

Quantum phase transition at non-zero doping in a random t - J model

CIFAR Quantum Materials mini-meeting
January 27, 2021
Subir Sachdev



Talk online: sachdev.physics.harvard.edu



Henry Shackleton

arXiv:2012.06589



Alexander Wietek



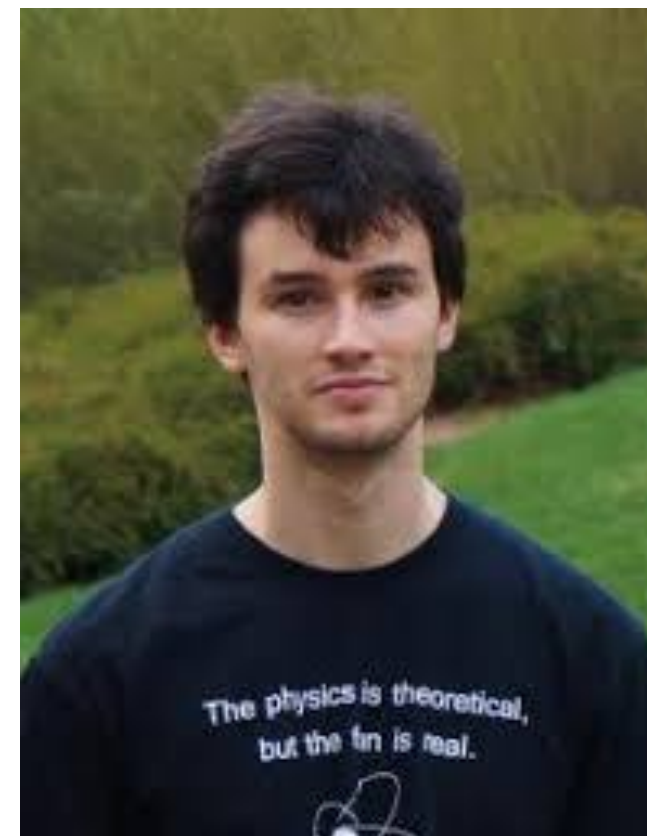
Antoine Georges



Maria Tikhanovskaya



Haoyu Guo



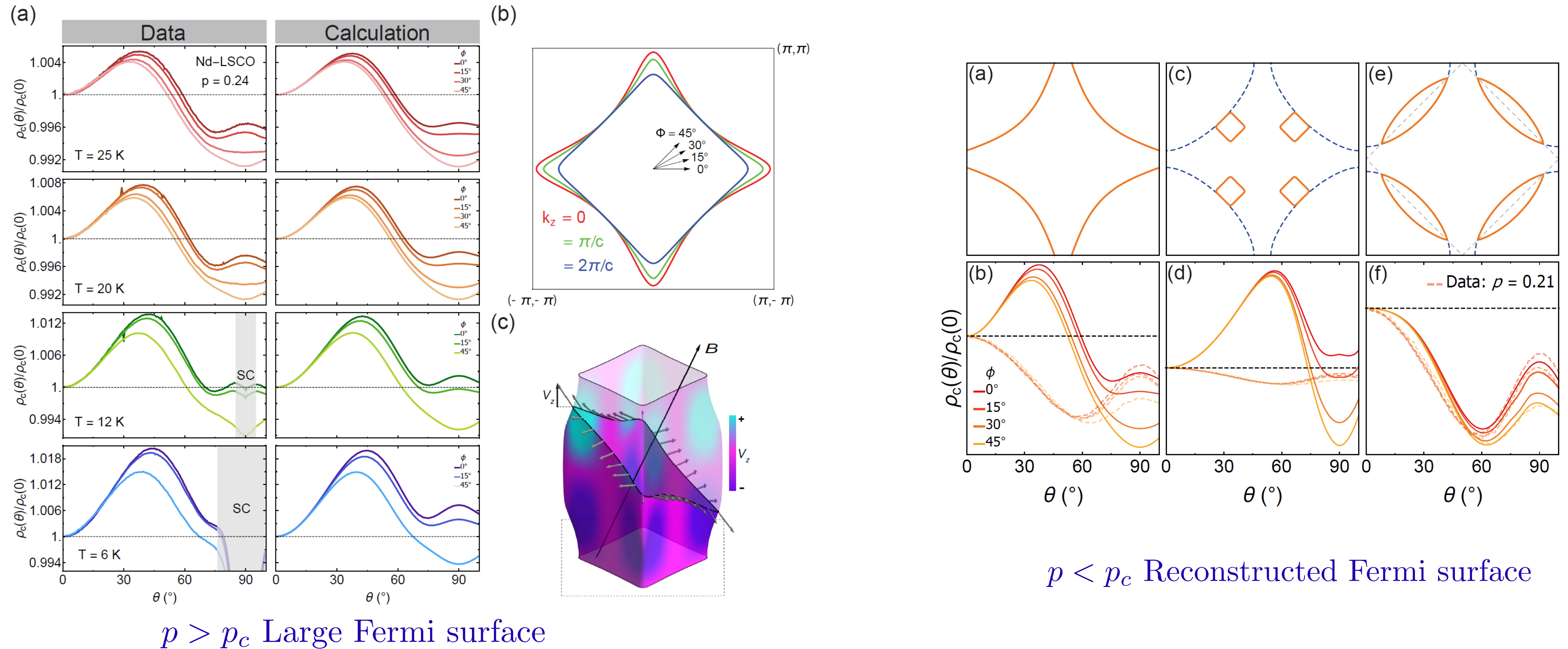
Grigory Tarnopolsky

arXiv:2010.09742
arXiv:2012.14449

Fermi surface transformation at the pseudogap critical point of a cuprate superconductor

Yawen Fang, Gaël Grissonnanche, Anaëlle Legros, Simon Verret, Francis Laliberté, Clément Collignon, Amirreza Ataei, Maxime Dion, Jianshi Zhou, David Graf, M. J. Lawler, Paul Goddard, Louis Taillefer, and B. J. Ramshaw, arXiv:2004.01725

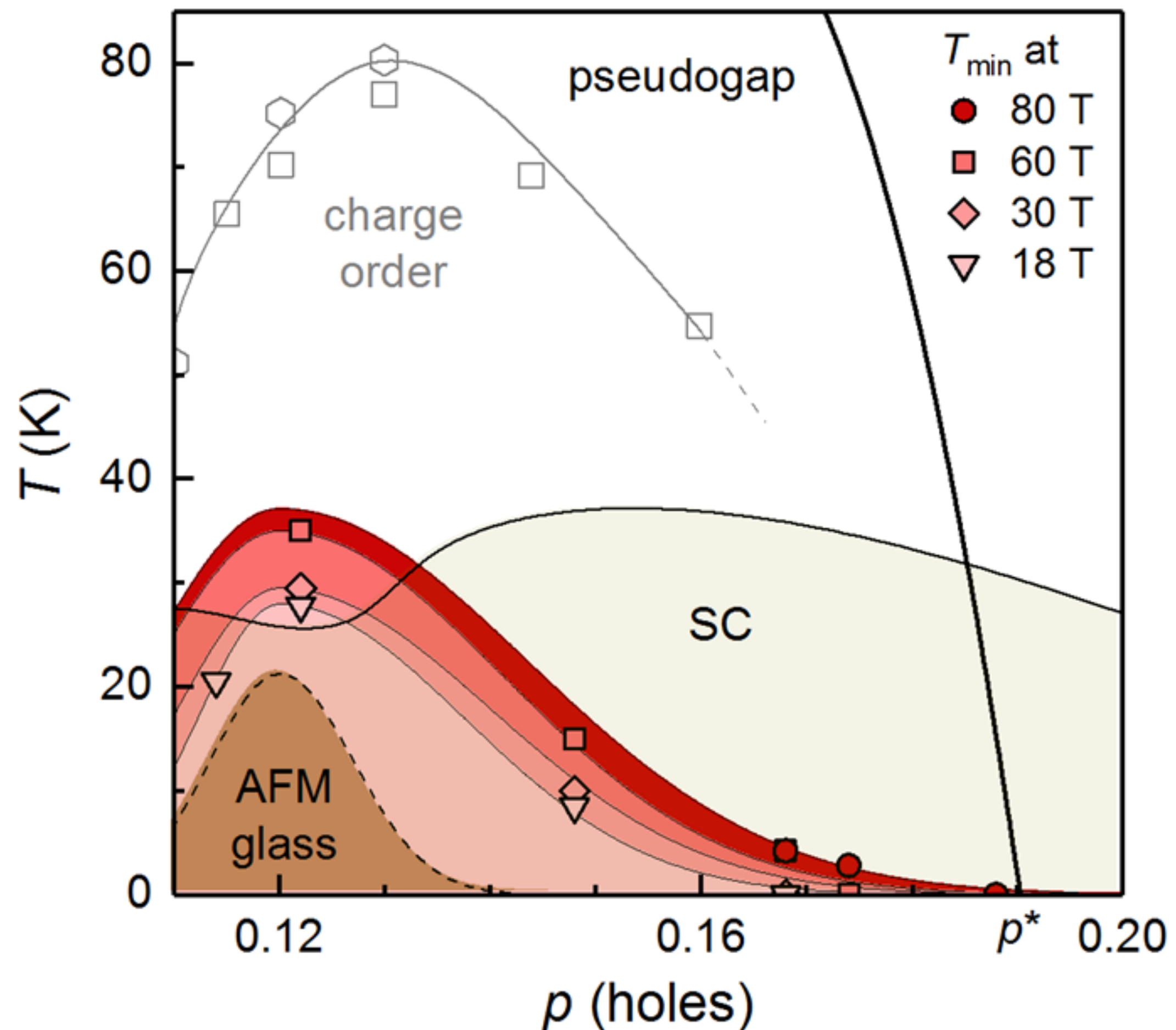
We use angle-dependent magnetoresistance (ADMR) to measure the Fermi surface of the cuprate $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$. Above the critical doping p^* — outside of the pseudogap phase — we find a Fermi surface that is in quantitative agreement with angle-resolved photoemission. Below p^* , however, the ADMR is qualitatively different, revealing a clear change in Fermi surface topology. We find that our data is most consistent with a Fermi surface that has been reconstructed by a $Q = (\pi, \pi)$ wavevector. While static $Q = (\pi, \pi)$ antiferromagnetism is not found at these dopings, our results suggest that this wavevector is a fundamental organizing principle of the pseudogap phase.



Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics doi: 10.1038/s41567-020-0950-5

Mehdi Frachet^{1†}, Igor Vinograd^{1†}, Rui Zhou^{1,2}, Siham Benhabib¹, Shangfei Wu¹, Hadrien Mayaffre¹, Steffen Krämer¹, Sanath K. Ramakrishna³, Arneil P. Reyes³, Jérôme Debray⁴, Tohru Kurosawa⁵, Naoki Momono⁶, Migaku Oda⁵, Seiki Komiya⁷, Shimpei Ono⁷, Masafumi Horio⁸, Johan Chang⁸, Cyril Proust¹, David LeBoeuf^{1*}, Marc-Henri Julien^{1*}



