

The interplay of antiferromagnetism and superconductivity: new results and open questions

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Erez Berg



Matthias Punk



Outline

1. Sign-problem free quantum Monte Carlo for the onset of antiferromagnetism in metals

2. Hole-doped cuprates:

Where are the electron pockets in the Brillouin zone ?

Is the pseudogap state an exotic metal with Fermi pockets which violate the Luttinger relation ? (Such a violation requires emergent gauge excitations: an example of such a metal is the Z_2 -FL* state)

Outline

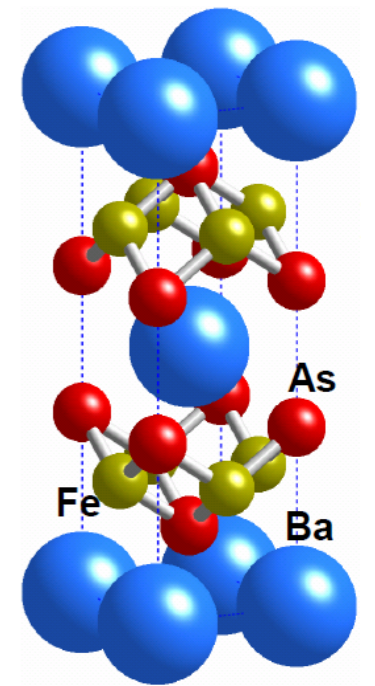
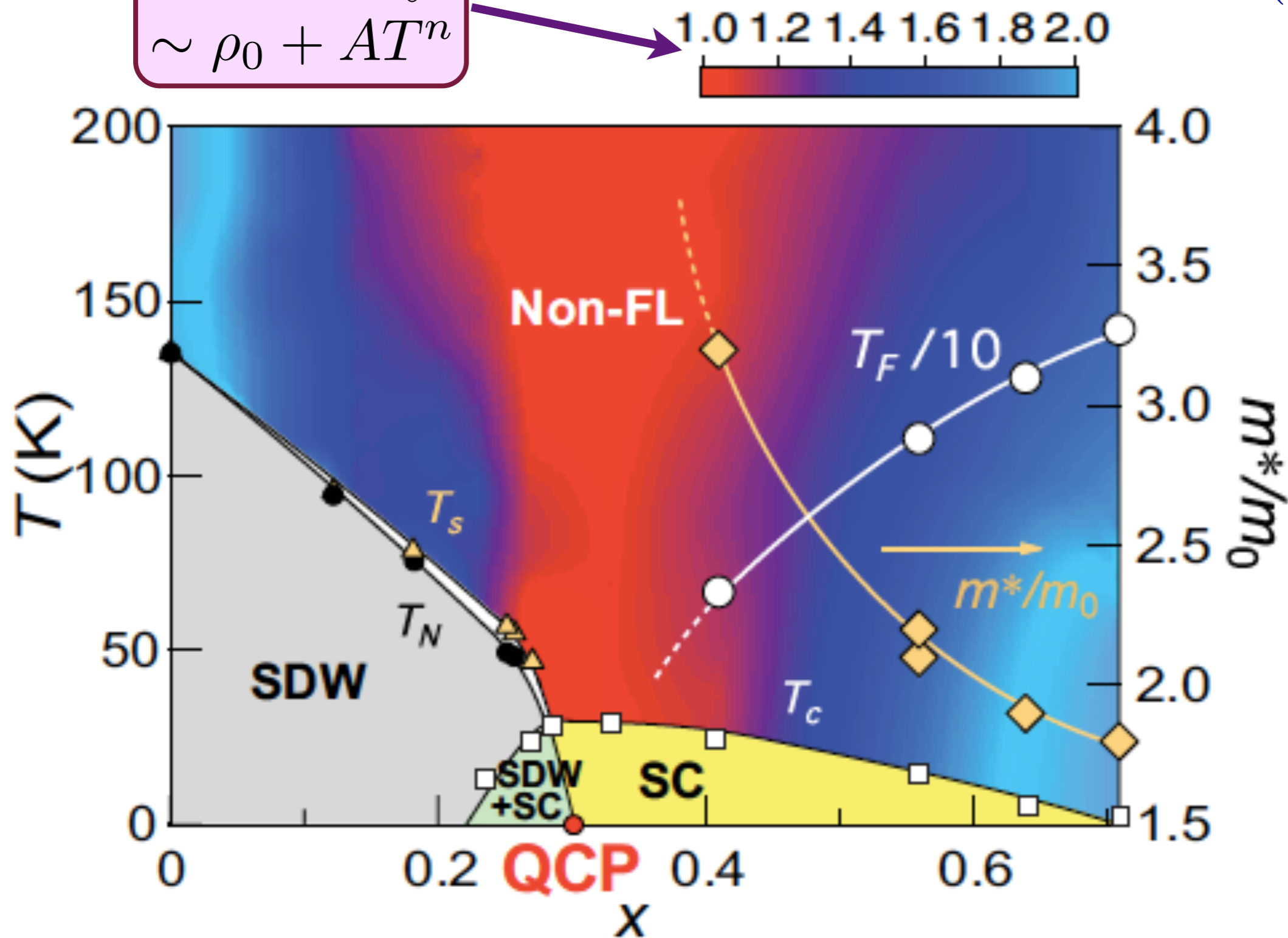
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2. Hole-doped cuprates:

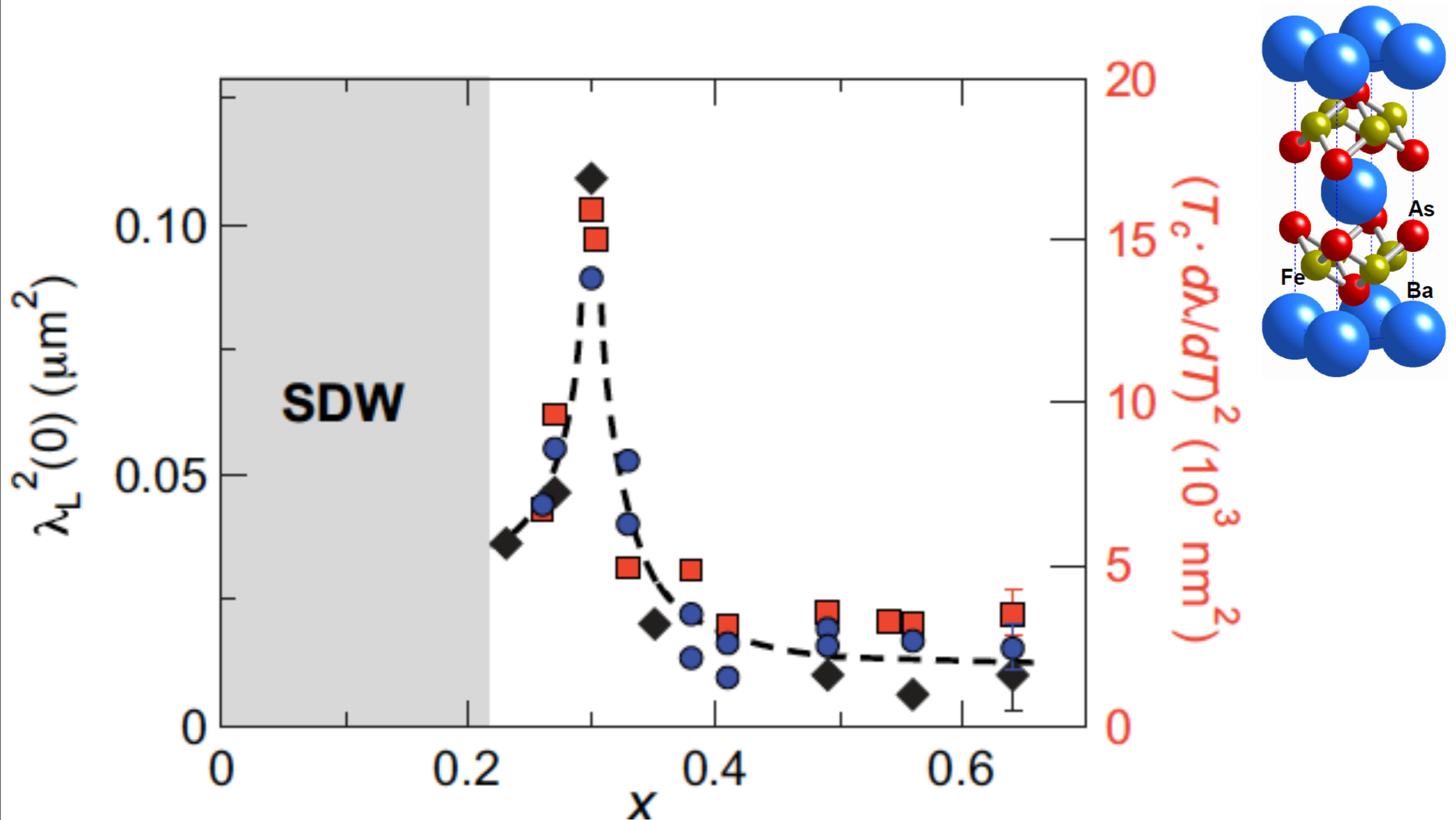
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Is the pseudogap state an exotic metal with Fermi pockets which violate the Luttinger relation ? (Such a violation requires emergent gauge excitations: an example of such a metal is the Z_2 -FL* state)

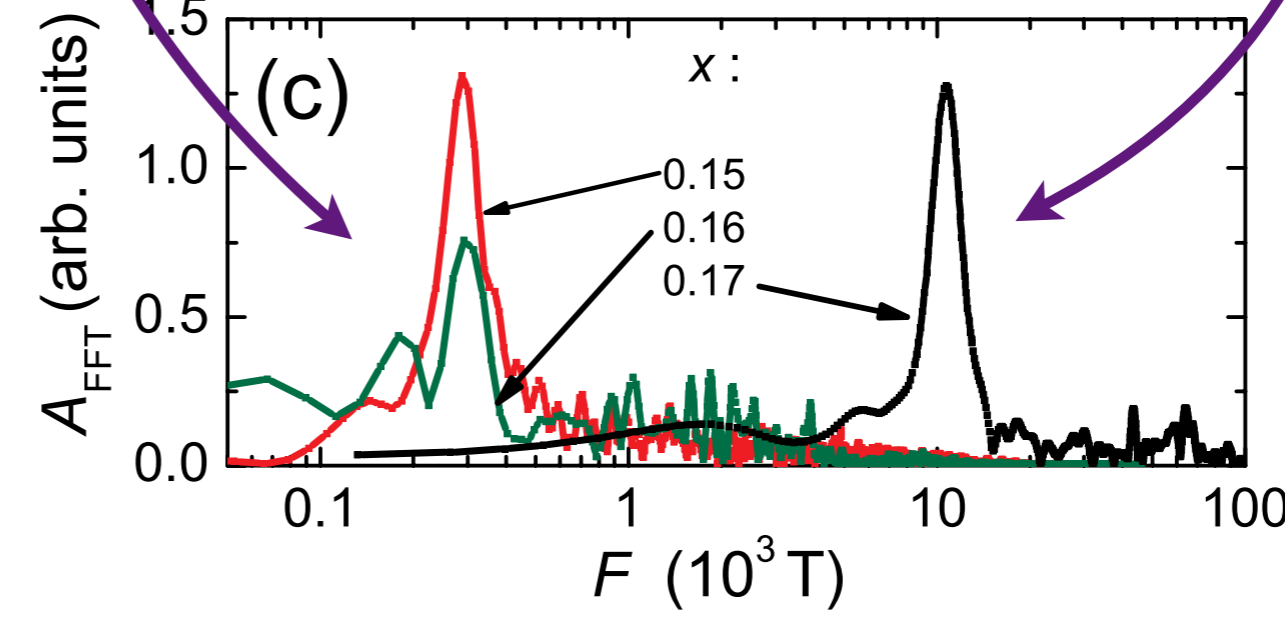
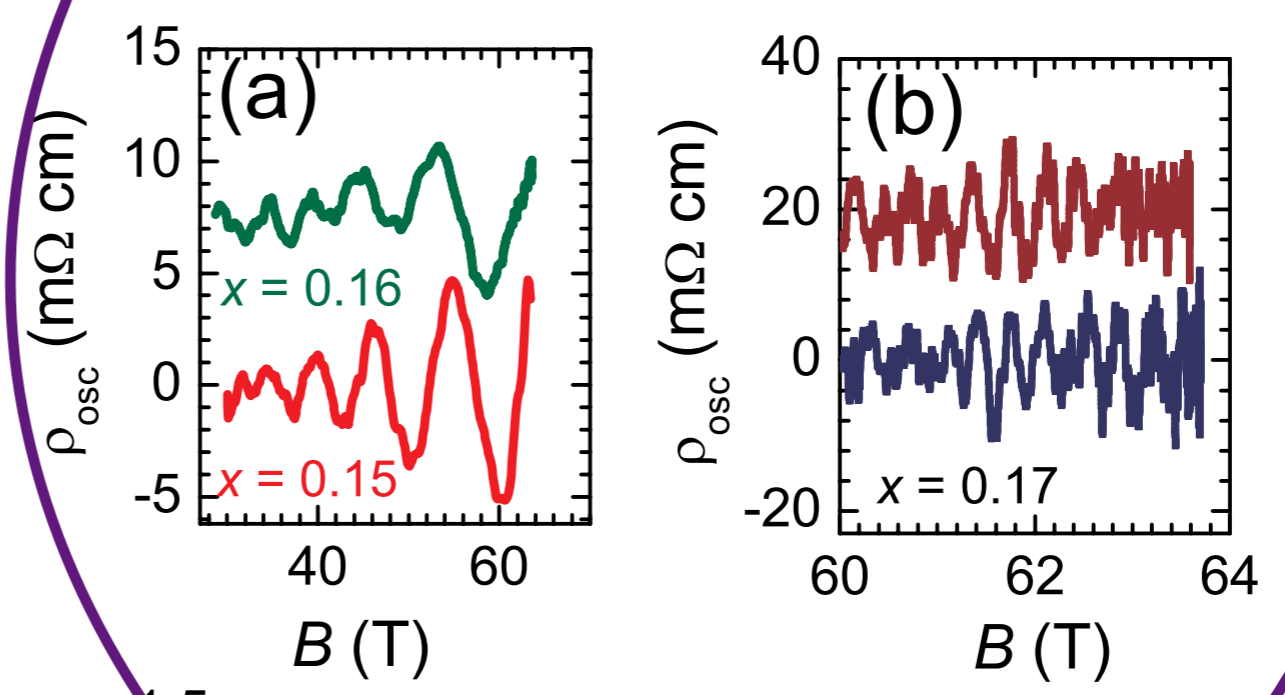
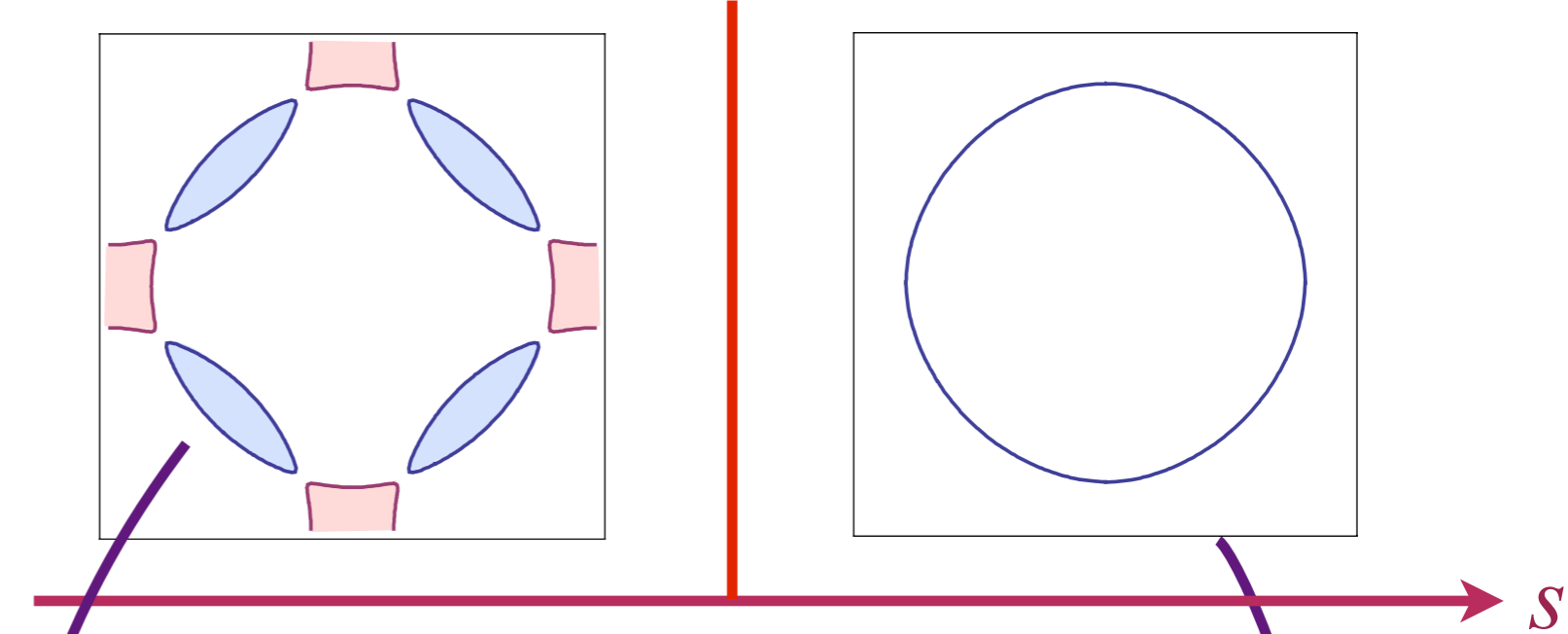
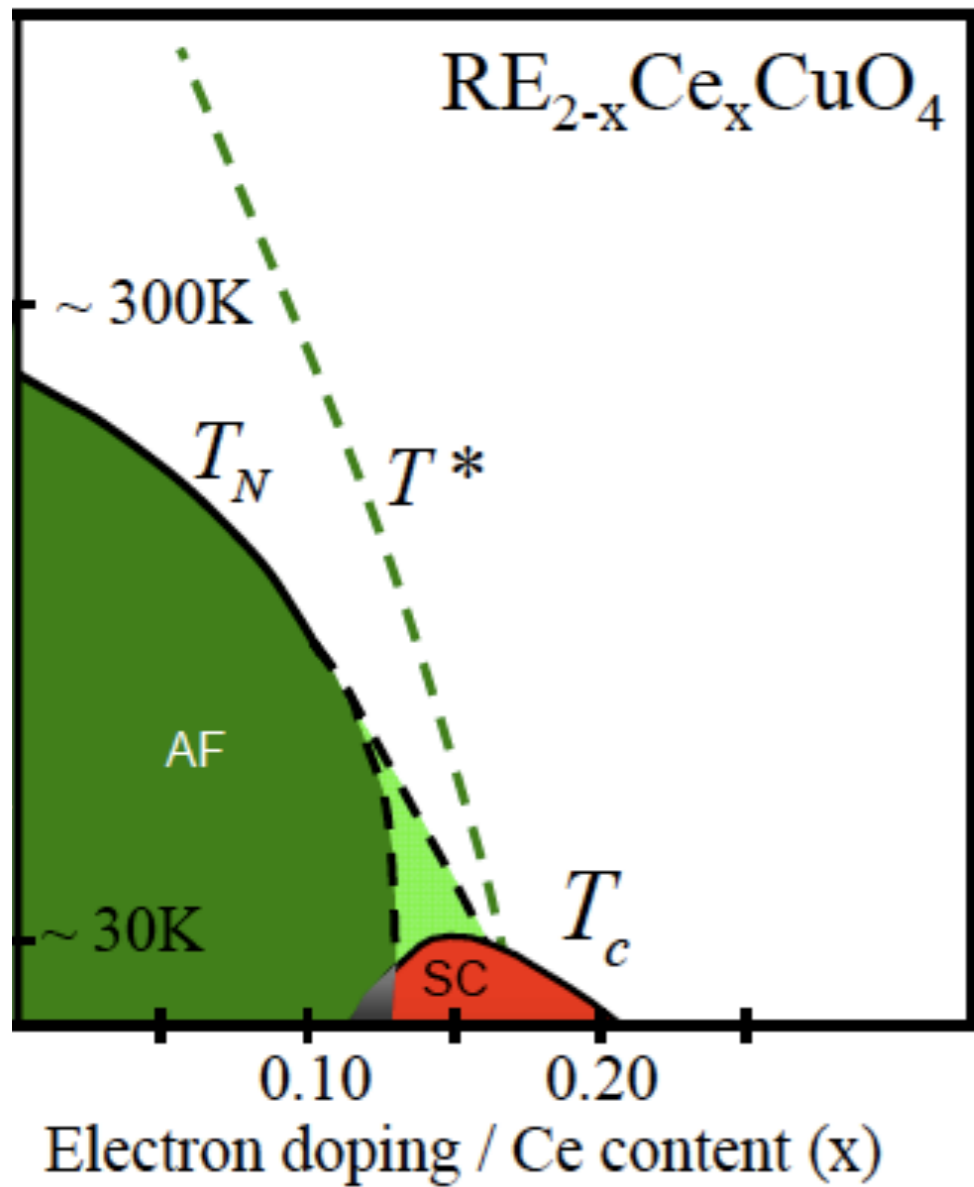
Resistivity
 $\sim \rho_0 + AT^n$



K. Hashimoto, K. Cho, T. Shibauchi, S. Kasahara, Y. Mizukami, R. Katsumata, Y. Tsuruhara, T. Terashima, H. Ikeda, M.A. Tanatar, H. Kitano, N. Salovich, R.W. Giannetta, P. Walmsley, A. Carrington, R. Prozorov, and Y. Matsuda, *Science* **336**, 1554 (2012).



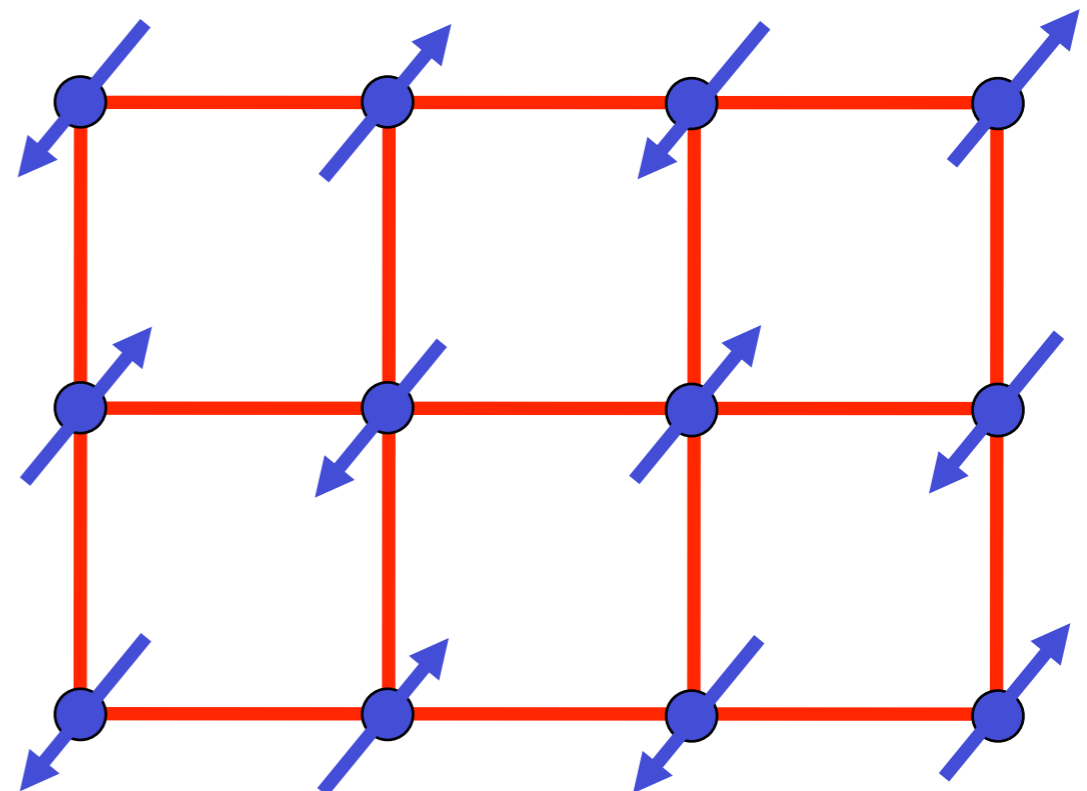
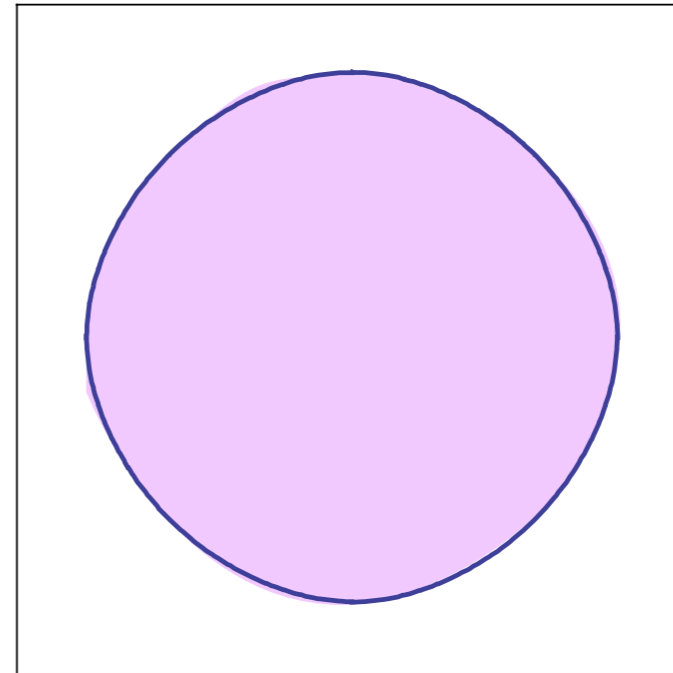
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T. Helm, M.V. Kartsovnik,
M. Bartkowiak, N. Bittner,
M. Lambacher, A. Erb, J. Wosnitza,
and R. Gross,
Phys. Rev. Lett. **103**, 157002 (2009).

