Resonant "side-jump" thermal Hall effect of phonons coupled to dynamical defects

CIFAR Quantum Materials Program Meeting

May 4, 2022

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Talk online: sachdev.physics.harvard.edu

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PHYSICAL REVIEW B **103**, 205115 (2021)

Extrinsic phonon thermal Hall transport from Hall viscosity

arXiv:2201.11681

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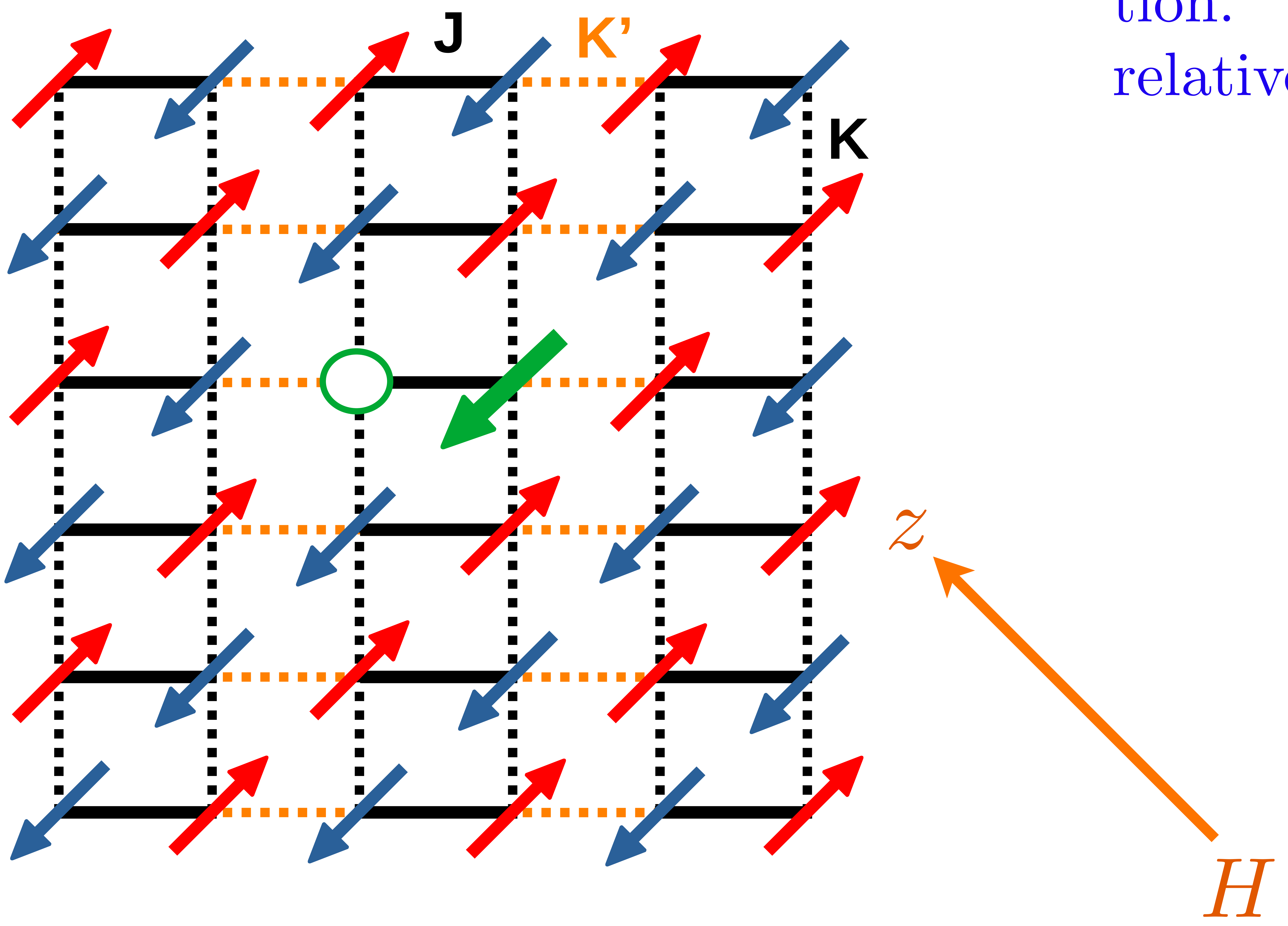
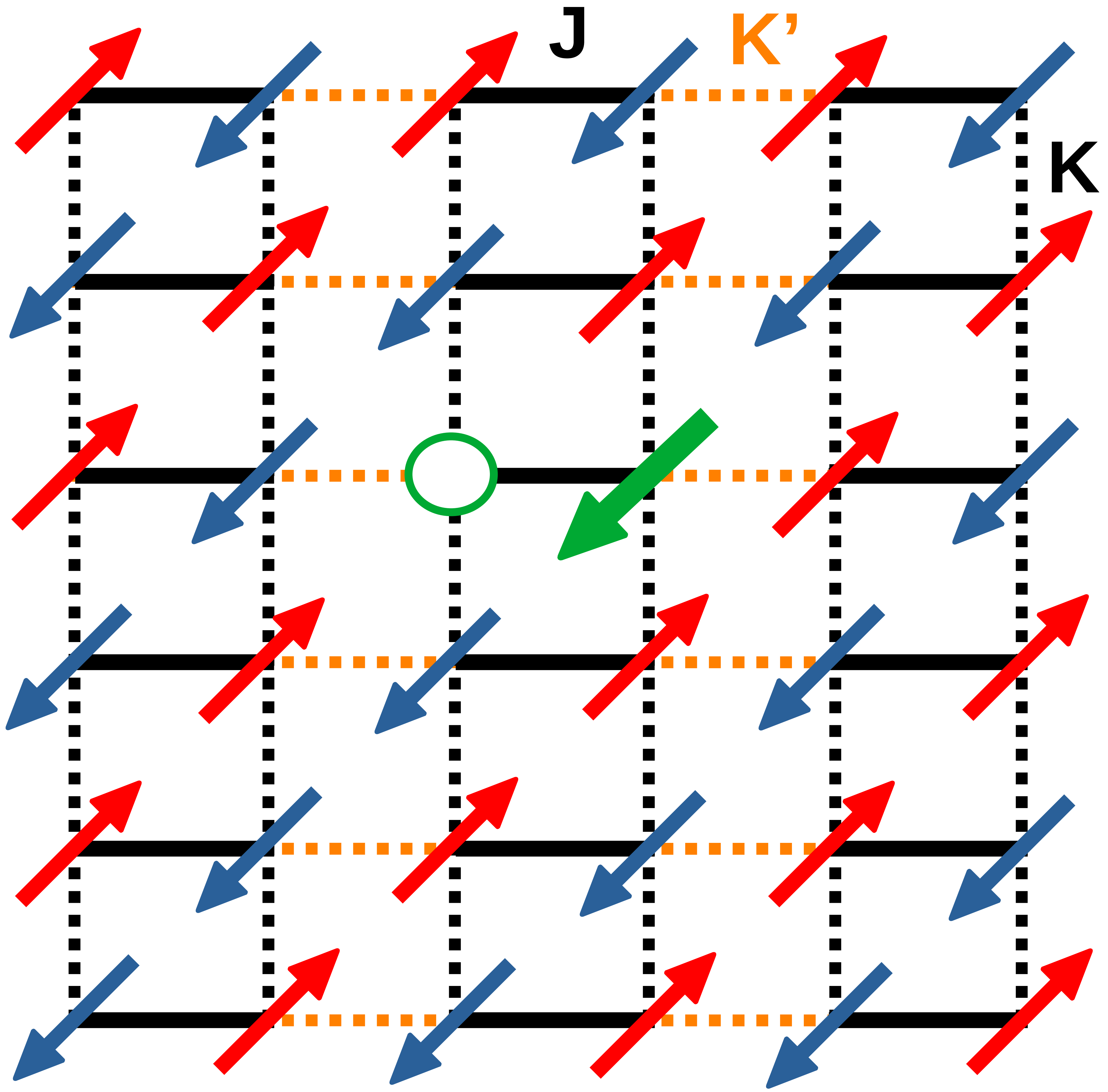
1. Spin-phonon model
2. Theory
3. Thermal Hall response
4. Metallic spin ice
5. Cuprates

