Quantum criticality of Fermi surfaces in two dimensions

Talk online: sachdev.physics.harvard.edu

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Yejin Huh, Harvard

Max Metlitski, Harvard

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Outline

1. Quantum criticality of Fermi points:

   Dirac fermions in d-wave superconductors

2. Quantum criticality of Fermi surfaces:

   Onset of spin density wave order in the cuprates

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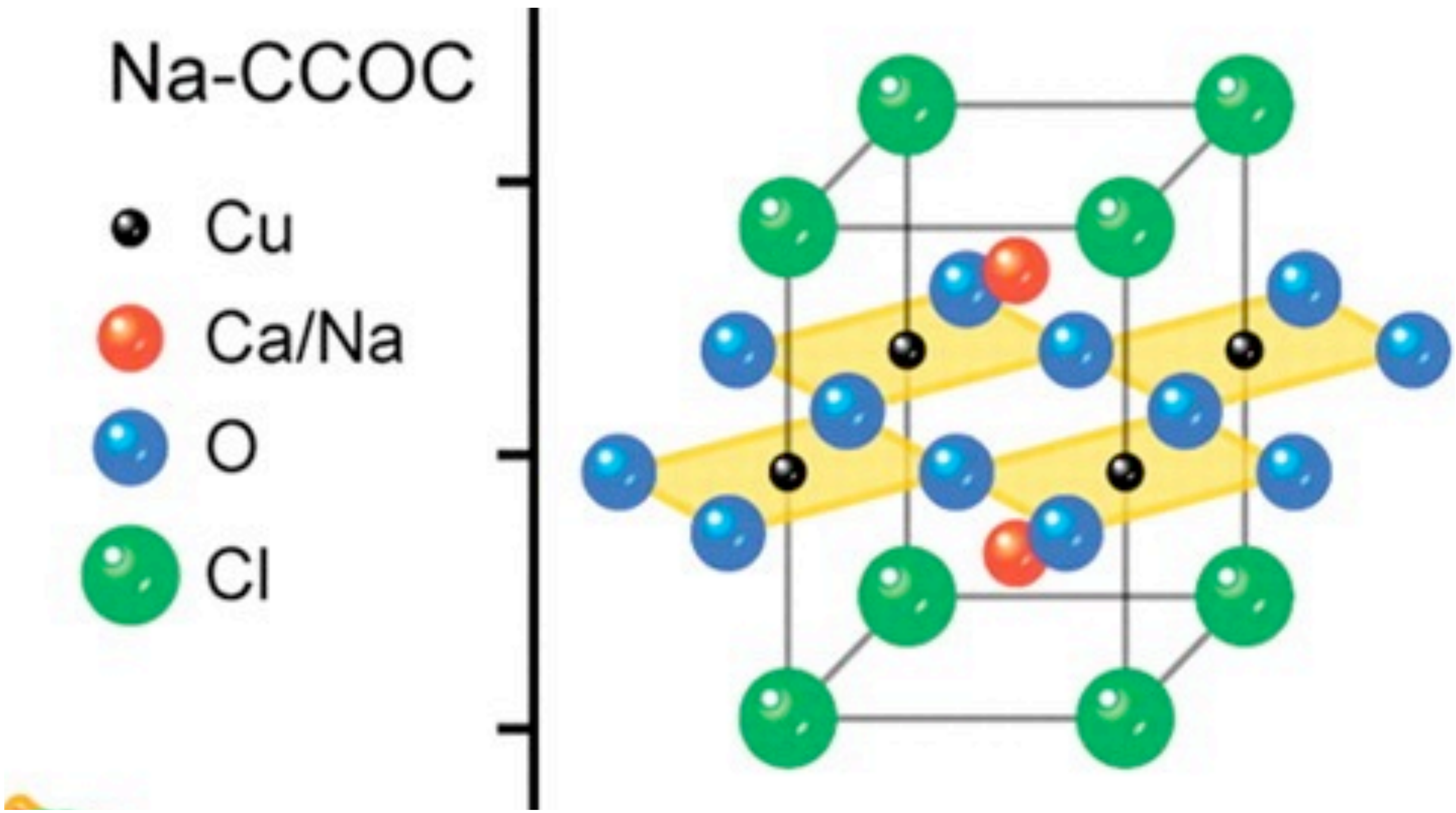
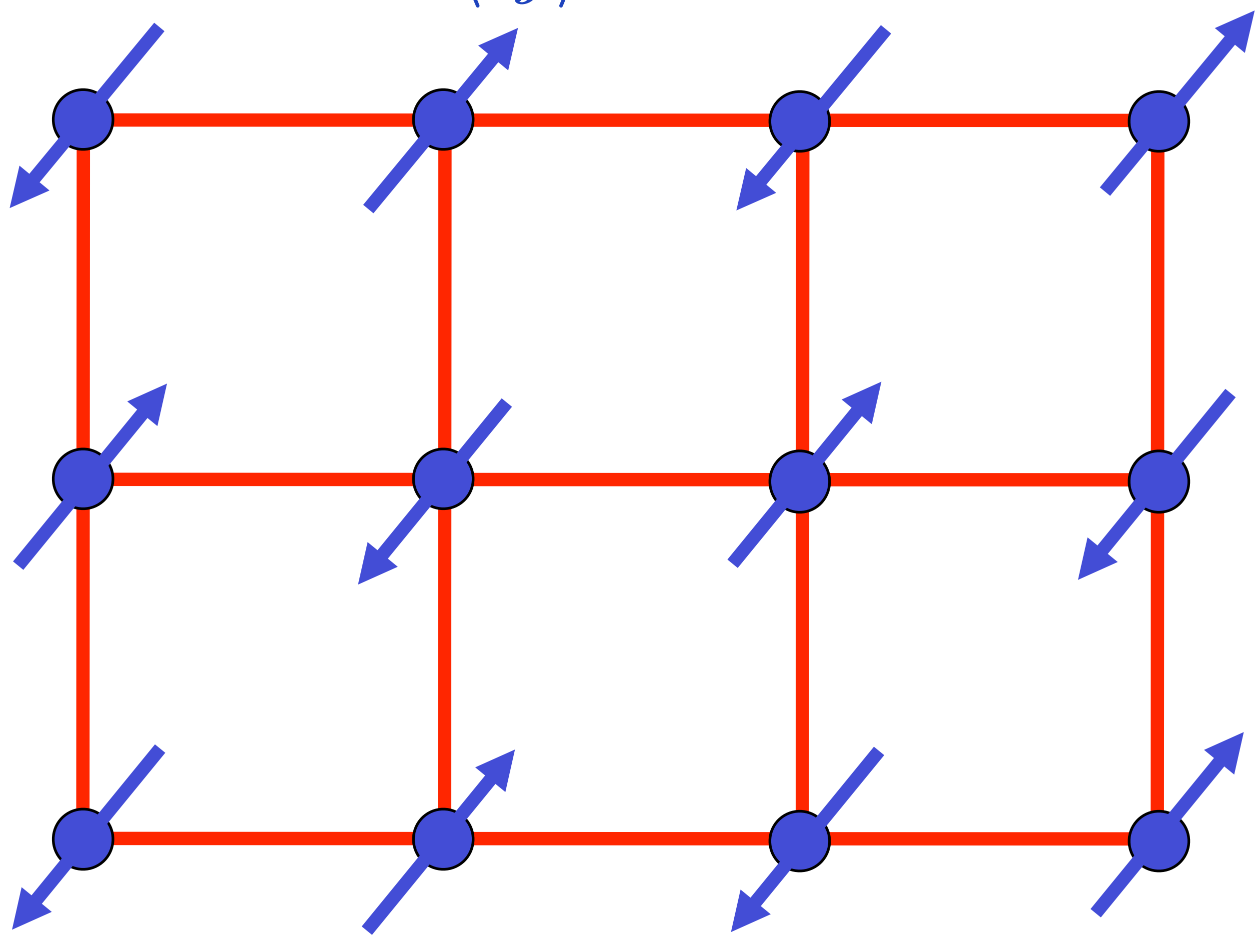
2. Quantum criticality of Fermi surfaces:
Onset of spin density wave order in the cuprates

The cuprate superconductors



Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



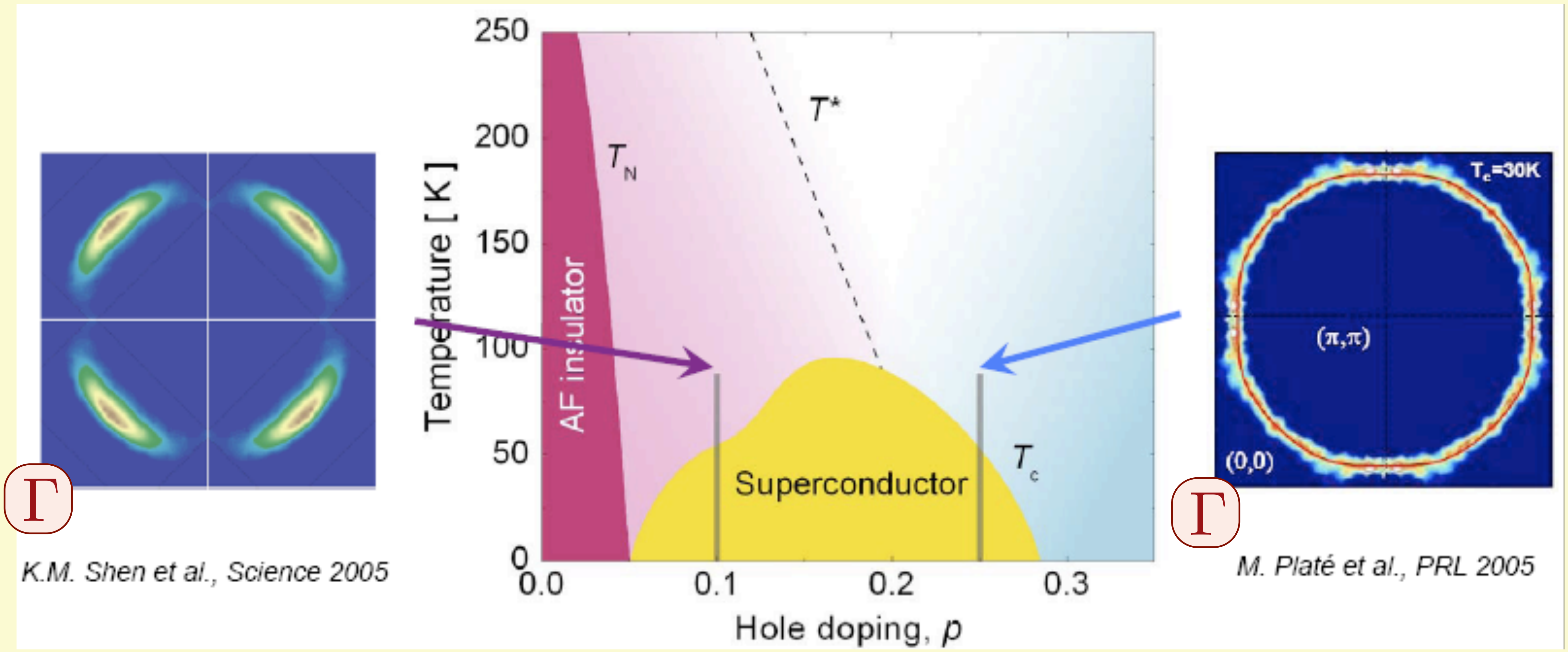
Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