Fermi surface+antiferromagnetism

Metal with “large”
Fermi surface

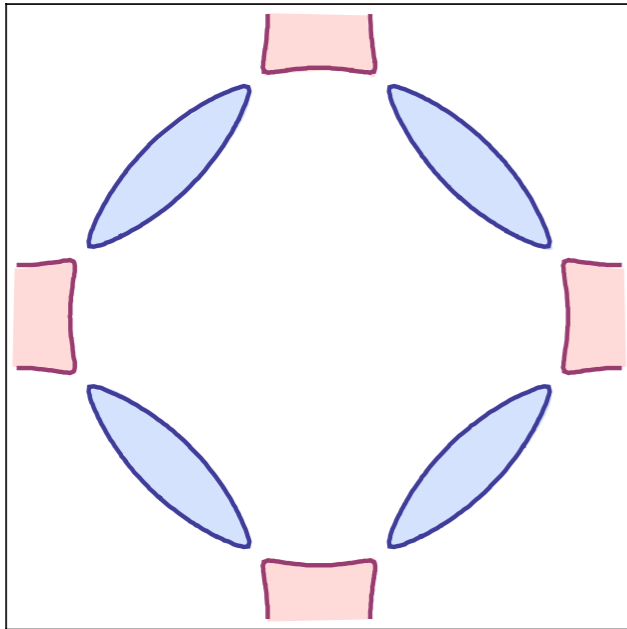


The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K} \cdot \mathbf{r}}$$

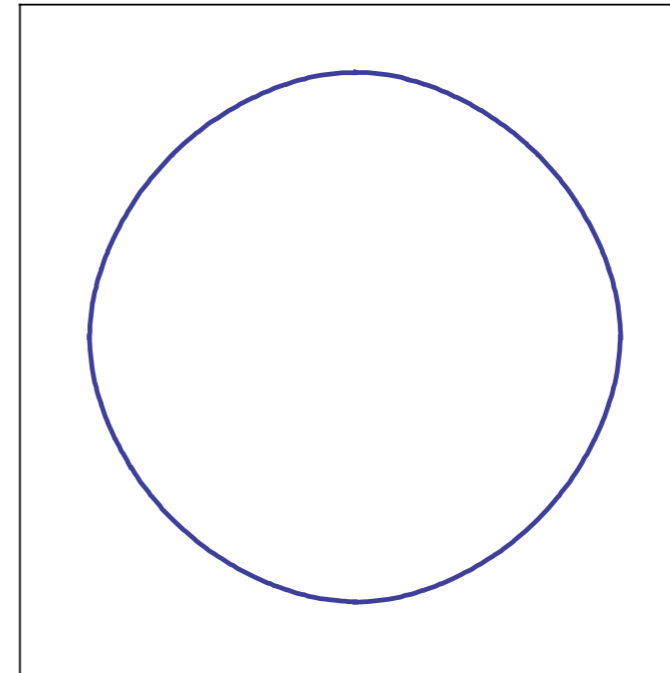
where \mathbf{K} is the ordering wavevector.

Fermi surface+antiferromagnetism



$$\langle \vec{\varphi} \rangle \neq 0$$

Antiferromagnetic
metal with electron
and hole pockets



$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large"
Fermi surface

← Increasing interaction

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A. V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Theory for onset of antiferromagnetism

$$\mathcal{S} = \int d^2r d\tau [\mathcal{L}_c + \mathcal{L}_\varphi + \mathcal{L}_{c\varphi}]$$

$$\mathcal{L}_c = c_a^\dagger \varepsilon (-i \nabla) c_a$$

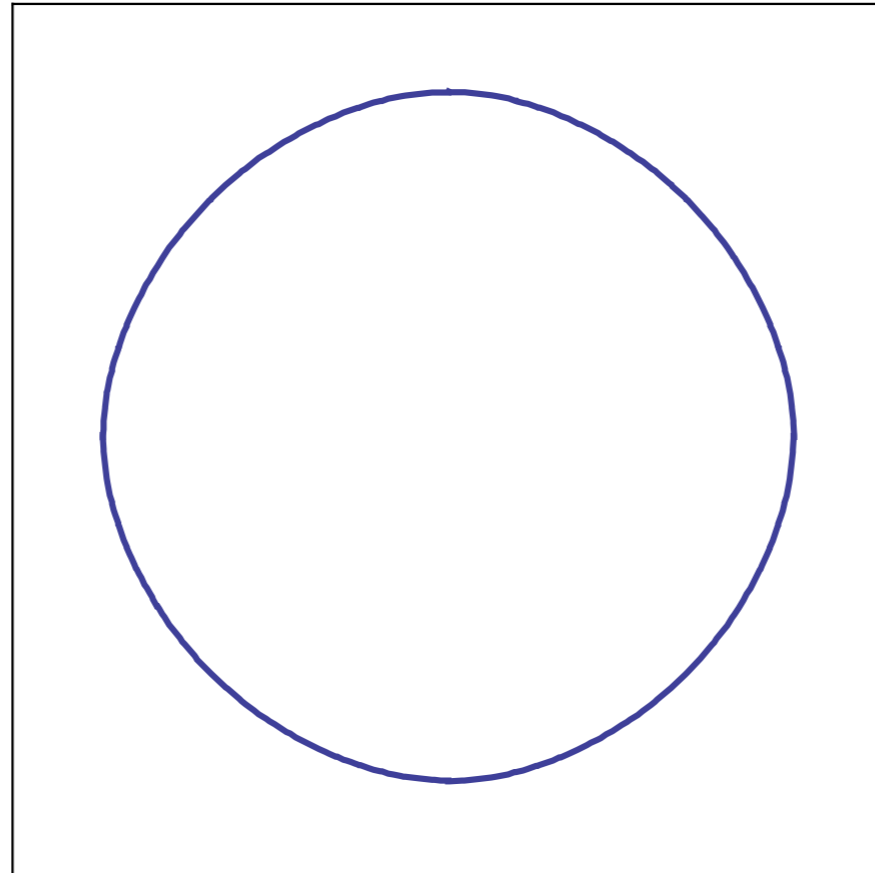
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla \varphi_\alpha)^2 + \frac{r}{2} \varphi_\alpha^2 + \frac{u}{4} (\varphi_\alpha^2)^2$$

$$\mathcal{L}_{c\varphi} = \lambda \varphi_\alpha e^{i\mathbf{K}\cdot\mathbf{r}} c_a^\dagger \sigma_{ab}^\alpha c_b.$$

“Yukawa” coupling between fermions and antiferromagnetic order:

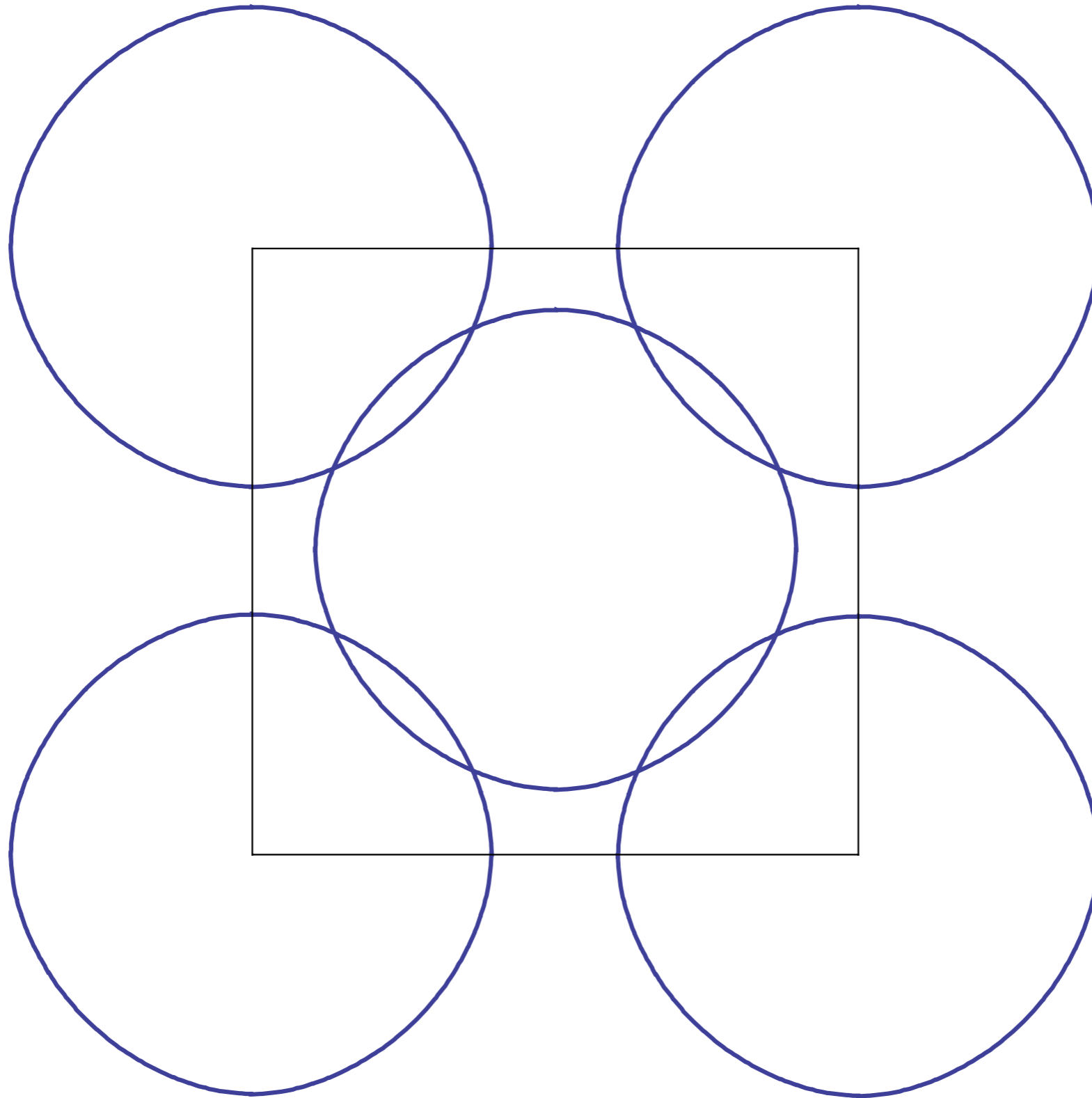
$$\lambda^2 \sim U, \text{ the Hubbard repulsion}$$

Fermi surface+antiferromagnetism



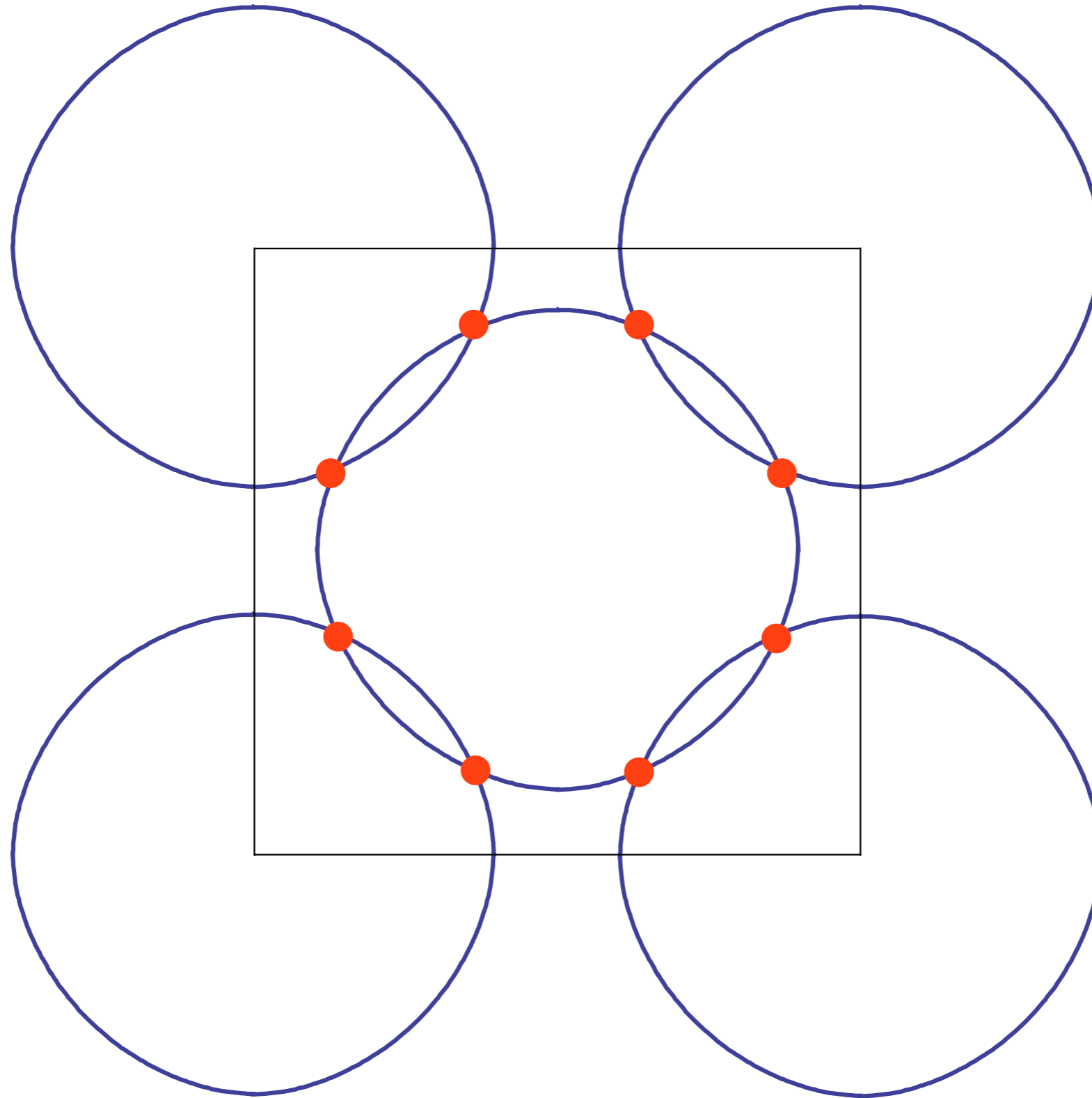
Metal with “large” Fermi surface

Fermi surface+antiferromagnetism



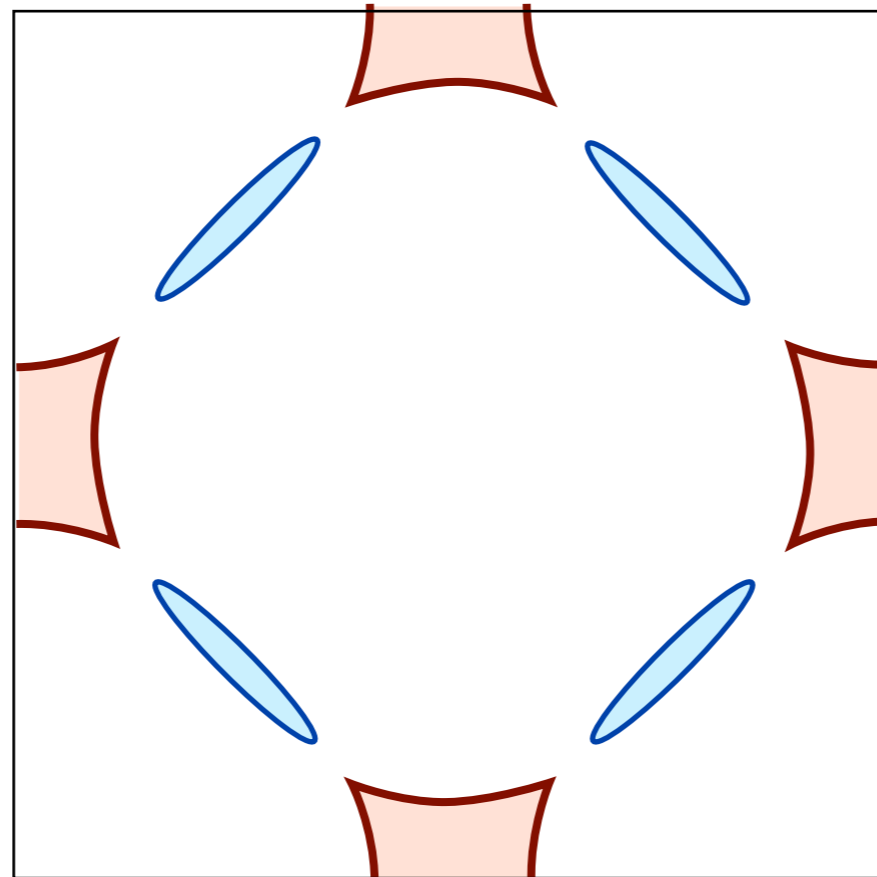
Fermi surfaces translated by $\mathbf{K} = (\pi, \pi)$.

Fermi surface+antiferromagnetism



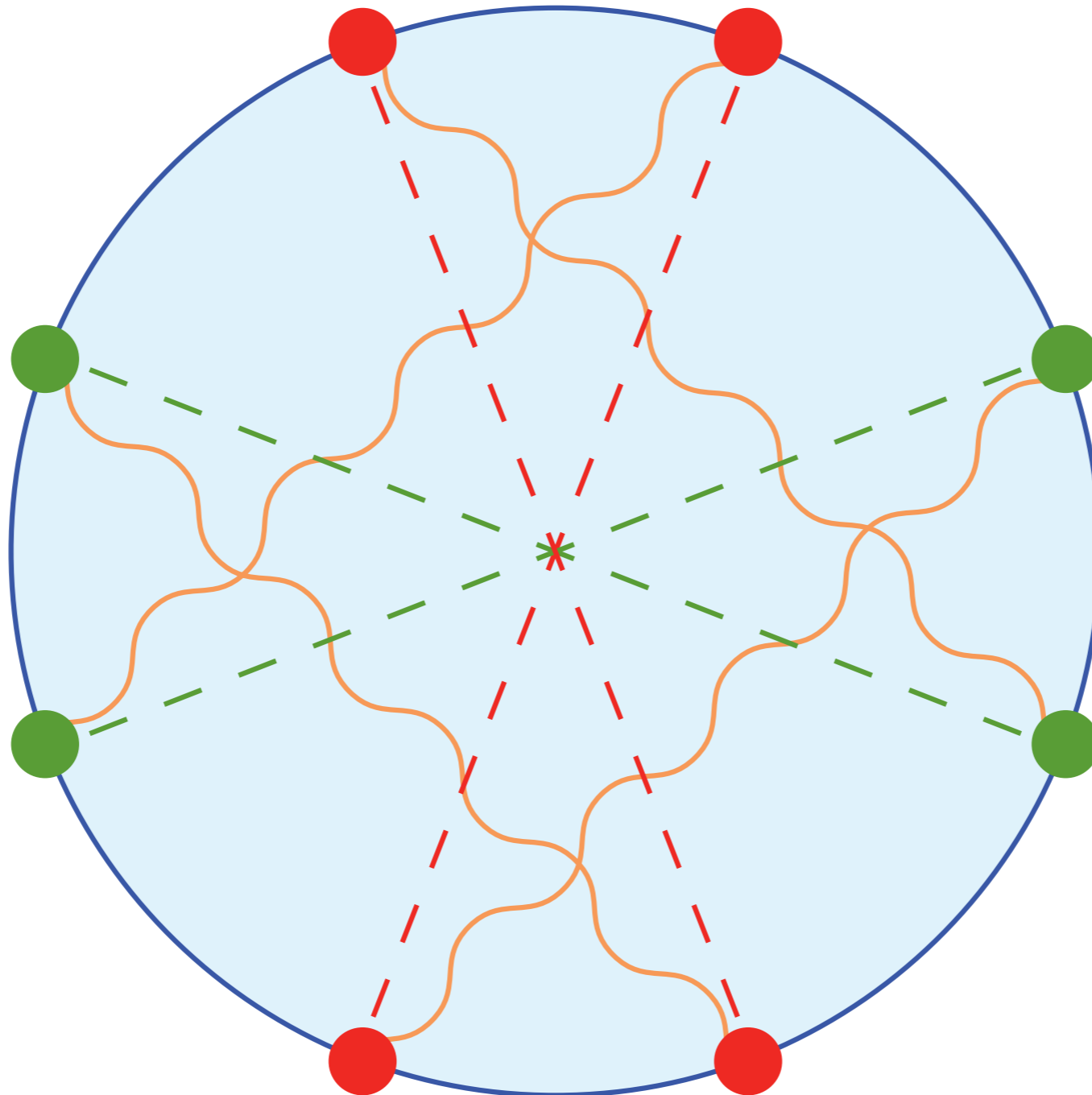
“Hot” spots

Fermi surface+antiferromagnetism



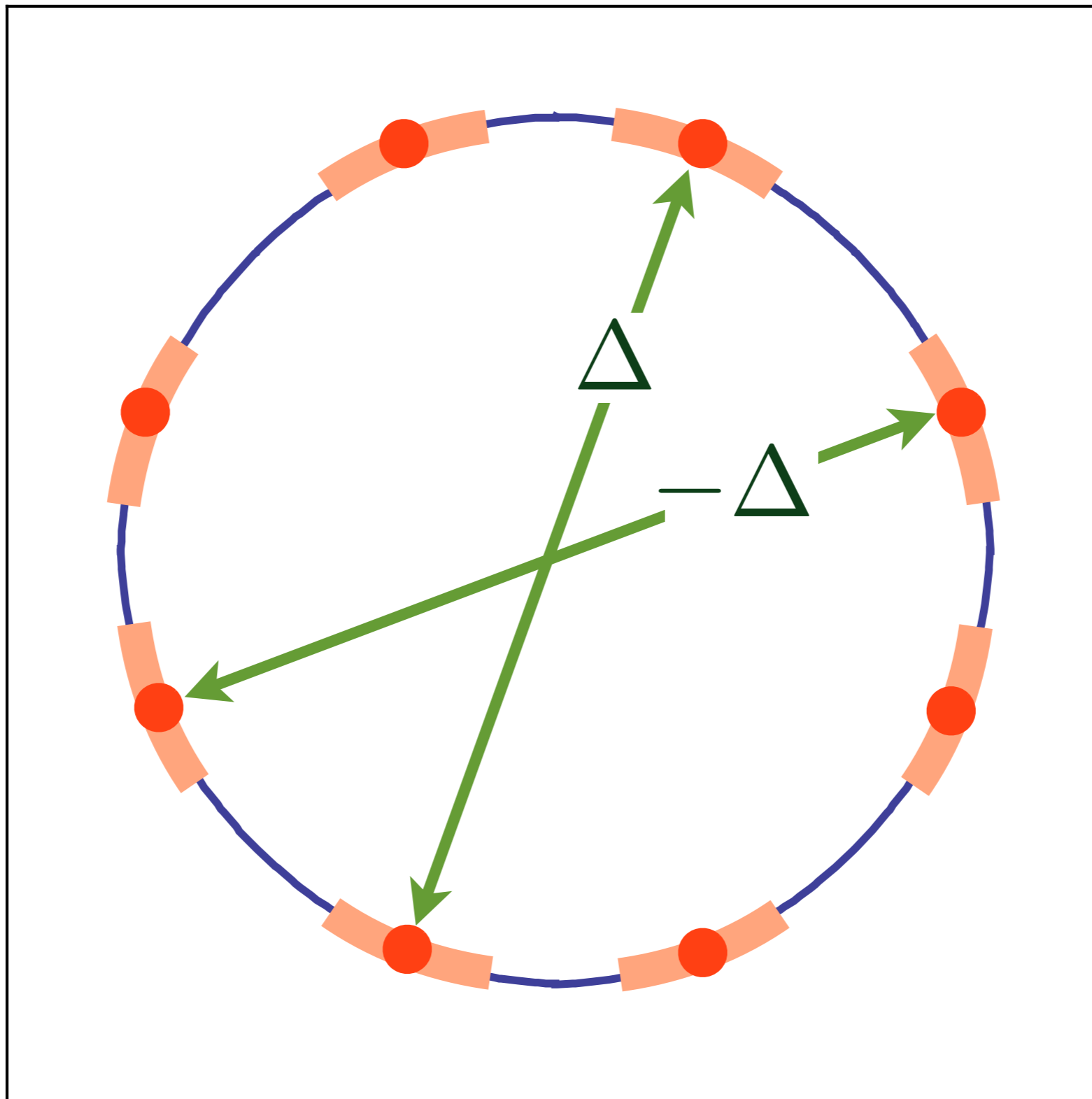
Electron and hole pockets in
antiferromagnetic phase with $\langle \vec{\varphi} \rangle \neq 0$