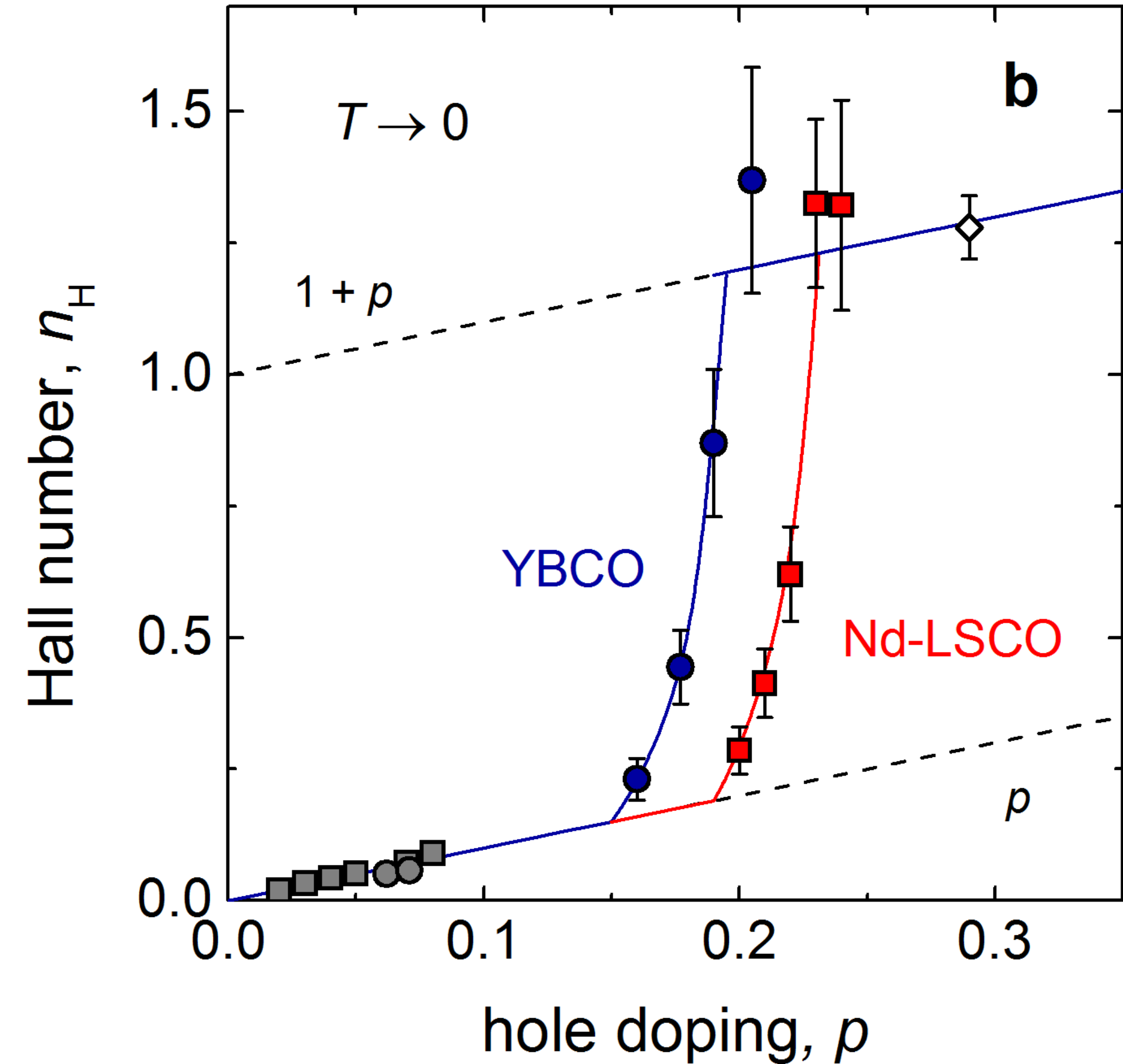
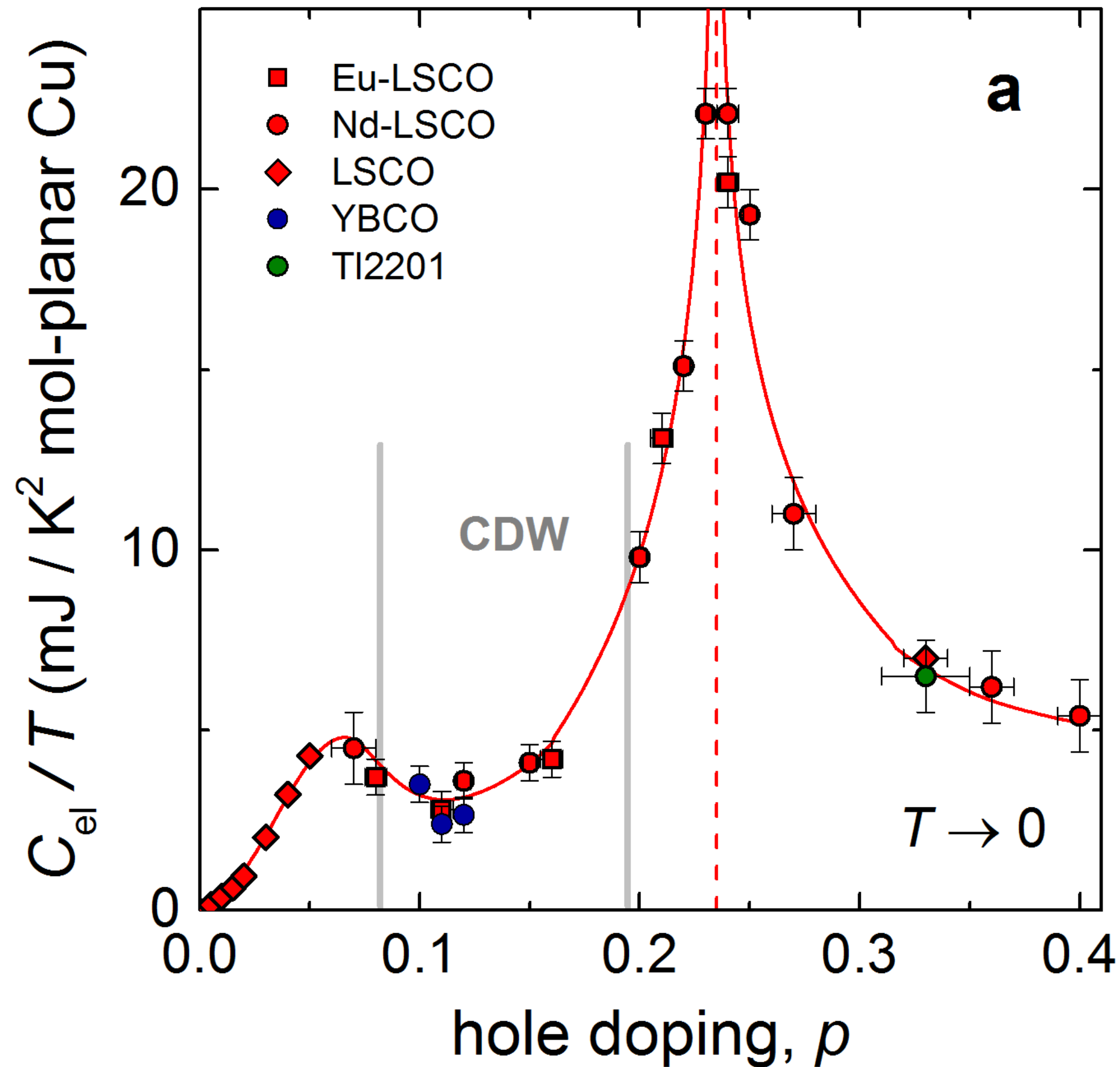
Quasi-static magnetism in the pseudogap state of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$.

Temperature – doping phase diagram representing T_{min} , the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

Hole doped cuprates

The remarkable underlying ground states of cuprate superconductors

Cyril Proust and Louis Taillefer, Annual Review Condensed Matter Physics **10**, 409 (2019)



Random t - J model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^N t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^N J_{ij} \vec{S}_i \cdot \vec{S}_j$$

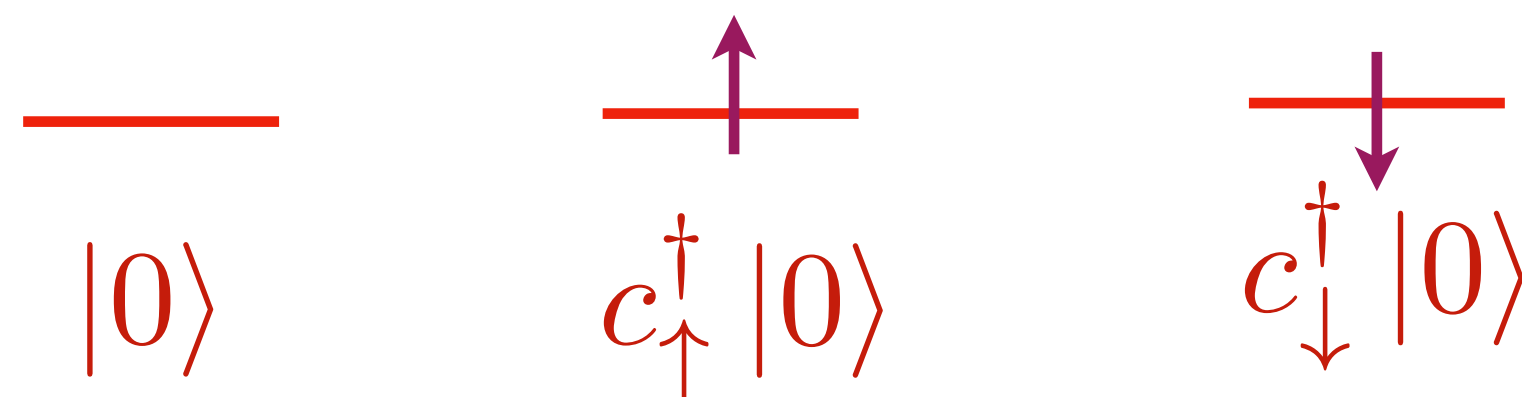
We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij} \delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_{\alpha} c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N} \sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random, } \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2$$