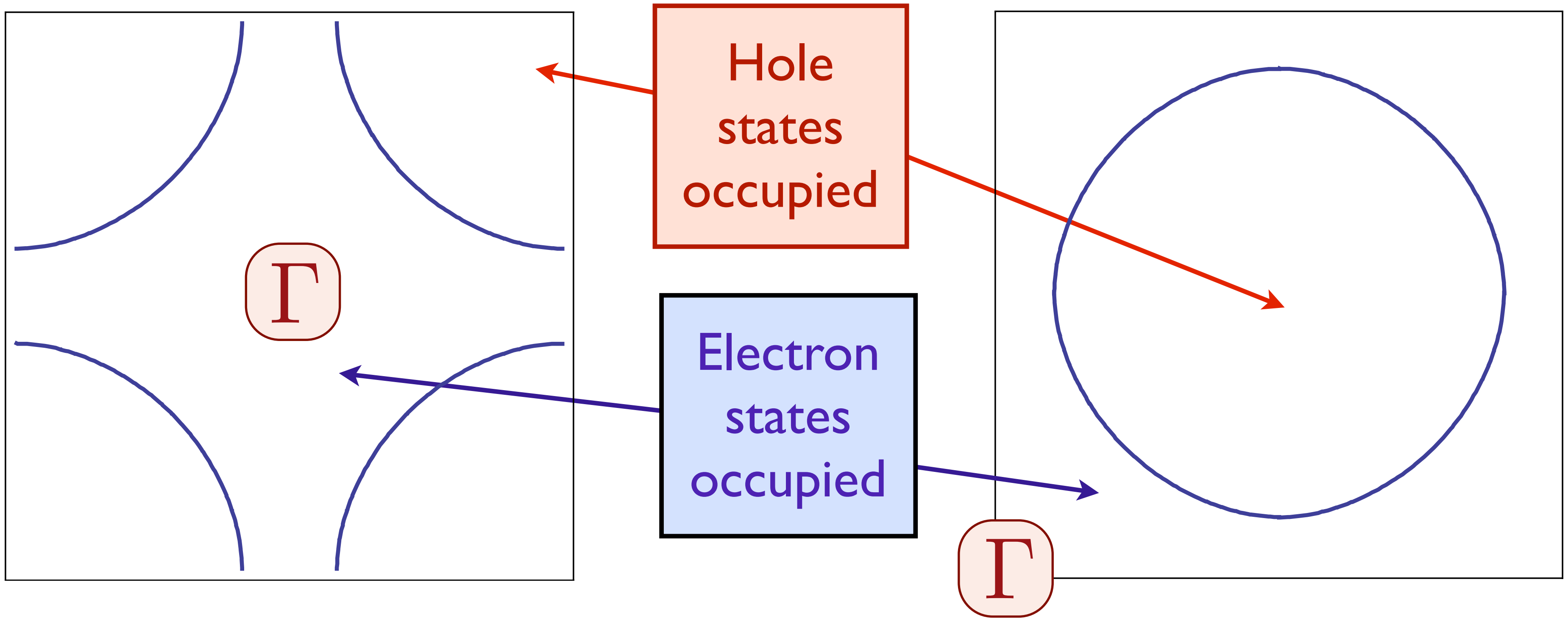
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

Large hole Fermi surface

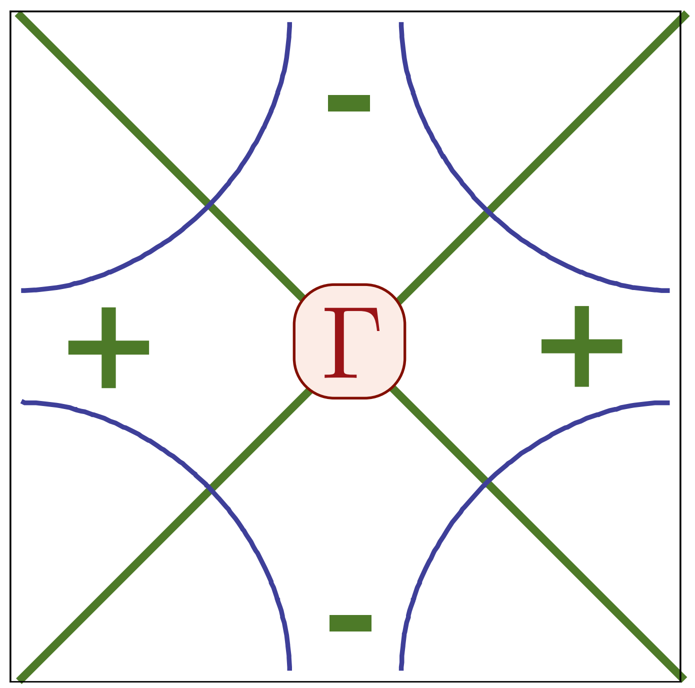
d-wave superconductivity in cuprates

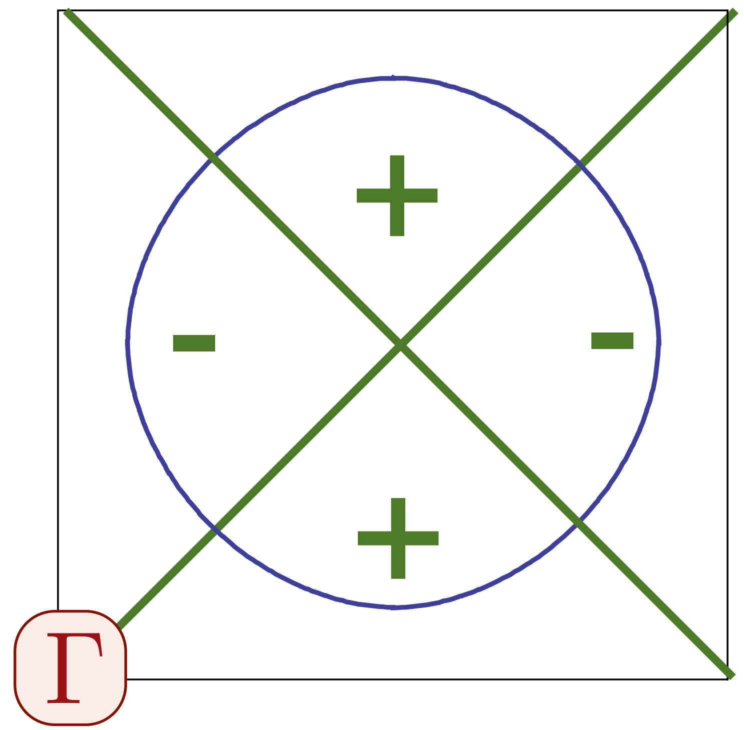


$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

- Begin with free electrons.

d-wave superconductivity in cuprates

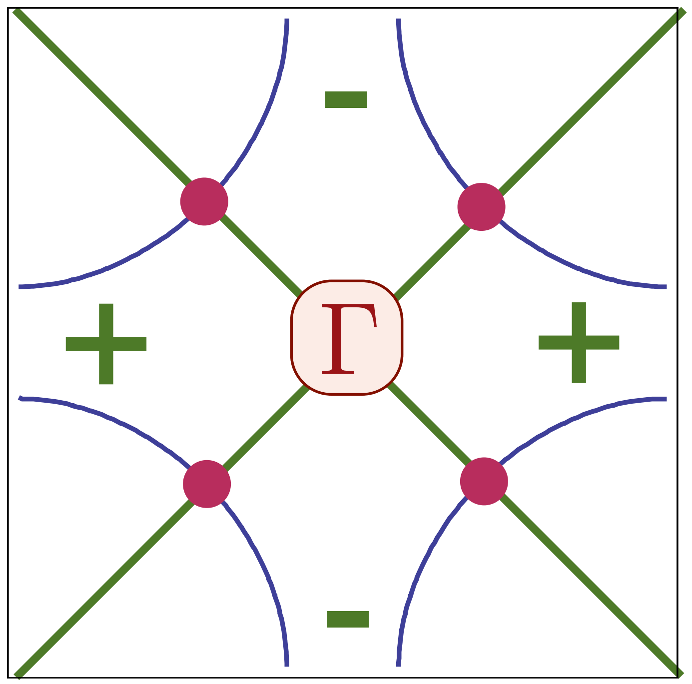


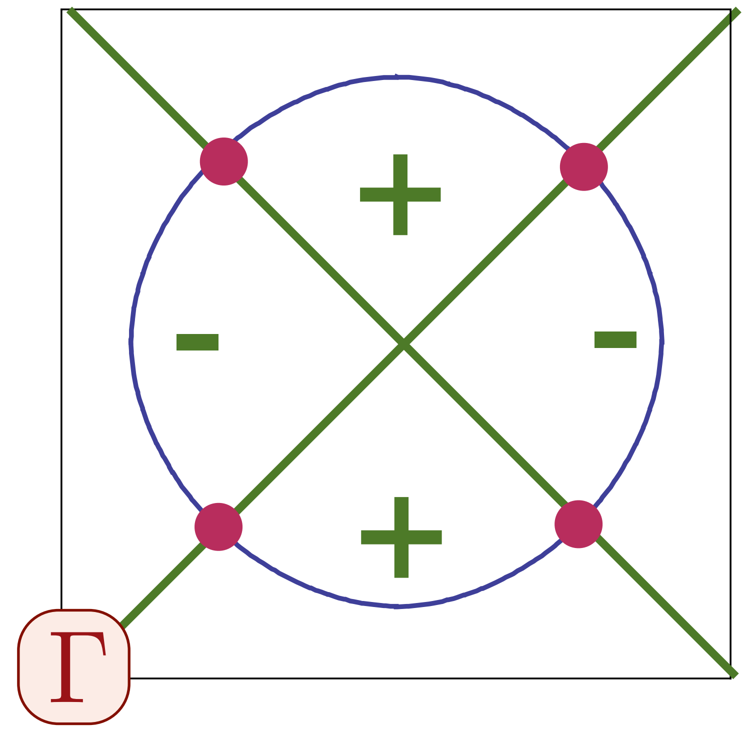


$$H = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\uparrow} c^{\dagger}_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

- Begin with free electrons.
- Add *d*-wave pairing interaction $\Delta_k \sim \cos k_x - \cos k_y$ which vanishes along diagonals

d-wave superconductivity in cuprates

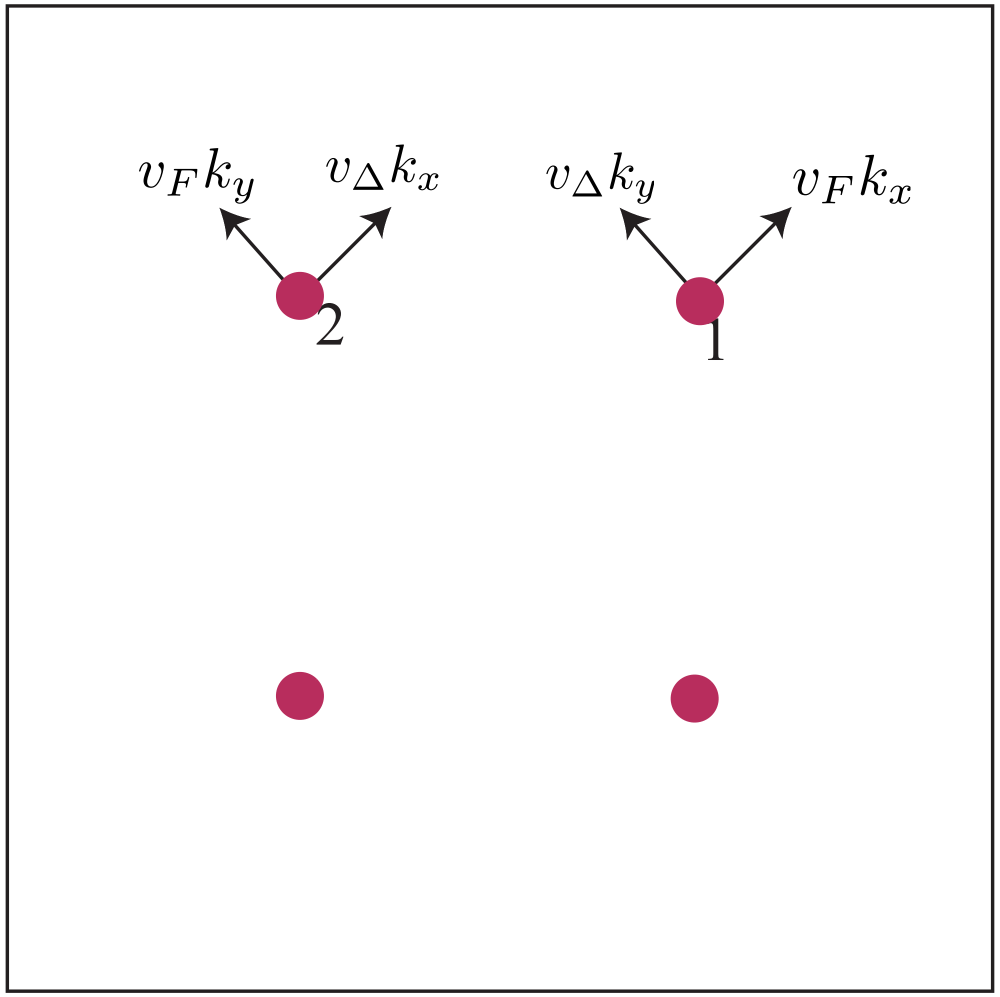




$$H = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

- Begin with free electrons.
- Add d-wave pairing interaction Δ_k which vanishes along diagonals
- Obtain Bogoliubov quasiparticles with dispersion $\sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2}$