Pairing “glue” from antiferromagnetic fluctuations



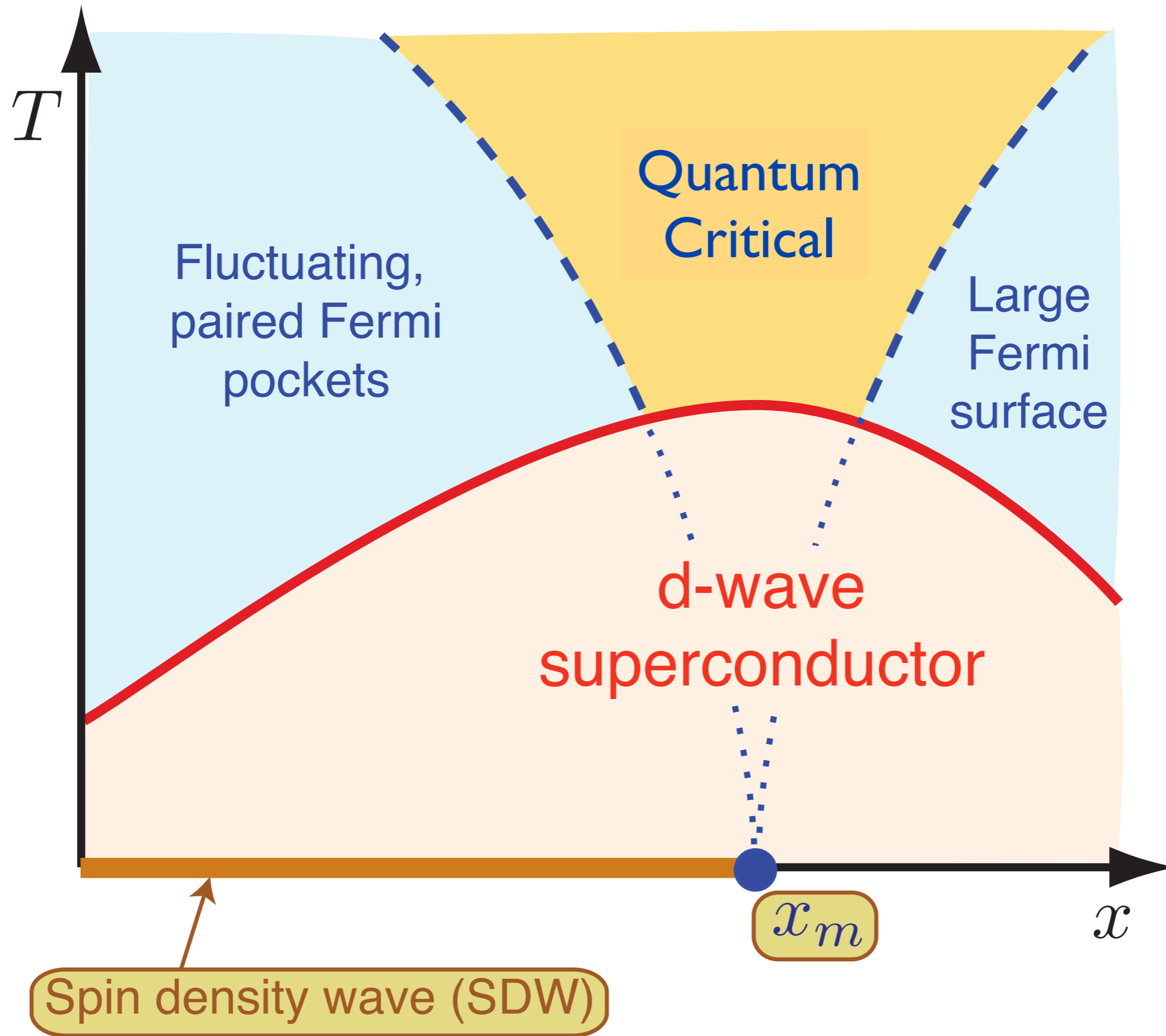
V. J. Emery, *J. Phys. (Paris) Colloq.* **44**, C3-977 (1983)
D. J. Scalapino, E. Loh, and J. E. Hirsch, *Phys. Rev. B* **34**, 8190 (1986)
K. Miyake, S. Schmitt-Rink, and C. M. Varma, *Phys. Rev. B* **34**, 6554 (1986)
S. Raghu, S. A. Kivelson, and D. J. Scalapino, *Phys. Rev. B* **81**, 224505 (2010)

$$\langle c_{\mathbf{k}\alpha}^\dagger c_{-\mathbf{k}\beta}^\dagger \rangle = \varepsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y)$$

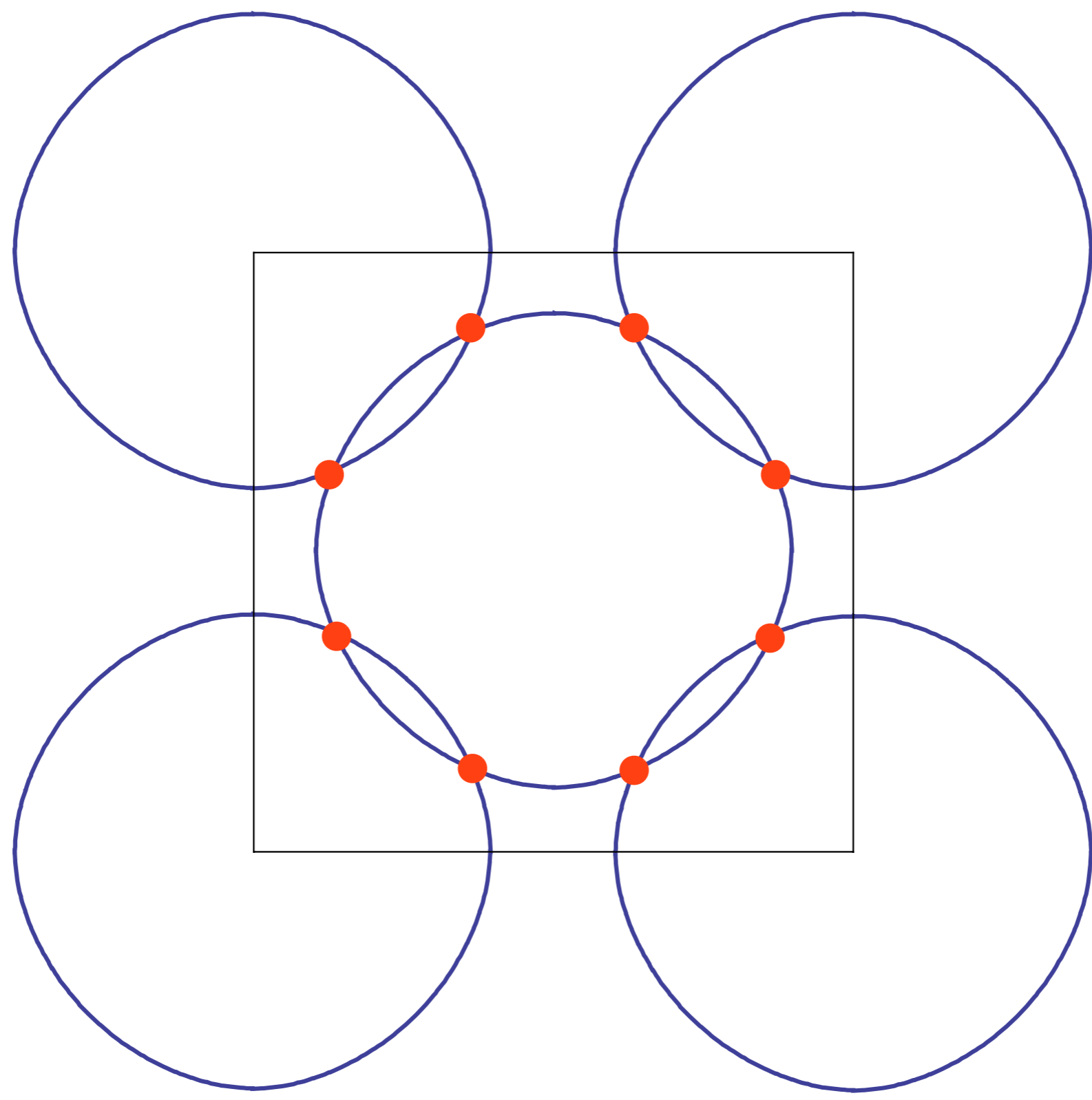


Unconventional pairing at and near hot spots

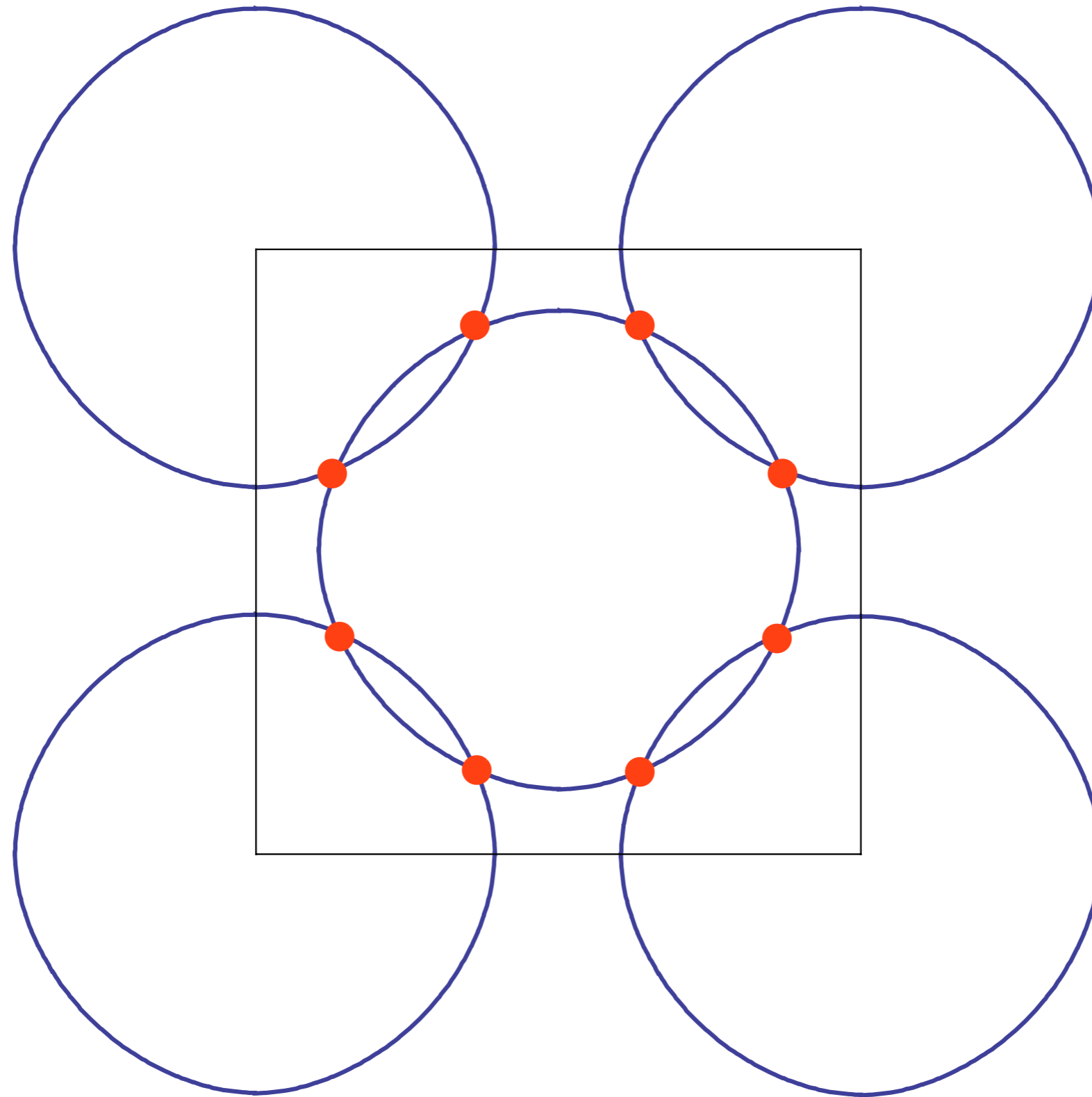
Fermi surface+antiferromagnetism



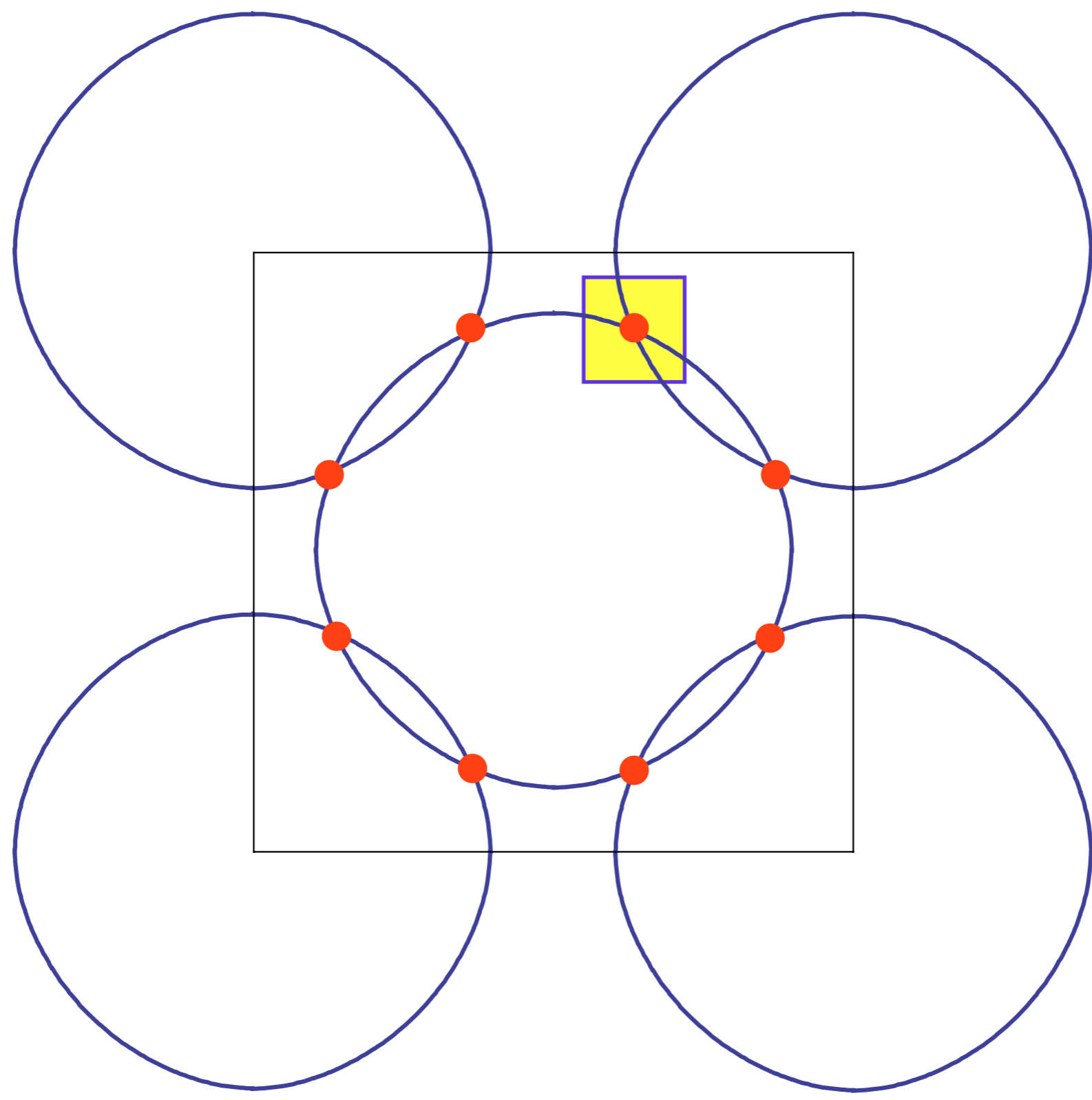
QCP for the onset of SDW order is actually within a superconductor



“Hot” spots

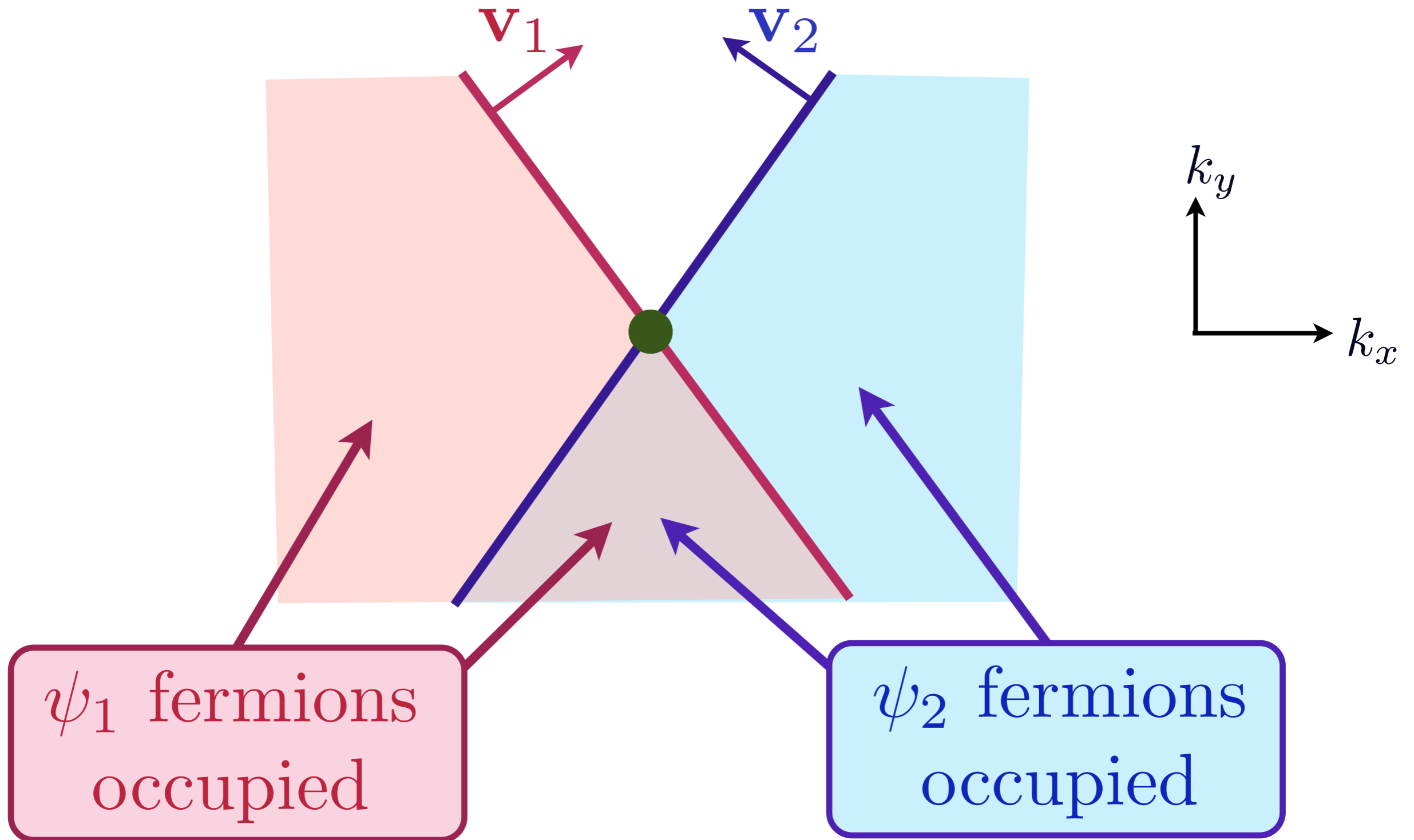


Low energy theory for critical point near hot spots



Low energy theory for critical point near hot spots

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$) and boson order parameter $\vec{\varphi}$, interacting with coupling λ



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and boson order parameter $\vec{\varphi}$,
interacting with coupling λ

$$\begin{aligned} \mathcal{L} = & \psi_{1\alpha}^\dagger (\partial_\tau - i\mathbf{v}_1 \cdot \nabla_r) \psi_{1\alpha} + \psi_{2\alpha}^\dagger (\partial_\tau - i\mathbf{v}_2 \cdot \nabla_r) \psi_{2\alpha} \\ & + \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \\ & - \lambda \vec{\varphi} \cdot \left(\psi_{1\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{2\beta} + \psi_{2\alpha}^\dagger \vec{\sigma}_{\alpha\beta} \psi_{1\beta} \right) \end{aligned}$$

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Note fermion spectrum has *lines* of zero energy
excitations in momentum space.

If fermions are replaced by massless Dirac
fermions with *points* of zero energy excitations,
then critical theory is well-understood

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$)
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M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)

S.A. Hartnoll, D.M. Hofman, M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **84**, 125115 (2011)

Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$)
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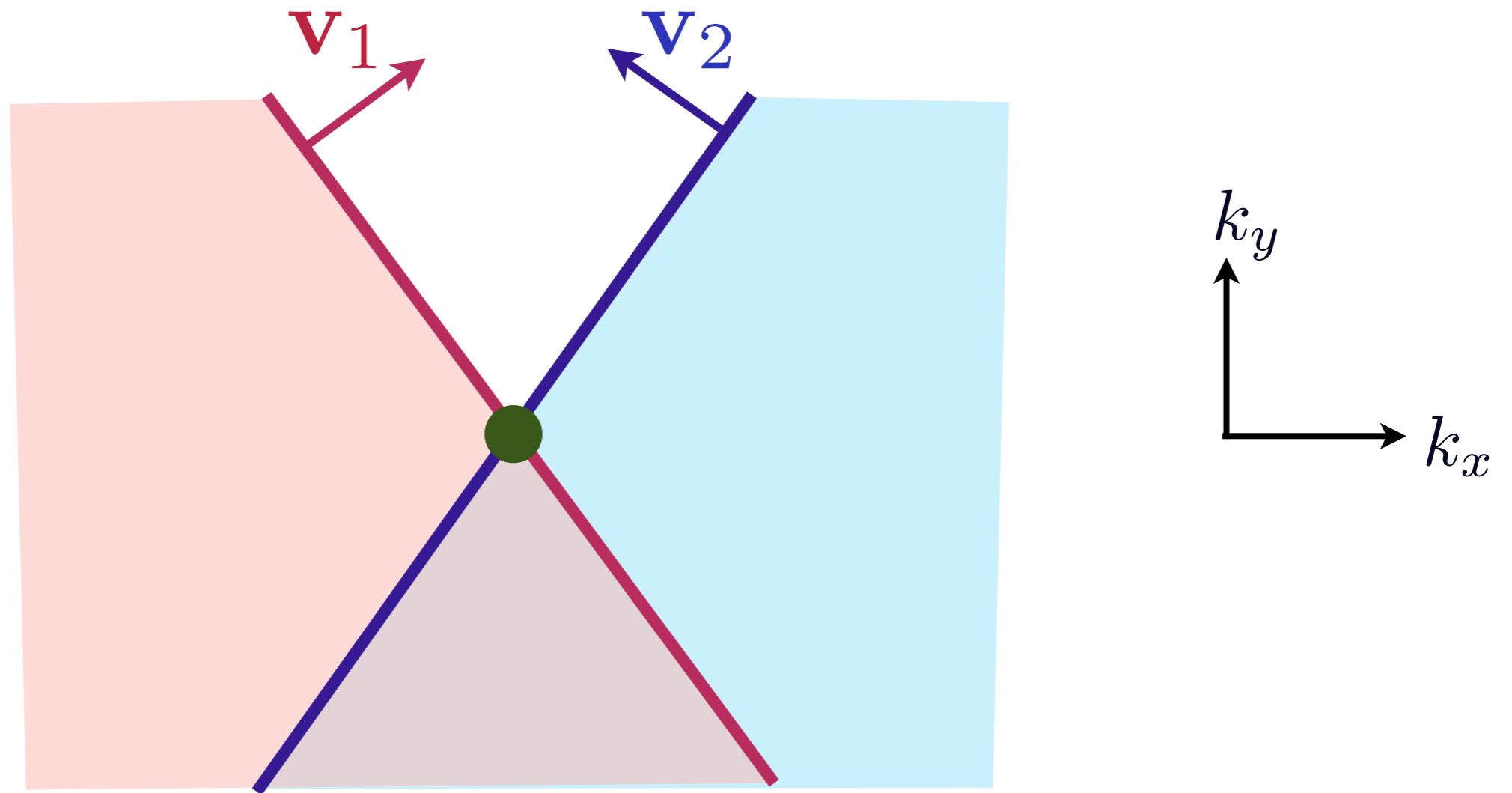
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Theory flows to
strong-coupling in $d = 2$.

