Statistical mechanics of strange metals and black holes

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Subir Sachdev

Talk online: qpt.physics.harvard.edu/talks

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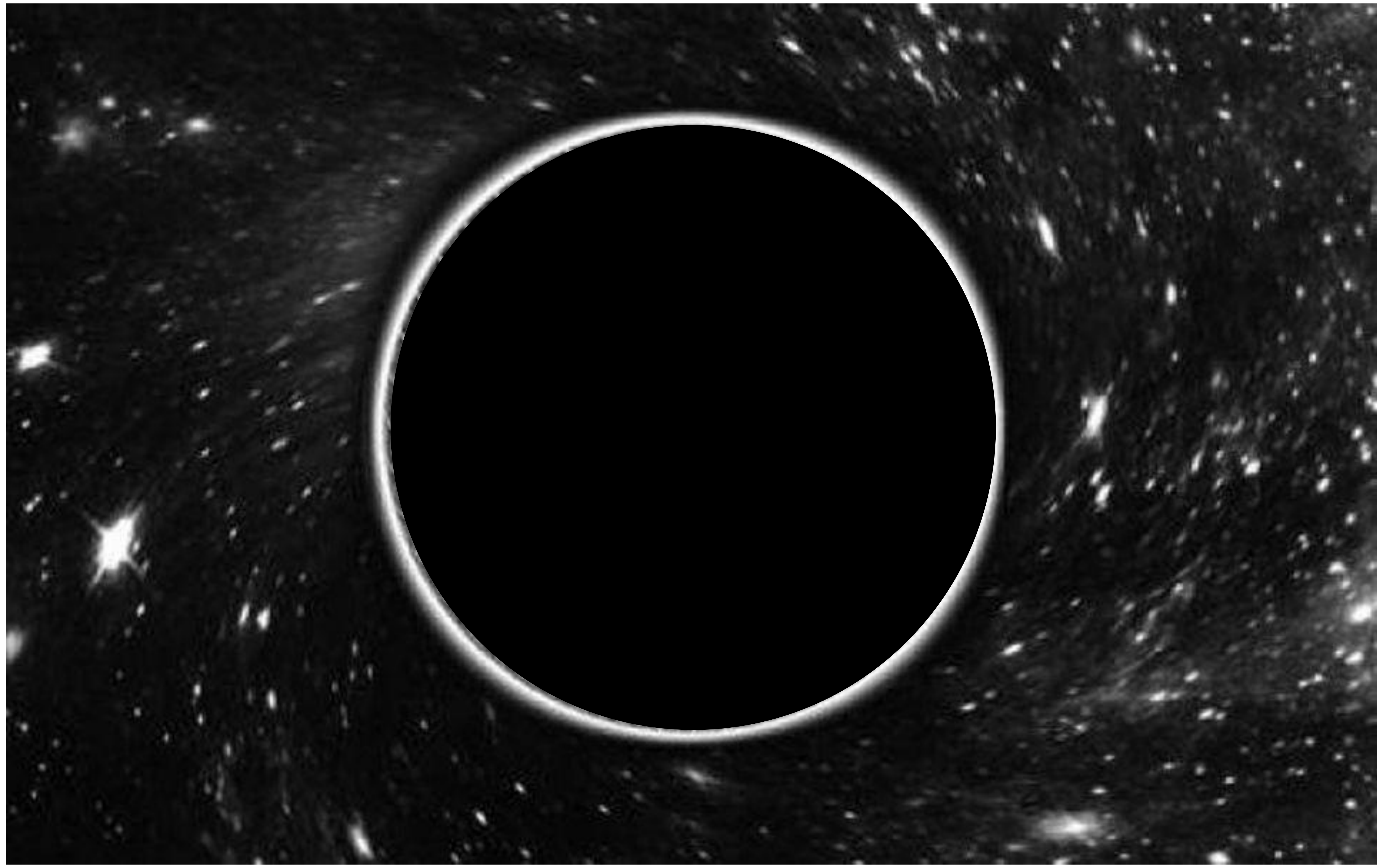
Quantum gravity

and

holography

Quantum Black holes

- Black holes have an entropy and a temperature, $T_H = \hbar c^3/(8\pi G M k_B)$.
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.



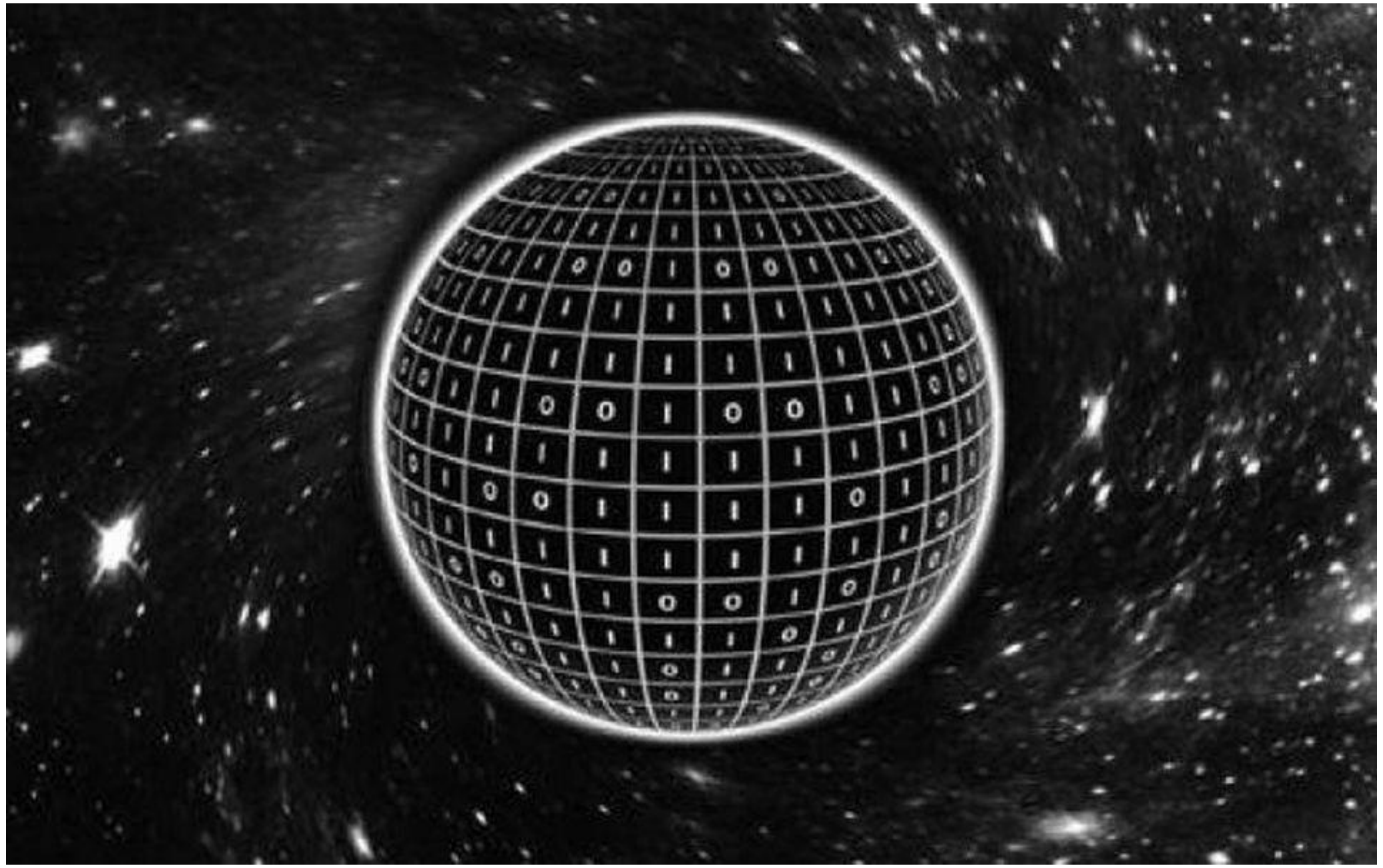
T.B. BAKKER / DR. J.P. VAN DER SCHAAR

J. D. Bekenstein, PRD **7**, 2333 (1973)
S. W. Hawking, Nature **248**, 30 (1974)

<u>Remarkable features:</u>

- Entropy is finite.
- Entropy is not proportional to volume

Quantum Black holes



T.B. BAKKER / DR. J.P. VAN DER SCHAAR

A quantum computer simulating a black hole must have:

- Number of qubits proportional to the surface area *i.e.* it is a ‘hologram’