d-wave superconductivity in cuprates



4 two-component Dirac fermions

$$
\begin{aligned}
S_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{1a} \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a} \\
&+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi^\dagger_{2a} \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_{2a}.
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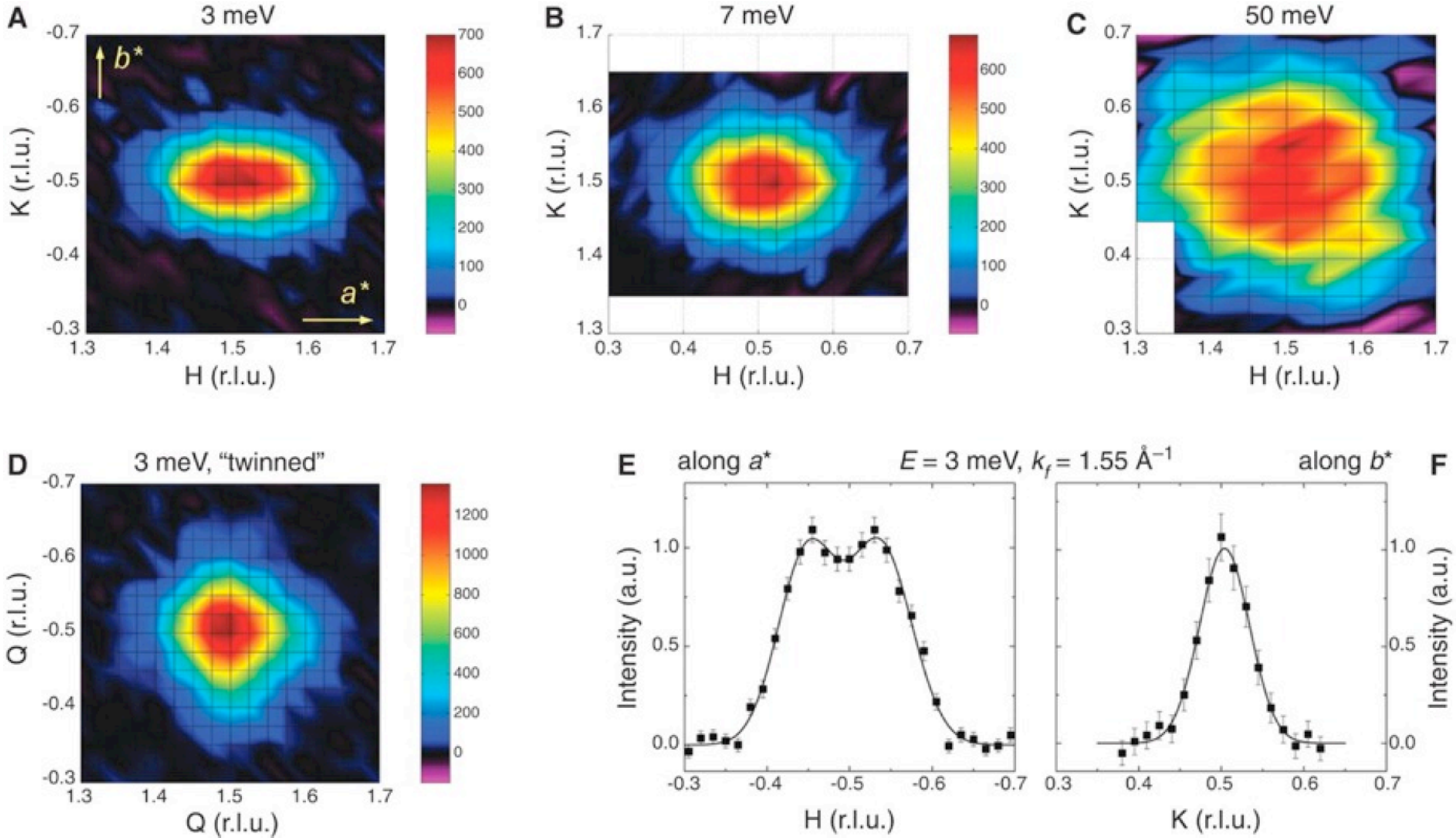
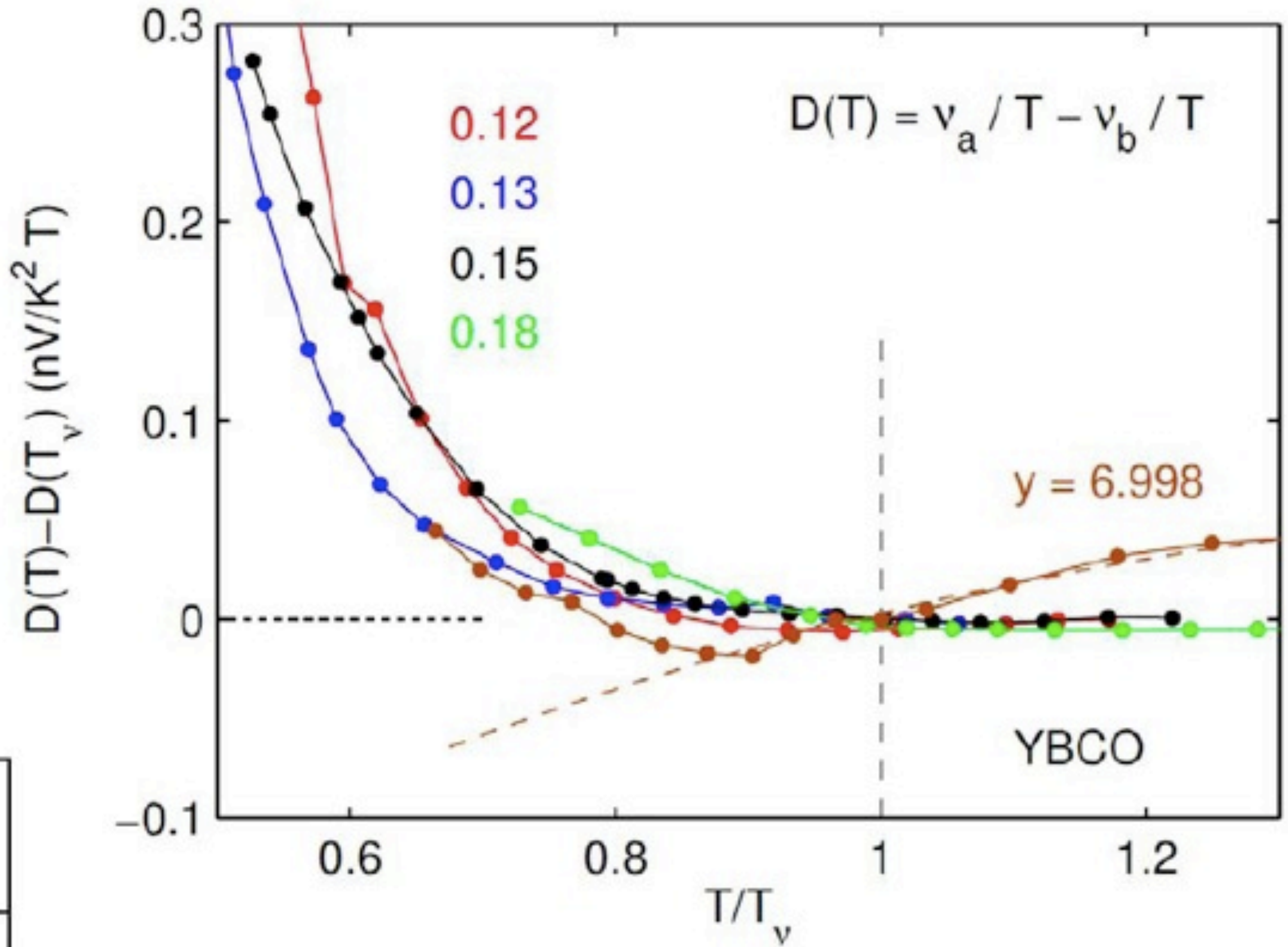


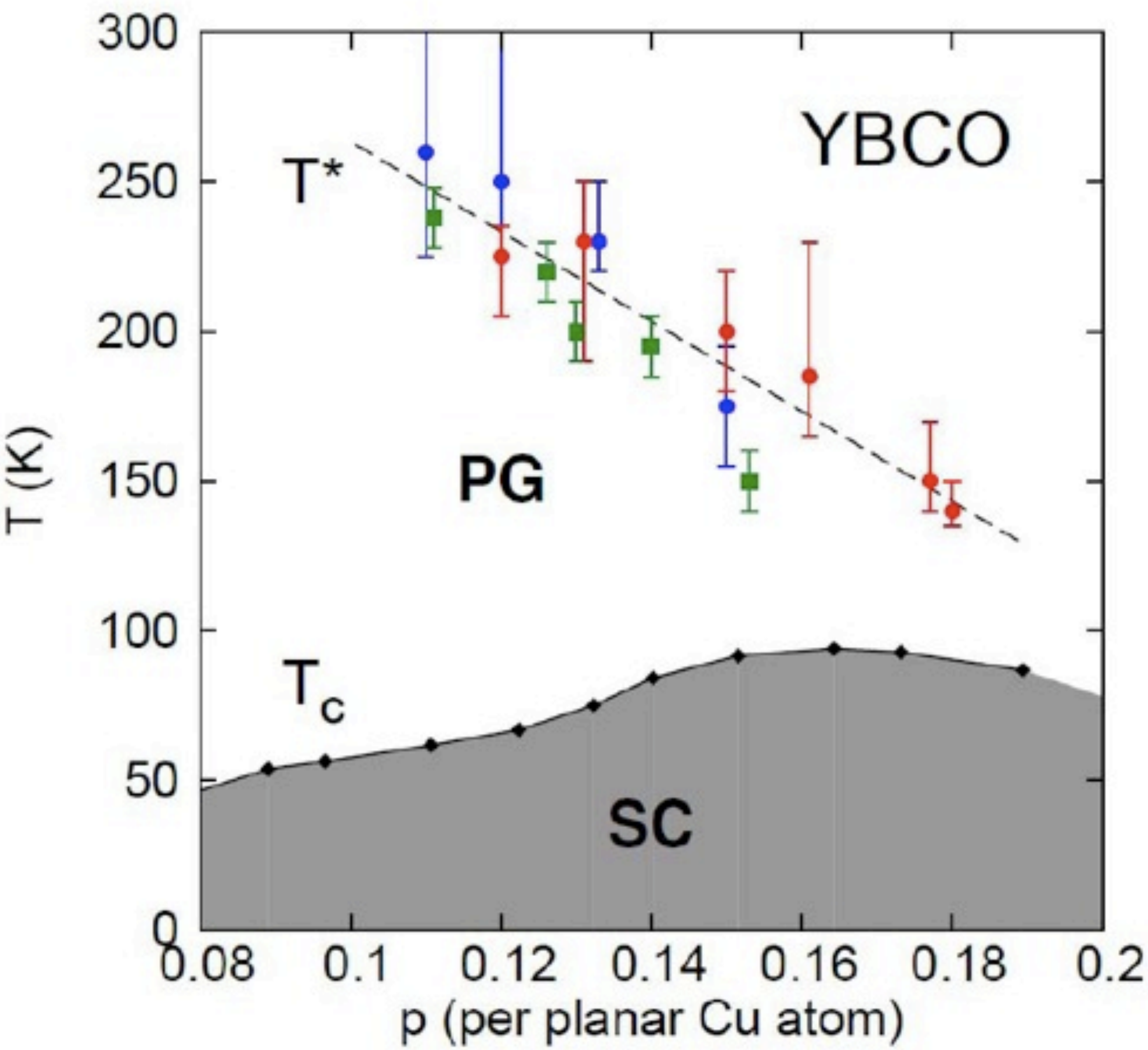
Nematic order in YBCO

V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer , *Science* **319**, 597 (2008)

Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D. A. Bonn, W. N. Hardy, and Louis Taillefer
arXiv: 0909.4430





S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).

d-wave superconductivity in cuprates

Now consider a discrete spontaneous symmetry breaking, with Ising symmetry, described by a real scalar field ϕ.
Two cases of experimental interest are:

- Break 4-fold lattice rotation symmetry to 2-fold lattice rotations: leads to a superconductor with **nematic** order: equivalent to $d_{x^2-y^2} + s$ pairing.

$$H = H_\phi + \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

$$H_\phi = \phi \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{c.c.}$$

d-wave superconductivity in cuprates

Now consider a discrete spontaneous symmetry breaking, with Ising symmetry, described by a real scalar field ϕ.
Two cases of experimental interest are:

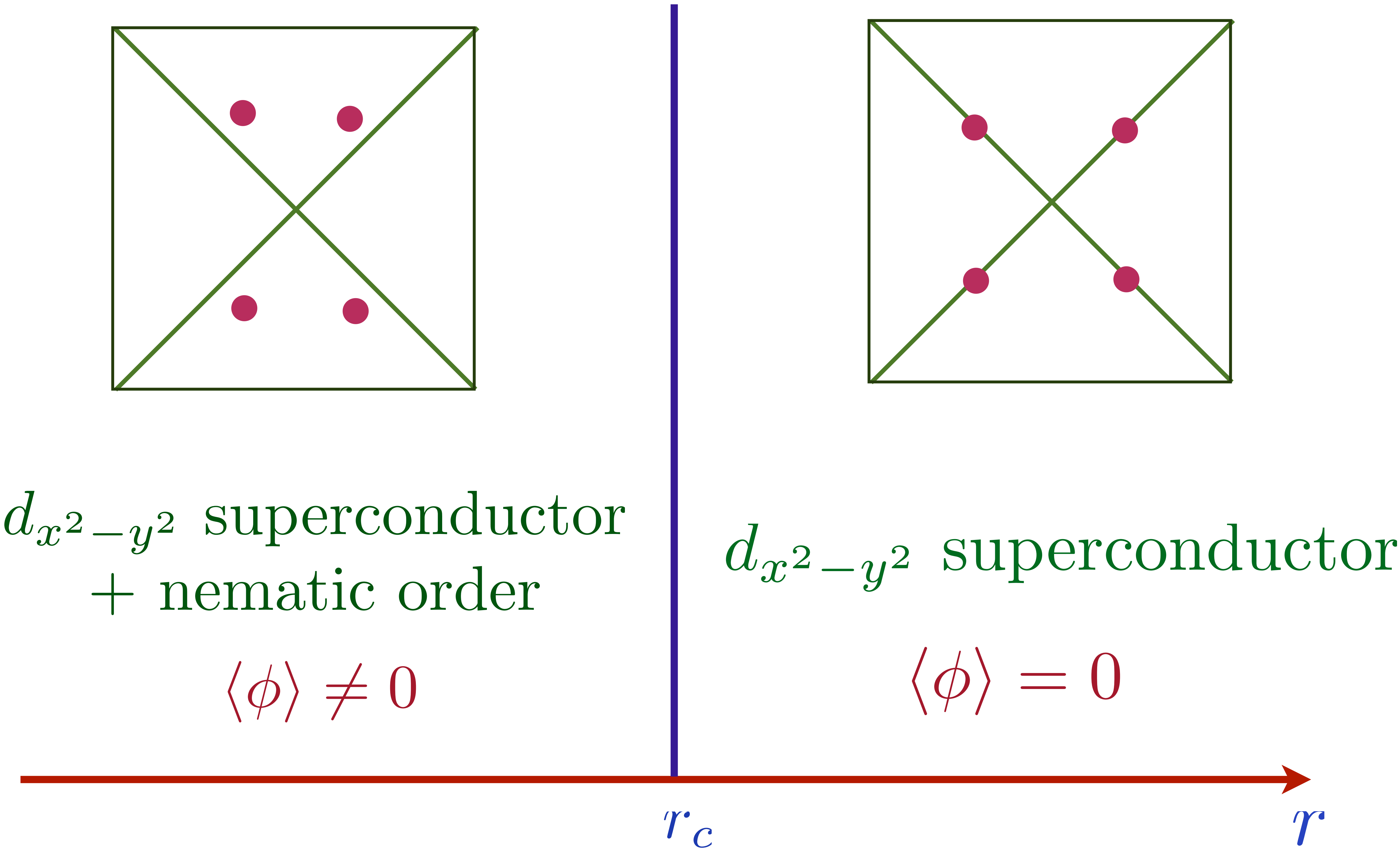
- Break 4-fold lattice rotation symmetry to 2-fold lattice rotations: leads to a superconductor with **nematic** order: equivalent to $d_{x^2-y^2} + s$ pairing.

- Time-reversal symmetry breaking: leads to a $d_{x^2-y^2} + id_{xy}$ superconductor, in which the Dirac fermions are massive

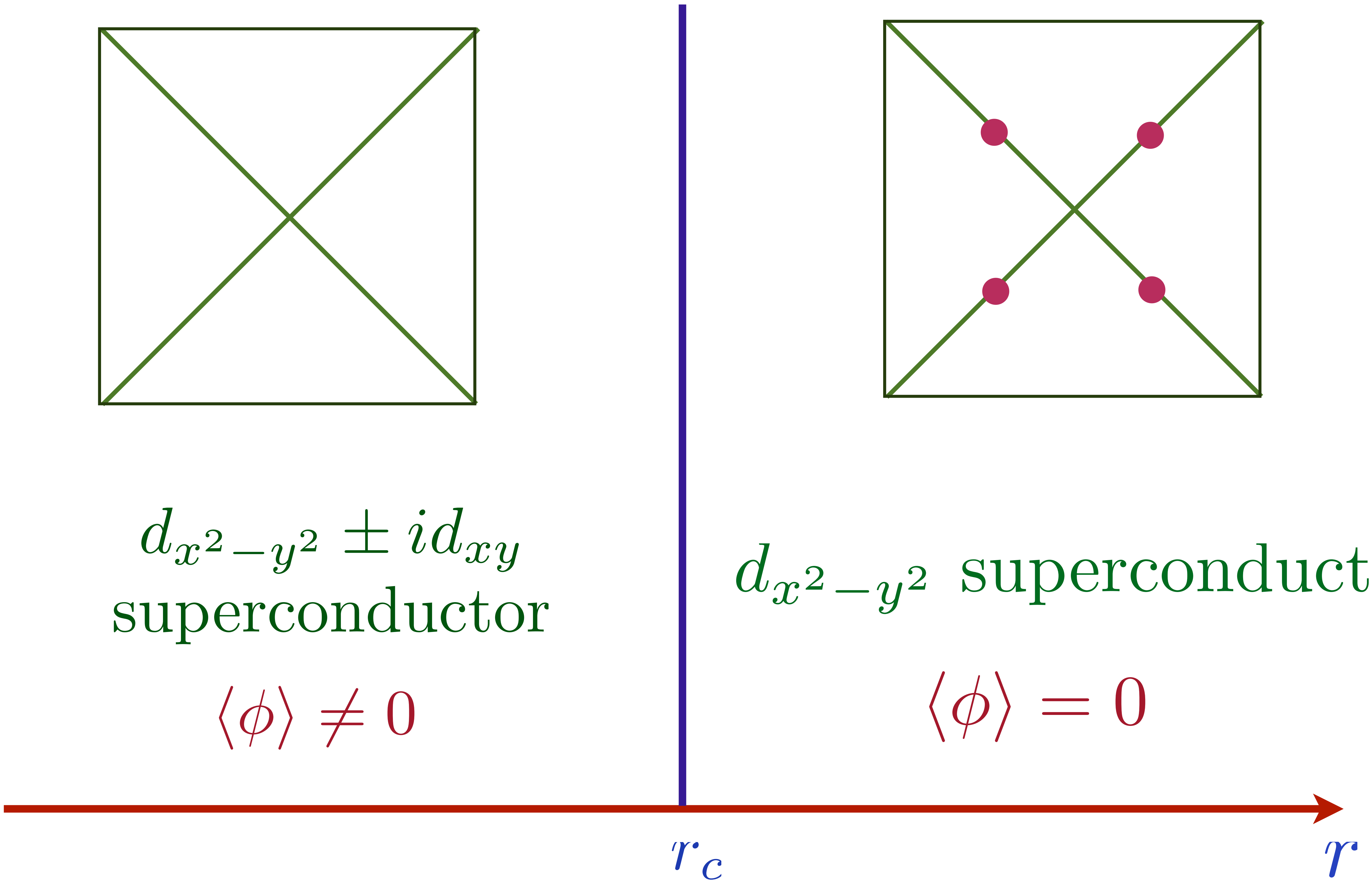
$$H = H_\phi + \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{c.c.} \right)$$