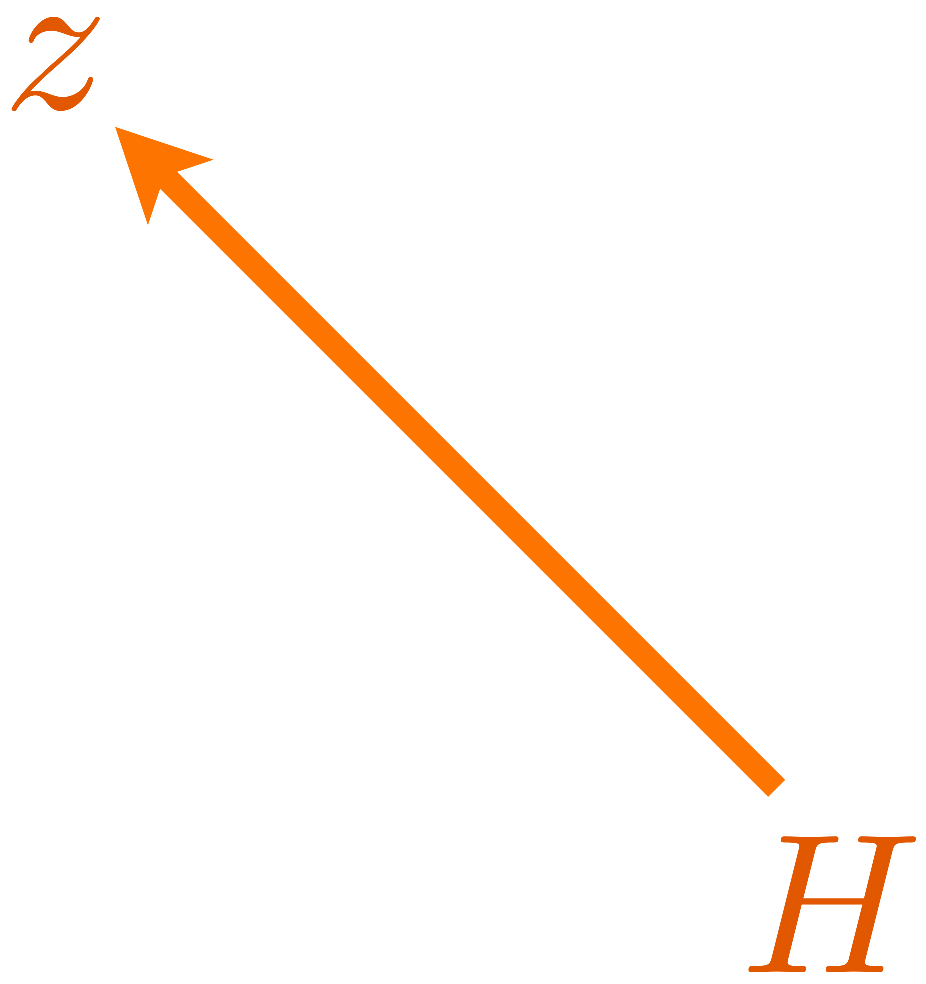
Model B



- Field is oriented along the 'z' direction. This could be any direction relative to the crystal.

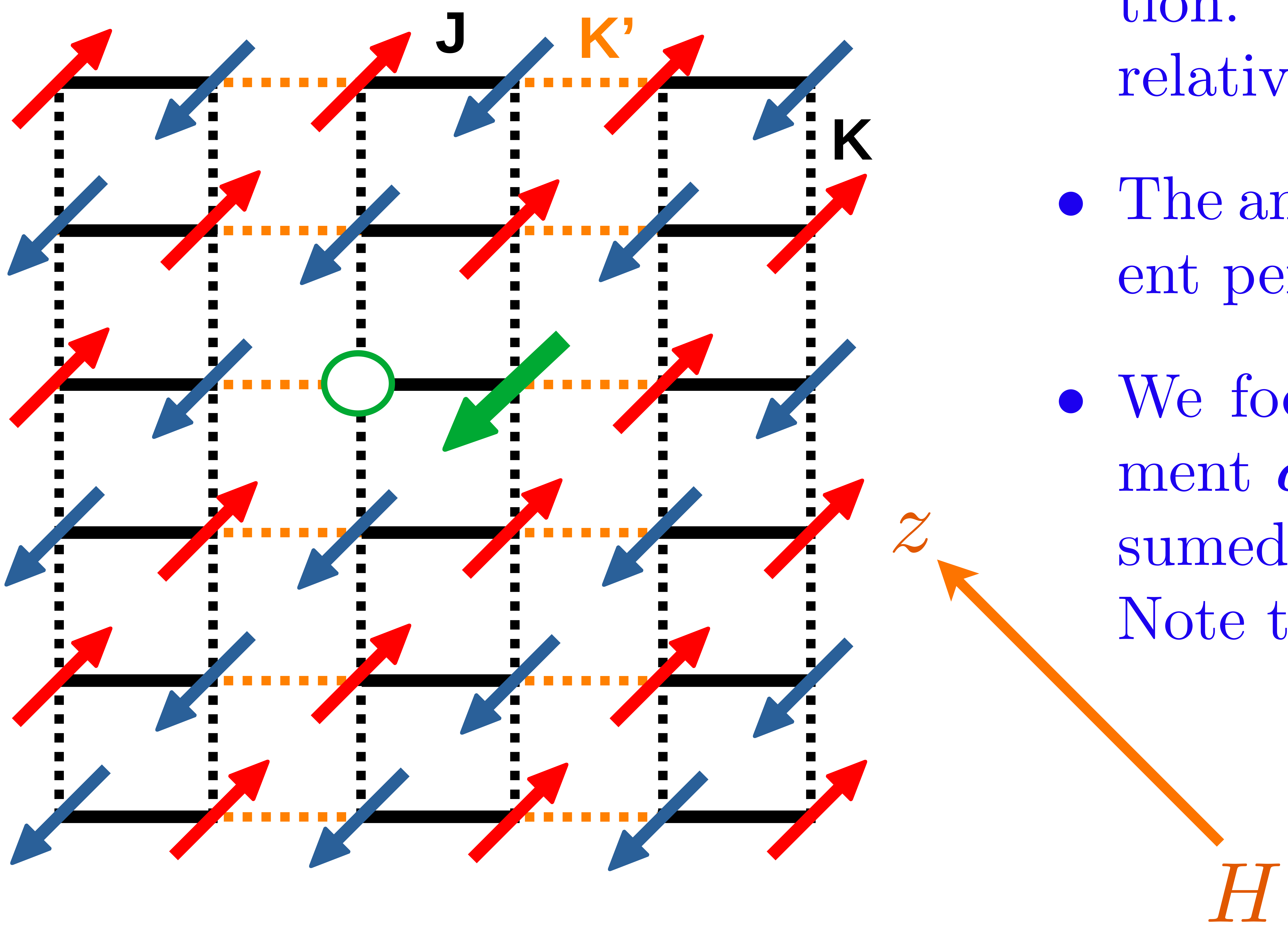
Model B





- Field is oriented along the 'z' direction. This could be any direction relative to the crystal.
- The antiferromagnetic order will orient perpendicular to the field.

Model B



- Field is oriented along the 'z' direction. This could be any direction relative to the crystal.
- The antiferromagnetic order will orient perpendicular to the field.
- We focus on an impurity spin moment $\boldsymbol{\sigma}$. The *local* field on $\boldsymbol{\sigma}$ is assumed to be along the '3' direction. Note that in general 'z' $\neq$ '3'.

Spin-phonon Hamiltonian

$$H_{\rm phonon} = \sum_p \frac{\pi_p^i \pi_p^i}{2m} + \frac{1}{2} \sum_{pq} u_p^i C_{pq}^{ij} u_q^j + H_{\rm dis} \,.$$

$i, j = x, y, z$ are Cartesian indices. p, q are site indices.
u_p^i is phonon displacement. π_p^i is phonon momentum.
$H_{\rm dis}$ leads to a phonon lifetime $\Gamma_{ph} \ll T$.

Spin-phonon Hamiltonian

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$i, j = x, y, z$ are Cartesian indices. p, q are site indices.
u_p^i is phonon displacement. π_p^i is phonon momentum.
H_{dis} leads to a phonon lifetime $\Gamma_{ph} \ll T$.

$$H_{\mathrm{imp}} = -\frac{\Delta}{2}\sigma^3\,.$$

The '3' axis sets the orientation of the *local* field.

Spin-phonon Hamiltonian

Model B

$$H_{\text{phonon}-\text{imp}} = K_{ij\alpha}\, \partial_i u^j_{\text{imp}}\, \sigma^\alpha\,.$$

$i, j = x, y, z$ are indices oriented by the *external* field along the 'z' axis.
$\alpha = 1, 2, 3$ is an index oriented the *local* field along the '3' axis.

$K_{ij\alpha}$ arises from bond-length dependence of exchange interactions, in the presence of background magnetic order.

