

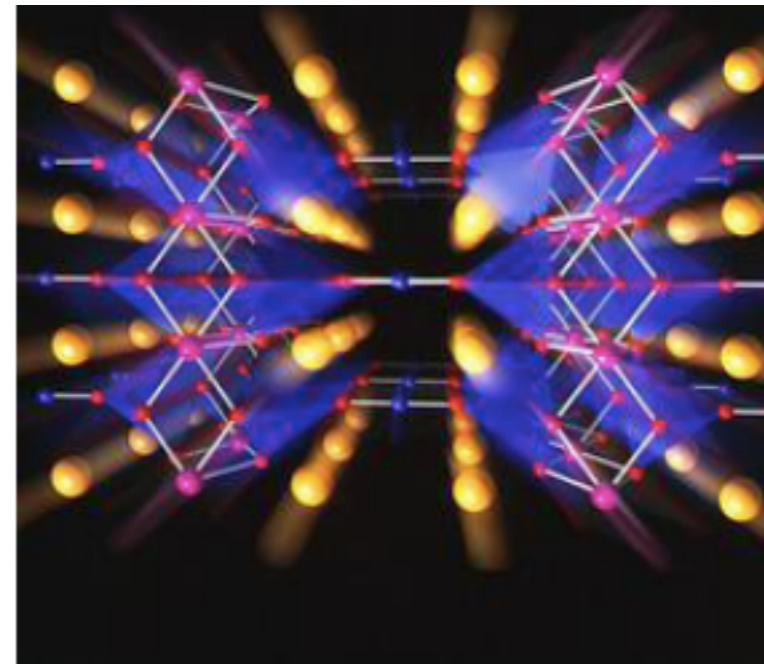
# Superconductors come of age

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**A South Korean company has placed by far the biggest commercial order for superconducting wires.**



Superconducting wires could soon help to light up Seoul.



YBCO superconductors are likely to be used in more power grids in future.

LS Cable, a South Korean company based in Anyang-si near Seoul, has ordered three million metres of superconducting wire from US firm American Superconductor in Devens, Massachusetts. Jason Fredette, managing director of corporate communications at the company, says that LS Cable will use the wire to make about 20 circuit kilometres of cable as part of a programme to modernize the South Korean electricity network starting in the capital, Seoul.

The superconducting wire is made using the ceramic compound yttrium barium copper oxide (YBCO), part of a family of 'high-temperature' superconducting ceramics that were first discovered in 1986.

# The onset of spin density wave order in metals

Talk online: [sachdev.physics.harvard.edu](http://sachdev.physics.harvard.edu)

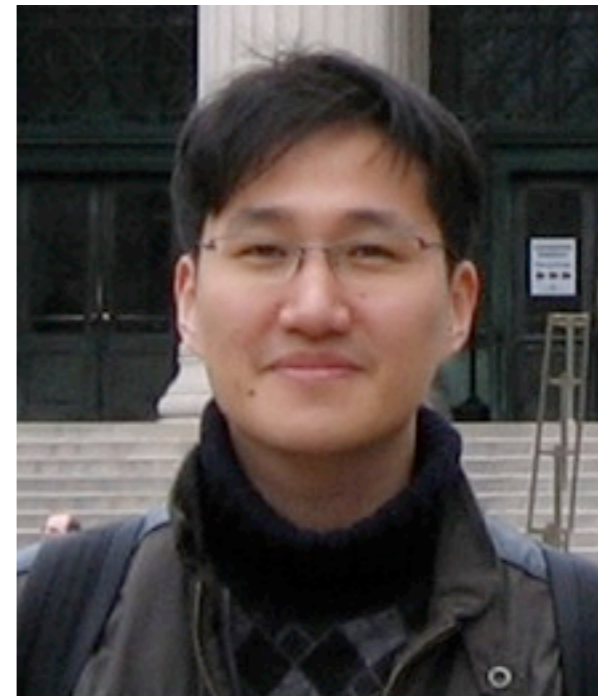
PHYSICS



HARVARD



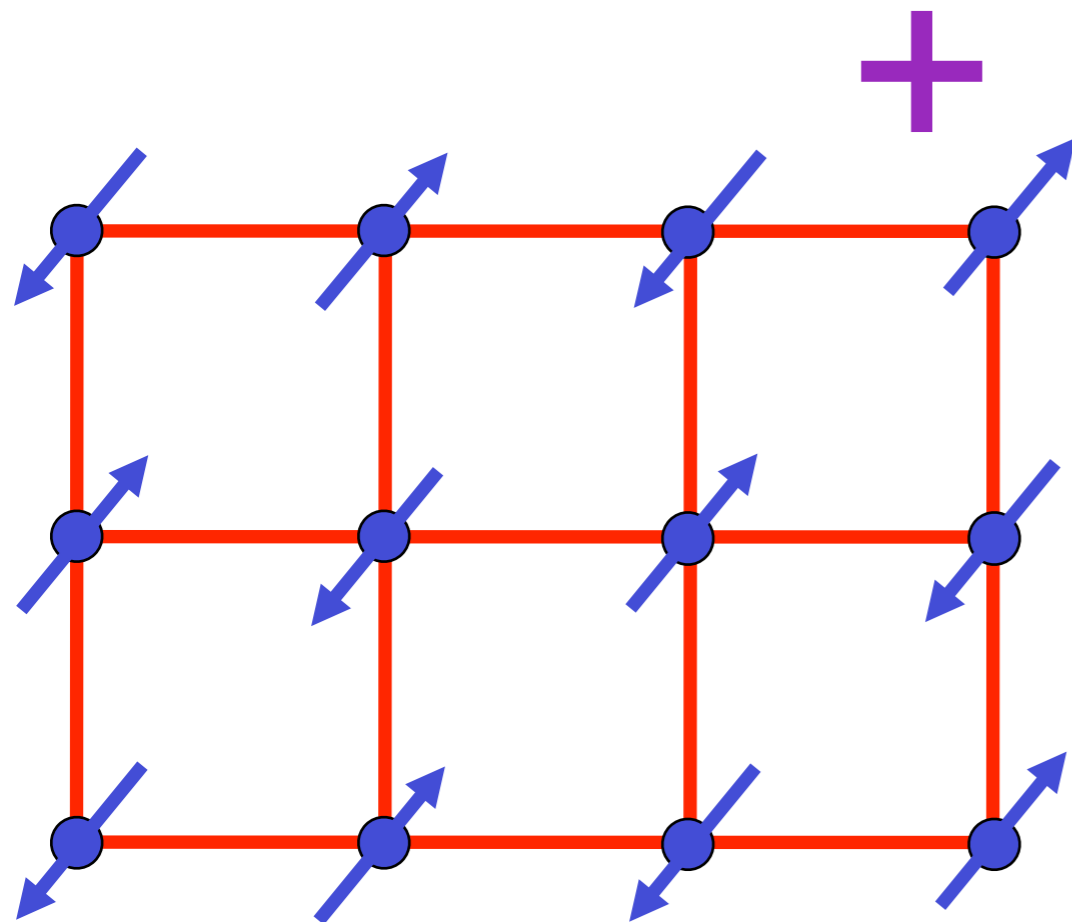
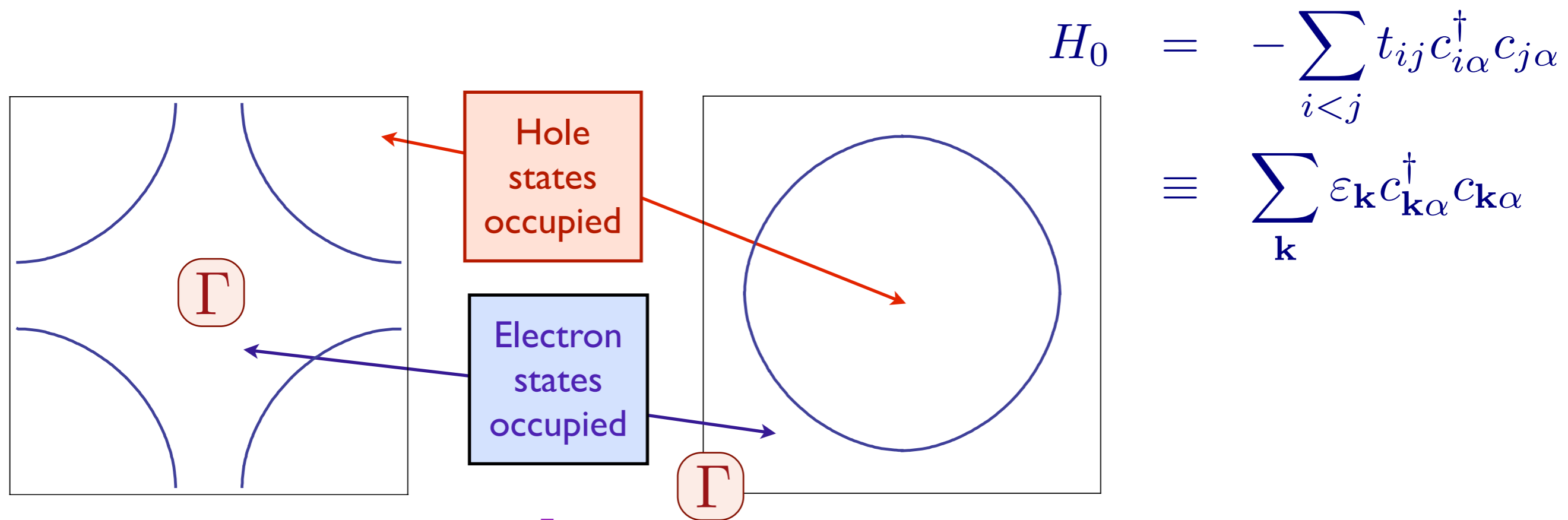
Max Metlitski, Harvard



Eun Gook Moon, Harvard



# Fermi surface+antiferromagnetism



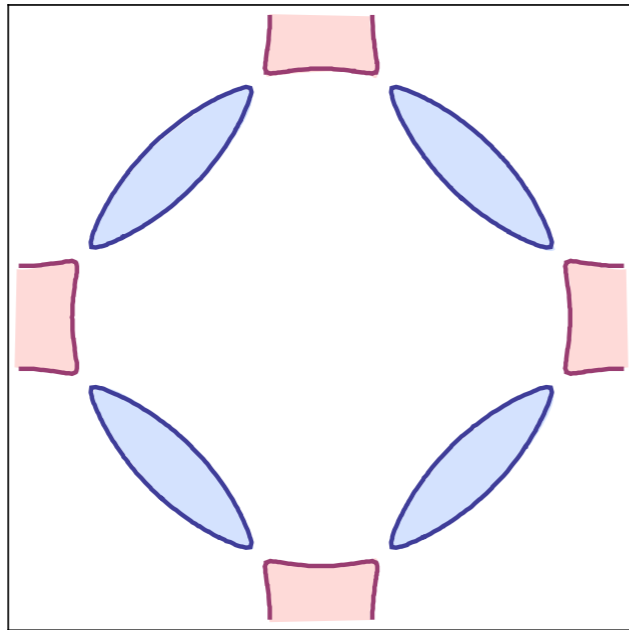
The electron spin polarization obeys

$$\langle \vec{S}(\mathbf{r}, \tau) \rangle = \vec{\varphi}(\mathbf{r}, \tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where  $\mathbf{K}$  is the ordering wavevector.