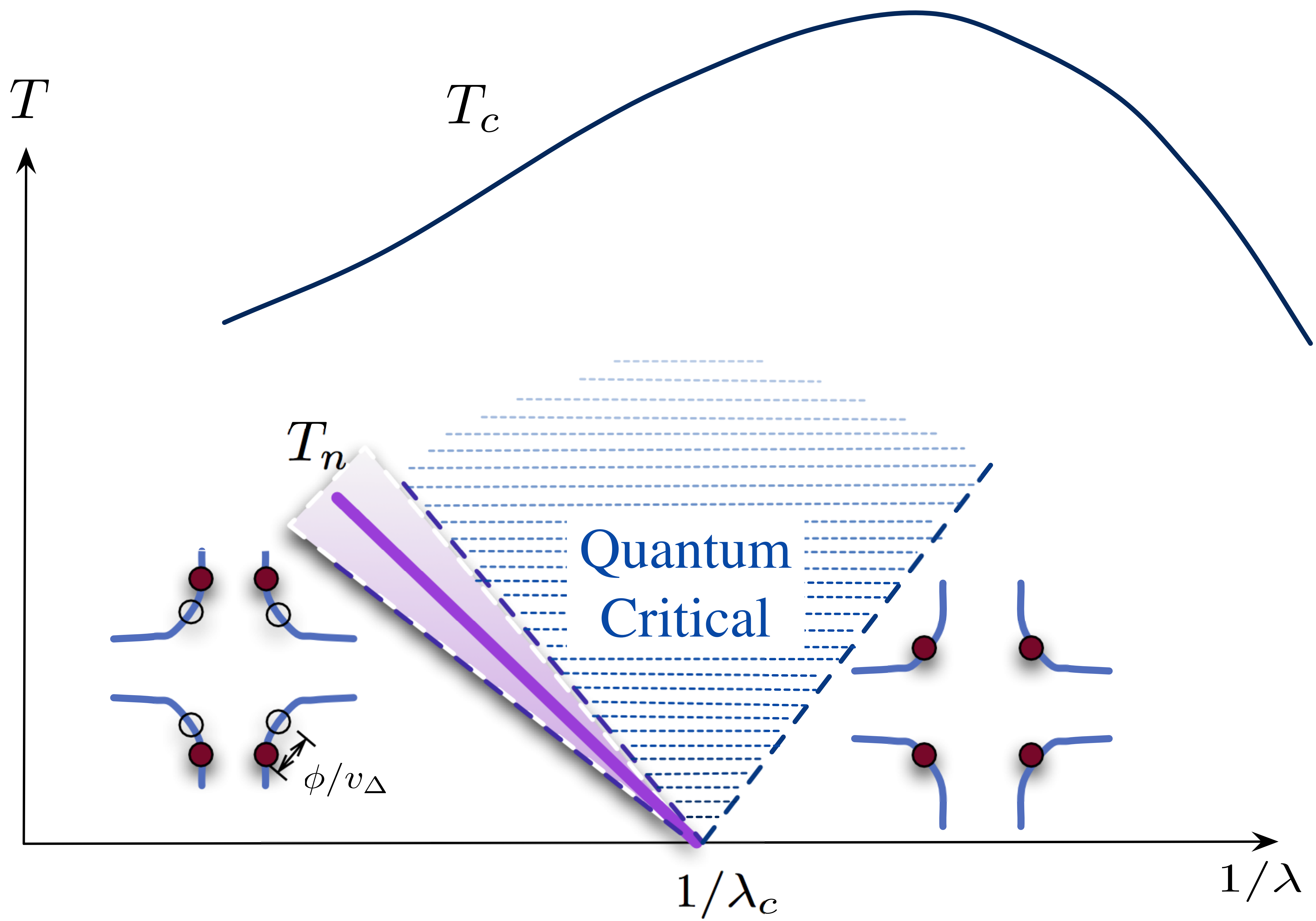
$$H_\phi = i\phi \sum_{\mathbf{k}} \sin k_x \sin k_y c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} + \text{c.c.}$$

Lattice rotation symmetry breaking



Time-reversal symmetry breaking





M. Vojta, Y. Zhang, and S. Sachdev, Phys. Rev. Lett. **85**, 4940 (2000)
E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, Phys. Rev. B **77**, 184514 (2008).

Discrete symmetry breaking in d-wave superconductors

Field theory for transition with Ising order described by a real scalar field ϕ:

$$\mathcal{S} = \mathcal{S}_\Psi + \mathcal{S}_\phi + \mathcal{S}_{\Psi\phi}$$

4 two-component Dirac fermions

$$\begin{aligned} \mathcal{S}_\Psi &= \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{1a}^\dagger \left(-i\omega_n + v_F k_x \tau^z + v_\Delta k_y \tau^x\right) \Psi_{1a} \\ &+ \int \frac{d^2k}{(2\pi)^2} T \sum_{\omega_n} \Psi_{2a}^\dagger \left(-i\omega_n + v_F k_y \tau^z + v_\Delta k_x \tau^x\right) \Psi_{2a}. \end{aligned}$$

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Ising field theory

$$\mathcal{S}_\phi = \int d^2x d\tau \left[\frac{1}{2}(\partial_\tau \phi)^2 + \frac{c^2}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \frac{u_0}{24}\phi^4\right];$$

Ising order and Dirac fermions couple via a "Yukawa" term.

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^x \Psi_{1a} + \Psi_{2a}^\dagger \tau^x \Psi_{2a} \right) \right],$$

Nematic ordering

$$S_{\Psi\phi} = \int d^2x d\tau \left[\lambda_0 \phi \left(\Psi_{1a}^\dagger \tau^y \Psi_{1a} + \Psi_{2a}^\dagger \tau^y \Psi_{2a} \right) \right]$$

Time reversal symmetry breaking

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

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Time reversal symmetry breaking

For the latter case *only*, with $v_F = v_\Delta = c$, theory reduces to relativistic Gross-Neveu model

M. Vojta, Y. Zhang, and S. Sachdev, Physical Review Letters **85**, 4940 (2000)

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the scalar order parameter

$$S_\phi = \frac{N_f}{v_\Delta v_F} \Gamma\left[\lambda_0 \phi(x,\tau); \frac{v_\Delta}{v_F}\right] + \frac{N_f}{2} \int d^2 x d\tau \Big(r \phi^2(x,\tau) \Big) + \text{irrelevant terms}$$

where Γ is a non-local and non-analytic functional of ϕ.

The theory has only 2 couplings constants: r and v_Δ / v_F.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Expansion in number of fermion spin components N_f

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$$S_\phi = \frac{N_f}{2}\int_{k,\omega}|\phi(k,\omega)|^2\Bigg[r + \frac{\lambda_0^2}{8v_F v_\Delta}\left(\frac{\omega^2+v_F^2k_x^2}{\sqrt{\omega^2+v_F^2k_x^2+v_\Delta^2k_y^2}} + (x\leftrightarrow y)\right)\Bigg]$$

$$+\text{higher order terms which cannot be neglected}$$

E.-A. Kim, M. J. Lawler, P. Oreto, S. Sachdev, E. Fradkin, S.A. Kivelson, arXiv:0705.4099

Expansion in number of fermion spin components N_f

Integrating out the fermions yields an effective action for the T-breaking order parameter

$$S_\phi = \frac{N_f}{2}\int_{k,\omega}|\phi(k,\omega)|^2\Bigg[r + \frac{\lambda_0^2}{8v_F v_\Delta}\left(\sqrt{\omega^2+v_F^2k_x^2+v_\Delta^2k_y^2}+(x\leftrightarrow y)\right)\Bigg]$$

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where Γ is a non-local and non-analytic functional of ϕ.

There is a systematic expansion in powers of $1/N_f$ for renormalization group equations and all critical properties.

Y. Huh and S. Sachdev, Physical Review B **78**, 064512 (2008).

Outline

1. Quantum criticality of Fermi points:

   Dirac fermions in d-wave superconductors

2. Quantum criticality of Fermi surfaces:

   Onset of spin density wave order in the cuprates

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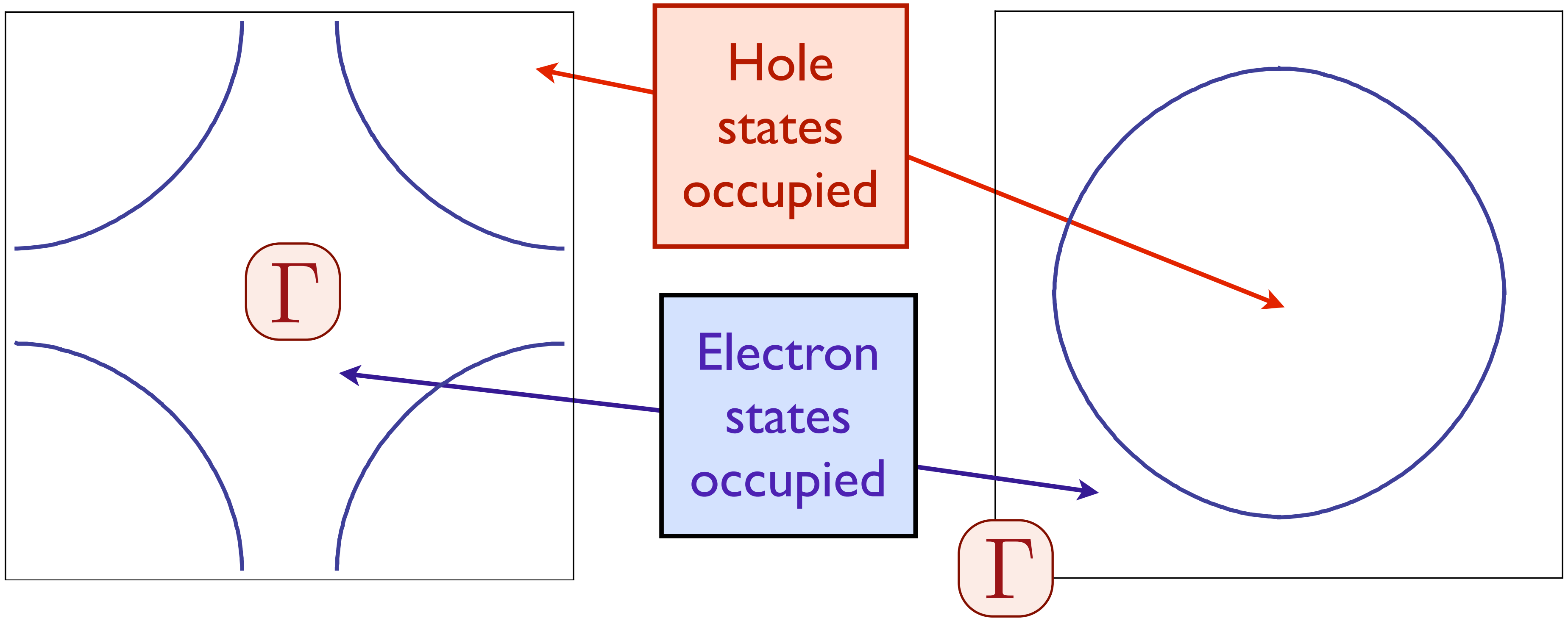
"Large" Fermi surfaces in cuprates



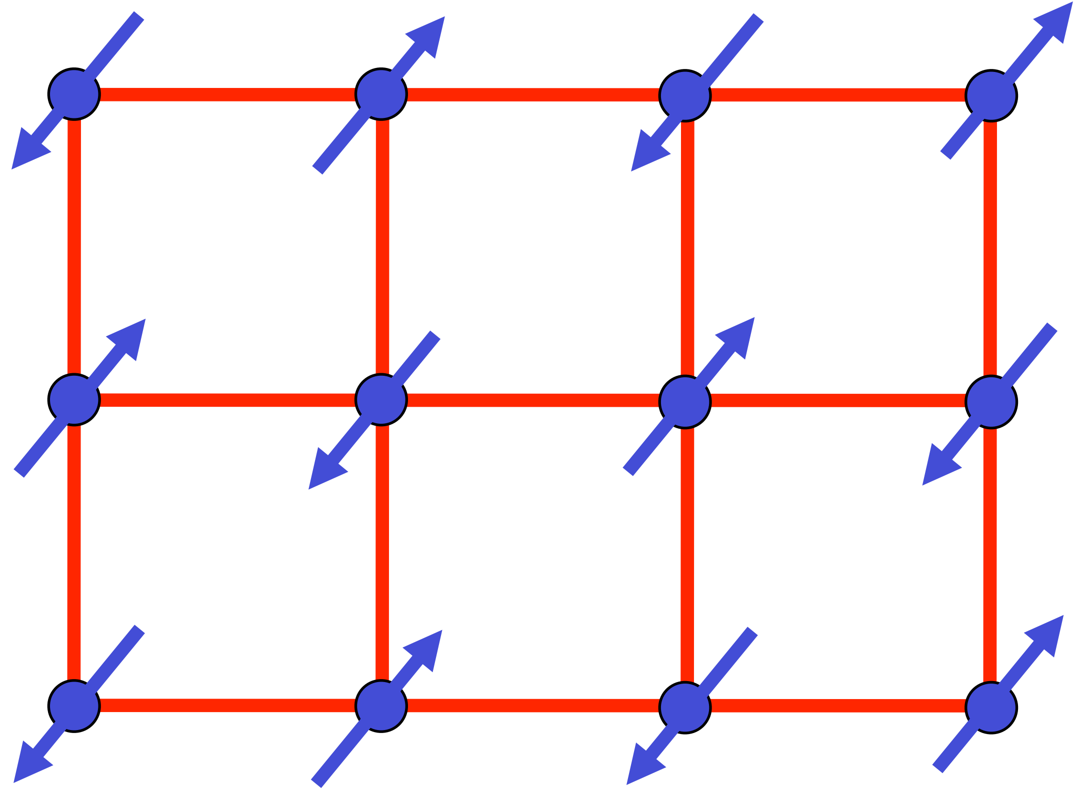
$$H_0 = -\sum_{i<j} t_{ij} c_{i\alpha}^{\dagger} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

