

# The underdoped cuprates as fractionalized Fermi liquids (FL\*)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil,  
*Physical Review B* **75**, 235122 (2007)

R. K. Kaul, Y. B. Kim, S. Sachdev, and T. Senthil, *Nature Physics* **4**, 28 (2008)

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

Y. Qi and S. Sachdev, *Physical Review B* **81**, 115129 (2010)

E.G. Moon and S. Sachdev, arXiv:1010.xxxx

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)

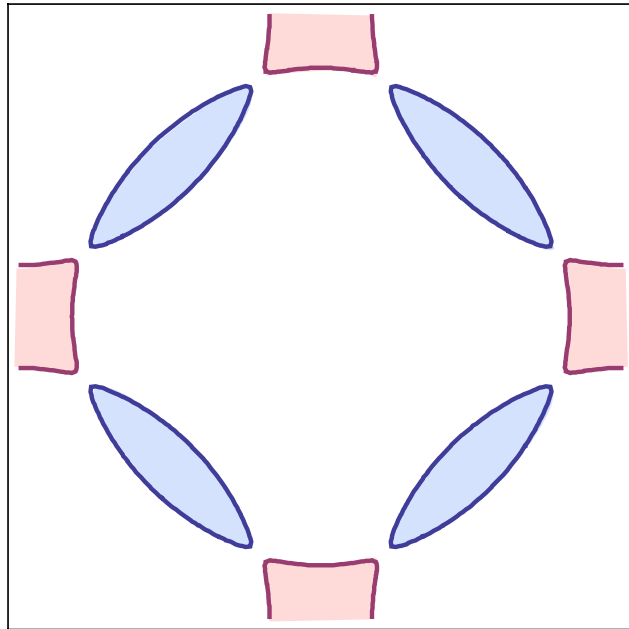
PHYSICS



HARVARD

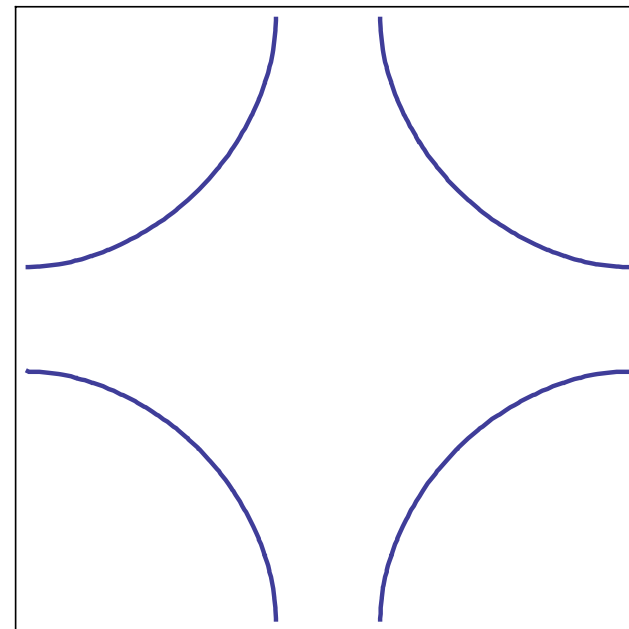
# Quantum criticality of the onset of antiferromagnetism in a metal

