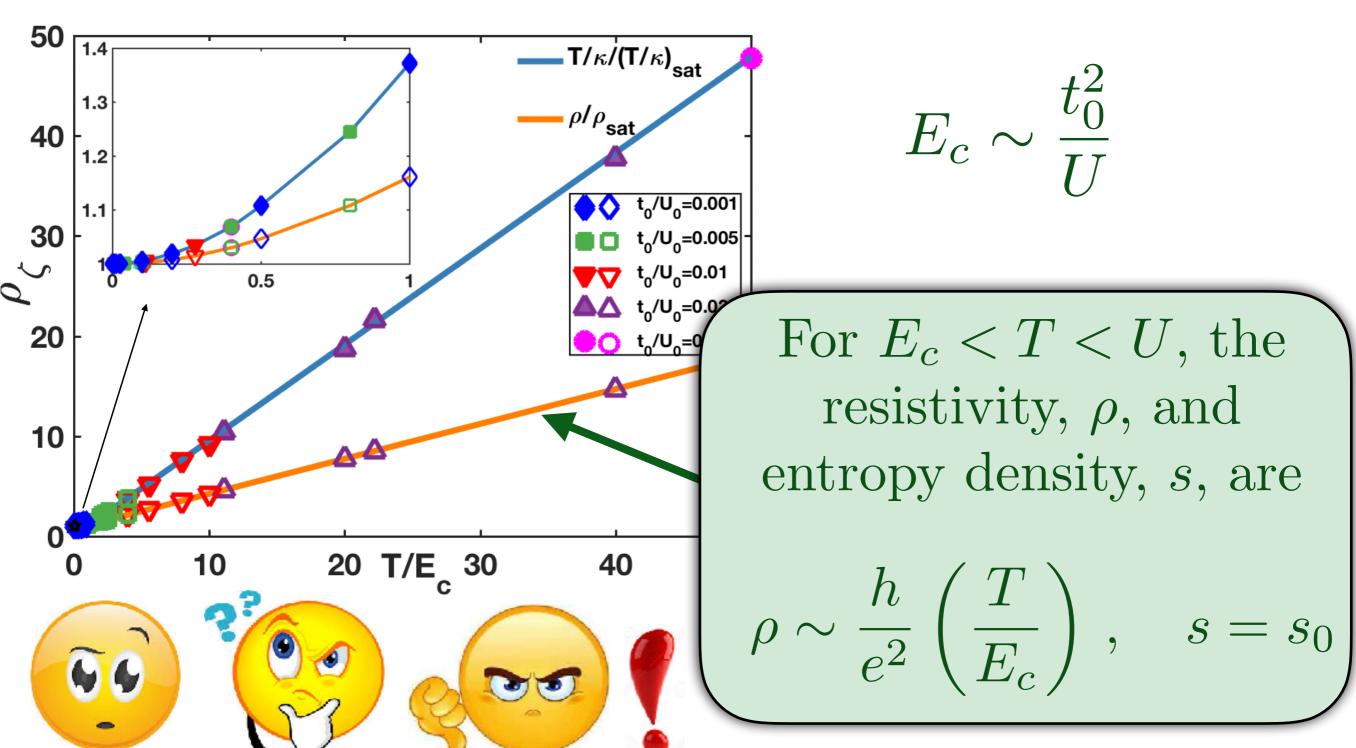
$$E_c \sim \frac{t_0^2}{U}$$

For $E_c < T < U$, the resistivity, ρ , and entropy density, s, are

$$\rho \sim \frac{h}{e^2} \left(\frac{T}{E_c} \right) , \quad s = s_0$$

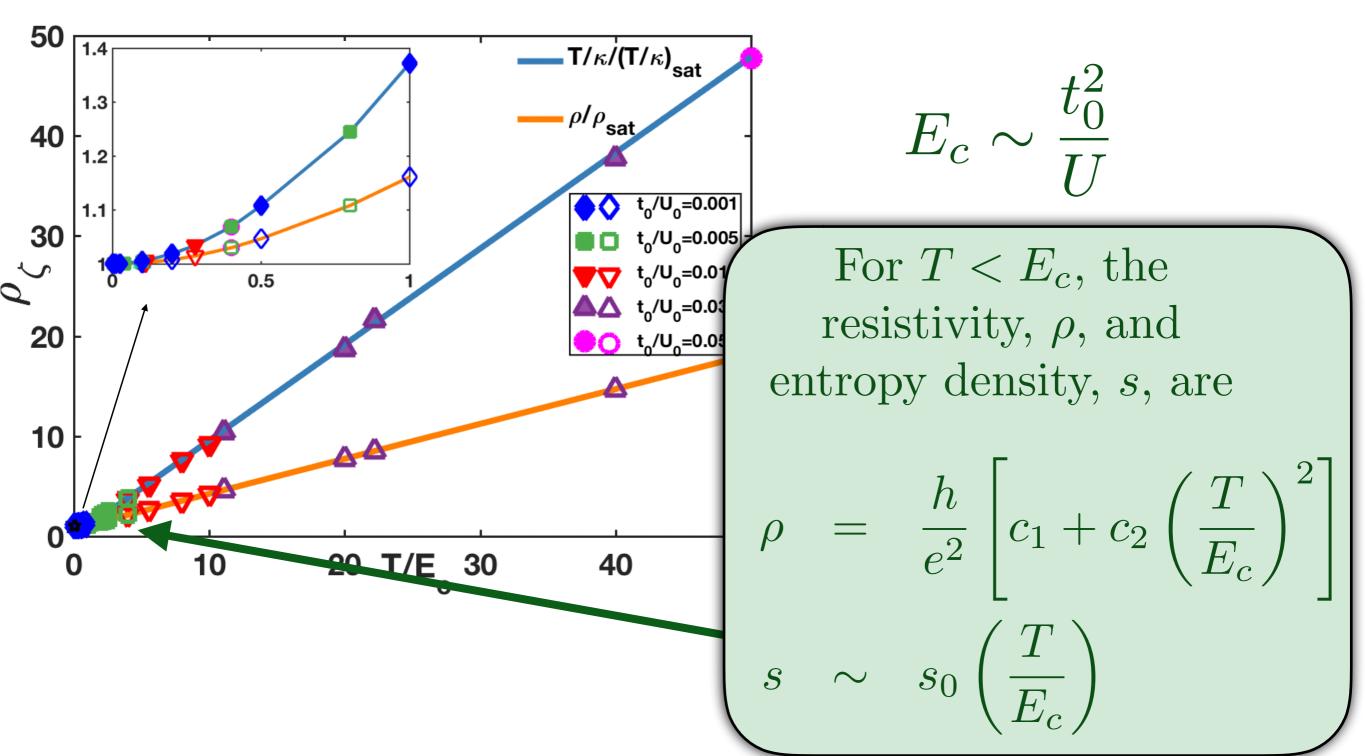