The area of the occupied electron/hole states:

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-x) & \text{for hole-doping } x \\ 2\pi^2(1+p) & \text{for electron-doping } p \end{cases}$$

$$\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$$

Spin density wave theory

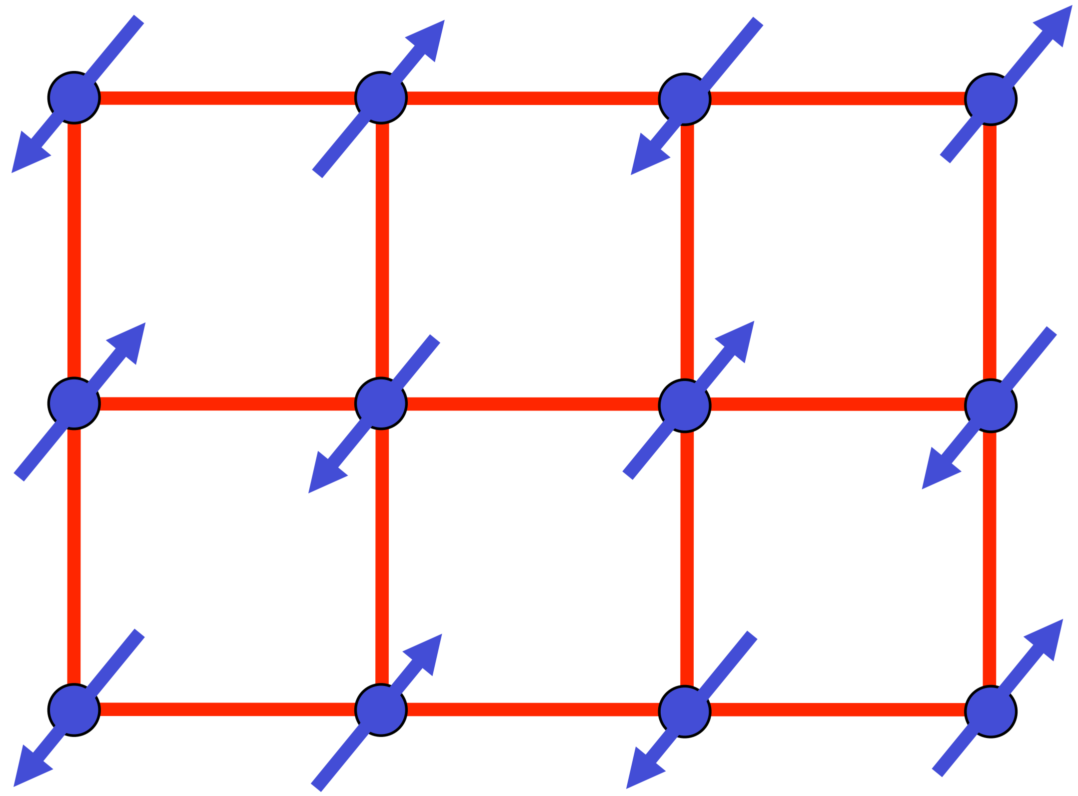


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\vec{\varphi}$ is the spin density wave (SDW) order parameter, and $\mathbf{K}$ is the ordering wavevector. For simplicity, we consider $\mathbf{K} = (\pi, \pi)$.

Spin density wave theory



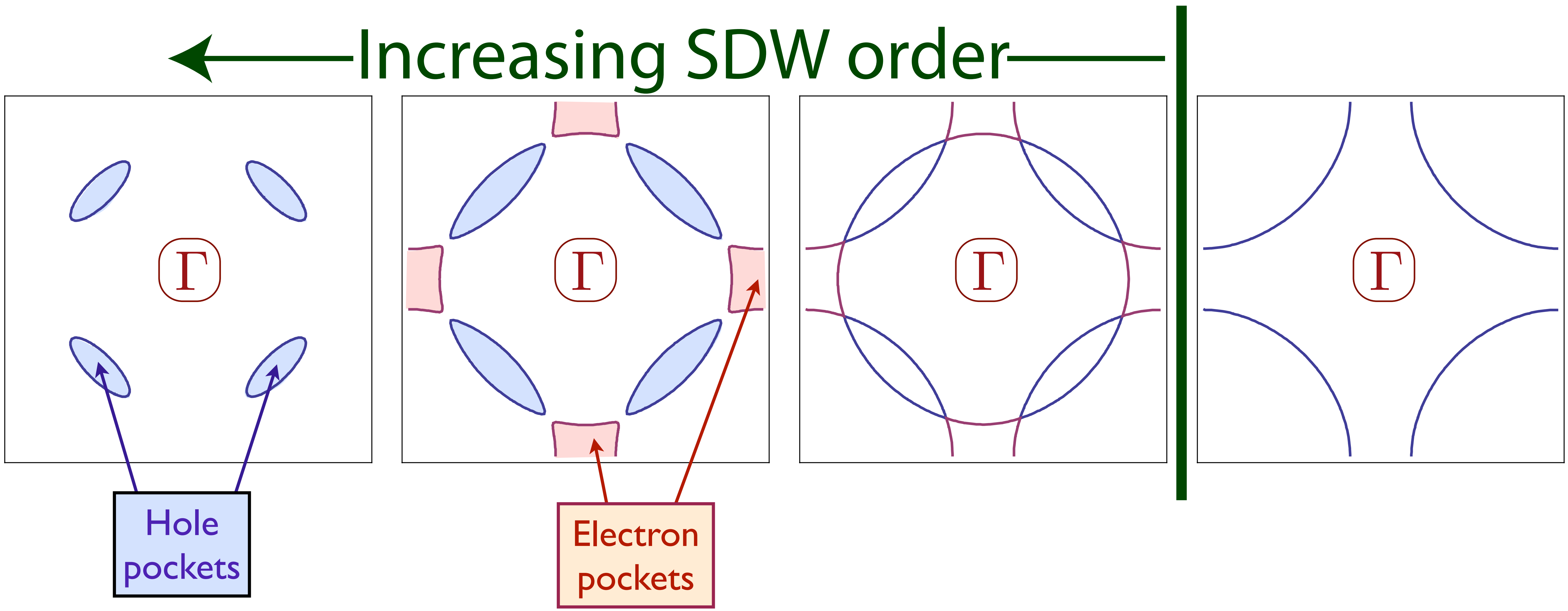
Spin density wave Hamiltonian

$$H_{\rm sdw} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c^\dagger_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

Diagonalize $H_0 + H_{\rm sdw}$ for $\vec{\varphi} = (0, 0, \varphi)$

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

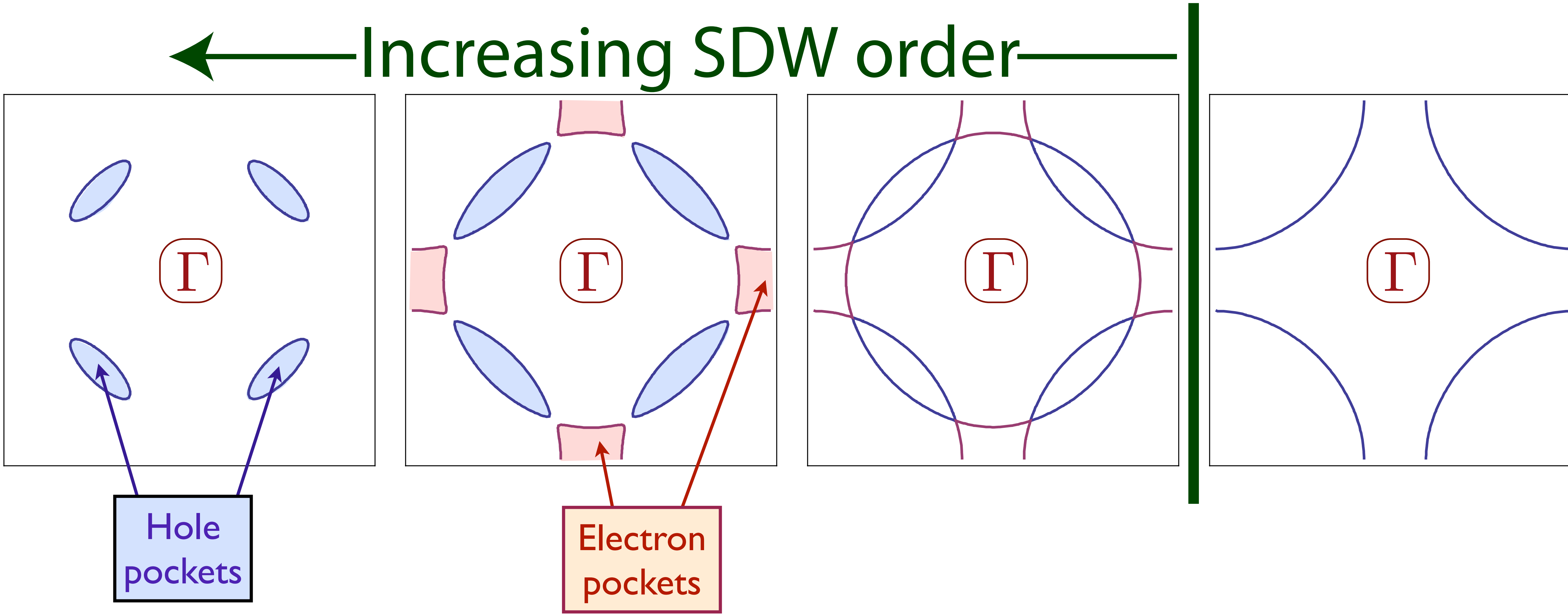
Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



