

**Many body quantum
entanglement:
from organic insulators
to ultracold atoms**

sachdev.physics.harvard.edu



Outline

1. Organic insulators:
Spin liquids on the triangular lattice
2. Ultracold atoms:
bosons in tilted Mott insulators
3. Ultracold atoms:
dynamics near quantum-critical points
and gauge-gravity duality

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Rudro Rana Biswas

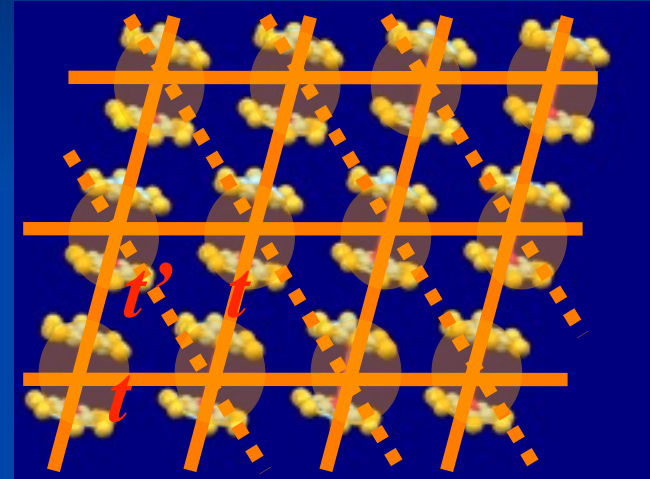
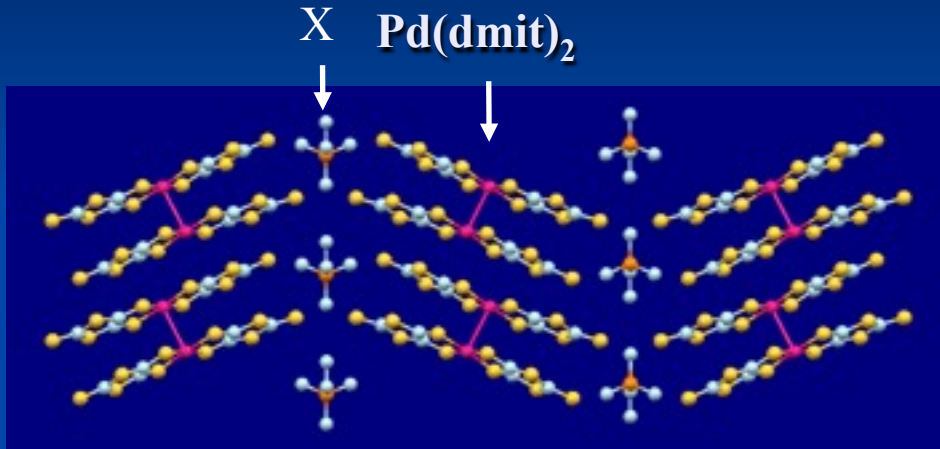
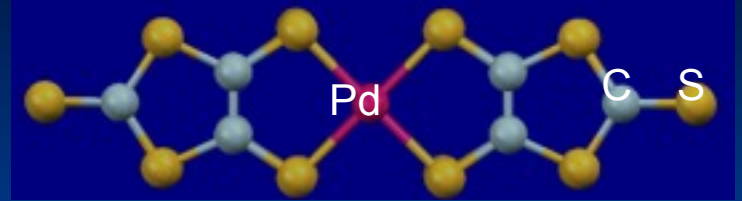


Liang Fu



Chris Laumann

R. R. Biswas, L. Fu, C. Laumann, and S. Sachdev, *Physical Review B* **83**, 245131 (2011)



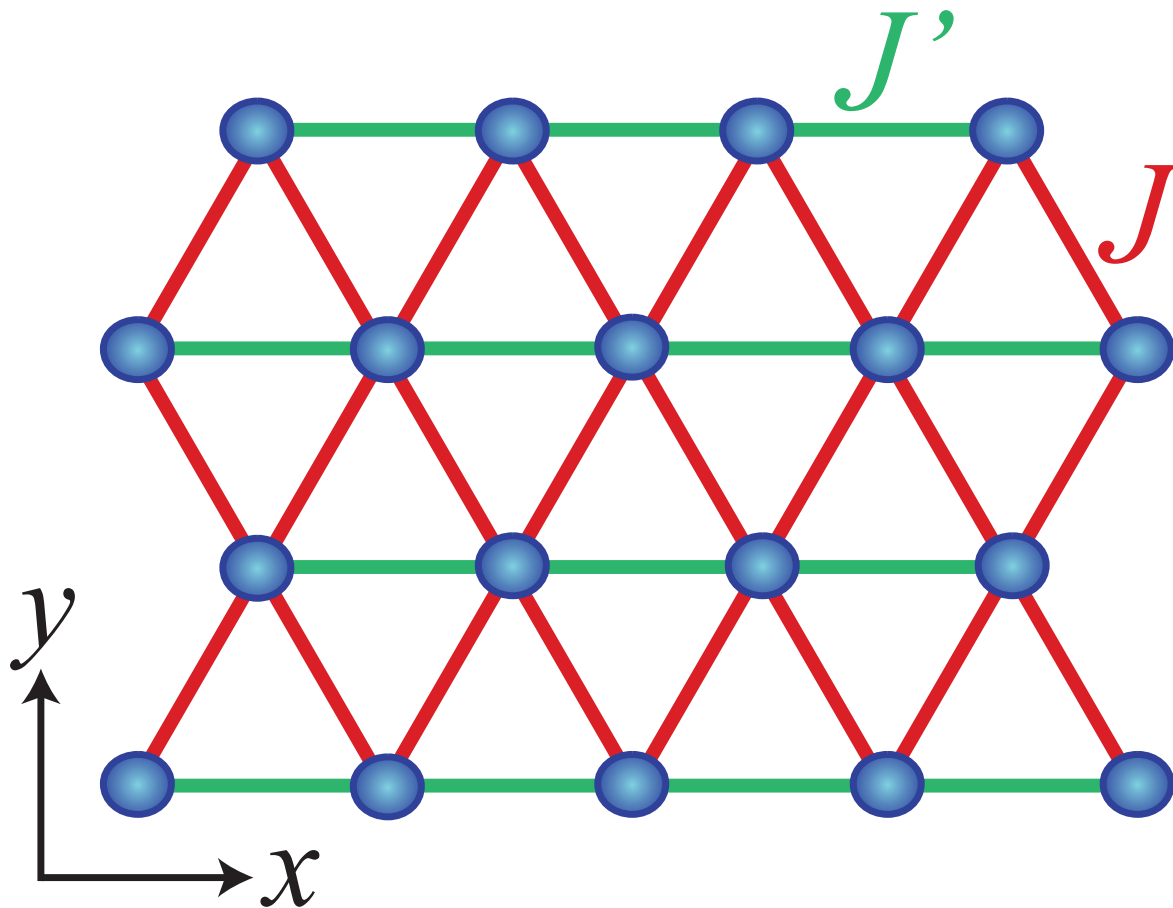
Half-filled band \rightarrow Mott insulator with spin $S = 1/2$

Triangular lattice of $[\text{Pd}(\text{dmit})_2]_2$

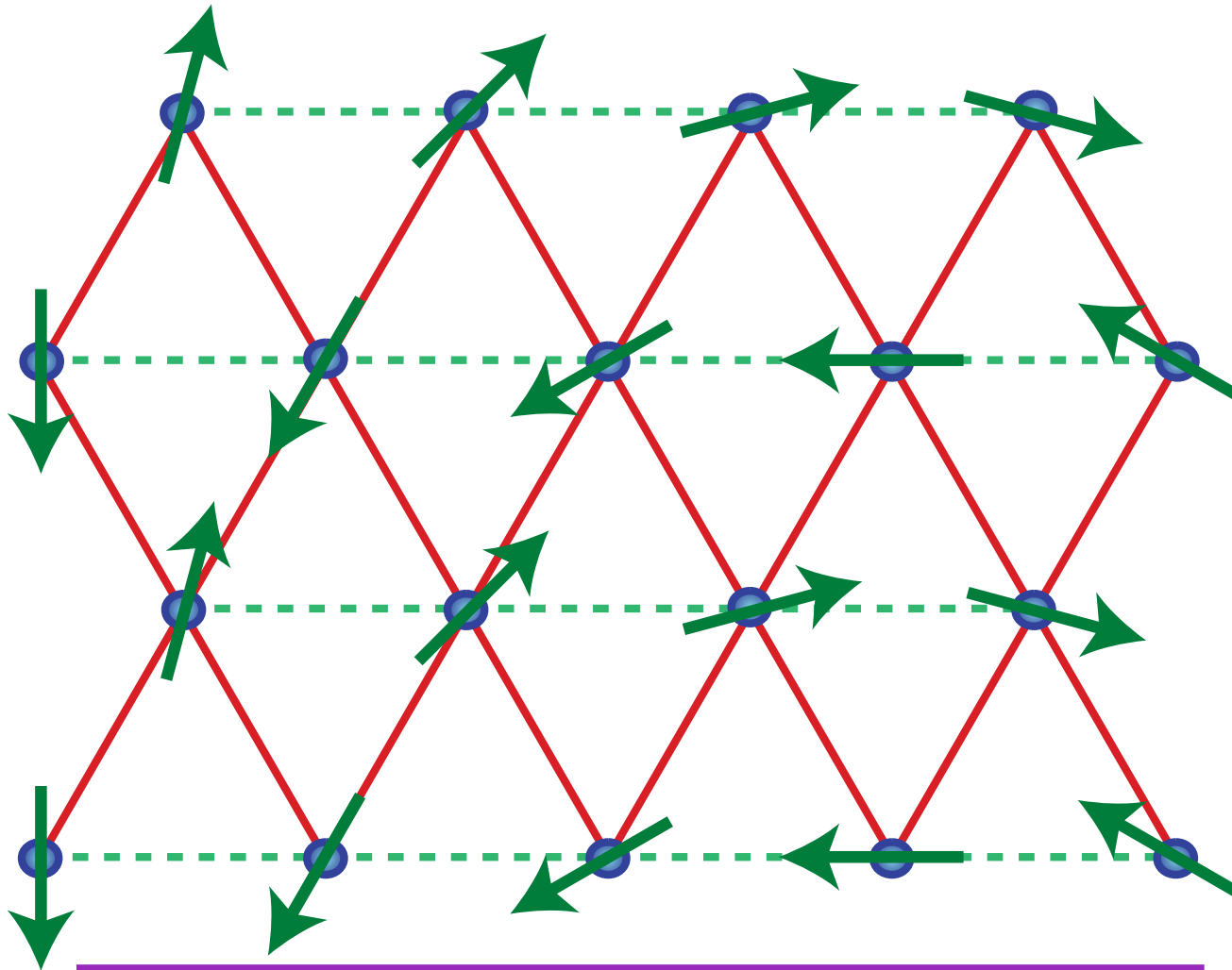
\rightarrow frustrated quantum spin system

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \dots$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



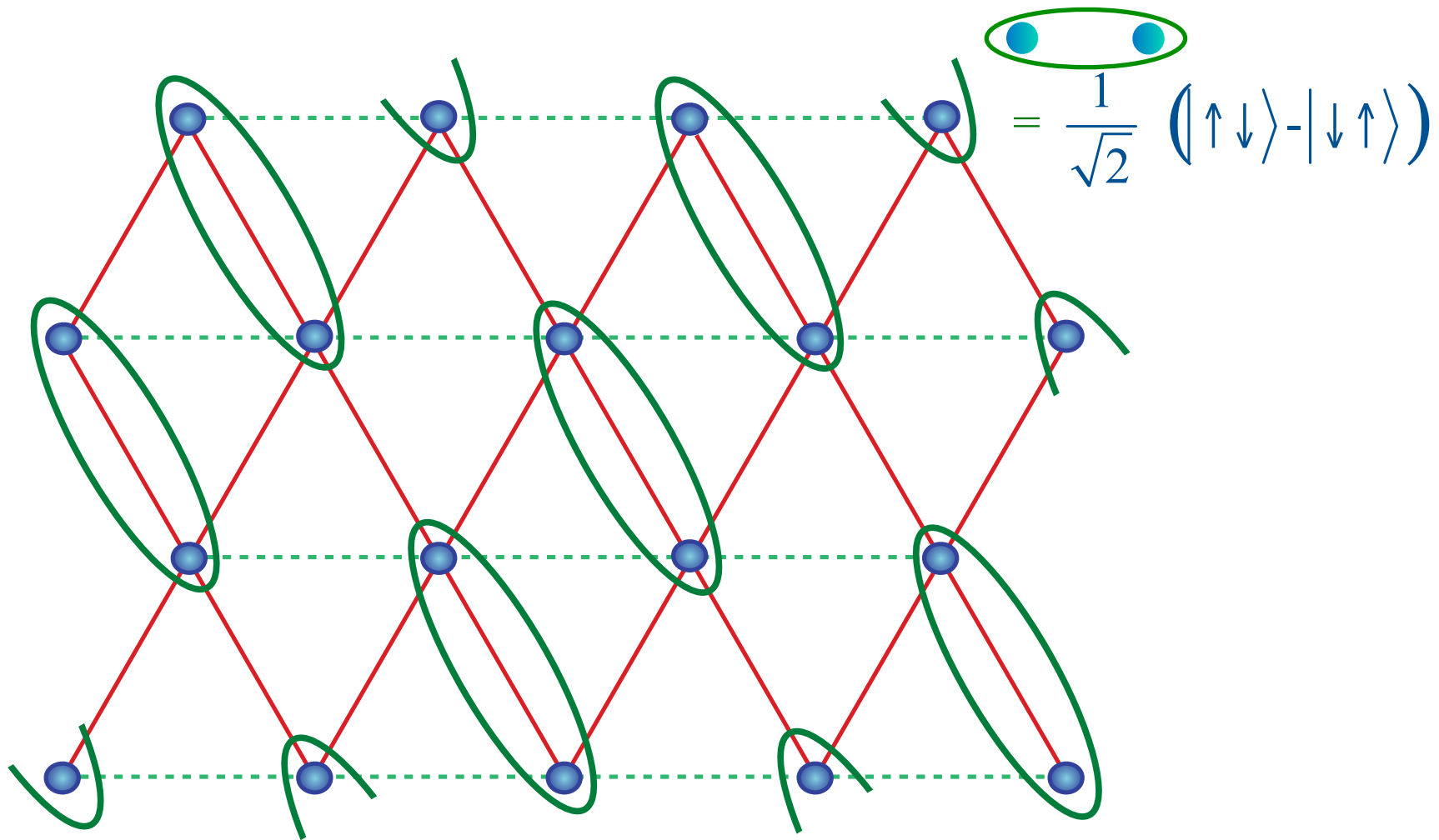
Anisotropic triangular lattice antiferromagnet