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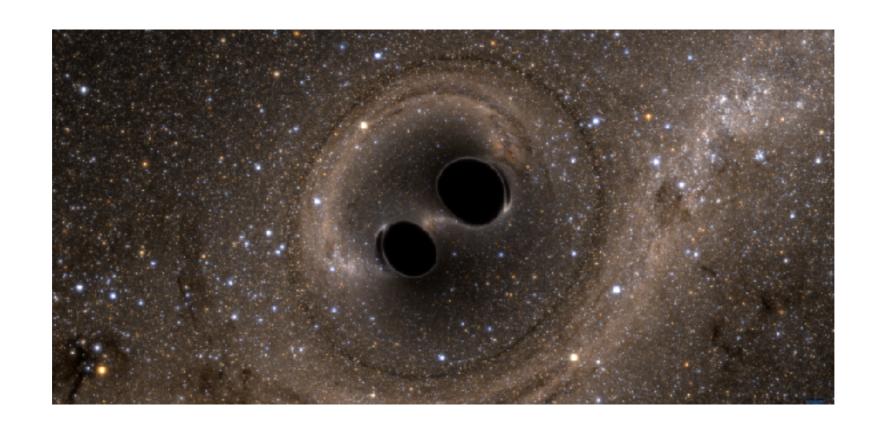
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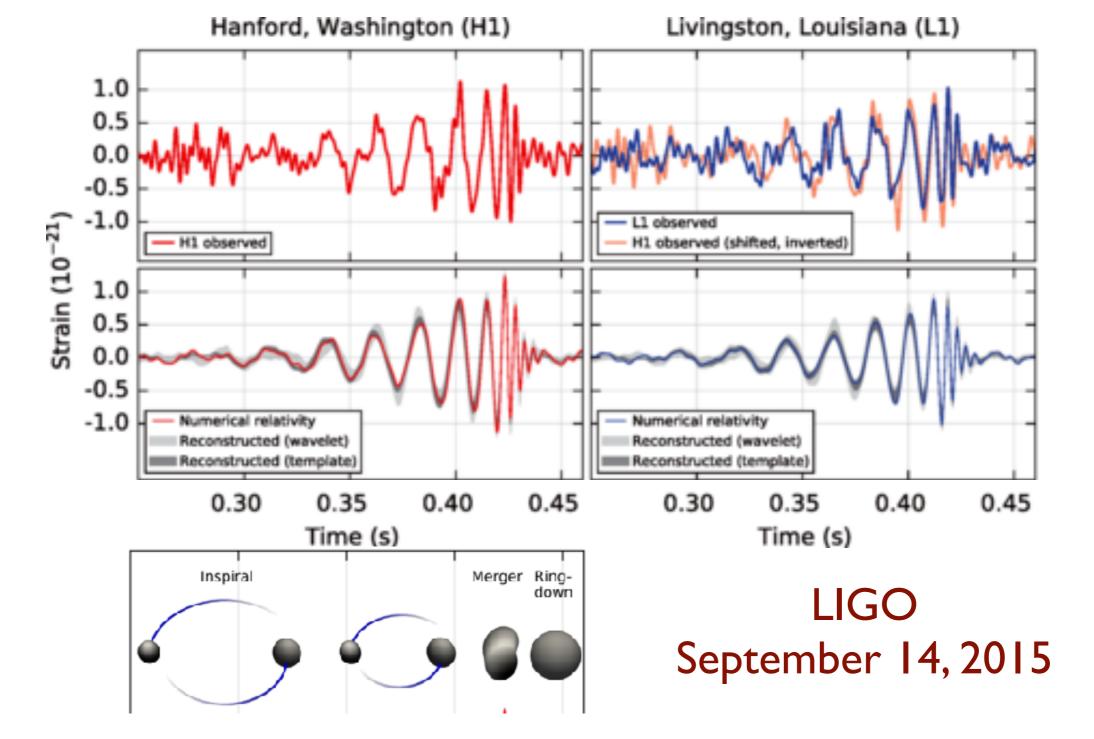
Low 'coherence' scale



Black

- Black holes have an entropy and a temperature, T_H .
- The entropy is proportional to their surface area.

