Deconfined criticality in a doped random quantum Heisenberg magnet

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Talk online: sachdev.physics.harvard.edu

PHYSICS
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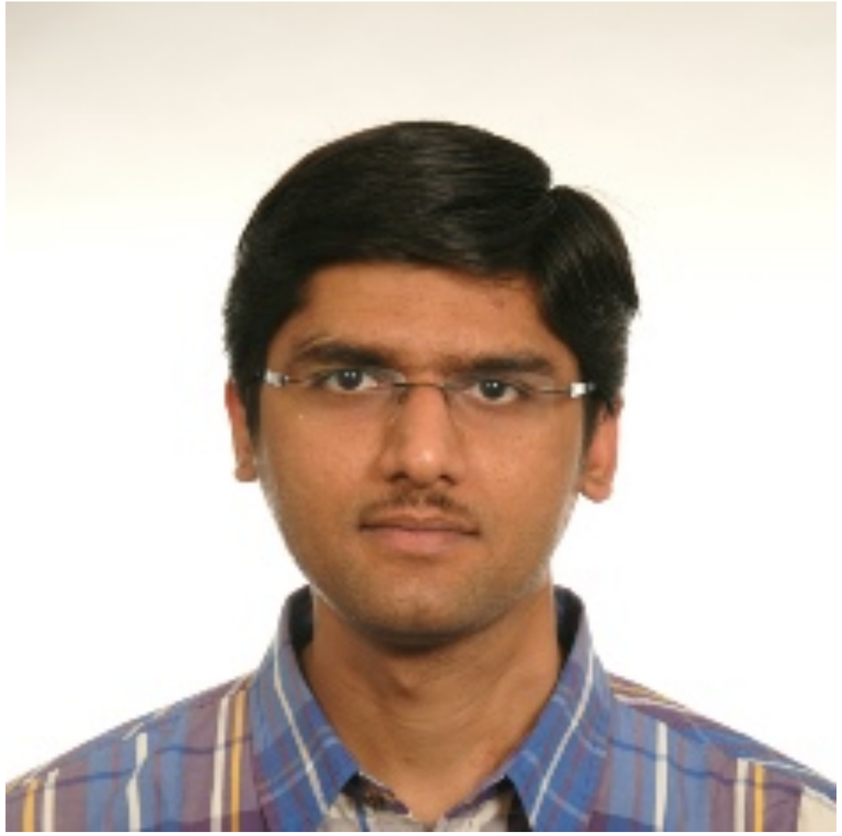

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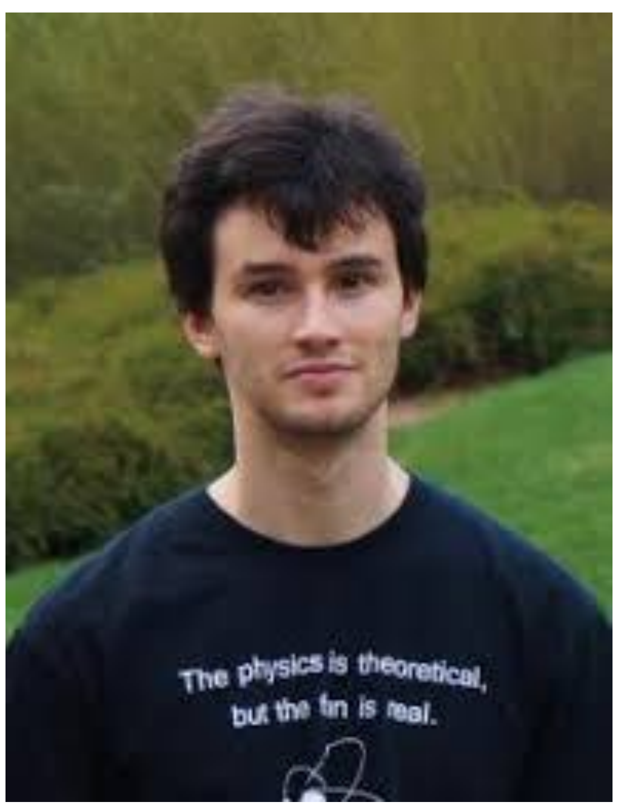
Chenyuan Li



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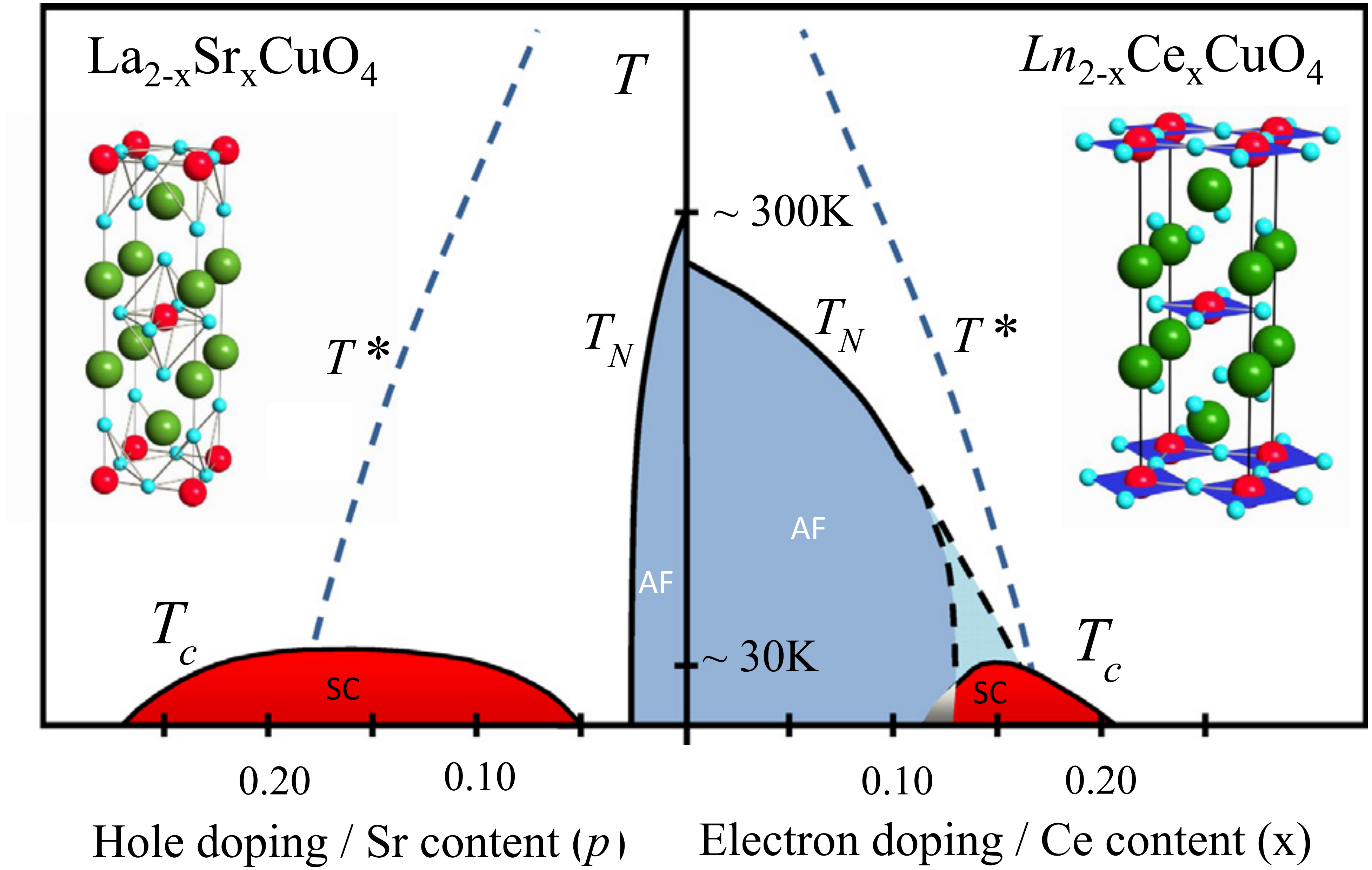
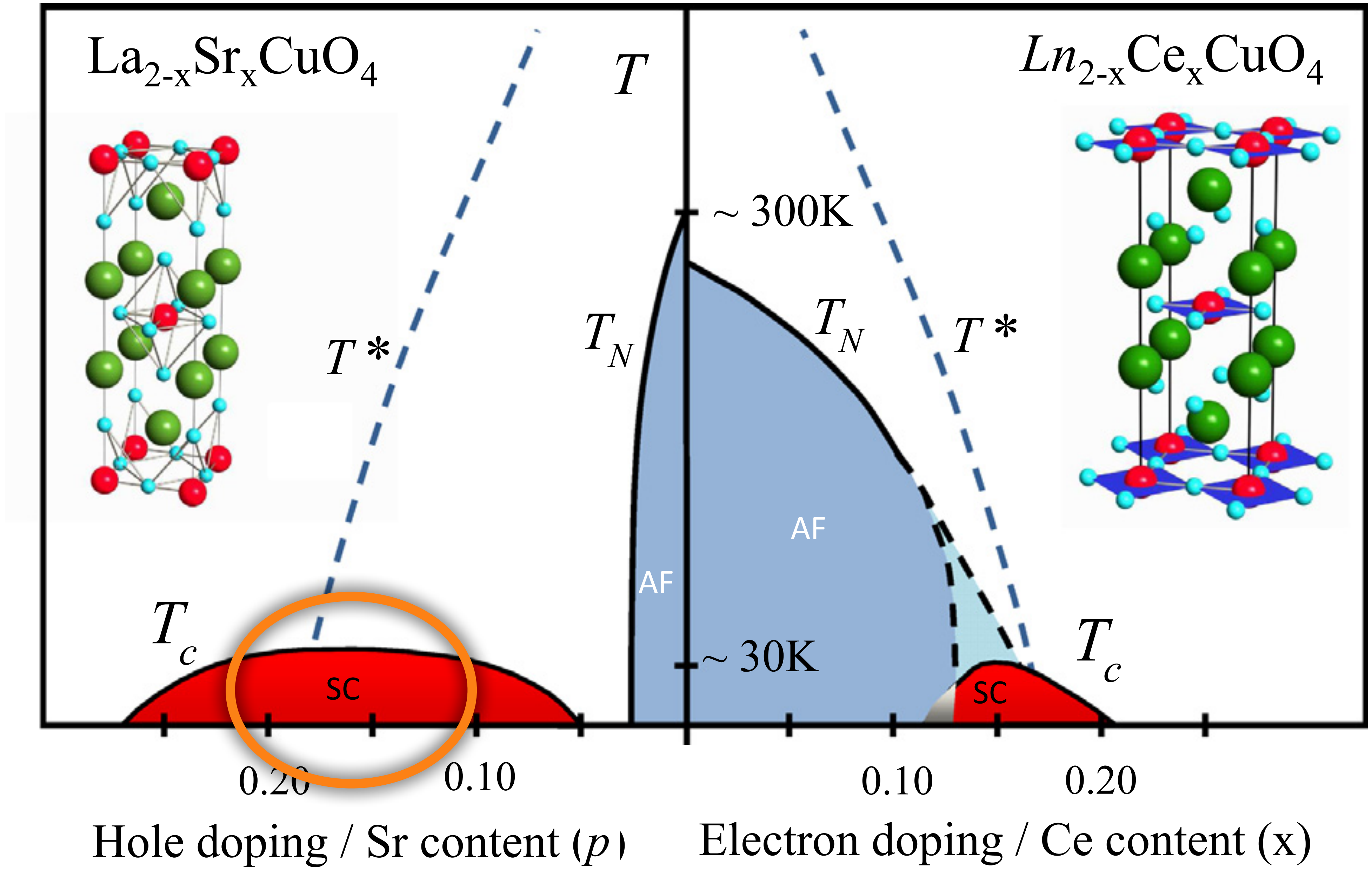


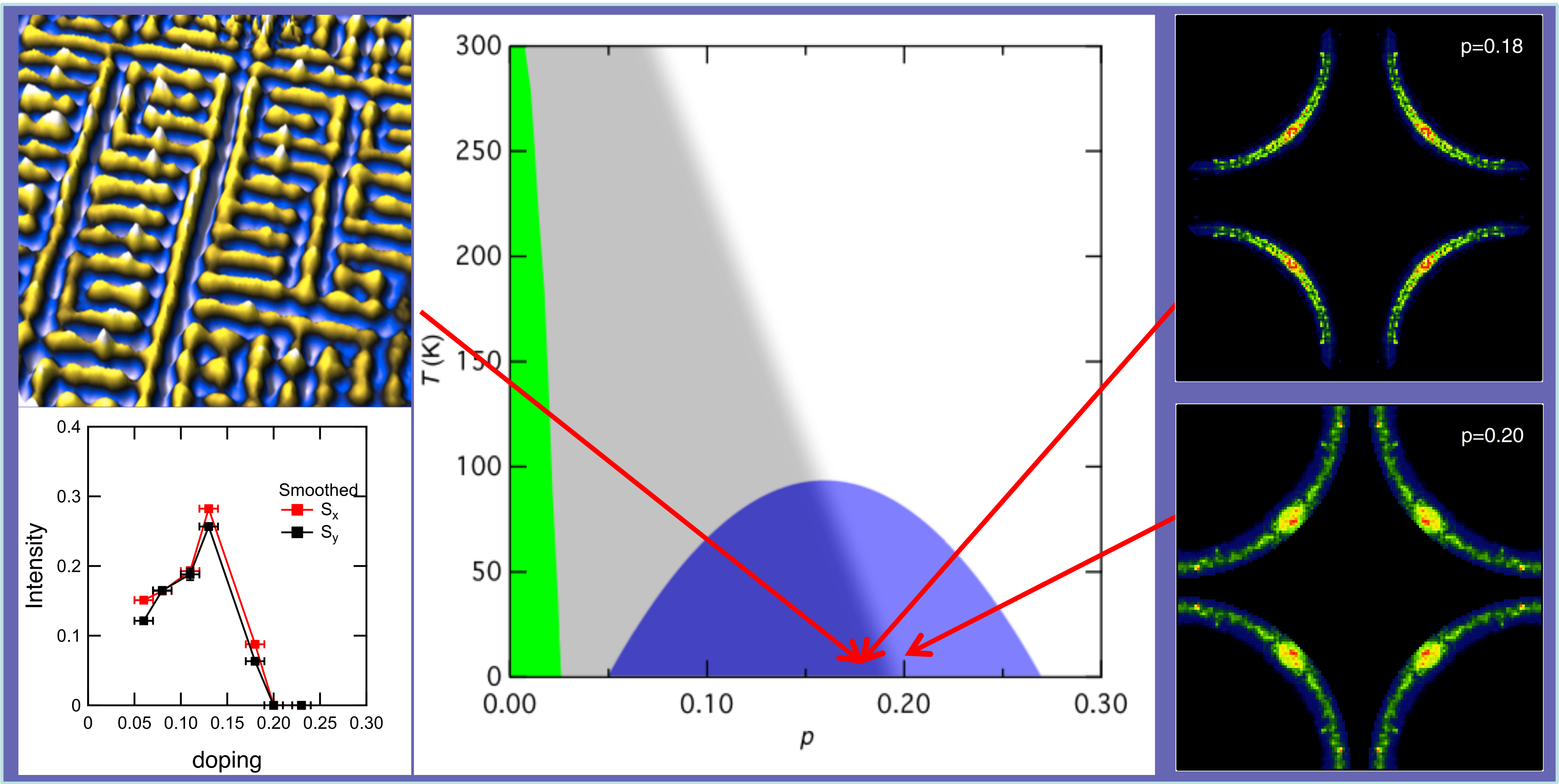
La2-xSrxCuO4
T
Ln2-xCexCuO4
~ 300K
T_N
T_N
T*
T*
AF
AF
T_c
T_c
~ 30K
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)