Classical ground state for large J'/J

Found in $\text{ET}_2\text{Me}_2\text{P}[\text{Pd}(\text{dmit})_2]_2$ and Cs_2CuCl_4

Anisotropic triangular lattice antiferromagnet



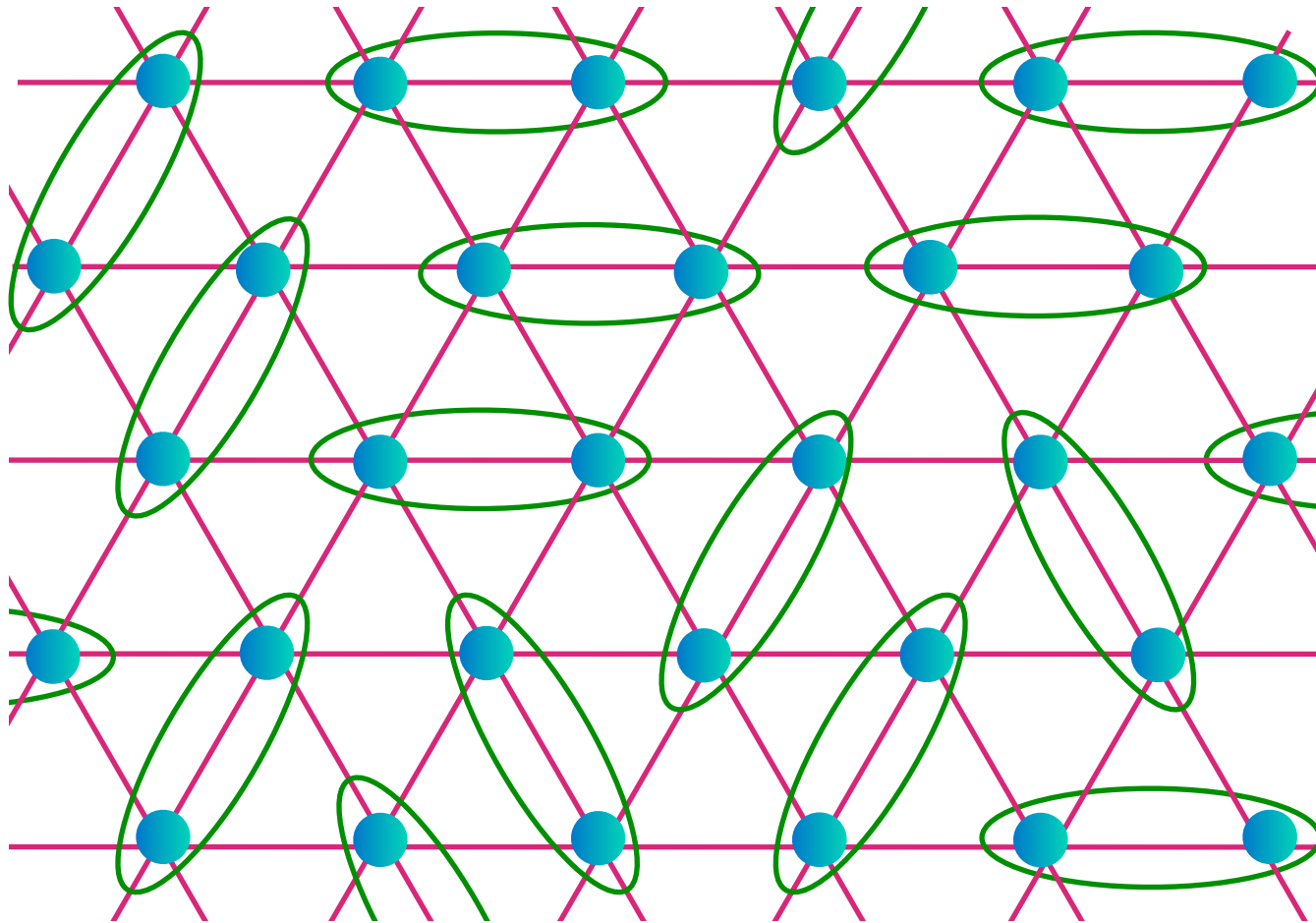
Valence bond solid

N. Read and S. Sachdev, *Phys. Rev. Lett.* **62**, 1694 (1989)

Found in $\text{ETMe}_3\text{P}[\text{Pd}(\text{dmit})_2]_2$

Triangular lattice antiferromagnet

Z_2 spin liquid

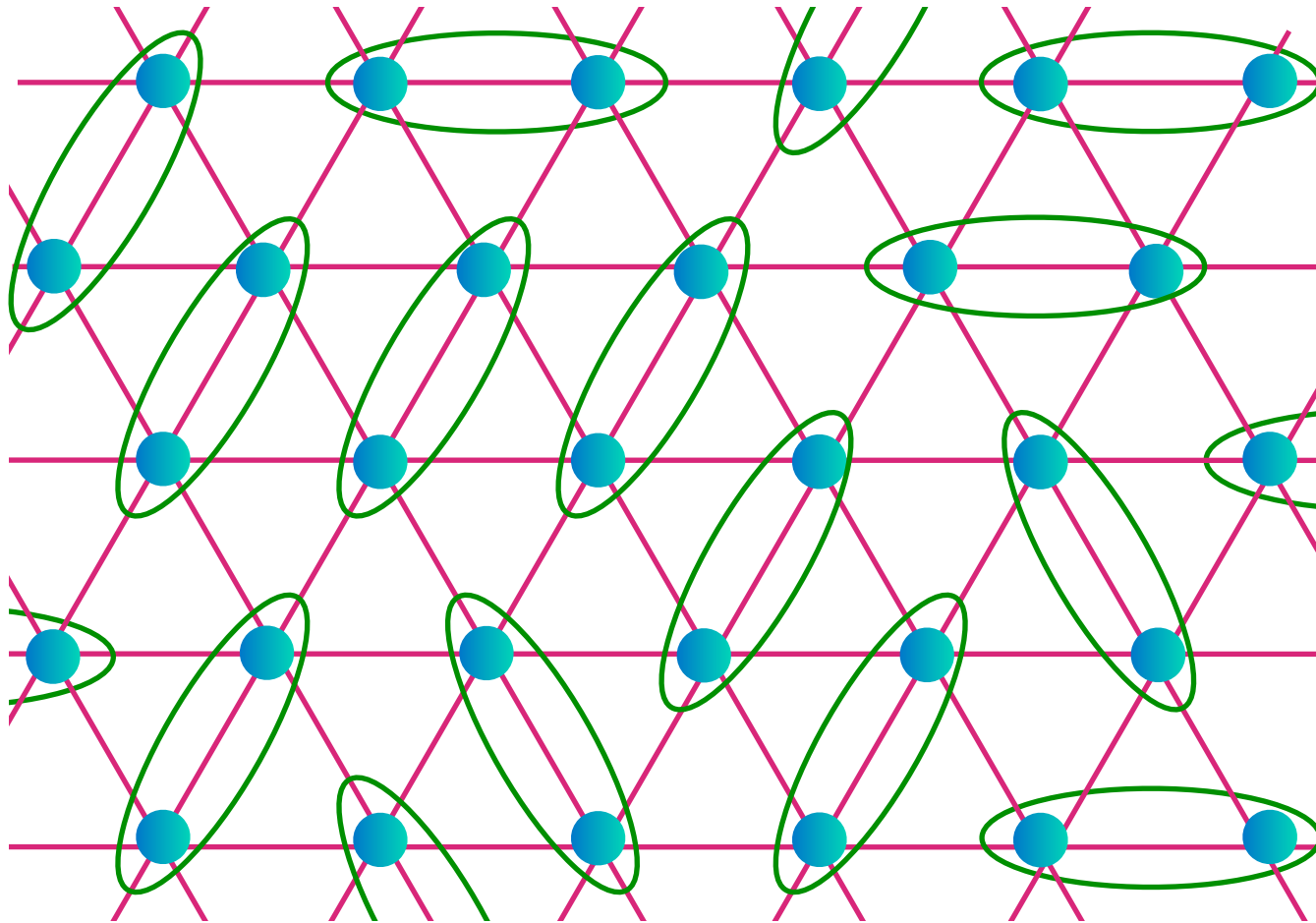


$$\begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ \text{---} \end{array} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Triangular lattice antiferromagnet

Z_2 spin liquid

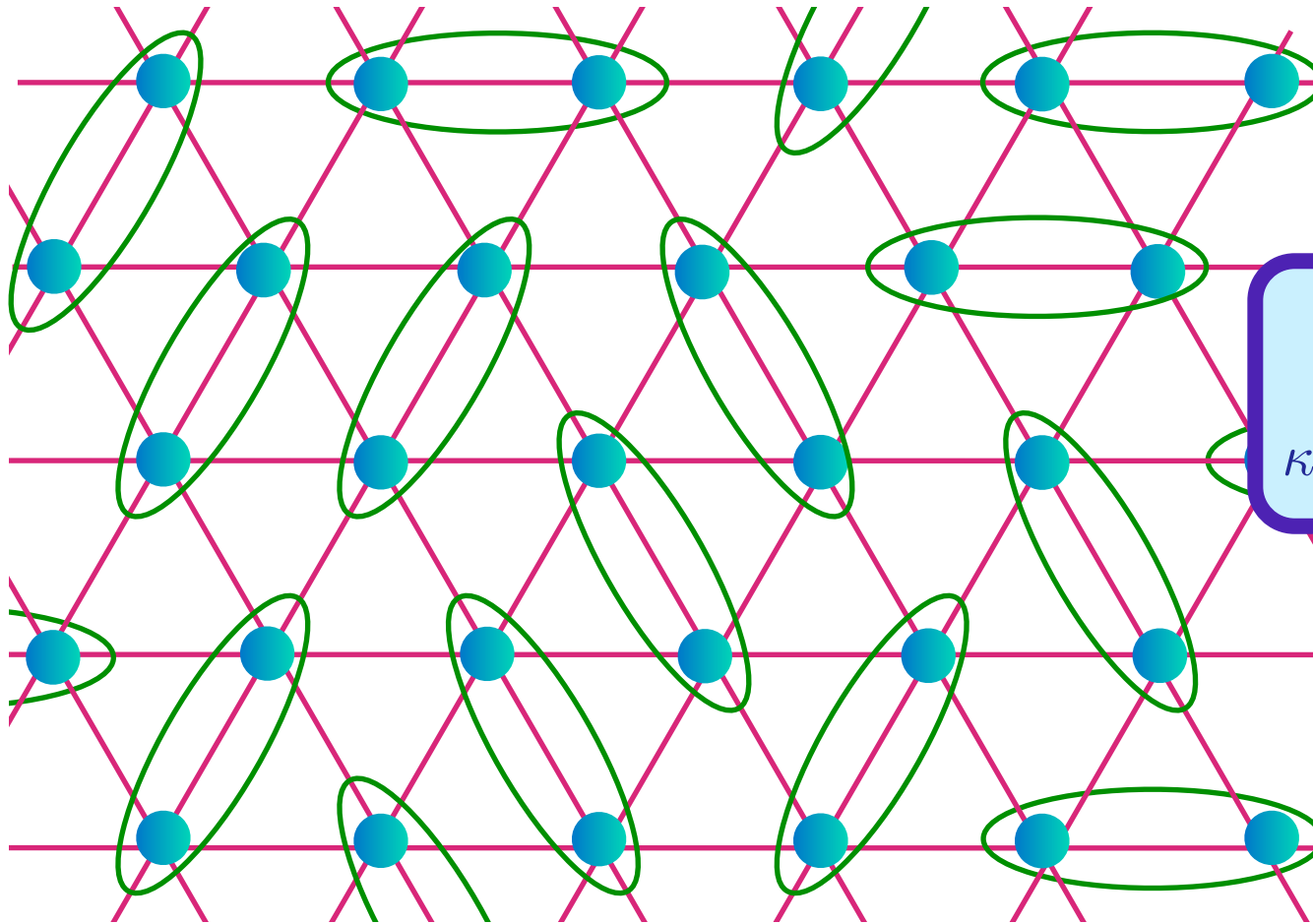


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N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Triangular lattice antiferromagnet

Z_2 spin liquid



$$\begin{array}{c} \text{---} \circ \text{---} \circ \text{---} \\ \text{---} \end{array} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Candidate for a
 Z_2 spin liquid:
 $\kappa\text{-(ET)}_2\text{Cu}_2(\text{CN})_3$

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Effective description of Z_2 spin liquids, their visons and valence bond solids

Quantum dimer model:

Hilbert space - set of dimer coverings of triangular/square lattice

$$H = V \left| \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right| + V \left| \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right|$$