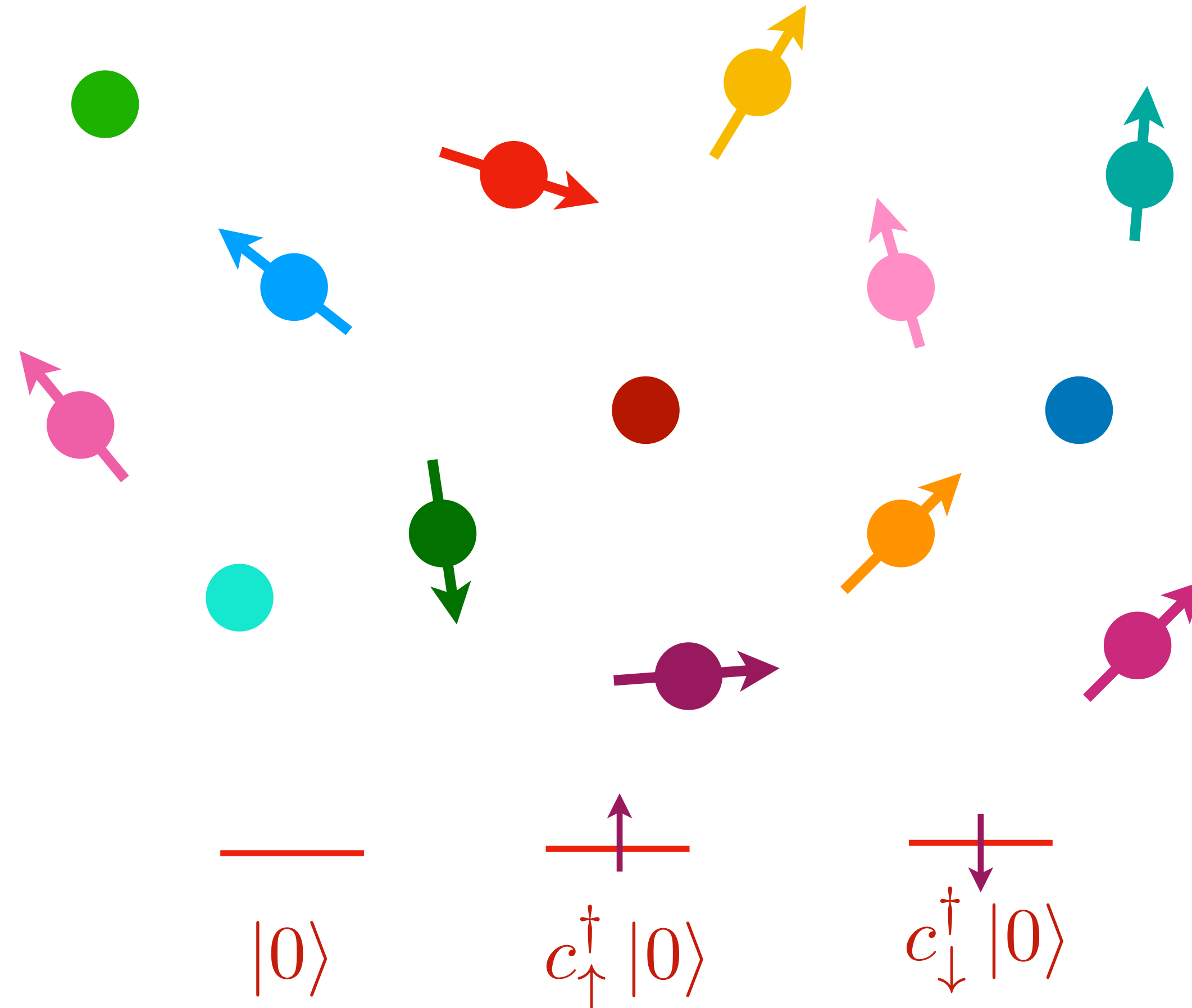
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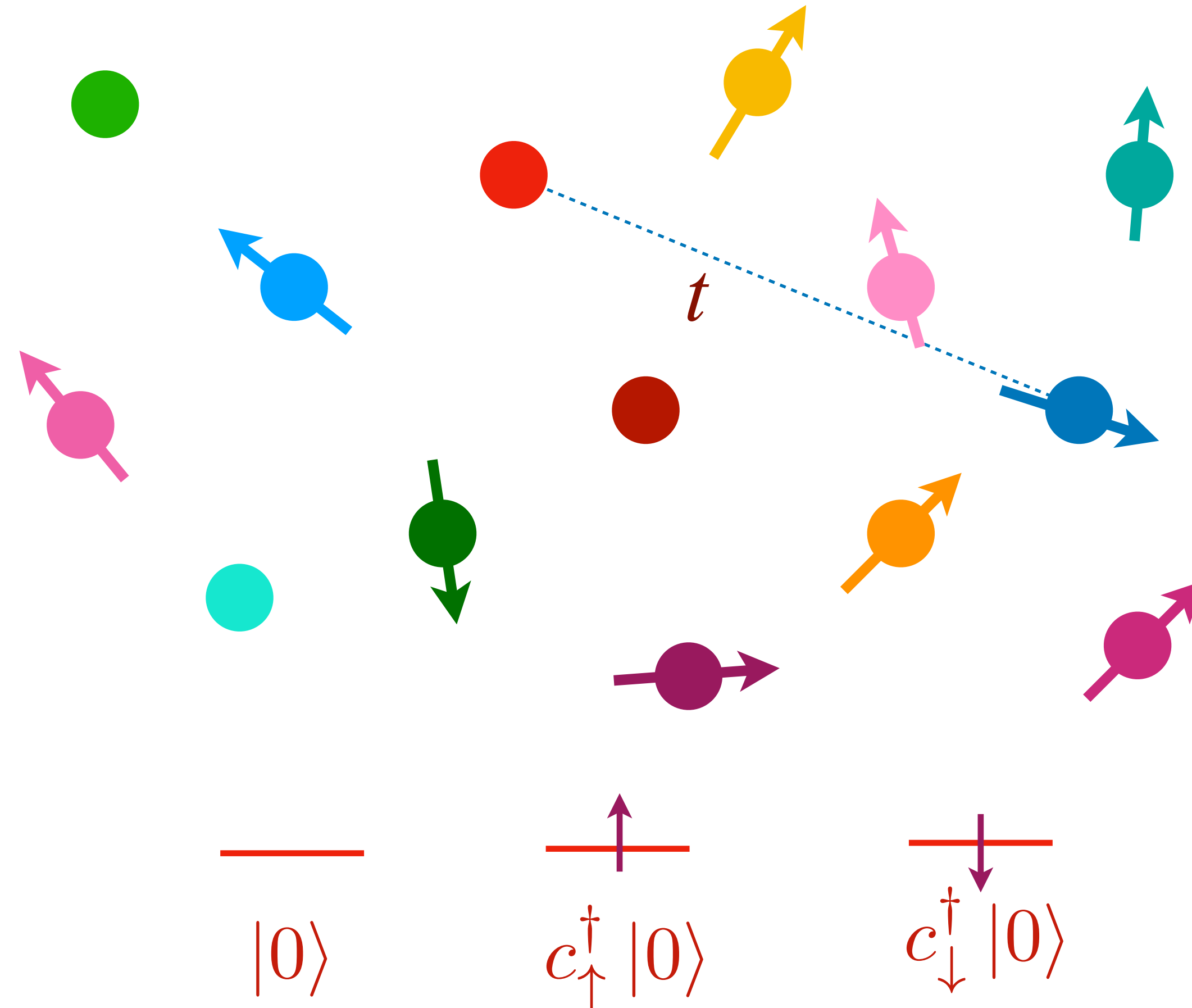
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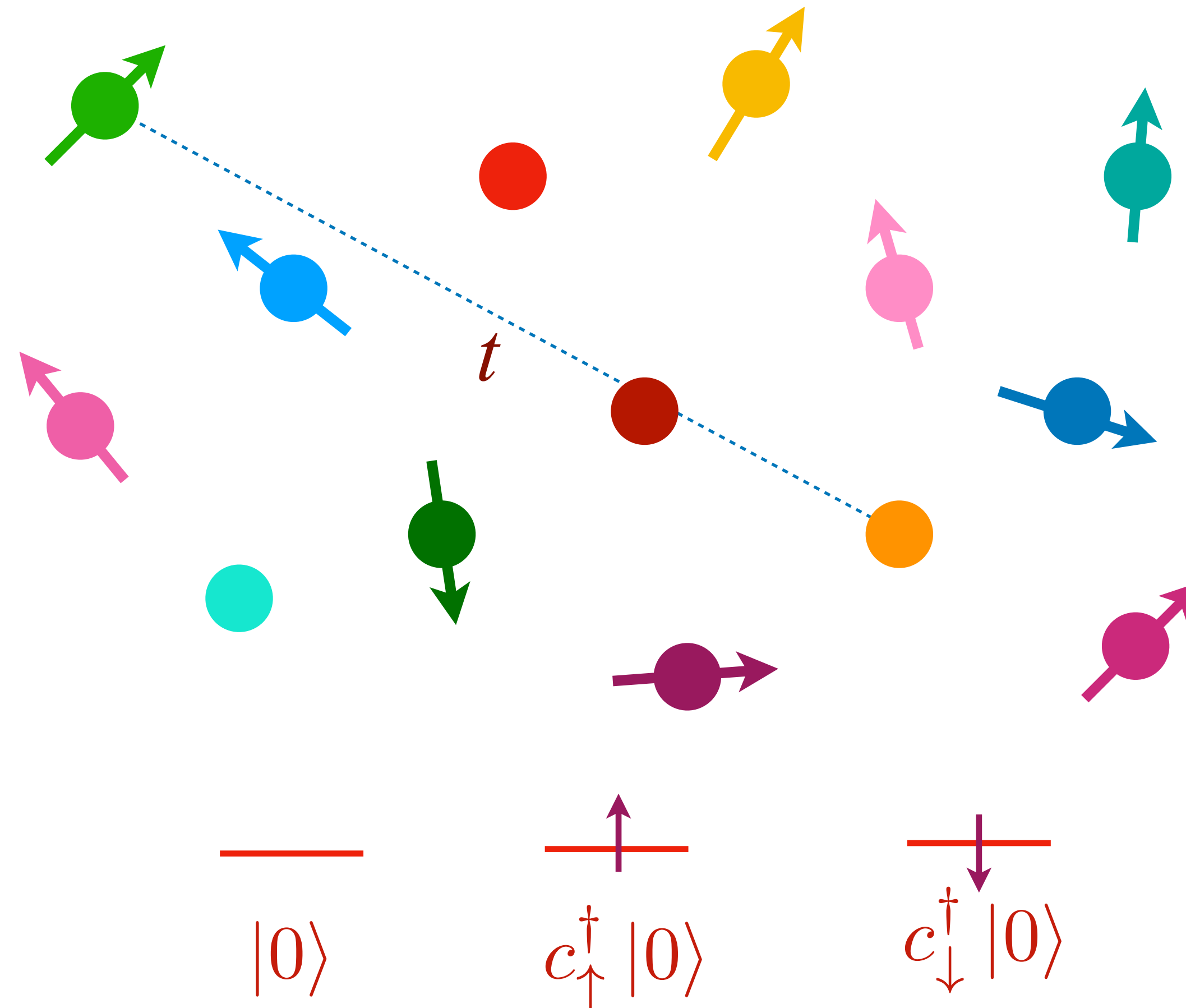
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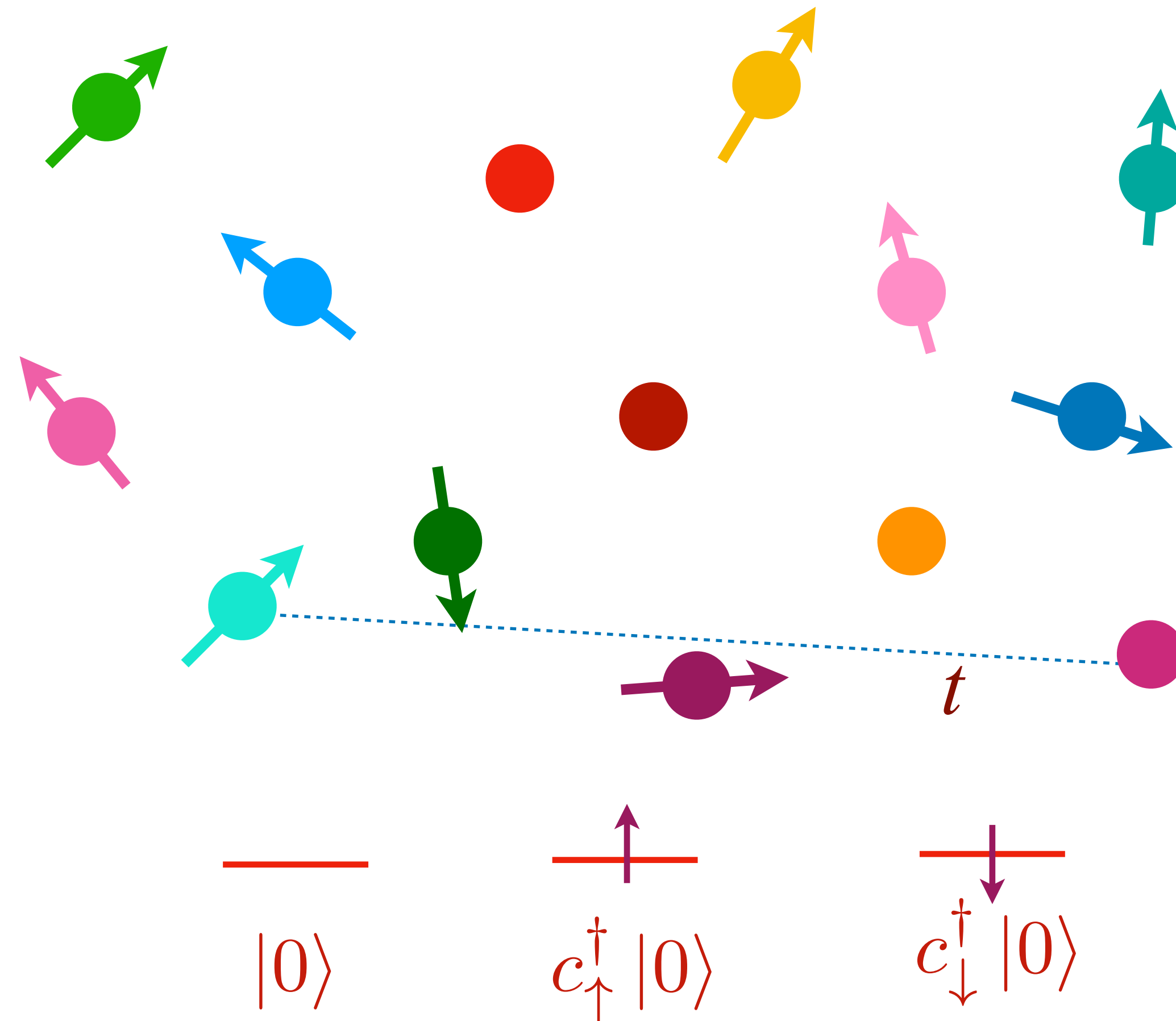
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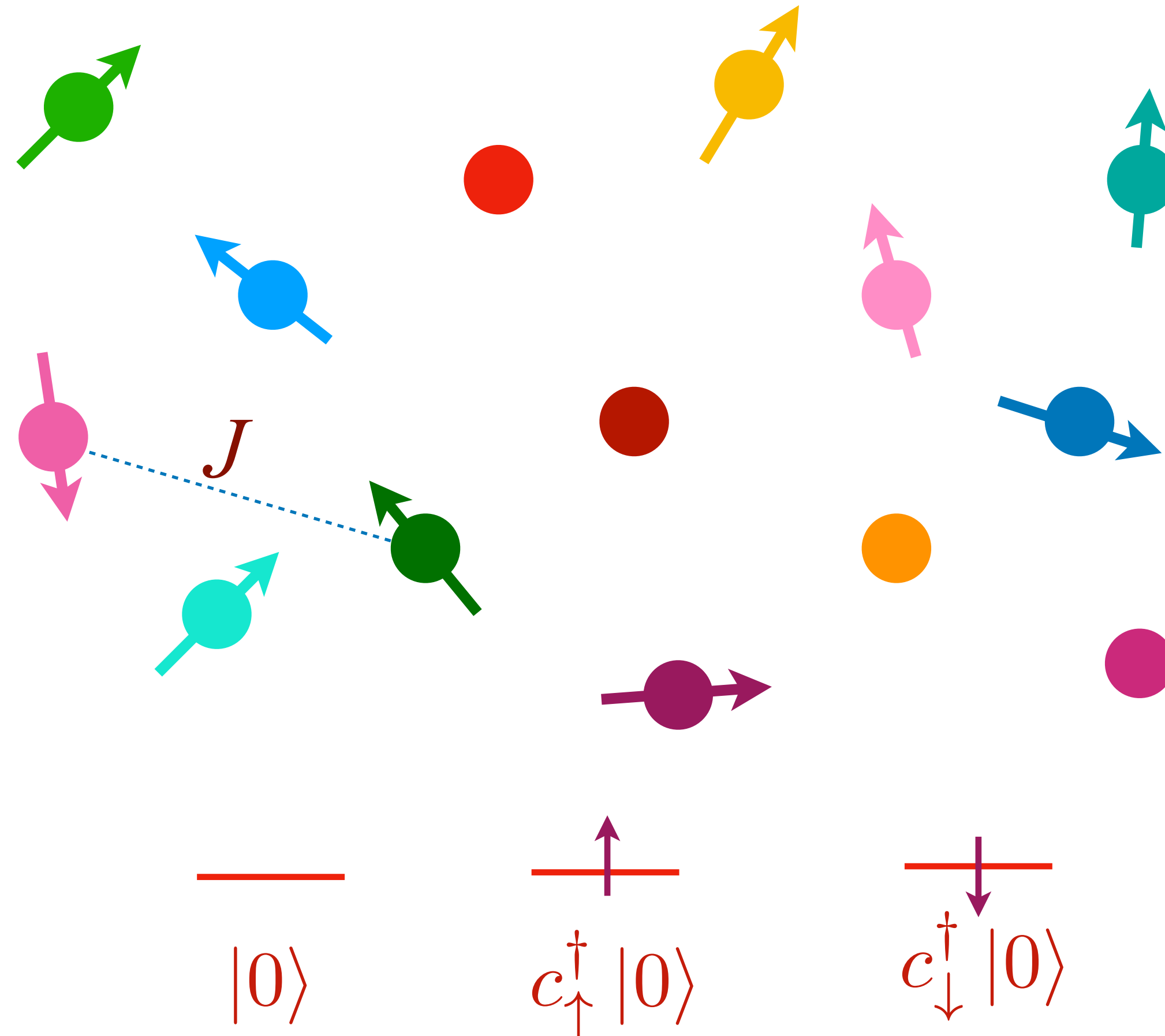
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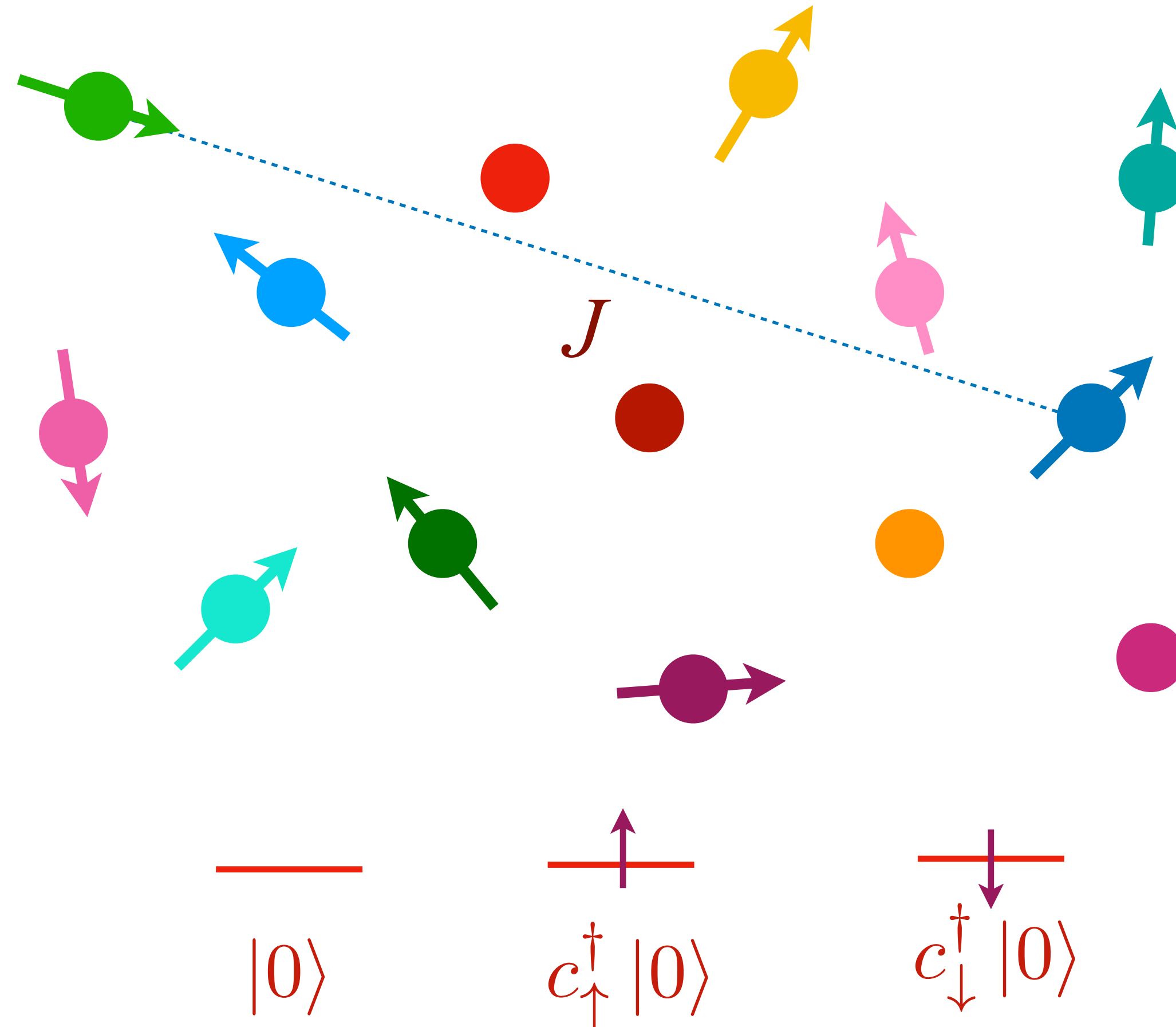