Quantum Black holes

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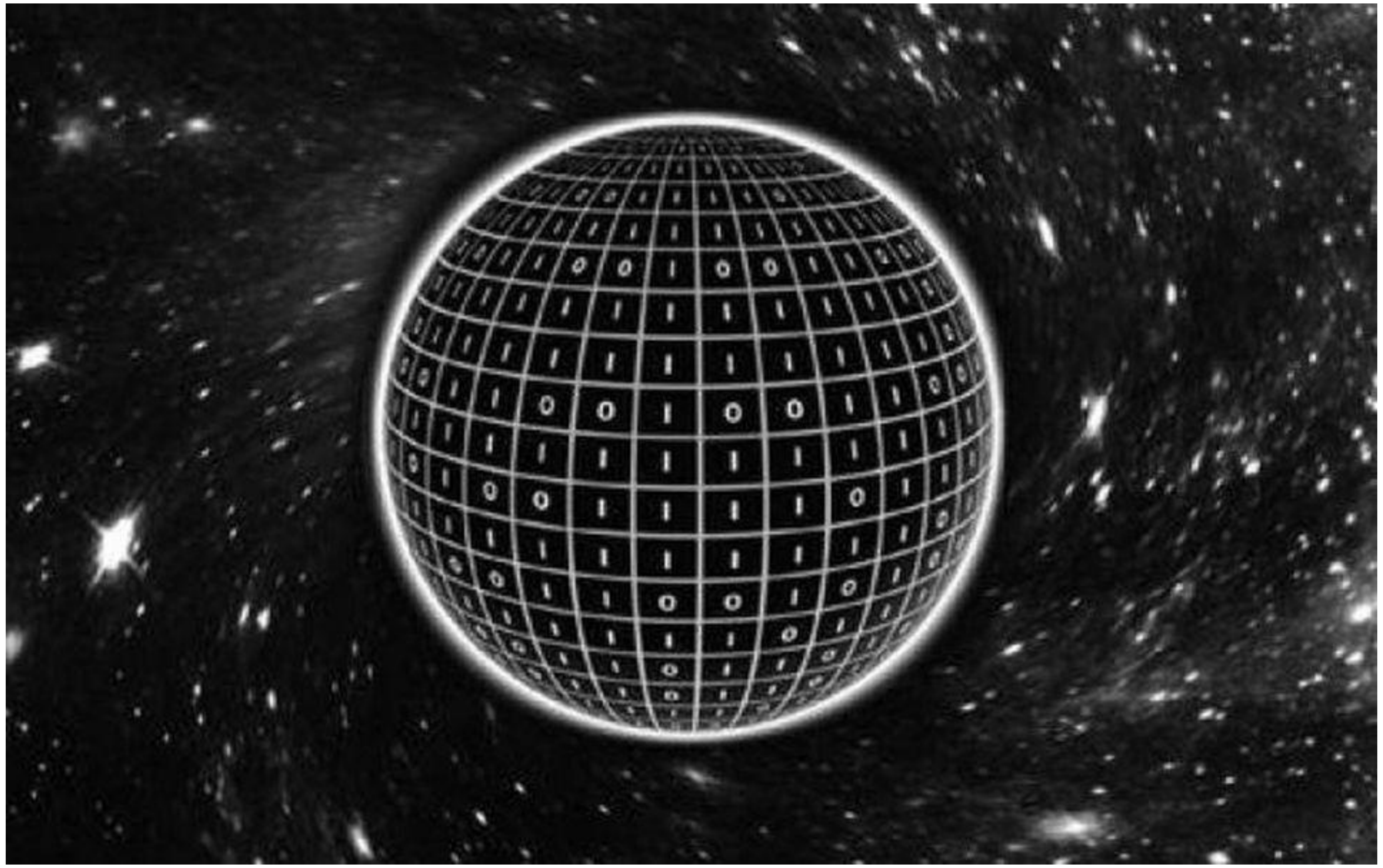
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3$



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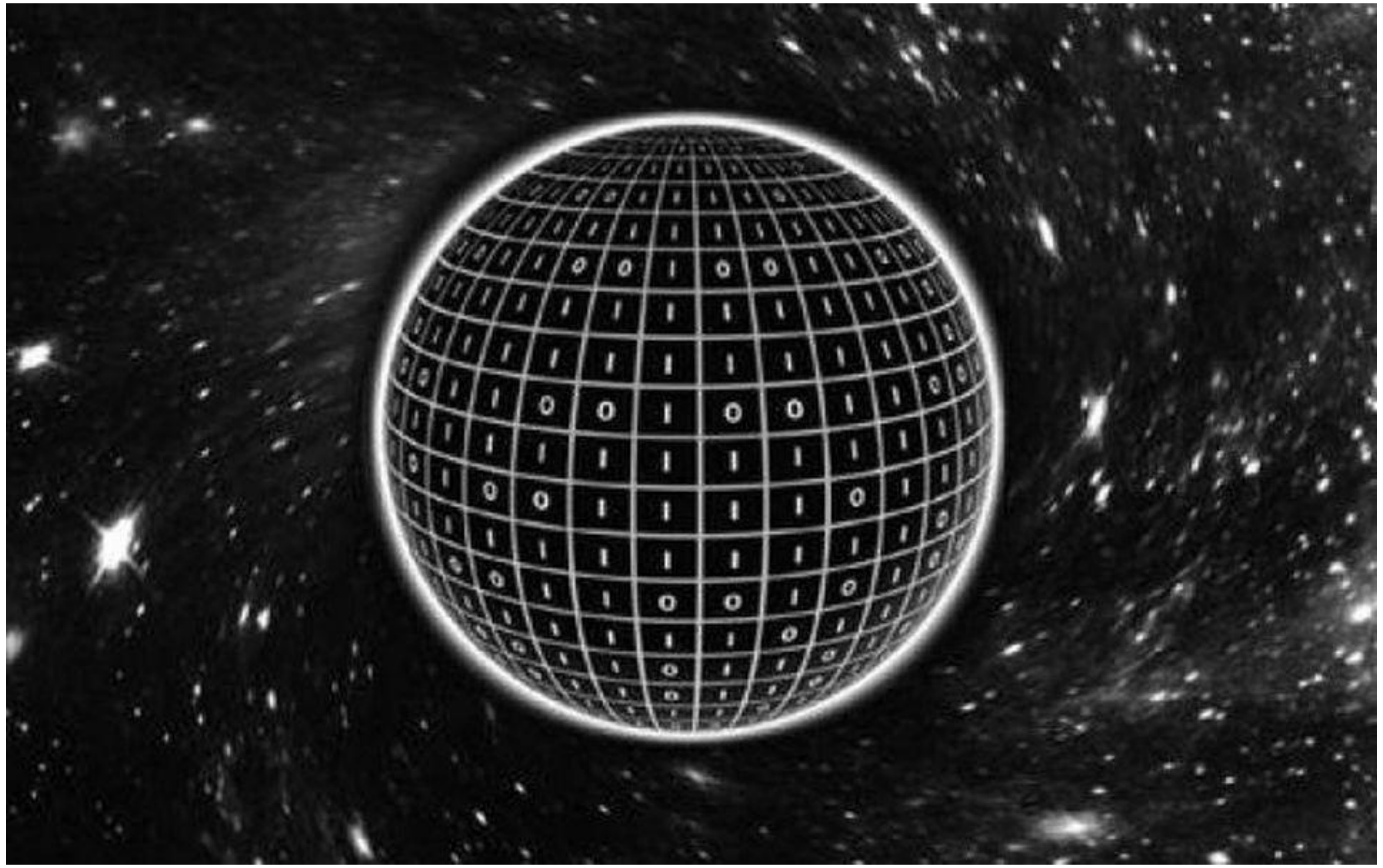
- The entropy is proportional to their surface area. $S = A k_B c^3/(4G\hbar)$.
- They relax to thermal equilibrium in a time $\sim 8\pi G M/c^3 = \hbar/(k_B T_H)$ which is Planckian!