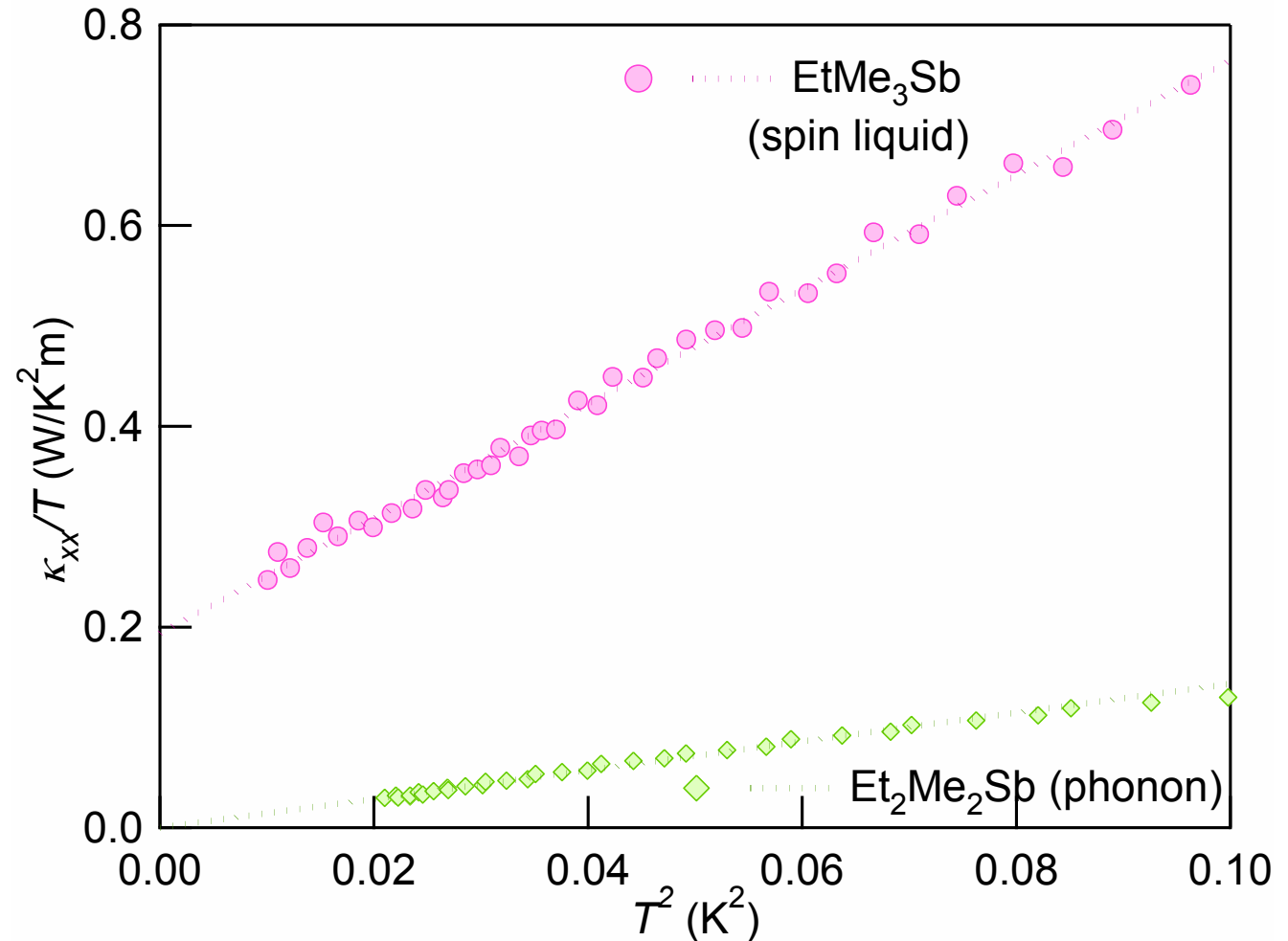
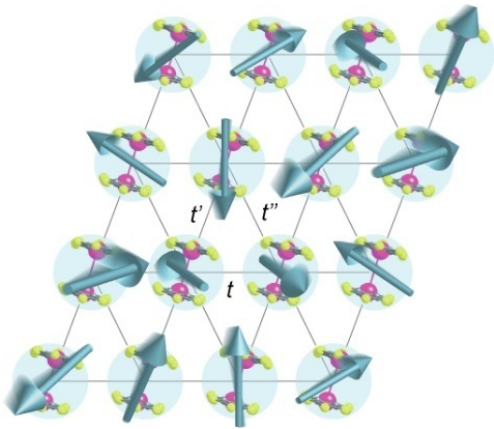
$$- J \left| \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right| - J \left| \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right\rangle \left\langle \begin{array}{c} \text{---} \langle \text{---} \\ \text{---} \langle \text{---} \end{array} \right|$$

D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)

R. Moessner and S. L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001)

Spin liquid in $\text{EtMe}_3\text{Sb}[\text{Pd}(\text{dmit})_2]_2$

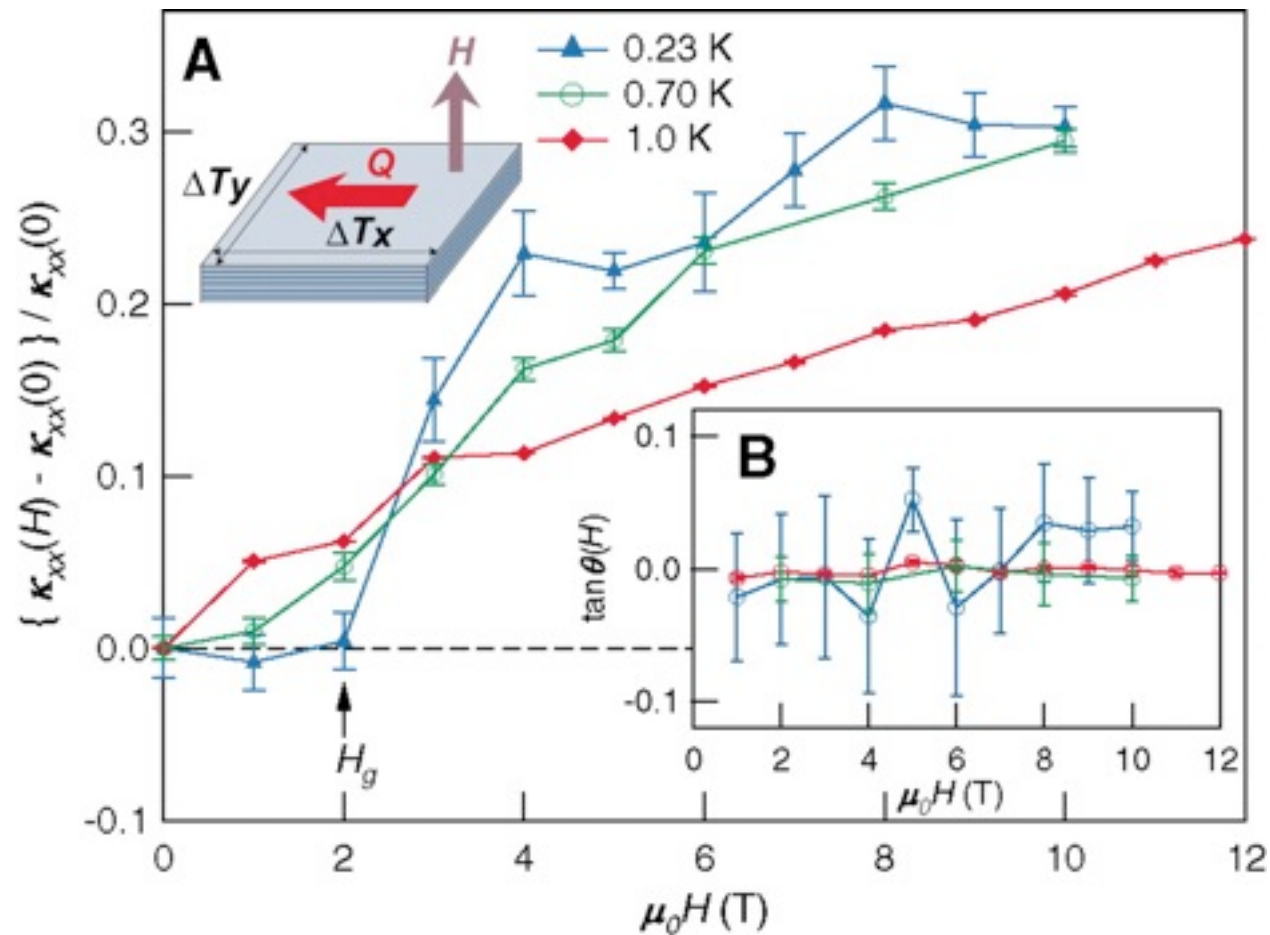
Minoru Yamashita, Norihito Nakata, Yoshinori Senshu, Masaki Nagata, Hiroshi M. Yamamoto, Reizo Kato, Takasada Shibauchi, Yuji Matsuda, Science 328, 1246 (2010).



An exotic compressible “metal” of spin excitations

Spin liquid in $\text{EtMe}_3\text{Sb}[\text{Pd}(\text{dmit})_2]_2$

Minoru Yamashita, Norihito Nakata, Yoshinori Senshu, Masaki Nagata, Hiroshi M. Yamamoto, Reizo Kato, Takasada Shibauchi, Yuji Matsuda, Science 328, 1246 (2010).

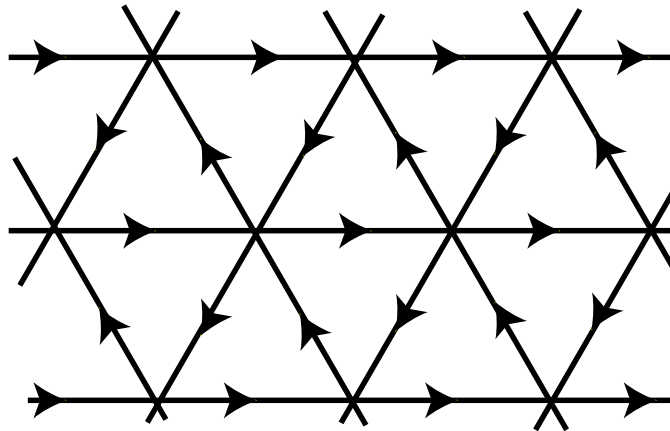


No thermal Hall effect - disagrees with simplest “spinon metal” theory

SU(2)-invariant spin liquids with neutral, S=1 Majorana excitations

$$H = -i \sum_{\alpha=x,y,z} \sum_{i<j} t_{ij} \gamma_{i\alpha} \gamma_{j\alpha}$$

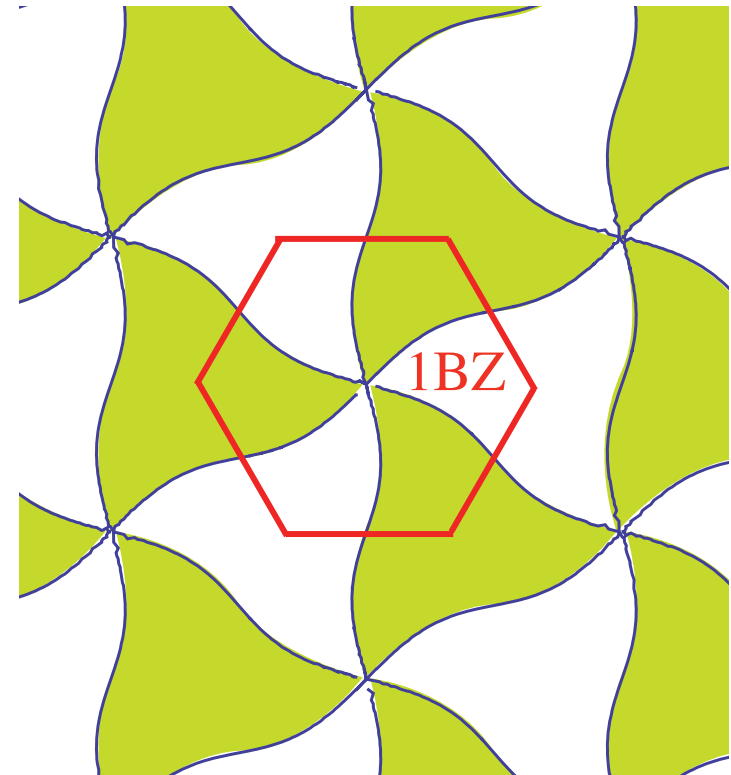
where the spins $S_{i\alpha} = (i/4)\epsilon_{\alpha\beta\gamma}\gamma_{i\beta}\gamma_{i\gamma}$, $\gamma_{i\alpha}$ is a $S = 1$ Majorana fermion, and t_{ij} is an anti-symmetric matrix with the following symmetry:



SU(2)-invariant spin liquids with neutral, S=1 Majorana excitations

The Majorana fermions generically have Fermi surfaces with the structure shown.

This leads to an insulating spin liquid with “metallic” thermal conductivity, and **no** thermal Hall effect.



