Quantum matter and gauge-gravity duality

University of Cincinnati
March 31, 2012

Subir Sachdev

PHYSICS
VE RI TAS
HARVARD

1. Strange metals

A. Field theory

B. Holography

2. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

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A. Field theory

B. Holography

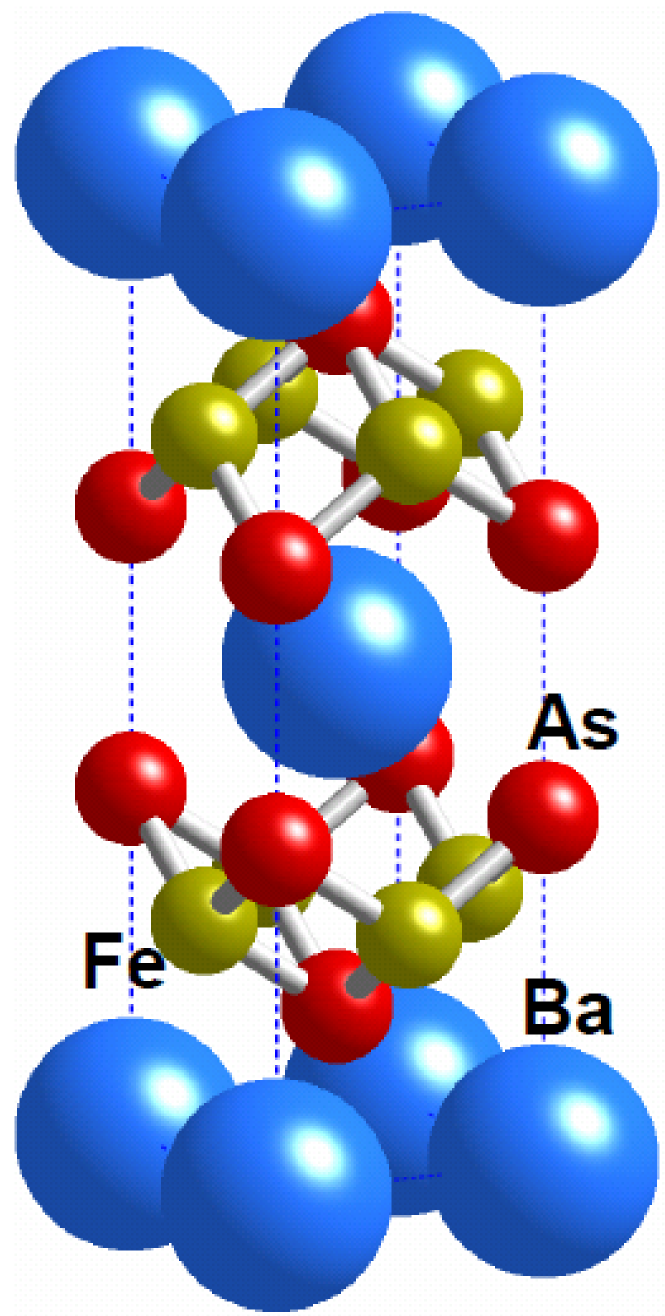
Liza Huijse

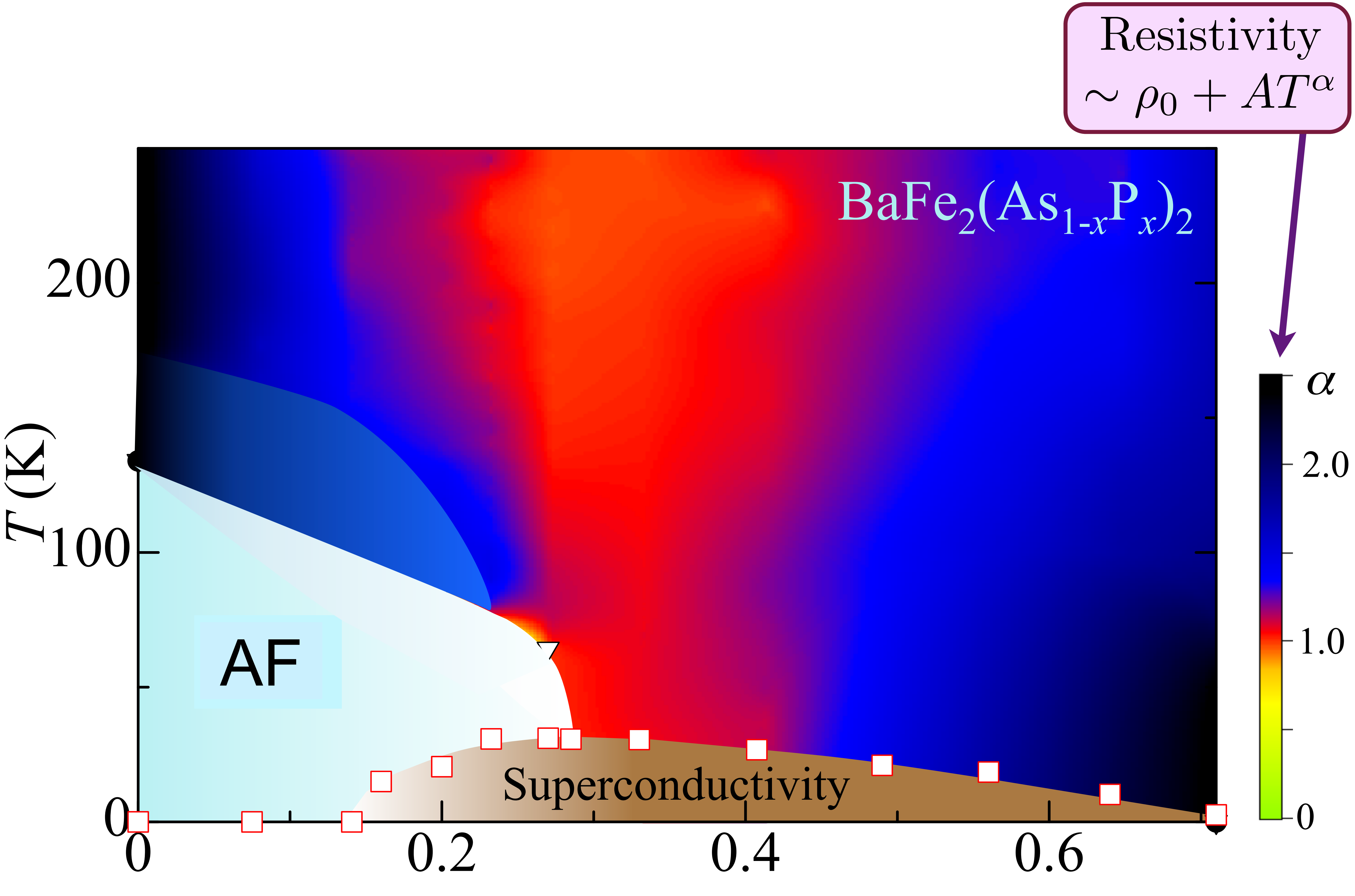
Max Metlitski

Brian Swingle

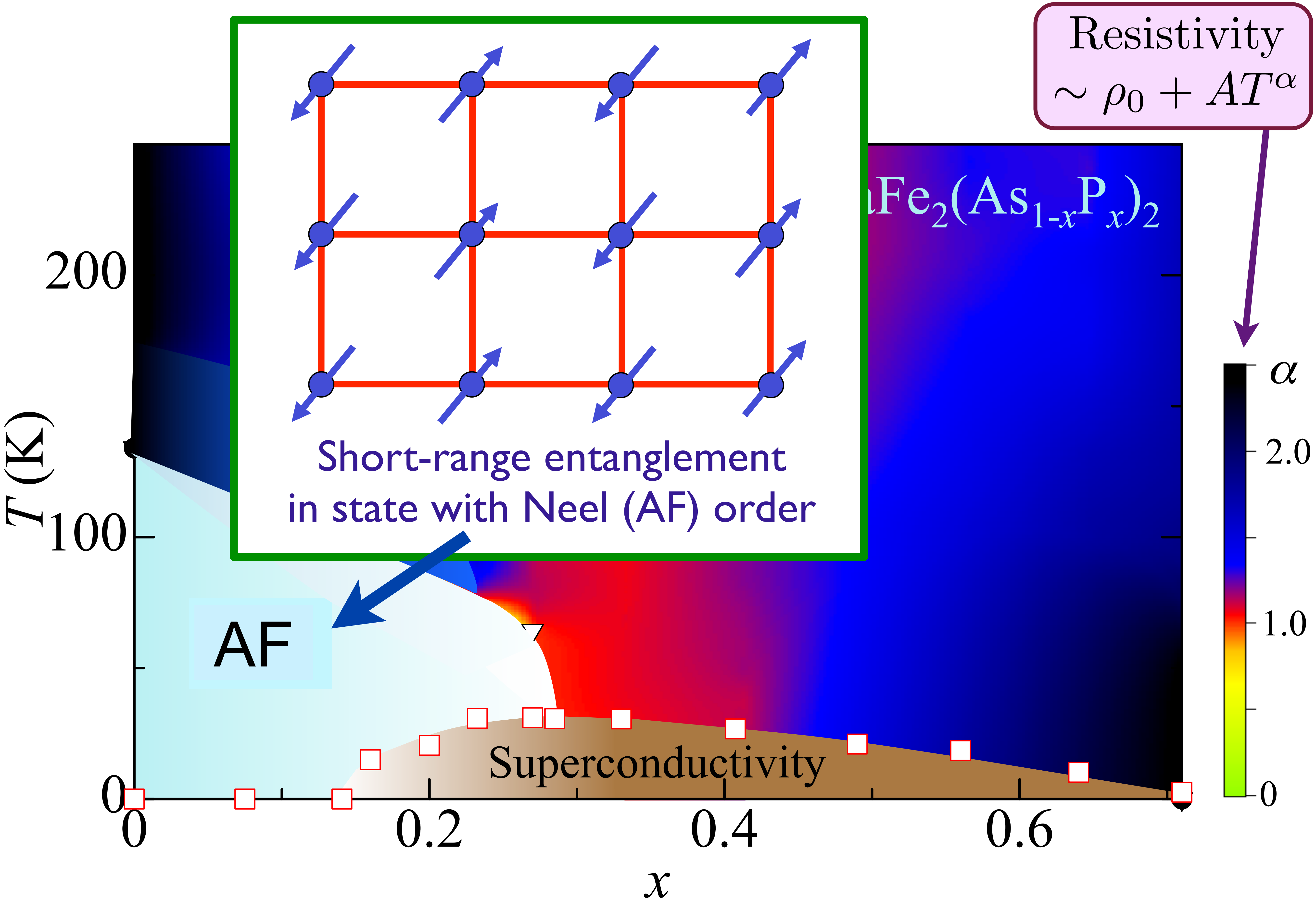
Iron pnictides:

a new class of high temperature superconductors





S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



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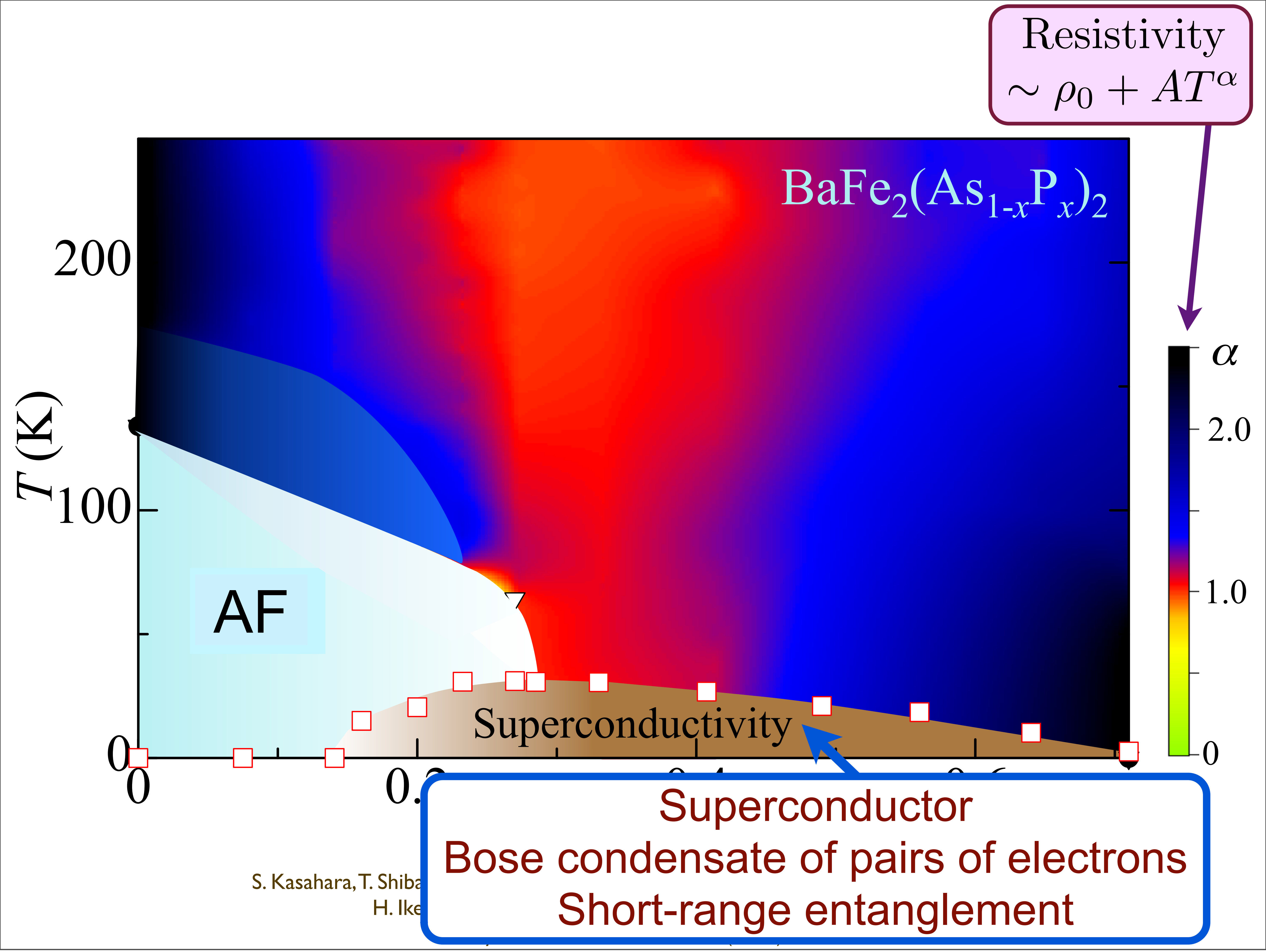
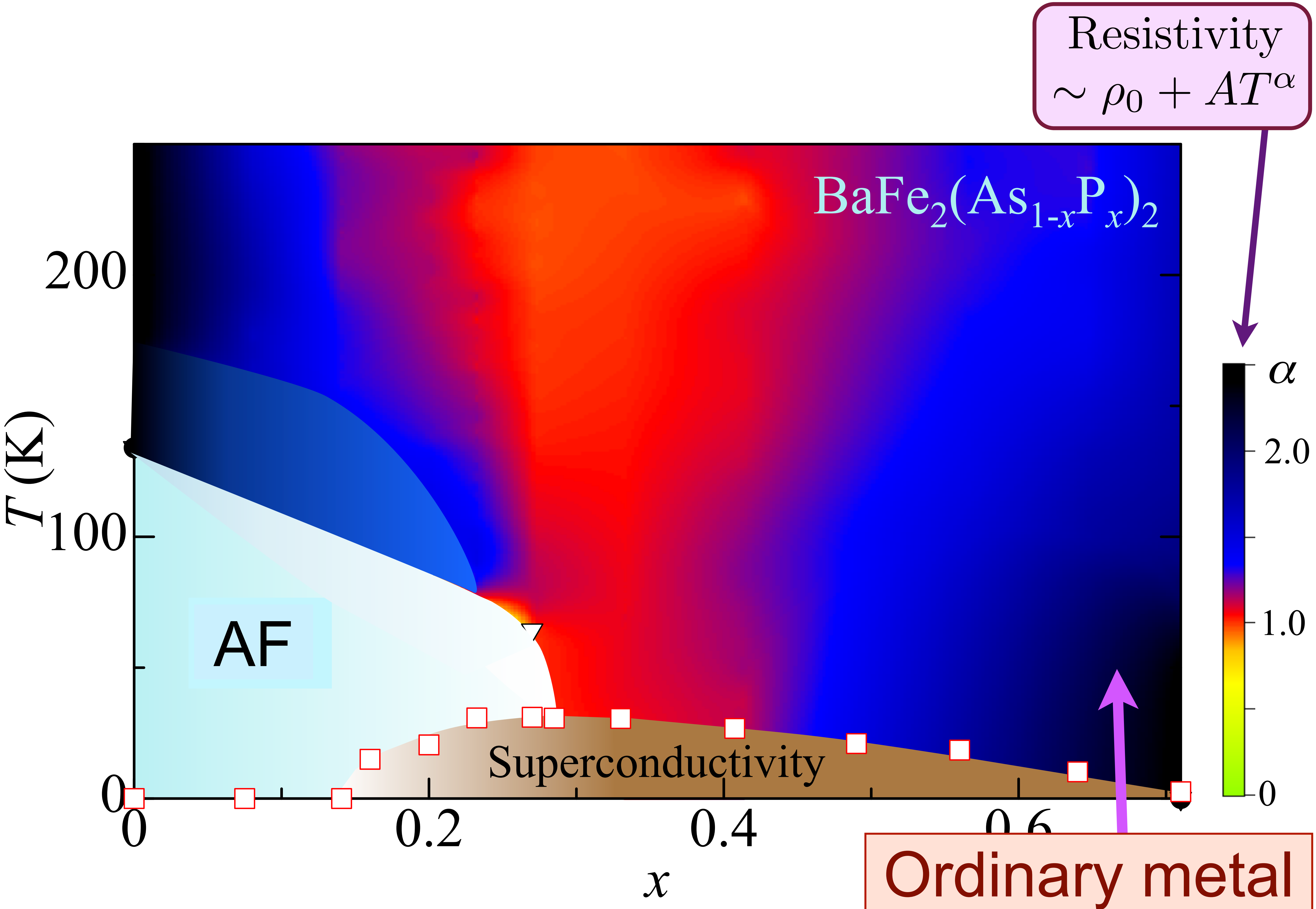

Resistivity
$\sim \rho_0 + AT^\alpha$
$BaFe_2(As_{1-x}P_x)_2$
200
100
0
T (K)
α
2.0
1.0
0
AF
Superconductivity
0
0.2
Superconductor
Bose condensate of pairs of electrons
Short-range entanglement
S. Kasahara, T. Shiba
H. Ike



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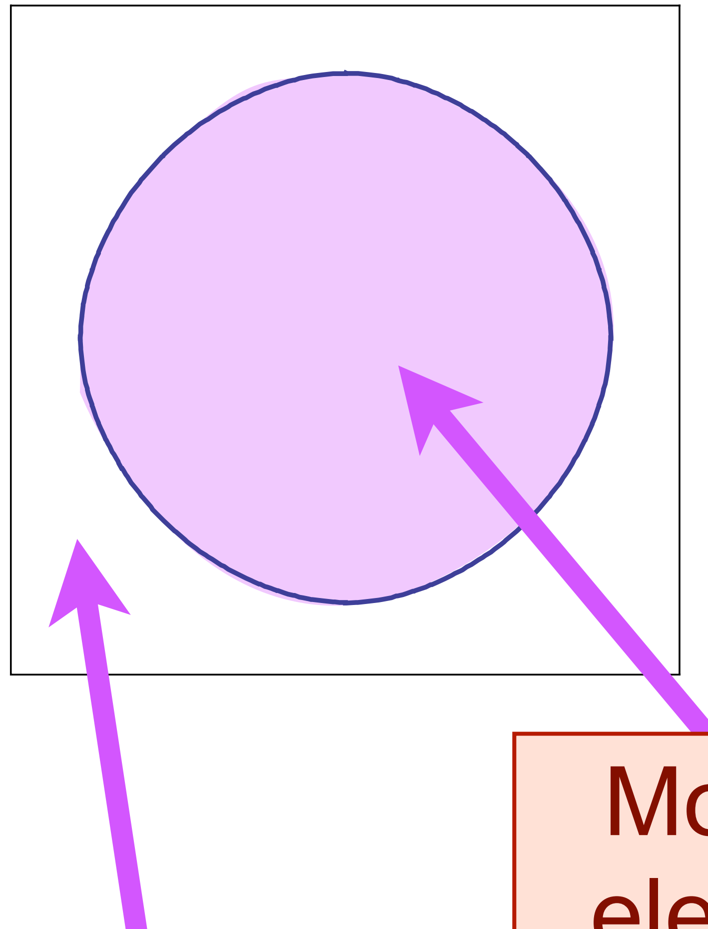
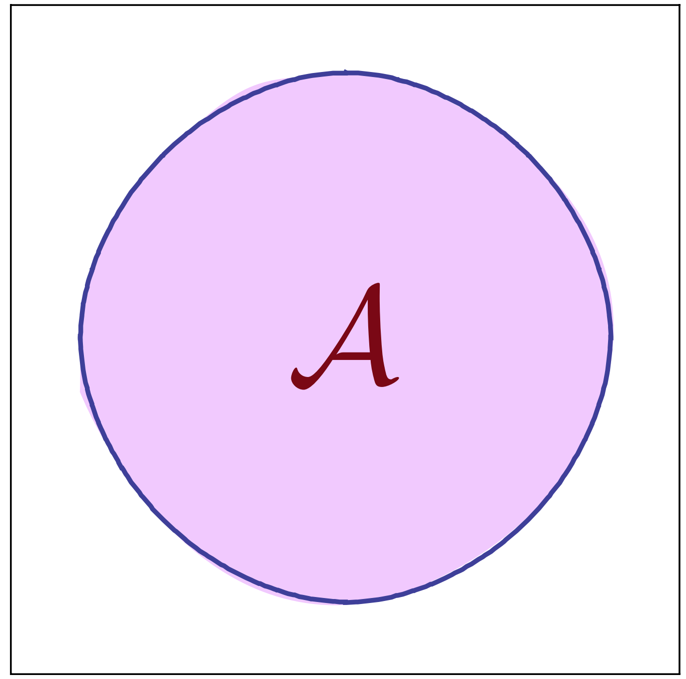
Sommerfeld-Bloch-Landau theory of ordinary metals



Momenta with electron states empty

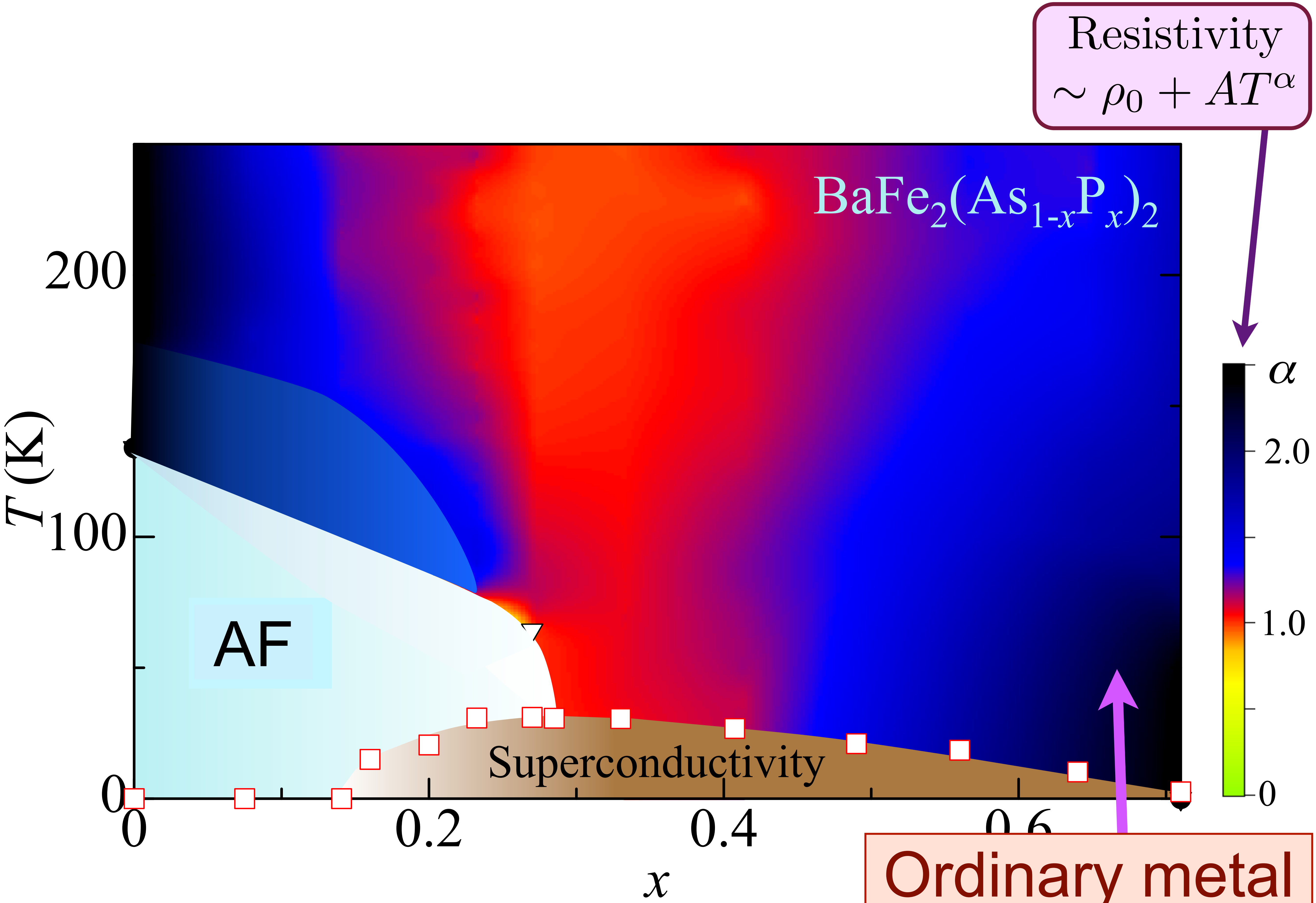
Momenta with electron states occupied

Sommerfeld-Bloch-Landau theory of ordinary metals



Key feature of the theory:
the **Fermi surface**

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the electron density
- Excitations near the Fermi surface are responsible for the familiar properties of ordinary metals, such as resistivity $\sim T^2$.



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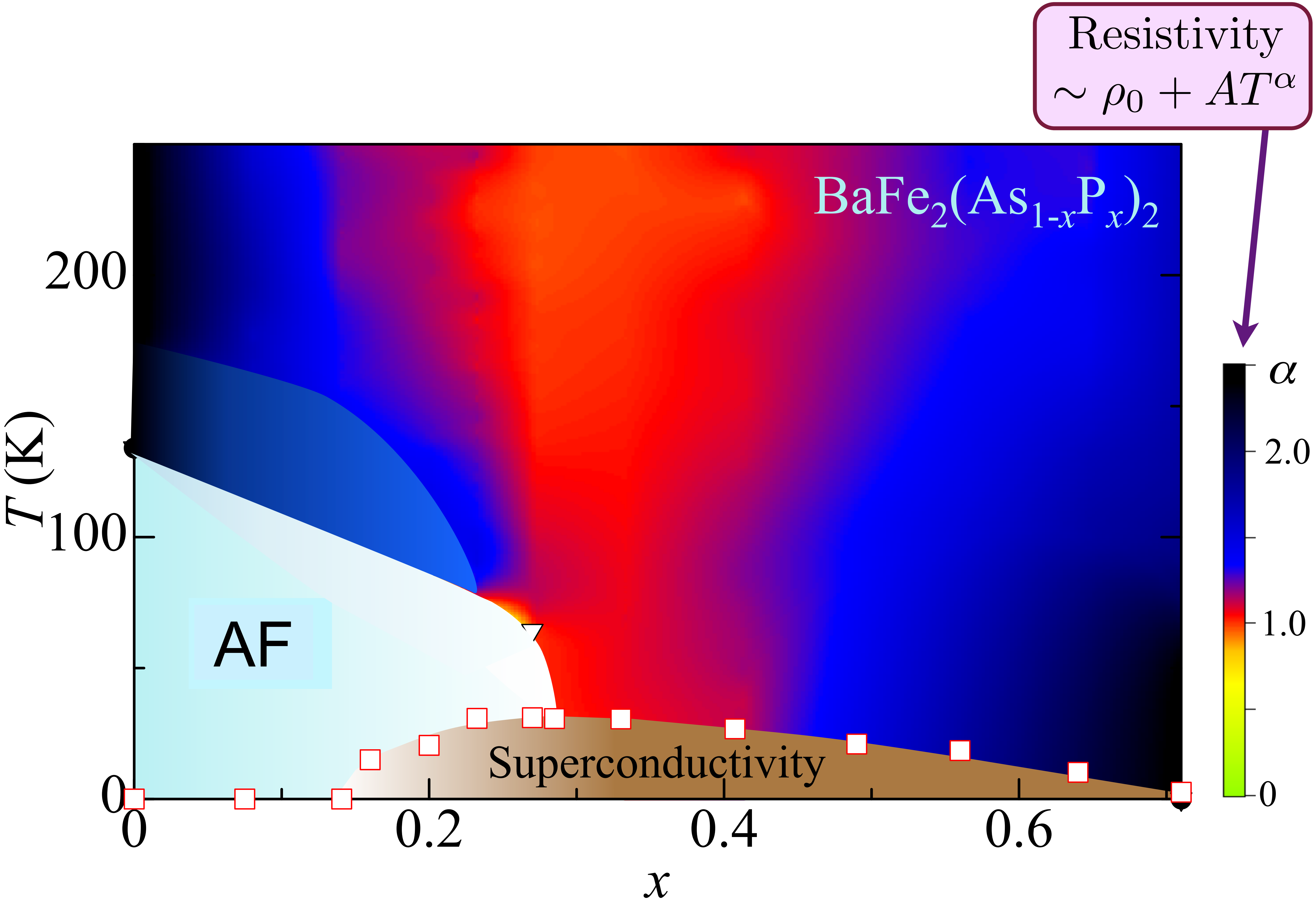