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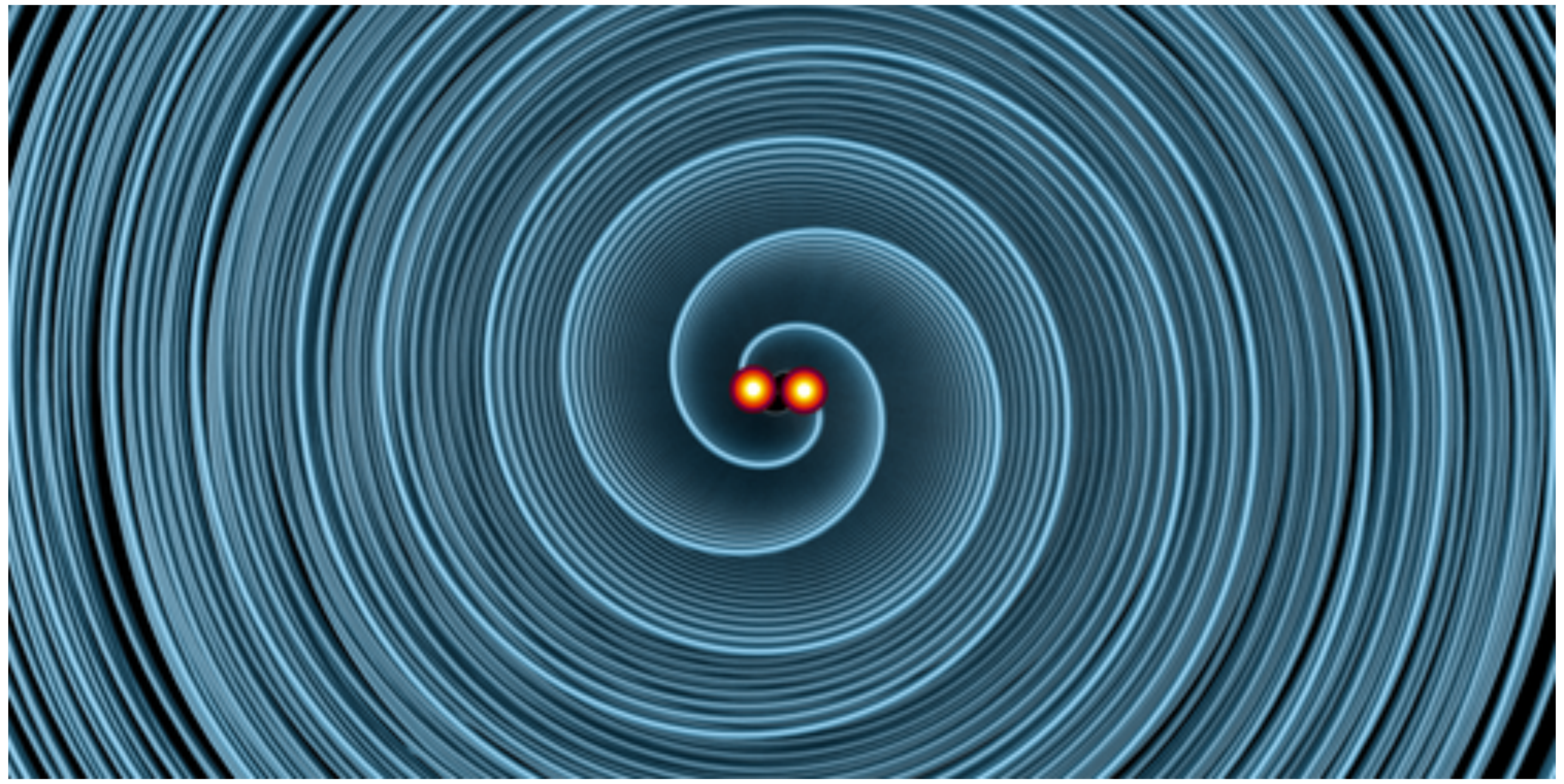
Black Holes Obey Information-Emission Limits

April 22, 2021 • *Physics* 14, s47 –Christopher Crockett

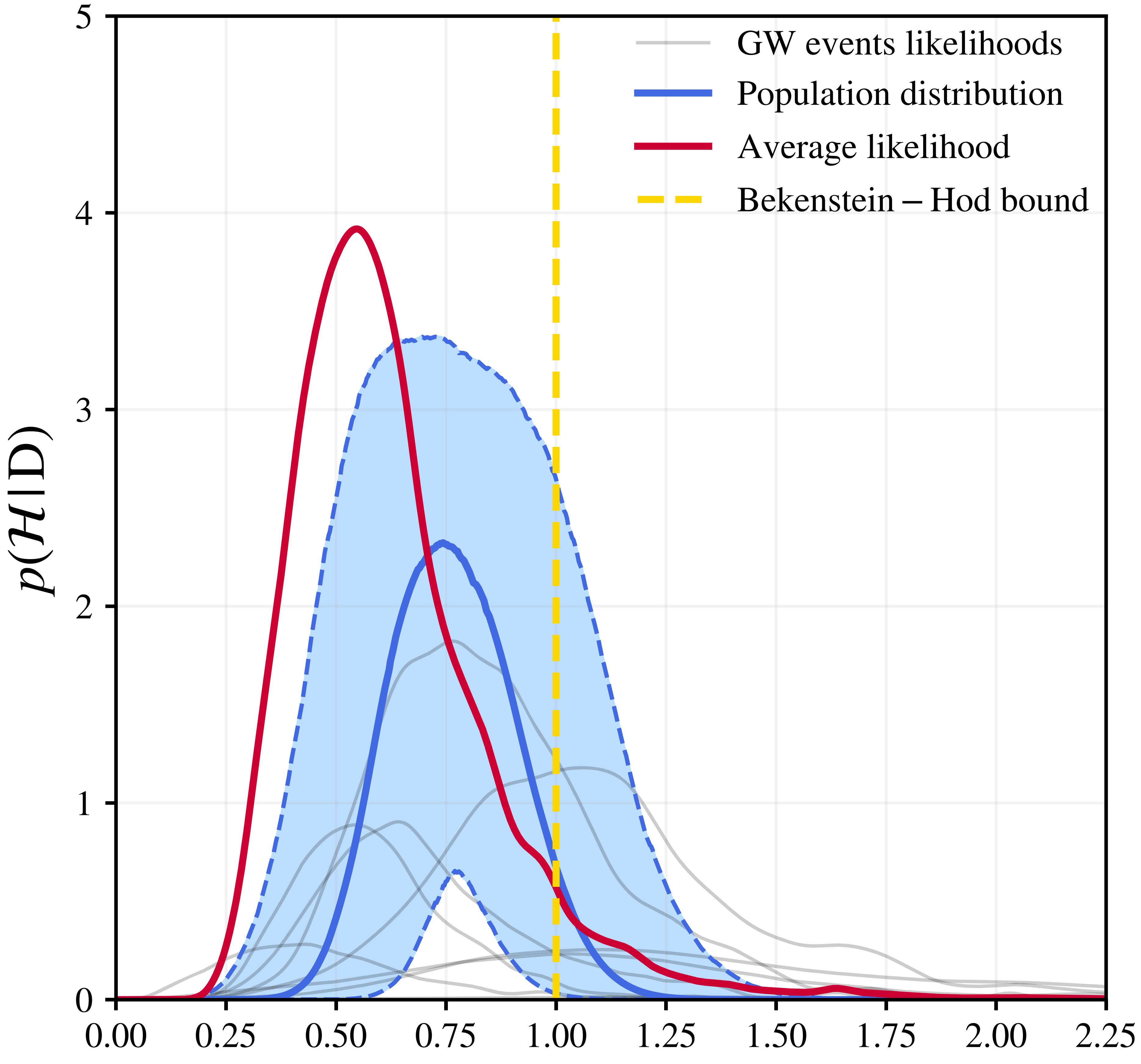
G. Carullo, D. Laghi, J. Veitch, W. Del Pozzo, Phys. Rev. Lett. **126**, 161102 (2021)

An analysis of the gravitational waves emitted from black hole mergers confirms that black holes are the fastest known information dissipaters.