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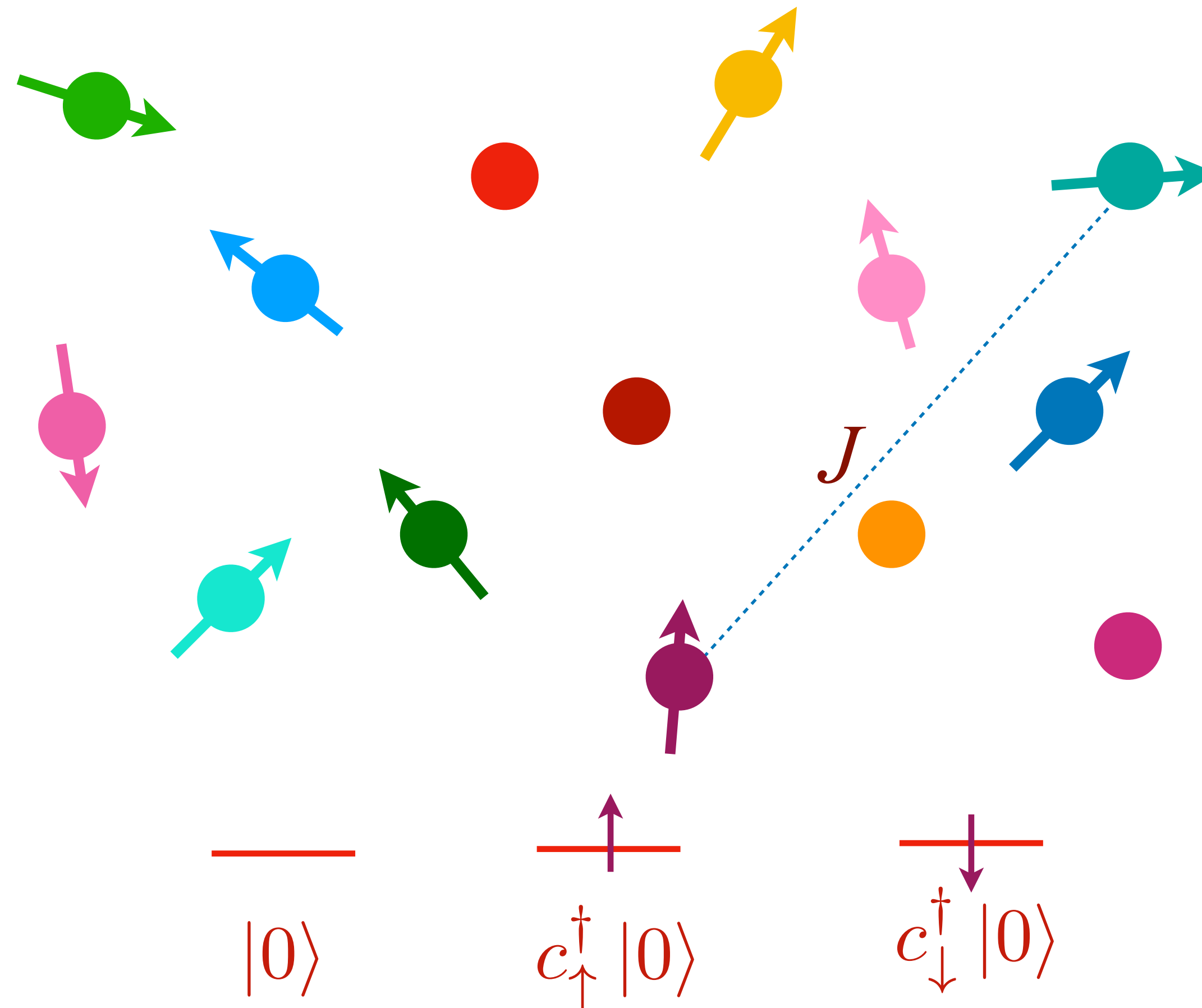
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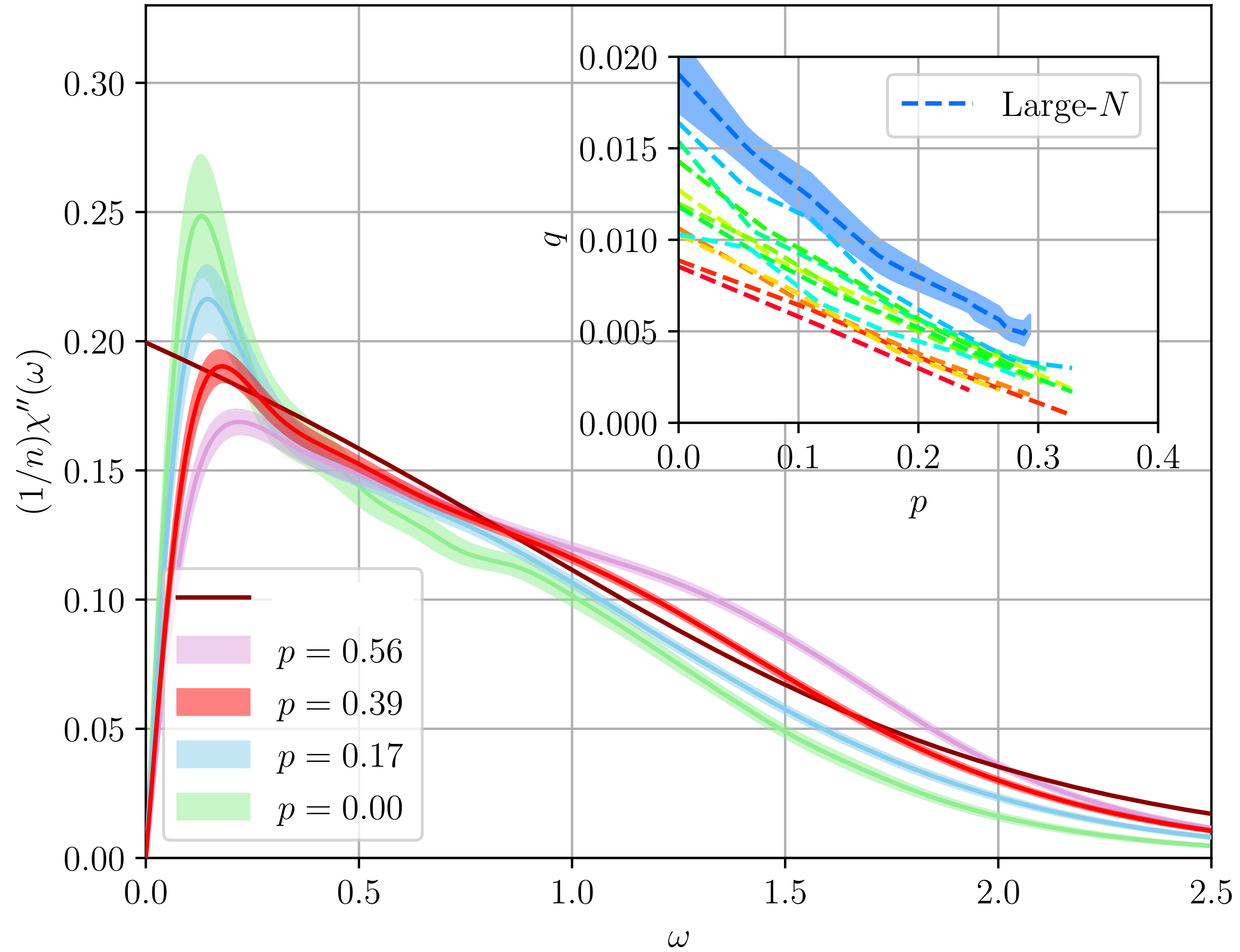
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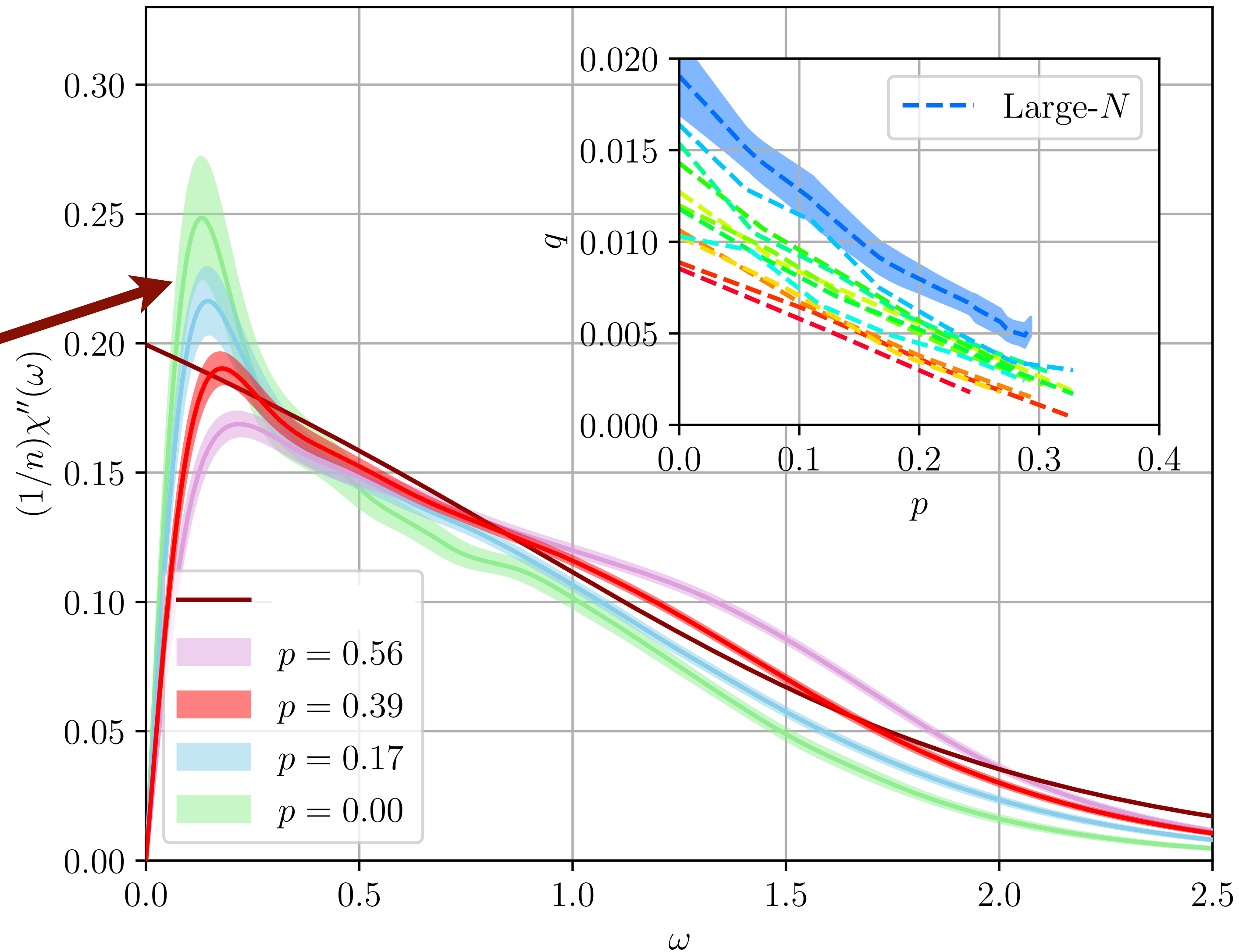
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- Introducing randomness removes the “distractions” of multiple competing orders
- Averaging over many samples allows smoother and faster approach to the thermodynamic limit from finite size studies.