$La_{2-x}Sr_xCuO_4$
$Ln_{2-x}Ce_xCuO_4$
T
~ 300K
~ 30K
T_N
T_N
T^*
T^*
T_c
T_c
AF
AF
SC
SC
0.20
0.10
0.10
0.20
Hole doping / Sr content (p)
Electron doping / Ce content (x)

Hole doped cuprates

Yang He, Yi Yin, M. Zech, A. Soumyanarayanan, I. Zeljkovic, M. M. Yee, M. C. Boyer, K. Chatterjee, W. D. Wise, Takeshi Kondo, T. Takeuchi, H. Ikuta, P. Mistark, R. S. Markiewicz, A. Bansil, S. Sachdev, E. W. Hudson, and J. E. Hoffman, Science **344**, 608 (2014)

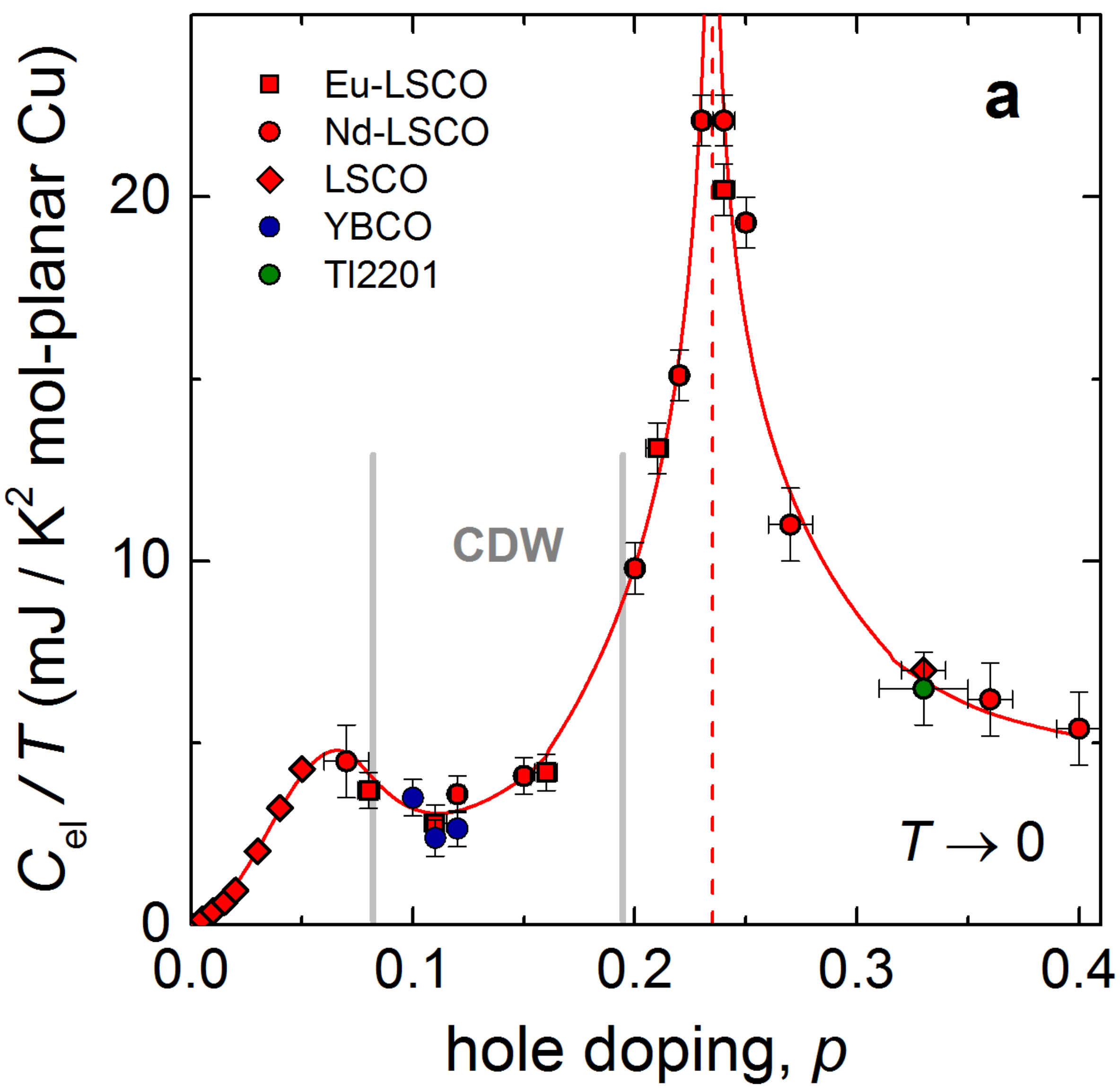
K. Fujita, Chung Koo Kim, Inhee Lee, Jinho Lee, M. H. Hamidian, I. A. Firmo, S. Mukhopadhyay, H. Eisaki, S. Uchida, M. J. Lawler, E.-A. Kim, J. C. Davis, Science **344**, 612 (2014)

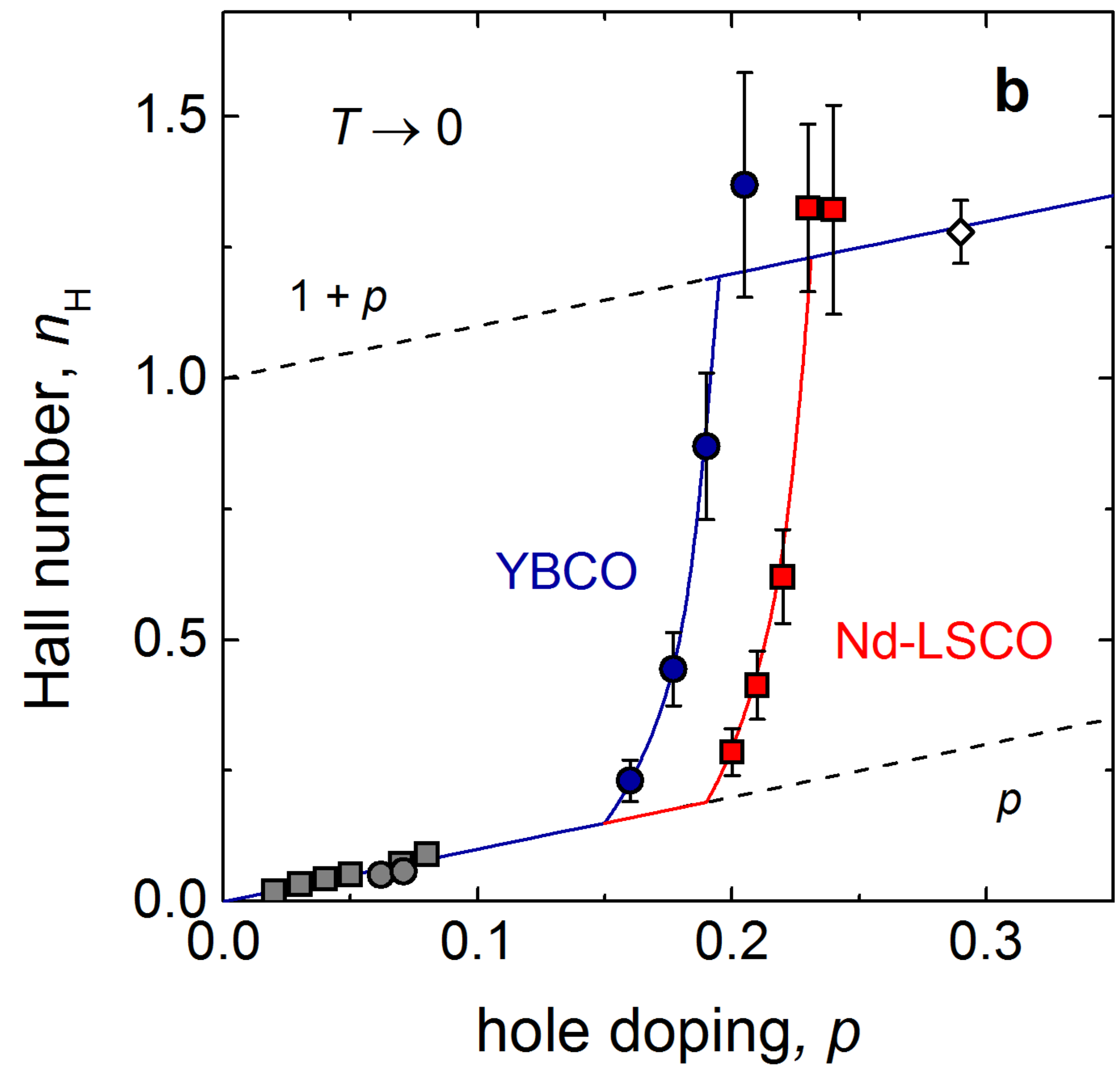


Hole doped cuprates

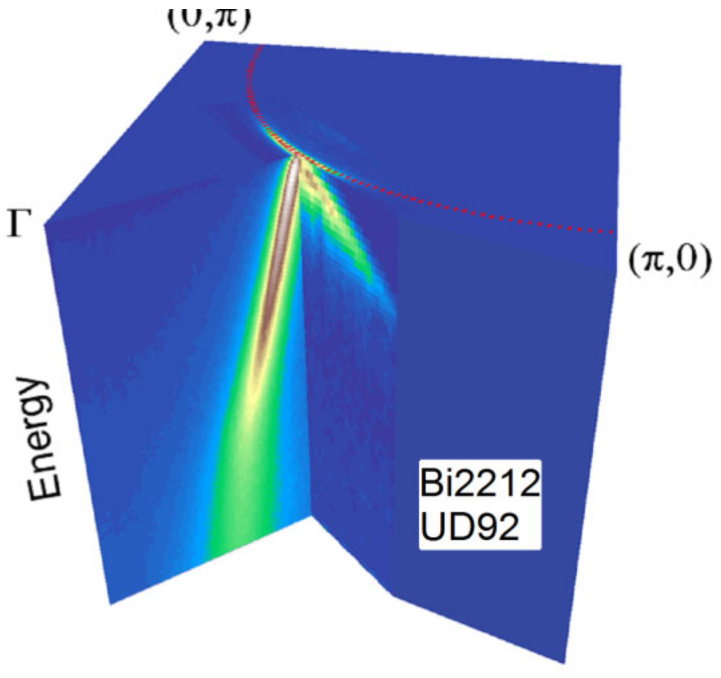
The remarkable underlying ground states of cuprate superconductors

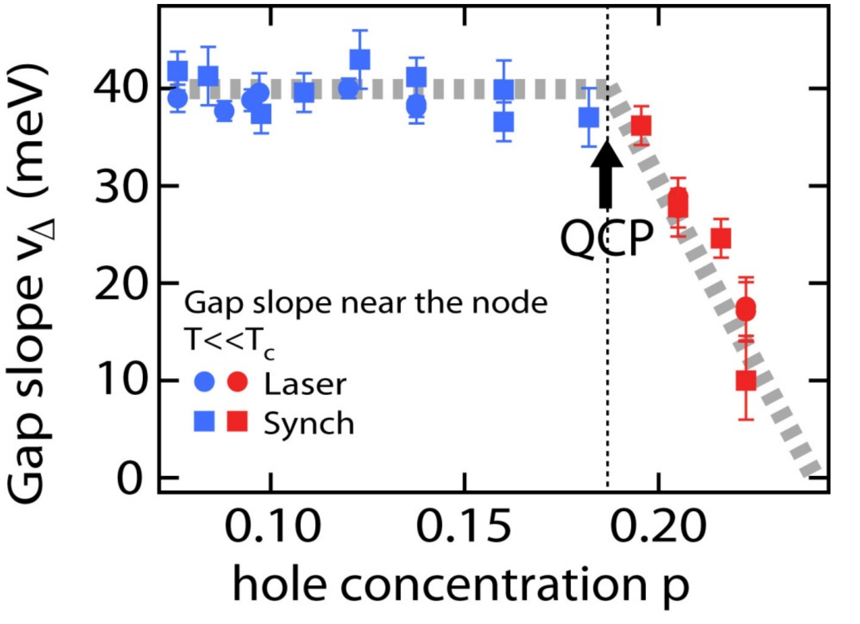
Cyril Proust and Louis Taillefer, arXiv:1807.0507