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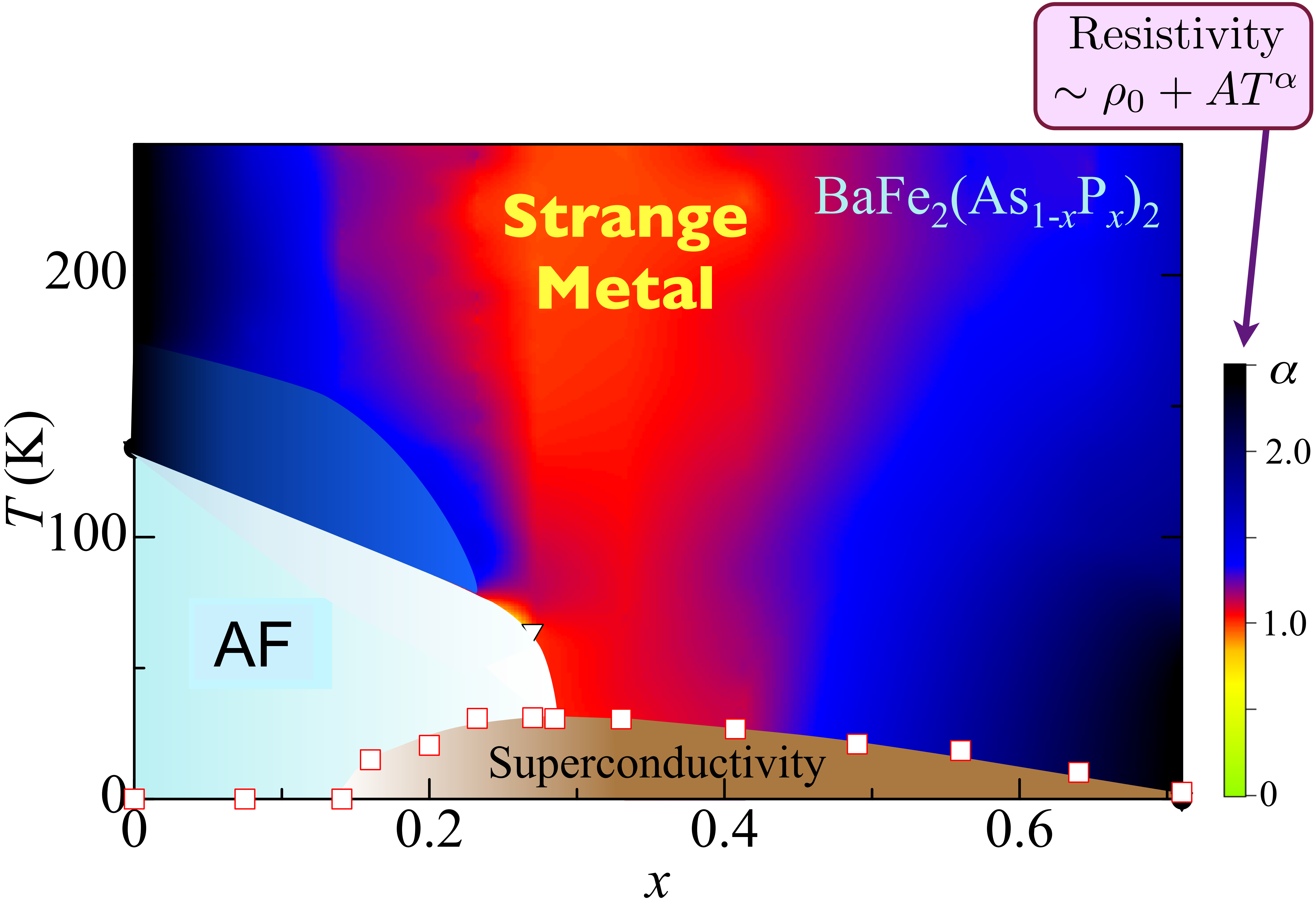

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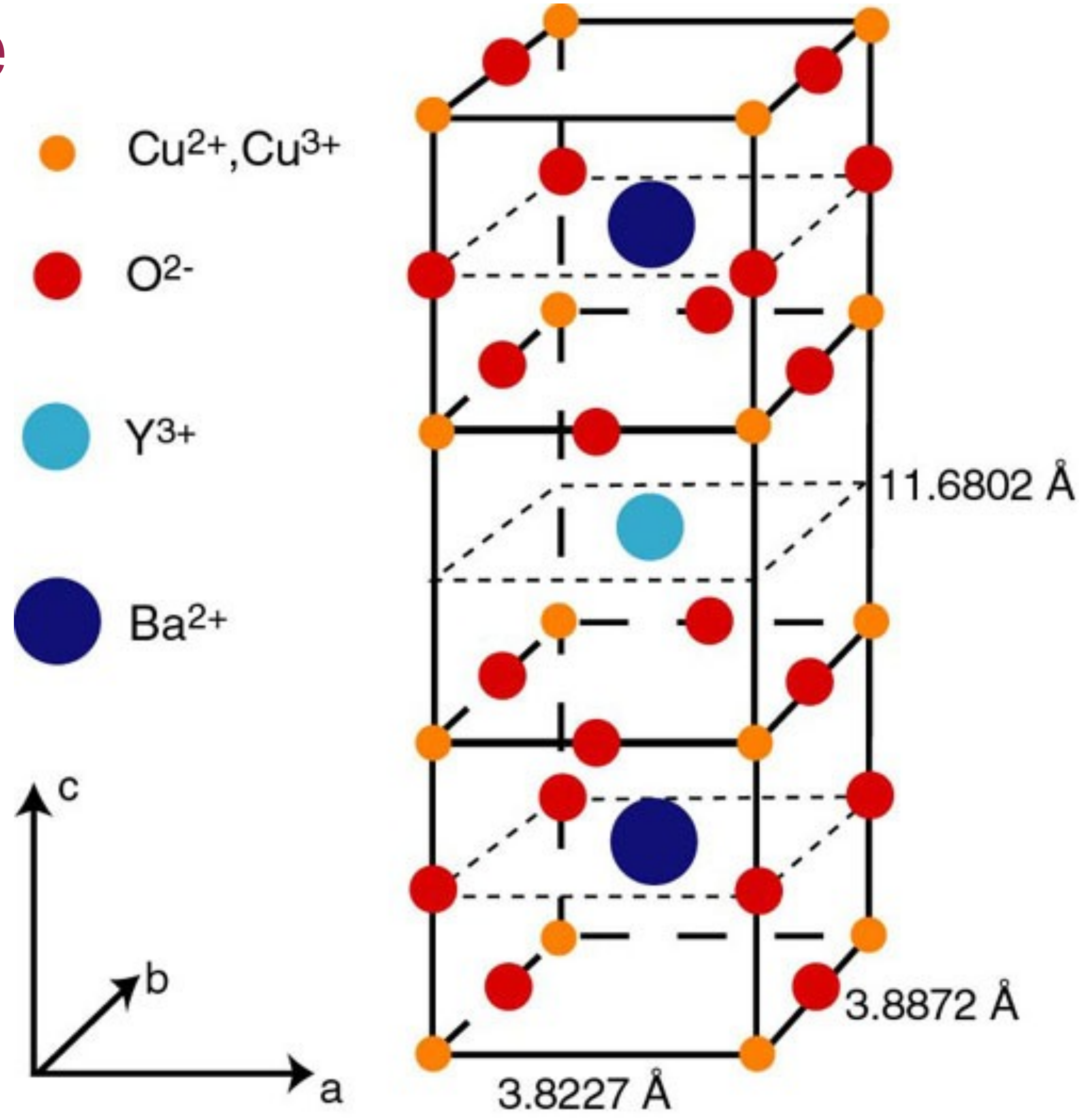
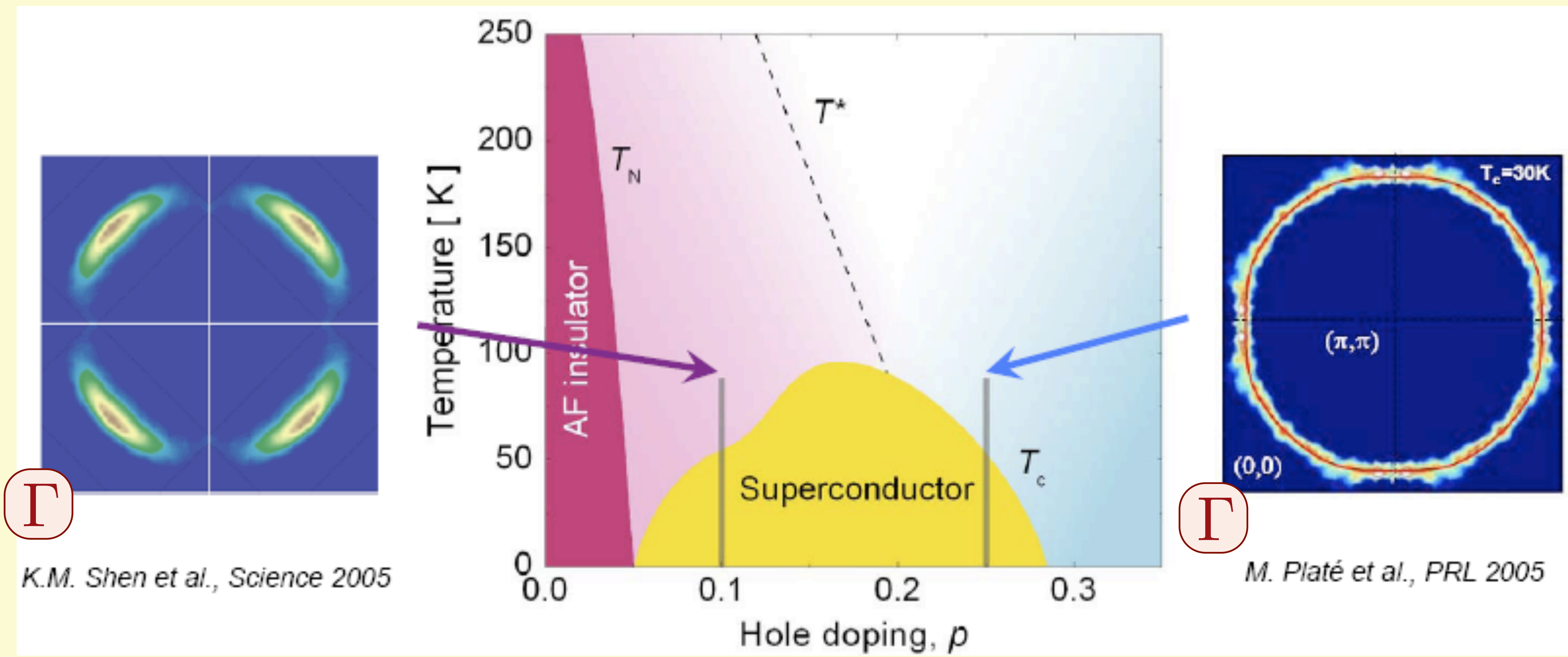
High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$

Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

Large hole Fermi surface

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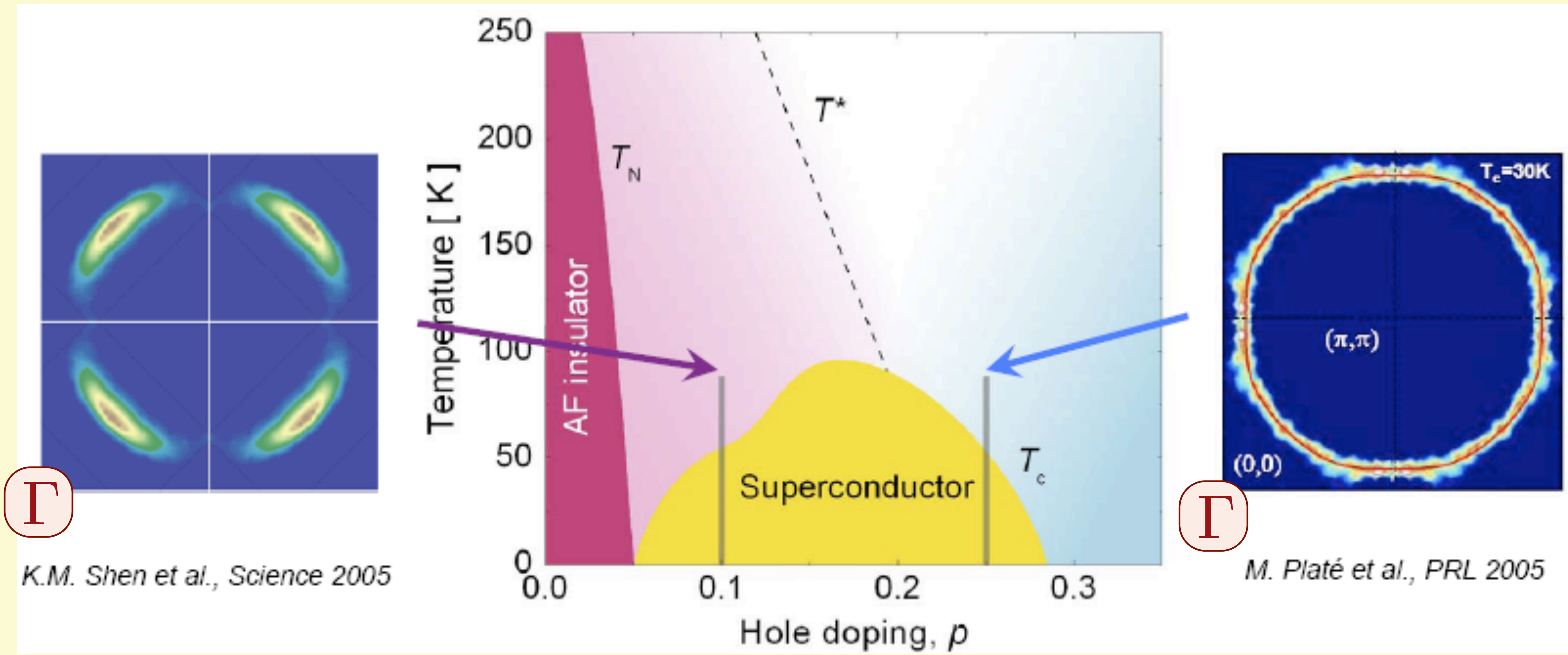

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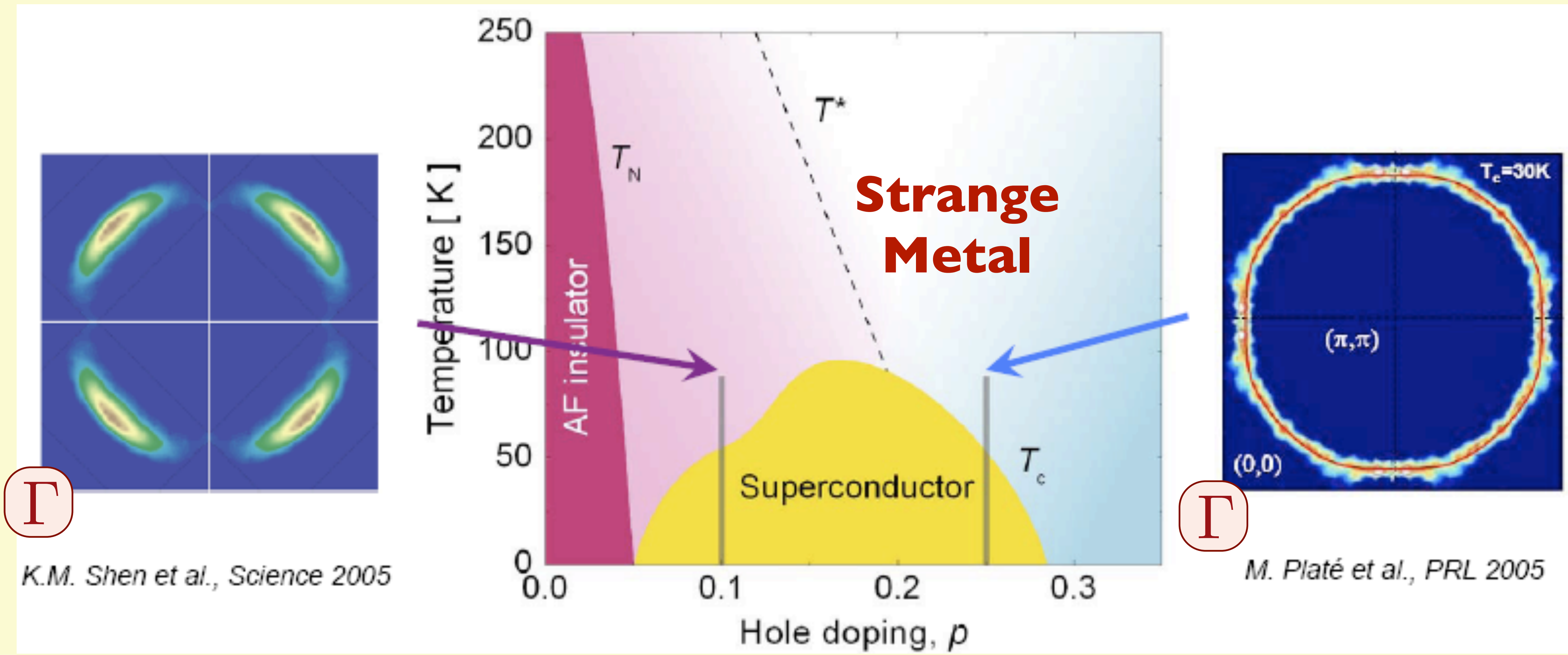

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu \mathcal{Q}$.

The only compressible phase of traditional condensed matter physics which does not break the translational or $\mathrm{U}(1)$ symmetries is the Landau Fermi liquid

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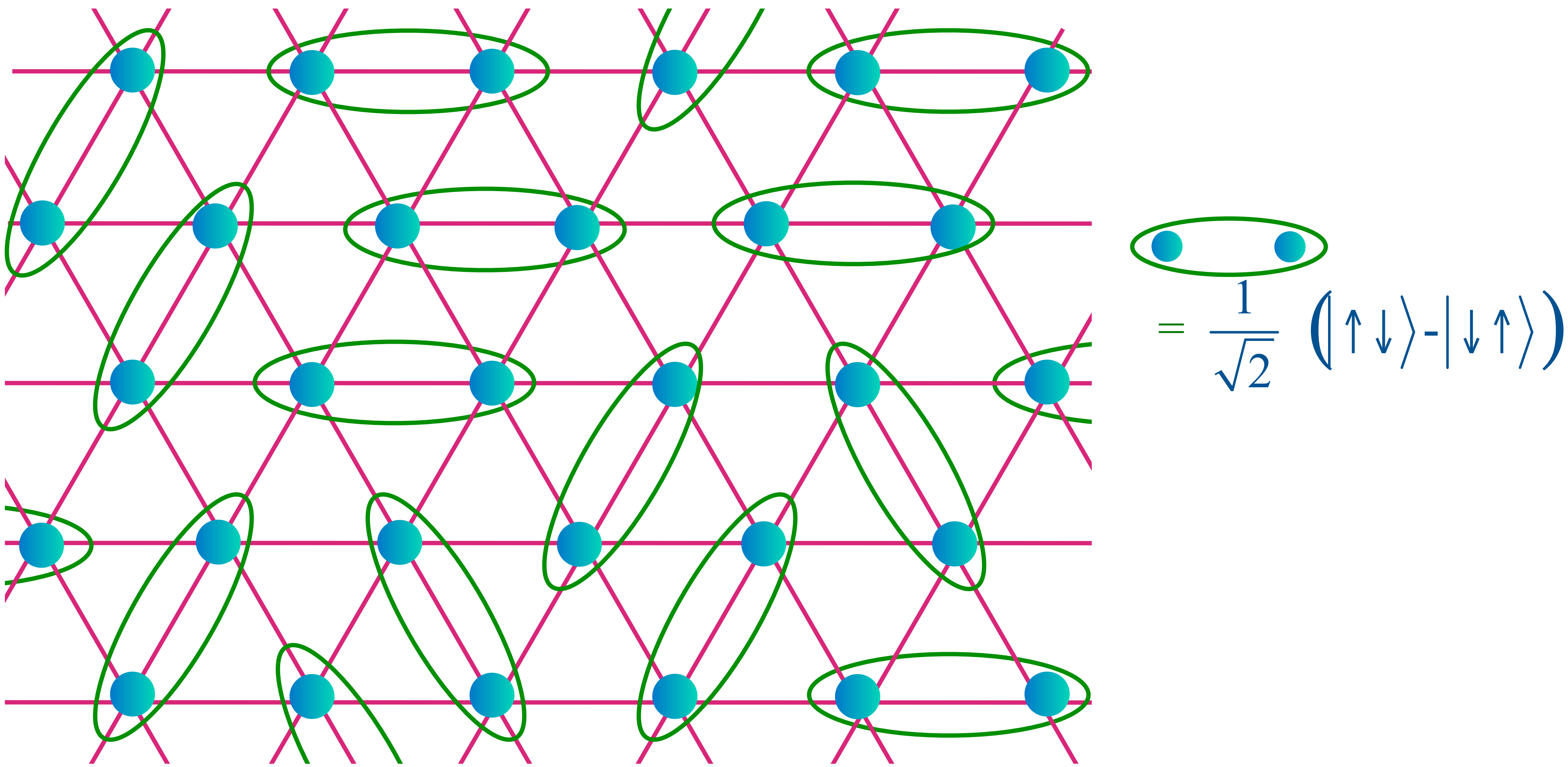
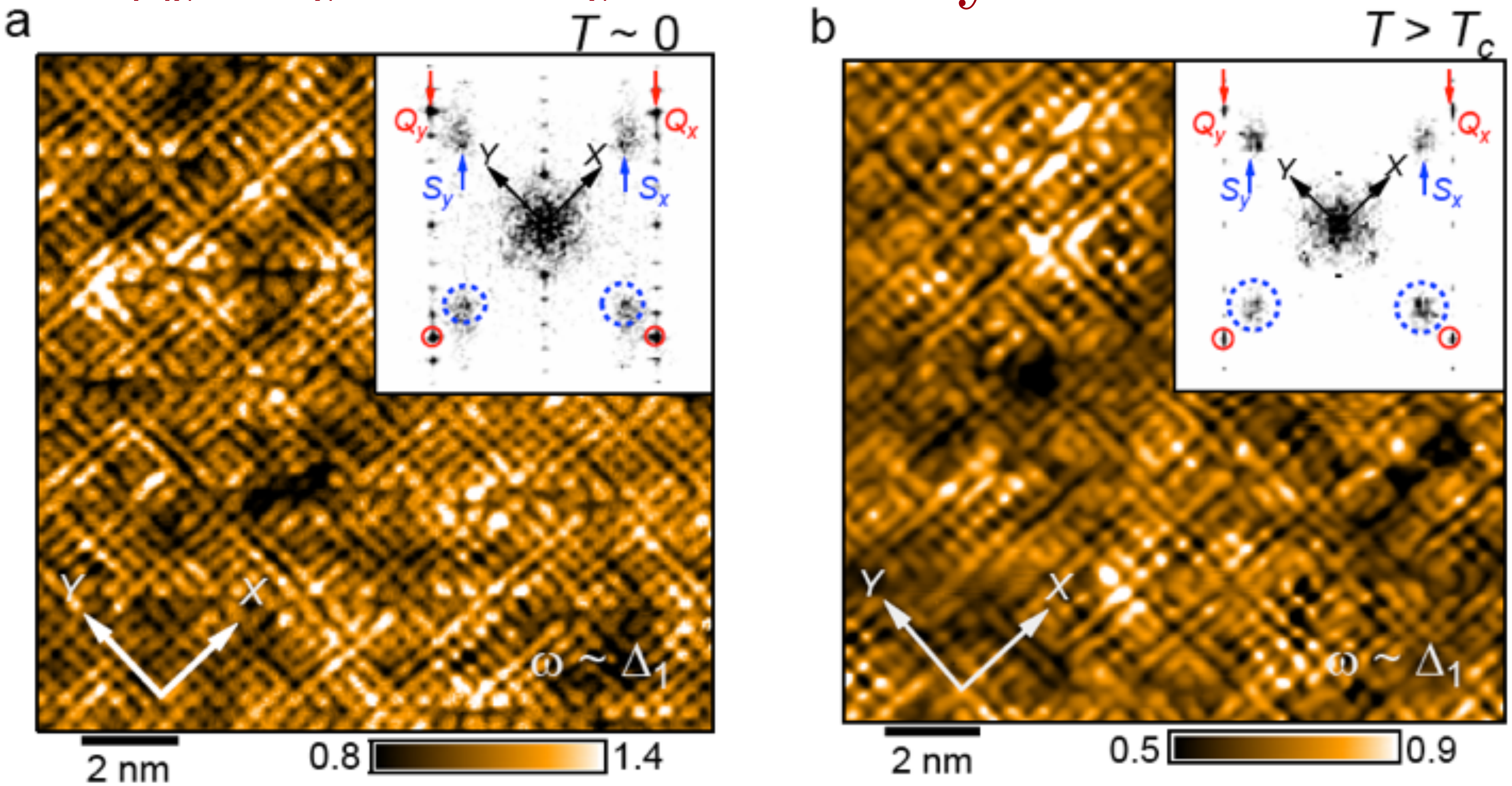
The Non-Fermi Liquid (NFL)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.

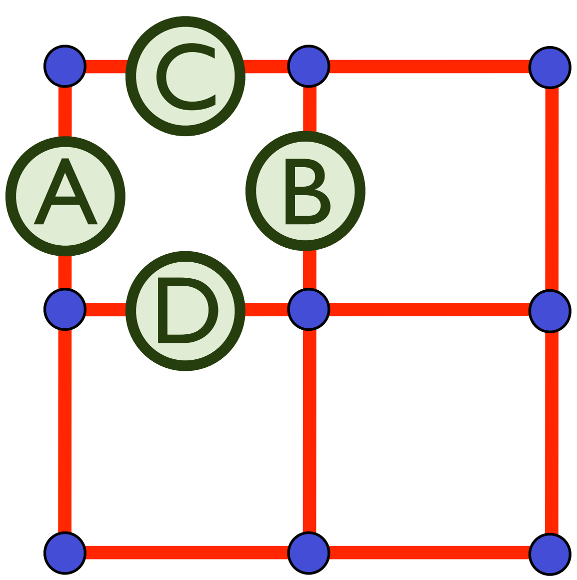


$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

STM measurements of $Z(r)$, energy asymmetry in density of states



M. J. Lawler, K. Fujita, Jhinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, preprint



$$O_N = Z_A + Z_B - Z_C - Z_D$$

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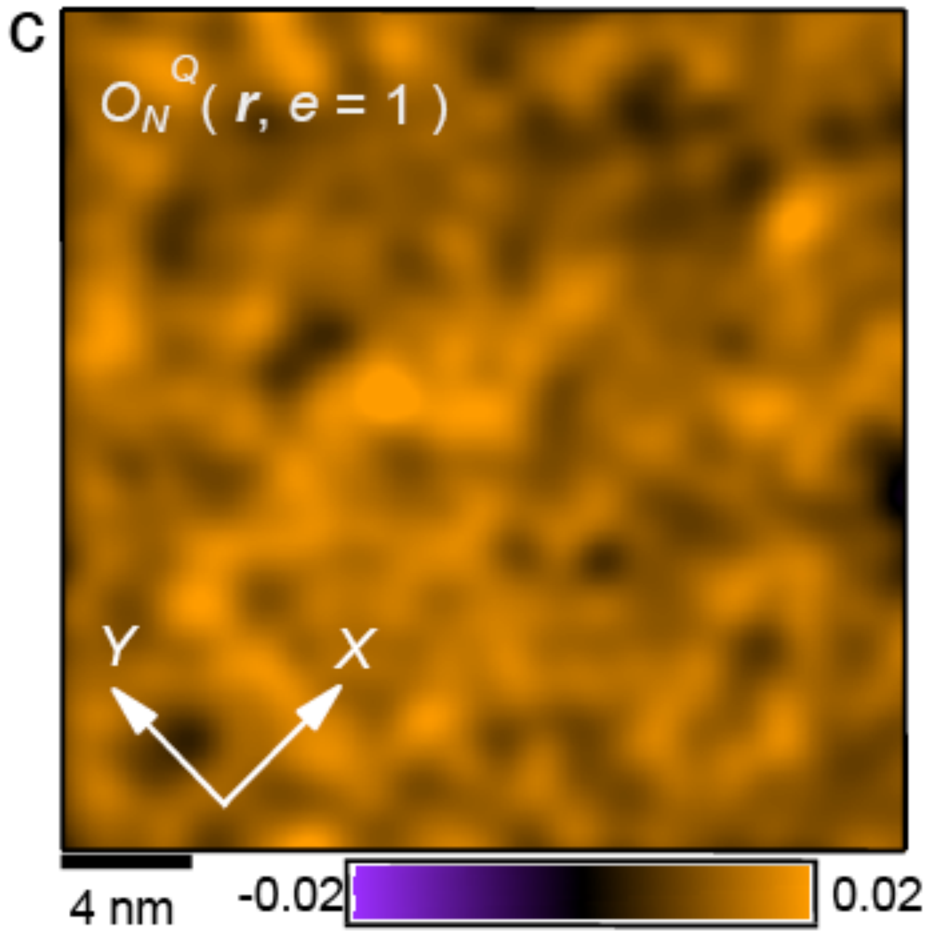

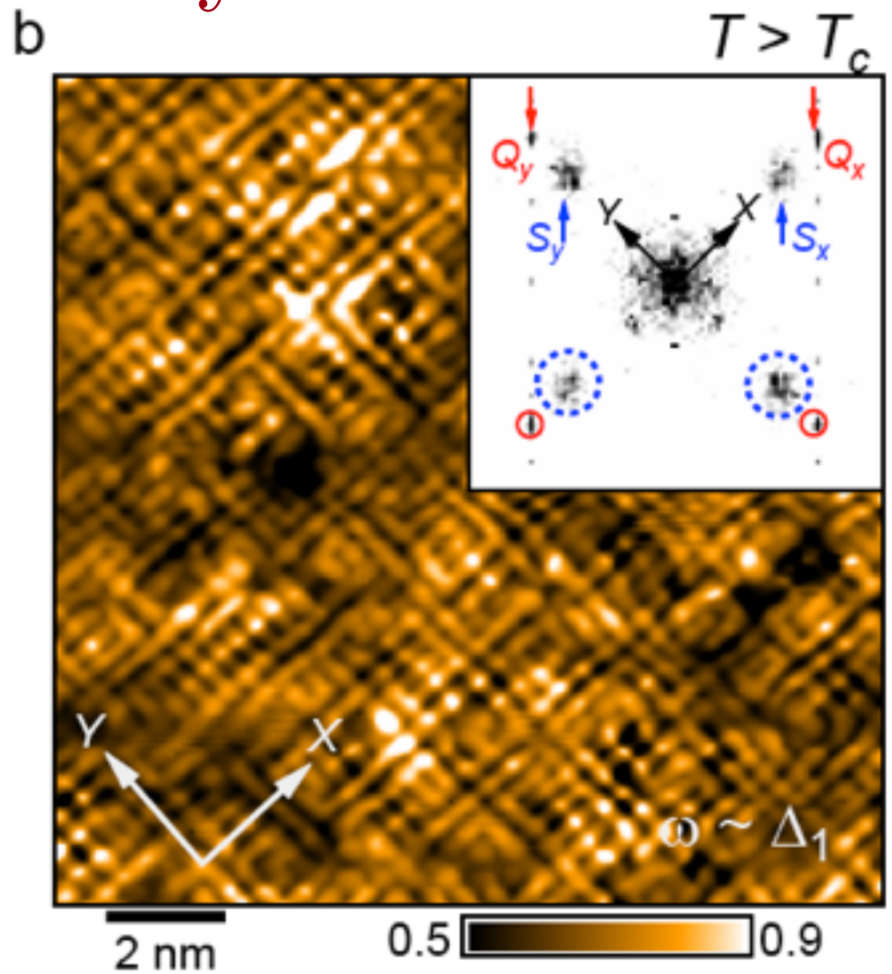

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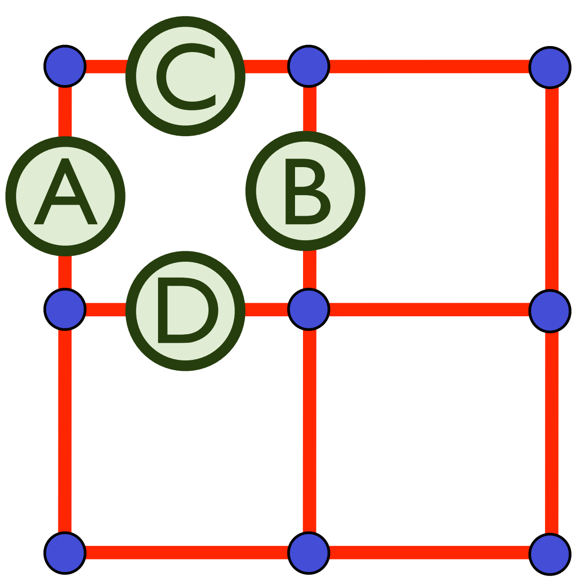
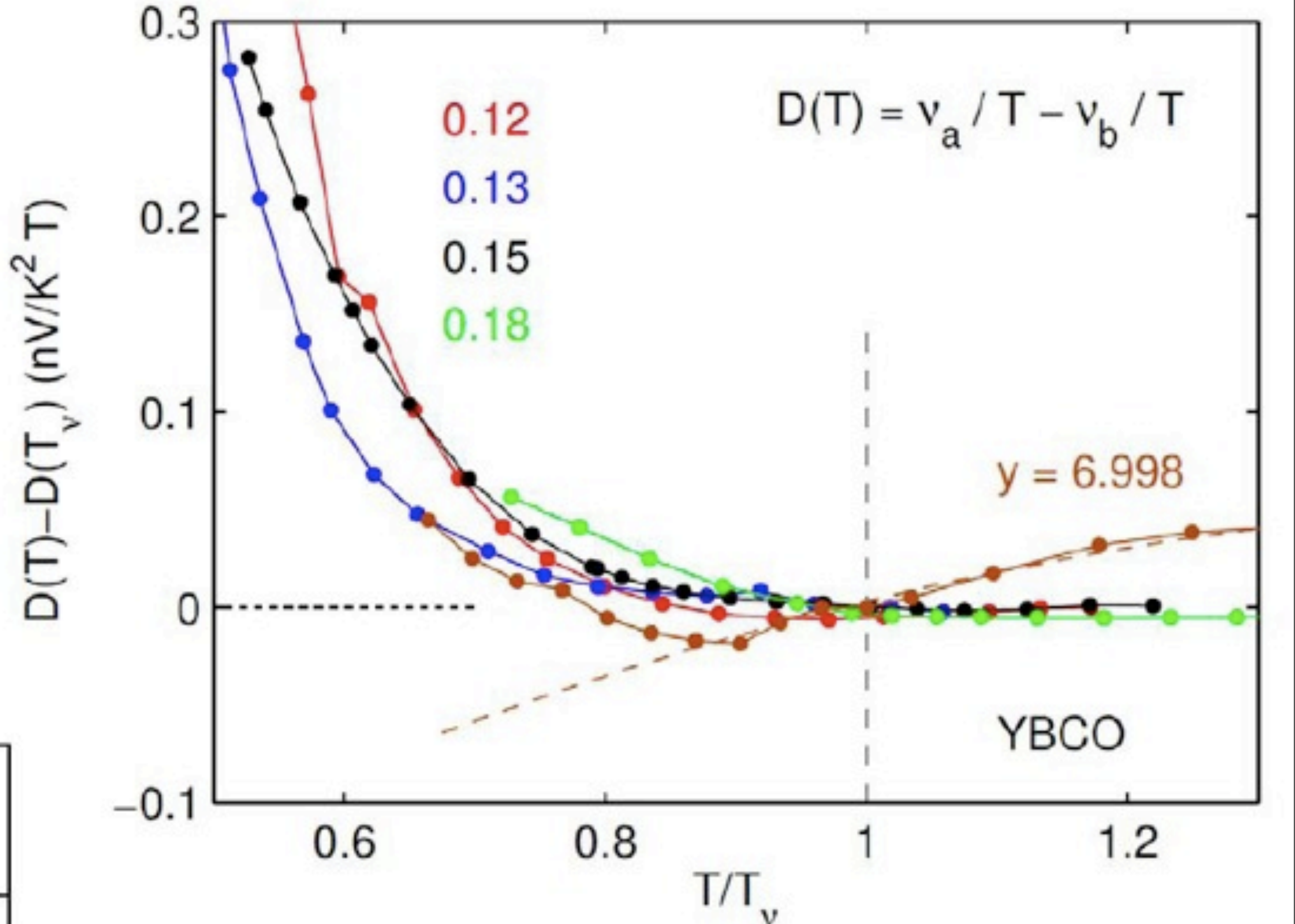


