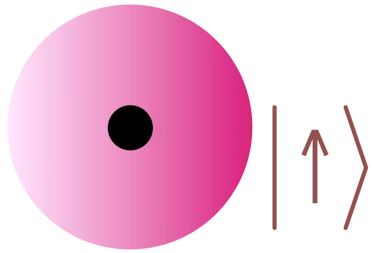
Quantum Criticality and Black Holes

Subir Sachdev

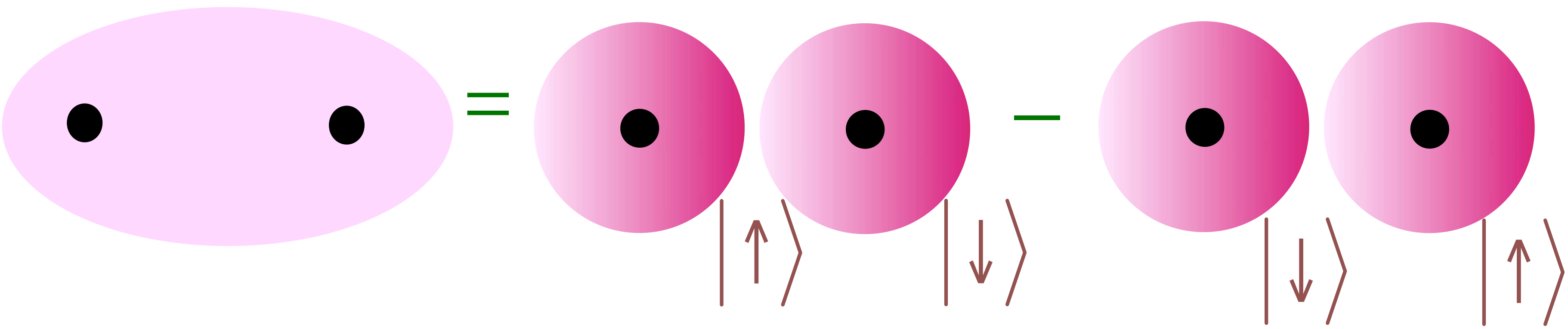
Talk online at http://sachdev.physics.harvard.edu

Quantum Entanglement

Hydrogen atom:



Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Superposition of two electron states leads to non-local correlations between spins

Quantum Phase Transition

Change in the nature of entanglement in a macroscopic quantum system.

Familiar phase transitions, such as water boiling to steam, also involve macroscopic changes, but in thermal motion

Quantum Criticality

The complex and non-local entanglement at the critical point between two quantum phases

Outline

1. Entanglement of spins

   Experiments on spin-gap insulators

2. Entanglement of valence bonds

   Deconfined criticality in antiferromagnets

3. Black Hole Thermodynamics

   Connections to quantum criticality

4. Nernst effect in the cuprate superconductors

   Quantum criticality and dyonic black holes

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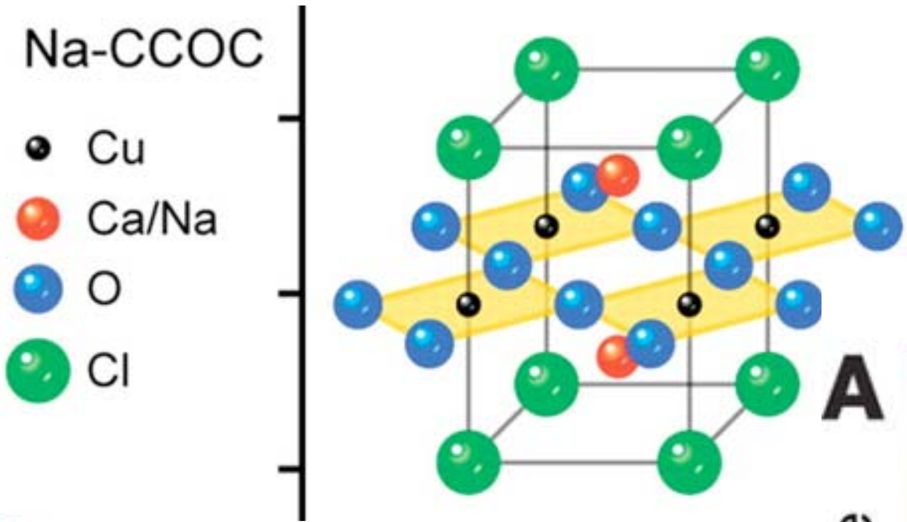
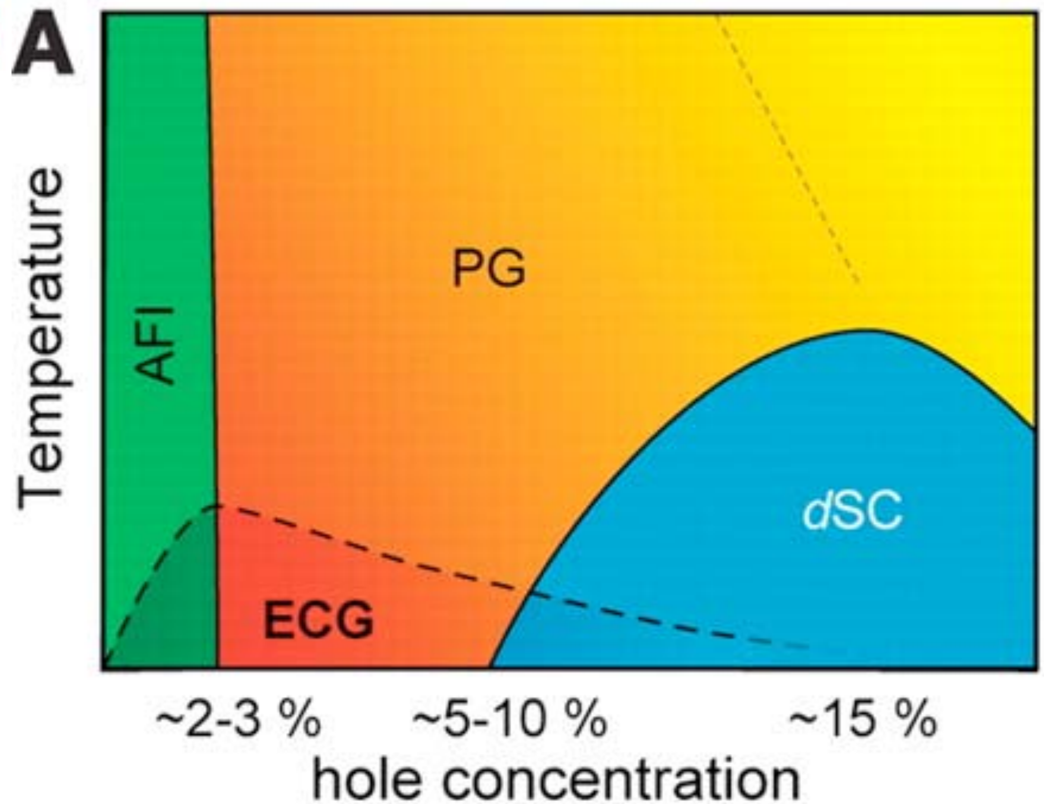
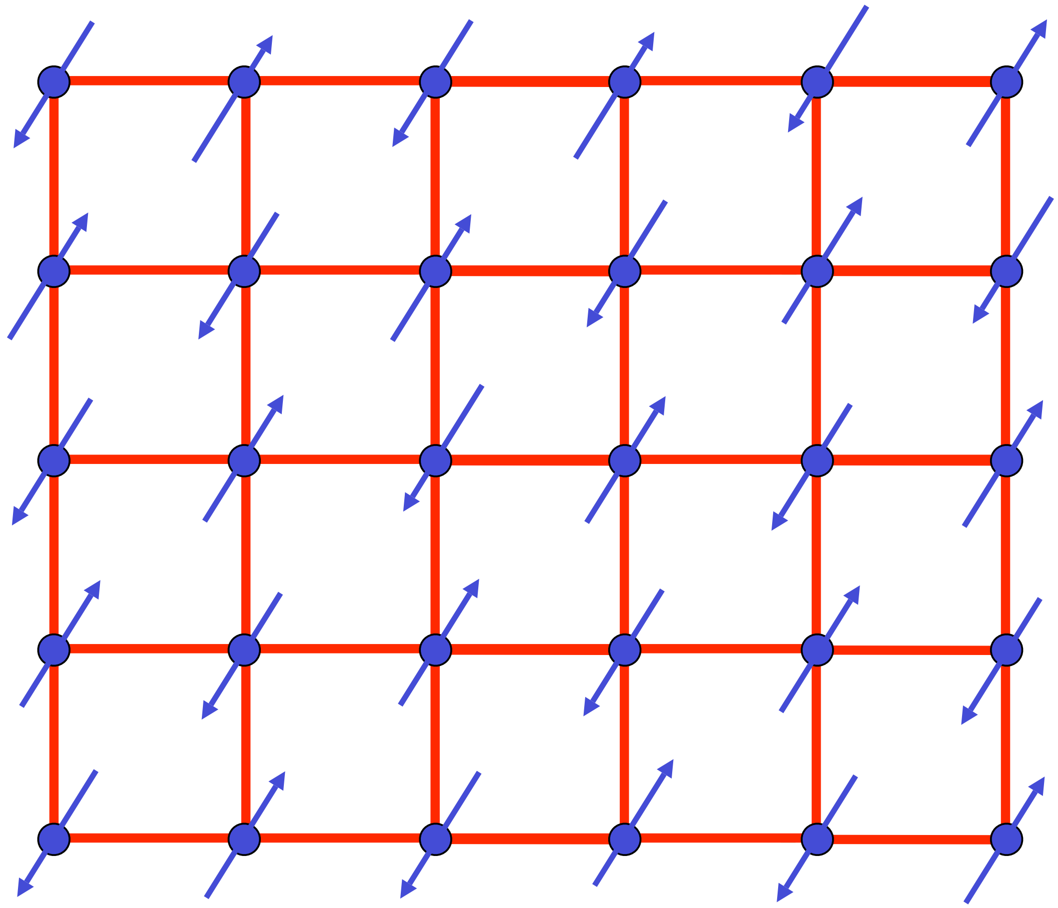
The cuprate superconductors

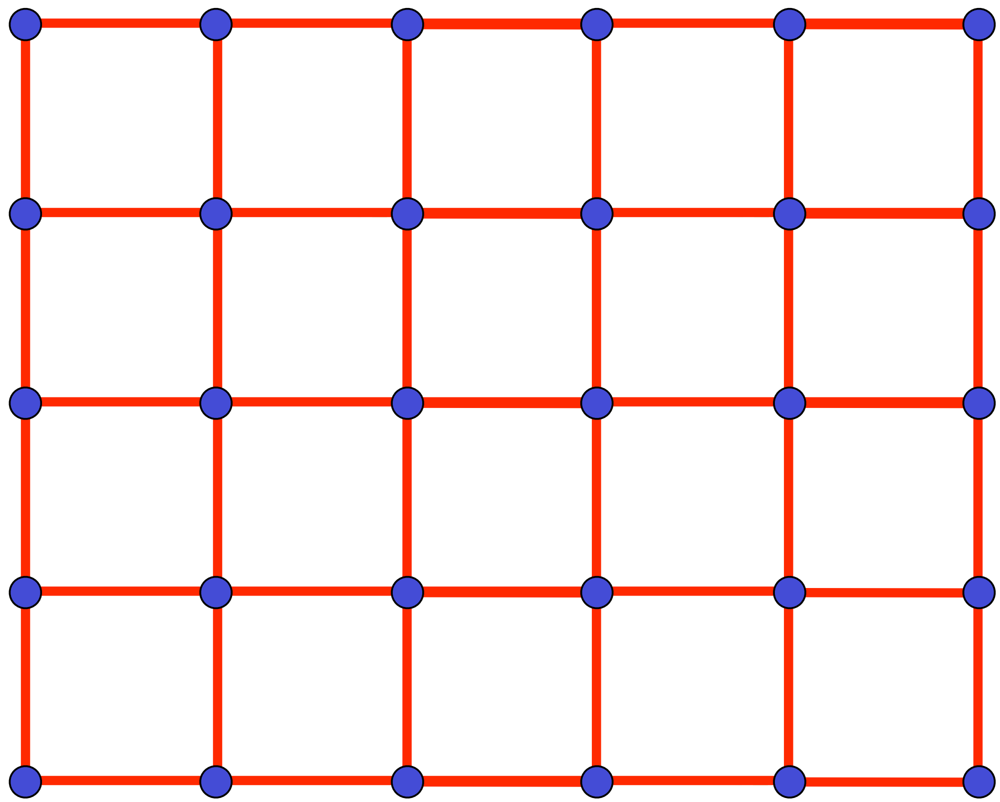




Antiferromagnetic (Neel) order in the insulator



No entanglement of spins



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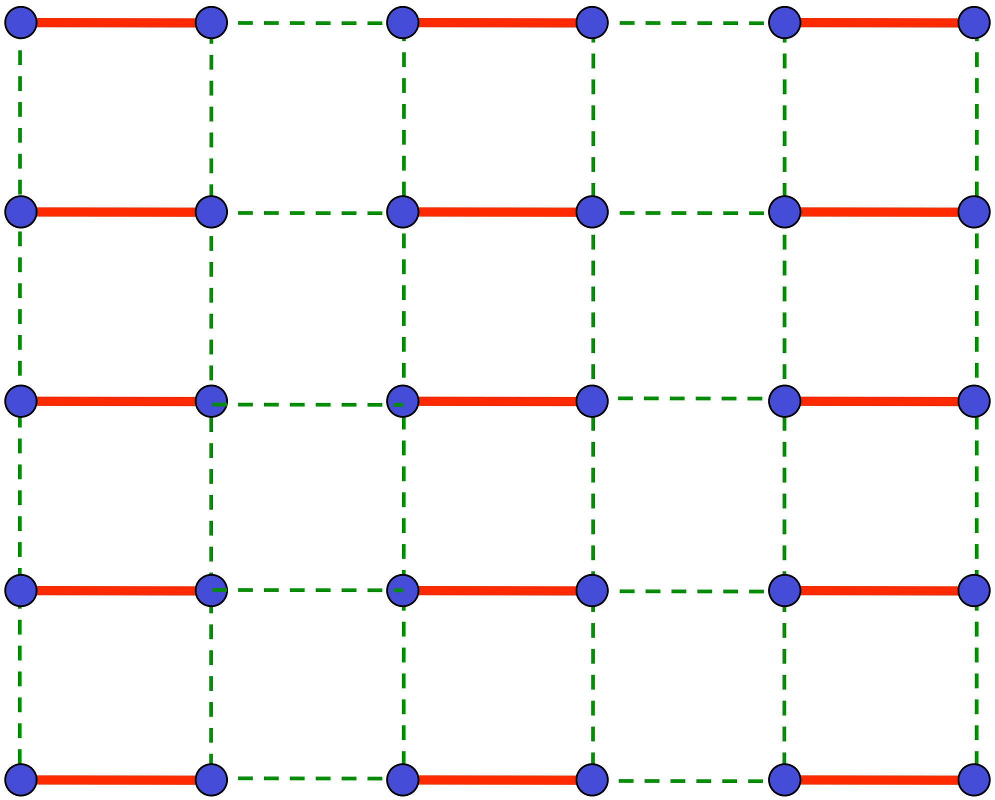
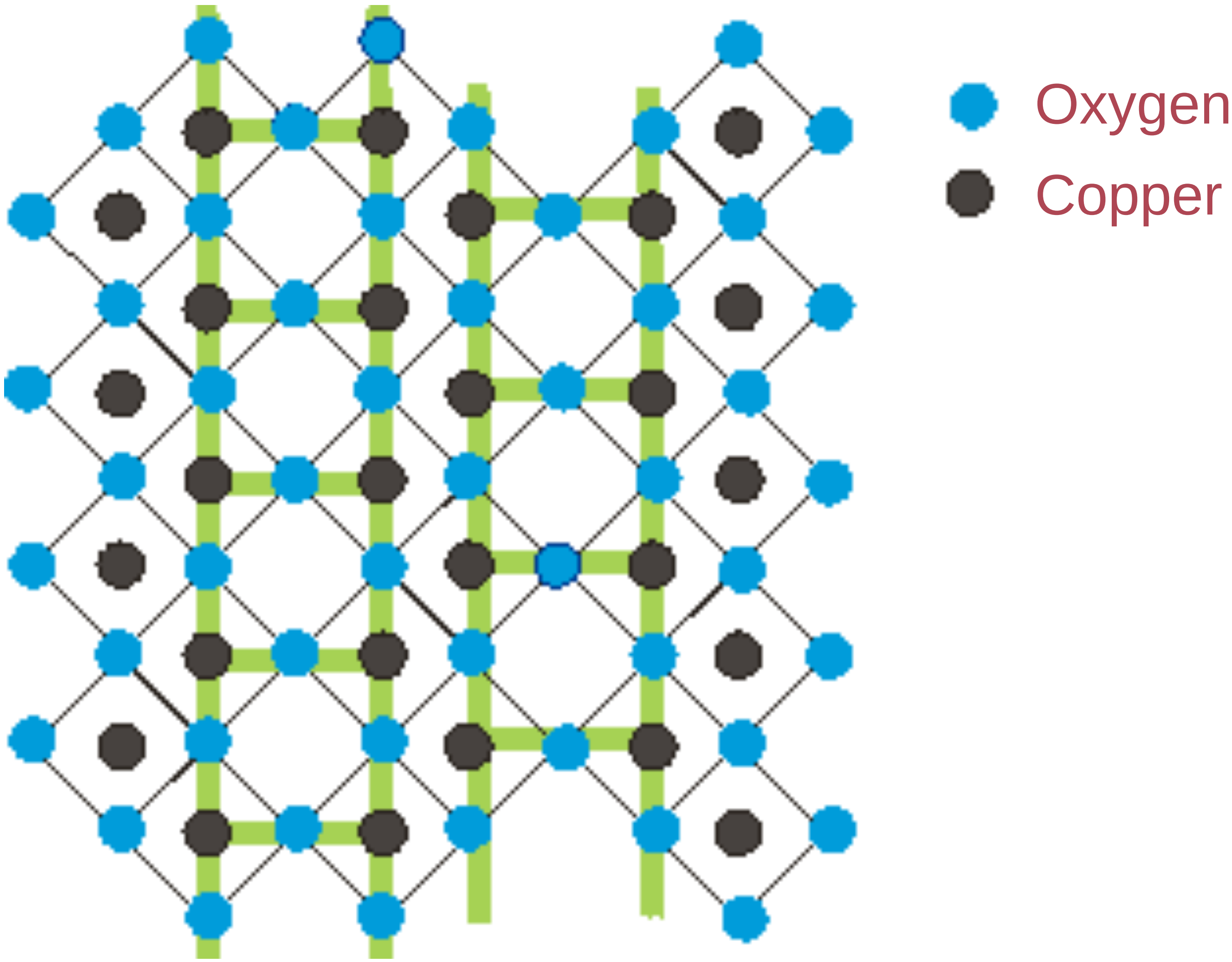
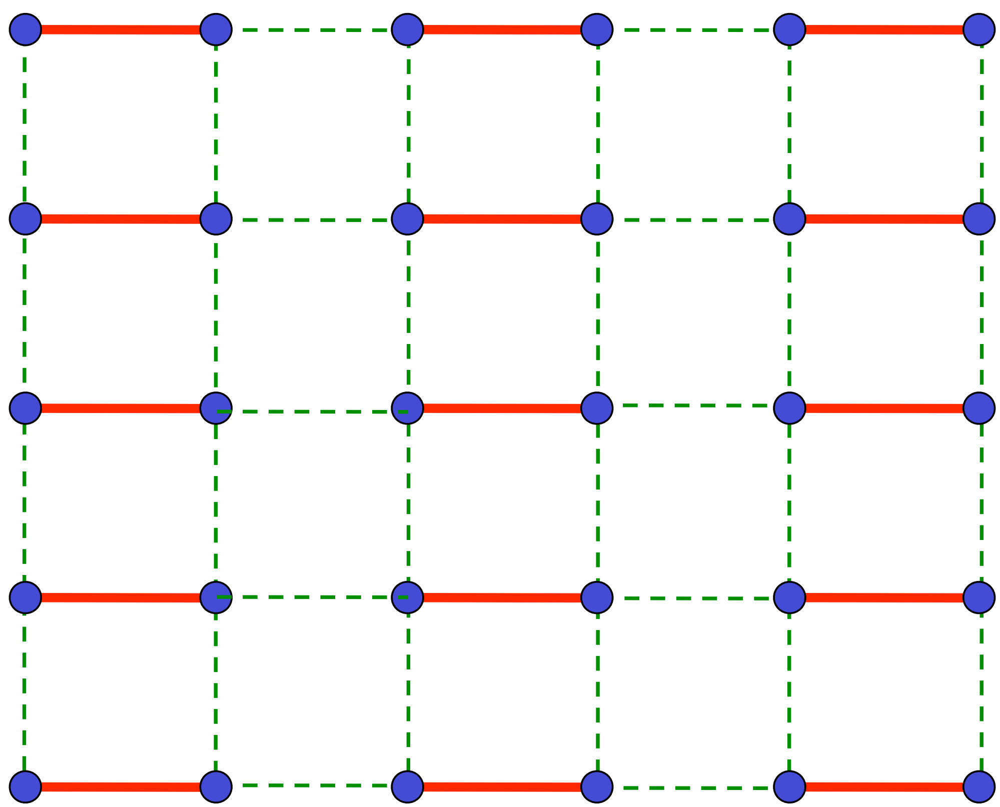
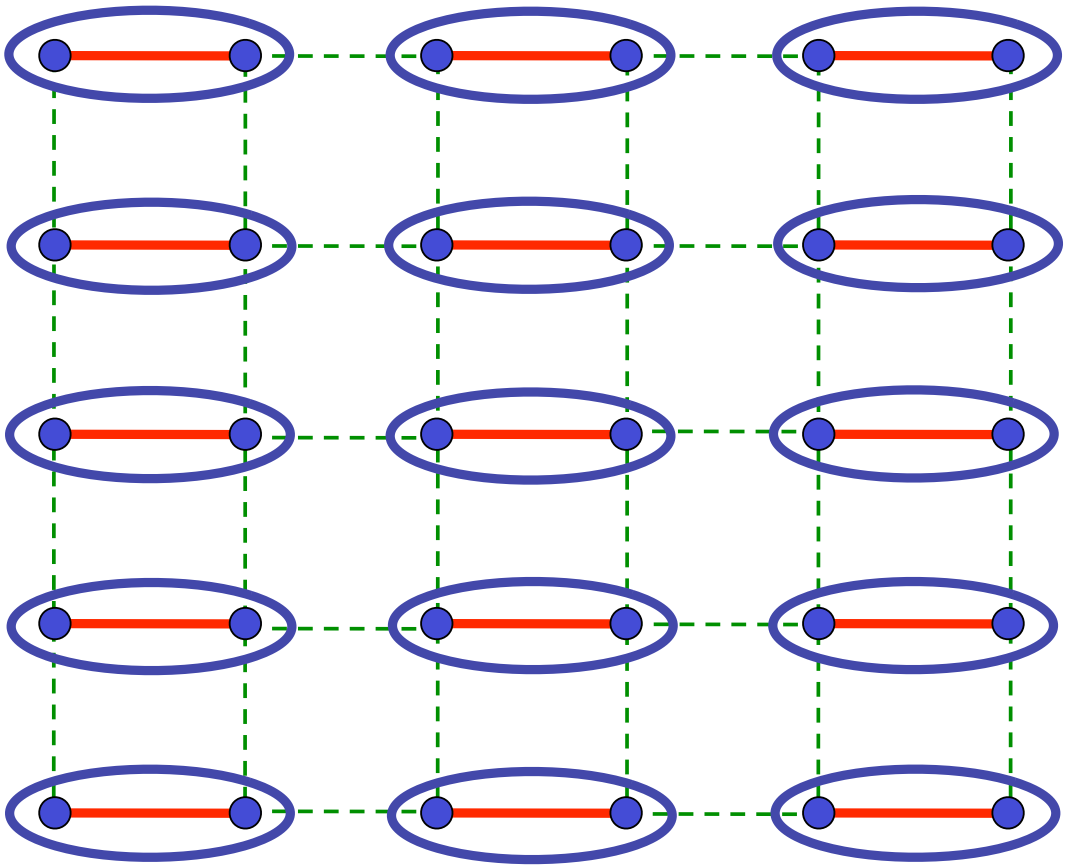


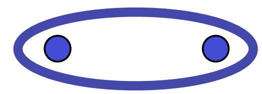
Weaken some bonds to induce spin entanglement in a new quantum phase

$SrCu_2O_3$





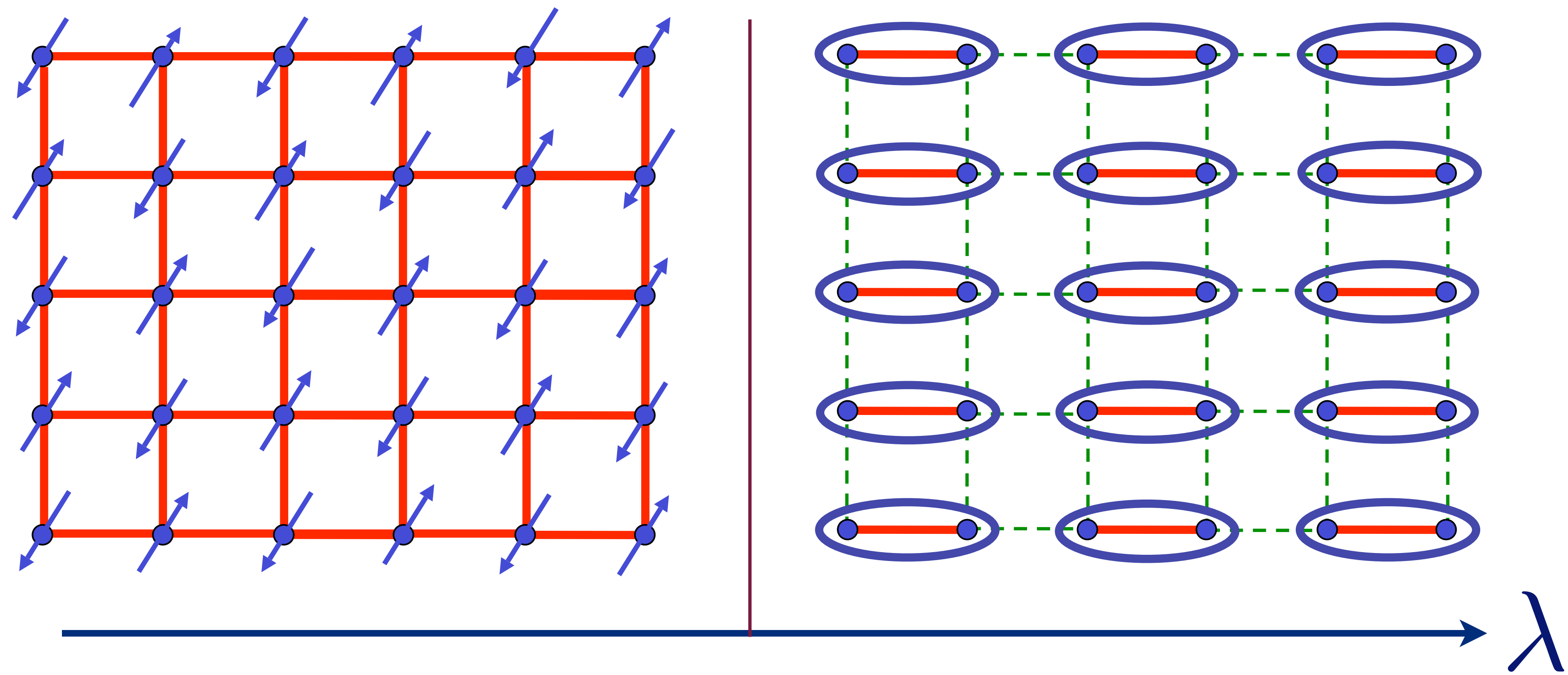


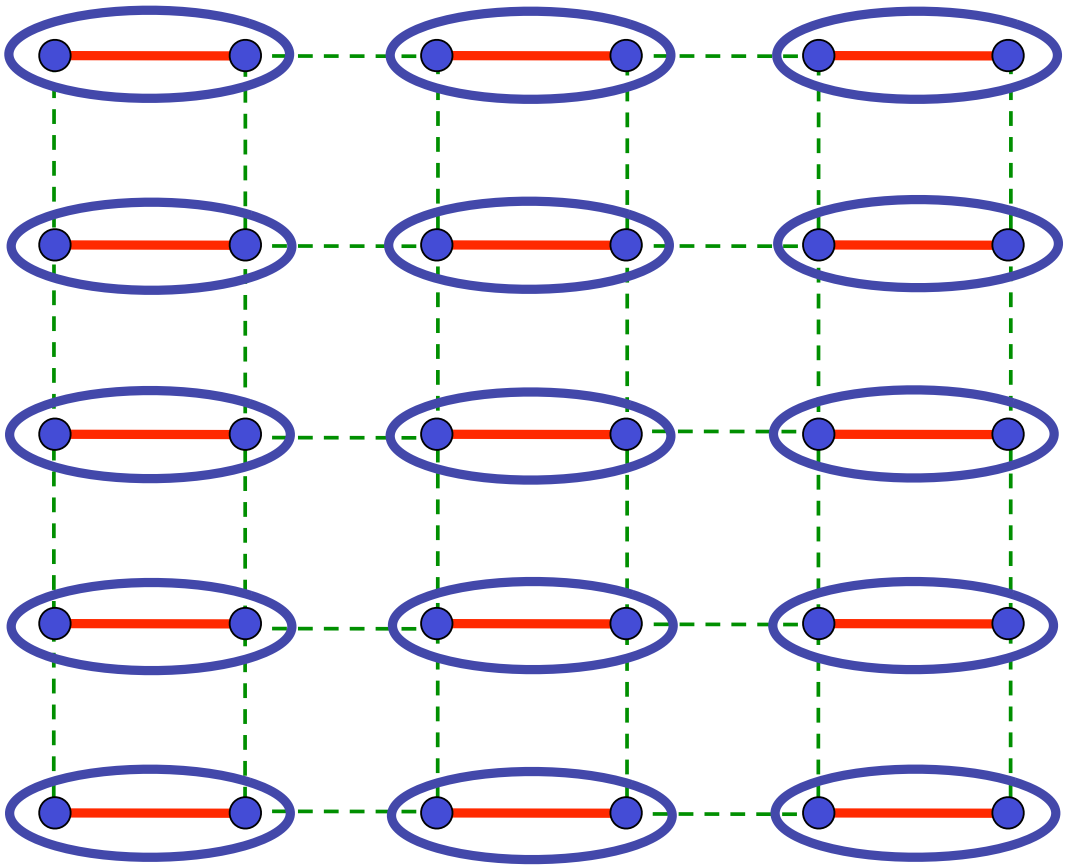


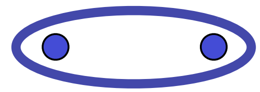
$$= \frac{1}{\sqrt{2}}\Big(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \Big)$$

Ground state is a product of pairs
of entangled spins.