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Susanne
Pielawa

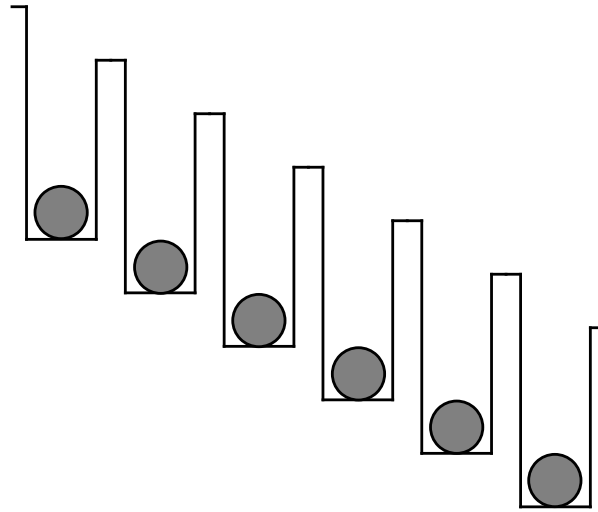


Takuya
Kitagawa



Erez
Berg

S. Sachdev, K. Sengupta, and S.M. Girvin, Phys. Rev. B **66**, 075128 (2002)
S. Pielawa, T. Kitagawa, E. Berg, S. Sachdev, Phys. Rev. B **83**, 205135 (2011)

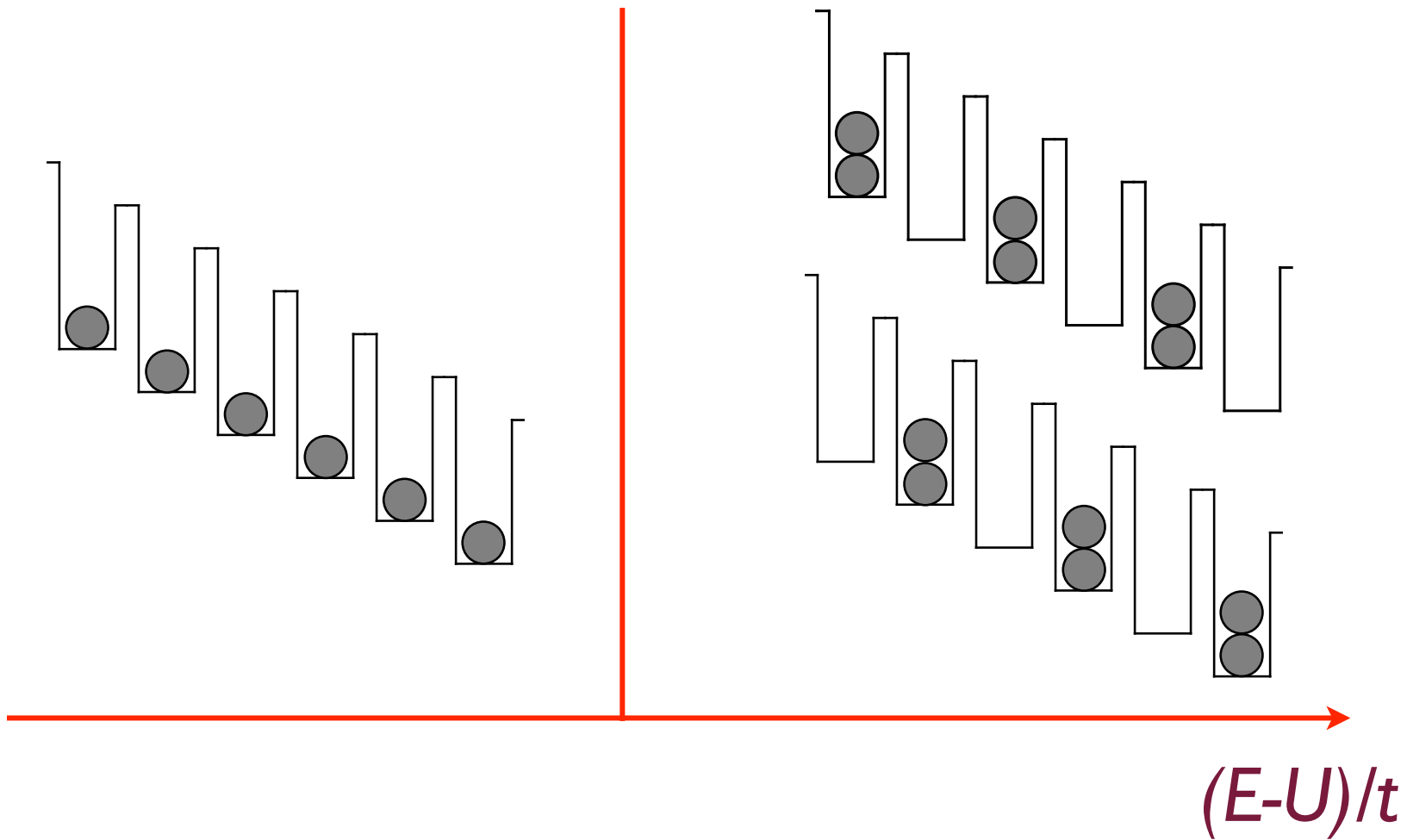


$$H = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \frac{U}{2} \sum_i n_i (n_i - 1) - \sum_i E \cdot r_i n_i$$

$$n_i = b_i^\dagger b_i$$

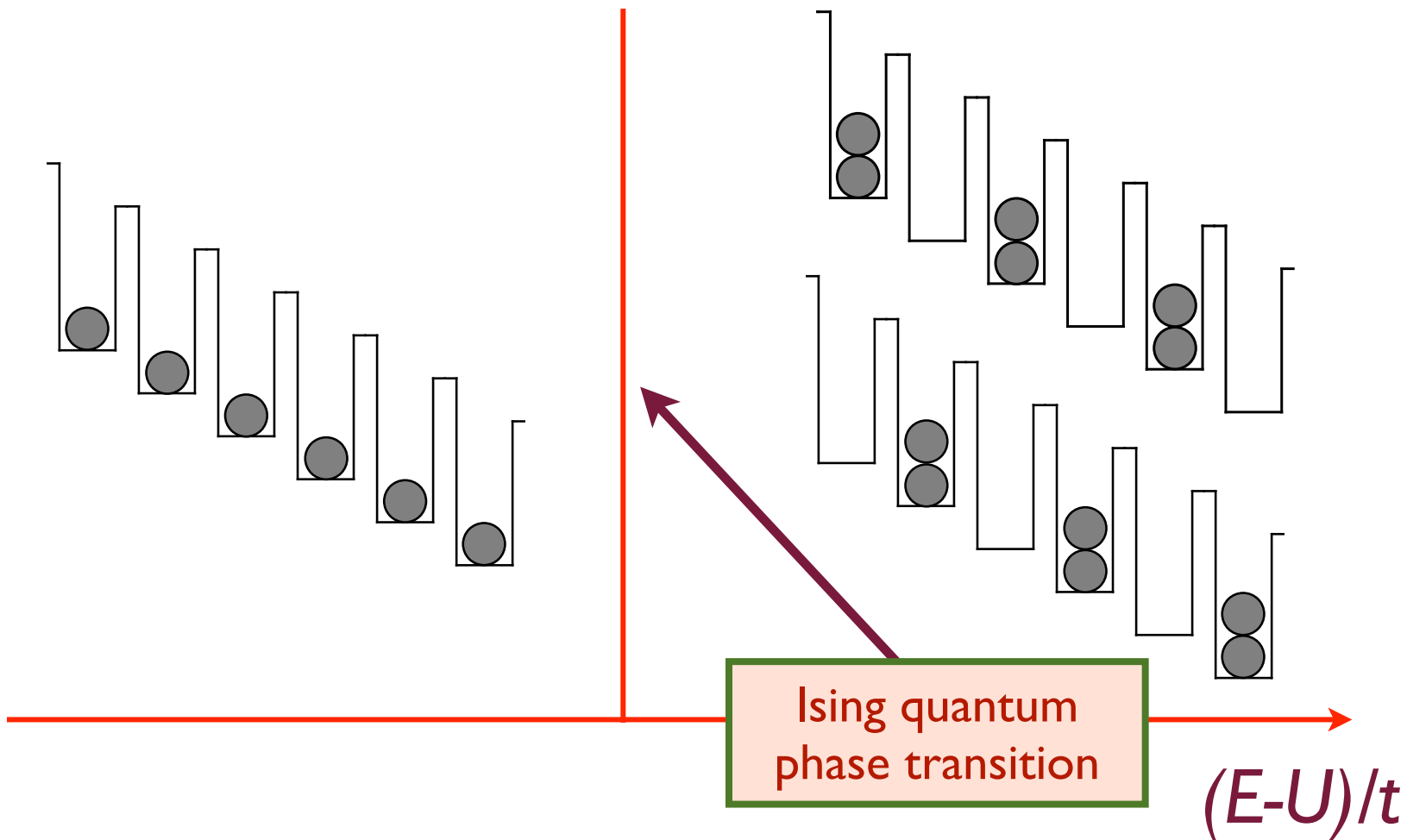
$$|U - E|, t \ll E, U$$

Phase diagram in one dimension



S. Sachdev, K. Sengupta, and S.M. Girvin, Phys. Rev. B 66, 075128 (2002)

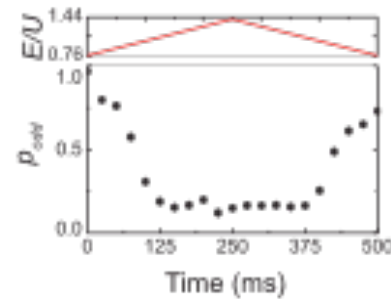
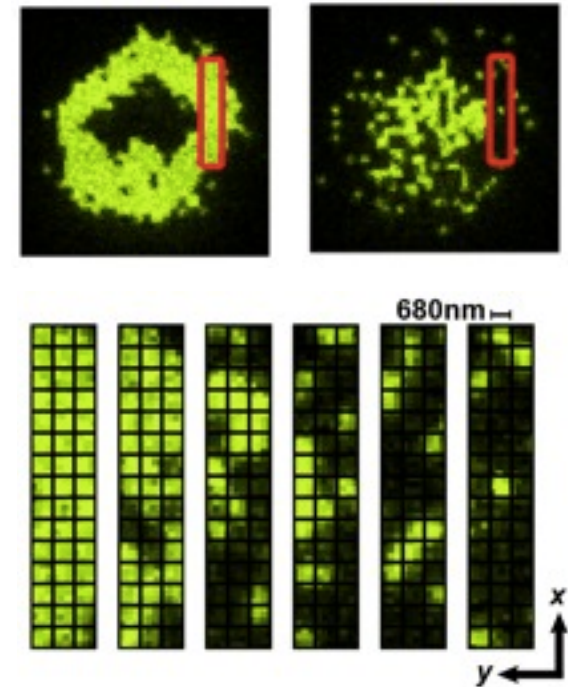
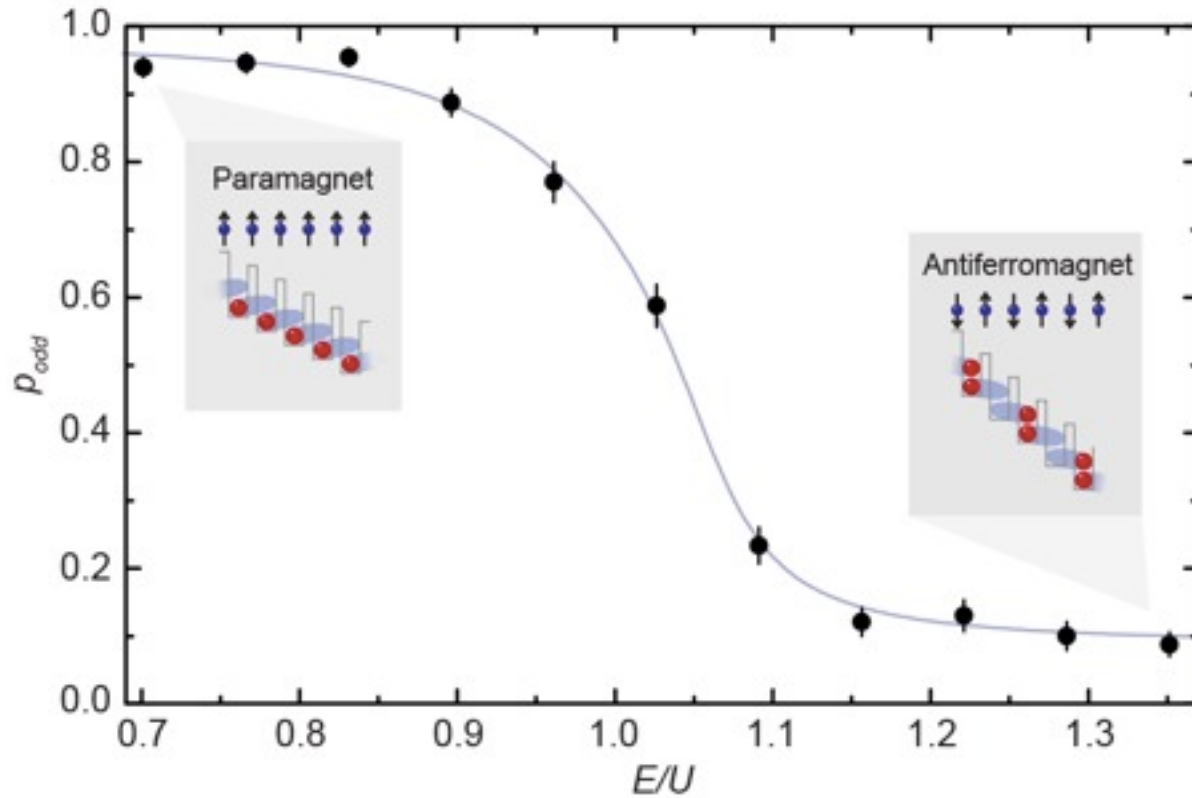
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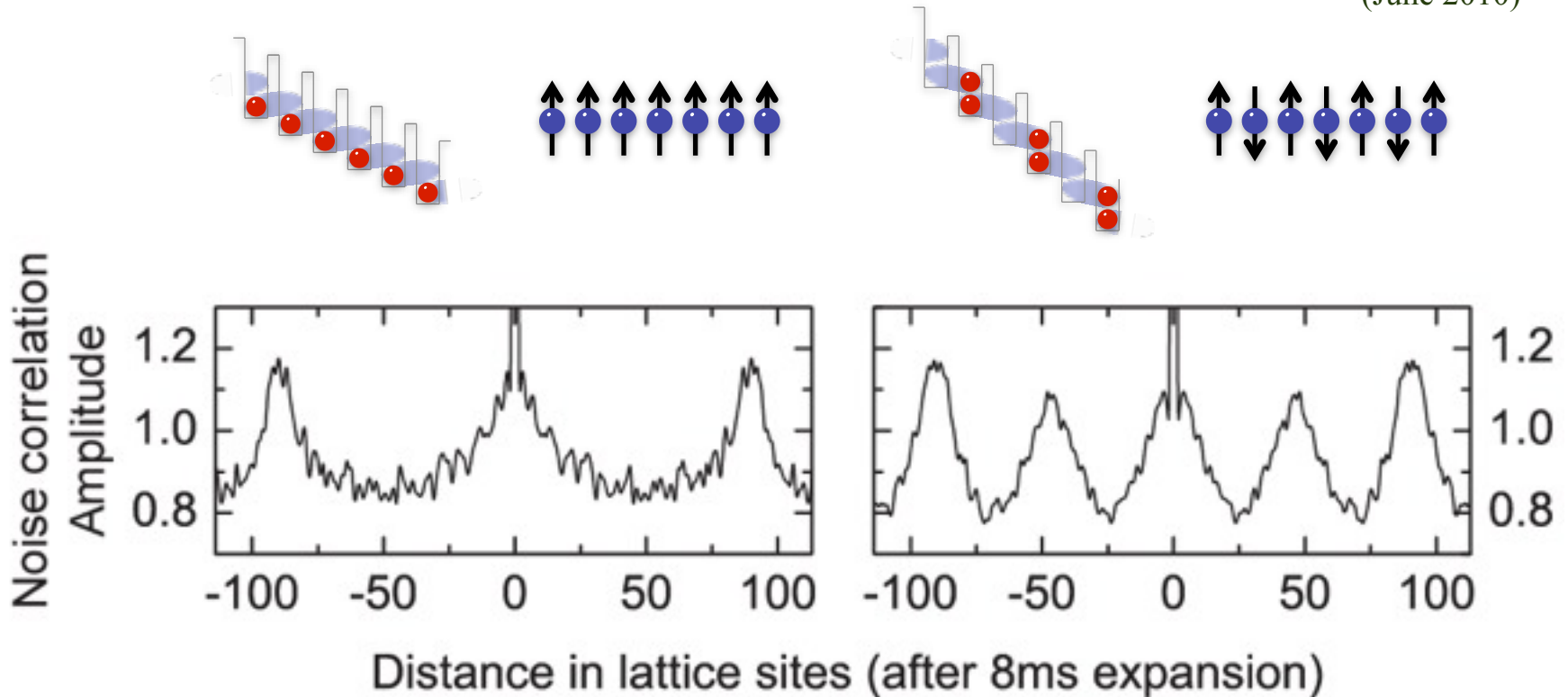
Quantum gas microscope