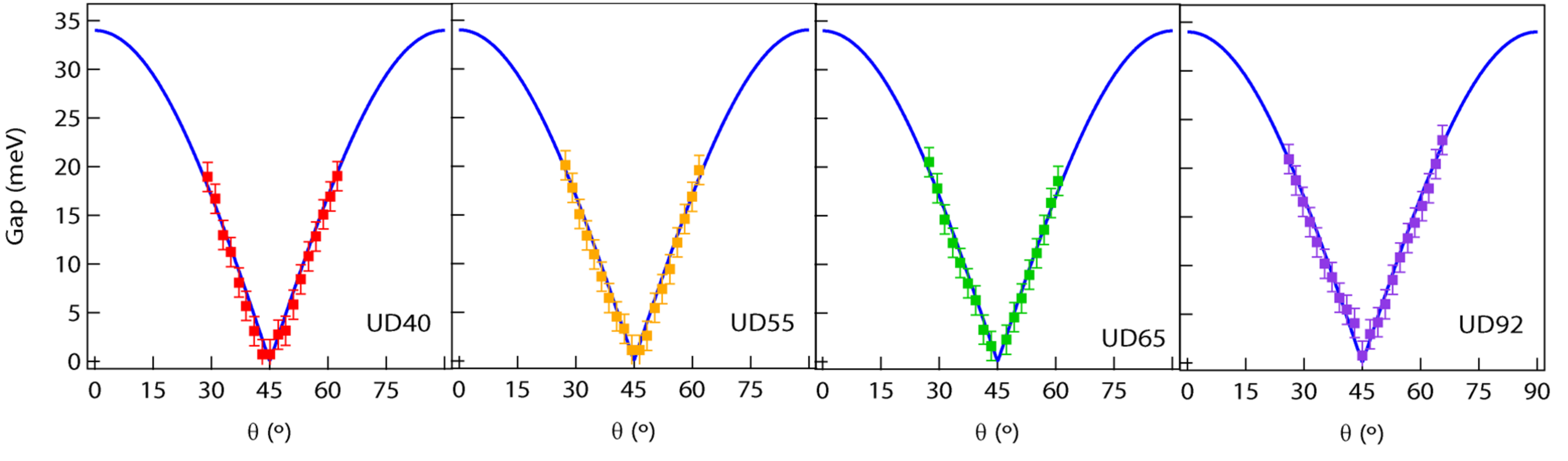




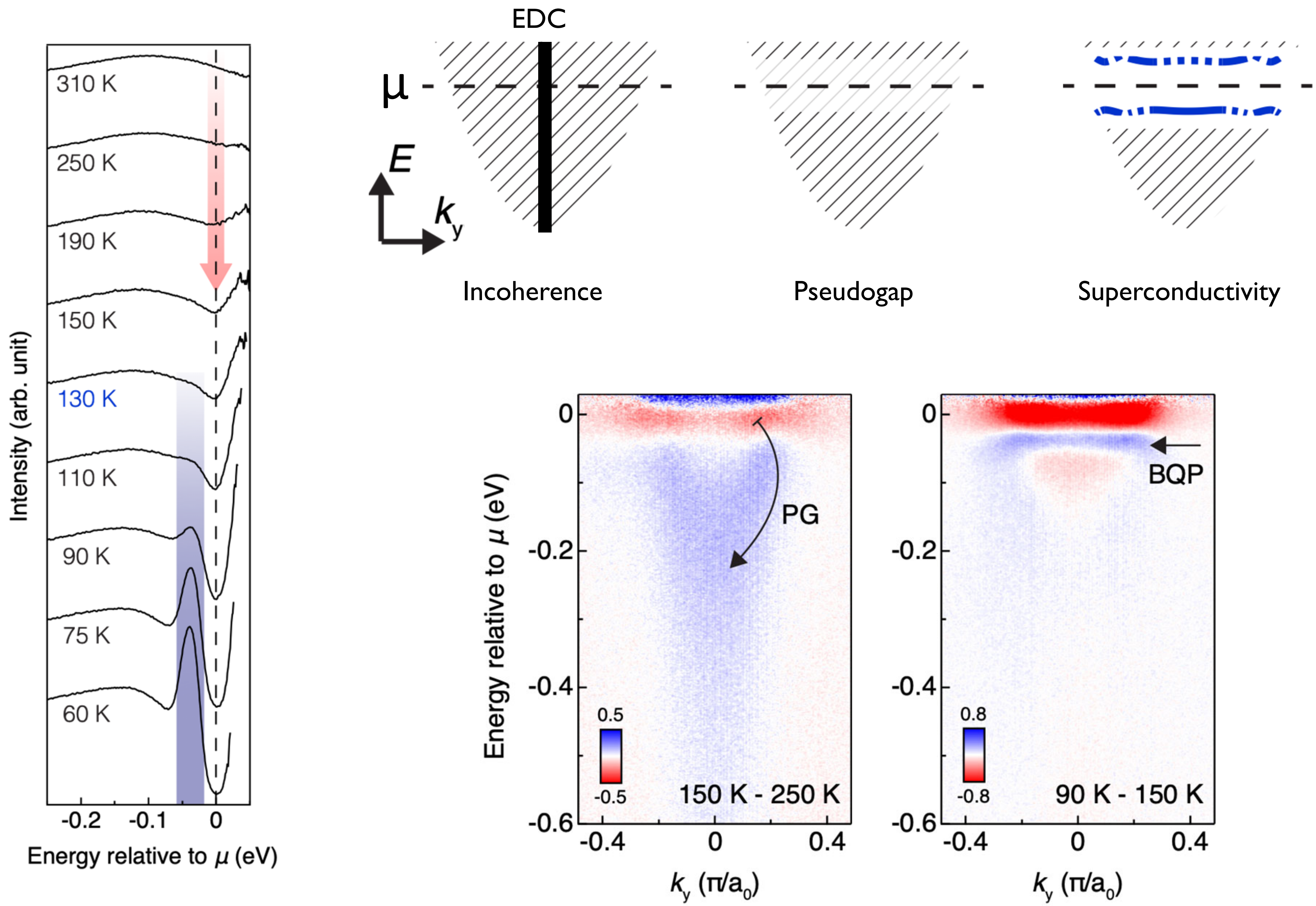
Precision Measurement of the Node





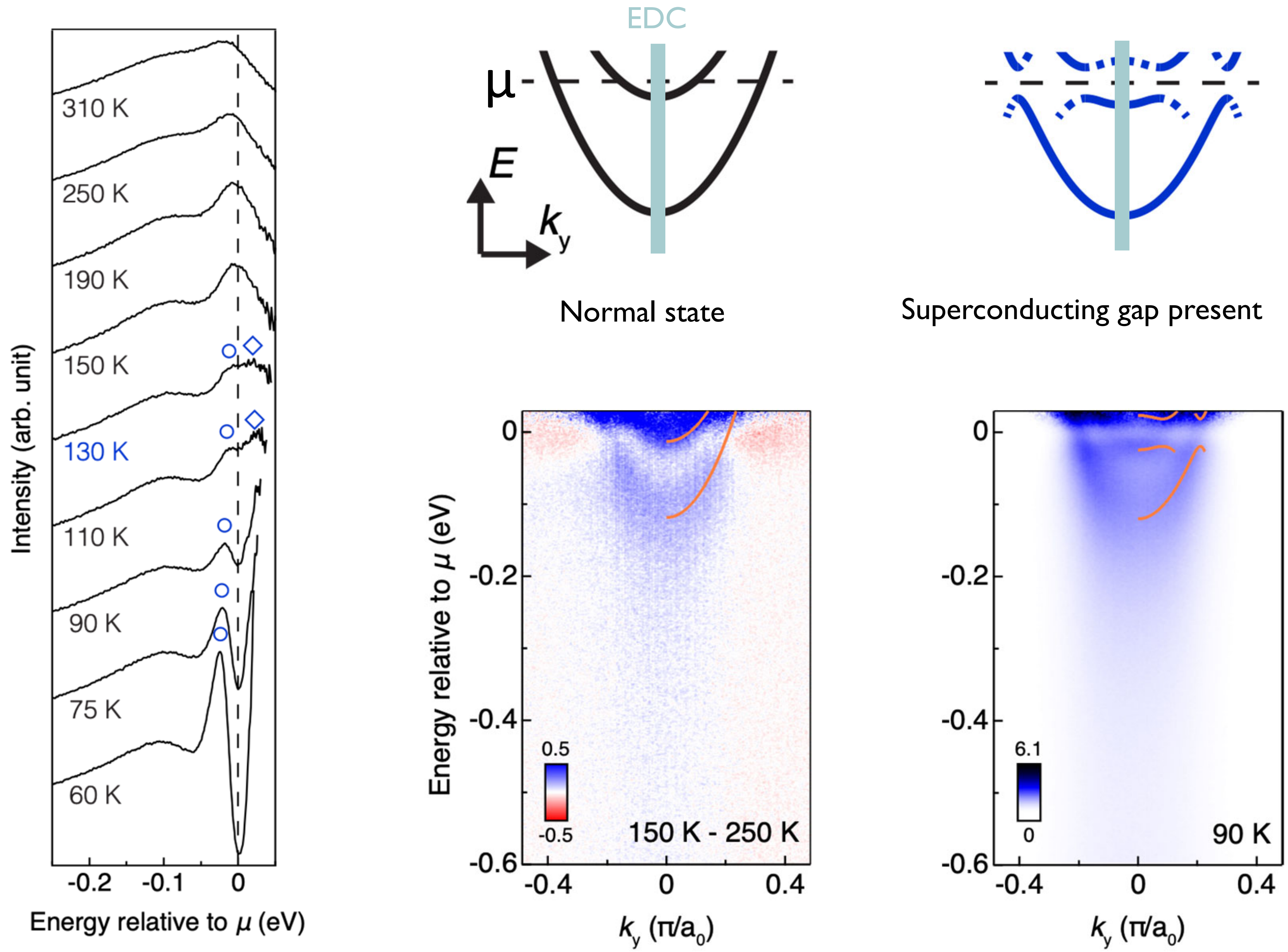


Two “gaps” for p < 0.19 (T_c~ 86 K)



S.D. Chen et al., Science 2019

One gap for p > 0.19 (T_c ~ 81 K)



S.D. Chen et al., Science 2019

1. Insulating random magnet

2. Deconfined criticality at non-zero doping

3. Phase diagram of disordered Hubbard model

1. Insulating random magnet

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3. Phase diagram of disordered Hubbard model

Insulating J model

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} = 1$$

$$J_{ij} \text{ random}, \; \overline{J_{ij}} = 0, \; \overline{J_{ij}^2} = J^2$$

Insulating J model

$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}}$$

$$\mathcal{S} = \int d\tau i\vec{A}(\vec{S}) \cdot \frac{d\vec{S}}{d\tau} - \frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau)\cdot\vec{S}(\tau')$$

$$\vec{\nabla}_{\vec{S}} \times \vec{A}(\vec{S}) = \frac{1}{2}\vec{S}.$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model

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$$\vec{\nabla}_{\vec{S}} \times \vec{A}(\vec{S}) = \frac{1}{2}\vec{S}\,.$$

From this action we compute

$$\overline{Q}(\tau - \tau') = \frac{1}{3}\left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \overline{Q}(\tau).$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Insulating J model: large M limit

Express the spin operator in terms of fermions $\vec{S} = (1/2) f_\alpha^\dagger \vec{\sigma}_{\alpha\beta} f_\beta$, and let $\alpha = 1 \ldots M$. The fermions obey the constraint

$$\sum_{\alpha=1}^{M} f_\alpha^\dagger f_\alpha = \frac{M}{2}$$

In the large M limit we obtain for the fermion Green's function G and self energy Σ (same as the SYK equations)

$$G(i\omega) = \frac{1}{i\omega - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -J^2 G^2(\tau) G(-\tau)$$

The solution is

$$G(\tau) \sim \frac{\text{sgn}(\tau)}{\sqrt{\tau}} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

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Insulating J model

$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

Numerical studies for SU(2) spin-1/2 show spin-glass order!

L. Arrachea and M. J. Rozenberg, PRB **65**, 224430 (2002)

Insulating J model: RG

We assume a power-law decay

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}}\,.$$

Ignore the self-consistency condition for now. We decouple the $\vec{S}(\tau) \cdot \vec{S}(0)$ interaction by introducing a bosonic (ϕ_a, $a = 1 \ldots 3$) bath.

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
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$$H_{\text{imp}} = \gamma_0 f_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} f_\beta \, \phi_a(0) + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right]$$

where π_a is canonically conjugate to the field ϕ_a, $\phi_a(0) \equiv \phi_a(x=0)$, and we have the constraint

$$f_\alpha^\dagger f_\alpha = 1\,.$$

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$$f_\alpha^\dagger f_\alpha = 1\,.$$

We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and it can be verified that this correlator decays as above.

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Insulating J model: RG

Schwinger fermions

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Insulating J model: RG

Schwinger bosons

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$$H_{\mathrm{imp}} = \gamma_0 \mathfrak{b}_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} \mathfrak{b}_\beta \, \phi_a(0) + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right]$$

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Insulating J model: RG

We can perform a RG analysis in a $\epsilon = 3-d$ expansion, while imposing the fermion constraint *exactly*. The two-loop β function is

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 - \gamma^5 + \ldots$$

This has a stable fixed point at $\gamma^{*2} = \epsilon/2 + \epsilon^2/4 + \ldots$.

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The scaling dimension of the spin operator is $\dim[\vec{S}] = \epsilon/2$, exact to *all* orders in ϵ. This implies the correlator

$$\overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|^{3-d}} .$$

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$$\overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|^{3-d}}\,.$$

Finally, we impose the self-consistency condition $Q(\tau) = \overline{Q}(\tau)$, and obtain the same self-consistent result as in the large M expansion

$$\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}\,.$$

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t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^{\dagger} c_{i\alpha} \leq 1$$

$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$
$$t_{ij} \text{ random, } \overline{t_{ij}} = 0, \ \overline{t_{ij}^2} = t^2$$

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We consider the hole-doped case, with no double occupancy. Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \quad , \quad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU$(1|2)$ superspin space.

t-J model

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$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

t-J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^{\dagger} \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_{\alpha}^{\dagger} \sigma_{\alpha\beta} \mathfrak{b}_{\beta} \\ \mathfrak{b}_{\alpha}^{\dagger} \mathfrak{b}_{\alpha} + \mathfrak{f}^{\dagger} \mathfrak{f} &= 1 \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

$$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$$

t-J model

$$\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}} \\
\mathcal{S} &= \int d\tau \left[f_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau} + s_0 + i\lambda\right) f_\alpha(\tau) + b^\dagger(\tau)\left(\frac{\partial}{\partial\tau} + i\lambda\right) b(\tau) - i\lambda \right] \\
&\quad - t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') + \text{H.c.} \\
&\quad - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \,.
\end{aligned}$$

SU(1|2) theory

t-J model

$$\mathcal{Z} = \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}}$$

$$\mathcal{S} = \int d\tau \left[f_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau} + s_0 + i\lambda\right) f_\alpha(\tau) + b^\dagger(\tau)\left(\frac{\partial}{\partial\tau} + i\lambda\right) b(\tau) - i\lambda \right]$$
$$- t^2 \int d\tau d\tau' R(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') + \text{H.c.}$$
$$- \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \, .$$

SU(1|2) theory

From this action we determined the correlators

$$\overline{R}(\tau - \tau') = -\frac{1}{2} \left\langle c_\alpha(\tau) c_\alpha^\dagger(\tau') \right\rangle_{\mathcal{Z}}$$
$$\overline{Q}(\tau - \tau') = \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and finally impose the self-consistency conditions

$$R(\tau) = \overline{R}(\tau) \quad , \quad Q(\tau) = \overline{Q}(\tau).$$

t-J model

$$\mathcal{Z} = \int \mathcal{D}\mathfrak{b}_\alpha(\tau)\mathcal{D}\mathfrak{f}(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}}$$

$$\mathcal{S} = \int d\tau \left[\mathfrak{b}^\dagger_\alpha(\tau)\left(\frac{\partial}{\partial\tau} + s_0 + i\lambda\right)\mathfrak{b}_\alpha(\tau) + \mathfrak{f}^\dagger(\tau)\left(\frac{\partial}{\partial\tau} + i\lambda\right)\mathfrak{f}(\tau) - i\lambda\right]$$
$$- t^2 \int d\tau d\tau' R(\tau - \tau')c^\dagger_\alpha(\tau)c_\alpha(\tau') + \text{H.c.}$$
$$- \frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.$$

SU(2|1) theory

From this action we determined the correlators

$$\overline{R}(\tau - \tau') = -\frac{1}{2}\left\langle c_\alpha(\tau)c^\dagger_\alpha(\tau')\right\rangle_{\mathcal{Z}}$$
$$\overline{Q}(\tau - \tau') = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}$$

and finally impose the self-consistency conditions

$$R(\tau) = \overline{R}(\tau) \quad , \quad Q(\tau) = \overline{Q}(\tau).$$

t-J model RG

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} \, .$$

We ignore the self-consistency condition for now. We decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths.