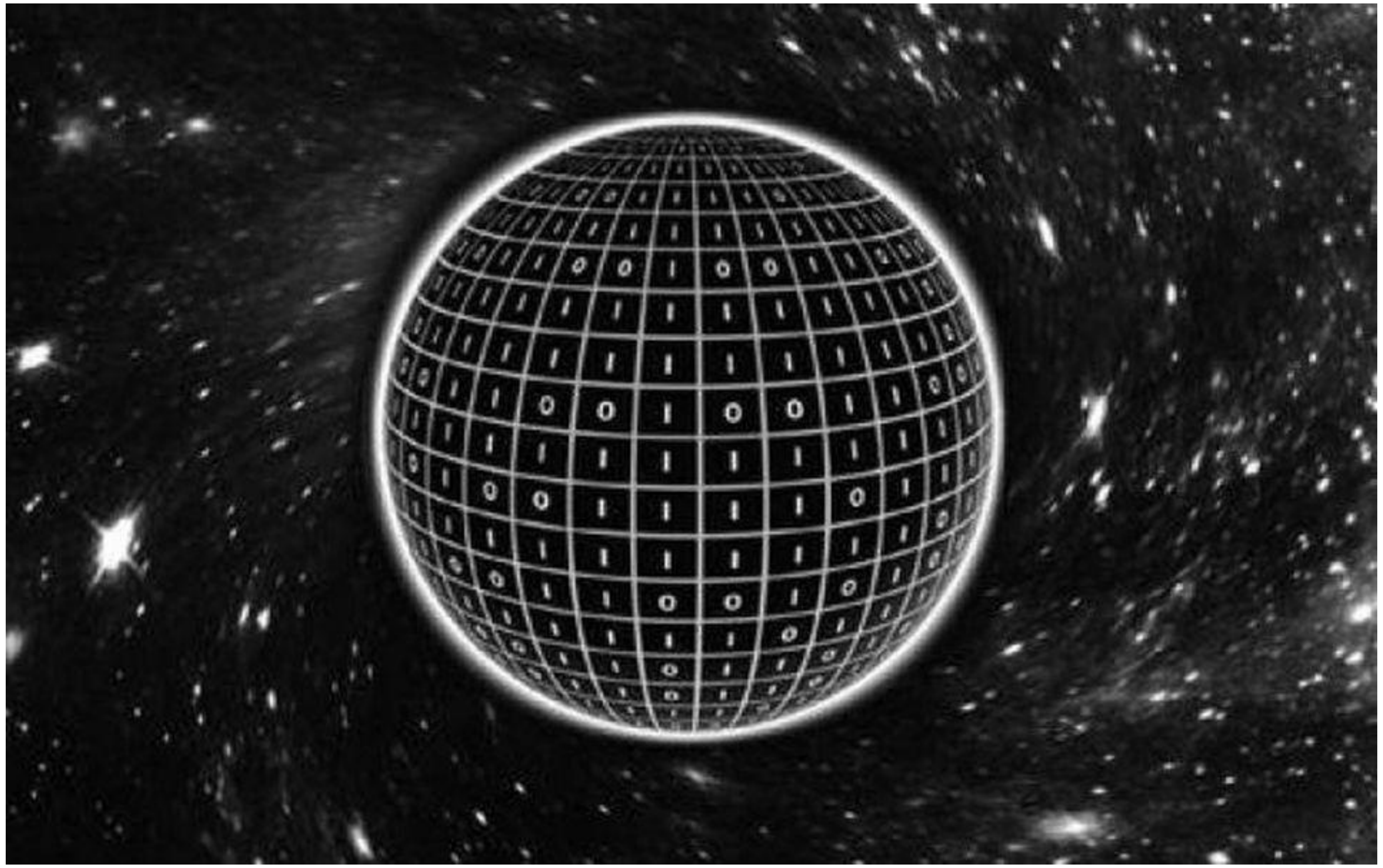
Gravity wave observations of 8 different black holes show a relaxation time

$$\tau \sim \frac{\hbar}{k_B T}$$



$$\mathcal{H} = \frac{1}{\pi}\frac{\hbar/\tau}{k_B T}$$

Quantum Black holes



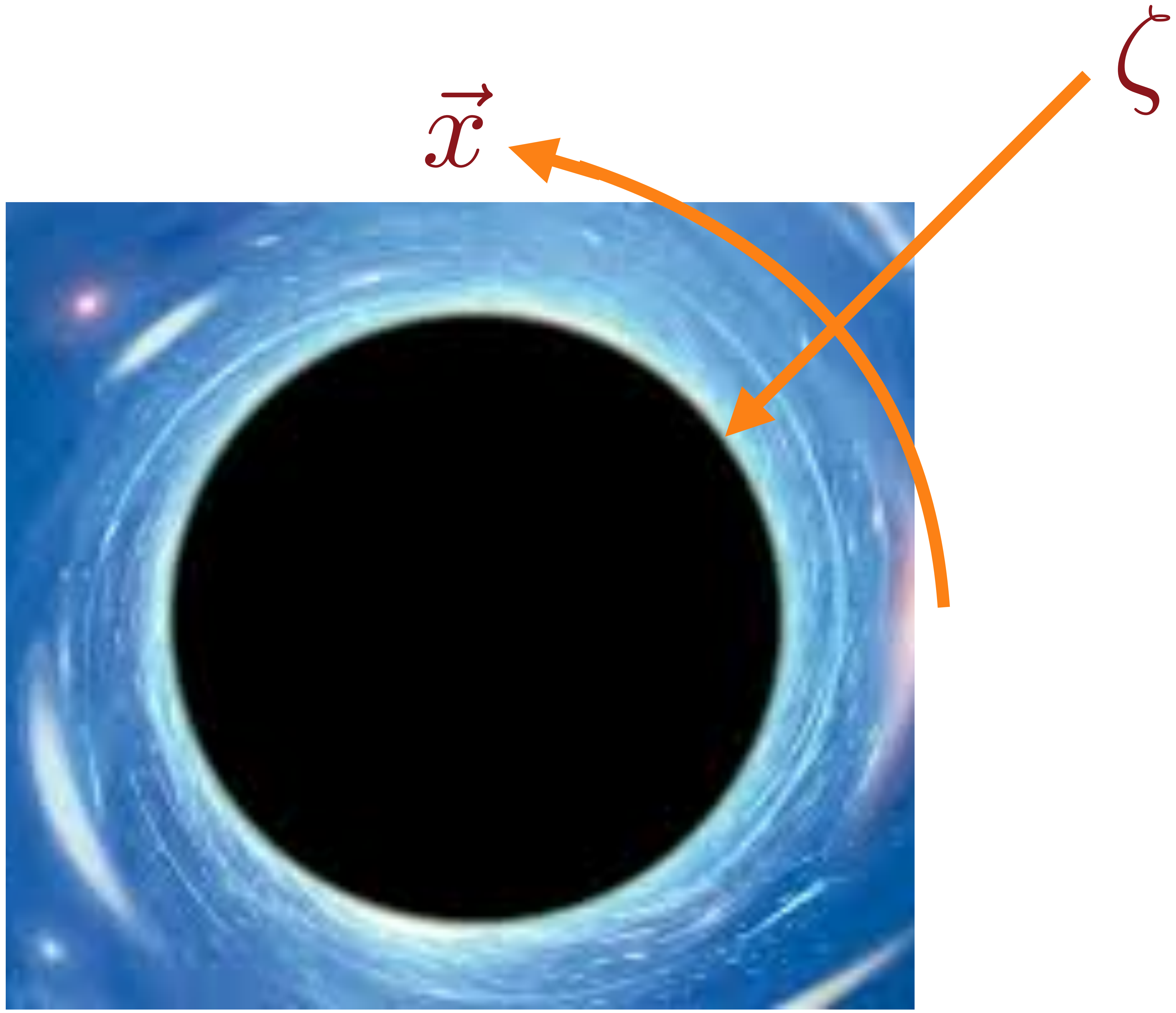
T.B. BAKKER / DR. J.P. VAN DER SCHAAR

A quantum computer simulating a black hole must have:

- Number of qubits proportional to the surface area *i.e.* it is a 'hologram'
- No quasiparticles and Planckian time relaxation to thermal equilibrium.