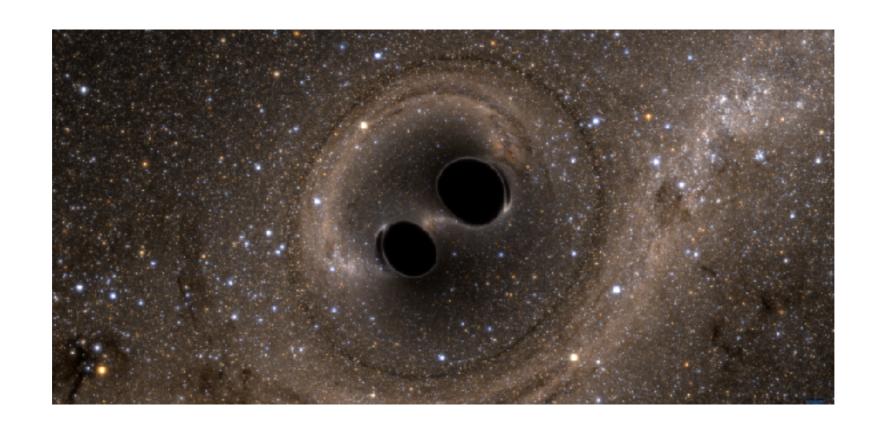




• The Hawking temperature, T_H influences the radiation from the black hole at the very last stages of the ring-down (not observed so far). The ring-down (approach to thermal equilibrium) happens very rapidly in a time $\sim \frac{\hbar}{k_B T_H} = \frac{8\pi GM}{c^3} \sim 8$ milliseconds.

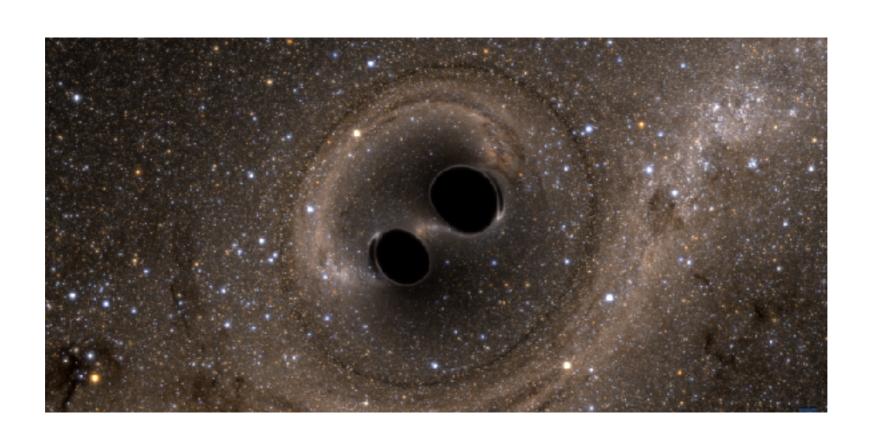
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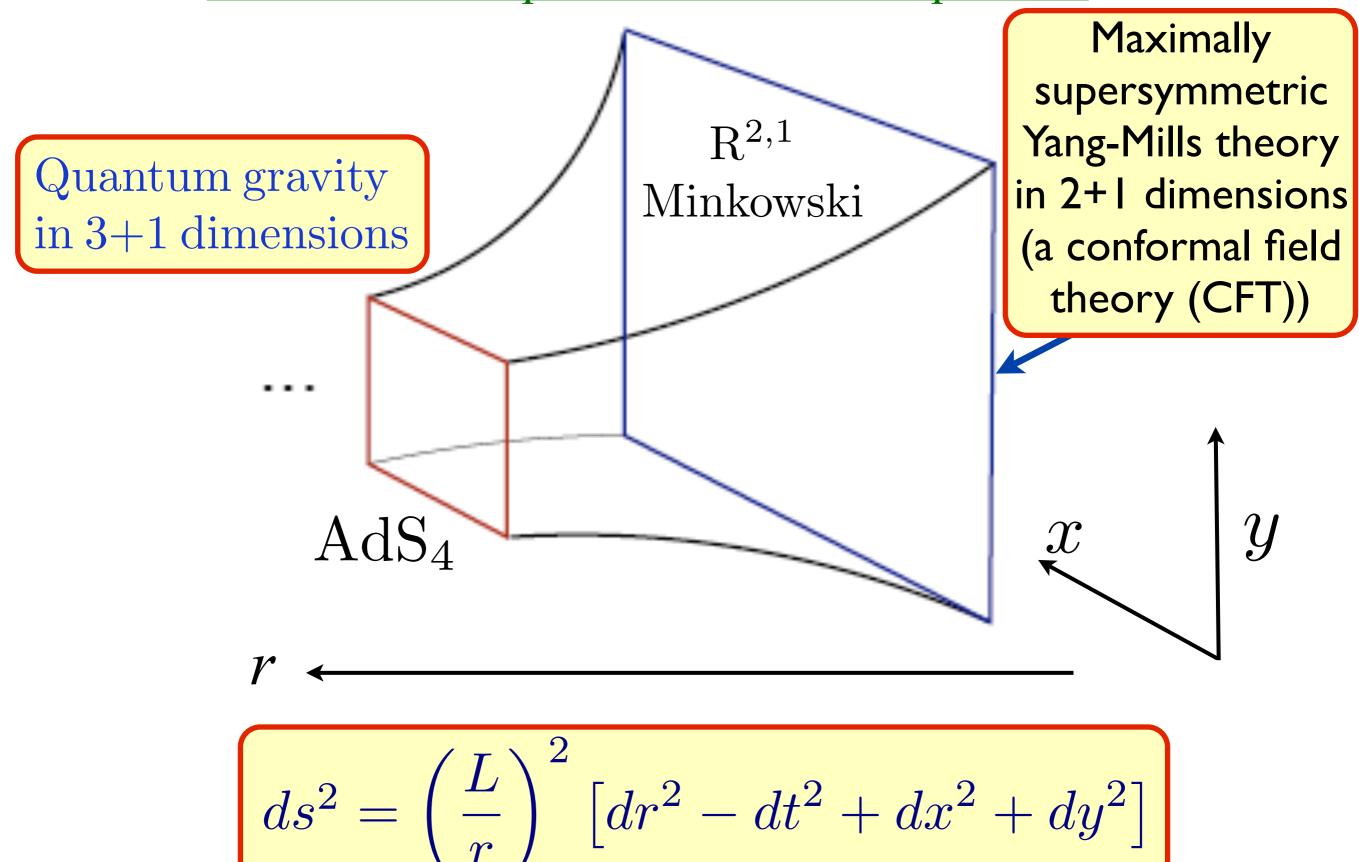