Phase diagram as a function of the ratio of exchange interactions, λ







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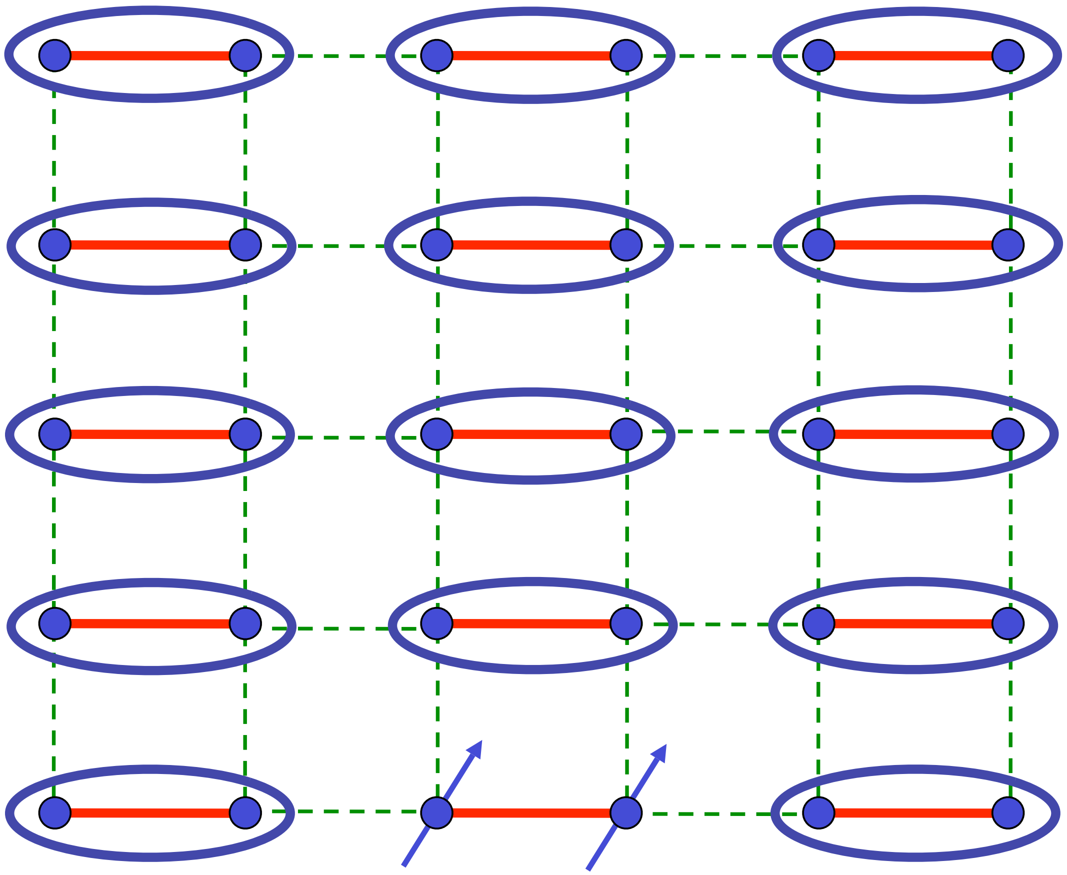

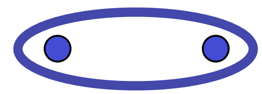

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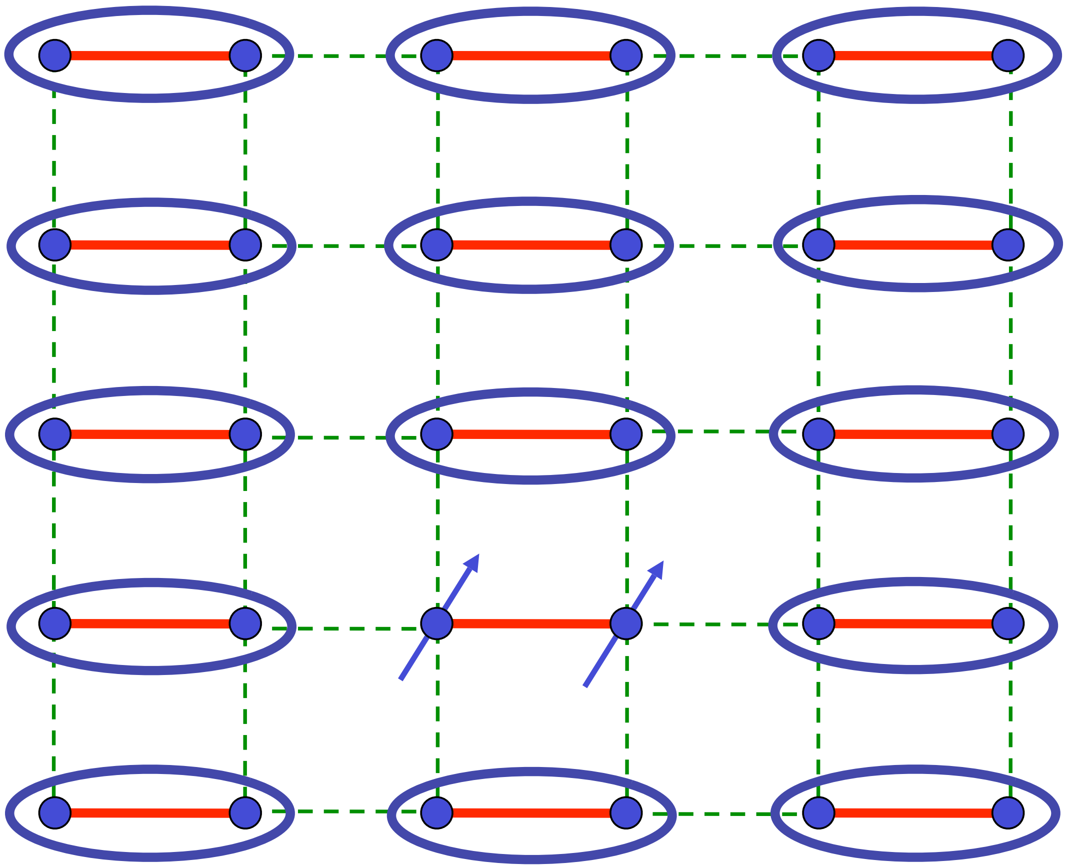
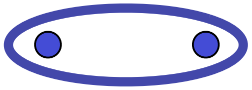
Excitation: S=1 ***triplon***





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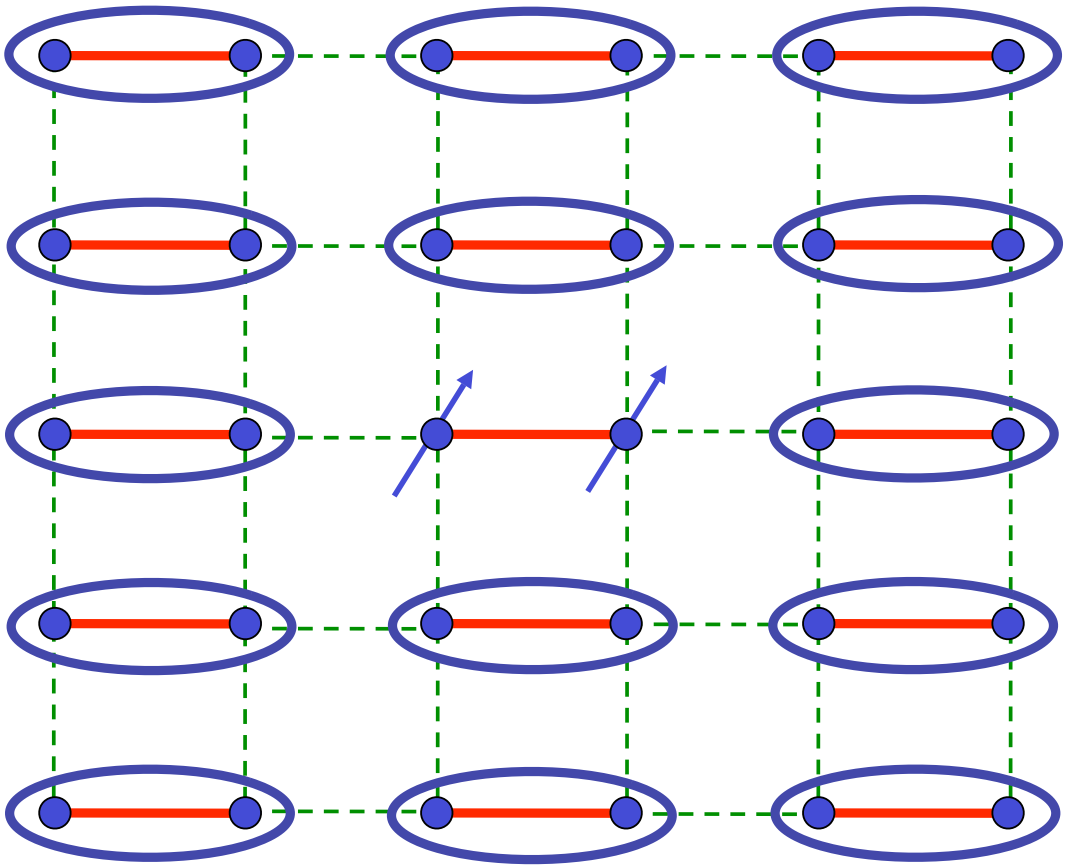

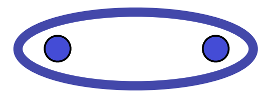

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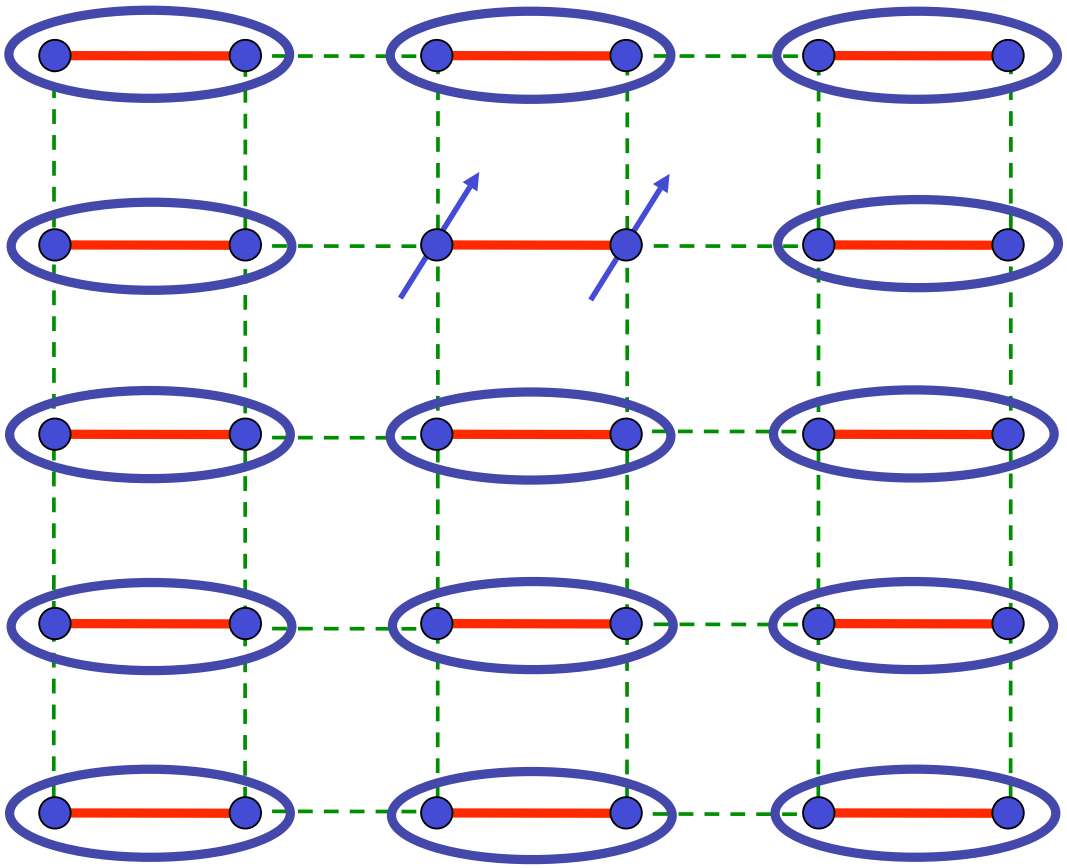

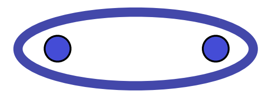

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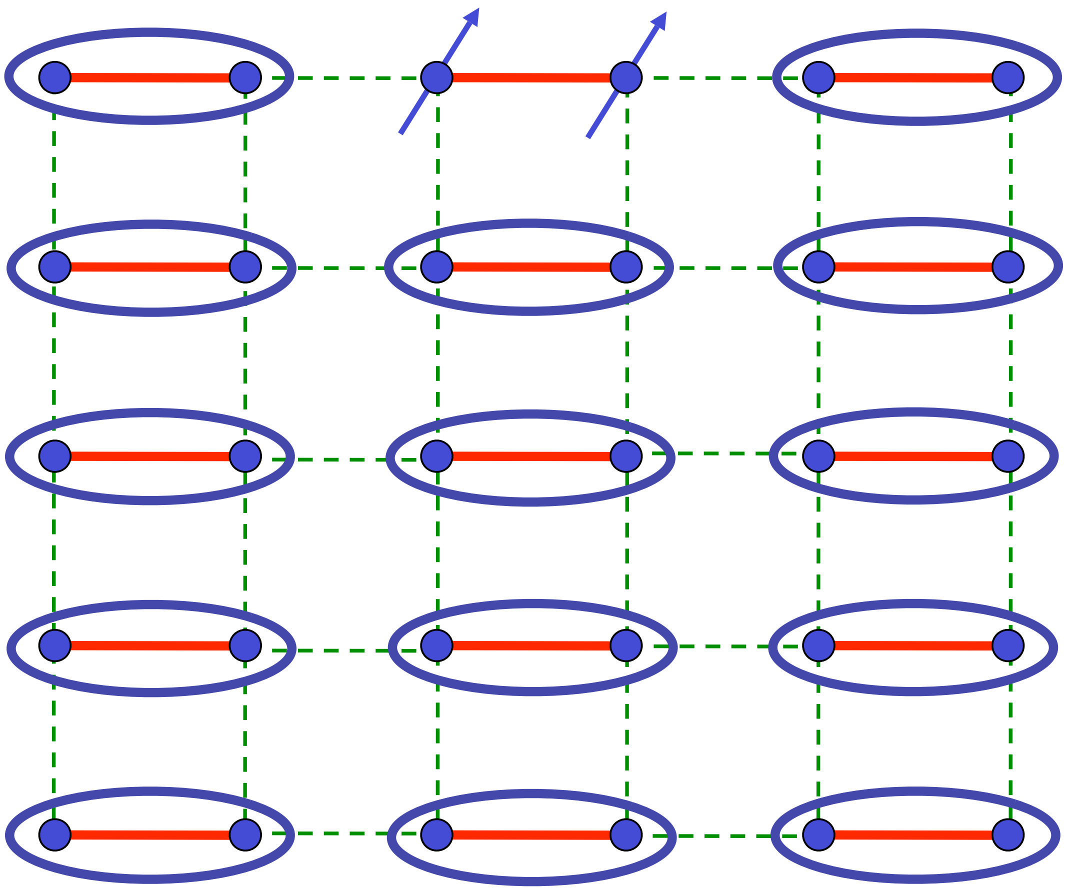

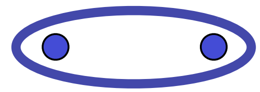

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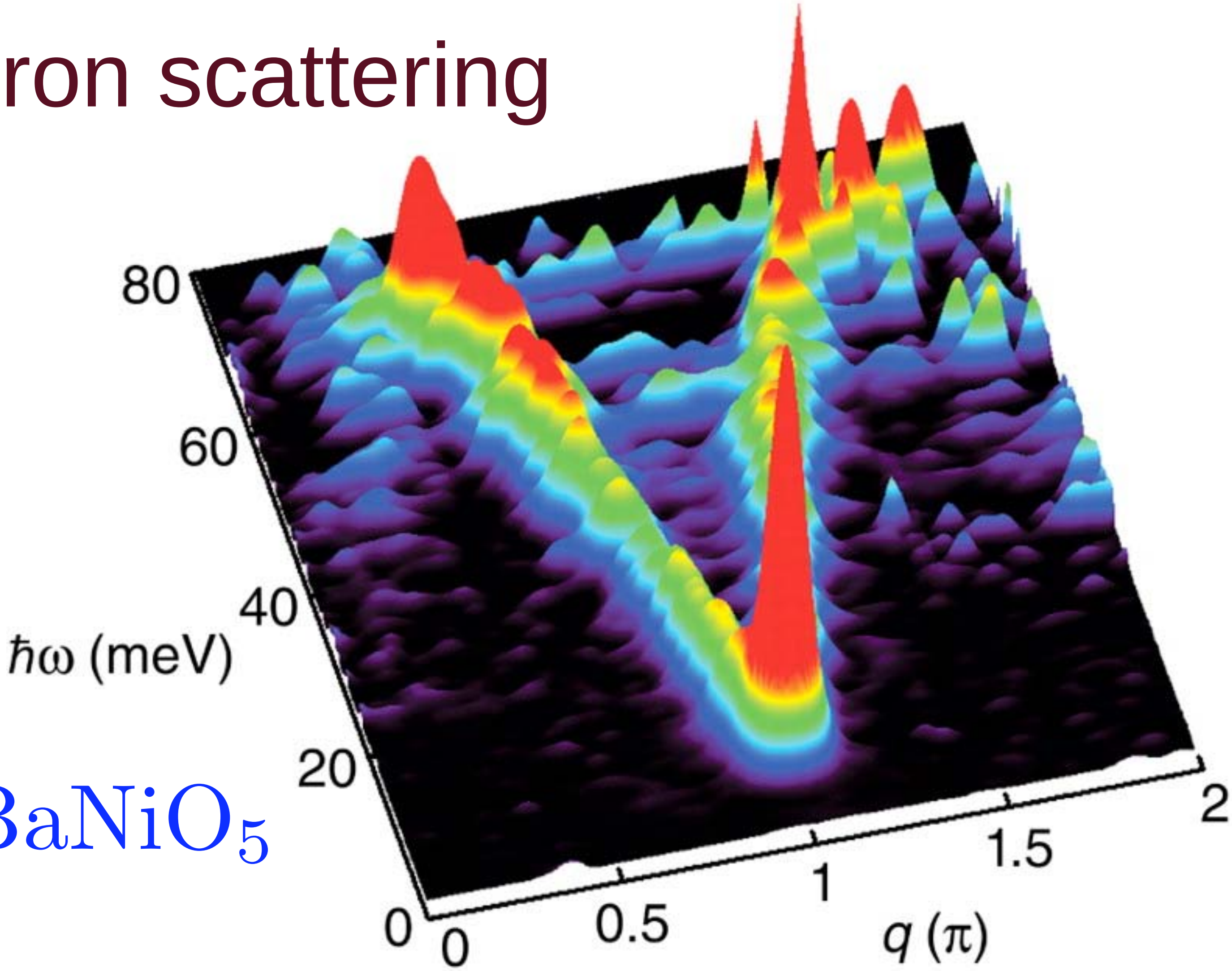
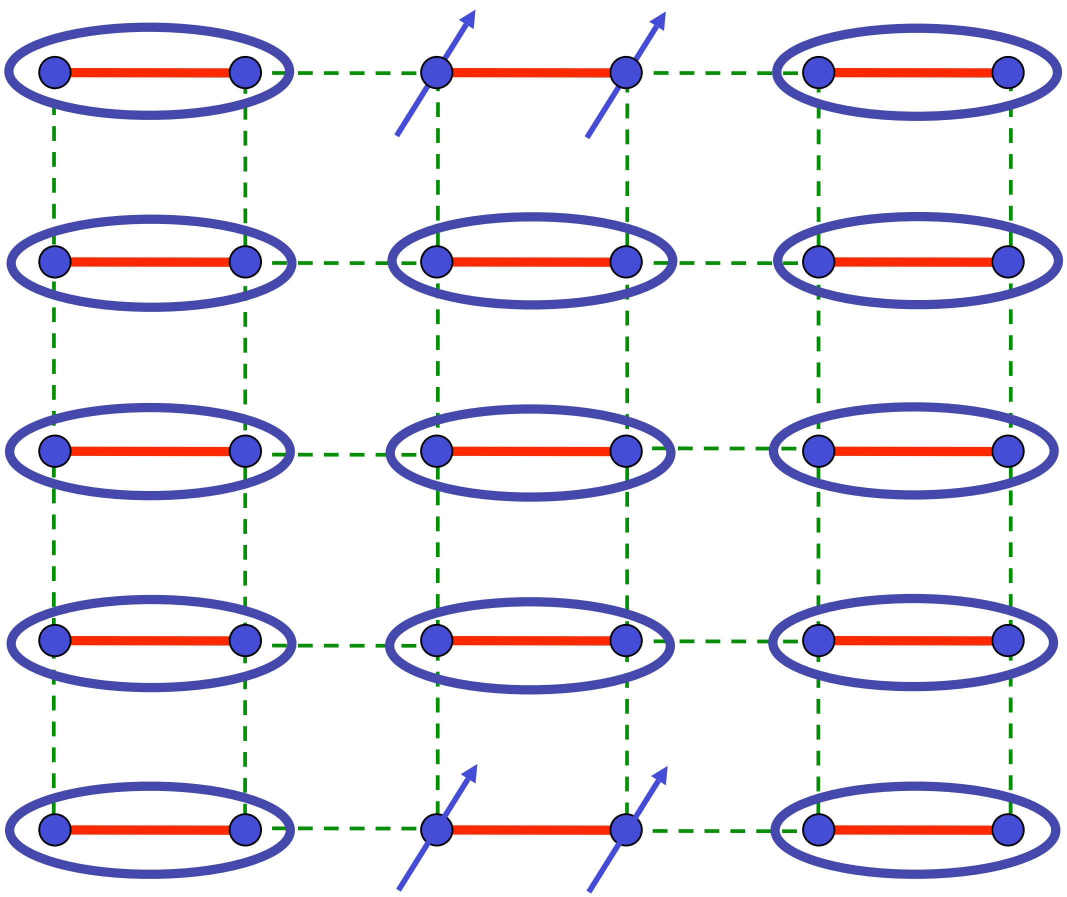
Neutron scattering

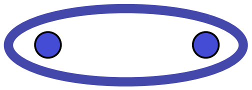


Y_2BaNiO_5

G. Xu, C. Broholm, Yeong-Ah Soh, G. Aeppli, J. F. DiTusa, Y. Chen, M. Kenzelmann, C. D. Frost, T. Ito, K. Oka, and H. Takagi, Science **317**, 1049 (2007).

Collision of triplons





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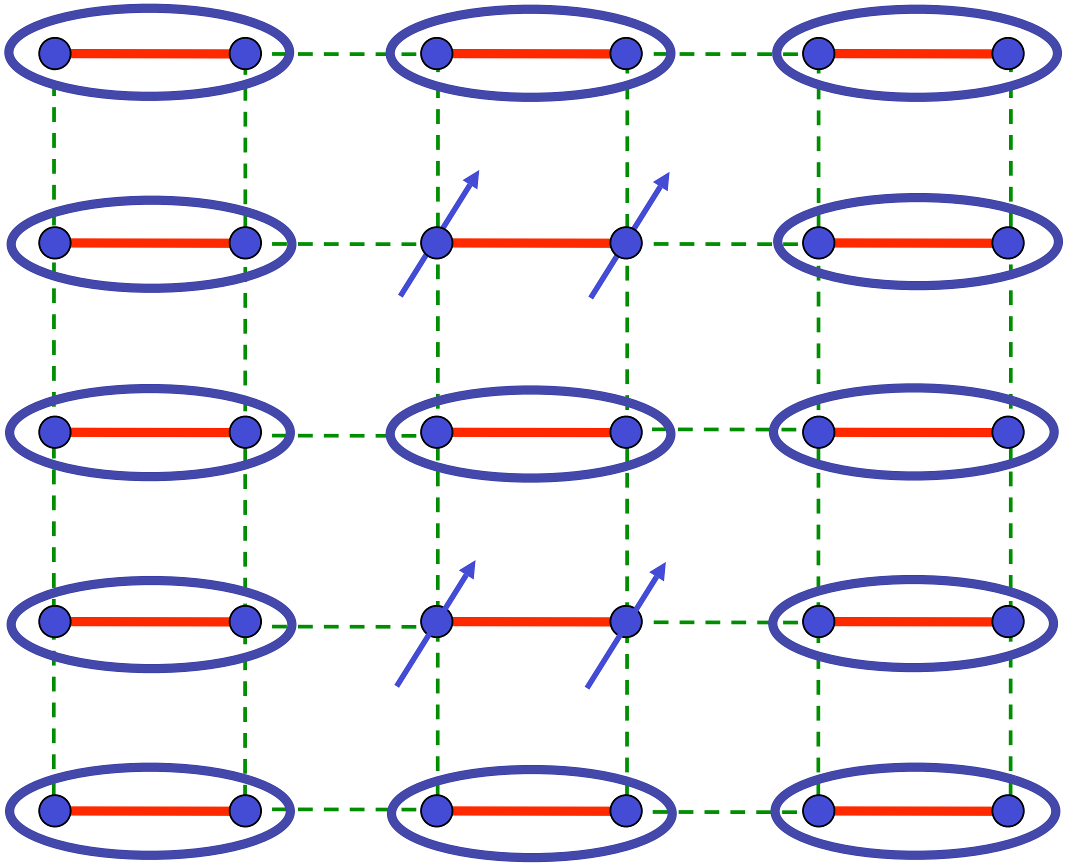

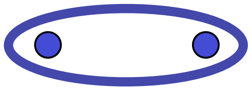

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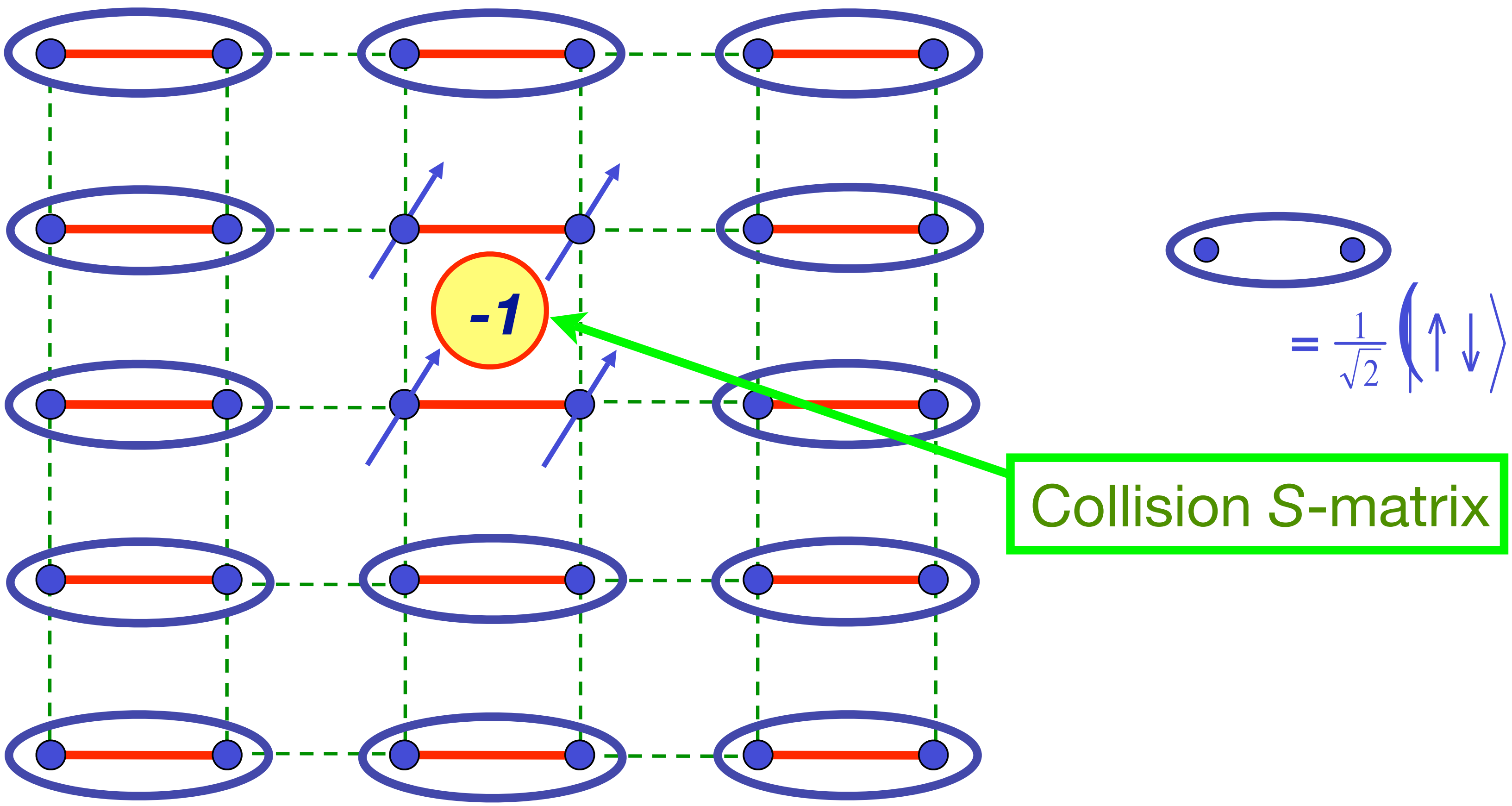

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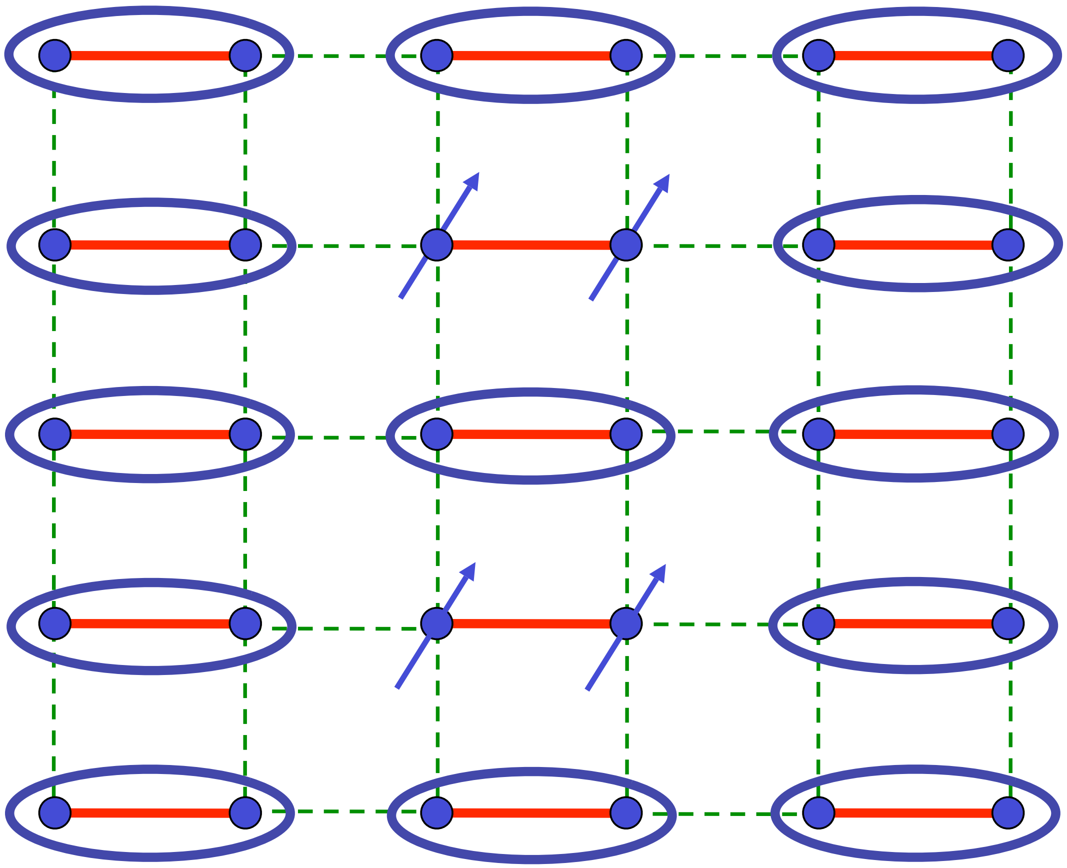

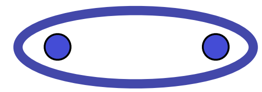

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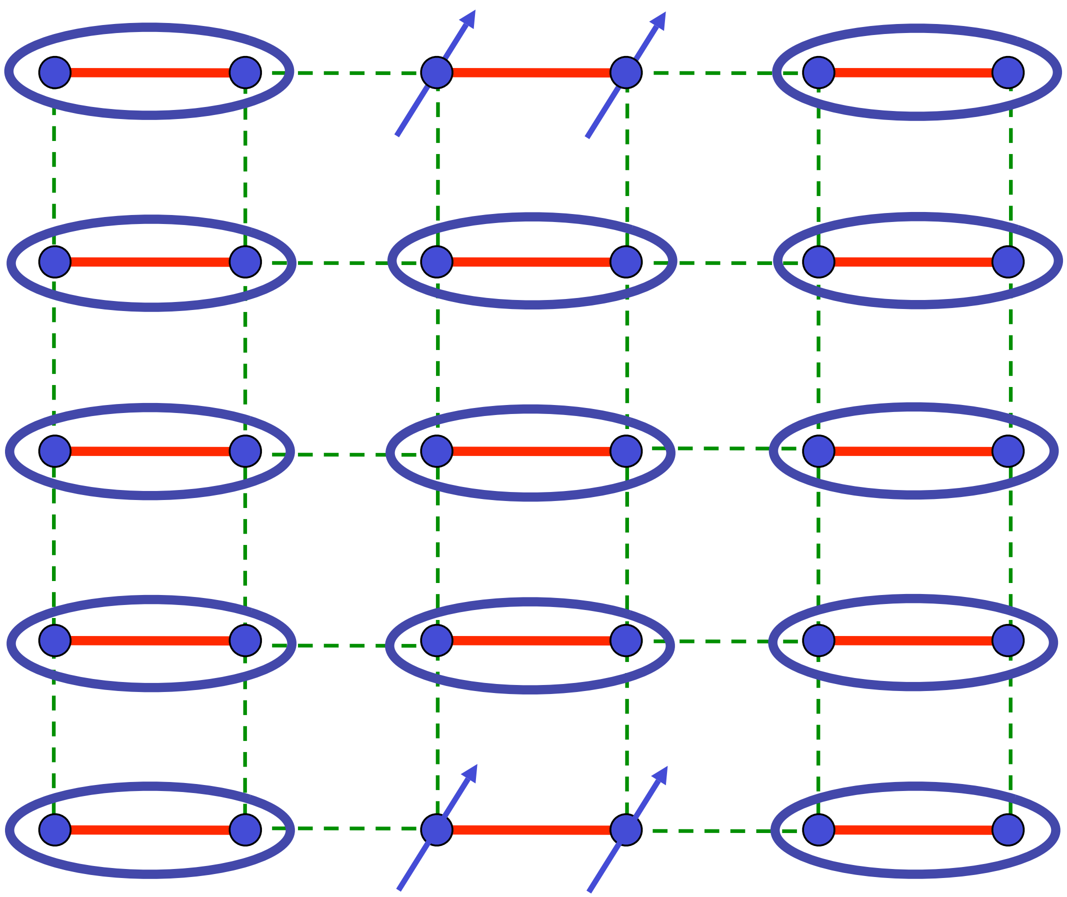

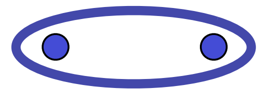

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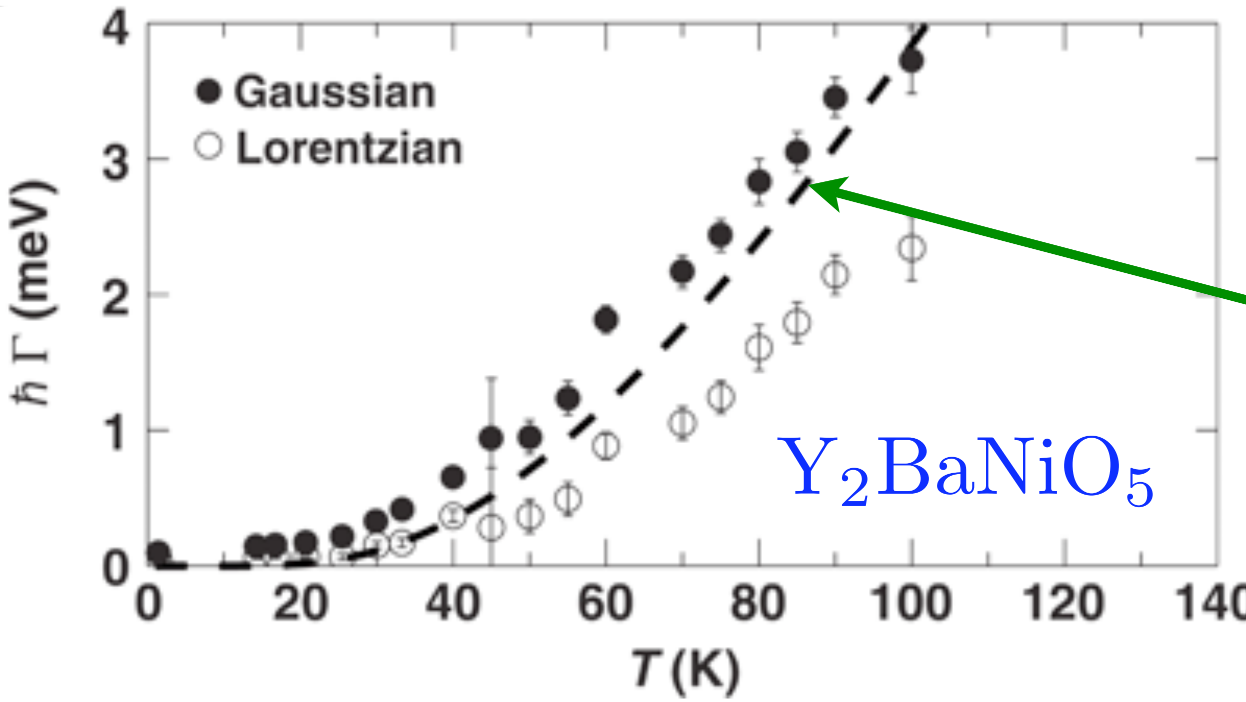
Neutron scattering linewidth



Parameter free prediction by K. Damle and S. Sachdev, Phys. Rev. B **57**, 8307 (1998) from multiple collisions with universal, quantum *S*-matrices

G. Xu, C. Broholm, Yeong-Ah Soh, G. Aeppli, J. F. DiTusa, Y. Chen, M. Kenzelmann, C. D. Frost, T. Ito, K. Oka, and H. Takagi, Science **317**, 1049 (2007).