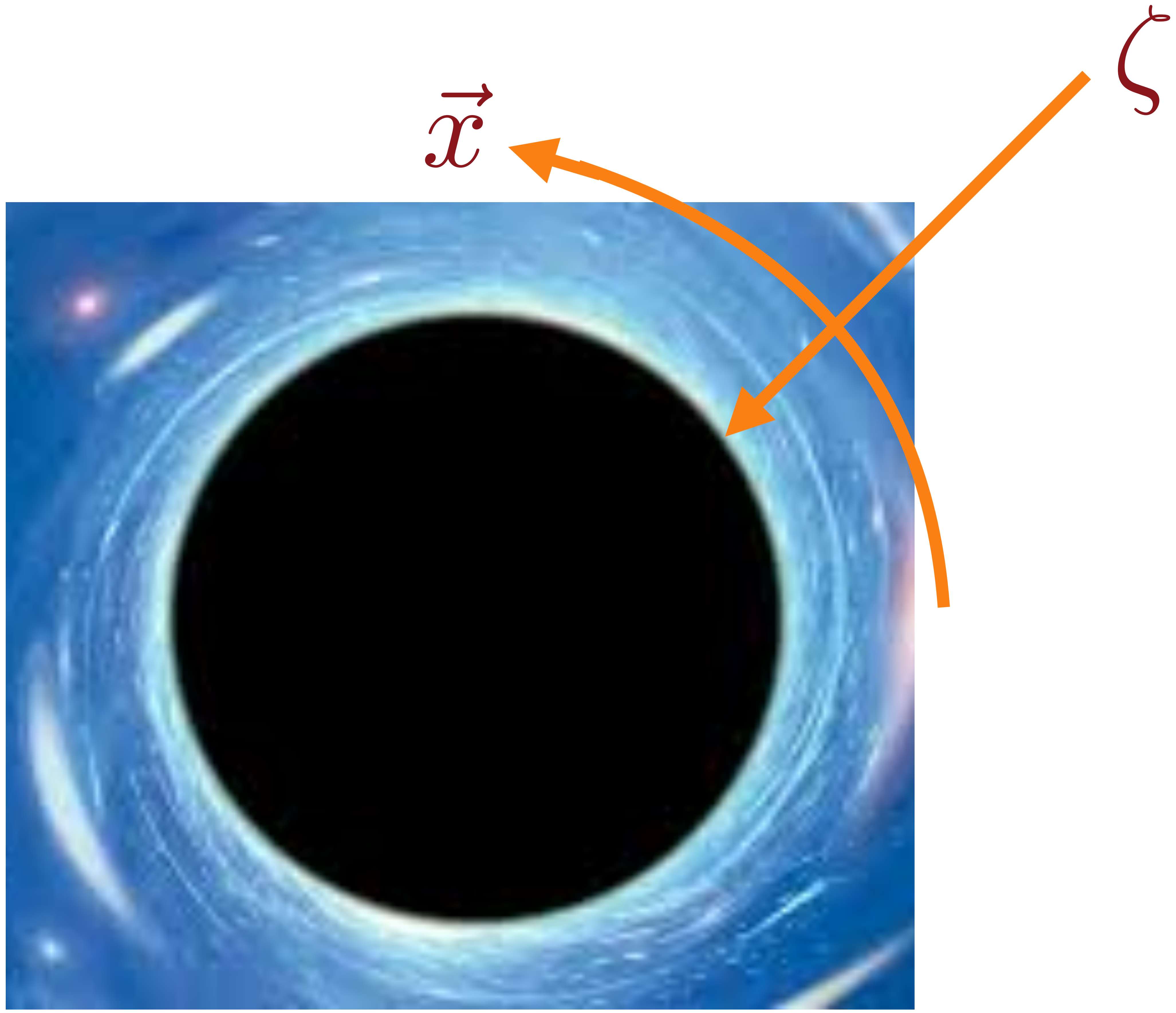
Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge

Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



Zooming into the near-horizon region of a charged black hole at low temperature, yields a quantum theory in one space (ζ) and one time dimension

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



This should be dual to a quantum computer in 0 space dimensions:

Maxwell's electromagnetism
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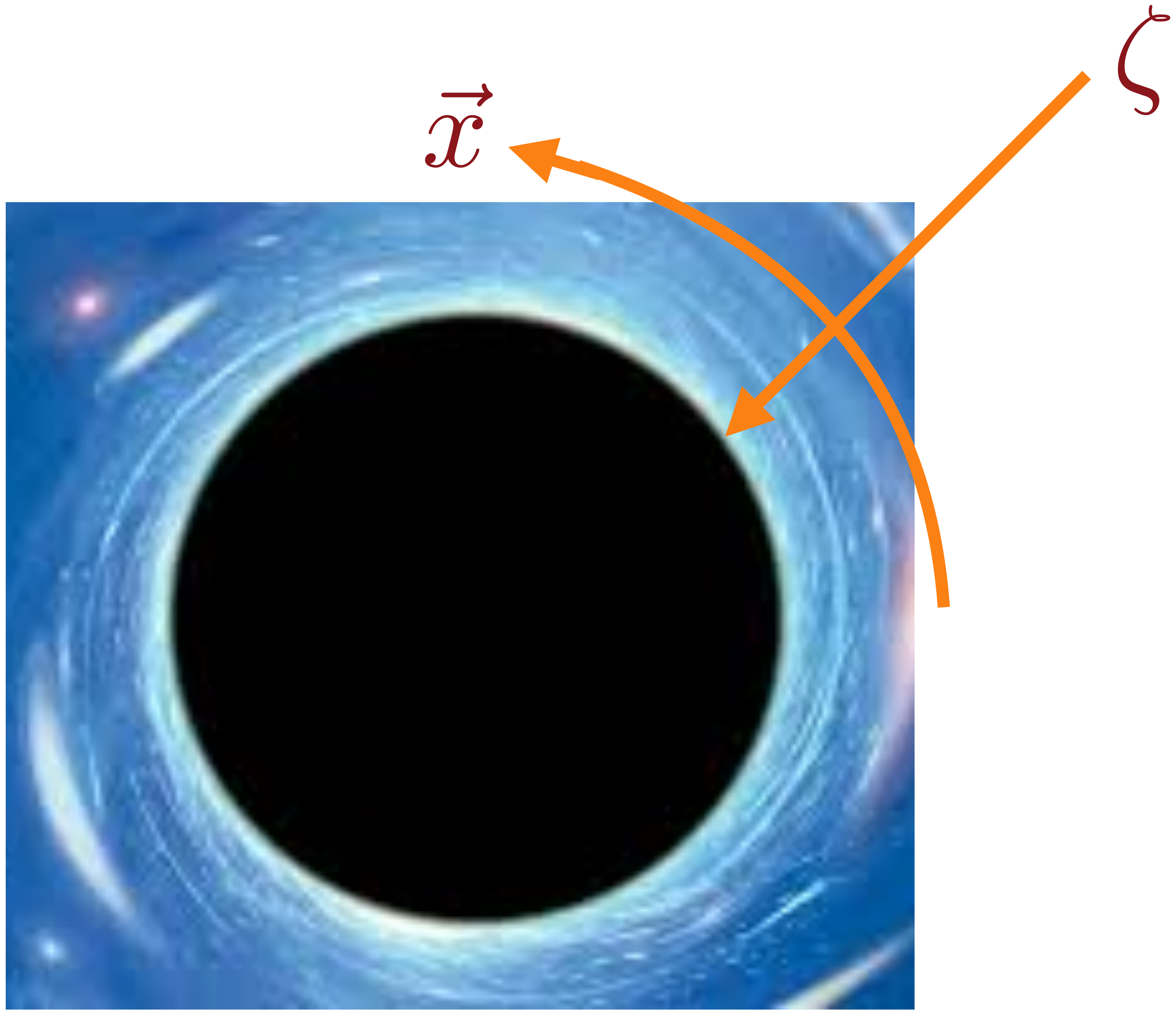
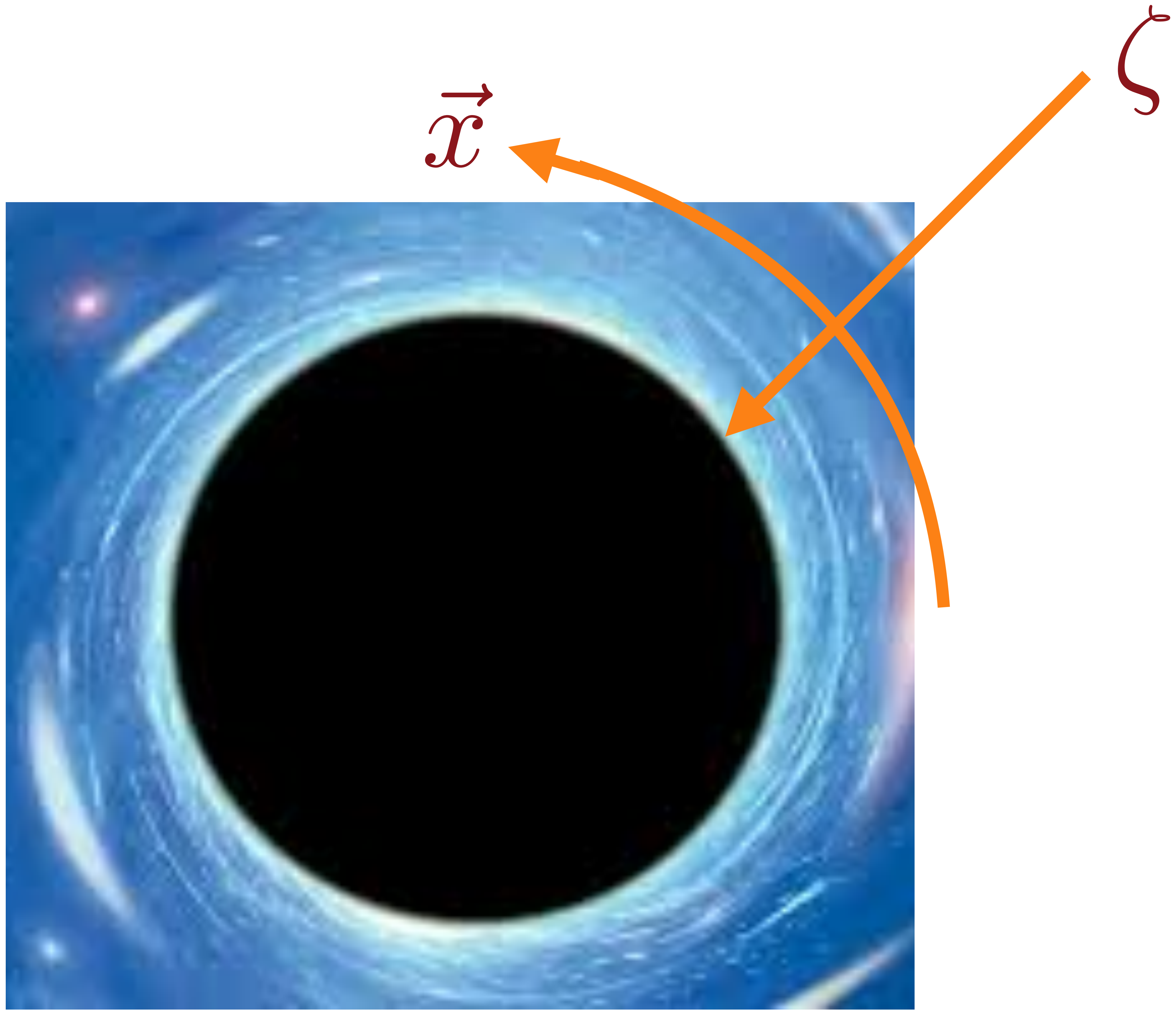