$$\langle \vec{\varphi} \rangle \neq 0$$



Metal with electron  
and hole pockets

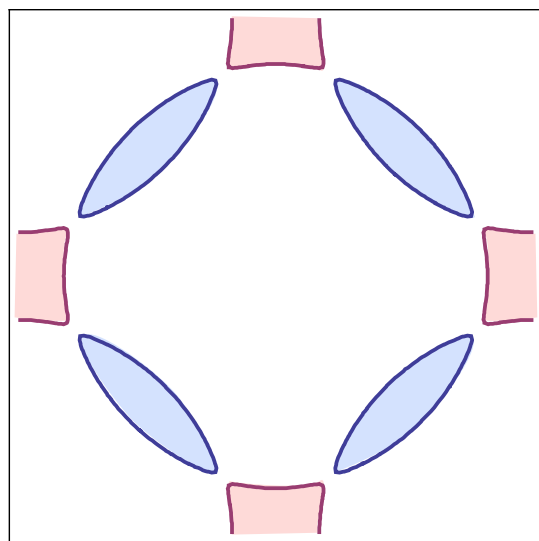
$$\langle \vec{\varphi} \rangle = 0$$



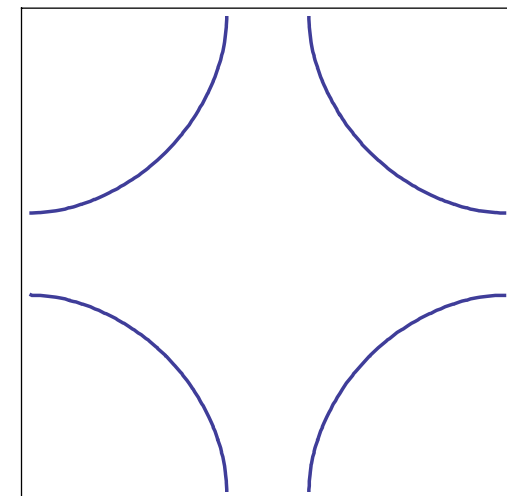
Metal with "large"  
Fermi surface

$S$

# SU(2) gauge theory: separating Fermi surface change from SDW order



SDW order  
small Fermi pockets



Fermi liquid  
large Fermi surface

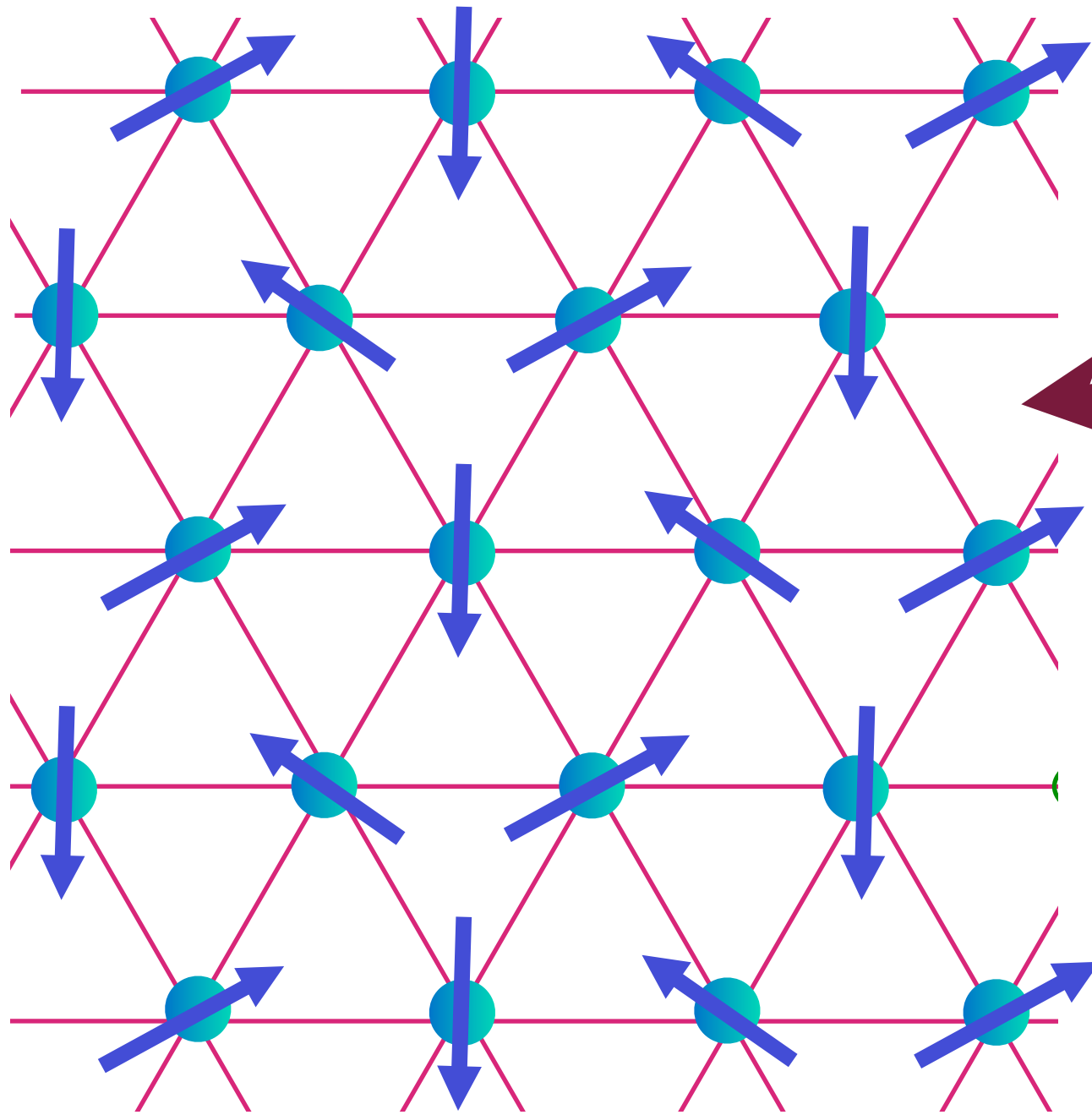
non-Fermi liquid  
Fermi pockets  
gapless U(1) photon

non-Fermi liquid  
large Fermi surface  
gapless SU(2) photons

Leads to a FL\* state: has Fermi pockets  
without translations symmetry breaking