Dynamic spin susceptibility

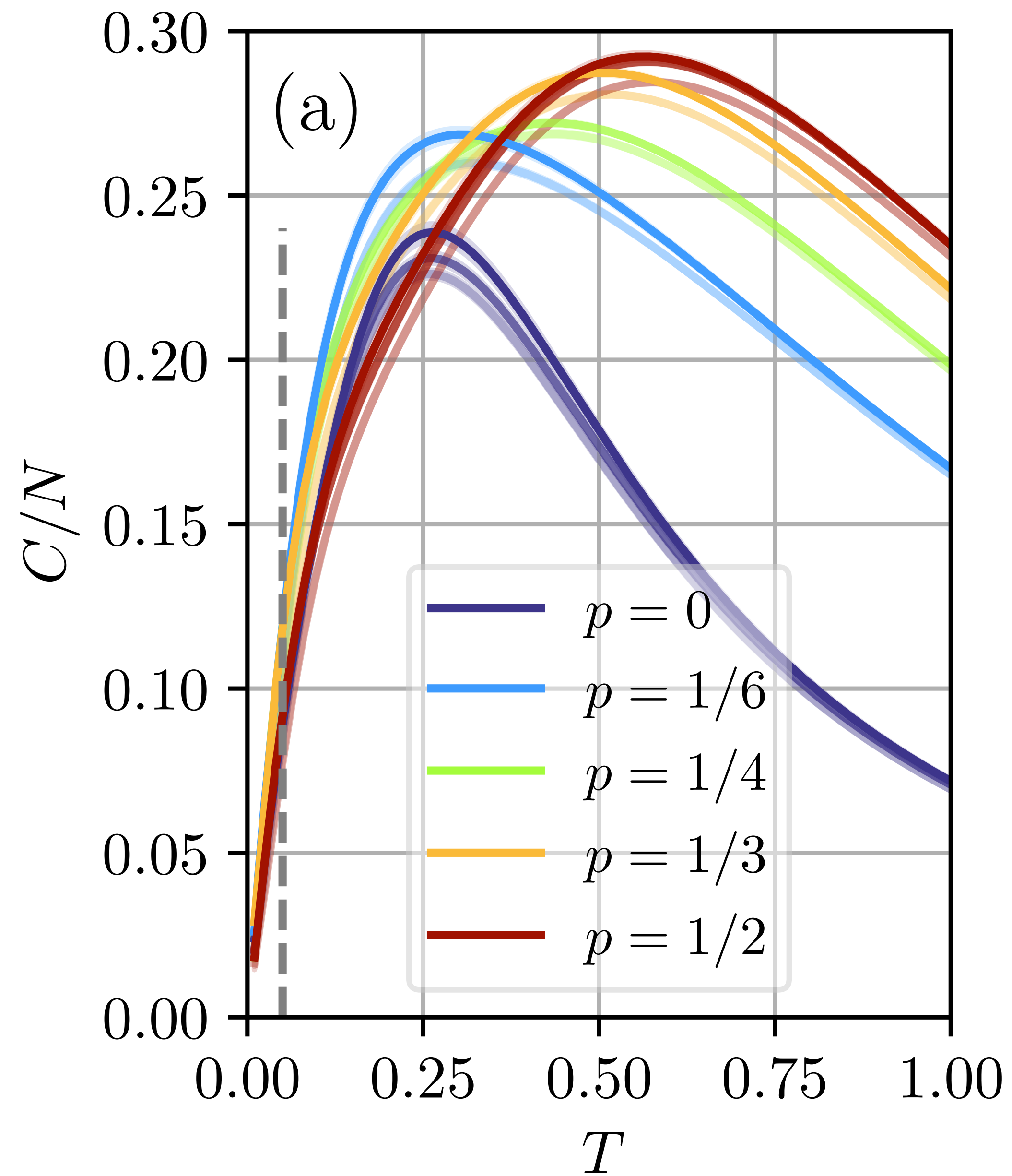


Dynamic spin susceptibility

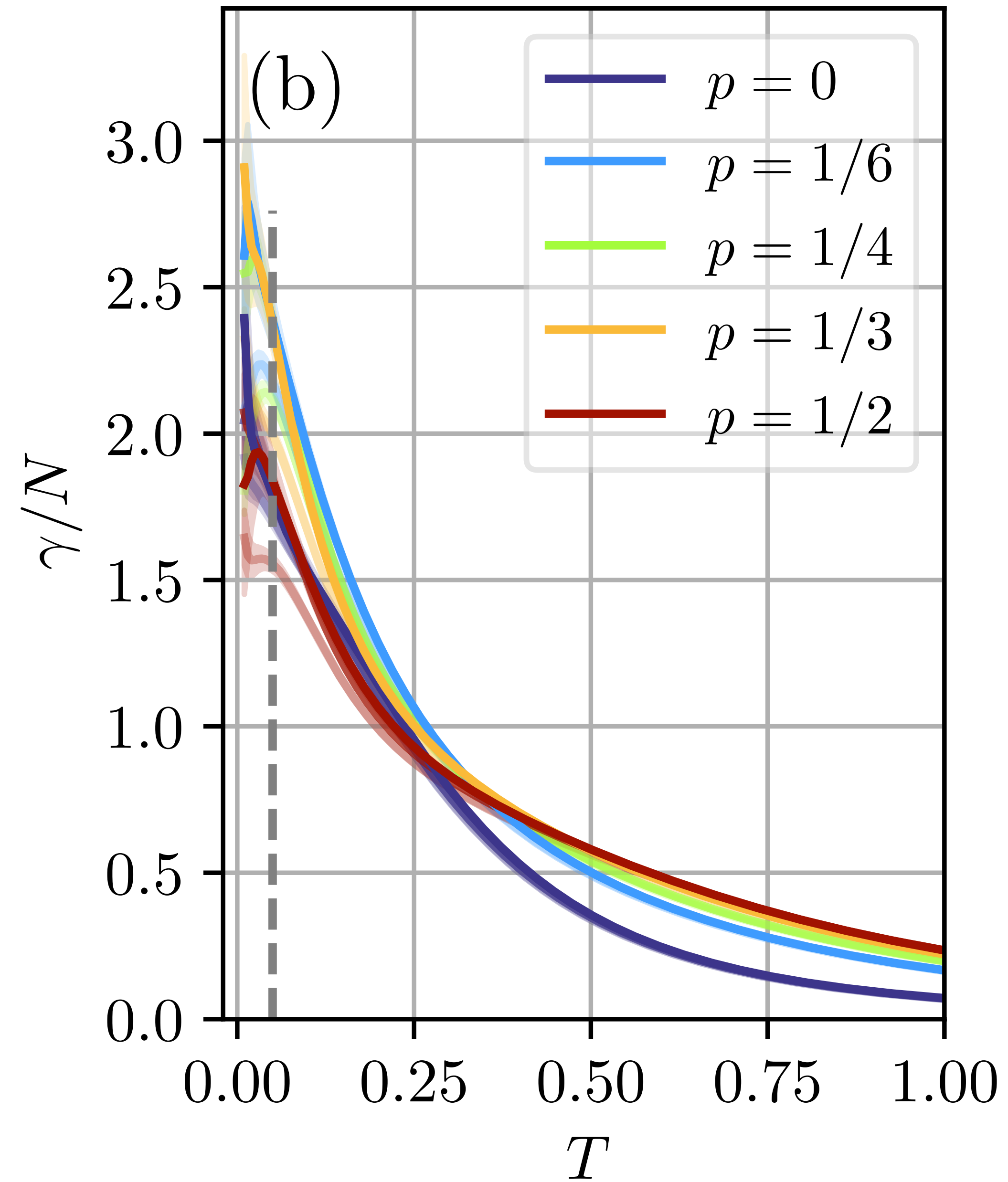


Evidence for a quantum critical point at $p = p_c \approx 0.3$.
Spin glass order q non-zero for $p < p_c$