This should be dual to a quantum computer in 0 space dimensions:

The SYK model!

Maxwell's electromagnetism and Einstein's general relativity allow black hole solutions with a net charge



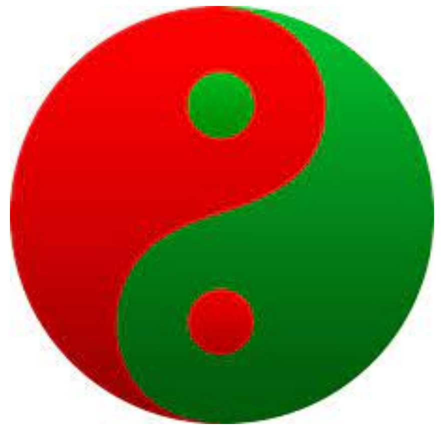
The quantum versions of Maxwell's and Einstein's equations in this two-dimensional spacetime are also the equations describing electron entanglement in the SYK model!

The Sachdev-Ye-Kitaev (SYK) model

The SYK model has a scale-invariant
entanglement structure:
i.e. electrons are entangled
at all distances

In one set of variables, it models the ***strange*** electrical properties of a material called YBCO

Sachdev, Ye (1993)



In a *dual* set of variables it describes
charged ***black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)

Thermodynamics of the SYK model and charged black holes

SYK model

$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T)$$

Charged black holes

$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right)$$

A_0 is the area of the horizon at $T = 0$.

S.S. 2010, 2015

SYK model

$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T)$$

$$G(\tau) \sim e^{-2\pi \mathcal{E} T \tau} \left(\frac{T}{\sin(\pi T \tau)} \right)^{2\Delta}$$

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SYK model	Charged black holes
$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T)$$	$$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2} c^2}{2}\frac{k_B T}{\hbar}\right)$$ A_0 is the area of the horizon at $T = 0$.
$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$	$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$
$$\lim_{T\to 0}\frac{1}{Nk_B}\frac{\partial S}{\partial Q} = 2\pi\mathcal{E}$$	$$\lim_{T\to 0}\frac{1}{k_B}\frac{\partial S}{\partial Q} = 2\pi\mathcal{E}$$