Bakr *et al.*, Nature 462, 74
(2009)
Bakr *et al.*, Science.1192368
(June 2010)



Quantum gas microscope

Bakr *et al.*, Nature 462, 74
(2009)
Bakr *et al.*, Science.1192368
(June 2010)



- Expand within each 1D tube, detect individual atoms, and calculate correlation function
- See Foelling *et al.*, Nature 434, 481–484 (2005)

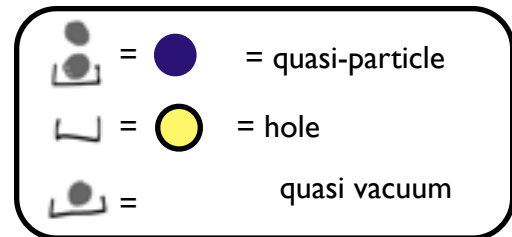
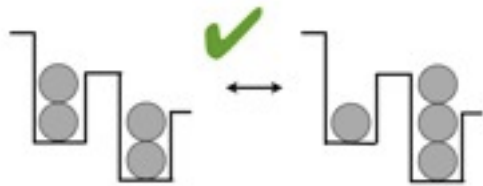
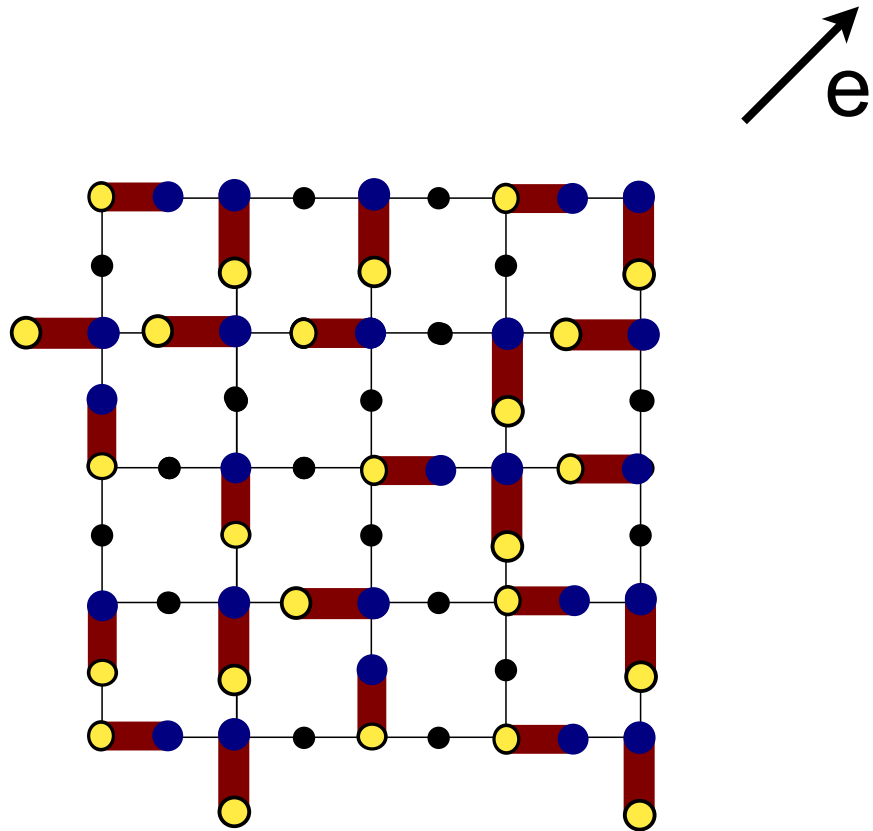
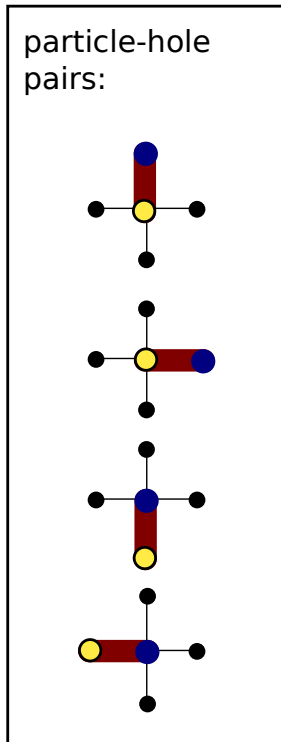
2d: phases, degeneracy and excitations depend on:

- lattice geometry
- tilt direction
- effective three-body interaction negligible?

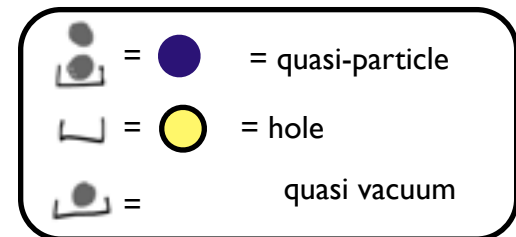
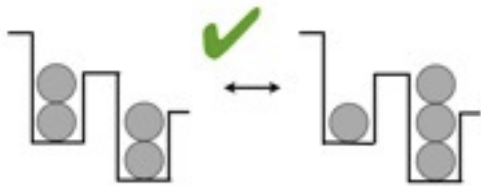
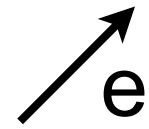
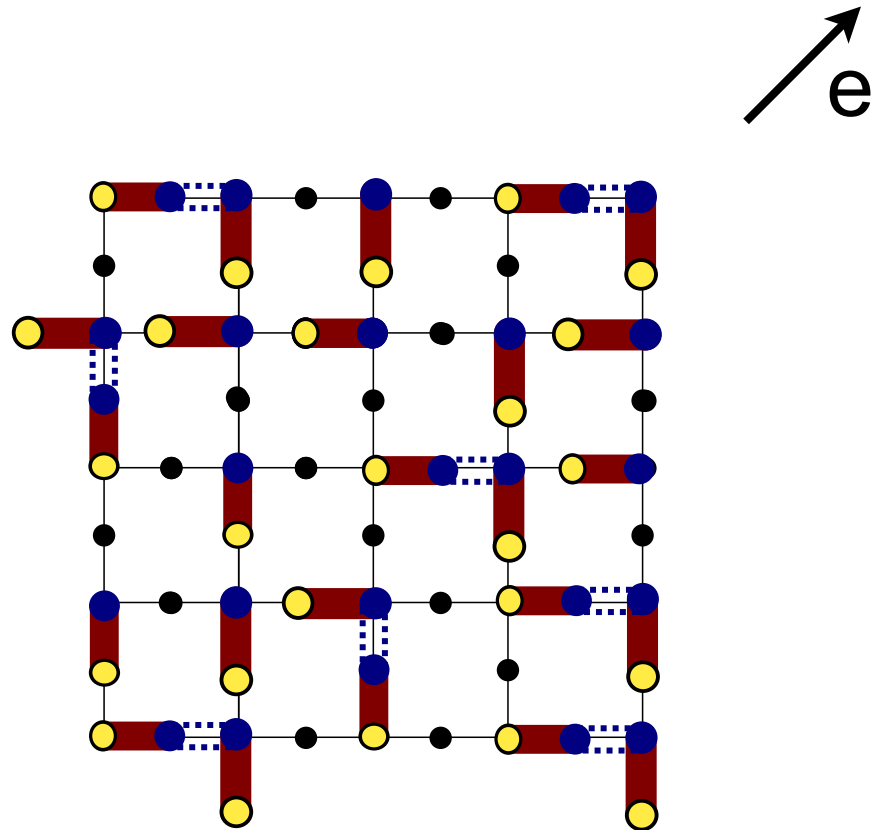
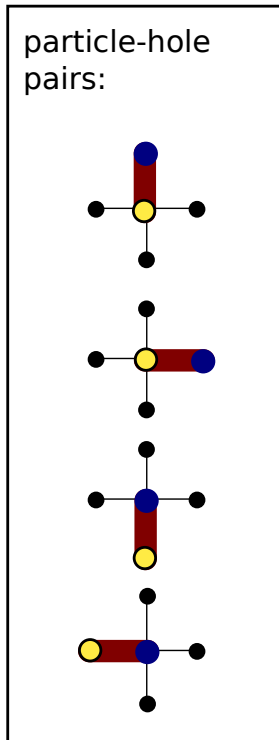


- filling of parent Mott insulator

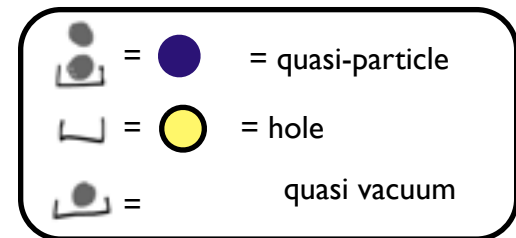
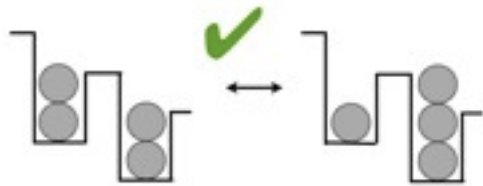
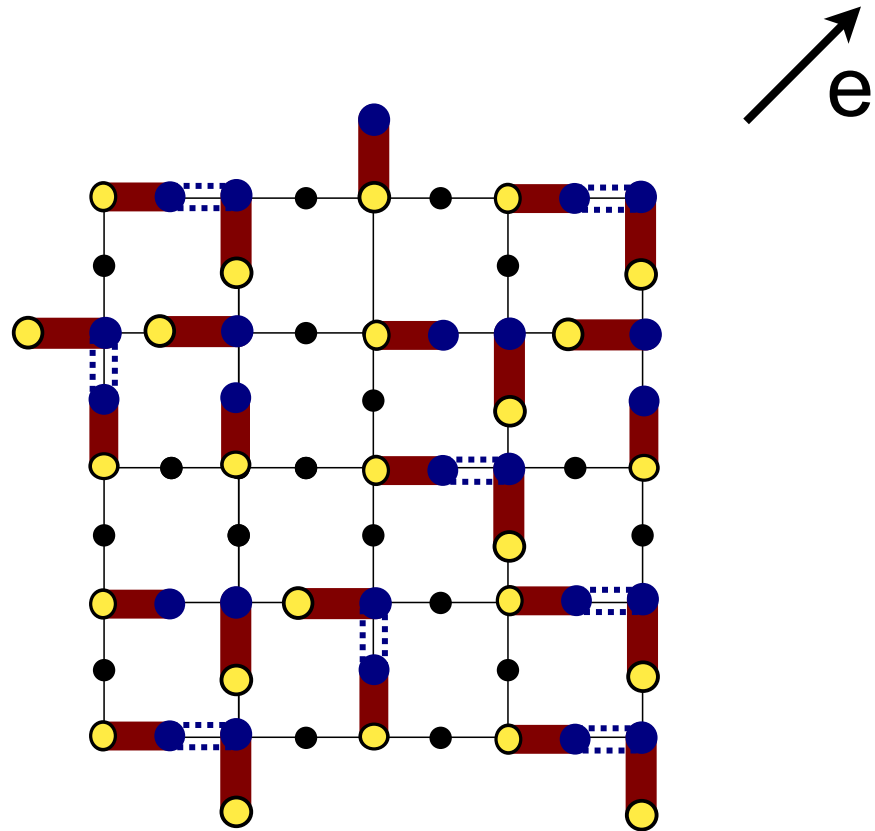
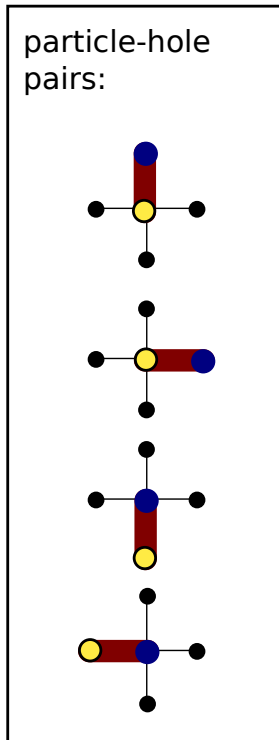
$$n_0 = 1$$