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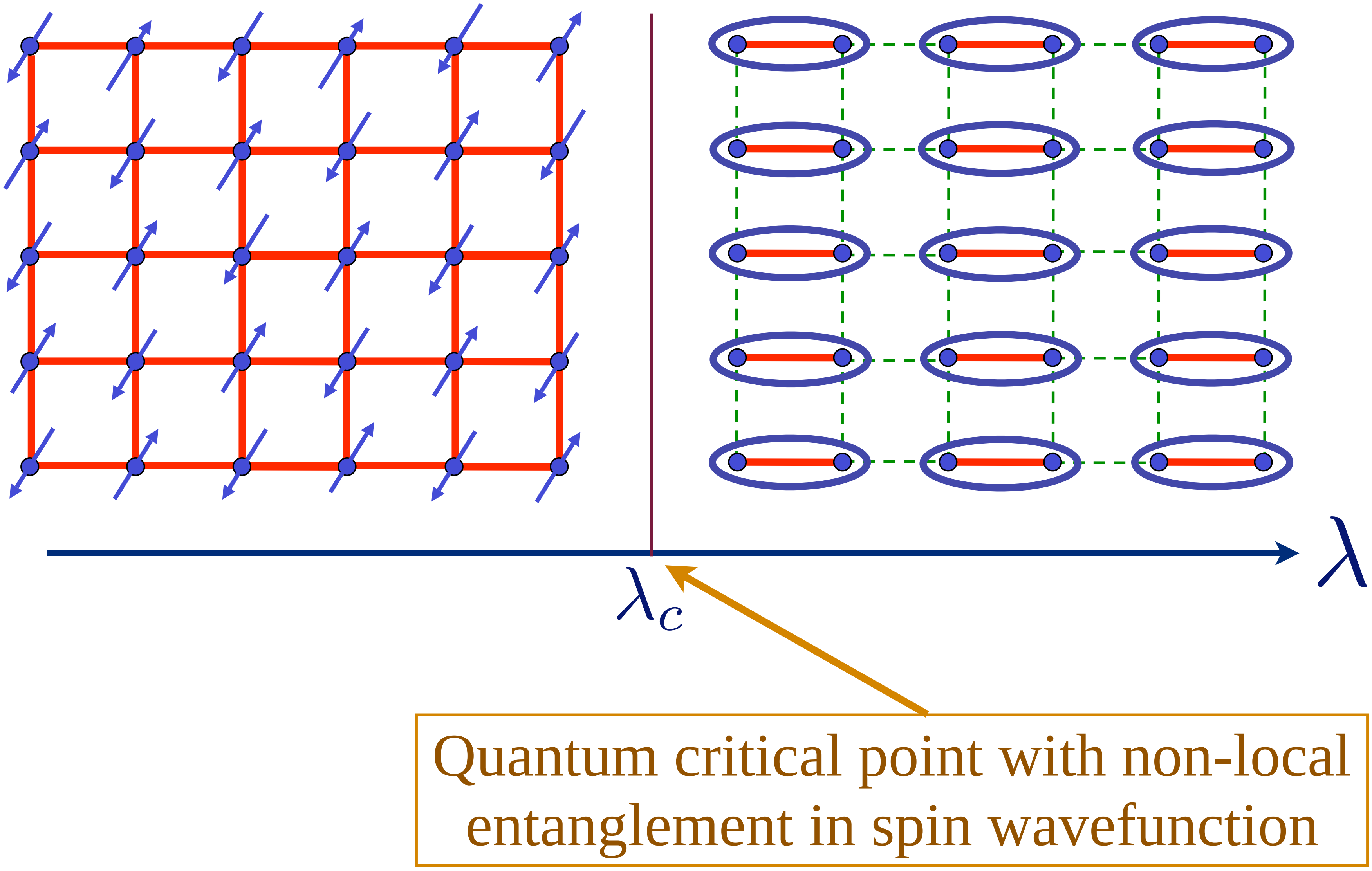

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   Experiments on spin-gap insulators

2. Entanglement of valence bonds

   Deconfined criticality in antiferromagnets

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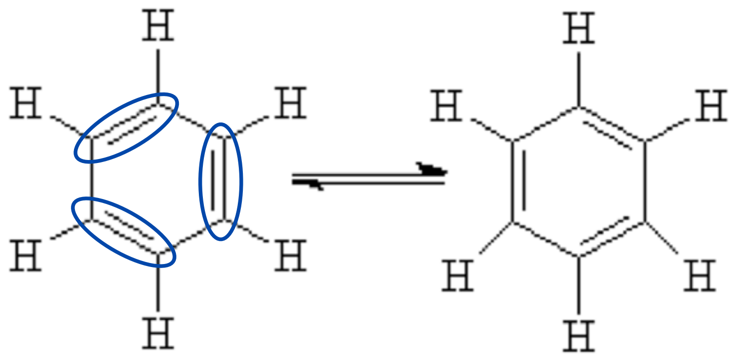
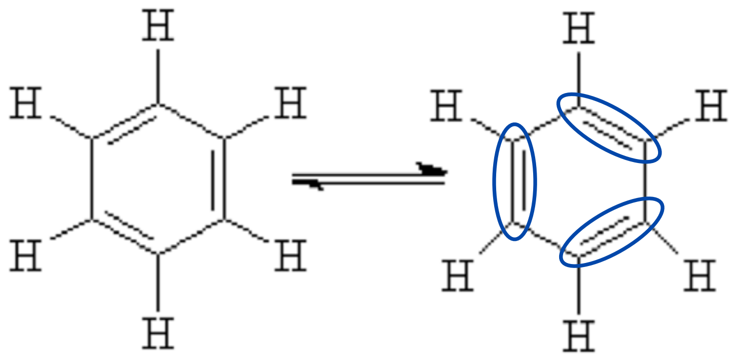


Resonance in benzene leads to a symmetric configuration of valence bonds
(*F. Kekulé, L. Pauling*)

Entanglement of valence bonds



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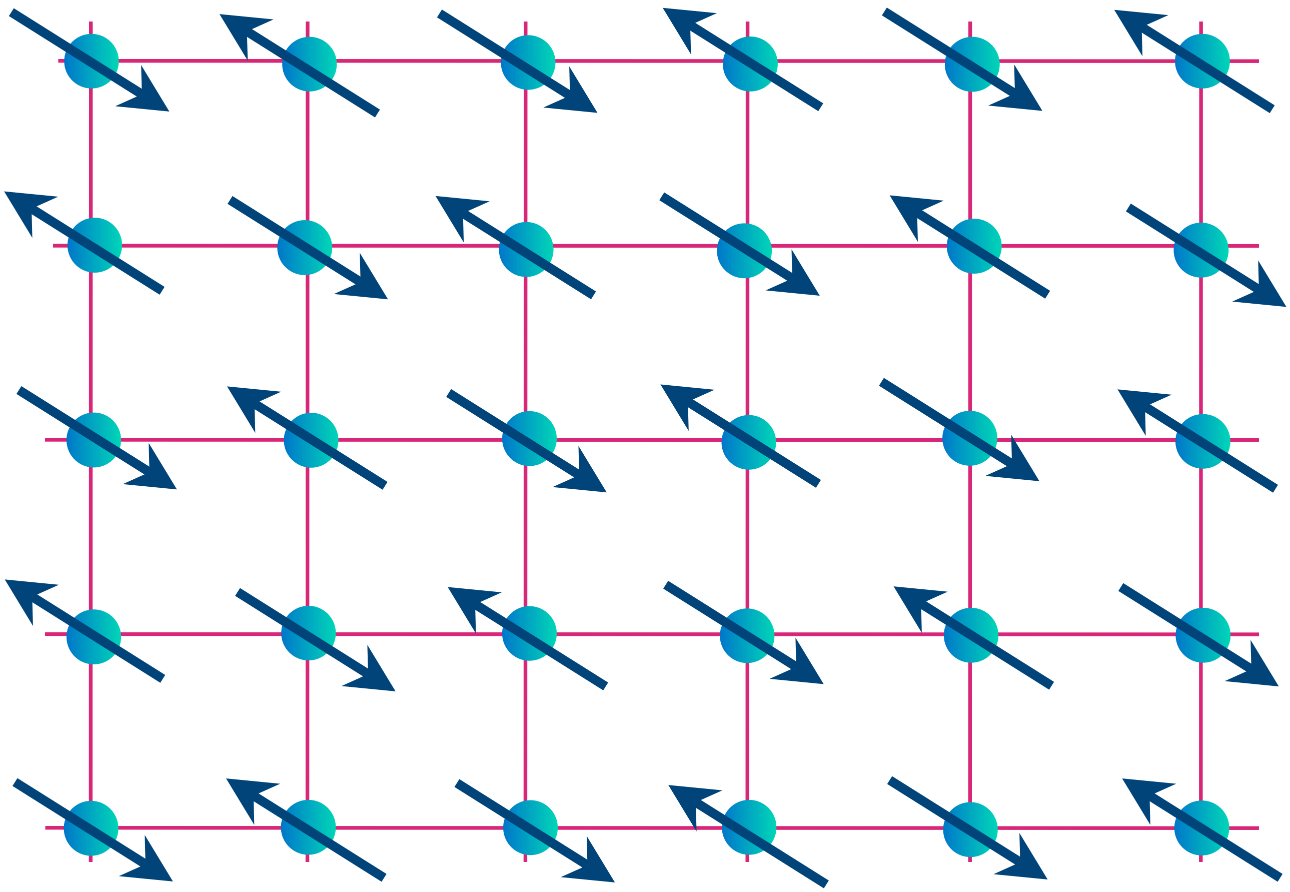
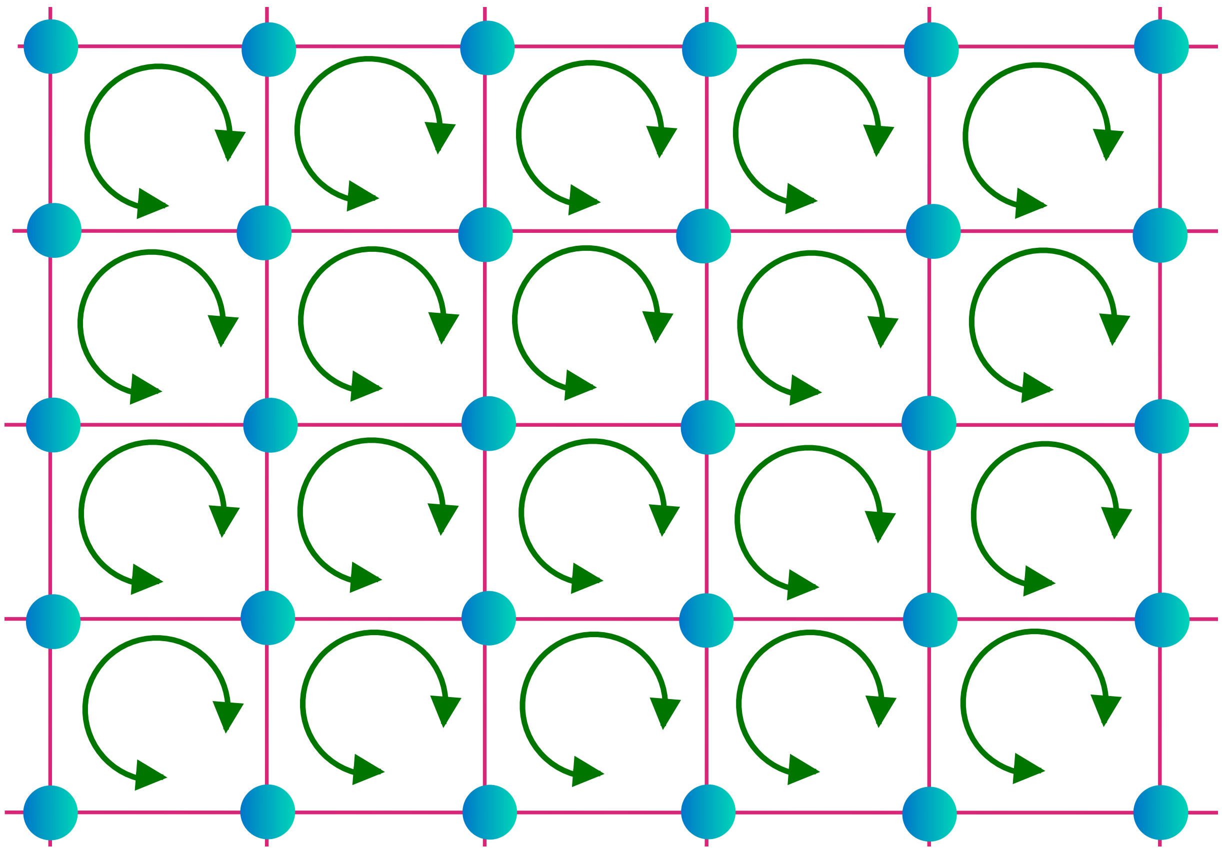


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j \quad ; \quad \vec{S}_i \Rightarrow \text{spin operator with } S = 1/2$$

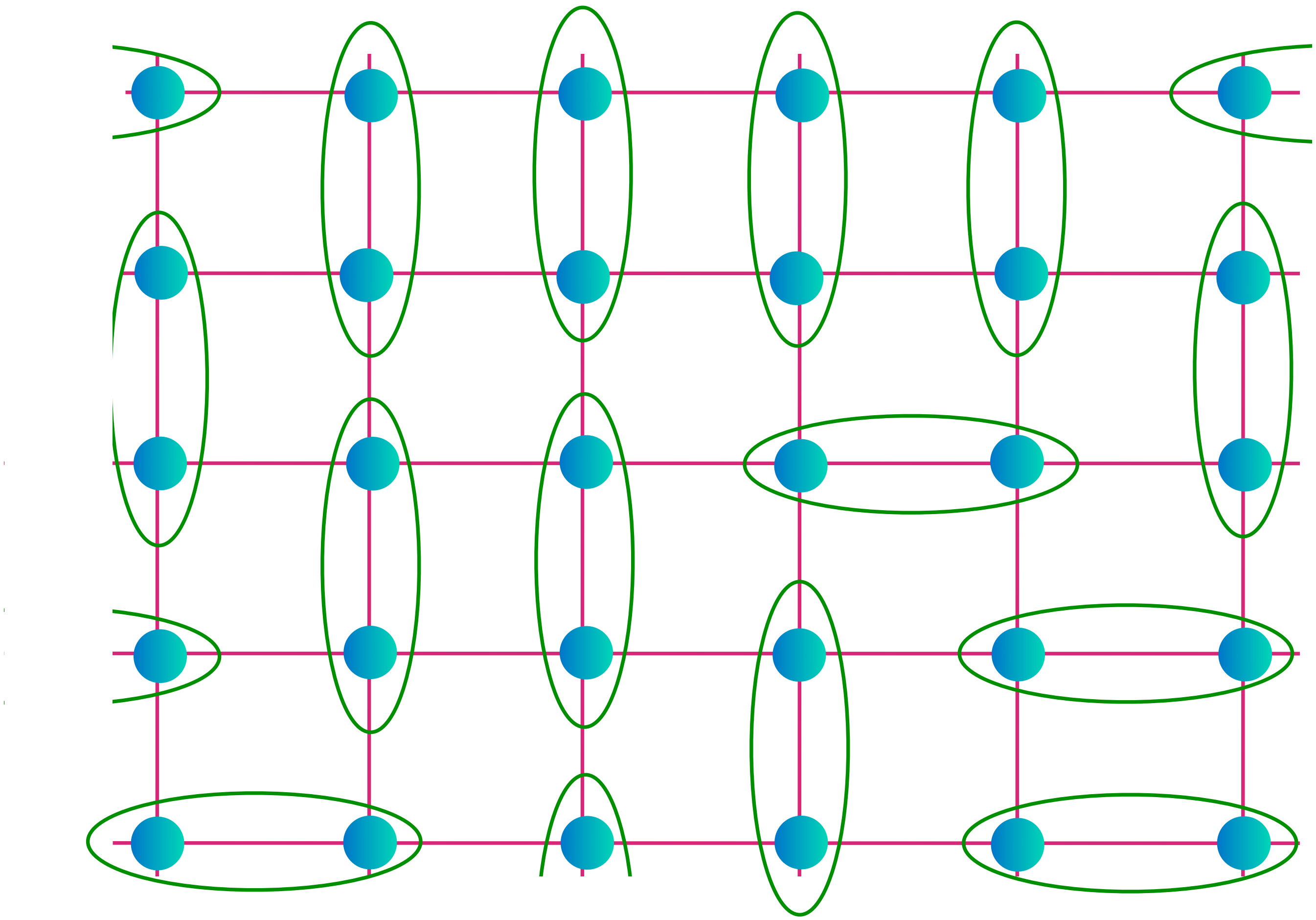
Induce formation of valence bonds by *e.g.* ring-exchange interactions

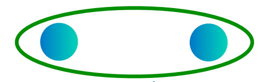


$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{four spin exchange}$$

A. W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007)

Valence bond entanglement in quantum spin systems





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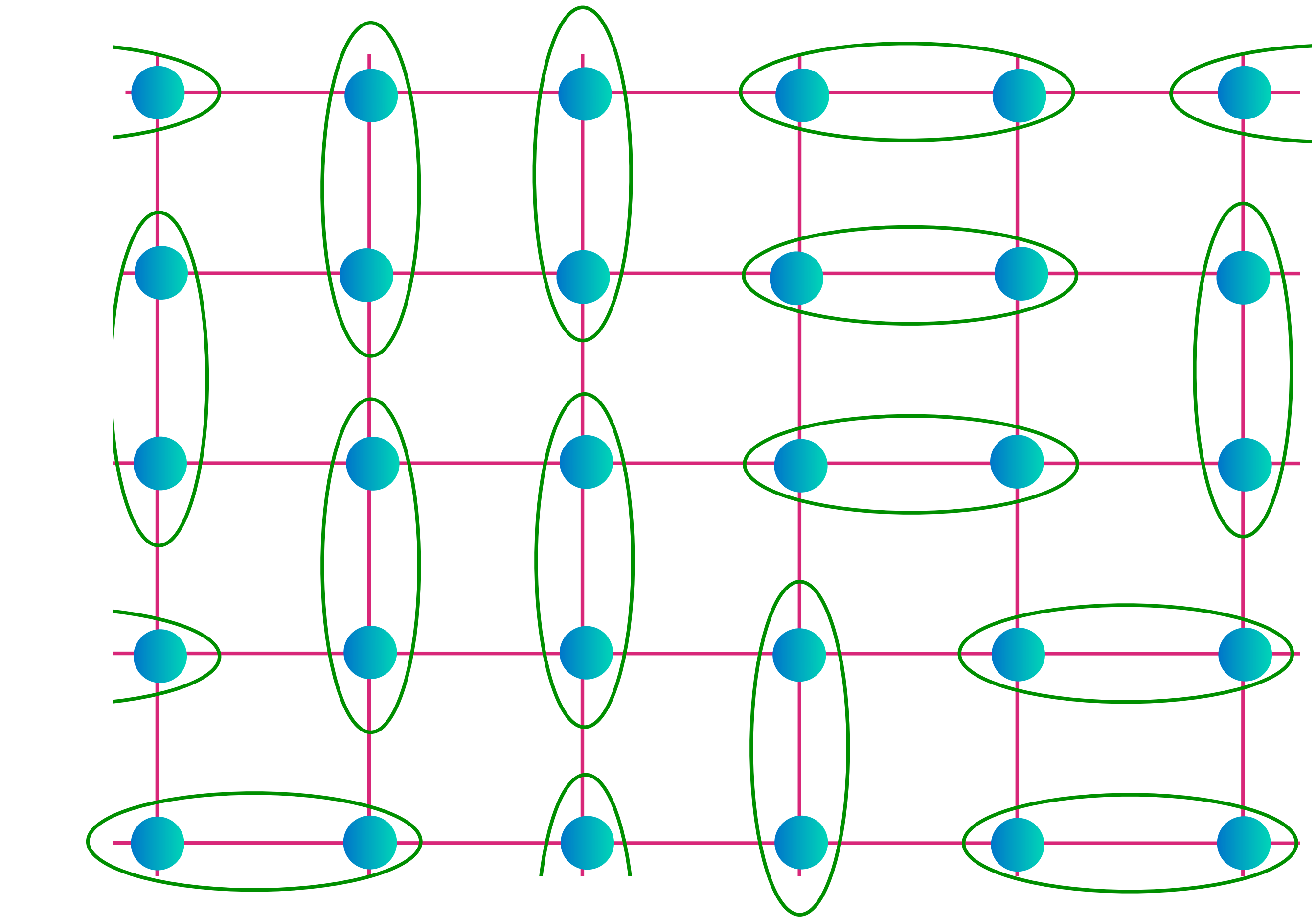

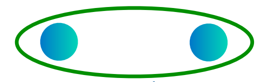

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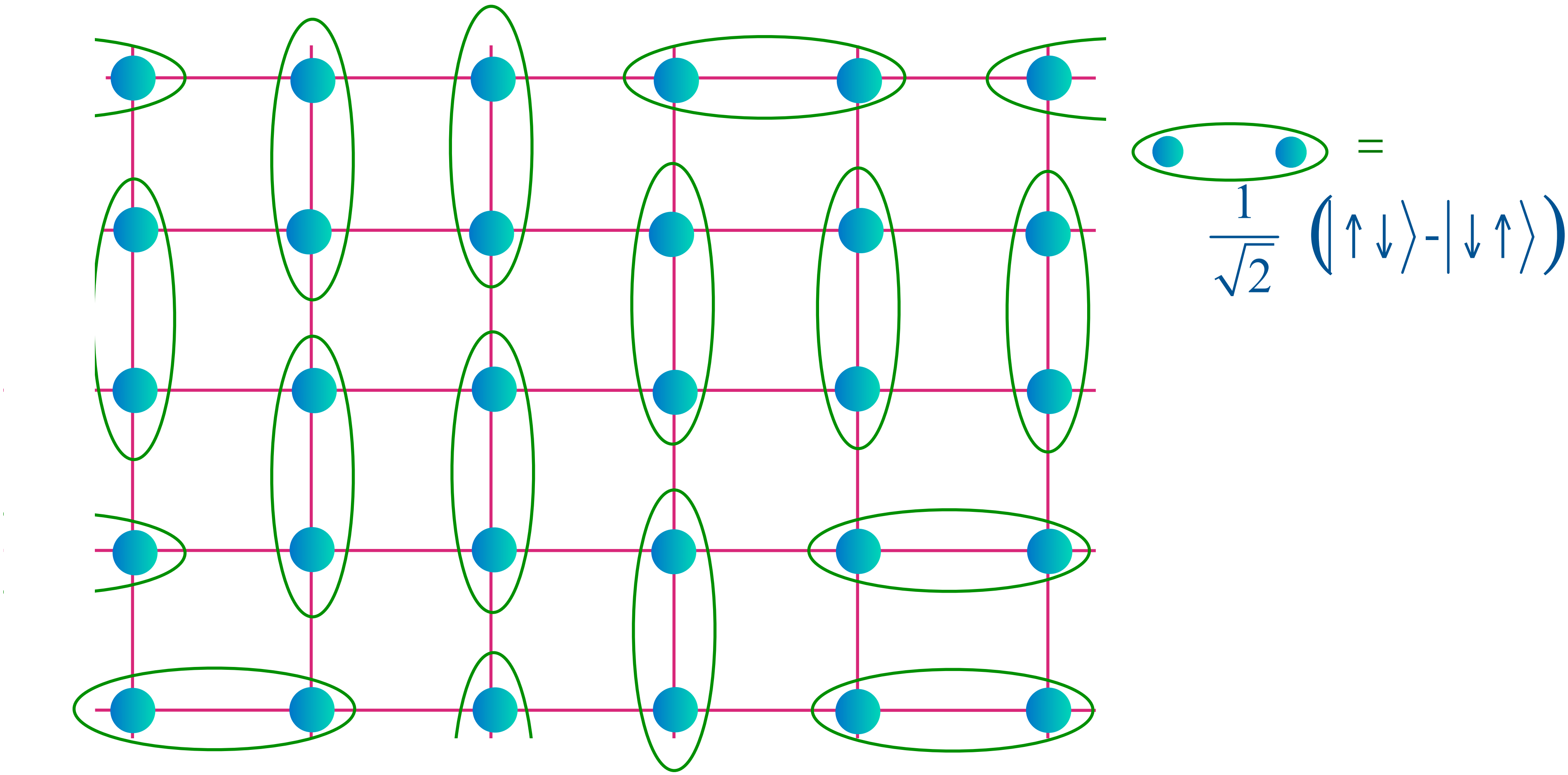

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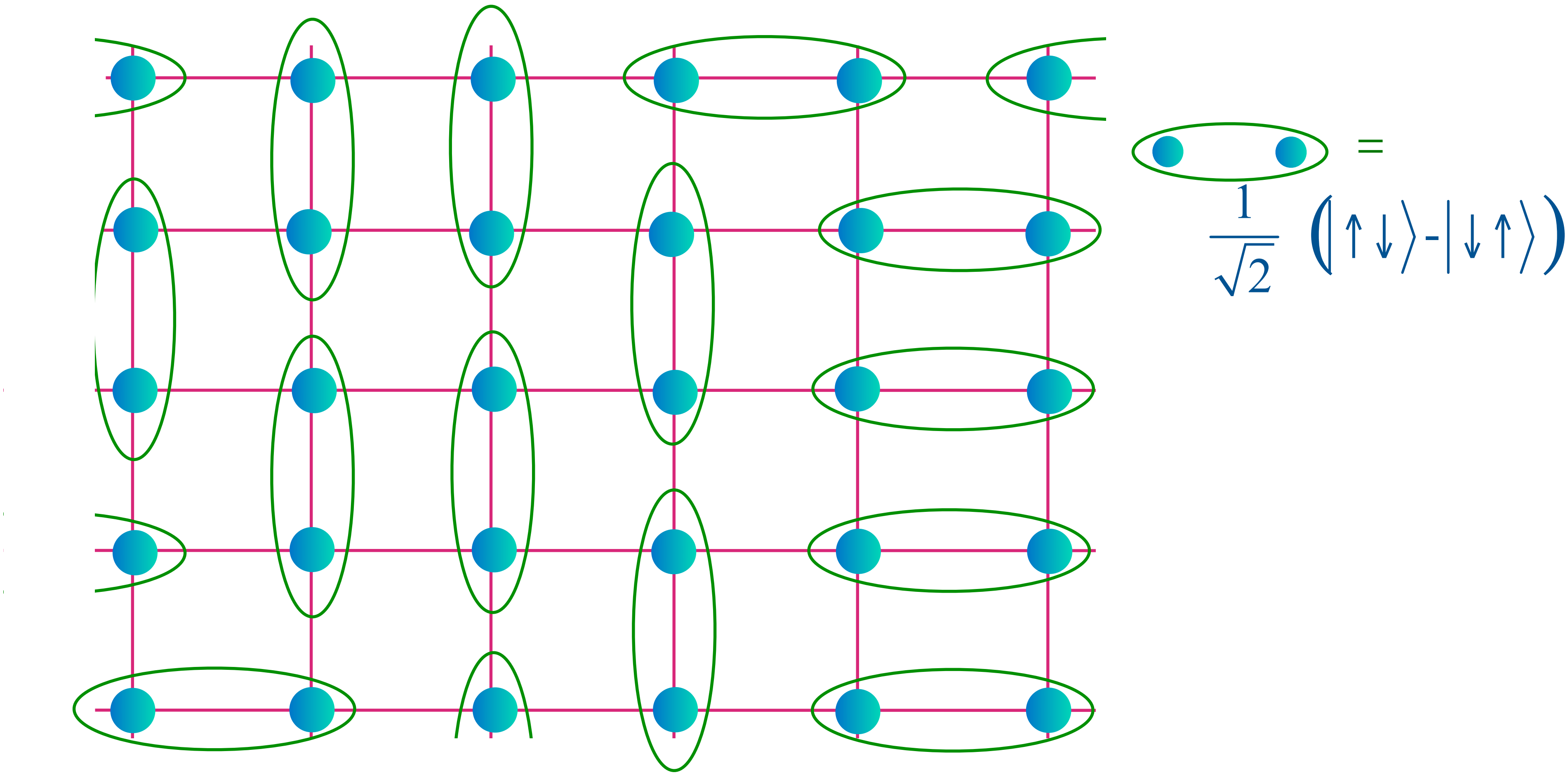