S. Sachdev, Physica C **357**, 78 (2001)
M. Vojta and L. Fritz, PRB **70**, 094502 (2004)

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$$H = (s_0 + \lambda) f_\alpha^\dagger f_\alpha + \lambda\, b^\dagger b + g_0 \left(f_\alpha^\dagger b\, \psi_\alpha(0) + \mathrm{H.c.} \right) + \gamma_0 f_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} f_\beta\, \phi_a(0)$$
$$+ \int |k|^r dk\, k\, \psi_{k\alpha}^\dagger \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right]$$

where $a = (x, y, z)$, σ^a are Pauli matrices, π_a is canonically conjugate to the field ϕ_a, and $\phi_a(0) \equiv \phi_a(x = 0)$, $\psi_\alpha(0) \equiv \int |k|^r dk\, \psi_{k\alpha}$.

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where $a = (x, y, z)$, σ^a are Pauli matrices, π_a is canonically conjugate to the field ϕ_a, and $\phi_a(0) \equiv \phi_a(x = 0)$, $\psi_\alpha(0) \equiv \int |k|^r dk \, \psi_{k\alpha}$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and $R(\tau)$ with the temporal correlator of $\psi_\alpha(0)$, and it can be verified that these correlators decay as above.

S. Sachdev, Physica C **357**, 78 (2001)
M. Vojta and L. Fritz, PRB **70**, 094502 (2004)

t-J model RG

SU(1|2) theory

We assume power-law decays

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} \,.$$

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The impurity superspin is coupled to
a fermionic bath by g_0,
and to a bosonic bath by γ_0,
and s_0 acts as a local field on the superspin -
a superKondo problem!

t-J model RG

SU(2|1) theory

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$$\begin{aligned} H \quad &= (s_0 + \lambda)\mathfrak{b}^\dagger_\alpha \mathfrak{b}_\alpha + \lambda \, \mathfrak{f}^\dagger \mathfrak{f} + g_0 \left(\mathfrak{b}^\dagger_\alpha \mathfrak{f} \, \psi_\alpha(0) + \mathrm{H.c.} \right) + \gamma_0 \mathfrak{b}^\dagger_\alpha \frac{\sigma^a_{\alpha\beta}}{2} \mathfrak{b}_\beta \, \phi_a(0) \\ &+ \int |k|^r dk \, k \, \psi^\dagger_{k\alpha} \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right] \end{aligned}$$

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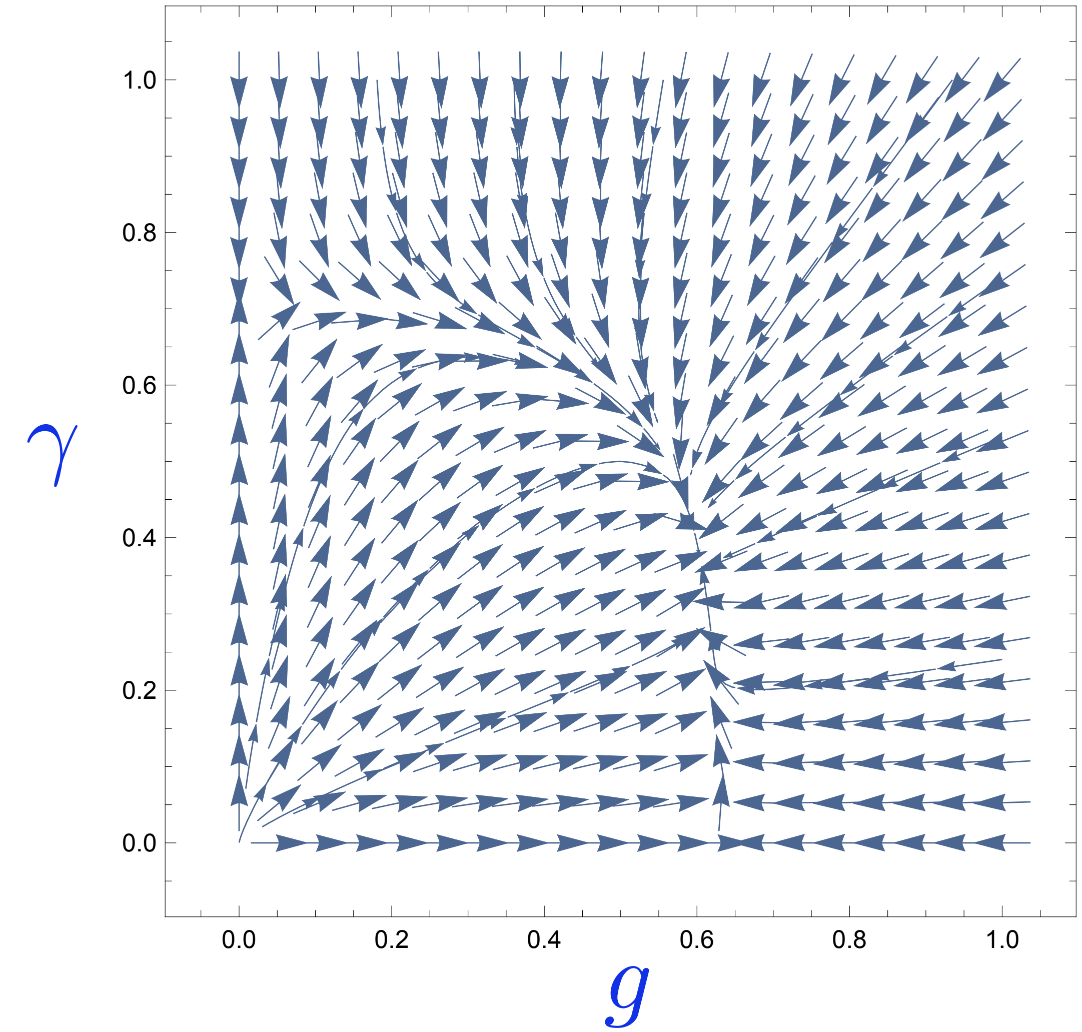
t-J model RG

We can perform a RG analysis for small $\epsilon = 3-d$ and $\bar{r} = (1-r)/2$, while imposing the local constraint *exactly*. The one-loop β functions are

$$\beta(g) = -\bar{r}g + \frac{3}{2}g^3 + \frac{3}{8}g\gamma^2\,,$$

$$\beta(\gamma) = -\frac{\epsilon}{2}\gamma + \gamma^3 + g^2\gamma\,.$$

$$\beta(s) = -s + 3g^2 s - g^2 + \frac{3}{4}\gamma^2\,.$$



These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of s to $\pm\infty$.

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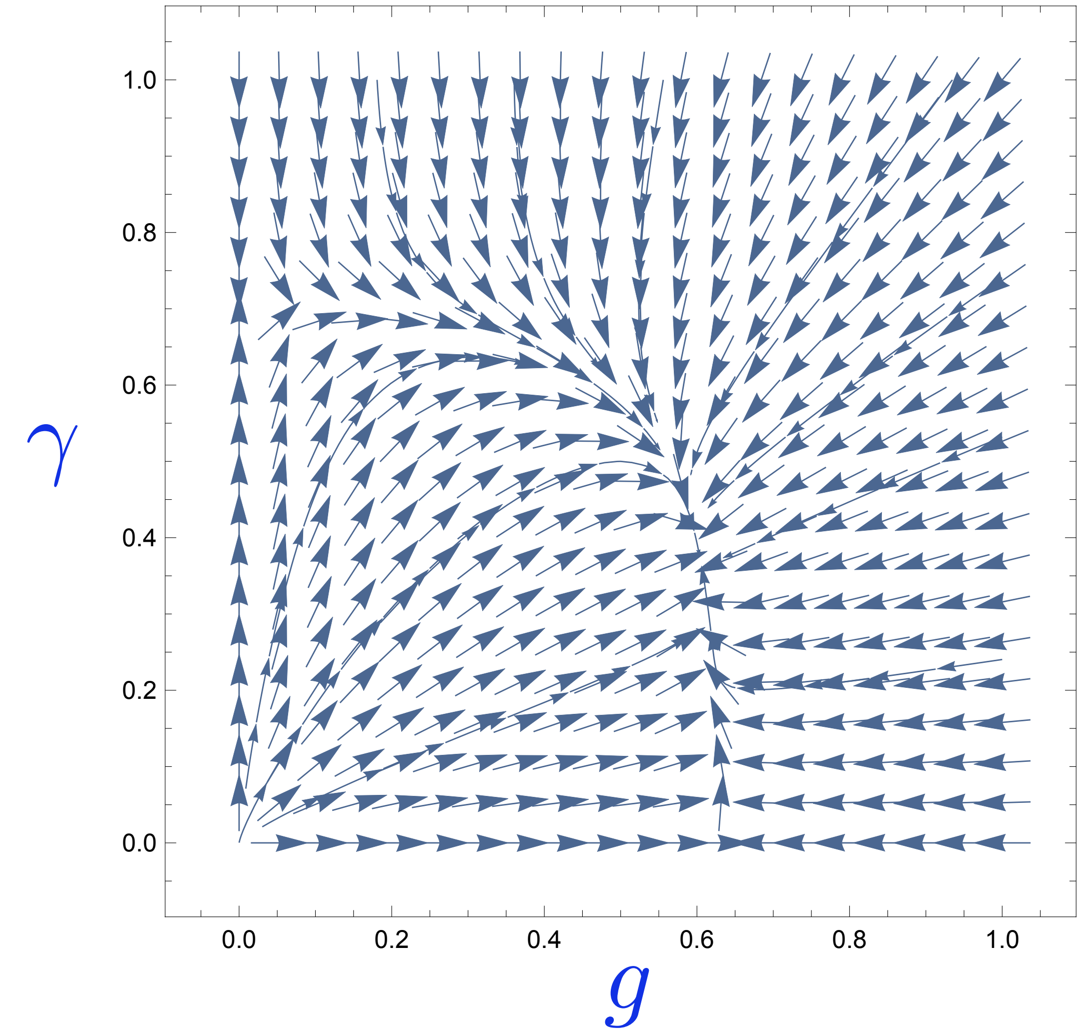

These equations have a fixed point with $s \approx 0$ with only one relevant direction, corresponding to the flow of s to $\pm\infty$. The 3 states of the superspin are nearly degenerate at the fixed point, and the flows away from the fixed point correspond to different orientations of the field on the superspin: one side (overdoped) favors the holon, and the other side (underdoped) favors the spinon.

t-J model RG

The scaling dimensions of the electron and spin operators can be determined to all orders in ϵ and $\bar{r}$ and these imply

$$\overline{R}(\tau) = -\frac{1}{2}\left\langle c_\alpha(\tau)c_\alpha^\dagger(0)\right\rangle \sim \frac{\text{sgn}(\tau)}{|\tau|^{1-r}} \quad , \quad \overline{Q}(\tau) = \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|^{3-d}}\,.$$

t-J model RG

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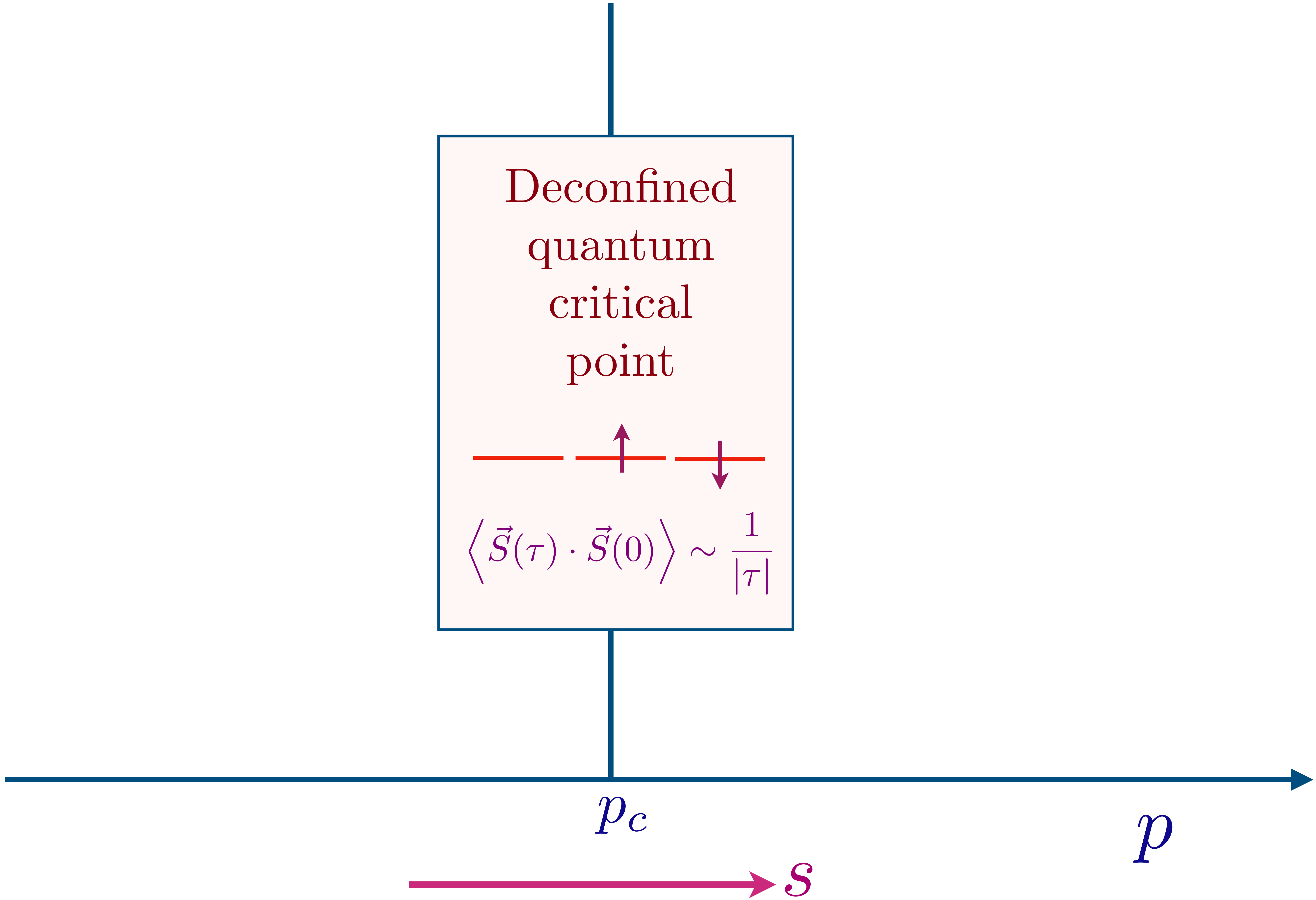
Finally, we impose the self-consistency conditions $R(\tau) = \overline{R}(\tau)$, $Q(\tau) = \overline{Q}(\tau)$ and obtain $r = 0$ ($\bar{r} = 1/2$) and $d = 2$ ($\epsilon = 1$), so that at the critical point we have