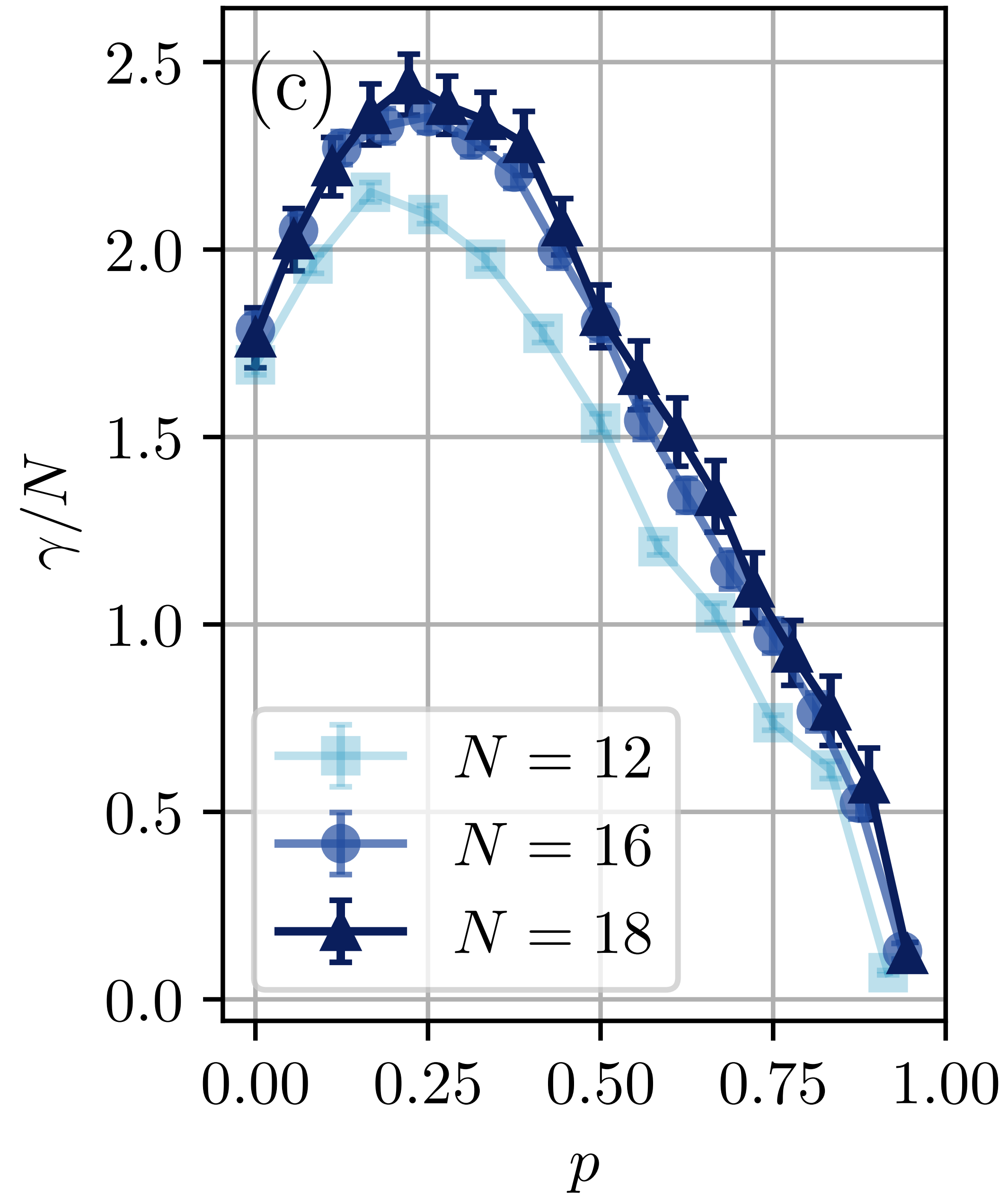
Specific heat



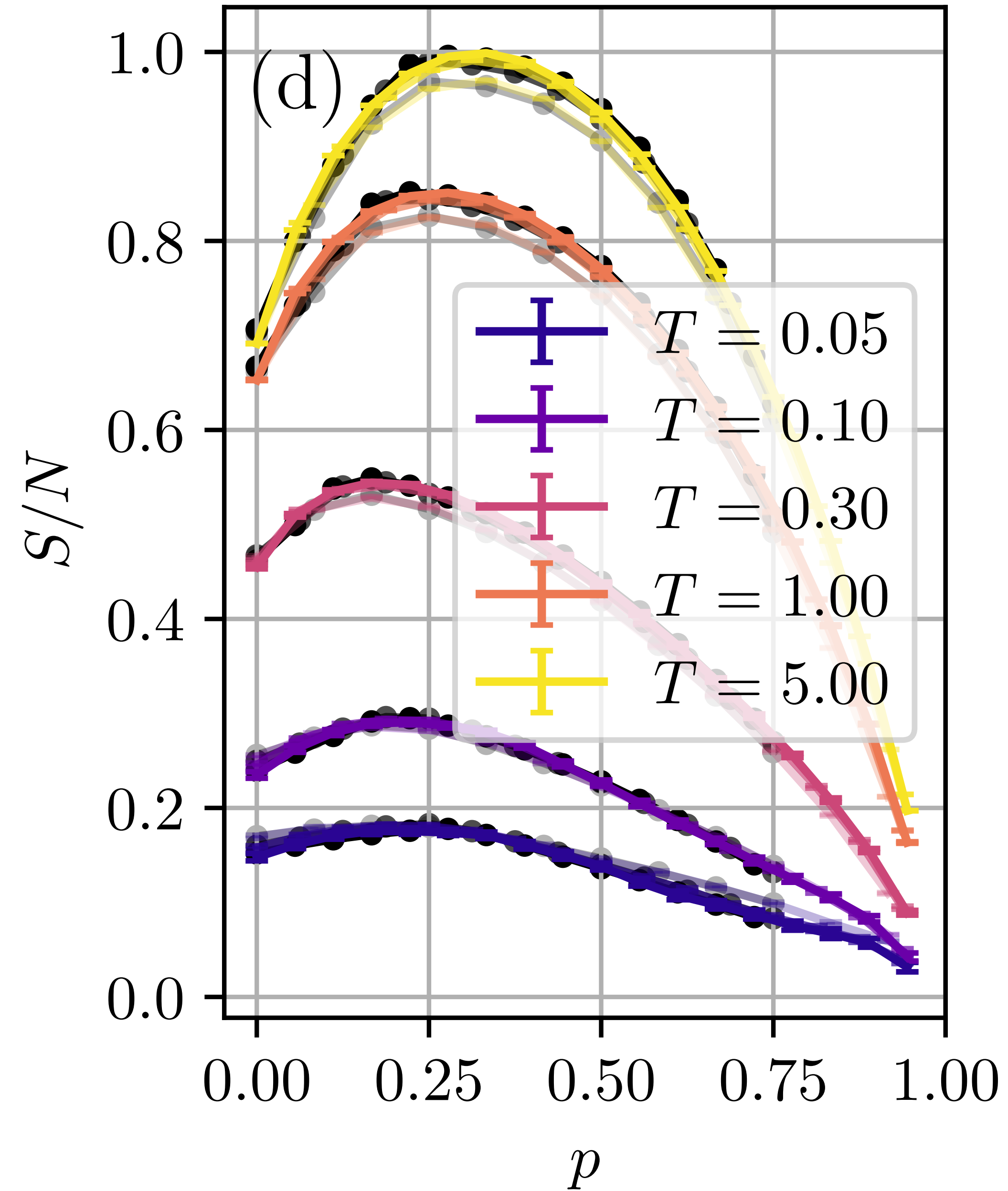
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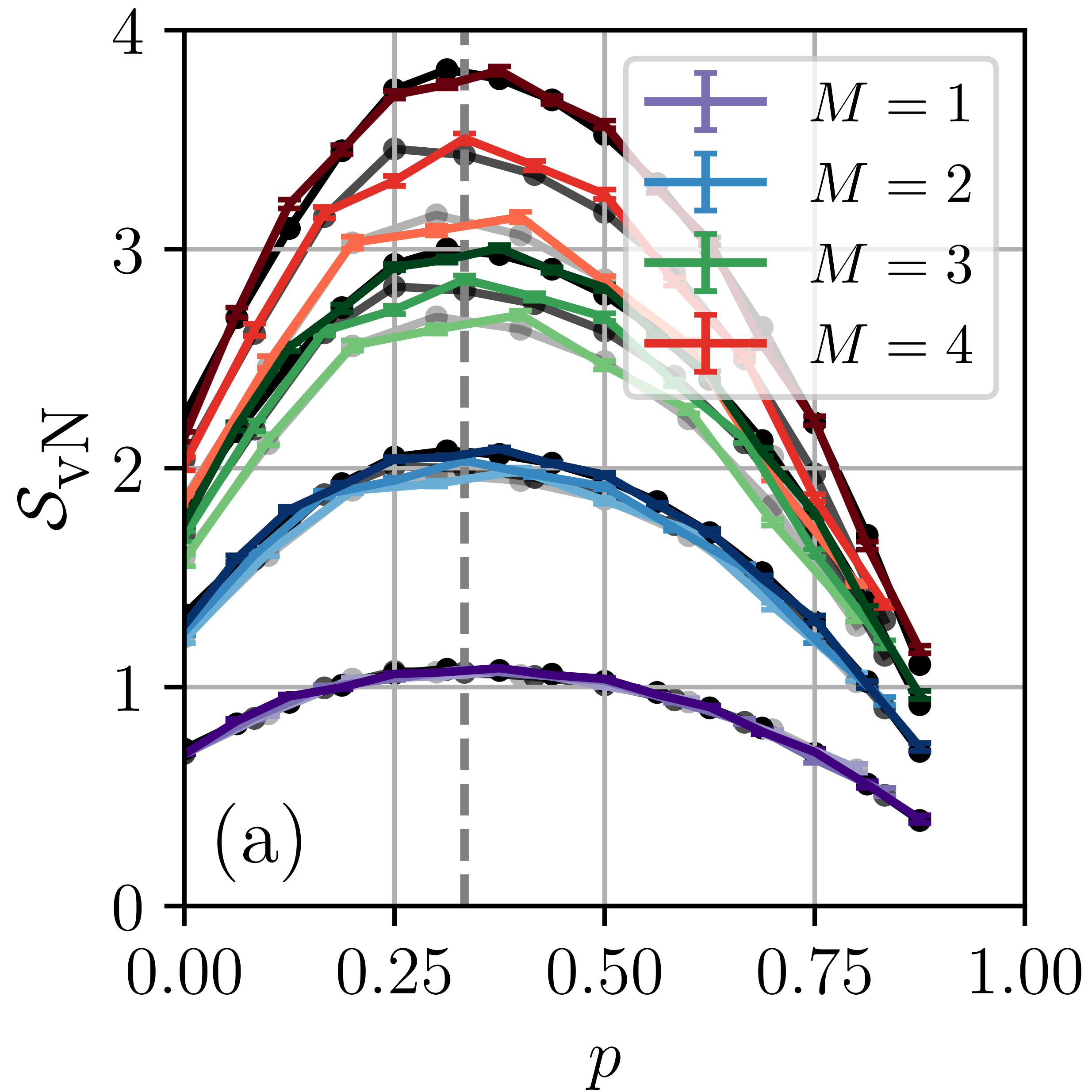
Specific heat



Entropy

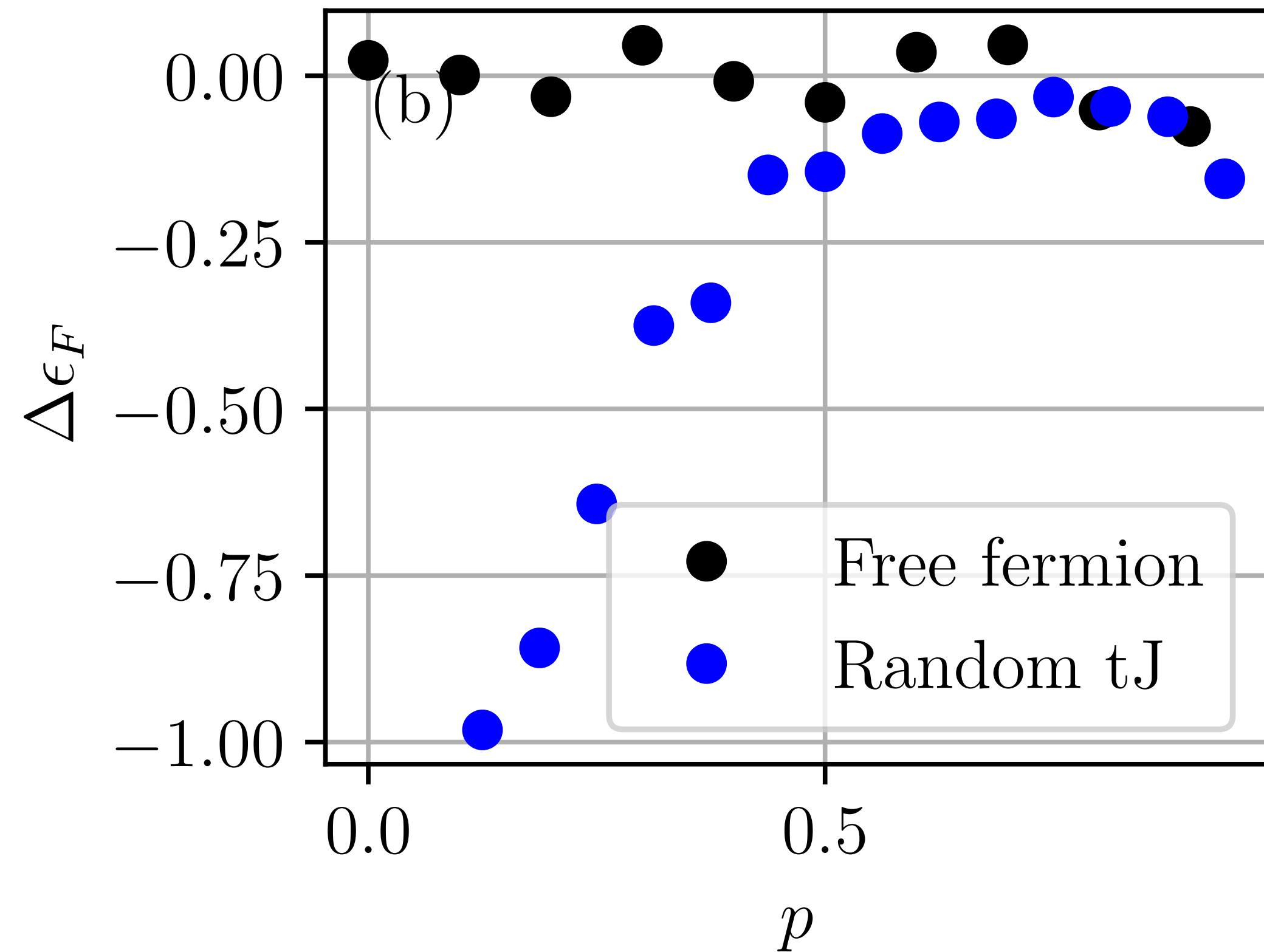
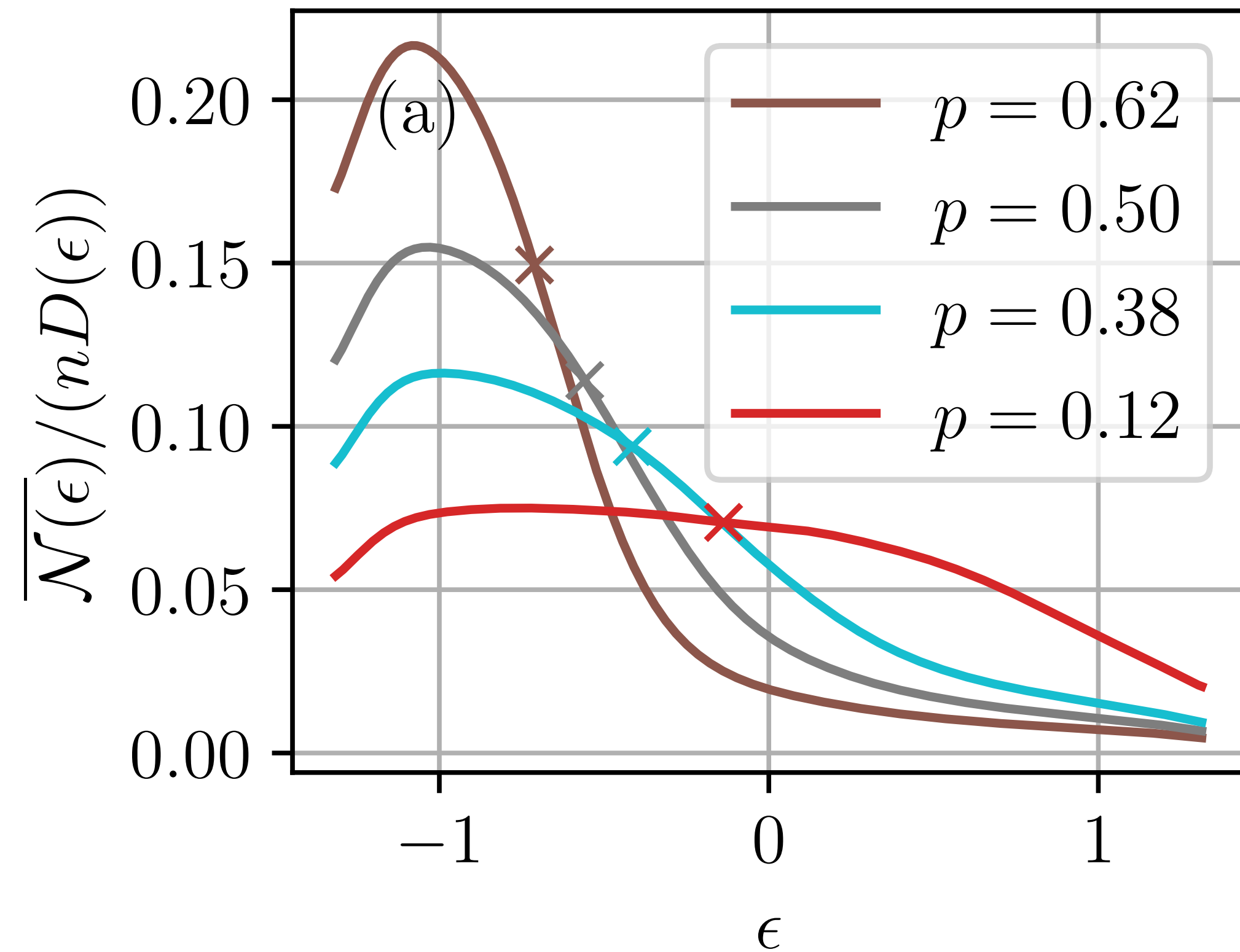