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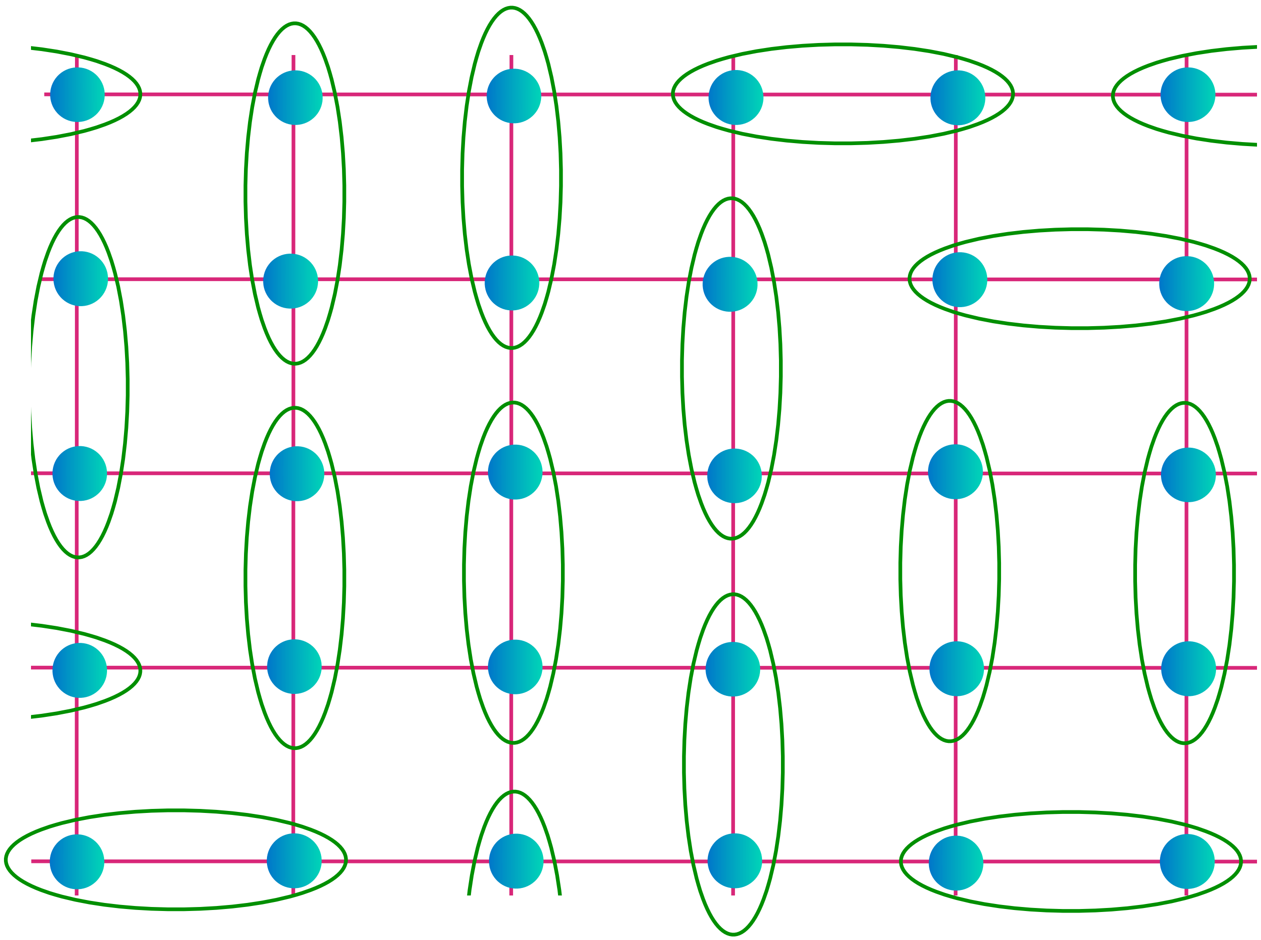
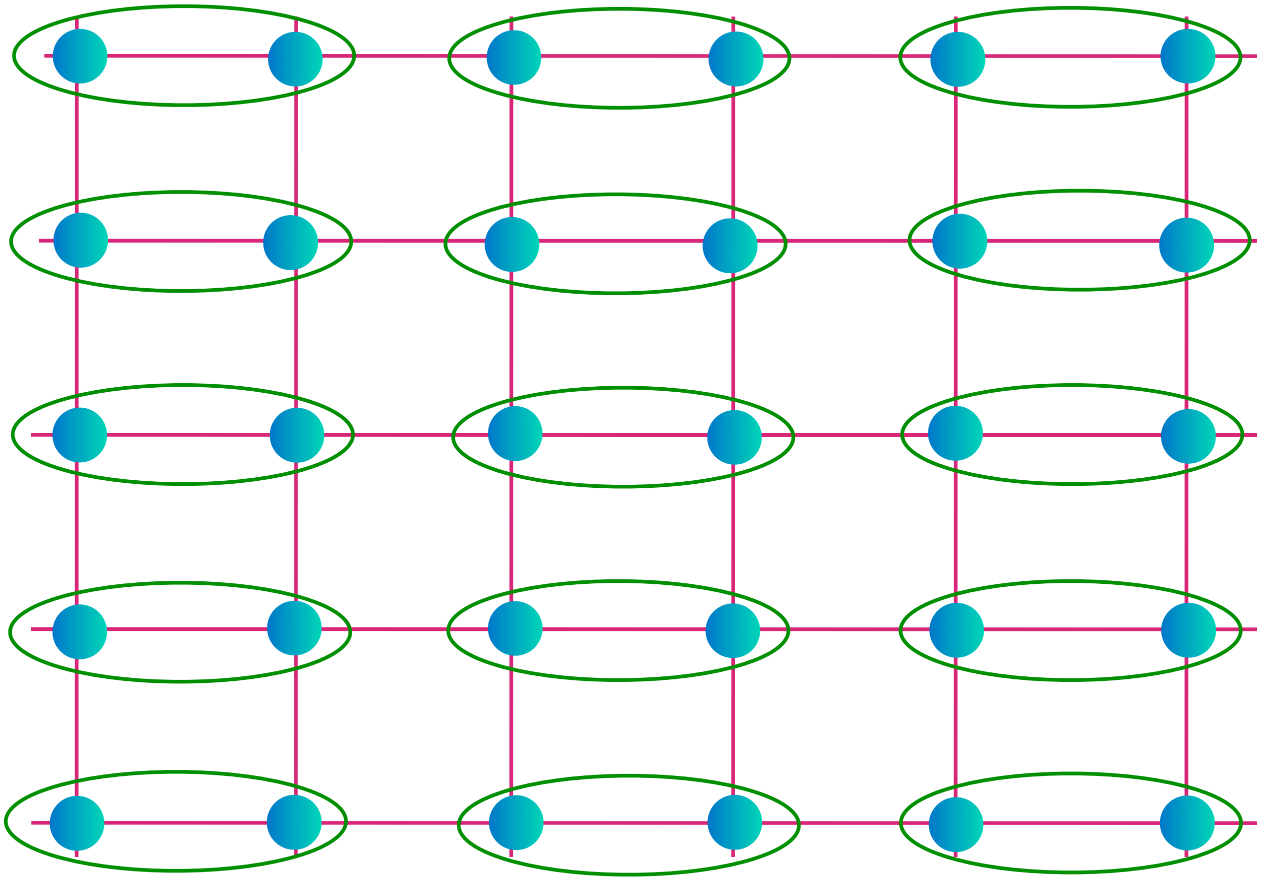


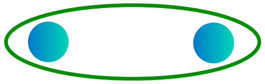
$$\text{(bond)} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Resonating valence bond (RVB) liquid

P. Fazekas and P.W. Anderson, *Phil Mag* **30**, 23 (1974).

Valence bond entanglement in quantum spin systems



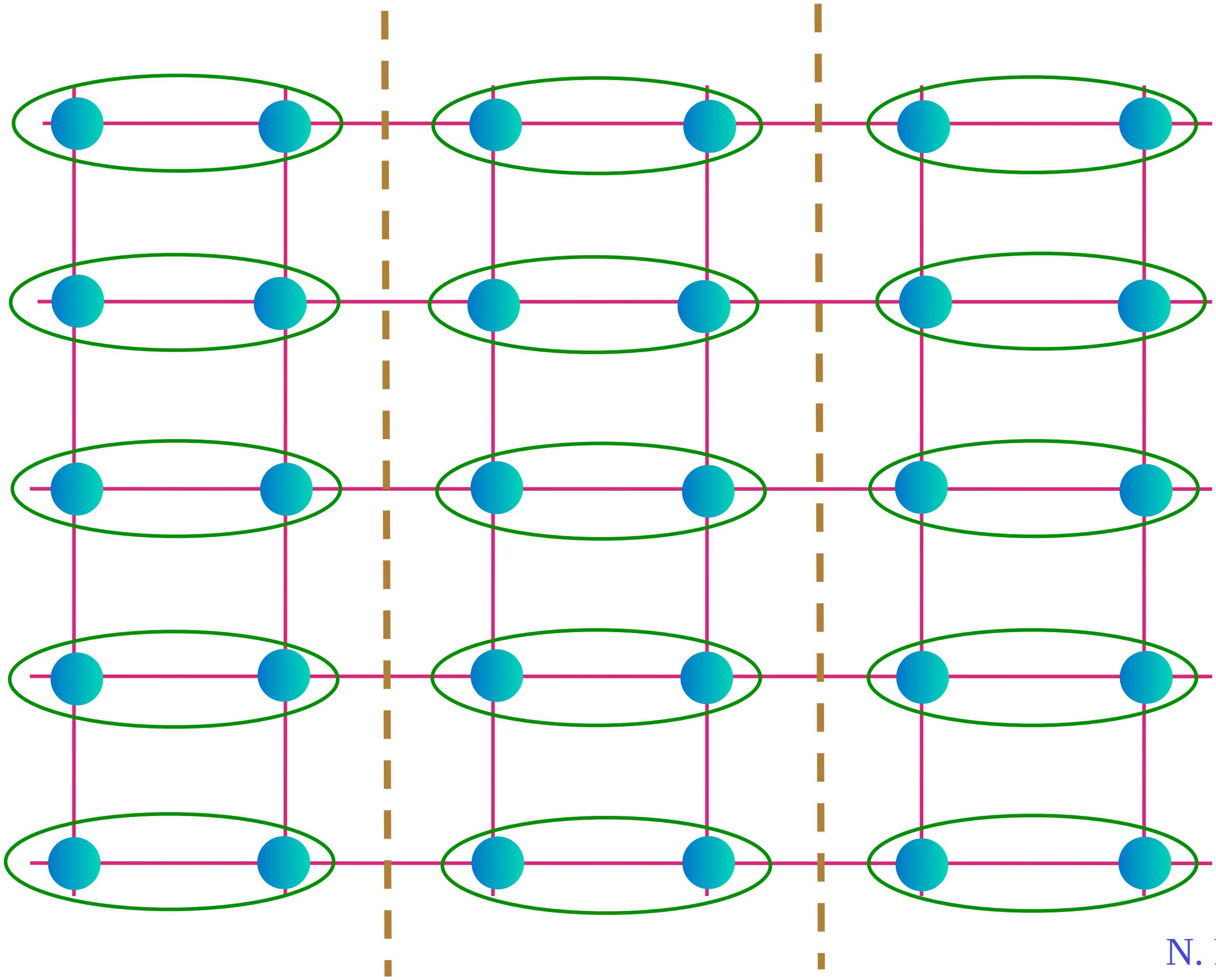
 =

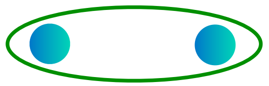
$$\frac{1}{\sqrt{2}}\left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$$

Valence Bond Solid (VBS)

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989).
R. Moessner and S. L. Sondhi, *Phys. Rev*. B **63**, 224401 (2001).

Valence bond entanglement in quantum spin systems





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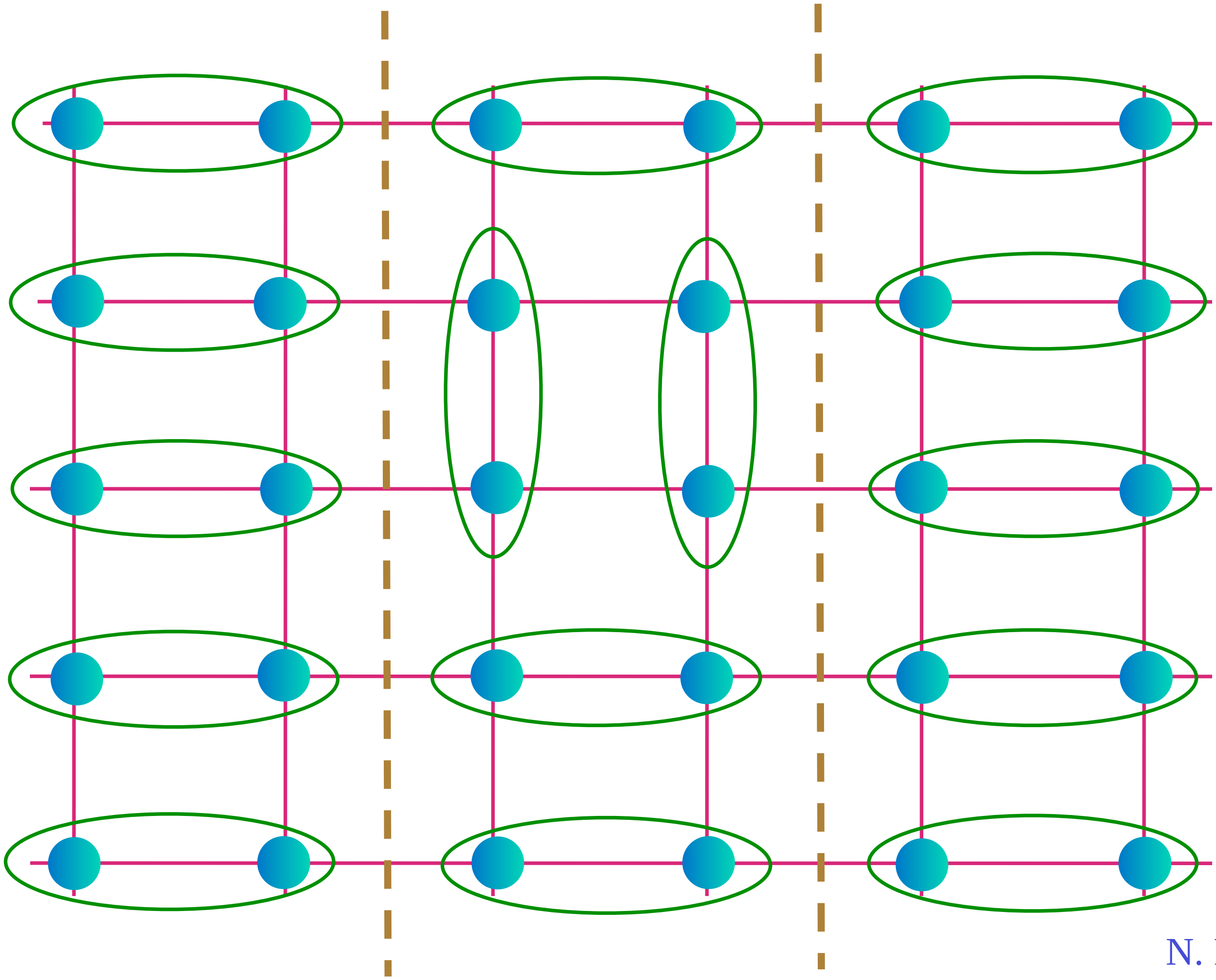

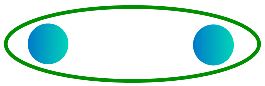

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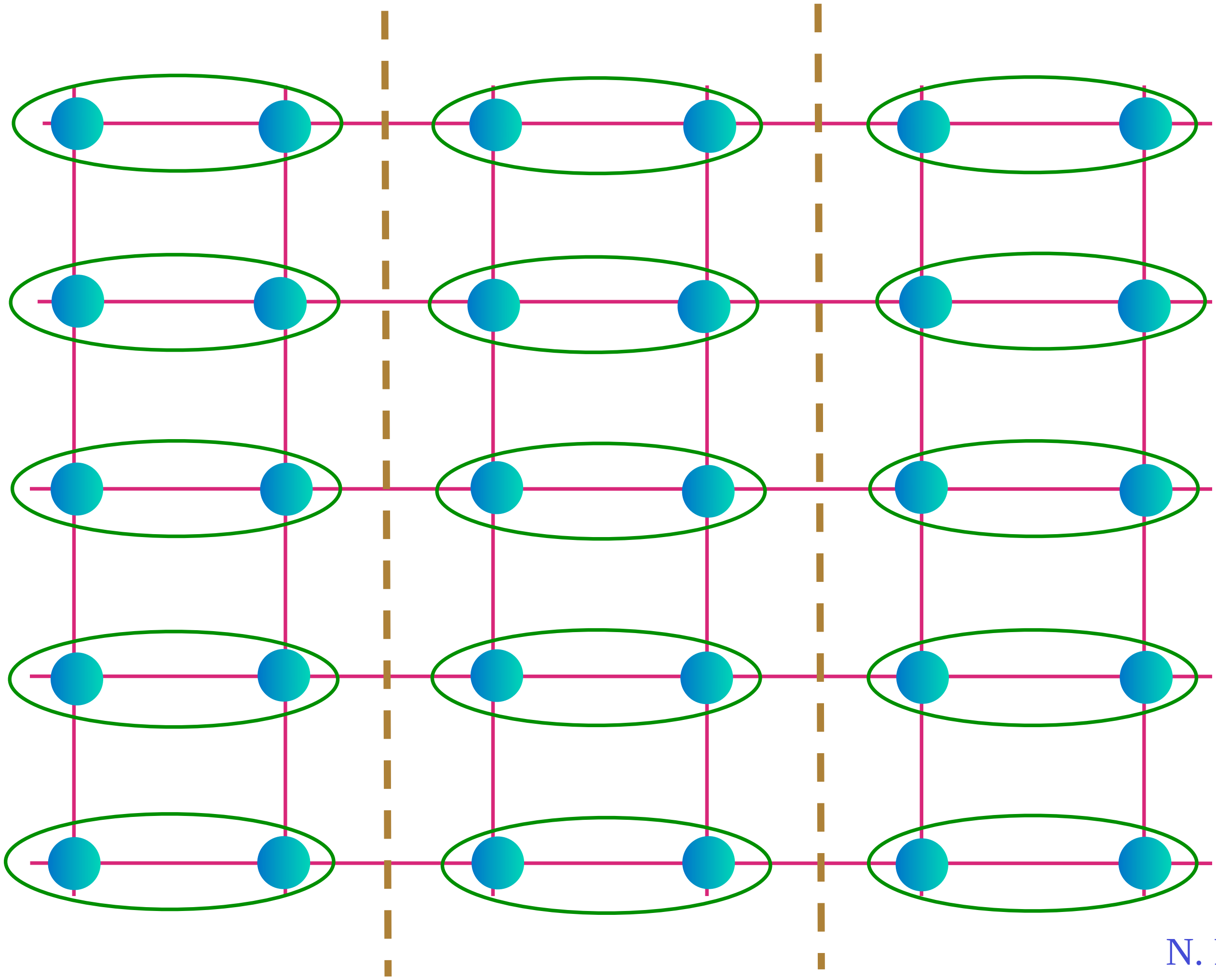

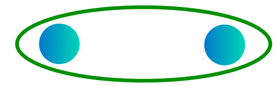

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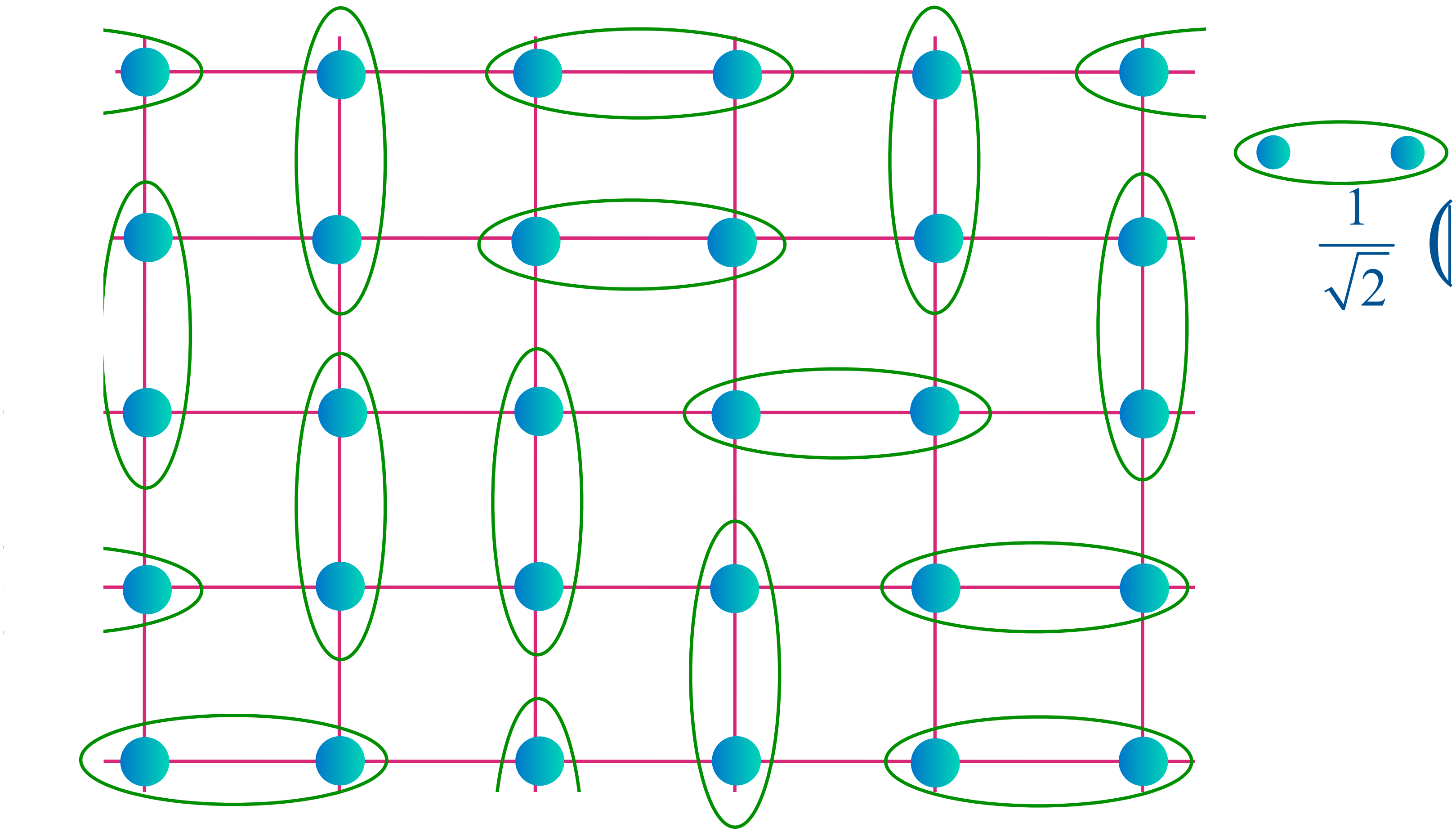
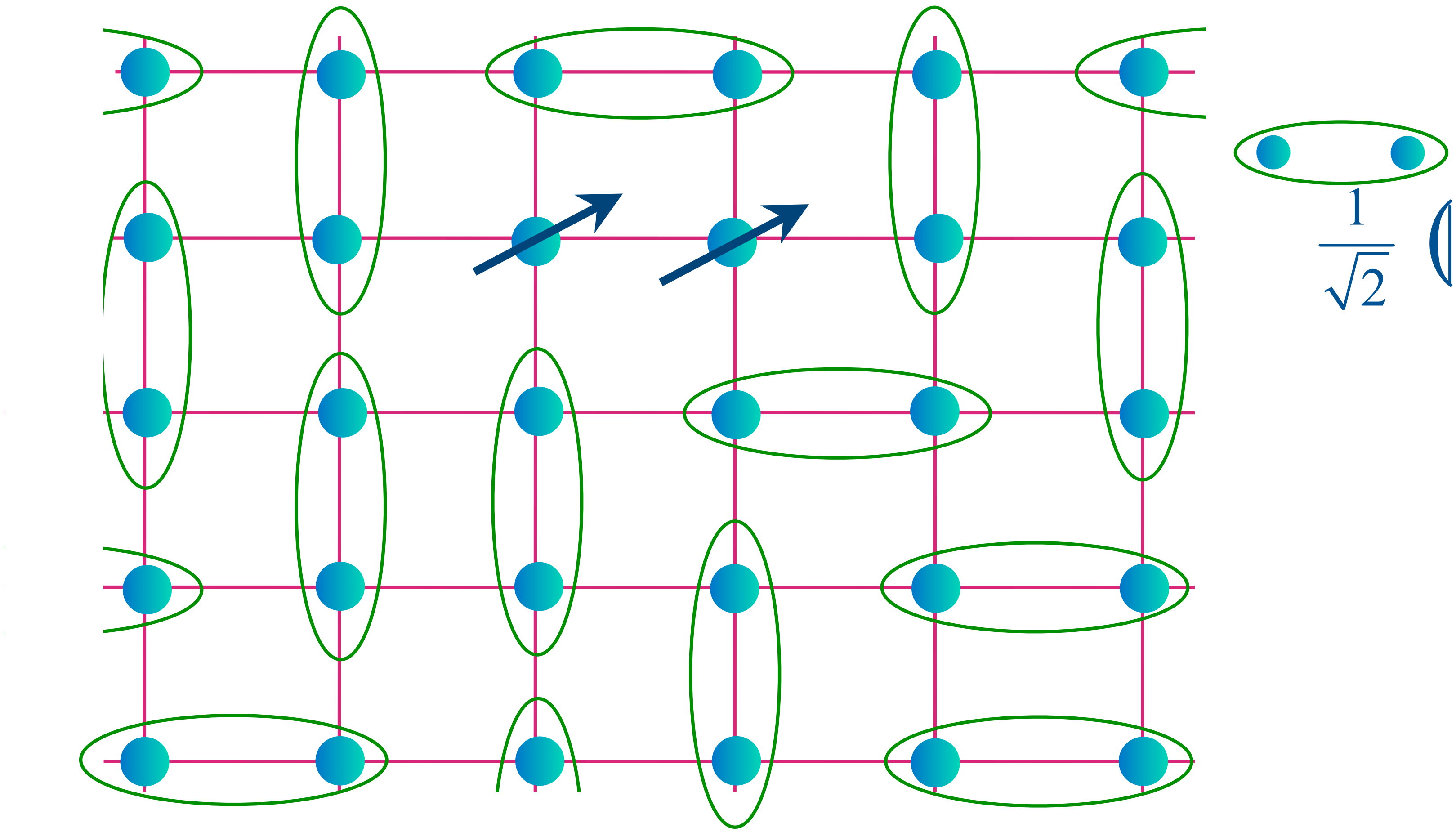
Excitations of the RVB liquid



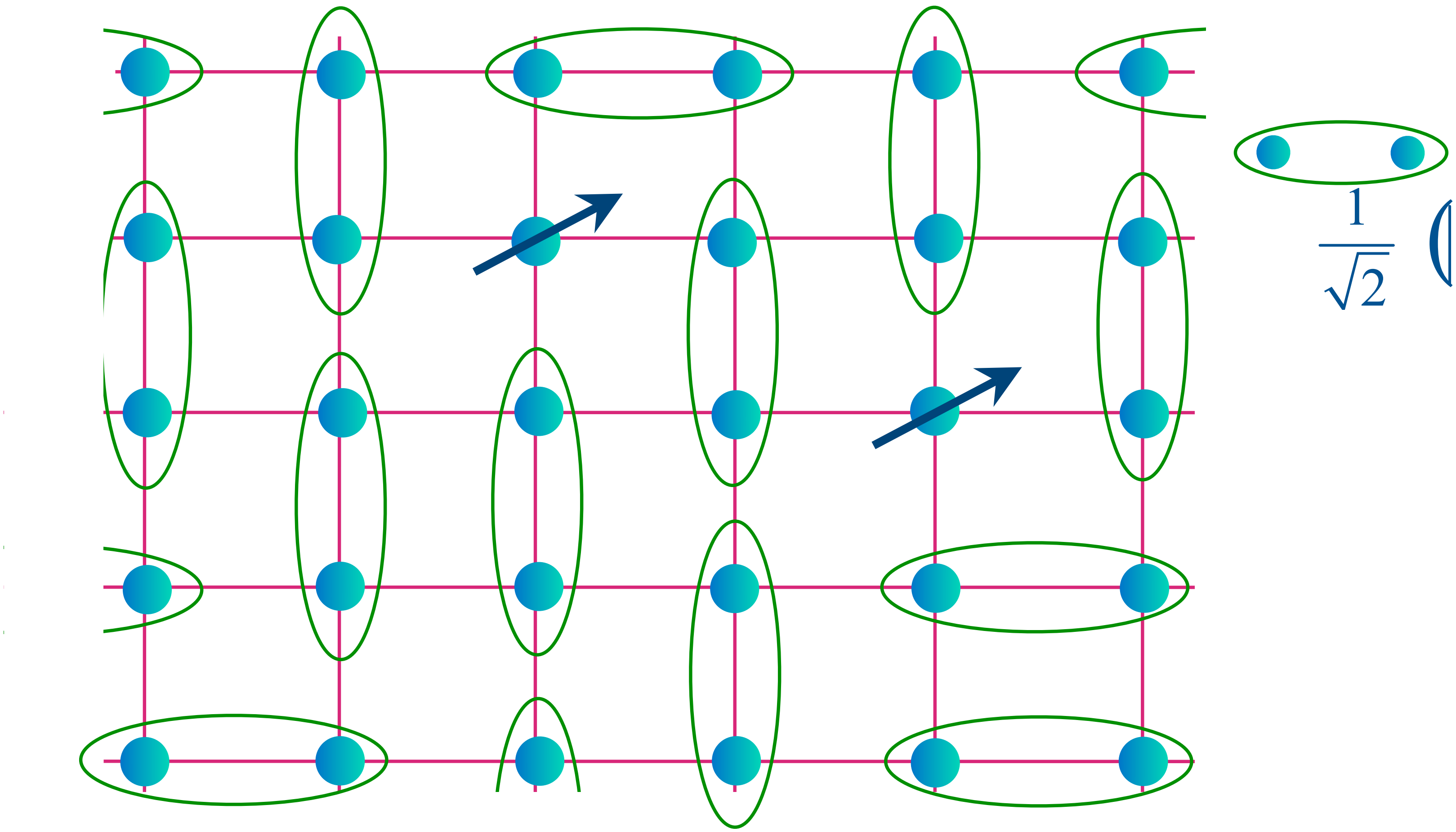
$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

Excitations of the RVB liquid



$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Excitations of the RVB liquid