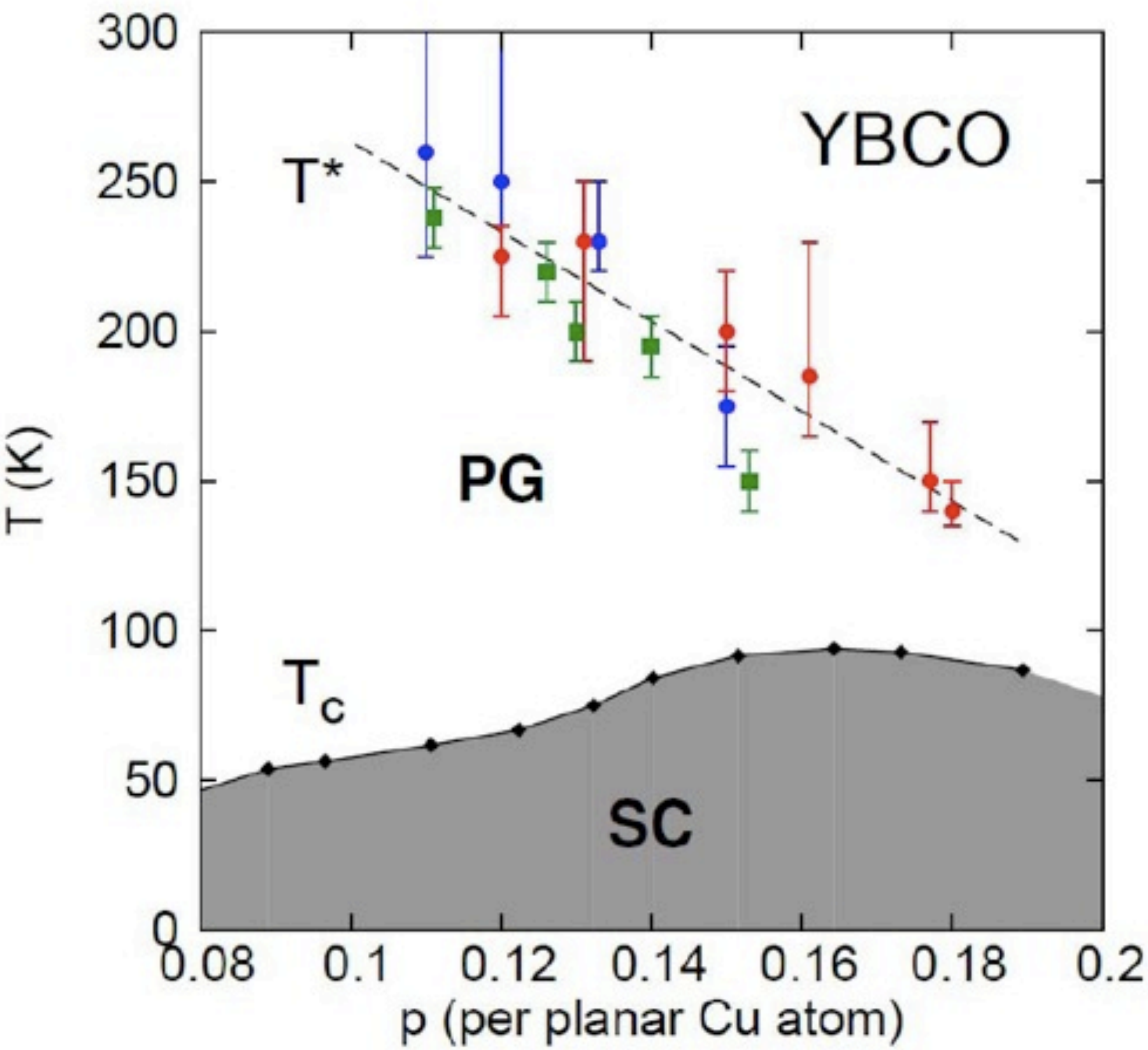
$$O_N = Z_A + Z_B - Z_C - Z_D$$

Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

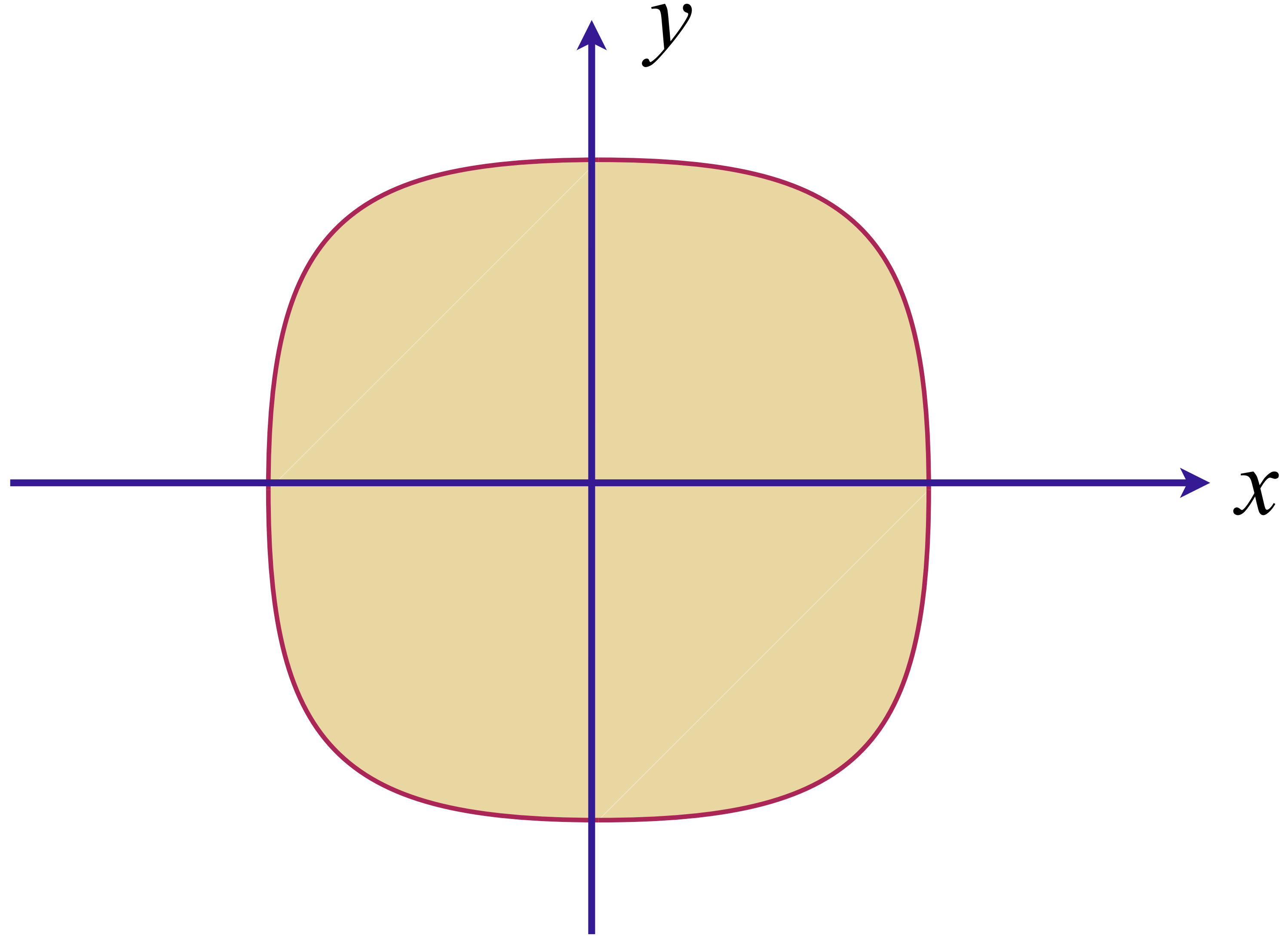
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D.A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).



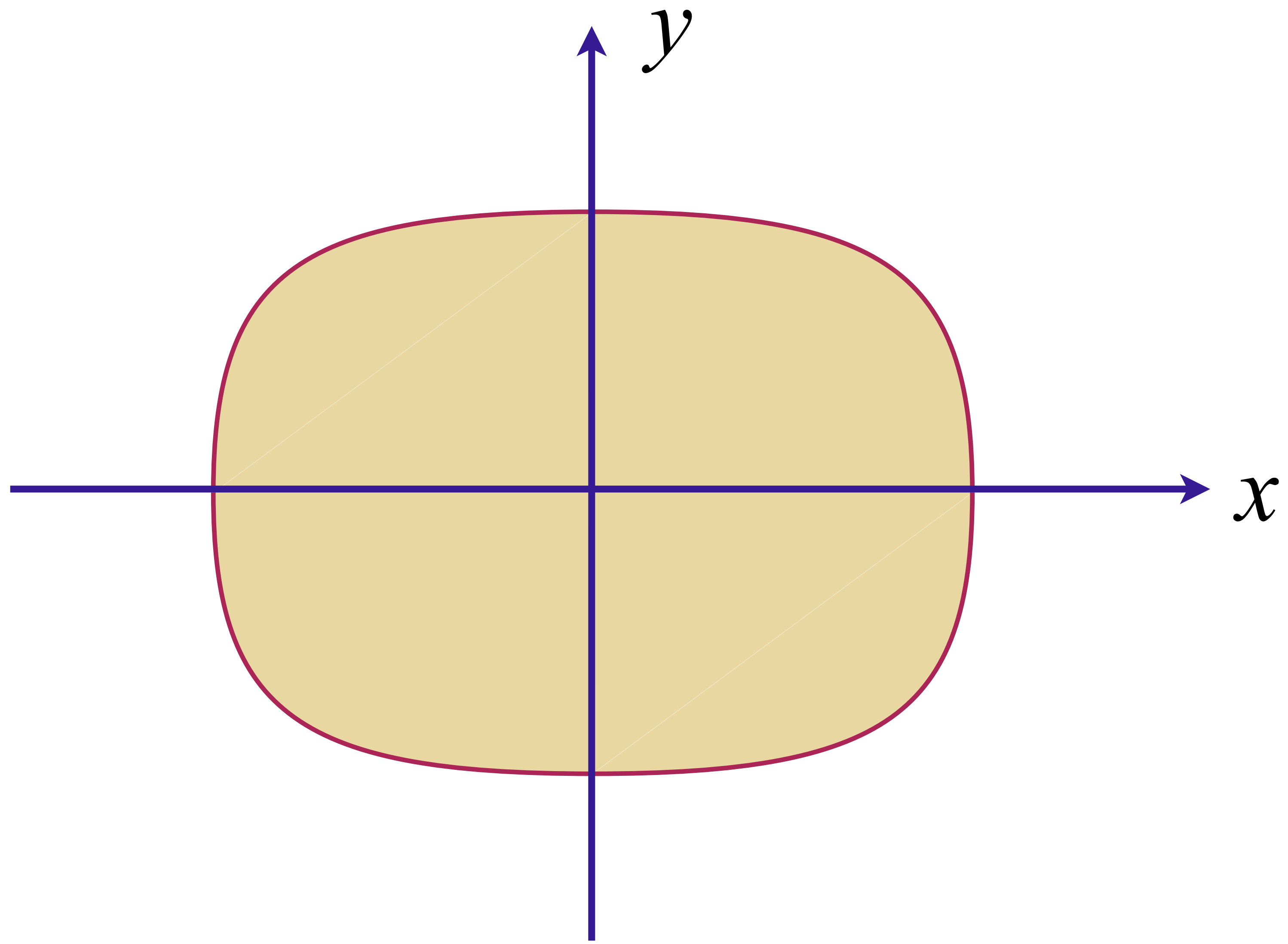


Quantum criticality of Ising-nematic ordering



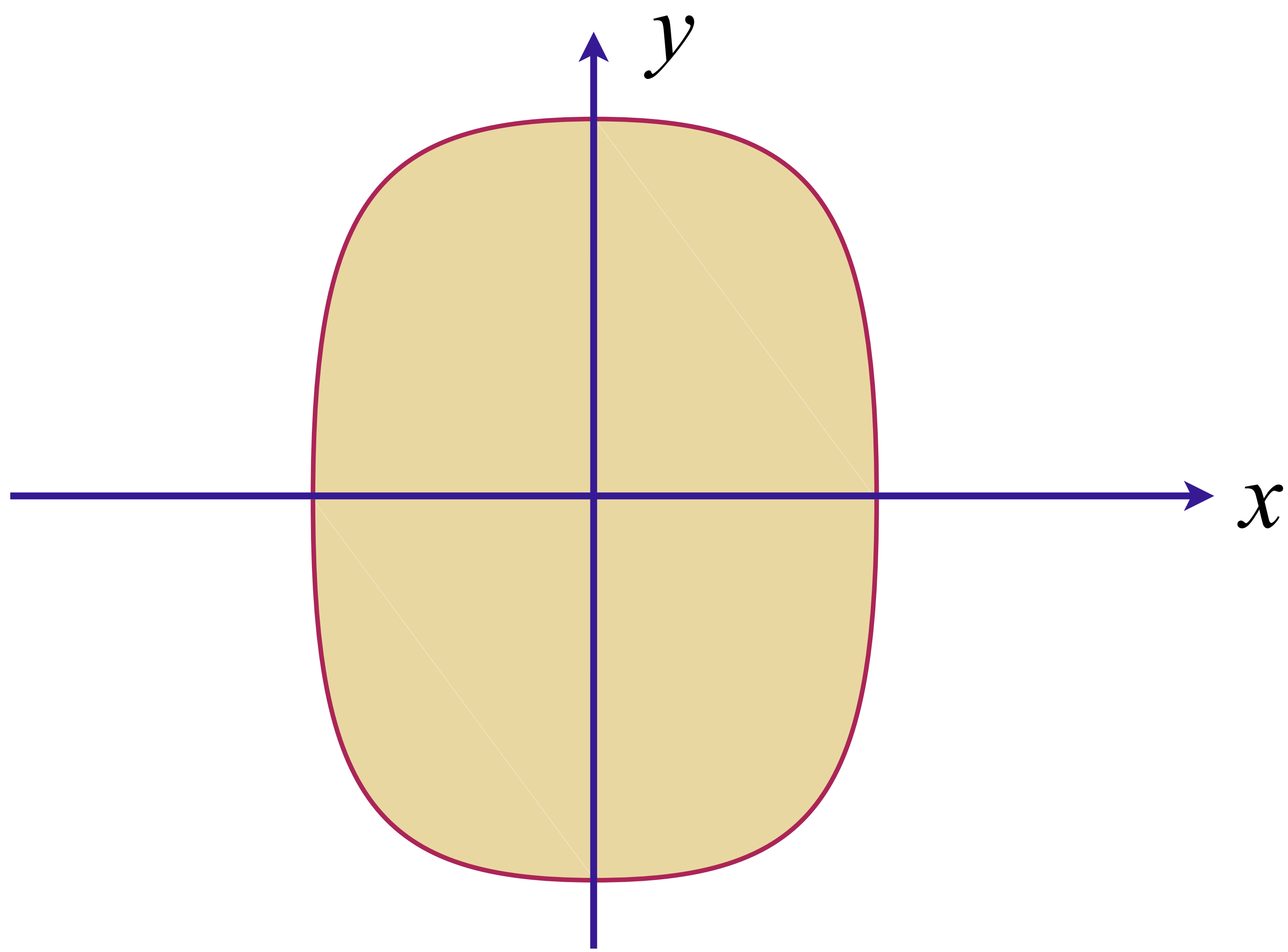
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2k \, (\cos k_x - \cos k_y) \, c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}$$

Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

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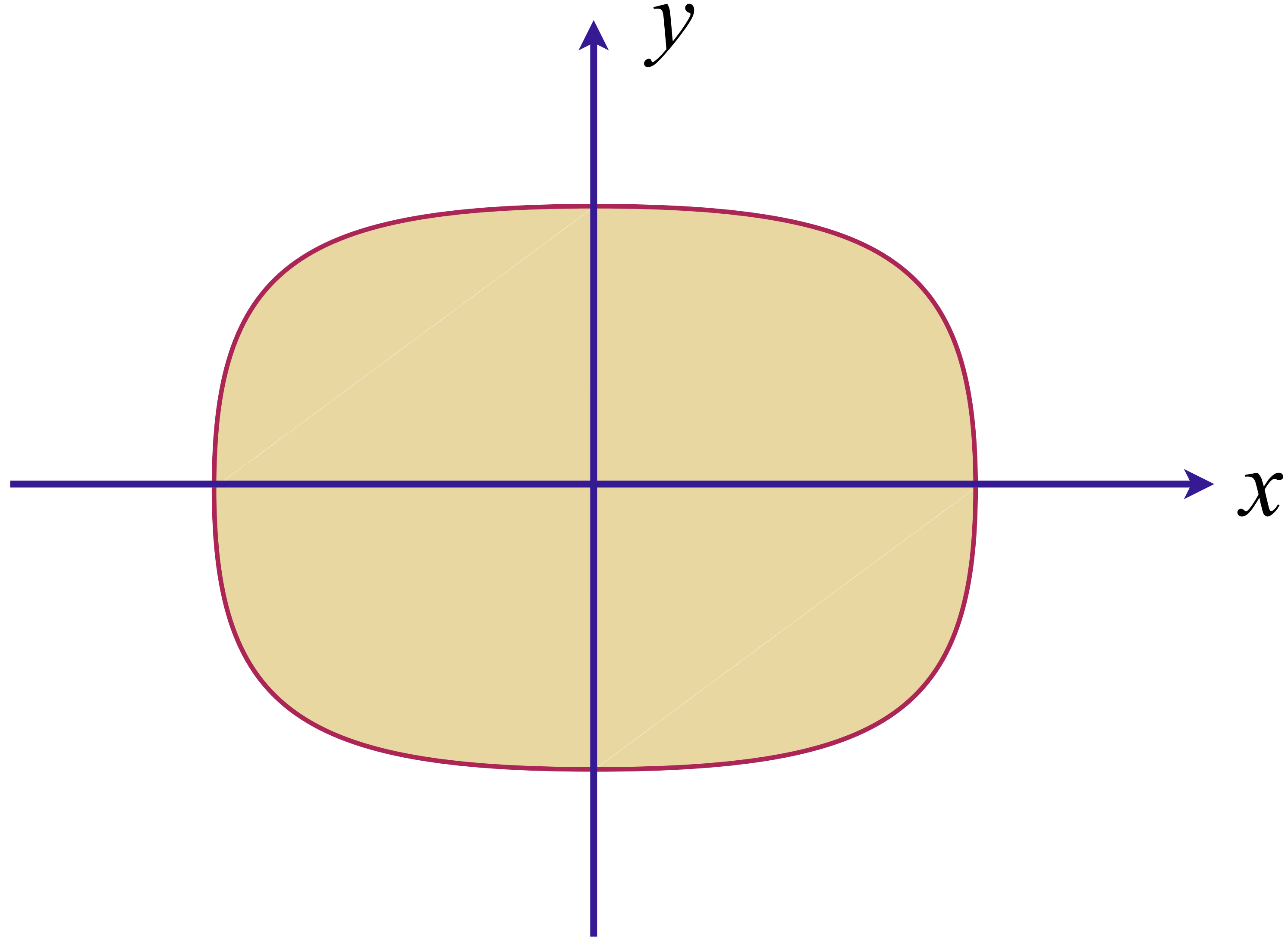

Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

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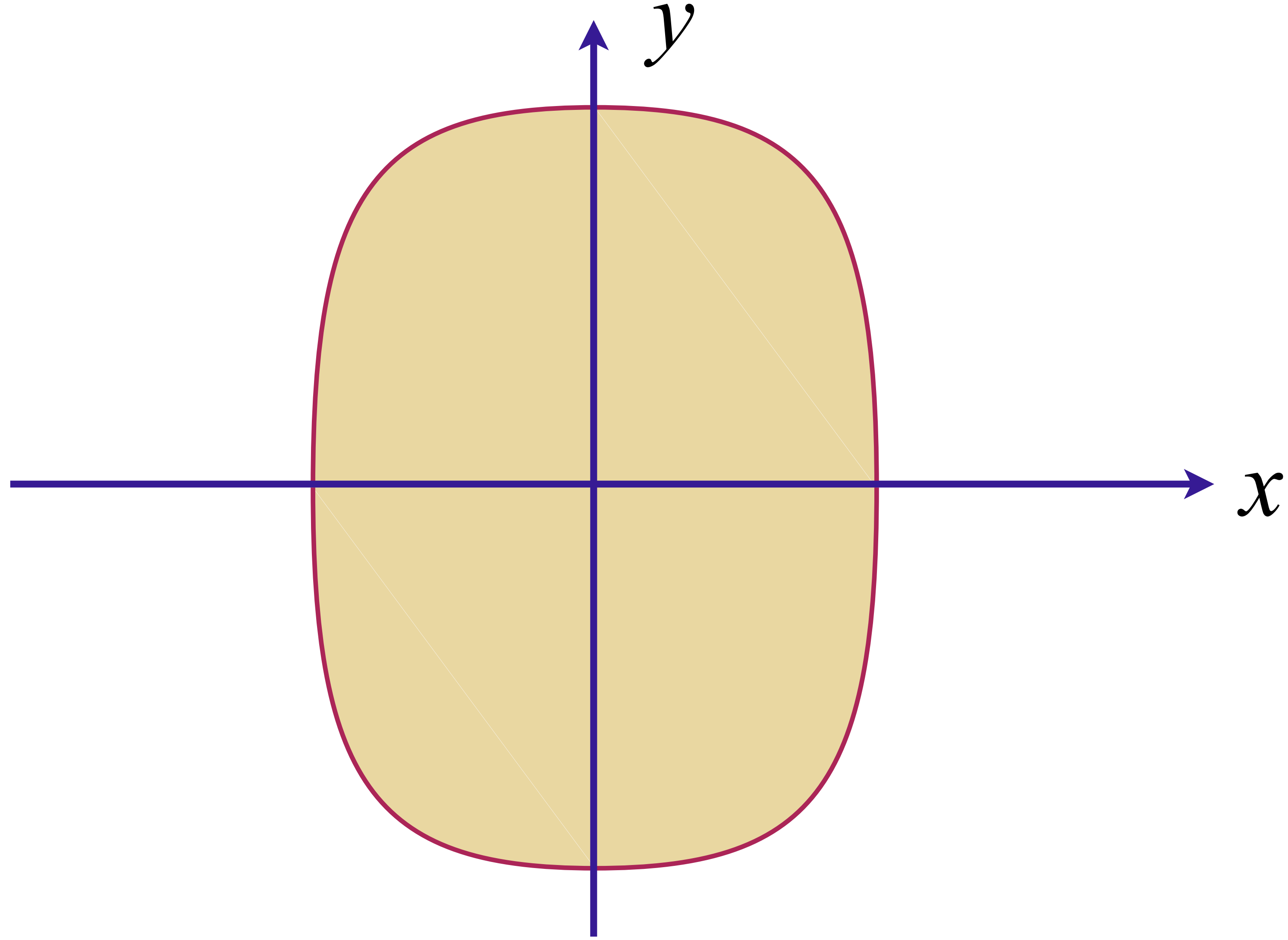
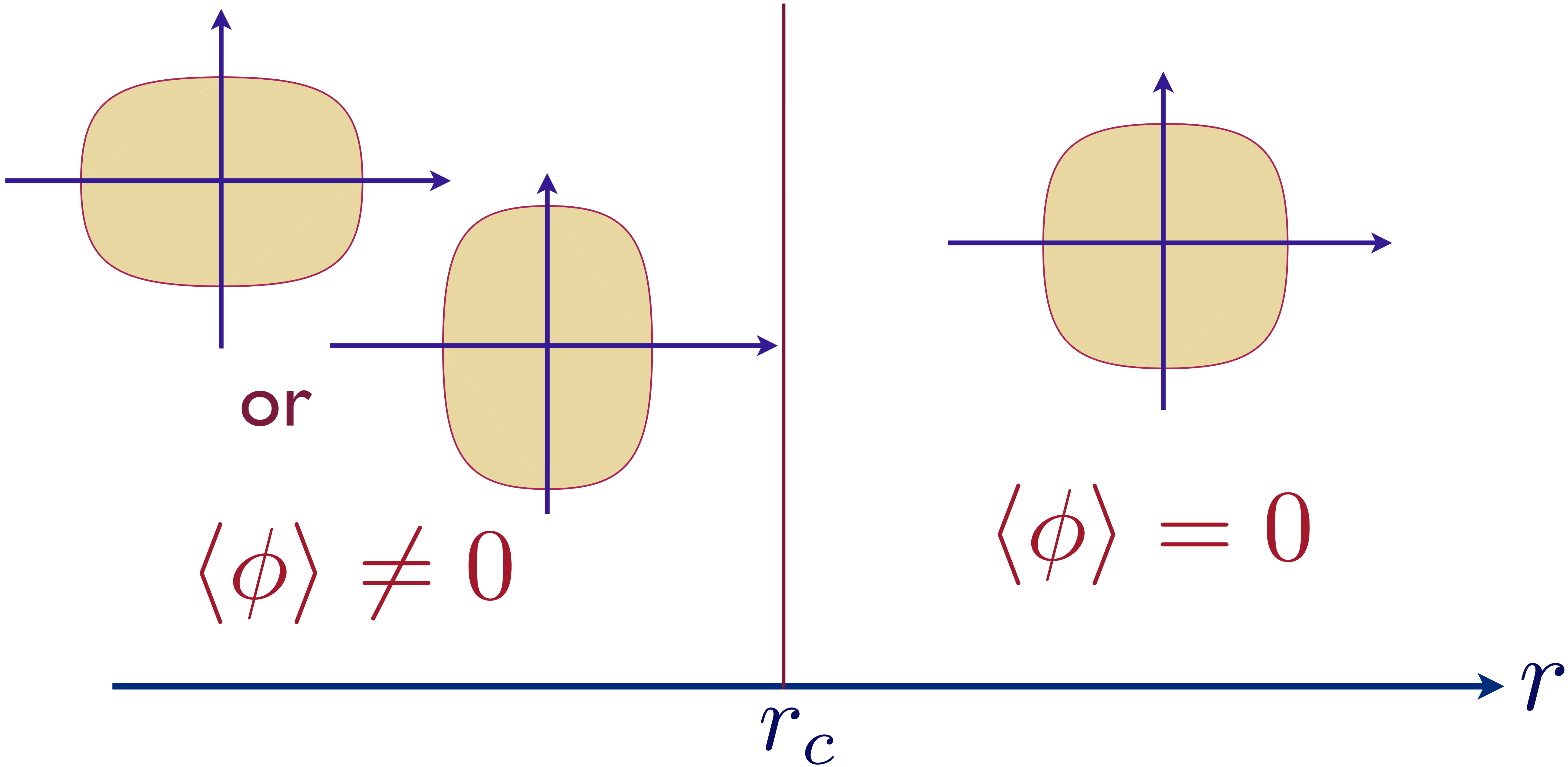