M.A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075128 (2010)

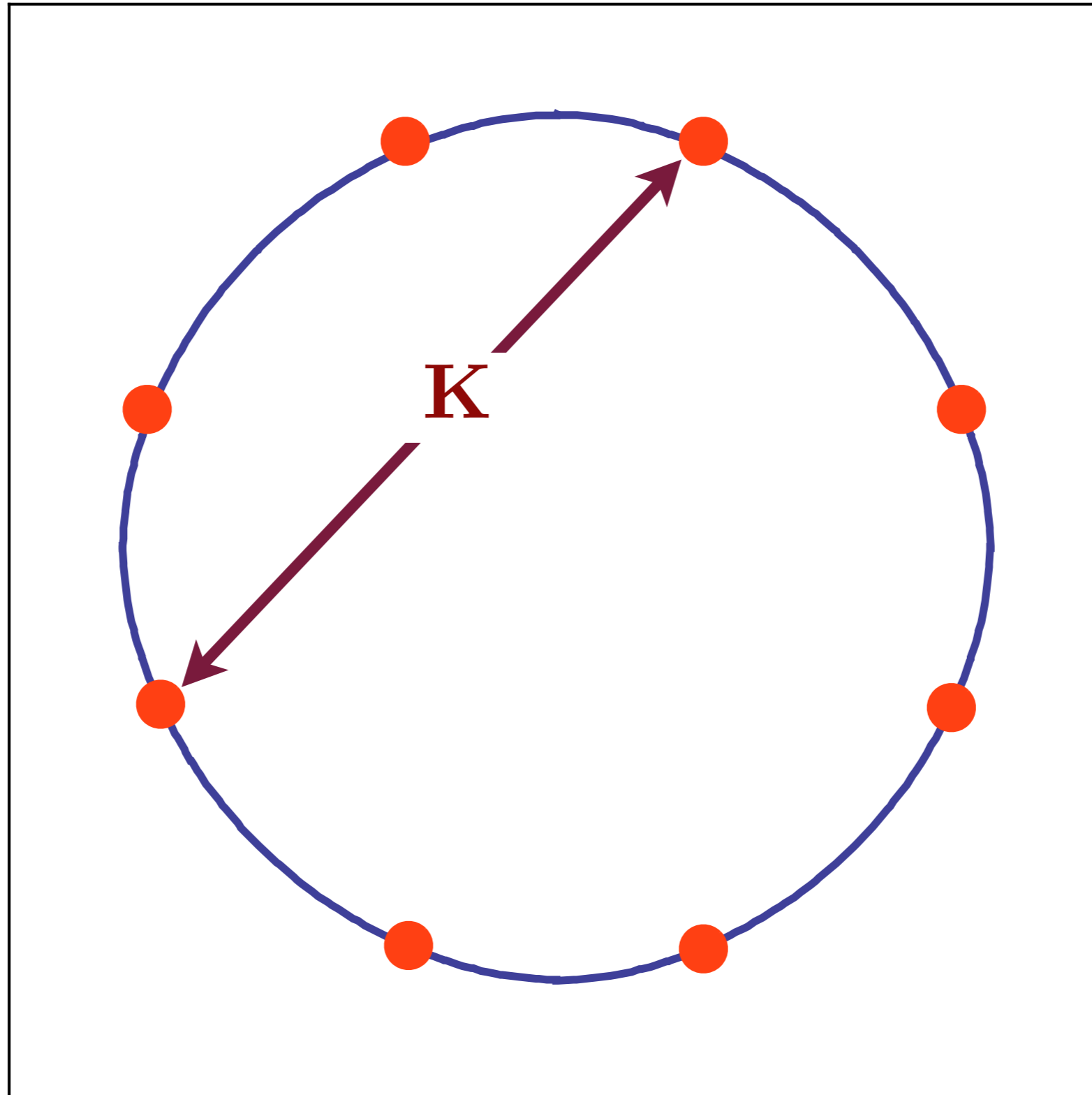
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Theory has fermions $\psi_{1,2}$ (with Fermi velocities $\mathbf{v}_{1,2}$)
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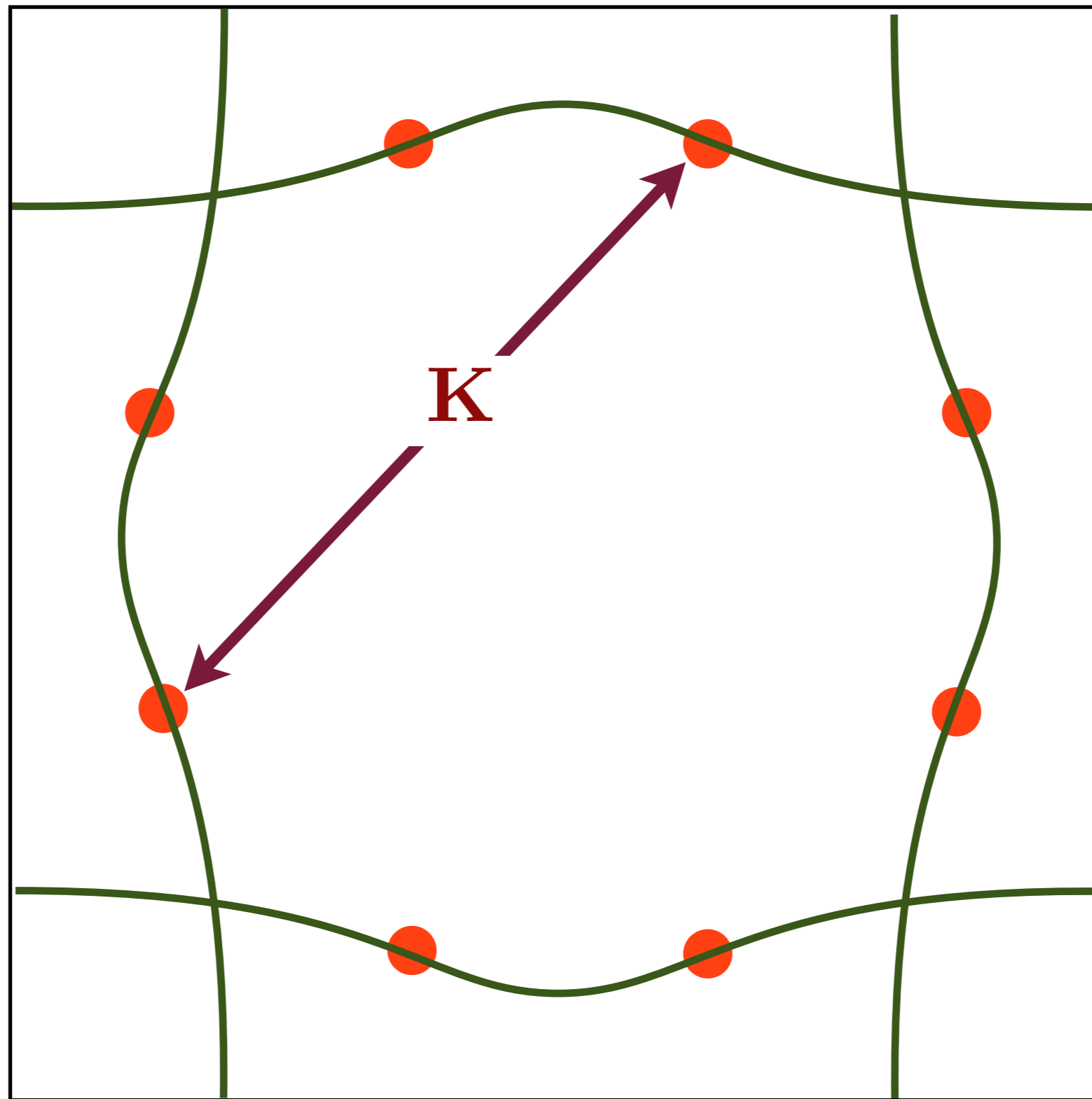
To faithfully realize low energy theory in quantum Monte Carlo,
we need a UV completion in which Fermi lines don't end
and all weights are positive.

QMC for the onset of antiferromagnetism



Hot spots in a single band model

QMC for the onset of antiferromagnetism

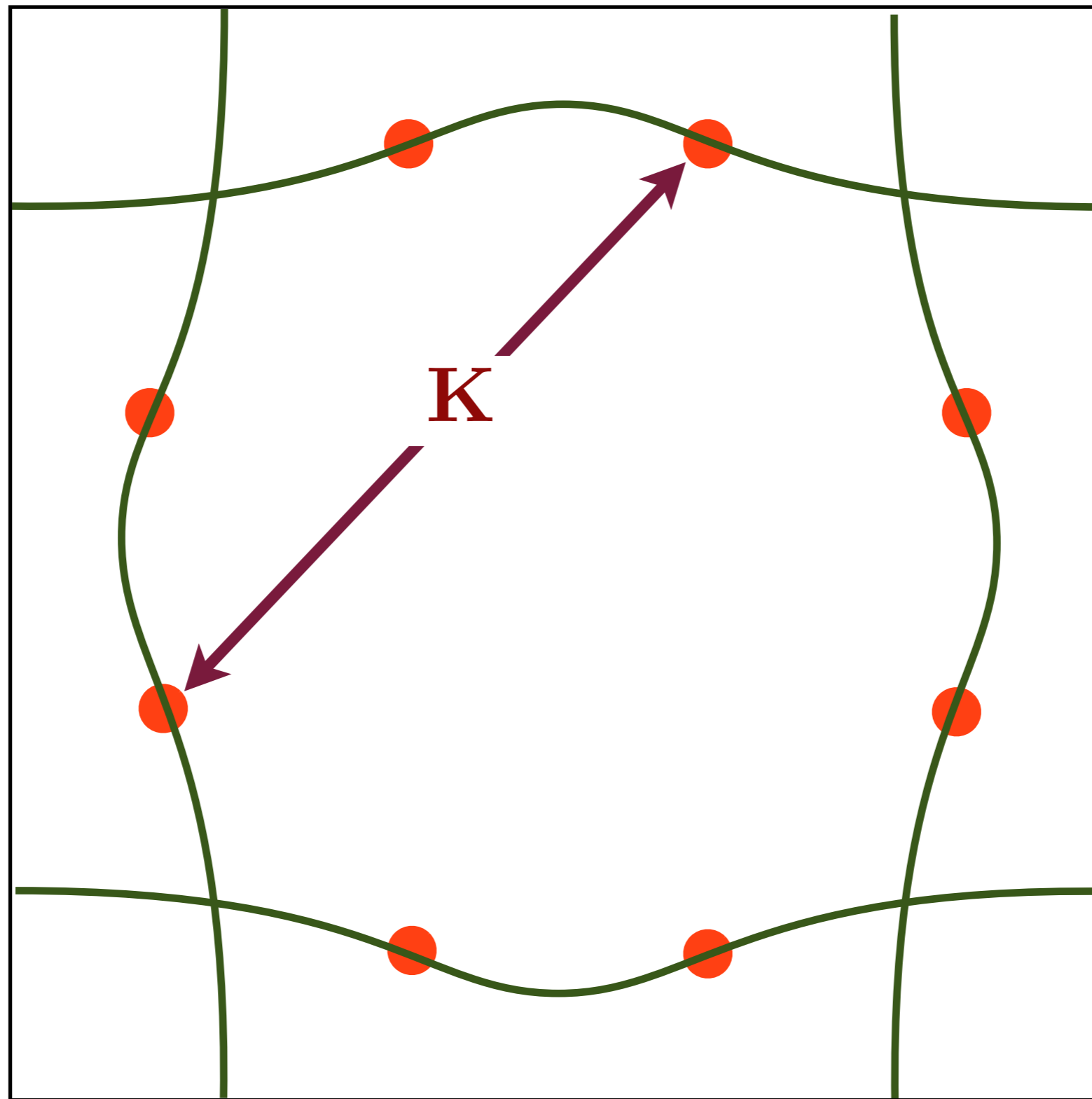


E. Berg,
M. Metlitski, and
S. Sachdev,
arXiv:1206.0742

Hot spots in a two band model

QMC for the onset of antiferromagnetism

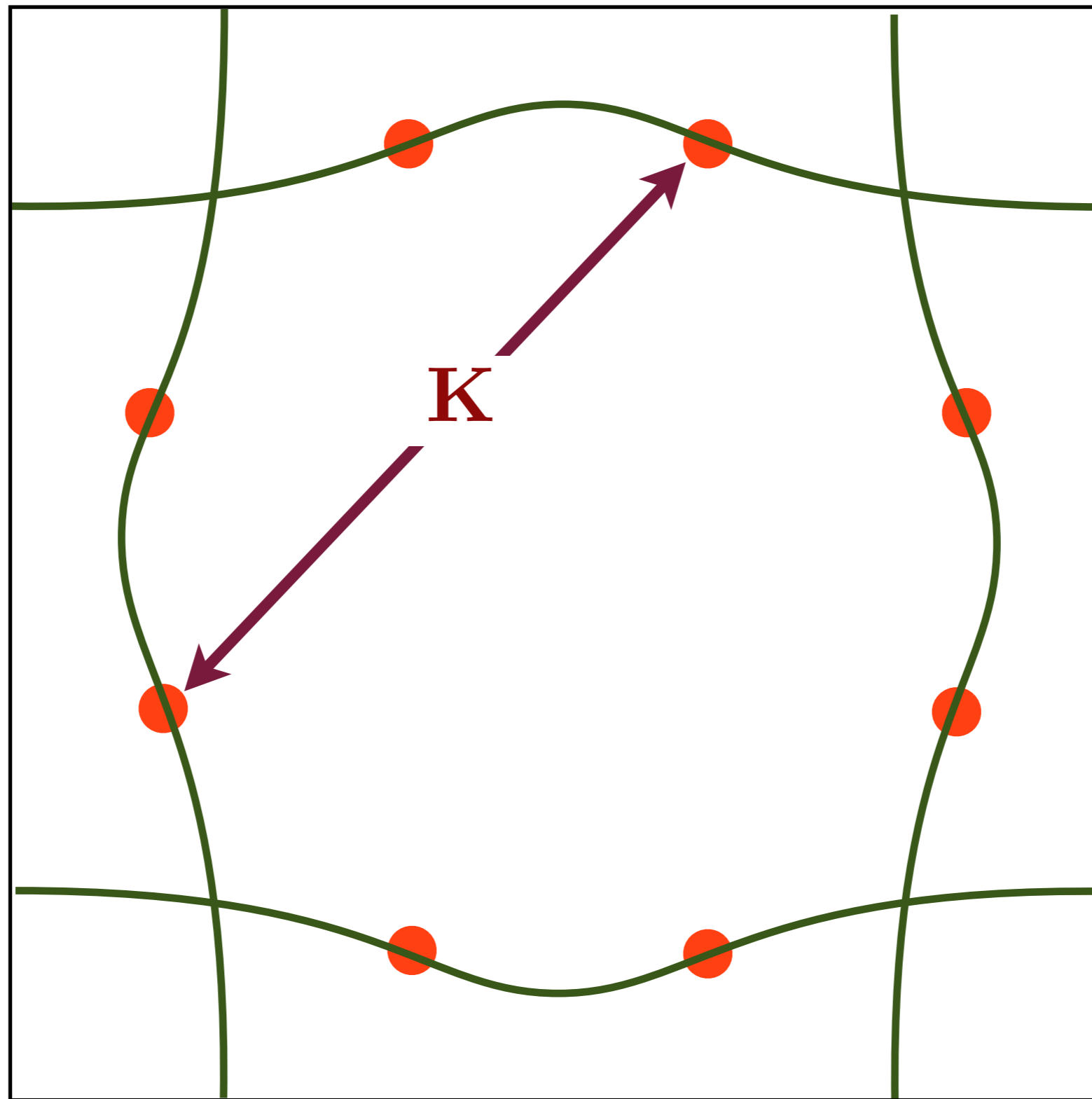
Faithful realization of the *generic* universal low energy theory for the onset of antiferromagnetism.



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Hot spots in a two band model

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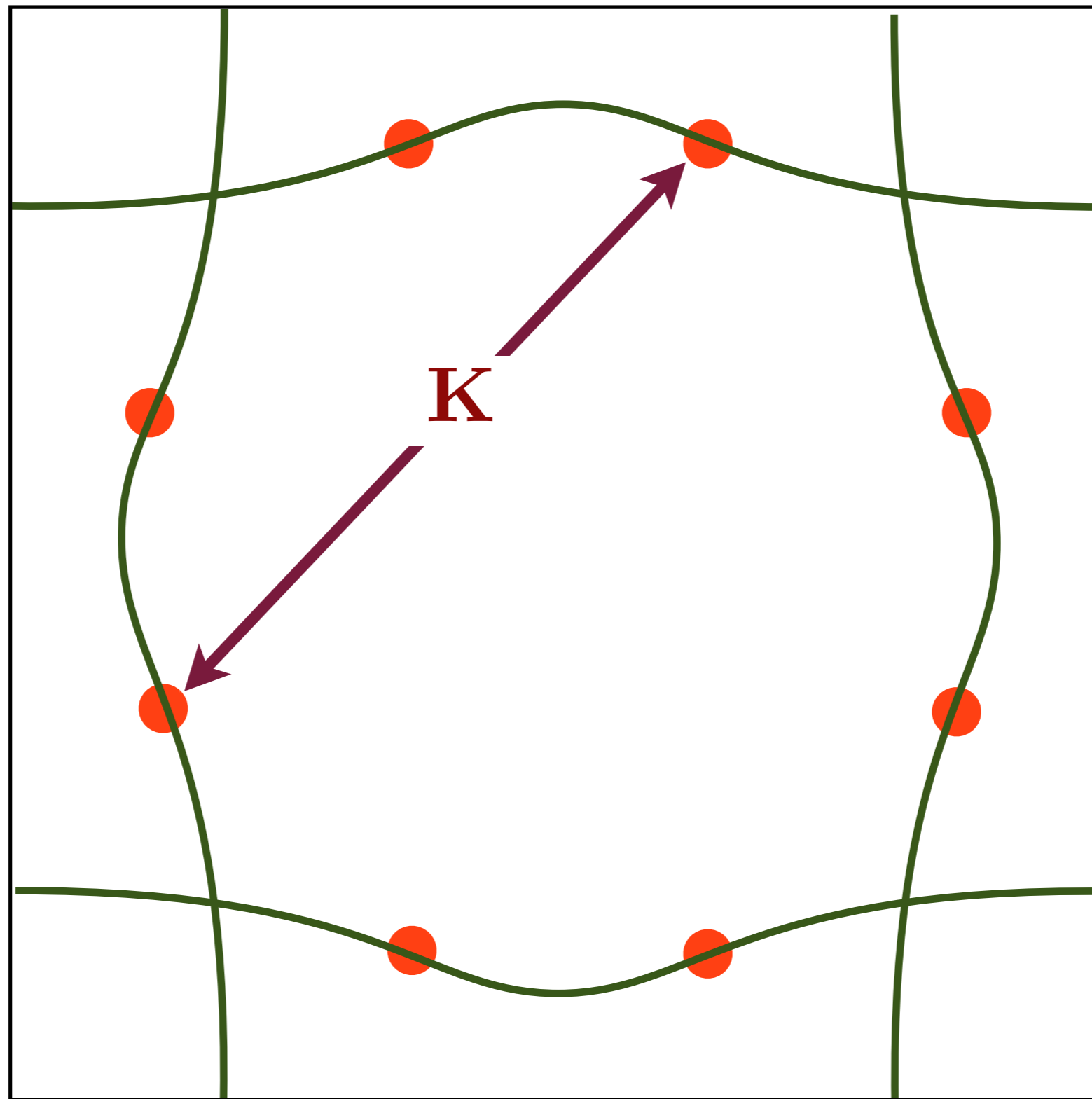


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Hot spots in a two band model

QMC for the onset of antiferromagnetism

Sign problem is absent as long as K connects hotspots in distinct bands

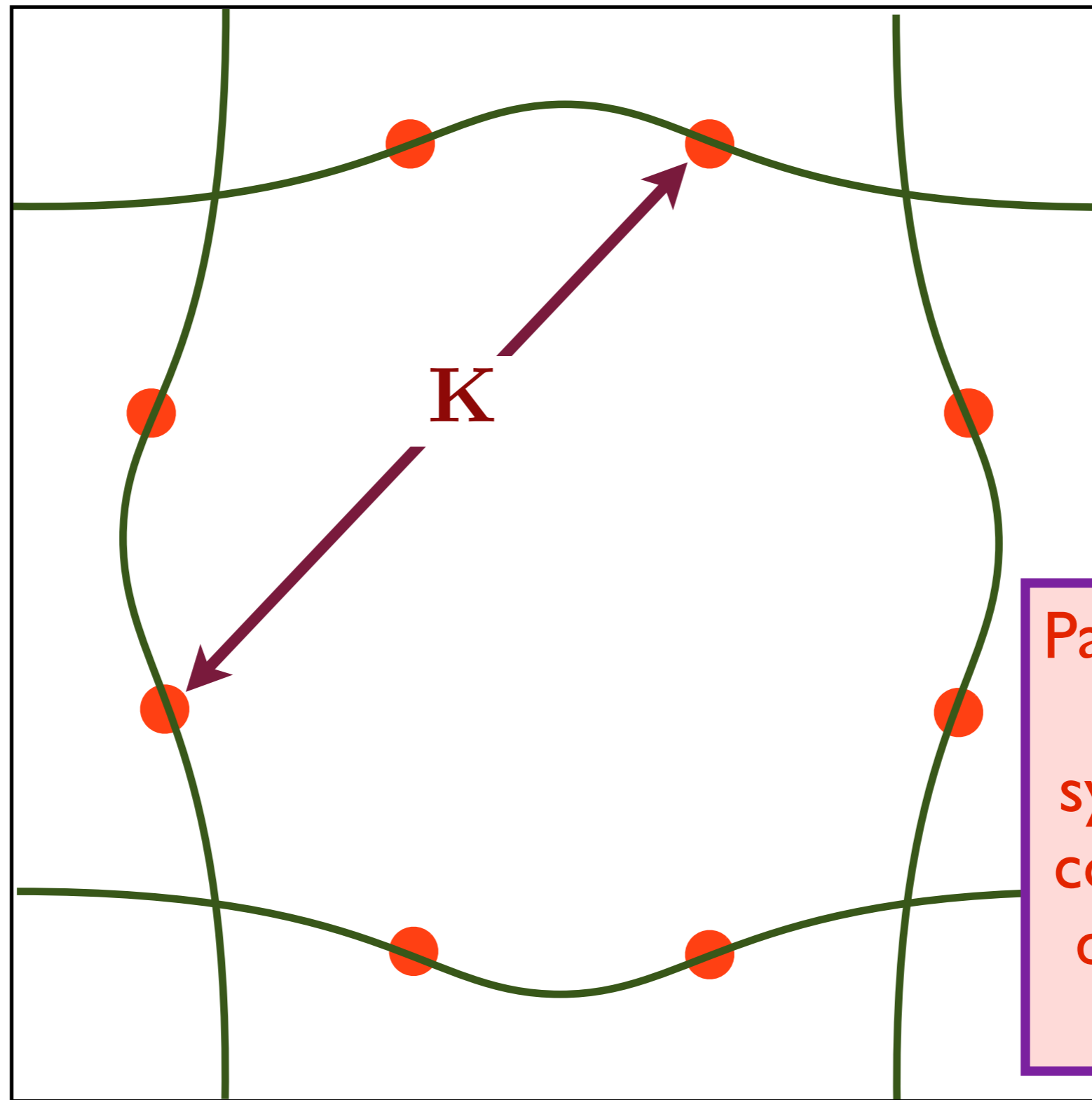


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Hot spots in a two band model

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Particle-hole or point-group symmetries or commensurate densities **not** required!

Hot spots in a two band model

QMC for the onset of antiferromagnetism

Electrons with dispersion $\varepsilon_{\mathbf{k}}$
interacting with fluctuations of the
antiferromagnetic order parameter $\vec{\varphi}$.

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}c_{\alpha} \mathcal{D}\vec{\varphi} \exp(-\mathcal{S}) \\ \mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha} \\ &+ \int d\tau d^2x \left[\frac{1}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \dots \right] \\ &- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta} \end{aligned}$$

QMC for the onset of antiferromagnetism

Electrons with dispersions $\varepsilon_{\mathbf{k}}^{(x)}$ and $\varepsilon_{\mathbf{k}}^{(y)}$ interacting with fluctuations of the antiferromagnetic order parameter $\vec{\varphi}$.