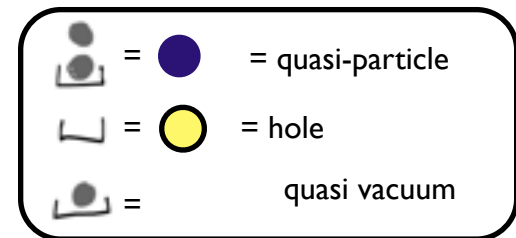
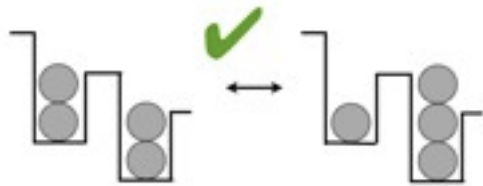
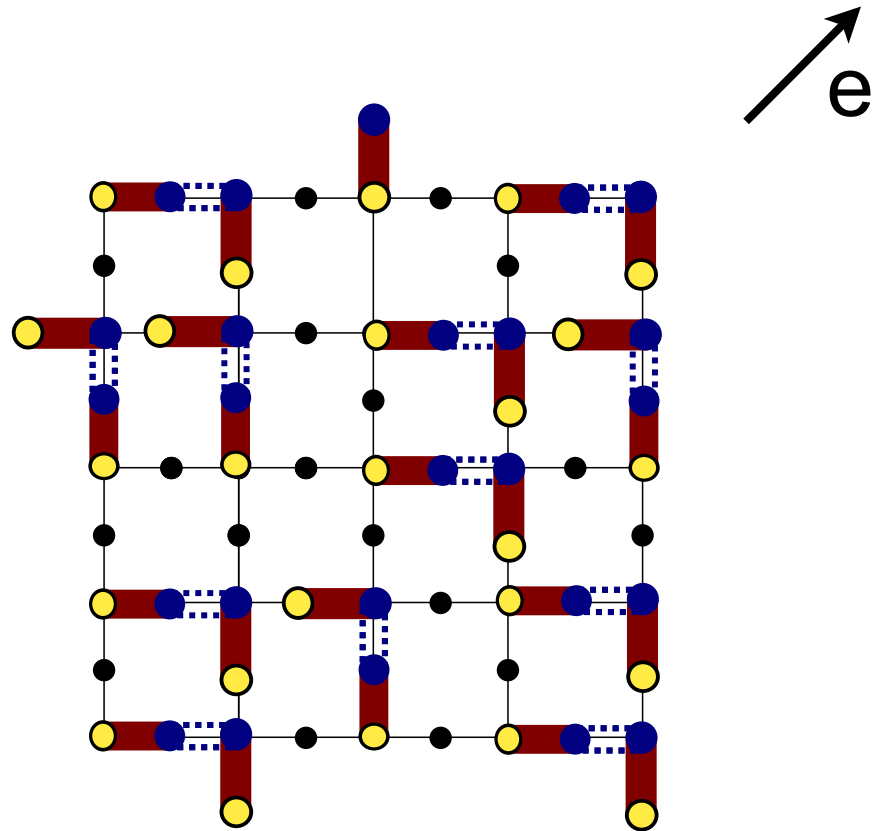
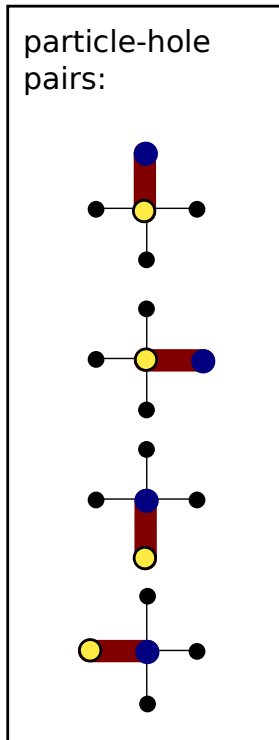
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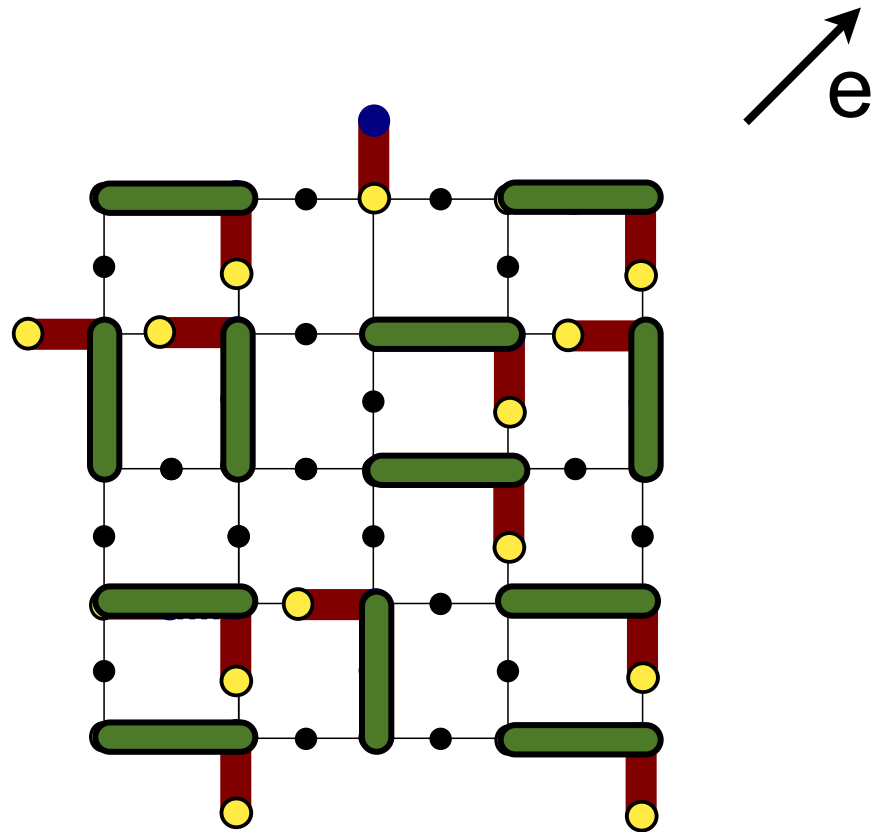
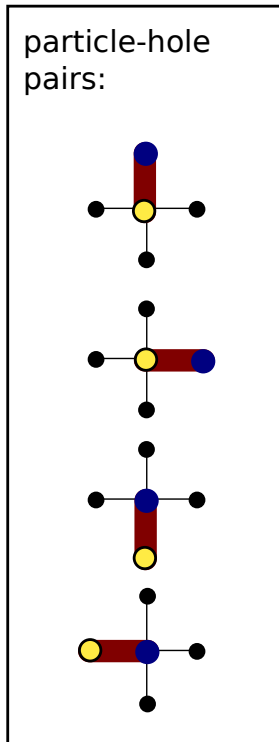
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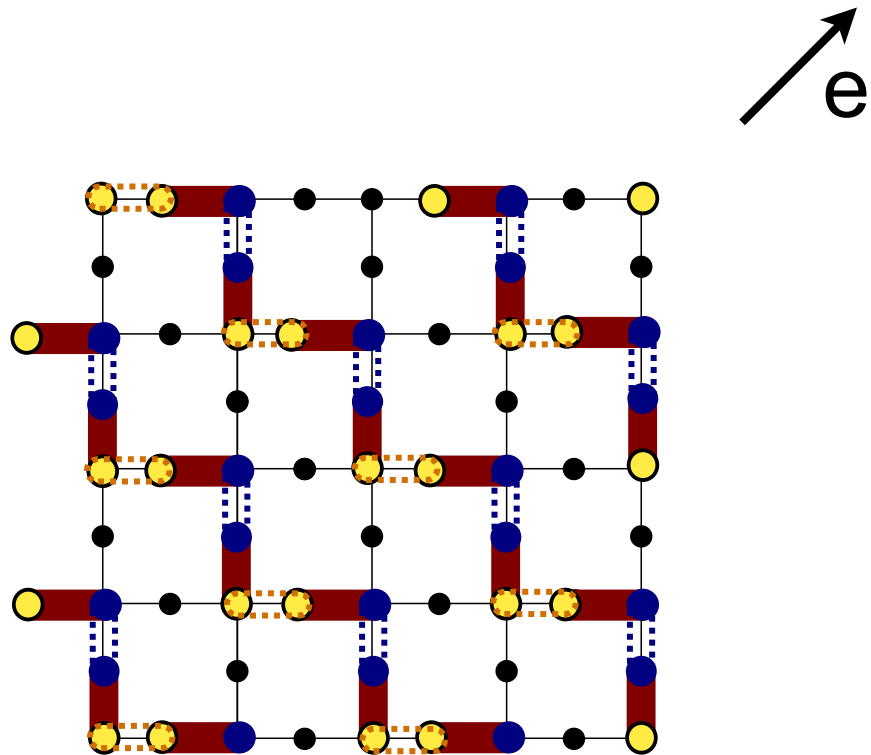
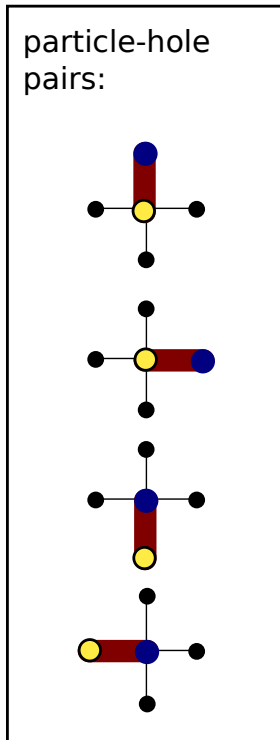


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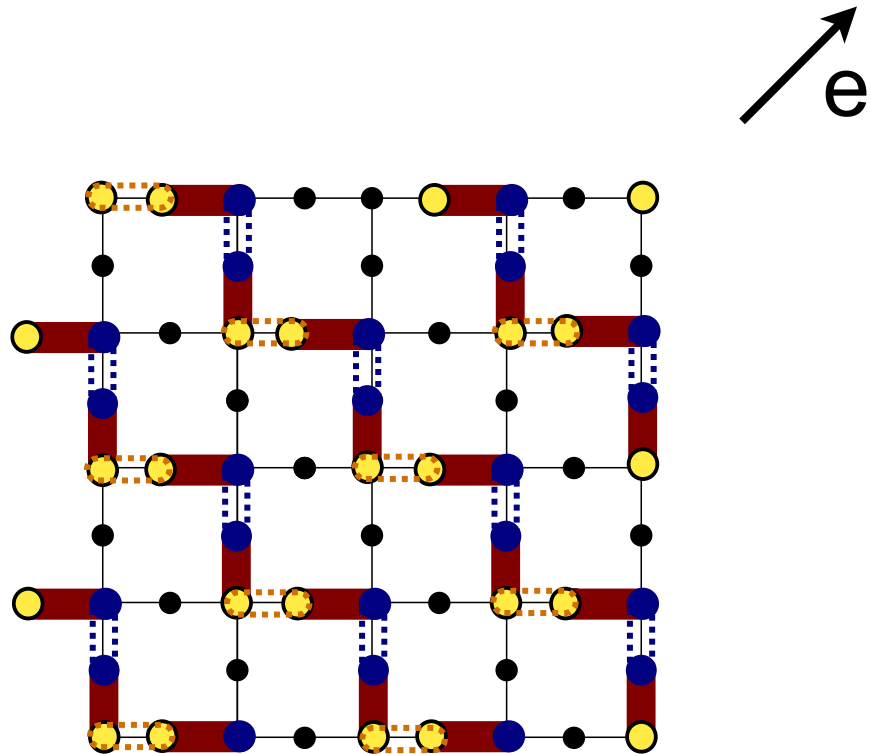
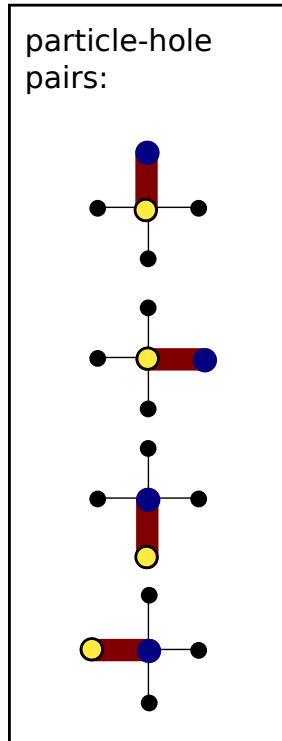