Entanglement Entropy



(a)

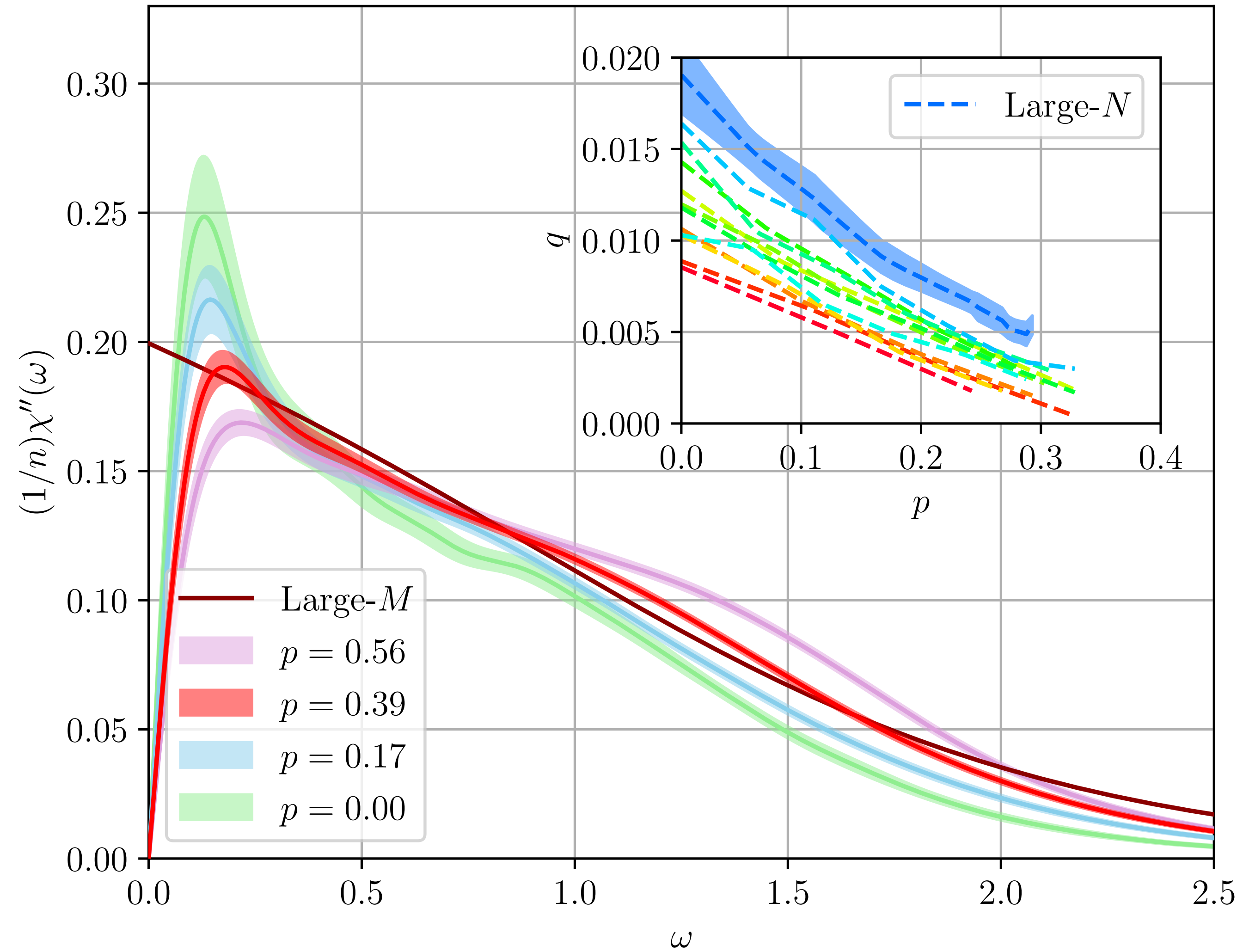
One particle energy distribution function



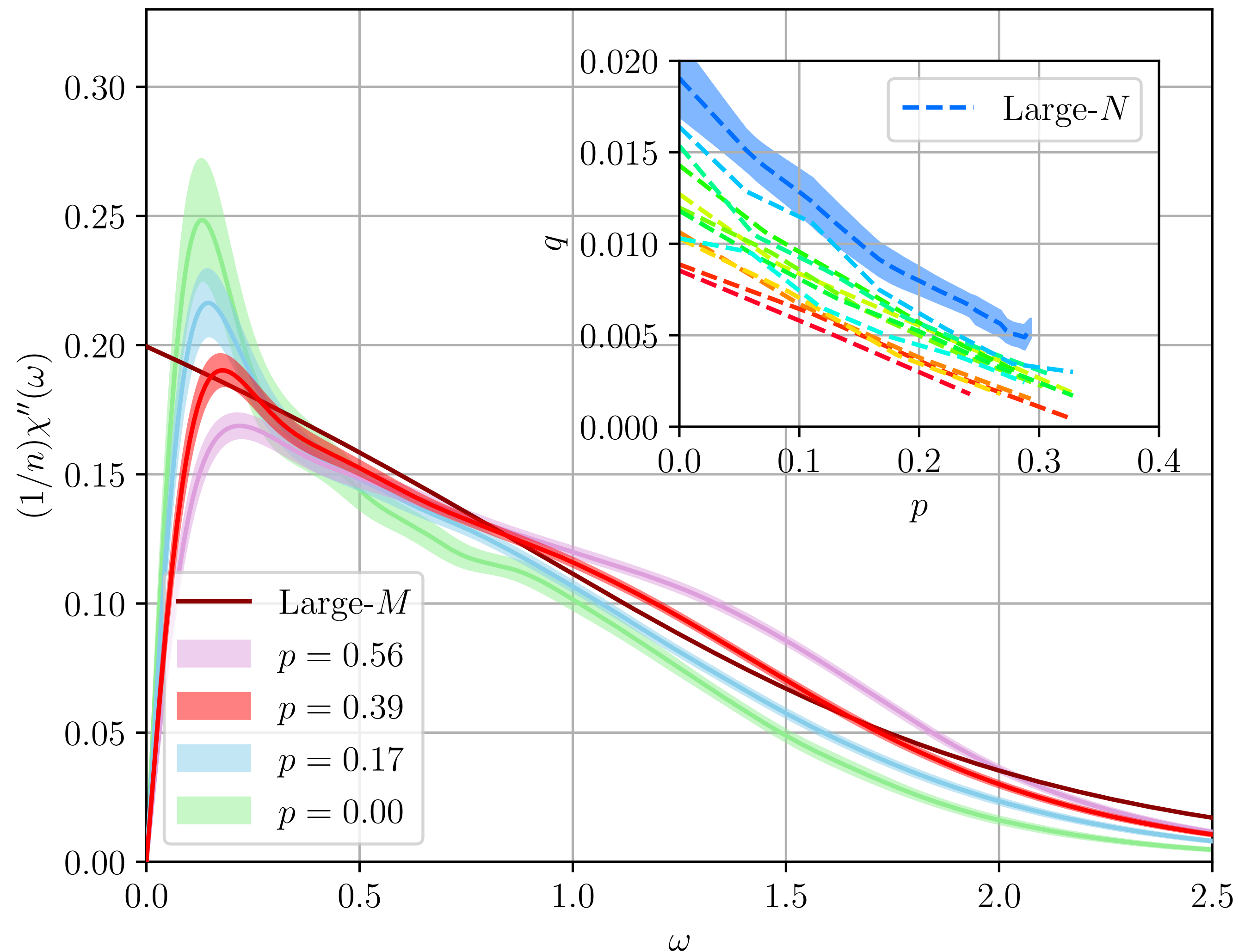
$$\mathcal{N}(\epsilon) = \frac{1}{N} \sum_{\lambda} \delta(\epsilon - \epsilon_{\lambda}) \sum_{ij\sigma} \langle \lambda | i \rangle \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle \langle j | \lambda \rangle$$

where $|\lambda\rangle$ are one-particle eigenstates of the t_{ij} . In a Fermi liquid, the Luttinger identity implies that $\mathcal{N}(\epsilon)$ has a discontinuity at the free particle Fermi energy ϵ_F . ($D(\epsilon)$ is the Wigner semi-circle density of states.)

Dynamic spin susceptibility



Dynamic spin susceptibility



Critical spin susceptibility matches the large M $SU(M)$ SYK model.

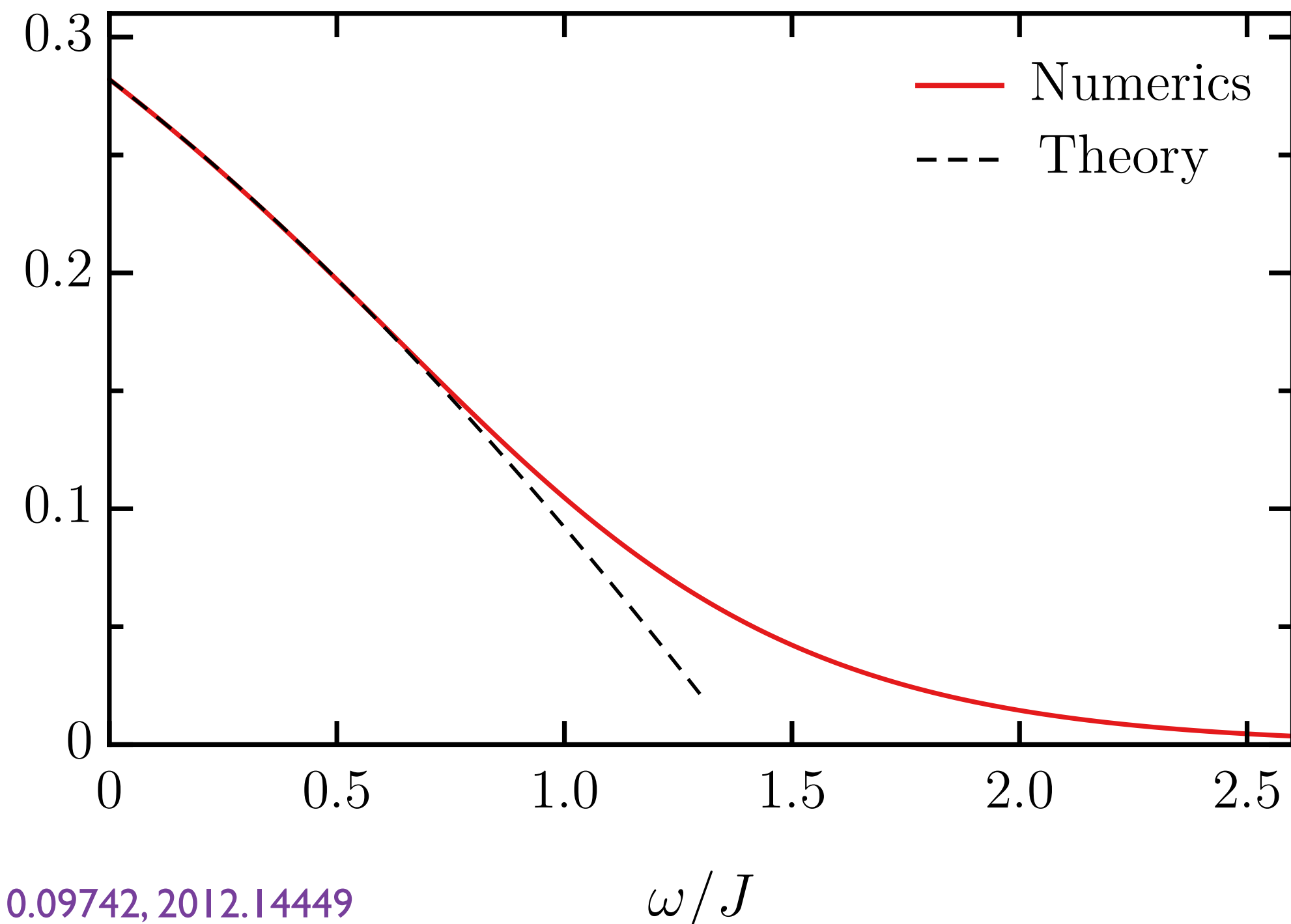
$\chi''(\omega) \sim \text{sgn}(\omega) [1 - \mathcal{C}\gamma|\omega| + \dots]$ has the ‘marginal’ $\text{sgn}(\omega)$ form, with a linear ω correction. Shown is the numerical solution of SYK equations (SY, PRL 1993), after rescaling J .