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}c_{\alpha}^{(x)} \mathcal{D}c_{\alpha}^{(y)} \mathcal{D}\vec{\varphi} \exp(-\mathcal{S}) \\ \mathcal{S} &= \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(x)\dagger} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(x)} \right) c_{\mathbf{k}\alpha}^{(x)} \\ &+ \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^{(y)\dagger} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}}^{(y)} \right) c_{\mathbf{k}\alpha}^{(y)} \\ &+ \int d\tau d^2x \left[\frac{1}{2} (\nabla_x \vec{\varphi})^2 + \frac{r}{2} \vec{\varphi}^2 + \dots \right] \\ &- \lambda \int d\tau \sum_i \vec{\varphi}_i \cdot (-1)^{\mathbf{x}_i} c_{i\alpha}^{(x)\dagger} \vec{\sigma}_{\alpha\beta} c_{i\beta}^{(y)} + \text{H.c.} \end{aligned}$$

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No sign problem !

QMC for the onset of antiferromagnetism

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Applies without changes to the microscopic band structure in the iron-based superconductors

QMC for the onset of antiferromagnetism

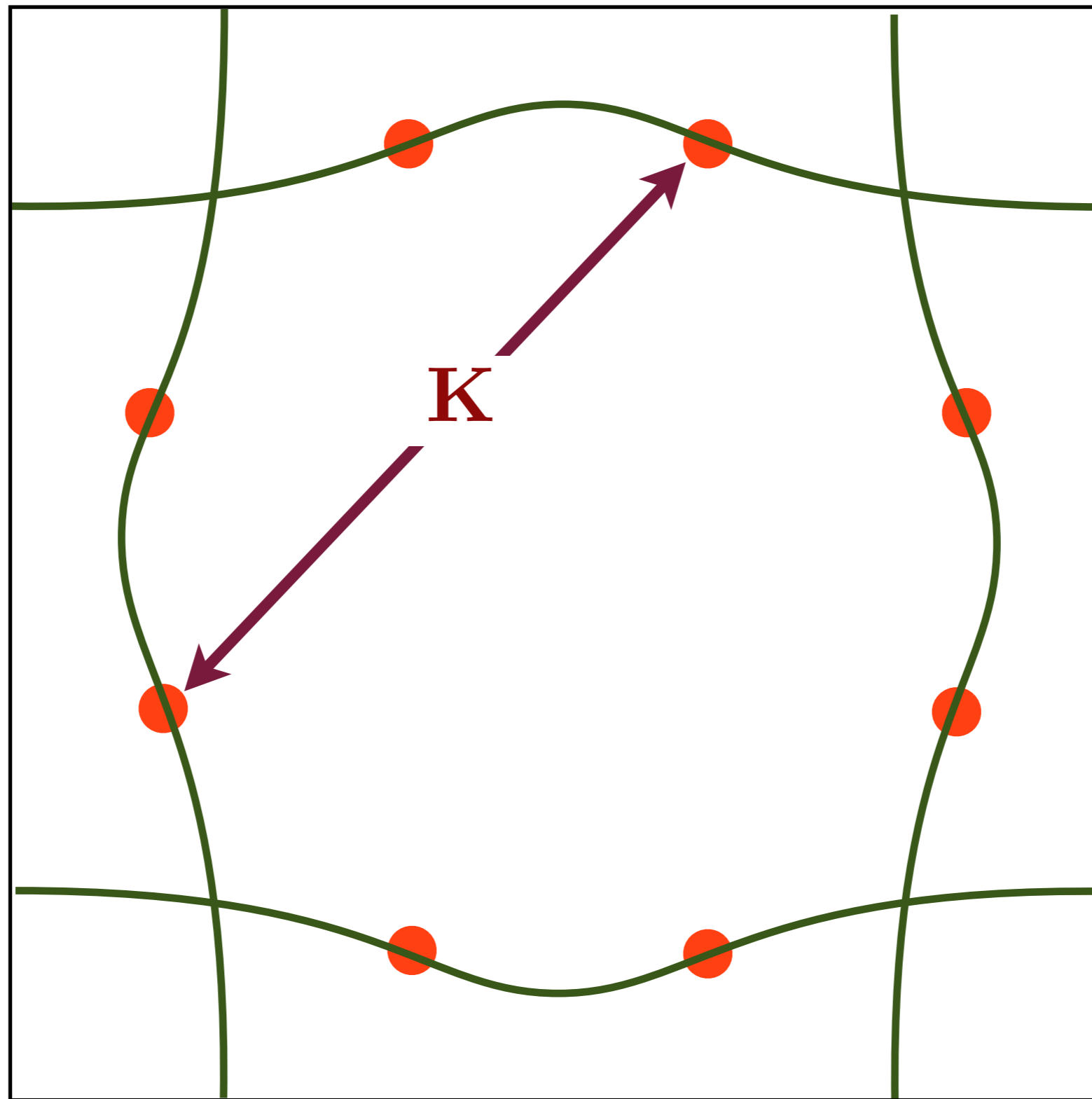
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Can integrate out $\vec{\varphi}$ to obtain an extended Hubbard model. The interactions in this model only couple electrons in separate bands.

QMC for the onset of antiferromagnetism

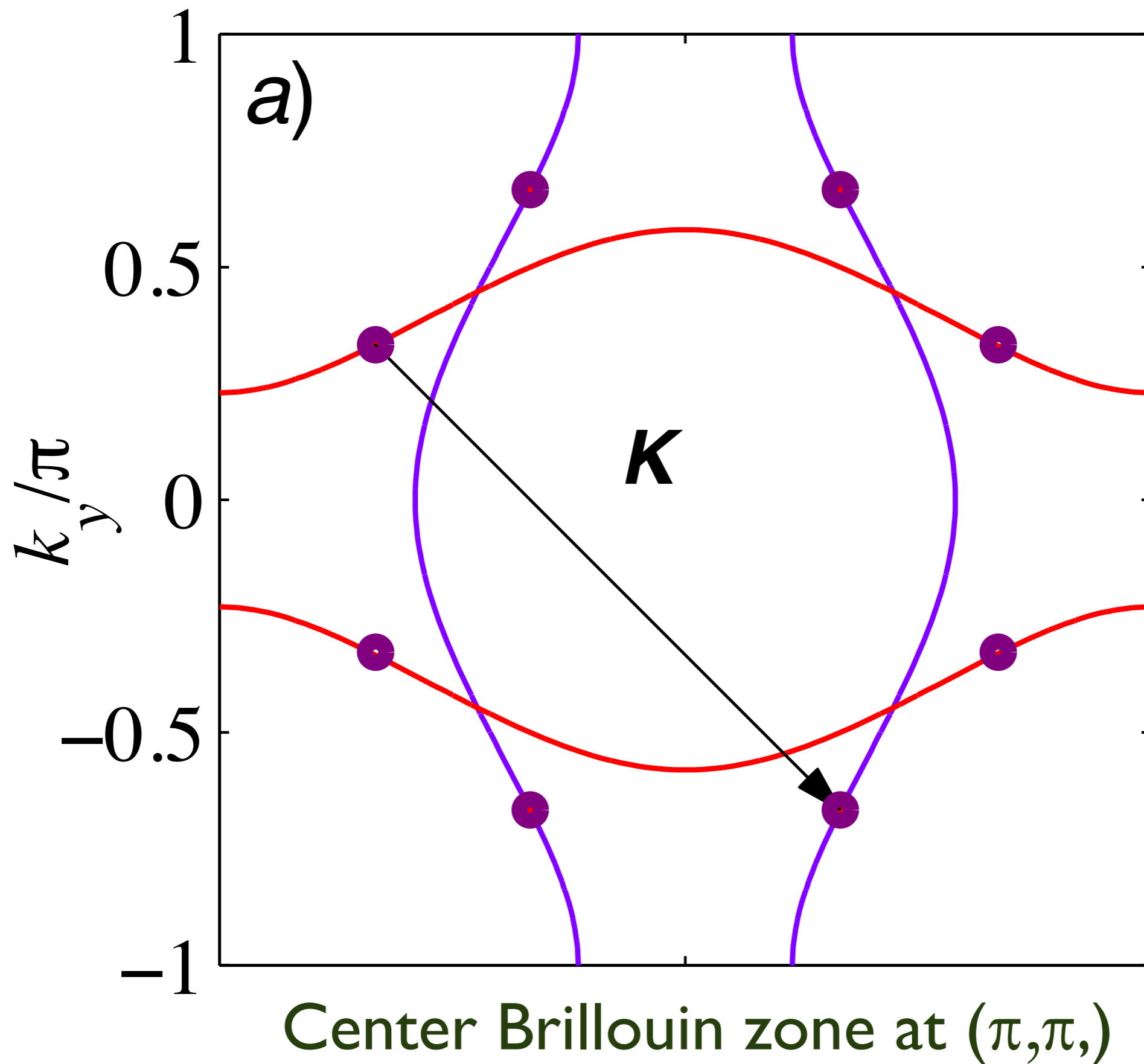


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Hot spots in a two band model

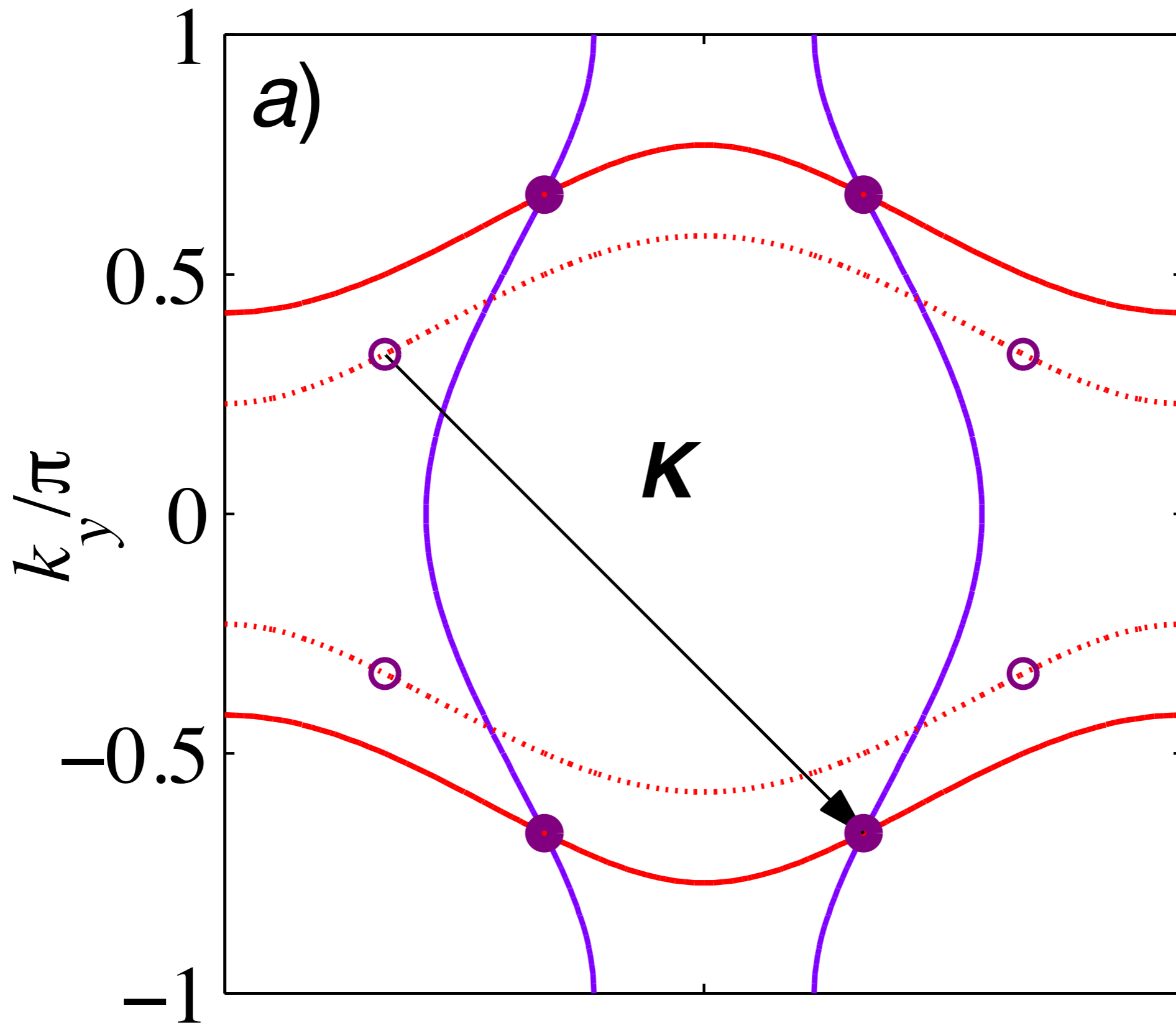
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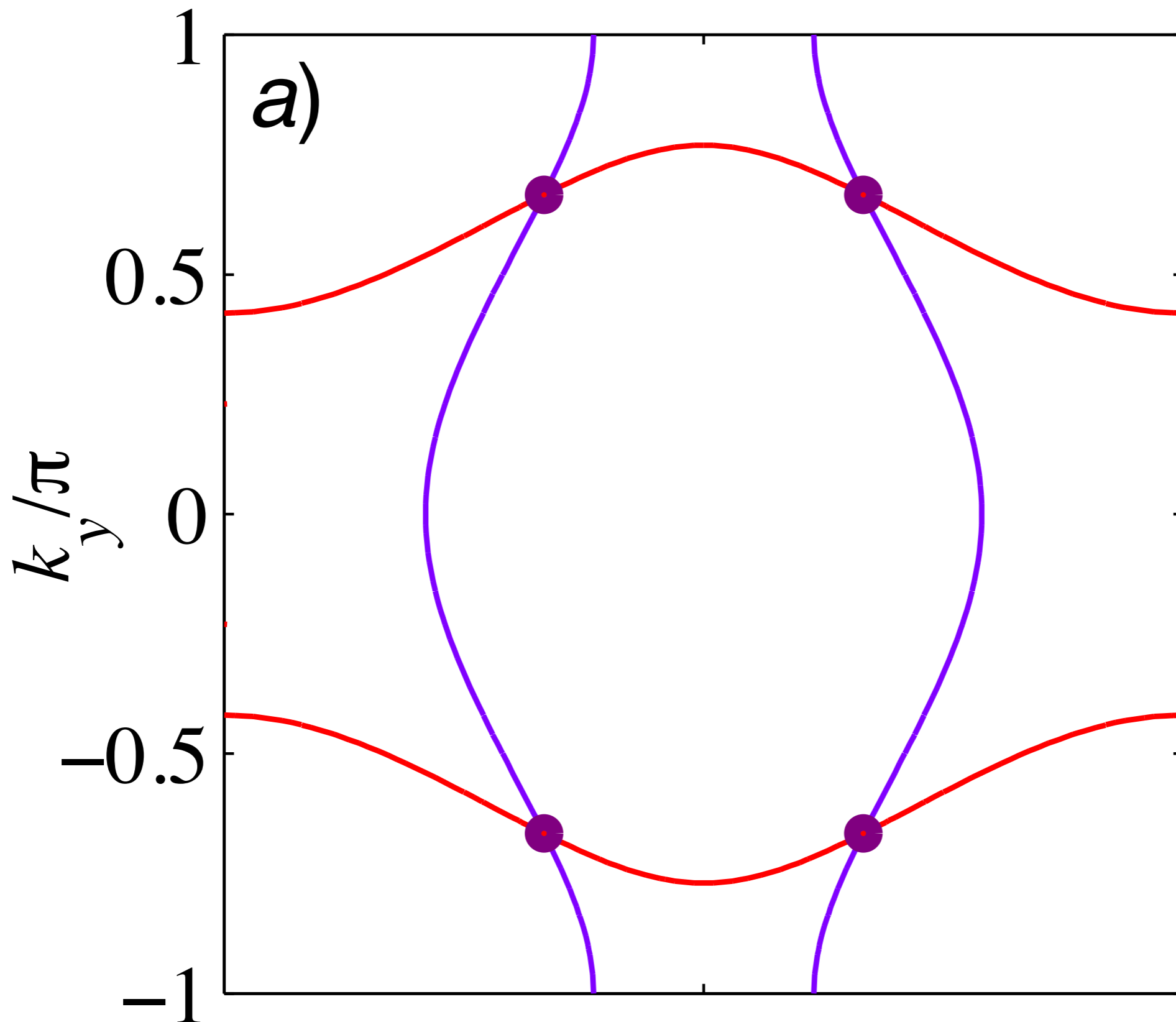
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Move one of the Fermi surface by (π, π)

QMC for the onset of antiferromagnetism

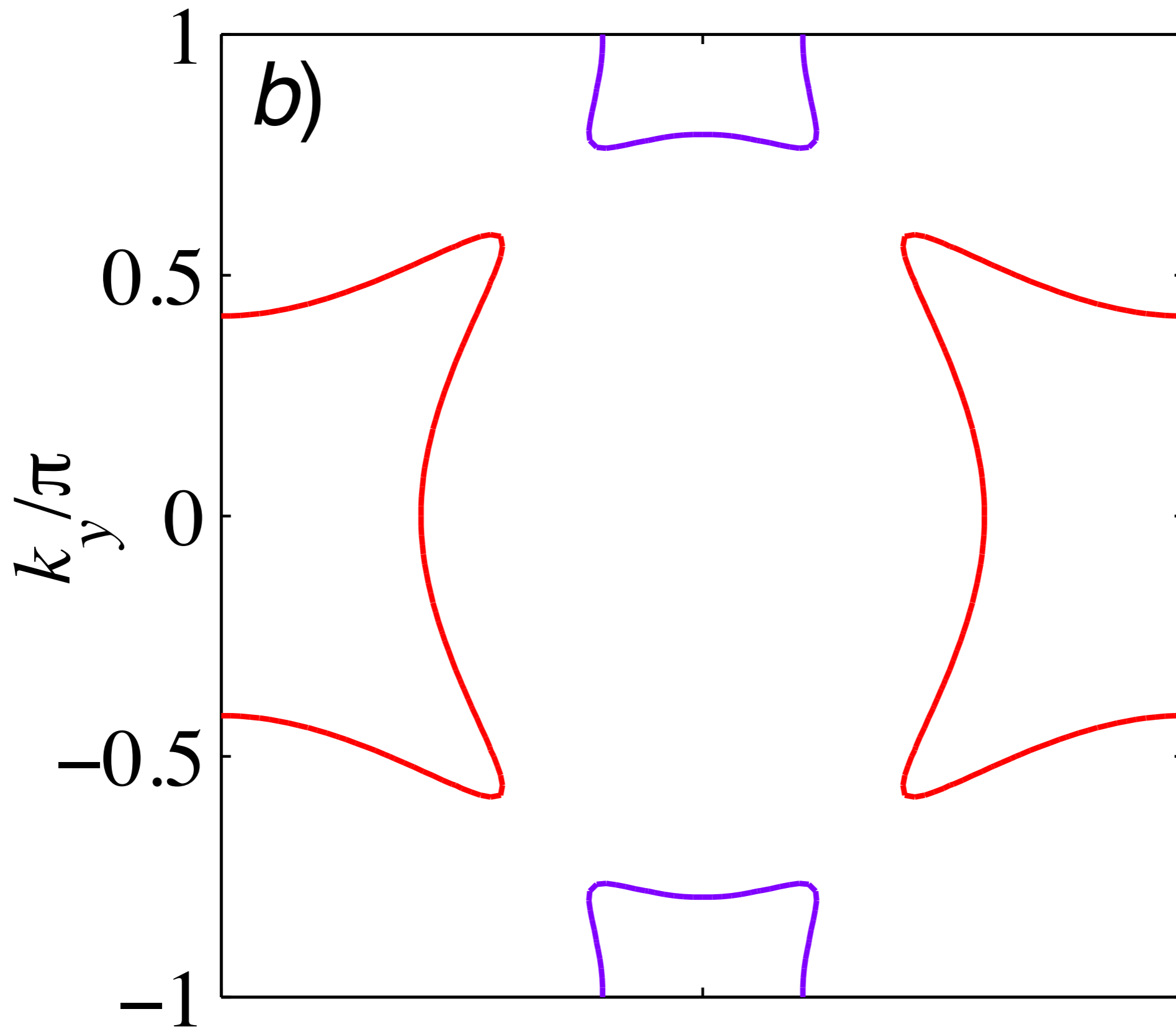


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Now hot spots are at Fermi surface intersections

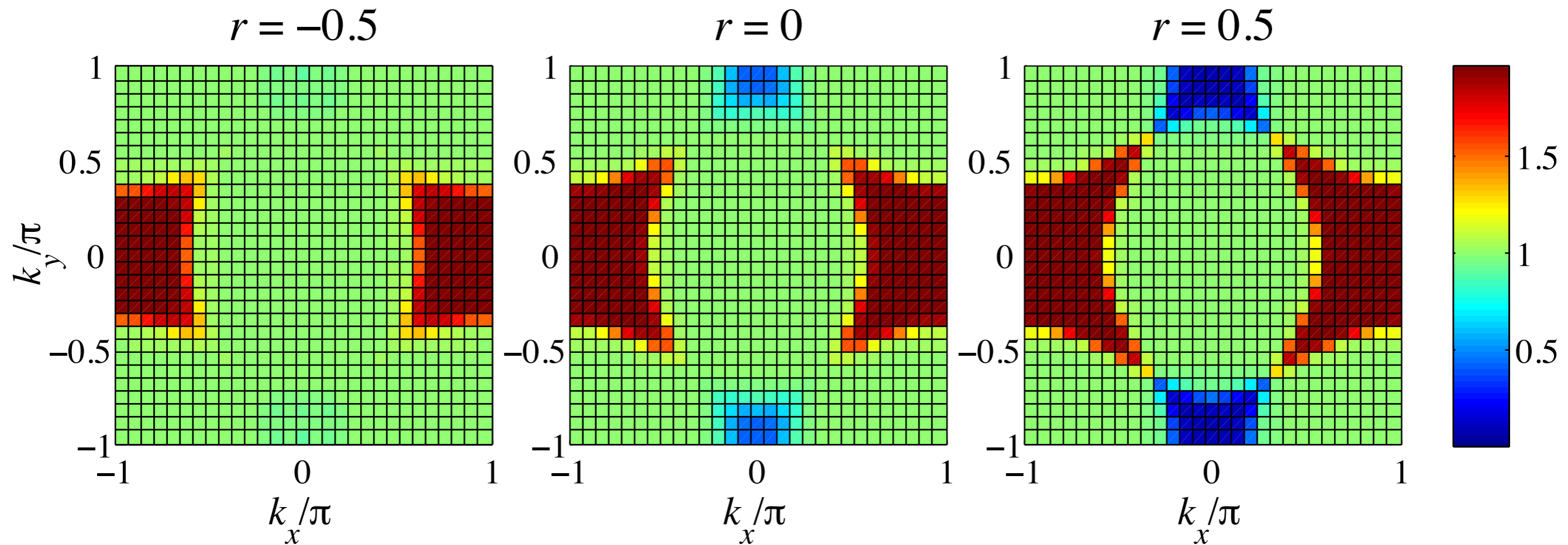
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Expected Fermi surfaces in the AFM ordered phase

QMC for the onset of antiferromagnetism

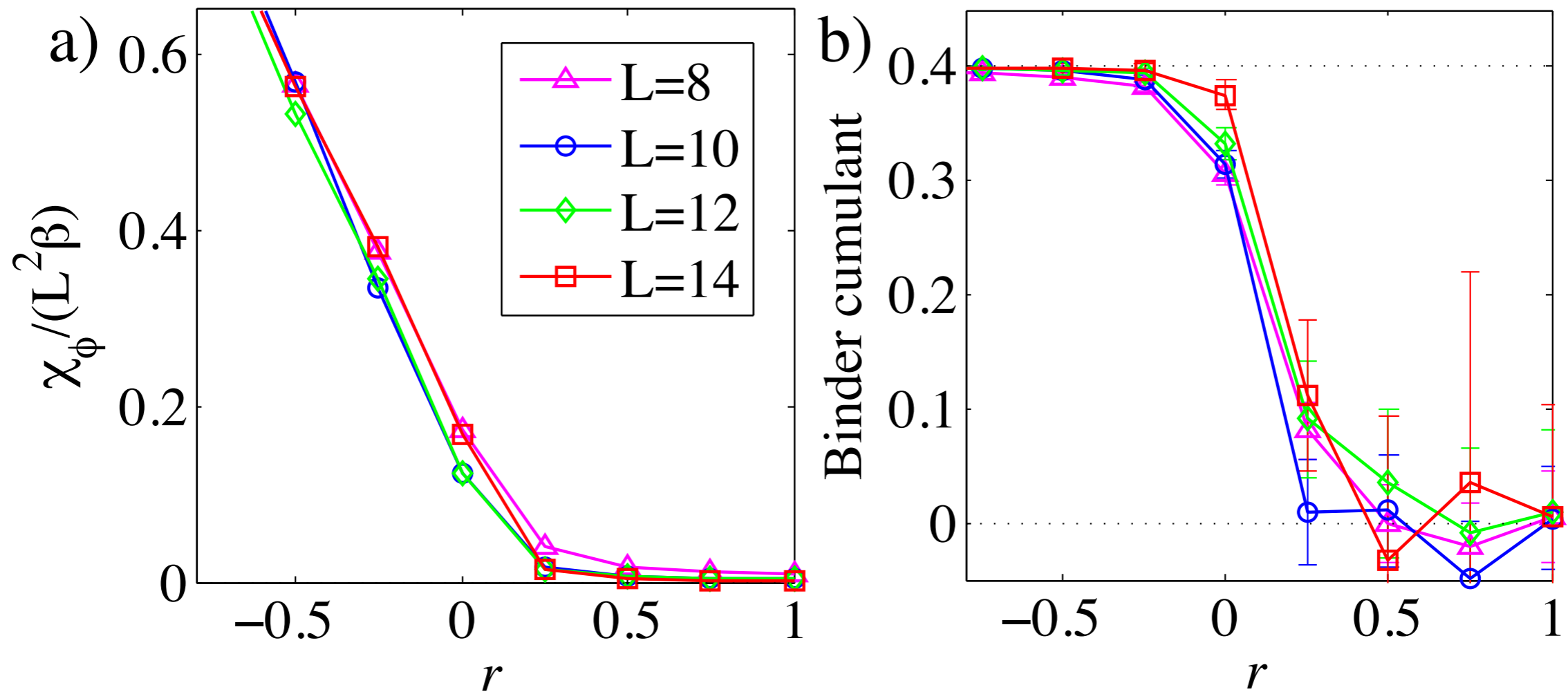


Electron occupation number $n_{\mathbf{k}}$
as a function of the tuning parameter r