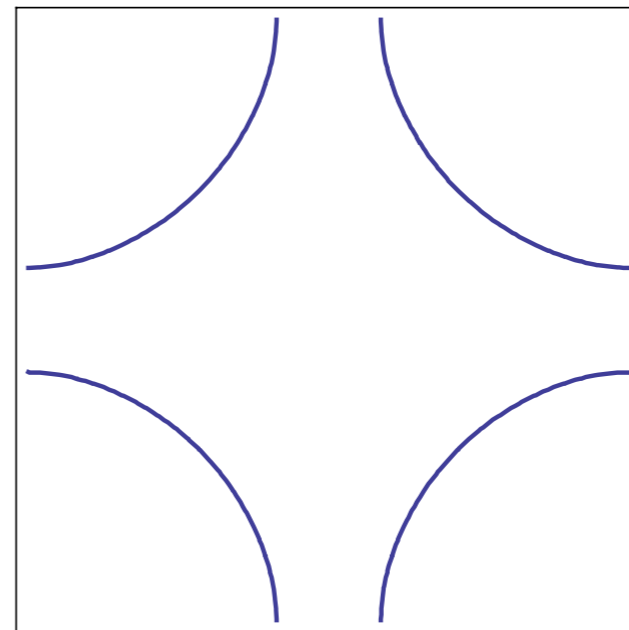
# Quantum criticality of the onset of antiferromagnetism in a metal

$$\langle \vec{\varphi} \rangle \neq 0$$



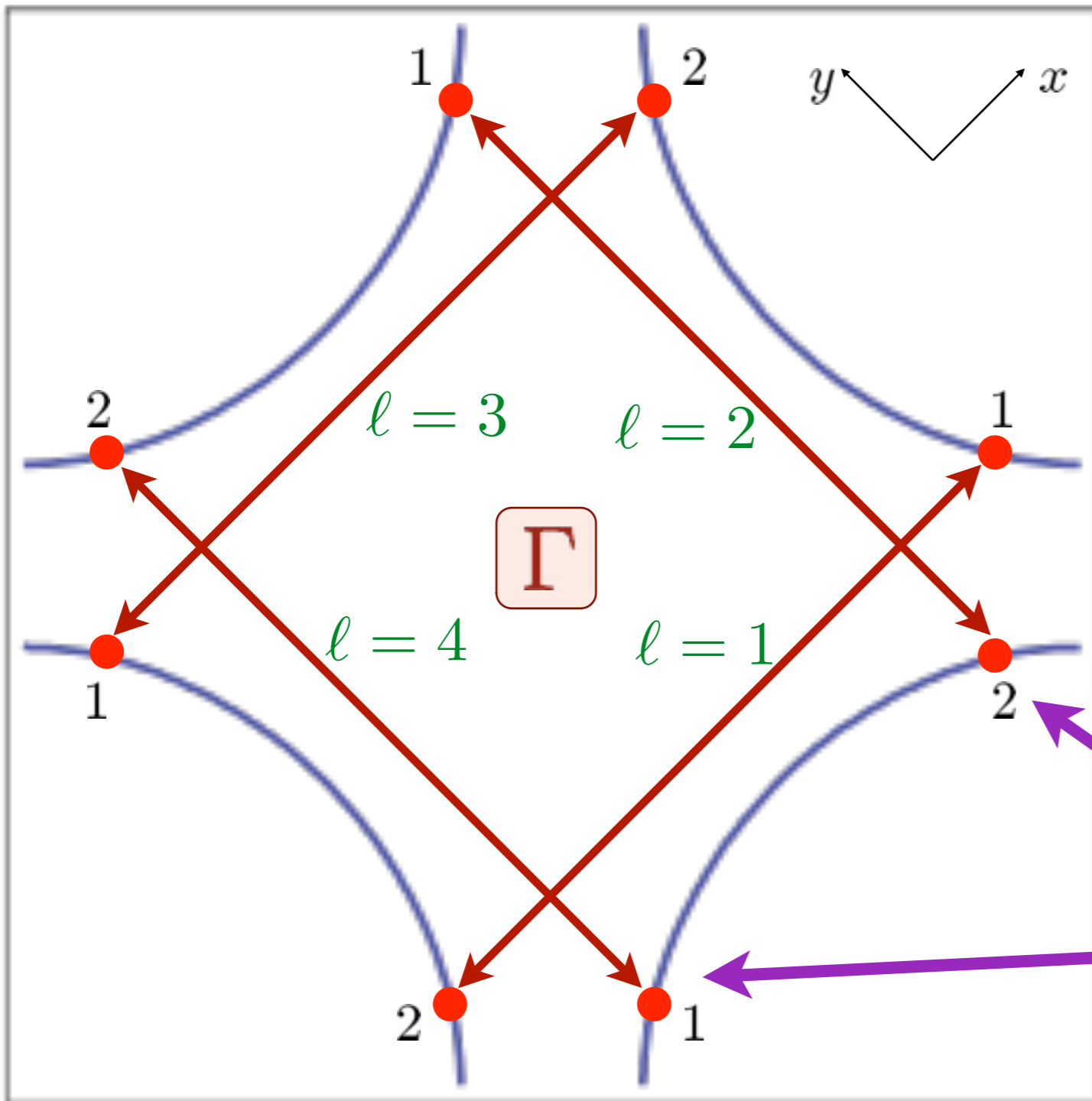
Metal with electron  
and hole pockets

$$\langle \vec{\varphi} \rangle = 0$$



Metal with "large"  
Fermi surface

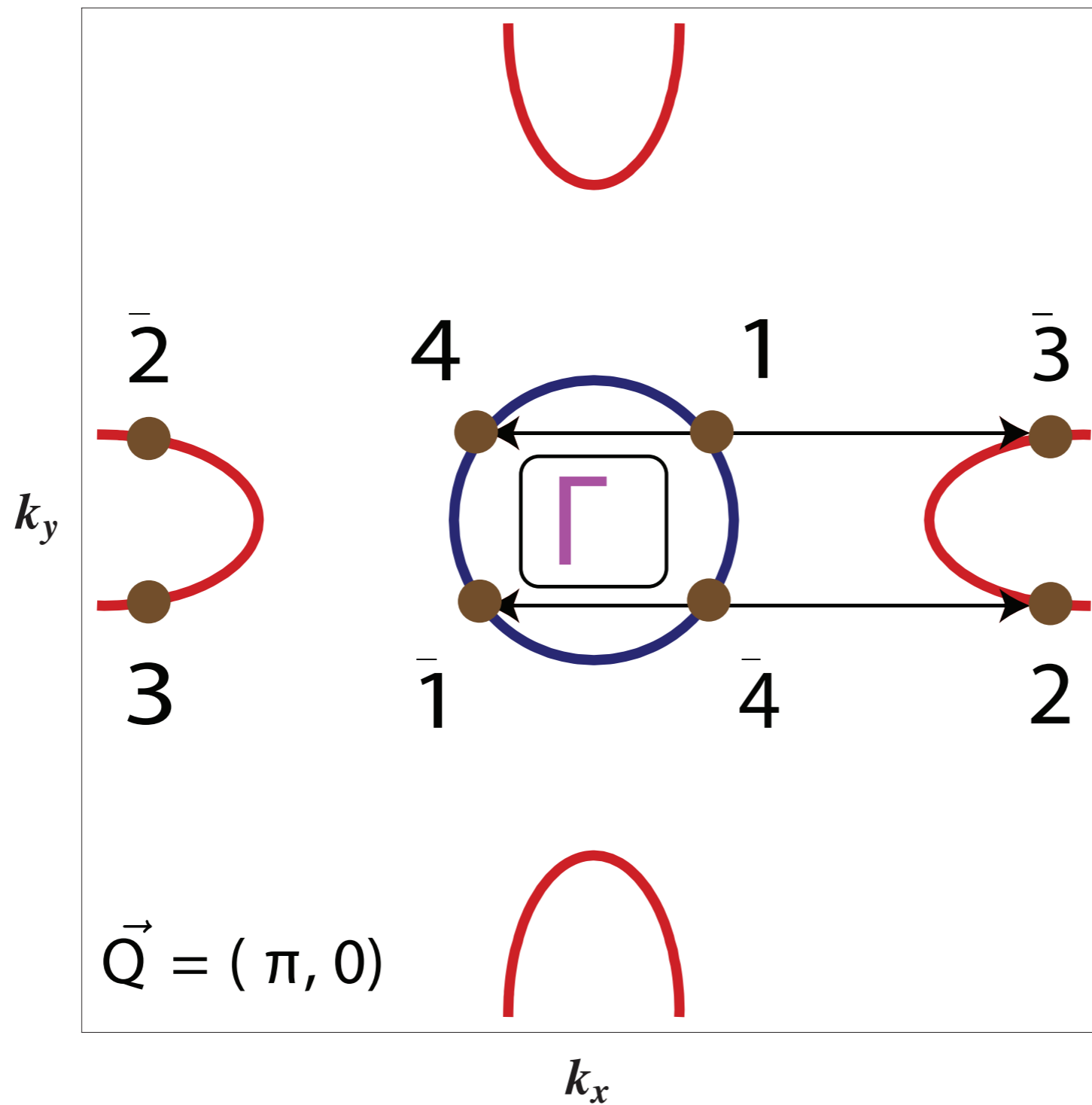
$S$



Low energy fermions  
at hot spots  $\mathbf{k} = \mathbf{k}_\ell$ :  
 $\psi_{1\alpha}^\ell, \psi_{2\alpha}^\ell$   
 $\ell = 1, \dots, 4.$   
 with  $c_{\mathbf{k}+\mathbf{k}_\ell} = \psi^\ell(\mathbf{k})$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

$$\mathbf{v}_1^{\ell=1} = (v_x, v_y), \quad \mathbf{v}_2^{\ell=1} = (-v_x, v_y)$$



Similar theory applies to the pnictides, and leads to  $s_{\pm}$  pairing.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter:  $\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$

“Yukawa” coupling:  $\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left( \psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$



## Results of RG analysis at 2+ loops

- The Hertz-Millis-Moriya procedure is valid in  $d = 3$ , but breaks down strongly in  $d = 2$ . (*cf.* Abanov-Chubukov)