S. Sachdev, M.A. Metlitski, Y. Qi, and C. Xu, *Physical Review B* **80**, 155129 (2009)

# Kondo lattice model



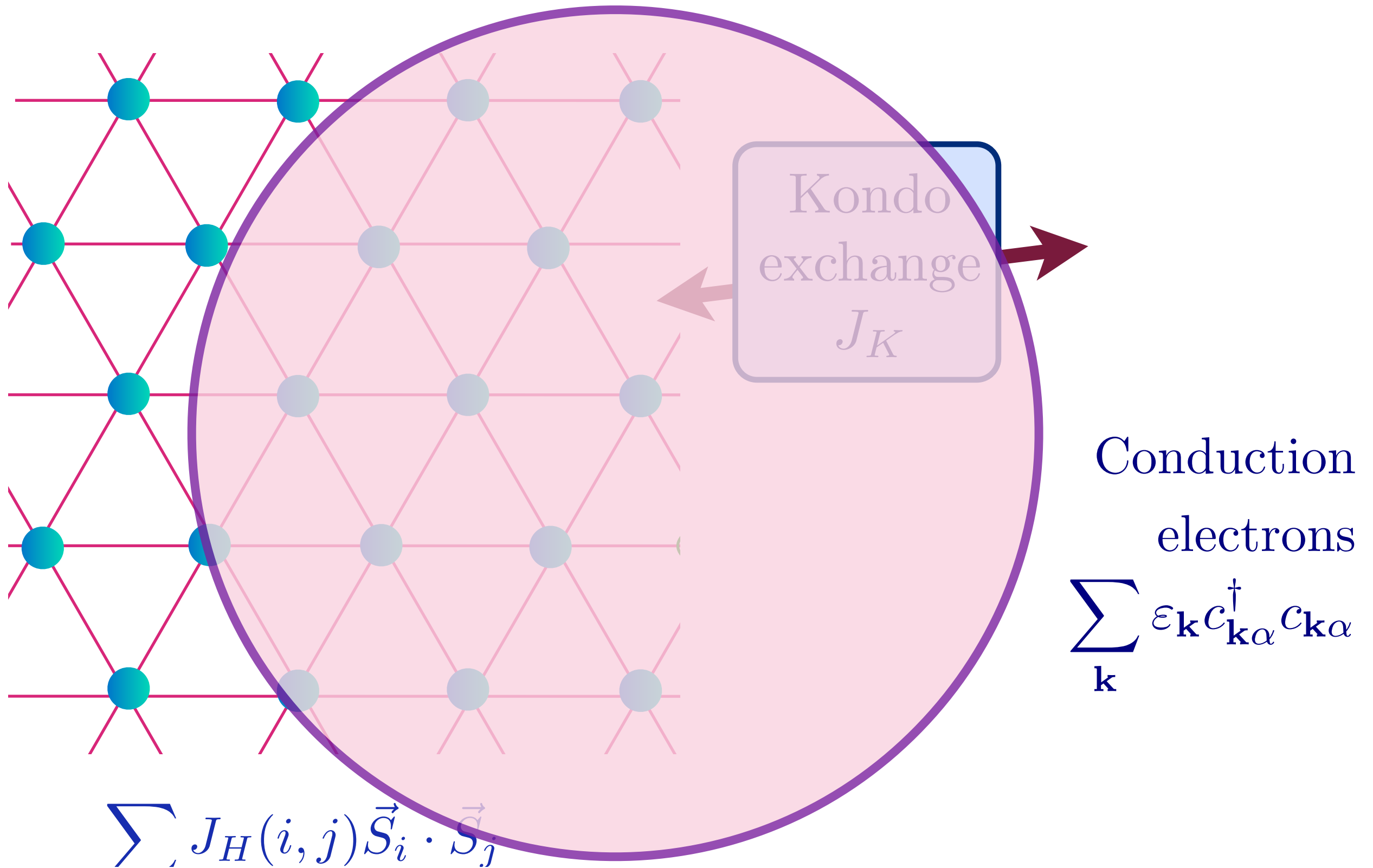
Kondo  
exchange  
 $J_K$

Conduction  
electrons

$$\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger c_{\mathbf{k}\alpha}$$

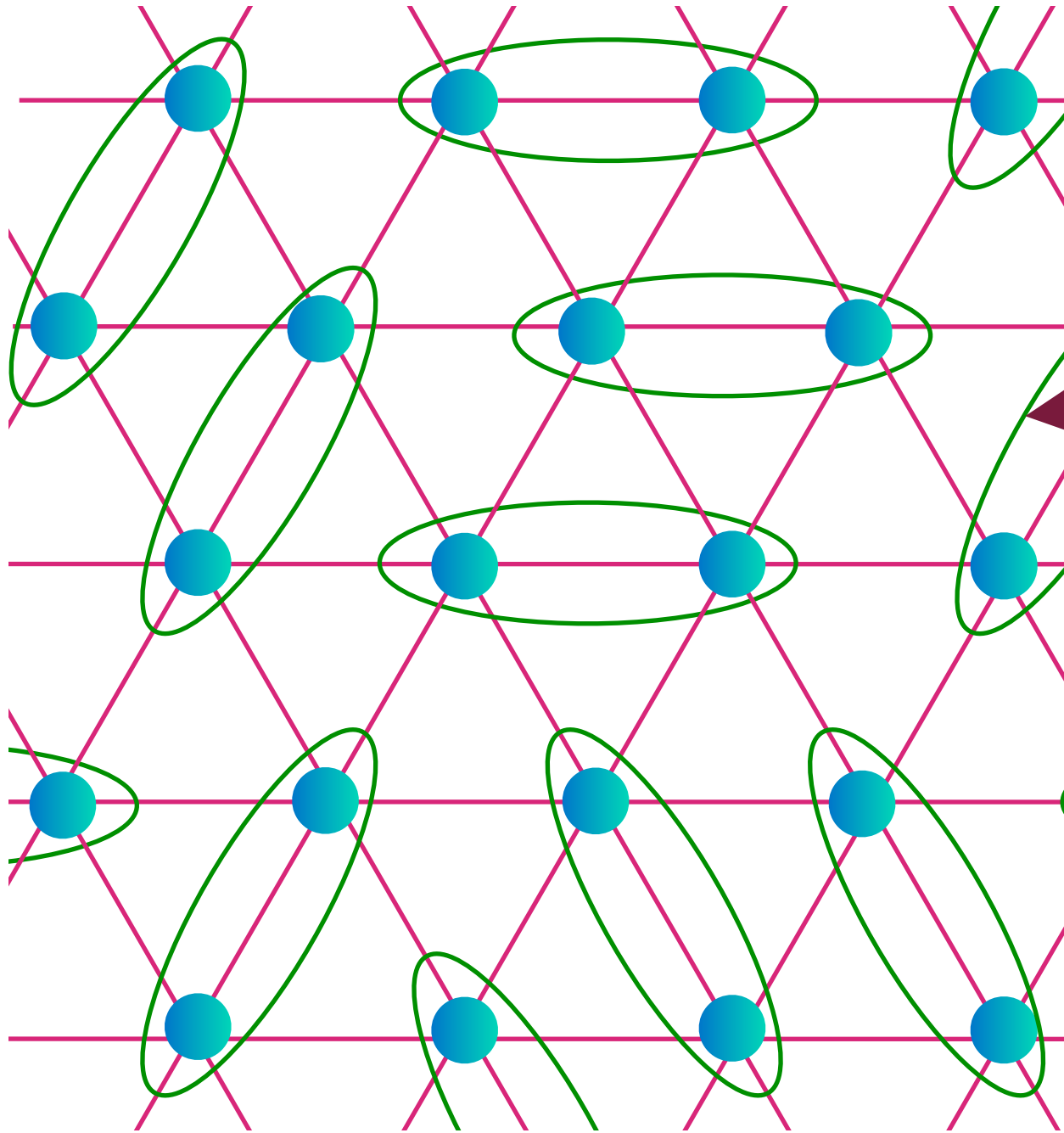
$$\sum_{i < j} J_H(i, j) \vec{S}_i \cdot \vec{S}_j$$