Spin-phonon Hamiltonian

Model B

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Model A

$$H_{\text{phonon}-\text{imp}} = K_{i\alpha}\, \pi^i_{\text{imp}}\, \sigma^\alpha\,.$$

Analog to Rashba term in the presence of spin-orbit coupling

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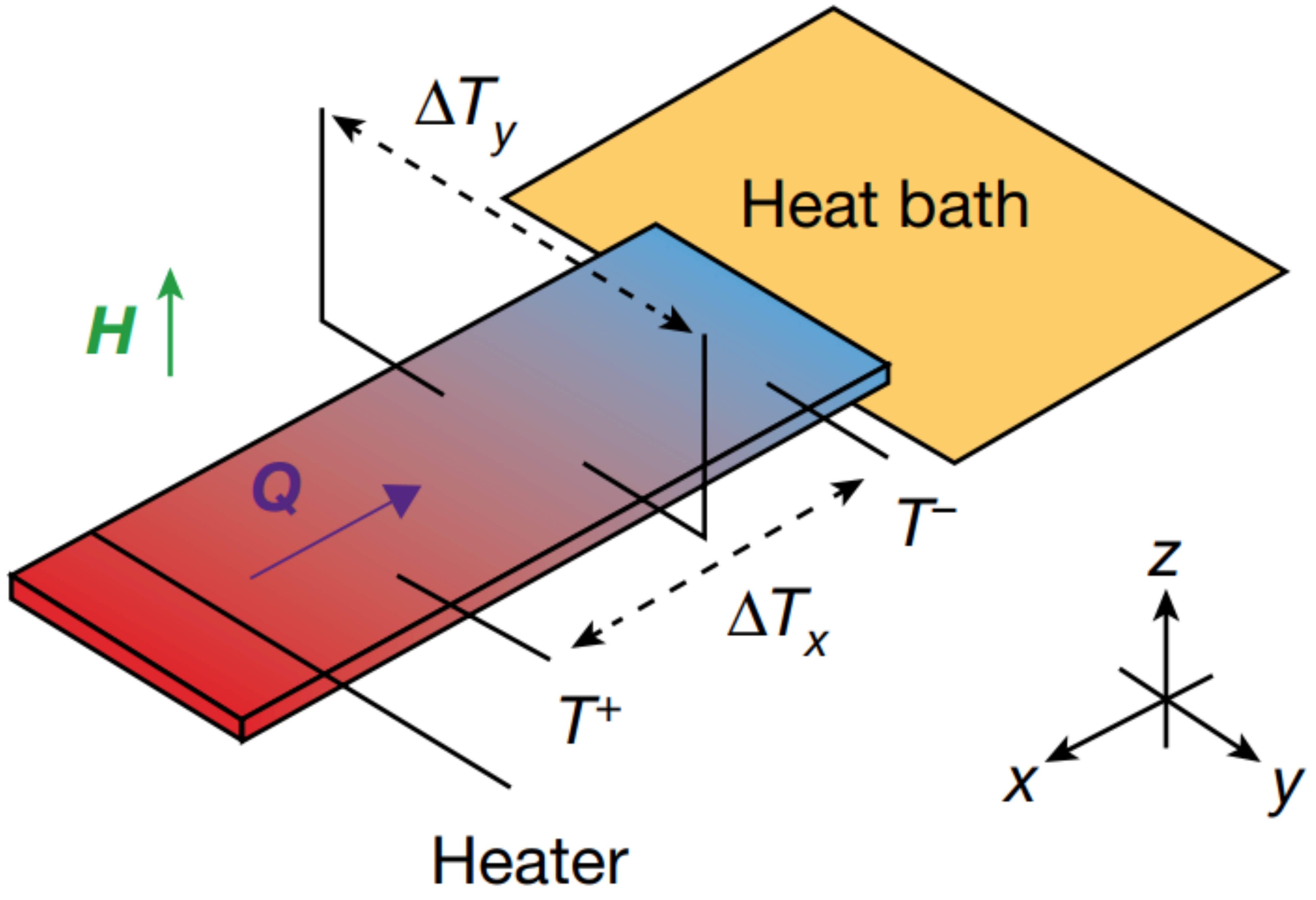
$$J_{Qx} = -\kappa_{xy} \partial_y T$$



G. Grissonnanche et al. *Nature* **571,** 376–380 (2019)

Microscopic formulas for thermoelectric transport coefficients in lattice systems

Anton Kapustin[*] and Lev Spodyneiko[†]
California Institute of Technology, Pasadena, California 91125, USA
PHYSICAL REVIEW B **104**, 035150 (2021)

- Hamiltonian on a lattice Λ can be decomposed into local terms (1-chain)

$$H = \sum_{p\in\Lambda} H_p \qquad [H_p, H_q] = 0 \text{ for } |p-q| > R$$

- Energy current found from Heisenberg equation (2-chain)

$$J^E_{pq} = -i[H_p, H_q]$$

- Current between two regions A,B: $A \cup B = \Lambda \quad A \cap B = \emptyset$

$$J^E(A, B) = \sum_{p\in A, q\in B} J^E_{pq}$$

Microscopic formulas for thermoelectric transport coefficients in lattice systems

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- Use this formalism to show that the "energy magnetization" corrections do not have an enhancement as $\Gamma_{ph} \to 0$.
- Thermal Hall response can be computed by direct application of Kubo formula

Feynman diagrams

$$D_{a\pm}^{R/A}(\omega, k) = \frac{1}{\omega \mp c_a k \pm i\Gamma/2}$$

- Solid line: phonon Green's function
- Dashed line: defect Green's function
- (a) phonon-interband coherence, similar to electron side jump
- (b) phonon-defect coherence, unique to energy transport and single-phonon process
- Perturb in phonon-defect coupling constant

(a) (b)

Side Jump: Exactly one pair of $D^R(\omega)D^A(\omega)$ of identical argument, yielding a factor of $1/\Gamma$

Feynman diagrams

$$D^{R/A}_{a\pm}(\omega, k) = \frac{1}{\omega \mp c_a k \pm i\Gamma/2}$$

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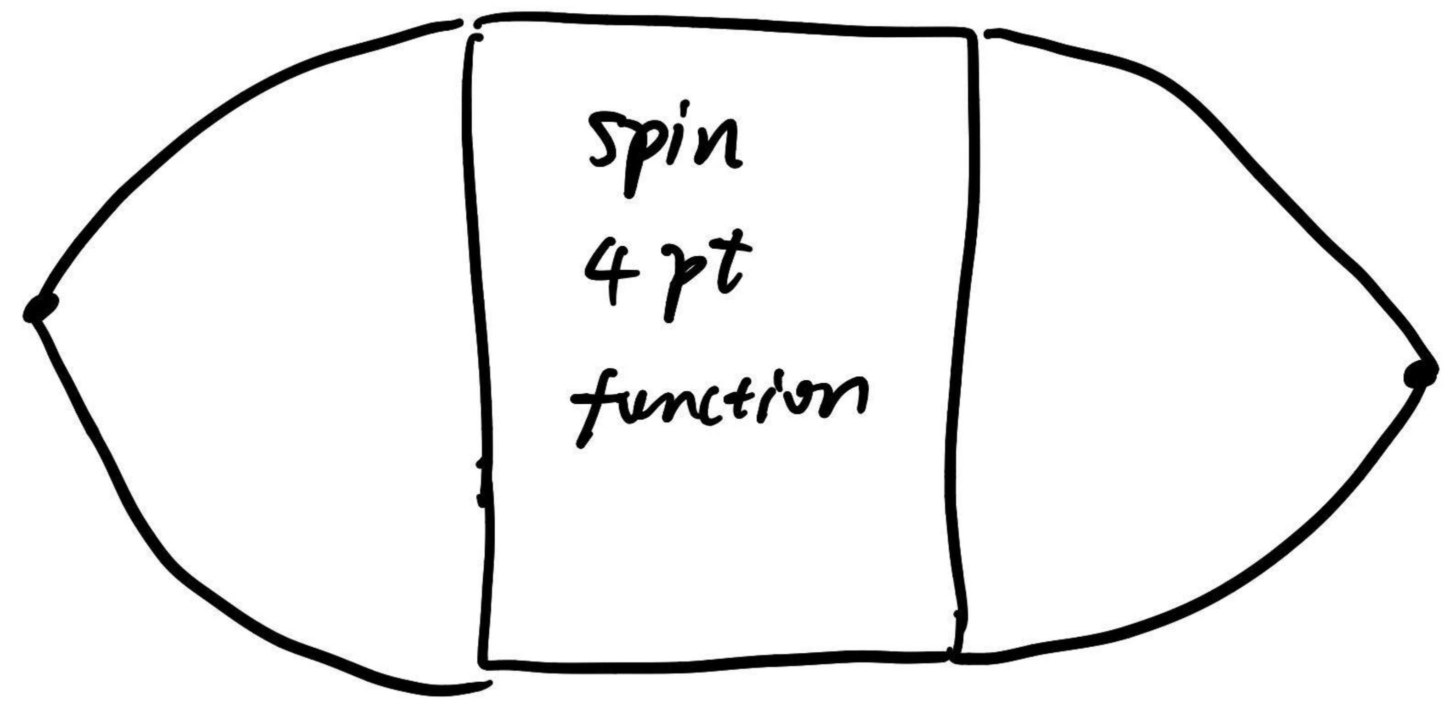