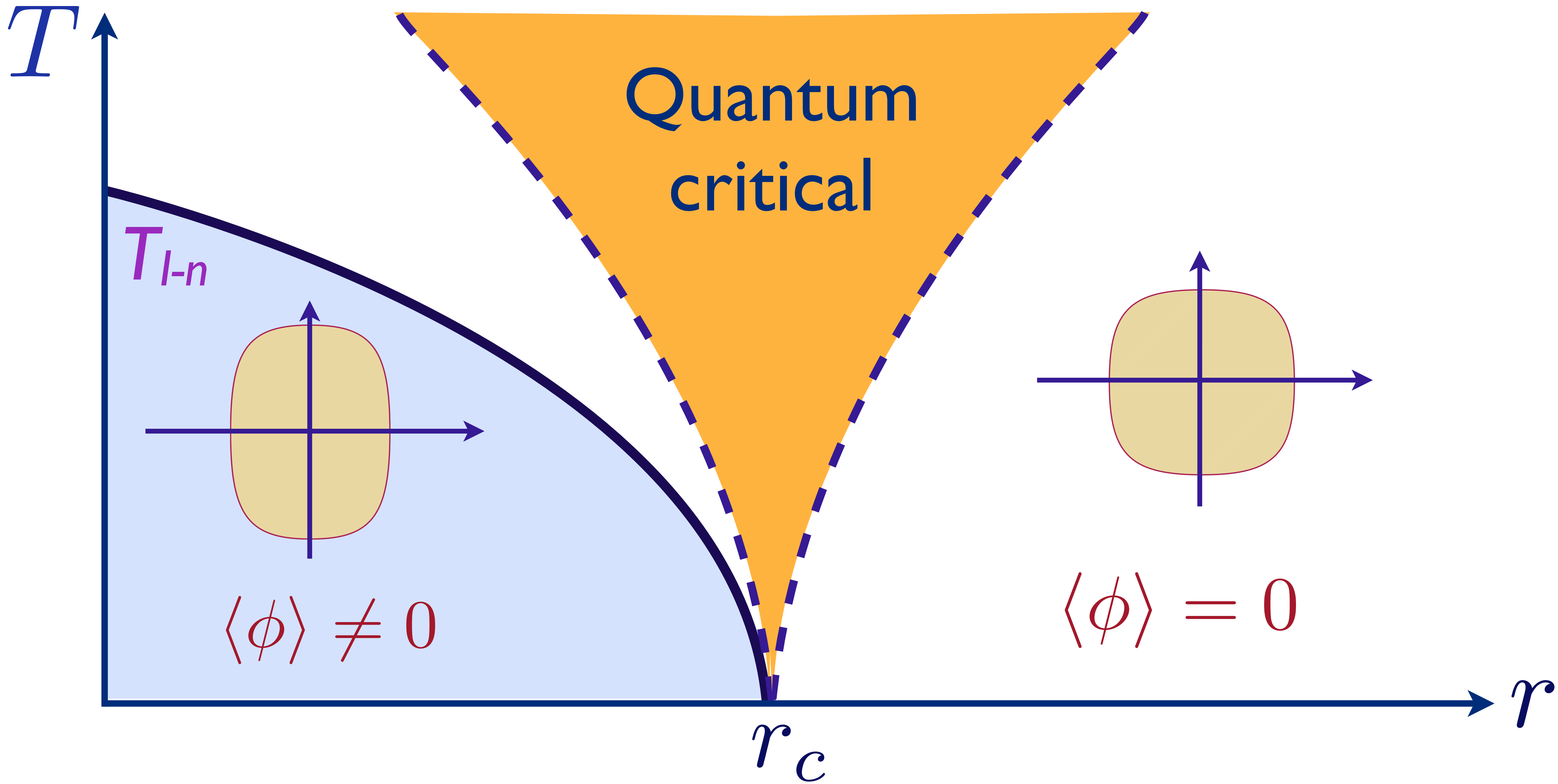
Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Ising-nematic ordering



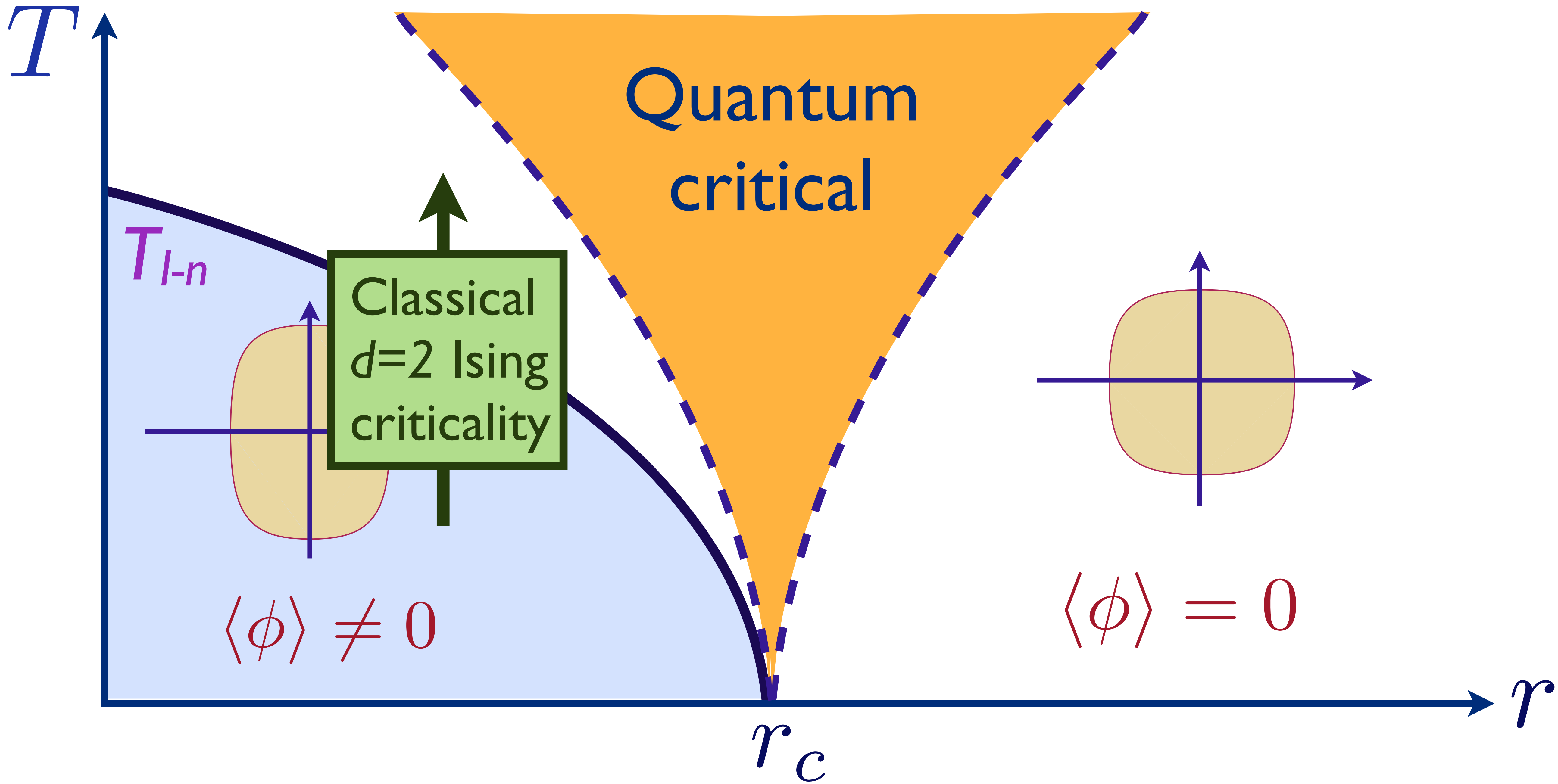
Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



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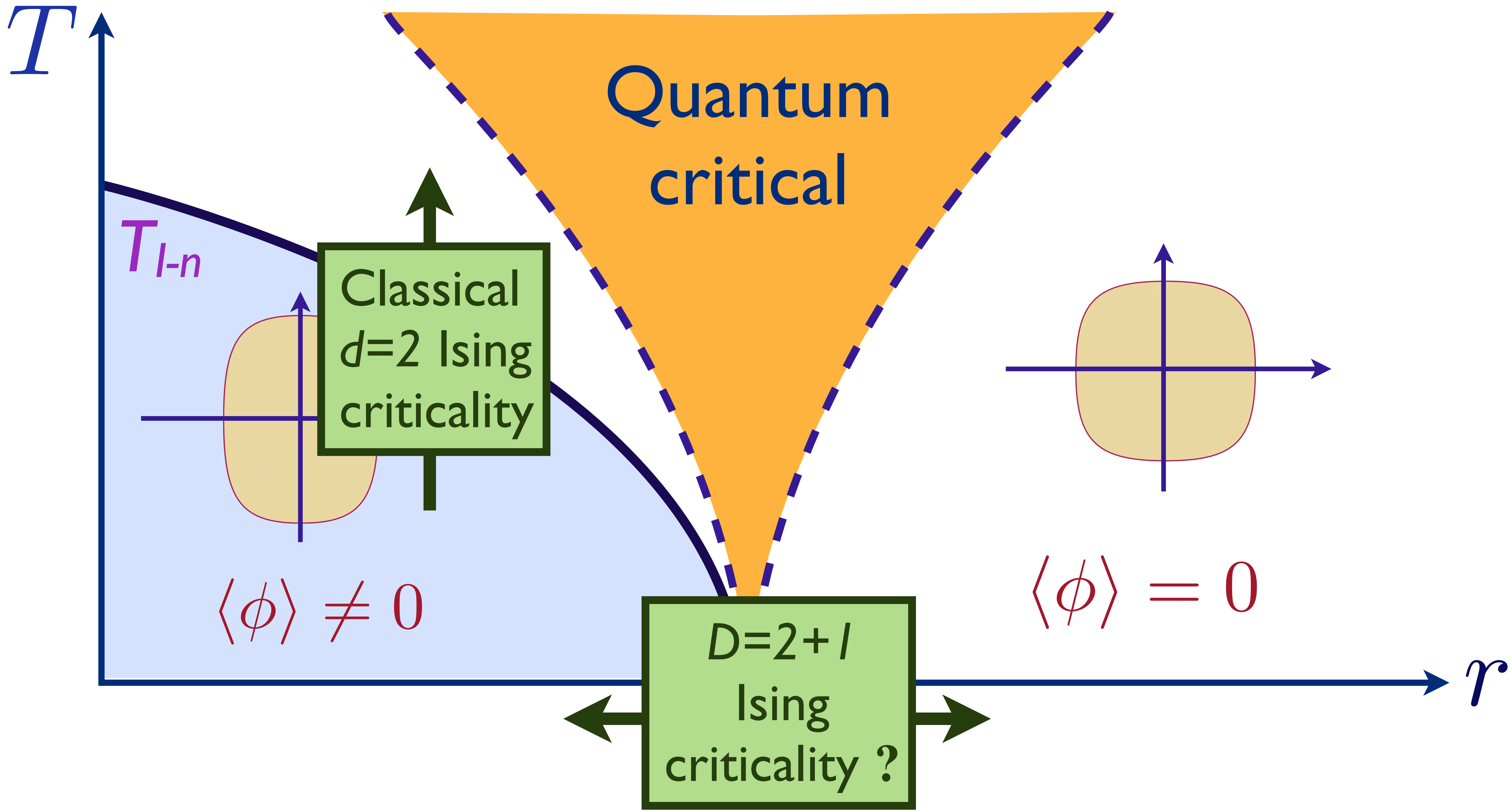

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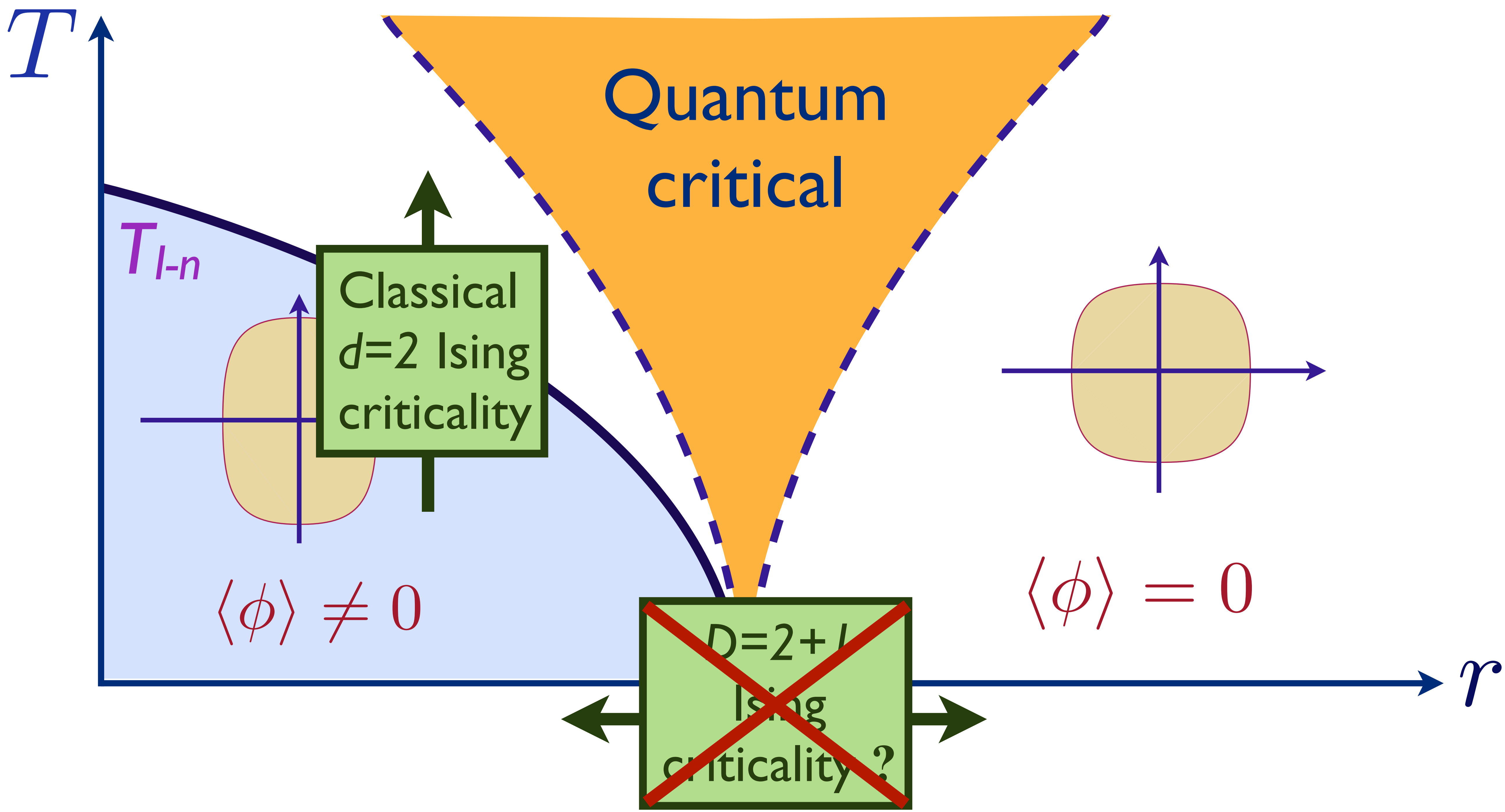

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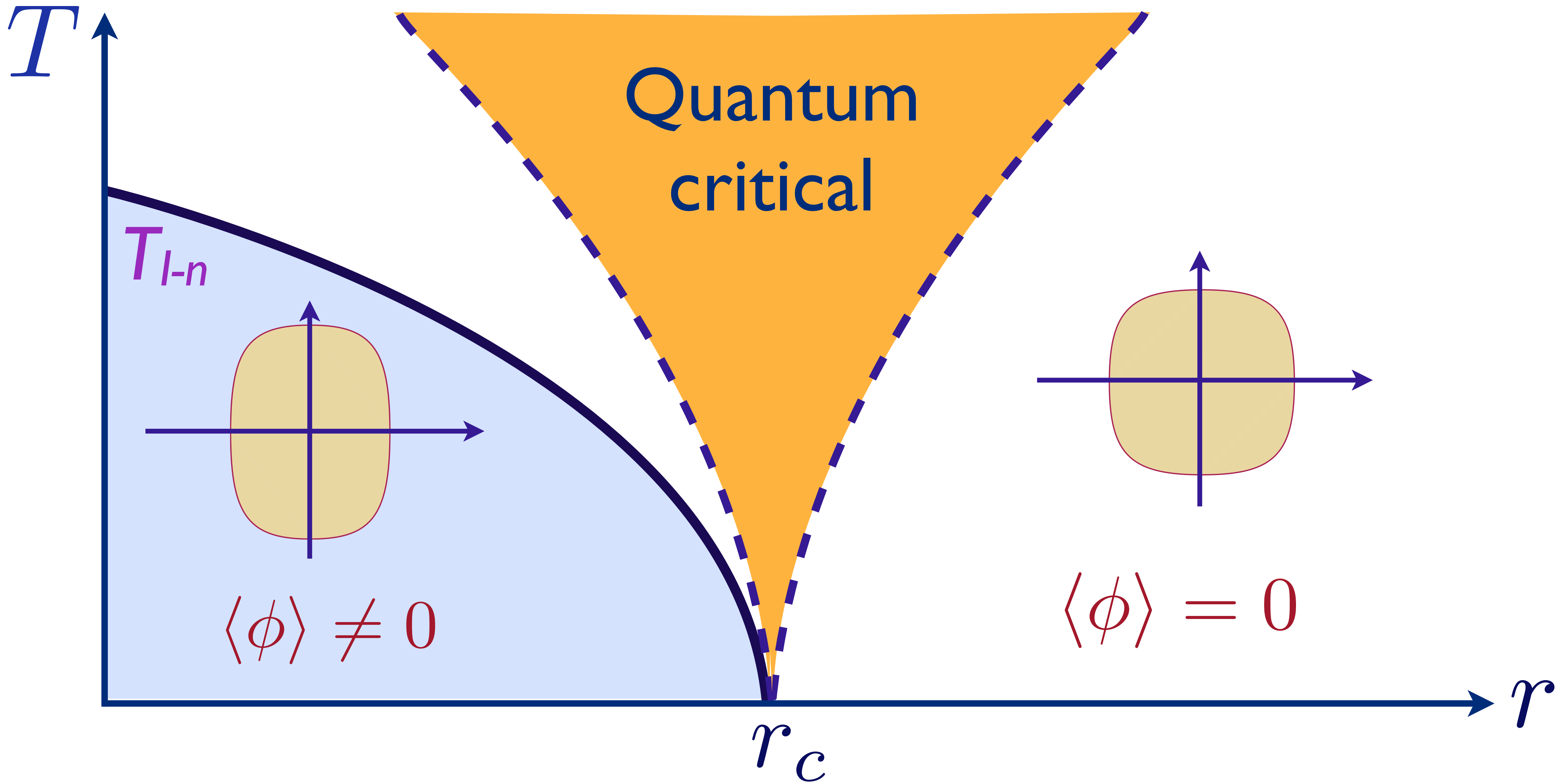

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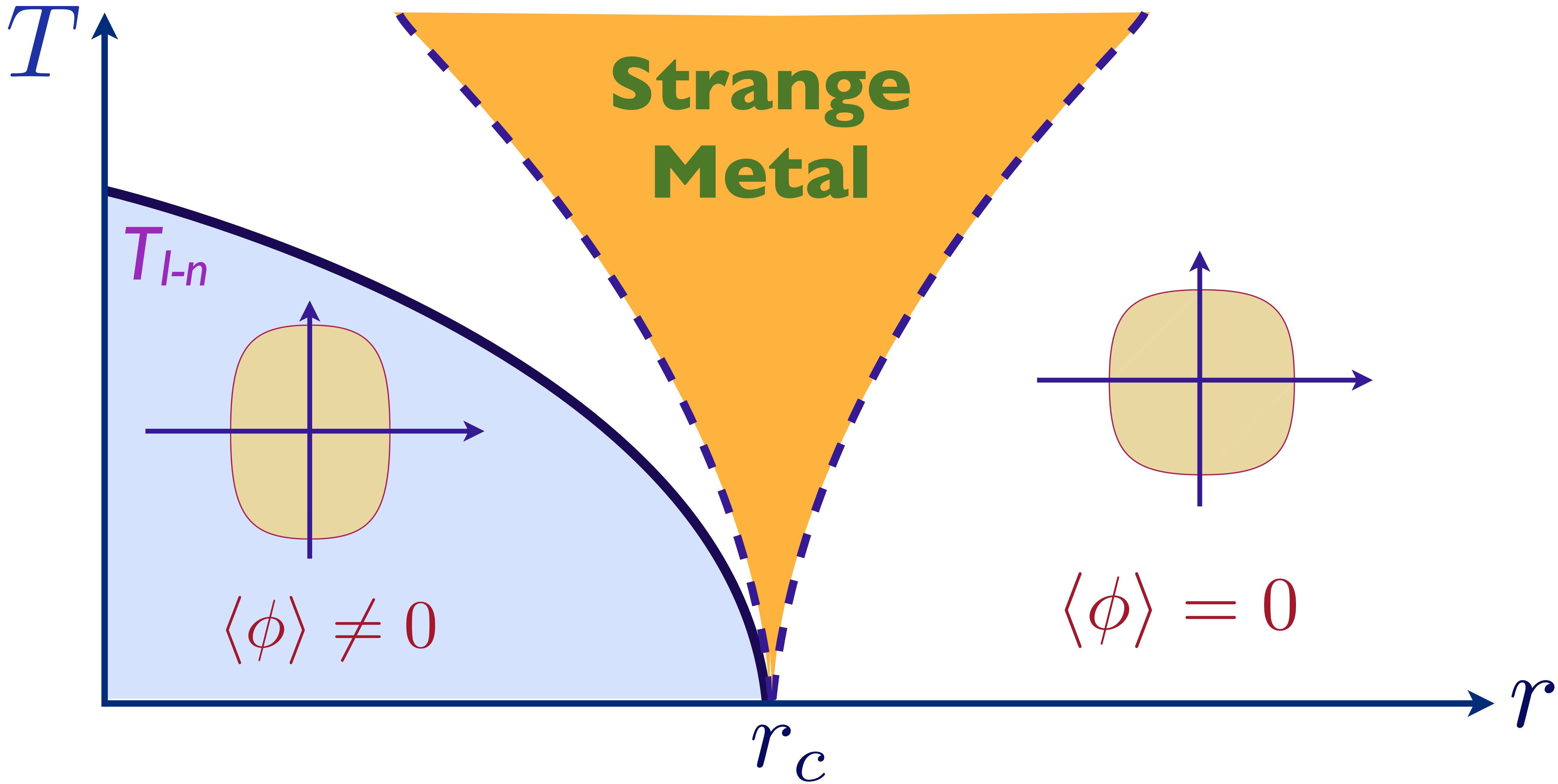

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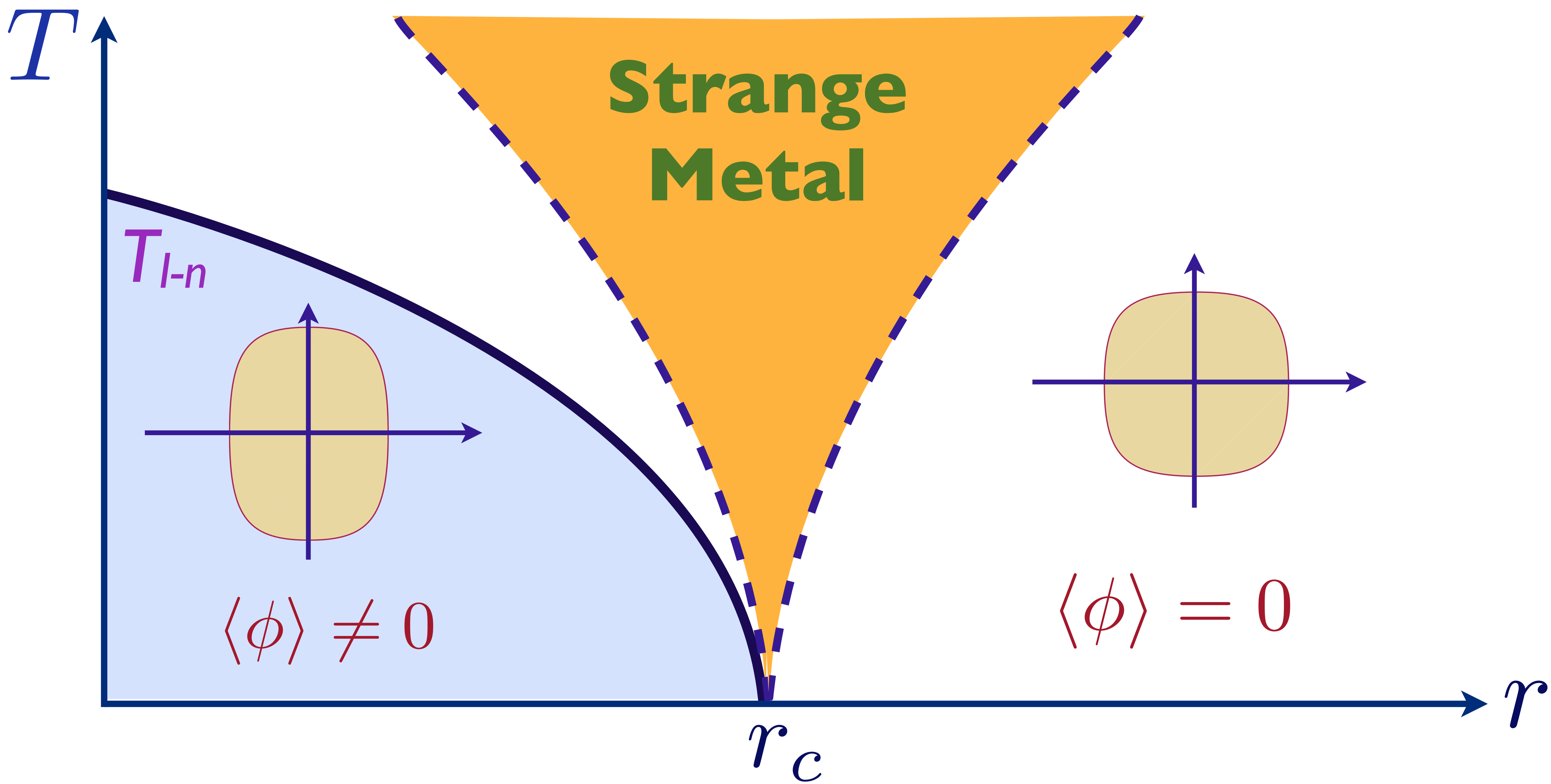

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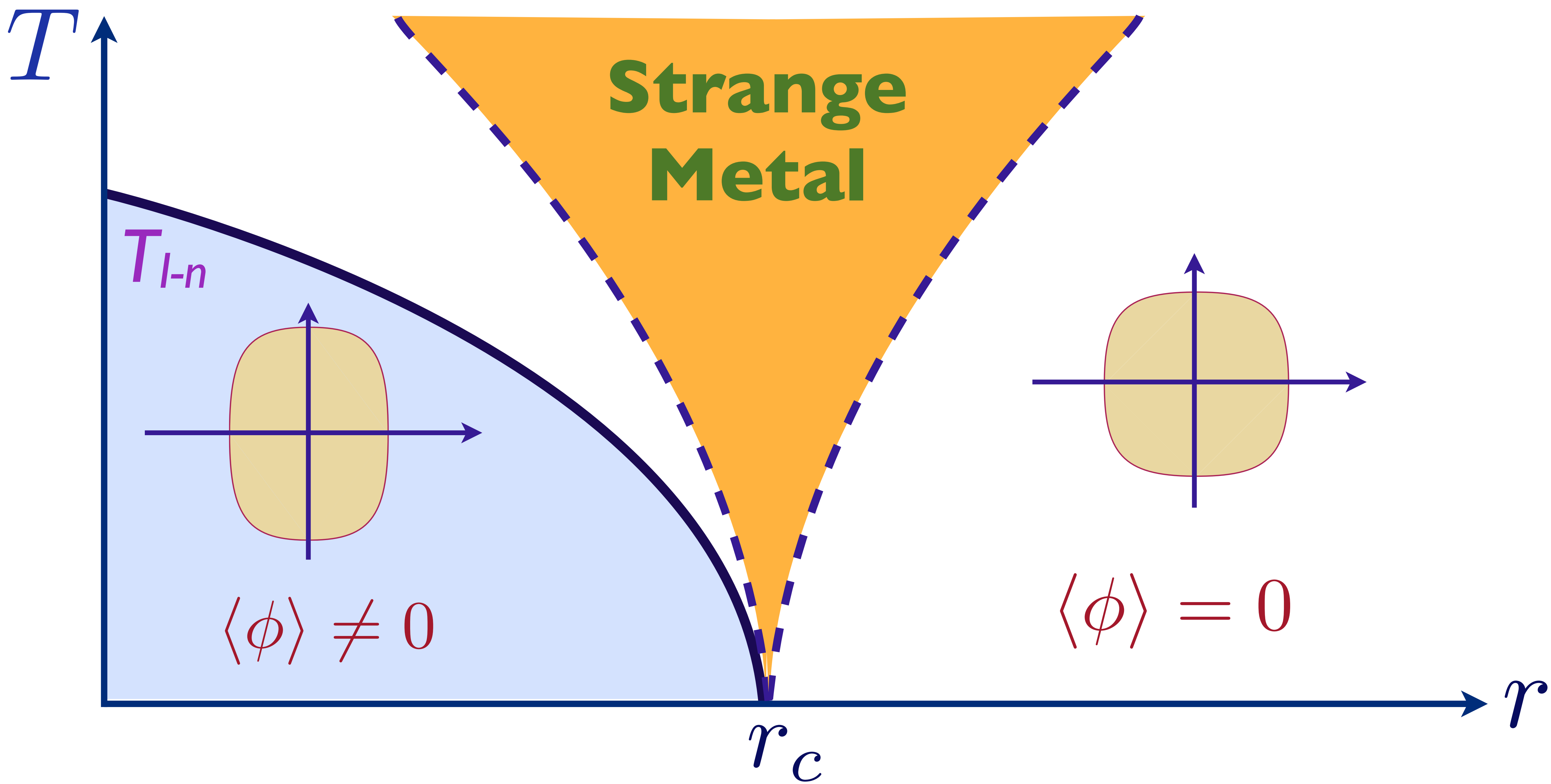
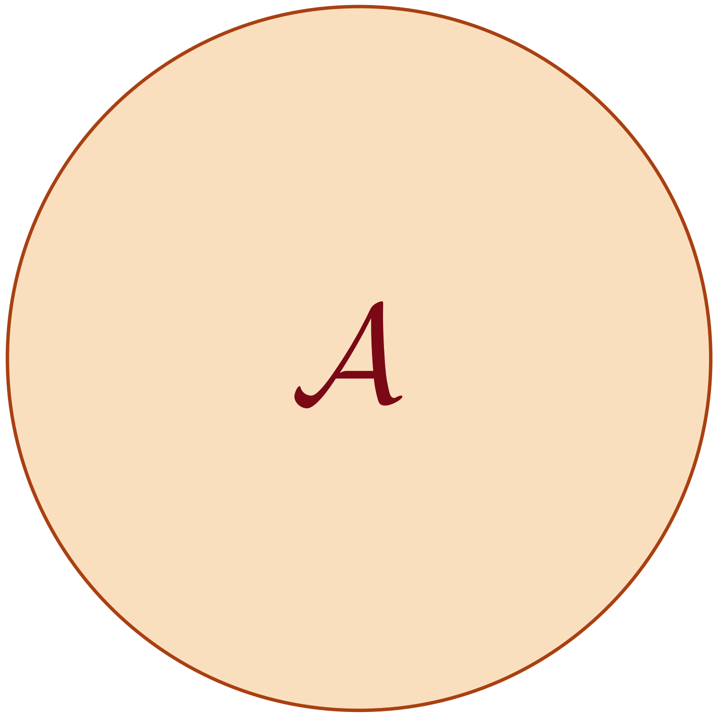