# Kondo lattice model



Large Fermi surface Fermi liquid (FL)

# Kondo lattice model



Kondo  
exchange  
 $J_K$



Conduction  
electrons

$$\sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

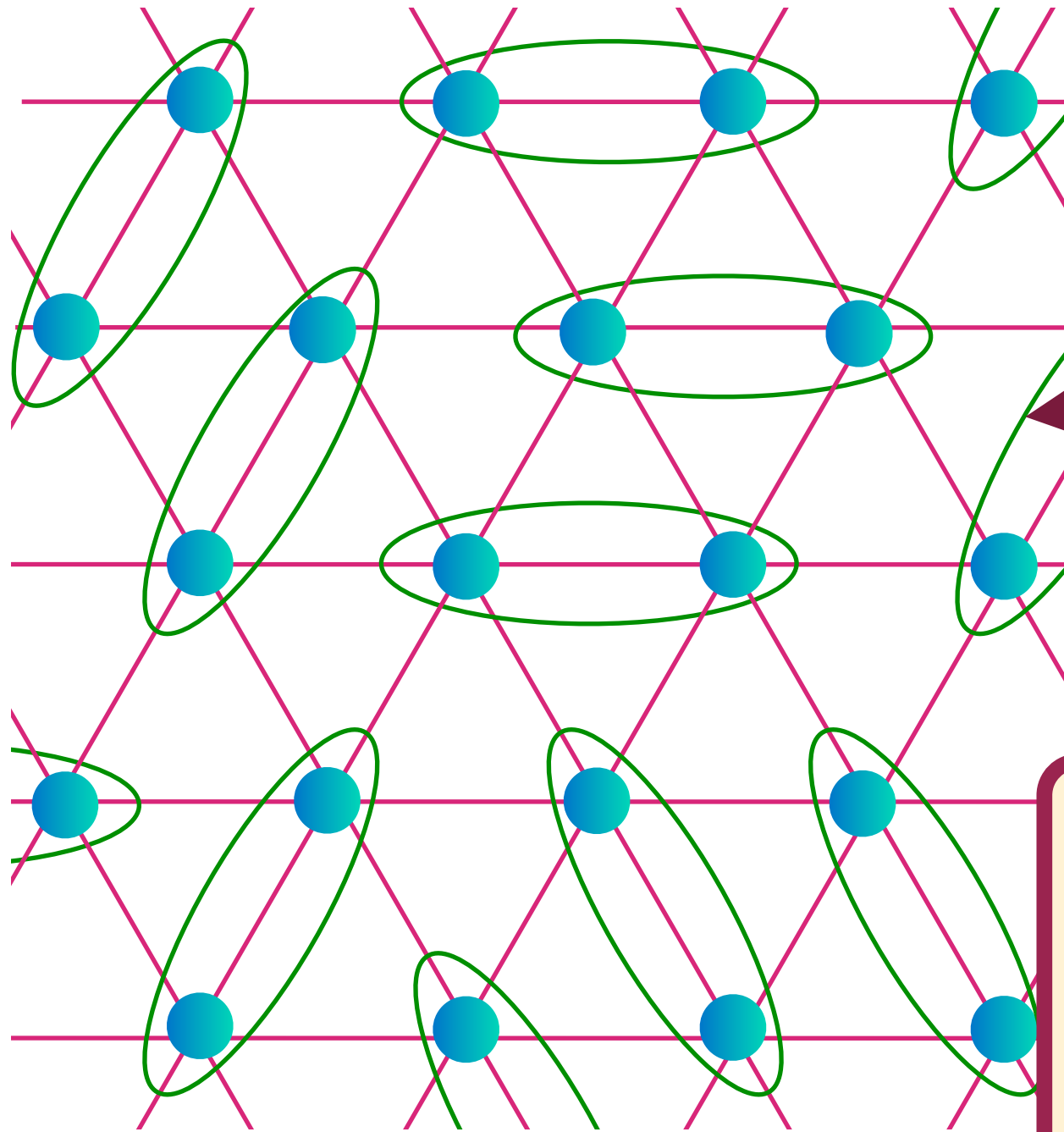
$$\sum_{i < j} J_H(i, j) \vec{S}_i \cdot \vec{S}_j$$

**Fractionalized  
Fermi liquid (FL\*)**

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).

# Kondo lattice model



Kondo  
exchange  
 $J_K$



Conduction

DMFT version:  
Orbitally Selective  
Mott Transition (OSMT):  
gauge fields of FL\*  
are missing

$$\sum_{i < j} J_H(i, j) \vec{S}_i \cdot \vec{S}_j$$

T. Senthil, S. Sachdev, and

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).

# FL\* from a one-band model of cuprates

Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order  $\hat{\varphi}$

$$\begin{pmatrix} c_{\uparrow} \\ c_{\downarrow} \end{pmatrix} = R \begin{pmatrix} \psi_{+} \\ \psi_{-} \end{pmatrix} ; \quad R^{\dagger} \hat{\varphi} \cdot \vec{\sigma} R = \sigma^z ; \quad R^{\dagger} R = 1$$

H. J. Schulz, *Physical Review Letters* **65**, 2462 (1990)

B. I. Shraiman and E. D. Siggia, *Physical Review Letters* **61**, 467 (1988).

J. R. Schrieffer, *Journal of Superconductivity* **17**, 539 (2004)



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$$\text{With } R = \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^{*} \\ z_{\downarrow} & z_{\uparrow}^{*} \end{pmatrix} \quad \text{or} \quad \hat{\varphi} = z_{\alpha}^{*} \vec{\sigma}_{\alpha\beta} z_{\beta}$$

we obtain a gauge theory of bosonic neutral spinons  $z_{\alpha}$ , spinless, charged fermions  $\psi_{\pm}$ , and an emergent gauge field.