E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

QMC for the onset of antiferromagnetism

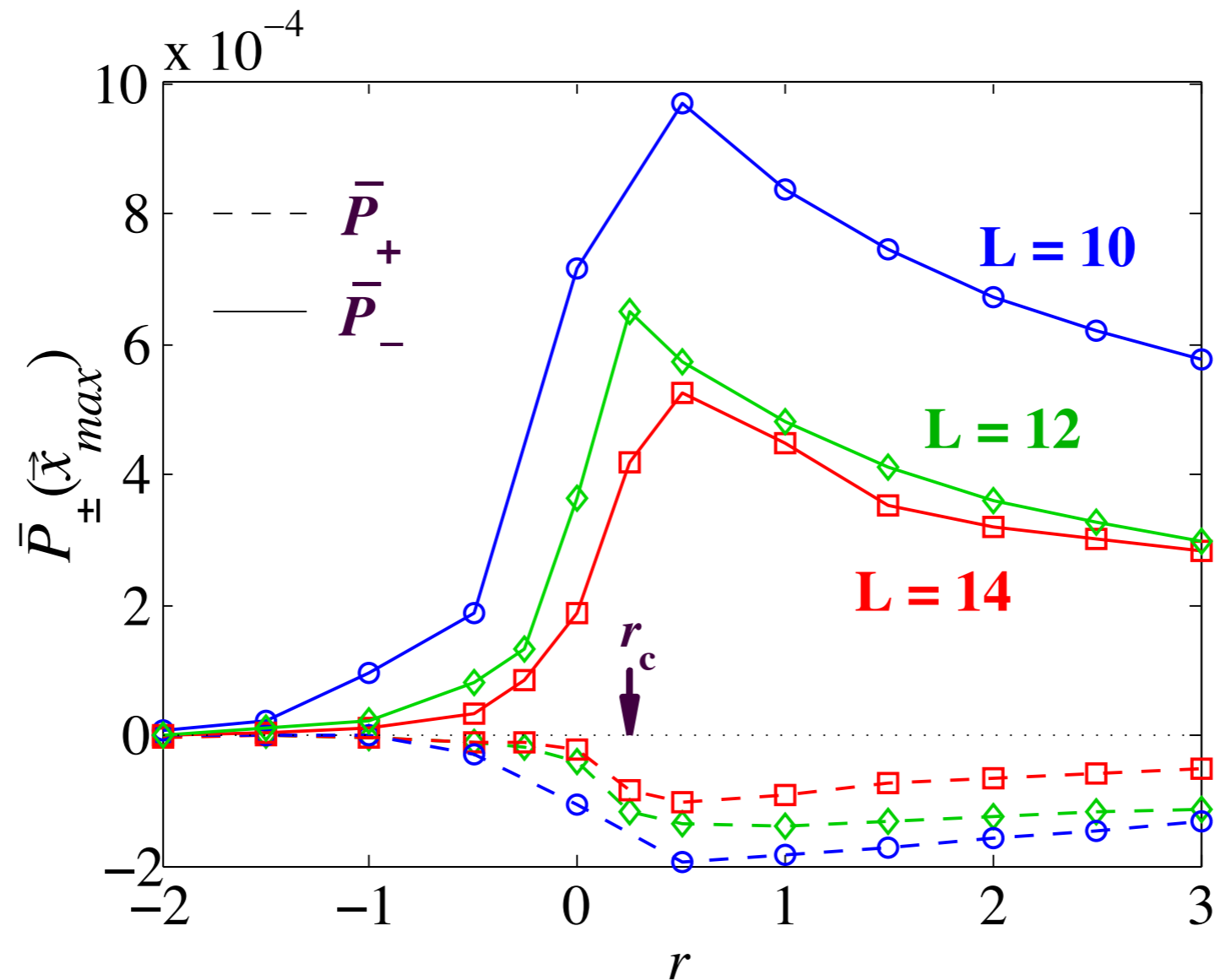


AF susceptibility, χ_ϕ , and Binder cumulant
as a function of the tuning parameter r



E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

QMC for the onset of antiferromagnetism

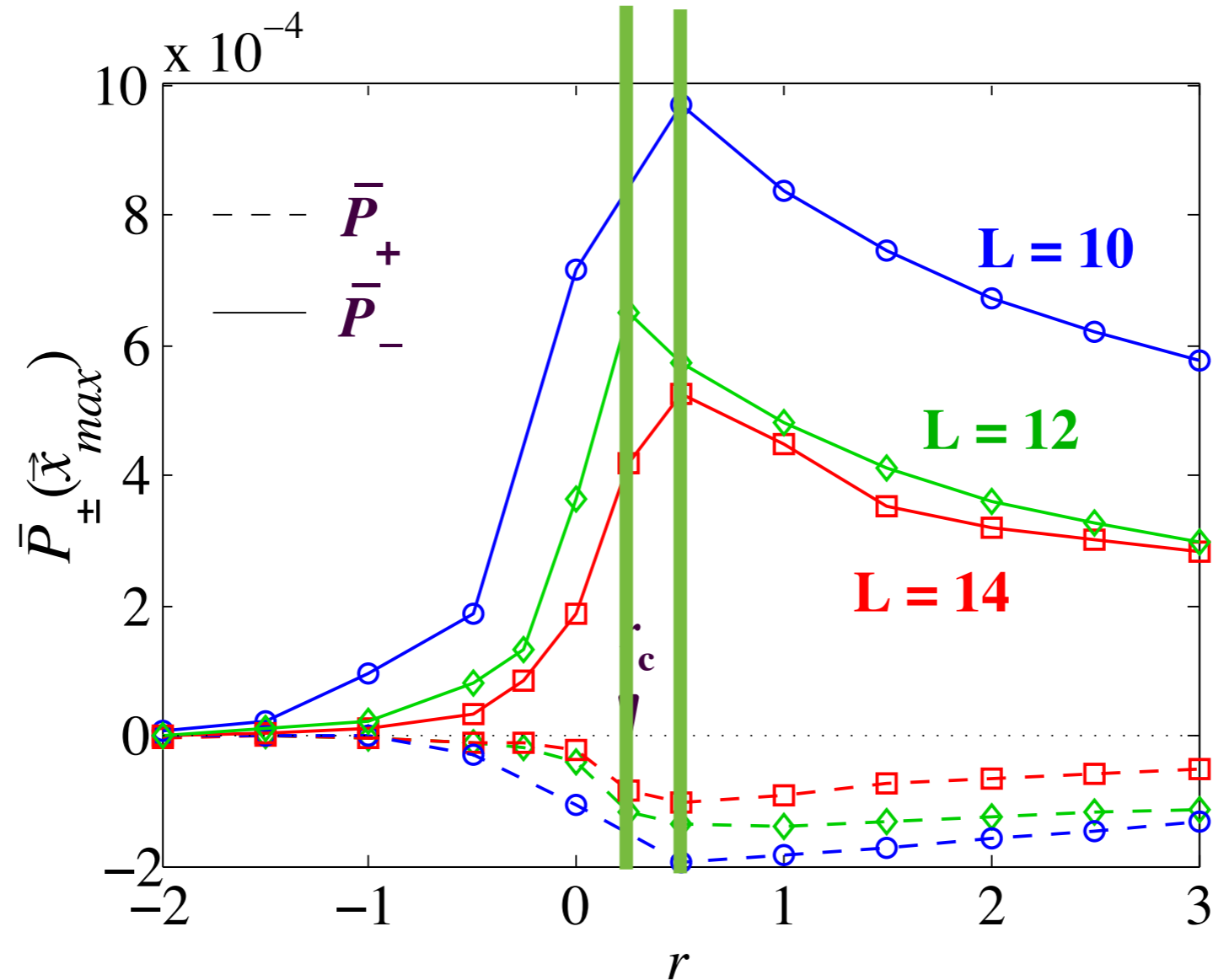


s/d pairing amplitudes P_+/P_-
as a function of the tuning parameter r



E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

QMC for the onset of antiferromagnetism



Notice shift between the position of the QCP in the superconductor, and the position of maximum pairing. This is found in numerous experiments.

E. Berg, M. Metlitski, and S. Sachdev, arXiv:1206.0742

Outline

1. Sign-problem free quantum Monte Carlo for the onset of antiferromagnetism in metals

2. Hole-doped cuprates:

Where are the electron pockets in the Brillouin zone ?

Is the pseudogap state an exotic metal with Fermi pockets which violate the Luttinger relation ? (Such a violation requires emergent gauge excitations: an example of such a metal is the Z_2 -FL* state)

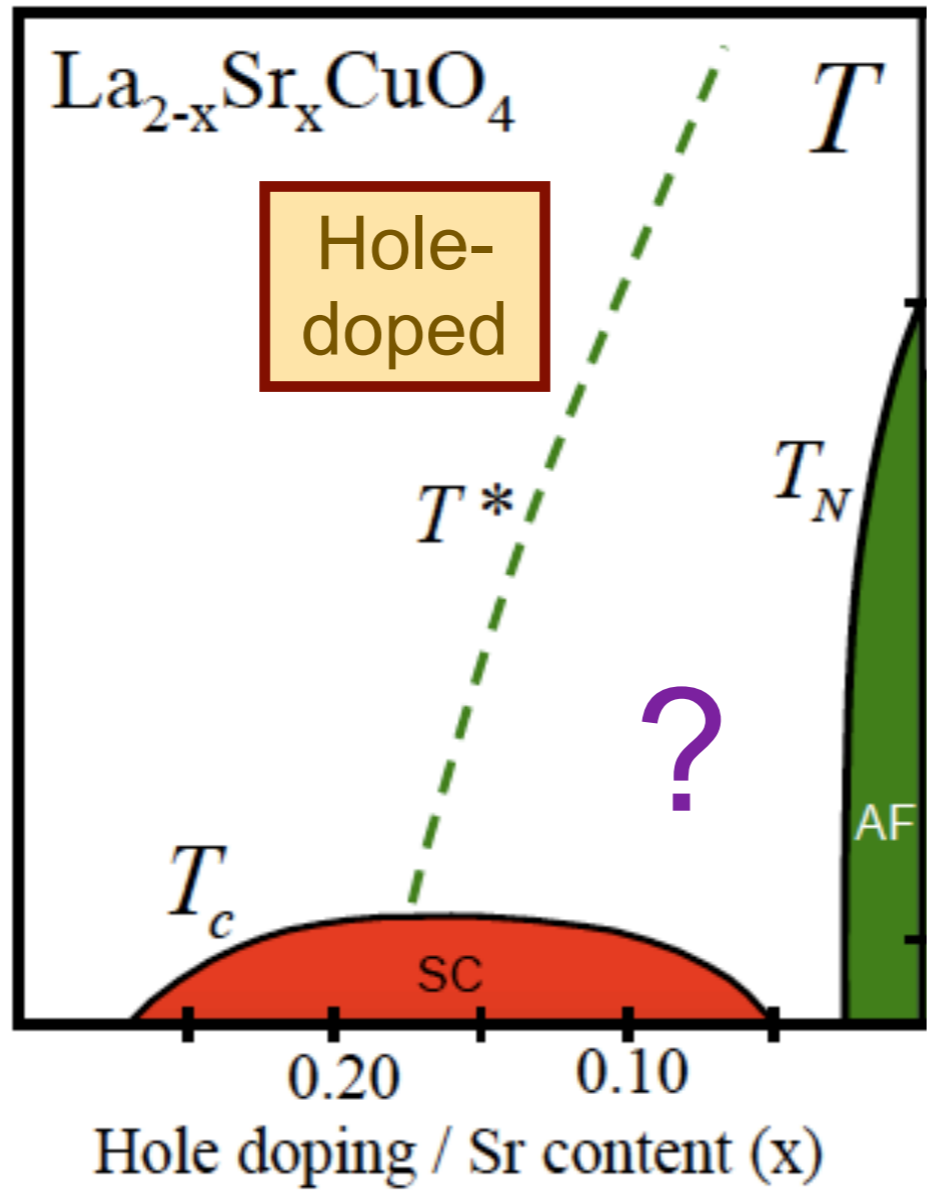
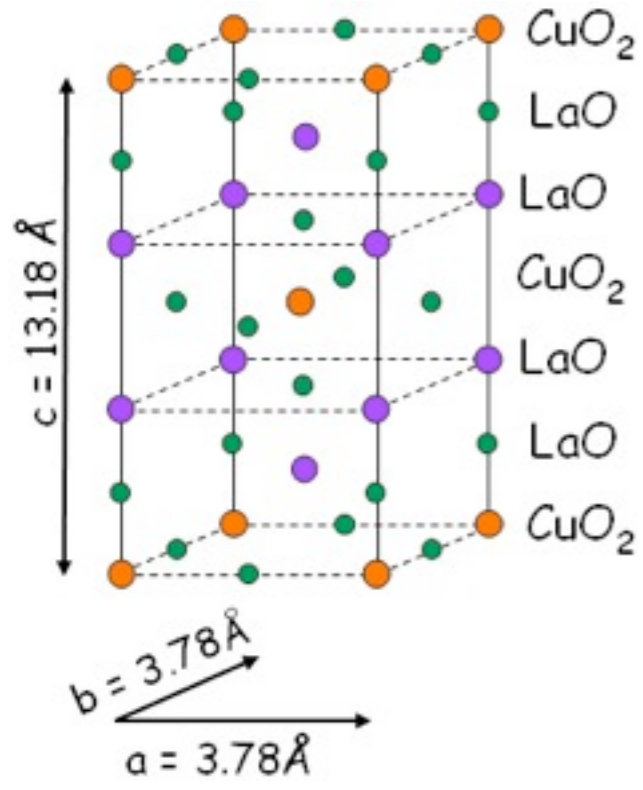
Outline

1. Sign-problem free quantum Monte Carlo for the onset of antiferromagnetism in metals

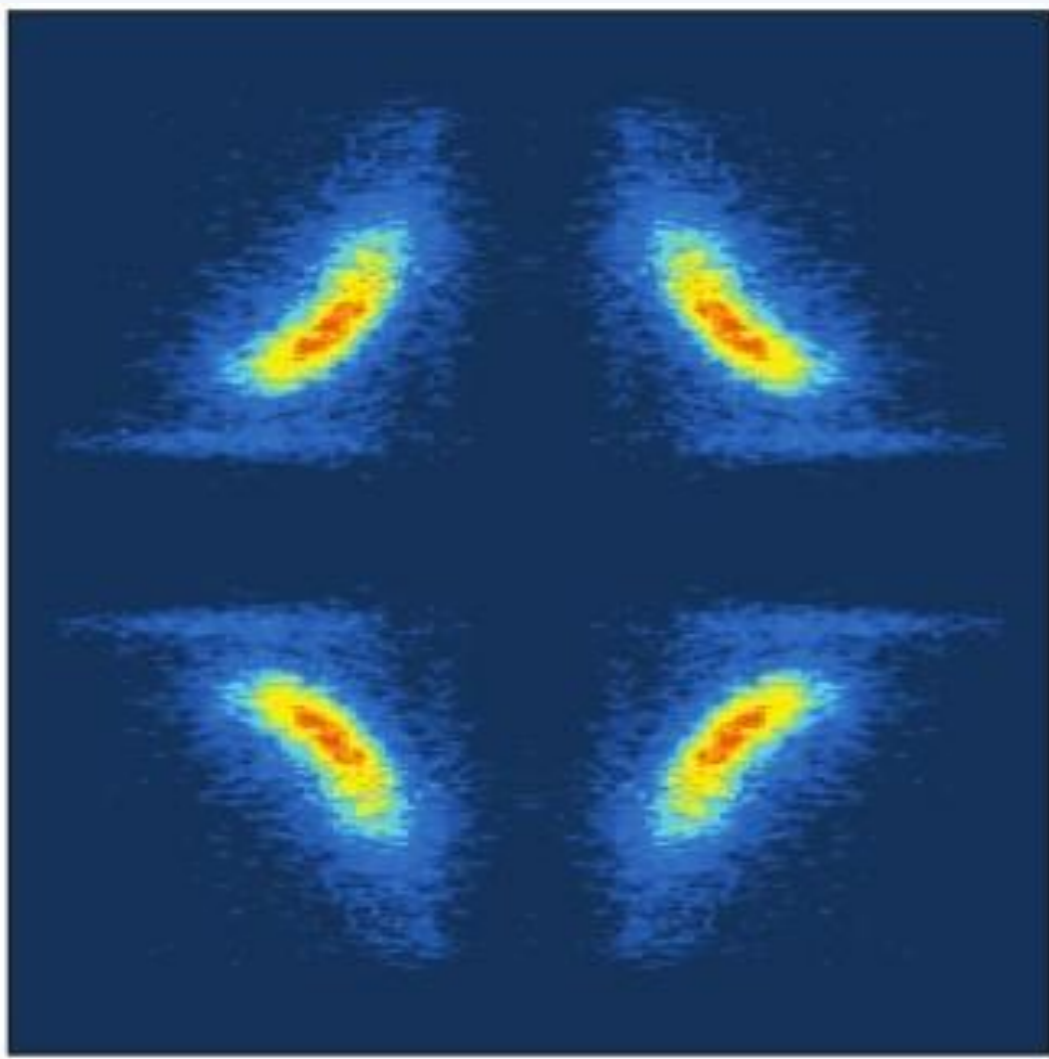
2. Hole-doped cuprates:

Where are the electron pockets in the Brillouin zone ?

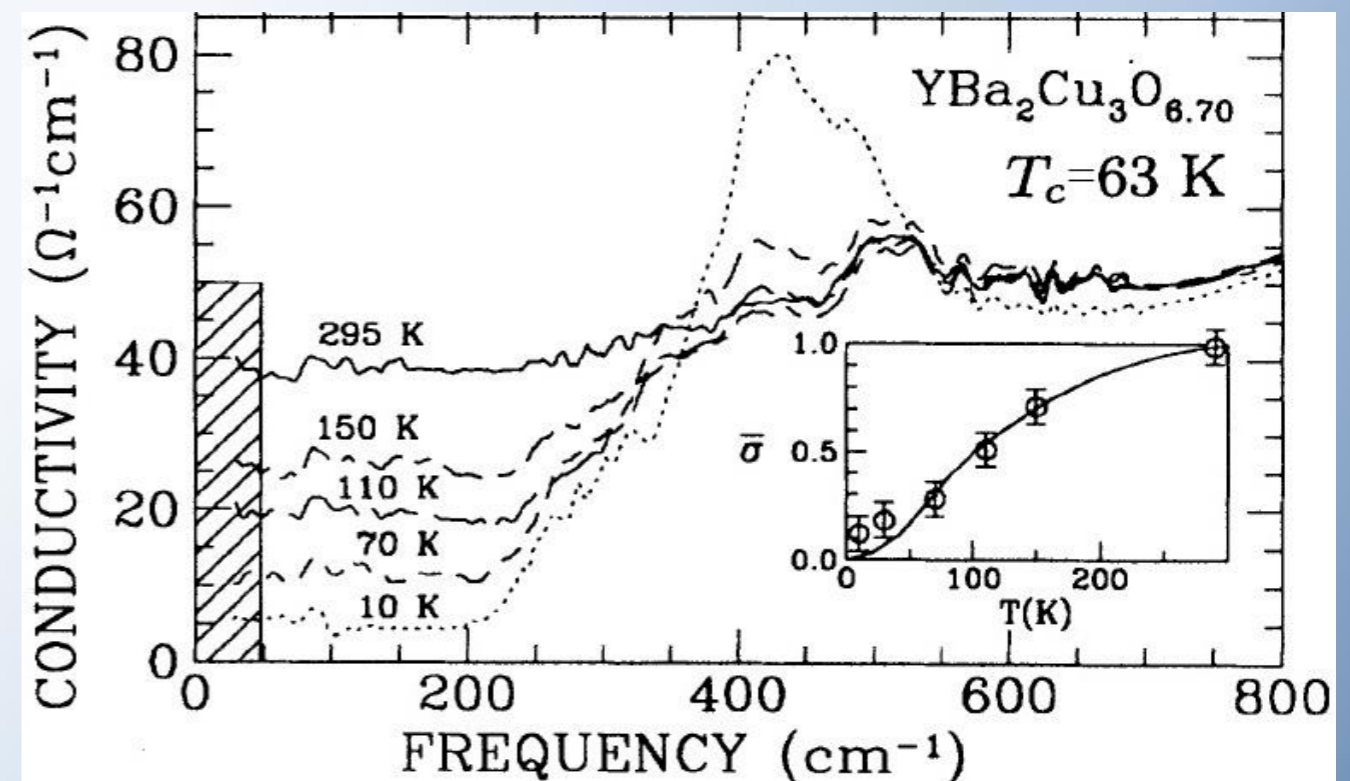
Is the pseudogap state an exotic metal with Fermi pockets which violate the Luttinger relation ? (Such a violation requires emergent gauge excitations: an example of such a metal is the Z_2 -FL* state)



Pseudogap state has no low energy fermions in the antinodal region



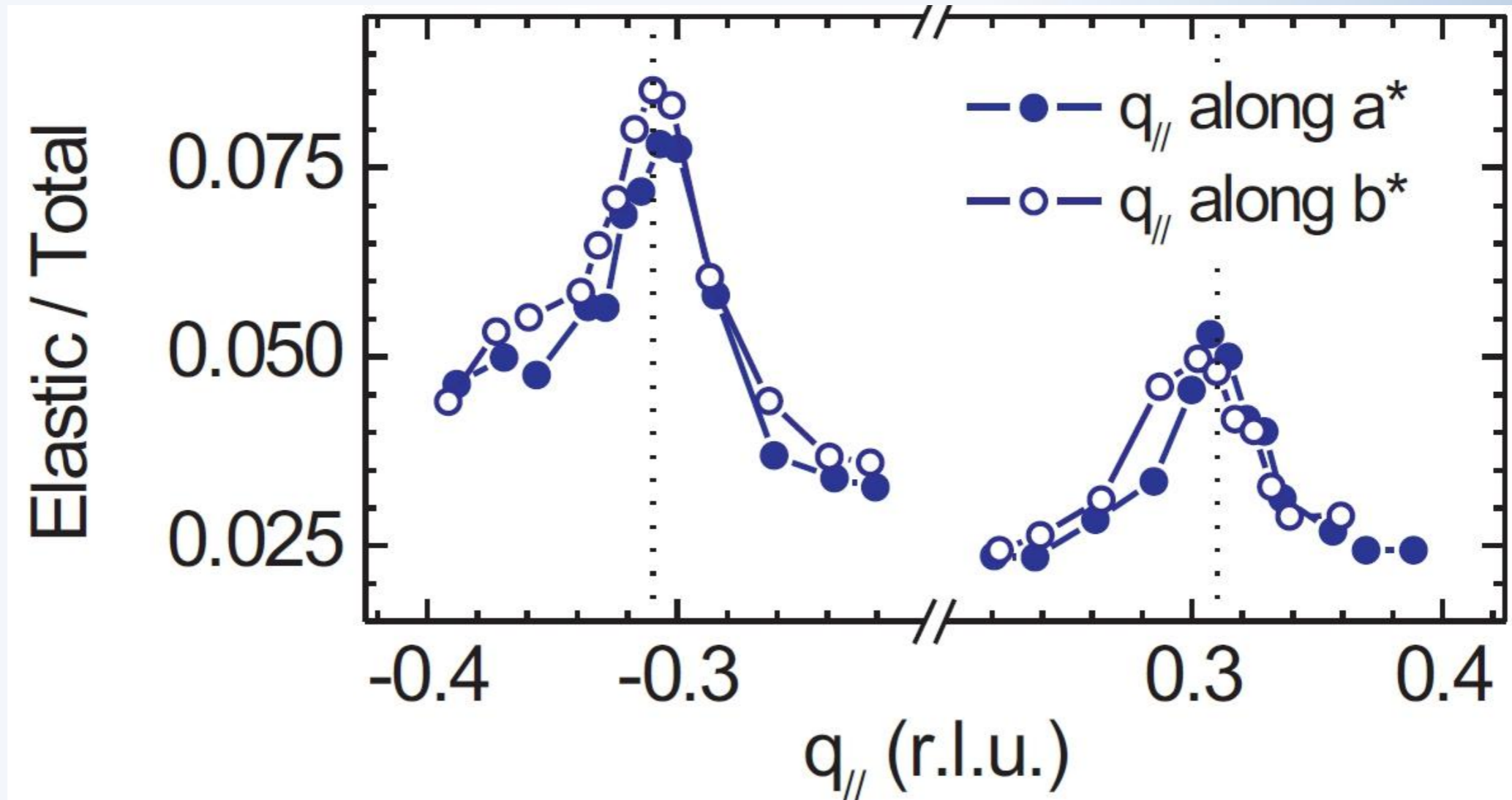
Peets D C... Damascelli A *New J. Phys.* **9** 28 (2007)