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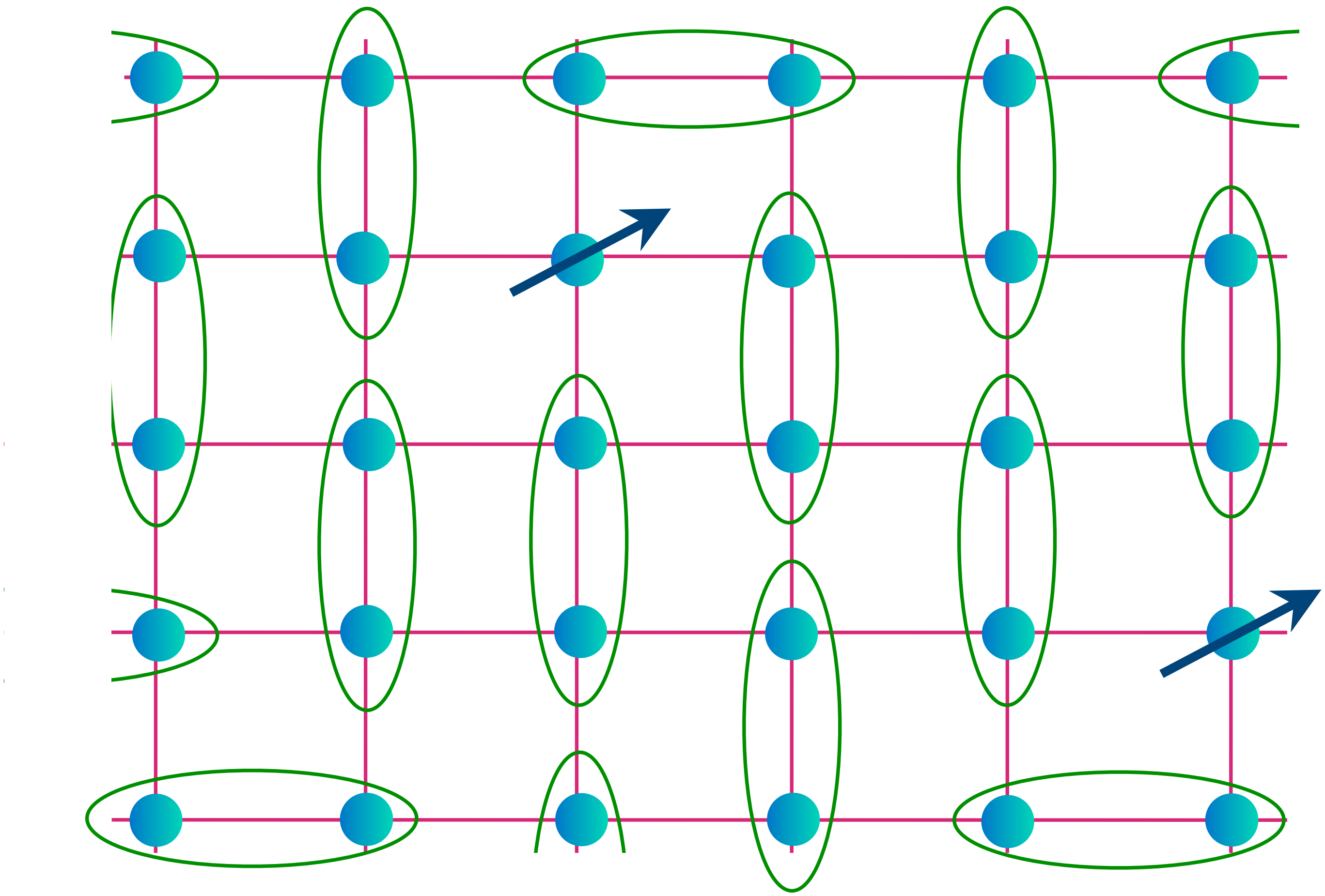
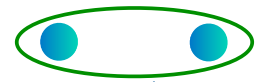
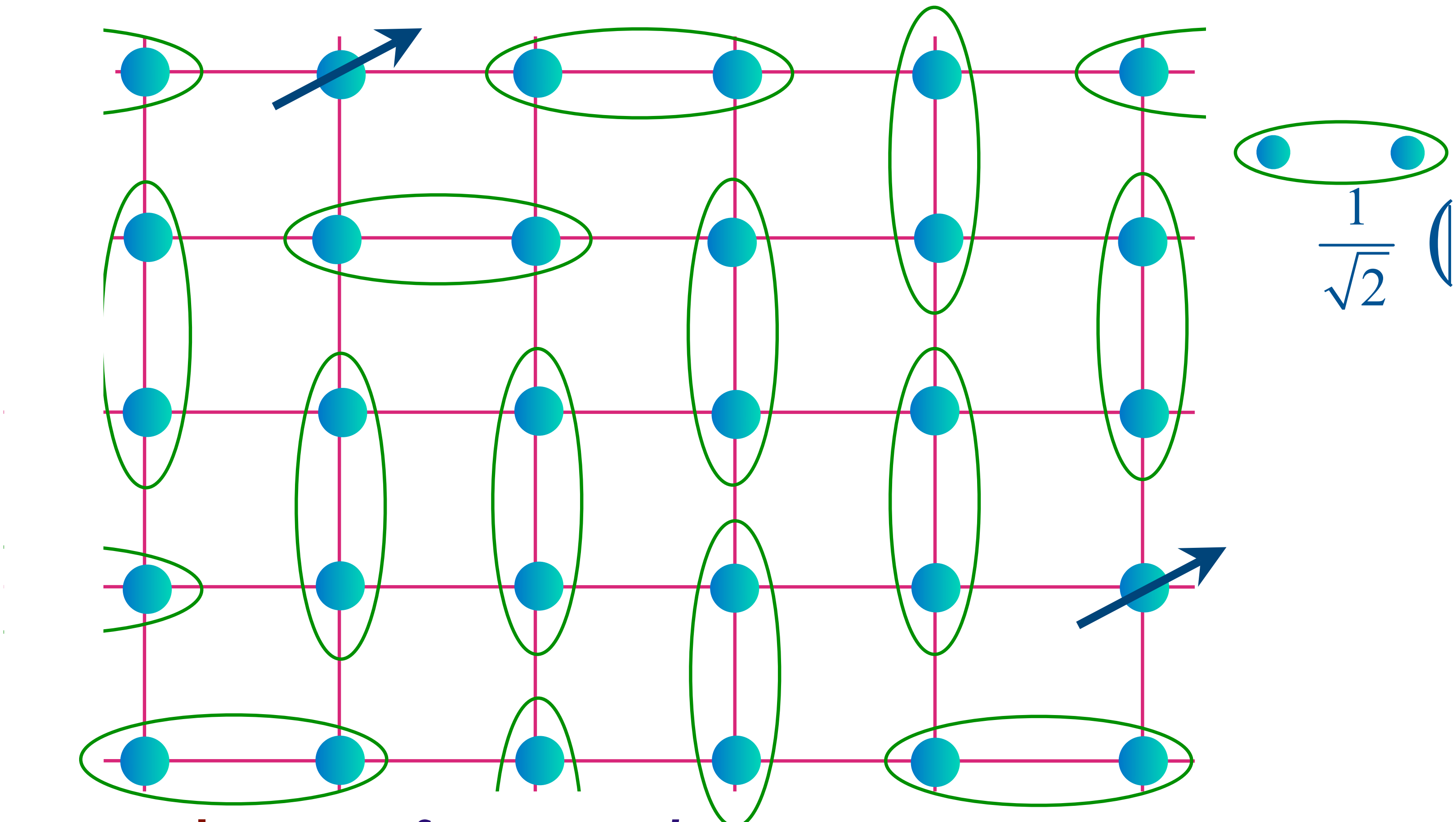
Excitations of the RVB liquid



 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

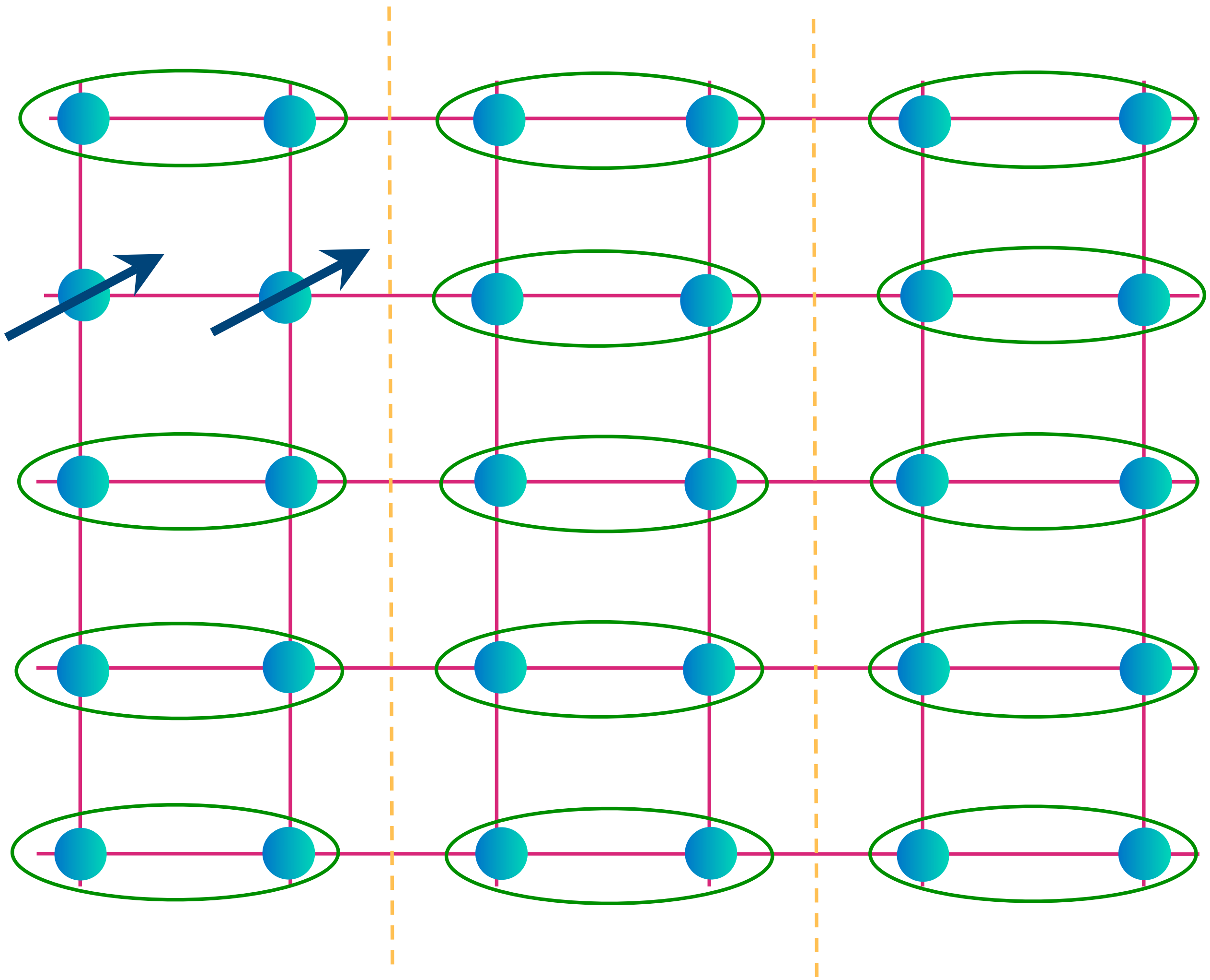
Excitations of the RVB liquid

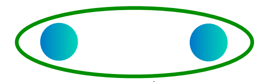


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Electron *fractionalization*: Excitations carry spin S=1/2 but no charge

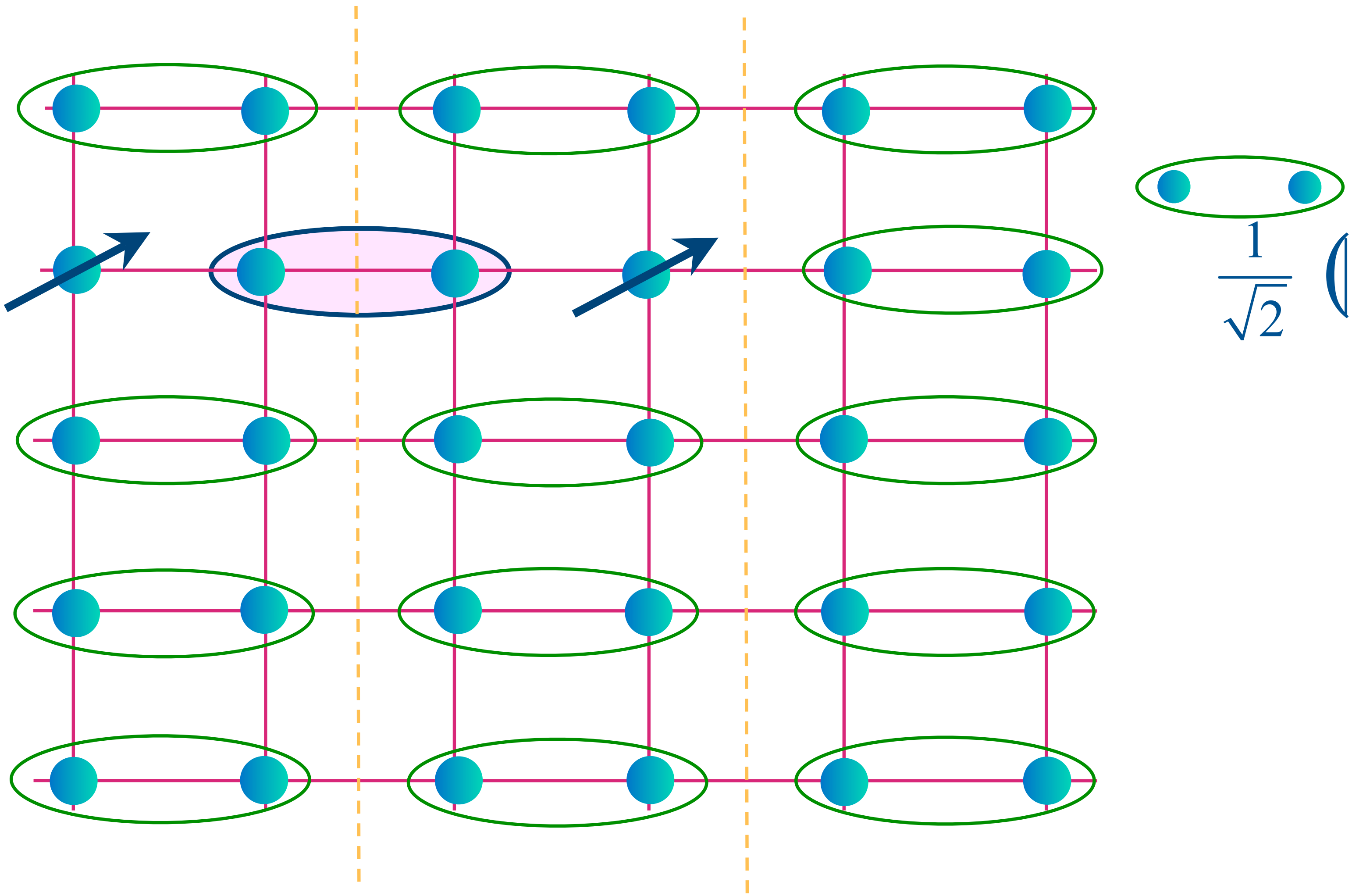
Excitations of the VBS



 =

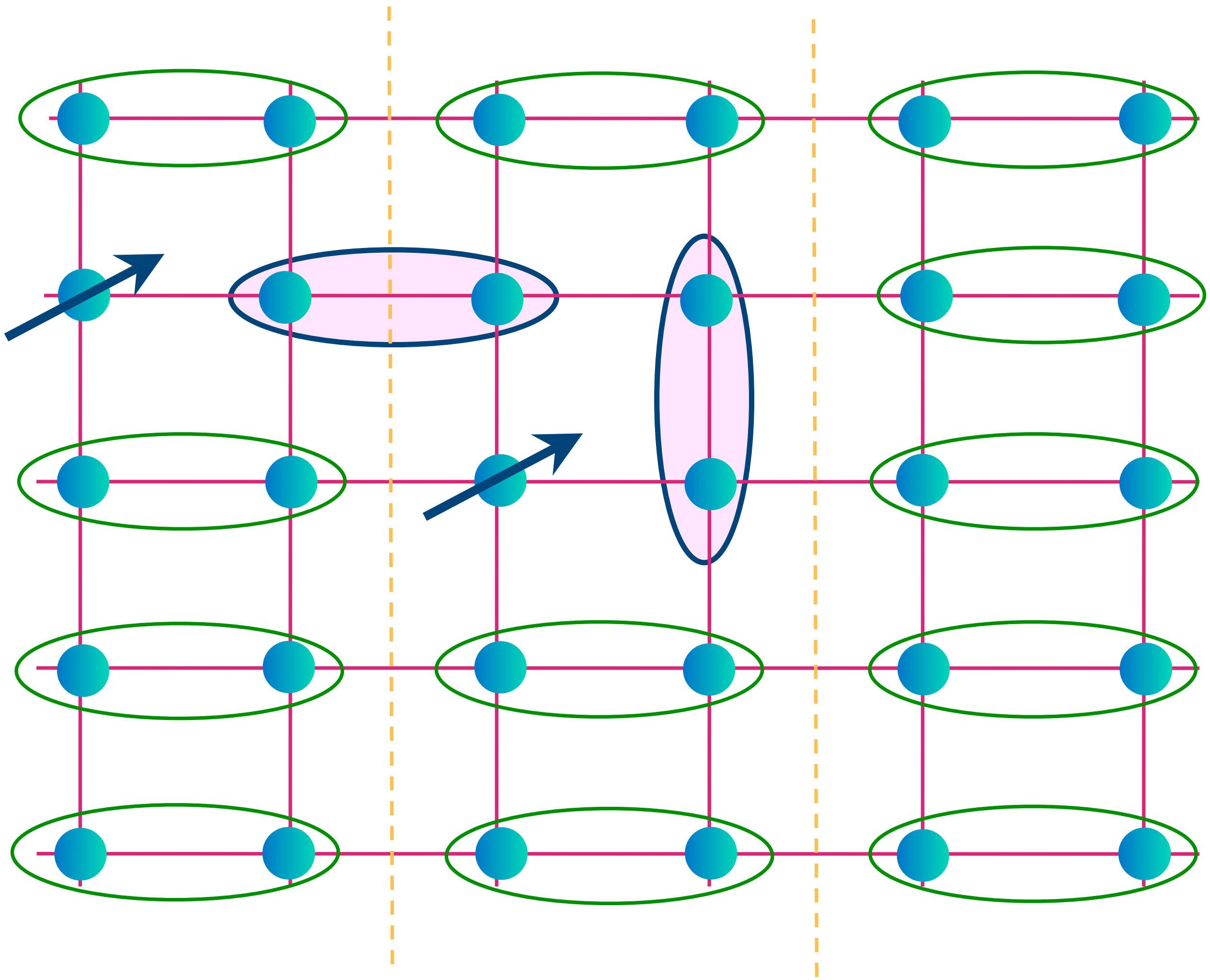
$$\frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Excitations of the VBS



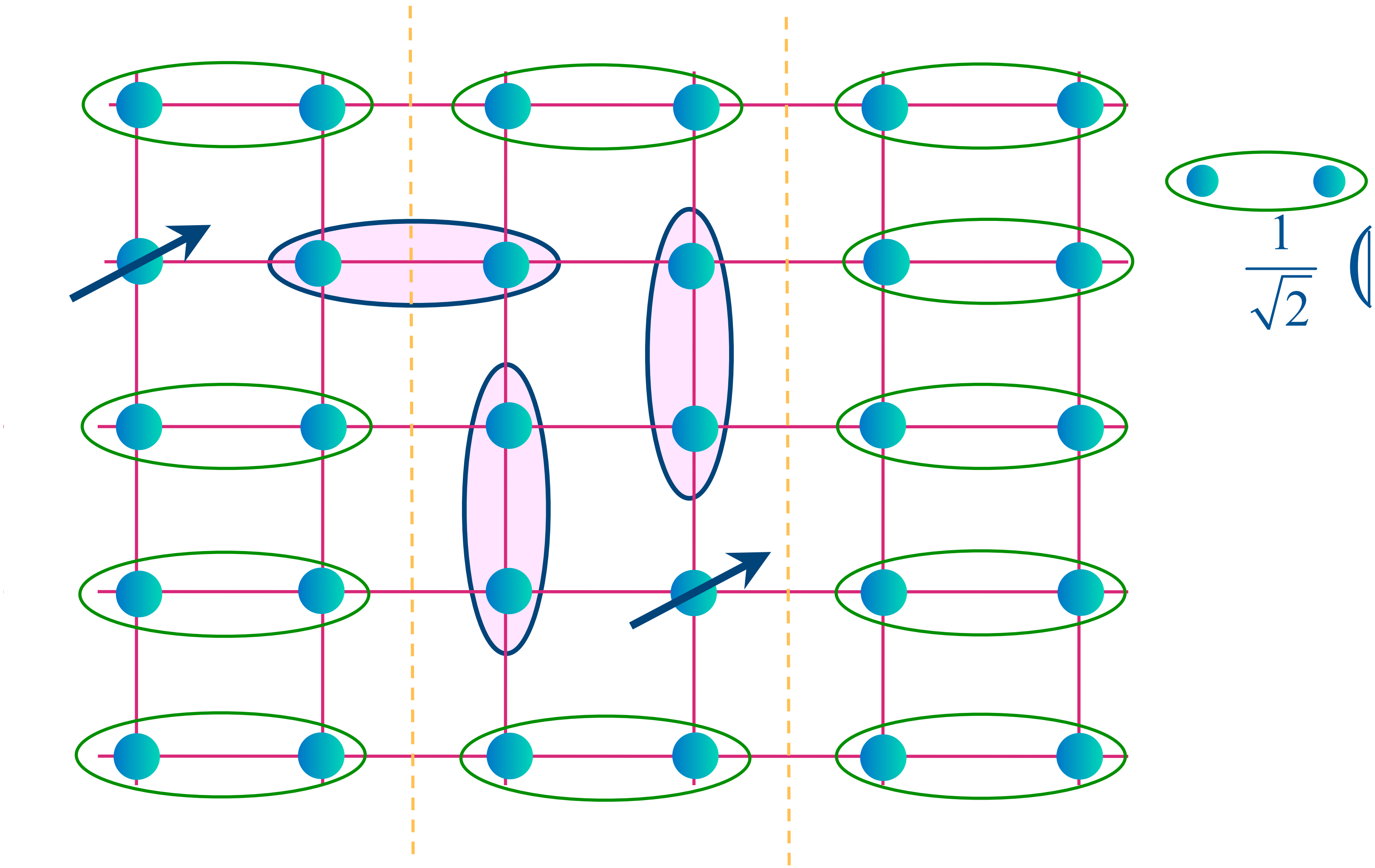
$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Excitations of the VBS



$$\text{(singlet pair)} = \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Excitations of the VBS



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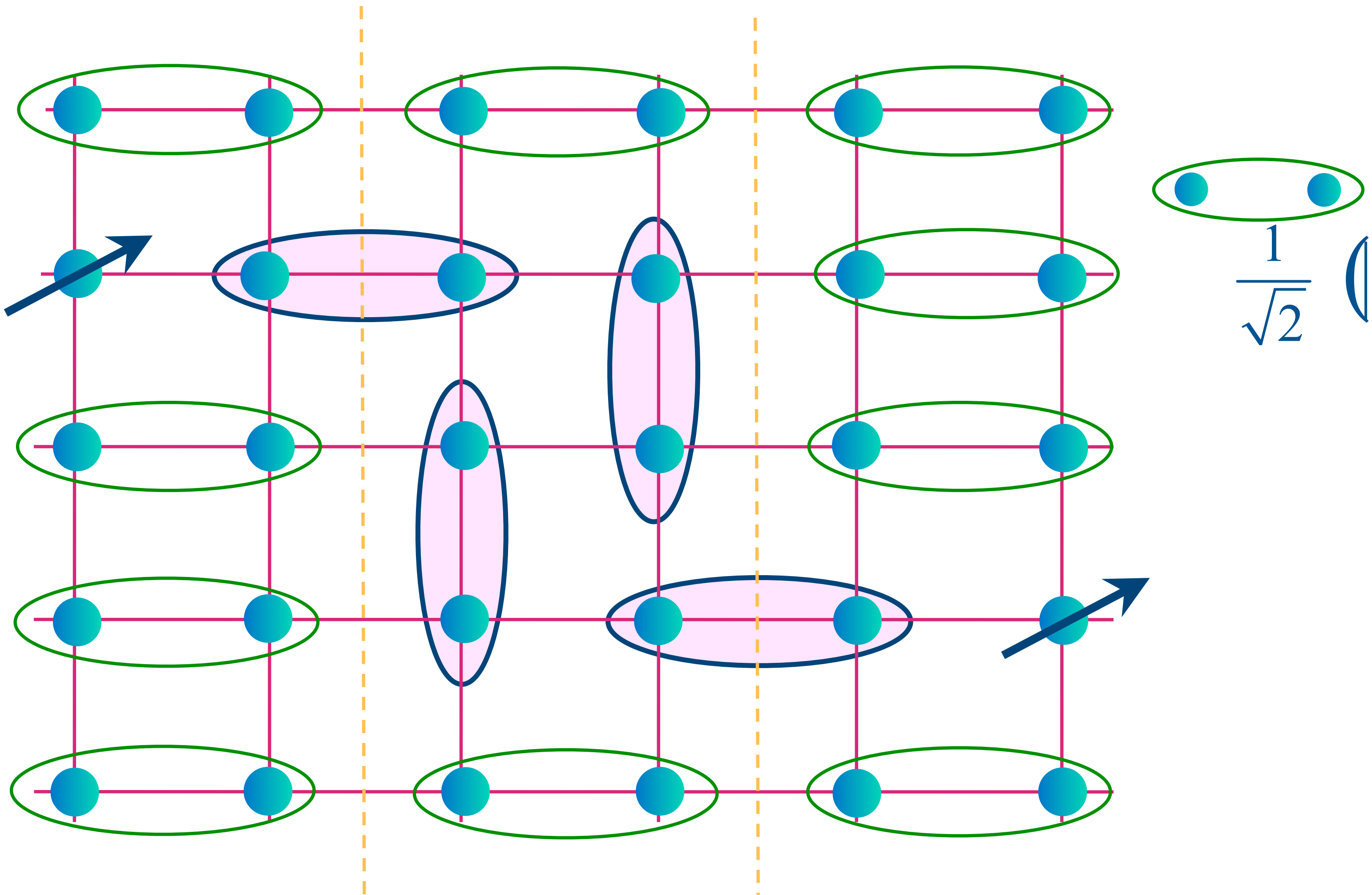
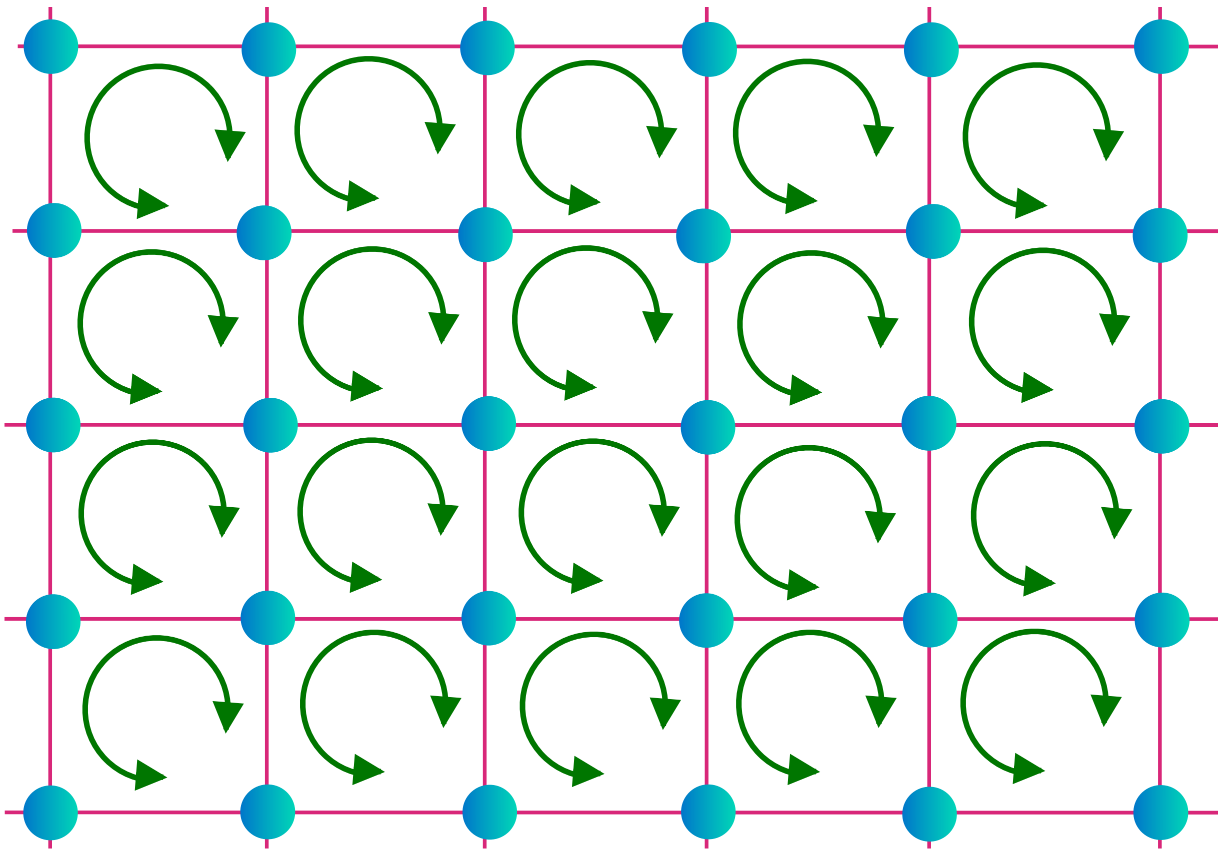


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

Free spins are unable to move apart: no fractionalization, but *confinement*

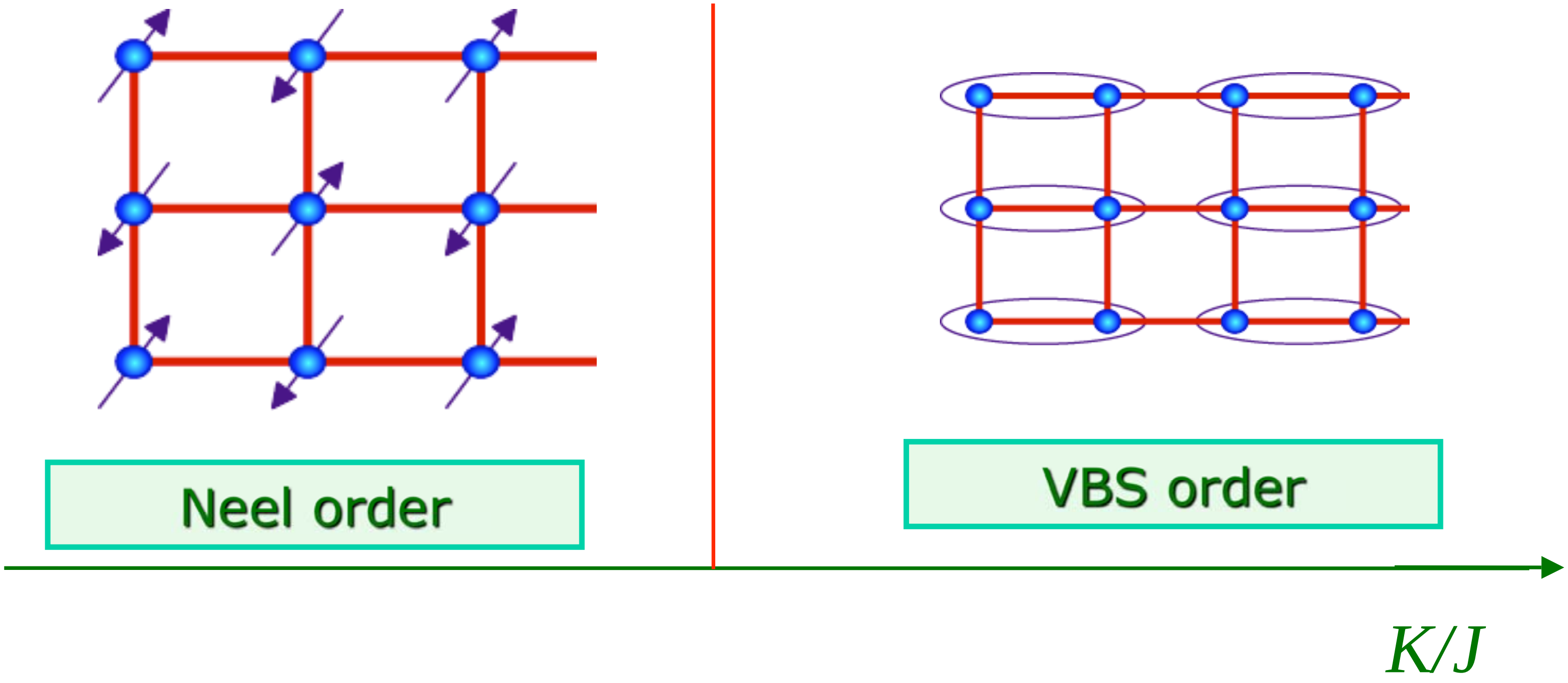
Phase diagram of square lattice antiferromagnet



$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{four spin exchange}$$

A. W. Sandvik, *Phys. Rev. Lett.* **98**, 227202 (2007)

Phase diagram of square lattice antiferromagnet



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T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