# FL\* from a one-band model of cuprates

Begin with SDW ordered state, and rotate to a frame polarized along the local orientation of the SDW order  $\hat{\varphi}$

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$$\text{With } R = \begin{pmatrix} z_{\uparrow} & -z_{\downarrow}^* \\ z_{\downarrow} & z_{\uparrow}^* \end{pmatrix} \quad \text{or} \quad \hat{\varphi} = z_{\alpha}^* \vec{\sigma}_{\alpha\beta} z_{\beta}$$

we obtain a gauge theory of bosonic neutral spinons  $z_{\alpha}$ , spinless, charged fermions  $\psi_{\pm}$ , and an emergent gauge field.

These particles can bind into two species of gauge neutral, electron-like quasiparticles:

$$F_{i\alpha} \sim z_{i\alpha} \psi_{i+} \quad , \quad G_{i\alpha} \sim \varepsilon_{\alpha\beta} z_{i\beta}^* \psi_{i-} .$$

This species doubling is a key characteristic of the “topological order” of the underlying U(1) spin liquid.

# FL\* from a one-band model of cuprates

Use symmetry and physical arguments to constrain the effective Hamiltonian for the  $F_\alpha$  and  $G_\alpha$

$$\begin{aligned} H_{\text{eff}} &= - \sum_{ij} t_{ij} (F_{i\alpha}^\dagger F_{j\alpha} + G_{i\alpha}^\dagger G_{j\alpha}) \\ &+ \lambda \sum_i (-1)^{i_x + i_y} (F_{i\alpha}^\dagger F_{i\alpha} - G_{i\alpha}^\dagger G_{i\alpha}) \\ &- \sum_{i < j} \tilde{t}_{ij} (F_{i\alpha}^\dagger G_{j\alpha} + G_{i\alpha}^\dagger F_{j\alpha}) \\ &+ \Delta (\cos k_x - \cos k_y) \text{ pairing in the superconductor} \end{aligned}$$

# FL\* from a one-band model of cuprates

Use symmetry and physical arguments to constrain the effective Hamiltonian for the  $F_\alpha$  and  $G_\alpha$


$$\begin{aligned} H_{\text{eff}} &= - \sum_{ij} t_{ij} (F_{i\alpha}^\dagger F_{j\alpha} + G_{i\alpha}^\dagger G_{j\alpha}) \\ &+ \lambda \sum_i (-1)^{i_x + i_y} \left( \text{Underlying band structure} \right) \\ &- \sum_{i < j} \tilde{t}_{ij} (F_{i\alpha}^\dagger G_{j\alpha} + G_{i\alpha}^\dagger F_{j\alpha}) \\ &+ \Delta (\cos k_x - \cos k_y) \text{ pairing in the superconductor} \end{aligned}$$

# FL\* from a one-band model of cuprates

Use symmetry and physical arguments to constrain the effective Hamiltonian for the  $F_\alpha$  and  $G_\alpha$

$$\begin{aligned} H_{\text{eff}} = & - \sum_{ij} t_{ij} (F_{i\alpha}^\dagger F_{j\alpha} + \\ & + \lambda \sum_i (-1)^{i_x+i_y} (F_{i\alpha}^\dagger F_{i\alpha} - G_{i\alpha}^\dagger G_{i\alpha}) \\ & - \sum_{i<j} \tilde{t}_{ij} (F_{i\alpha}^\dagger G_{j\alpha} + G_{i\alpha}^\dagger F_{j\alpha}) \\ & + \Delta(\cos k_x - \cos k_y) \text{ pairing in the superconductor} \end{aligned}$$