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0)\right\rangle \sim \frac{\text{sgn}(\tau)}{|\tau|} \quad , \quad \left\langle \vec{S}(\tau)\cdot\vec{S}(0)\right\rangle \sim \frac{1}{|\tau|}\,.$$

t-J model phase diagram



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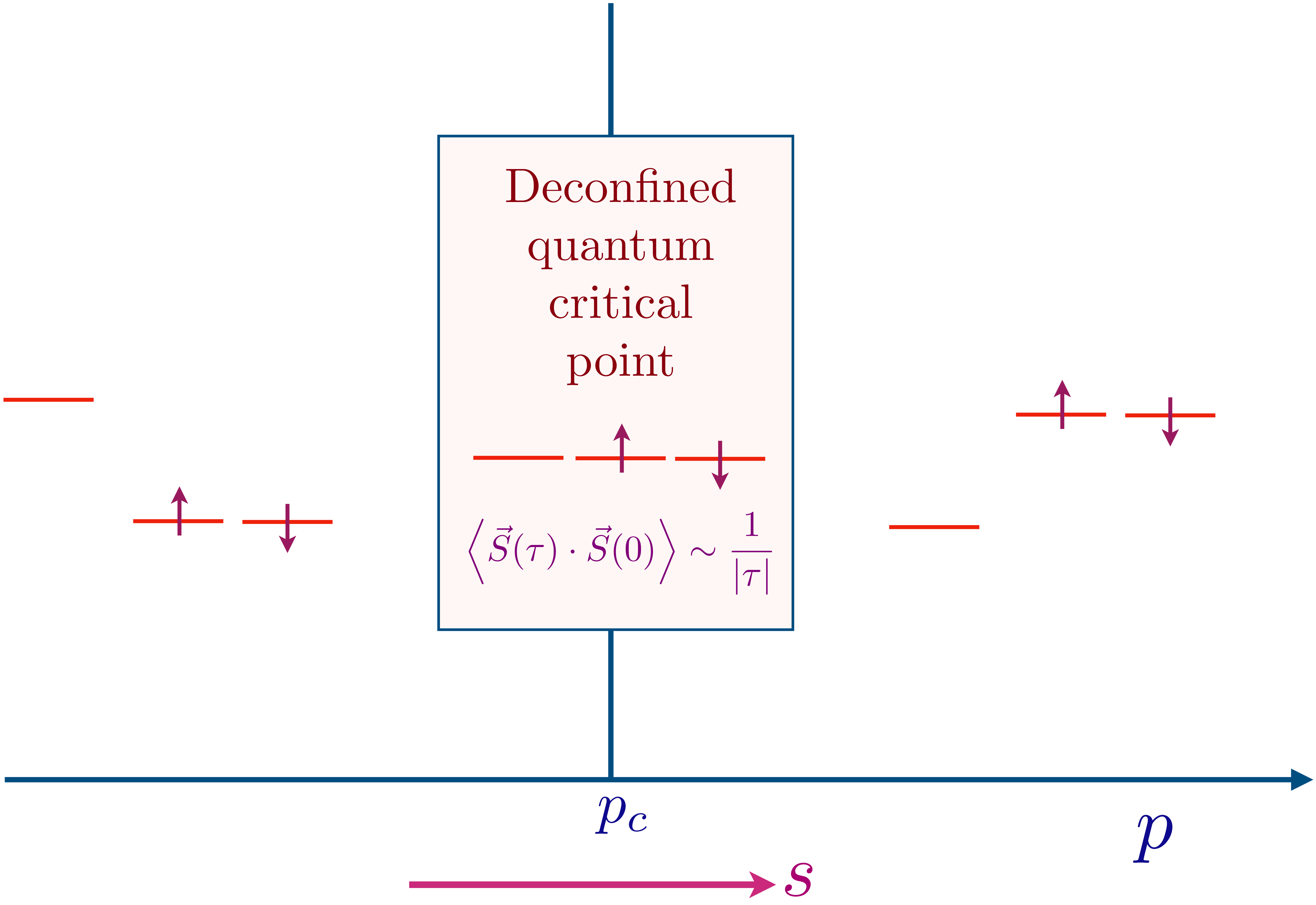

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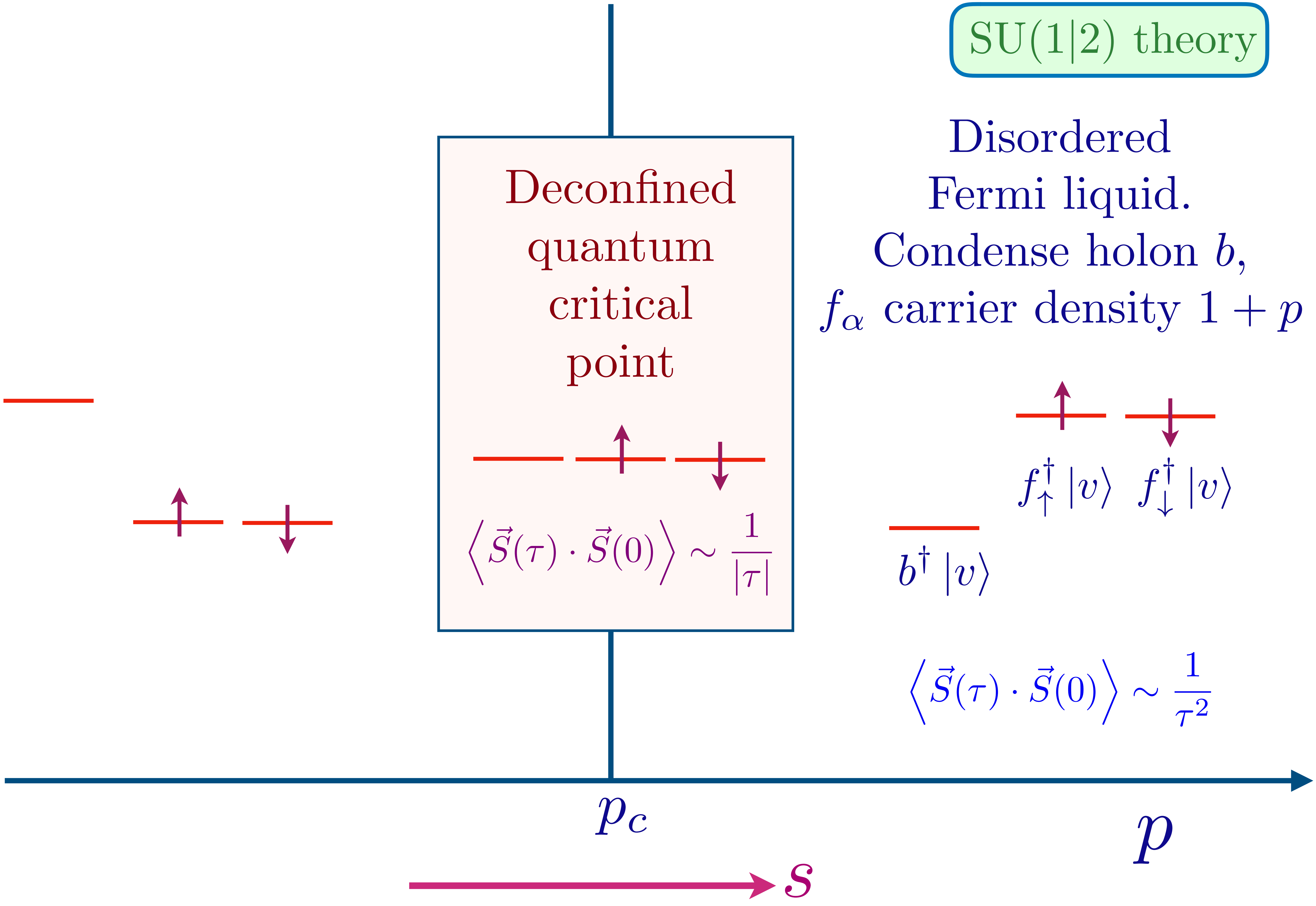

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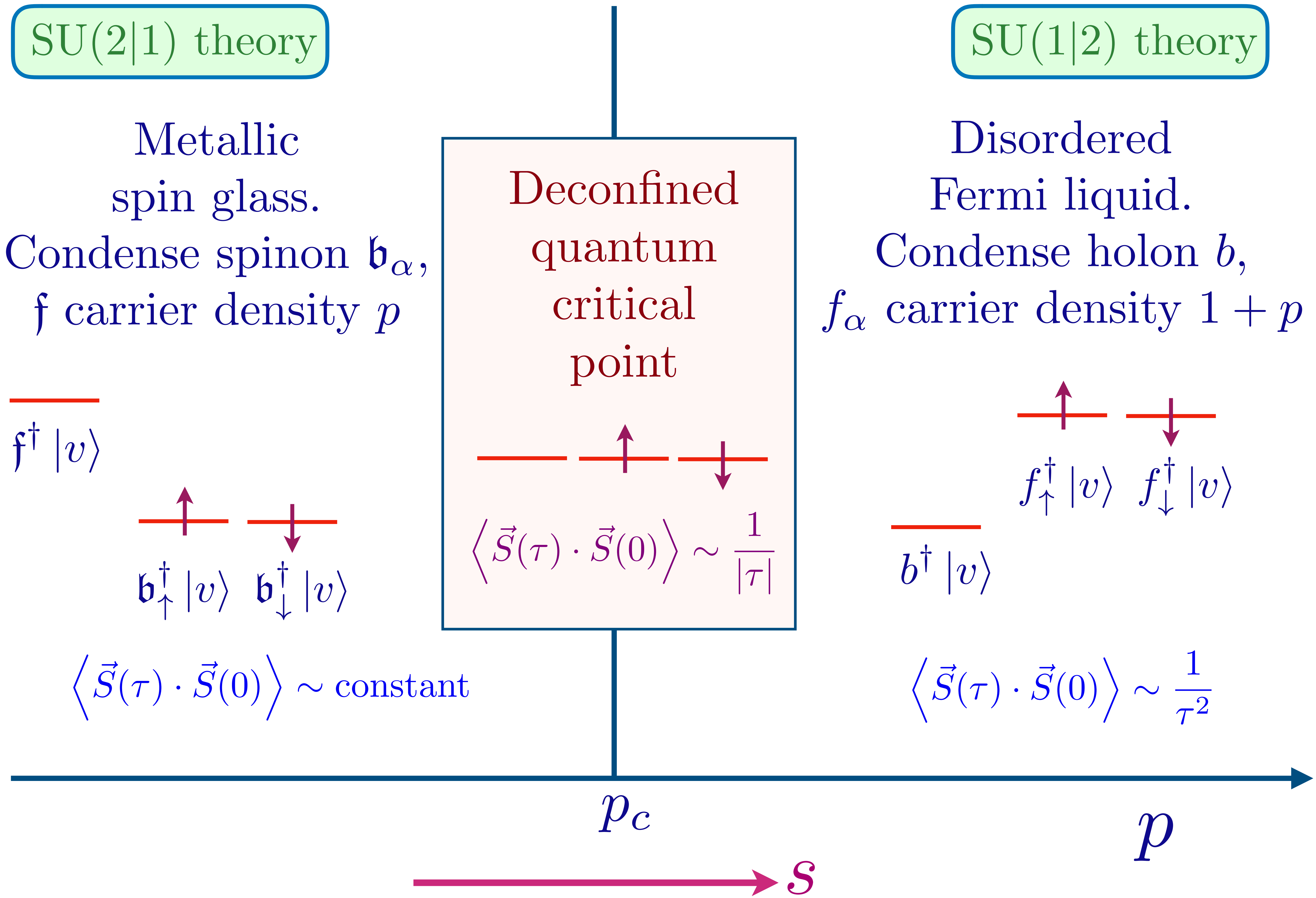

t-J model large M

Each site has 3 states which we map to the space of a boson b (the holon) and a fermion f_α (the spinon):

$$|0\rangle \Rightarrow b^\dagger |v\rangle \qquad , \qquad c_\alpha^\dagger |0\rangle \Rightarrow f_\alpha^\dagger |v\rangle$$
$$c_\alpha = f_\alpha b^\dagger \qquad , \qquad f_\alpha^\dagger f_\alpha + b^\dagger b = 1$$

To obtain a large M limit, let $\alpha = 1 \ldots M$, endow the boson with an 'orbital' index $a = 1 \ldots M'$ and send $M \to \infty$ at fixed $k = M'/M$. Then

$$c_{a\alpha} = f_\alpha b_a^\dagger \qquad , \qquad f_\alpha^\dagger f_\alpha + b_a^\dagger b_a = \frac{M}{2}$$

t-J model large M

Assuming the bosons are not condensed, we obtain SYK-like equations for the boson and fermion Green's functions:

$$
\begin{aligned}
G_b(i\omega_n) &= \frac{1}{i\omega_n + \mu_b - \Sigma_b(i\omega_n)} \\
\Sigma_b(\tau) &= -t^2 G_f(\tau) G_f(-\tau) G_b(\tau) \\
G_f(i\omega_n) &= \frac{1}{i\omega_n + \mu_f - \Sigma_f(i\omega_n)} \\
\Sigma_f(\tau) &= -J^2 G_f^2(\tau) G_f(-\tau) + k\, t^2 G_f(\tau) G_b(\tau) G_b(-\tau)
\end{aligned}
$$

Here μ_f and μ_b are chemical potentials chosen to satisfy

$$
\left\langle f^\dagger f \right\rangle = \frac{1}{2} - kp \quad , \quad \left\langle b^\dagger b \right\rangle = p\,.
$$

t-J model large M

These equations have critical solutions with

$$G_f(z) = C_f \frac{e^{-i(\pi\Delta_f + \theta_f)}}{z^{1-2\Delta_f}}, \quad G_b(z) = C_b \frac{e^{-i(\pi\Delta_b + \theta_b)}}{z^{1-2\Delta_b}}, \quad \mathrm{Im}(z) > 0$$

$$\Delta_f + \Delta_b = \frac{1}{2}$$

$$\frac{\theta_f}{\pi} + \left(\frac{1}{2} - \Delta_f\right) \frac{\sin(2\theta_f)}{\sin(2\pi\Delta_f)} = kp$$

$$\frac{\theta_b}{\pi} + \left(\frac{1}{2} - \Delta_b\right) \frac{\sin(2\theta_b)}{\sin(2\pi\Delta_b)} = \frac{1}{2} + p\,.$$

The last two are analogs of 'Luttinger' relations, which follow from an anomaly matching argument (Yingei Gu, A. Kitaev, S. Sachdev, G. Tarnopolsky arXiv:1910.14099).

t-J model large M

The critical solution which is self-consistent in both the t and J terms has $\Delta_b = \Delta_f = 1/2$, implying

$$\left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \begin{cases} \dfrac{A_+}{|\tau|} & , \quad \tau > 0 \\ -\dfrac{A_-}{|\tau|} & , \quad \tau < 0 \end{cases} \quad , \quad \left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \, .$$

The same exponents are obtained to all orders in the ϵ, $\overline{r}$ expansion, but with $A_+ = A_-$.