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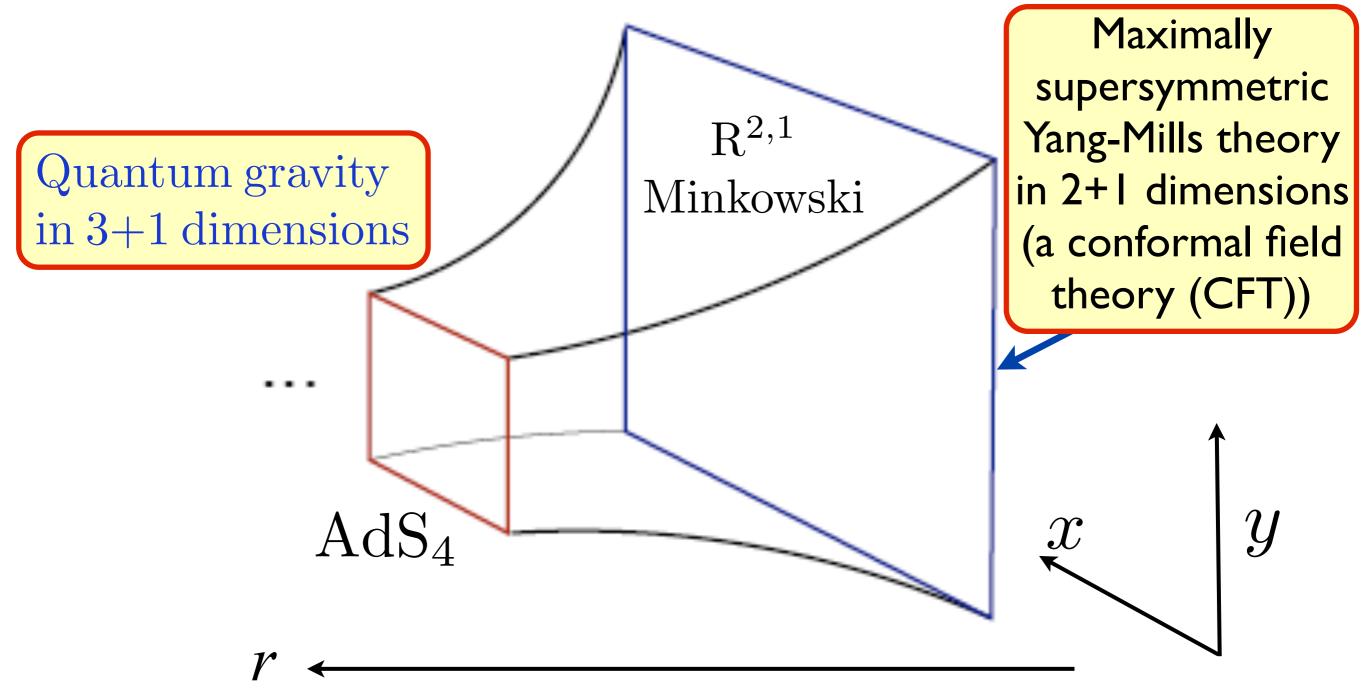
- Black holes have an entropy and a temperature, T_H .
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.