**Potential from local antiferromagnetism**



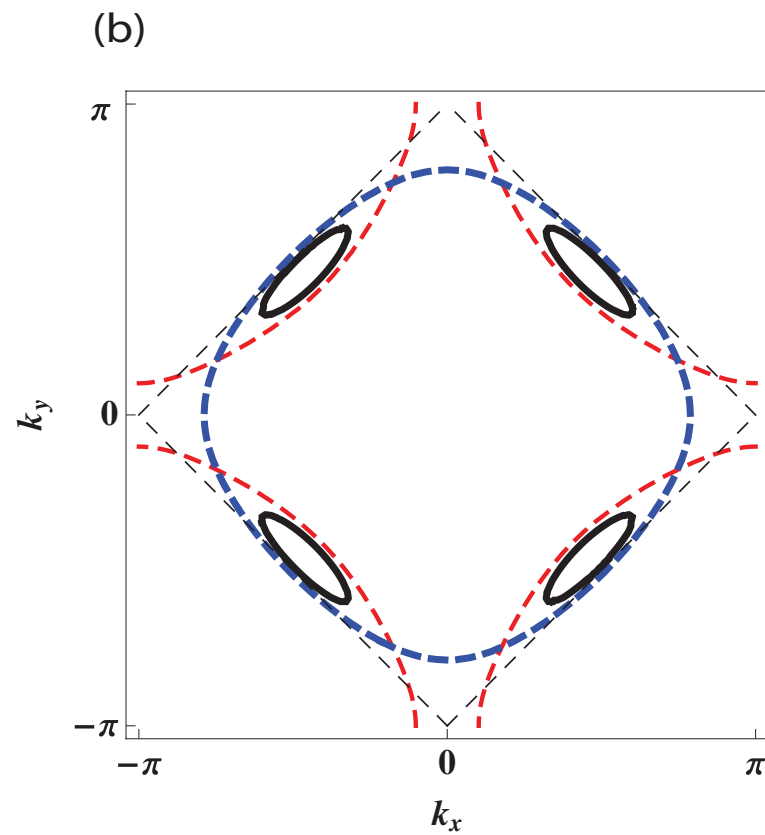
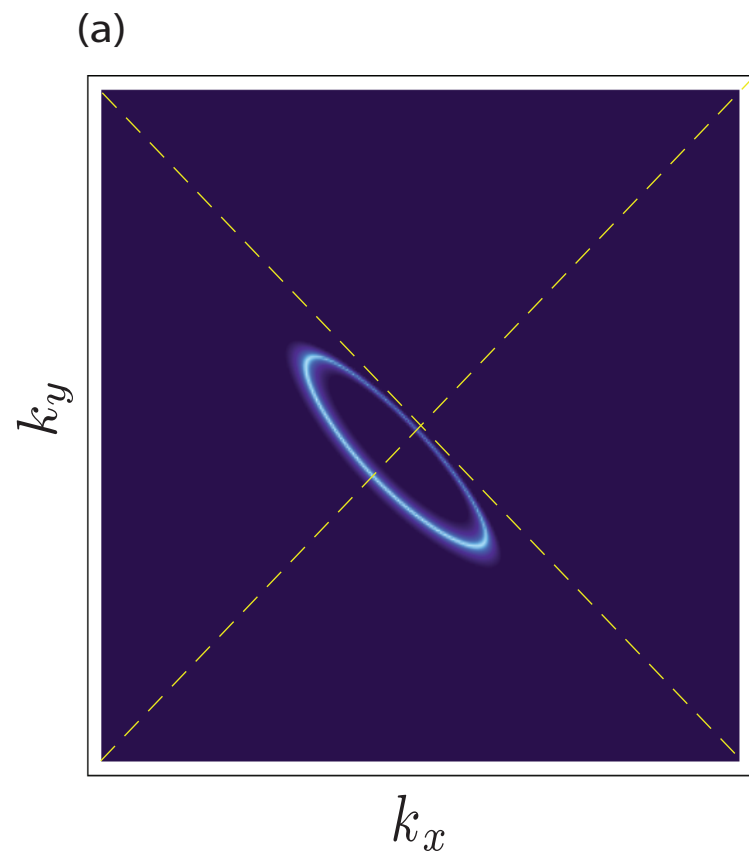
# FL\* from a one-band model of cuprates

Use symmetry and physical arguments to constrain the effective Hamiltonian for the  $F_\alpha$  and  $G_\alpha$

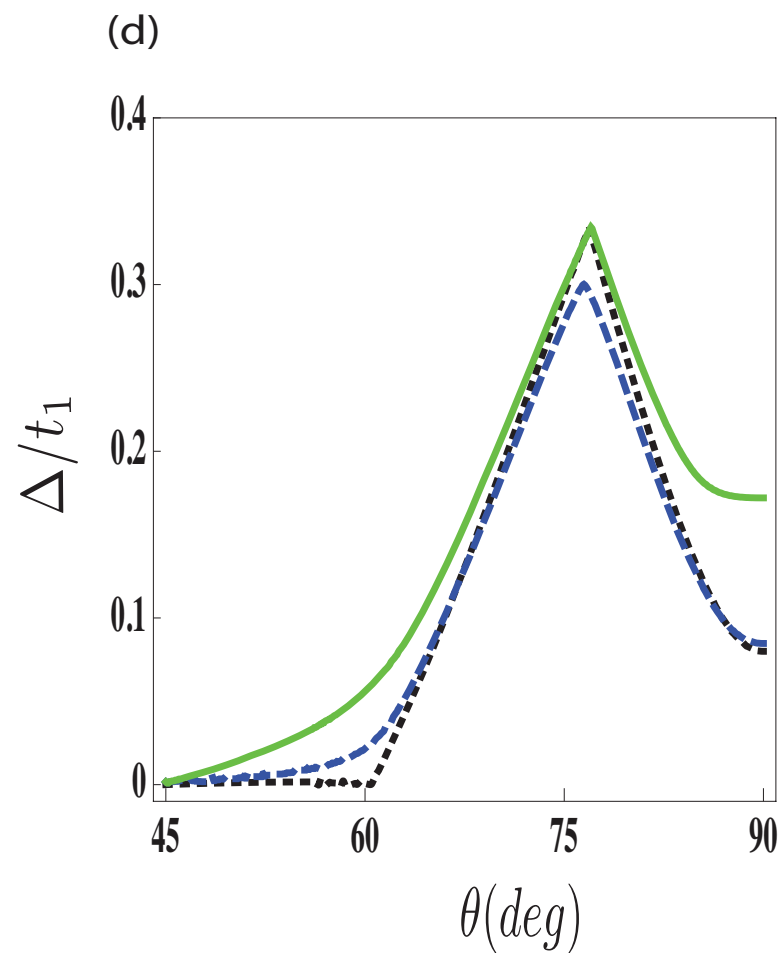
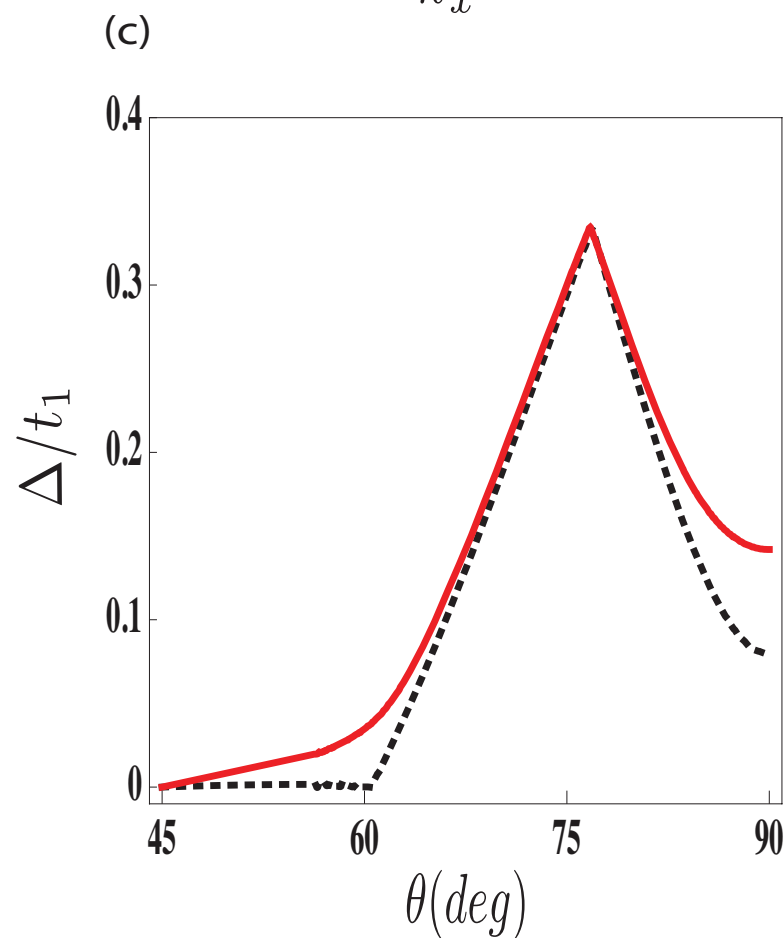
$$\begin{aligned} H_{\text{eff}} = & - \sum_{ij} t_{ij} (F_{i\alpha}^\dagger F_{j\alpha} \\ & + \lambda \sum_i (-1)^{i_x + i_y} (F_{i\alpha}^\dagger G_{i\alpha} \\ & - \sum_{i < j} \tilde{t}_{ij} (F_{i\alpha}^\dagger G_{j\alpha} + G_{i\alpha}^\dagger F_{j\alpha}) \\ & + \Delta (\cos k_x - \cos k_y) \text{ pairing in the superconductor} \end{aligned}$$