Skew scattering: fourth order in phonon-defect coupling, yielding a factor of $1/\Gamma^2$

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Thermal Hall co-efficient

Model A

To second order in $K_{i\alpha}$, the thermal Hall response is

$$\kappa_H = -\frac{m}{6\pi N_{\rm sys}} \frac{\Delta^4}{\Gamma_{ph} T^2 \sinh(\Delta/T)} \left(\frac{1}{c_L} + \frac{1}{c_T} \right) (K_{x1}K_{y2} - K_{x2}K_{y1})$$

Thermal Hall co-efficient

Model B

To second order in $K_{ij\alpha}$, the thermal Hall response is

$$\kappa_H = \frac{1}{30\pi m N_{\rm sys}} \frac{\Delta^4}{\Gamma_{ph} T^2 \sinh(\Delta/T)} \left(c_L^{-3} K_L + c_T^{-3} K_T\right)$$

Thermal Hall co-efficient

Model B

To second order in $K_{ij\alpha}$, the thermal Hall response is

$$\kappa_H = \frac{1}{30\pi m N_{\rm sys}} \frac{\Delta^4}{\Gamma_{ph} T^2 \sinh(\Delta/T)} \left(c_L^{-3} K_L + c_T^{-3} K_T \right) , \quad \text{where}$$

$$\begin{aligned} K_T = & -\frac{5}{2} \left[\left(K_{xx1} - K_{yy1} \right) \left(K_{xy2} + K_{yx2} \right) - \left(K_{xx2} - K_{yy2} \right) \left(K_{xy1} + K_{yx1} \right) \right] \\ & + \frac{1}{2} \left[\left(K_{xx1} + K_{yy1} \right) \left(K_{xy2} - K_{yx2} \right) - \left(K_{xx2} + K_{yy2} \right) \left(K_{xy1} - K_{yx1} \right) \right] \\ & + K_{zz1} \left(K_{yx2} - K_{xy2} \right) + K_{zz2} \left(K_{xy1} - K_{yx1} \right) - 4 K_{xz1} K_{yz2} + 4 K_{xz2} K_{yz1} \\ & - K_{zx1} K_{zy2} + K_{zx2} K_{zy1} , \end{aligned}$$

$$\begin{aligned} K_L = & -2 \left(K_{xx1} - K_{yy1} \right) \left(K_{xy2} + K_{yx2} \right) + 2 \left(K_{xx2} - K_{yy2} \right) \left(K_{xy1} + K_{yx1} \right) \\ & - \left(K_{xz1} + K_{zx1} \right) \left(K_{yz2} + K_{zy2} \right) + \left(K_{xz2} + K_{zx2} \right) \left(K_{yz1} + K_{zy1} \right) . \end{aligned}$$

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Phonon thermal Hall effect in a metallic spin ice

arXiv:2202.12149

Taiki Uehara,[1] Takumi Ohtsuki,[2] Masafumi Udagawa,[1]

Satoru Nakatsuji,[2,3,4] and Yo Machida[1]

It has become common knowledge that phonons can generate thermal Hall effect in a wide variety of materials, although the underlying mechanism is still controversial. We study longitudinal κ_{xx} and transverse κ_{xy} thermal conductivity in $Pr_2Ir_2O_7$, which is a metallic analogue of spin ice. Despite the presence of mobile charge carriers, we find that both κ_{xx} and κ_{xy} are dominated by phonons. A T/H scaling of κ_{xx} unambiguously reveals that longitudinal heat current is substantially impeded by resonant scattering of phonons on paramagnetic spins.

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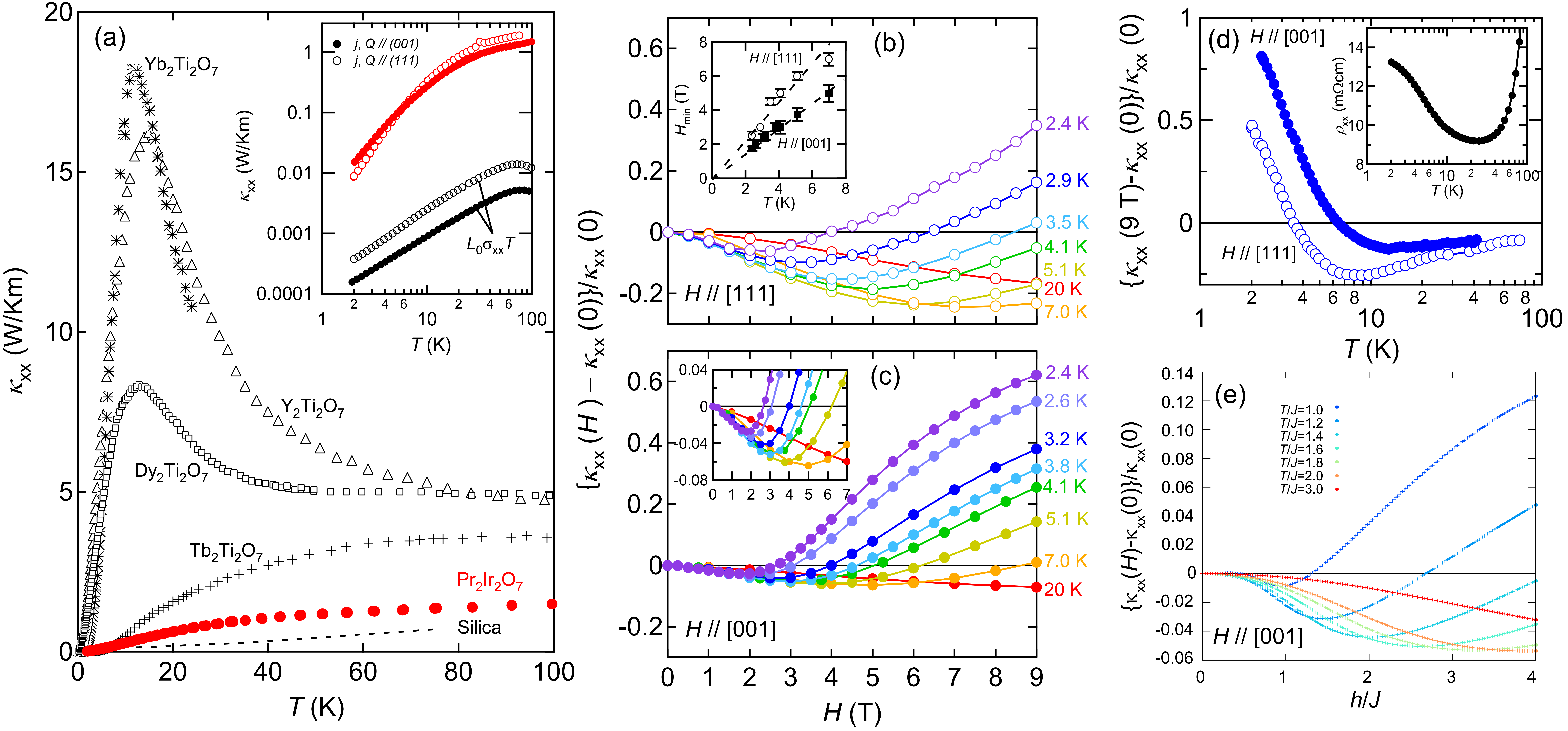
Upon cooling, the resonant scattering is strongly affected by a development of spin ice correlation and κ_{xx} deviates from the scaling in an anisotropic way with respect to field directions. Strikingly, a set of the κ_{xx} and κ_{xy} data clearly shows that κ_{xy} correlates with κ_{xx} in its response to magnetic field including a success of the T/H scaling and its failure at low temperature. This remarkable correlation provides solid evidence that an indispensable role is played by spin-phonon scattering not only for hindering the longitudinal heat conduction, but also for generating the transverse response.