Hilbert space and effective Hamiltonian of quantum dimer model




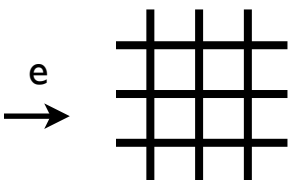
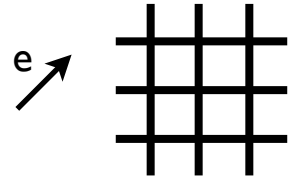
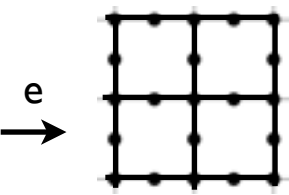
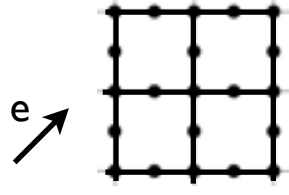
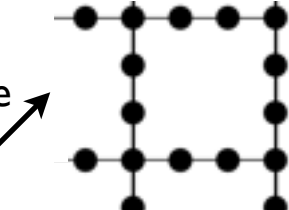

$$n_0 = 2$$



$$n_0 = 2$$



density wave order, two fold degenerate ground state

Lattice and tilt configuration 	U_3 important 	U_3 negligible 
square lattice tilted along principal lattice direction 	Ising order + transverse superfluid	“Tetris-Runaway-Instability” ?
square lattice diagonal tilt 	Ising order + transverse superfluid	“Tetris-Runaway-Instability” ?
decorated square tilt along lattice direction 	decoupled 1D systems	“Tetris-Runaway-Instability” ?
decorated square diagonal tilt 	quantum liquid state	$n_0 = 1$: quantum dimer model $n_0 > 1$: density wave order
doubly decorated square diagonal tilt 	quantum dimer model	 Susanne Pielawa

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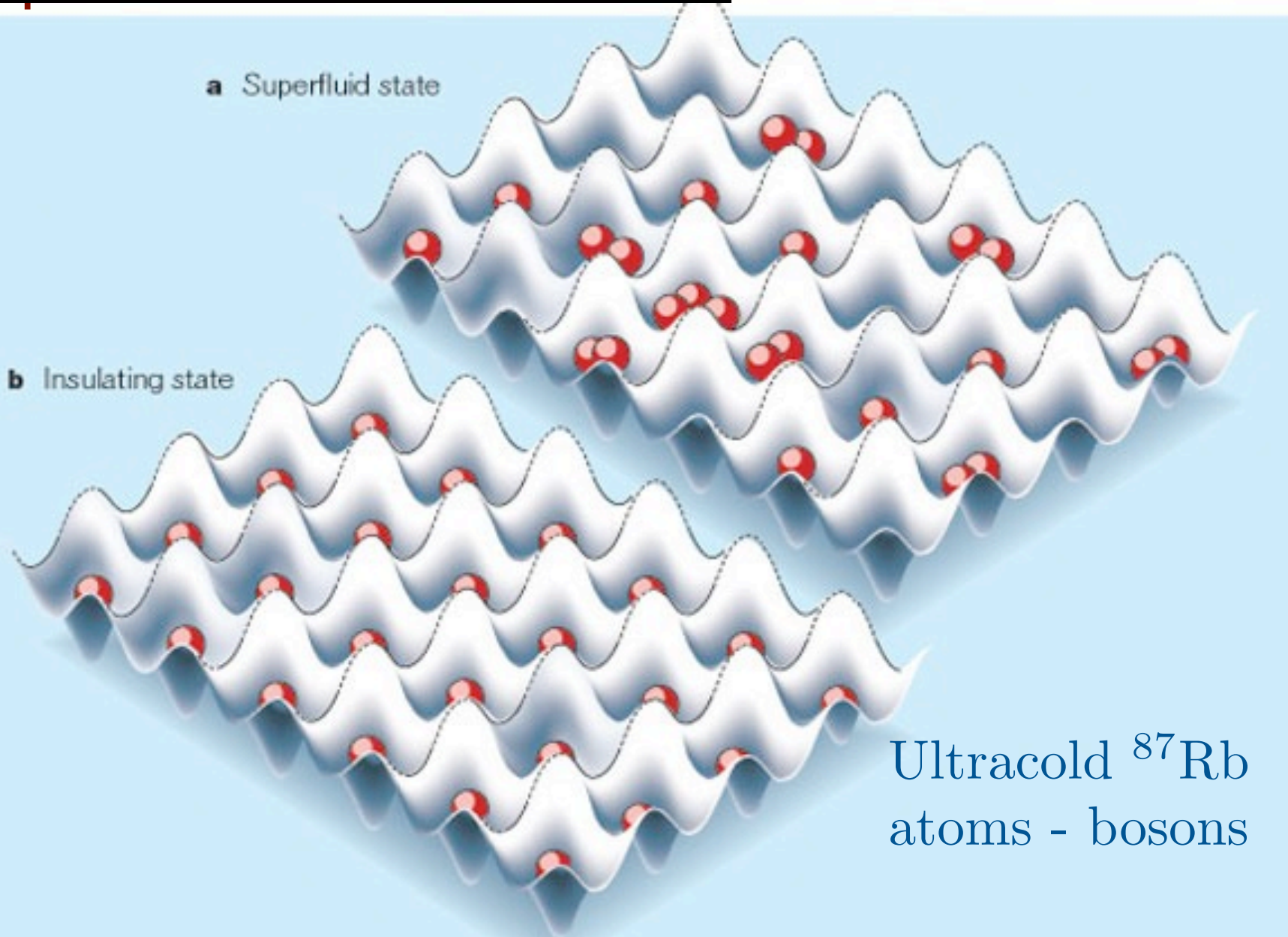
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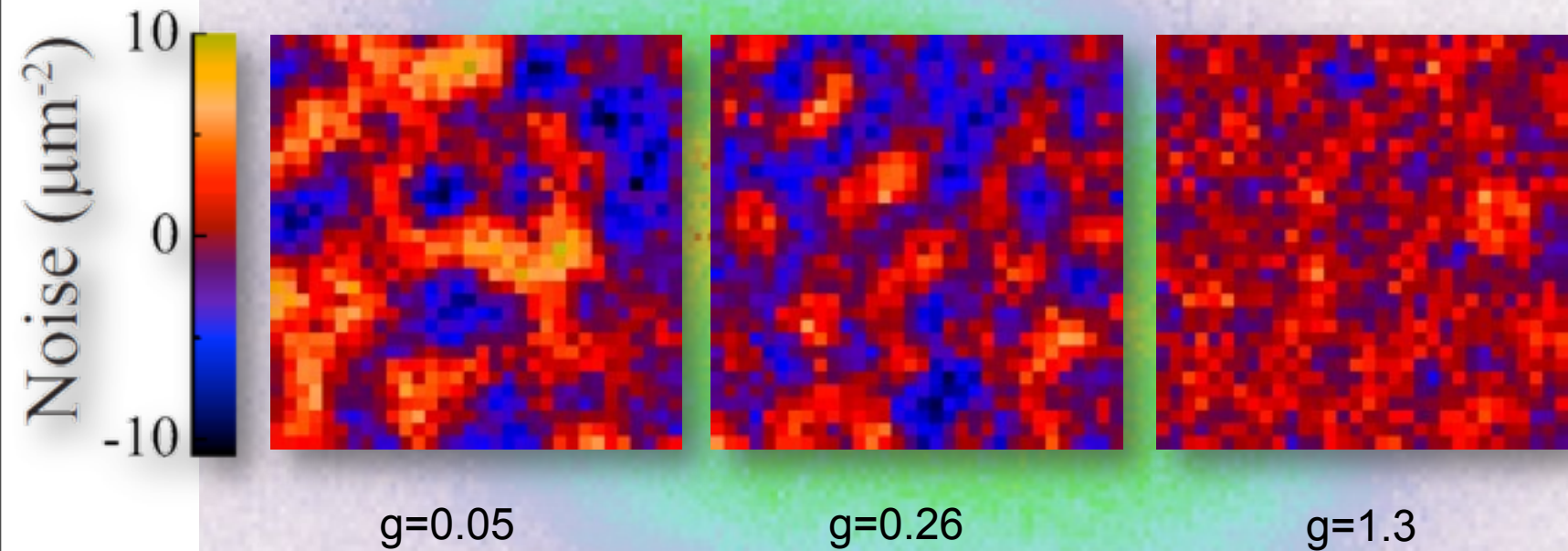
Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

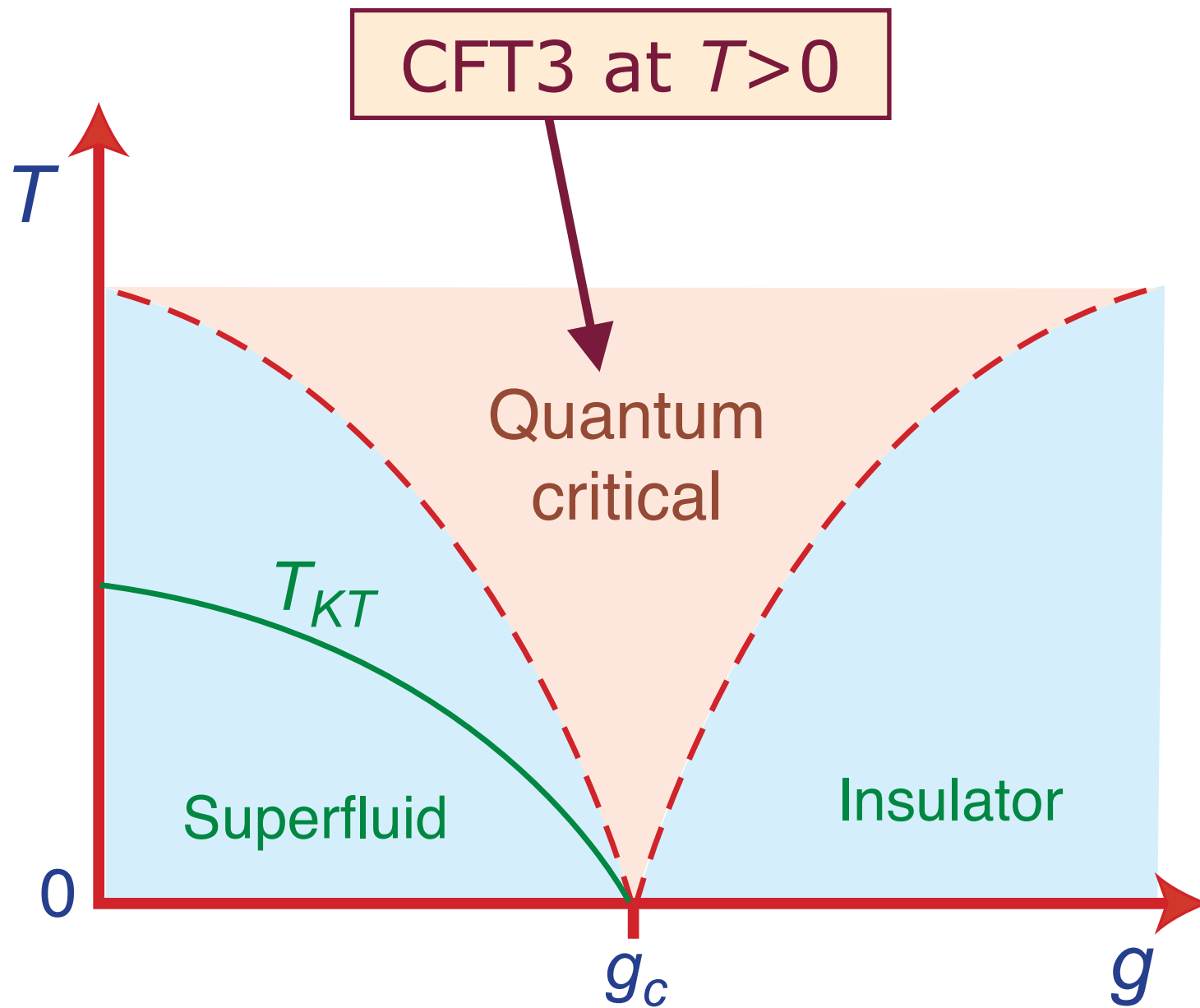
Quantum Criticality and Quantum Dynamics in 2D Gases



doi:10.1038/nature09722

Observation of scale invariance and universality in two-dimensional Bose gases

Chen-Lung Hung¹, Xibo Zhang¹, Nathan Gemelke^{1†} & Cheng Chin¹



AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son,
Phys. Rev. D **75**, 085020 (2007).

AdS₄ theory of “nearly perfect fluids”

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS₄):

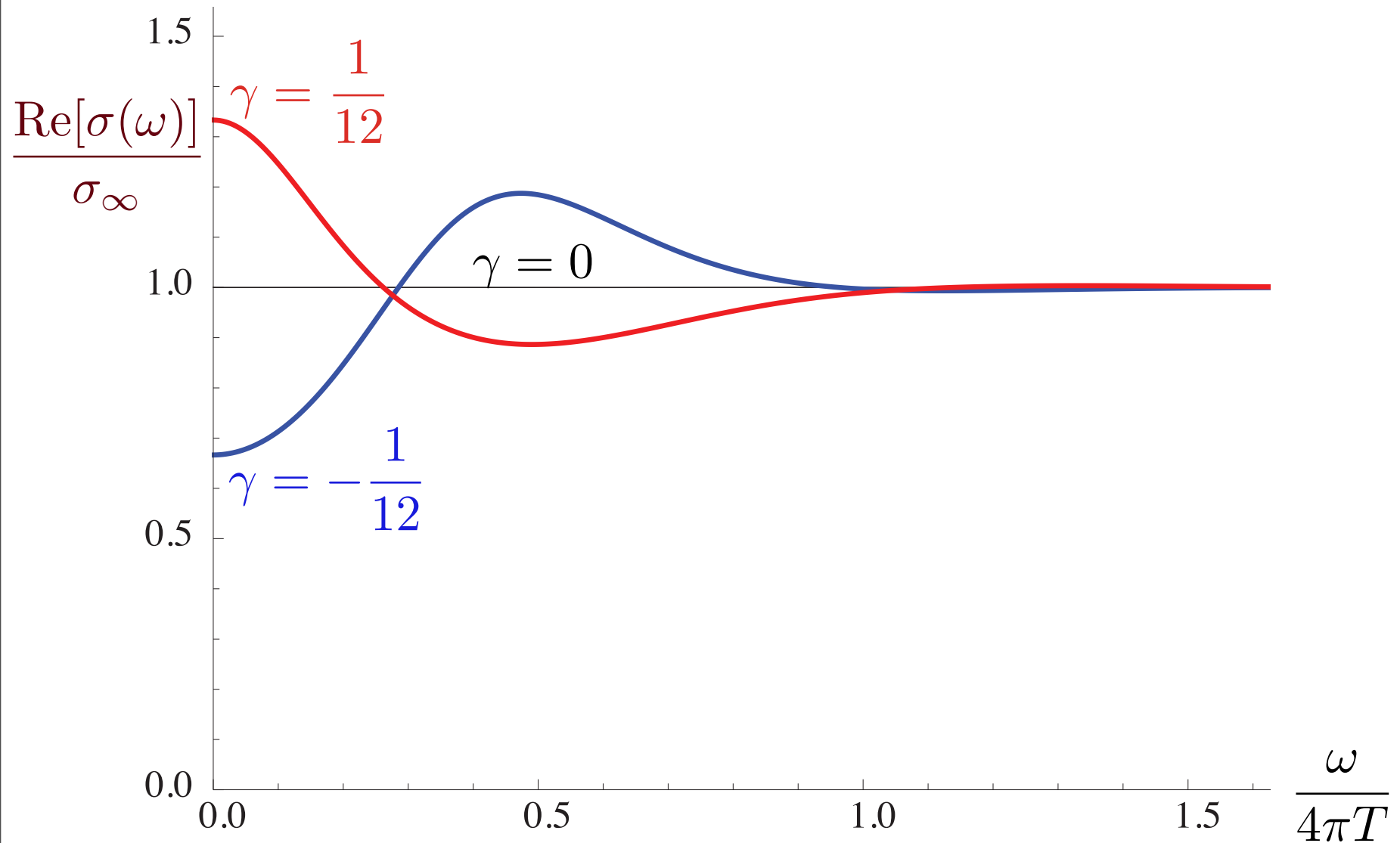
$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right],$$

where C_{abcd} is the Weyl curvature tensor.

