Compressible quantum matter and gauge-gravity duality

Review: arXiv:1203.4565

Gravity, black holes, and condensed matter,
Kavli Royal Society Center, Chicheley_Hall
A Royal Society International Seminar, April 23-24, 2012

Subir Sachdev

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Talk online at sachdev.physics.harvard.edu


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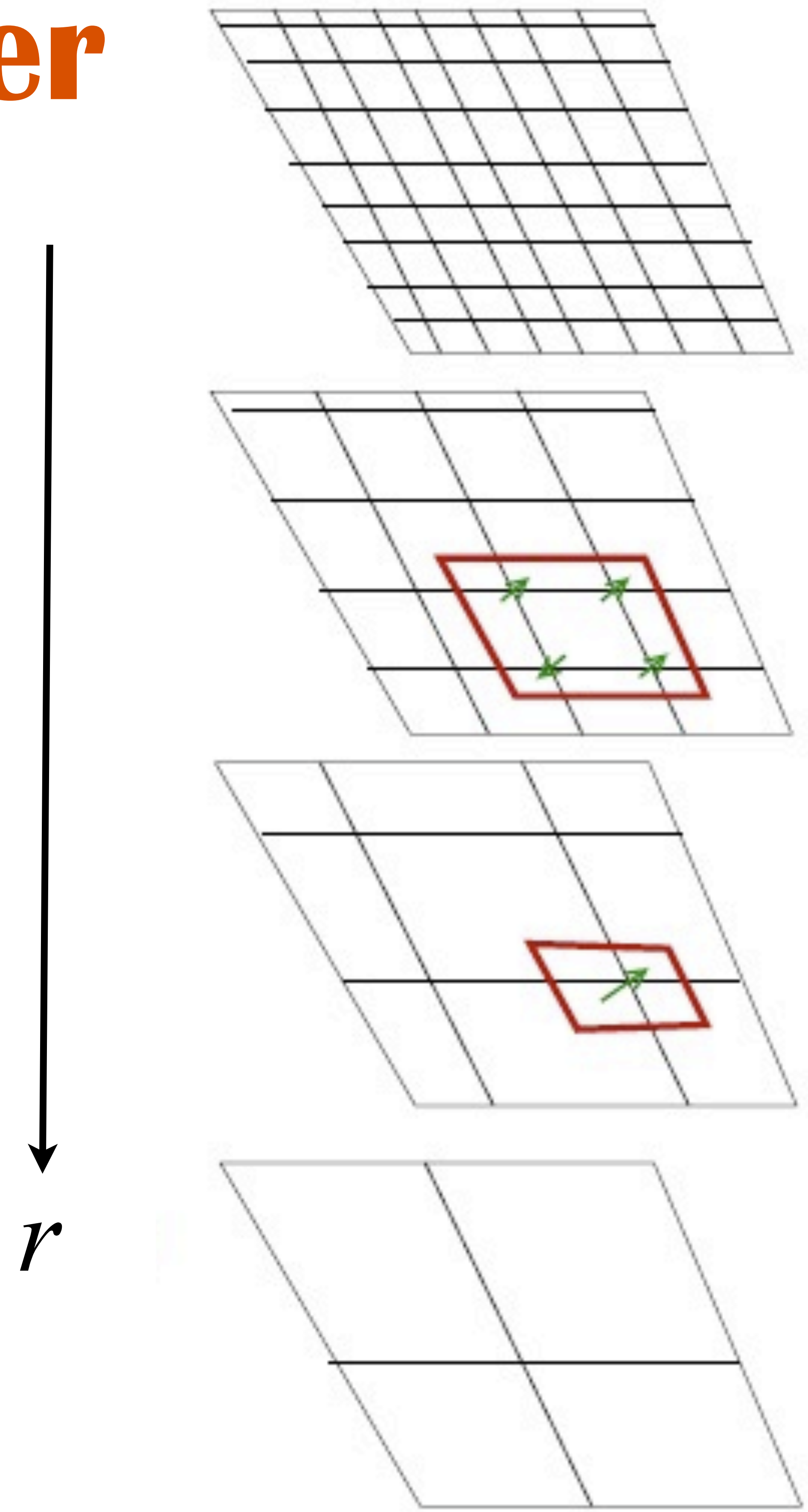

J. McGreevy, arXiv0909.0518

Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

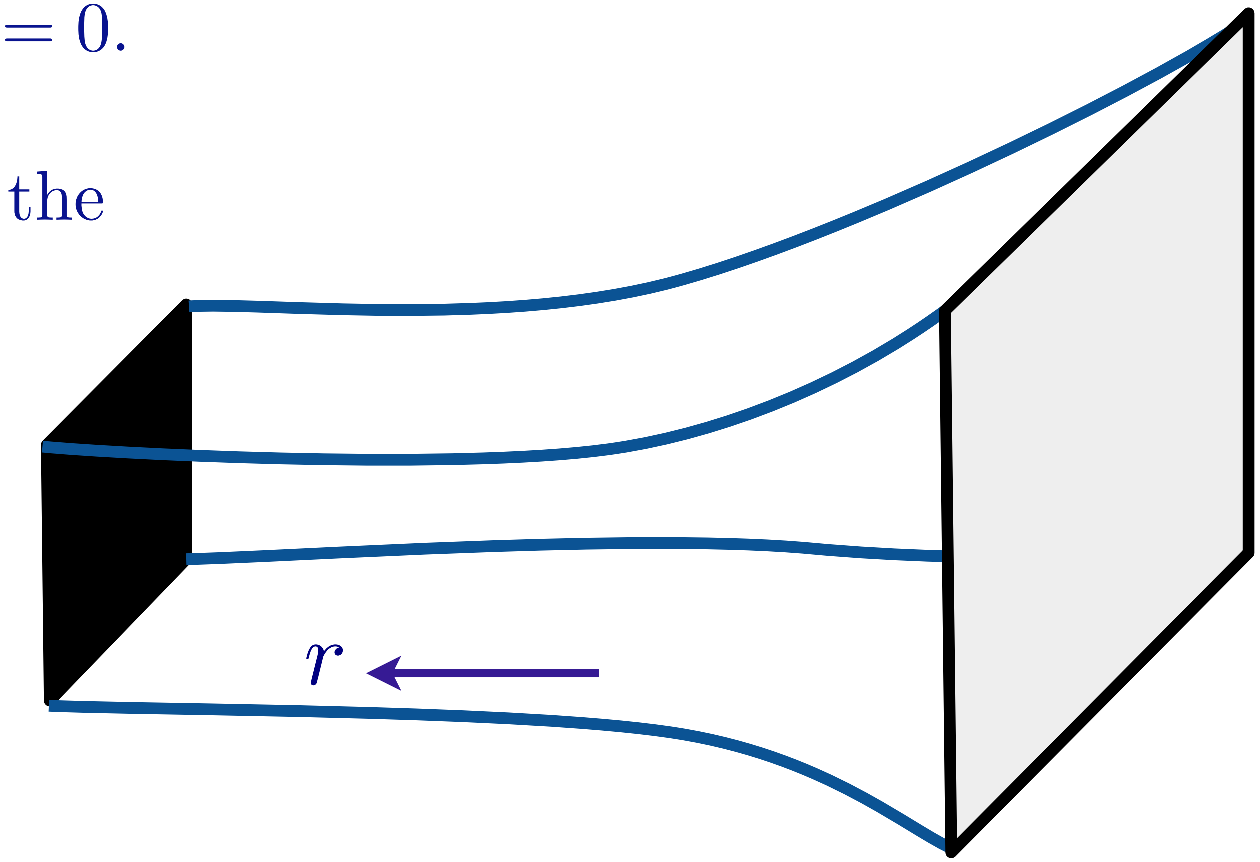
We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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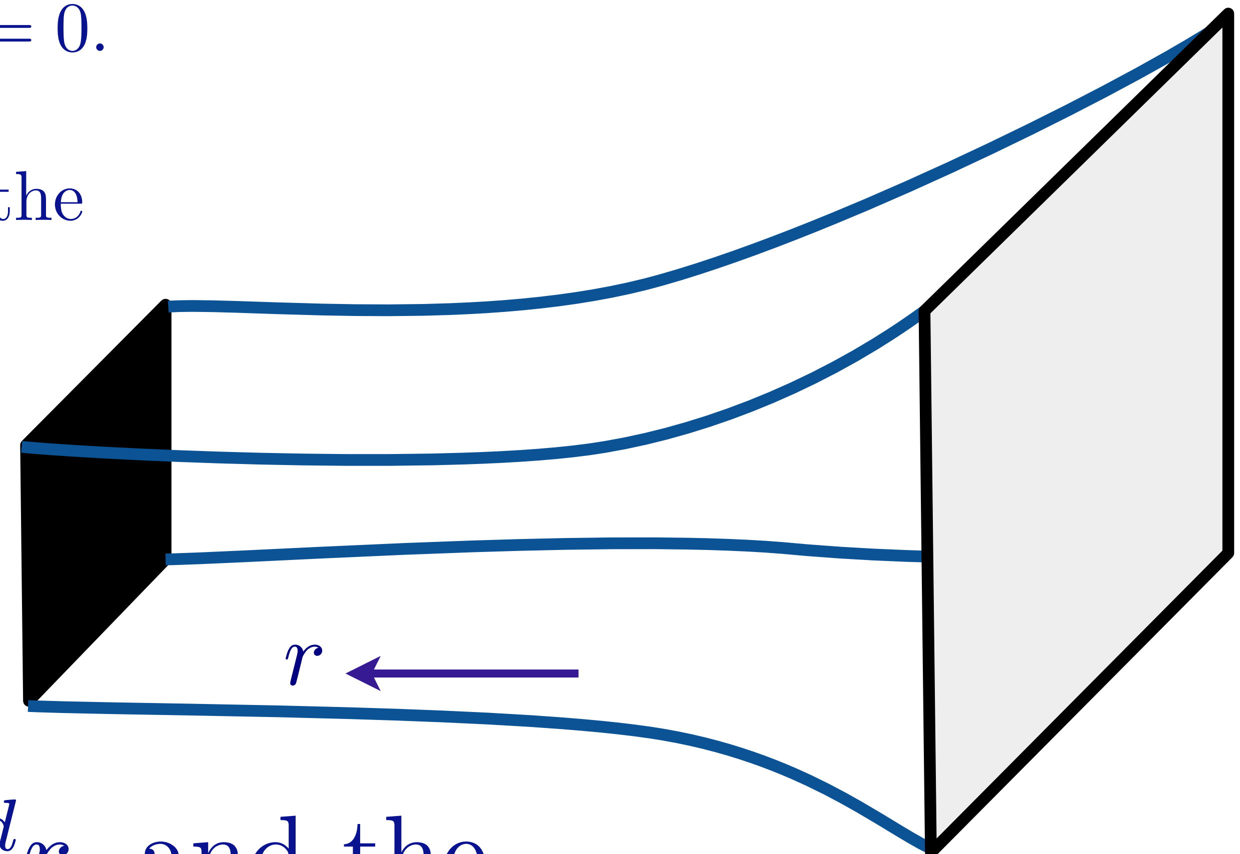


Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\text{eff}}/z}$$

where $\theta = d - d_{\text{eff}}$ measures "dimension deficit" in the phase space of low energy degrees of a freedom.

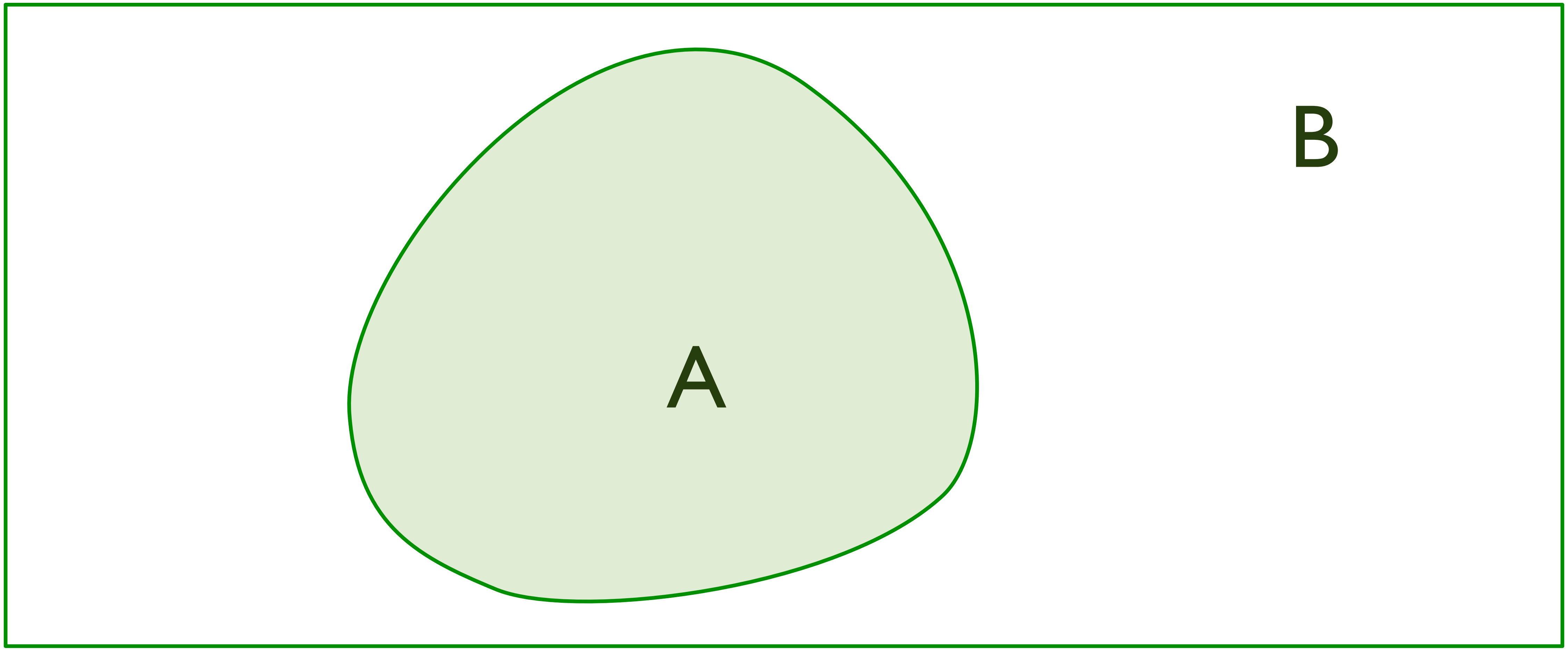
$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

Entanglement entropy

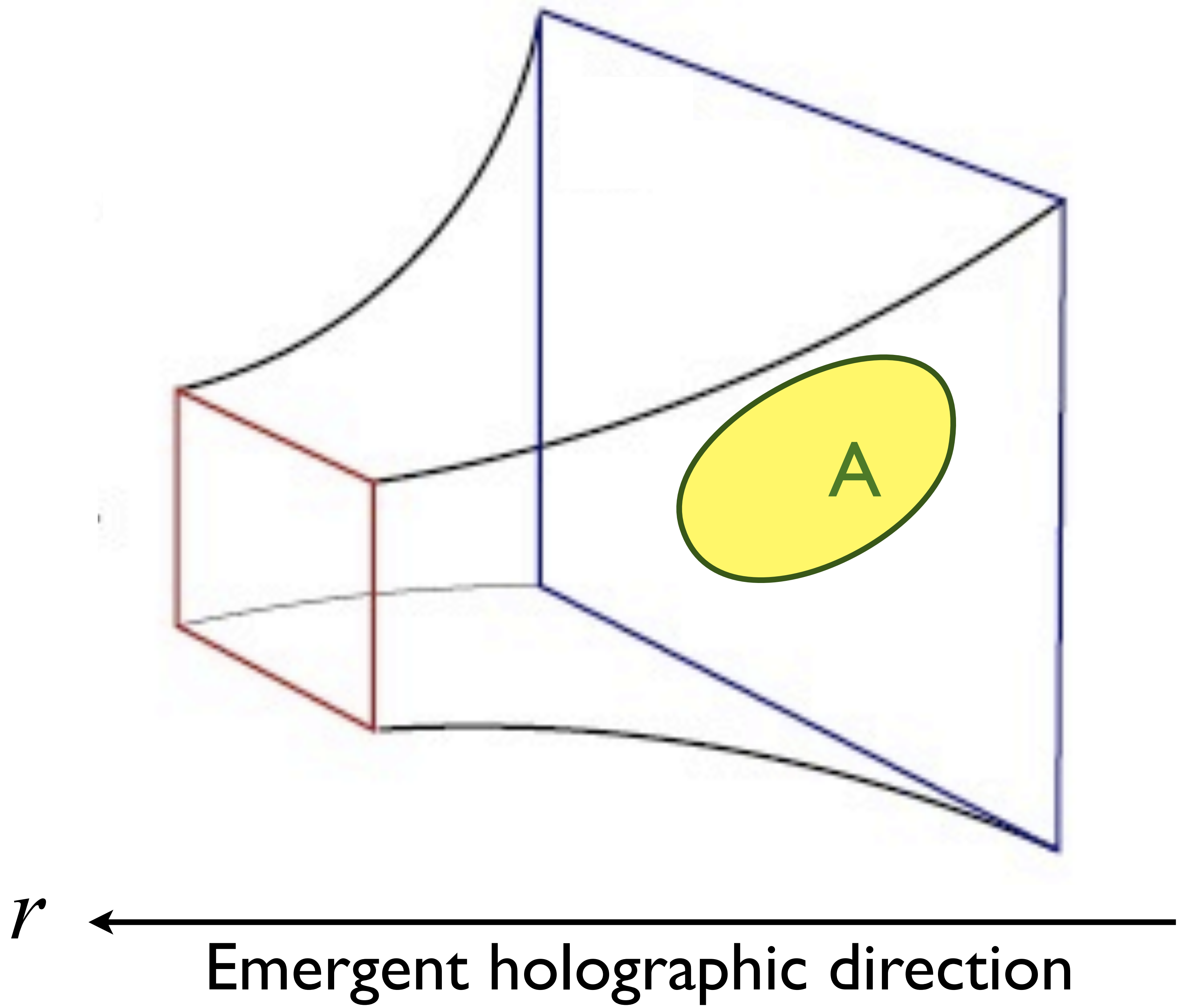


Measure strength of quantum entanglement of region A with region B.

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Holographic entanglement entropy



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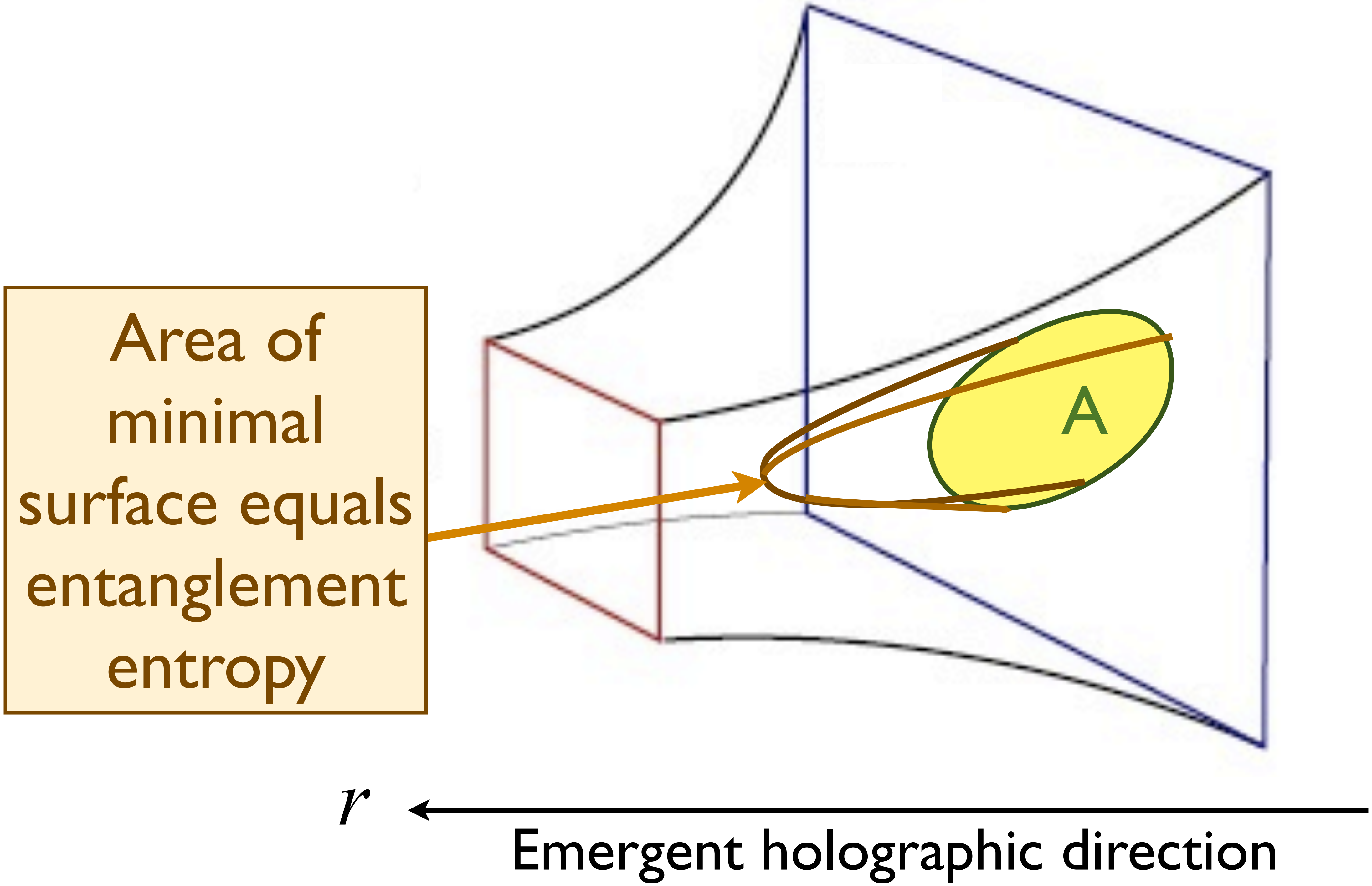

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

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- The entanglement entropy, S_E, of an entangling region with boundary surface 'area' Σ scales as

$$S_E \sim \begin{cases} \Sigma & , \quad \text{for } \theta < d-1 \\ \Sigma \ln \Sigma & , \quad \text{for } \theta = d-1 \\ \Sigma^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases}$$

  All local quantum field theories obey the "area law" (upto log violations) and so $\theta \leq d-1$.

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AdS$_2\times$R^d corresponds to $\theta = d(1-1/z)$ and $z \to \infty$.

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Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

- Describe *zero temperature* phases where $d\langle \mathcal{Q} \rangle / d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H - \mu\mathcal{Q}$.

Compressible quantum matter

The only compressible phase of traditional condensed matter physics which does not break the translational or $\mathrm{U}(1)$ symmetries is the Landau Fermi liquid

Compressible quantum matter

Challenge to string theory:

Classify and understand non-Fermi liquid phases of compressible quantum matter, *i.e.* strange metals

Strange metals

A. Field theory

B. Holography

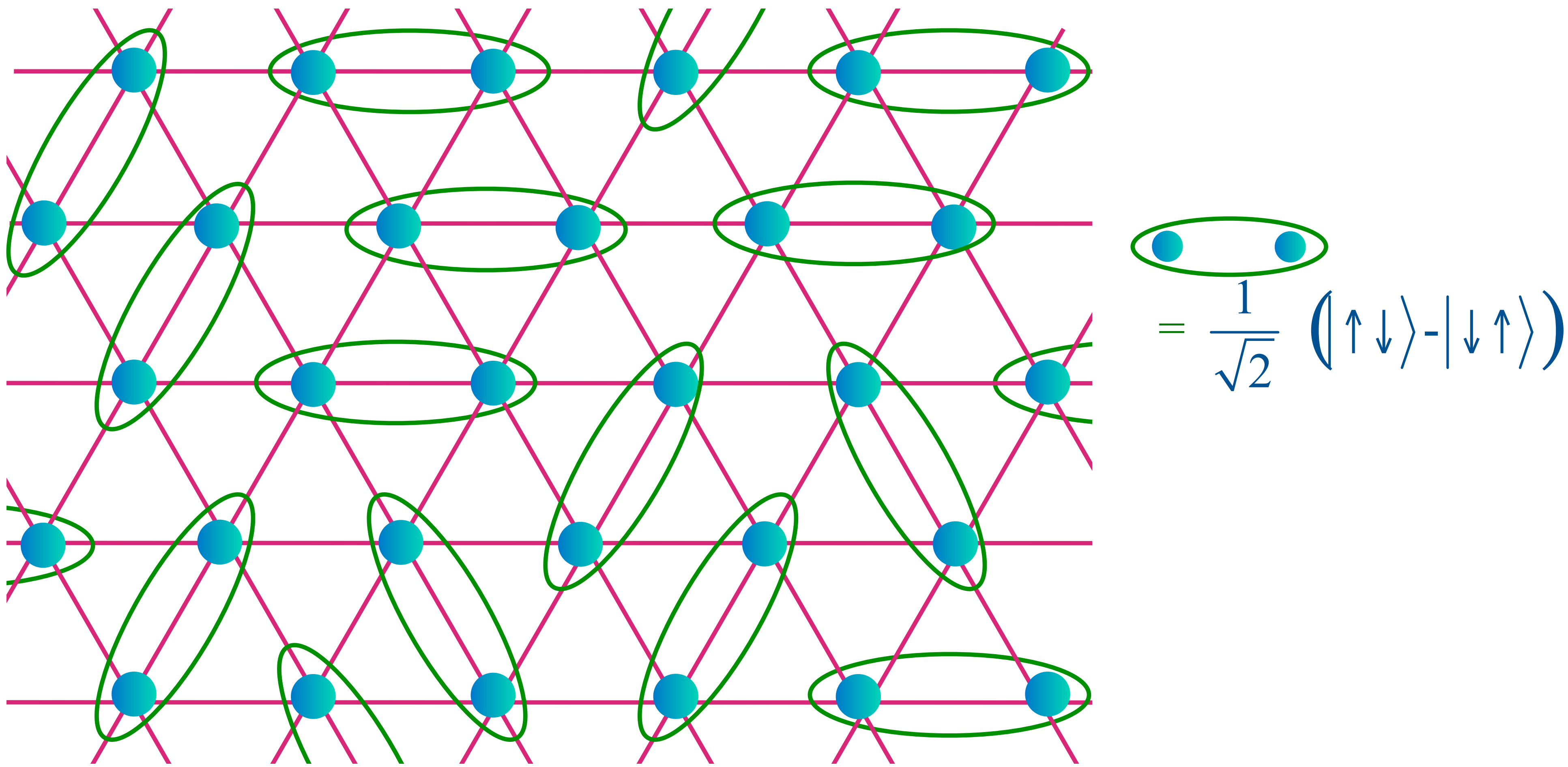
Strange metals

A. Field theory

B. Holography

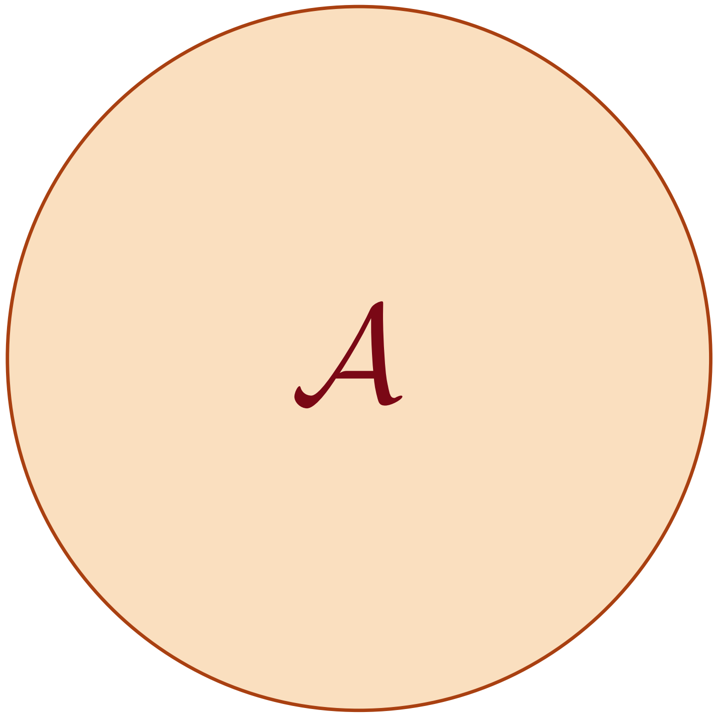
The Non-Fermi Liquid (NFL)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

Fermi surface of an ordinary metal



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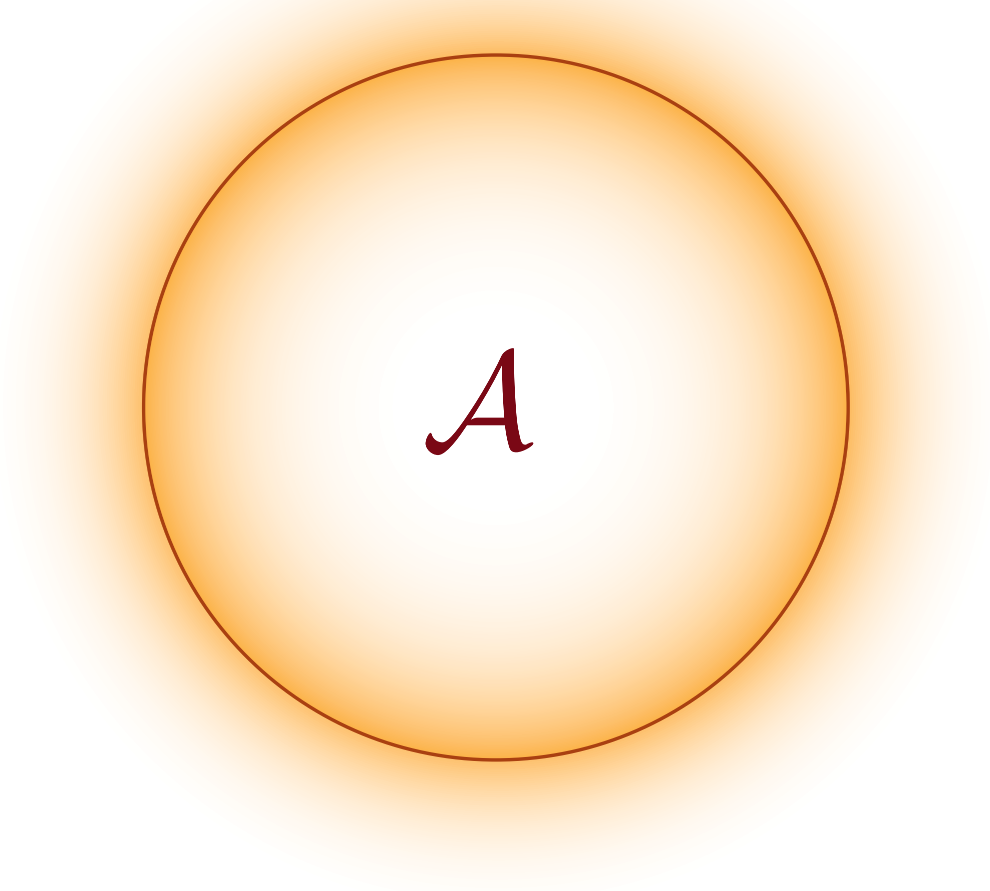
Fermions coupled to a gauge field



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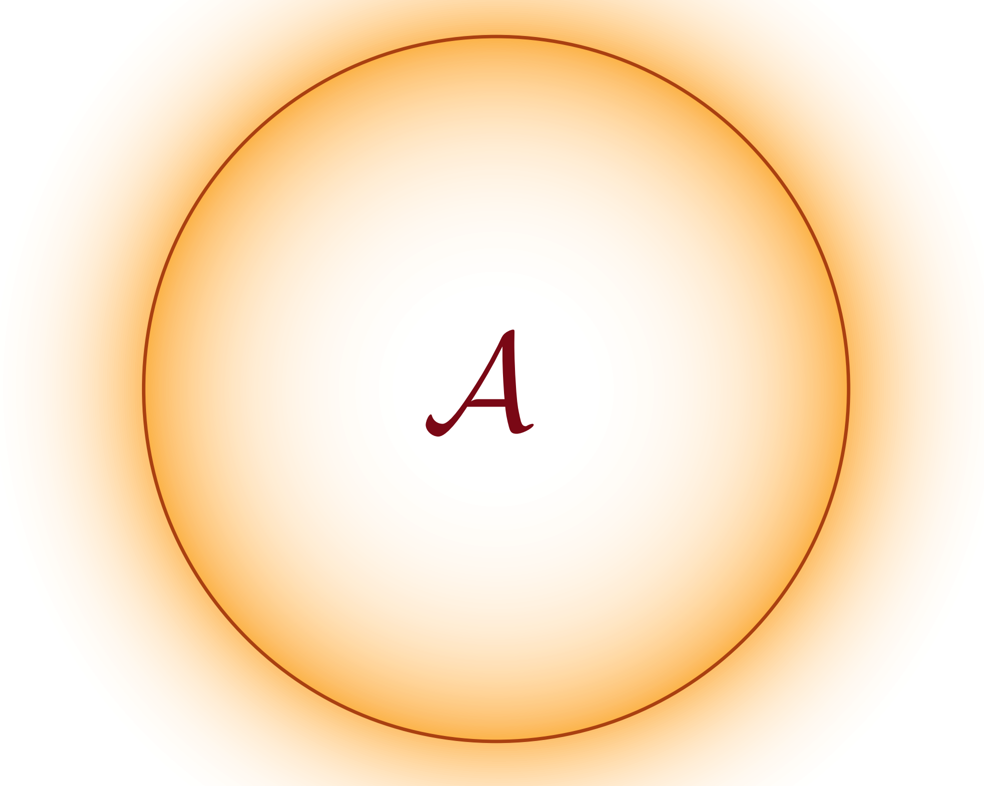
Properties of this strange metal



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

- There is a sharp Fermi surface defined by the (gauge-dependent) fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$. This Green's function is not measurable, and so the Fermi surface is "*hidden*".

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

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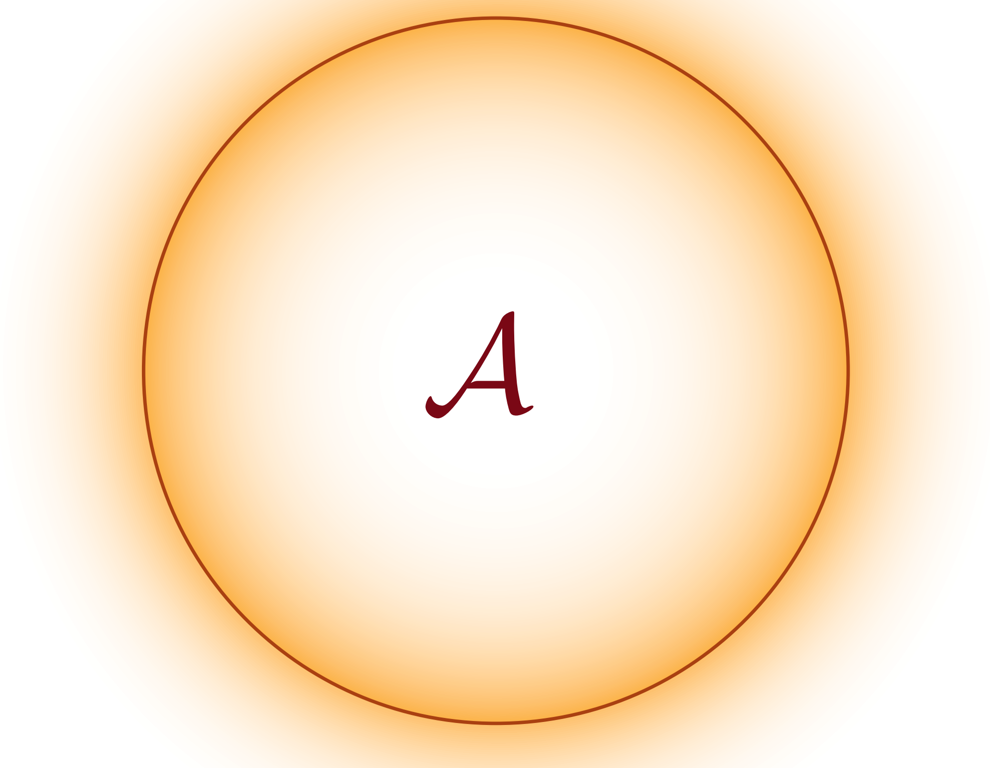

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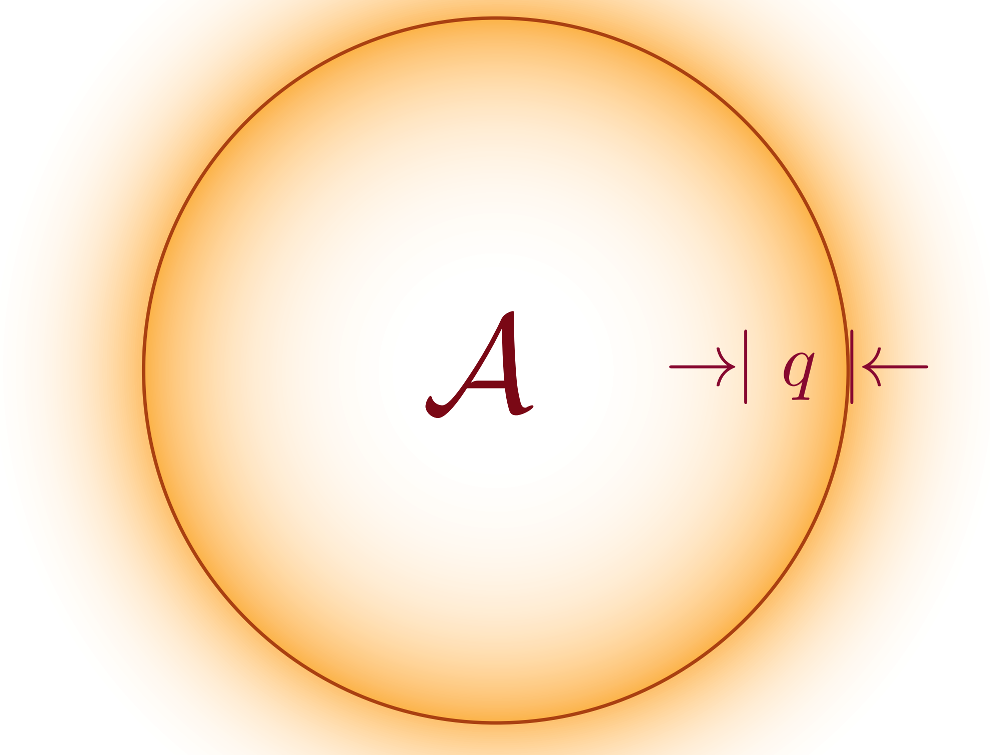

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- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

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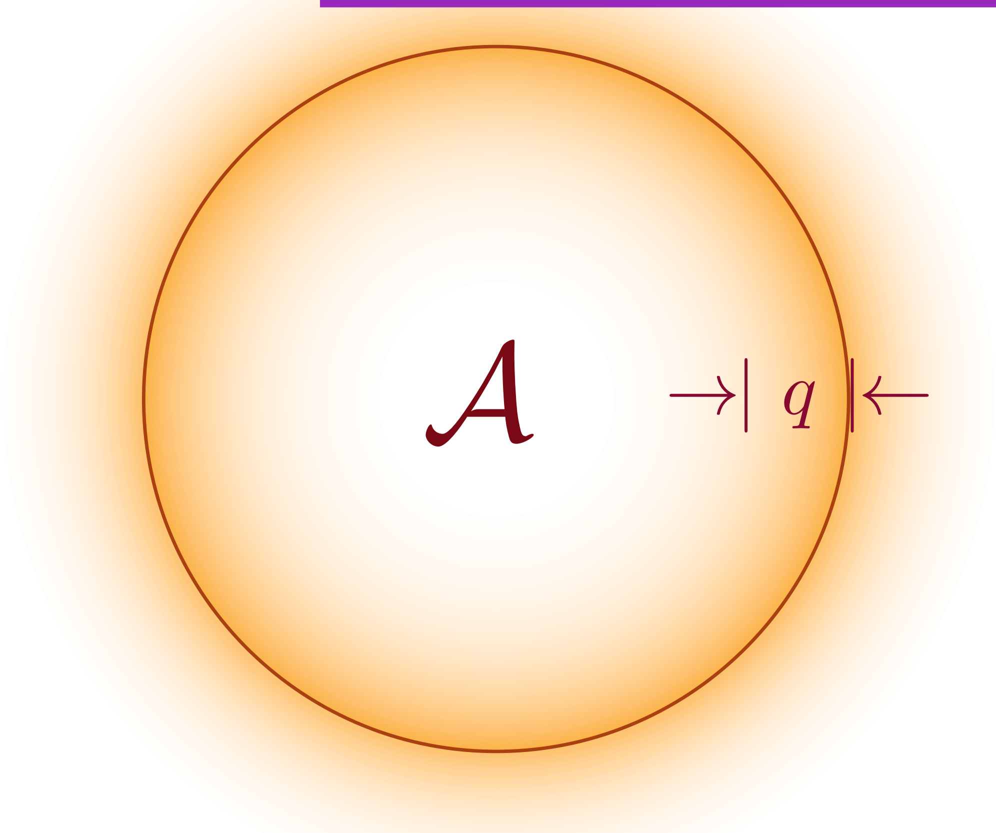

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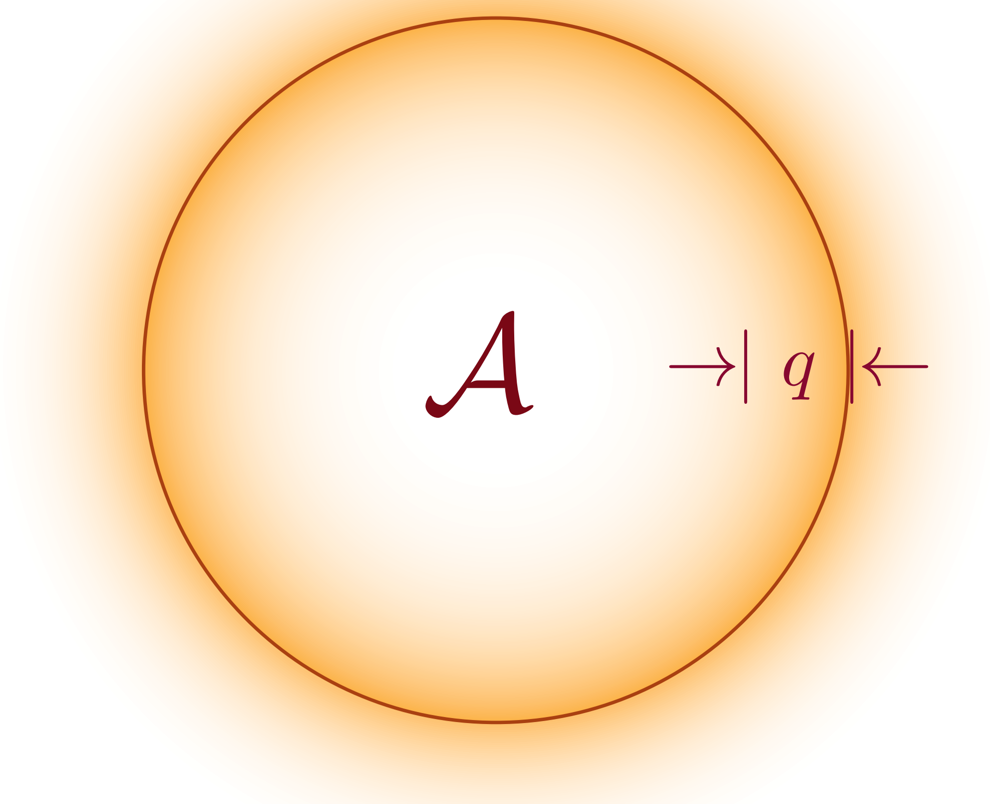

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- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\rm eff}/z}$ with $d_{\rm eff} = 1$.

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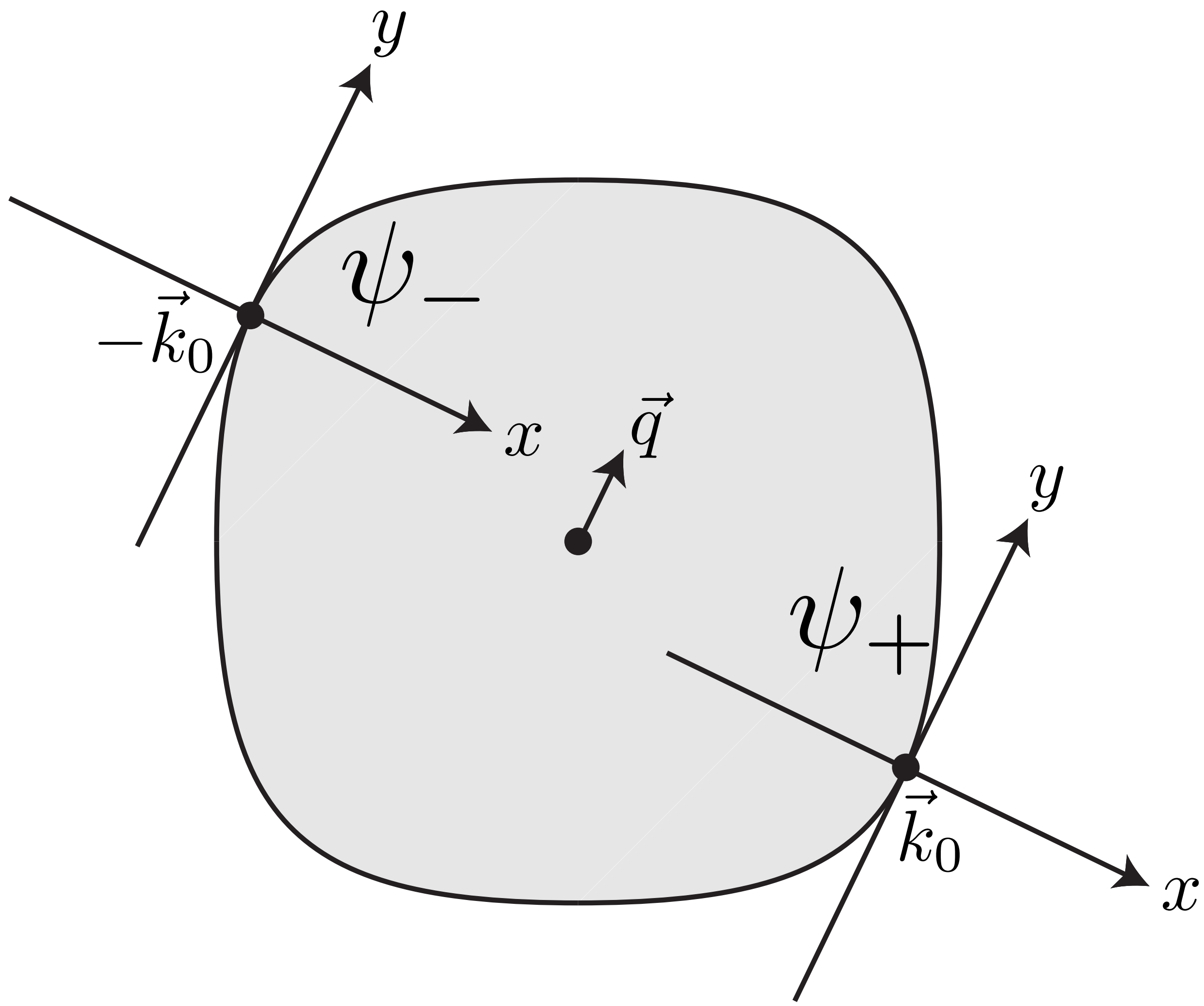
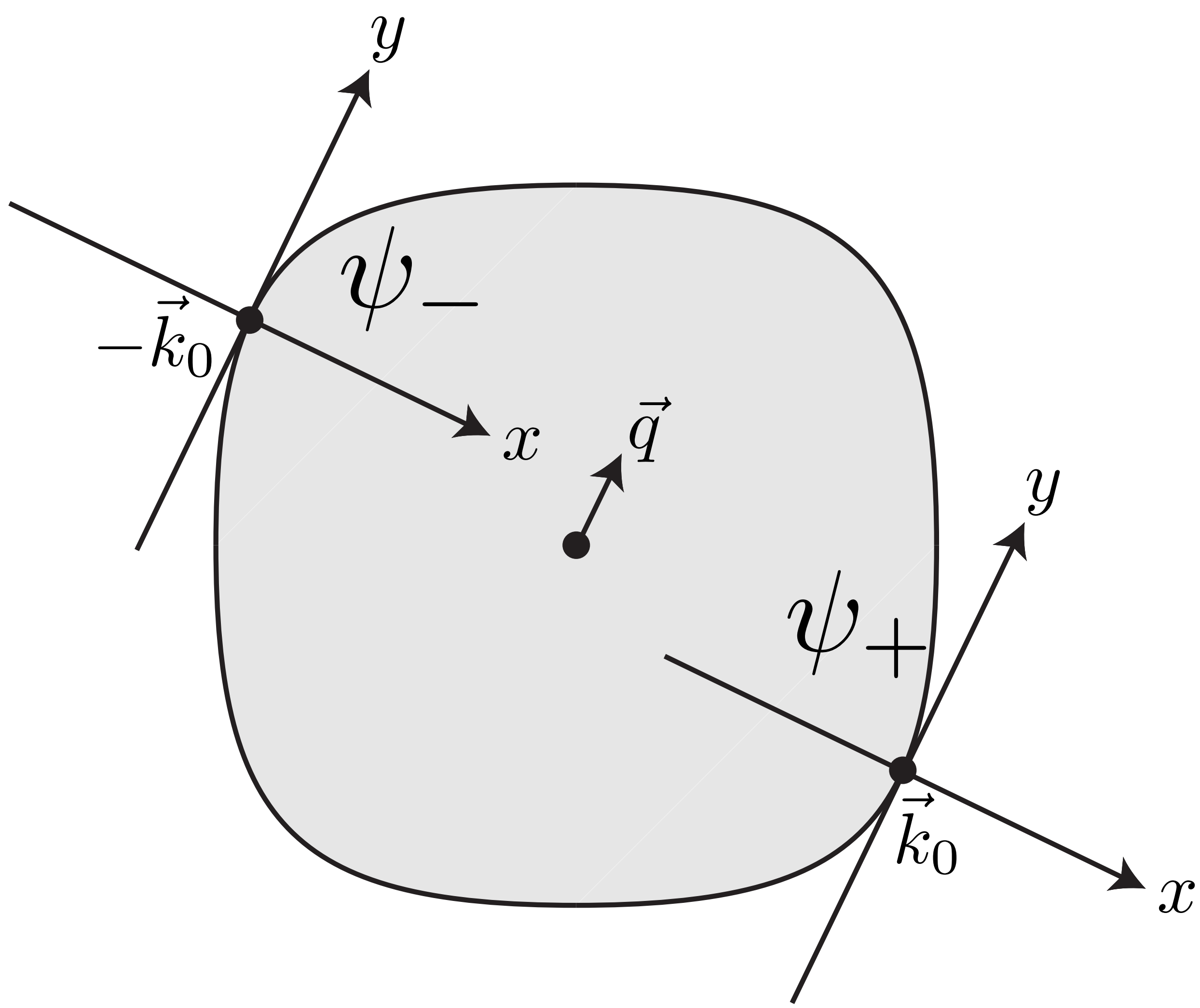
Field theory of this strange metal



- Gauge fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.
- Expand fermion kinetic energy at wavevectors about $\vec{k}_0$.
- In Landau gauge, only need the component of the gauge field, a, orthogonal to $\vec{q}$.

Field theory of this strange metal



$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - a\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

Simple scaling argument for $z = 3/2$.

Field theory of this strange metal

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Simple scaling argument for $z = 3/2$.

Perturbative computations show that the $\psi_\pm^\dagger \partial_\tau \psi_\pm$ terms are irrelevant

Field theory of this strange metal

$$\mathcal{L}_{\rm scaling} = \psi_+^\dagger \left(-i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(+i\partial_x - \partial_y^2\right) \psi_- - g\, a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2} \left(\partial_y a\right)^2$$

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Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} a &\to a\,s^{(2z+1)/4} \\ \psi &\to \psi\,s^{(2z+1)/4} \\ g &\to g\,s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

Fermions and bosons coupled to a gauge field

$$\begin{aligned}\mathcal{L} &= f^\dagger \left(\partial_\tau - iA_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f \\ &+ b^\dagger \left(\partial_\tau + iA_\tau - \frac{(\nabla + i\mathbf{A})^2}{2m_b} - \mu_b \right) b + s|b|^2 - g\, b^\dagger f^\dagger f b + \ldots\end{aligned}$$

Fermions and bosons coupled to a gauge field

$$
\begin{aligned}
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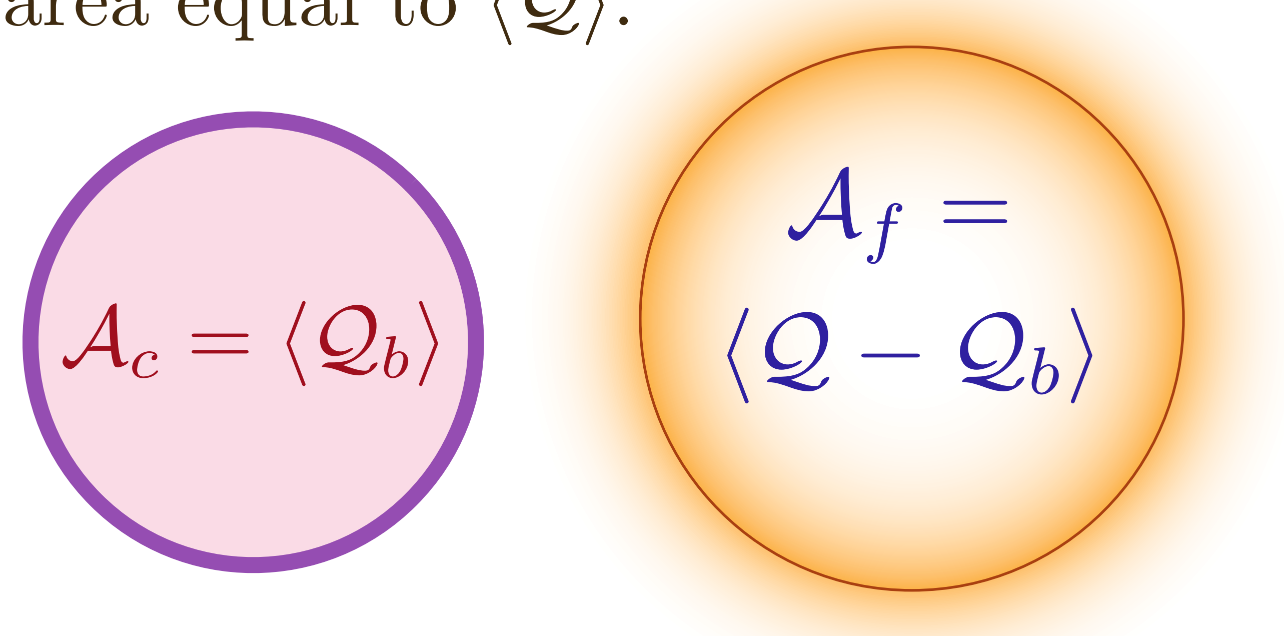
\end{aligned}
$$

Another strange metal: the fractionalized Fermi liquid (FL*)

Bosons can bind with fermions to form a gauge-neutral fermion $c \sim b\, f$. The result FL* phase has *partial confinement* and 2 Fermi surfaces: the gauge-neutral Fermi surface of c, and the gauge-charged Fermi surface of f. They enclose a *combined* area equal to $\langle \mathcal{Q} \rangle$.



T. Senthil, M. Vojta, and S. Sachdev, *Physical Review* B **69**, 035111 (2004)

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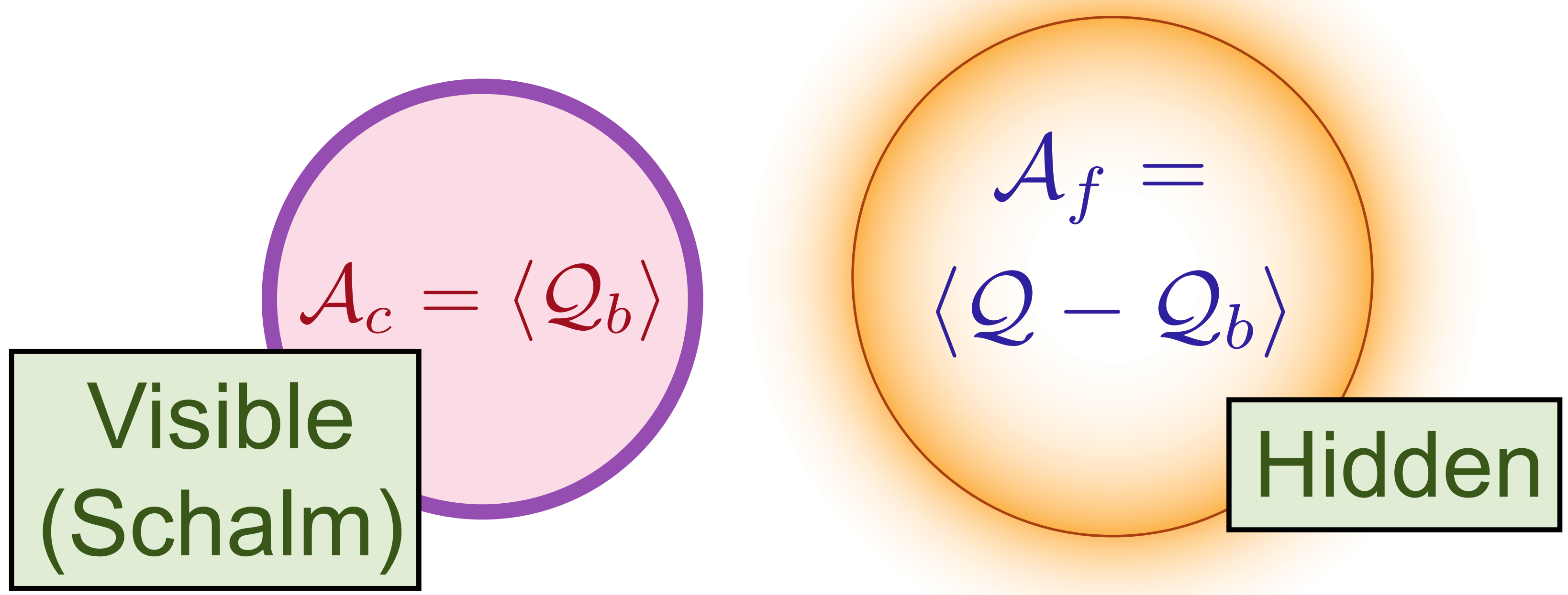
Another strange metal: the fractionalized Fermi liquid (FL*)

In holography:
the c Fermi surface is that of the "probe" fermion;
the fractionalized f Fermi surface is "hidden" past the horizon.



S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)

Kondo lattice model

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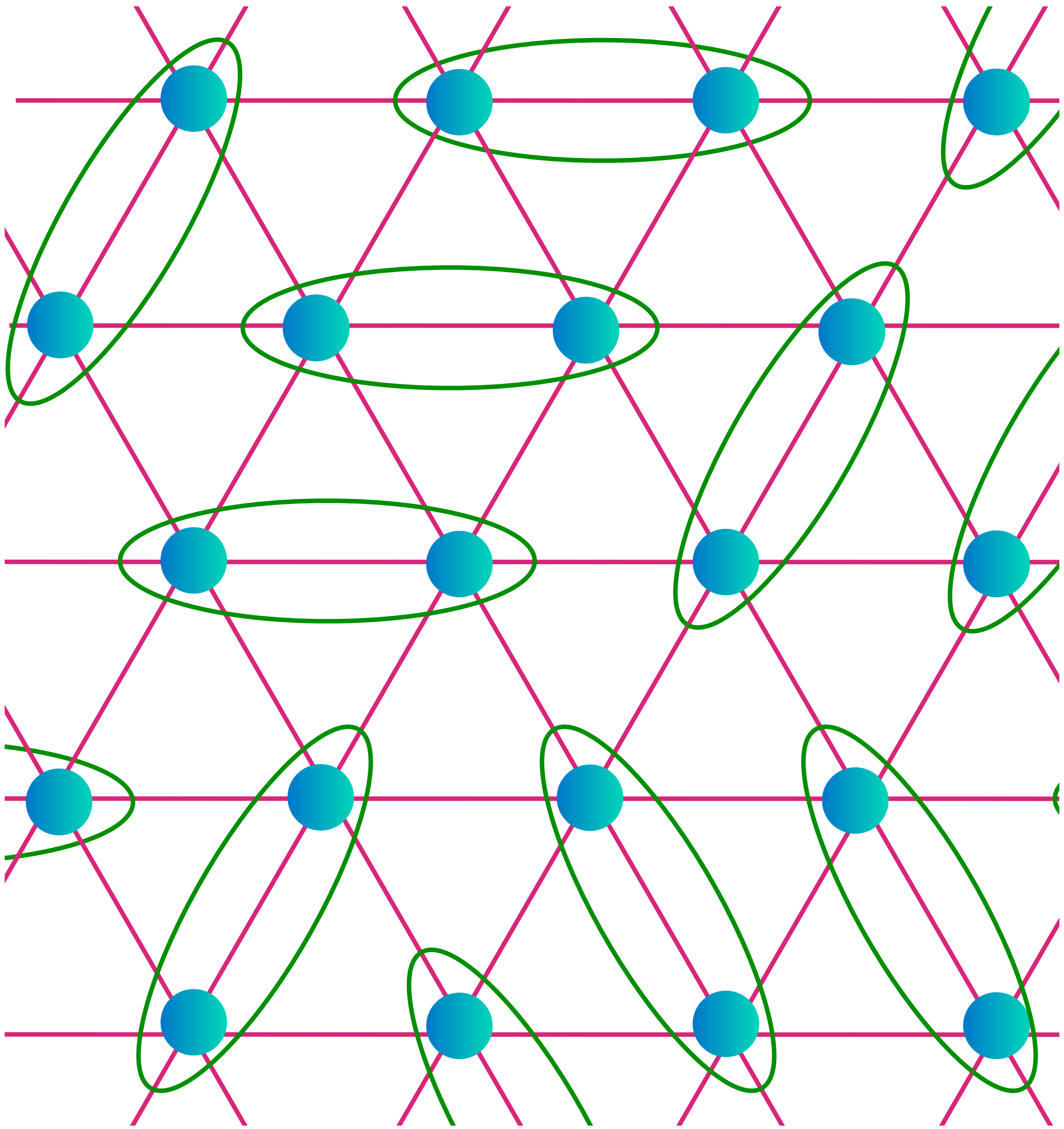

Spin liquid of *f* electrons

Fermi surface of *c* conduction electrons

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Strange metals

A. Field theory

B. Holography

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d.
  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface.

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- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d.
  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface.

- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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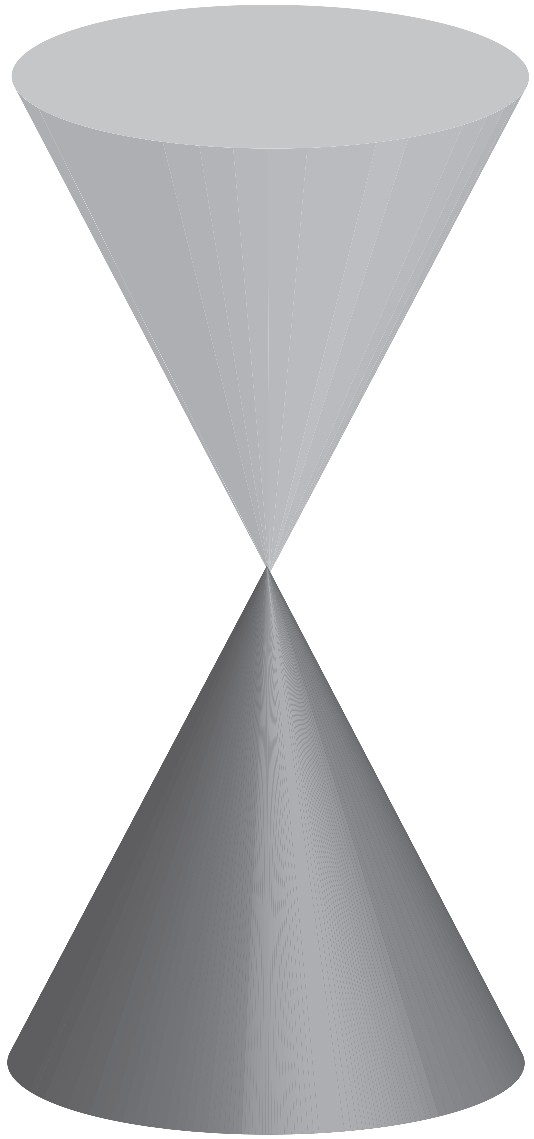
- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

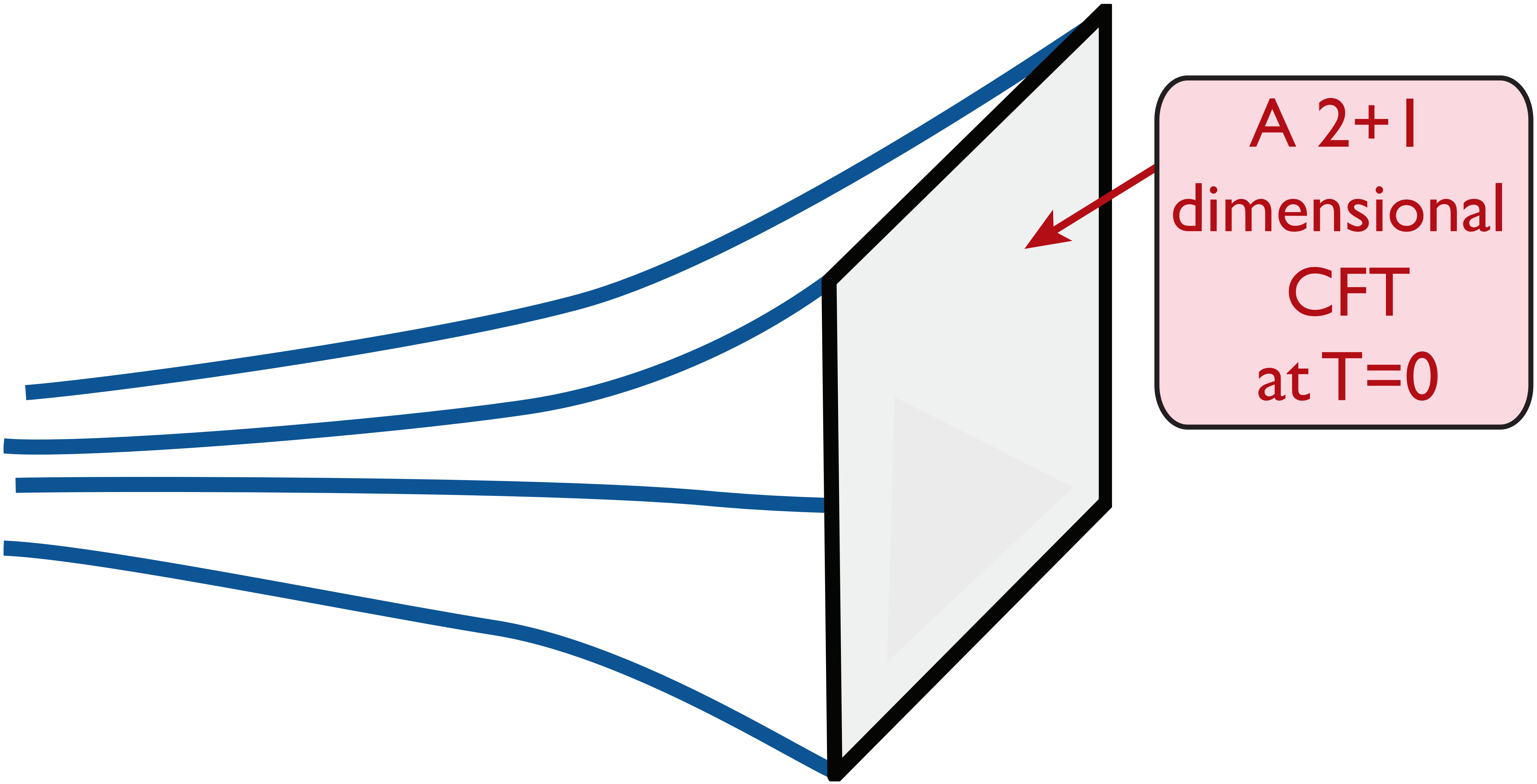
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Begin with a CFT



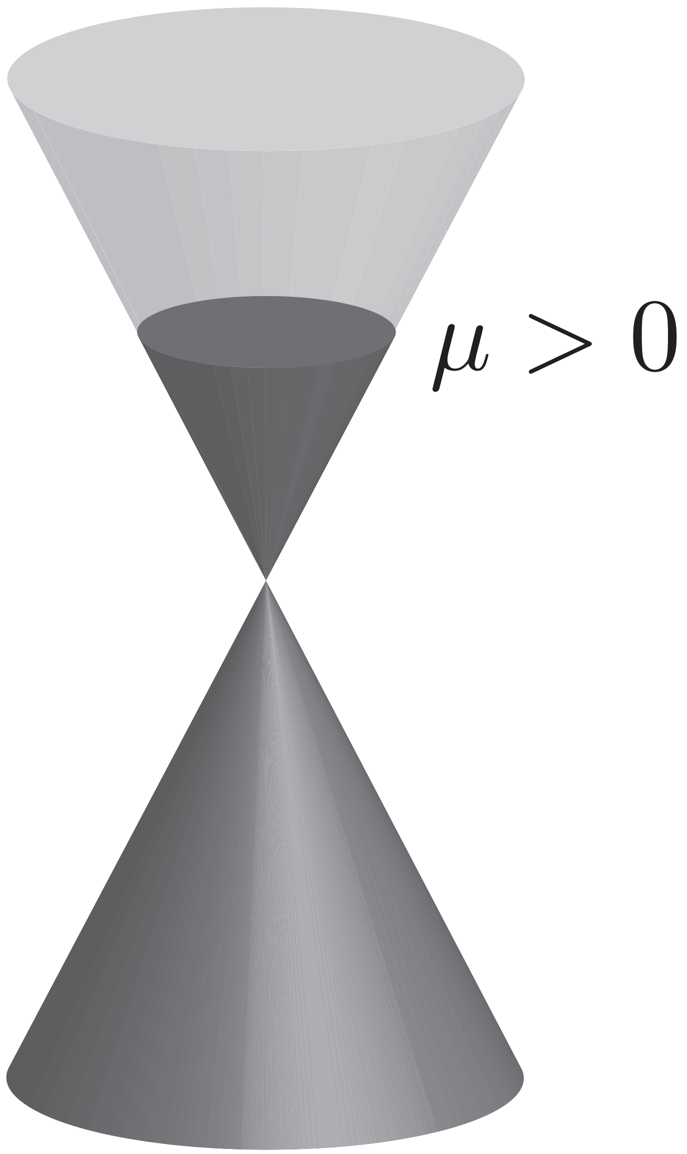
Dirac fermions + gauge field + ...

Holographic representation: AdS$_4$

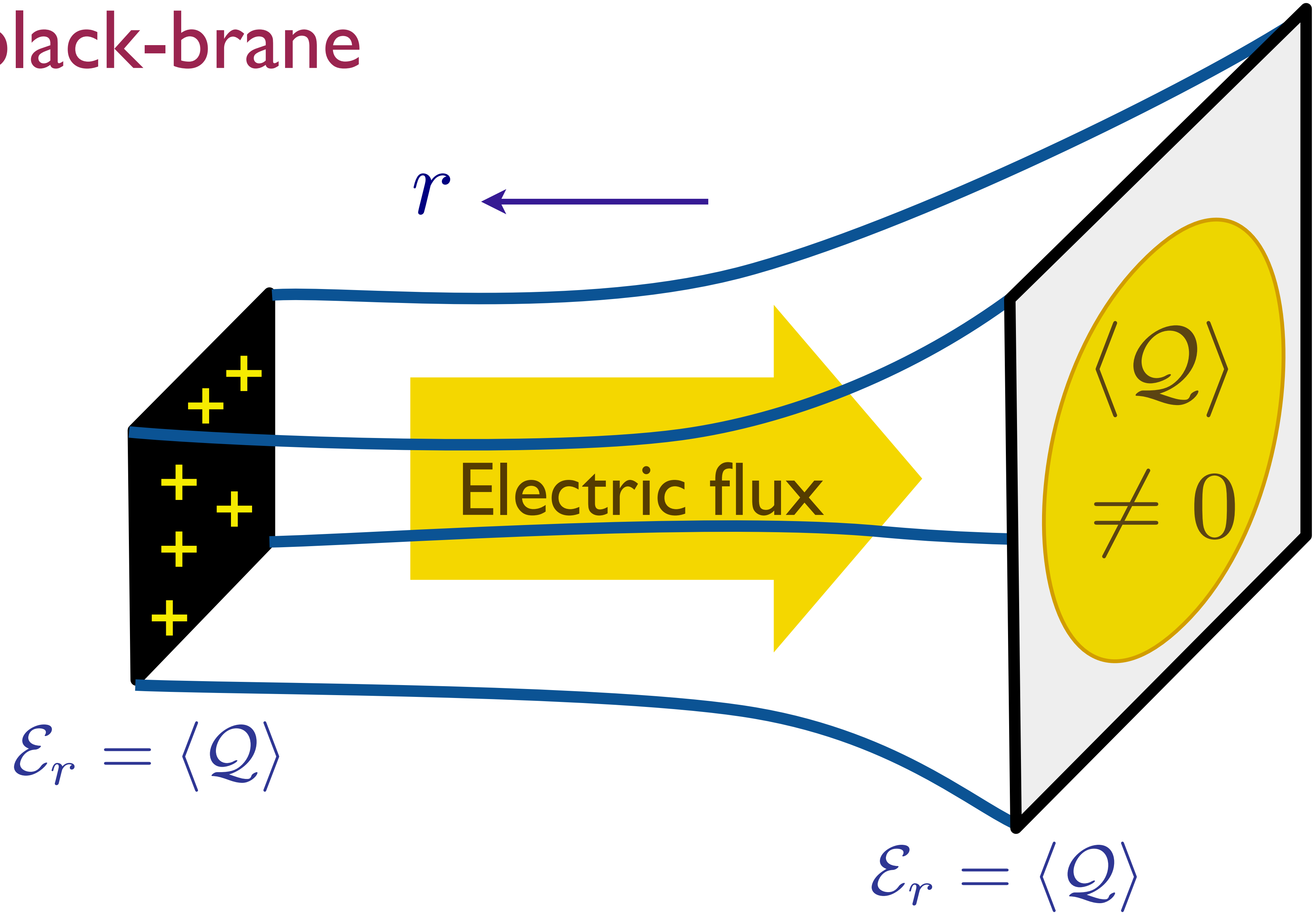


$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Apply a chemical potential to the “deconfined” CFT



The Maxwell-Einstein theory of the applied chemical potential yields a AdS_4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

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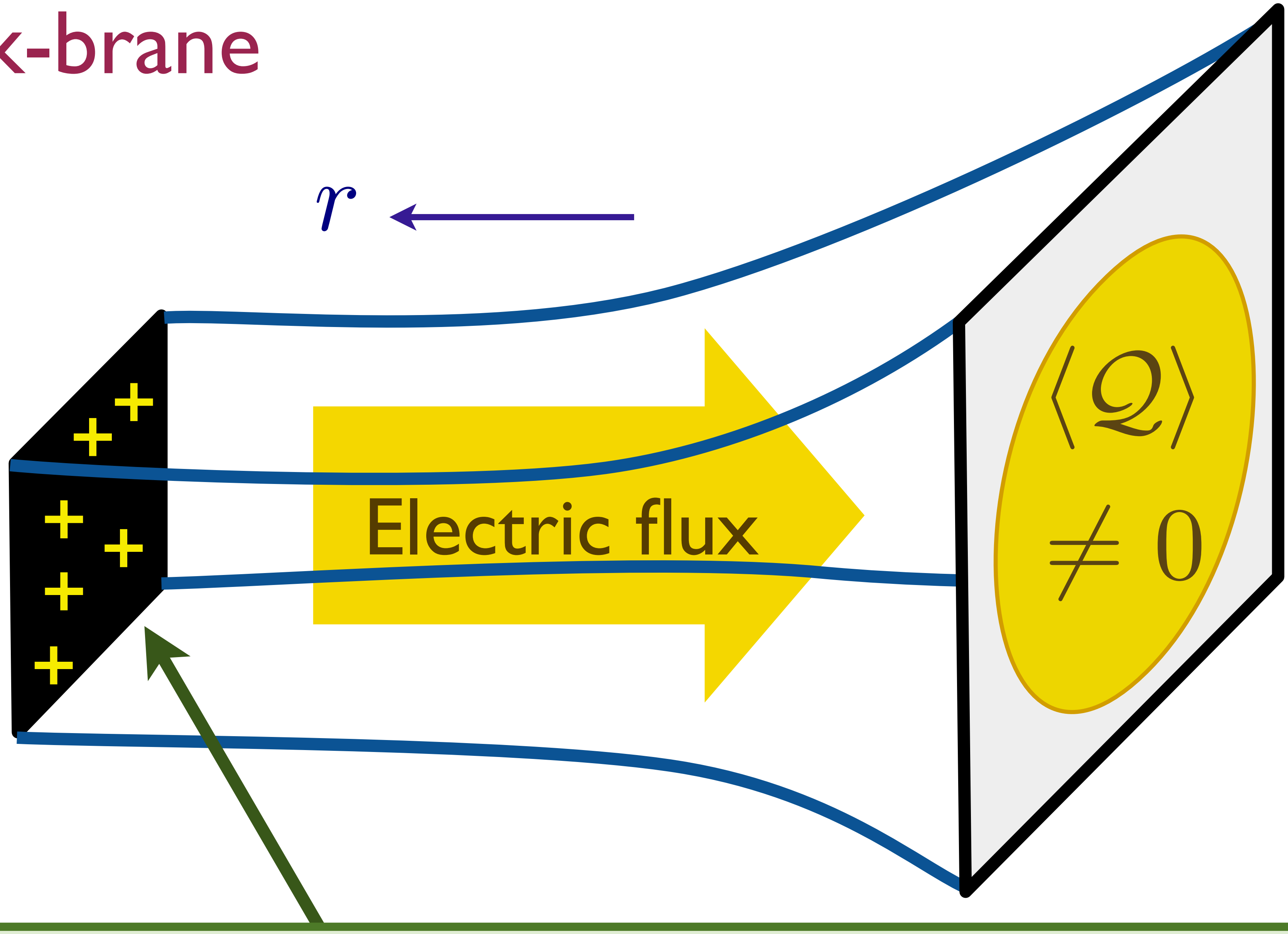


At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\text{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Artifacts of $AdS_2 \times R^2$

- Corresponds to $\theta \to d$ and $z \to \infty$. This implies non-zero entropy density at $T = 0$, and "volume" law for entanglement entropy.

- Green's function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface

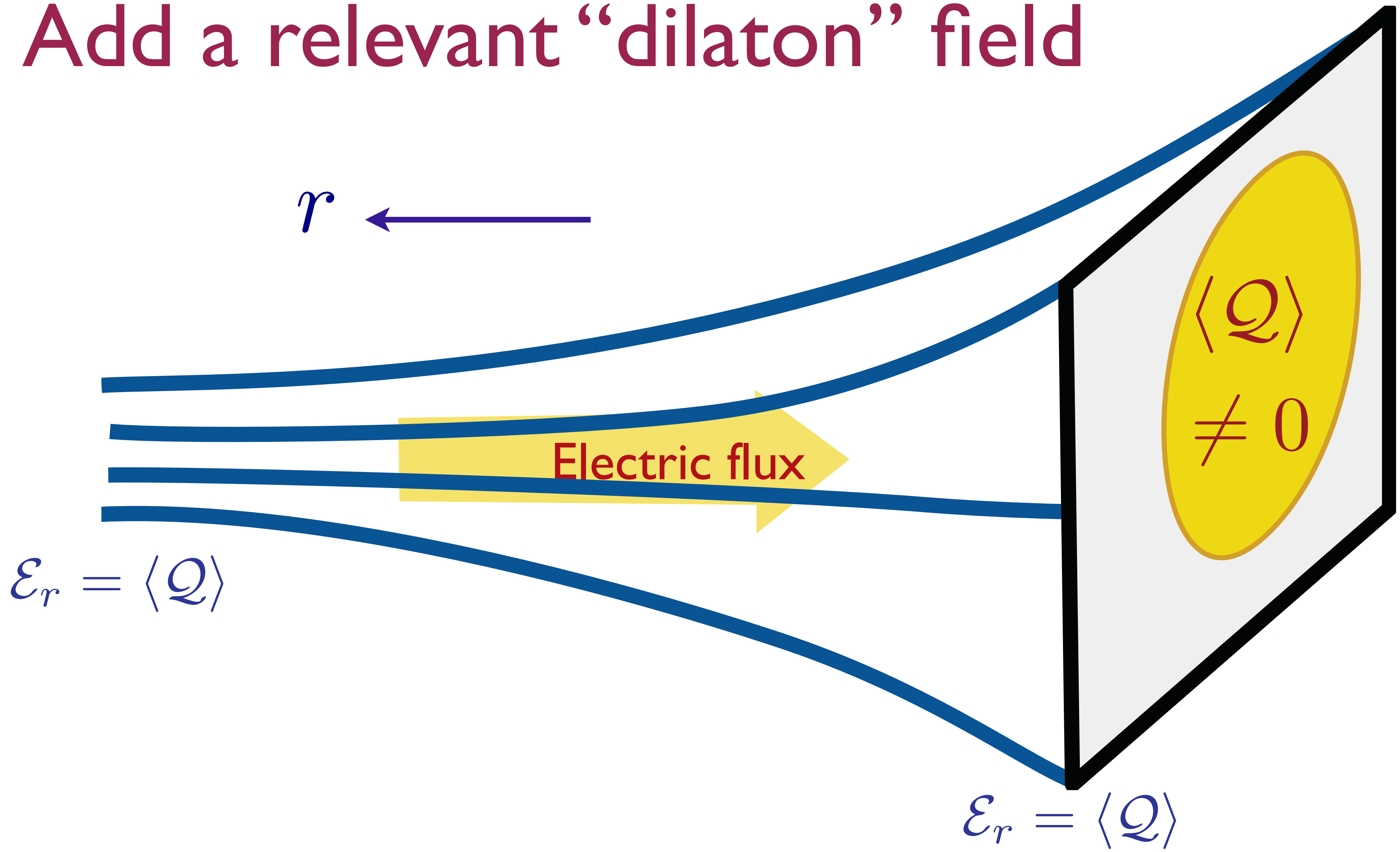
- Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with "hidden Fermi surfaces" of gauge-charged particles (the *quarks*).

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant "dilaton" field



$$\mathcal{S} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2}\right) - \frac{Z(\Phi)}{4e^2}F_{ab}F^{ab}\right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, arXiv:1105.1162 [hep-th].

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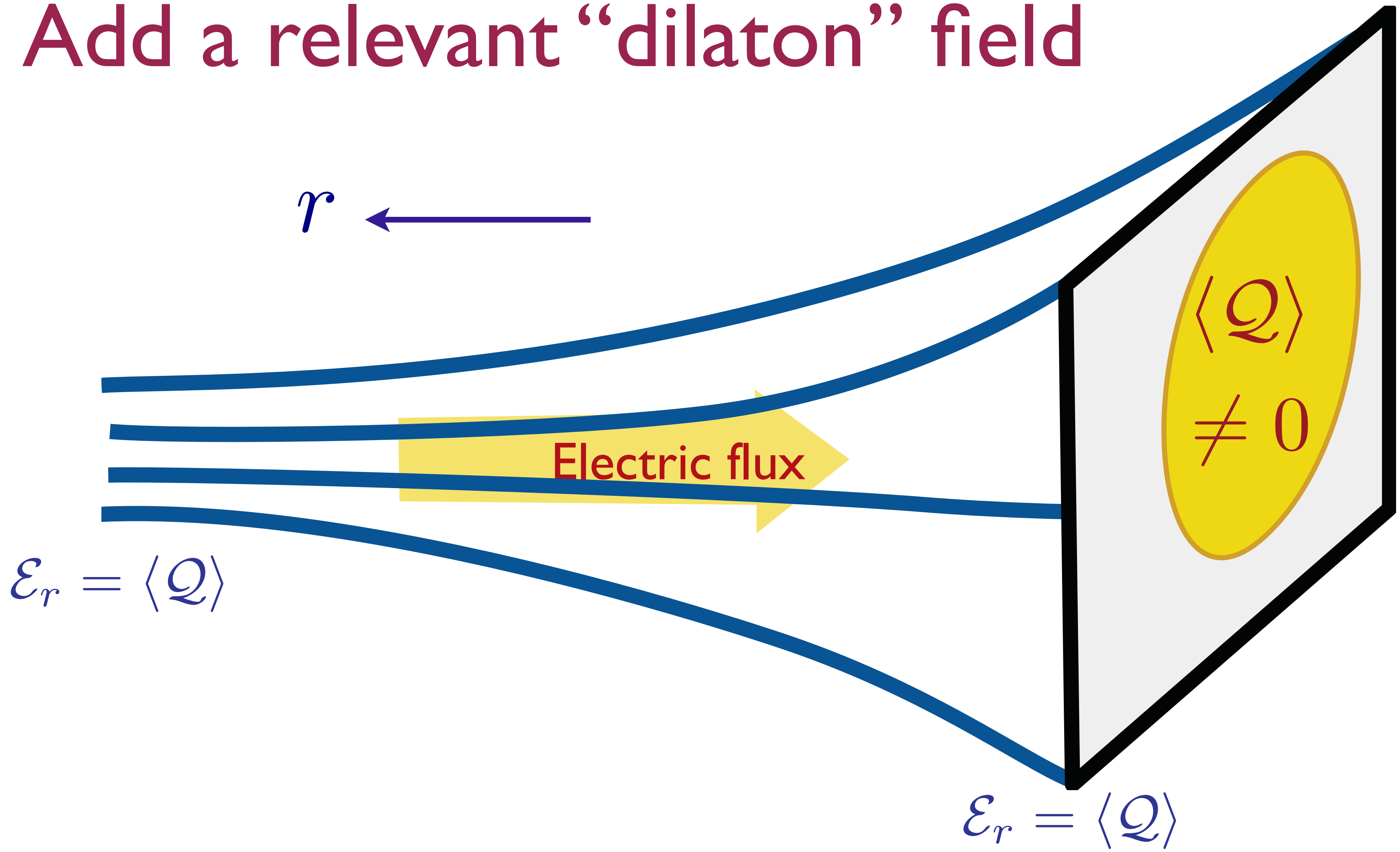

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This is a "bosonization" of the Fermi surface

Holographic theory of a non-Fermi liquid (NFL)

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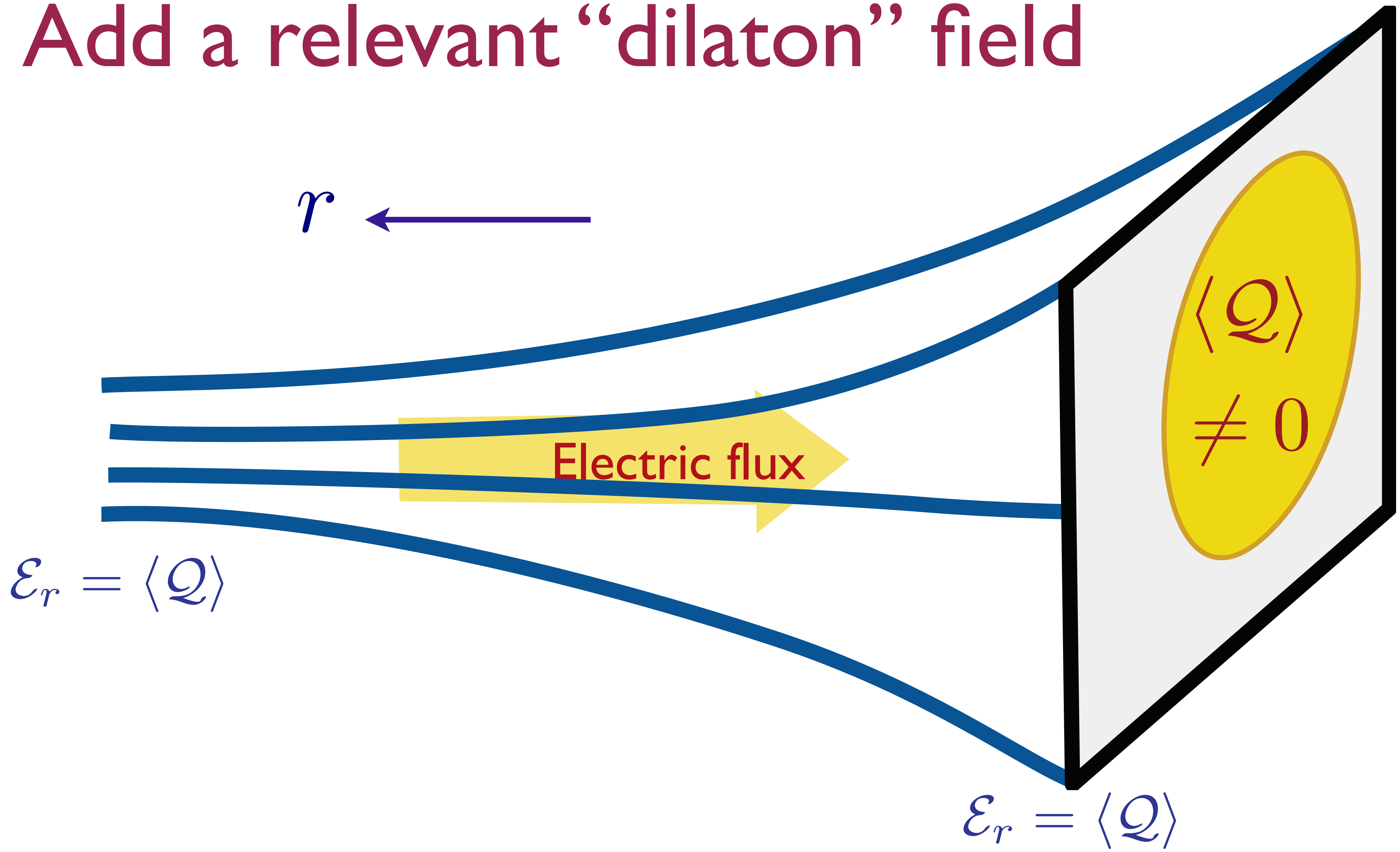

Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$

with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^{\delta}$, $\Phi(r) \sim \ln(r)$ as $r \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
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Holographic theory of a non-Fermi liquid (NFL)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The $r \to \infty$ metric has the above form with

$$\begin{aligned}\theta &= \frac{d^2\beta}{\alpha + (d-1)\beta} \\ z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta)+\theta)^2}{d^2(d-\theta)\alpha^2}.\end{aligned}$$

Note $z \geq 1 + \theta/d$.

Holographic theory of a non-Fermi liquid (NFL)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The solution also specifies the missing numerical prefactors in the metric. In general, these depend upon the details on the UV boundary condition as $r \to 0$. However, the coefficient of dx_i^2/r^2 turns out to be *independent* of the UV boundary conditions, and proportional to $\mathcal{Q}^{2\theta/(d(d-\theta))}$.

The square-root of this coefficient is the prefactor of the log divergence in the entanglement entropy for $\theta = d-1$.

Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

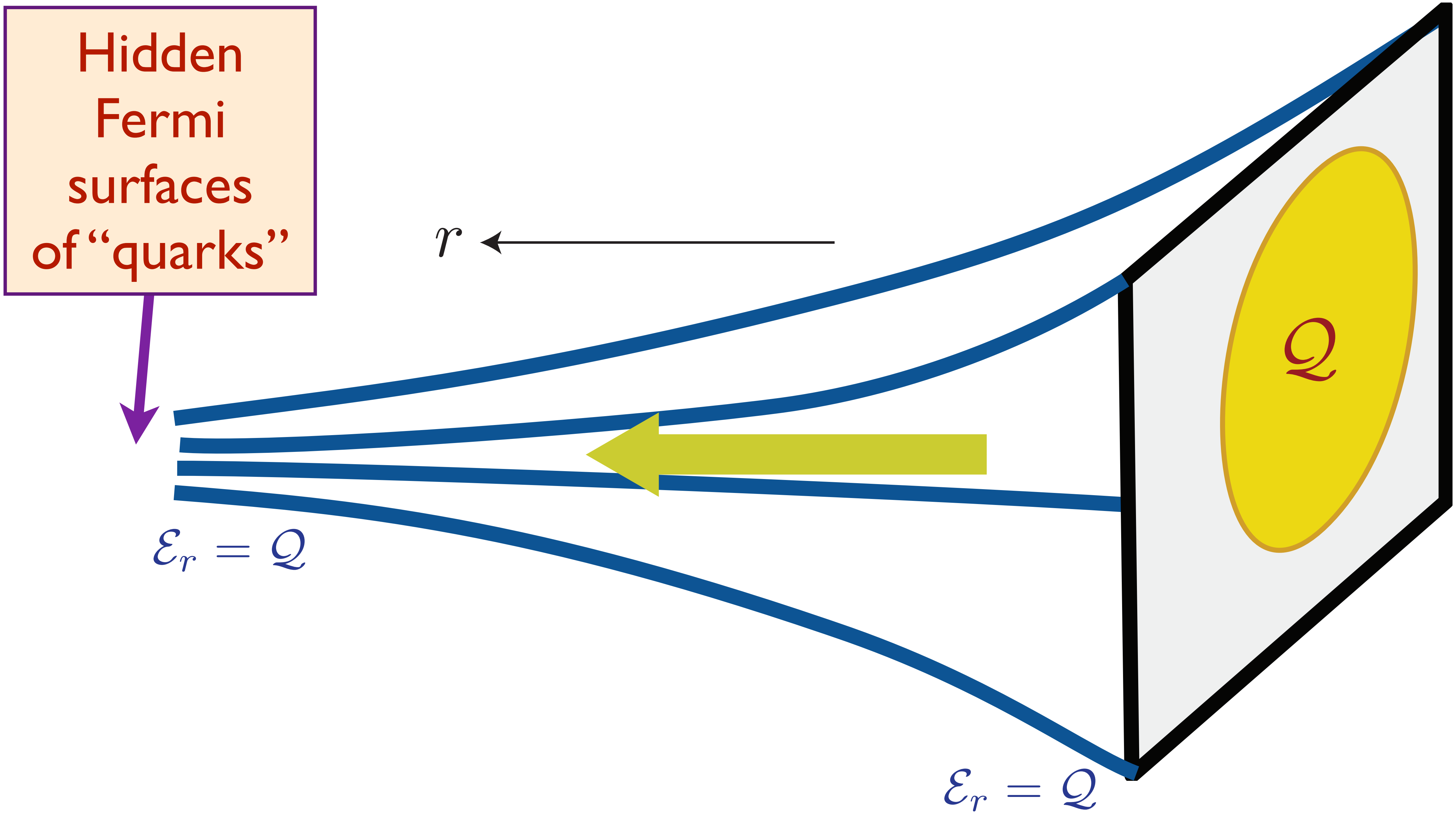
- The entanglement entropy has log-violation of the area law

$$S_E = \Xi\, \mathcal{Q}^{(d-1)/d}\Sigma\, \ln\left(\mathcal{Q}^{(d-1)/d}\Sigma\right).$$

where Σ is surface area of the entangling region, and Ξ is a dimensionless constant which is independent of all UV details, of $\mathcal{Q}$, and of any property of the entangling region. Note $\mathcal{Q}^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

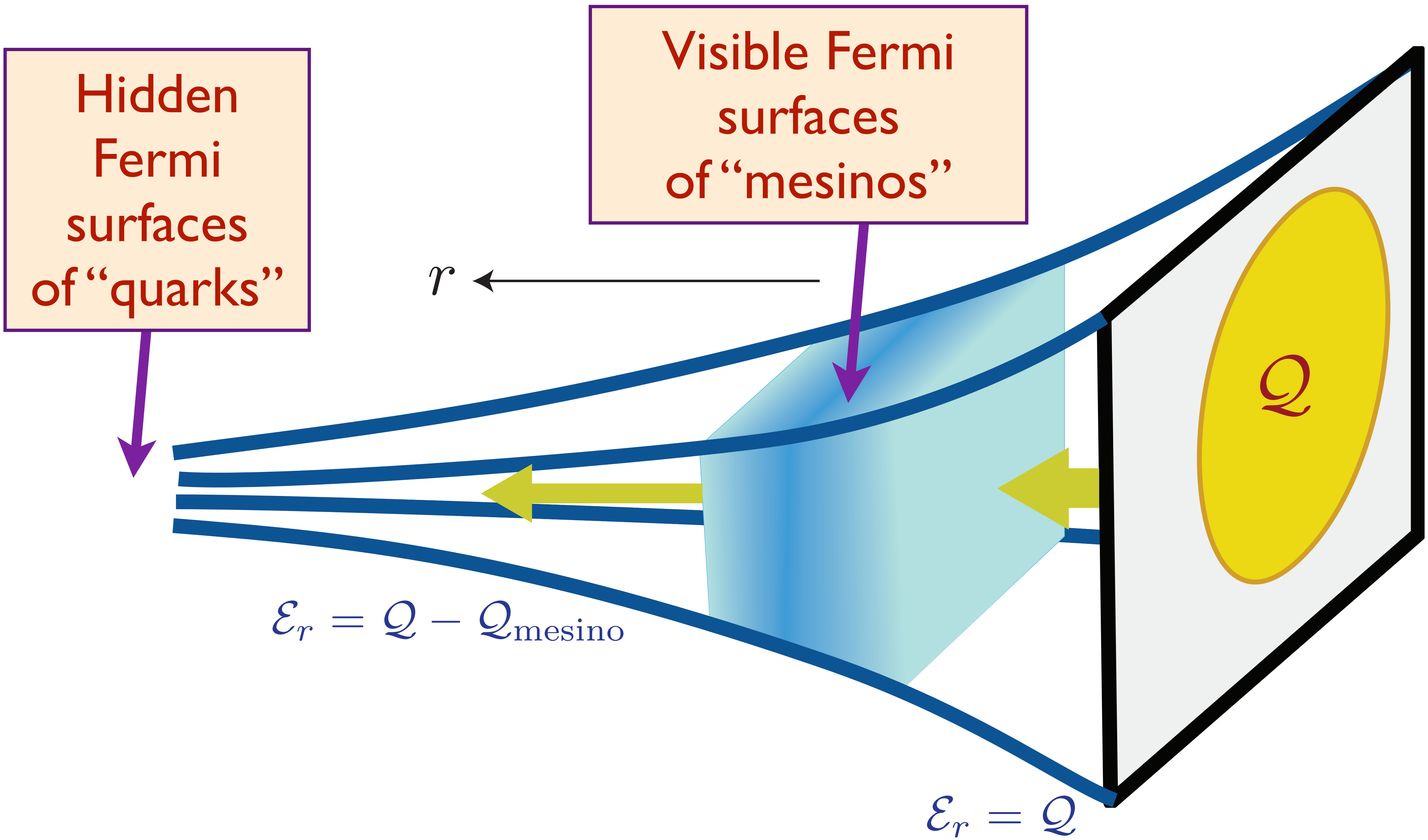
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holographic theory of a non-Fermi liquid (NFL)



Gauss Law and the "attractor" mechanism
$\Leftrightarrow$ Luttinger theorem on the boundary

Holographic theory of a fractionalized-Fermi liquid (FL*)



A state with *partial* confinement

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)
S. Sachdev, *Physical Review D* **84**, 066009 (2011)

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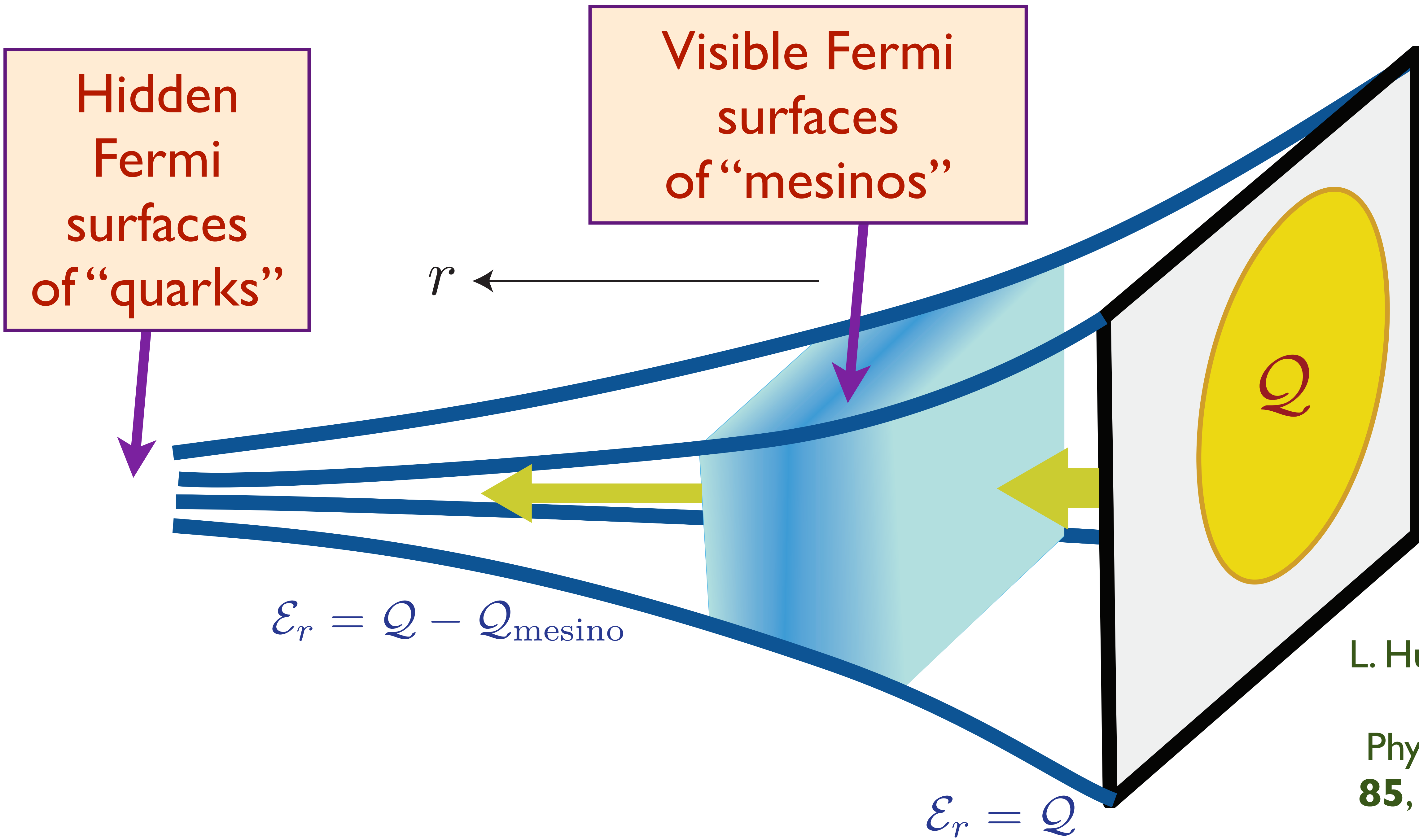
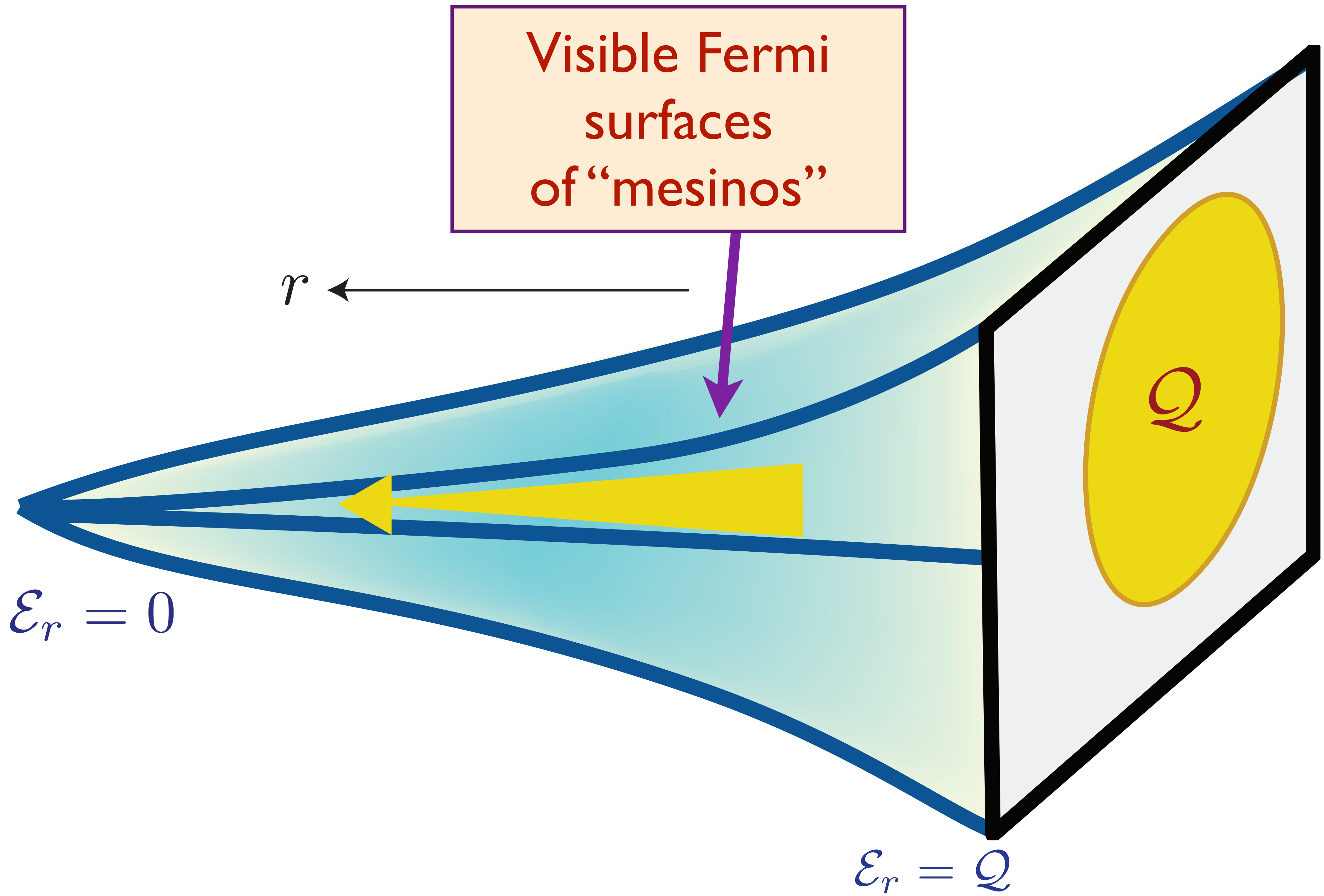


L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

- Now the entanglement entropy implies that the Fermi momentum of the hidden Fermi surface is given by $k_F^d \sim \mathcal{Q} - \mathcal{Q}_{\rm mesino}$, just as expected by the extended Luttinger relation. Also the probe fermion quasiparticles are sharp for $\theta = d - 1$, as expected for a FL* state.

Holographic theory of a Fermi liquid (FL)



- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)

Conclusions

Compressible quantum matter

- Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

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- Evidence for Luttinger theorem in prefactor of S_E.

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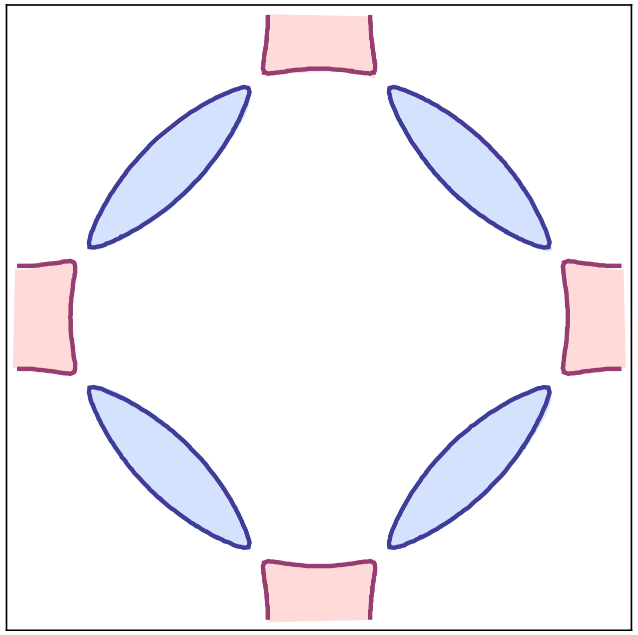
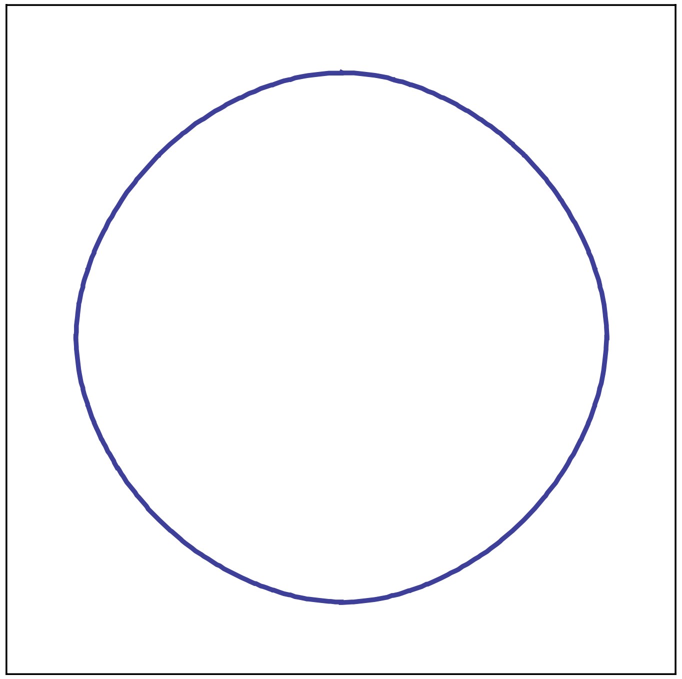
- Hidden Fermi surfaces can co-exist with Fermi surfaces of mesinos, leading to a state with *partial confinement*: the fractionalized Fermi liquid (FL*)

Quantum phase transition with Fermi surface reconstruction



$\langle \vec{\varphi} \rangle \neq 0$

Metal with electron and hole pockets

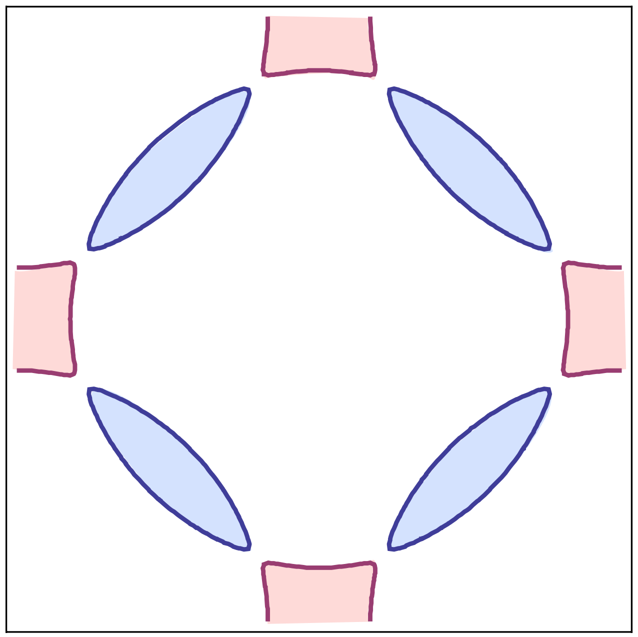


$\langle \vec{\varphi} \rangle = 0$

Metal with "large" Fermi surface

Pnictides, electron-doped cuprates ....

Proposed phase diagram for the hole-doped cuprates



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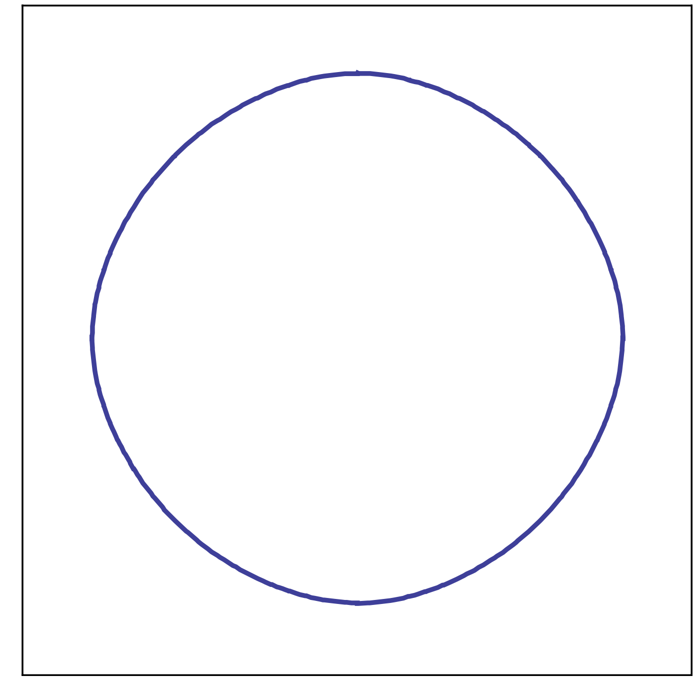
Metal with electron and hole pockets

Electron and/or hole Fermi pockets form in "local" SDW order, but quantum fluctuations destroy long-range SDW order

$\langle \vec{\varphi} \rangle = 0$

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface



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M. Punk and S. Sachdev, arXiv:1202.4023

E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).