(a)
Yb$_2$Ti$_2$O$_7$
Y$_2$Ti$_2$O$_7$
Dy$_2$Ti$_2$O$_7$
Tb$_2$Ti$_2$O$_7$
Pr$_2$Ir$_2$O$_7$
Silica
κ_{xx} (W/Km)
T (K)
j, Q // (001)
j, Q // (111)
$L_0\sigma_{xx}T$
(b)
H_{min} (T)
H // [111]
H // [001]
$\{\kappa_{xx}(H) - \kappa_{xx}(0)\}/\kappa_{xx}(0)$
2.4 K
2.9 K
3.5 K
4.1 K
5.1 K
20 K
7.0 K
(c)
2.6 K
3.2 K
3.8 K
H (T)
(d)
$\{\kappa_{xx}(9\text{ T})-\kappa_{xx}(0)\}/\kappa_{xx}(0)$
ρ_{xx} (mΩcm)
(e)
T/J=1.0
T/J=1.2
T/J=1.4
T/J=1.6
T/J=1.8
T/J=2.0
T/J=3.0
h/J

Haoyu Guo

- Assume Γ_{ph} is dominated by spin-phonon scattering. Resonant scattering at 4th-order in spin-phonon coupling yields $\Gamma_{ph} \sim \Delta^{4/3}$

- Then

$$\kappa_{xy} \sim \frac{\Delta^{8/3}}{T^2 \sinh(\Delta/T)}$$

Haoyu Guo

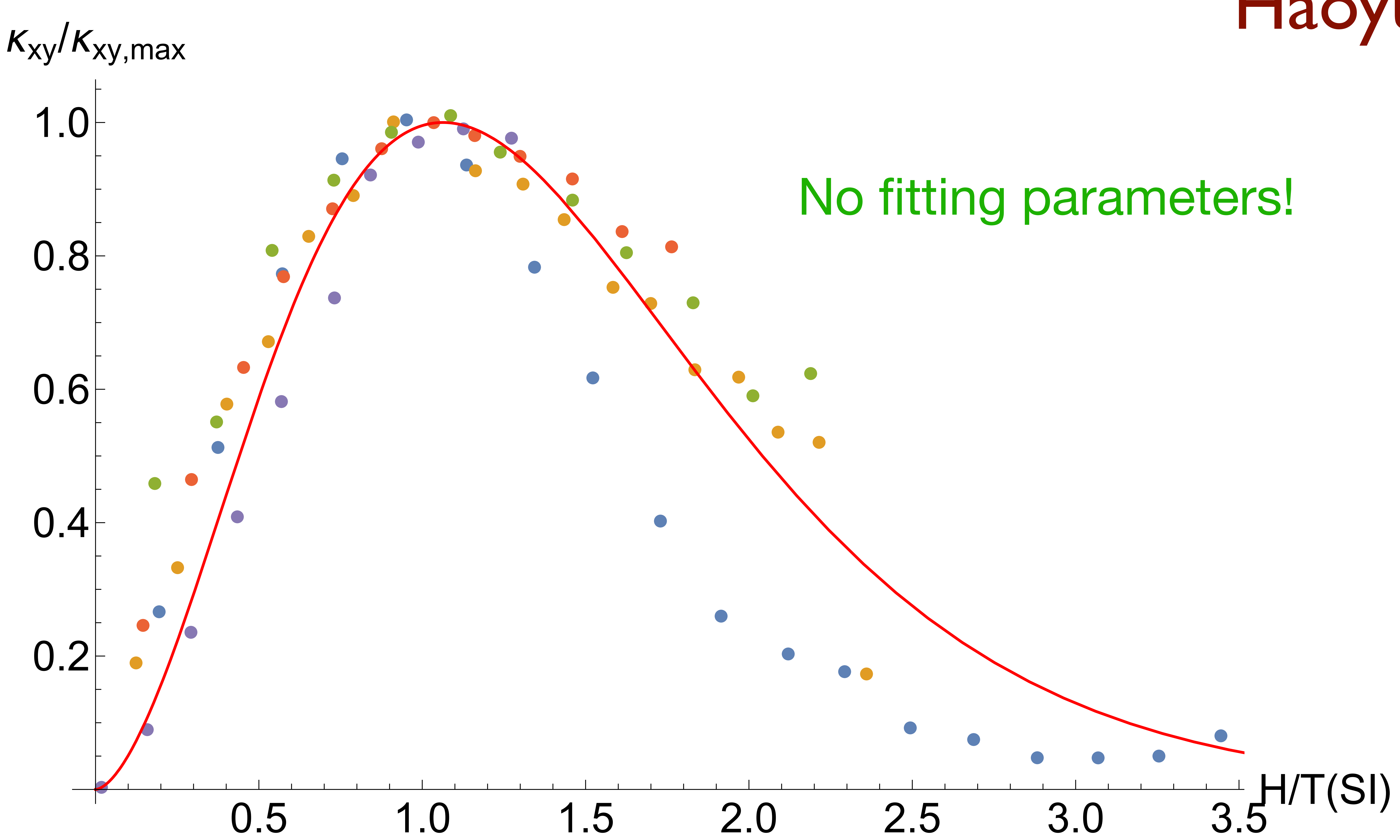


FIG. 1: Comparison between experiment from Fig.4(b) of [7], and our theory Eq.(11.16). The theoretical curve is plotted with $\beta\Delta = 2.482(H/T)$ (magnetic moment of free Pr^{3+}ion) in SI units, computed from microscopic data of $Pr_2Ir_2O_7$.

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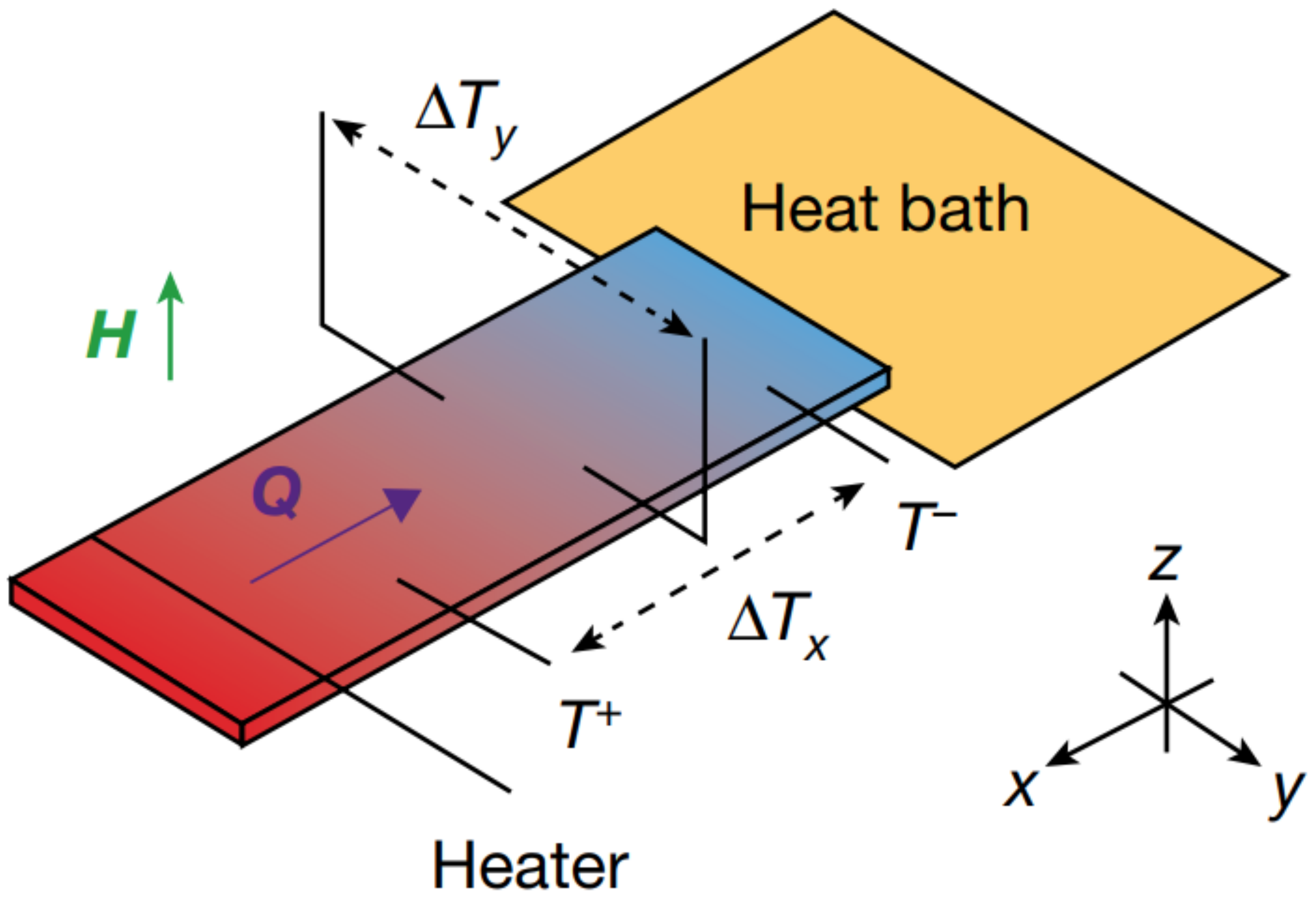
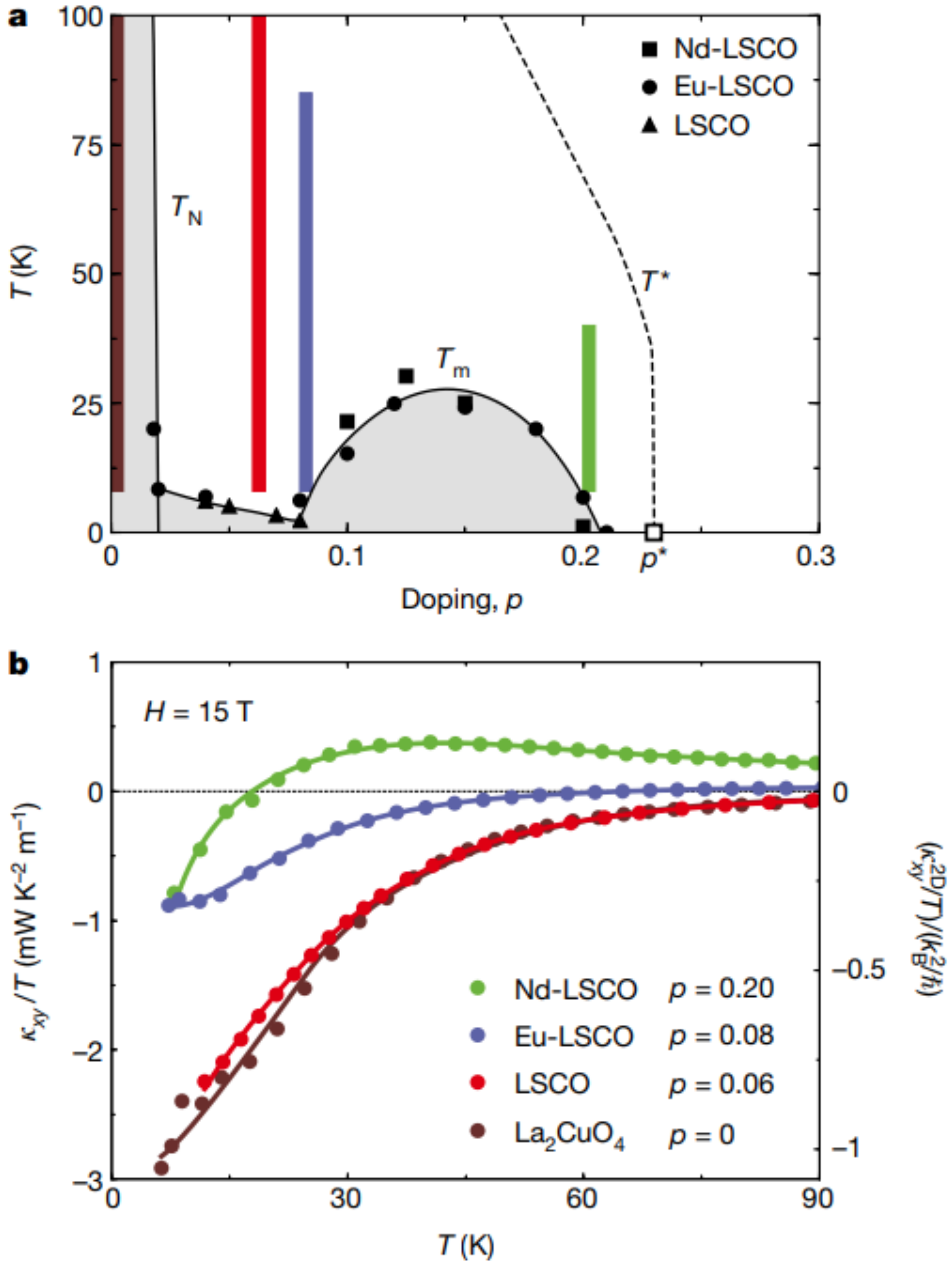
Giant thermal Hall signal in cuprates