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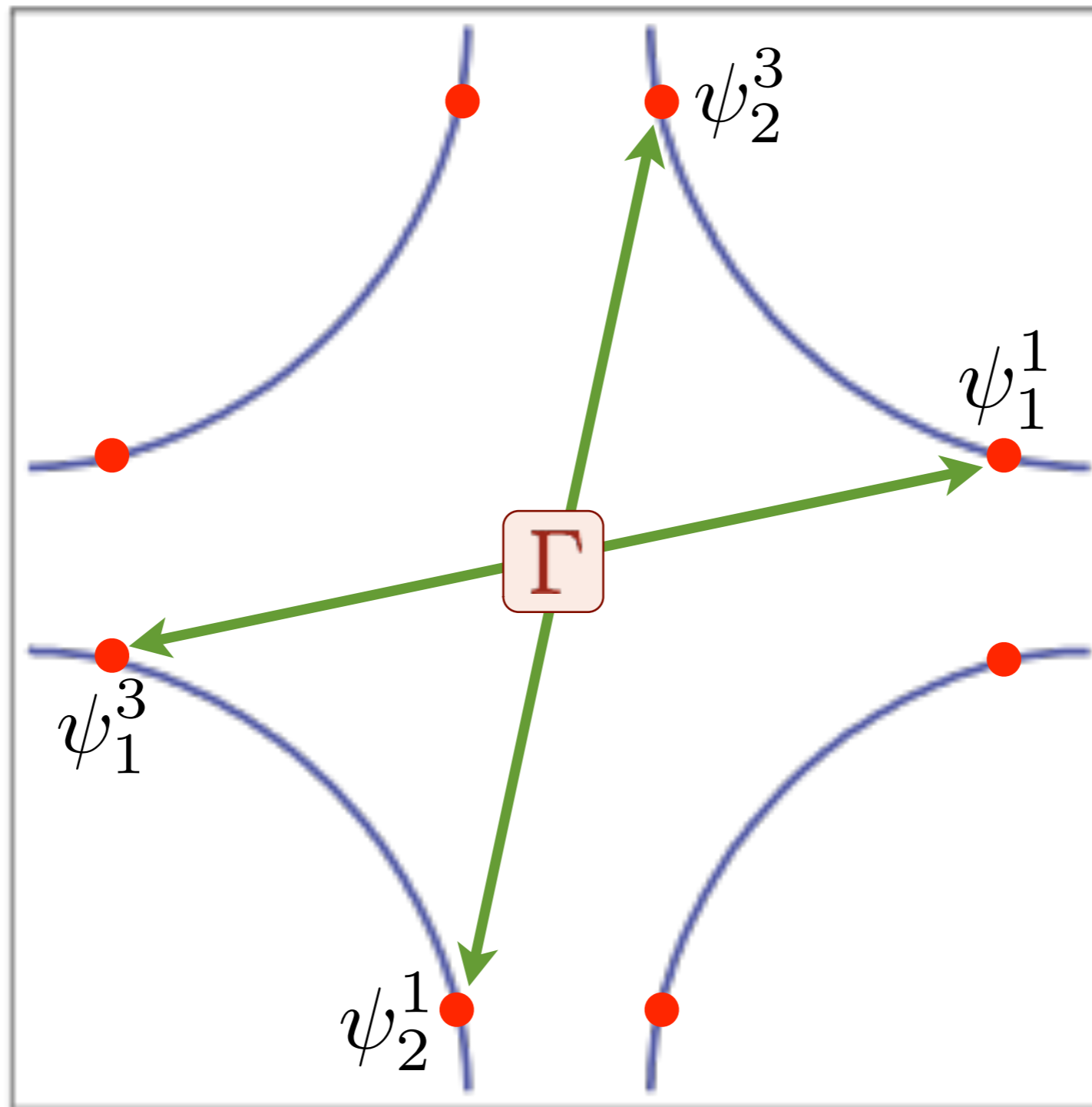
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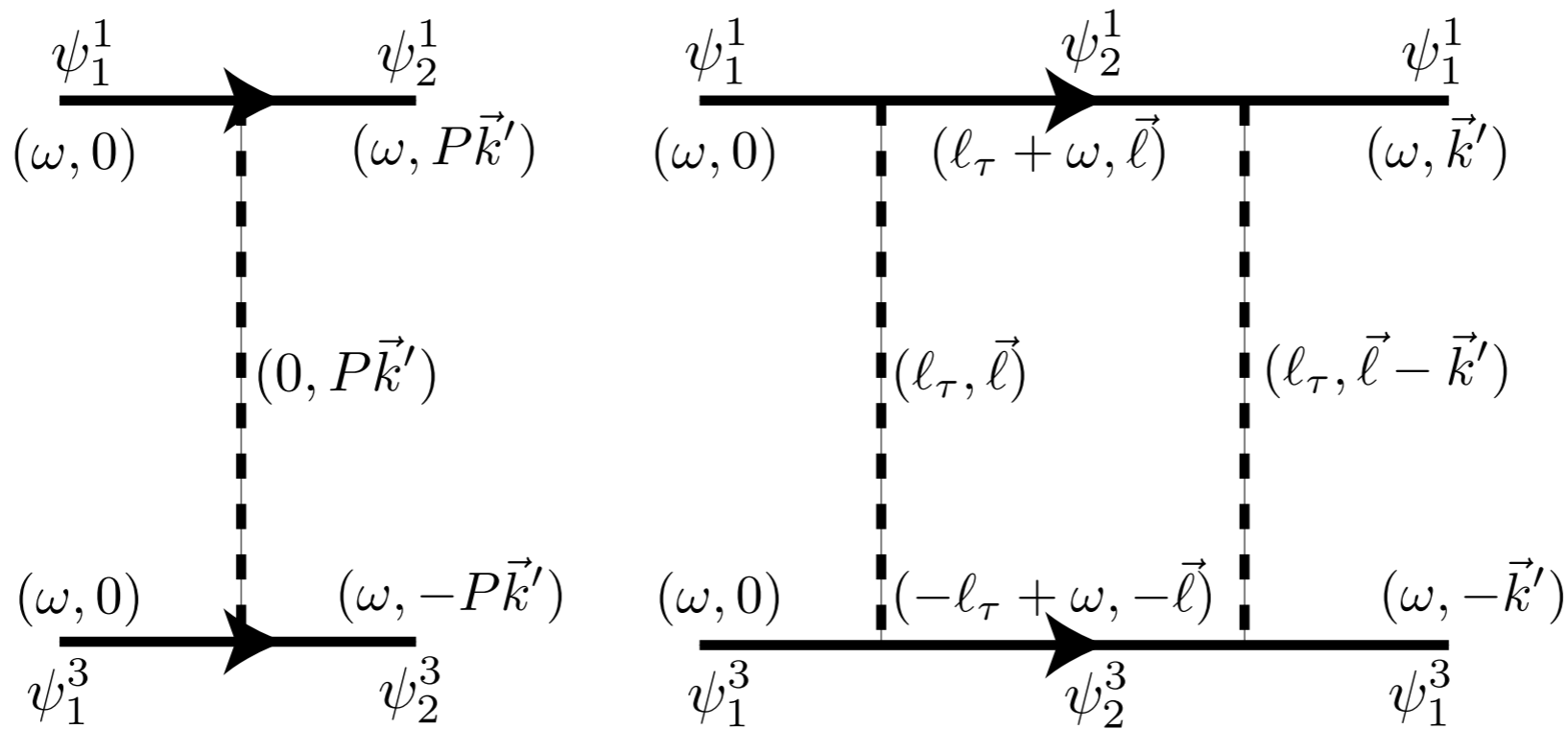
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- There is a universal “log-squared” instability to unconventional (*i.e.*  $d$ -wave like) superconductivity with a coupling of order unity.

# *d*-wave pairing



Pairing order parameter:  $\varepsilon^{\alpha\beta} \left( \psi_{1\alpha}^3 \psi_{1\beta}^1 - \psi_{2\alpha}^3 \psi_{2\beta}^1 \right)$



Need fermion Green's functions on Fermi surface near hot spots:

$$G(\omega, \vec{p}) \sim \frac{\mathcal{Z}(p_{\parallel})}{i\omega - v_F(p_{\parallel})p_{\perp}}.$$

Near the hot spot we have  $v_F \sim \mathcal{Z} \sim p_{\parallel}$ . The pairing interaction is enhanced at one loop by the factor

$$\left[ 1 + \frac{\alpha}{\pi(\alpha^2 + 1)} \log^2 \frac{\vec{k}'^2}{\gamma|\omega|} \right]$$

where  $\alpha = v_y/v_x$  is of order unity.

M. A. Metlitski and S. Sachdev,  
*Physical Review B* **82**, 075127 (2010)

## Results of RG analysis at 2+ loops

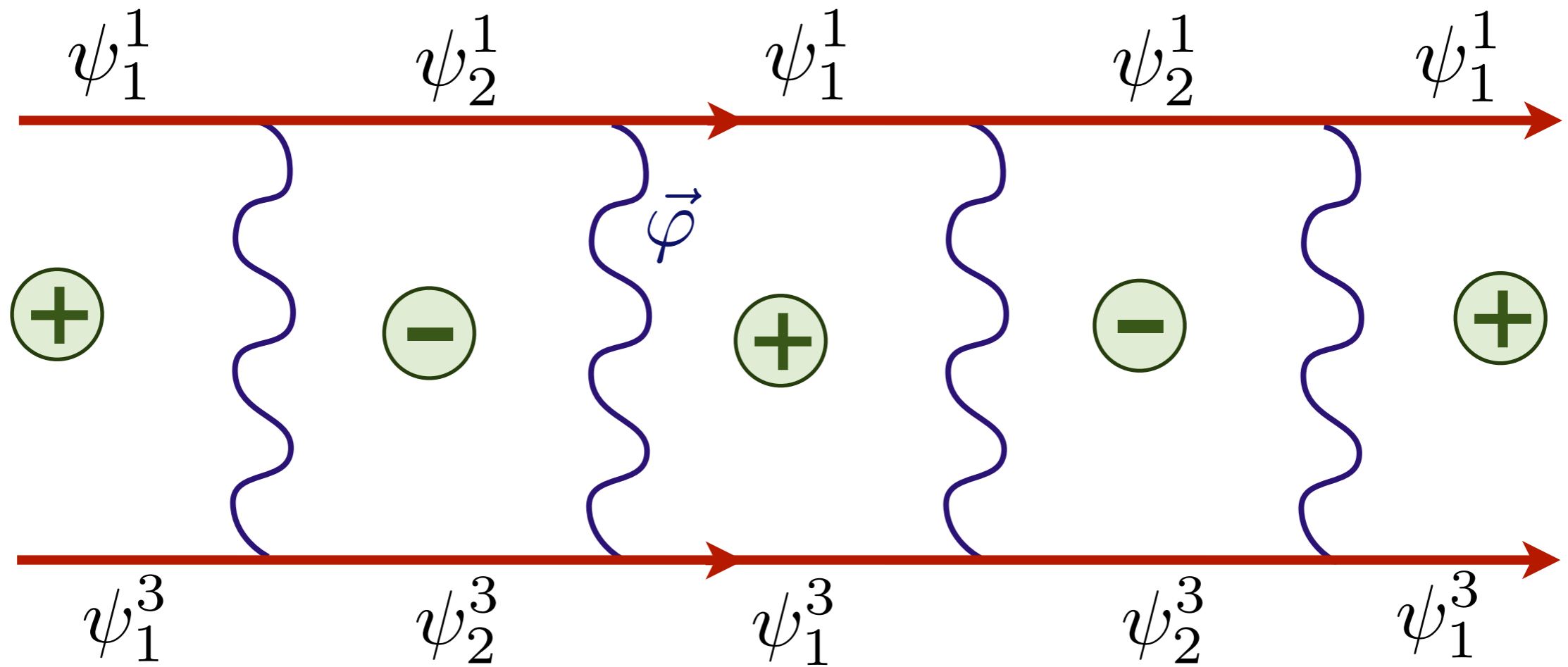
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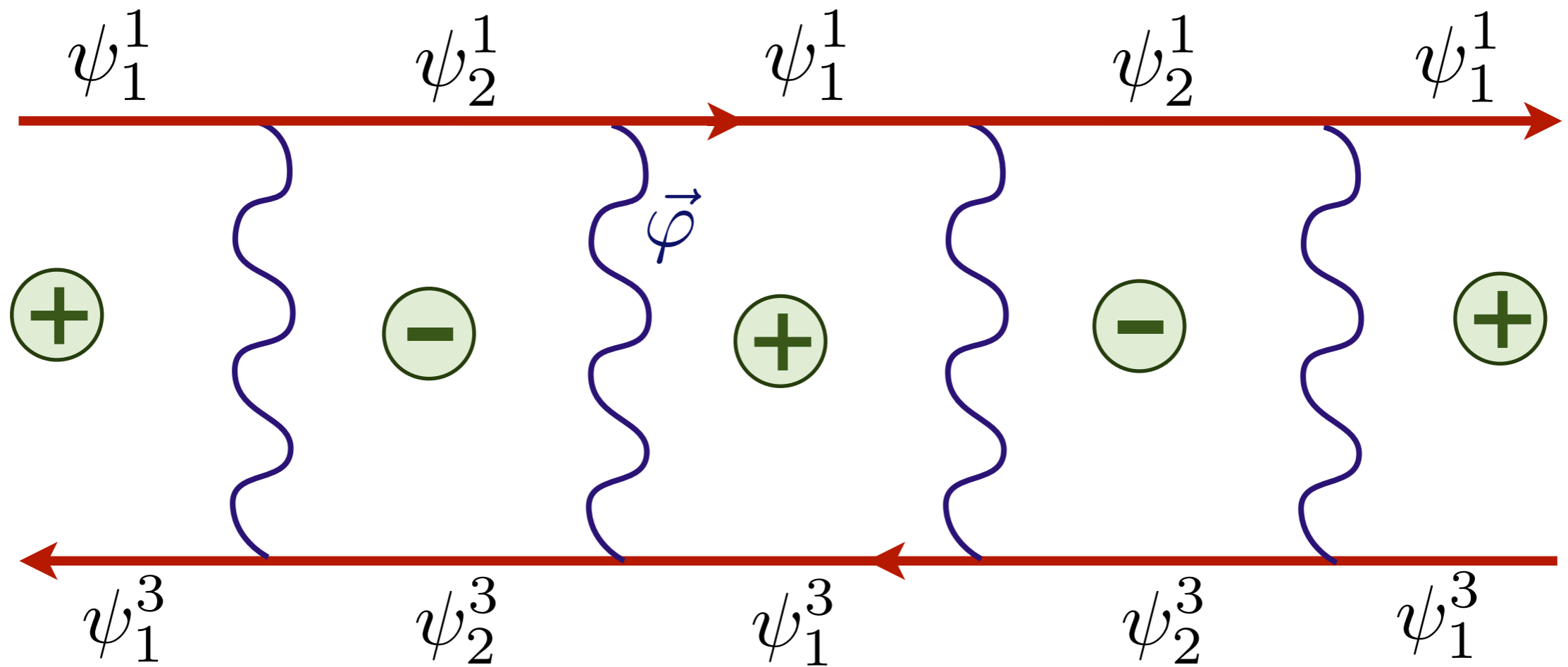
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- There is a universal “log-squared” instability to unconventional (*i.e.*  $d$ -wave like) superconductivity with a coupling of order unity.
- There is a sub-dominant “log-squared” instability to a modulated bond order, which locally has a Ising-nematic character.

M. A. Metlitski and S. Sachdev,  
*Physical Review B* **82**, 075127 (2010)



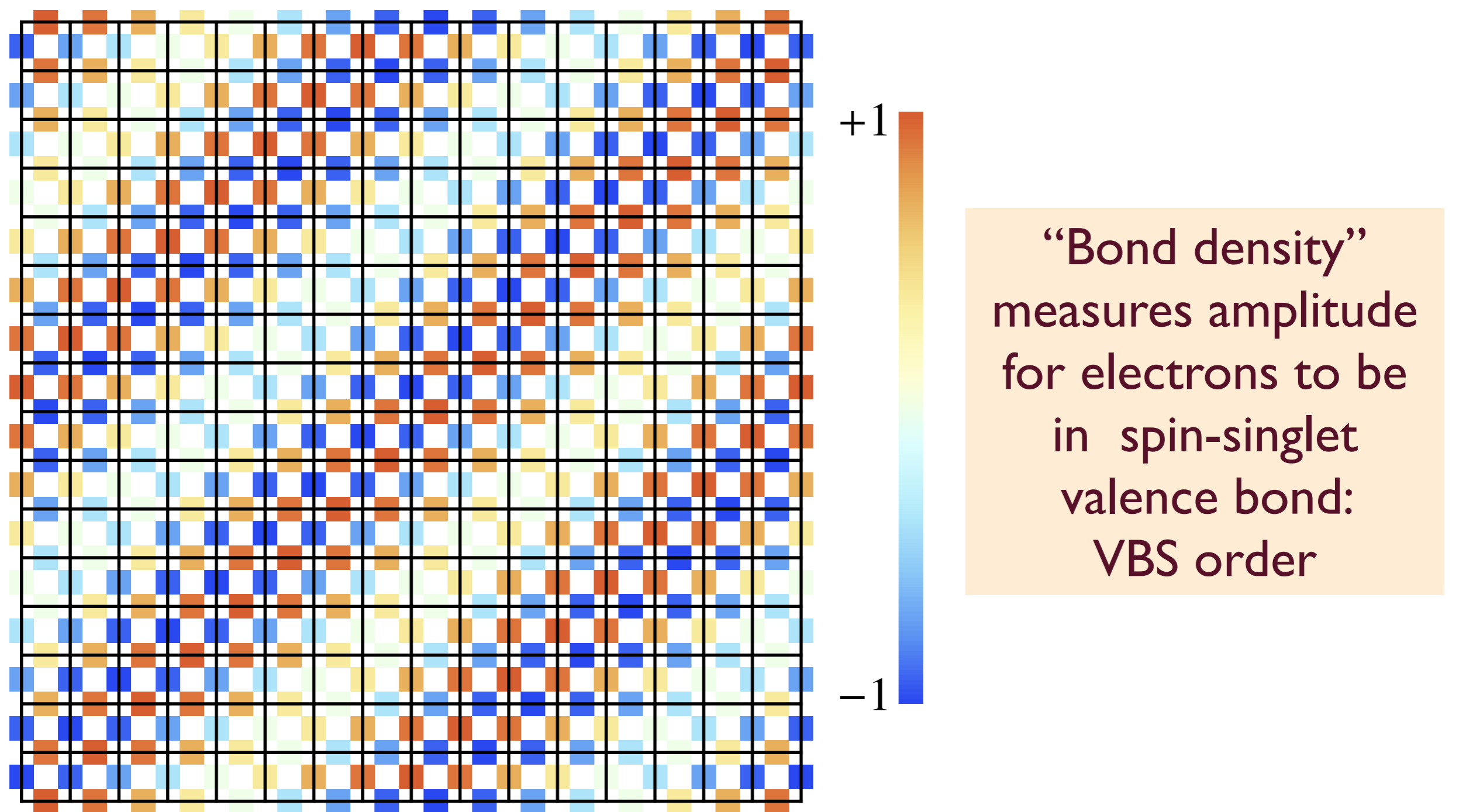


*d*-wave pairing instability in particle-particle channel



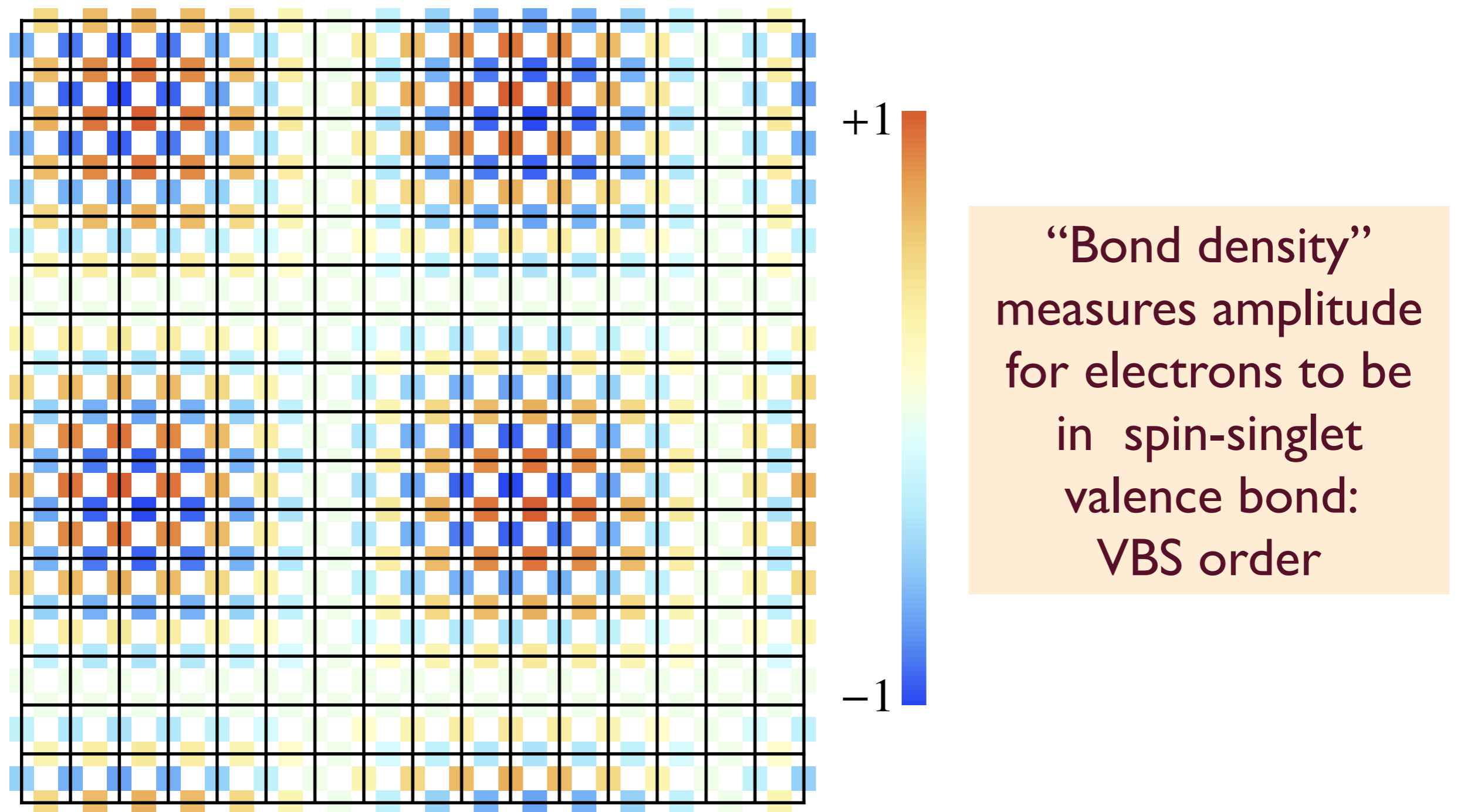
Bond density wave (with local Ising-nematic order) instability in particle-hole channel.

Nearly as strong as pairing instability because of a pseudospin symmetry of low energy theory



No modulations on sites:  $\langle c_{\mathbf{r}\alpha}^\dagger c_{\mathbf{s}\alpha} \rangle$  is non-zero only for  $\mathbf{r} \neq \mathbf{s}$ . Modulated bond-density wave with local Ising-nematic ordering:

$$\left\langle c_{\mathbf{k}-\mathbf{Q}/2,\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



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The co-existence of  
spin density wave order  
and d-wave superconductivity

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \nabla_r) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} (\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \nabla_r) \psi_{2\alpha}^\ell$$

Order parameter: 
$$\mathcal{L}_\varphi = \frac{1}{2} (\nabla_r \vec{\varphi})^2 + \frac{\tilde{\zeta}}{2} (\partial_\tau \vec{\varphi})^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$$

“Yukawa” coupling: 
$$\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left( \psi_{1\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger} \vec{\sigma}_{\alpha\beta} \psi_{1\beta}^\ell \right)$$

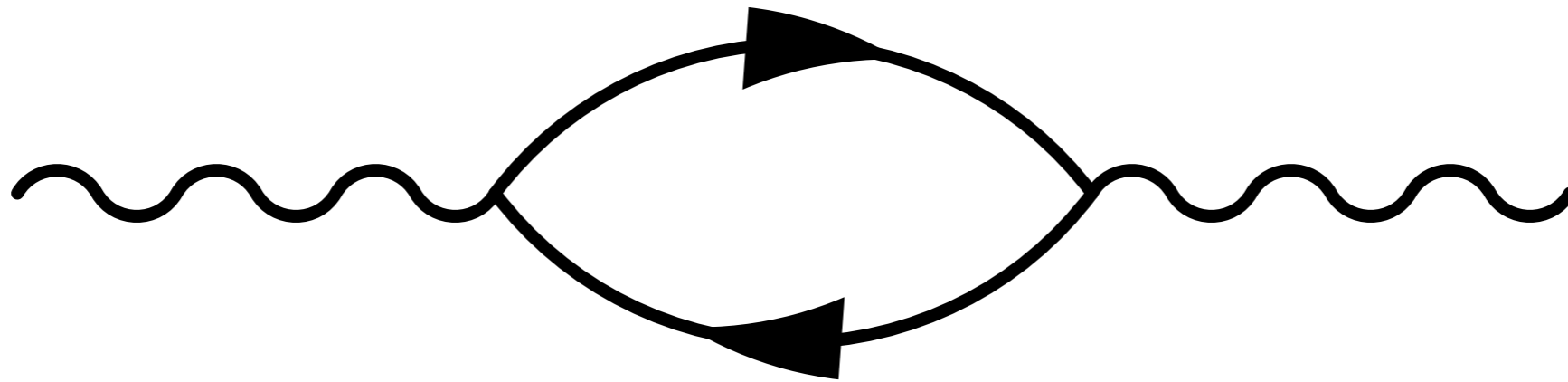
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Pairing: 
$$\mathcal{L}_\Delta = \Delta \varepsilon^{\alpha\beta} \left( \psi_{1\alpha}^3 \psi_{1\beta}^1 - \psi_{2\alpha}^3 \psi_{2\beta}^1 - \psi_{1\alpha}^4 \psi_{1\beta}^2 - \psi_{2\alpha}^4 \psi_{2\beta}^2 \right) + \text{H.c.}$$

Include the possibility of pairing in the metal.  
 And then compute the shift in the critical value  
 of the SDW transition,  $s_c - s_c^0$  due to a non-zero  $\Delta$ .



Compute the SDW susceptibility,  $\chi$ , in the superconducting state. As  $\Delta \rightarrow 0$ , we find

$$\chi(\Delta) = \chi(0) - C|\Delta|$$

where  $C$  is a universal constant dominated by the vicinity of the hot spots.

The weak-coupling theory with equivalent hotspots yields  $C = 0$  - there is an exact cancellation of competition between SDW and SC at the hot spots, and attraction between SDW and SC away from the hot spot.

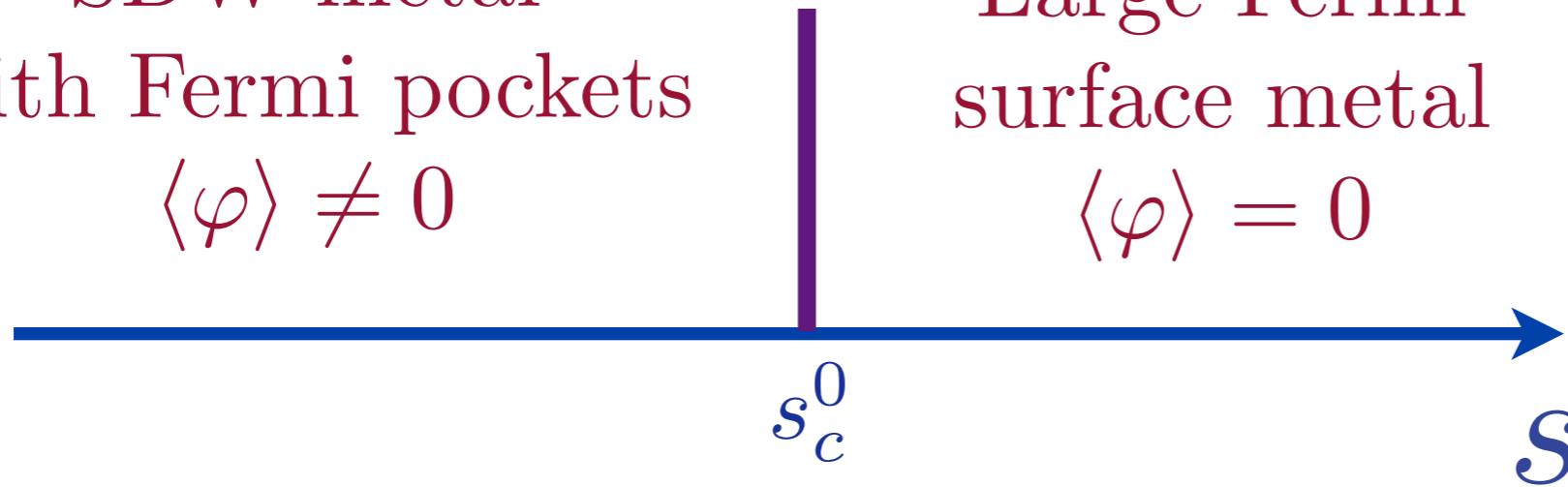
For inequivalent hot spots (as in pnictides, or with incommensurate order in the cuprates) in weak-coupling theory, or in a strong-coupling analysis, we generically find  $C > 0$ .



# Fermi surface theory of competing orders

SDW metal  
with Fermi pockets  
 $\langle \varphi \rangle \neq 0$

Large Fermi  
surface metal  
 $\langle \varphi \rangle = 0$

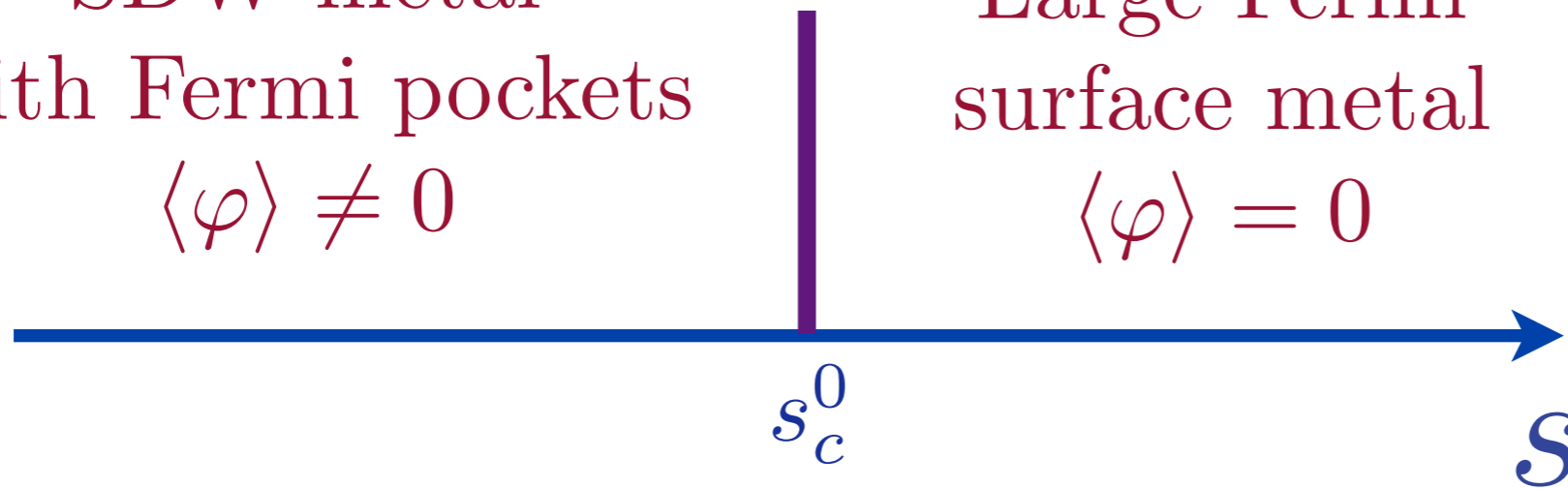


No SC  
 $\Delta = 0$

# Fermi surface theory of competing orders

SDW metal  
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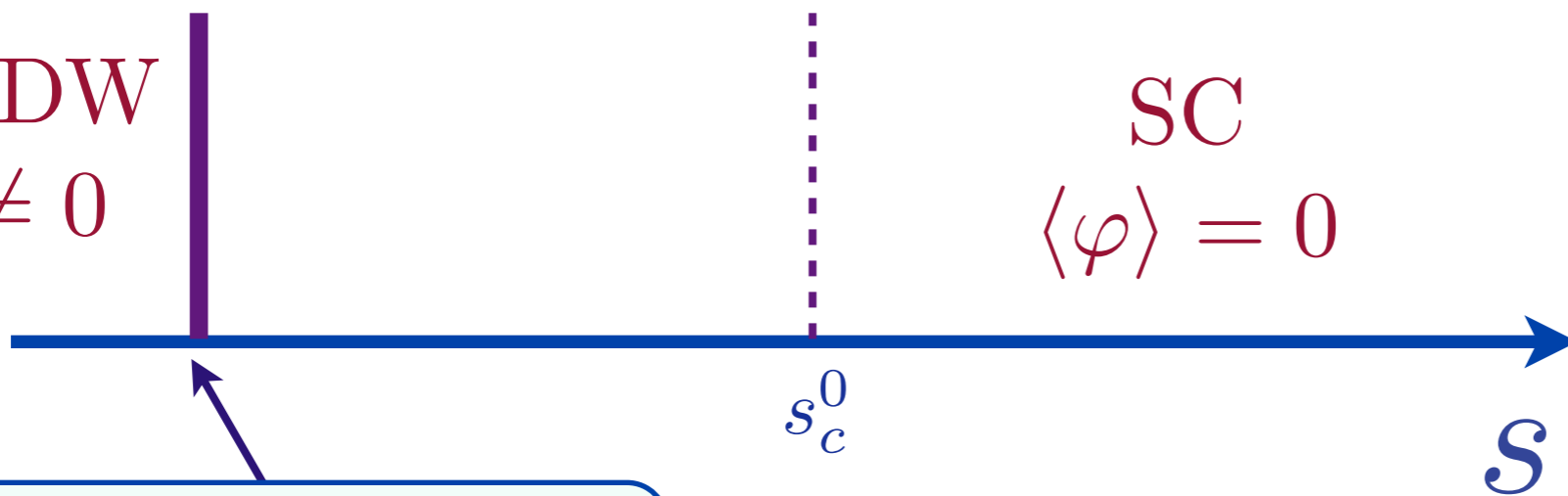
Large Fermi  
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No SC  
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SC+SDW  
 $\langle \varphi \rangle \neq 0$

SC  
 $\langle \varphi \rangle = 0$

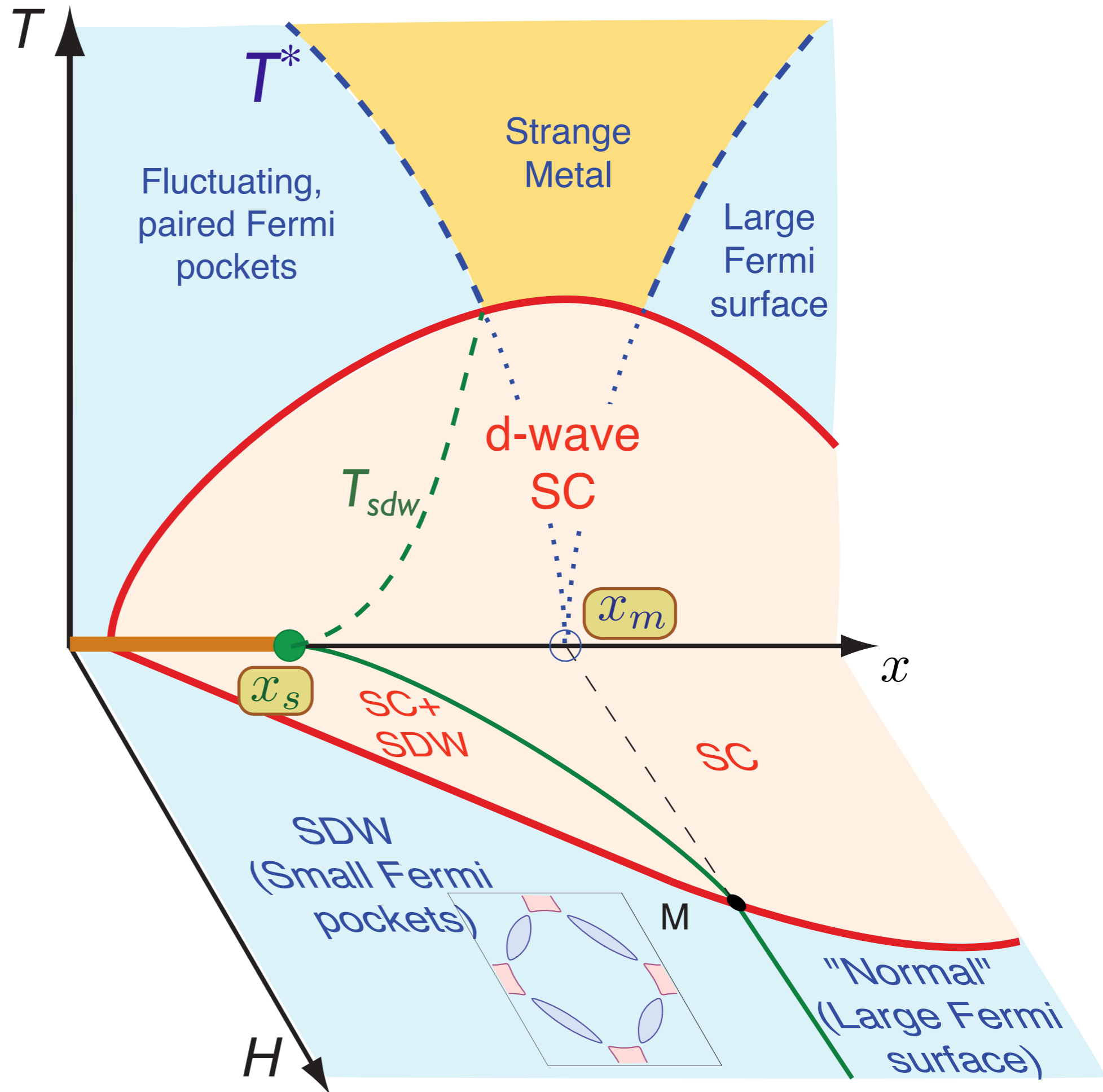


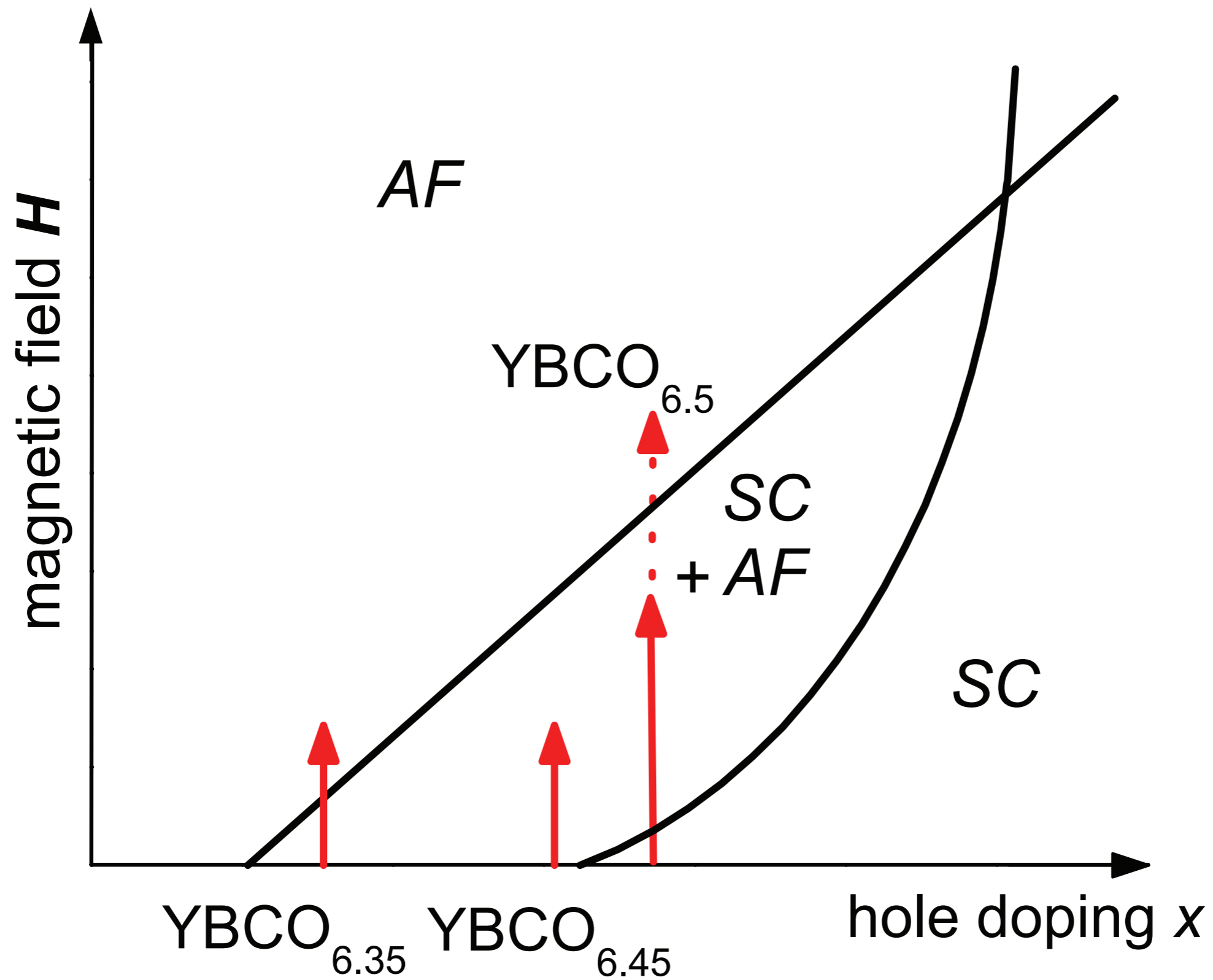
SC  
 $\Delta \neq 0$

$$s_c^0 - s_c \sim |\Delta|$$

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

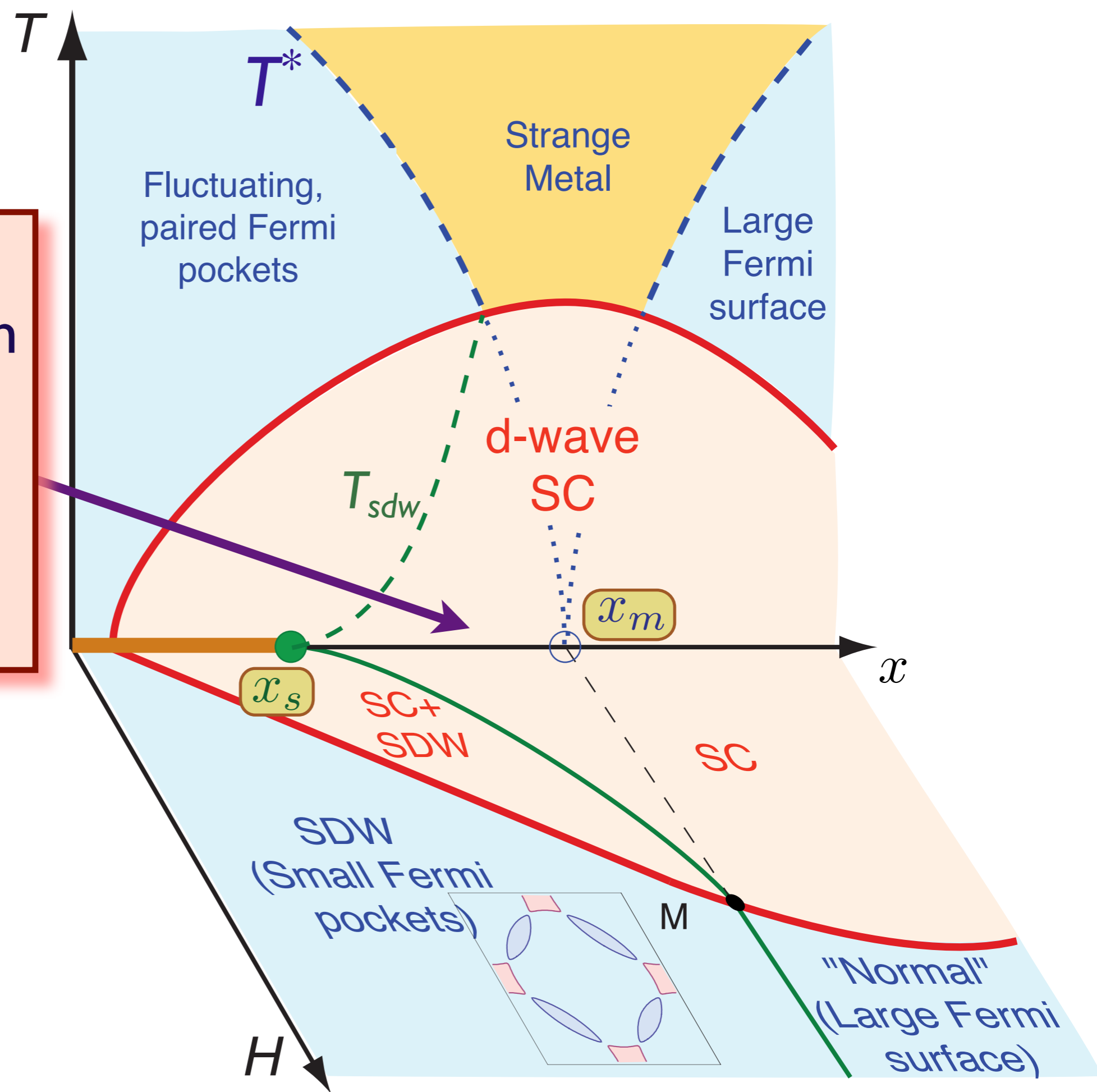
E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

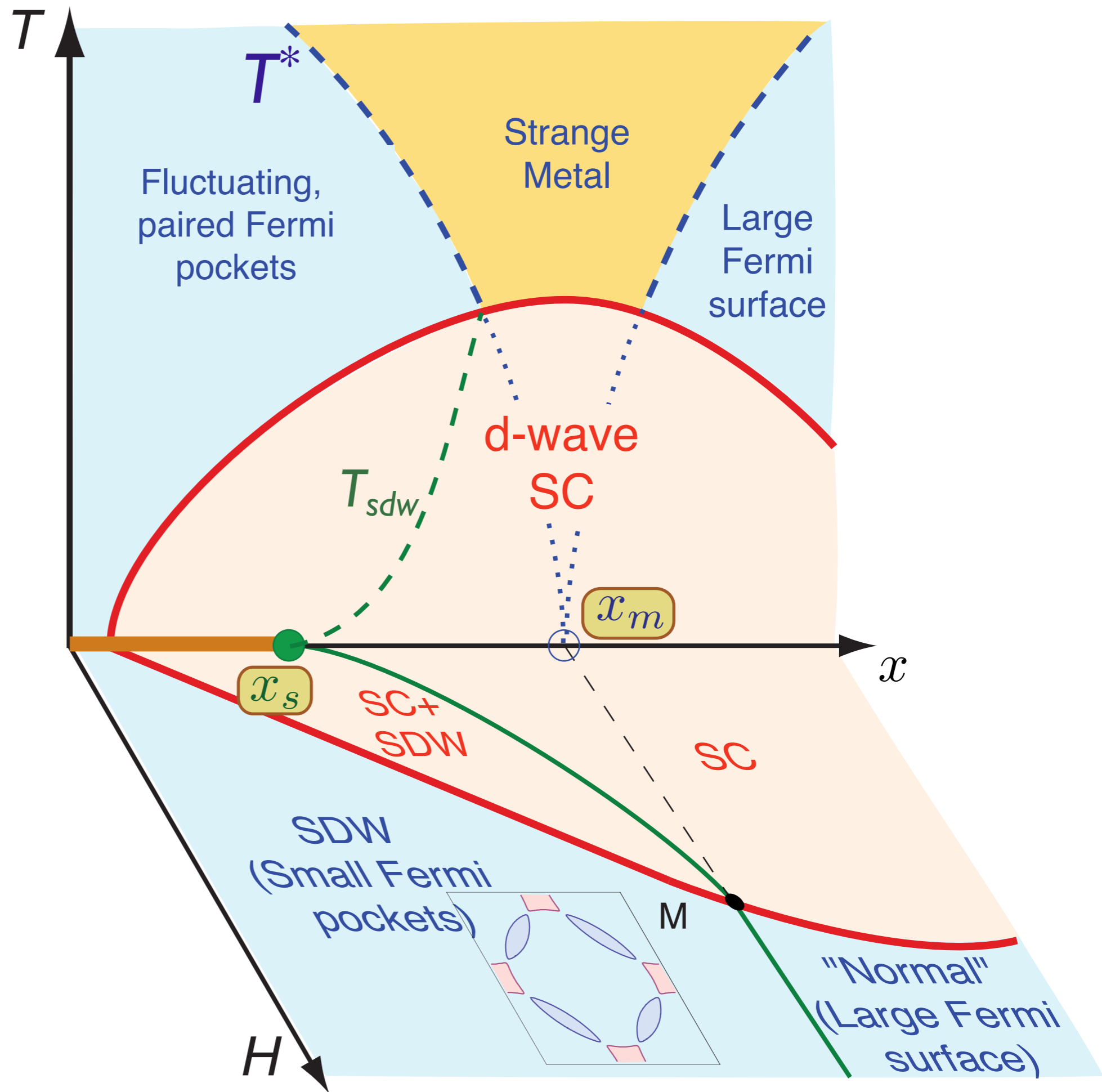




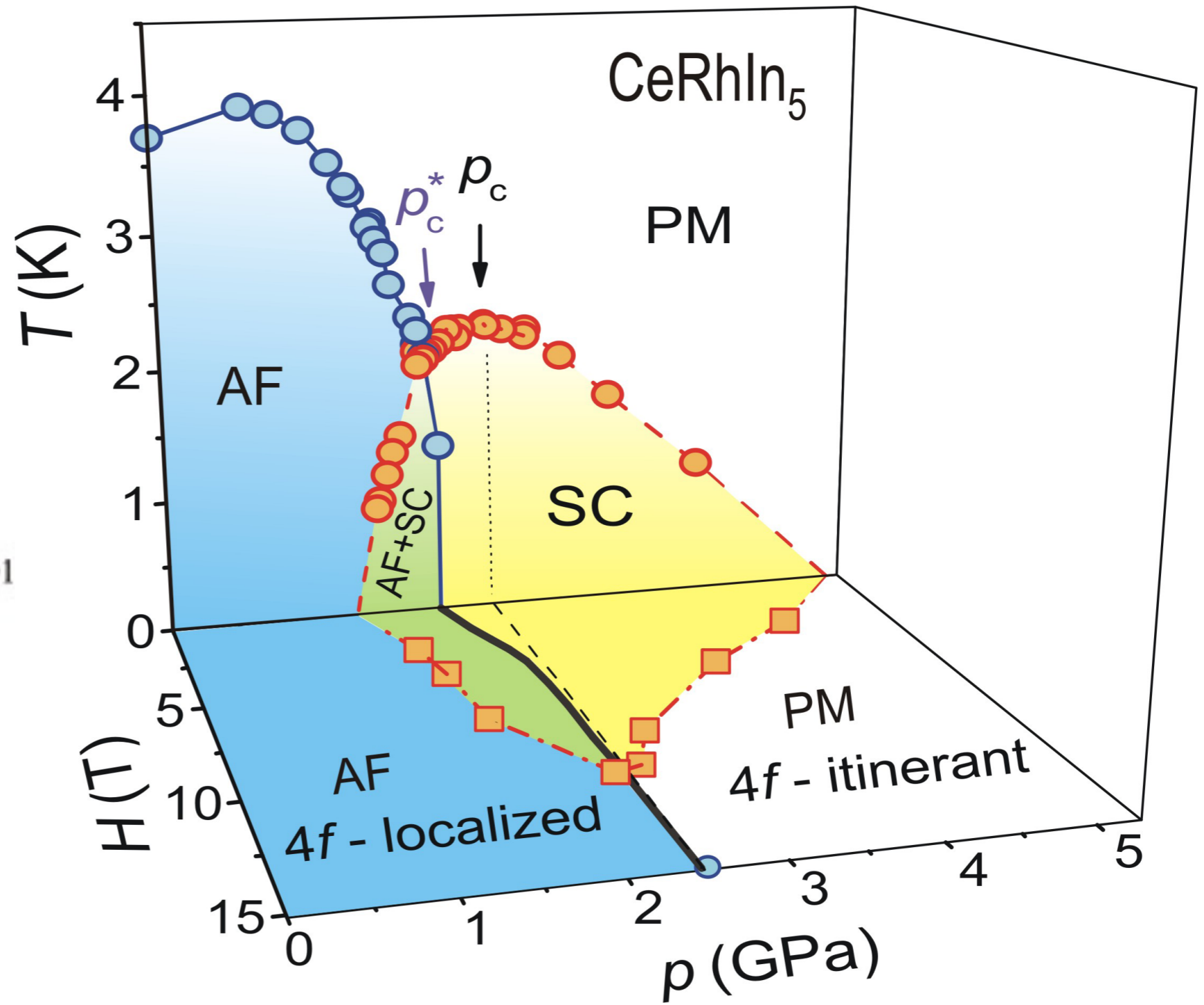
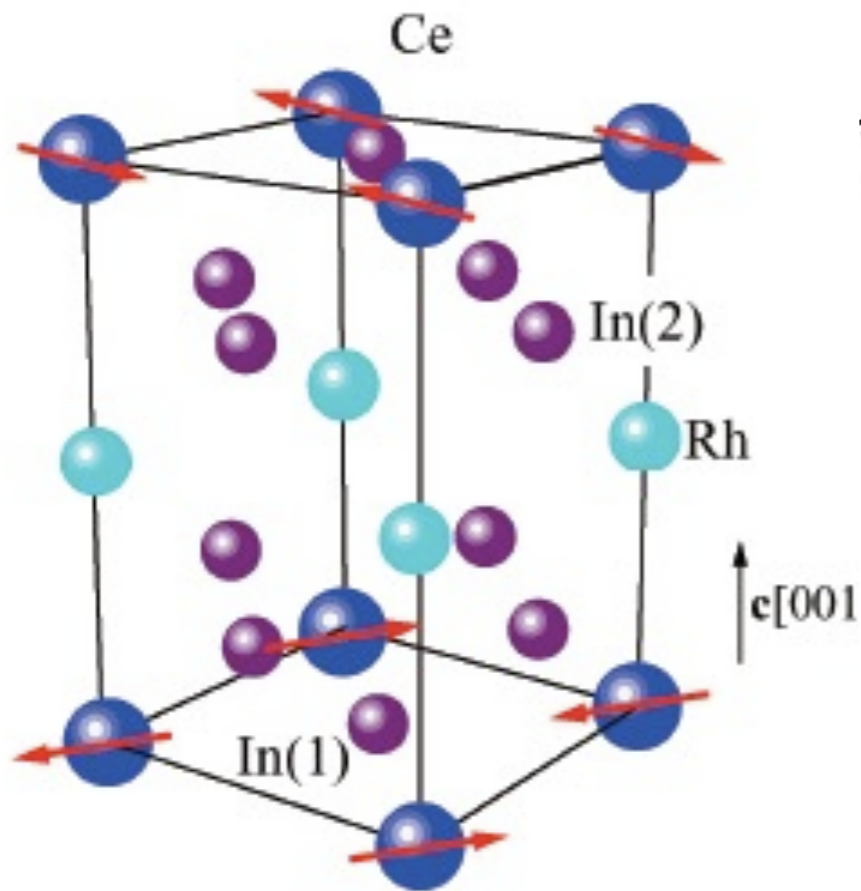
D. Haug, V. Hinkov, Y. Sidis, P. Bourges, N. B. Christensen, A. Ivanov, T. Keller, C. T. Lin, and B. Keimer, arXiv:1008.4298

Other orders appear between  $x_s$  and  $x_m$  e.g. nematic ordering, VBS, or even SC\*



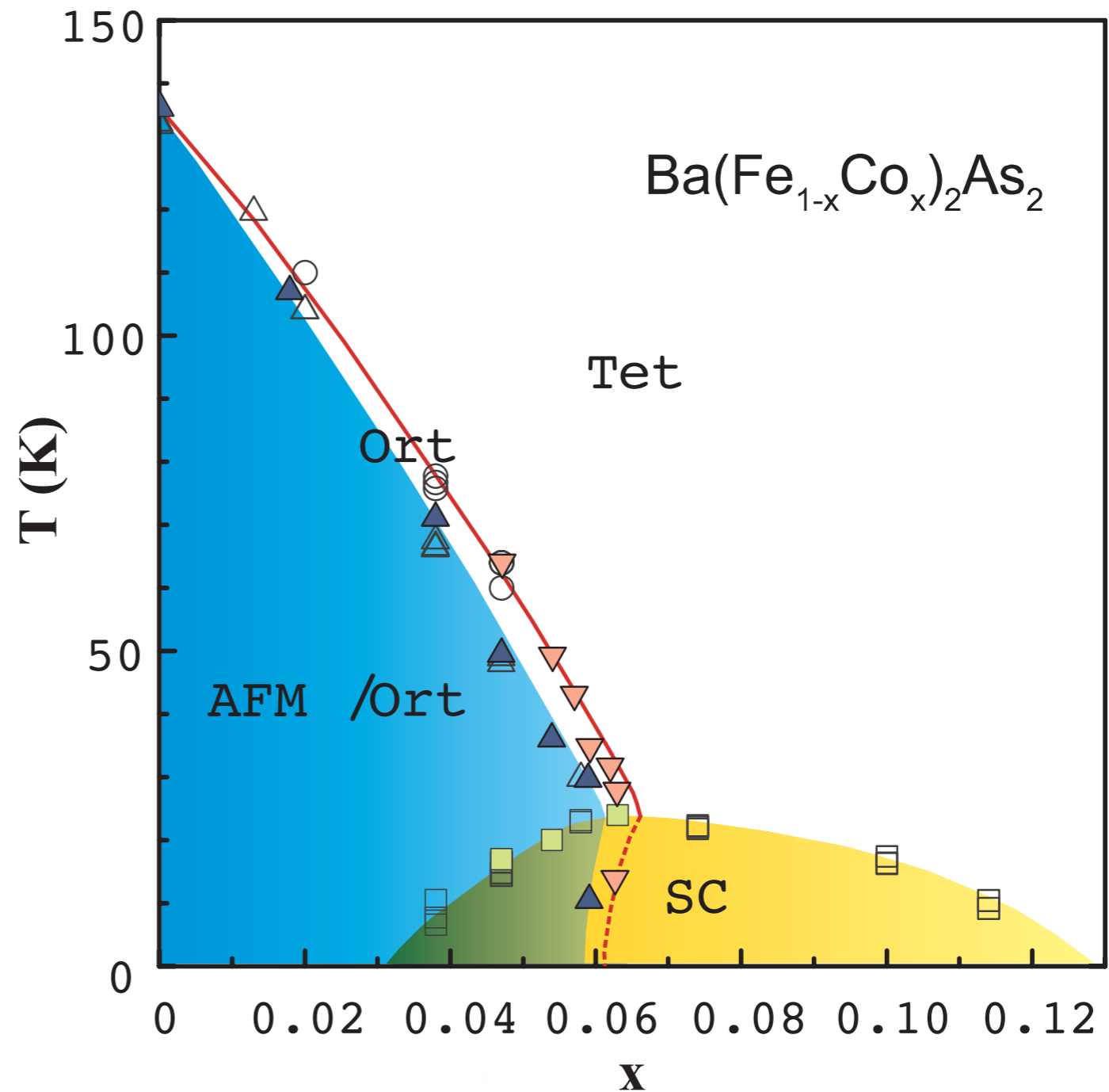
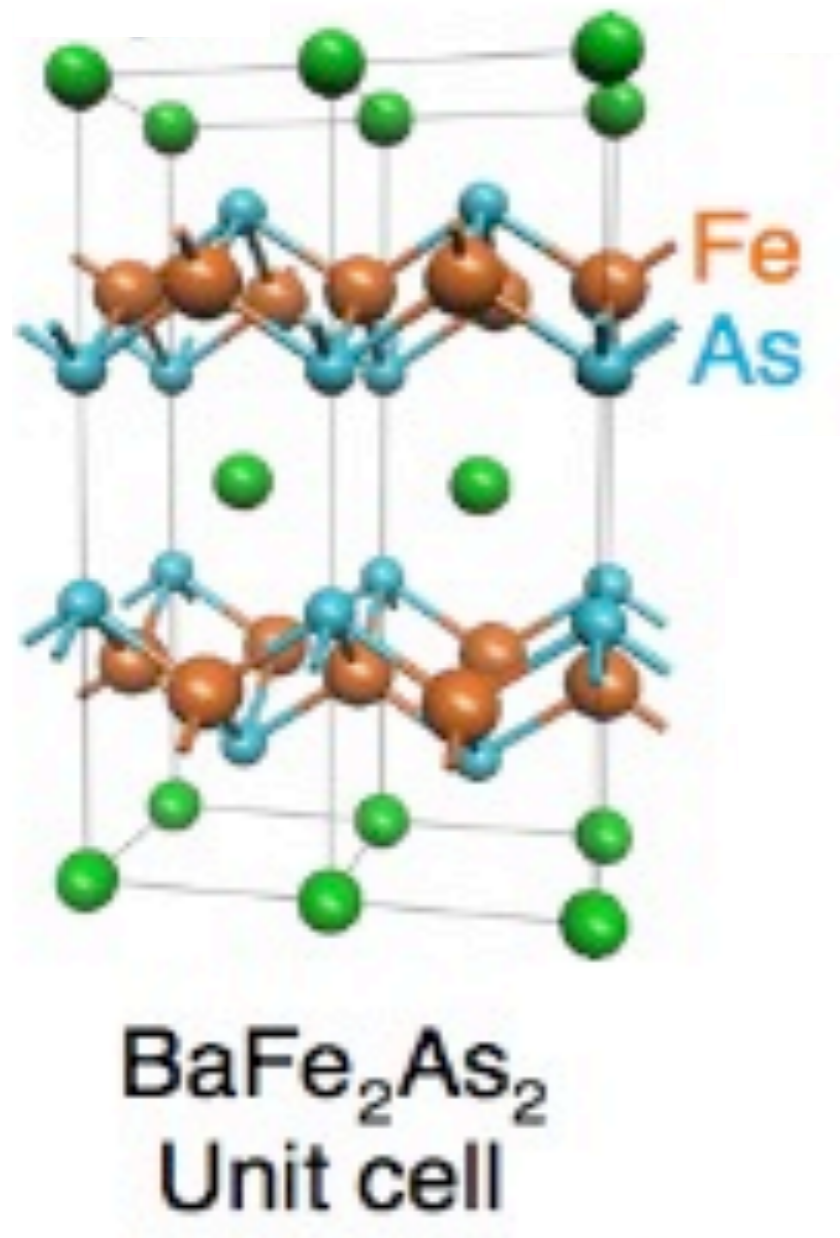


# Similar phase diagram for CeRhIn<sub>5</sub>



G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

# Similar phase diagram for the pnictides

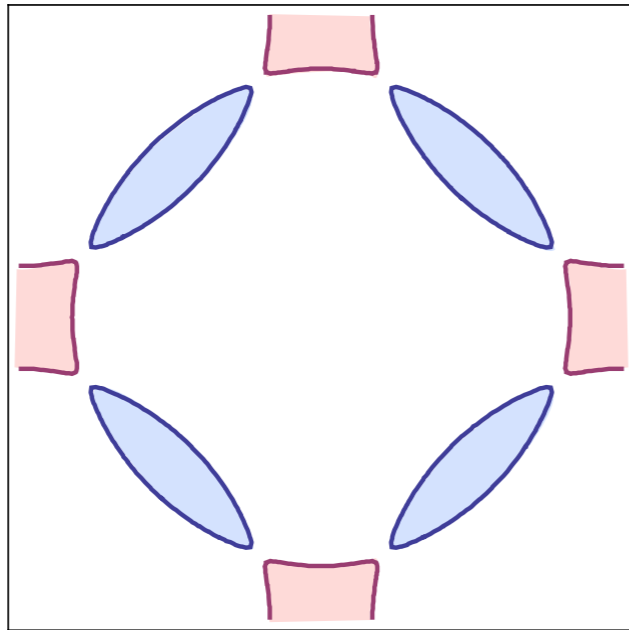


S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, and A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010)



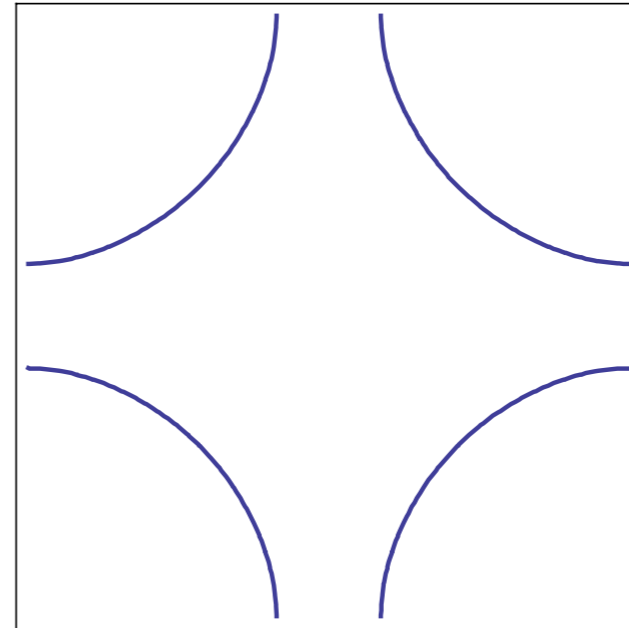
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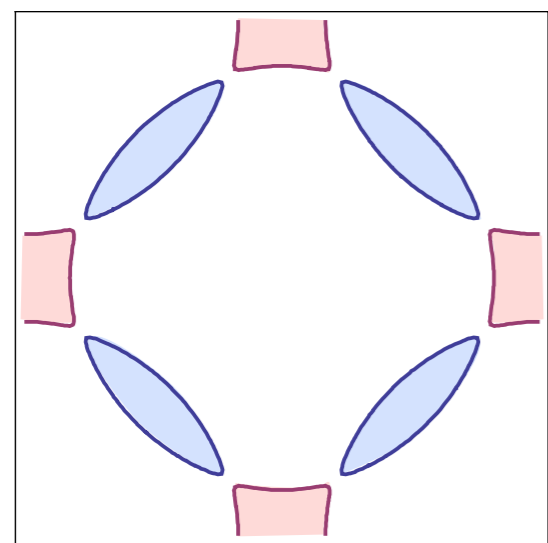
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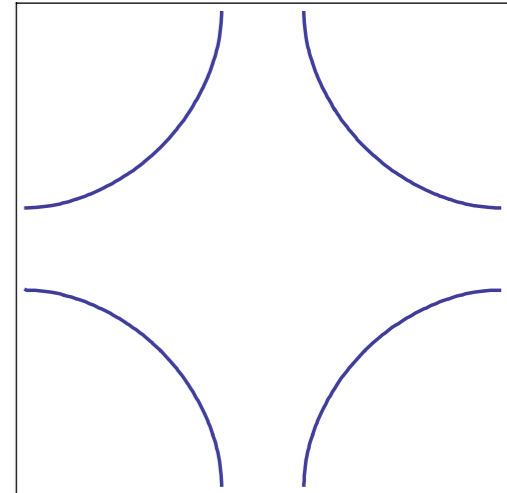
Metal with "large"  
Fermi surface

$S$

# SU(2) gauge theory: separating Fermi surface change from SDW order



SDW order  
small Fermi pockets

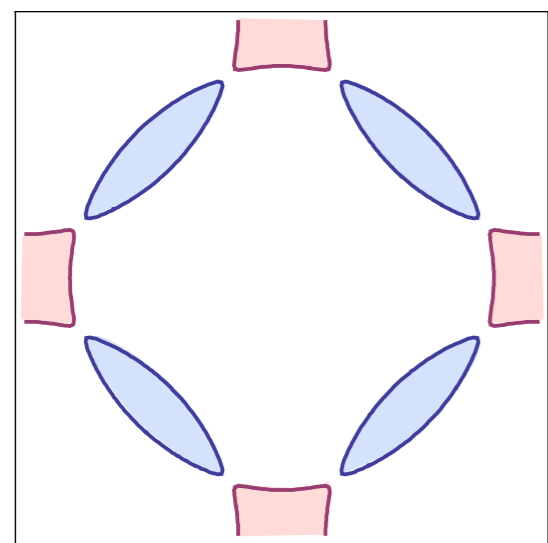


Fermi liquid  
large Fermi surface

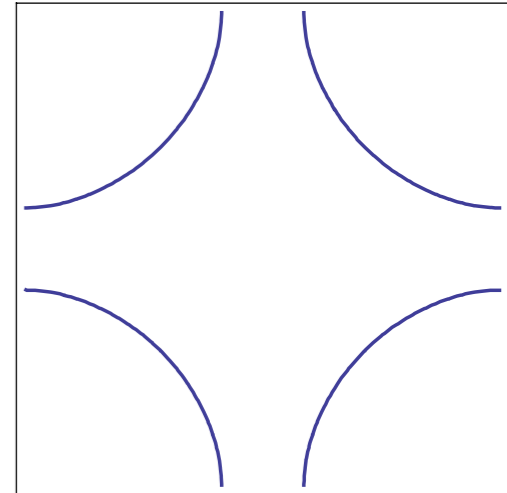
non-Fermi liquid  
Fermi pockets  
gapless U(1) photon

non-Fermi liquid  
large Fermi surface  
gapless SU(2) photons

# SU(2) gauge theory: separating Fermi surface change from SDW order



SDW order  
small Fermi pockets



Fermi liquid  
large Fermi surface

non-Fermi liquid  
Fermi pockets  
gapless U(1) photon

non-Fermi liquid  
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Leads to a FL\* state: has Fermi pockets without translations symmetry breaking