Stability and causality constraints restrict $|\gamma| < 1/12$.

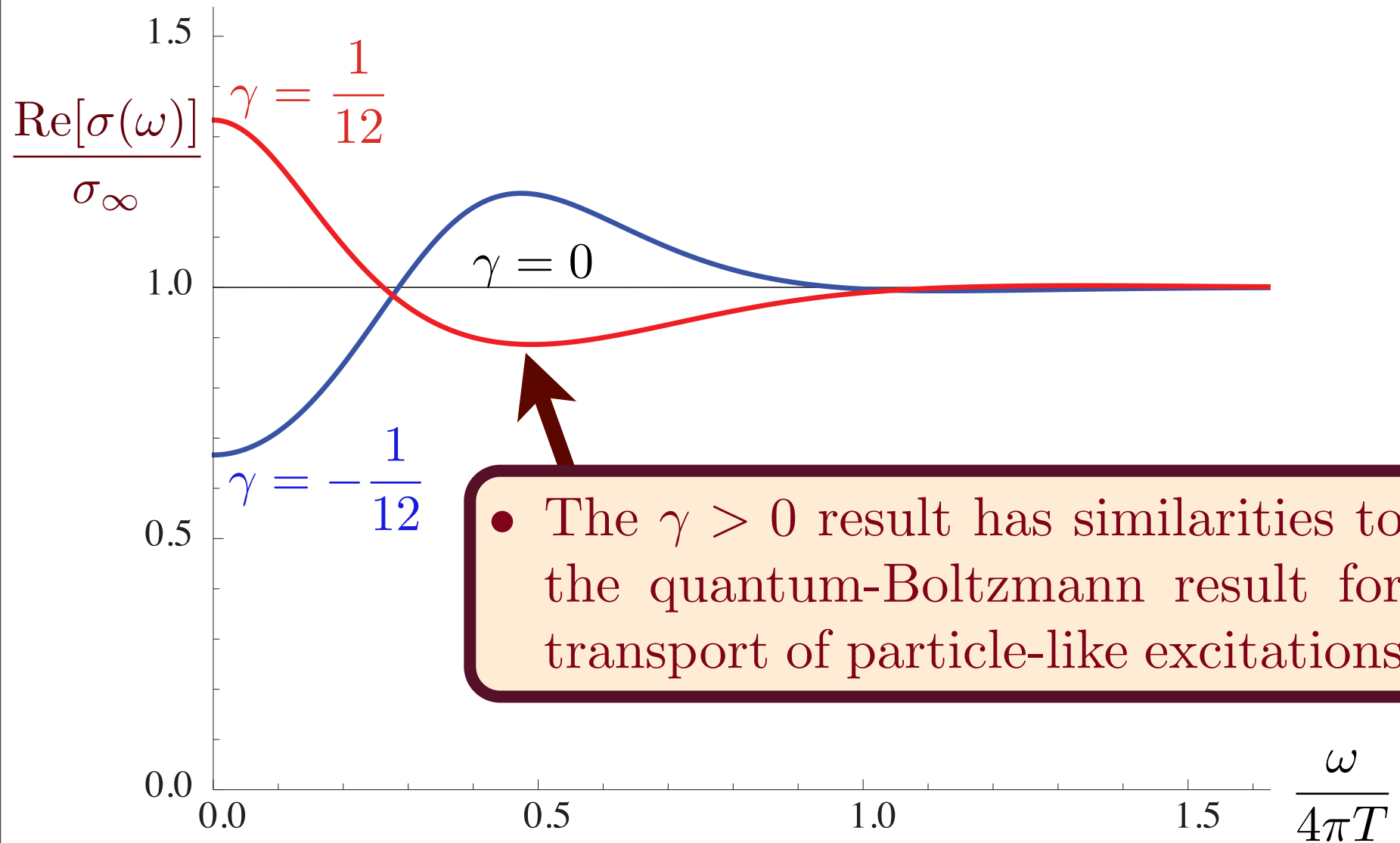
The value of γ can be fixed by matching to a direct computation in the CFT3 at $T = 0$.

AdS₄ theory of strongly interacting “perfect fluids”



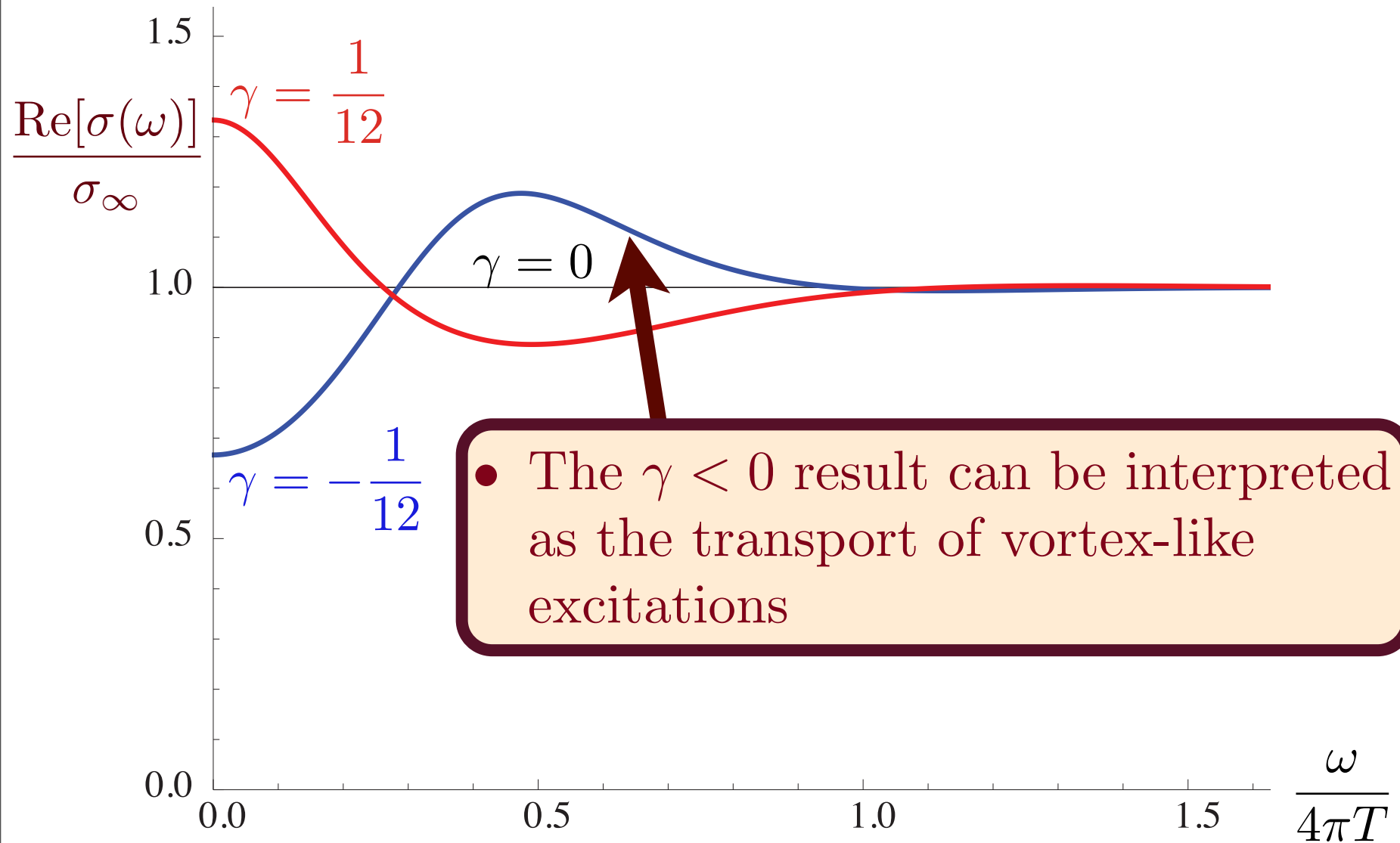
R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS₄ theory of strongly interacting “perfect fluids”



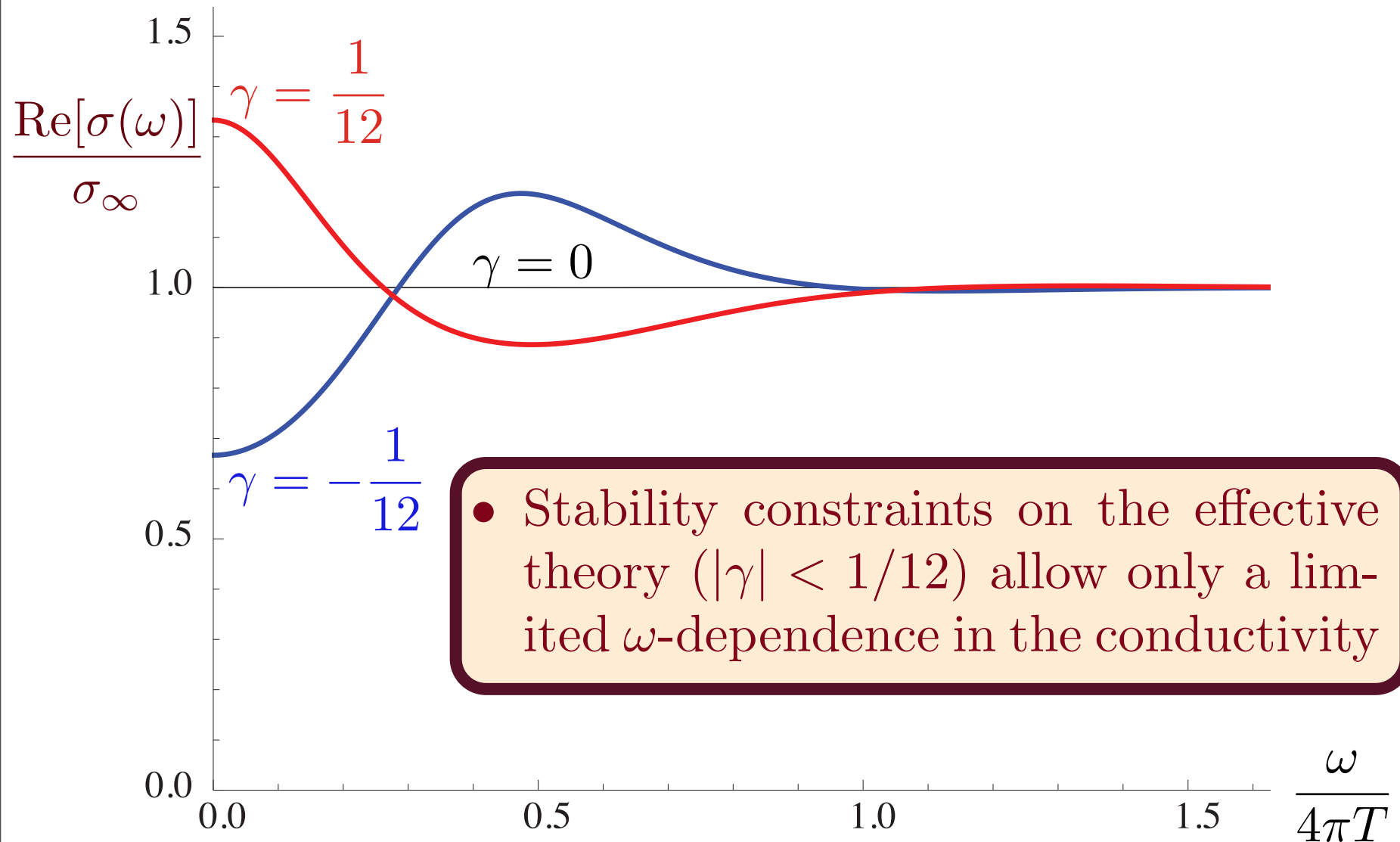
R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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AdS₄ theory of strongly interacting “perfect fluids”



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Quantum criticality and gauge-gravity duality

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Outline

1. Organic insulators:
Spin liquids on the triangular lattice
2. Ultracold atoms:
bosons in tilted Mott insulators
3. Ultracold atoms:
dynamics near quantum-critical points
and gauge-gravity duality