Mixing between 2 species:  
descends from the  
Shraiman-Siggia term

# Weaker local antiferromagnetism



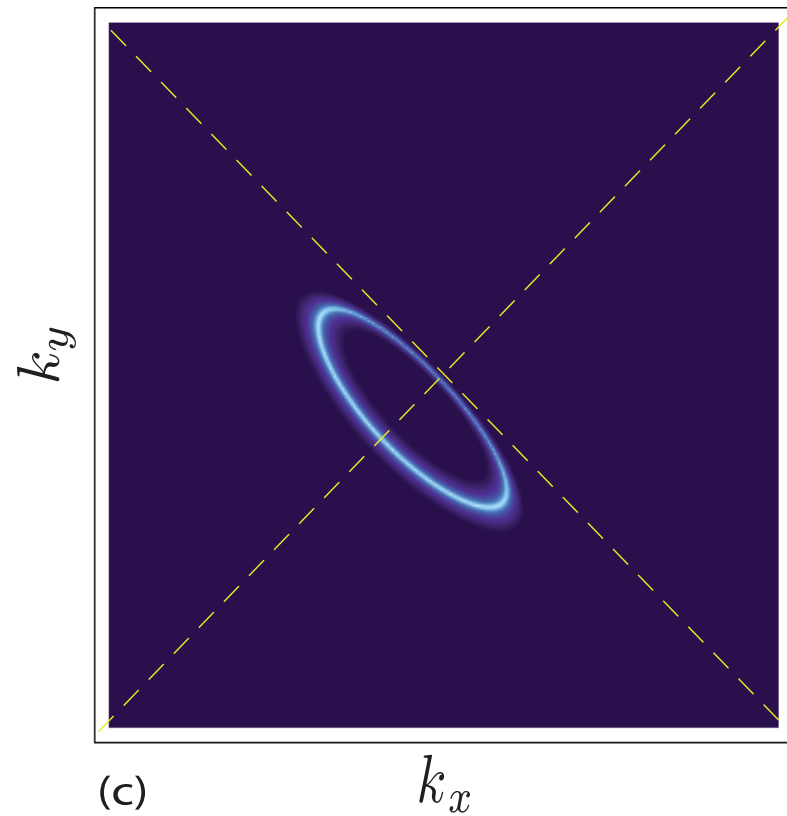
Normal state  
electron spectrum



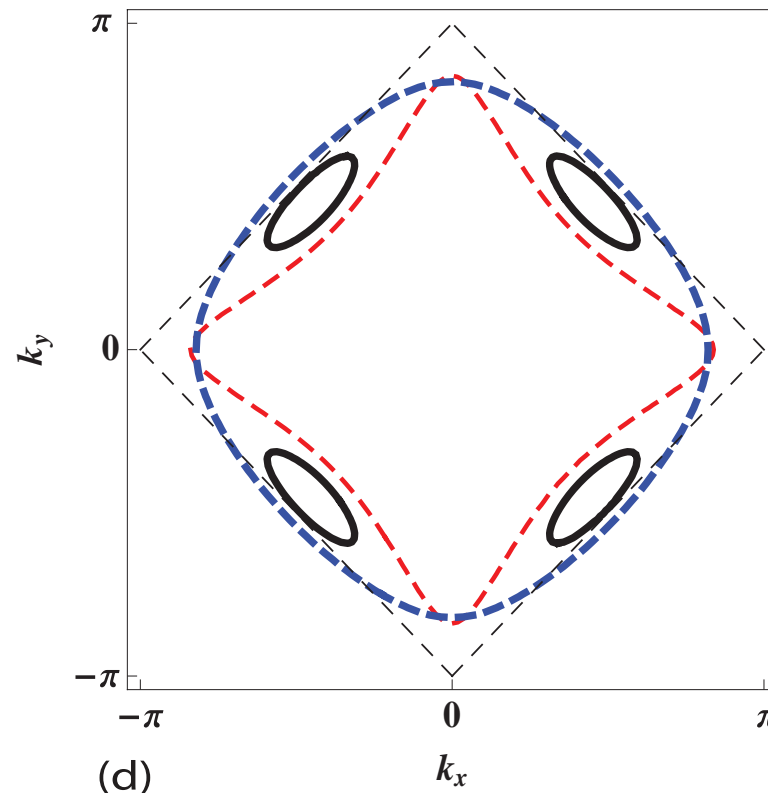
Angular  
dependence of  
electronic gap in  
normal state and  
in superconductor