Phase diagram as a function of T and r

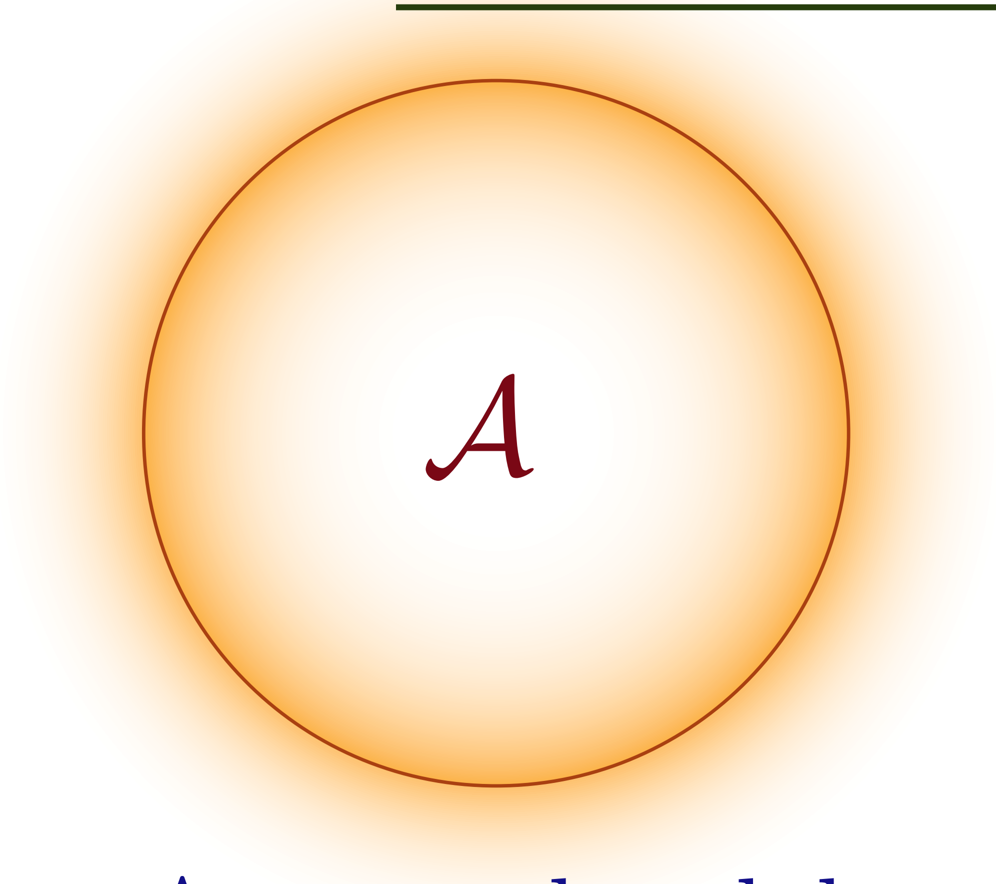
Low energy theory of this strange metal is essentially identical to that of a Fermi surface coupled to a gauge field

Fermi surface of an ordinary metal



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma$$

Fermions coupled to a gauge field



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - i A_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
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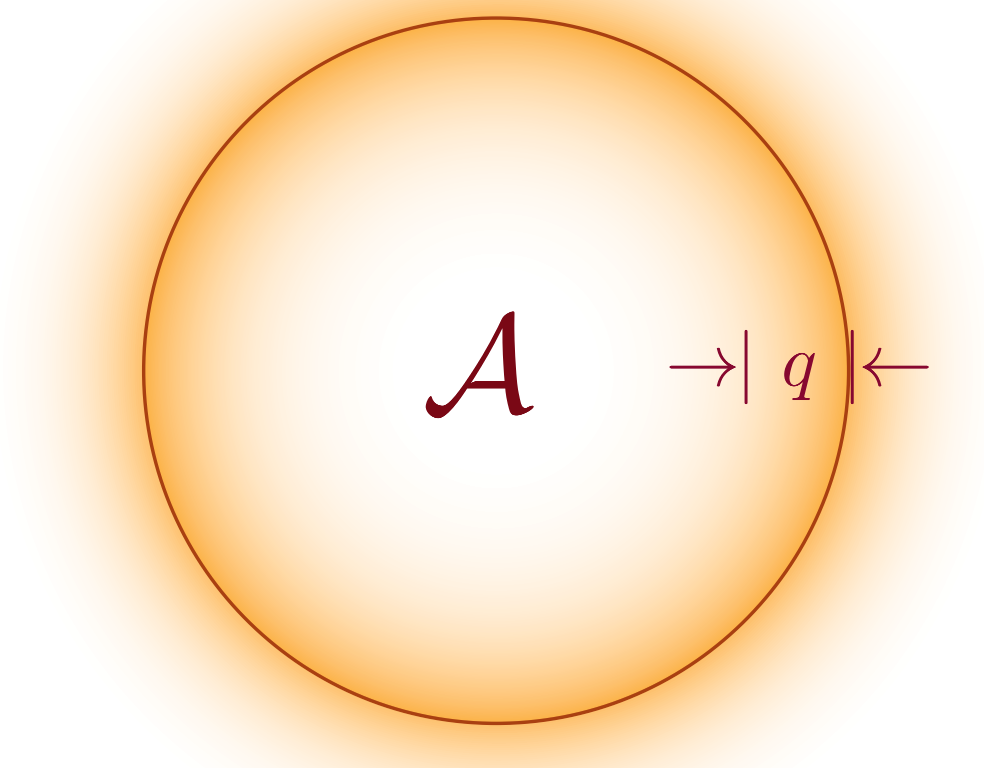

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- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

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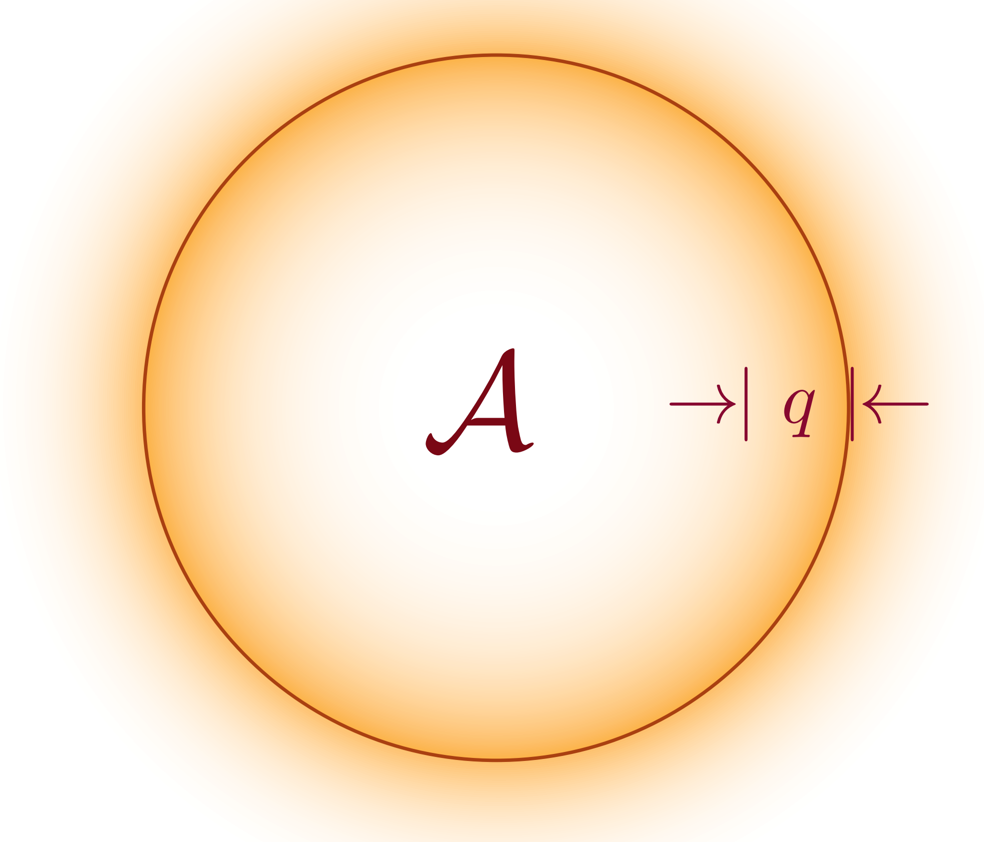

$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - i A_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

- Gauge-dependent Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.

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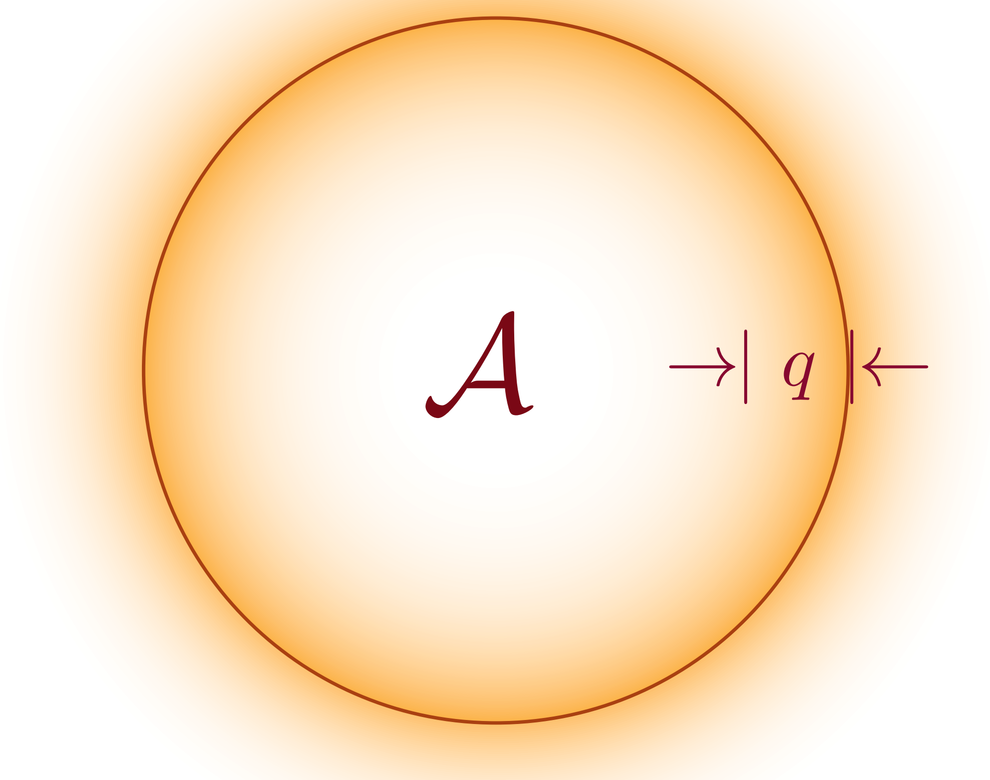

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- Gauge-dependent Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\rm eff}/z}$ with $d_{\rm eff} = 1$.

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M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
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Study the large N_c limit of a SU(N_c) Yang-Mills gauge field coupled to adjoint (matrix) fermions at a non-zero chemical potential

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- 'tHooft's argument for the large N_c limit of pure Yang-Mills theory applies unchanged. Feynman diagrams acquire a factor of $1/N_c^g$ where g is genus of the surface defined by the Feynman diagram.

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- The pairing instability to superconducting phases is subdominant in the $1/N_c$ expansion.

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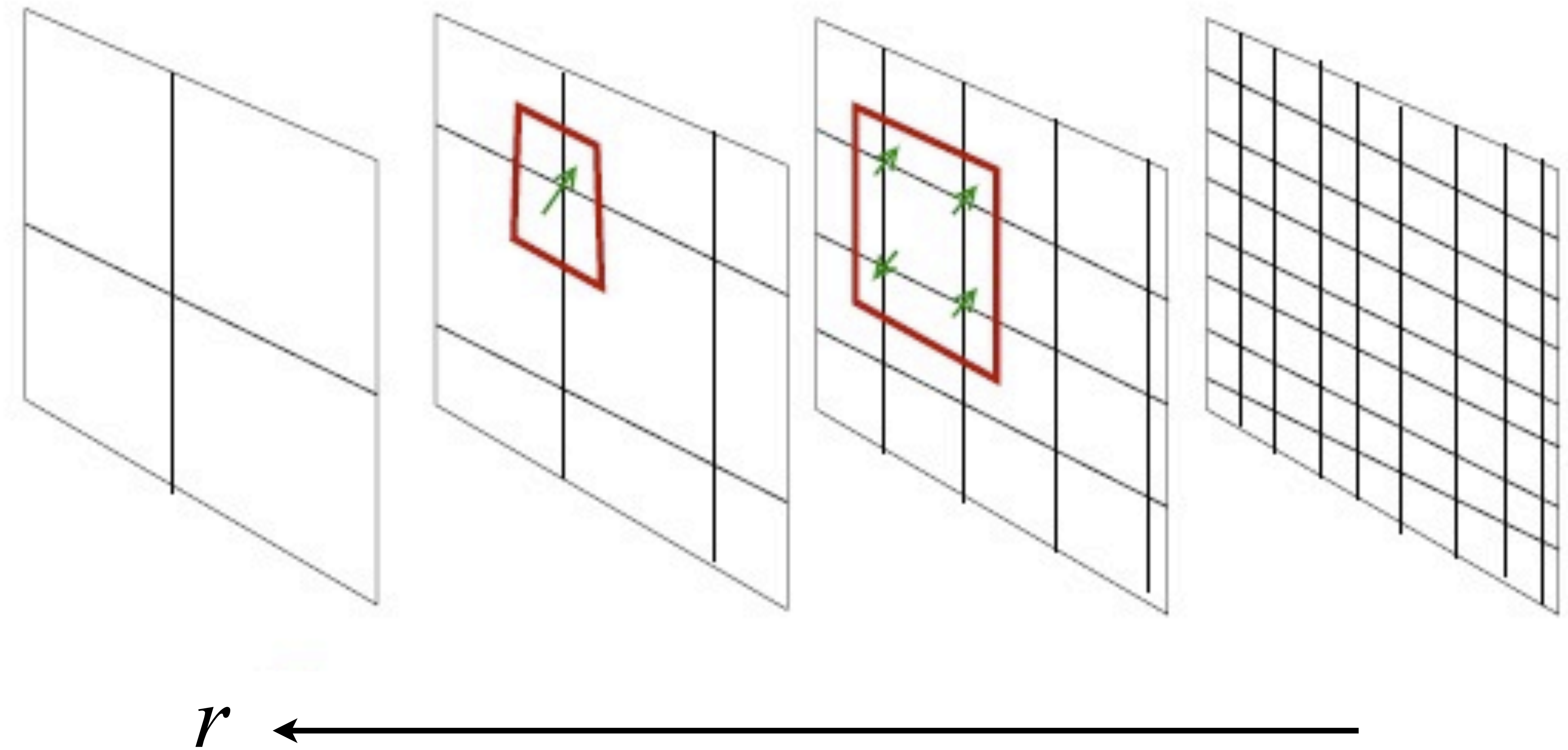
- 'tHooft's argument for the large N_c limit of pure Yang-Mills theory applies unchanged. Feynman diagrams acquire a factor of $1/N_c^g$ where g is genus of the surface defined by the Feynman diagram.
- The $N_c \to \infty$ limit requires summation of all "planar" graphs.
- There is Landau damping of gauge bosons, and non-Fermi liquid damping of fermions, in the planar limit, and hence the possibility of a deconfined compressible phase with a scale-invariant structure at low energies.
- The pairing instability to superconducting phases is subdominant in the $1/N_c$ expansion.
- We will now present a conjectured gravity dual of this theory.



J. McGreevy, arXiv0909.0518

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

Consider the following (most) general metric for the holographic theory

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

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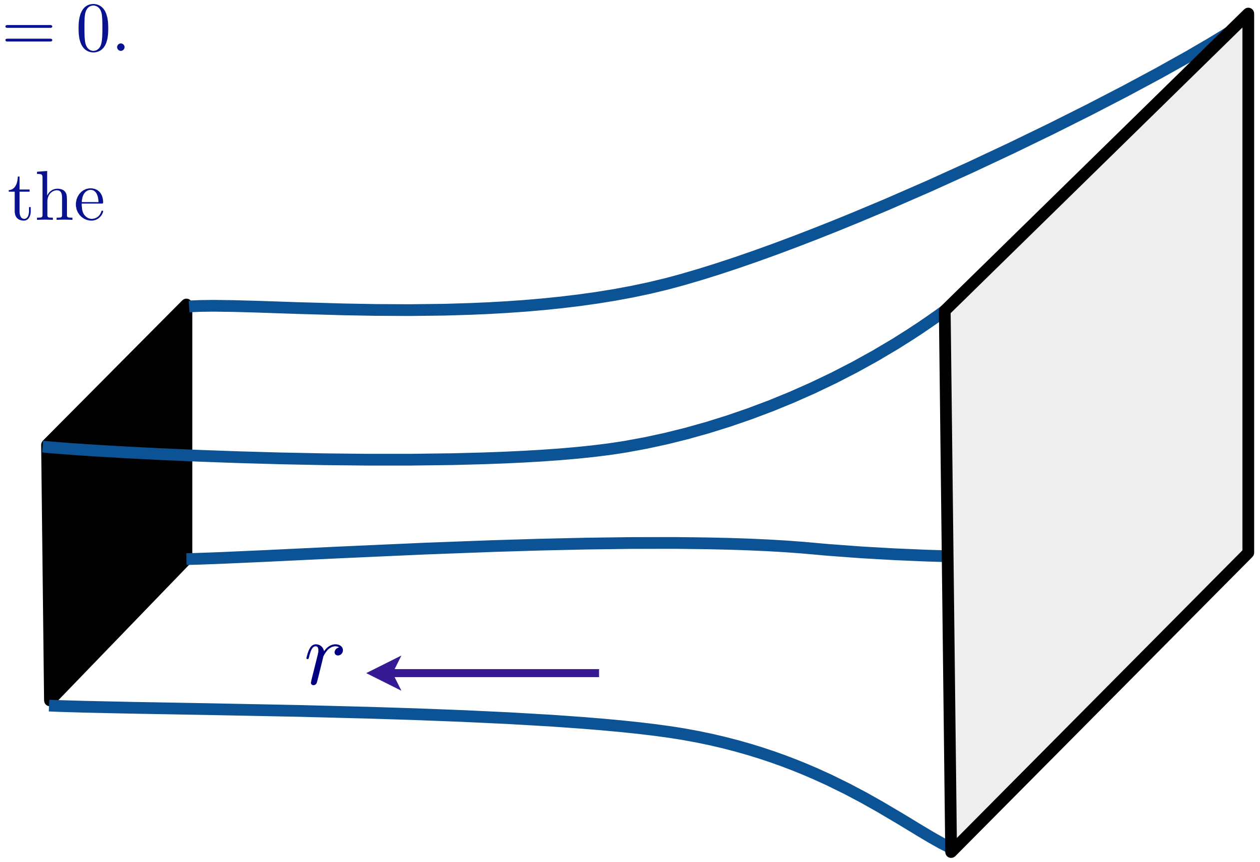
What is θ ? ($\theta = 0$ for "relativistic" quantum critical points).

At $T > 0$, there is a “black-brane” at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the “area” of the horizon, and so $S \sim r_h^{-d}$



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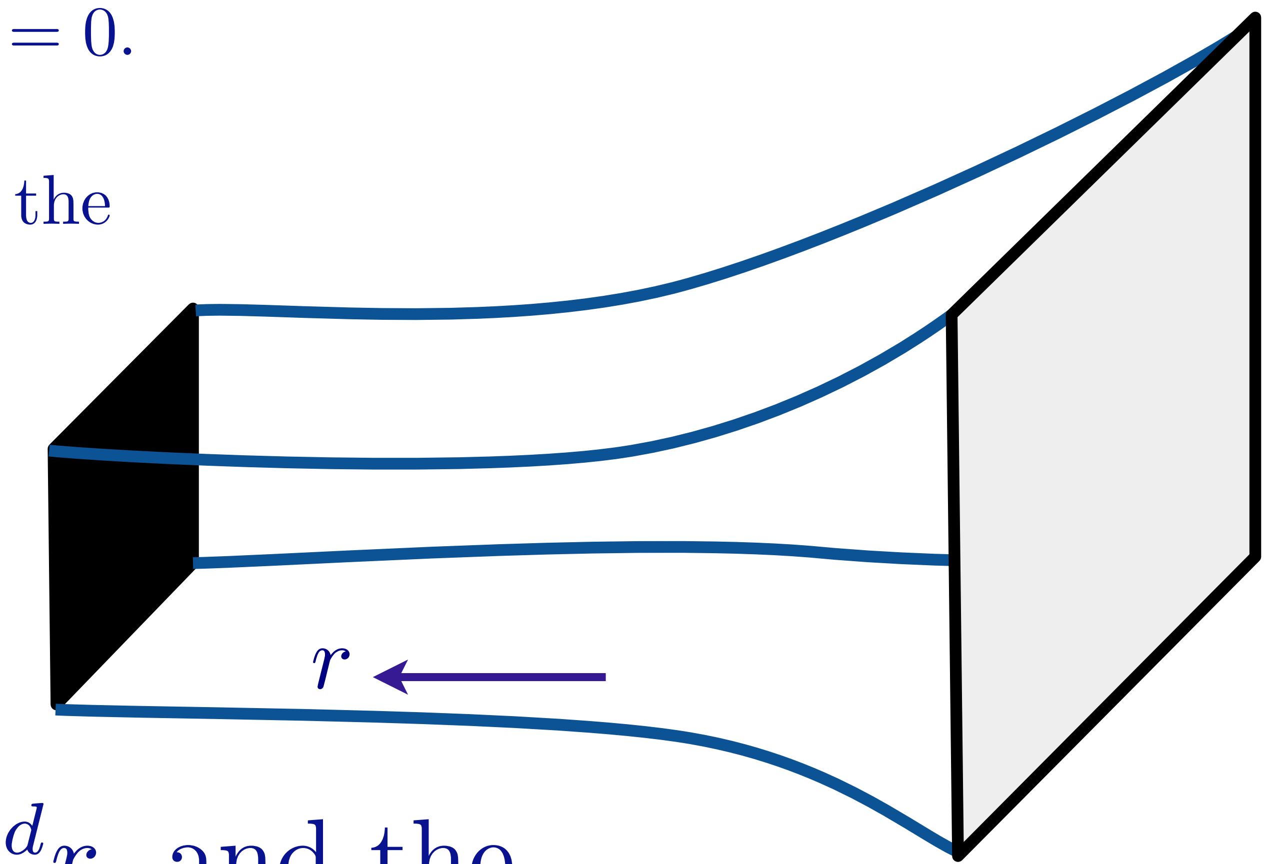


Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures "dimension deficit" in the phase space of low energy degrees of a freedom.

Holography of non-Fermi liquids

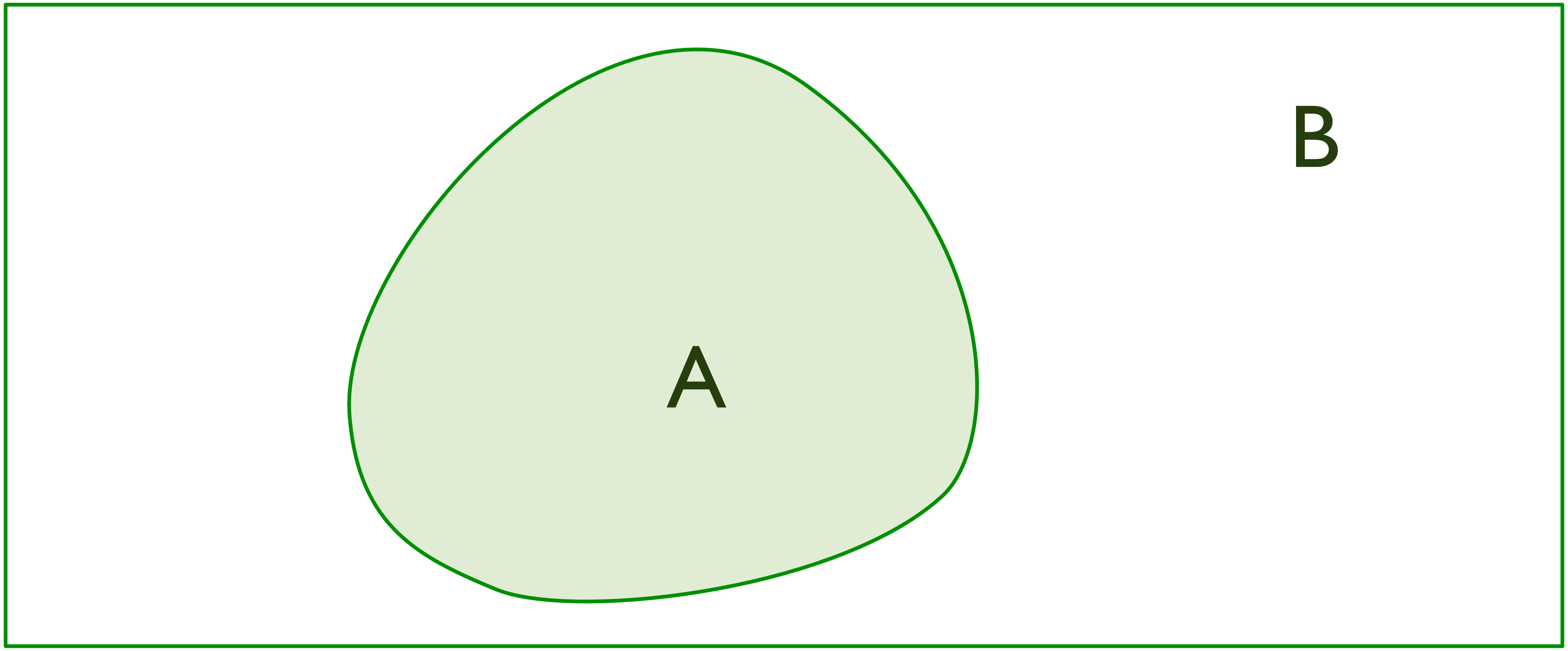
$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent z, with entropy density $\sim T^{1/z}$. So we expect compressible quantum states to have $d_{\text{eff}} = 1$, or

$$\theta = d - 1$$

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Entanglement entropy

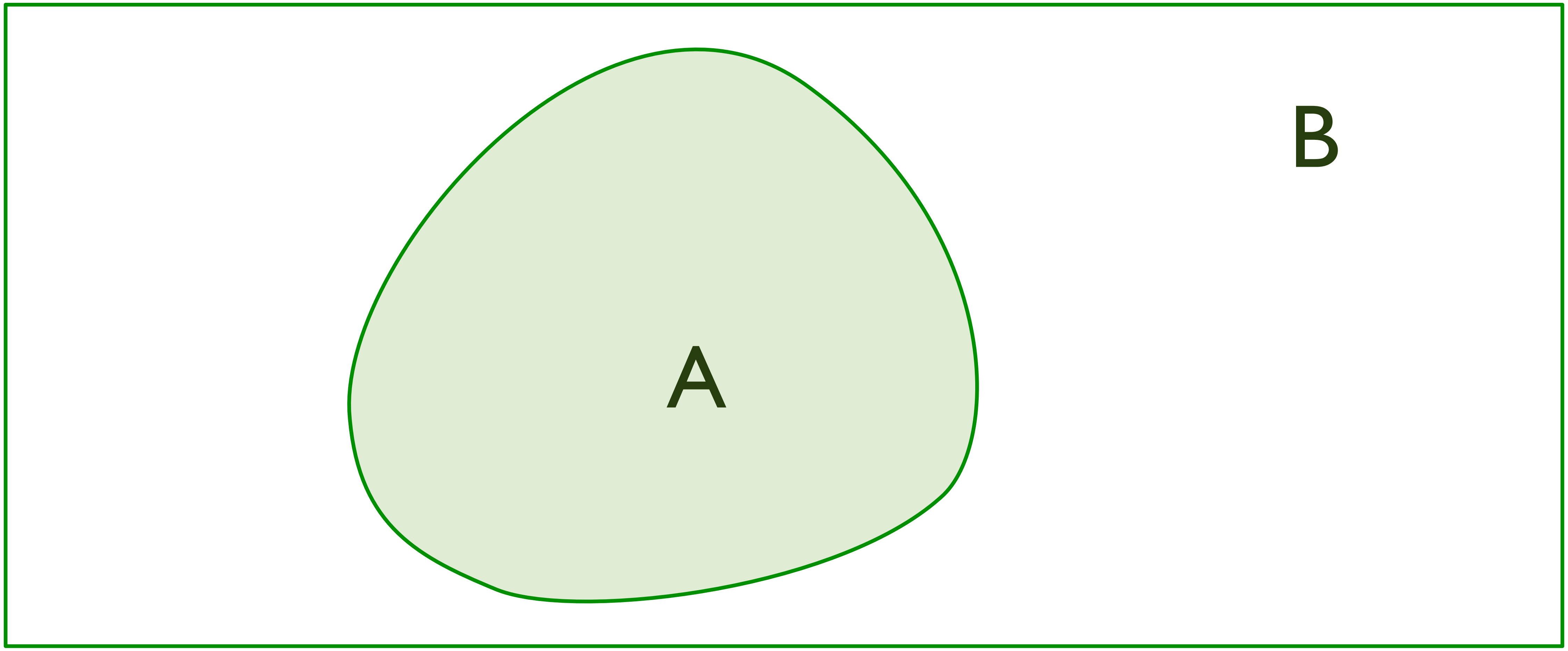


Measure strength of quantum entanglement of region A with region B.