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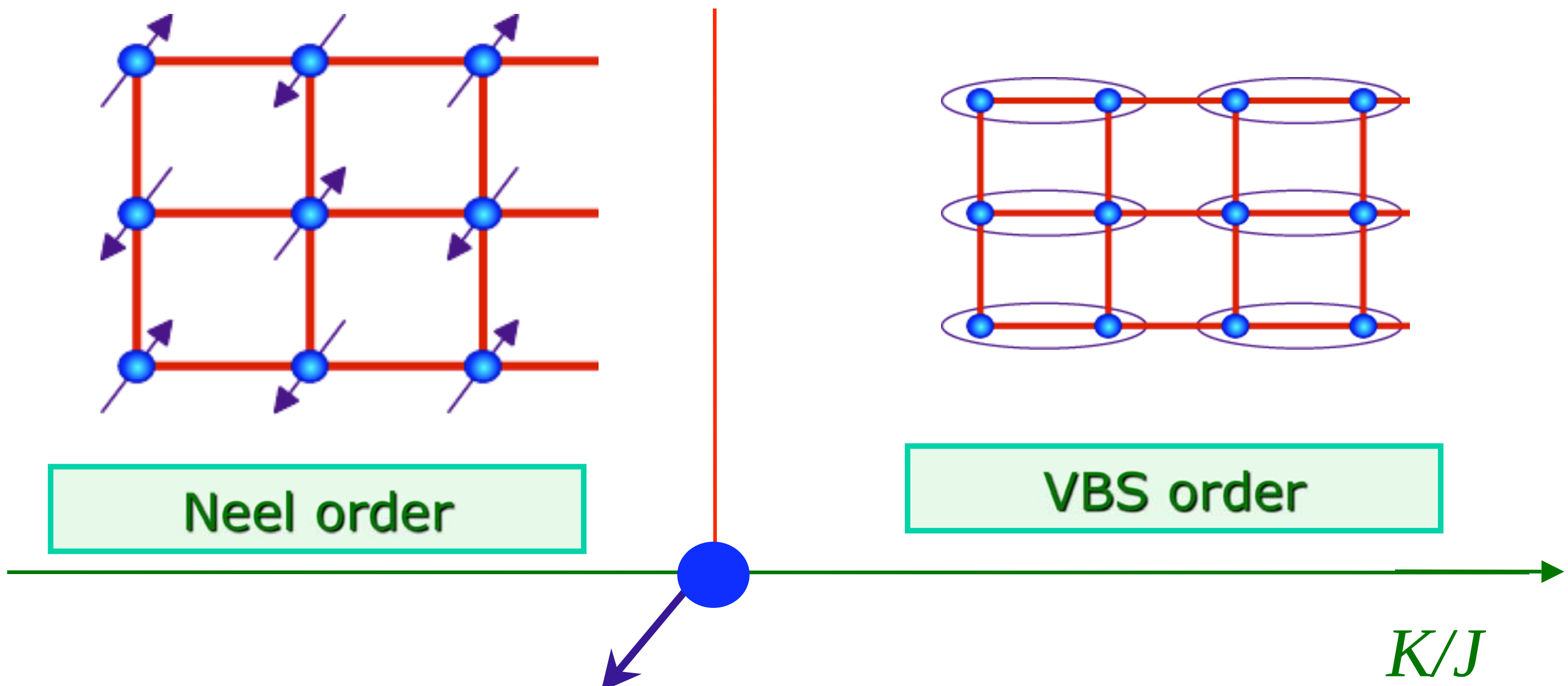
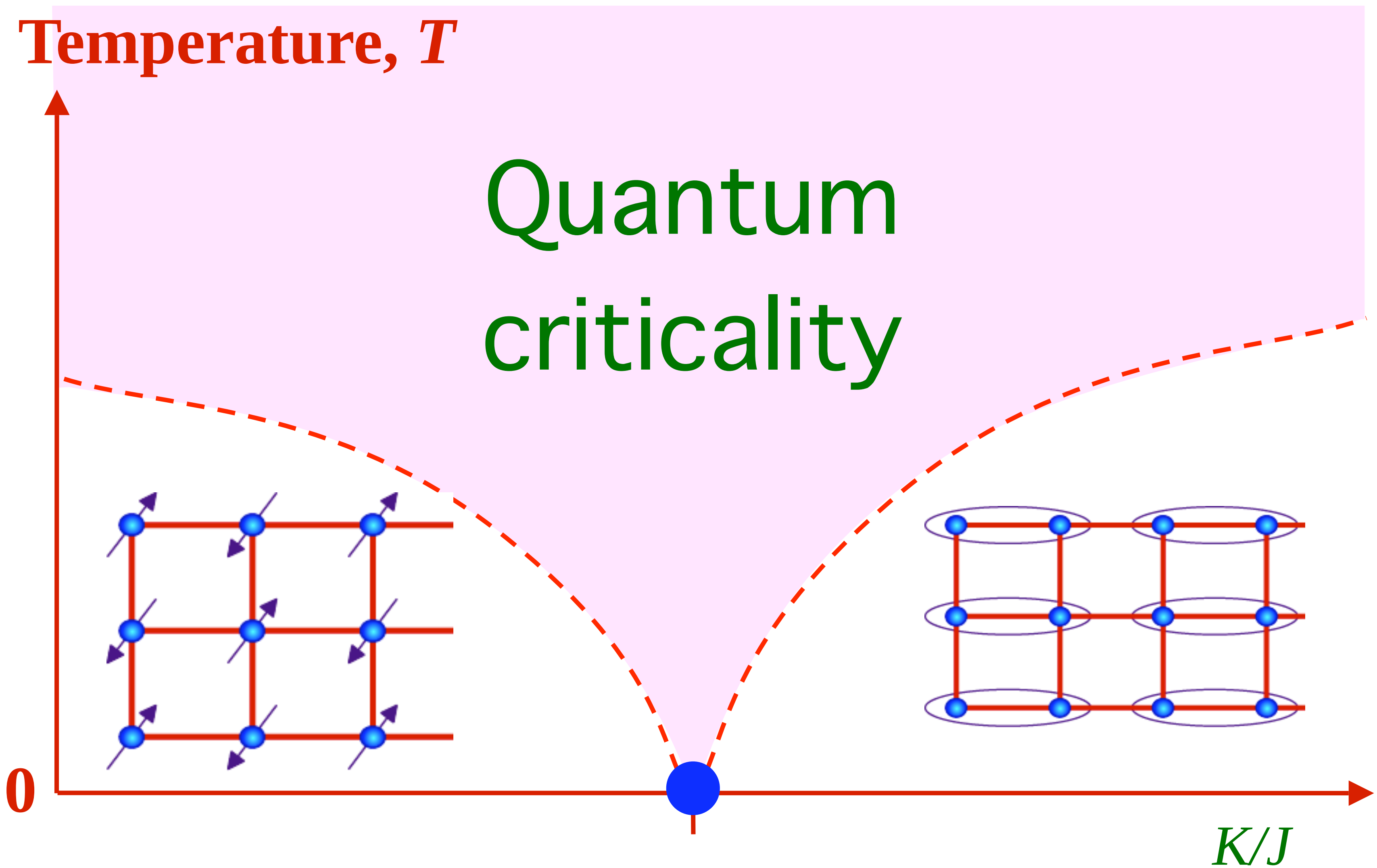


Quantum critical point with RVB-like entanglement: "*deconfined criticality*"

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\square} \text{four spin exchange}$$

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Why should we care about the entanglement at an isolated critical point in the parameter space ?



Temperature, *T*
Quantum criticality
0
K/J

Outline

1. Entanglement of spins
   Experiments on spin-gap insulators

2. Entanglement of valence bonds
   Deconfined criticality in antiferromagnets

3. Black Hole Thermodynamics
   Connections to quantum criticality

4. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

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Black Holes

Objects so massive that light is gravitationally bound to them.

Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \frac{k_B A}{4\ell_P^2}$

where A is the area of the horizon, and

$\ell_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$

Black Hole Thermodynamics

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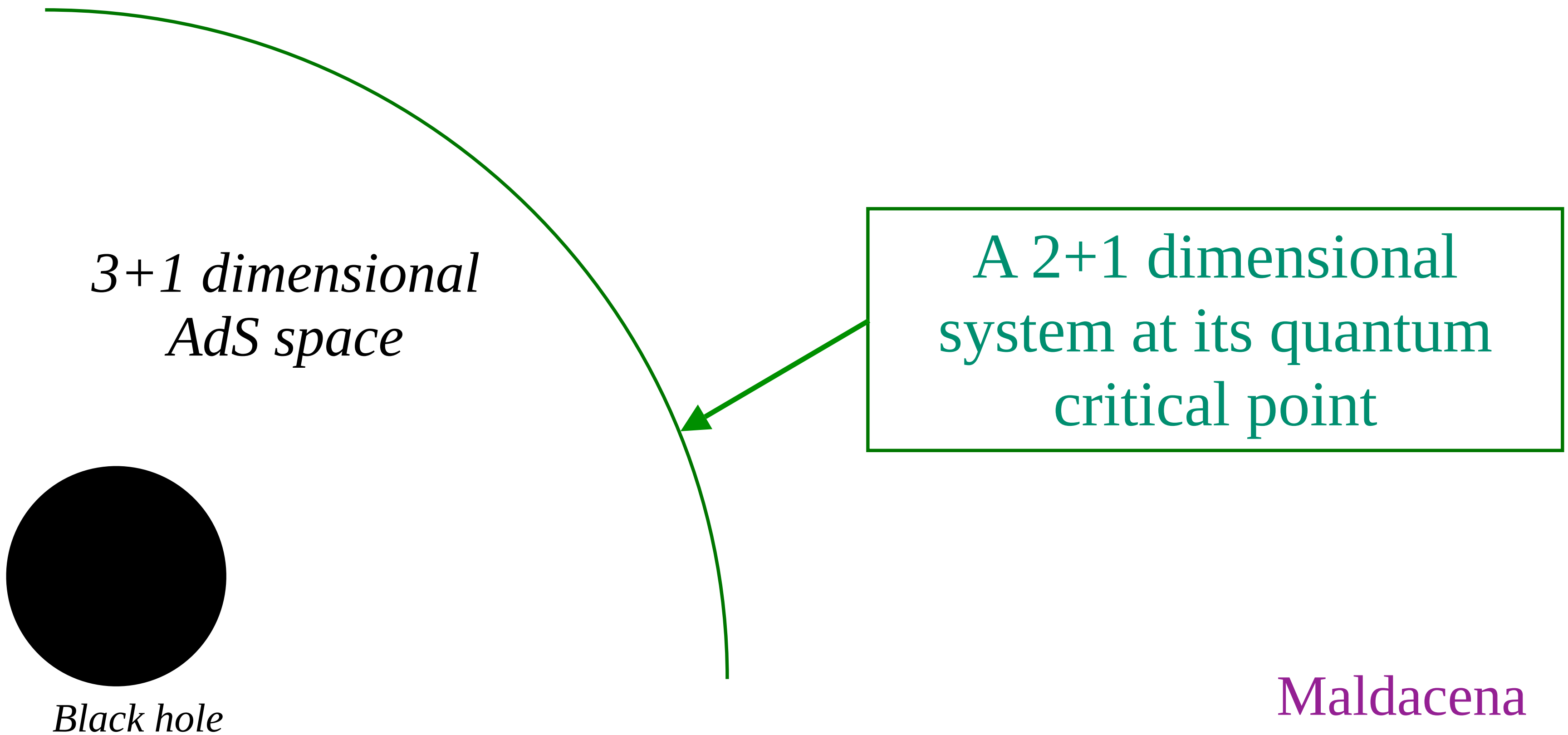
$$\text{Horizon temperature: } 4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$$

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena

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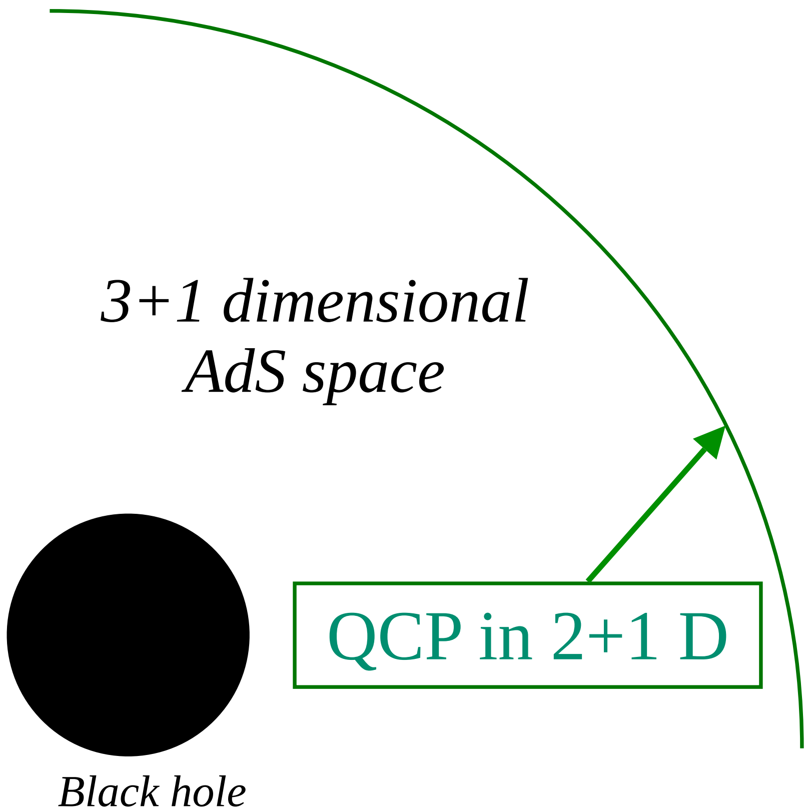

Black hole temperature = temperature of quantum criticality

Strominger, Vafa

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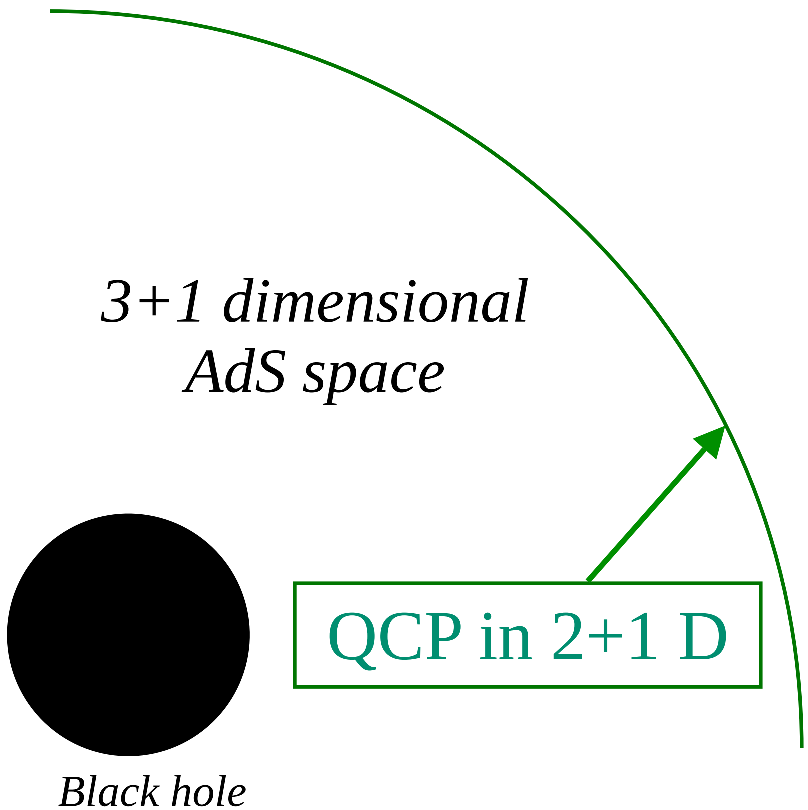

Black hole entropy = entropy of quantum criticality in 2+1 dimensions

Strominger, Vafa

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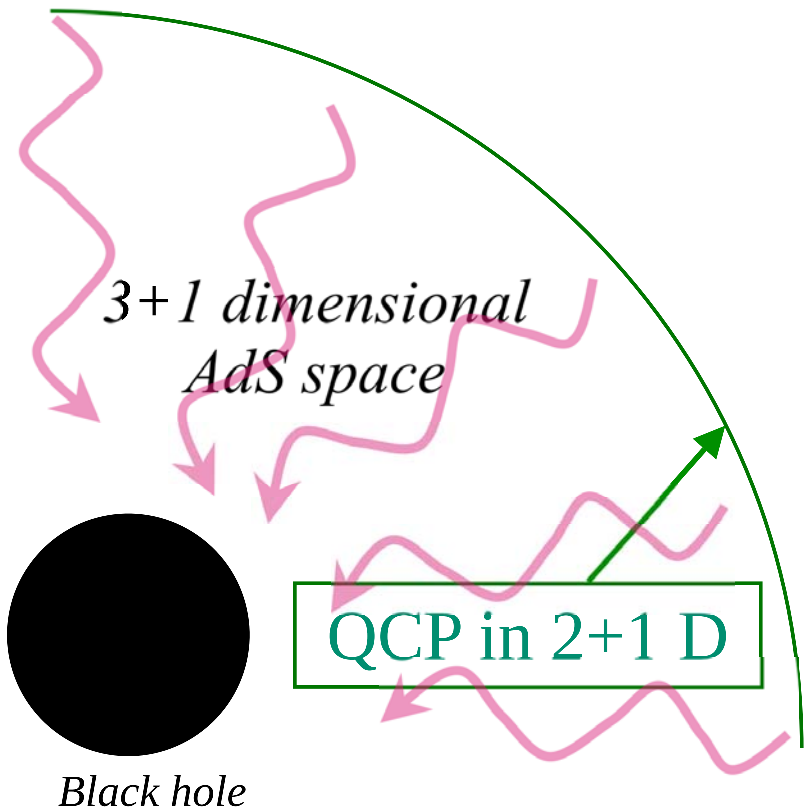

Dynamics of quantum criticality = waves in curved gravitational background

Maldacena

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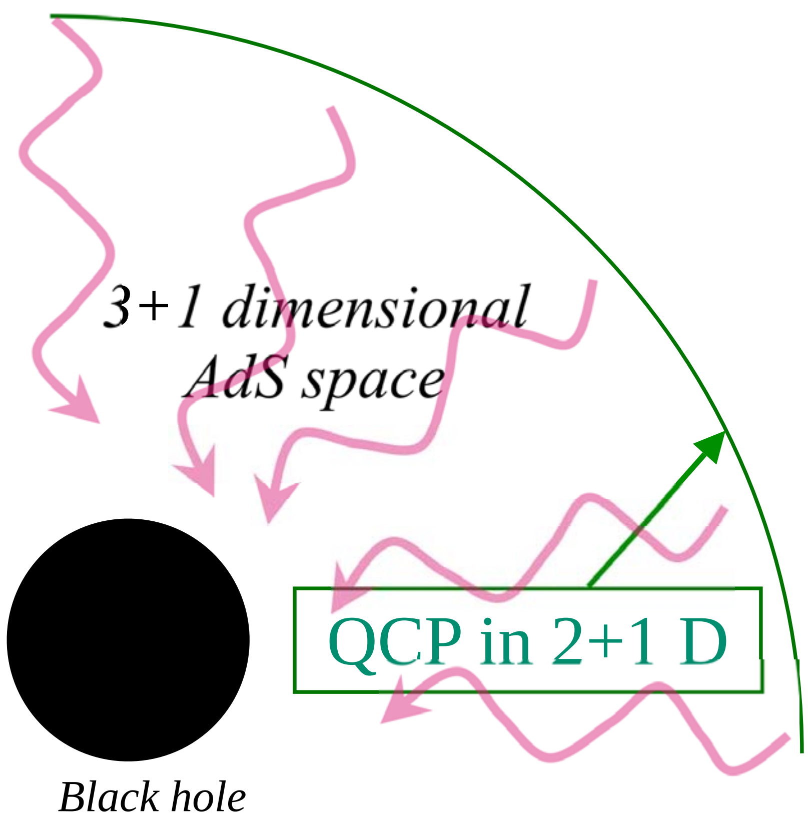

"Friction" of quantum critical dynamics = black hole absorption rates

Son

Outline

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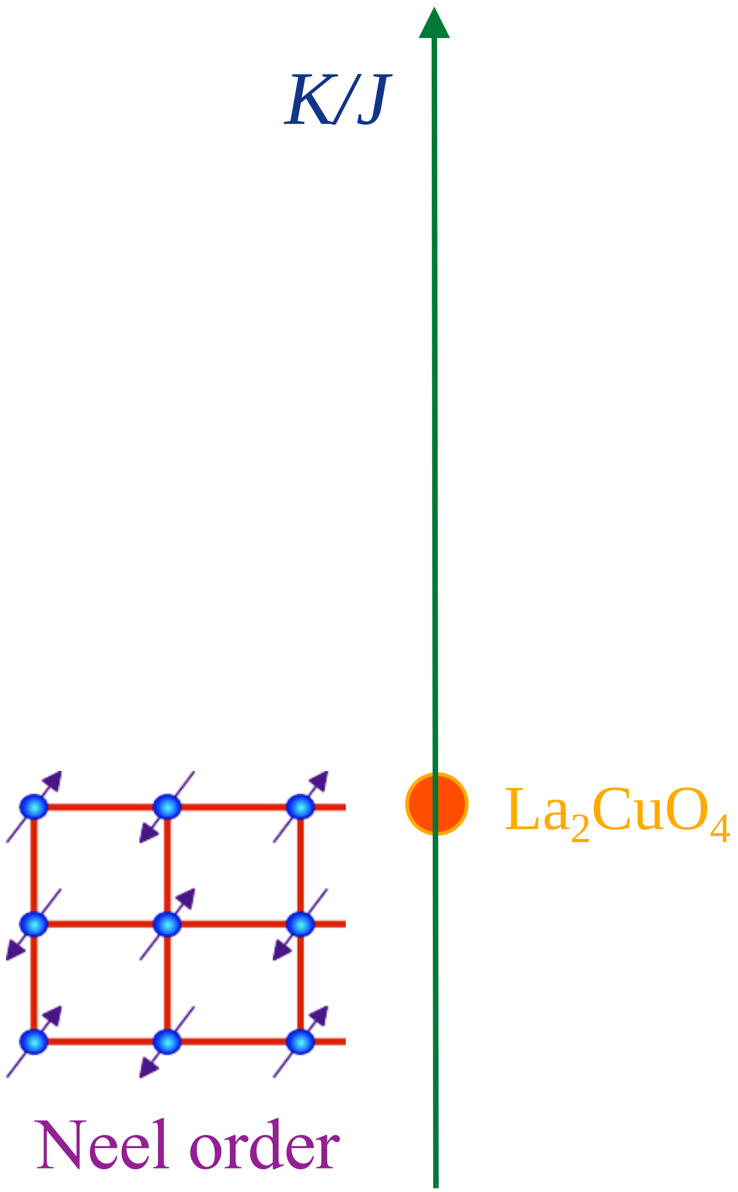
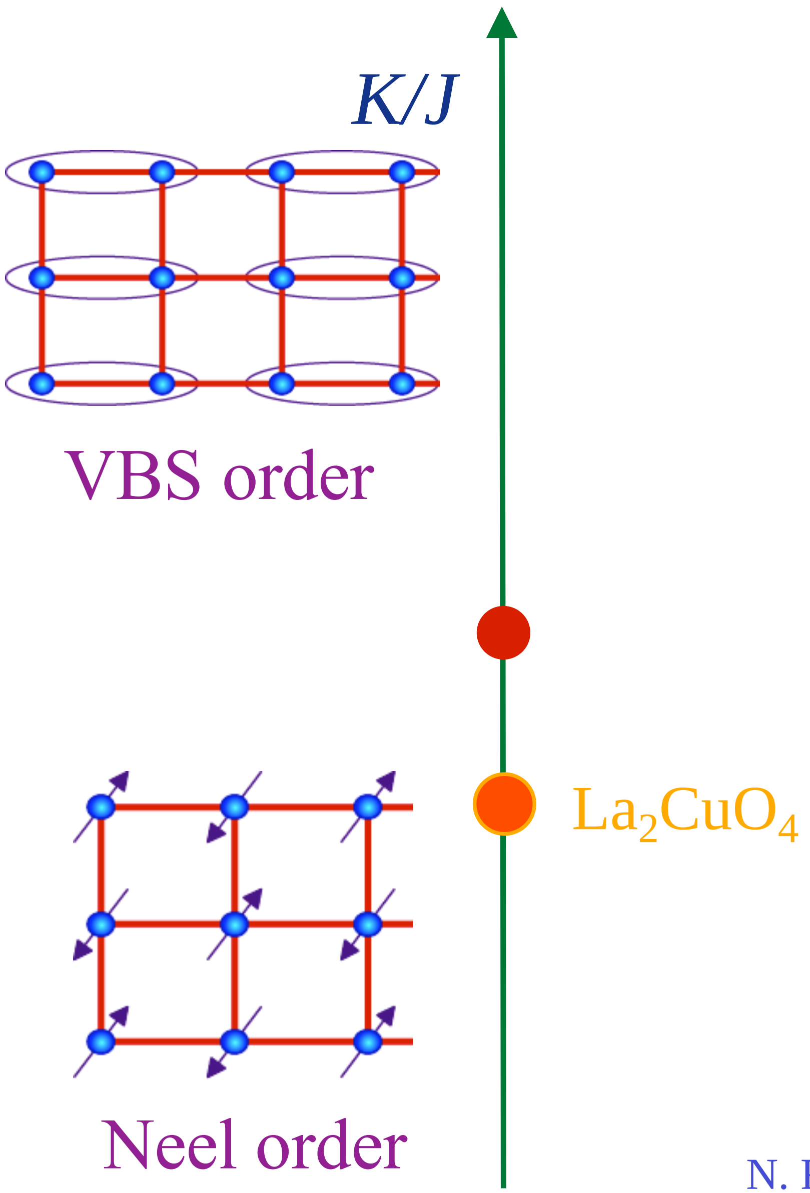
Phase diagram of doped antiferromagnets



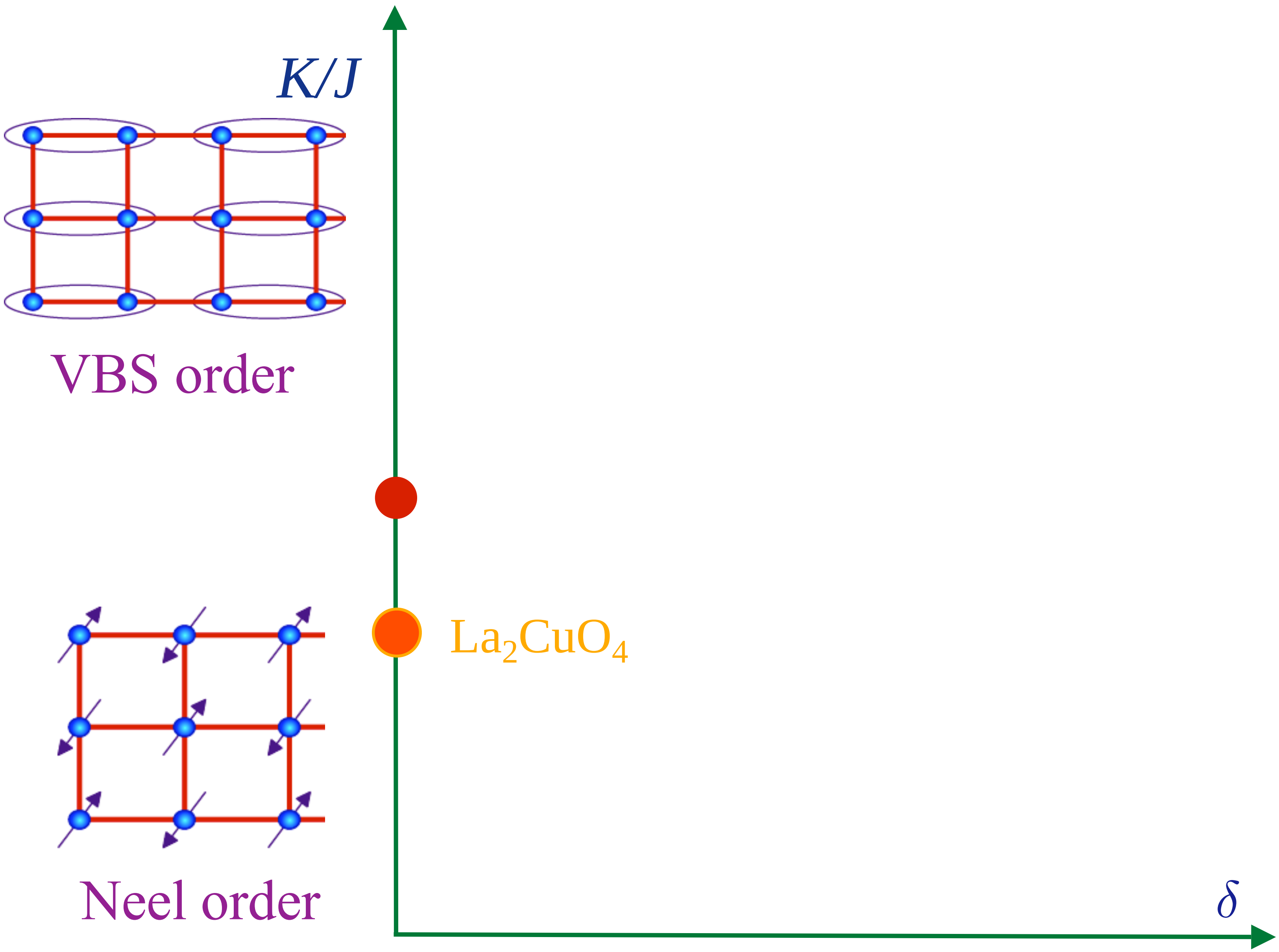
Phase diagram of doped antiferromagnets

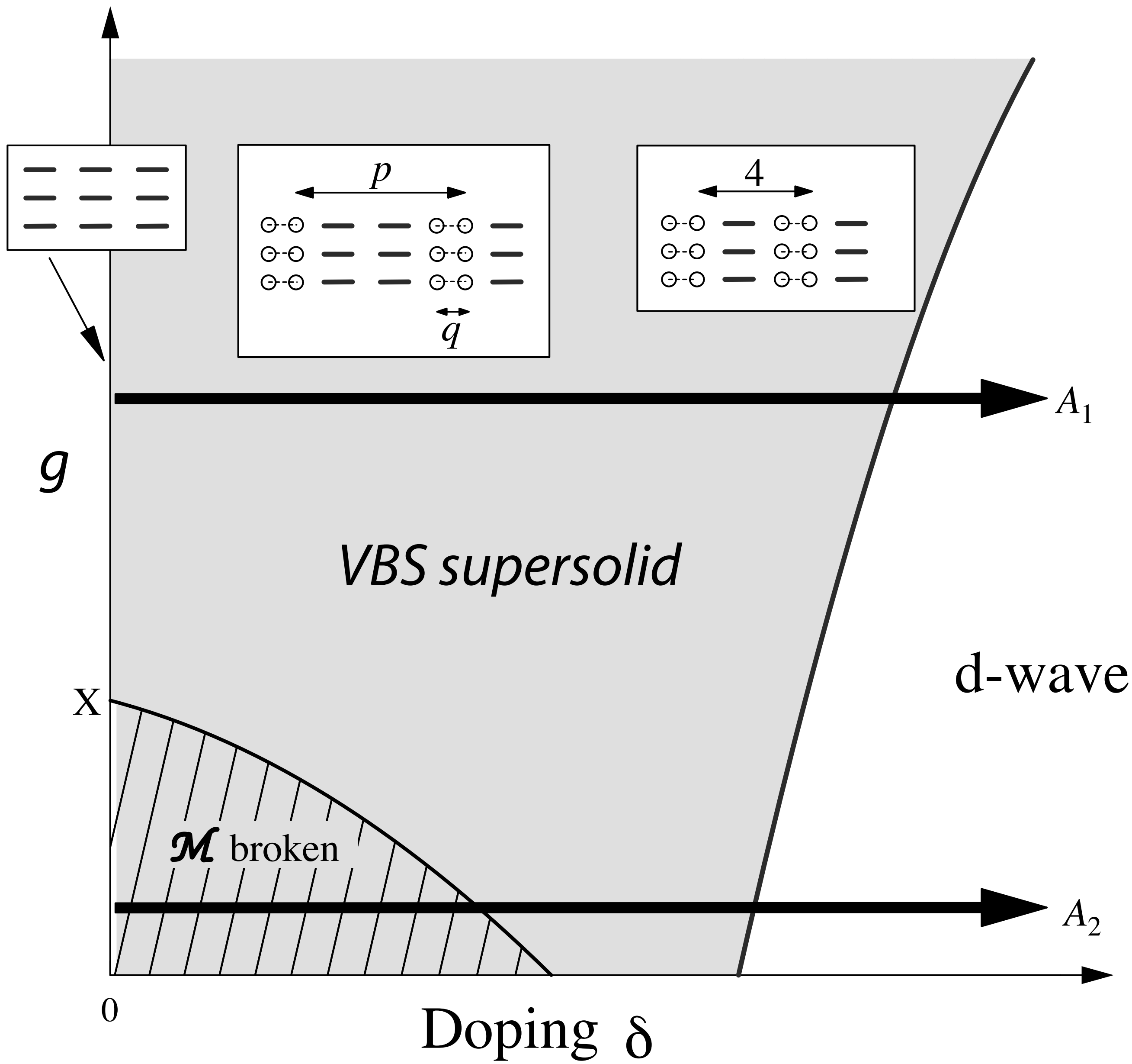


N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989).