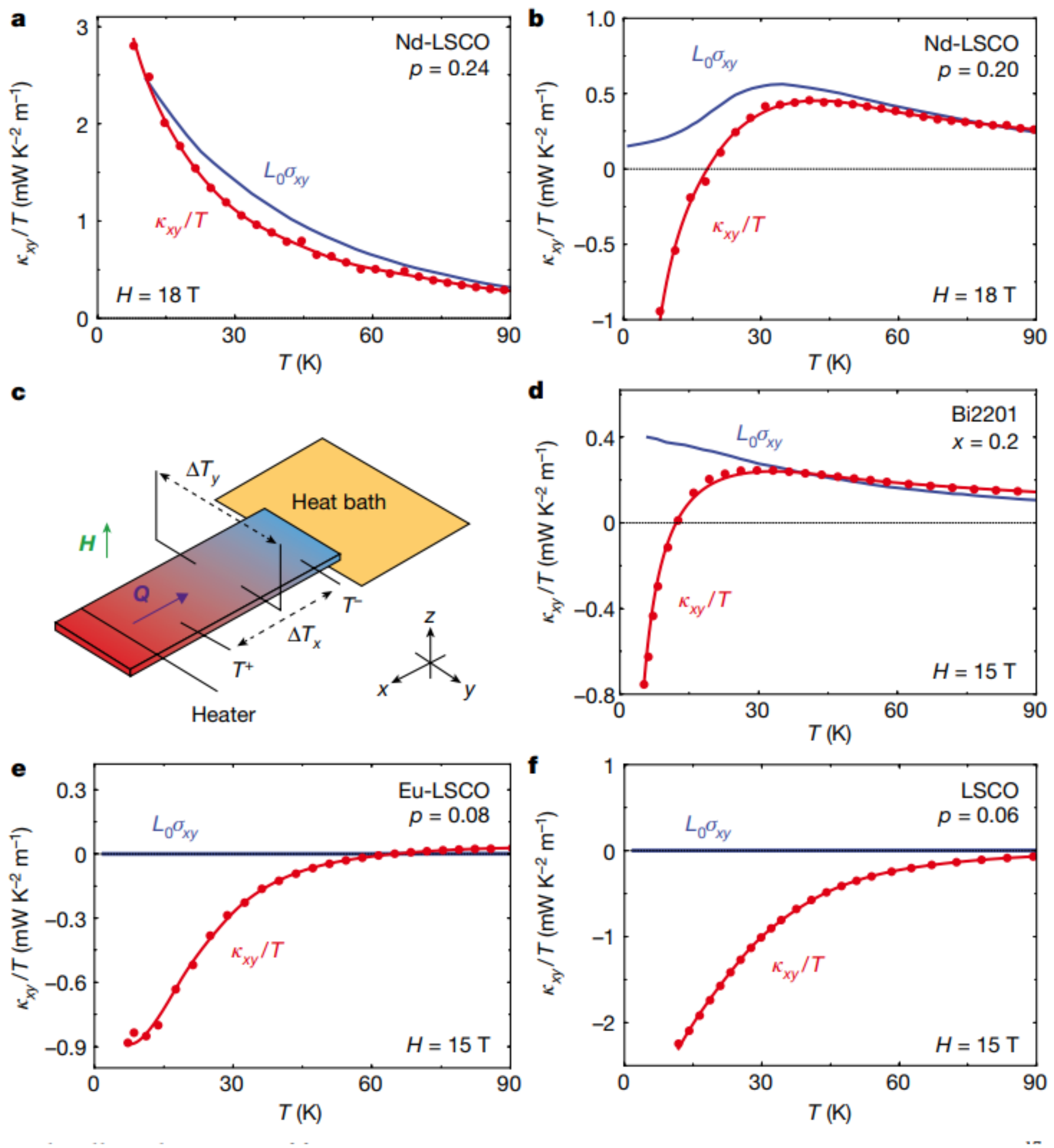
$$J_{Qx} = -\kappa_{xy}\partial_y T$$





G. Grissonnanche et al. *Nature* **571,** 376–380 (2019)

Violation of Wiedemann–Franz law



Violates Wiedemenn-Franz law for $p < p_*$

It has the opposite sign to σ_{xy}

So the mysterious heat carrier isn't charged

It appears outside the AF phase

So it's not due to magnons

Emergent excitation or phonons?