AdS/CFT correspondence at zero temperature



AdS/CFT correspondence at zero temperature



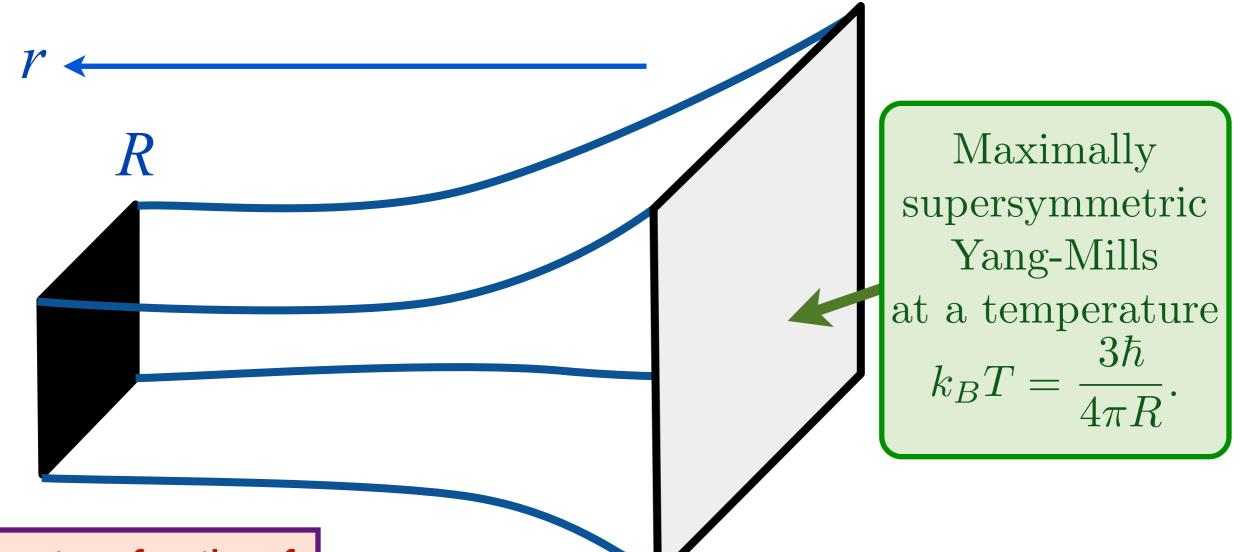
This spacetime is a solution of Einstein gravity with a negative cosmological constant

$$S_E = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

, Polyakov, Witten

AdS/CFT correspondence at non-zero temperatures

AdS₄-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

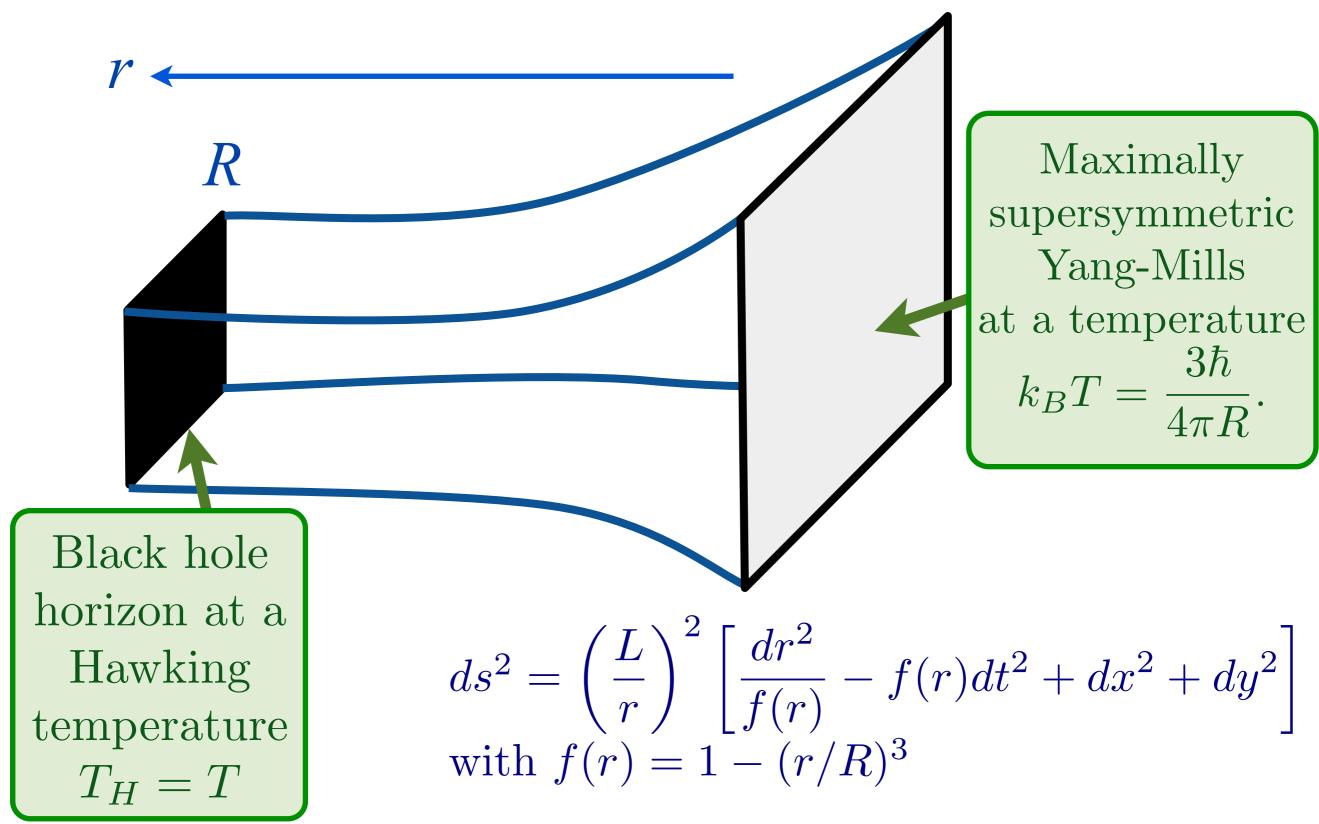
$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence at non-zero temperatures