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

Entanglement entropy $S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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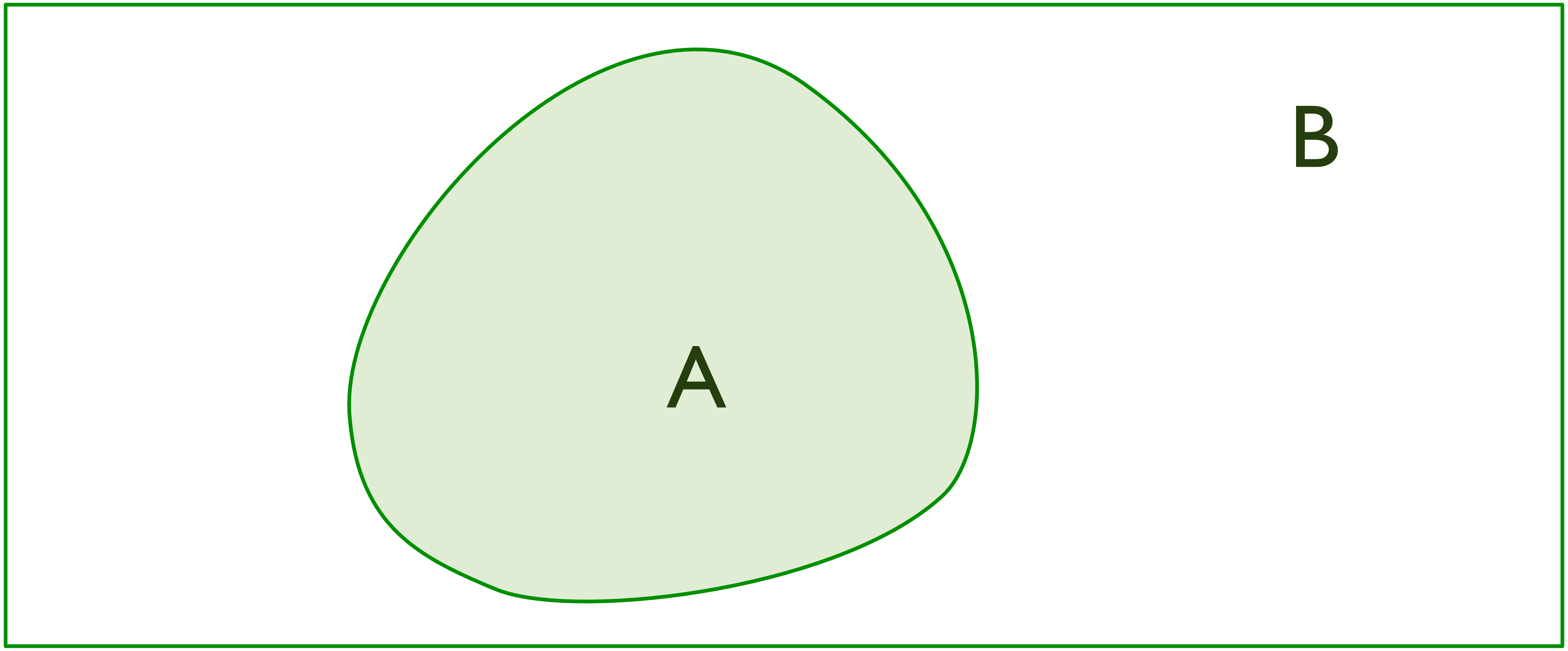

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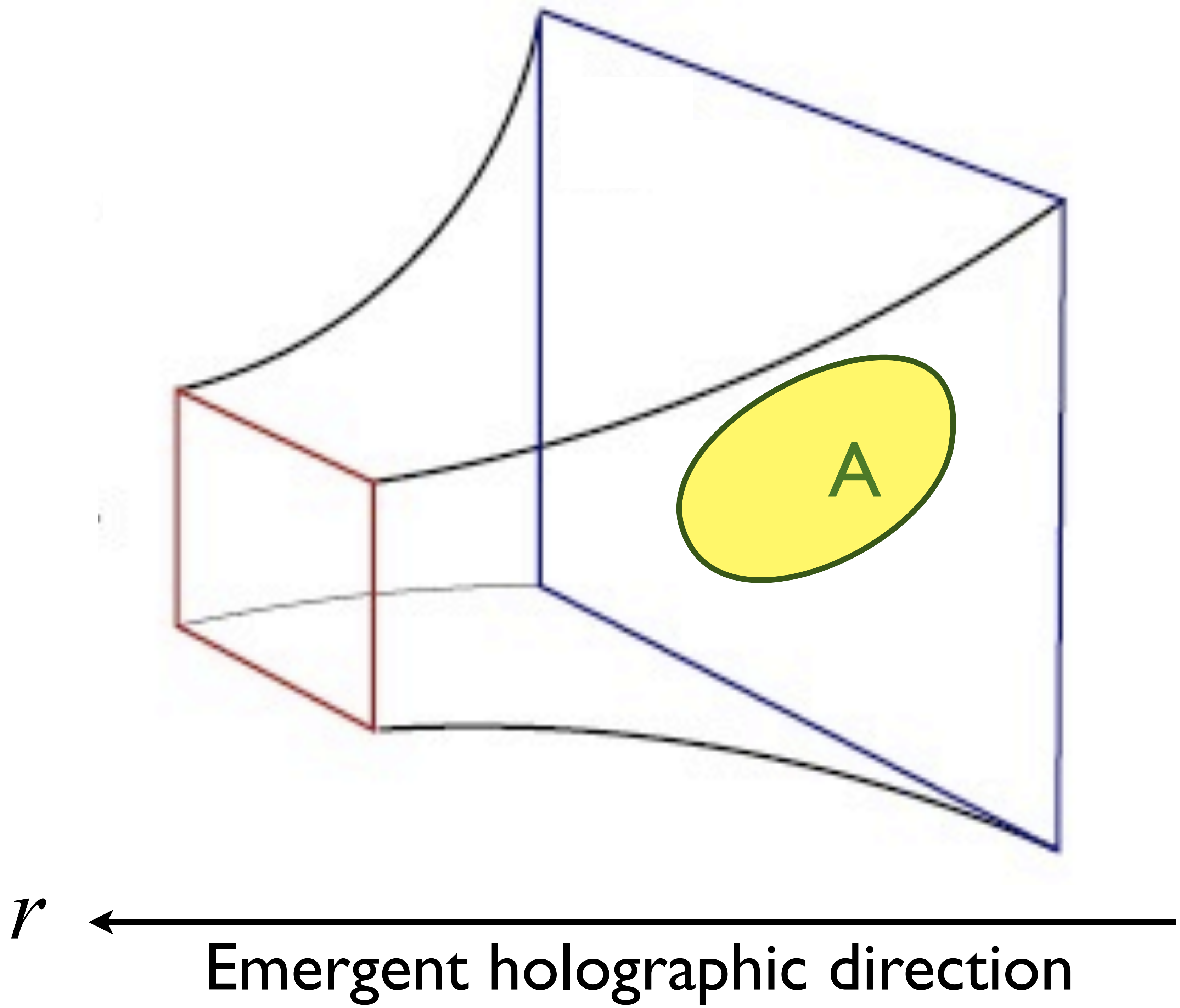
Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Holographic entanglement entropy



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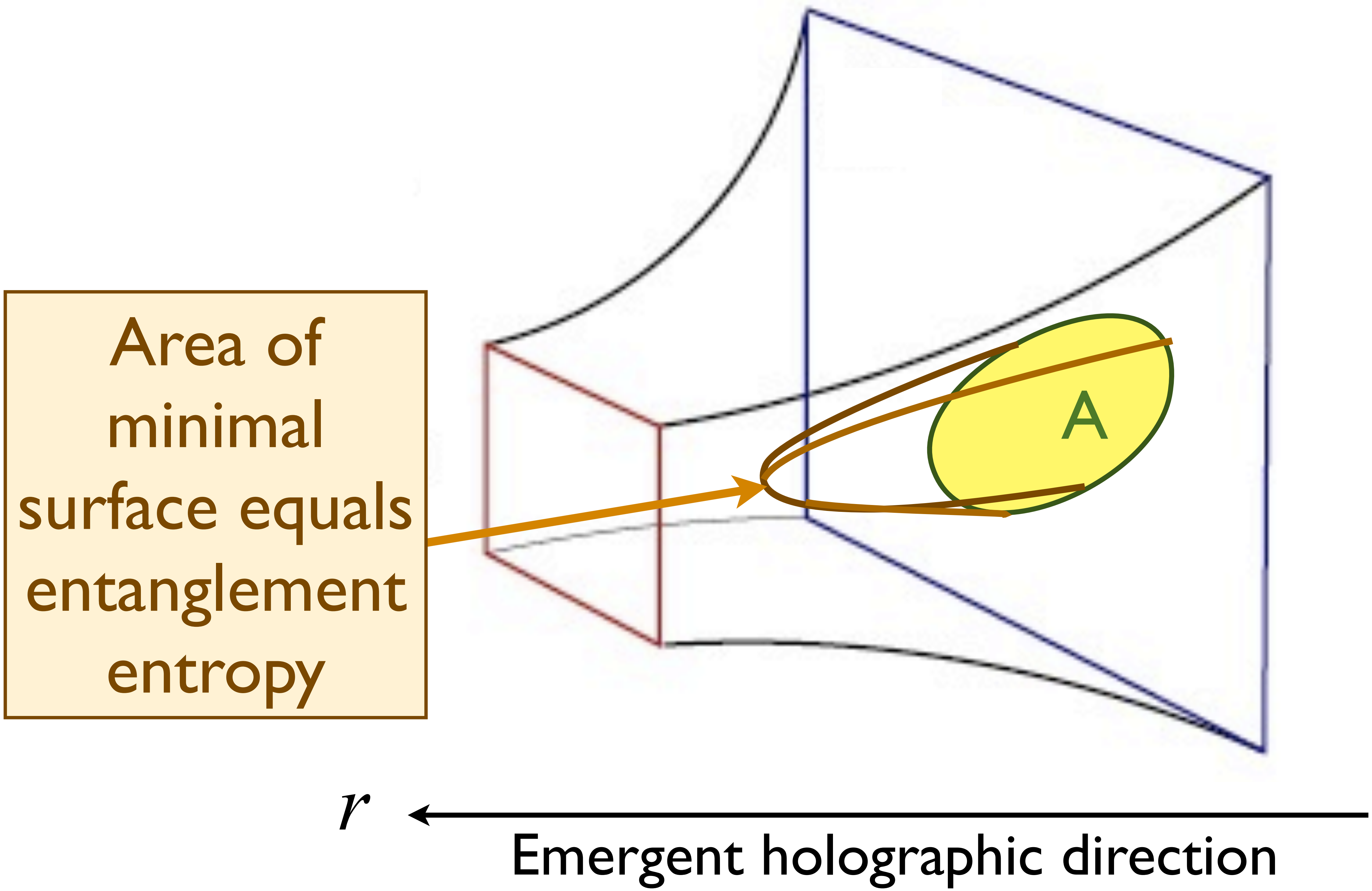

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

Holography of non-Fermi liquids

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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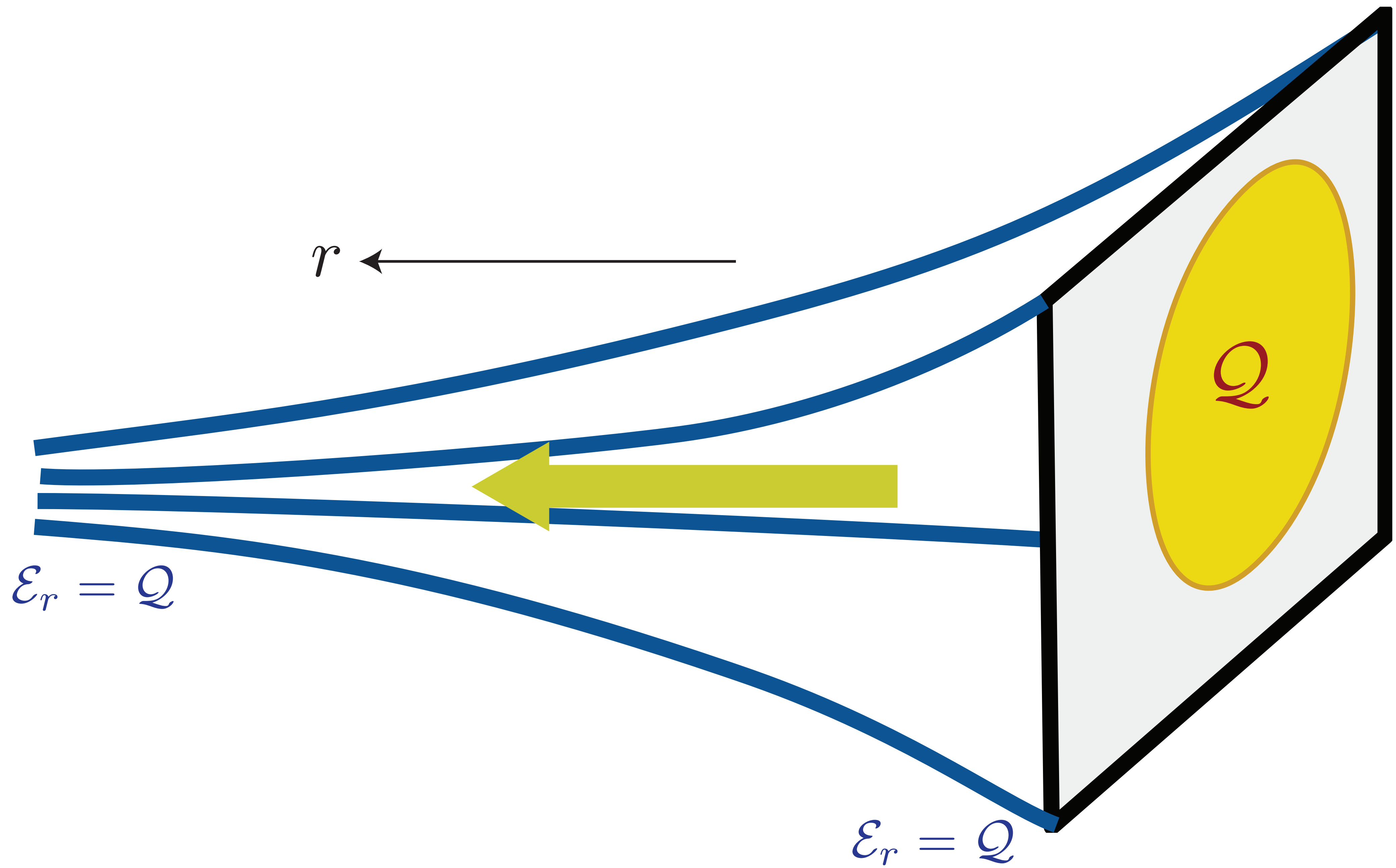
K. Goldstein, S. Kachru, S. Prakash, and S. P. Trivedi JHEP **1008**, 078 (2010)

Holographic theory of a non-Fermi liquid (NFL)



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L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The two methods of deducing with fermion density, from the electric flux as $r \to \infty$ and from the entanglement entropy, are consistent with the Luttinger relation !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Inequalities

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The area law of entanglement entropy is obeyed for

$$\theta \leq d - 1.$$

The "null energy condition" of the gravity theory yields

$$z \geq 1 + \frac{\theta}{d}.$$

Remarkably, for $d = 2$, $\theta = d - 1$ and $z = 1 + \theta/d$, we obtain $z = 3/2$, the same value associated with the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
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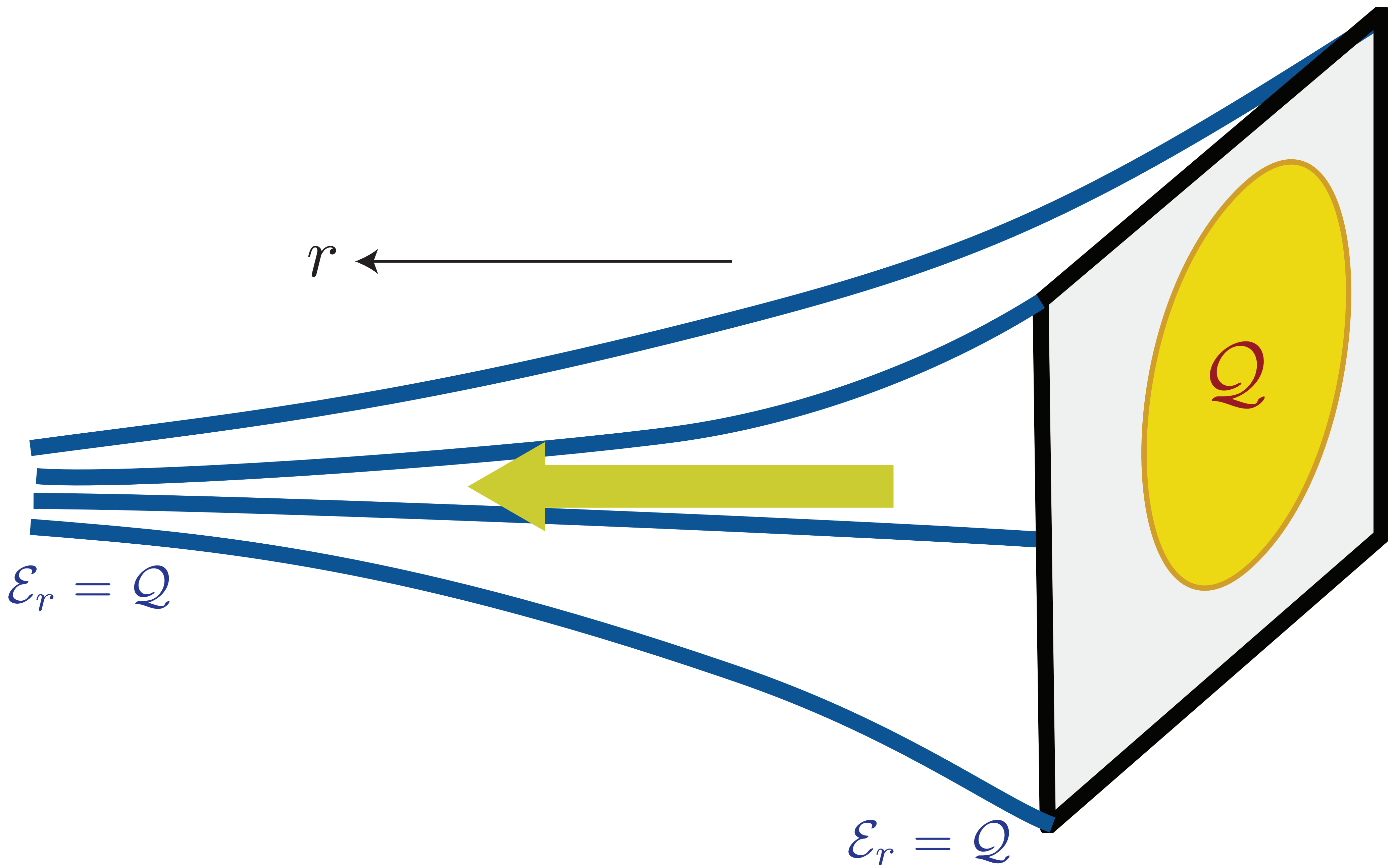
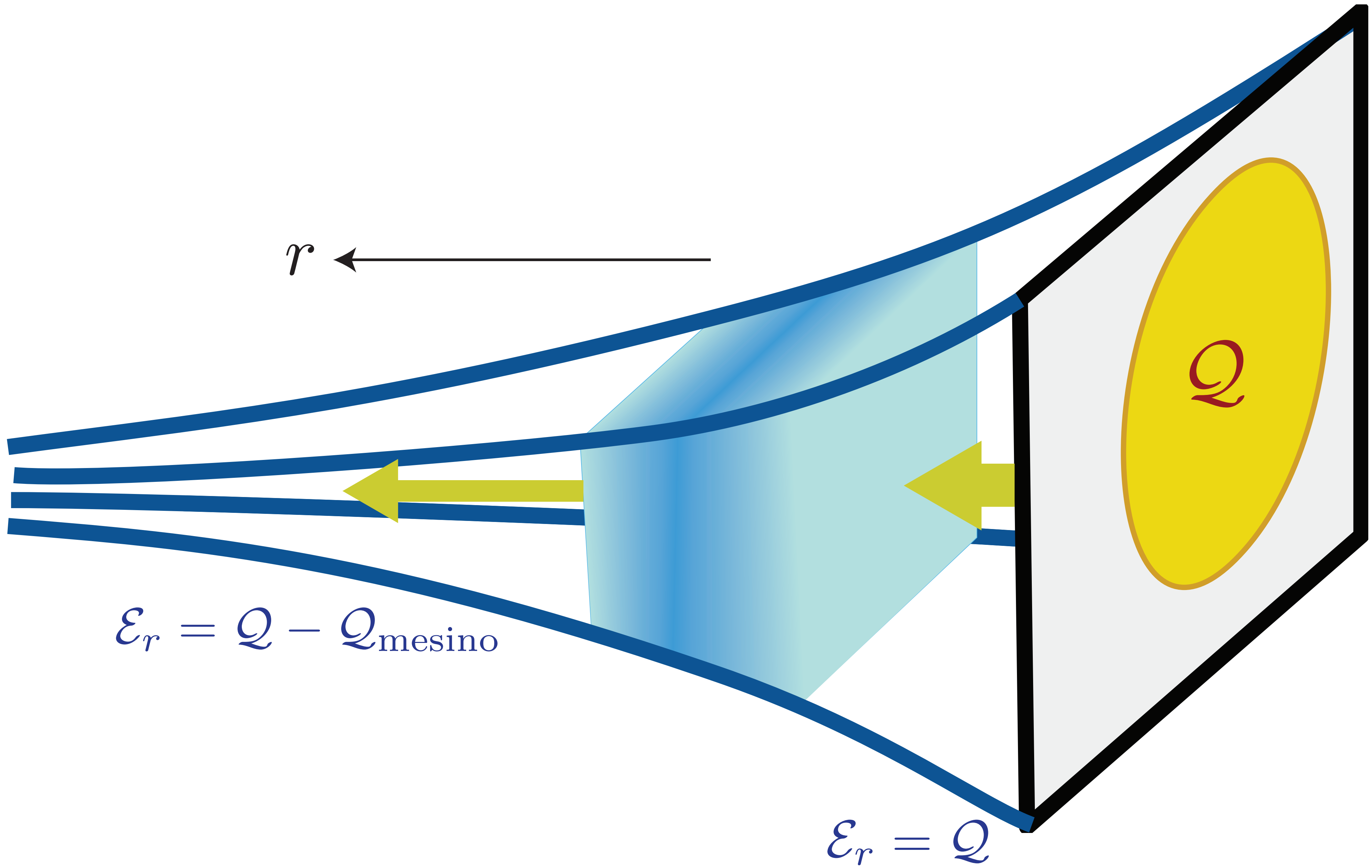


Gauss Law in the bulk

$\Leftrightarrow$ Luttinger theorem on the boundary

Holographic theory of a fractionalized-Fermi liquid (FL*)



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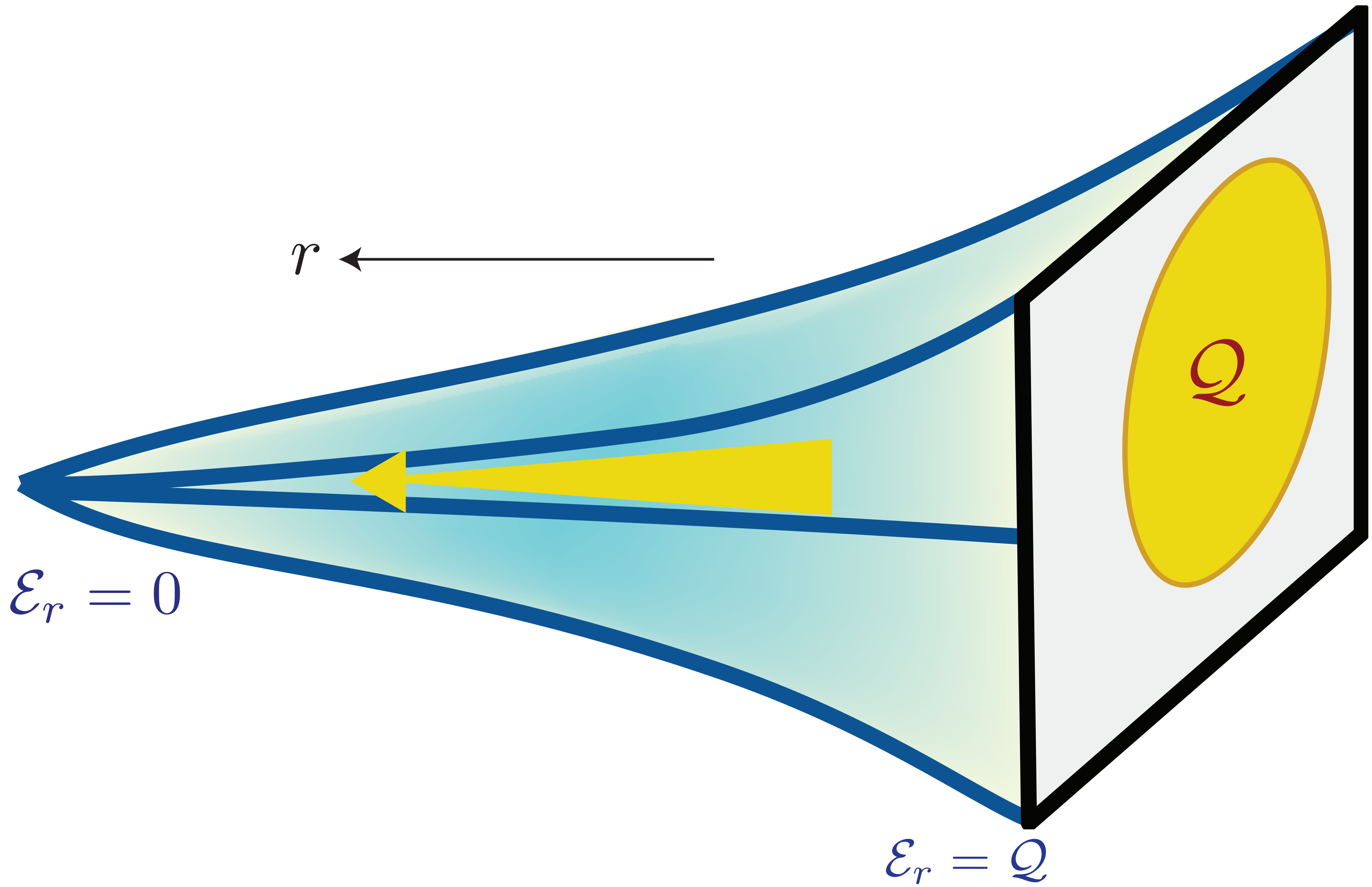

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Theory of a non-Fermi liquid (NFL)

Field theory	Holography
A gauge-dependent Fermi surface of overdamped gapless fermions.	Fermi surface is hidden.

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Thermal entropy density $S \sim T^{1/z}$ in $d = 2$, where z is the dynamic critical exponent.	Thermal entropy density $S \sim T^{1/z}$ in all d for hyperscaling violation exponent $\theta = d - 1$, and z the dynamic critical exponent.

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Logarithmic violation of area law of entanglement entropy, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.	Logarithmic violation of area law of entanglement entropy for $\theta = d - 1$, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.

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Gauge neutral 'mesinos' reduce the volume enclosed by Fermi surfaces of gauge-charged fermions to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.	Gauge neutral 'mesinos' reduce the volume enclosed by hidden Fermi surfaces to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.

1. Strange metals

A. Field theory

B. Holography

2. The superfluid-insulator quantum phase transition

A. Field theory

B. Holography

1. Strange metals

A. Field theory

B. Holography

2. The superfluid-insulator quantum phase transition

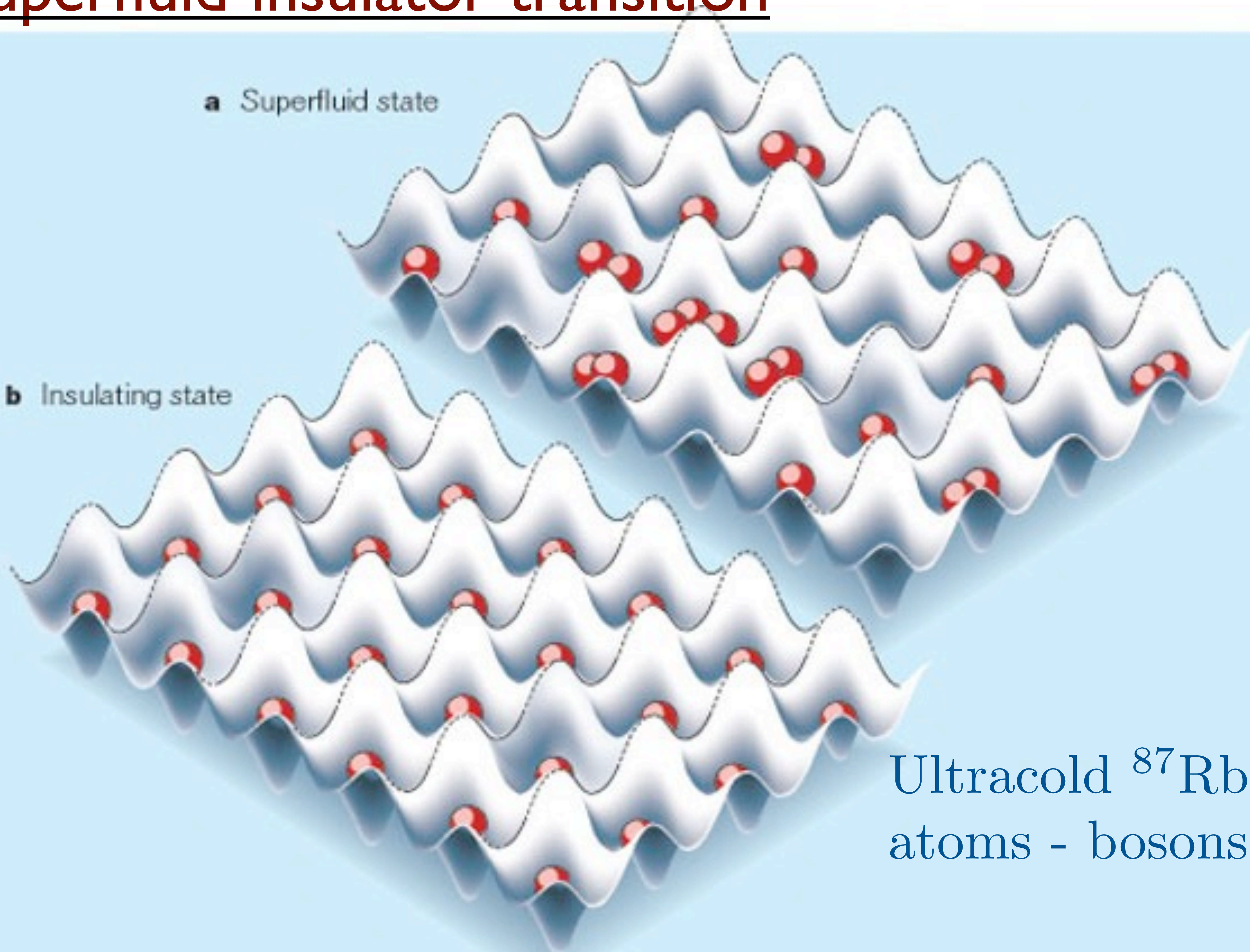
A. Field theory

B. Holography

Rob Myers

Ajay Singh

Superfluid-insulator transition



Ultracold ^{87}Rb
atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

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A. Field theory

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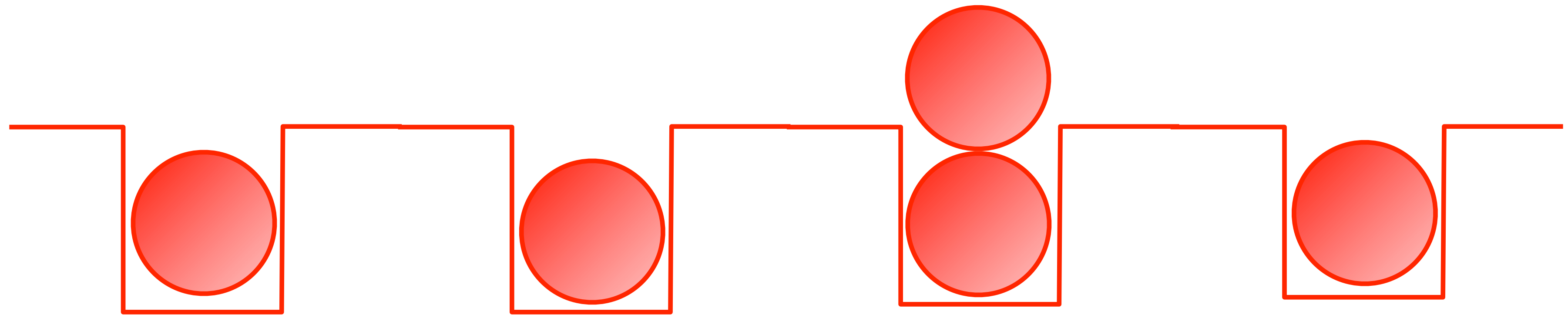
Excitations of the insulator:



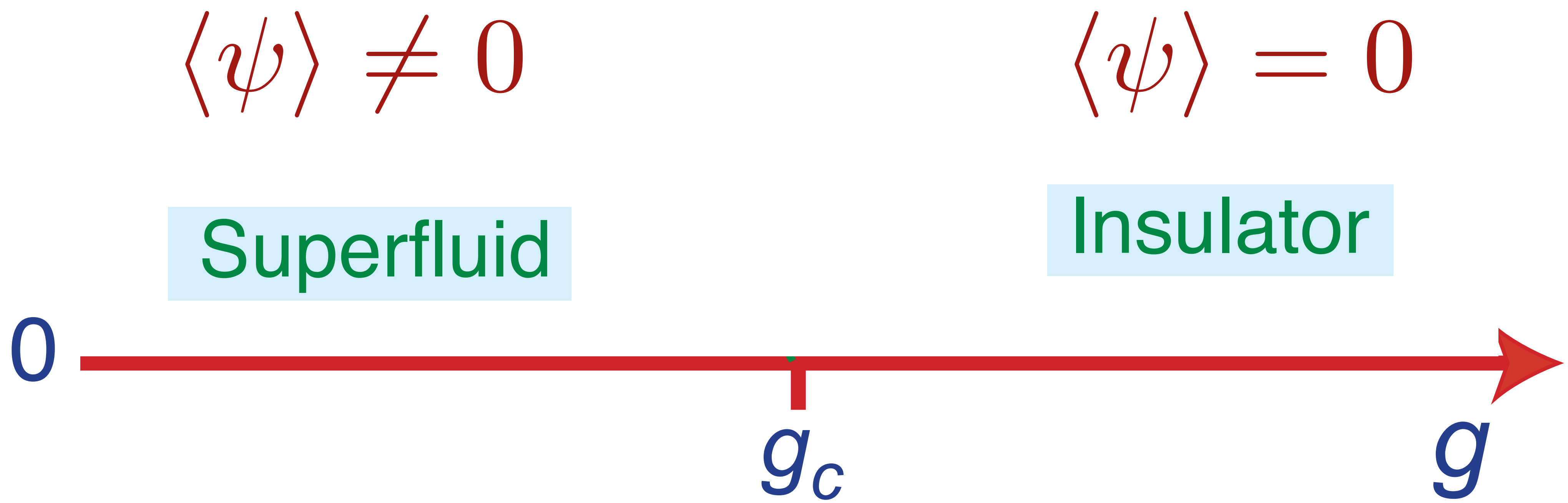
Particles ~ $\psi^\dagger$

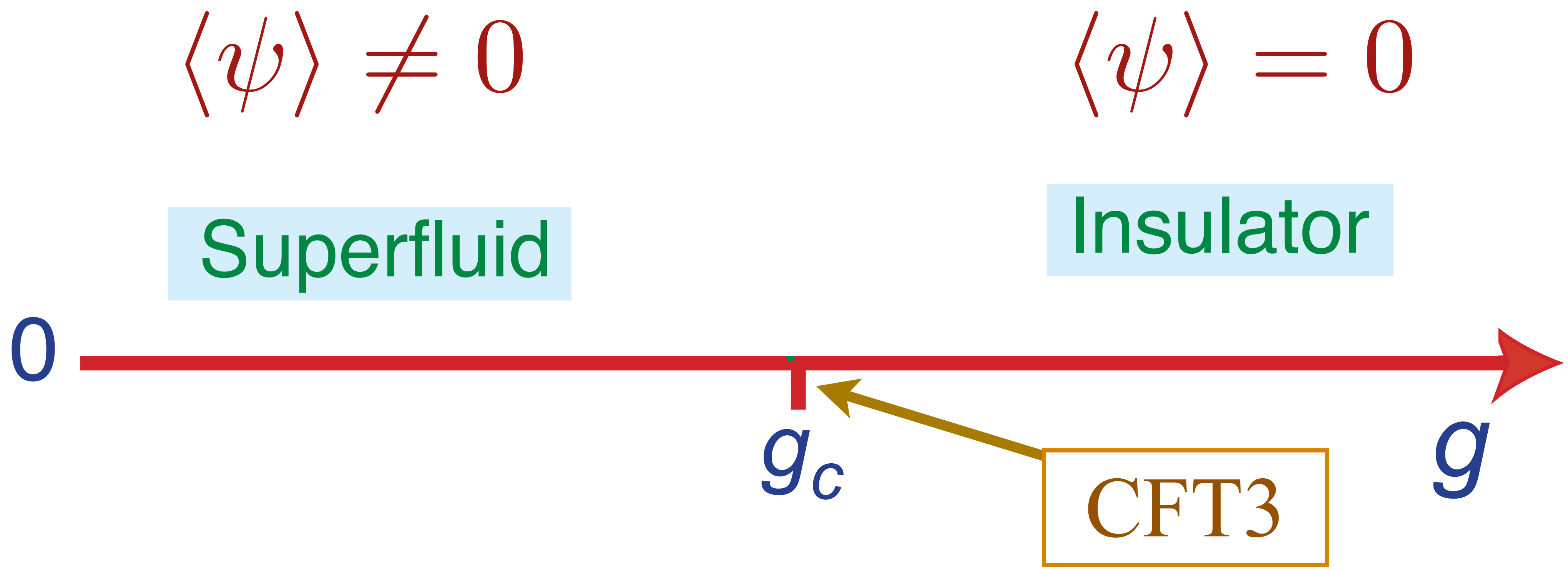
Holes ~ ψ

Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

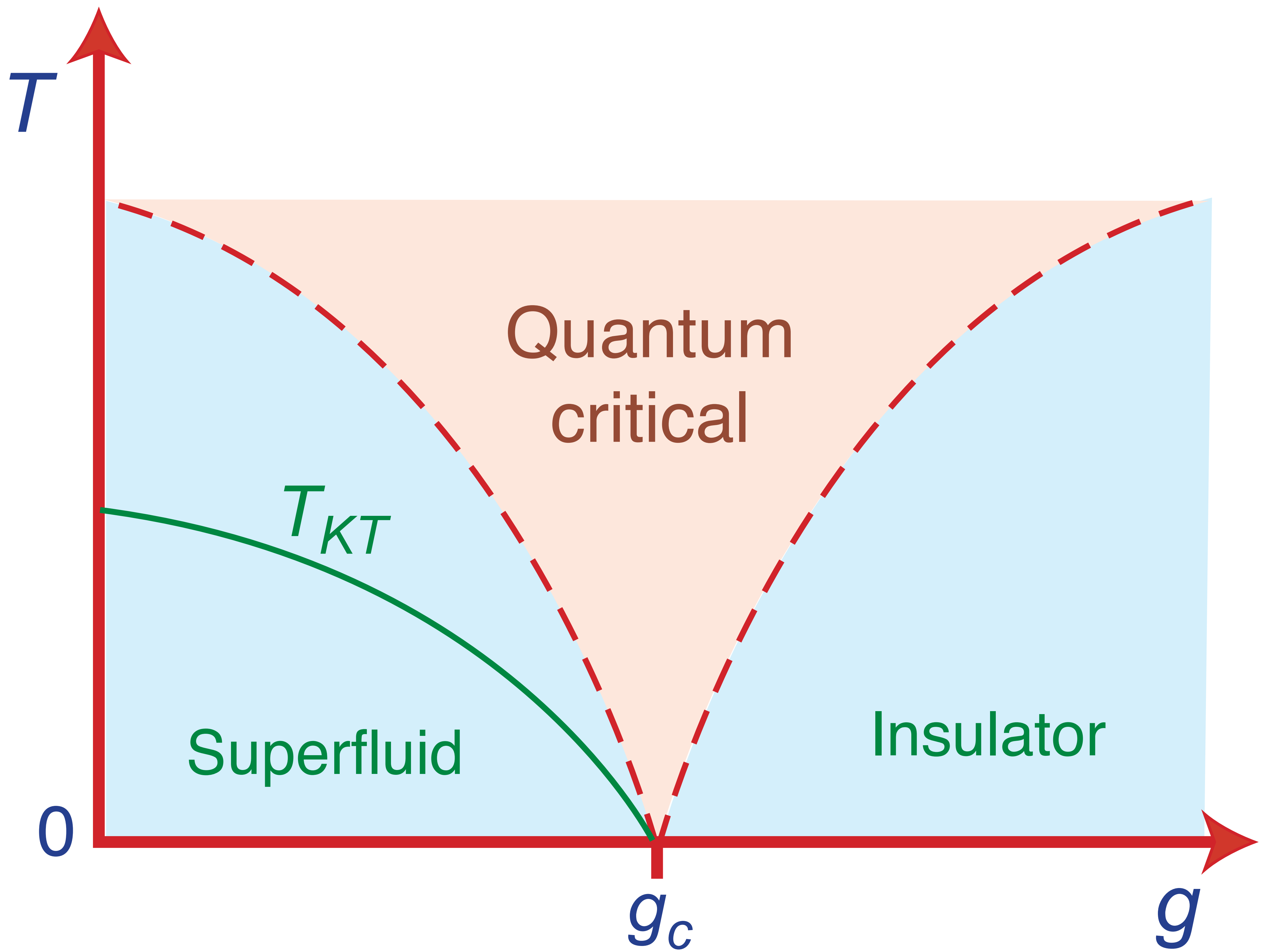
$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

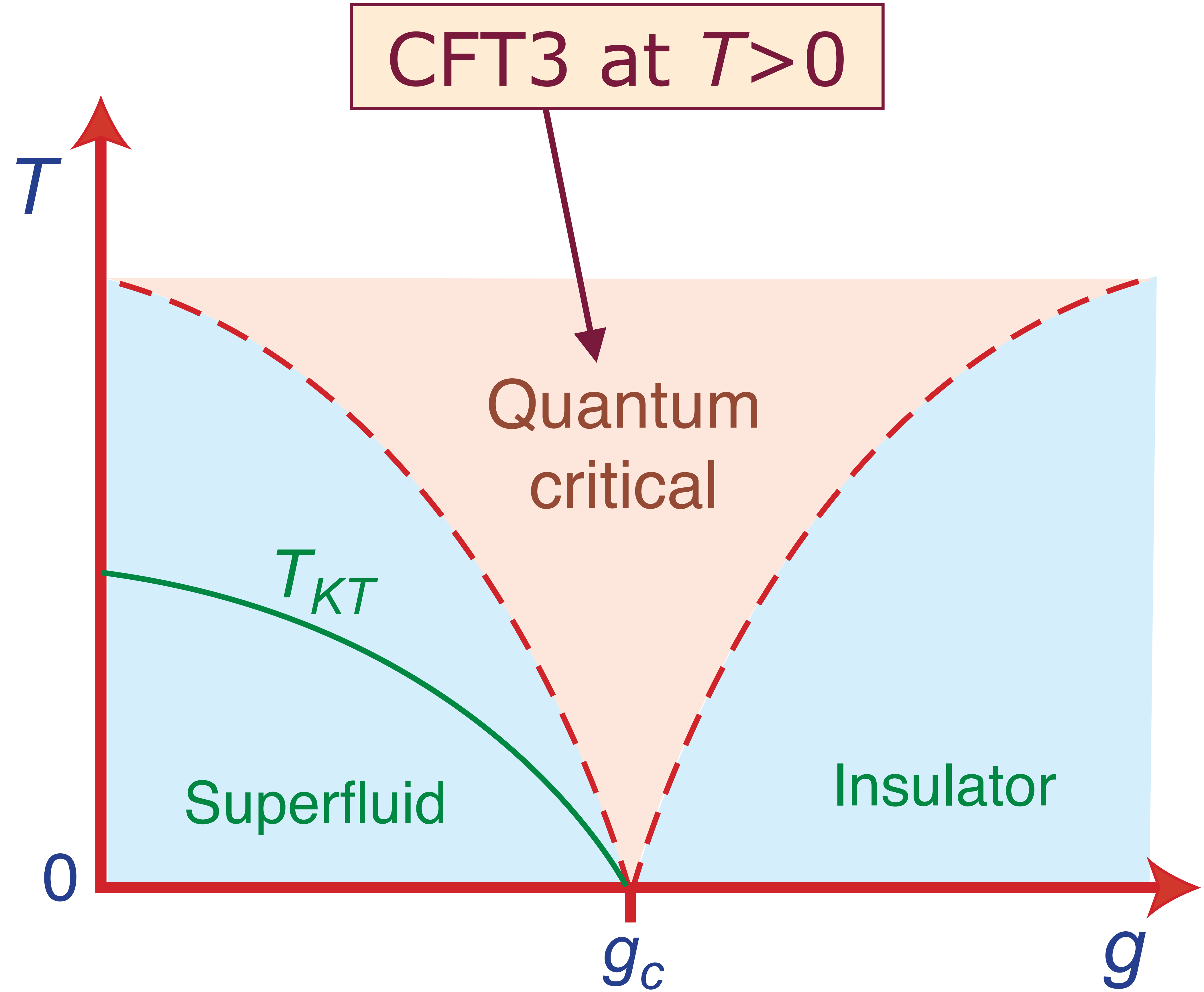

$\langle\psi\rangle \neq 0$
$\langle\psi\rangle = 0$
Superfluid
Insulator
0
g_c
g



$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
CFT3
g



T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g


CFT3 at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, τ_{eq}

$$\tau_{\text{eq}} = \mathcal{C}\,\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$

$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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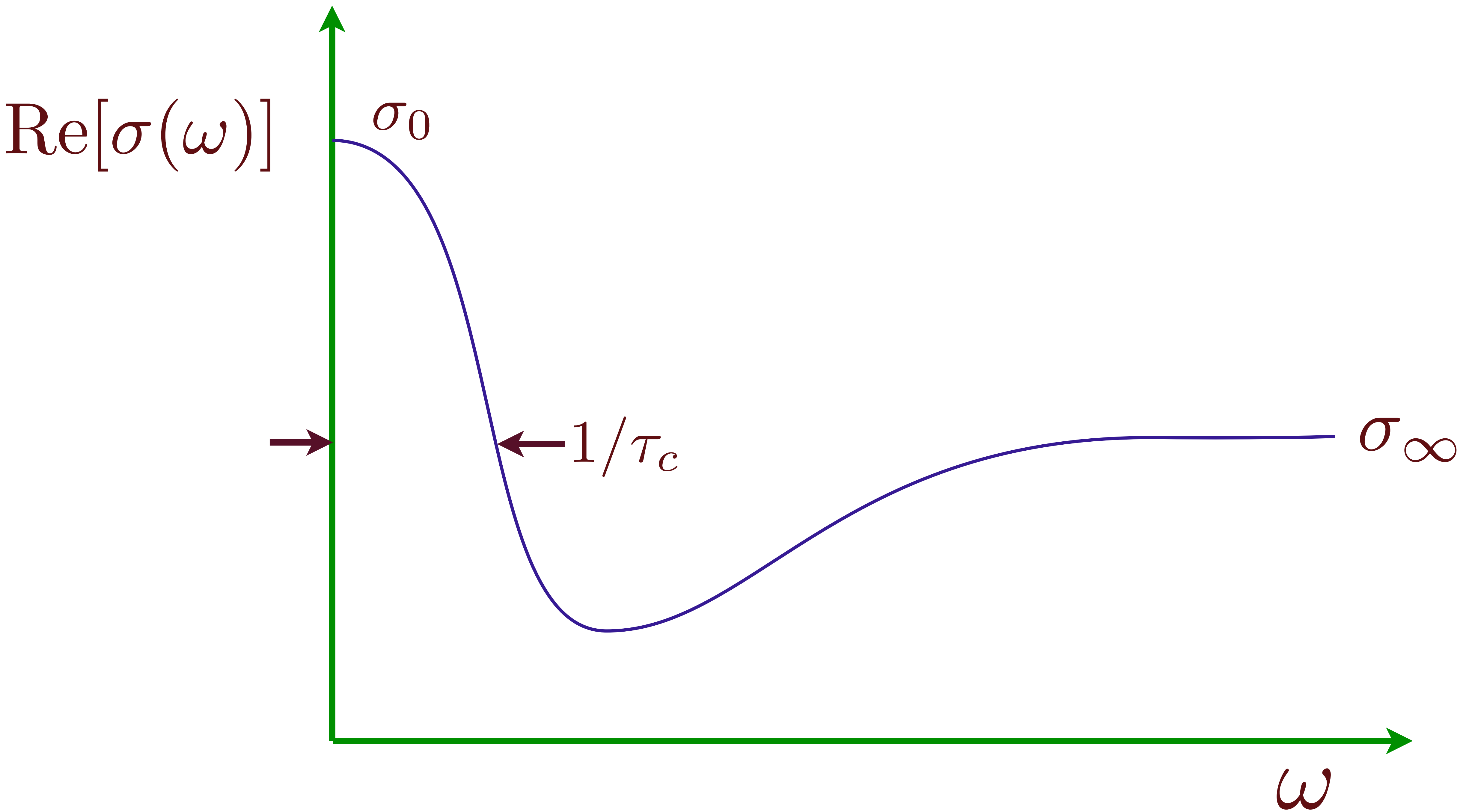
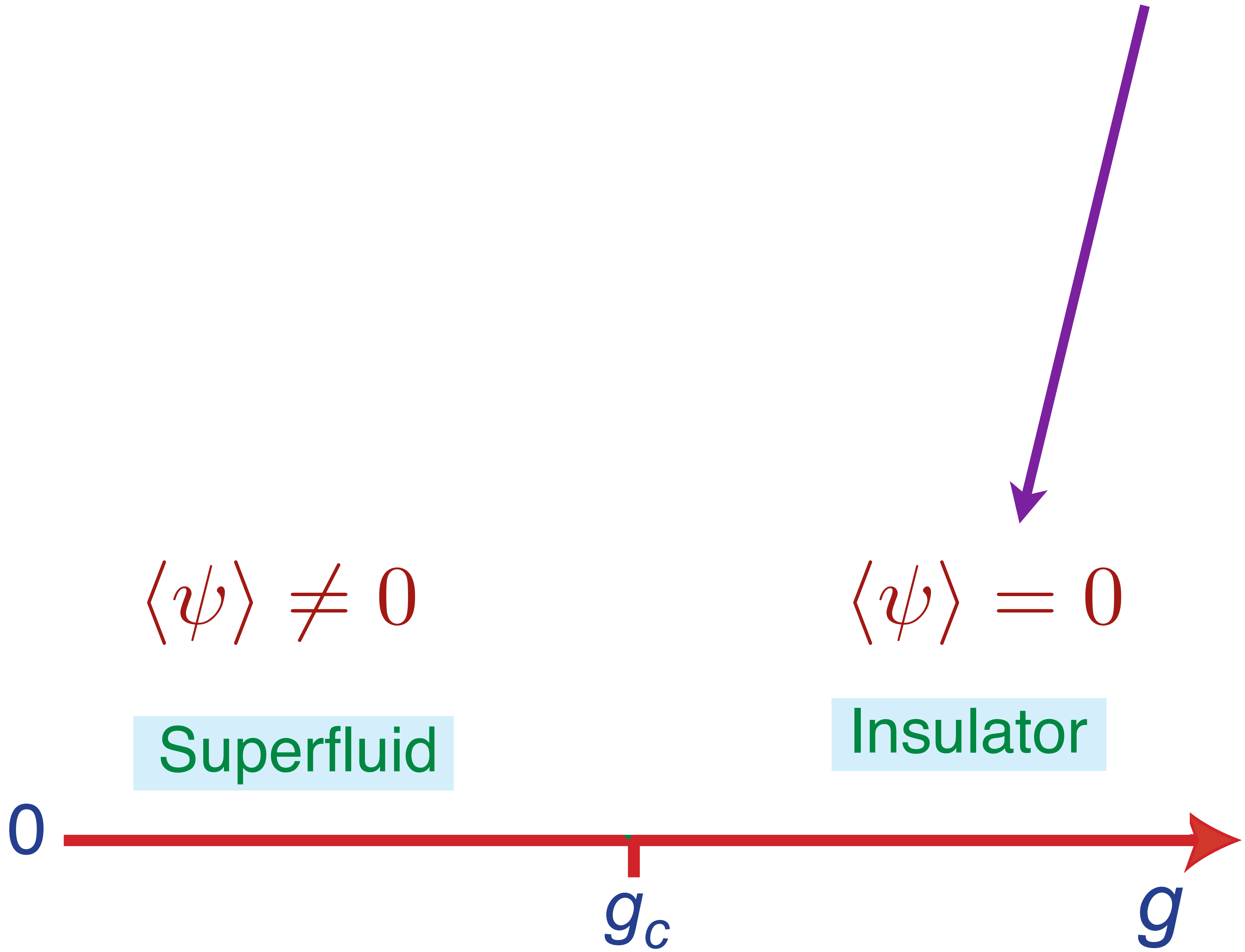
Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

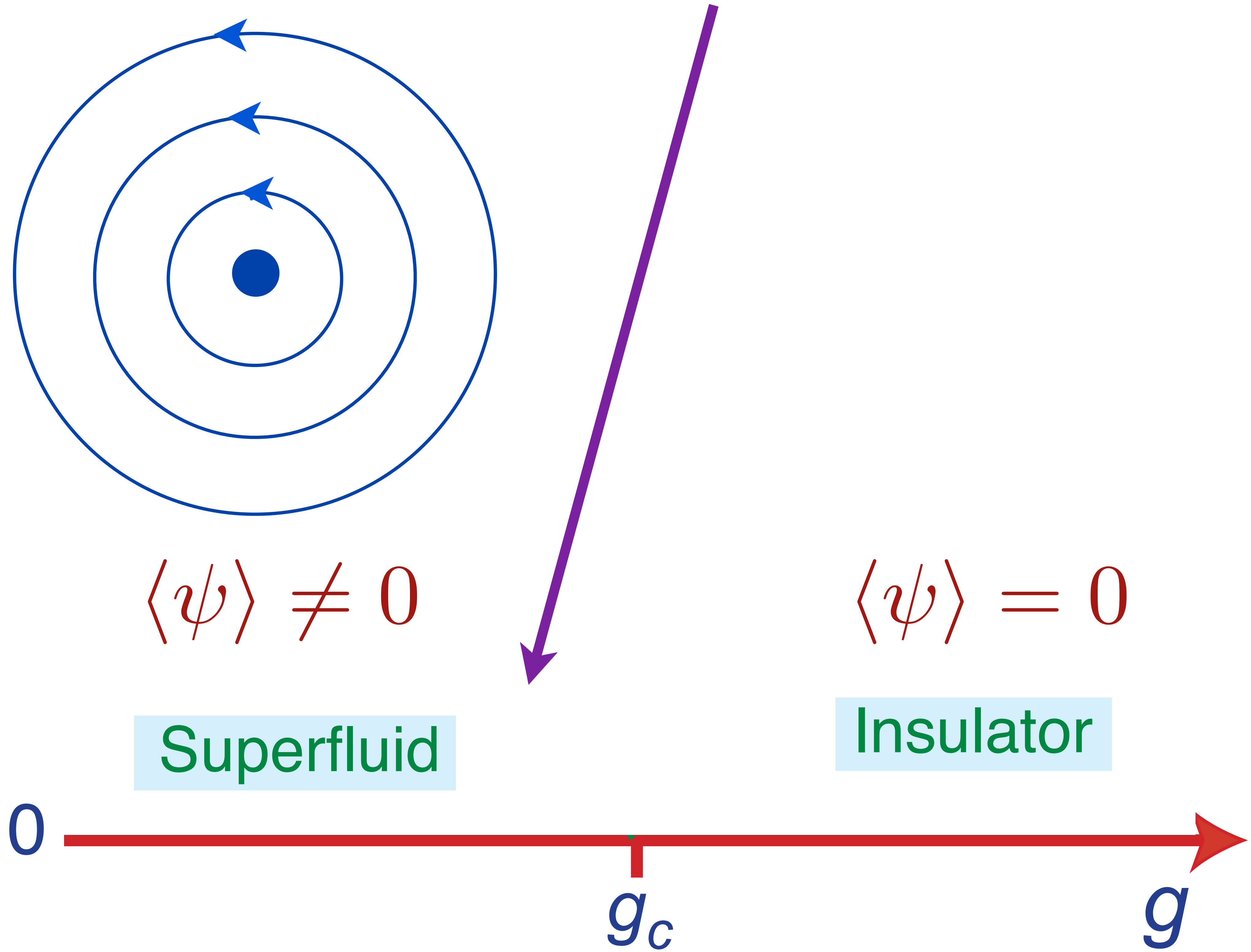
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



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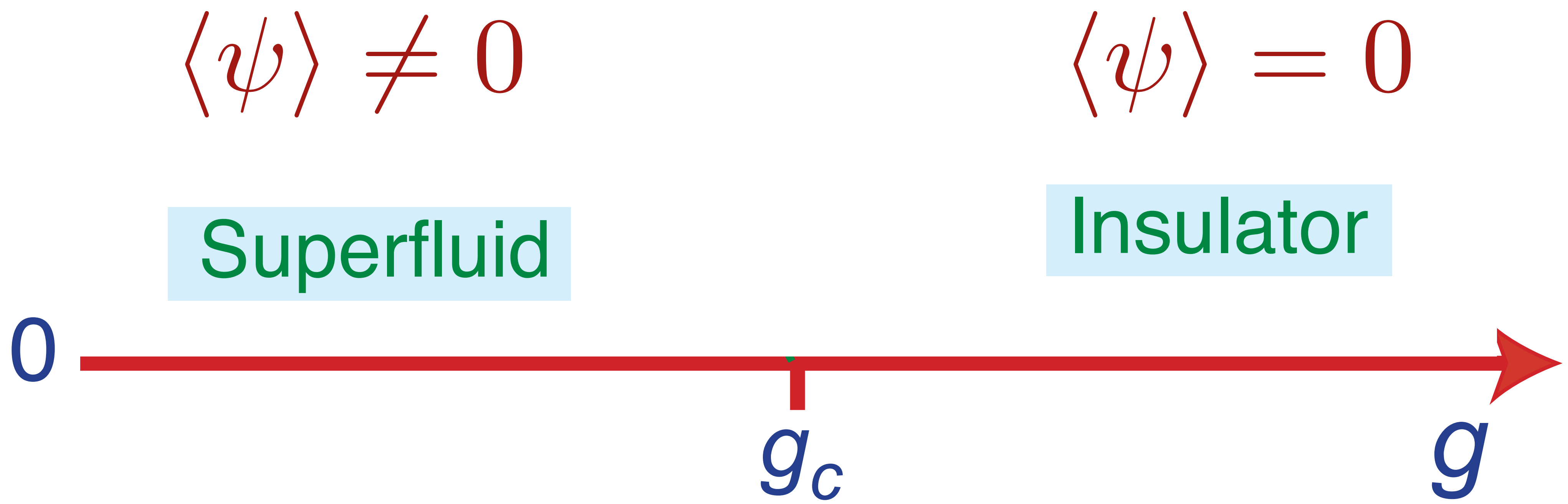
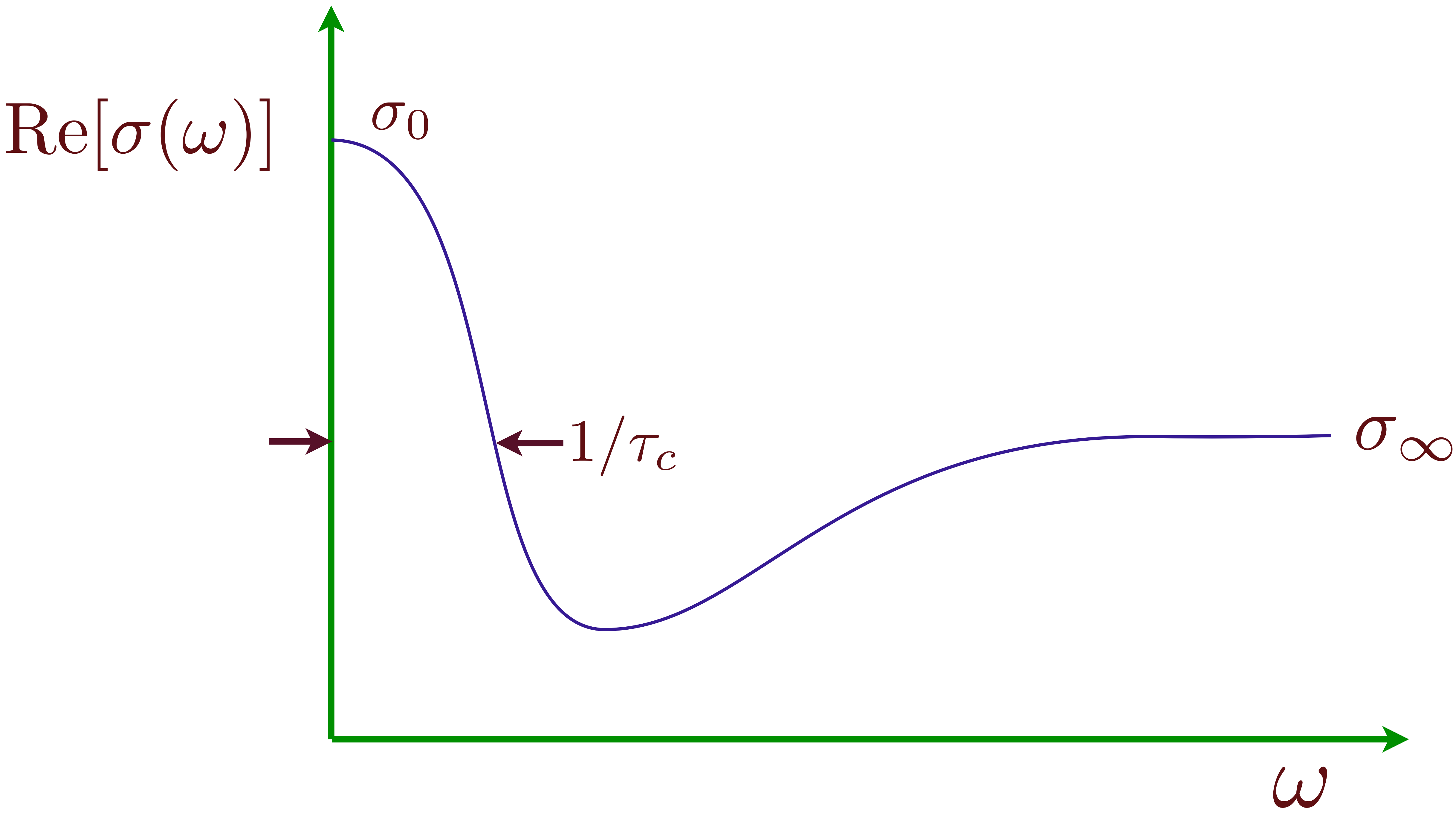
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

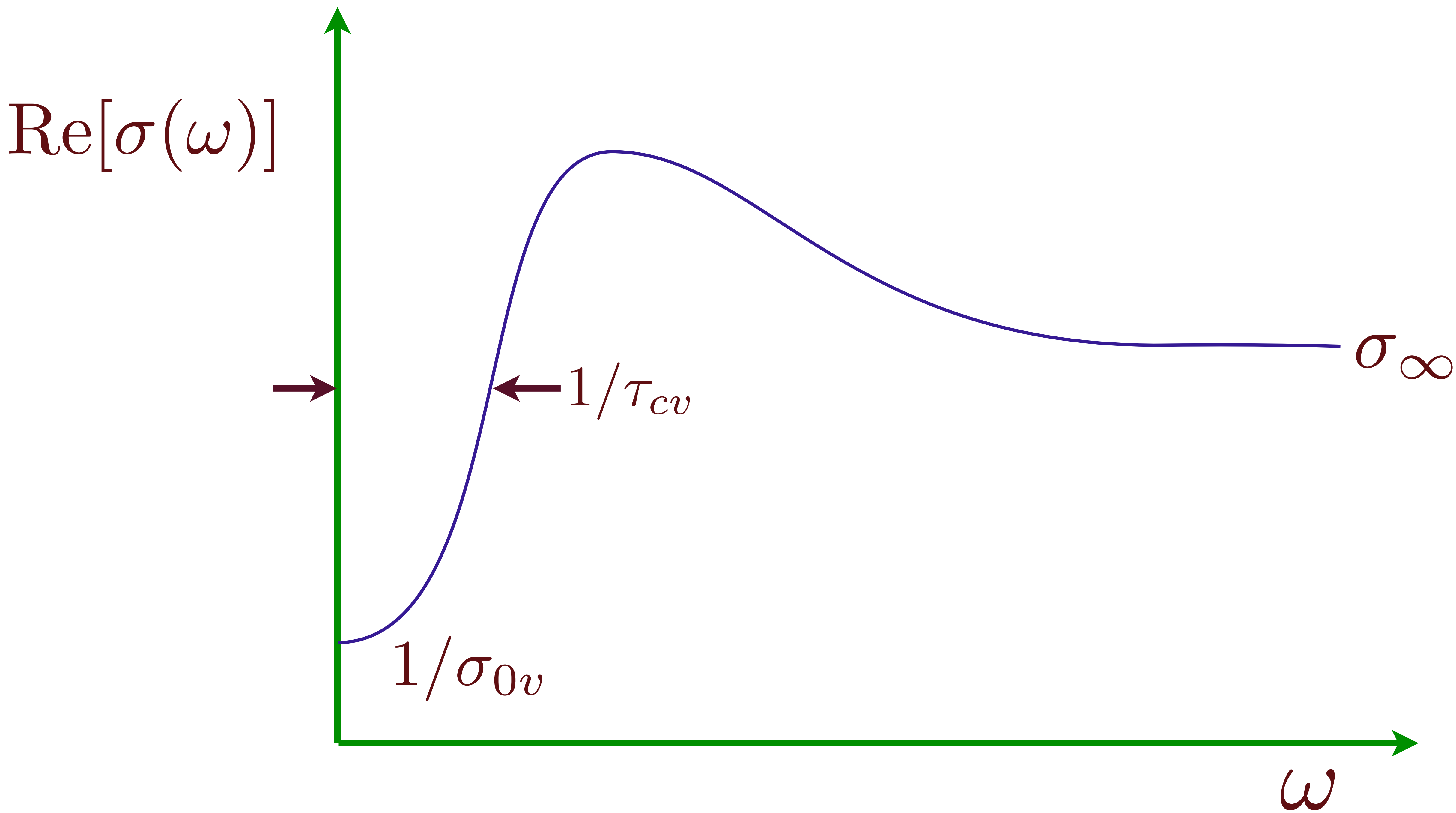
$$\text{Conductivity} = \text{Resistivity of vortices}$$