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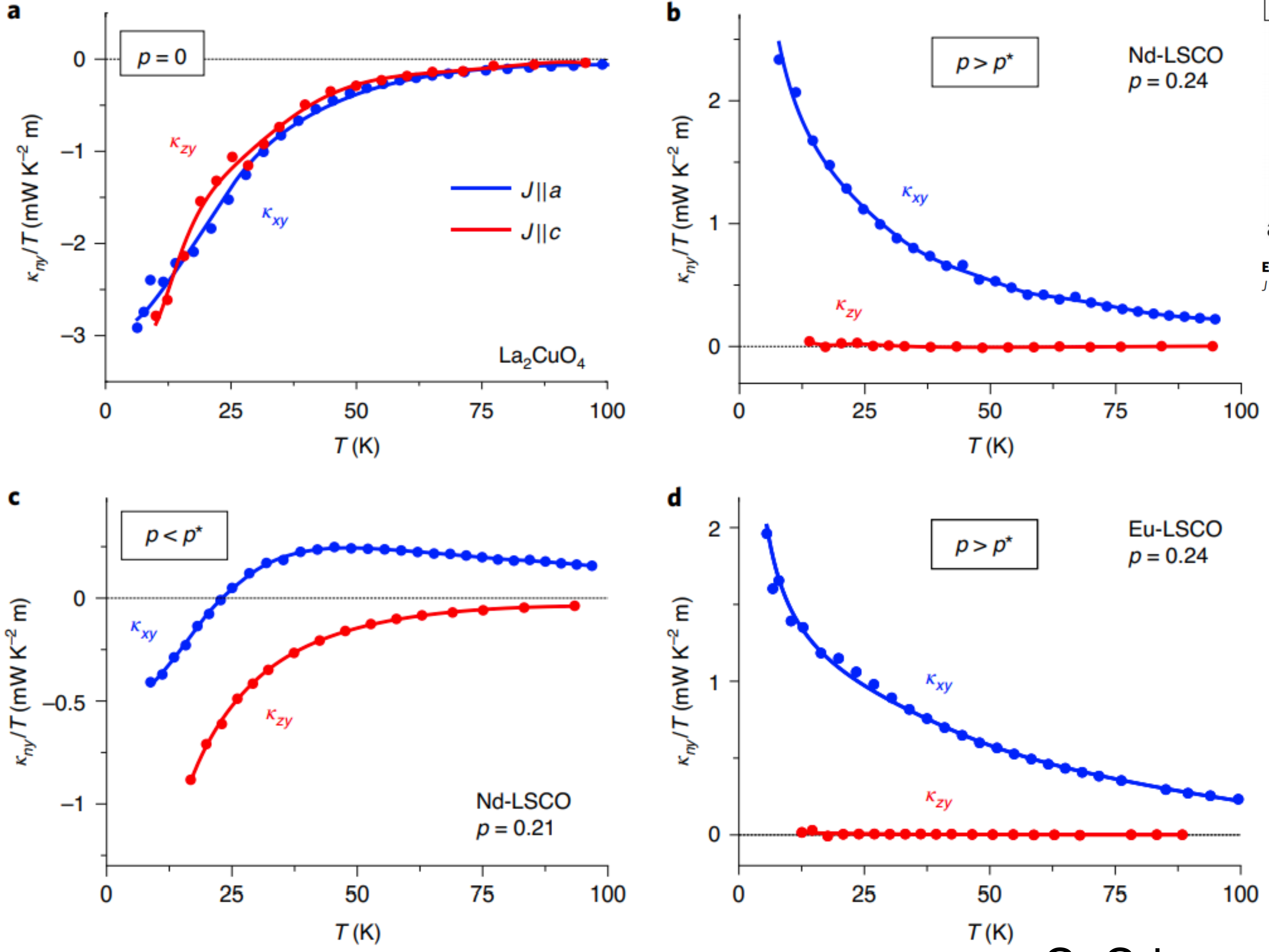
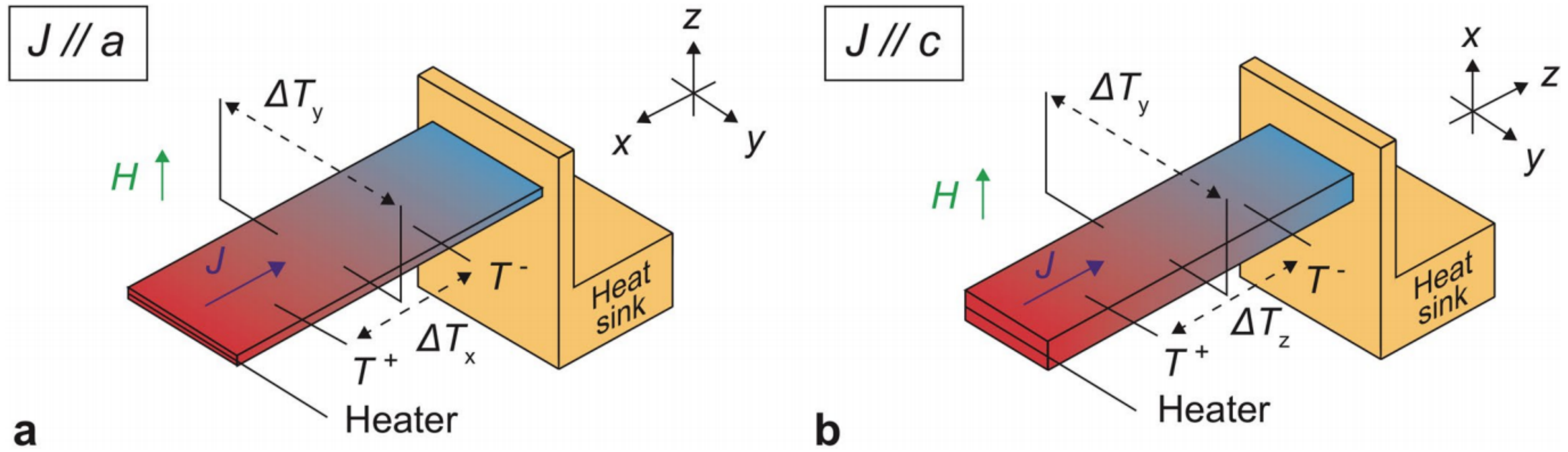
Experiment points to Phonons



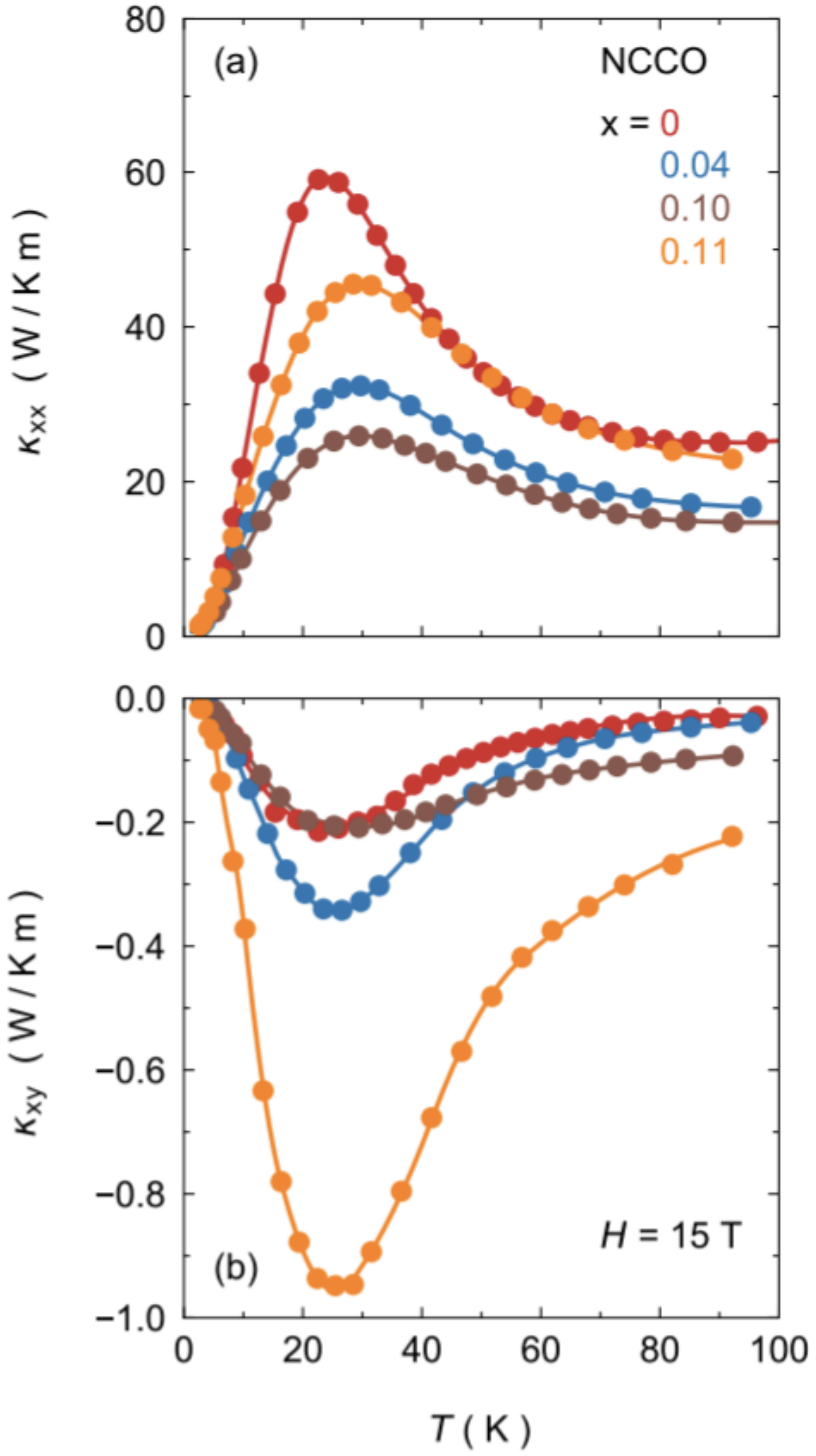


Extended Data Fig. 1 | Current and field orientation for κ_{xy} and κ_{zy} measurements. Sketch of the thermal Hall measurement setup for **a)** *J // a // -x* and **b)** *J // c // z*. The Cartesian coordinate system is defined in the same way for the two samples.