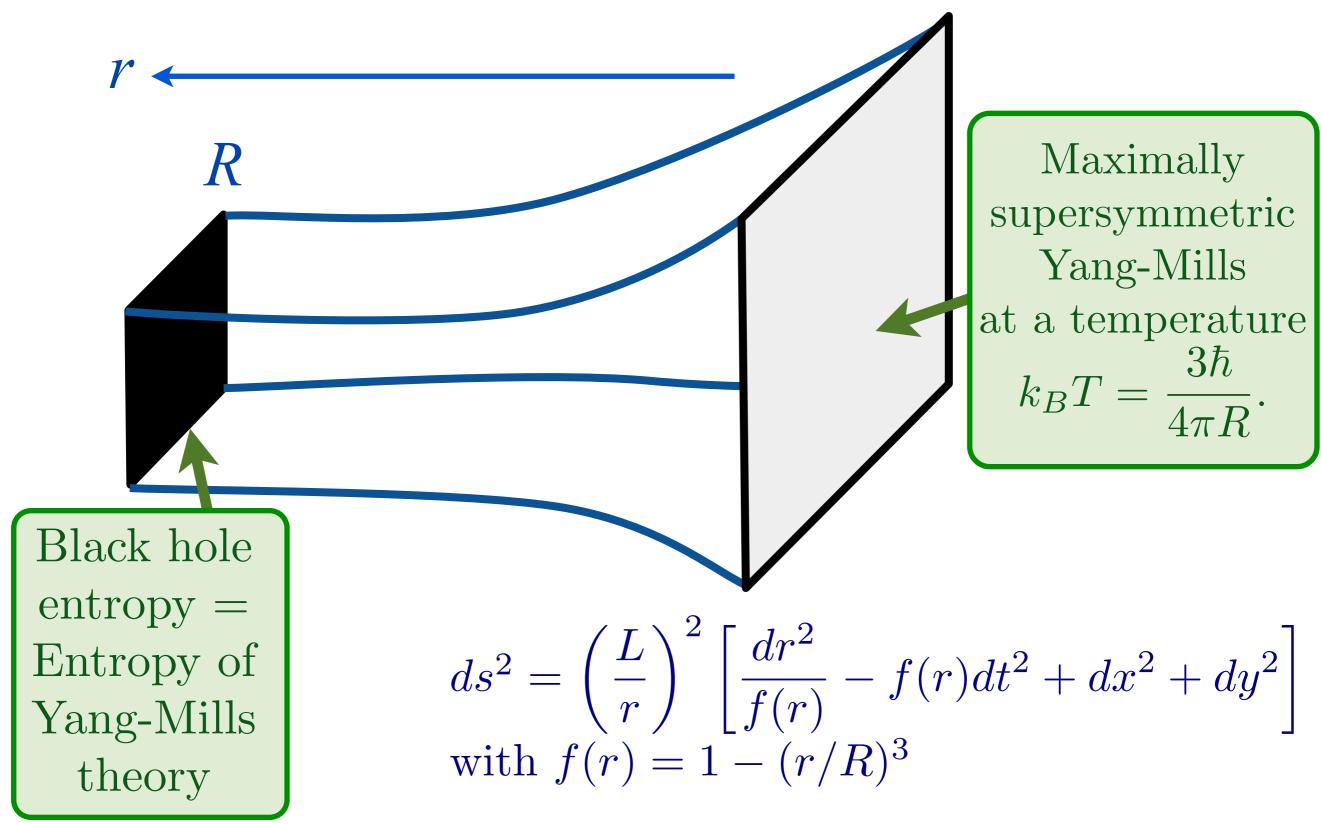
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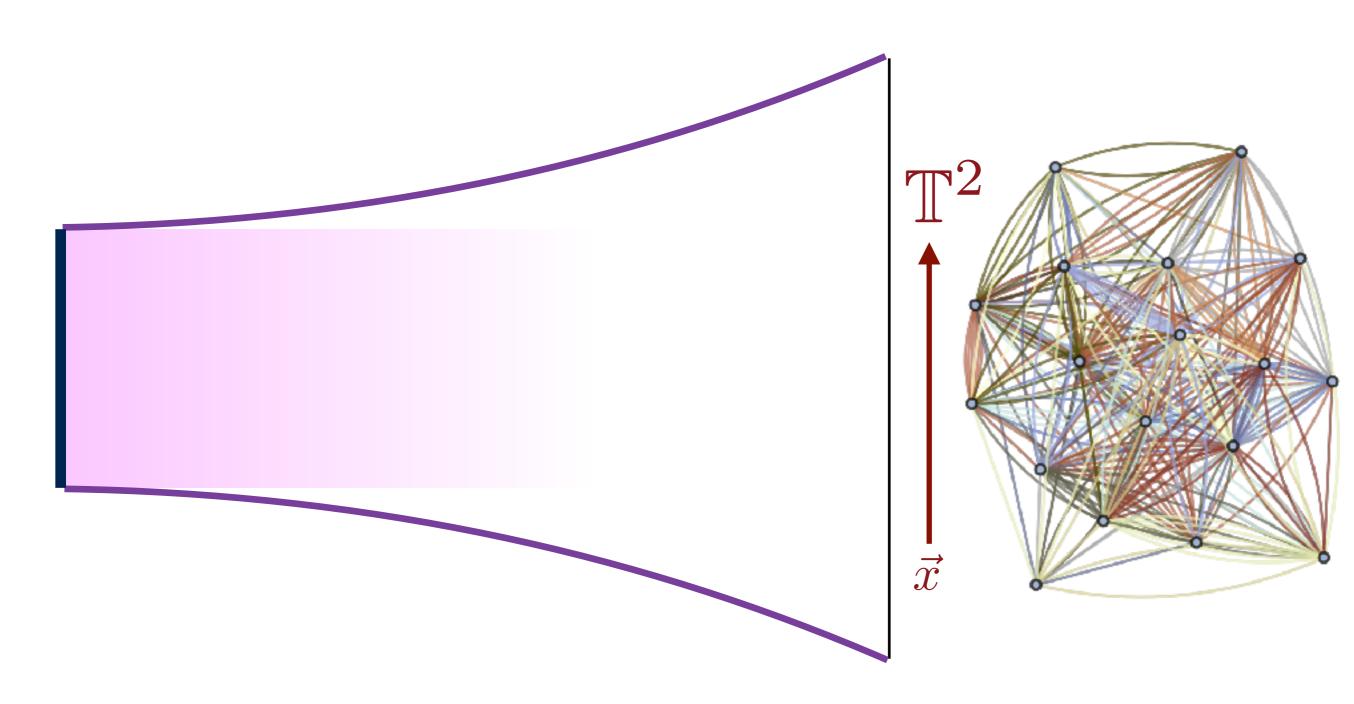
Maldacena, Gubser, Klebanov, Polyakov, Witten