S.S. 2010, 2015

SYK model	Charged black holes
$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T) - \frac{3}{2} \ln \left(\frac{N^{1/3} U}{k_B T} \right) + \ldots$	$\frac{S(T)}{k_B} = \frac{1}{\hbar G} \left(\frac{A_0 c^3}{4} + \frac{\sqrt{\pi} A_0^{3/2} c^2}{2} \frac{k_B T}{\hbar} \right) - \frac{3}{2} \ln \left(\frac{c^2}{A_0^{1/6} (\hbar G)^{1/3} (k_B T/\hbar)} \right) + \ldots$
$G(\tau) \sim e^{-2\pi \mathcal{E} T \tau} \left(\frac{T}{\sin(\pi T \tau)} \right)^{2\Delta}$	$G(\tau) \sim e^{-2\pi \mathcal{E} T \tau} \left(\frac{T}{\sin(\pi T \tau)} \right)^{2\Delta}$
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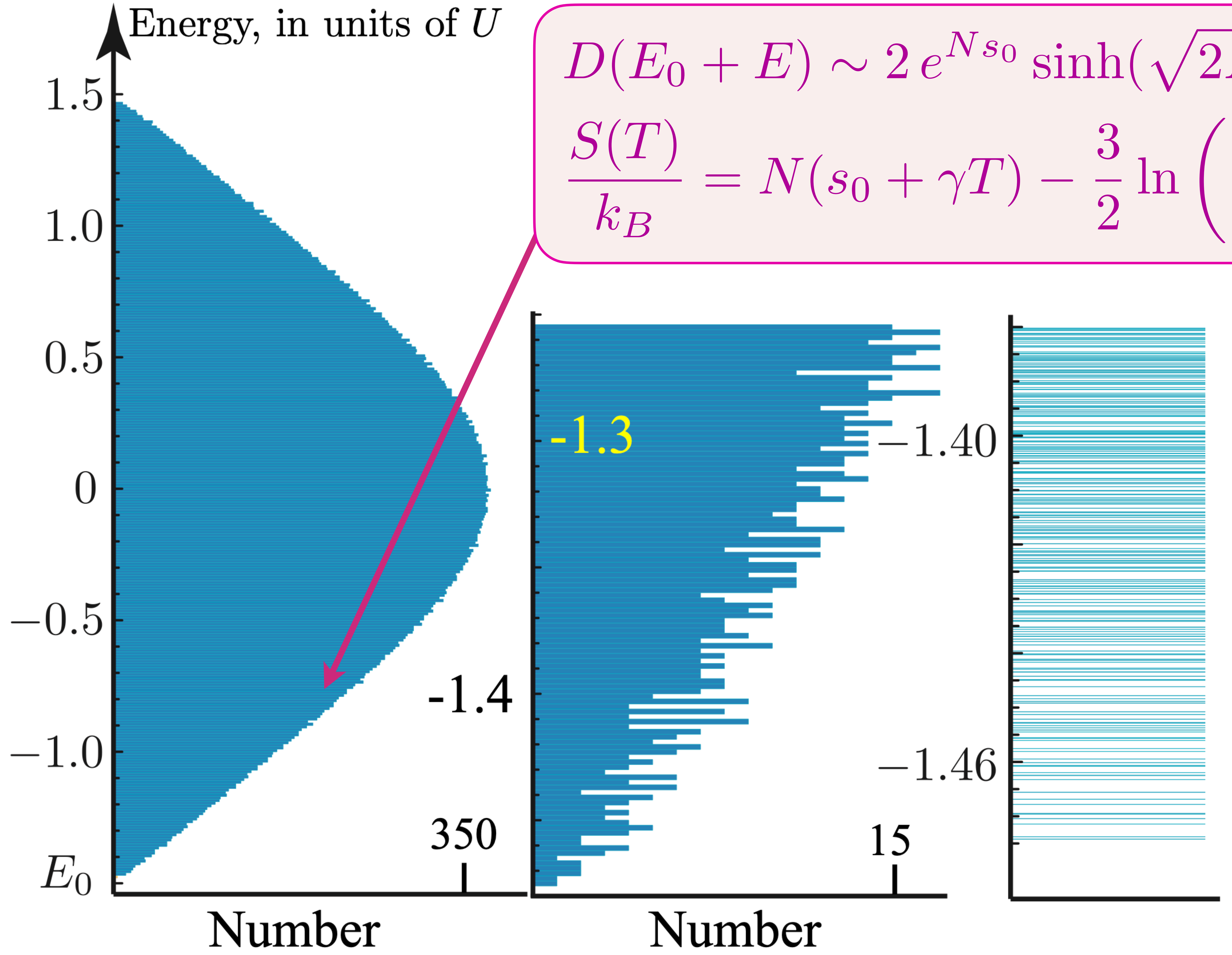
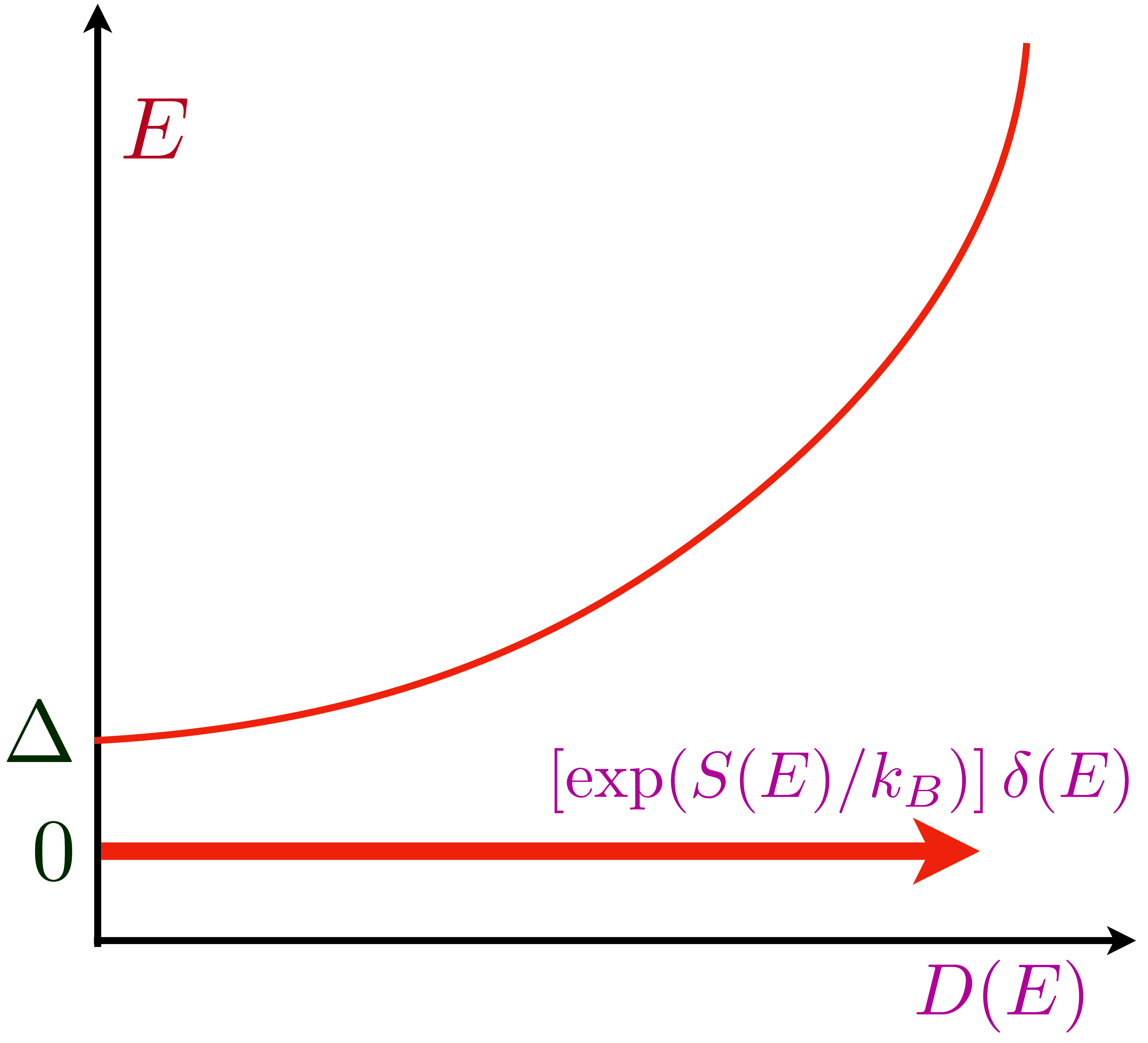
SYK model	Charged black holes
$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T) - \frac{3}{2}\ln\left(\frac{N^{1/3}U}{k_B T}\right) + \ldots$	$\frac{S(T)}{k_B} = \frac{1}{\hbar G}\left(\frac{A_0 c^3}{4} + \frac{\sqrt{\pi}A_0^{3/2}c^2}{2}\frac{k_B T}{\hbar}\right) - \frac{3}{2}\ln\left(\frac{c^2}{A_0^{1/6}(\hbar G)^{1/3}(k_B T/\hbar)}\right) + \ldots$
$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$	$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$
$\lim_{T\to 0}\frac{1}{Nk_B}\frac{\partial S}{\partial Q} = 2\pi\mathcal{E}$	$\lim_{T\to 0}\frac{1}{k_B}\frac{\partial S}{\partial Q} = 2\pi\mathcal{E}$
$D(E) \sim \exp(Ns_0)\sinh\left(\sqrt{2N\gamma E}\right)$	$D(E) \sim \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\sinh\left(\left[\sqrt{\pi}A_0^{3/2}\frac{c^3}{\hbar G}\frac{E}{\hbar c}\right]^{1/2}\right)$

$$D(E_0 + E) \sim 2\, e^{N s_0} \sinh(\sqrt{2N\gamma E})$$

$$\frac{S(T)}{k_B} = N(s_0 + \gamma T) - \frac{3}{2} \ln\left(\frac{U}{T}\right)$$



SYK model
or
charged black hole



charged black hole
in string theory
or
supersymmetric SYK model
or
supersymmetric CFT

Corrections to scaling at the SYK saddle point

SYK model

$$\frac{S(T)}{k_B} = N(s_0 + \gamma\, k_B T) - \frac{3}{2}\ln\left(\frac{N^{1/3}U}{k_B T}\right) + \ldots$$

$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$

$$\lim_{T\to 0}\frac{1}{Nk_B}\frac{\partial S}{\partial Q} = 2\pi\mathcal{E}$$

$$D(E) \sim \exp(Ns_0)\sinh\left(\sqrt{2N\gamma E}\right)$$

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$$G(\tau) \sim e^{-2\pi\mathcal{E}T\tau}\left(\frac{T}{\sin(\pi T\tau)}\right)^{2\Delta}$$

$$\lim_{T\to 0}\frac{1}{k_B}\frac{\partial S}{\partial Q} = 2\pi\mathcal{E}$$

$$D(E) \sim \exp\left(\frac{A_0 c^3}{4\hbar G}\right)\sinh\left(\left[\sqrt{\pi}A_0^{3/2}\frac{c^3}{\hbar G}\frac{E}{\hbar c}\right]^{1/2}\right)$$

Conformal Perturbation theory

$$S = S_{\rm CFT} + \sum_h \lambda_h \int_0^\beta d\tau O_h(\tau)$$

where $G_{\rm CFT} = G_* \sim \mathrm{sgn}(\tau)/\sqrt{|\tau|}$ and $\langle O_h(\tau) O_h(0)\rangle \sim 1/|\tau|^{2h}$

$$G(\tau) \sim \frac{\mathrm{sgn}(\tau)}{\sqrt{|\tau|}} \left(1 + \sum_h \frac{g_h}{|\tau|^{h-1}} + \dots\right) \quad , \quad S(T) = N\left[s_0 + \sum_h s_h T^{h-1} + \dots\right]$$