Consequences of 2D-gravity for the dynamic spin susceptibility of SYK model

$$\chi_L(\omega) = \sum_n |\langle 0 | X_i | n \rangle|^2 \delta(\hbar\omega - E_n + E_0), \text{ (at } T = 0)$$

$$\text{Im}\chi_L(\omega) \sim \text{sgn}(\omega) \left[1 - \mathcal{C}\gamma|\omega| - \frac{7}{16}(\mathcal{C}\gamma)^2|\omega|^2 - \mathcal{C}'|\omega|^{2.77354\dots} + \frac{37}{48}(\mathcal{C}\gamma)^3|\omega|^3 - \dots \right]$$

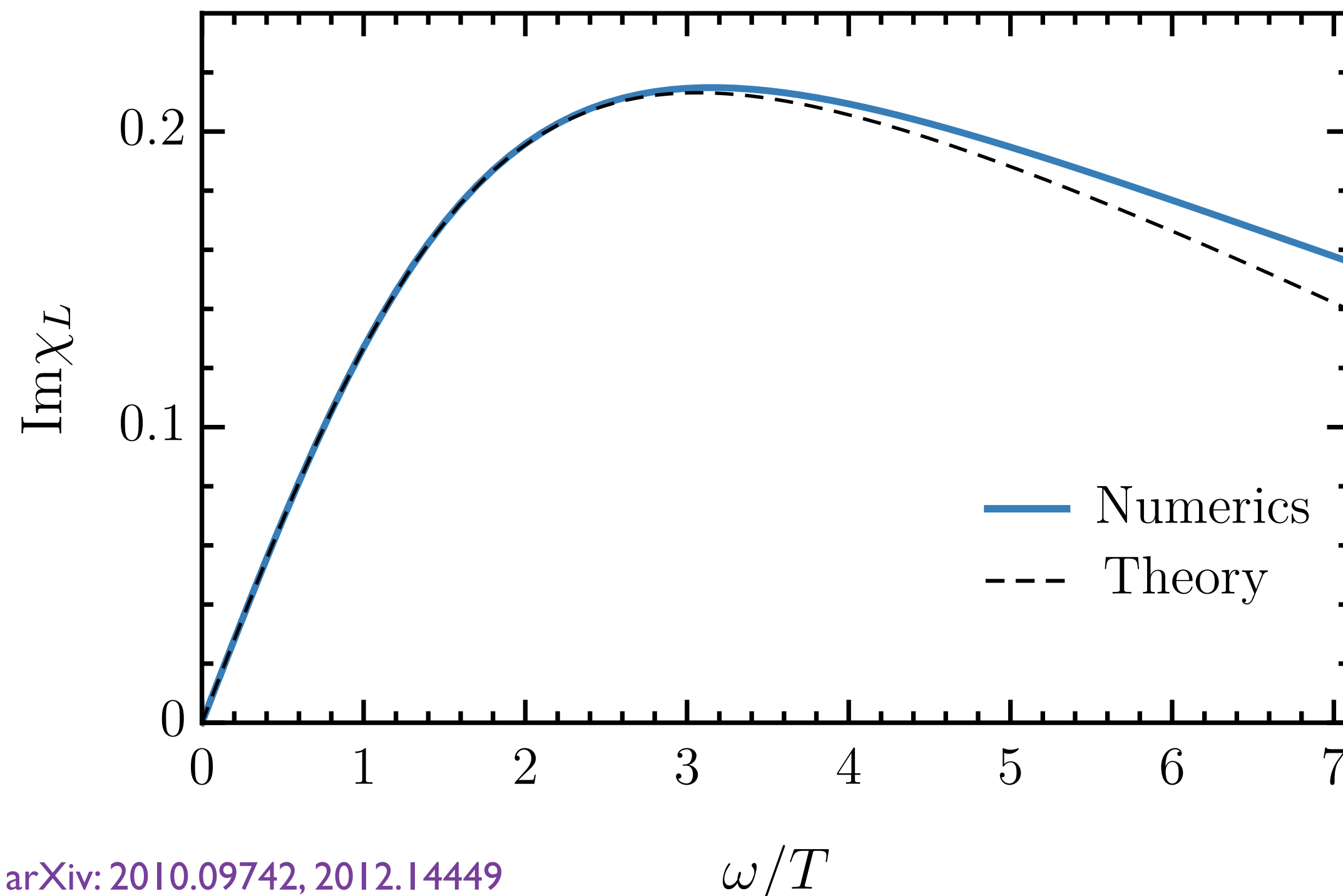
Numerical solution of SYK equations (SY, PRL 1993), compared with conformal perturbation theory.
 \mathcal{C} is the co-efficient of the action for the ‘boundary graviton’ in holographic dual.



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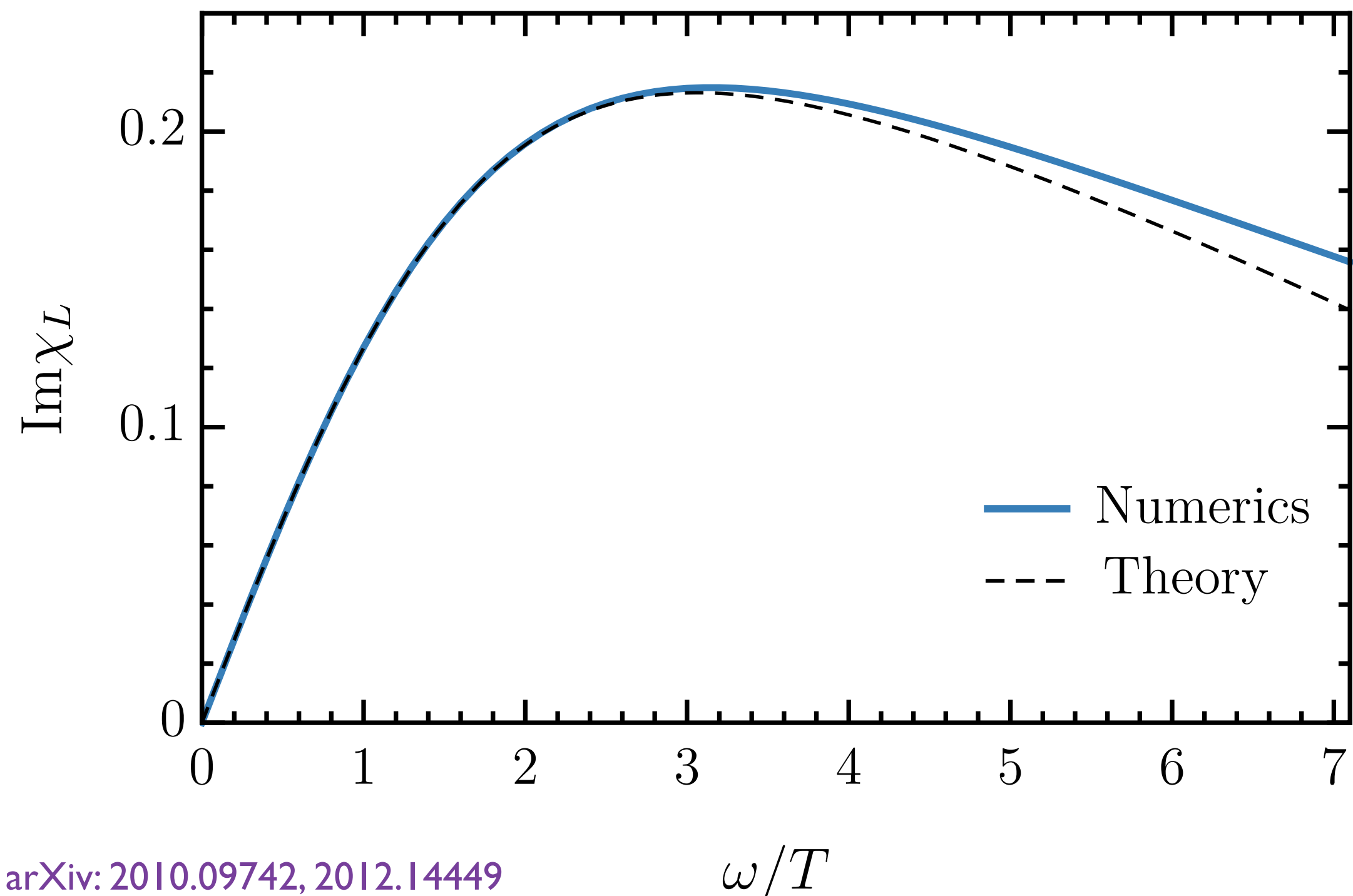
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Conformally (SL(2,R))
invariant result with
characteristic dissipative
time $\sim \hbar/(k_B T)$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)

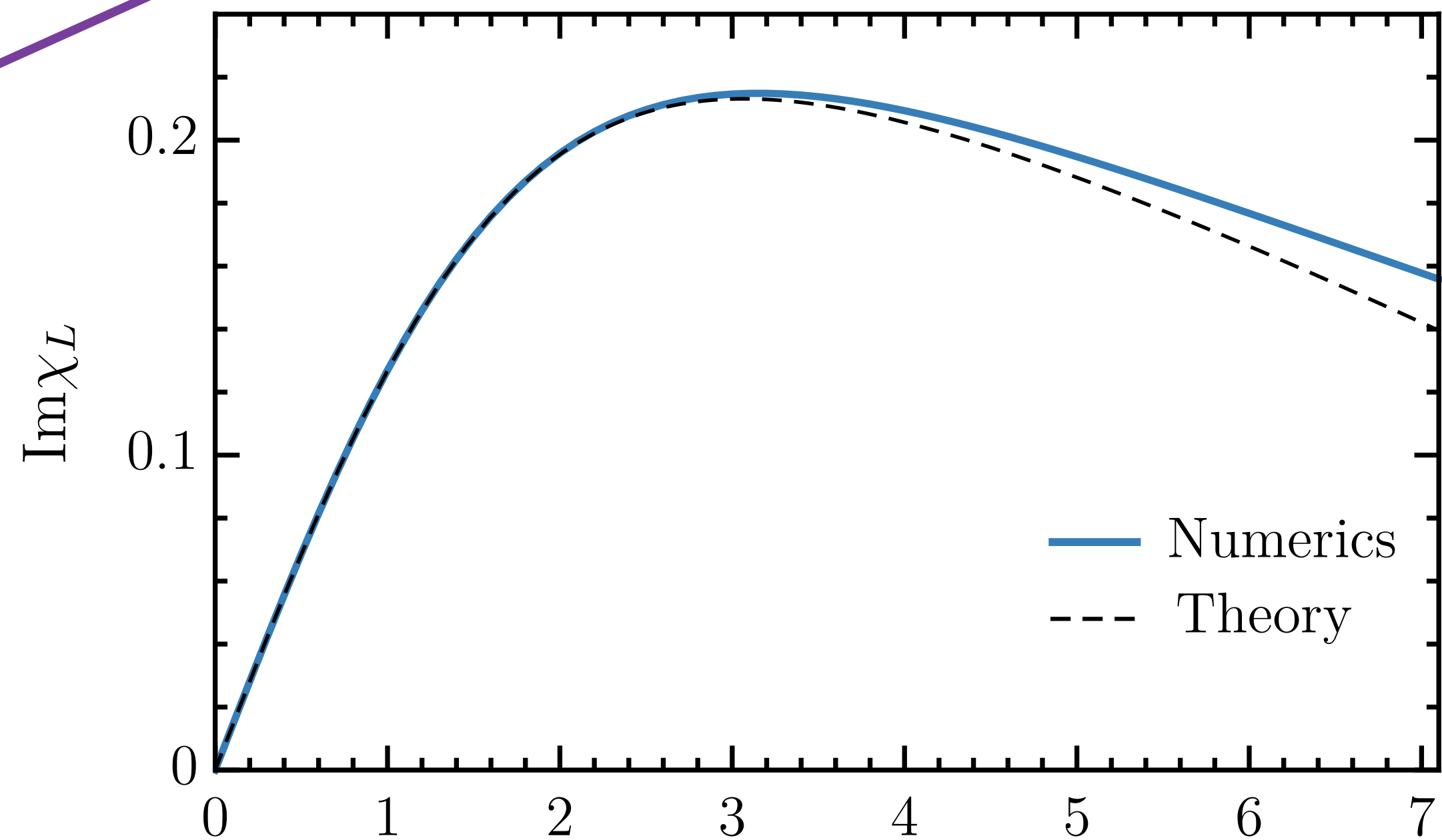


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Correction from
the boundary
graviton



The random t - J model has

- Spin glass order for $p < p_c$.
- Fermi liquid with Luttinger volume Fermi surface for $p > p_c$
- Maxima in entropy, specific heat, and entanglement entropy near $p = p_c$
- SYK-Planckian criticality near p_c .
- ‘Marginal’ spin susceptibility near criticality, with boundary graviton correction ‘observed’ in SU(2) model.