AdS/CFT correspondence at non-zero temperatures

AdS₄-Schwarzschild black-brane



Maldacena, Gubser, Klebanov, Polyakov, Witten



Is there a holographic quantum gravity dual of the SYK model?

• Low energy, many-body density of states $\rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E-E_0)N\gamma})$

- Low temperature entropy $S = Ns_0 + N\gamma T + \dots$
- T=0 fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ . (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

- T > 0 Green's function has conformal invariance $G \sim (T/\sin(\pi k_B T \tau/\hbar))^{1/2}$
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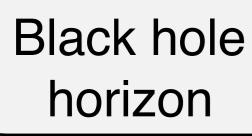
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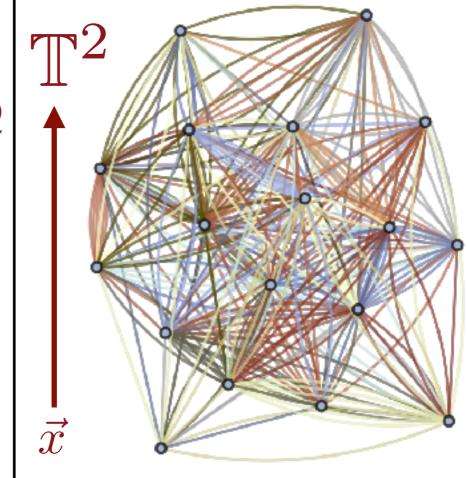
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- The ls there any black hole which holographically had matches these properties?