C. C. Holmes et al. *Phys. Rev. Lett.* **71**, 1645 (1993)

Resonant x-ray scattering

Surprise – wavevectors $\approx 3.3a$, biaxial



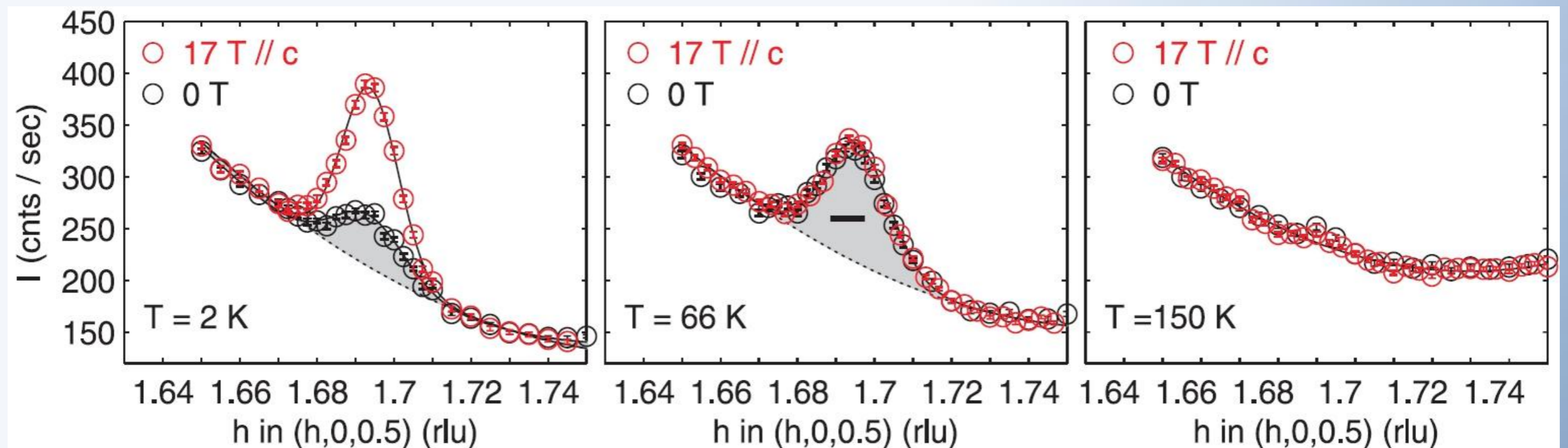
$E \approx 931$ eV

$Q = (0.31, 0)$ and $(0, 0.31)$
Correlation length $\xi \approx 60\text{\AA}$

$\text{YBa}_2\text{Cu}_3\text{O}_{6.6}$

$Q_{\text{CDW}} \neq 2Q_{\text{SDW}}$

Hard x-ray scattering in agreement with results from resonant x-ray scattering



$E \approx 100$ keV

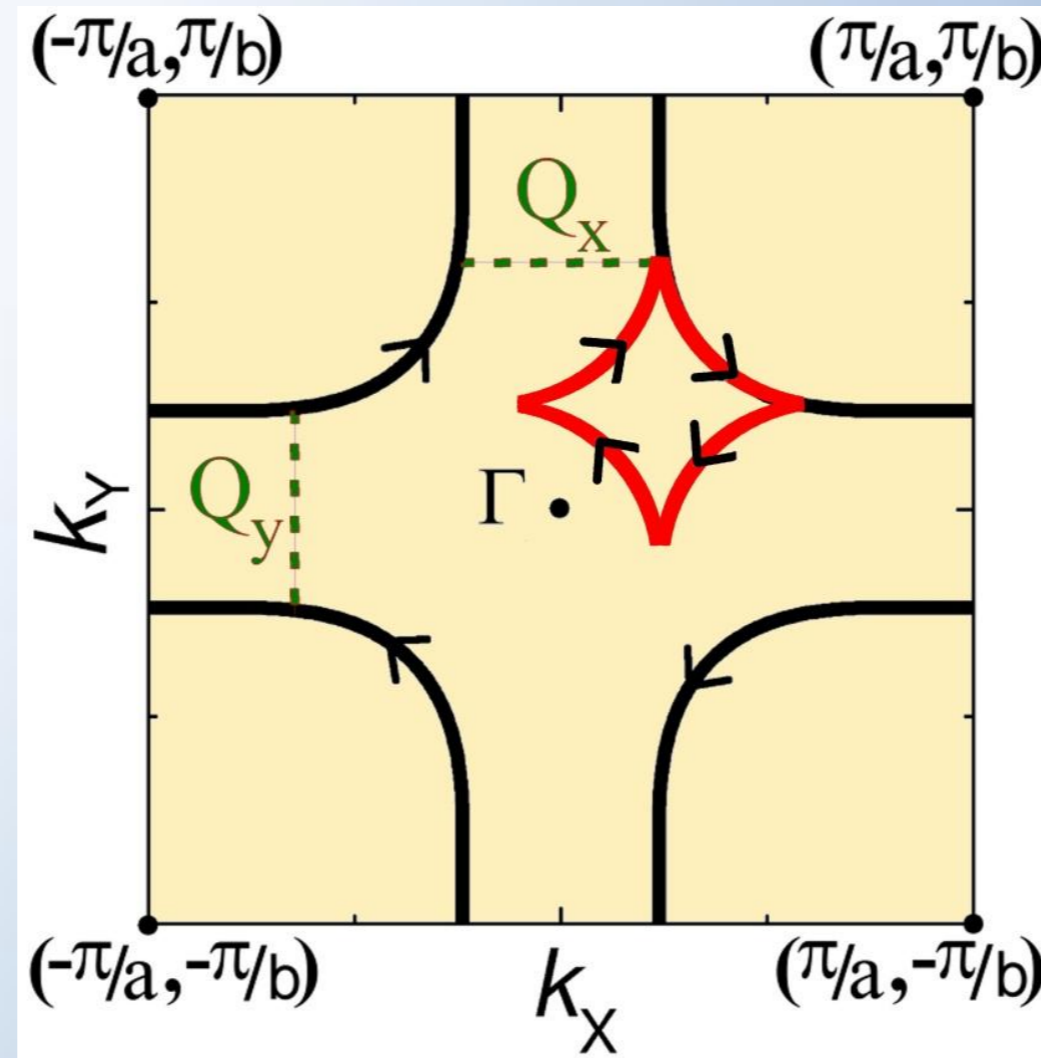
J. Chang et al. arXiv: 1206.4333

$\text{YBa}_2\text{Cu}_3\text{O}_{6.67}$ (ortho-VIII)

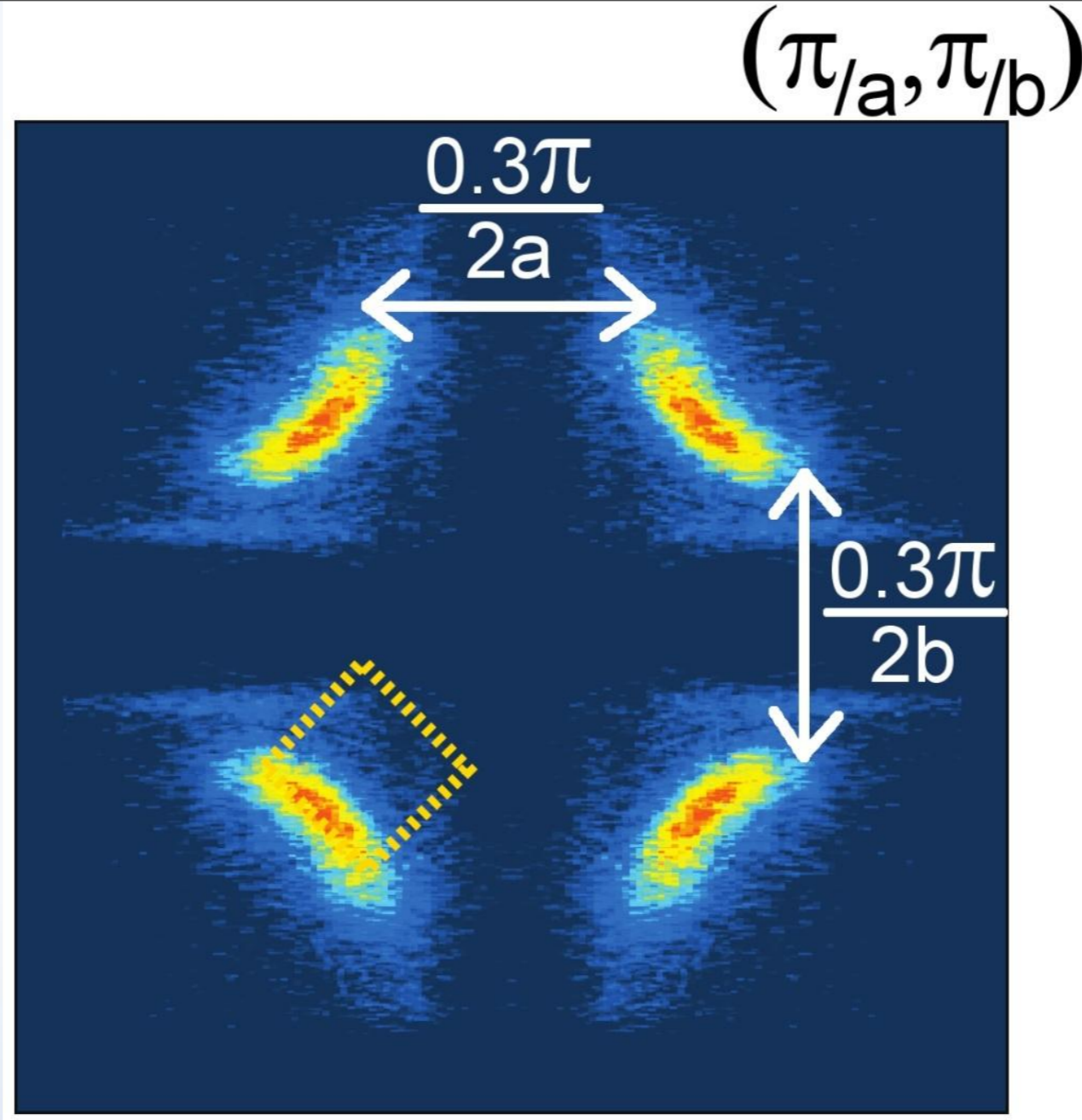
Correlation length $\xi \approx 98 \text{ \AA}$ (at 2K and 17T)

$\mathbf{q}_{\text{CDW}} = \mathbf{q}_1 = (\delta_1, 0, 0.5)$ and $\mathbf{q}_2 = (0, \delta_2, 0.5)$, where $\delta_1 \approx 0.30$ and $\delta_2 \approx 0.31$.

Nodal pocket with electron-like direction of cyclotron motion



N. Harrison & SES Phys. Rev. Lett. 106, 226402 (2011)
S. E. Sebastian et al., Rep. Prog. Physics (in press, 2012)

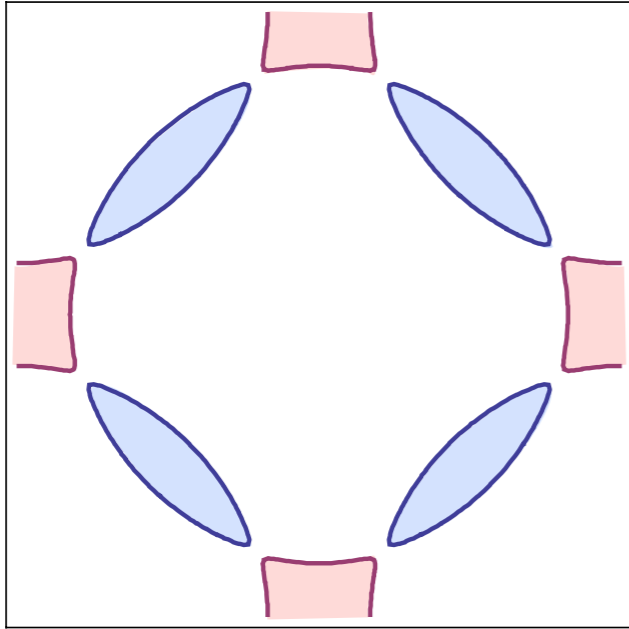


S. E. Sebastian et al., Rep. Prog. Physics (in press, 2012)

In reality, such an electron pocket is a reconstruction of a parent state which has no Fermi surface in the anti-nodal region. So if we accept this picture, the parent state is very exotic !

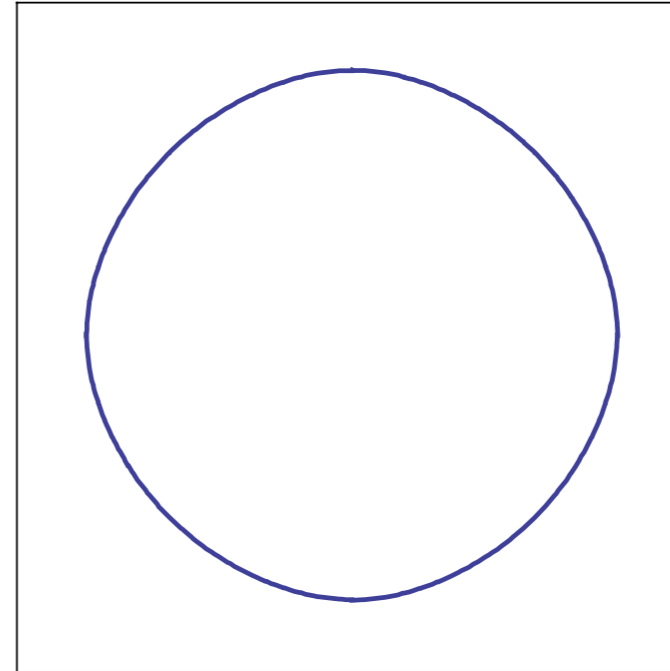
A metal with Fermi surfaces only near the nodes, and no broken symmetry, which violates the Luttinger relation.

Quantum phase transition with Fermi surface reconstruction



$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron
and hole pockets

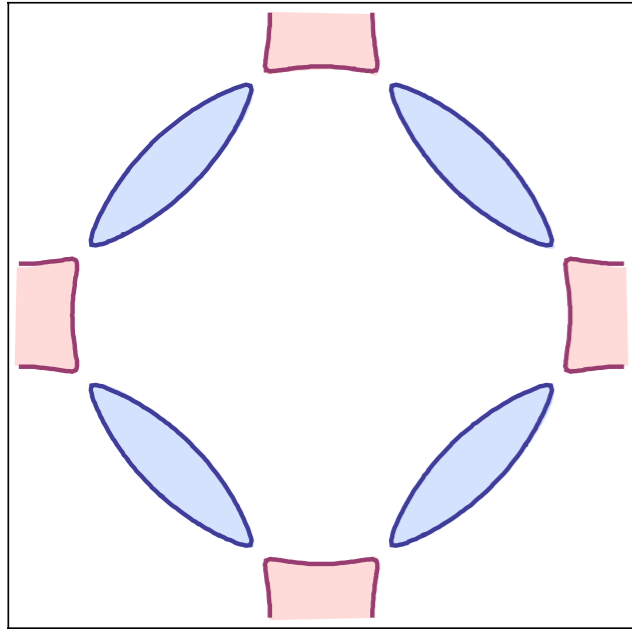


$$\langle \vec{\varphi} \rangle = 0$$

Metal with “large”
Fermi surface

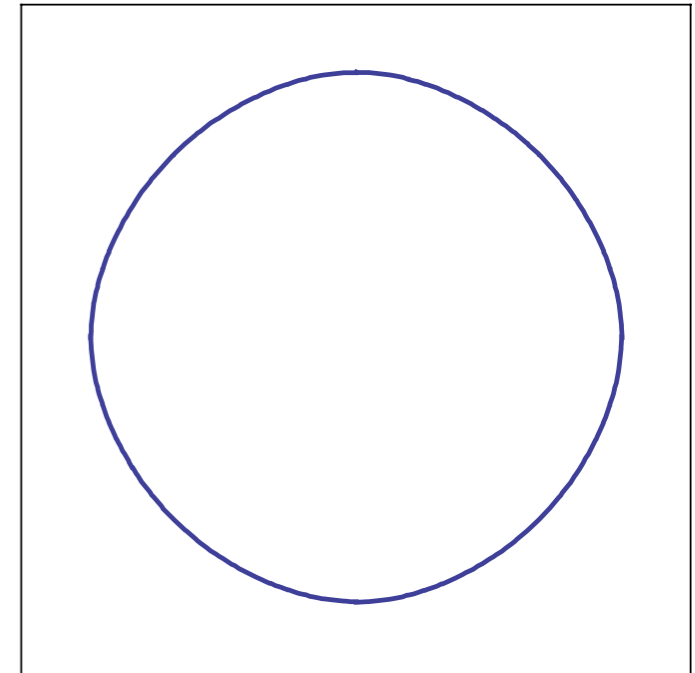


Separating onset of SDW order and Fermi surface reconstruction



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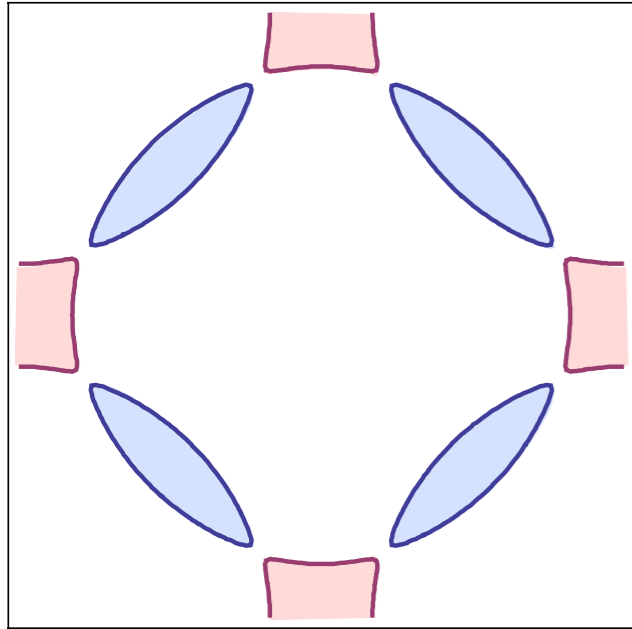


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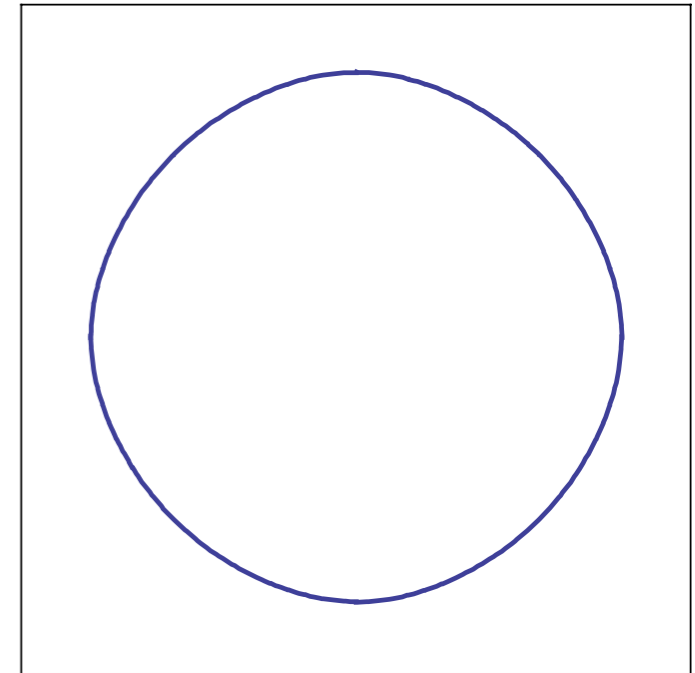
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Electron and/or hole
Fermi pockets form in
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destroy long-range
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$$\langle \vec{\varphi} \rangle = 0$$

Fractionalized Fermi
liquid (FL*) phase
with no symmetry
breaking and “small”
Fermi surface

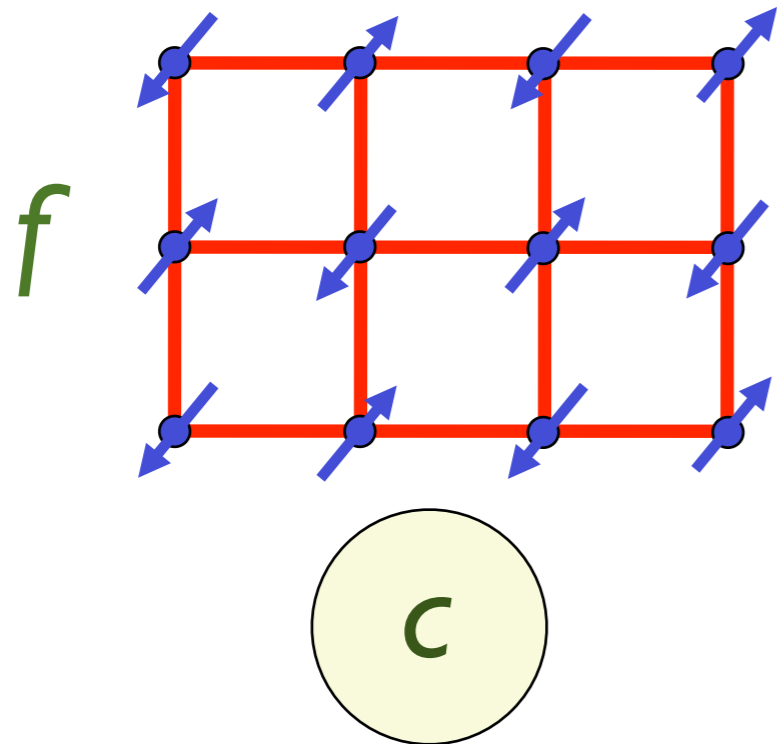


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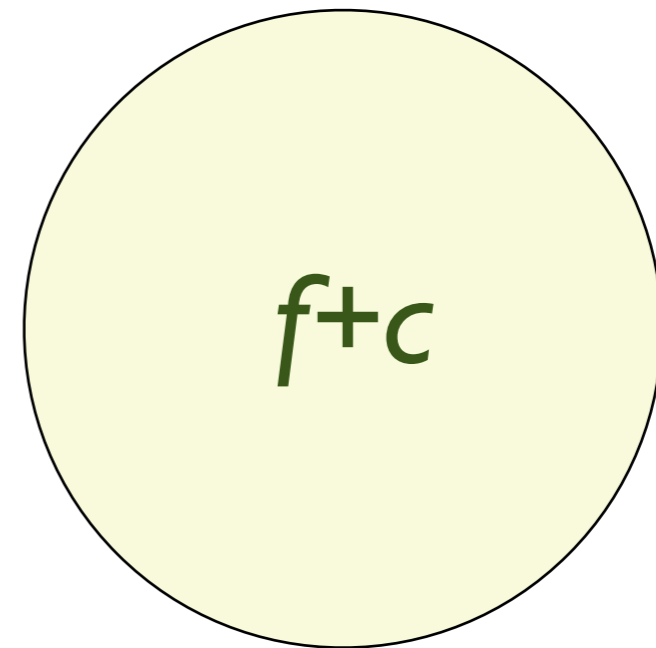
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Magnetic order and the heavy Fermi liquid in the Kondo lattice



$$\langle \vec{\varphi} \rangle \neq 0$$

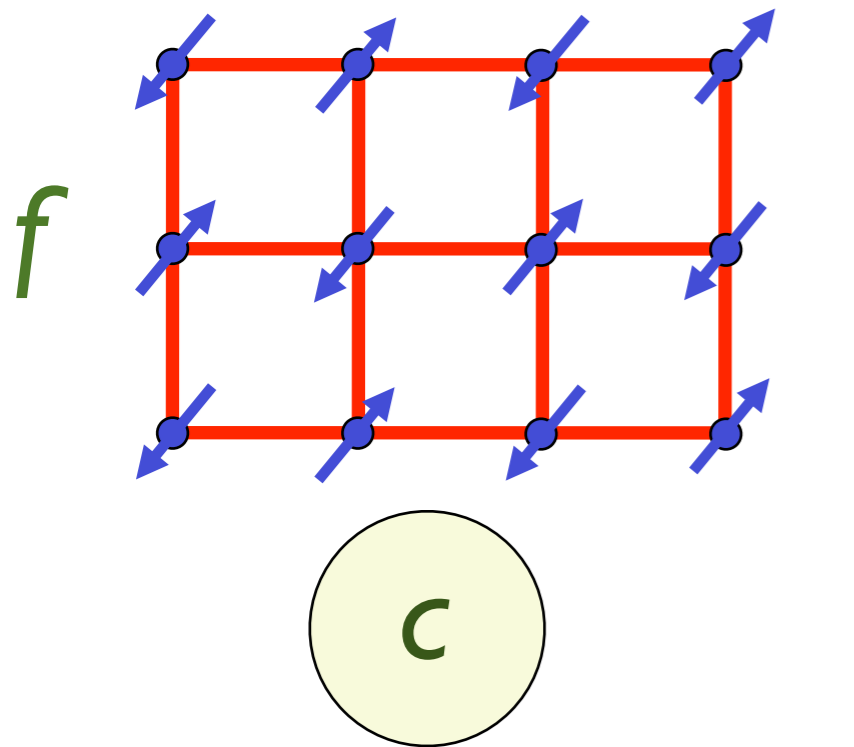
Magnetic Metal:
f-electron moments
and
c-conduction electron
Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