Conformal Perturbation theory

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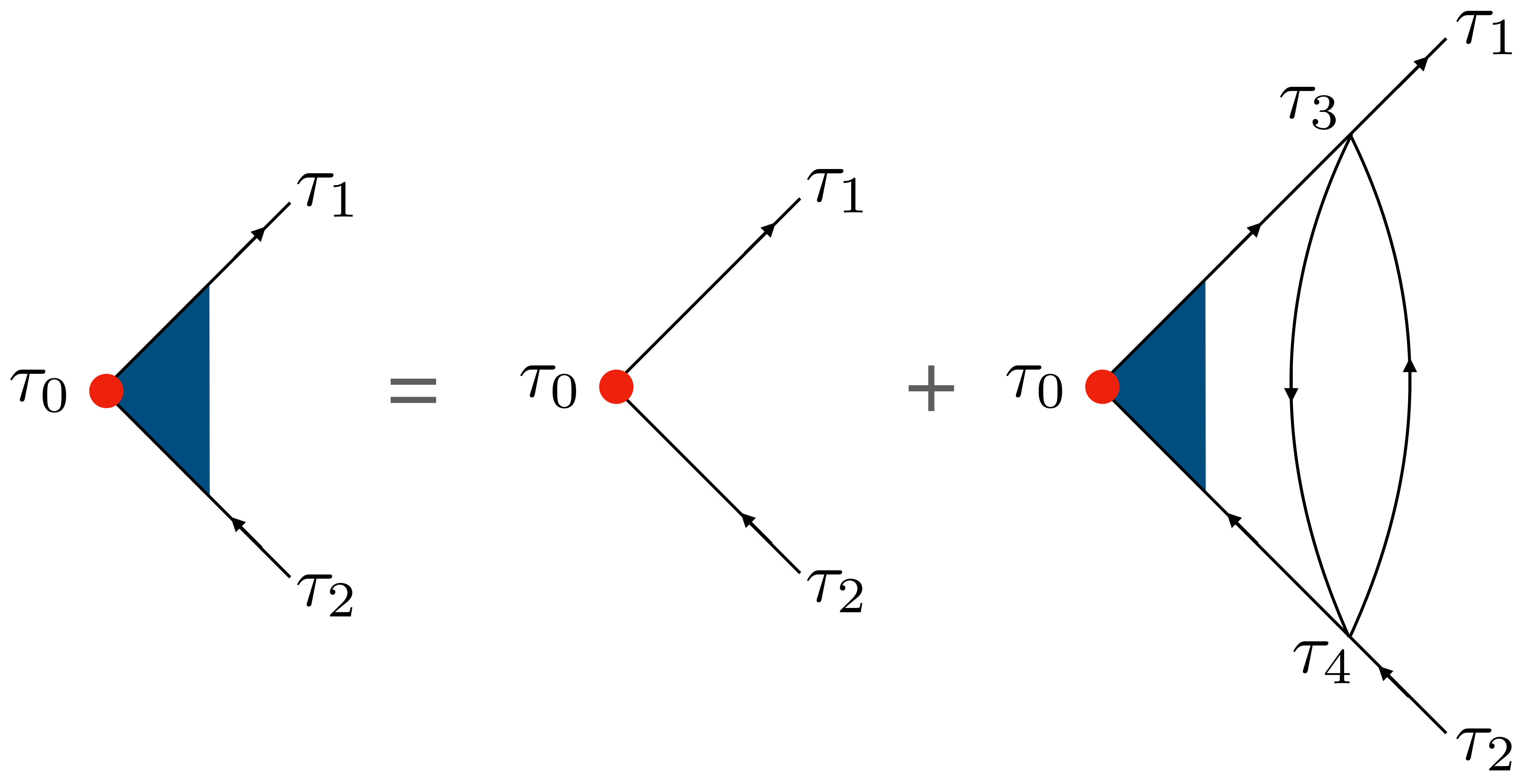

Gross, Rosenhaus (2017)
Klebanov, Tarnopolsky (2017)

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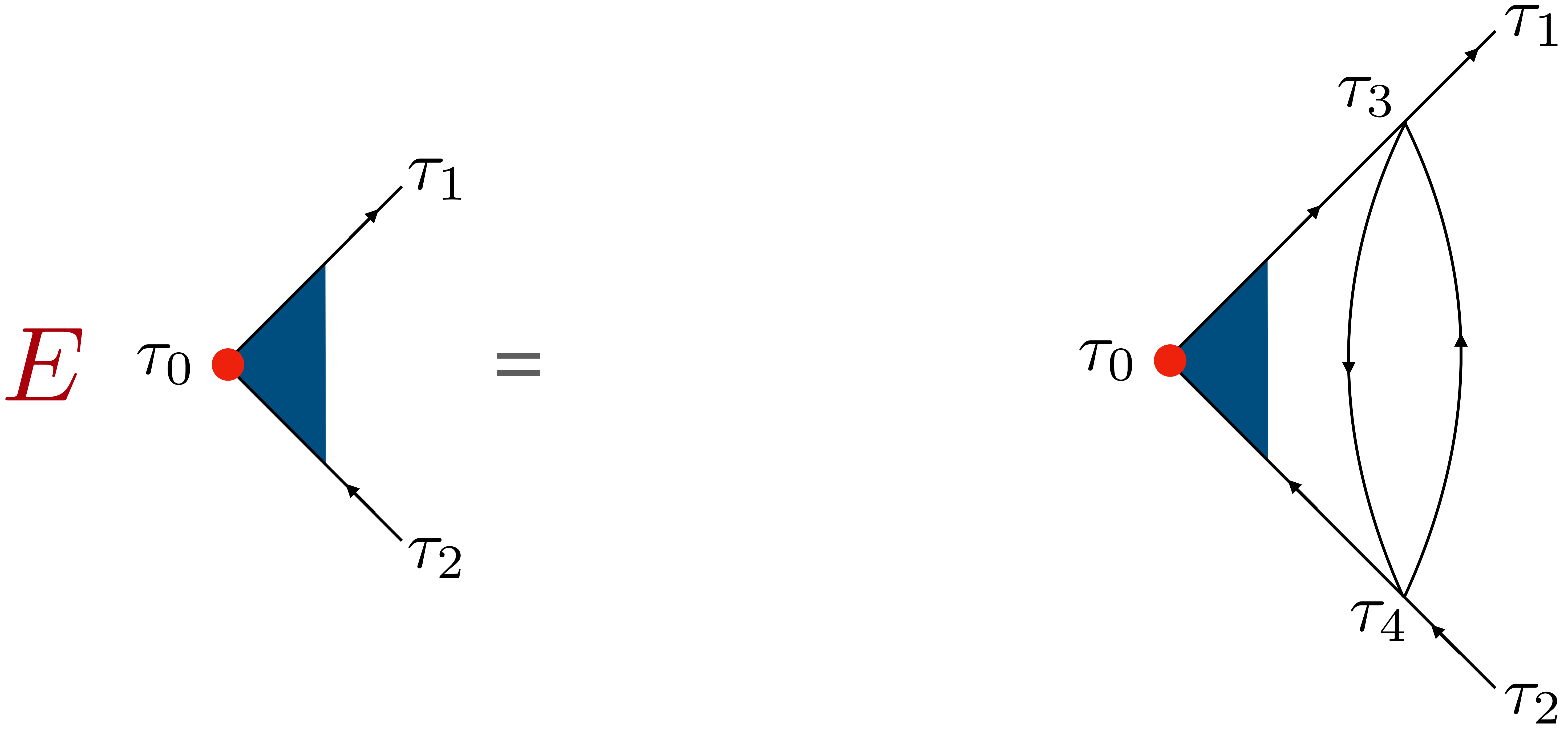

Solution of eigenvalue equation with $E = 1$ yields a tower of O_h.

Gross, Rosenhaus (2017)
Klebanov, Tarnopolsky (2017)

Conformal Perturbation theory

We define the three point function

$$v_h(\tau_1, \tau_2, \tau_0) = \langle c(\tau_1) c^\dagger(\tau_2) O_h(\tau_0) \rangle \, .$$

In the long time scaling limit, we can drop the bare first time on the right hand side, and obtain the eigenvalue equation

$$E v(\tau_1, \tau_2, \tau_0) = \int d\tau_3 d\tau_4 K(\tau_1, \tau_2; \tau_3, \tau_4) v_h(\tau_3, \tau_4, \tau_0) \, ,$$

where the kernel K is

$$K(\tau_1, \tau_2; \tau_3, \tau_4) = -3U^2 G_*(\tau_{13}) G_*(\tau_{24}) G_*(\tau_{34})^2 \, ,$$

with $\tau_{ij} \equiv \tau_i - \tau_j$, and we are interested in the eigenvalue $E = 1$.

Conformal Perturbation theory

It is sufficient to solve the eigenvalue equation as $\tau_0 \to \infty$. Then, we can use the operator product expansion to write

$$c(\tau_1)c^\dagger(\tau_2) \sim \mathrm{sgn}(\tau_{12}) \left[\frac{1}{|\tau_{12}|^{1/2}} + \sum_h \frac{c_h}{|\tau_{12}|^{1/2-h}} O_h(\tau_1) + \ldots \right]$$

Inserting this into the defintion of v, we conclude that $v \sim \mathrm{sgn}(\tau_{12})/|\tau_{12}|^{1/2-h}$ as $\tau_0 \to \infty$. Then the eigenvalue equation simplifies to

$$E = -\frac{3\tan(\pi h/2 - \pi/4)}{2h-1} = 1\,.$$

There are an infinite number of solutions, and the lowest values are $h = 2, 3.77354\ldots, 5.567946\ldots, 7.63197\ldots, \ldots$. Consequently, the low T behavior of the entropy is

$$S(T) = N\left[s_0 + \gamma\, T + \gamma_2\, T^{2.77354\ldots} + \ldots\right]\,.$$

We will have a particular interest in the $h=2$ operator in the remaining discussion.