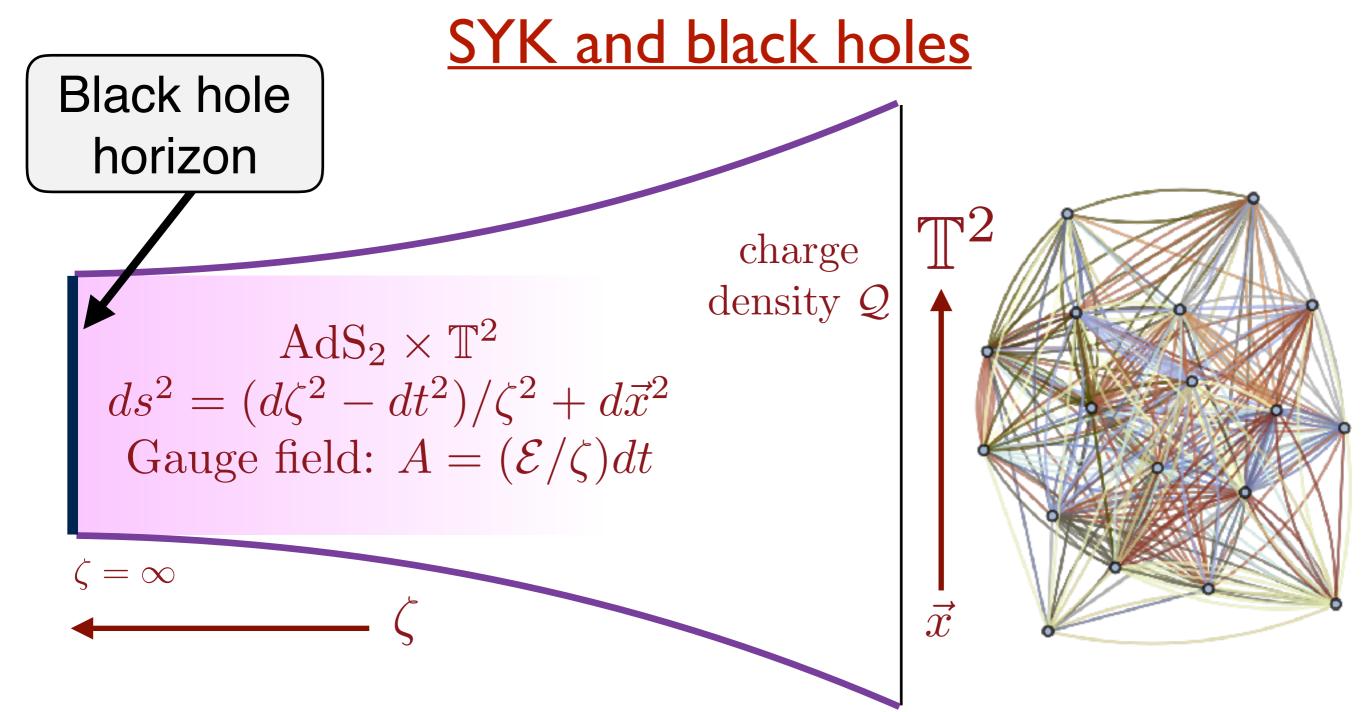


 $\begin{array}{c} \text{charge} \\ \text{density } \mathcal{Q} \end{array}$



$$S = \int d^4x \sqrt{-\hat{g}} \left(\hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

Yes, a charged black hole in Einstein-Maxwell theory



The leading low temperature properties of an AdS-Reissner-Nordstrom black hole (as computed by T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694) match those of the SYK model. The mapping applies when temperature $\ll 1/(\text{size of }\mathbb{T}^2)$.

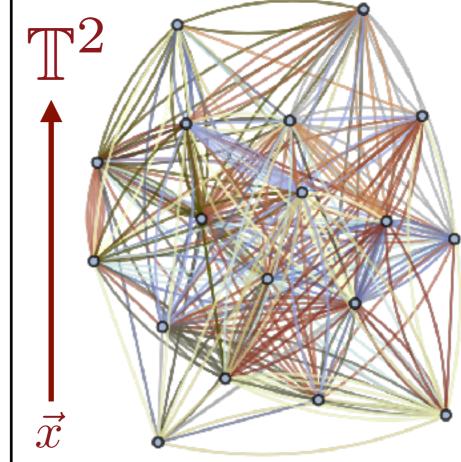
Black hole horizon

 $\begin{array}{c} \text{charge} \\ \text{density } \mathcal{Q} \end{array}$

$$AdS_2 \times \mathbb{T}^2$$

$$ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$$
 Gauge field: $A = (\mathcal{E}/\zeta)dt$

$$\zeta = \infty$$



Quantum gravity on the I+I dimensional spacetime AdS₂ (when embedded in AdS₄) is holographically matched to the 0+I dimensional SYK model

• Low energy, many-body density of states $\rho(E) \sim e^{Ns_0}$

- Low temperature entropy $S = Ns_0$
- T=0 fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ . (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

- T > 0 Green's function has conformal invariance
 - All these properties of the SYK model match those of the AdS₂ horizon in Einstein-Maxwell theory

• Low energy, many-body density of states $\rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E-E_0)N\gamma})$

- Low temperature entropy $S = Ns_0 + N\gamma T + \dots$
- T=0 fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ . (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

• T > 0 Green's function has conformal invariance

Schwarzian theory of quantum gravity fluctuations also matches these corrections

A Kitaev, unpublished, J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857; D. Stanford and E. Witten, arXiv:1703.04612

Many-body quantum chaos

• Using holographic analogies, Shenker and Stanford introduced the "Lyapunov time", τ_L , the time over which a generic many-body quantum system loses memory of its initial state.

S. Shenker and D. Stanford, arXiv:1306.0622

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• The SYK model, and black holes in Einstein gravity, saturate the bound on the Lyapunov time

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A. Kitaev, unpublished J. Maldacena and D. Stanford, arXiv:1604.07818

• No quasiparticle decomposition of low-lying states:

$$E \neq \sum_{\alpha} n_{\alpha} \varepsilon_{\alpha} + \sum_{\alpha,\beta} F_{\alpha\beta} n_{\alpha} n_{\beta} + \dots$$

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- These are also characteristics of black holes in quantum gravity.