1. Insulating random magnet

2. Deconfined criticality at non-zero doping

3. Phase diagram of disordered Hubbard model

Random *t-J-U* model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha},$$

$$t_{ij}, J_{ij} \ \text{random}, \quad U > 0$$

Large N limit

$$\begin{aligned}\mathcal{Z} &= \int \mathcal{D}c_\alpha(\tau) e^{-\mathcal{S}} \\ \mathcal{S} &= \int d\tau \left[c_\alpha^\dagger(\tau) \left(\frac{\partial}{\partial \tau} - \mu \right) c_\alpha(\tau) + U n_\uparrow(\tau) n_\downarrow(\tau) \right] \\ & \quad - t^2 \int d\tau d\tau' R^*(\tau - \tau') c_\alpha^\dagger(\tau) c_\alpha(\tau') \\ & \quad - \frac{J^2}{2} \int d\tau d\tau' Q(\tau - \tau') \vec{S}(\tau) \cdot \vec{S}(\tau') \,.\end{aligned}$$

From this action we determine the correlators

$$\begin{aligned}\overline{R}(\tau - \tau') &= -\frac{1}{2} \left\langle c_\alpha^\dagger(\tau) c_\alpha(\tau') \right\rangle_{\mathcal{Z}} \\ \overline{Q}(\tau - \tau') &= \frac{1}{3} \left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}\end{aligned}$$

and finally impose the self-consistency conditions

$$R(\tau) = \overline{R}(\tau) \quad , \quad Q(\tau) = \overline{Q}(\tau).$$

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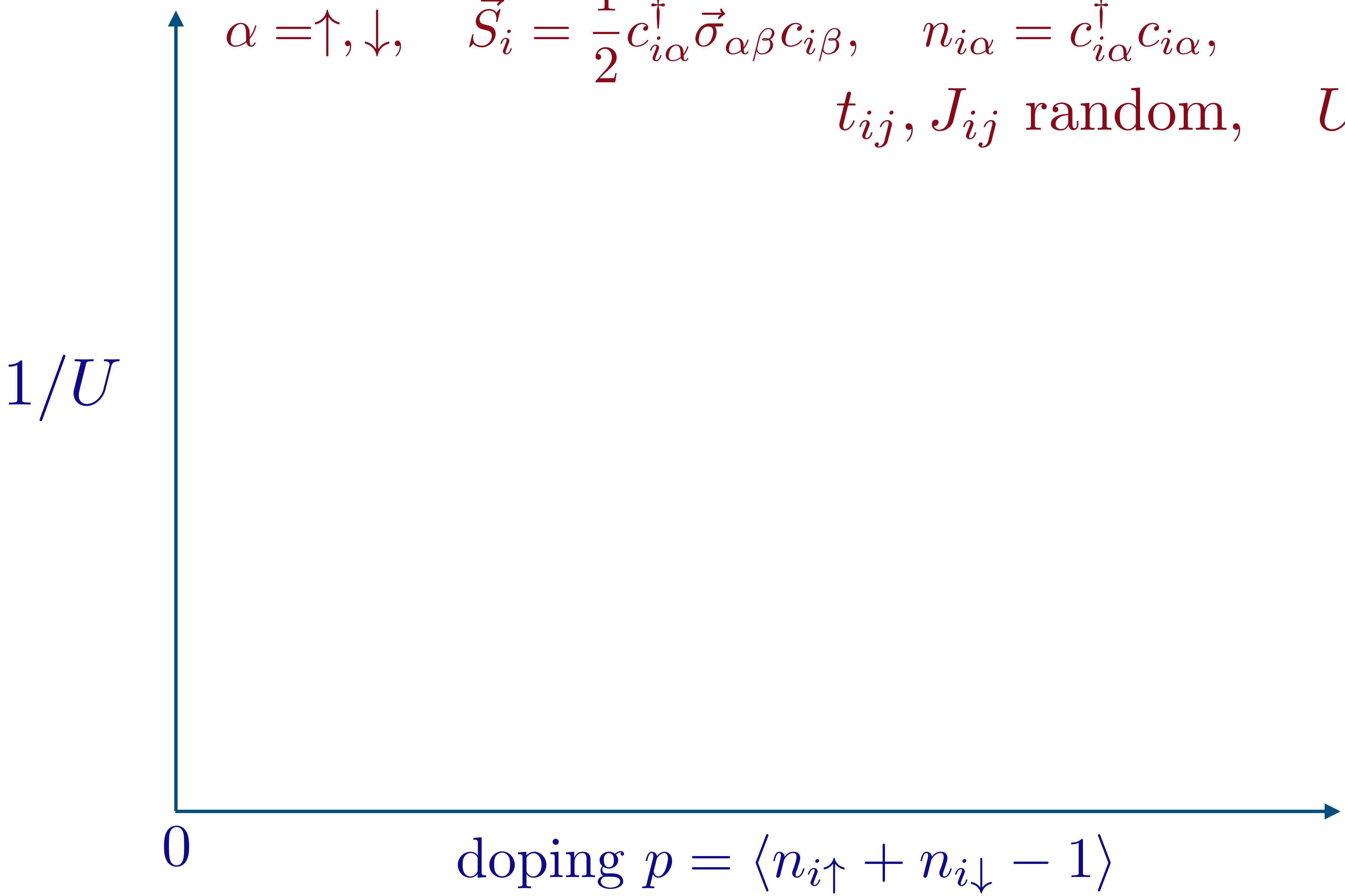

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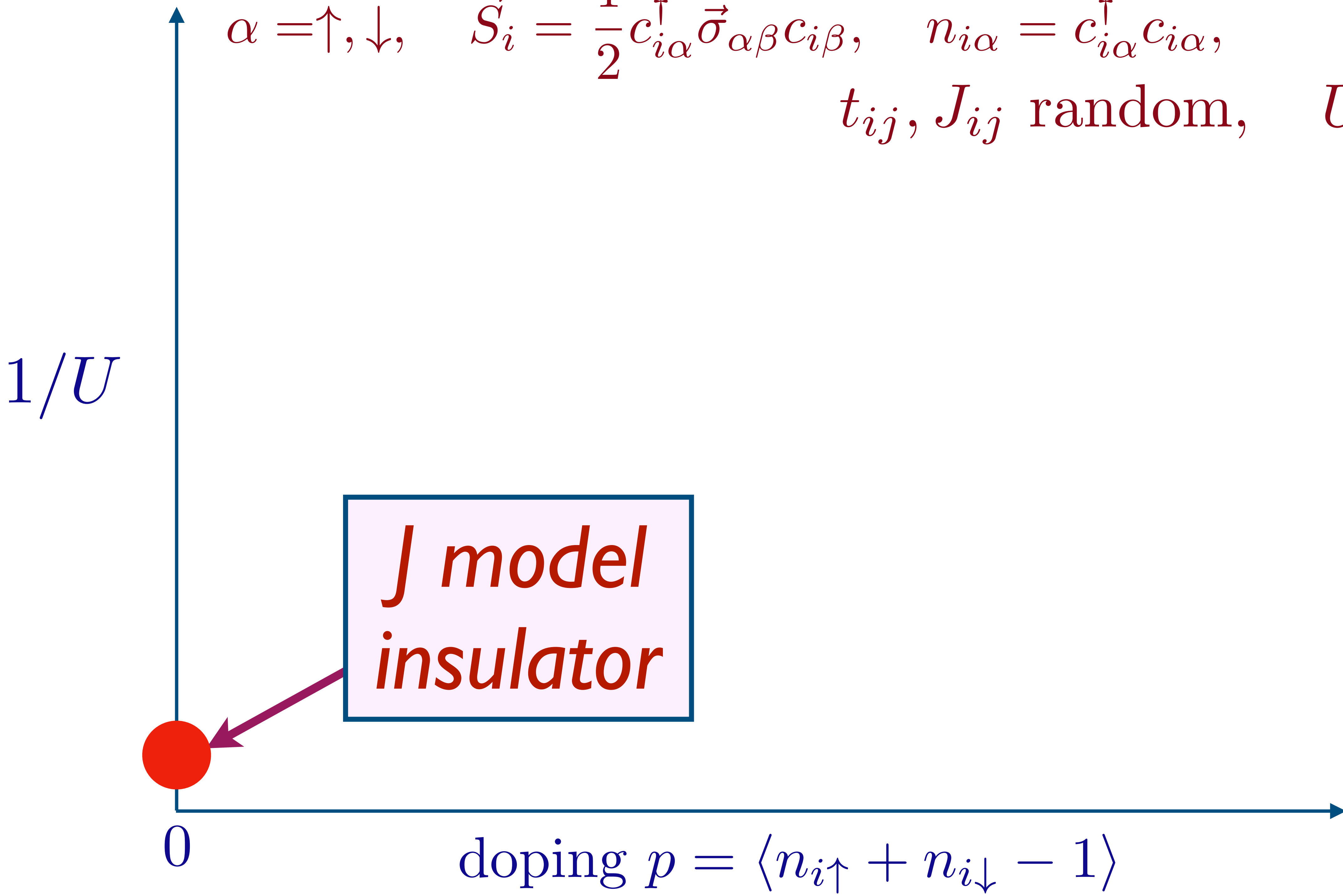

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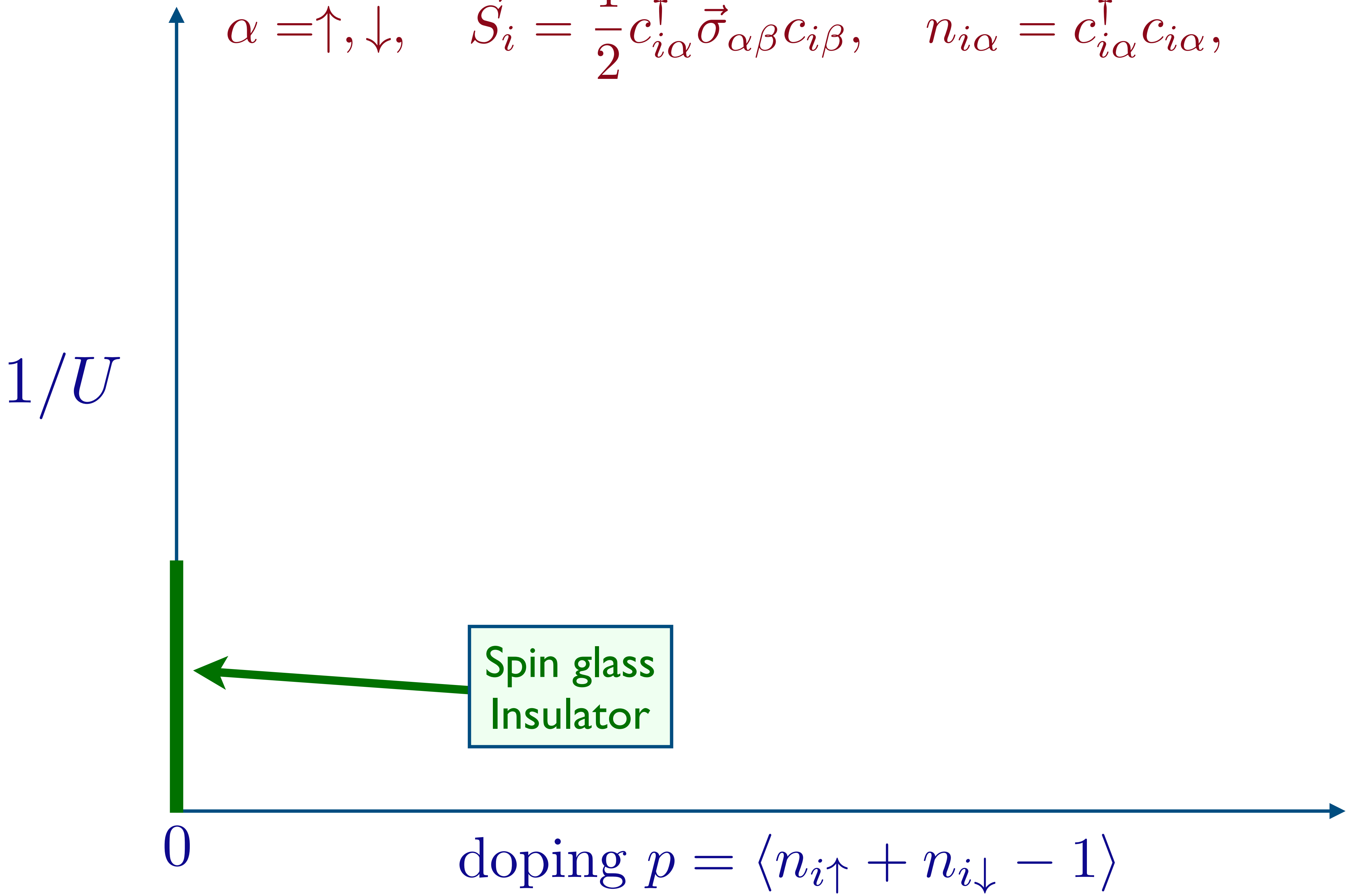