T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

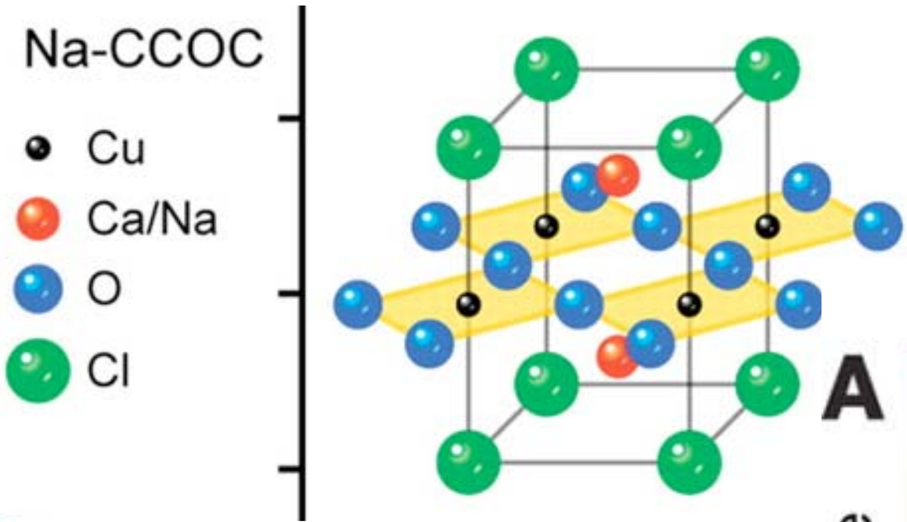
Phase diagram of doped antiferromagnets

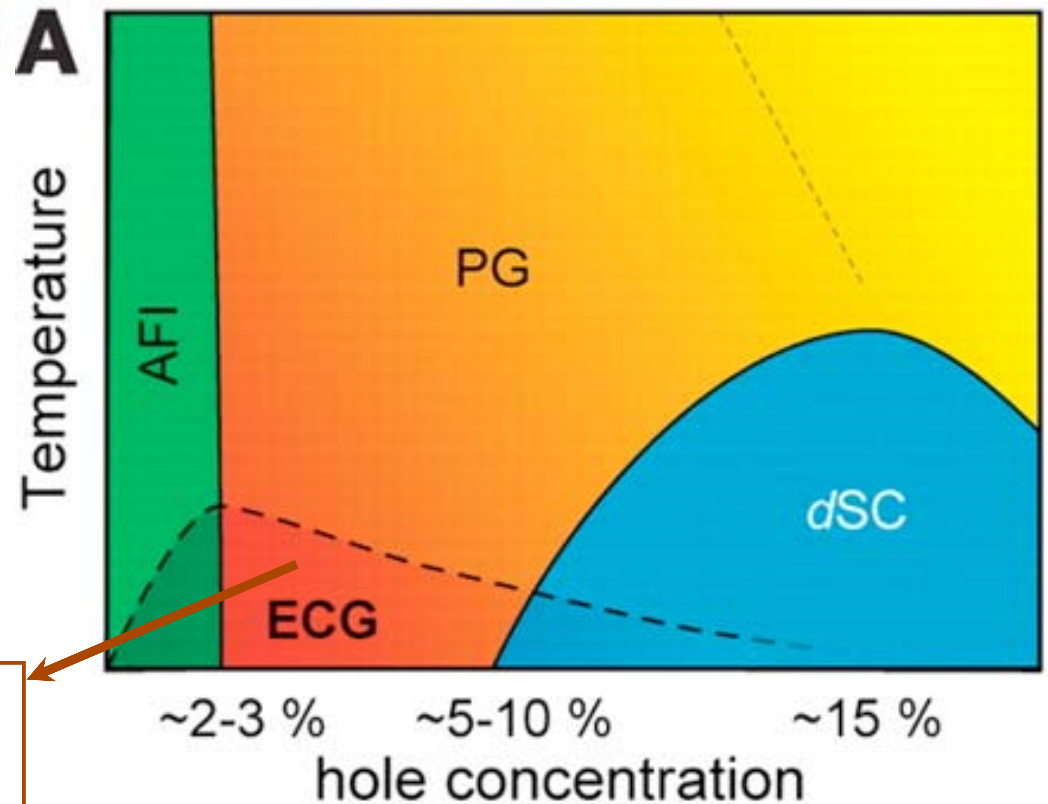




M. Vojta and S. Sachdev, *Phys. Rev. Lett.* **83**, 3916 (1999)

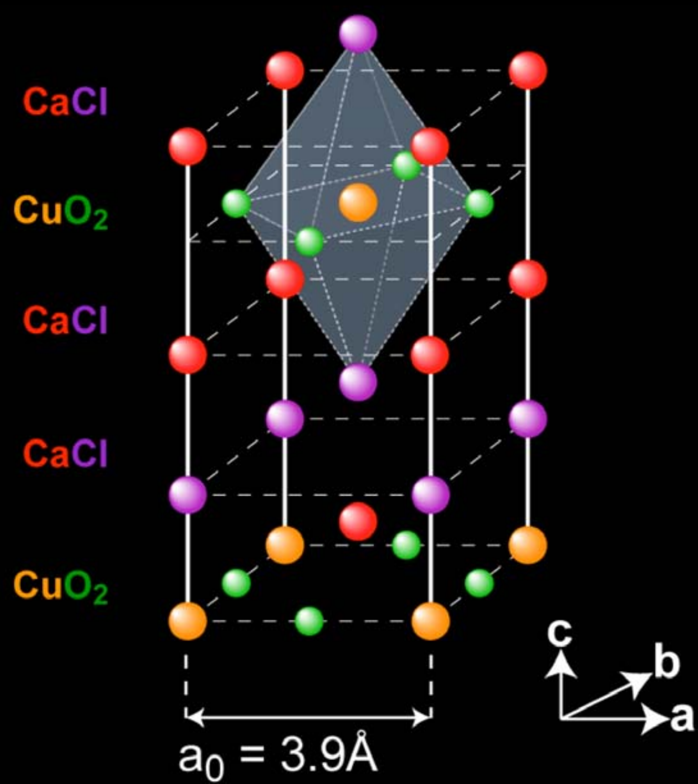
Temperature-doping phase diagram of the cuprate superconductors



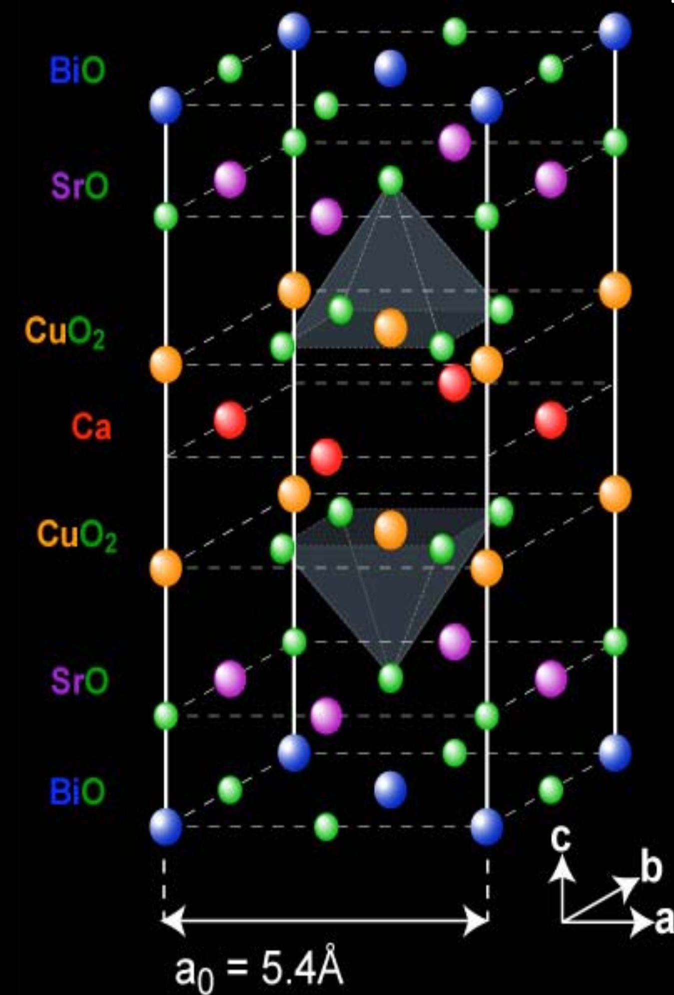


Scanning Tunneling Microscopy experiments

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$

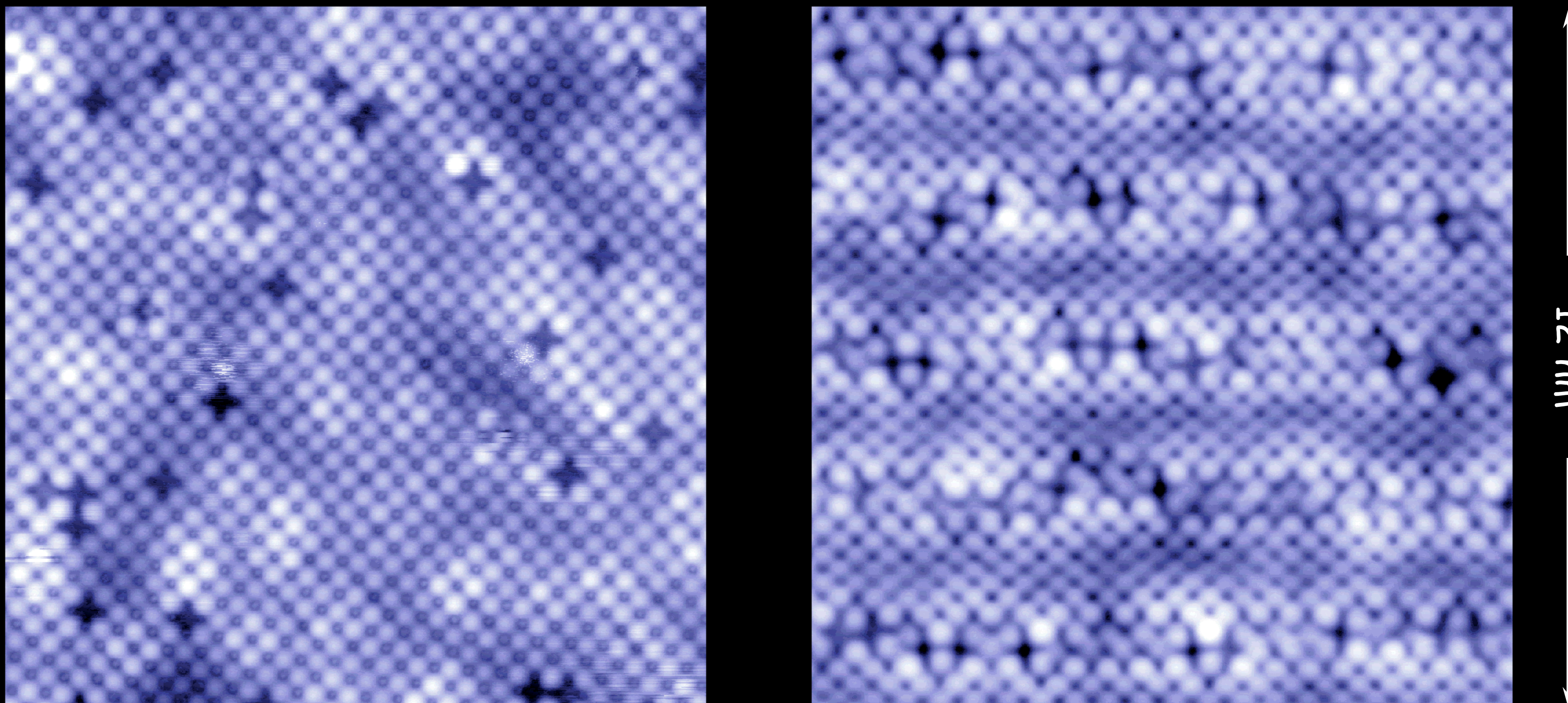


$Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



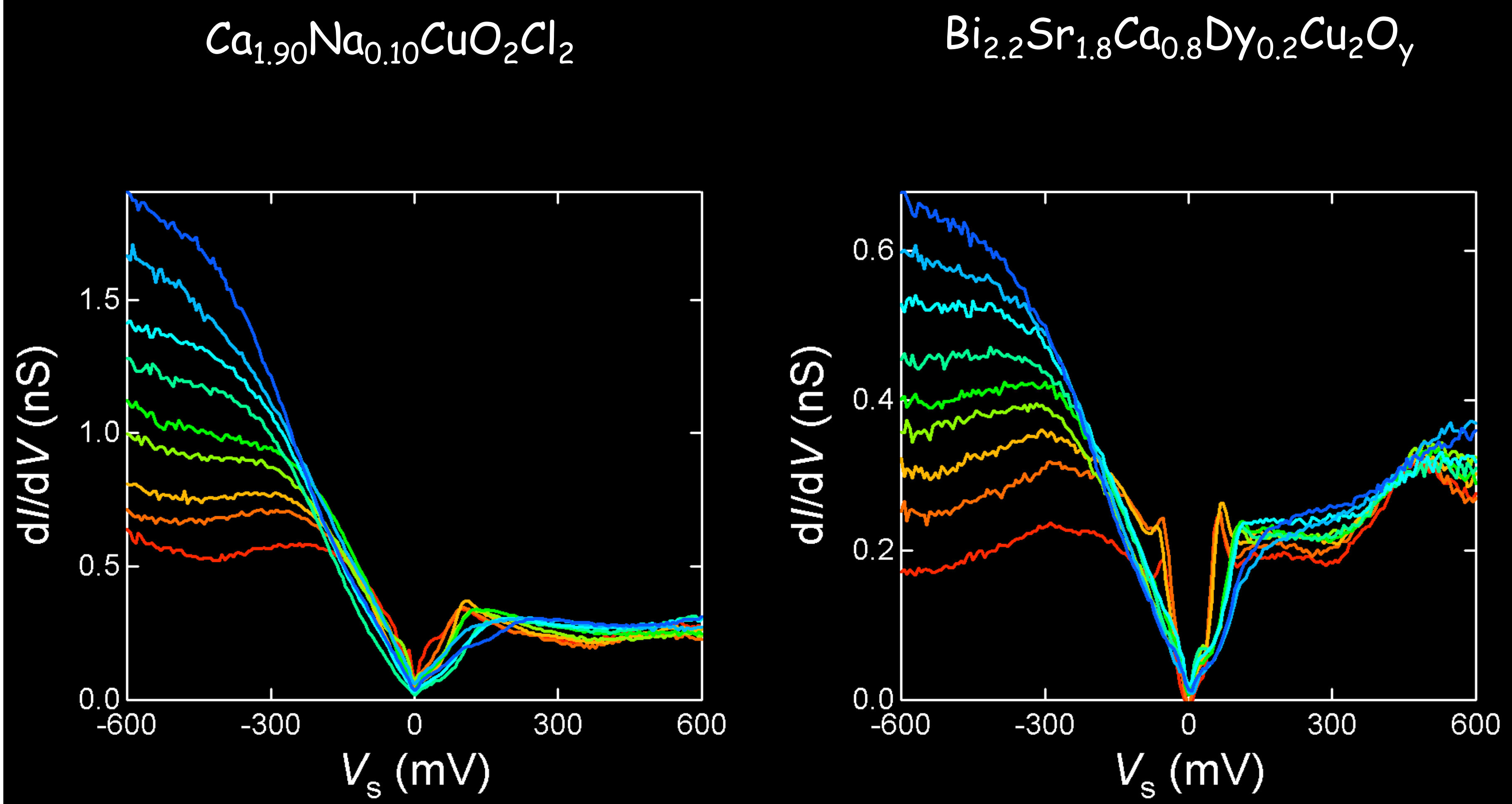
Topograph

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$ $\quad$ $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$



Science 315, 1380 (2007)

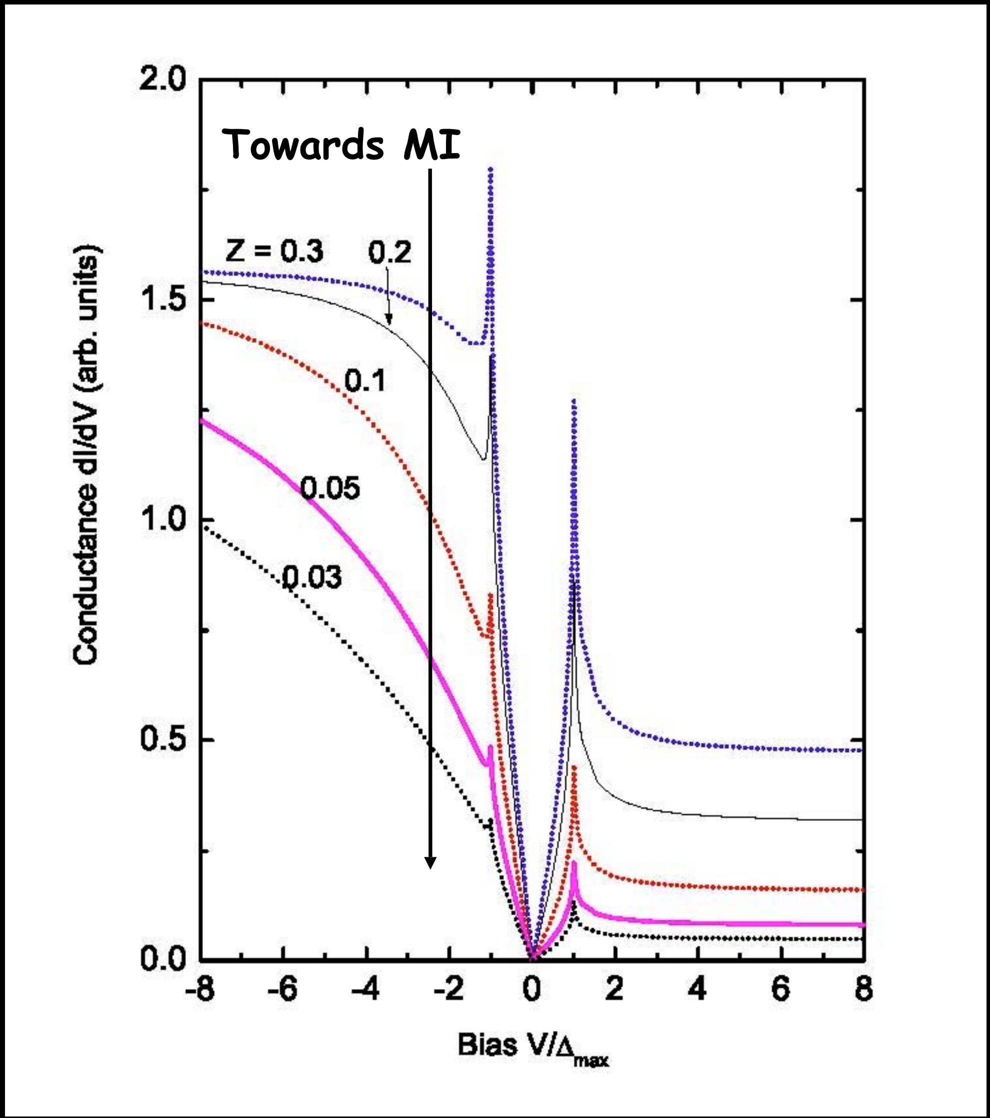
dI/dV Spectra



Intense Tunneling-Asymmetry (TA)
variation are highly similar

Science 315, 1380 (2007)

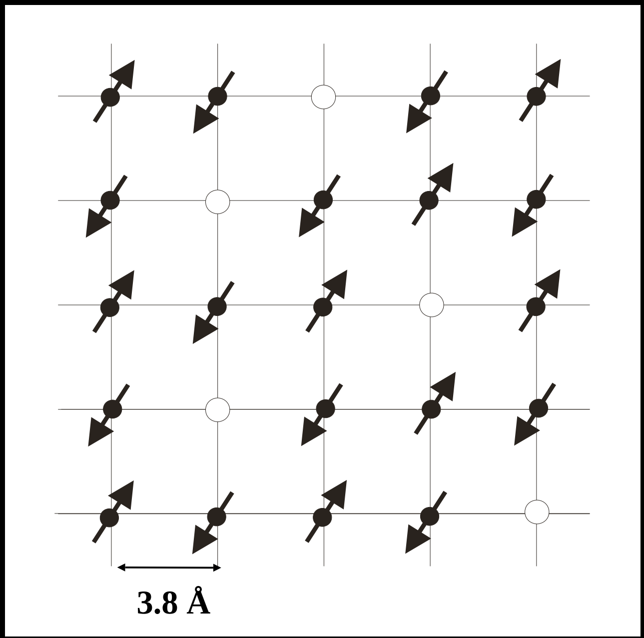
Tunneling Asymmetry is related to hole density



See also M.B.J. Meinders, H. Eskes and G.A. Sawatzky, PRB 48,3916 (1993).

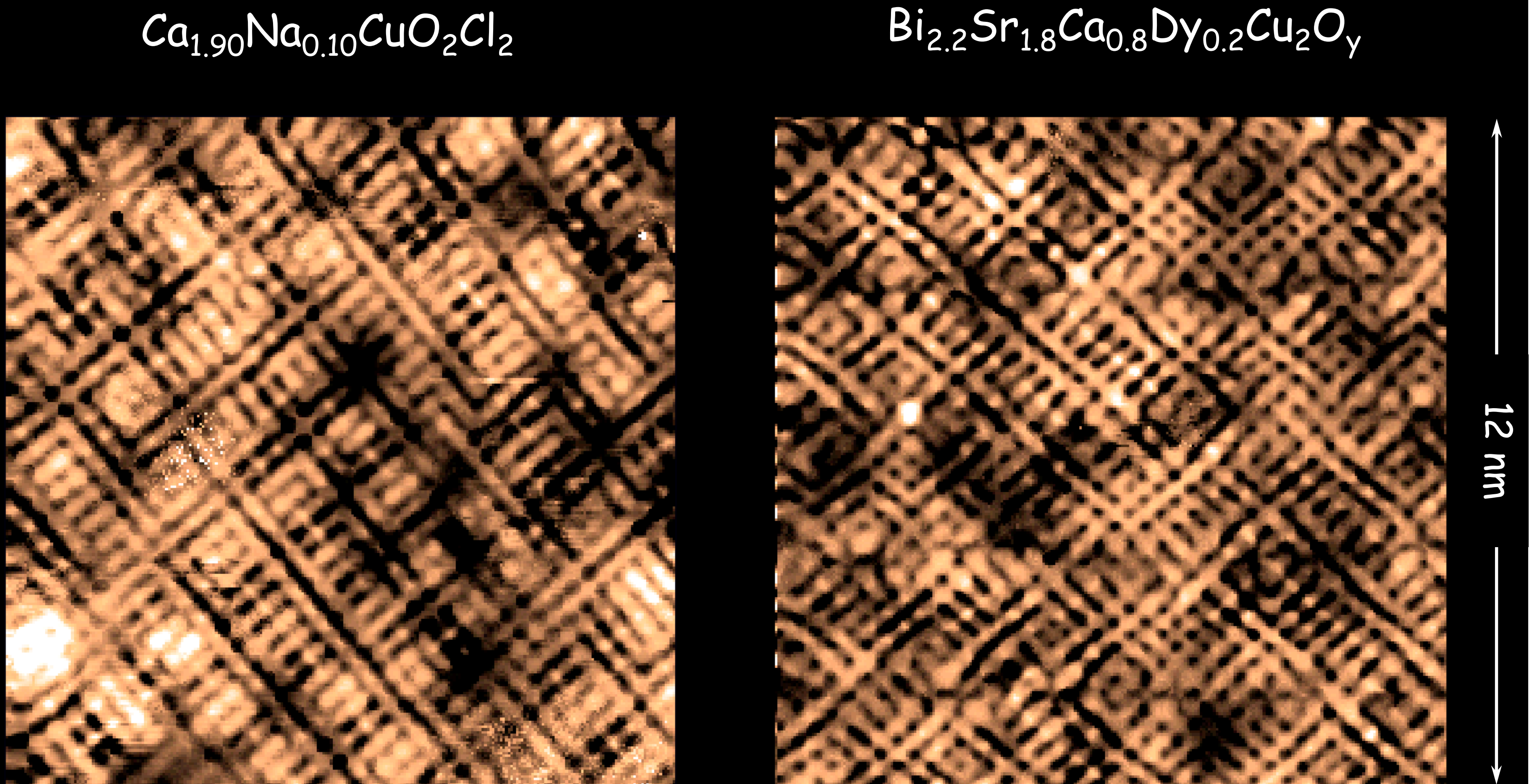
M. Randeria, N. Trivedi & FC Zhang
PRL **95**, 137001 (2005)

$$R = \frac{\int_0^{\infty} dE\, N(E)}{\int_{-\infty}^{0} dE\, N(E)} = \frac{2p}{(1-p)}$$



p= # holes per CuO_2

R-map at E=150meV

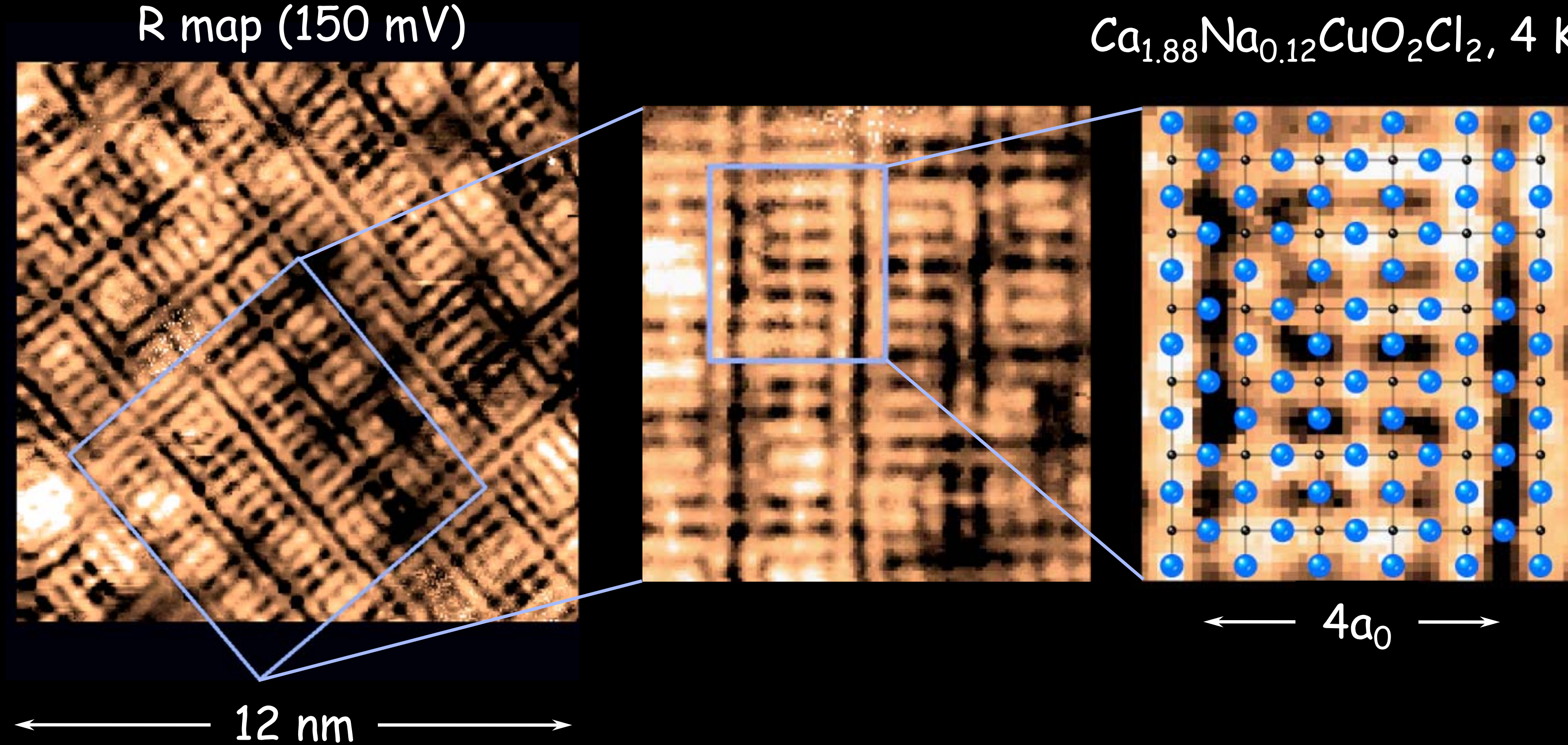


Indistinguishable bond-centered TA contrast

with disperse $4a_0$-wide nanodomains

Y. Kohsaka et al. Science 315, 1380 (2007)

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)



Y. Kohsaka et al. Science 315, 1380 (2007)

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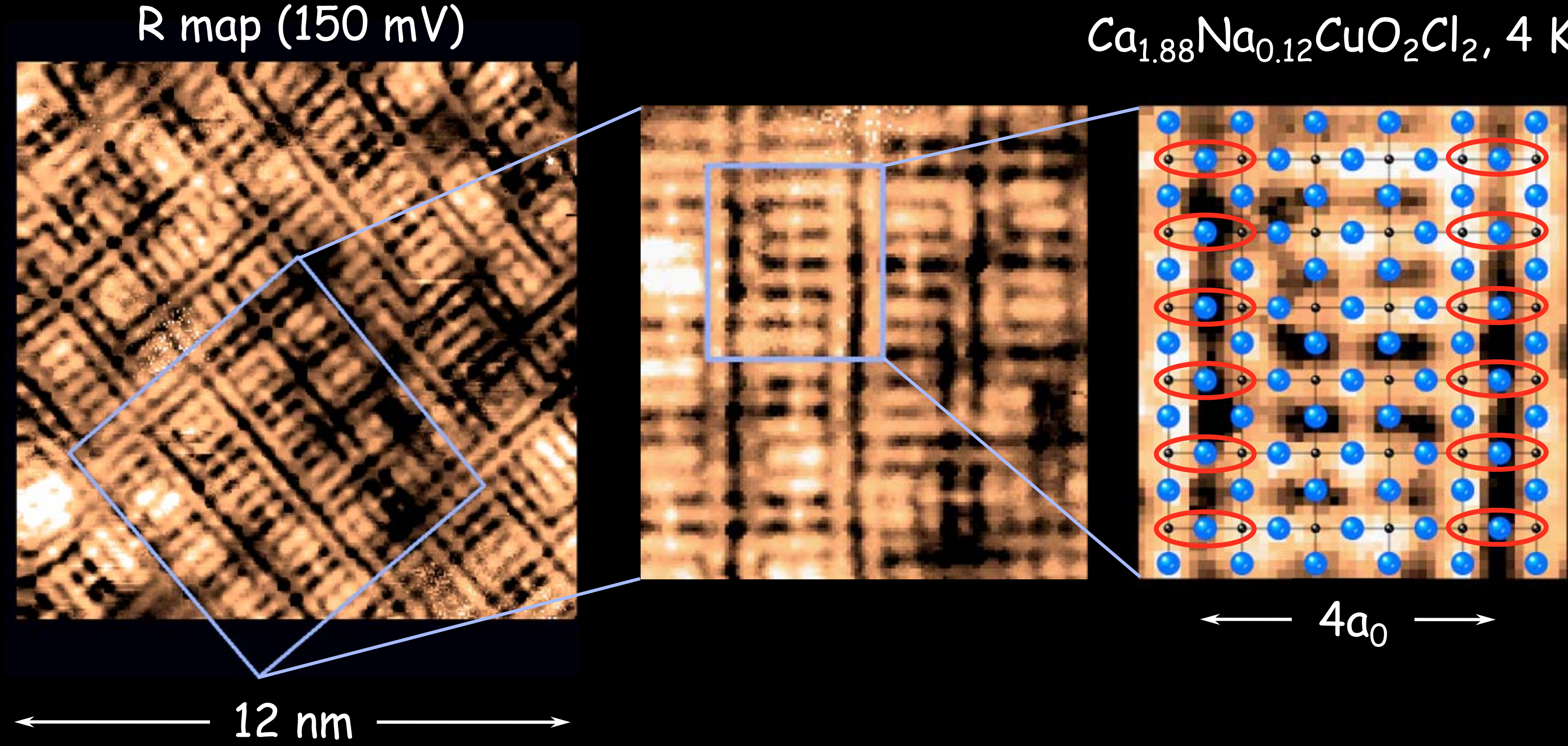
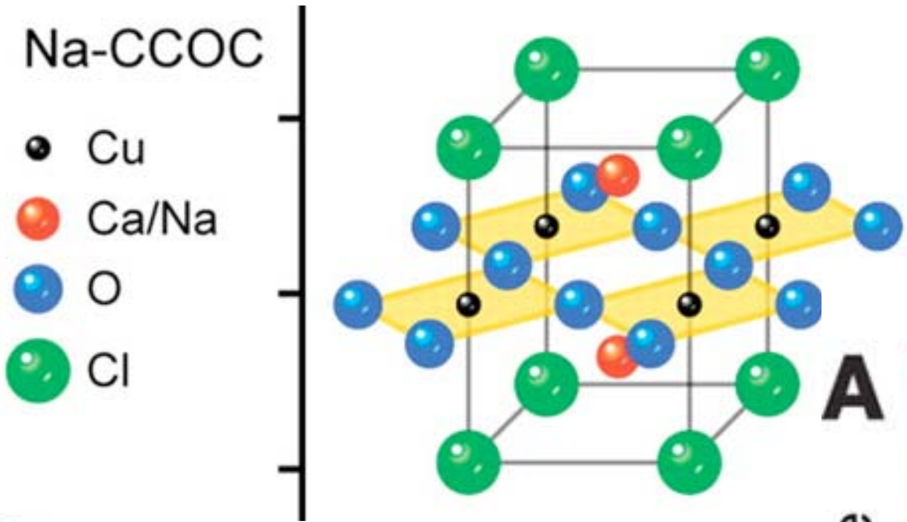
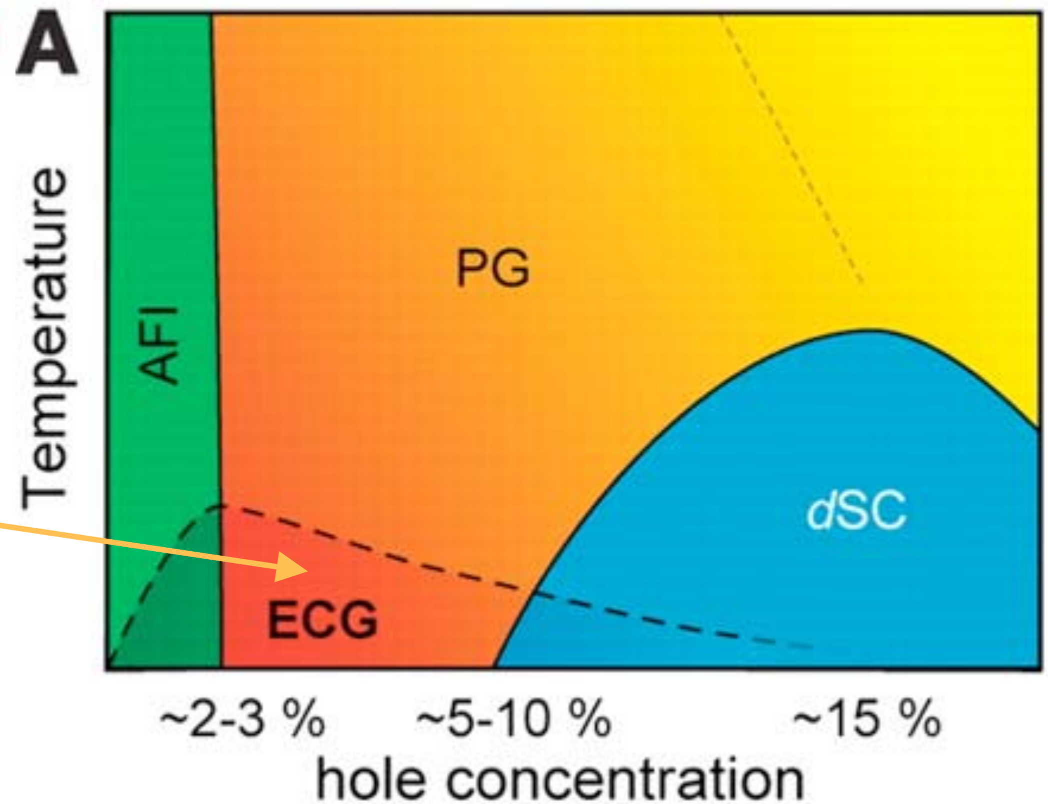