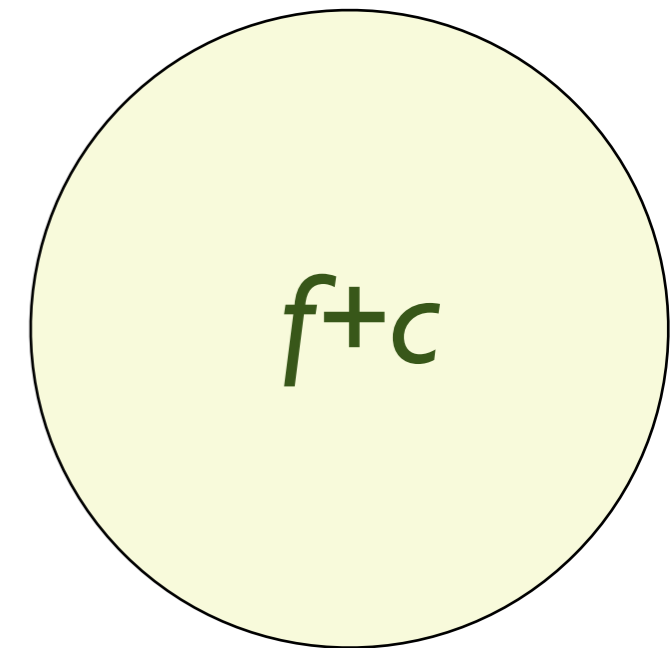
Heavy Fermi liquid
with “large” Fermi
surface of
hybridized f and
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Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



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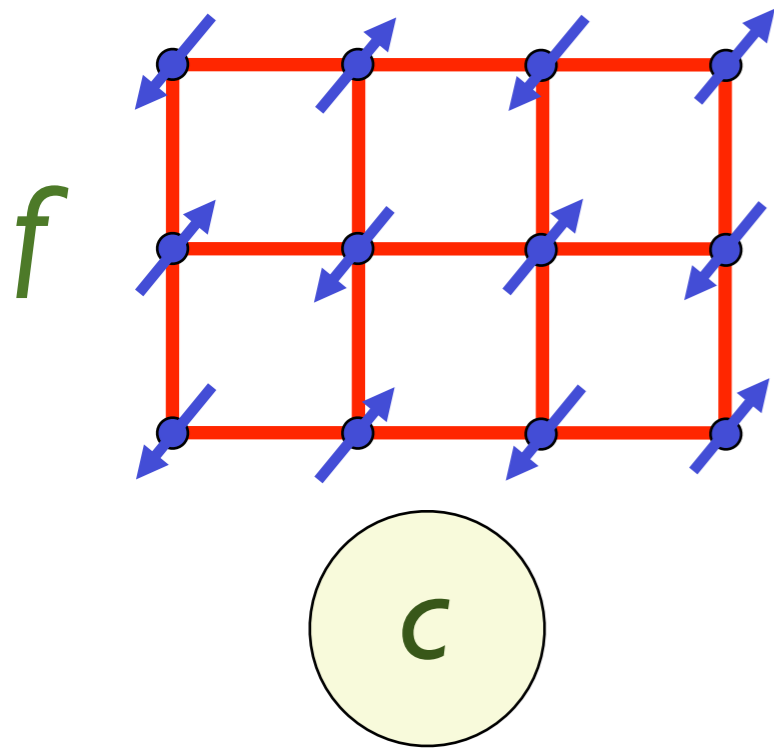


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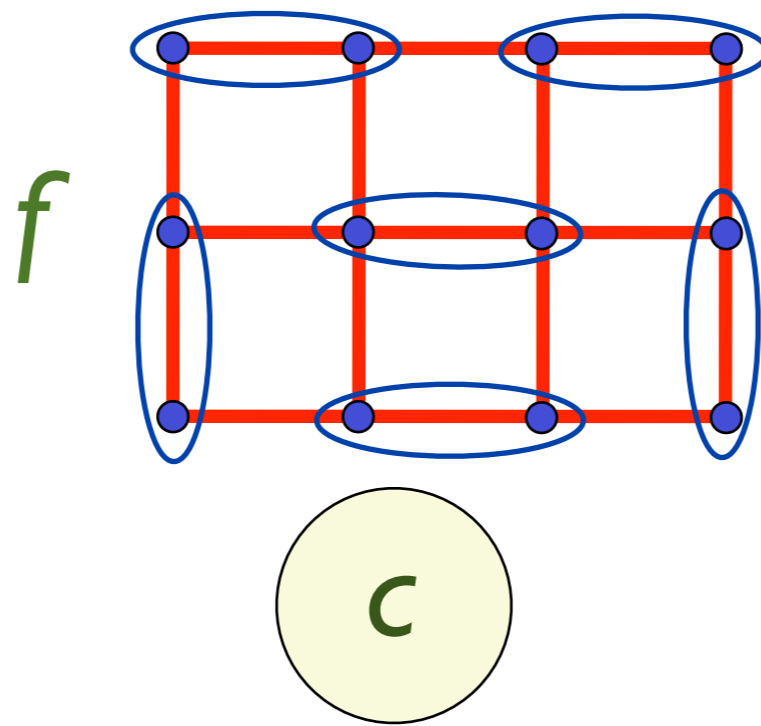
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Separating onset of SDW order and the heavy Fermi liquid in the Kondo lattice



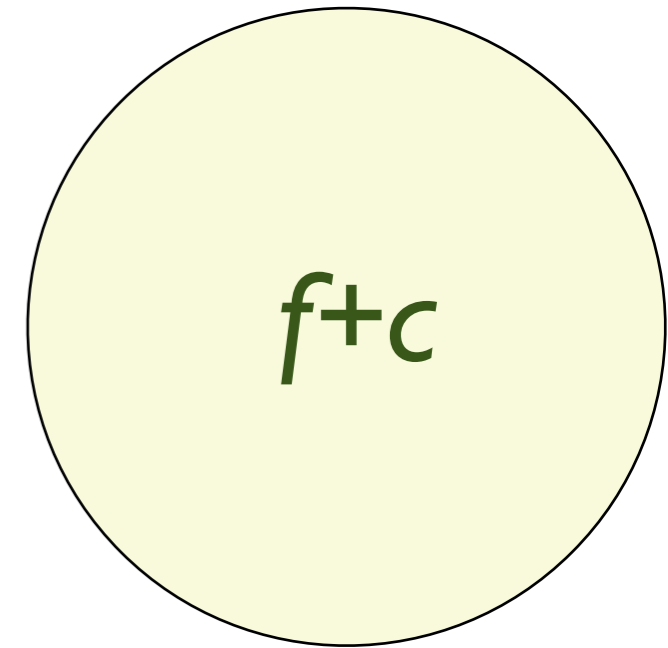
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$$\langle \vec{\varphi} \rangle = 0$$

Conduction electron
Fermi surface
and
spin-liquid of
f-electrons

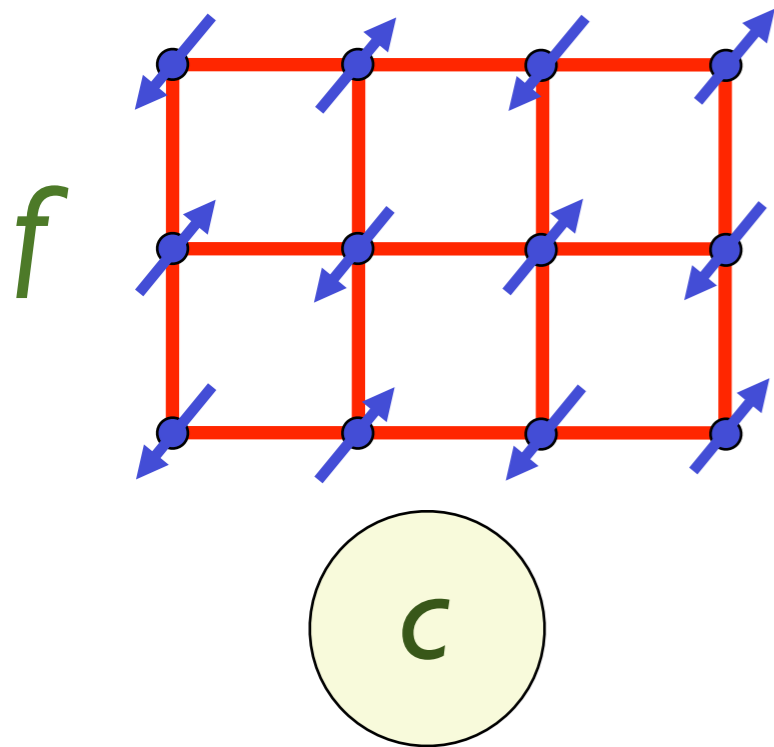


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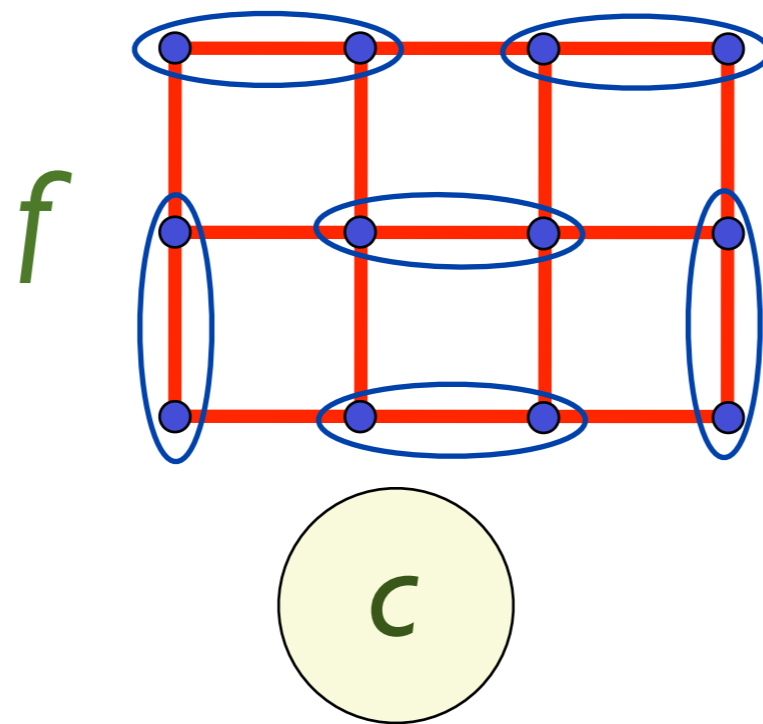
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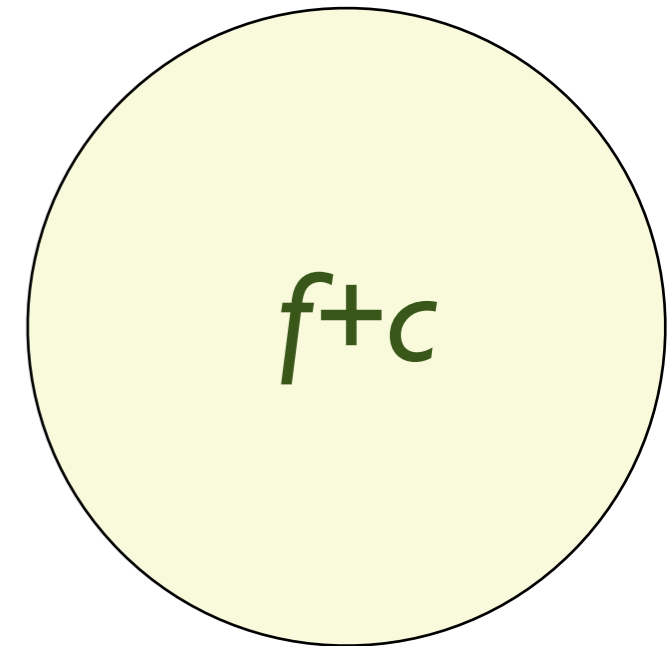
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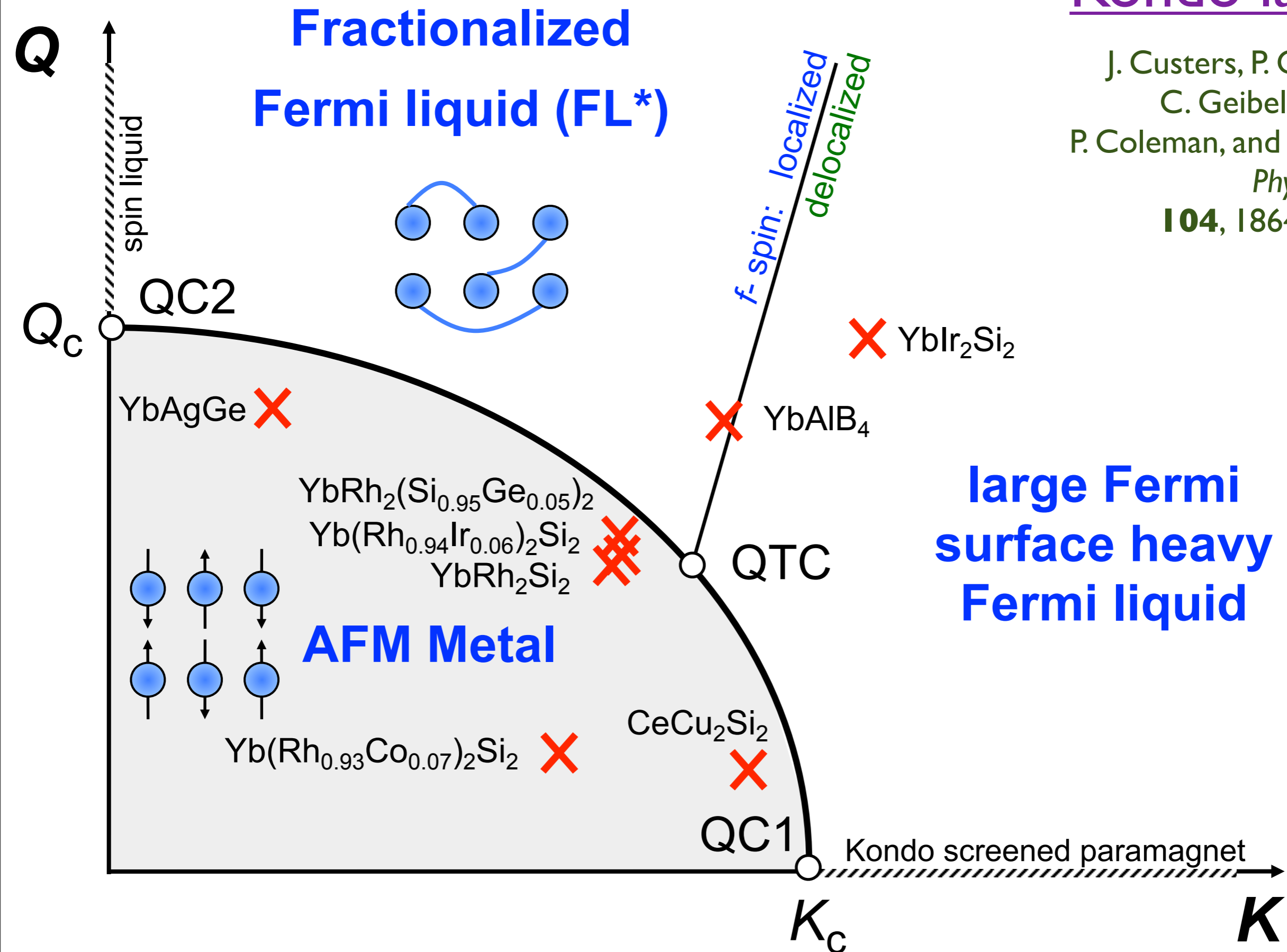
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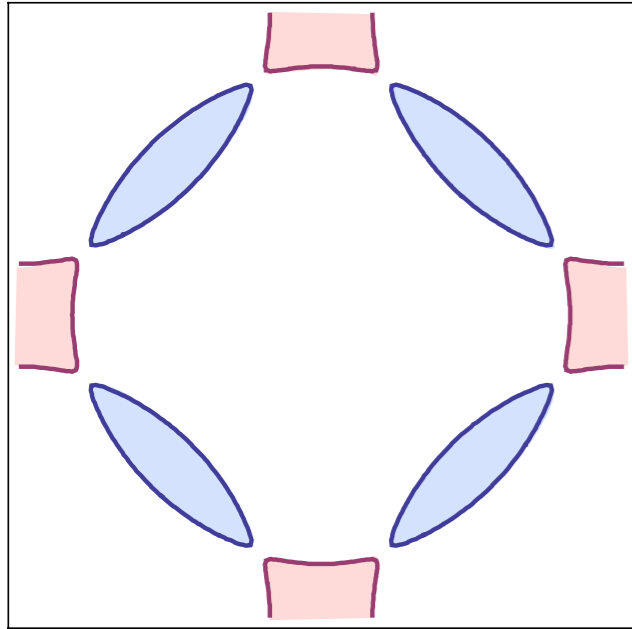
T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Experimental perspective on same phase diagrams of Kondo lattice

J. Custers, P. Gegenwart,
C. Geibel, F. Steglich,
P. Coleman, and S. Paschen,
Phys. Rev. Lett.
104, 186402 (2010)



Separating onset of SDW order and Fermi surface reconstruction



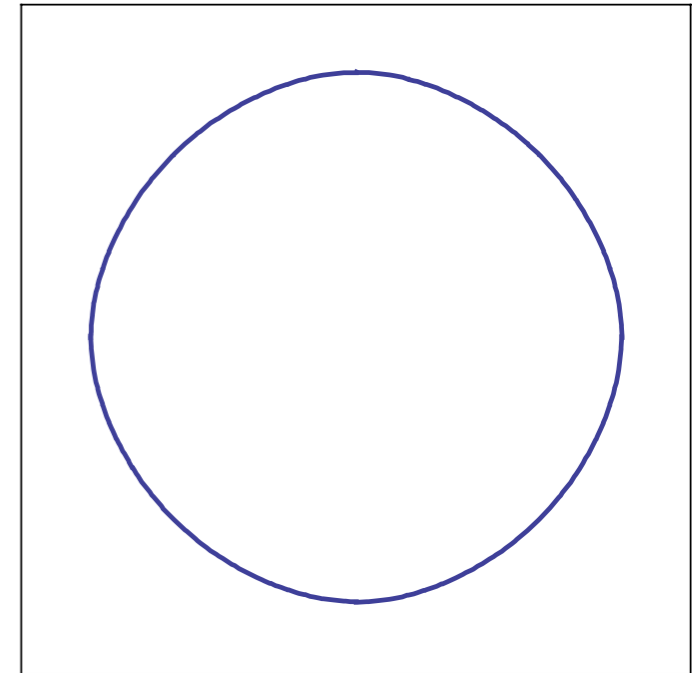
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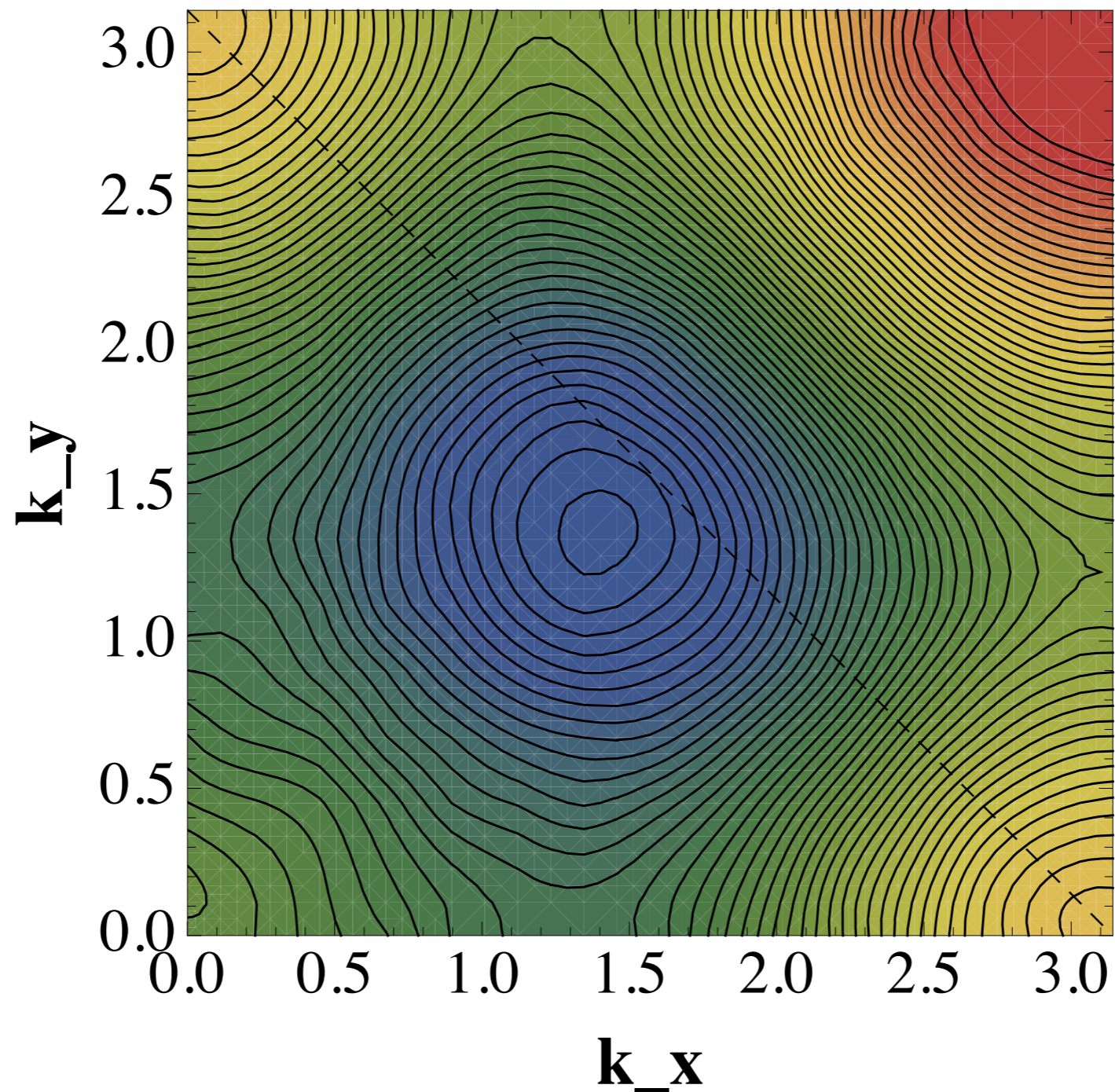
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T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)



Hole pocket of a \mathbb{Z}_2 -FL* phase
in a *single-band* t - J model

M. Punk and S. Sachdev, *Phys. Rev. B* **85**, 195123 (2012)

Characteristics of FL* phase

- Fermi surface volume does not count all electrons.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Characteristics of FL* phase

- Fermi surface volume does not count all electrons.
- Such a phase *must* have neutral $S = 1/2$ excitations (“spinons”), and collective spinless gauge excitations (“topological” order).

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Characteristics of FL* phase

- Fermi surface volume does not count all electrons.
- Such a phase *must* have neutral $S = 1/2$ excitations (“spinons”), and collective spinless gauge excitations (“topological” order).
- These topological excitations are needed to account for the deficit in the Fermi surface volume, in M. Oshikawa’s proof of the Luttinger theorem.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003)

Conclusions

Solved sign-problem for generic universal theory for the onset of antiferromagnetism in two-dimensional metals.

Good prospects for studying non-Fermi liquid physics at non-zero temperature

Conclusions

Pseudo-gap phase of hole-doped cuprates as a “fractionalized Fermi liquid”:

Hole pockets which violate the Luttinger theorem, in a phase with “topological” excitations.