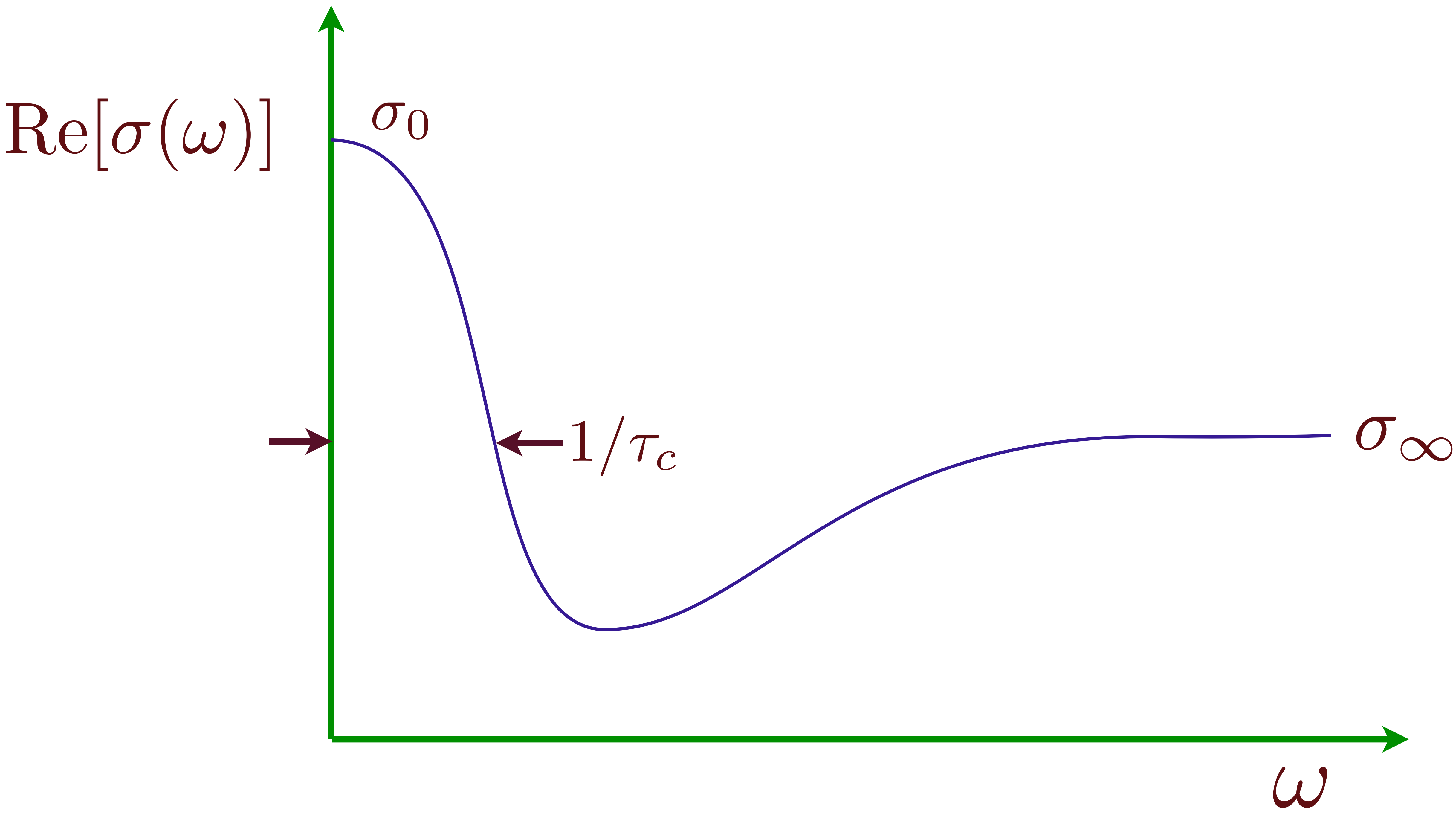
Boltzmann theory of bosons



Boltzmann theory of vortices

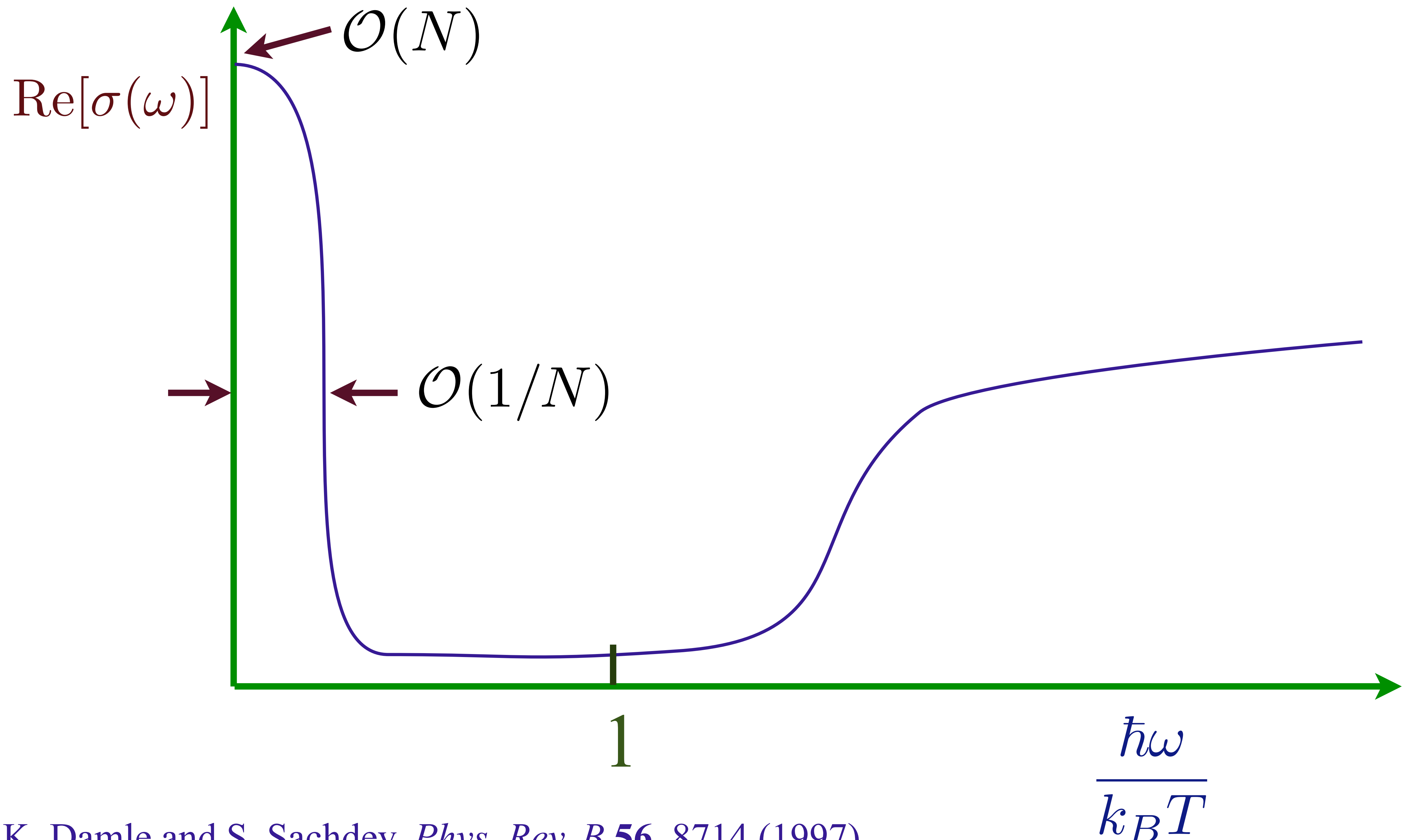


Boltzmann theory of bosons



Vector large N expansion for CFT3

$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right); \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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A. Field theory

B. Holography

2. The superfluid-insulator quantum phase transition

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1. Strange metals

A. Field theory

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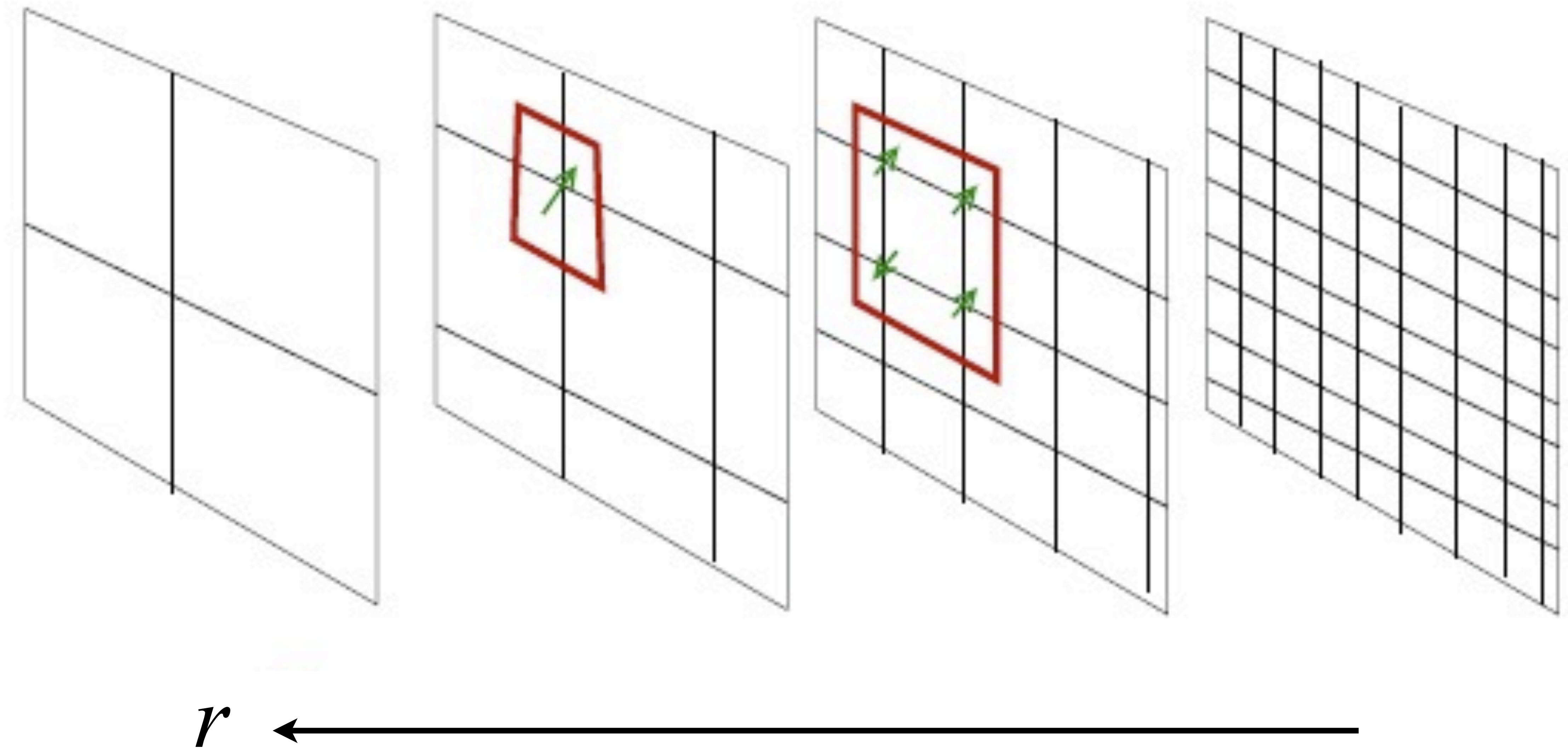

J. McGreevy, arXiv0909.0518

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \dots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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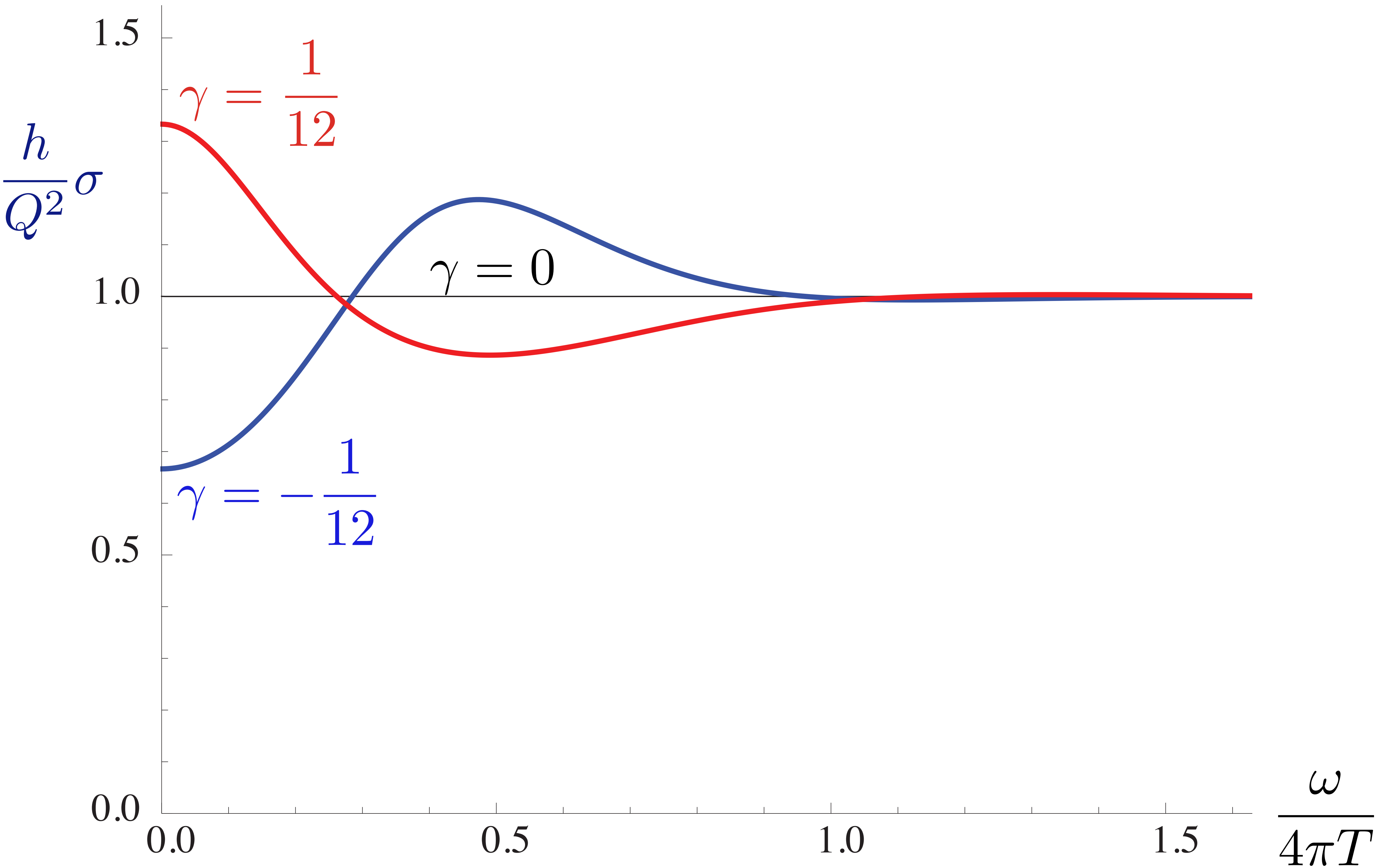
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

$$\mathcal{S}_{EM} = \int d^4x\sqrt{-g}\left[-\frac{1}{4e^2}F_{ab}F^{ab} + \frac{\gamma L^2}{e^2}C_{abcd}F^{ab}F^{cd}\right],$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

AdS$_4$ theory of strongly interacting "perfect fluids"



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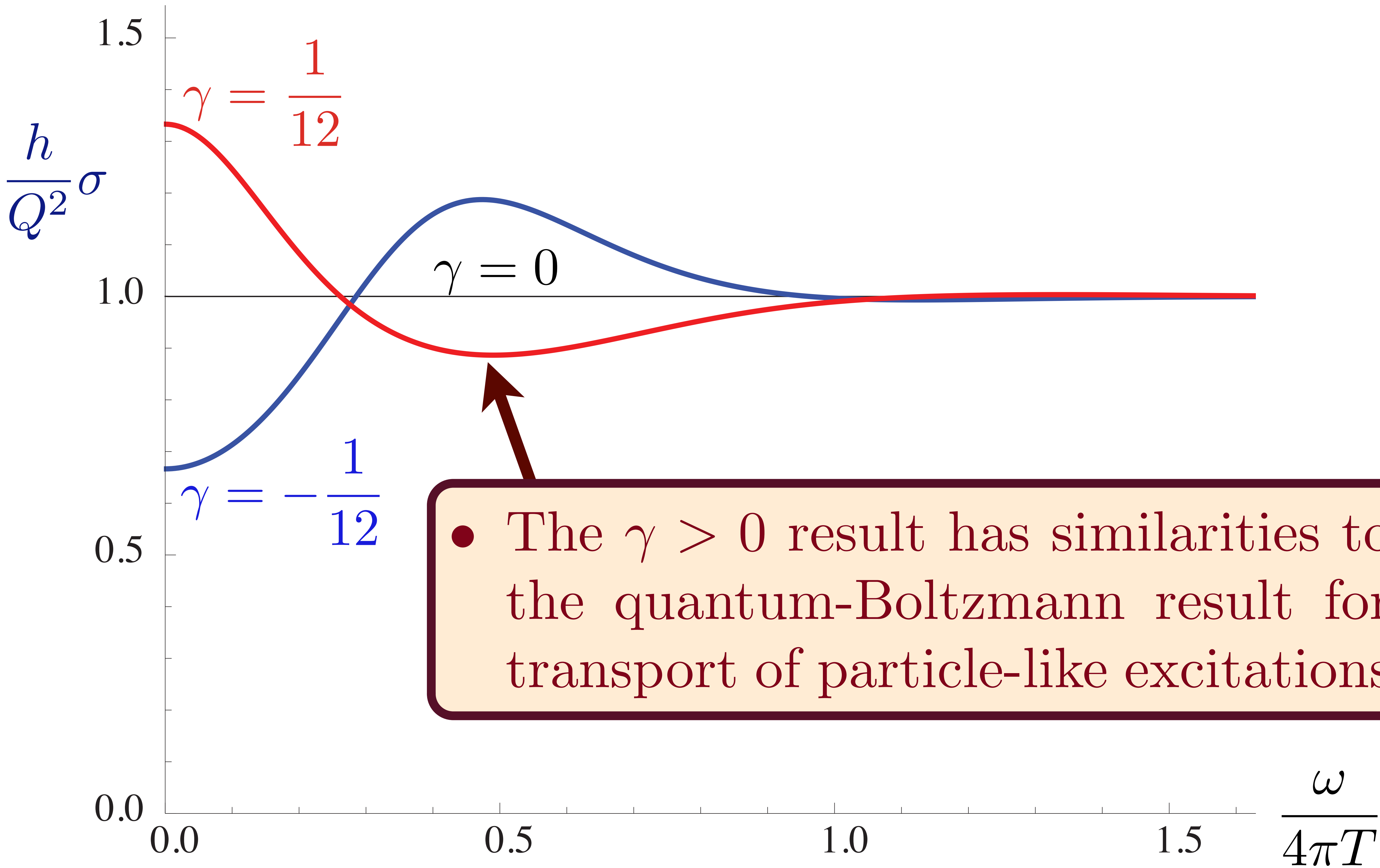

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

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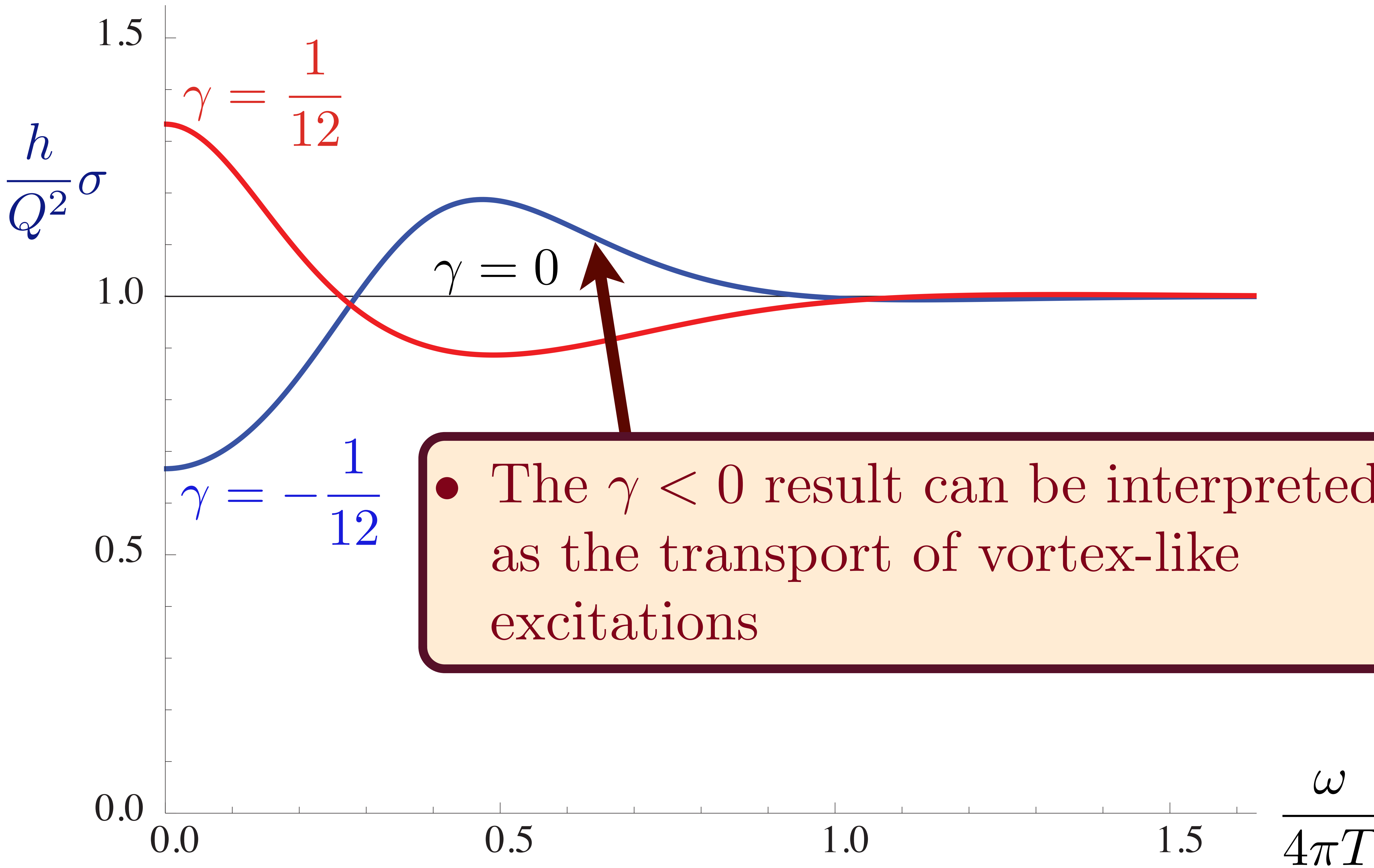

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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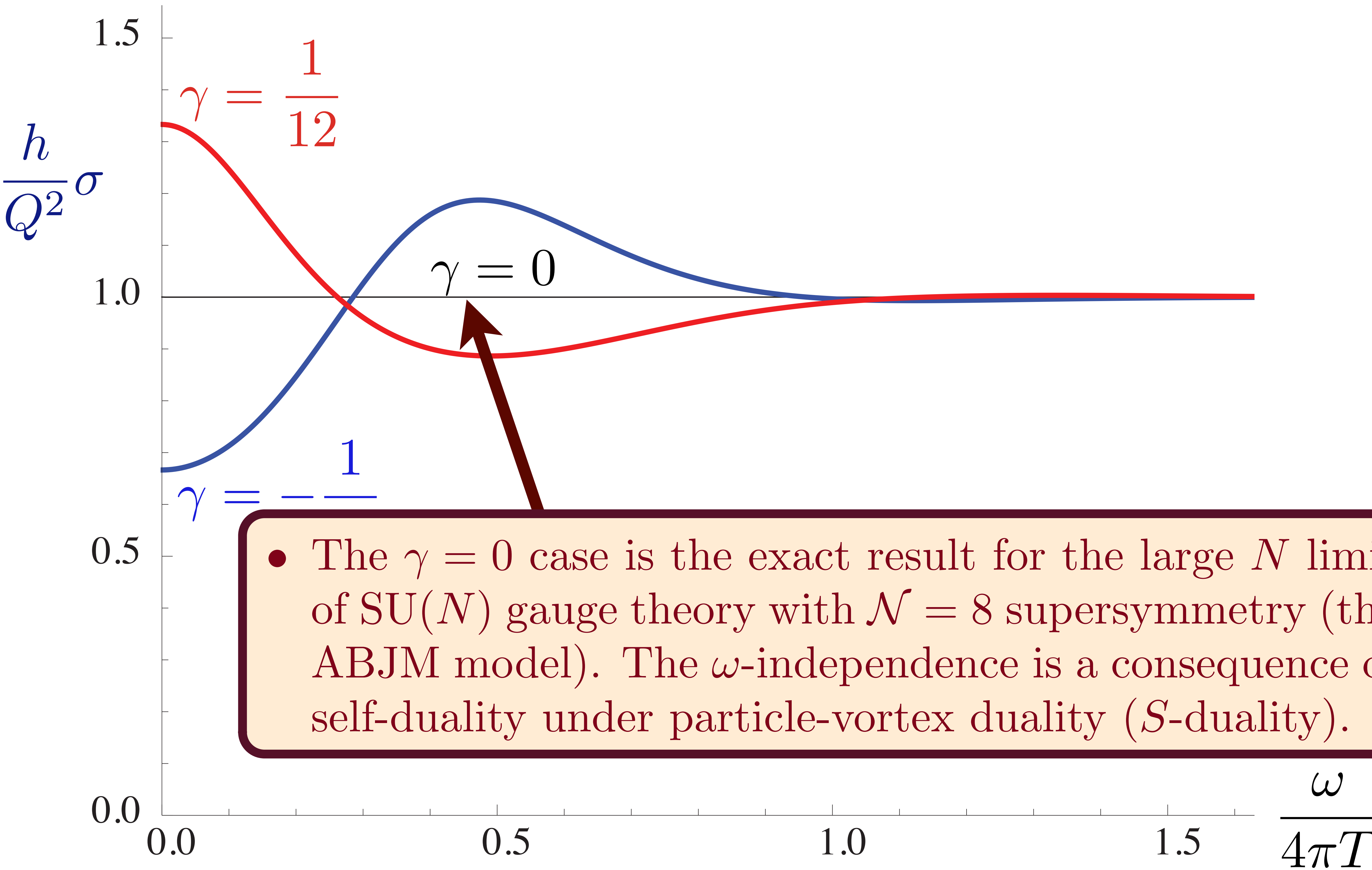

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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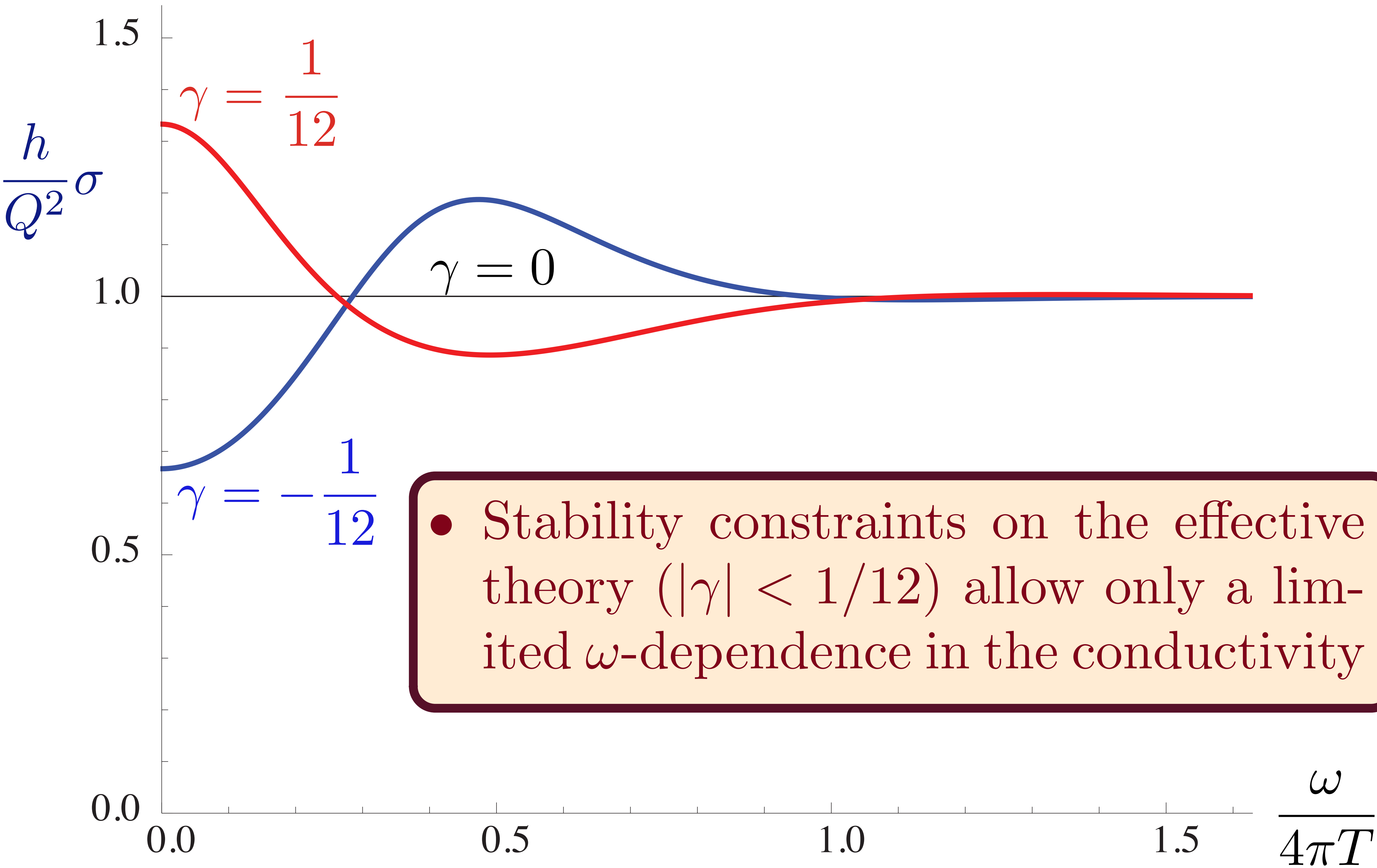

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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Theory for transport of conserved quantities in CFT3s:

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab}F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

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where C_{abcd} is the Weyl curvature tensor.

General approach:

- Theory has 2 free dimensionless parameters: e^2 and γ. We match these to correlators of the CFT3 of interest at $\omega \gg T$: e^2 is determines the current correlator $\langle J_\mu J_\nu \rangle$, while γ determines the 3-point function $\langle T_{\mu\nu} J_\rho J_\sigma \rangle$, where $T_{\mu\nu}$ is the stress-energy tensor.

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- We use $\mathcal{S}_{EM}$ to extrapolate to transport properties for $\omega \ll T$. This step is traditionally carried out by descendants of the Boltzmann equation.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Conclusions

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- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport