Statistical mechanics of strange metals and black holes

May 17, 24, 31, June 7, 2022

Subir Sachdev

Talk online: qpt.physics.harvard.edu/talks

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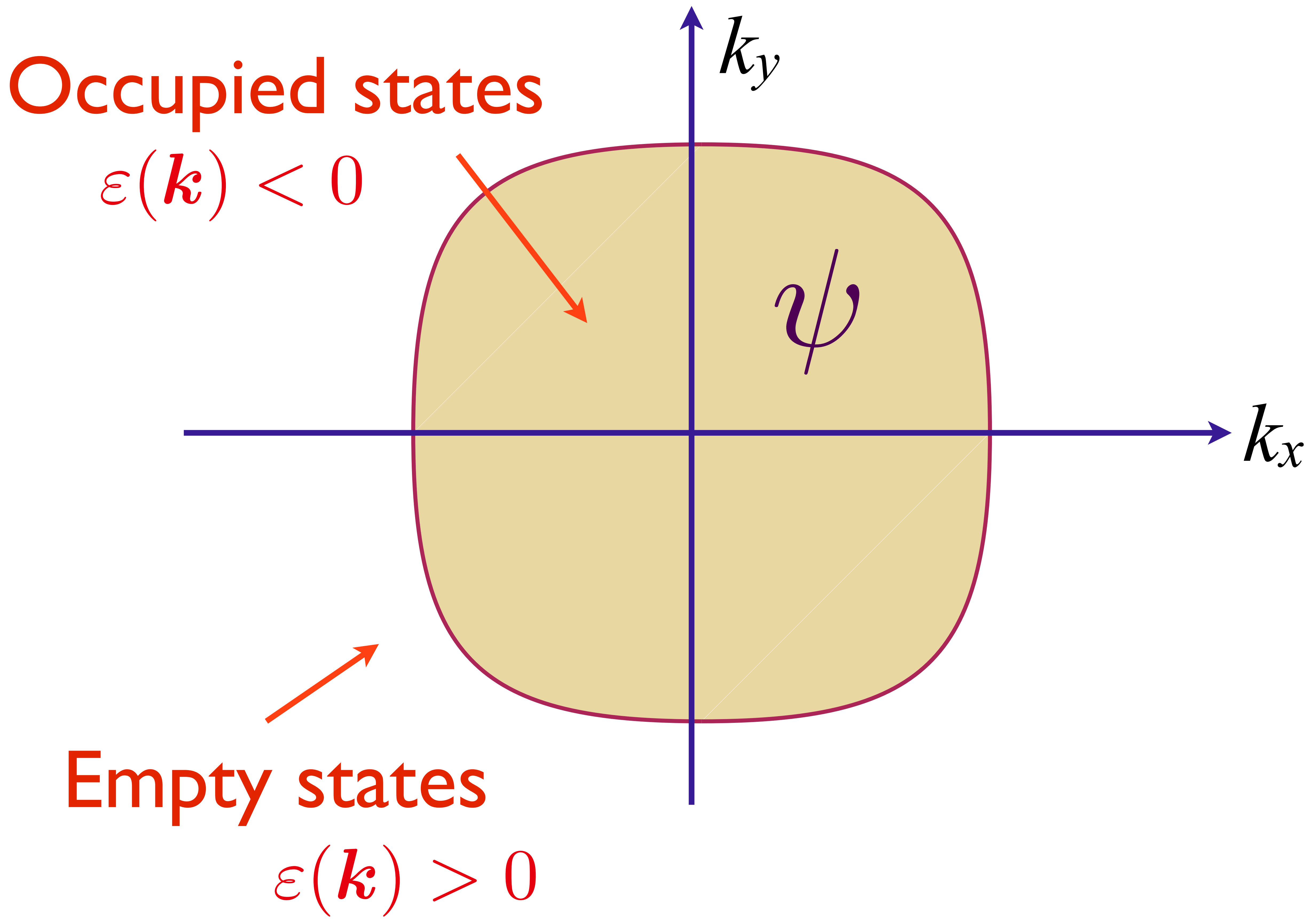
Yukawa-SYK models and a large N theory of a critical Fermi surface in two spatial dimensions

Fermi surface coupled to a critical boson



$+$

a
critical
boson
ϕ

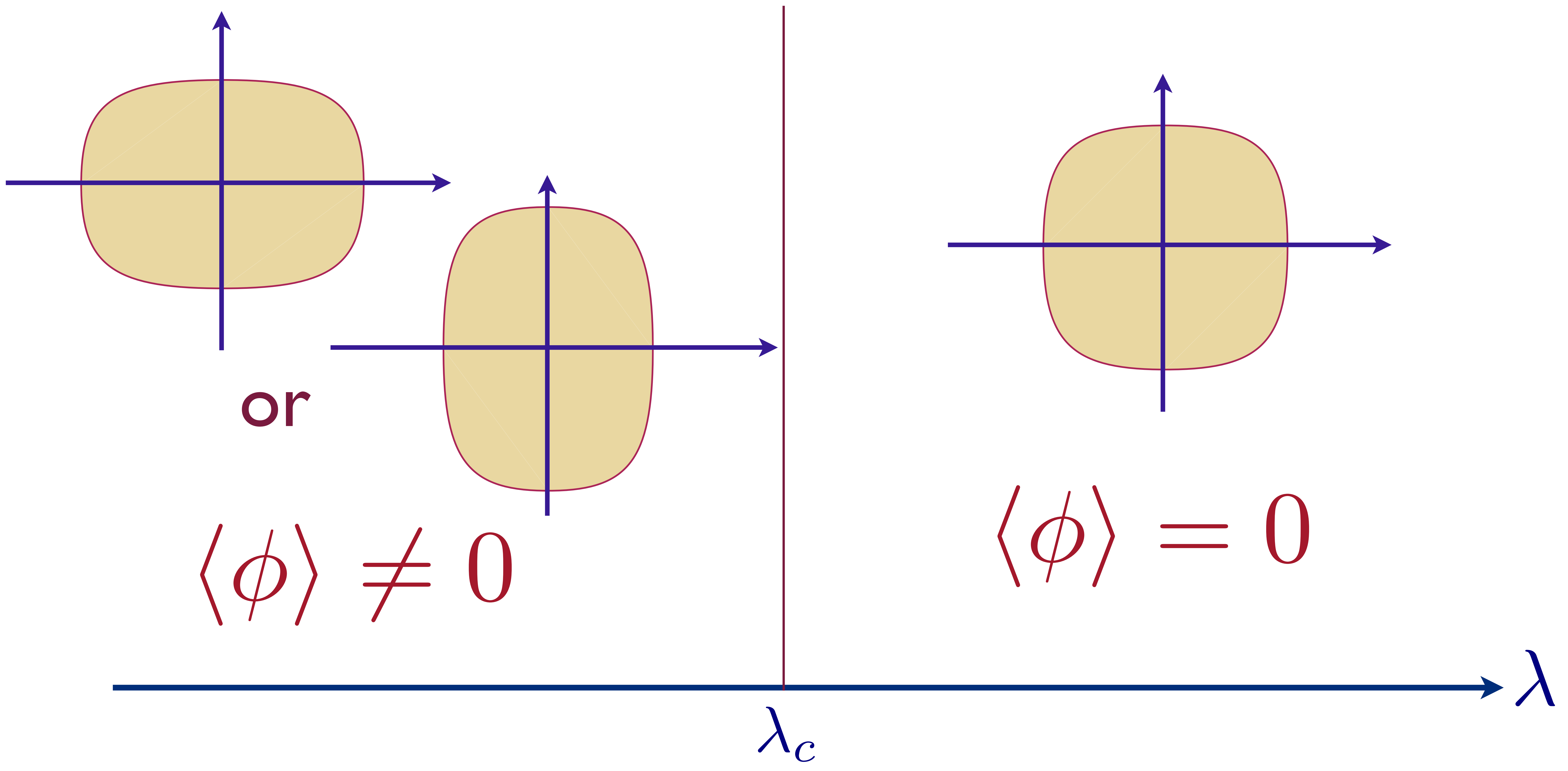
Yukawa-SYK models

$$H = \sum_{ij} t_{ij} \psi_i^\dagger \psi_j + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_\ell^2 \phi_\ell^2\right) + \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

Leads to fully self-consistent Migdal-Eliashberg equations $\Sigma_\psi \sim g^2 G_\psi G_\phi$, $\Sigma_\phi \sim g^2 G_\psi G_\psi$ in a SYK-like large N limit.

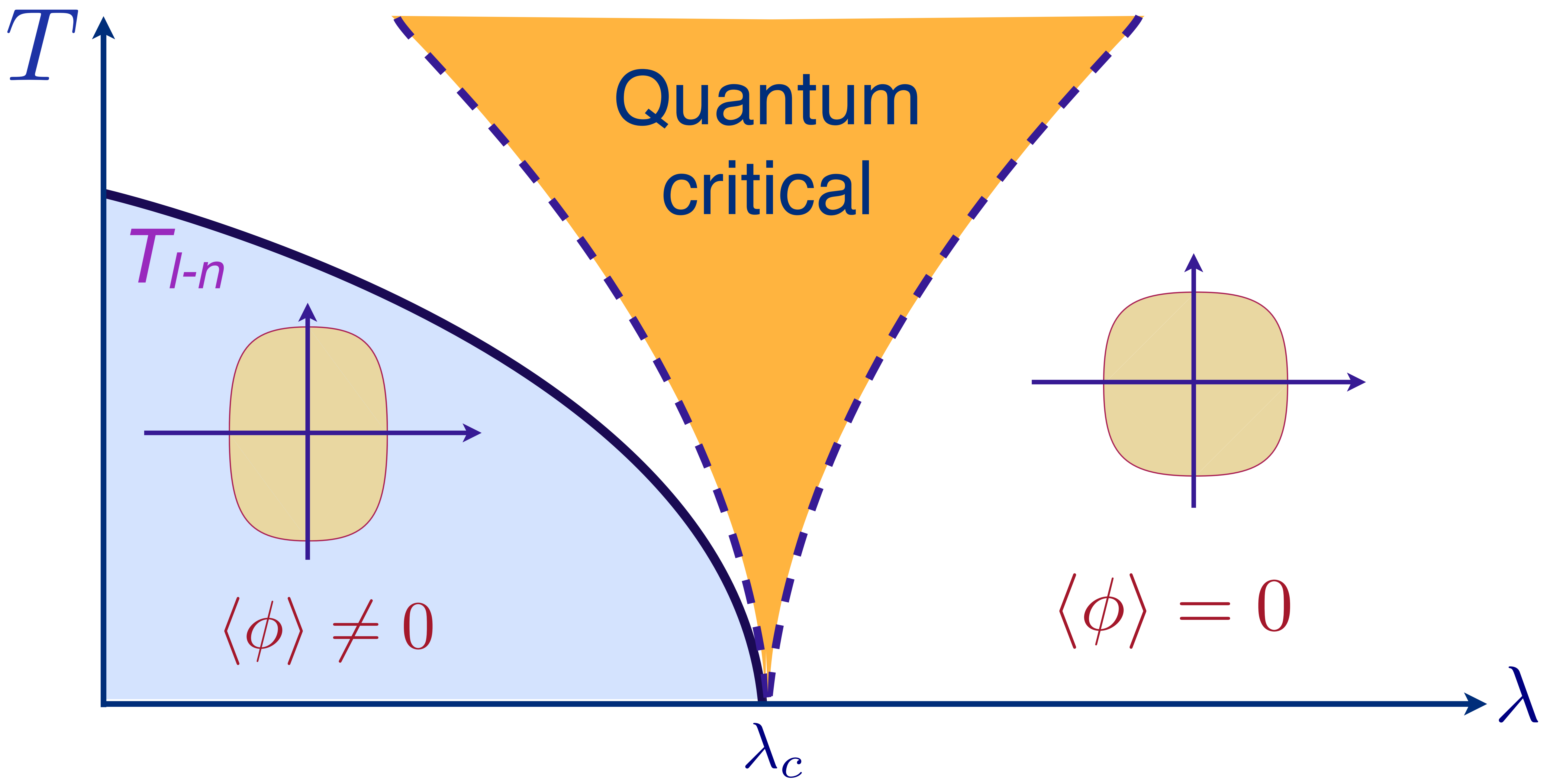
W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

Quantum criticality of Ising-nematic ordering in a metal



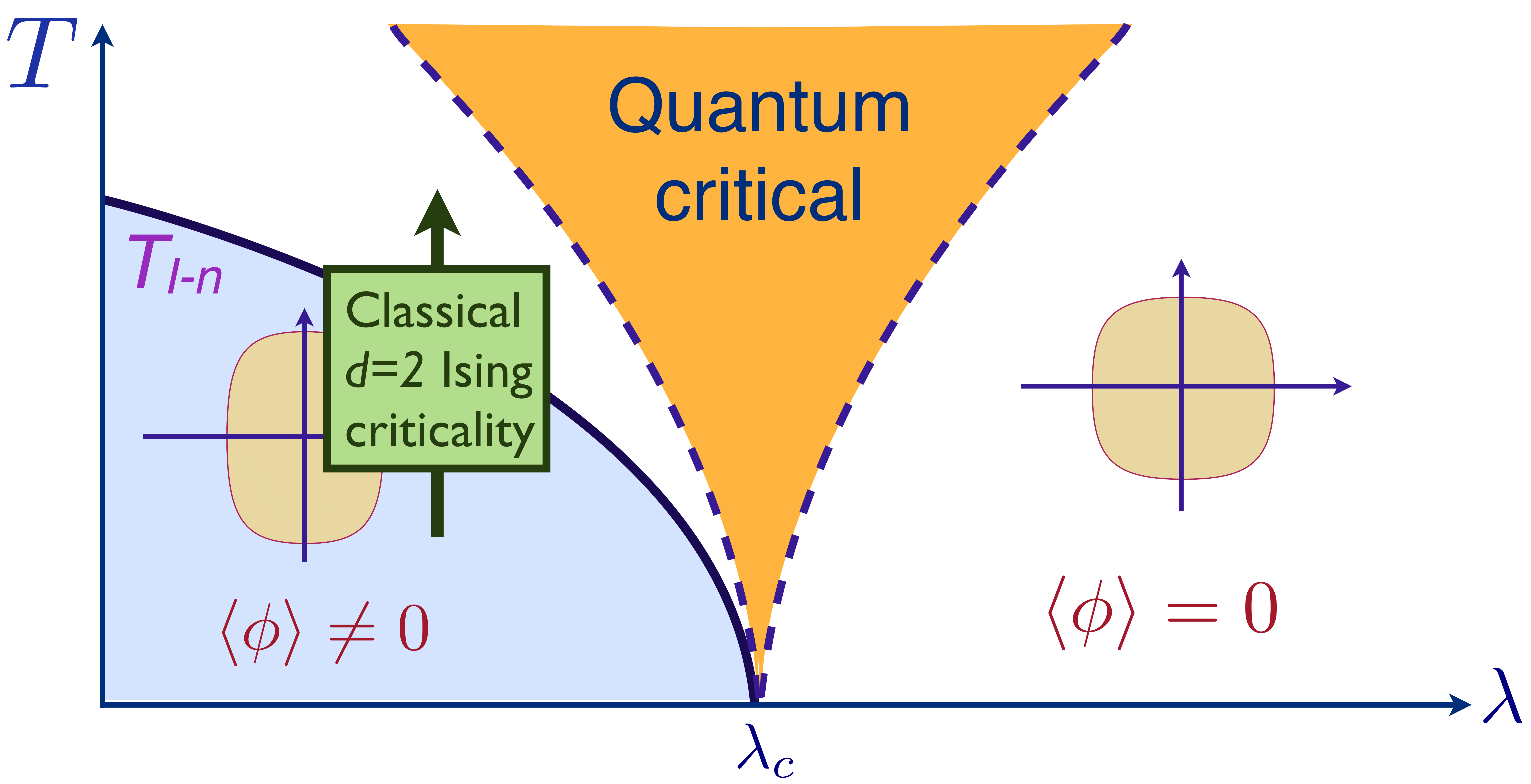
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



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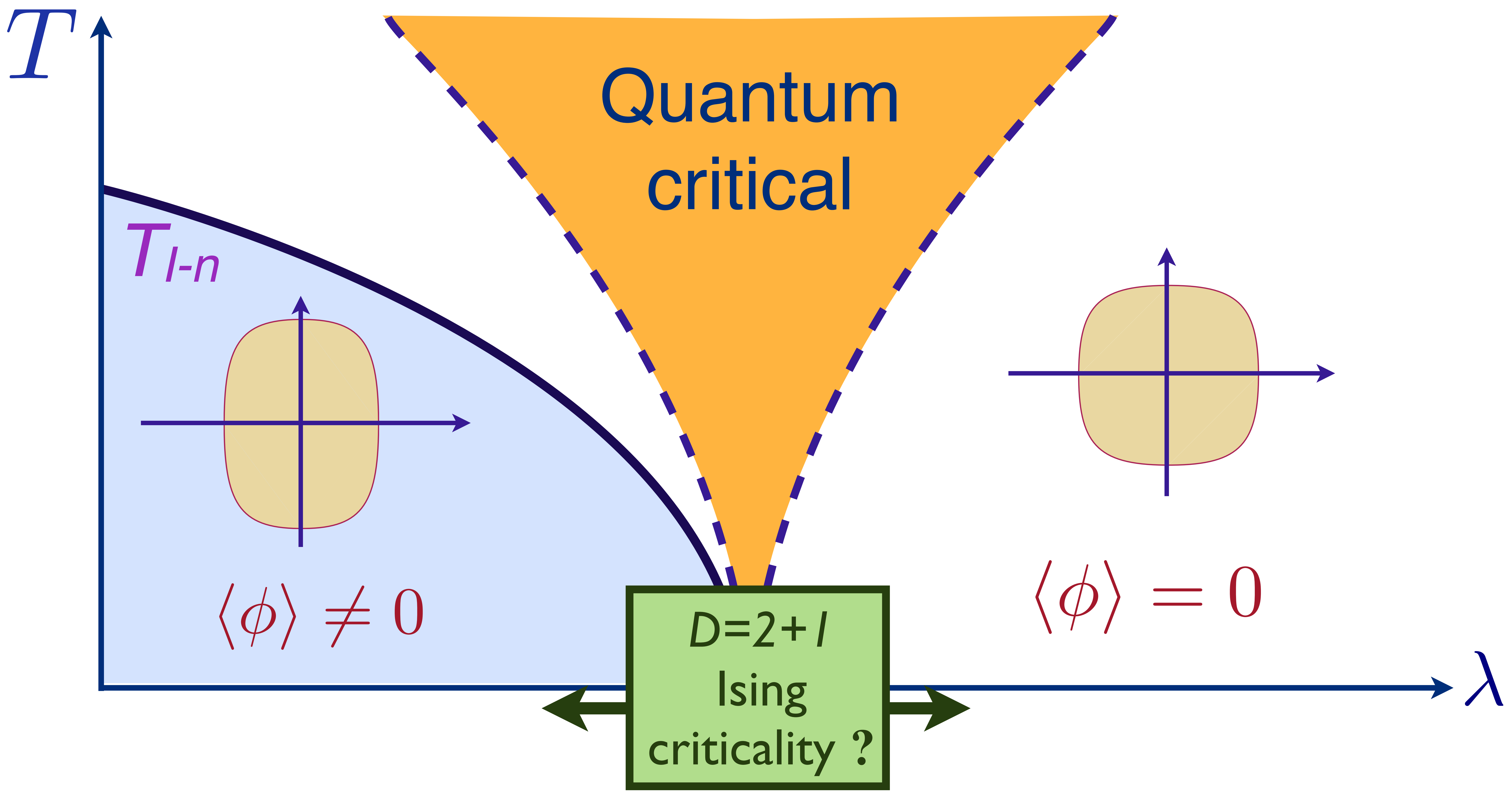

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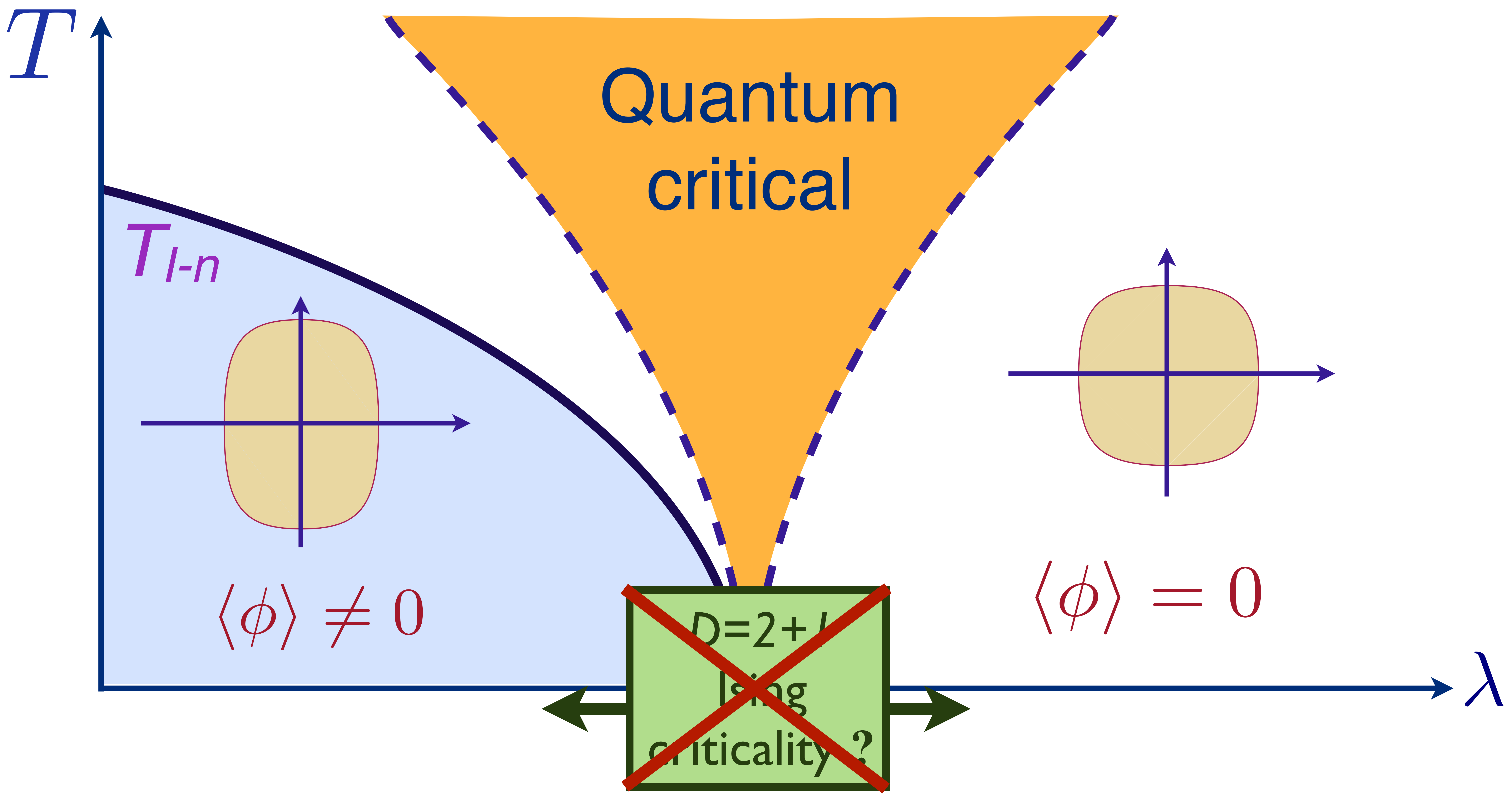

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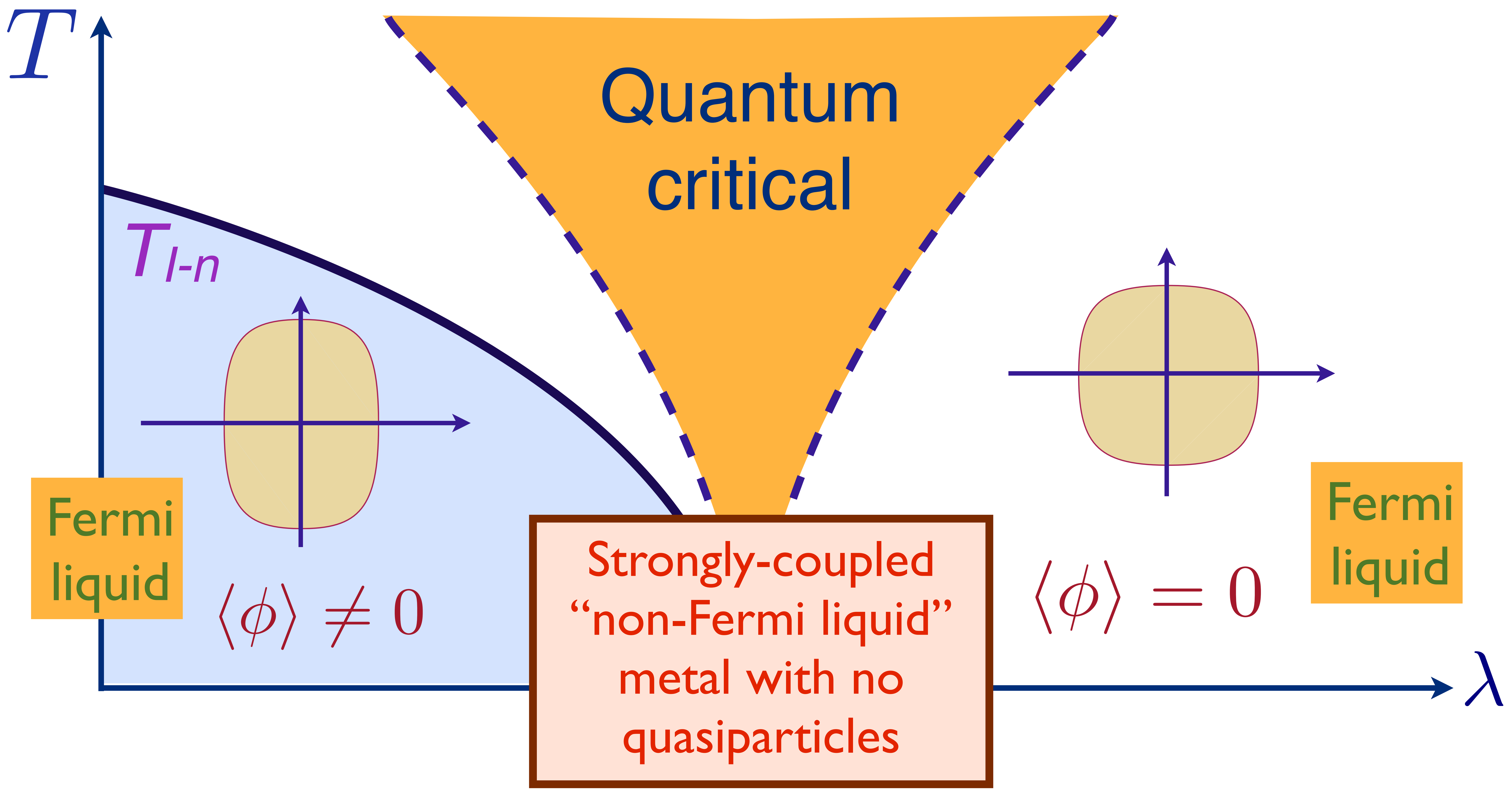

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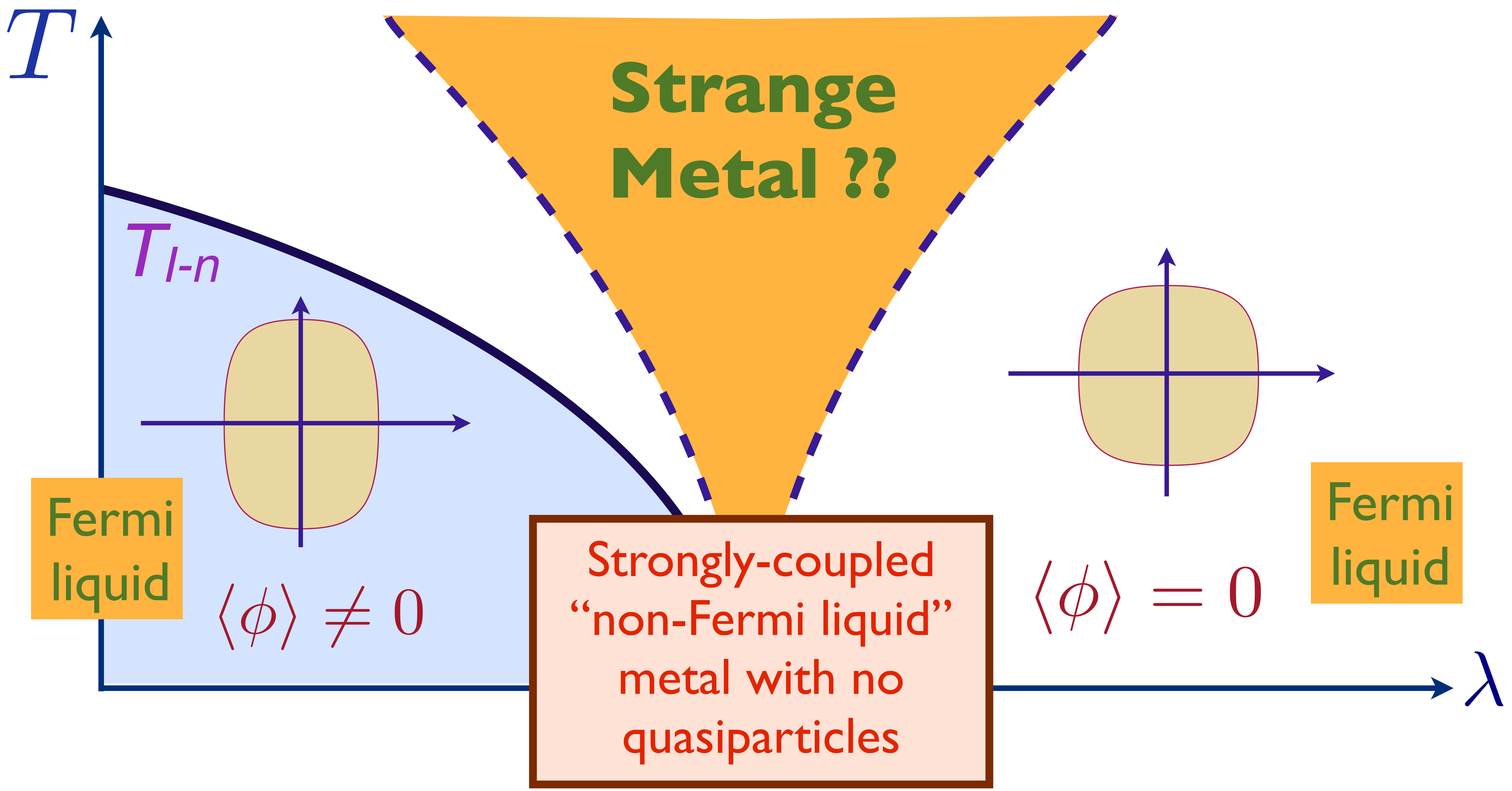

Phase diagram as a function of T and λ

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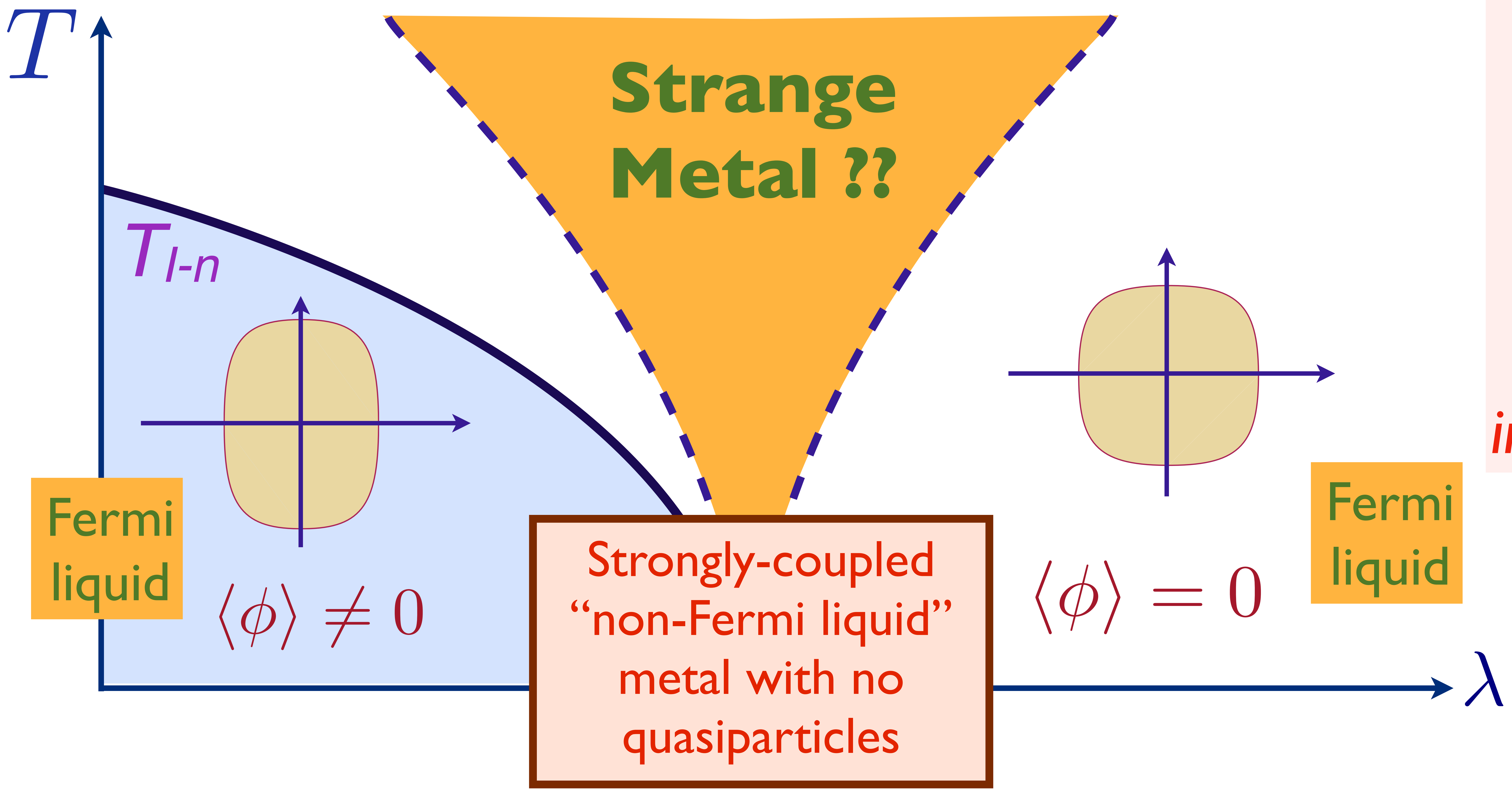
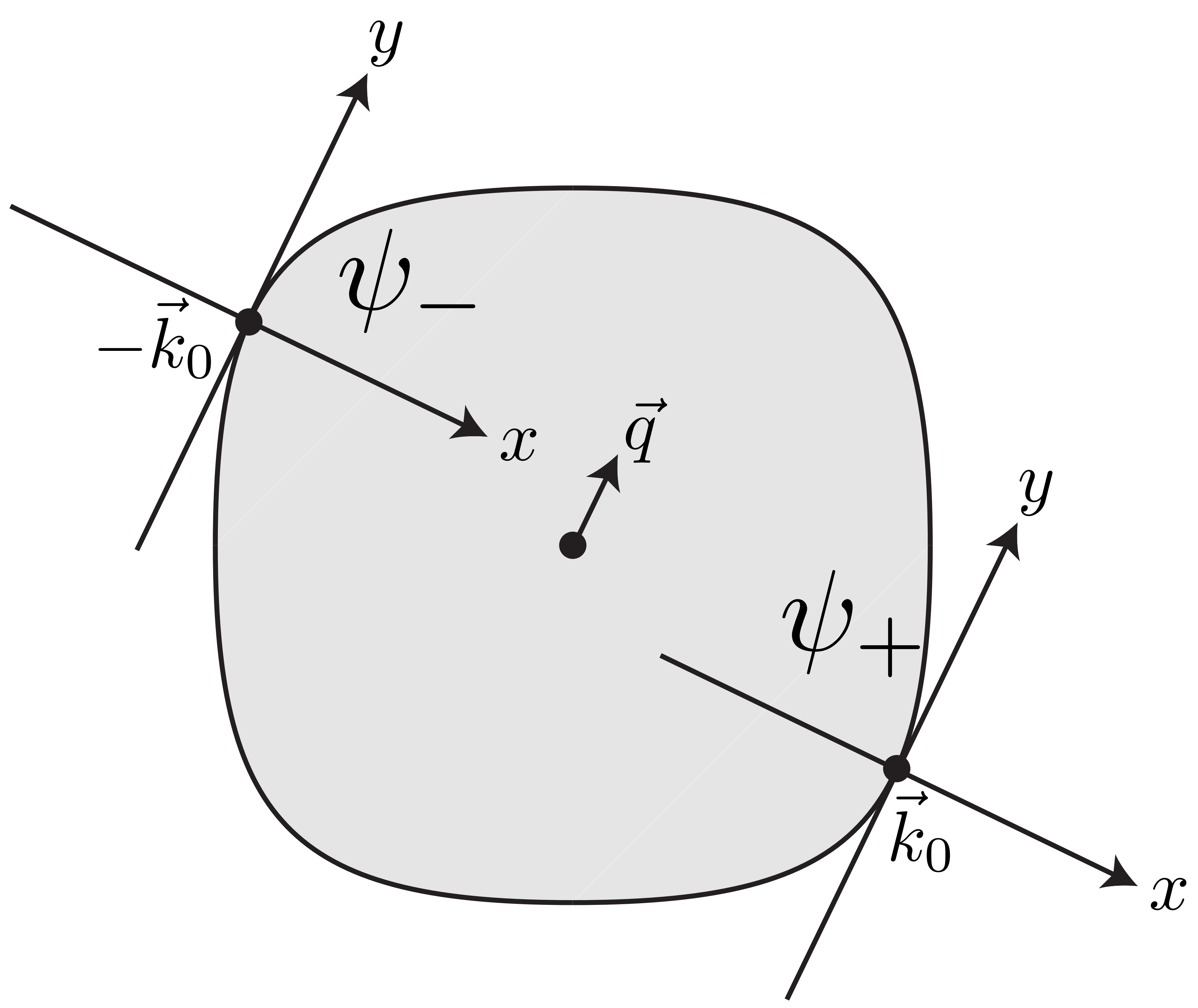


No.

Need spatial *disorder* in the *interactions*

Phase diagram as a function of T and λ

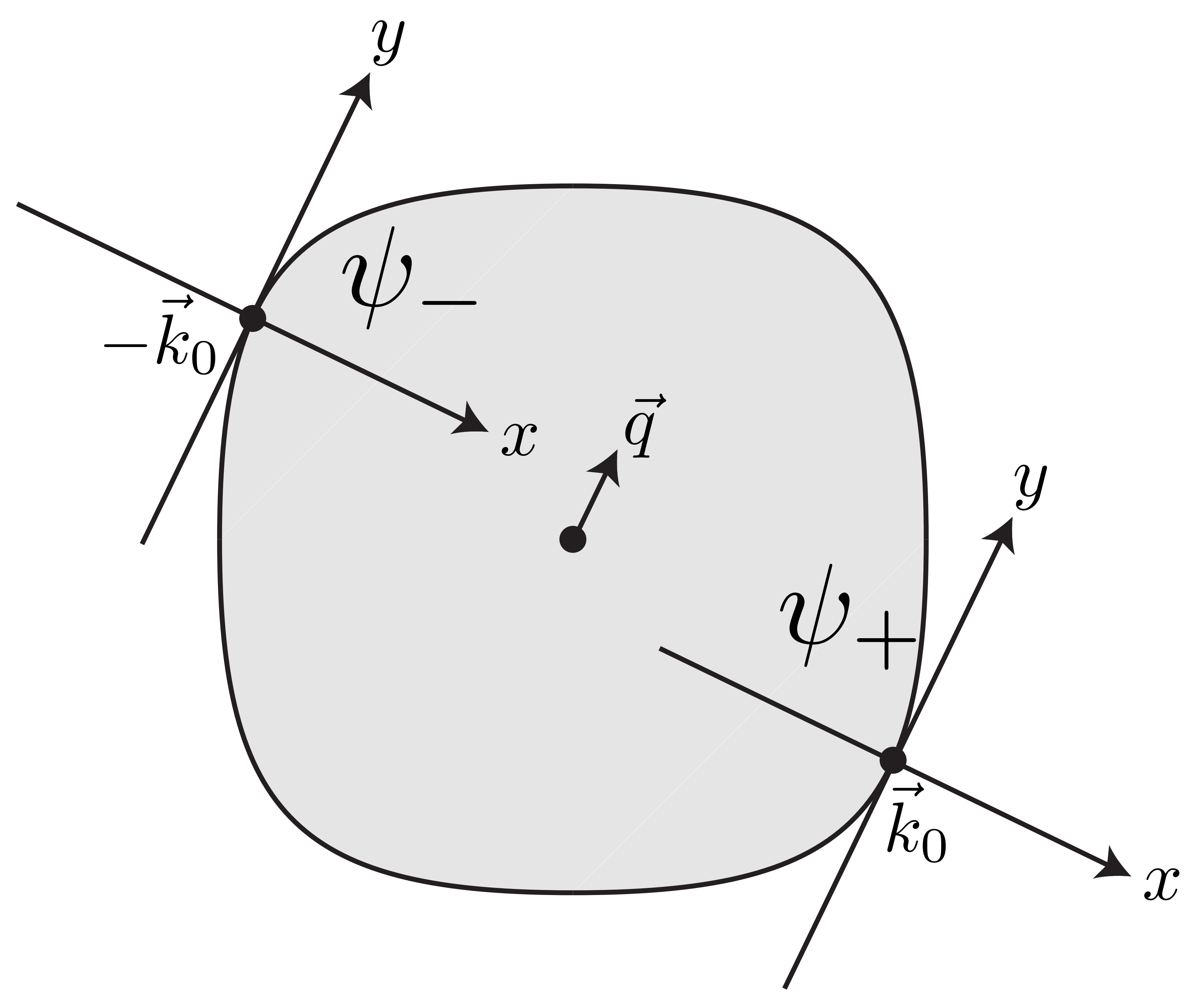
Quantum criticality of Ising-nematic ordering in a metal



- Nematic fluctuation at wavevector $\boldsymbol{q}$ couples most efficiently to fermions near $\pm\boldsymbol{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\boldsymbol{k}_0$.

Quantum criticality of Ising-nematic ordering in a metal



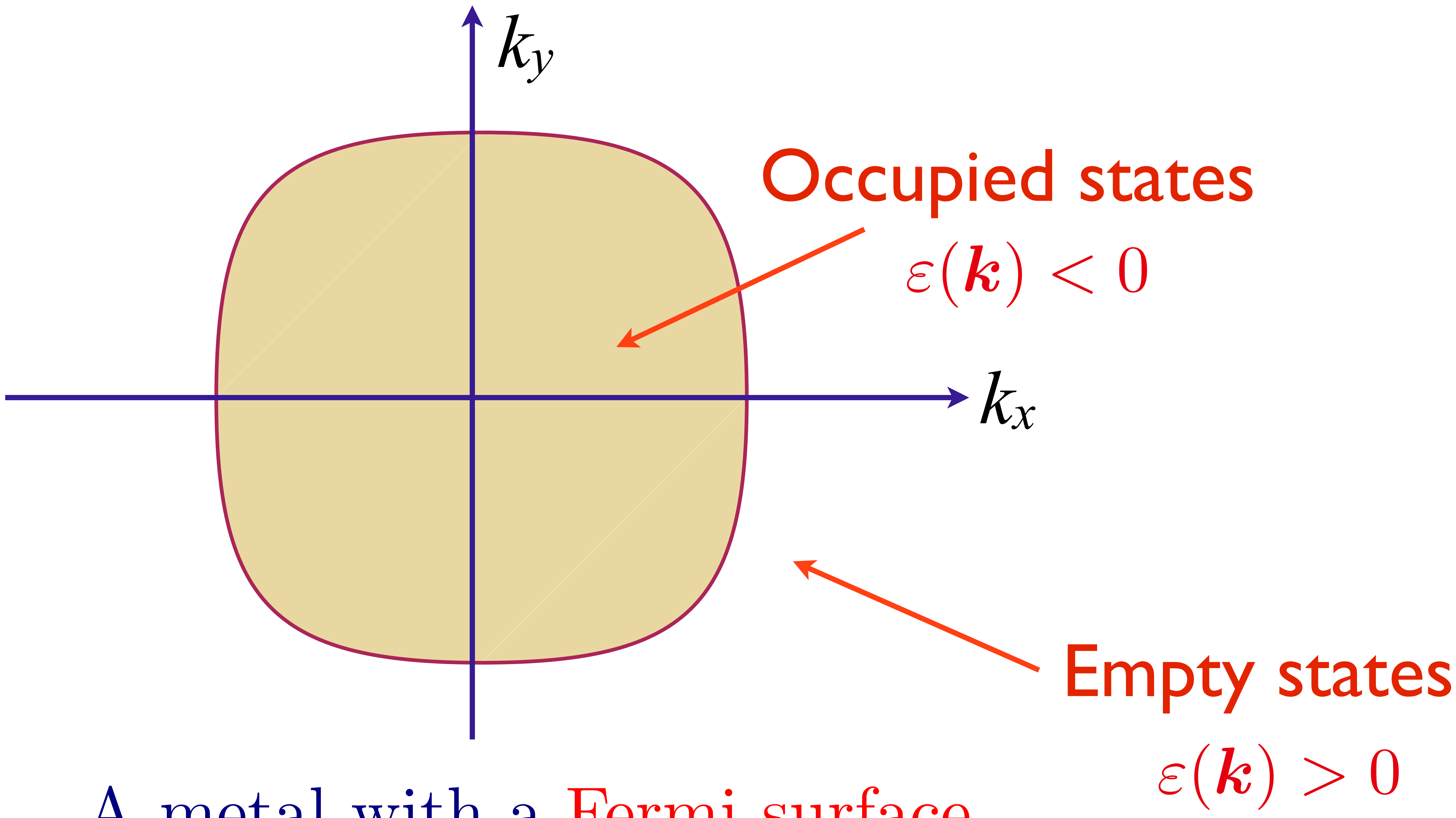
$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-g\,\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev,
Phys. Rev. B **82**, 075127 (2010)

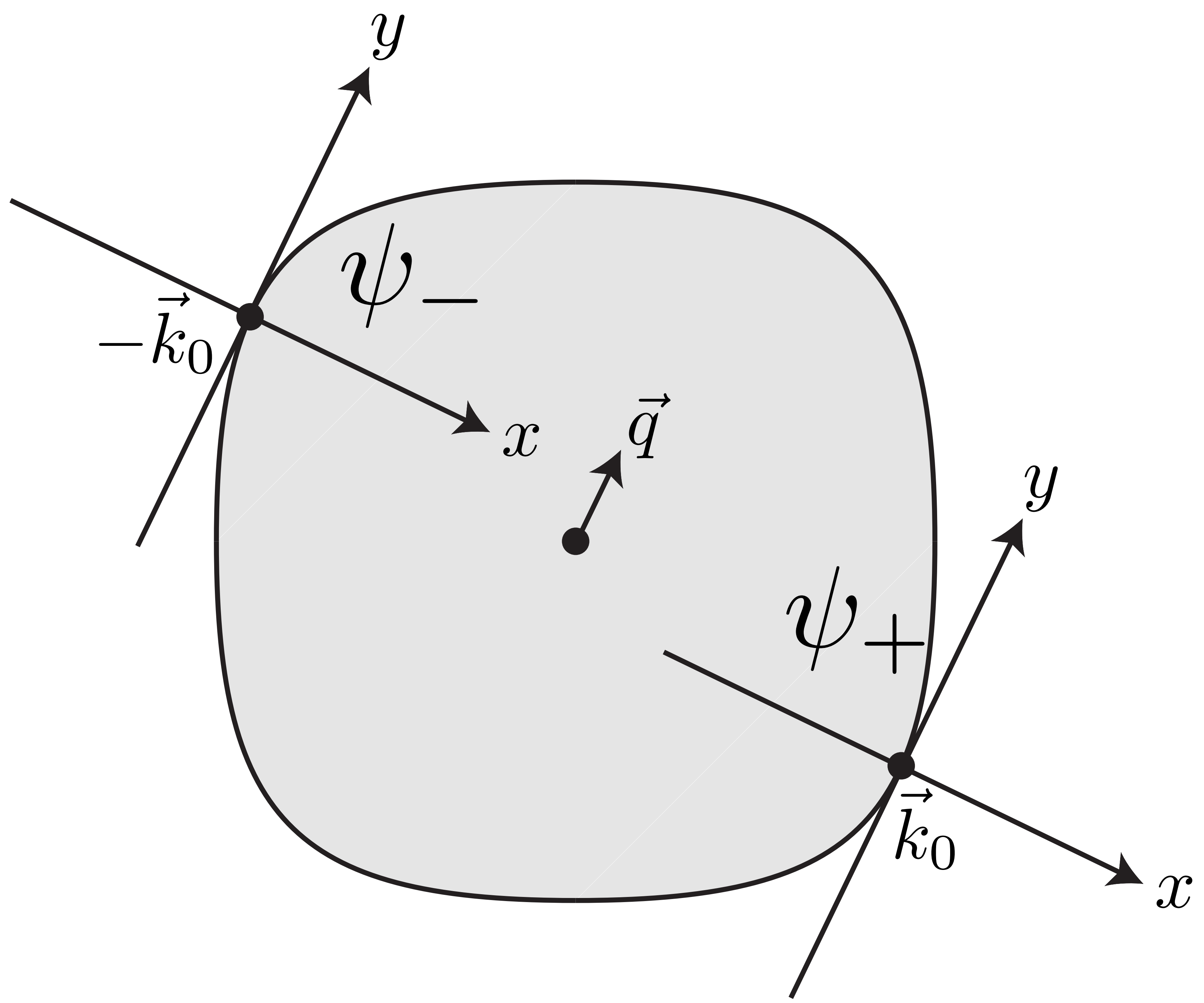
Fermi surface coupled to a gauge field



A metal with a Fermi surface
minimally coupled to a gauge field $\boldsymbol{A}$

$$\mathcal{L} = c_{\boldsymbol{k}}^{\dagger} \left(\frac{\partial}{\partial \tau} + \varepsilon(-i\boldsymbol{\nabla} - g\boldsymbol{A}) - \mu \right) c_{\boldsymbol{k}} + \frac{1}{2}(\boldsymbol{\nabla} \times \boldsymbol{A})^2$$

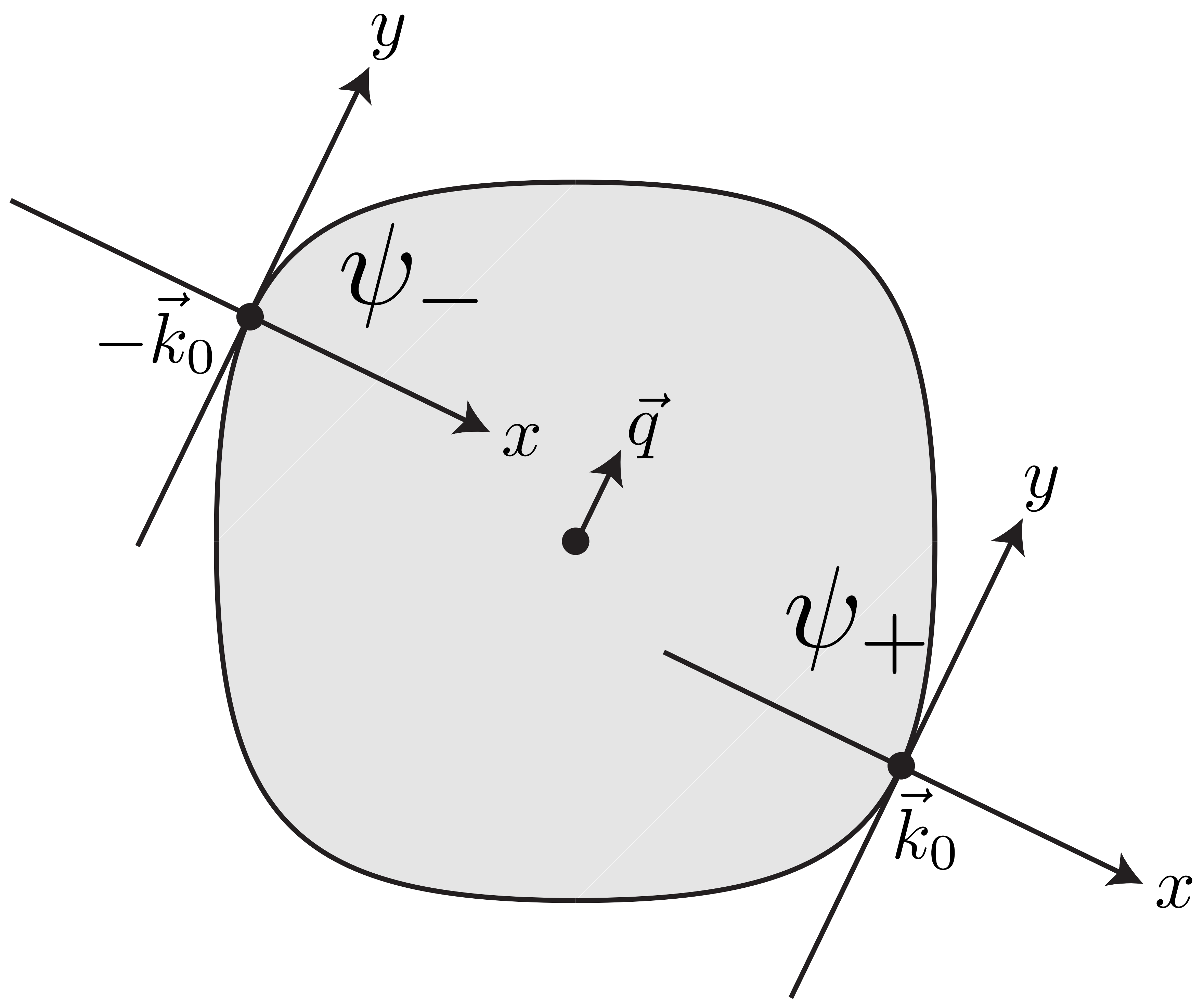
Fermi surface coupled to a gauge field



- Gauge fluctuation at wavevector $\boldsymbol{q}$ couples most efficiently to fermions near $\pm\boldsymbol{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\boldsymbol{k}_0$. In Landau gauge $\boldsymbol{A} = (a, 0)$.

Fermi surface coupled to a gauge field



$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-g\, a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2} \left(\partial_y a\right)^2$$

M. A. Metlitski and S. Sachdev,
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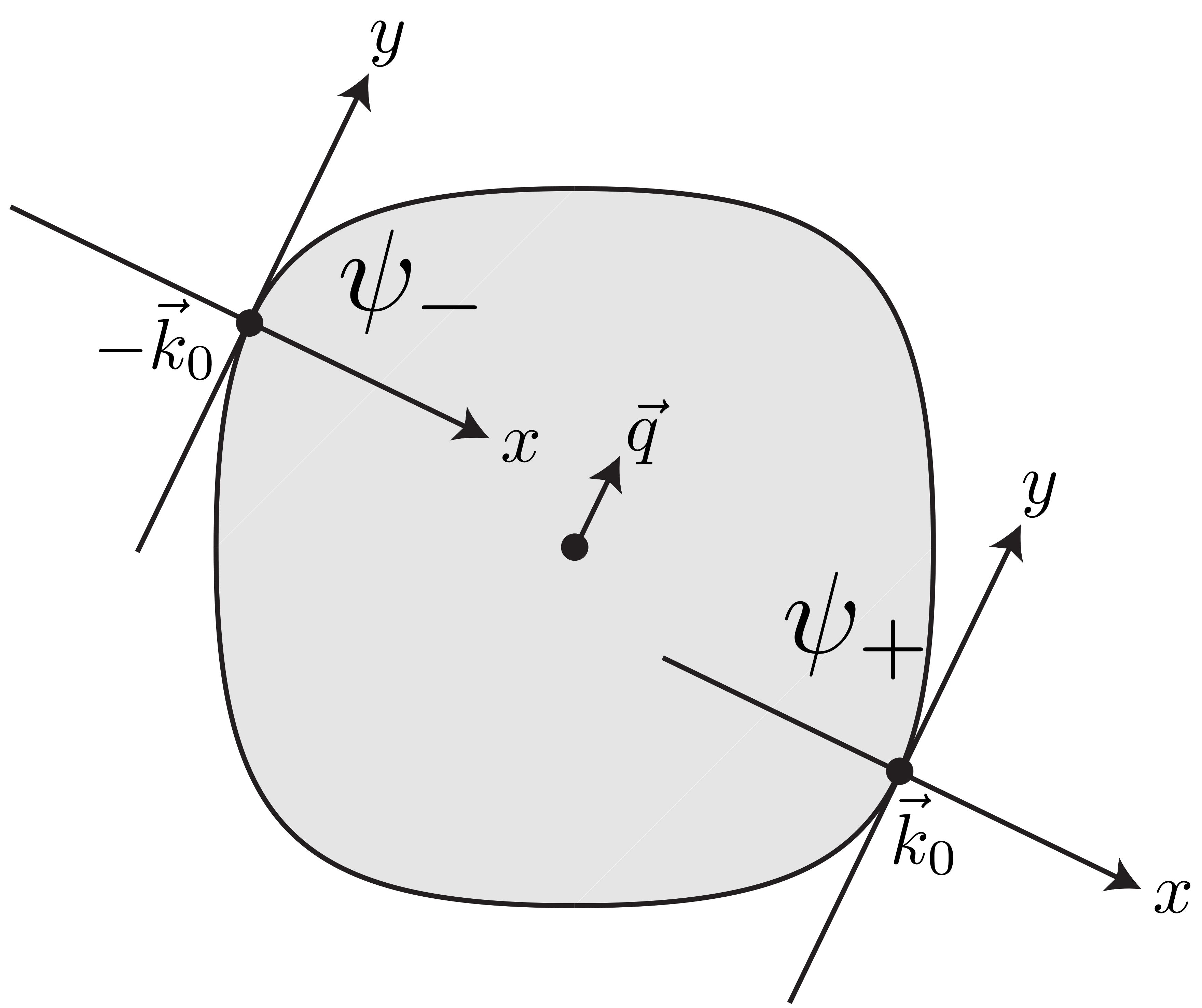

$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

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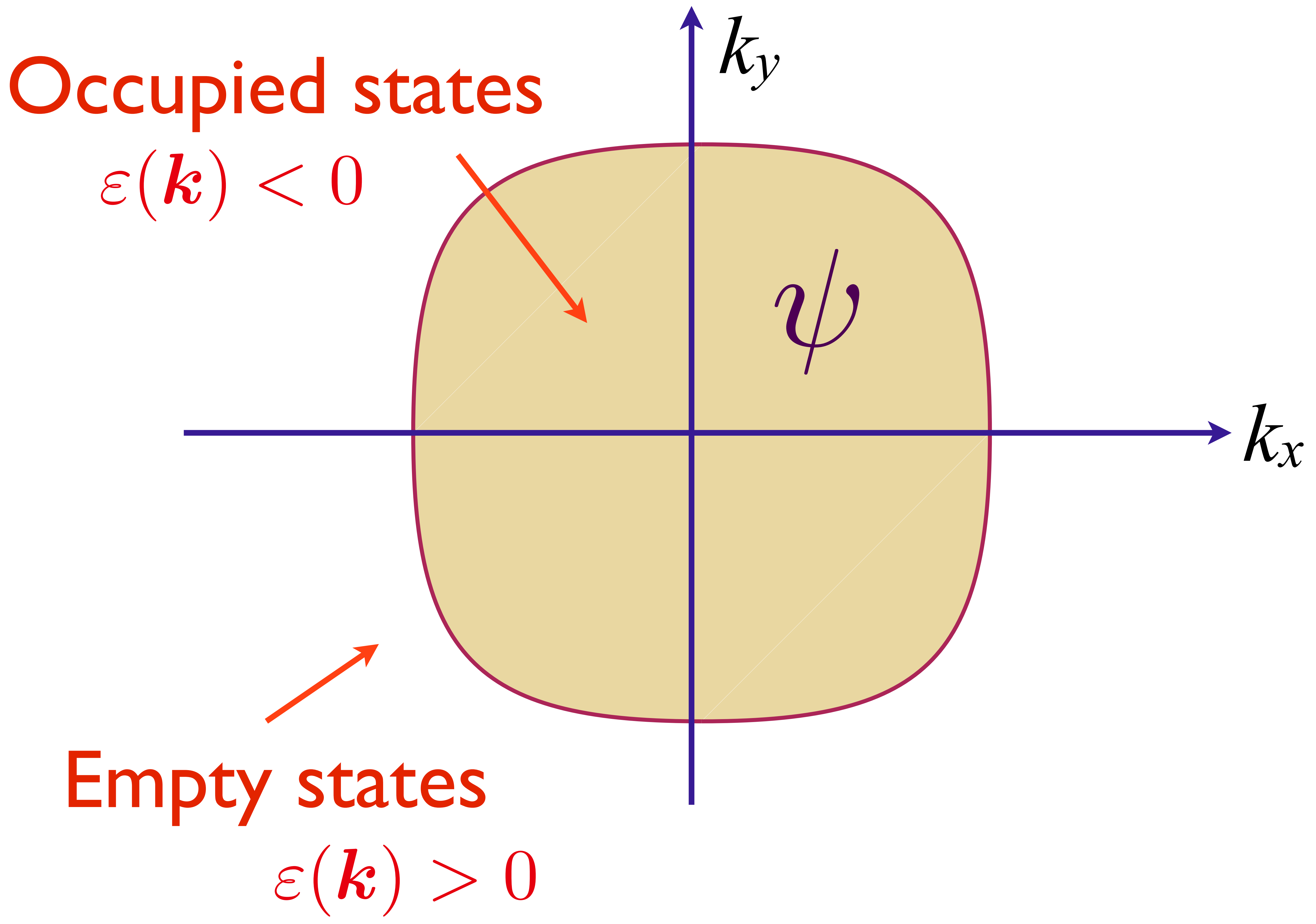
Fermi surface coupled to a critical boson



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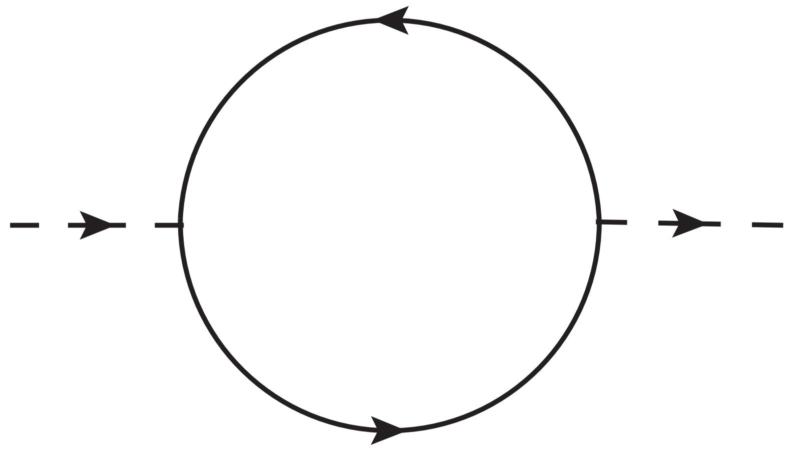
Fermi surface coupled to a critical boson

"Yukawa" coupling: $g \int d^2 r d\tau \, \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)$

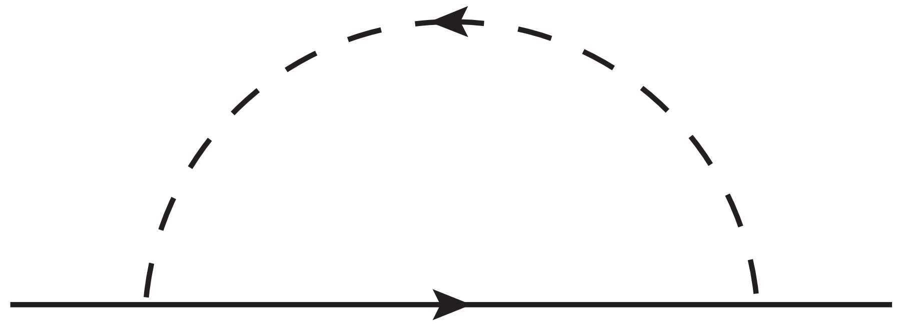
Boson self energy $\Pi(q, i\Omega) \sim -g^2 \dfrac{|\Omega|}{q}$ (Landau damping)



Boson Green's function $D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|/q}$

Fermion self energy $\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\mathrm{sgn}(\omega)|\omega|^{2/3}$

Fermion Green's function $G(\boldsymbol{k}, i\omega) = \dfrac{1}{i\omega \mp k_x - k_y^2 - \Sigma(\hat{\boldsymbol{k}}, i\omega)}$



P.A. Lee (1989)

Yields a state without quasiparticle excitations, but the theory is not systematic at large N

Sung-Sik Lee (2009)

Fermi surface coupled to a gauge field

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\ &\quad - g\, a \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2}\left(\partial_y a\right)^2\end{aligned}$$

Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} a &\to a\, s^{(2z+1)/4} \\ \psi &\to \psi\, s^{(2z+1)/4} \\ g &\to g\, s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

M. A. Metlitski and S. Sachdev,
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Fermi surface coupled to a gauge field

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\cancel{\partial_\tau} - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger\left(\cancel{\partial_\tau} + i\partial_x - \partial_y^2\right)\psi_- \\ &\quad - g\, a\left(\psi_+^\dagger\psi_+ - \psi_-^\dagger\psi_-\right) + \frac{1}{2}\left(\partial_y a\right)^2\end{aligned}$$

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Fermi surface coupled to a critical boson

"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N} \displaystyle\int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$

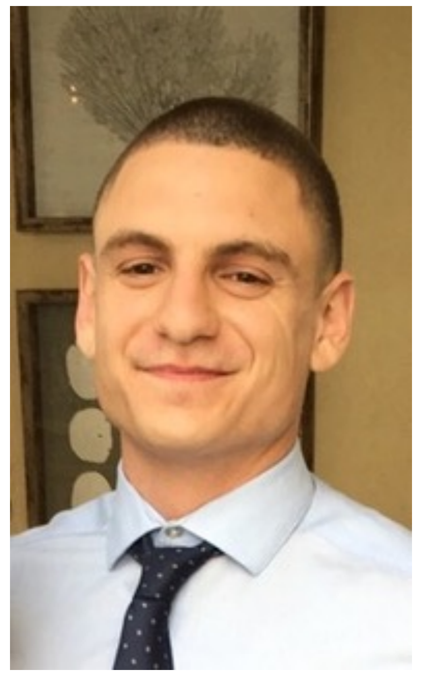
Main idea:

Introduce N flavors of fermions and bosons, and examine an *ensemble* of theories with different Yukawa couplings. In the large N limit, every member of the ensemble is expected to have the same critical properties, and so it is easier to study the average theory.

Ilya Esterlis, J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A.V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory

The theory self-averages, and the average partition function can be written exactly as a 'G-Σ' theory involving a path integral over *bilocal in spacetime*. We introduce the spacetime co-ordinate $X \equiv (\tau, x, y)$, and all Green's functions and self energies in the path integral are functions of two spacetime co-ordinates X_1 and X_2.



$$\overline{\mathcal{Z}} = \int \mathcal{D}G(X_1, X_2)\mathcal{D}\Sigma(X_1, X_2)\mathcal{D}D(X_1, X_2)\mathcal{D}\Pi(X_1, X_2) \exp\left[-NI(G, \Sigma, D, \Pi)\right] .$$

The G-Σ-D-Π action is now

$$\begin{aligned} I(G, \Sigma, D, \Pi) &= \frac{g^2}{2}\mathrm{Tr}\left(G \cdot [GD]\right) - \mathrm{Tr}(G \cdot \Sigma) + \frac{1}{2}\mathrm{Tr}(D \cdot \Pi) \\ &- \ln\det\left[(\partial_{\tau_1} + \varepsilon(-i\boldsymbol{\nabla}_1))\,\delta(X_1 - X_2) + \Sigma(X_1, X_2)\right] \\ &\quad + \frac{1}{2}\ln\det\left[\left(-\partial_{\tau_1}^2 - \boldsymbol{\nabla}_1^2 + s\right)\delta(X_1 - X_2) - \Pi(X_1, X_2)\right] . \end{aligned}$$

where we have introduced notation

$$\mathrm{Tr}\left(f \cdot g\right) \equiv \int dX_1 dX_2\, f(X_2, X_1) g(X_1, X_2) .$$

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

G-Σ-D-Π Theory

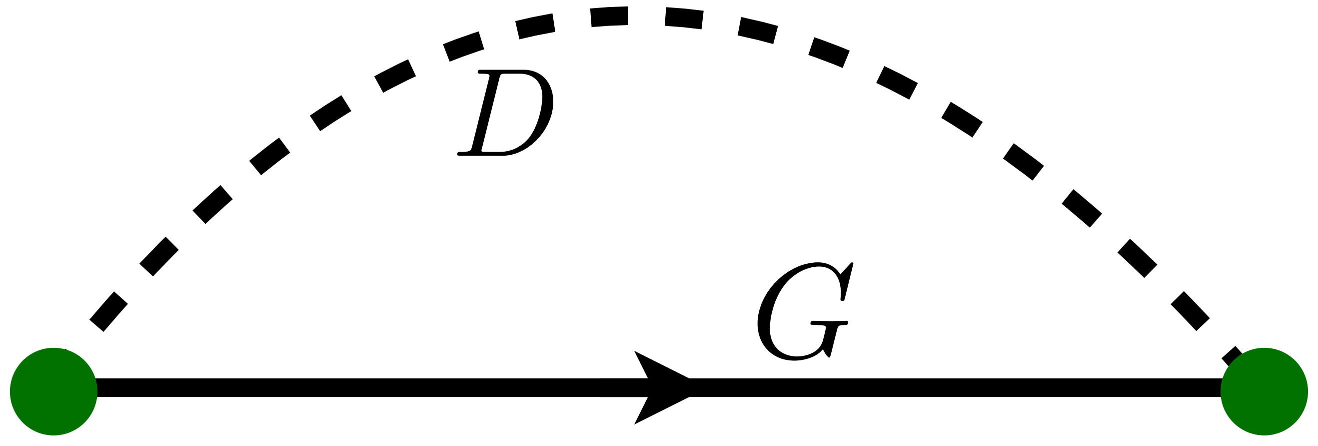
The saddle point equations are

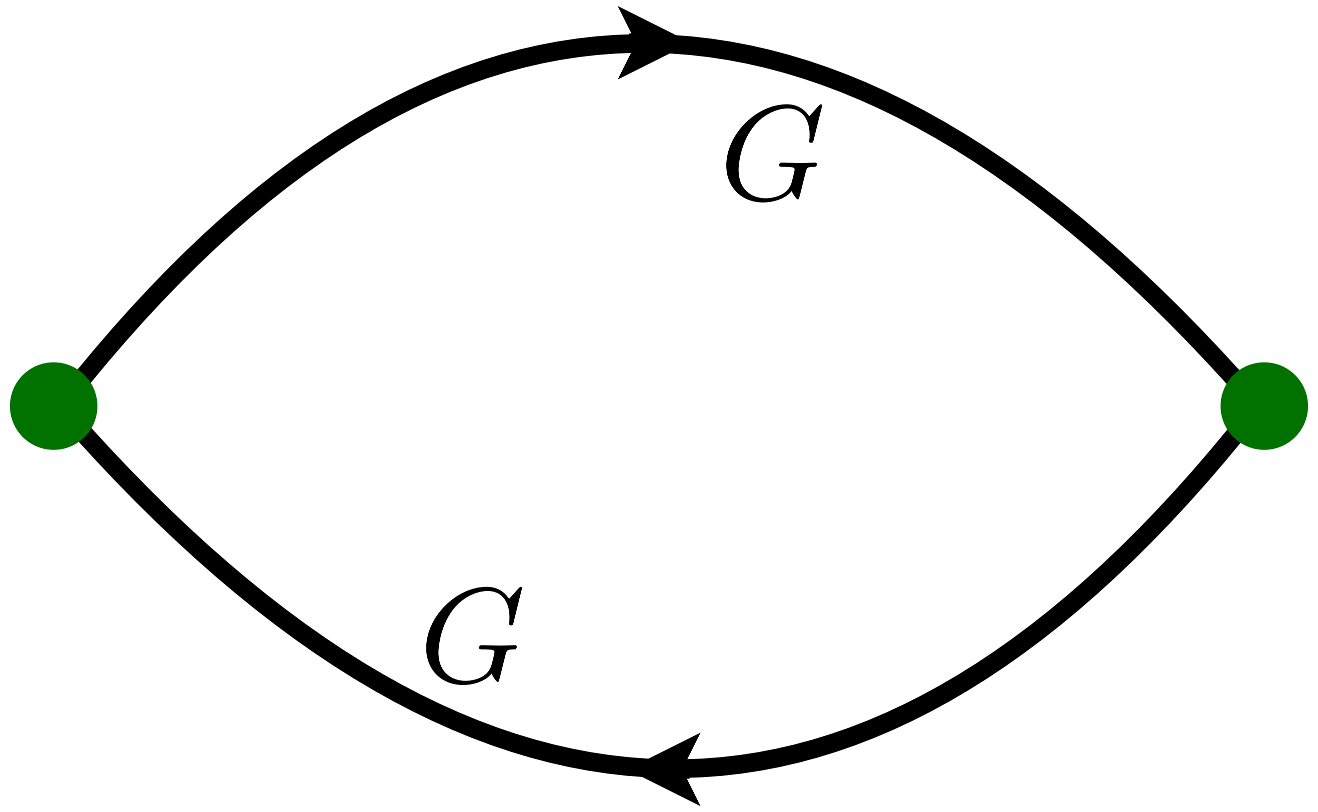
$$\Sigma(\boldsymbol{r},\tau) = g^2\lambda D(\boldsymbol{r},\tau)G(\boldsymbol{r},\tau),$$

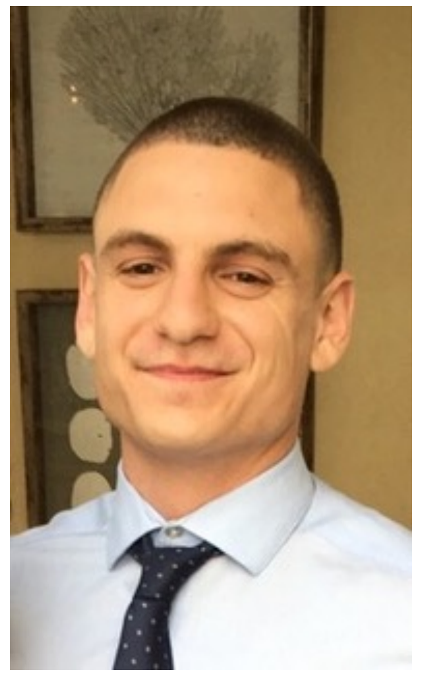
$$\Pi(\boldsymbol{r},\tau) = -g^2 G(-\boldsymbol{r},-\tau)G(\boldsymbol{r},\tau),$$

$$G(\boldsymbol{k}, i\omega_n) = \frac{1}{i\omega_n - \varepsilon(\boldsymbol{k}) - \Sigma(\boldsymbol{k}, i\omega_n)},$$

$$D(\boldsymbol{q}, i\Omega_m) = \frac{1}{\Omega_m^2 + q^2 + s - \Pi(\boldsymbol{q}, i\Omega_m)}.$$

$\Sigma =$ 

$\Pi =$ 



Exact Solution at small ω:

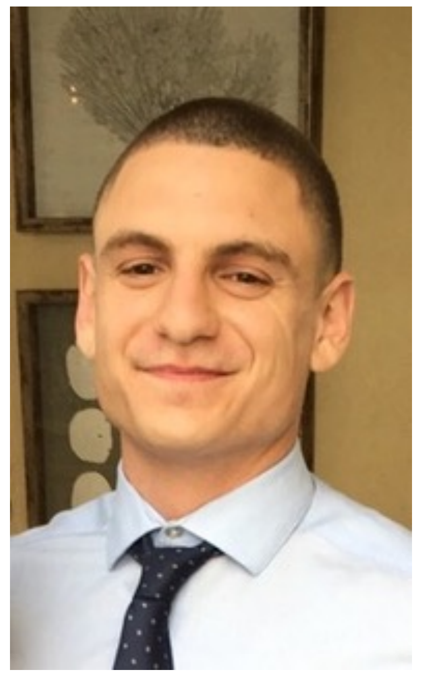
$$\Sigma(\hat{\boldsymbol{k}}, i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{2/3}\,, \quad G(\boldsymbol{k}, i\omega) = \frac{-1}{\varepsilon(\boldsymbol{k}) + \Sigma(\hat{\boldsymbol{k}}, i\omega)}$$

where the co-efficient is known exactly in terms of the Fermi velocity and Fermi surface curvature at the Fermi surface point along the direction $\hat{\boldsymbol{k}}$.

Ilya Esterlis, Haoyu Guo, Aavishkar Patel, S.S. PRB **103**, 235129 (2021)

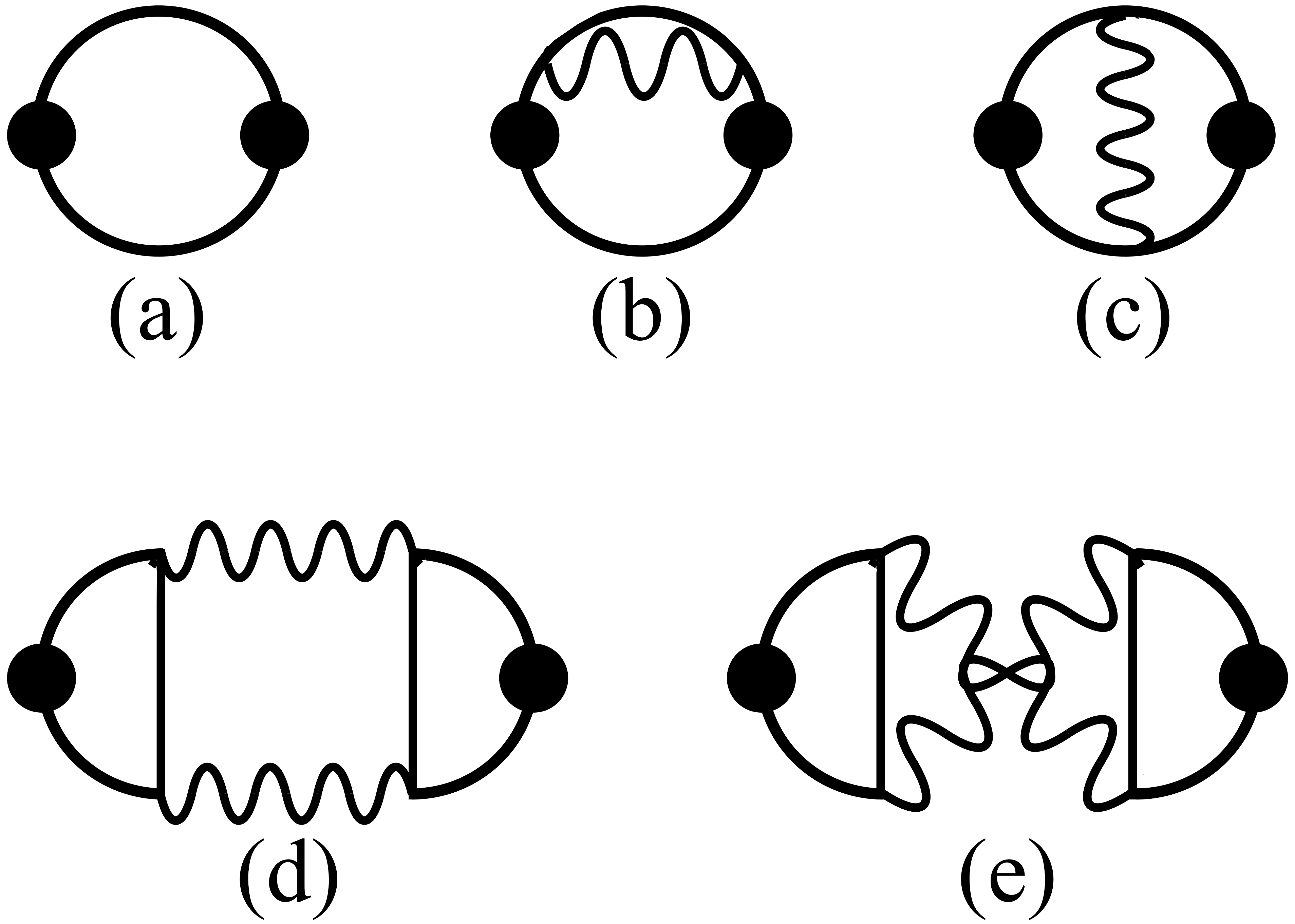
Strange metal
in two spatial dimensions from
spatially random interactions

Fermi surface coupled to a critical boson


"Yukawa" coupling: $\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{|g_{ijl}|^2} = g^2$$



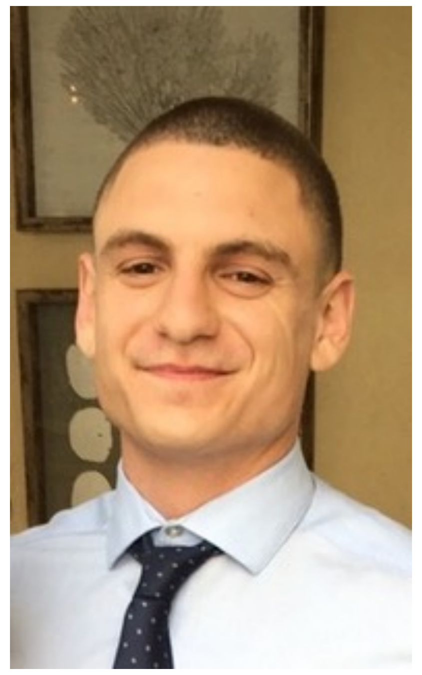
+ all ladders and bubbles…..

Yong Baek Kim, A. Furusaki, Xiao-Gang Wen, P. A. Lee, PRB **50**, 17917 (1994) examined these graphs and concluded that the d.c. resistivity $\rho(T) \sim T^{4/3}$ and $\sigma(\omega \gg T) \sim \omega^{-2/3}$.

These results do not account for conservation of total momentum *i.e.* 'boson drag'.

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson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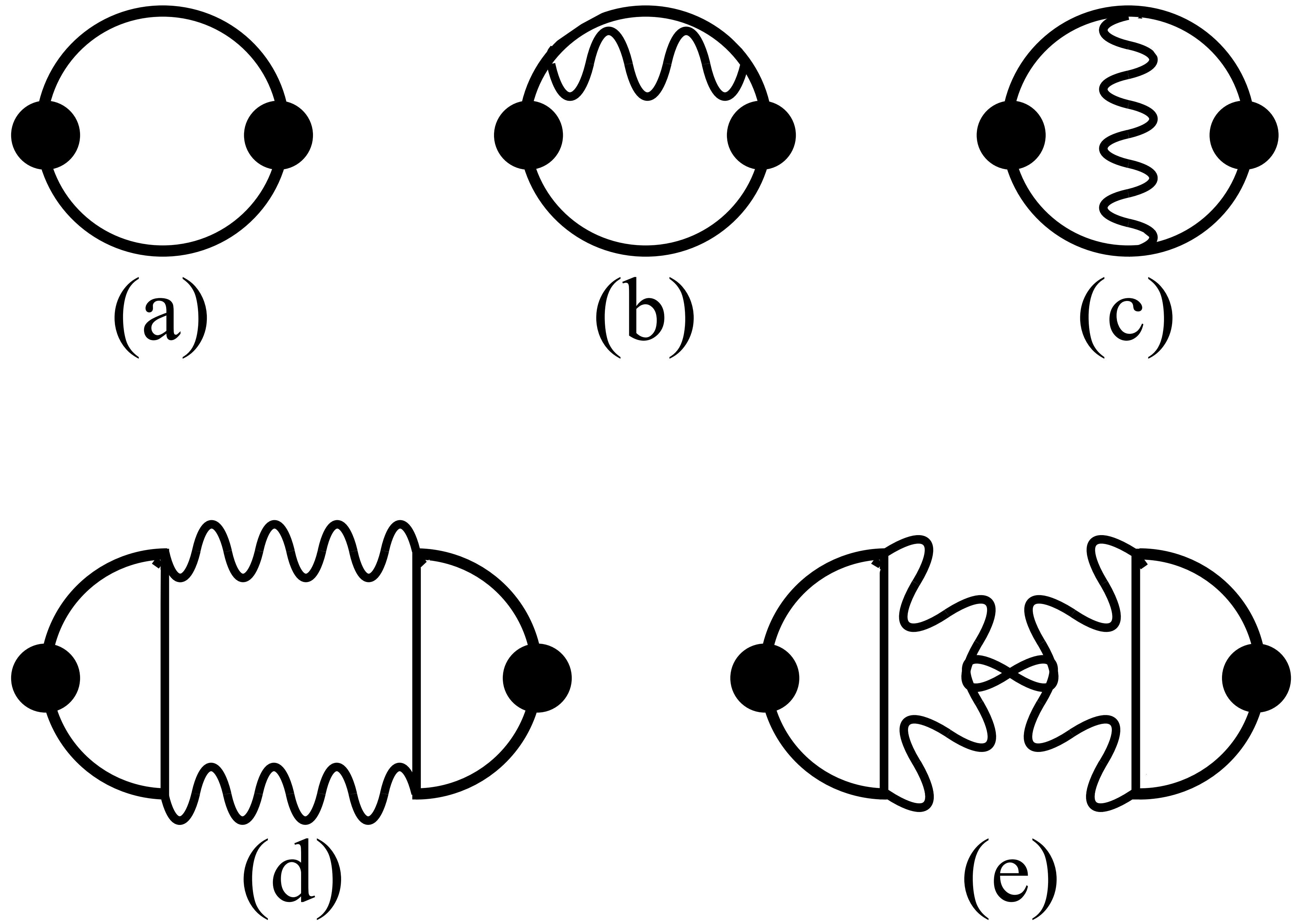

+ all ladders and bubbles…..

Conservation of momentum implies the d.c. conductivity is infinite

$$\mathrm{Re}\,\sigma(\omega) = D\delta(\omega) + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Muller, and S.S. PRB **76**, 144502 (2007)
S. A. Hartnoll, R. Mahajan, M. Punk, and S.S. PRB **89**, 155130 (2014)
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

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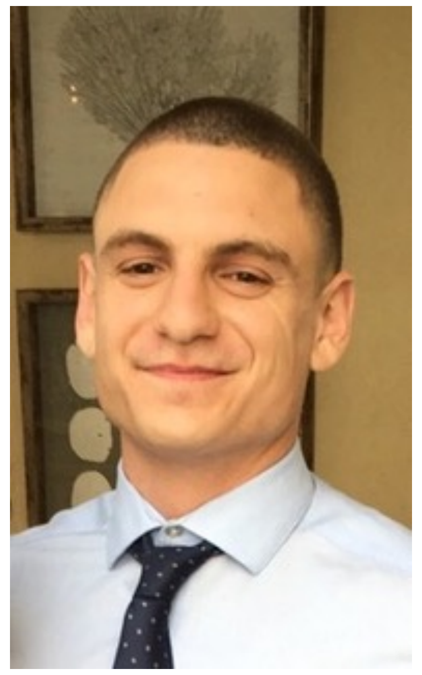


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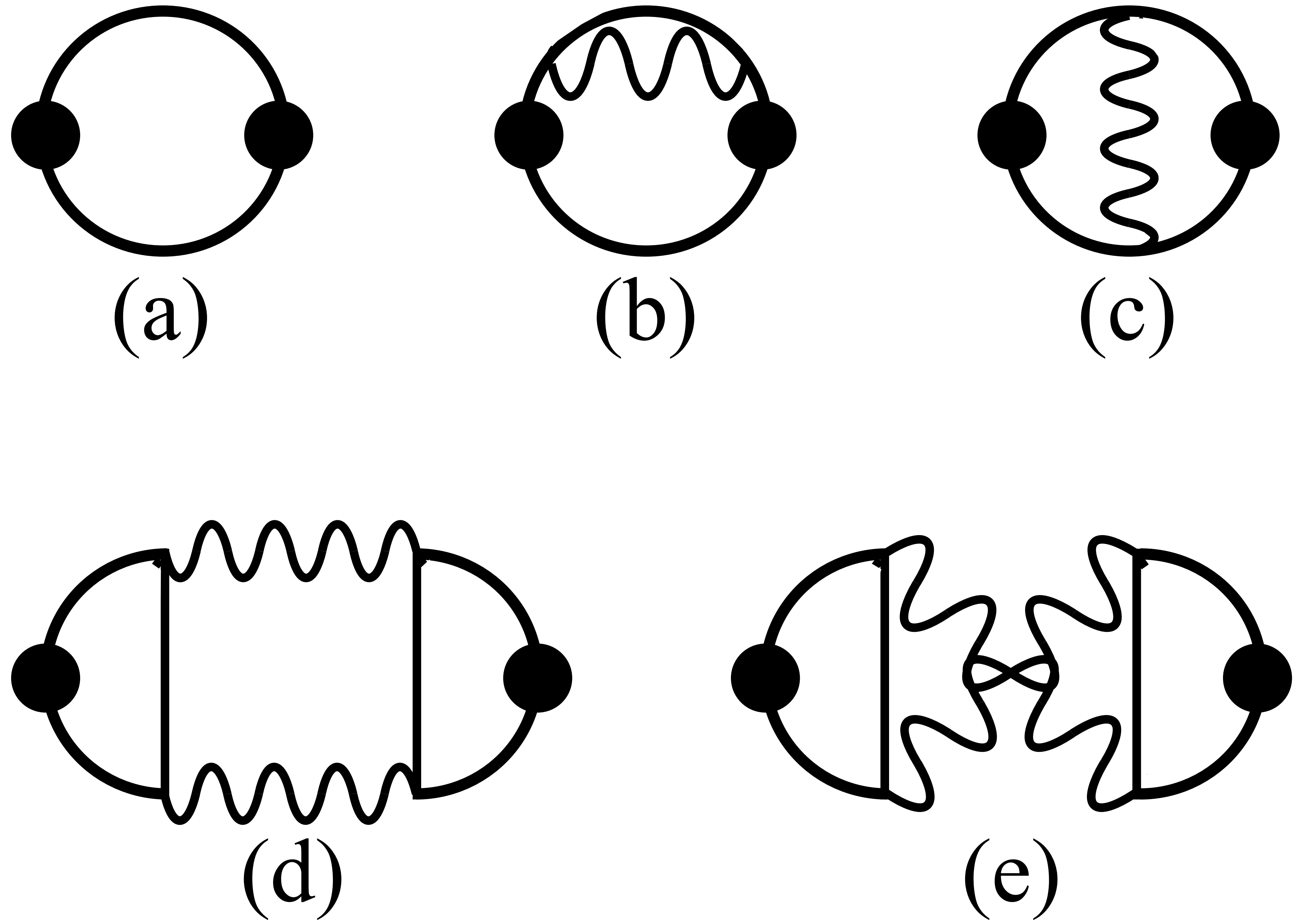

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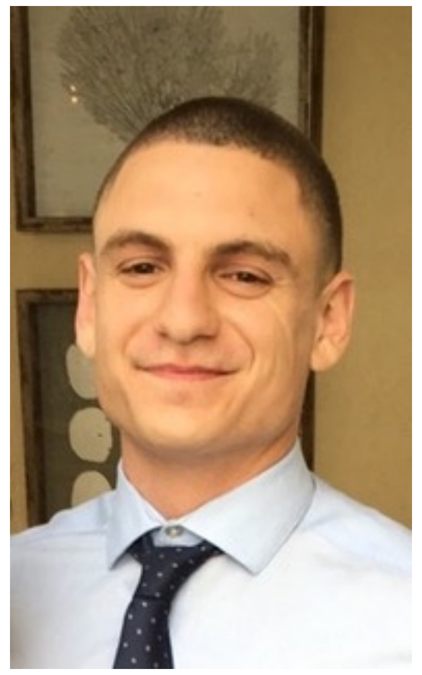
A. Eberlein, I. Mandal, and S. S. PRB **94**, 045133 (2016)

$$\sigma(\omega) \sim \frac{1}{-i\omega + \omega^2}$$

Zhengyan Darius Shi, Hart Goldman, Dominic V. Else, T. Senthil arXiv:2204.07585

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder

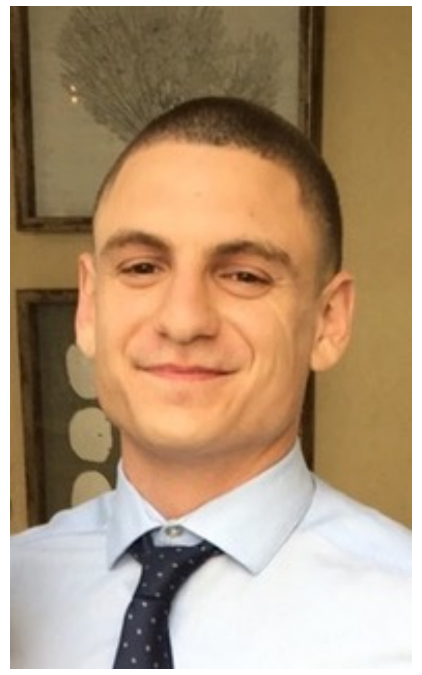

$$\text{“Yukawa” coupling:} \quad \frac{g_{ij\ell}}{N} \int d^2r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

$$\text{Random potential:} \quad + \frac{1}{\sqrt{N}} \int d^2r d\tau \, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g^*_{ijl} g_{abc}} = g^2 \, \delta_{ia}\delta_{jb}\delta_{lc} \quad , \quad \overline{v_{ij}(r)} = 0 \quad , \quad \overline{v^*_{ij}(r) v_{lm}(r')} = v^2 \, \delta(r - r')\delta_{il}\delta_{jm}$$

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder



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Random potential: $+\dfrac{1}{\sqrt{N}} \displaystyle\int d^2r d\tau \, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$

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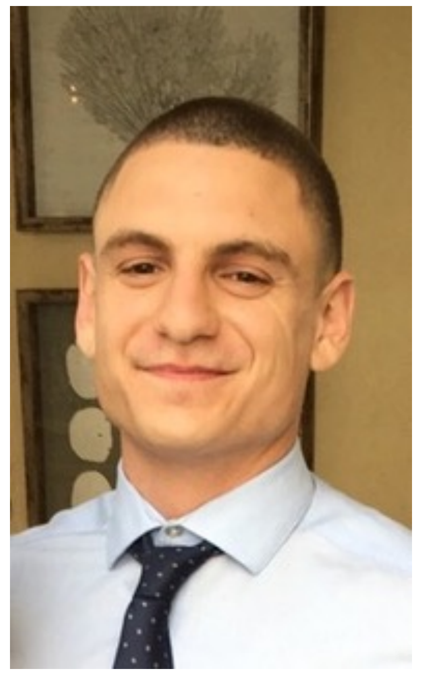
Boson self energy: $\Pi \sim -\dfrac{g^2}{v^2}|\Omega|, \qquad D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|}$

Fermion self energy: $\Sigma(i\omega) \sim -iv^2 \mathrm{sgn}(\omega) - i\dfrac{g^2}{v^2}\omega \ln(1/|\omega|)$

Marginal Fermi liquid self energy and $T \log T$ specific heat

Aavishkar Patel, Haoyu Guo, Ilya Esterlis, S.S. arXiv:2203.04990

Fermi surface coupled to a critical boson with spatial disorder


"Yukawa" coupling: $\frac{g_{ij\ell}}{N}\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

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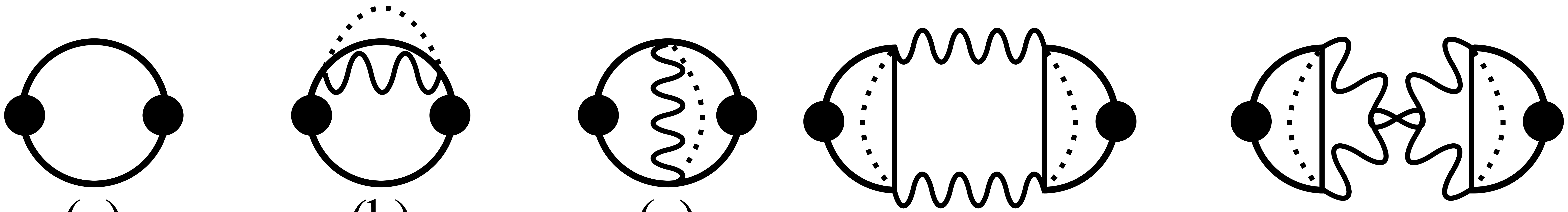
Conductivity:



(a) σ_v

(b) $\frac{\sigma_{\Sigma,g}}{2}, \frac{\sigma_{\Sigma,g'}}{2}$

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(d)

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+ all ladders and bubbles…..

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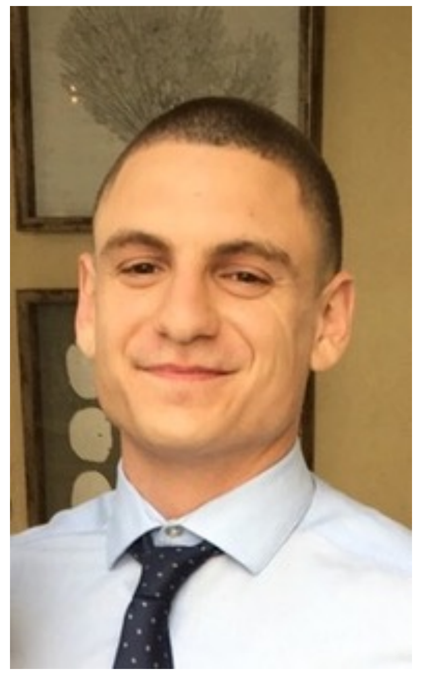

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The g^2 log term does not contribute to transport

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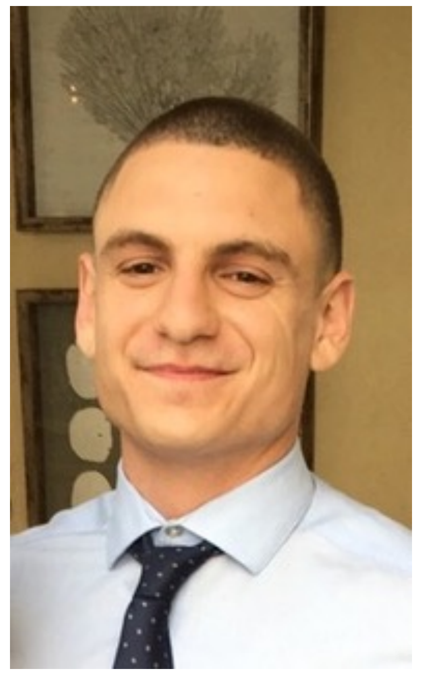
Fermi surface coupled to a critical boson with spatial disorder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With g and v non-zero, we obtain a non-zero residual resistivity and Fermi liquid like corrections

$$\rho(T) = \rho(0) + AT^2 + \ldots.$$

with $1/\rho(0) \sim 1/\tau_{\text{trans}} \sim v^2$.

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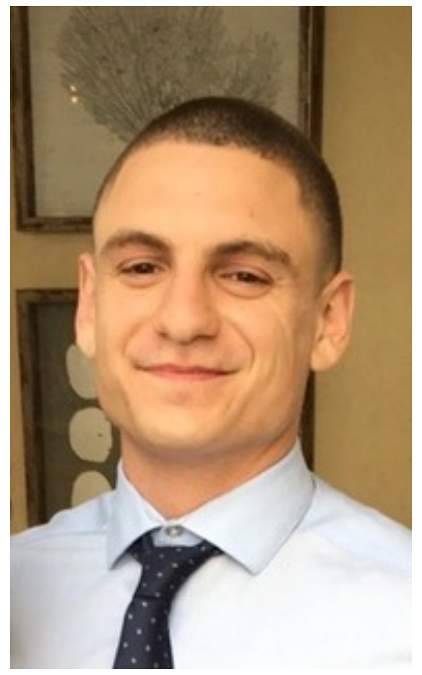


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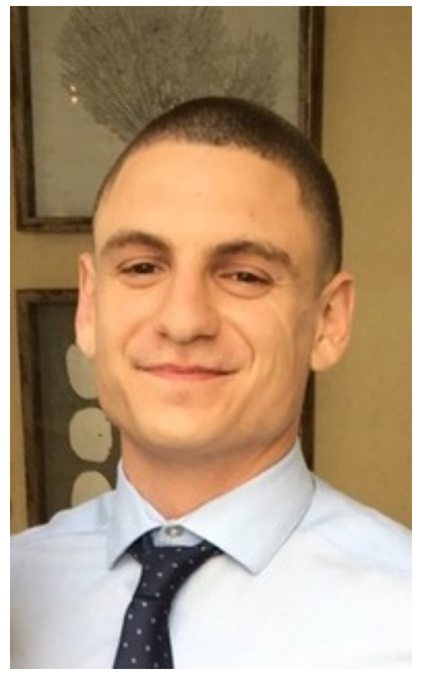

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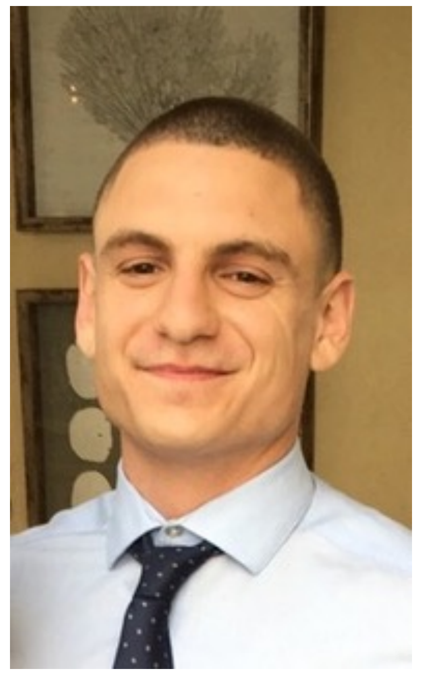

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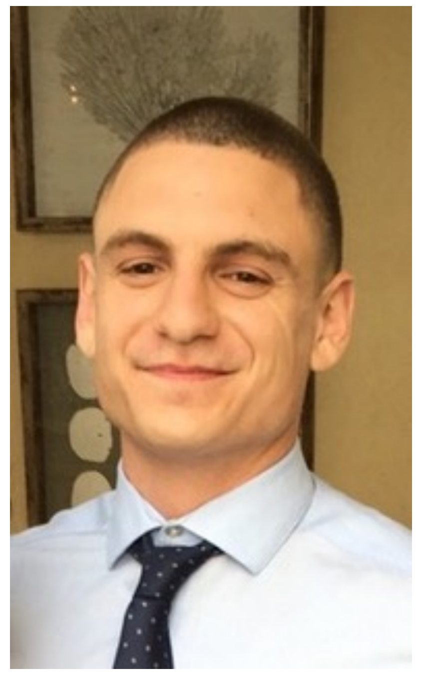


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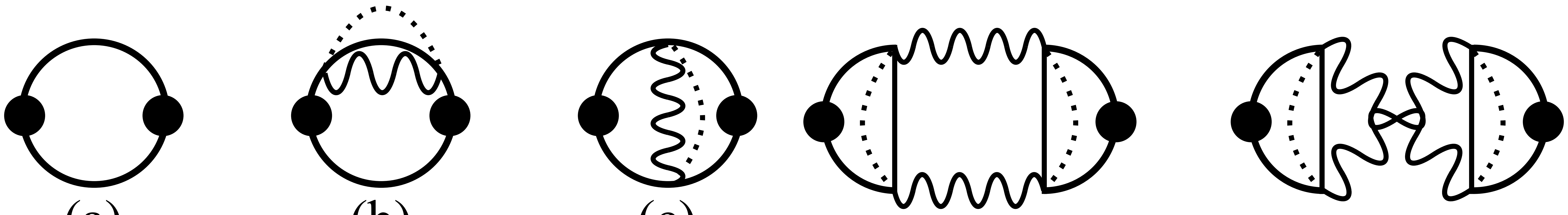
Conductivity:



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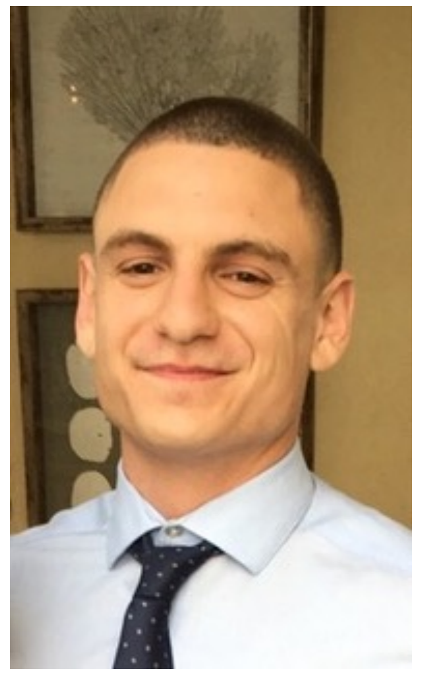

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Conductivity:

The g^2 log term does not contribute to transport but the g'^2 log term does!

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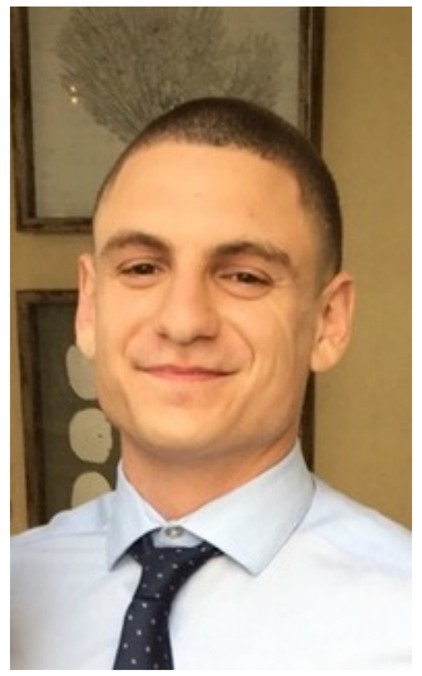


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$$\text{Conductivity: } \sigma(\omega) \sim [1/\tau_{\text{trans}}(\omega) - i\omega\, m^*(\omega)/m]^{-1}$$

$$\frac{1}{\tau_{\text{trans}}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Residual resistivity is determined by v^2; Linear-in-T resistivity determined by g'^2.

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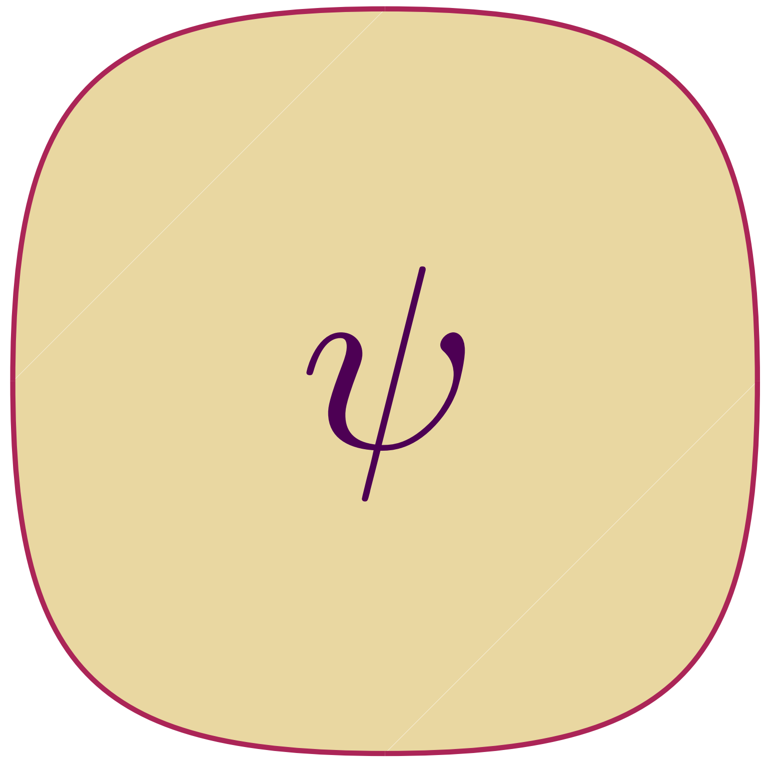
Strange metal from a Yukawa-SYK model



$+$ a critical boson ϕ

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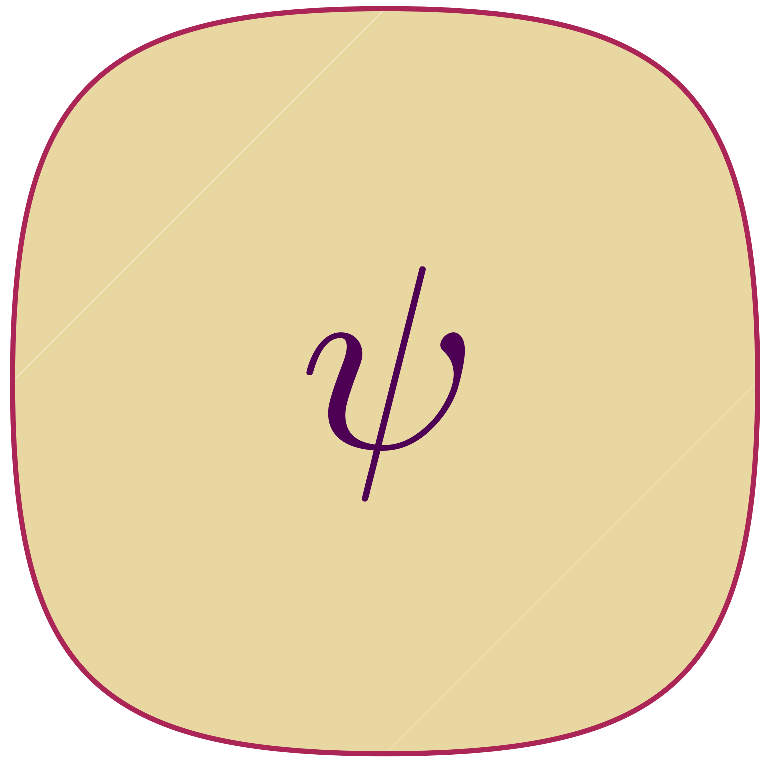

$+$ a critical boson ϕ

Non-Fermi liquid with $T^{2/3}$ specific heat,
but conductivity $\sigma(\omega) \sim \delta(\omega)$

"Yukawa" coupling: $$\frac{g_{ij\ell}}{N} \int d^2 r d\tau \, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$$

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Strange metal from a Yukawa-SYK model

ψ + a critical boson ϕ

MFL self-energy, $T\ln(1/T)$ specific heat,
but T-independent 'residual' resistivity,
and negligible optical conductivity

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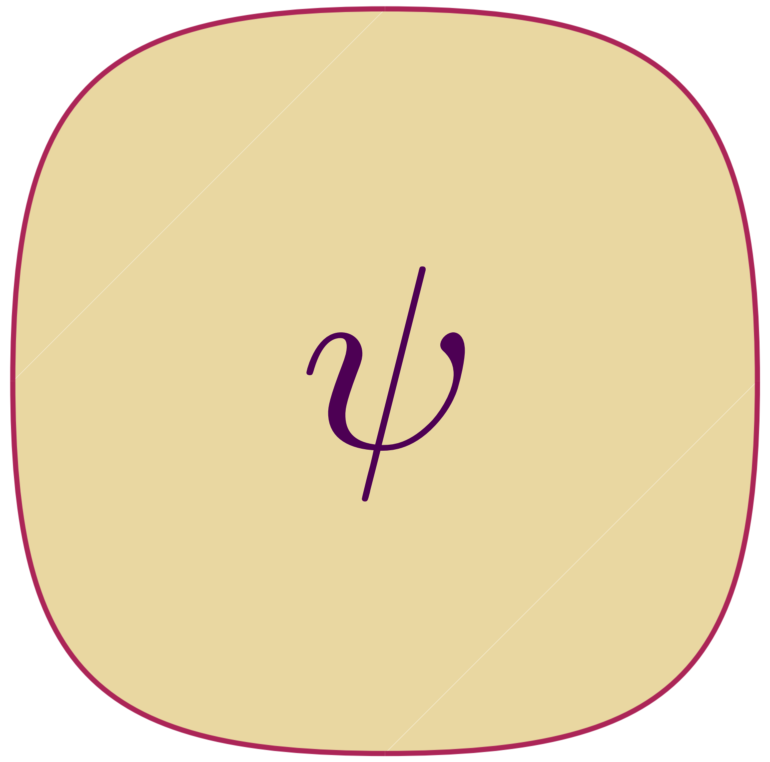

$+$ a critical boson ϕ

MFL self-energy, $T\ln(1/T)$ specific heat, linear-T resistivity and $1/[\omega - i(2\omega/\pi)\ln(\Lambda/\omega)]$ optical conductivity

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