# Stronger local antiferromagnetism

(a)

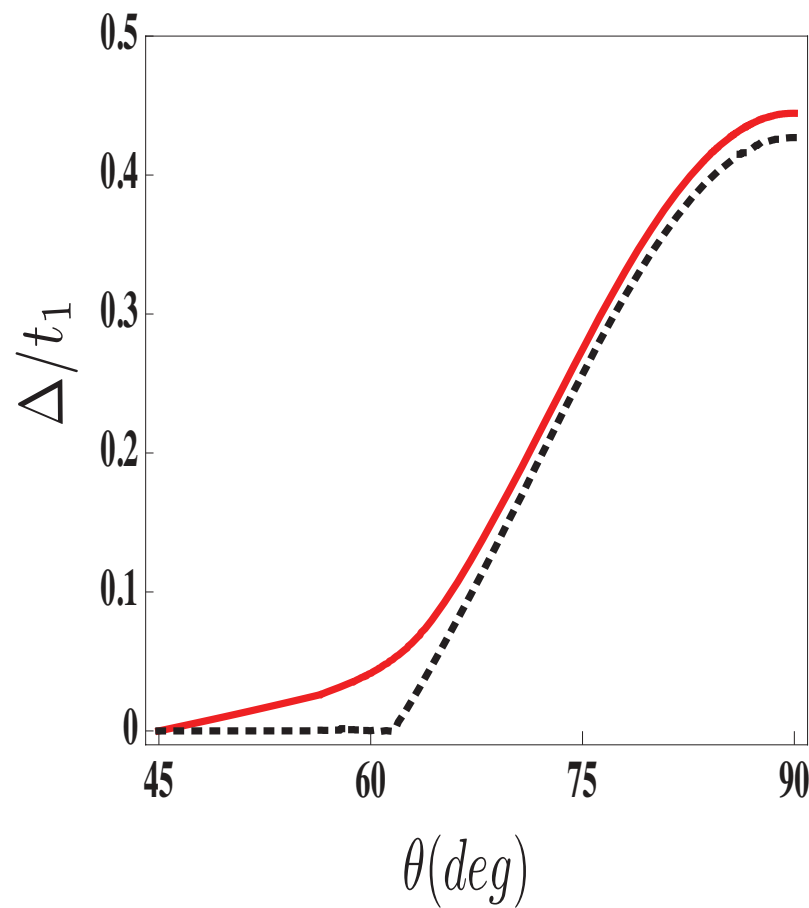


(b)

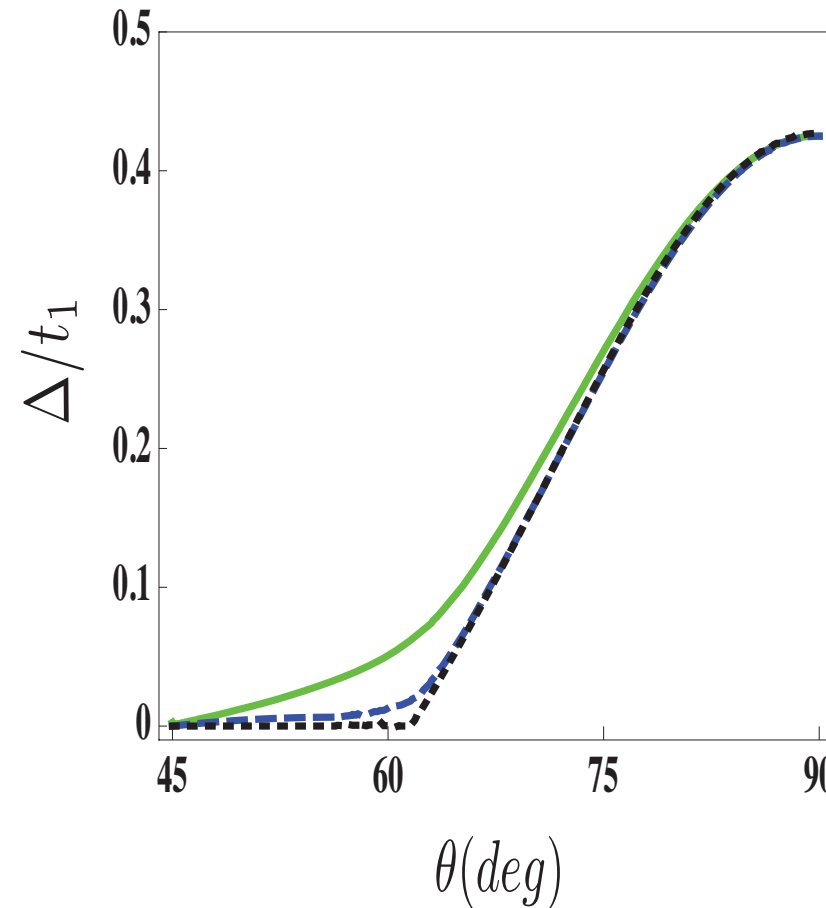


Normal state electron spectrum

(c)



(d)



Angular dependence of electronic gap in normal state and in superconductor



# Normal state with electron pockets