Y. Kohsaka et al. Science 315, 1380 (2007)

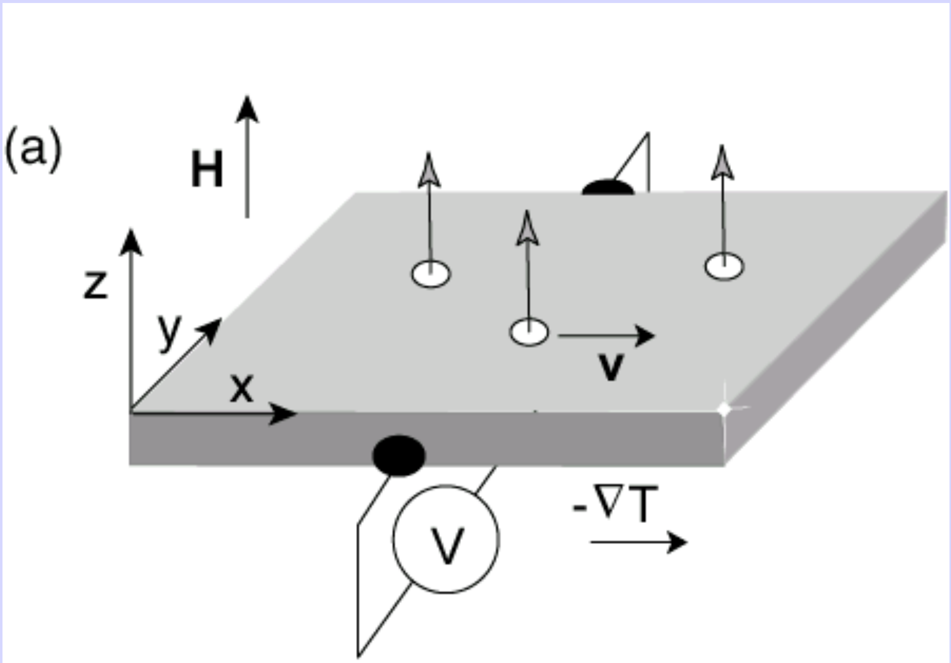
Temperature-doping phase diagram of the cuprate superconductors

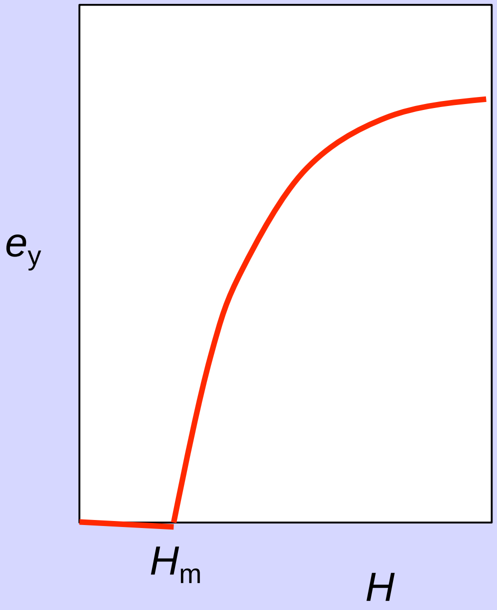




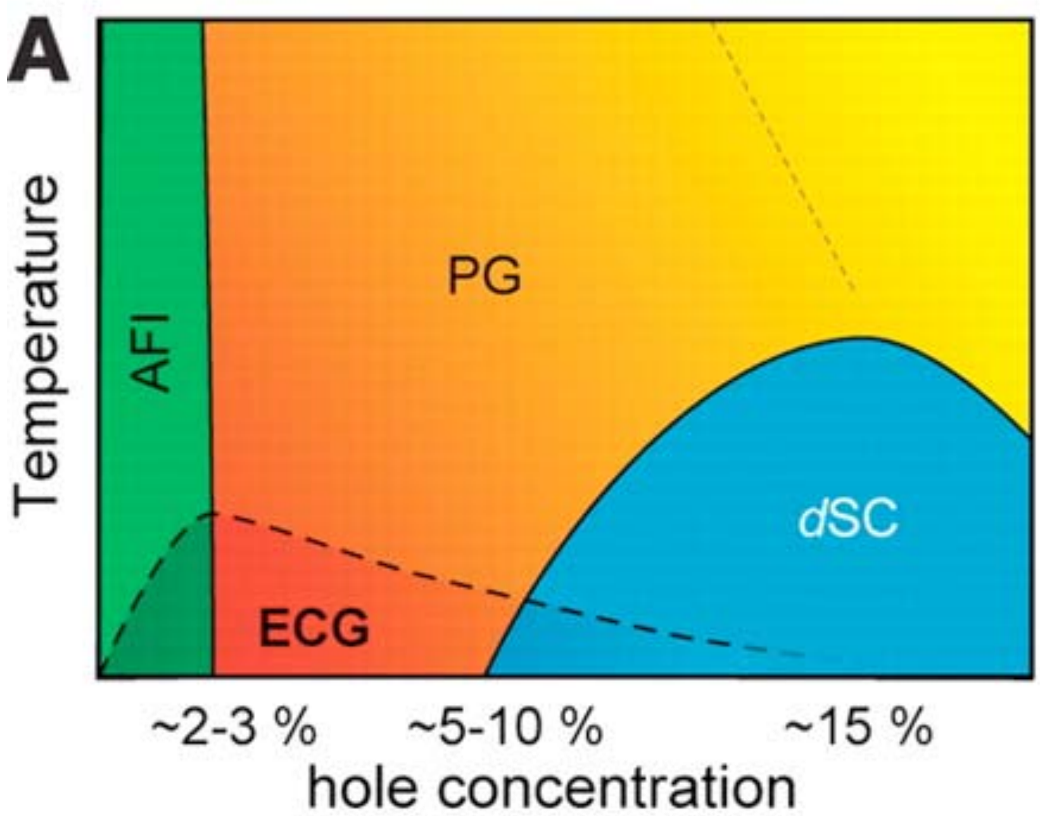
"Glassy" Valence Bond Solid (VBS)

Nernst experiment

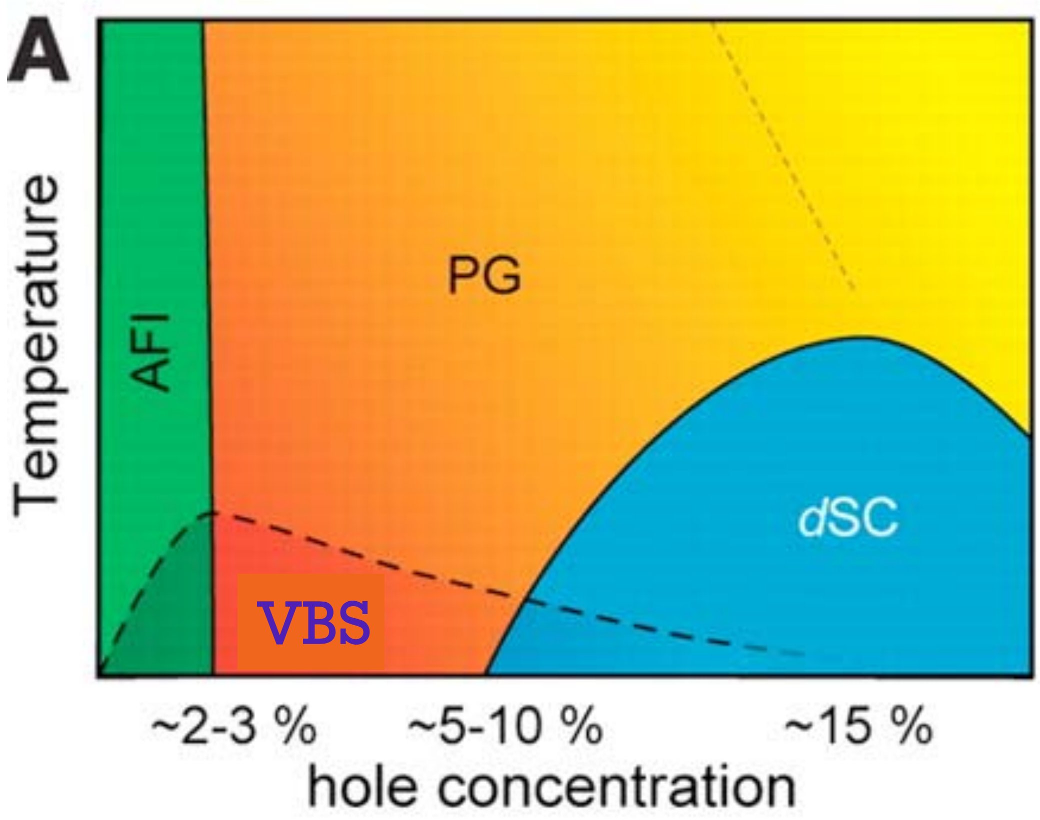




Non-zero temperature phase diagram



Non-zero temperature phase diagram



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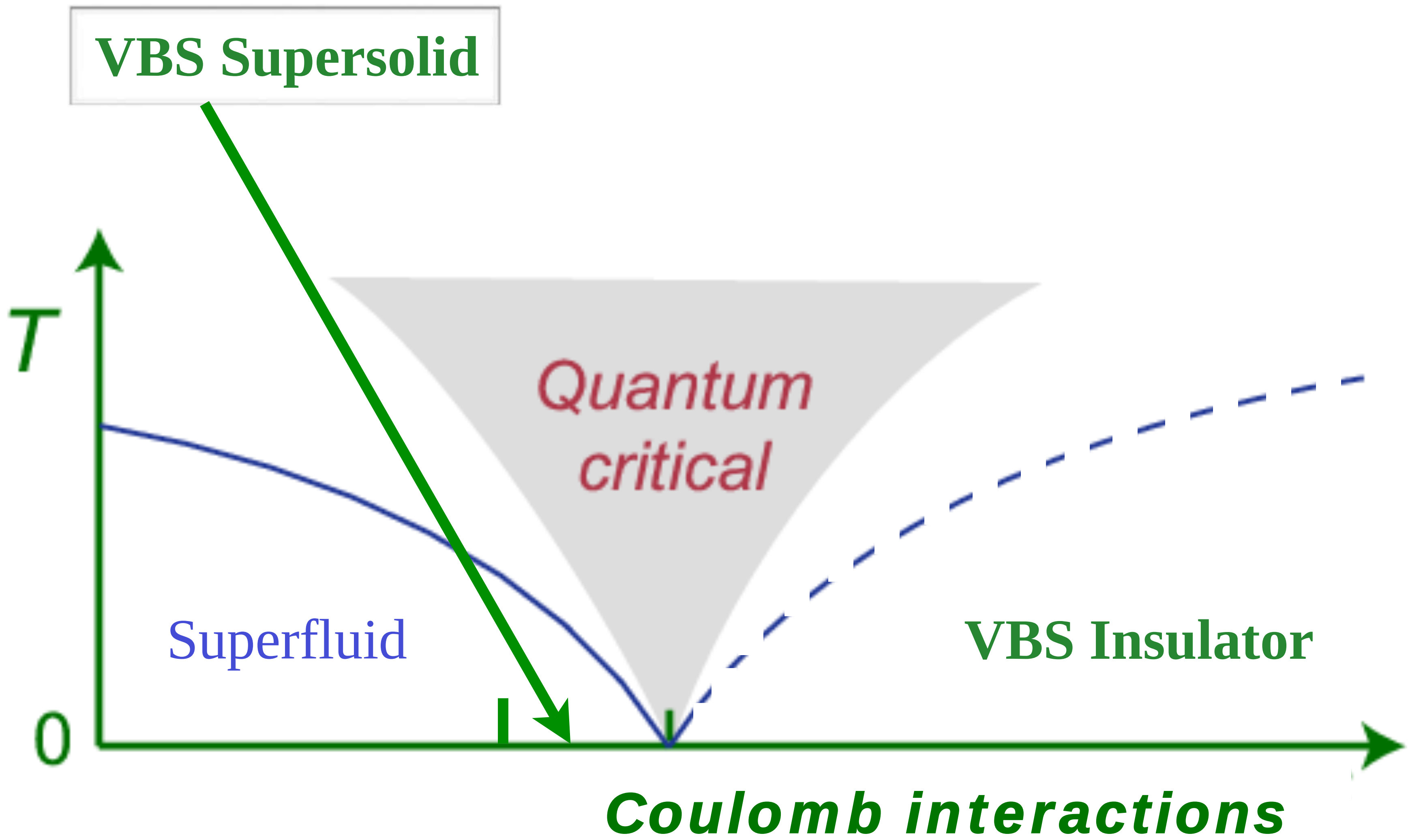

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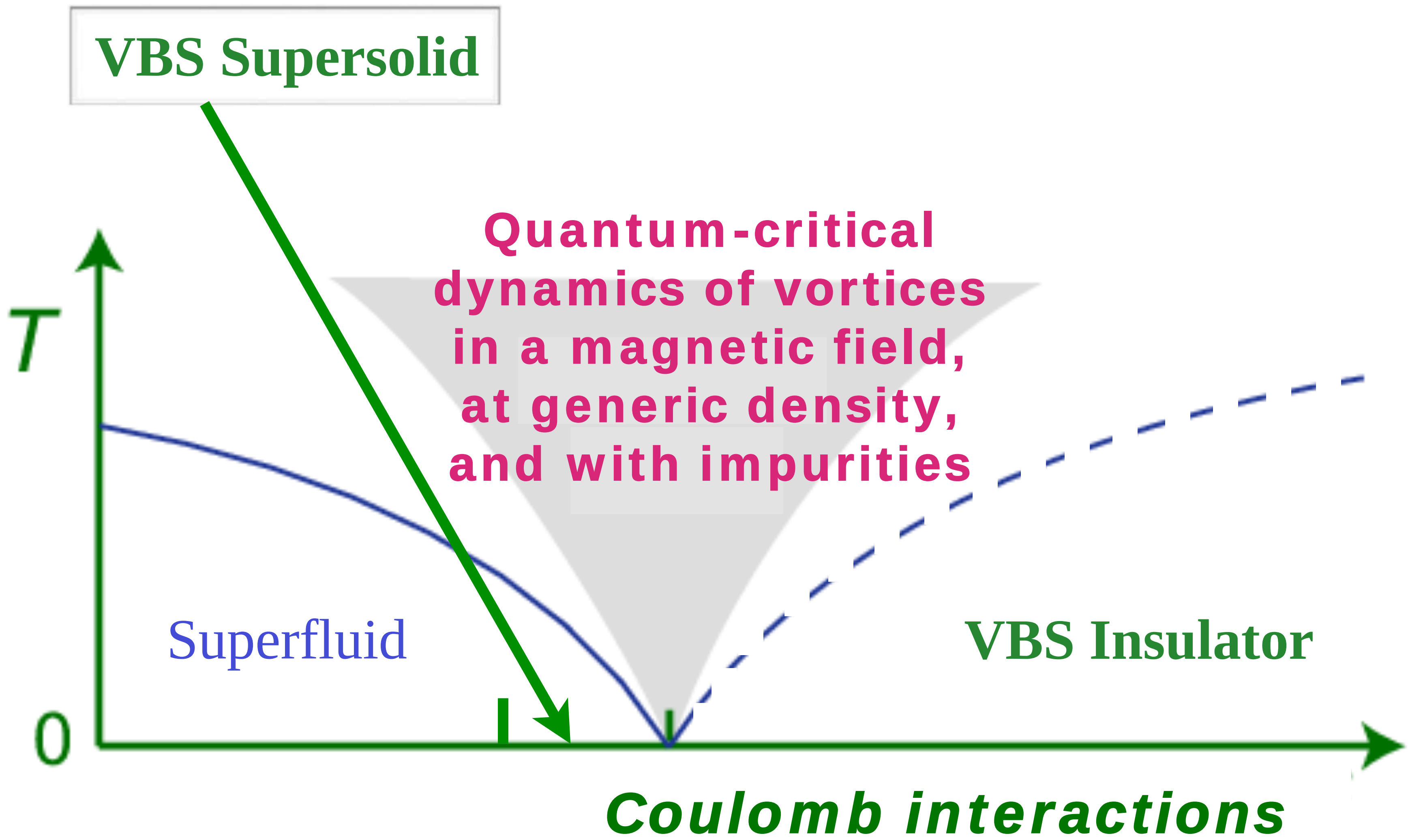


To the CFT of the quantum critical point, we add

- A chemical potential μ
- A magnetic field B

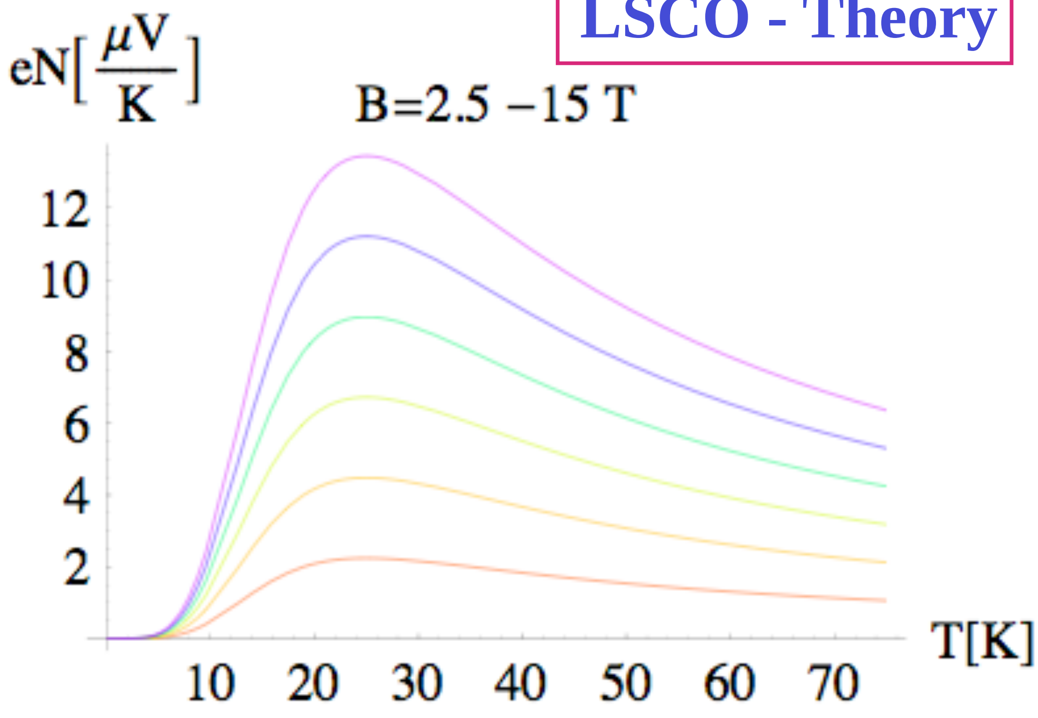
After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

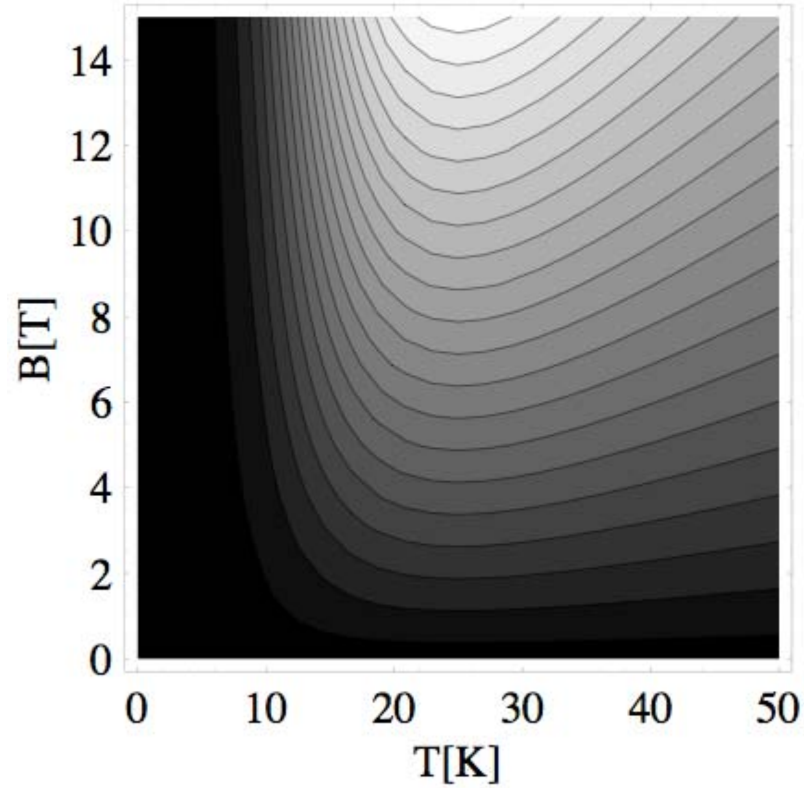
- An electric charge
- A magnetic charge

A precise correspondence is found between general hydrodynamics of vortices near quantum critical points and solvable models of black holes with electric and magnetic charges

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, Phys. Rev. B (2007)

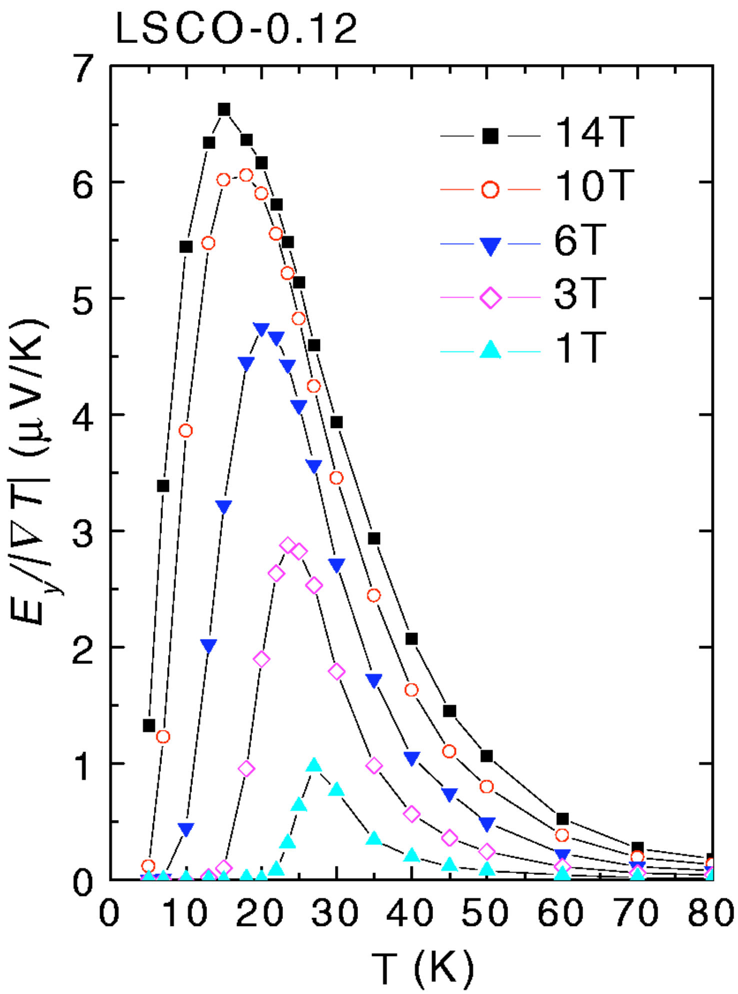
LSCO - Theory

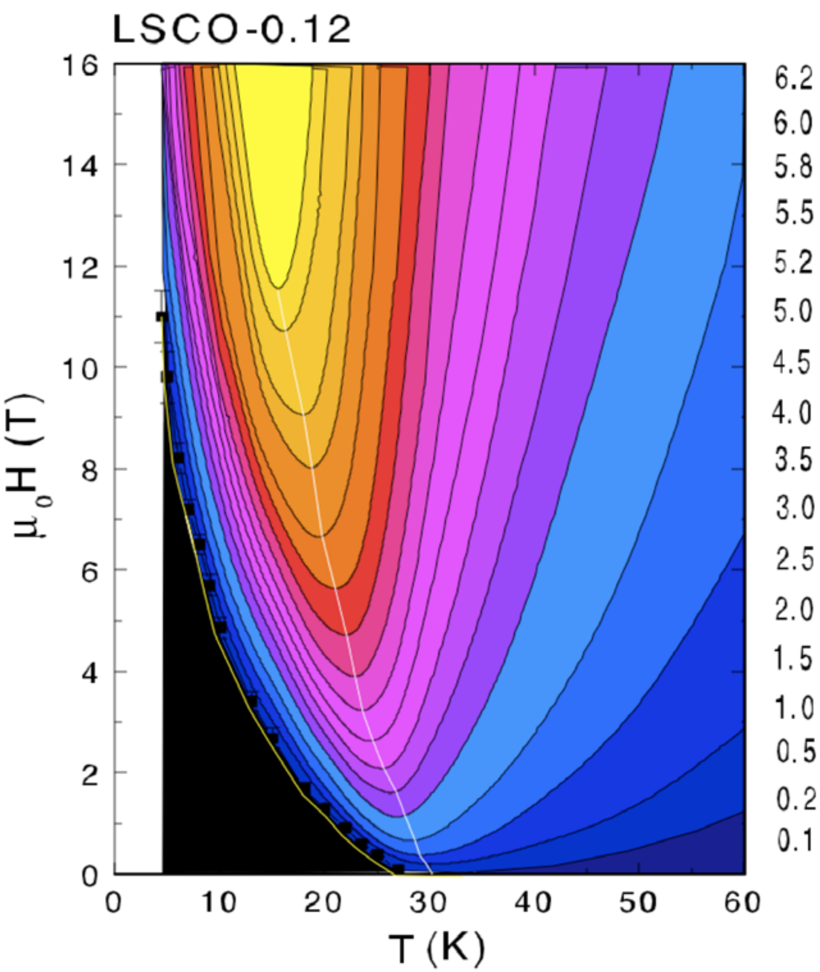




S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, Phys. Rev. B (2007)

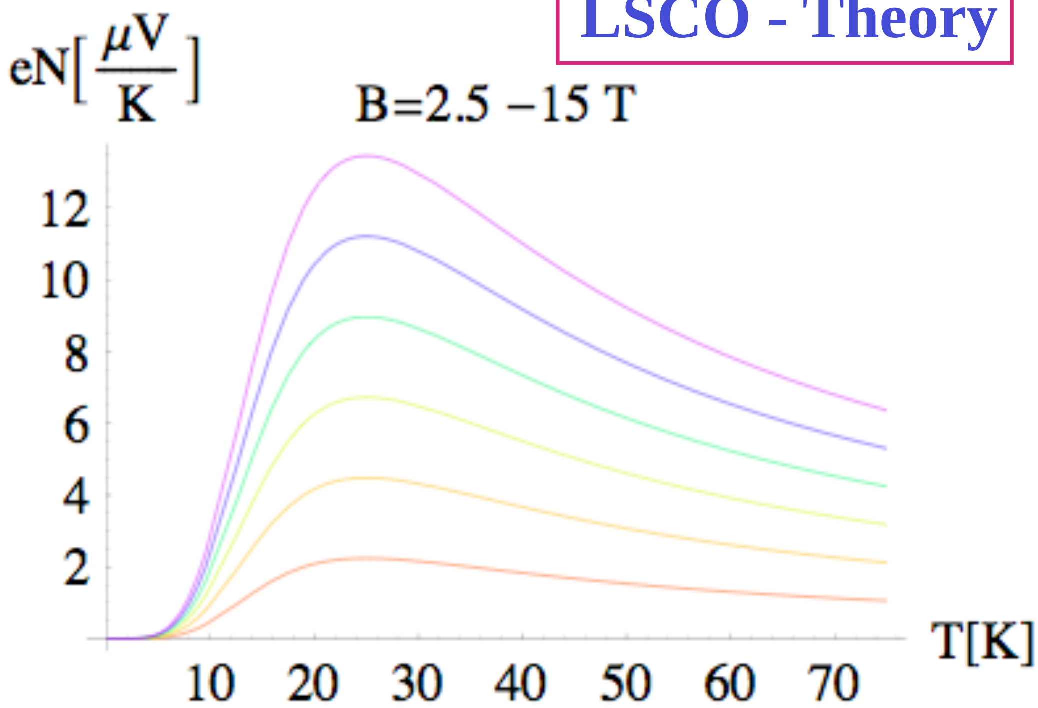
LSCO - Experiments

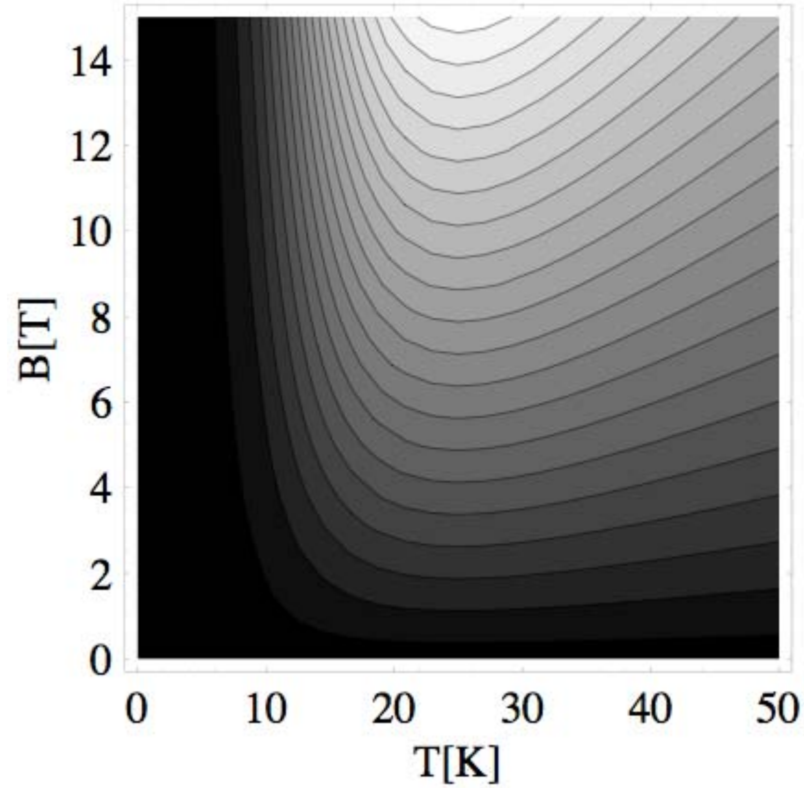




N. P. Ong *et al.*

LSCO - Theory





Only input parameters

$$\hbar v = 47 \text{ meV Å}$$

$$\tau_{\text{imp}} \approx 10^{-12} \text{ s}$$

Output

$$\omega_c = 6.2\text{GHz} \cdot \frac{B}{1\text{T}} \left(\frac{35\text{K}}{T}\right)^3$$

Similar to velocity estimates by
A.V. Balatsky and Z-X. Shen, *Science* **284**, 1137 (1999).

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, Phys. Rev. B (2007)

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Conclusions

- Studies of new materials and trapped ultracold atoms are yielding new quantum phases, with novel forms of quantum entanglement.
- Some materials are of technological importance: e.g. high temperature superconductors.
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.
- Tabletop "laboratories for the entire universe": quantum mechanics of black holes, quark-gluon plasma, neutrons stars, and big-bang physics.