Only phonons can move in z-direction

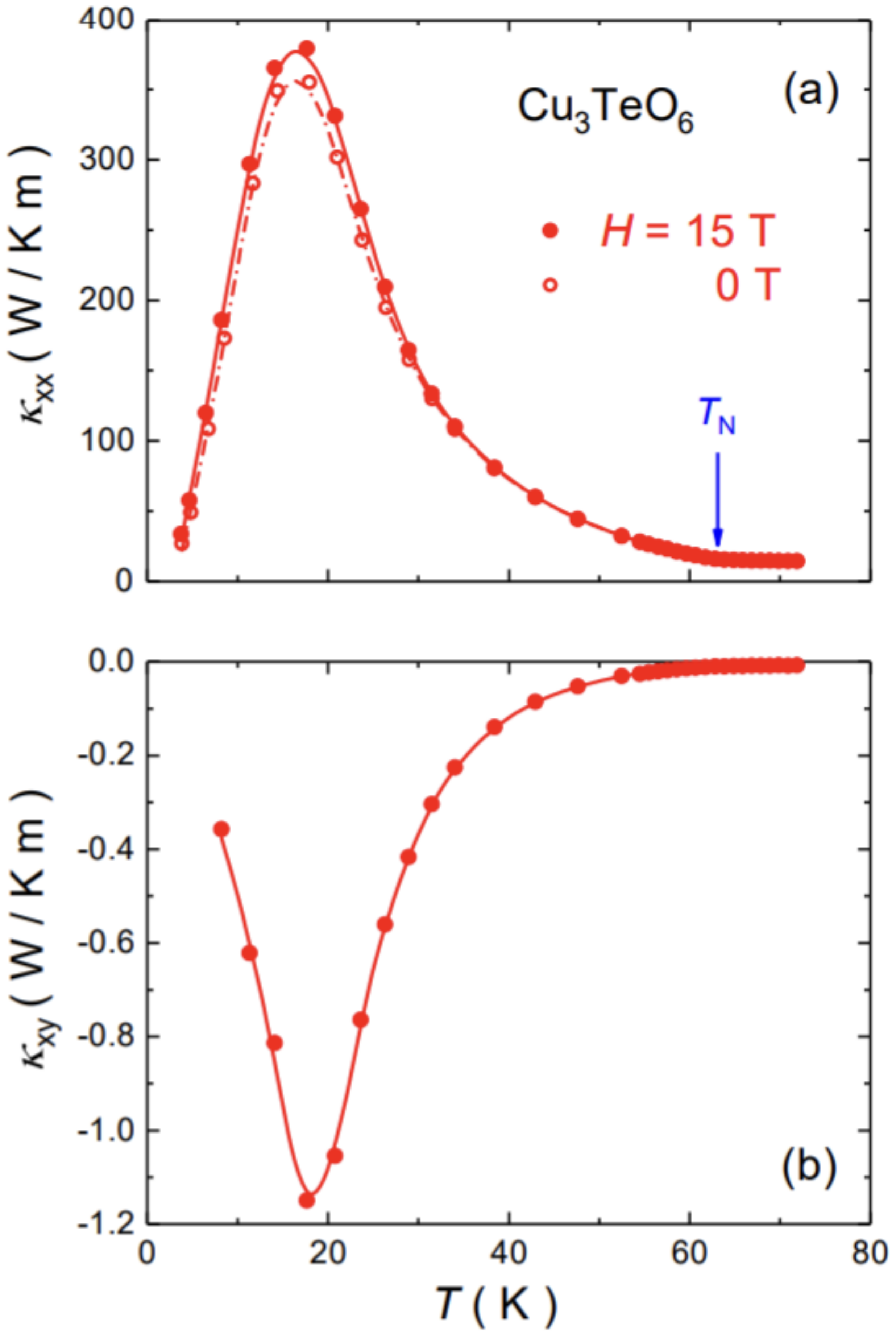
G. Grissonnanche et al. *Nat. Phys.* **16,** 1108–1111 (2020)

Thermal Hall in various materials



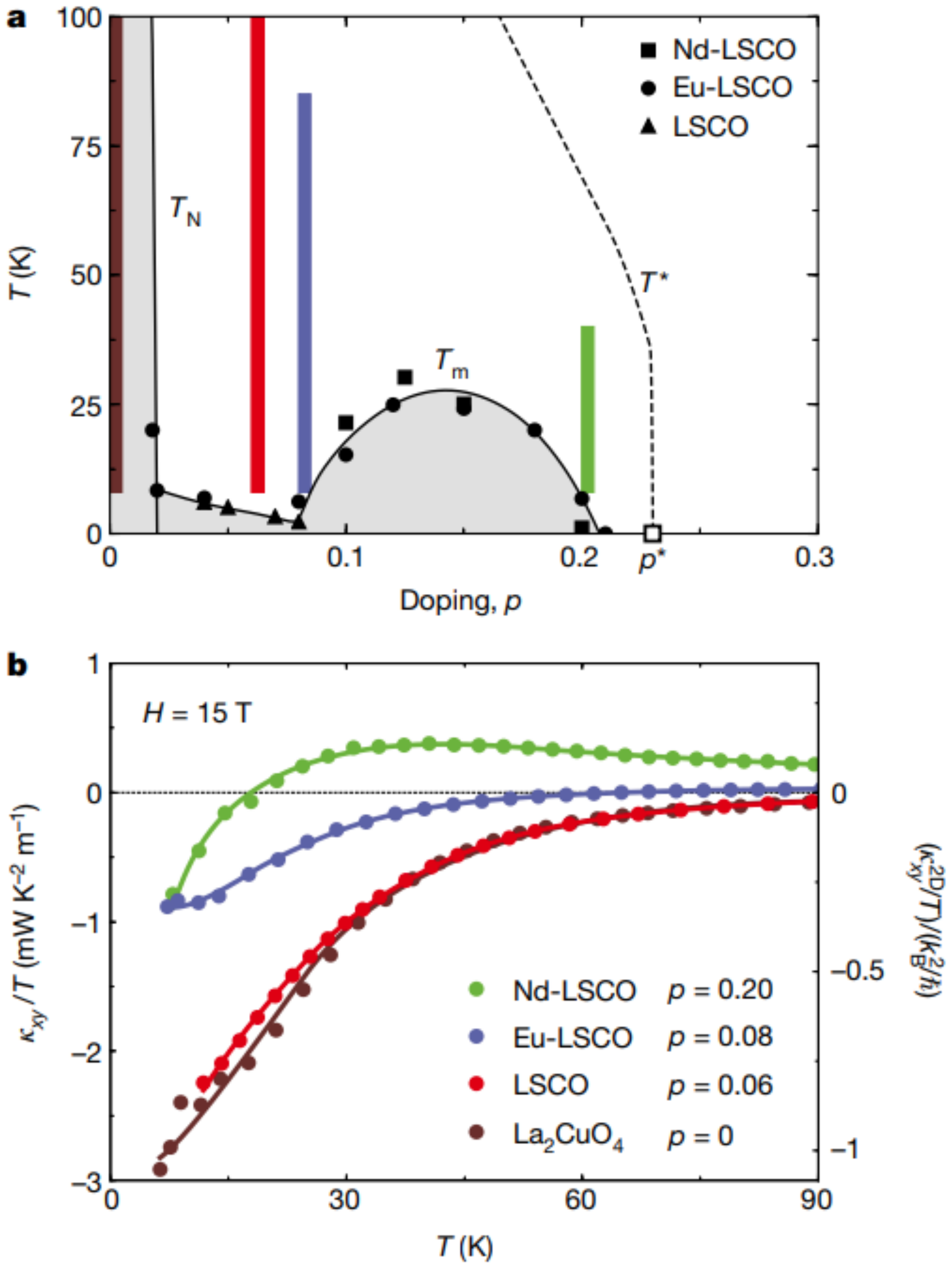
Electron-doped Cuprate

M. Boulanger et al. *arXiv:2112.09187*



Antiferromagnetic Insulator

Lu Chen et al. *arXiv:2110.13277*



Hole-doped Cuprate

G. Grissonnanche et al. *Nature* **571,** 376–380 (2019)