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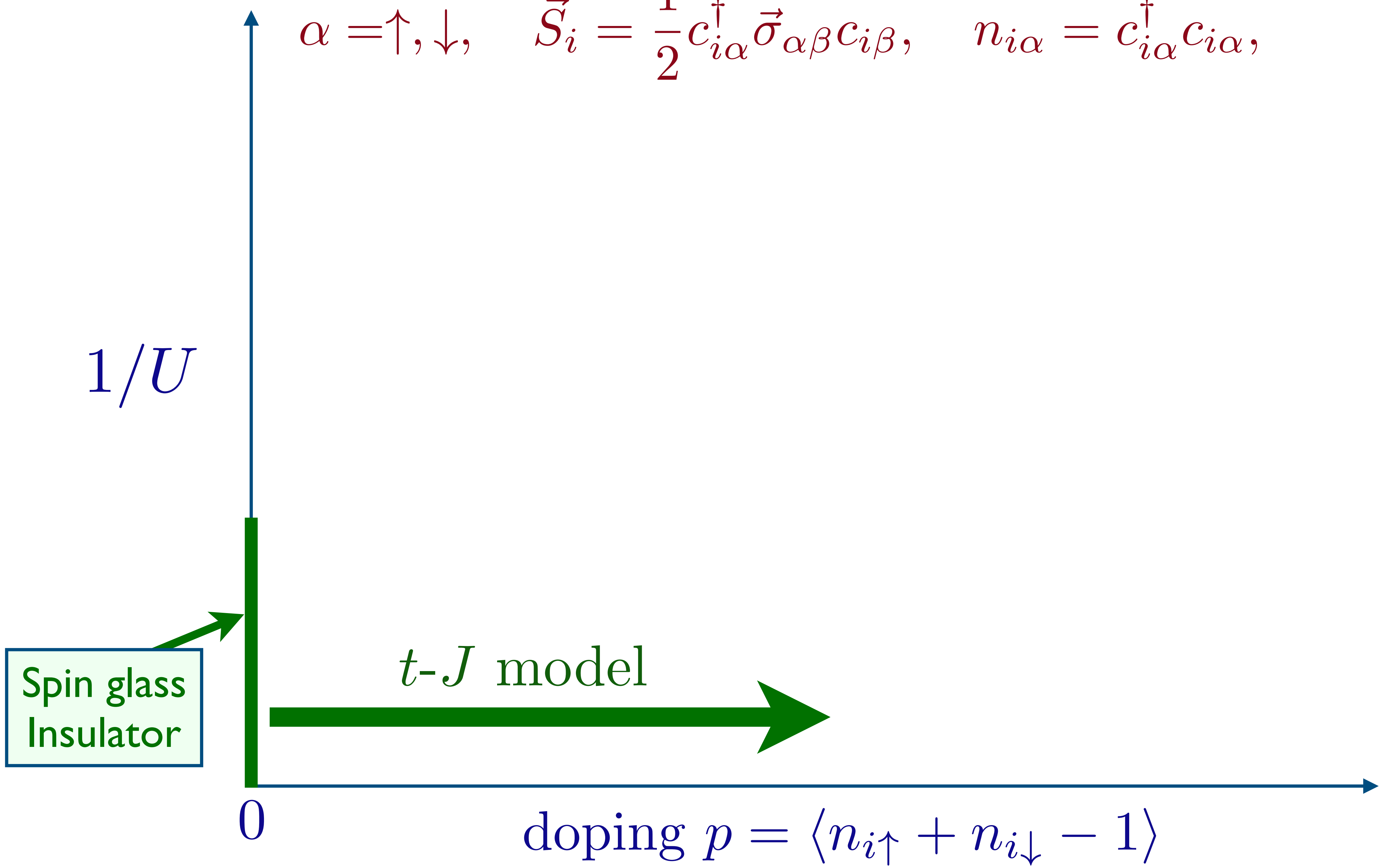

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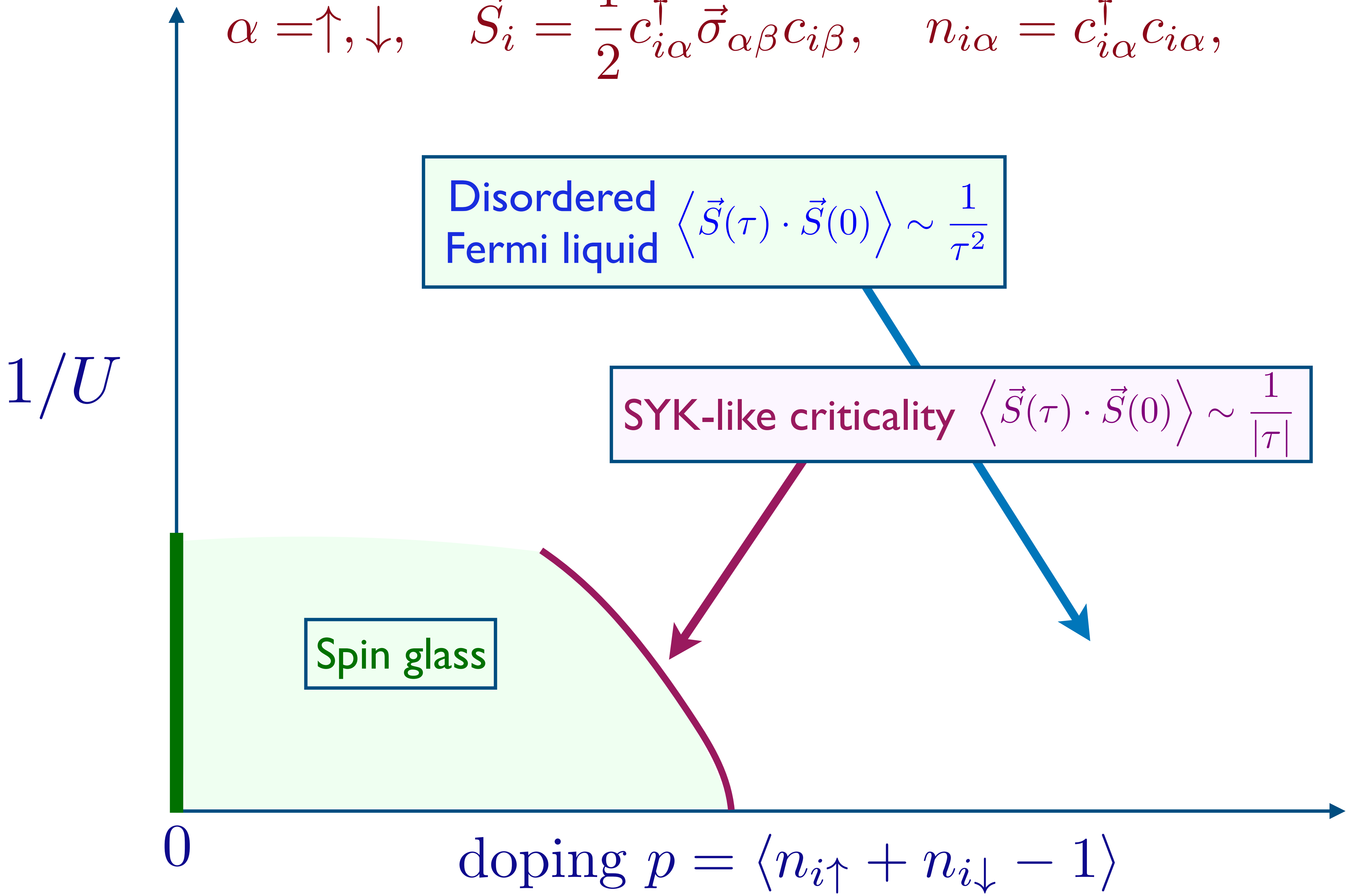
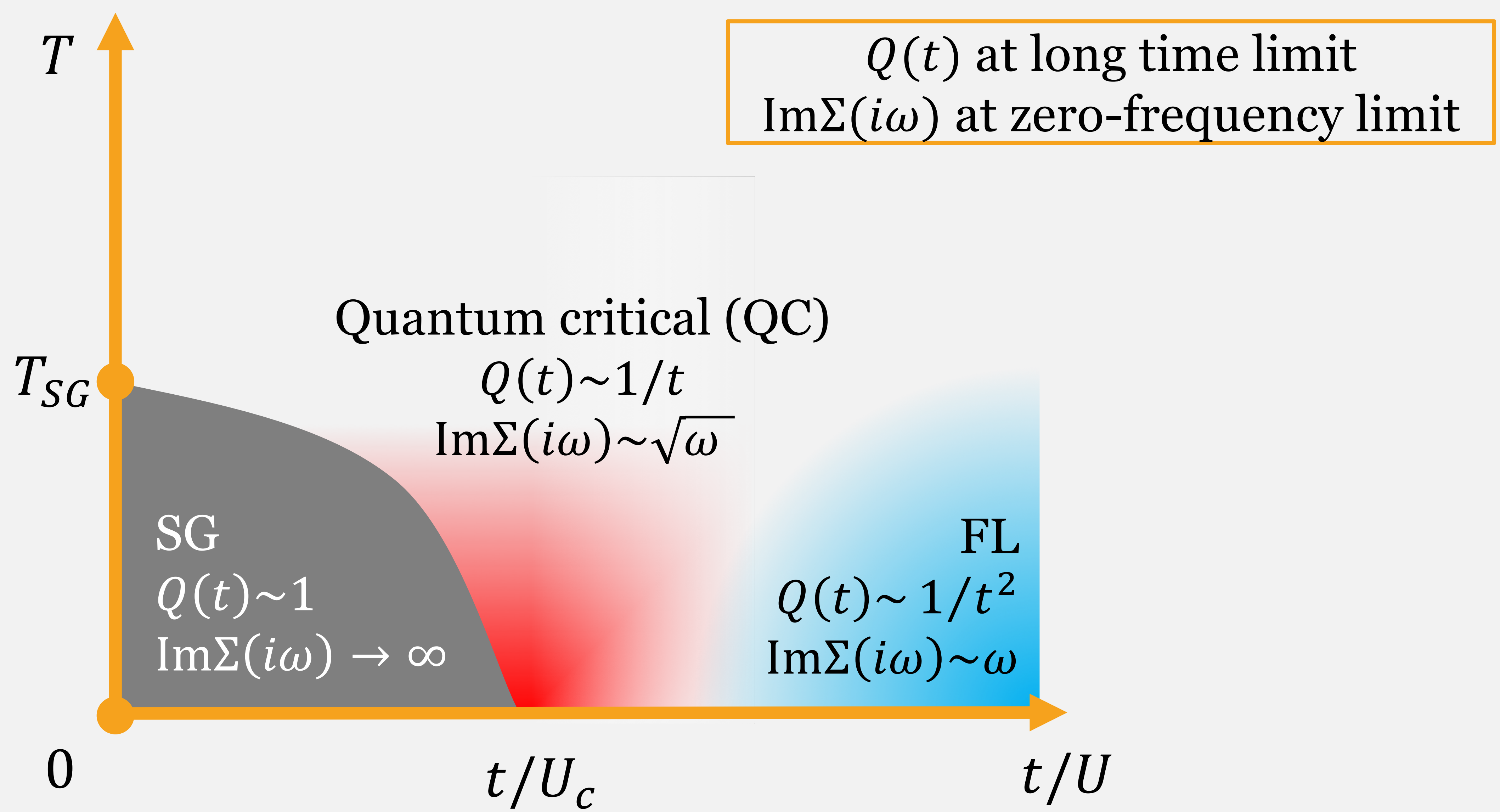


Quantum Critical Point between a FL Metal and a Spin Glass Mott Insulator



Peter Cha, Nils Wentzell, Olivier Parcollet, Antoine Georges, Eun-ah Kim,
APS March meeting 2019

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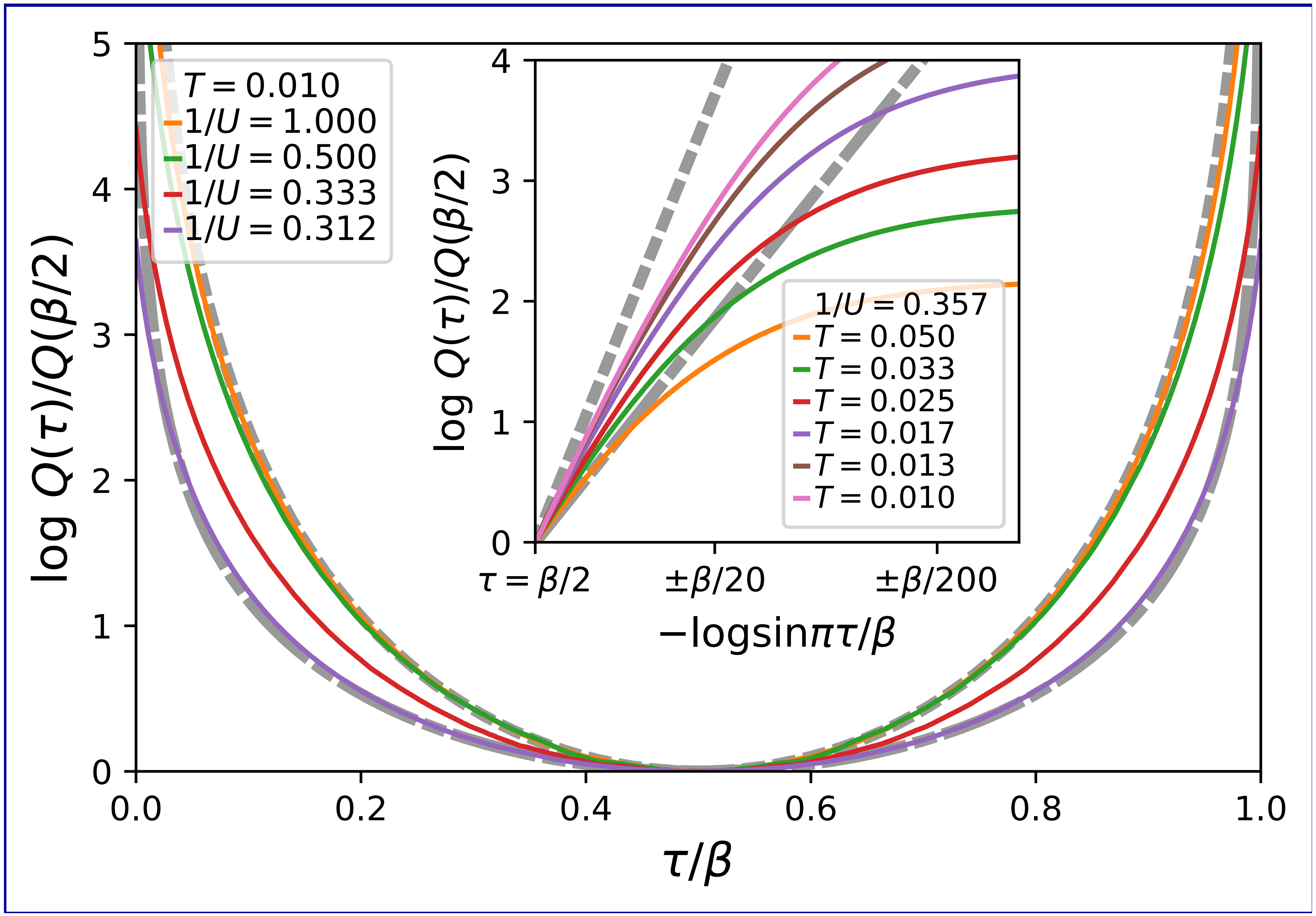

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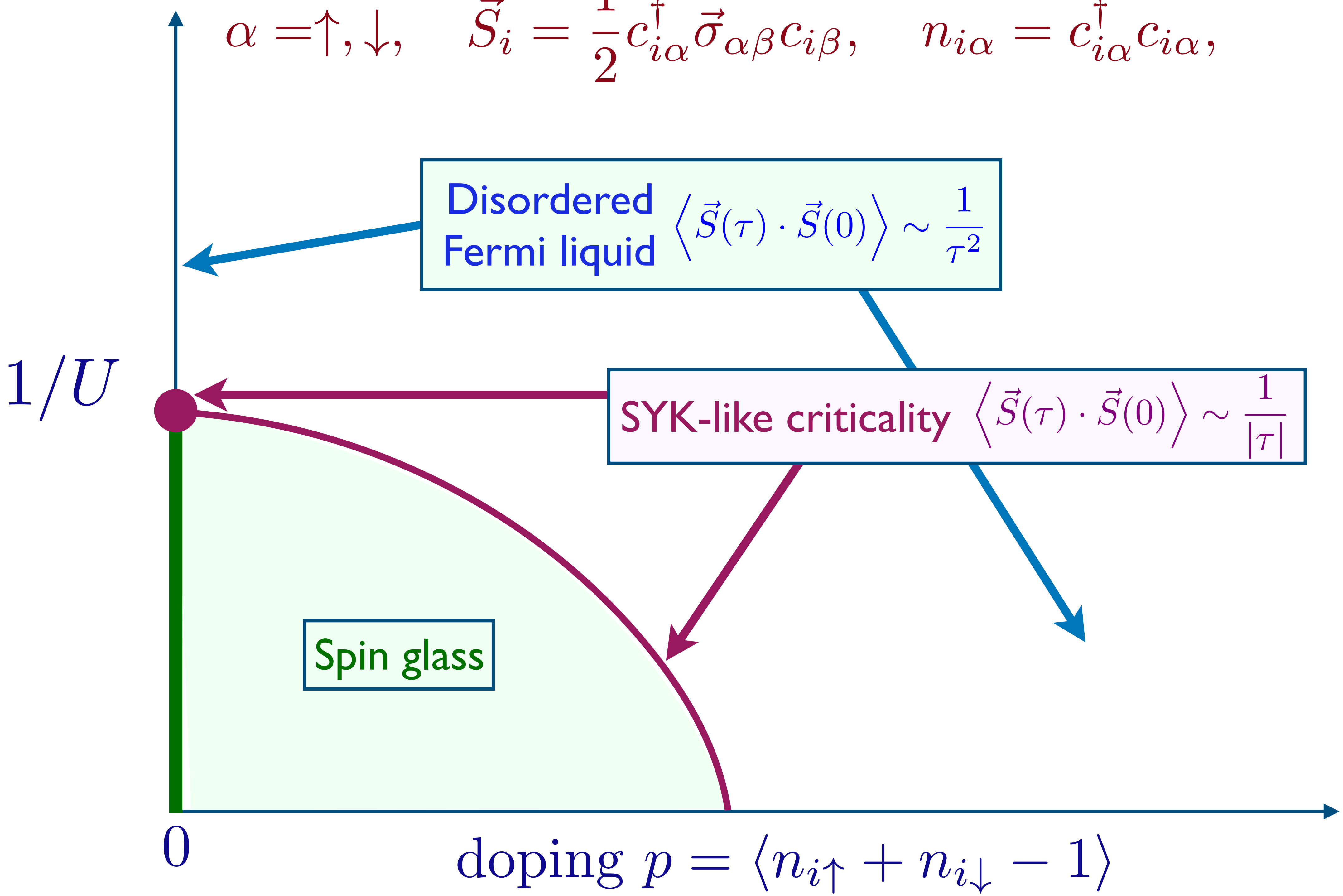

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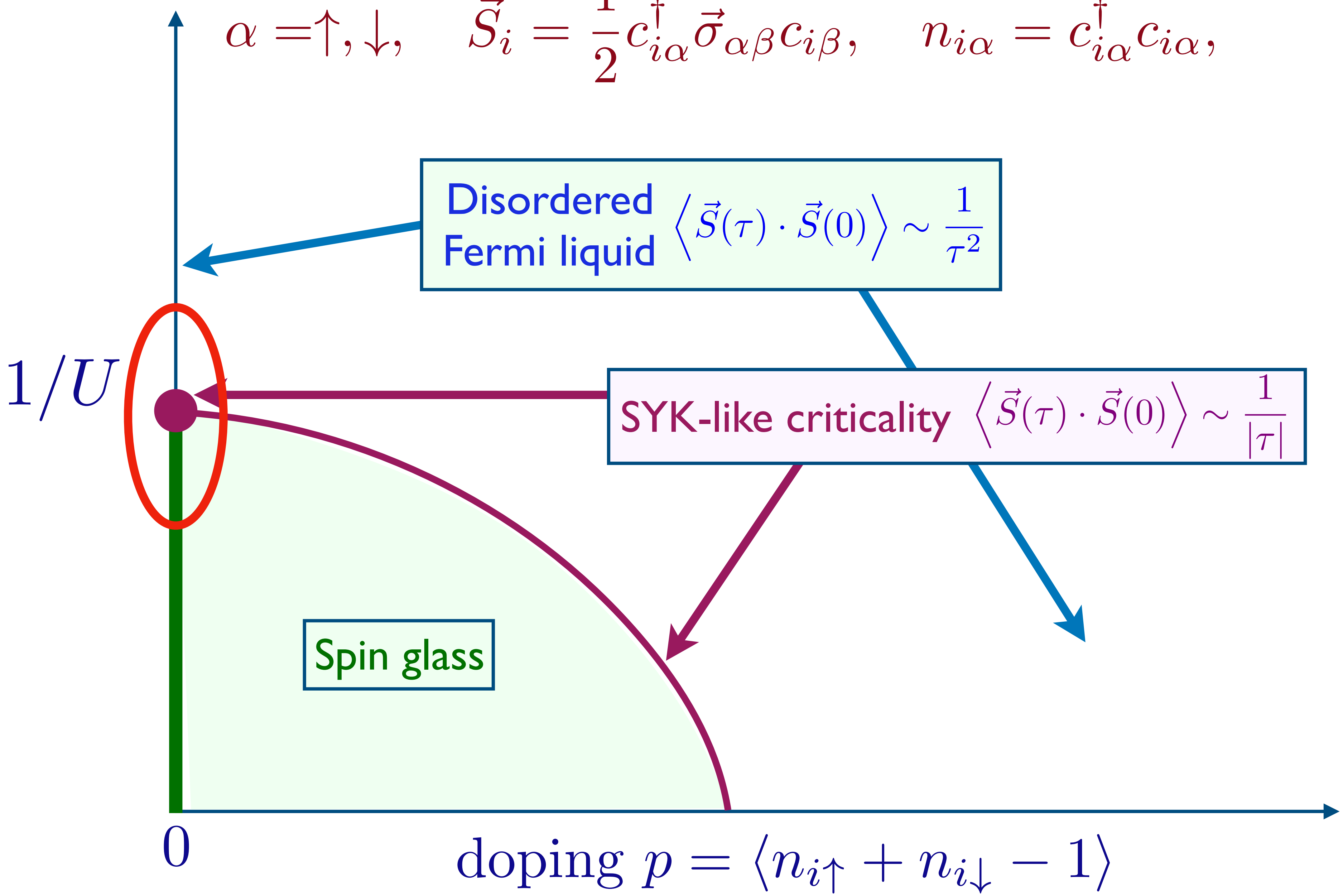

Half-filled *t-J-U* model RG

Let us focus on the critical point at half-filling, and assume a power-law decay

$$Q(\tau) \sim \frac{1}{|\tau|^{d-1}} \quad , \quad R(\tau) \sim \frac{\text{sgn}(\tau)}{|\tau|^{r+1}} \, .$$

We ignore the self-consistency condition for now. We also decouple the last two terms by introducing bosonic (ϕ_a, $a = 1 \ldots 3$) and fermionic (ψ_α) baths. Then the problem reduces to the Hamiltonian

$$\begin{aligned} H_{\text{imp}} &= \frac{U}{2} \left(c_\alpha^\dagger c_\alpha - 1 \right)^2 + V_0 c_\alpha^\dagger \psi_\alpha(0) + \text{H.c.} + \zeta [\phi_\alpha(0)]^2 \\ &+ \gamma_0 c_\alpha^\dagger \frac{\sigma^a_{\alpha\beta}}{2} c_\beta \, \phi_a(0) + \int |k|^r dk \, k \, \psi^\dagger_{k\alpha} \psi_{k\alpha} + \frac{1}{2} \int d^d x \left[\pi_a^2 + (\partial_x \phi_a)^2 \right] \end{aligned}$$

where $a = (x, y, z)$, σ^a are Pauli matrices, π_a is canonically conjugate to the field ϕ_a, and $\phi_a(0) \equiv \phi_a(x = 0)$, $\psi_\alpha(0) \equiv \int |k|^r dk \, \psi_{k\alpha}$. We identify $Q(\tau)$ with temporal correlator of $\phi_a(0)$, and $R(\tau)$ with the temporal correlator of $\psi_\alpha(0)$, and it can be verified that these correlators decay as above.

L. Fritz and M. Vojta, PRB **70**, 214427 (2004)

J. H. Pixley, S. Kirchner, K. Ingersent, and Qimiao Si, PRB **88**, 245111 (2013)

Half-filled *t-J-U* model RG

Now a RG analysis is possible for $d \approx 2$ and $r \approx 1/2$, which is perturbative in U. The one-loop equations are

$$\beta_\gamma = \frac{(1-2r)}{2}\gamma + \frac{(d-2)}{2}\gamma + \frac{\gamma U}{\pi (i A_0)^2} + \frac{\zeta\gamma}{2\pi}\,,$$

$$\beta_U = (1-2r)U - \frac{\gamma^2}{8\pi}\,,$$

$$\beta_\zeta = (d-2)\zeta + \frac{\zeta^2}{2\pi} + \frac{\gamma^2}{2\pi A_0^2}\,.$$

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$$\beta_\zeta = (d-2)\zeta + \frac{\zeta^2}{2\pi} + \frac{\gamma^2}{2\pi A_0^2} \,.$$

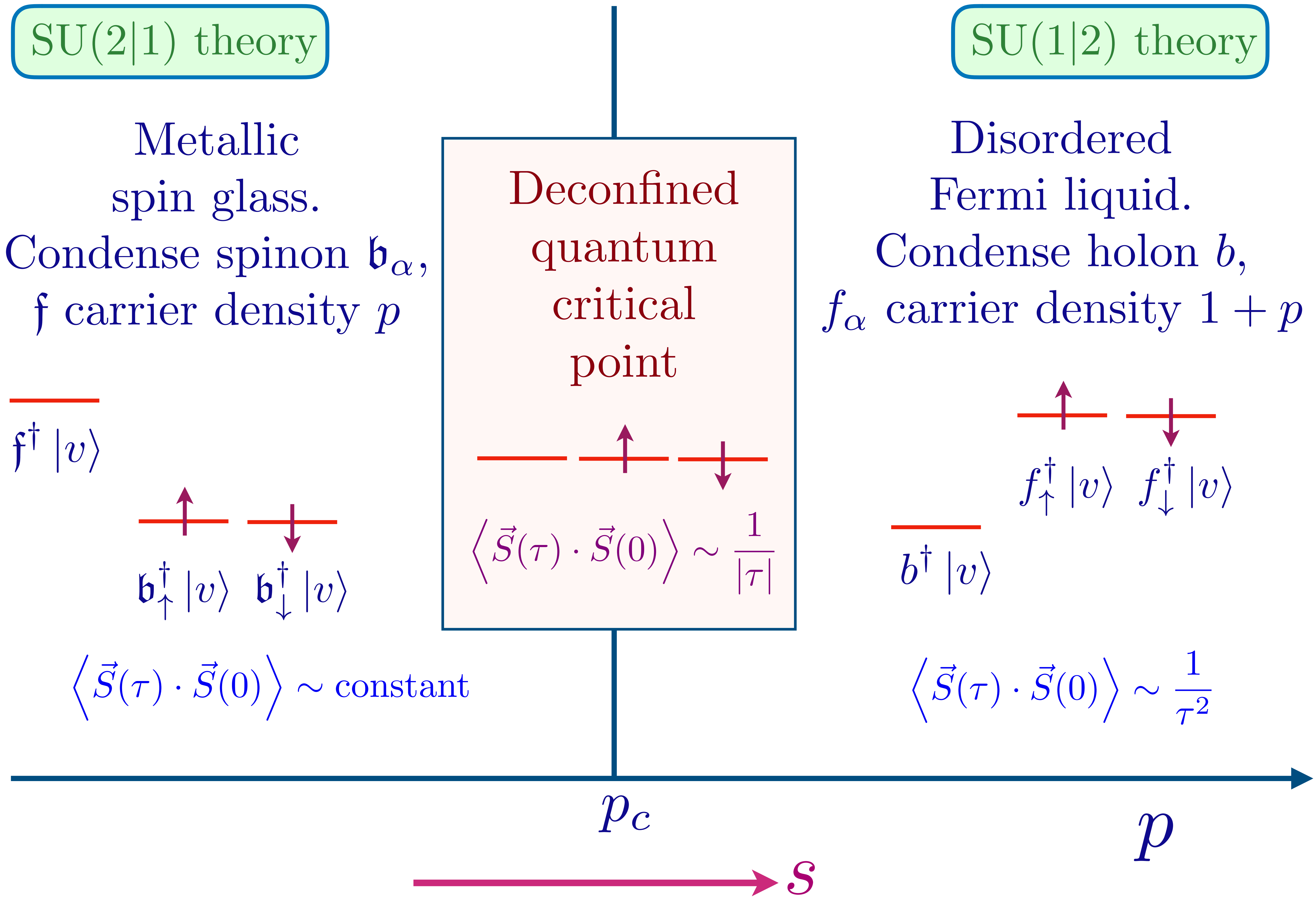
The electron operator c_α does not receive any wavefunction renormalization. This implies the correlator

$$\overline{R}(\tau) = -\frac{1}{2}\left\langle c_\alpha(\tau) c_\alpha^\dagger(0)\right\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{1-r}} \,.$$

The self-consistent value of r is therefore $r = 0$.

There is a non-trivial renormalization of the spin operator $\vec{S}$ at a fixed point with $\zeta \neq 0$. Determining the self-consistent value of d requires computation ...

t-J model phase diagram



Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, arXiv:1807.0507