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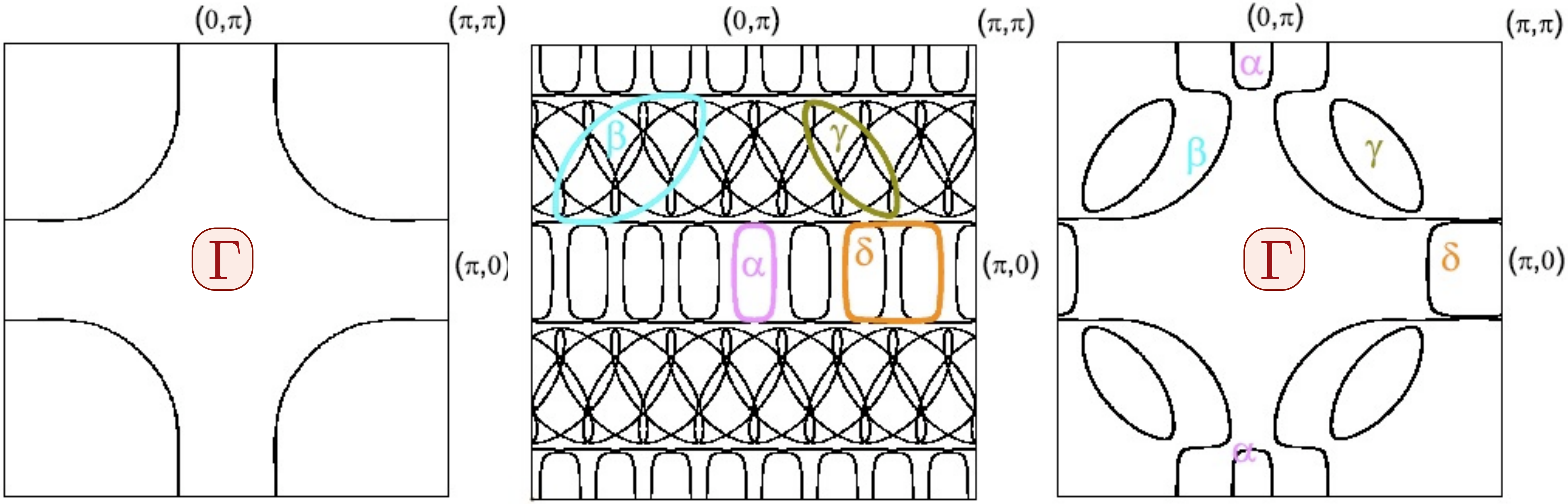
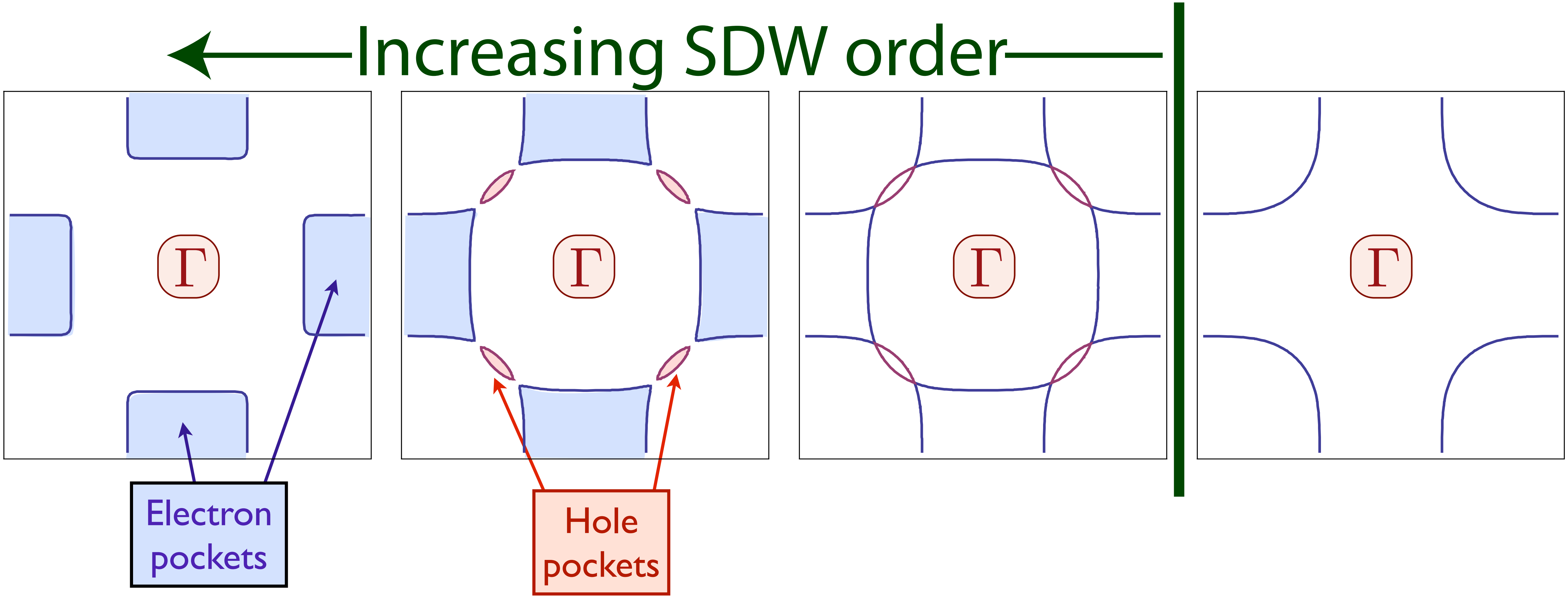
Spin density wave theory in hole-doped cuprates



Incommensurate order in $YBa_2Cu_3O_{6+x}$

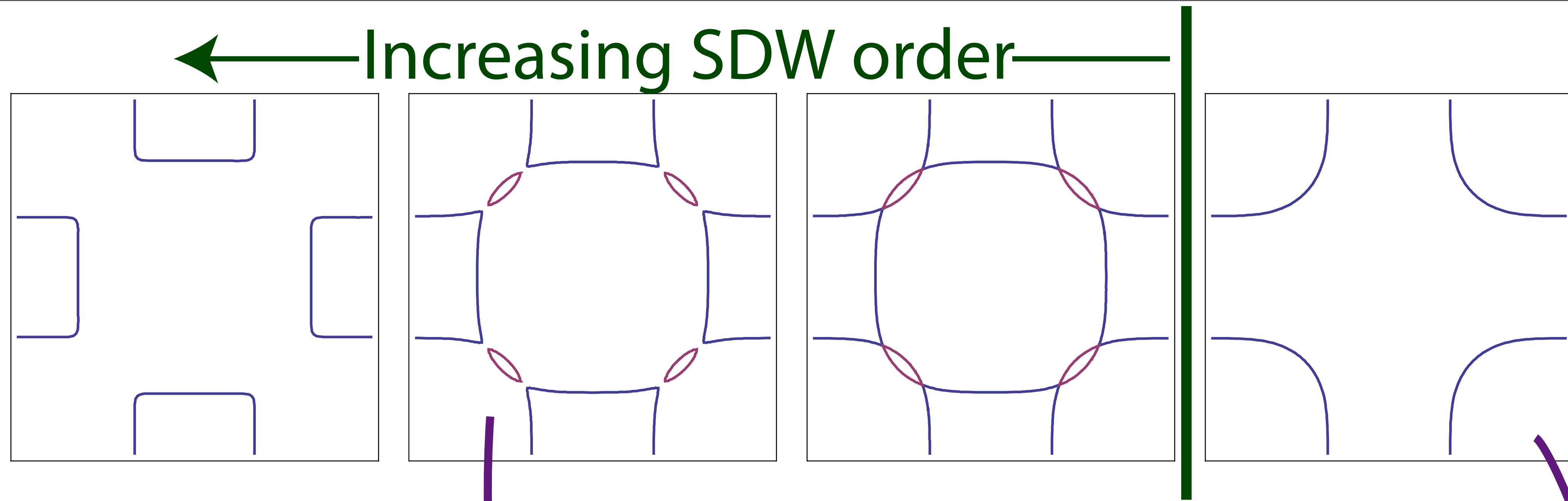
A. J. Millis and M. R. Norman, *Physical Review B* **76**, 220503 (2007).
N. Harrison, *Physical Review Letters* **102**, 206405 (2009).

Electron-doped cuprates



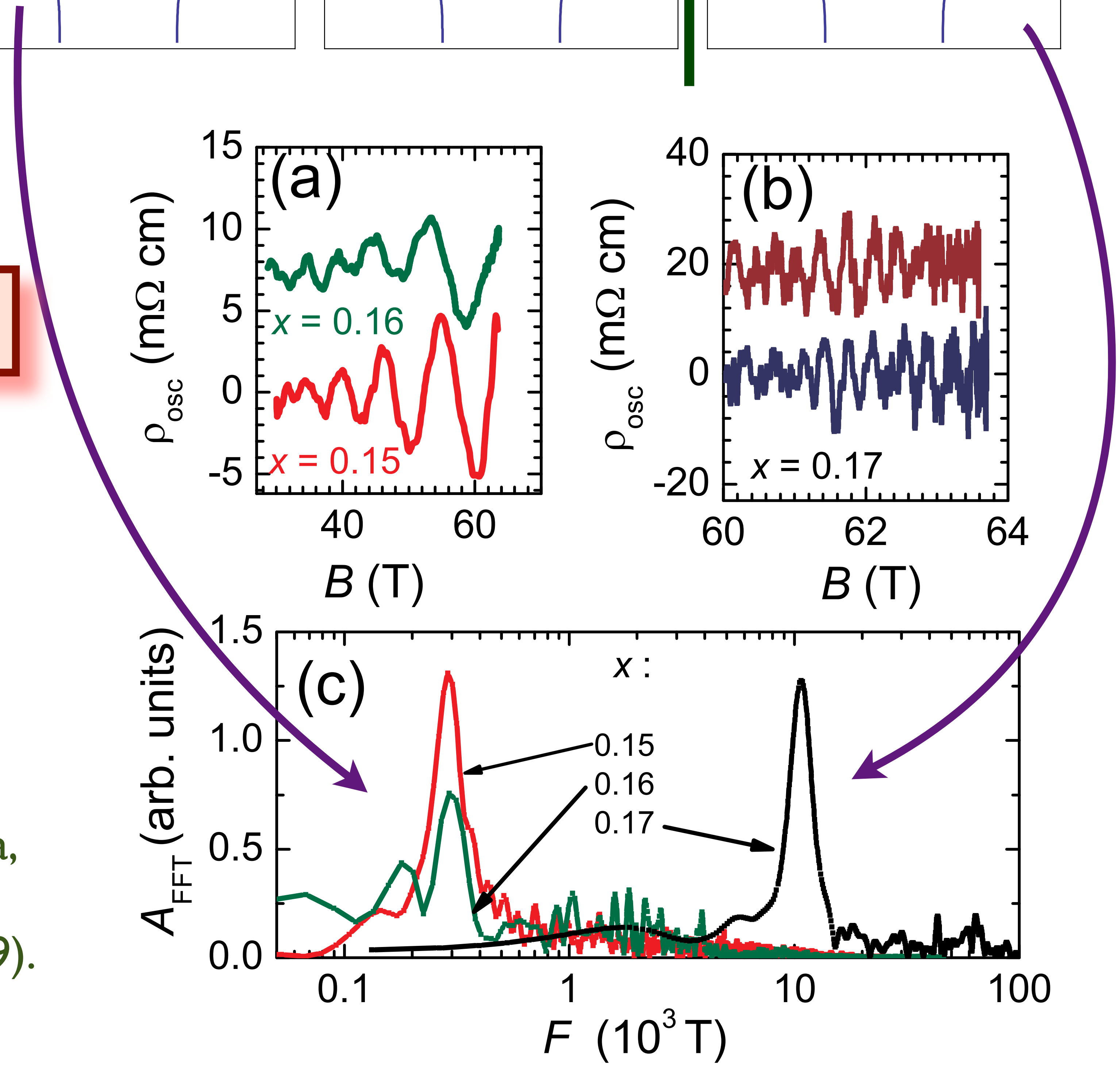
Large Fermi surface breaks up into electron and hole pockets

D. Senechal and A.-M. S. Tremblay, *Physical Review Letters* **92**, 126401 (2004)
J. Lin, and A. J. Millis, *Physical Review B* **72**, 214506 (2005).



Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$



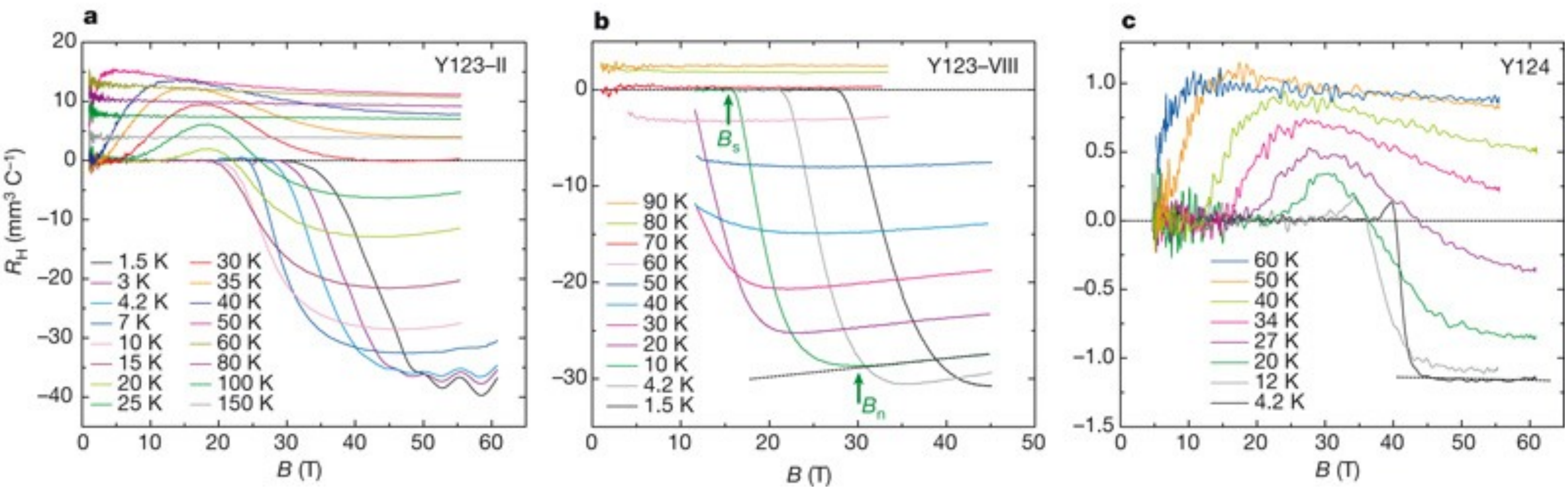
T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).

Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



Quantum oscillations

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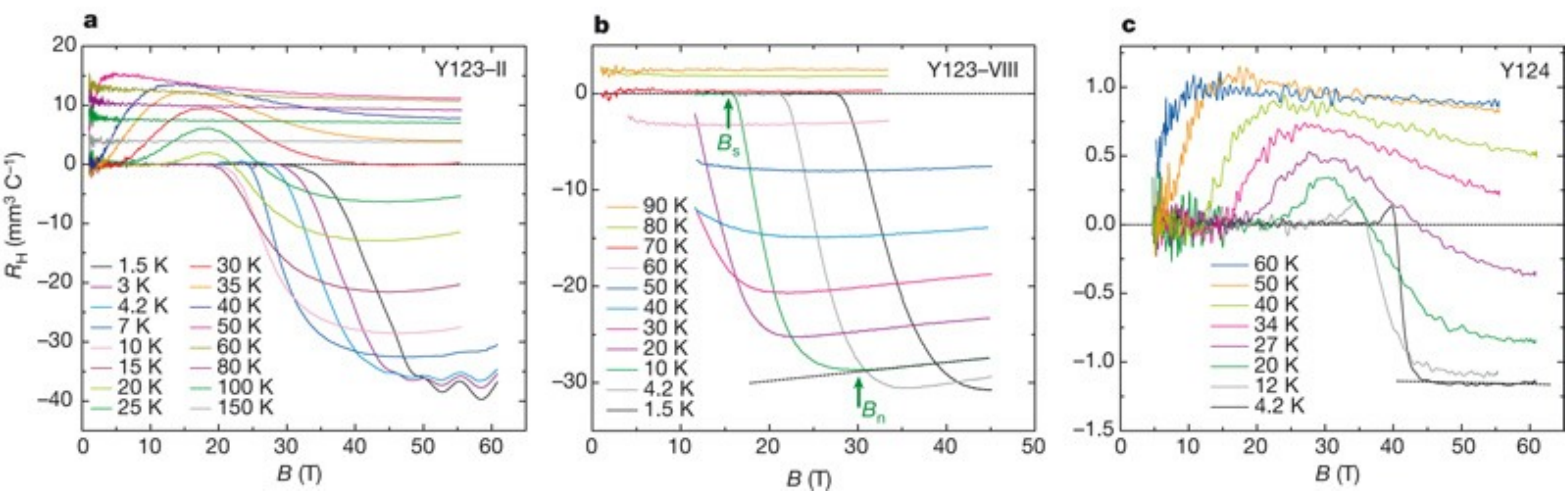
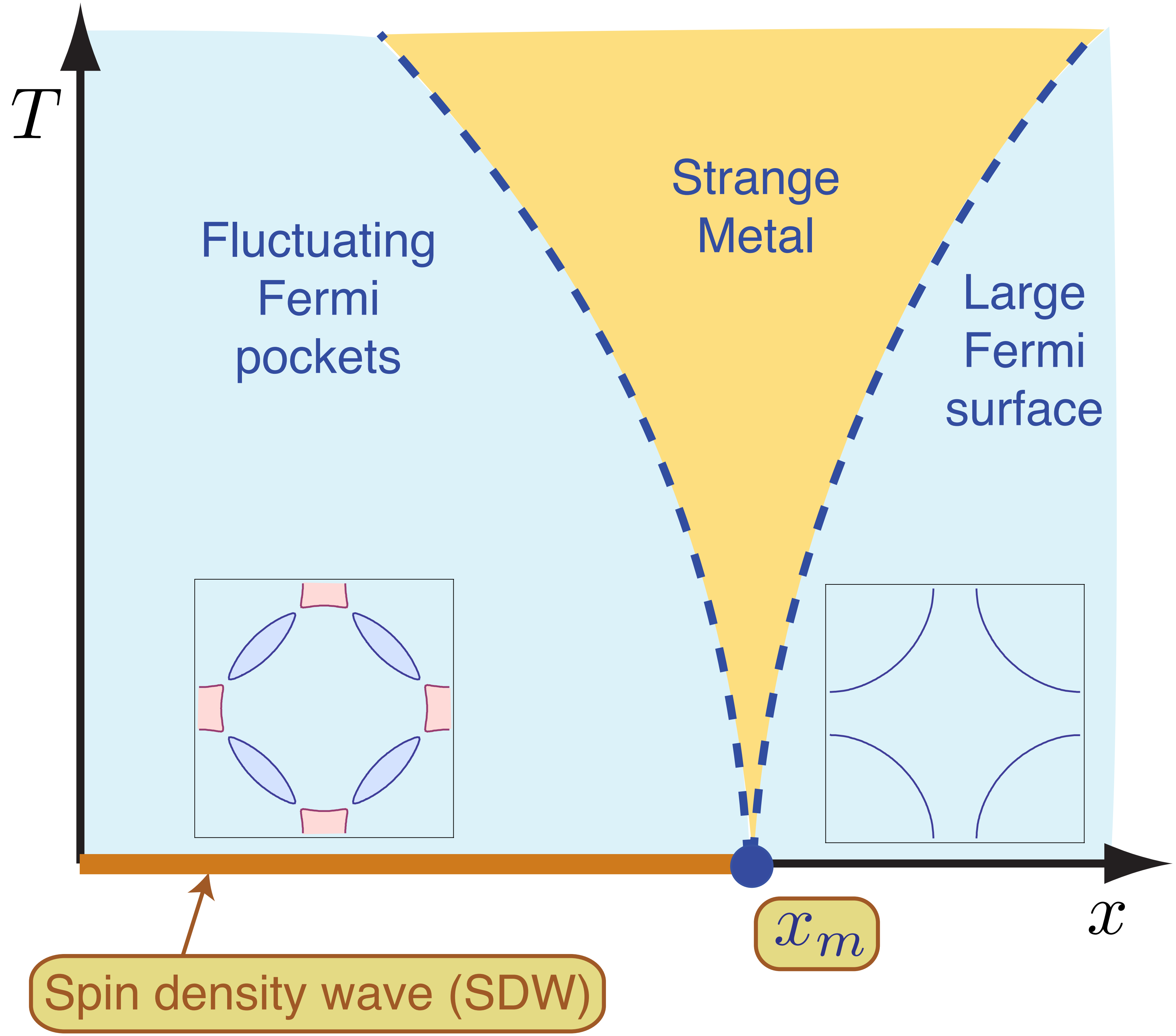
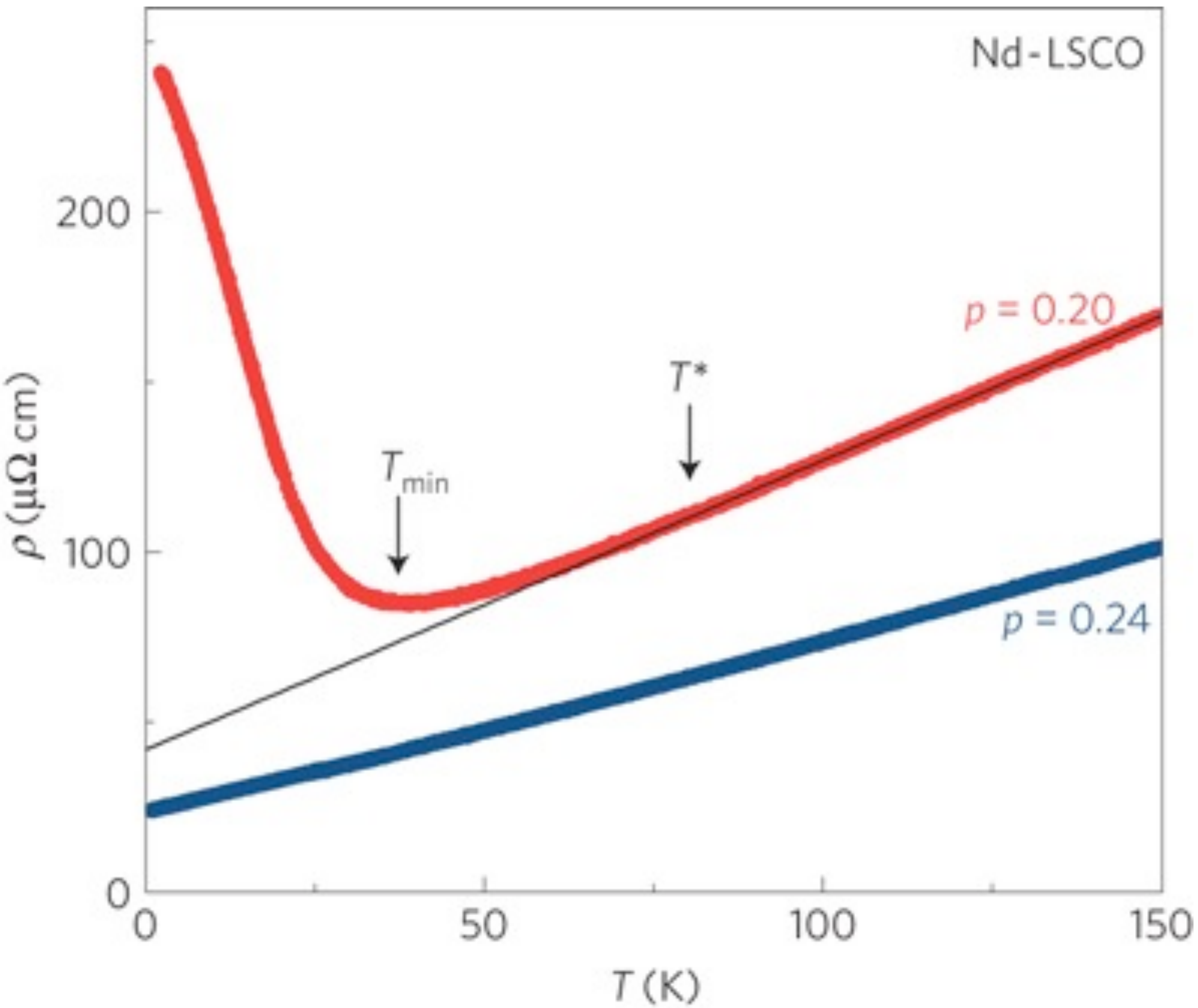


Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

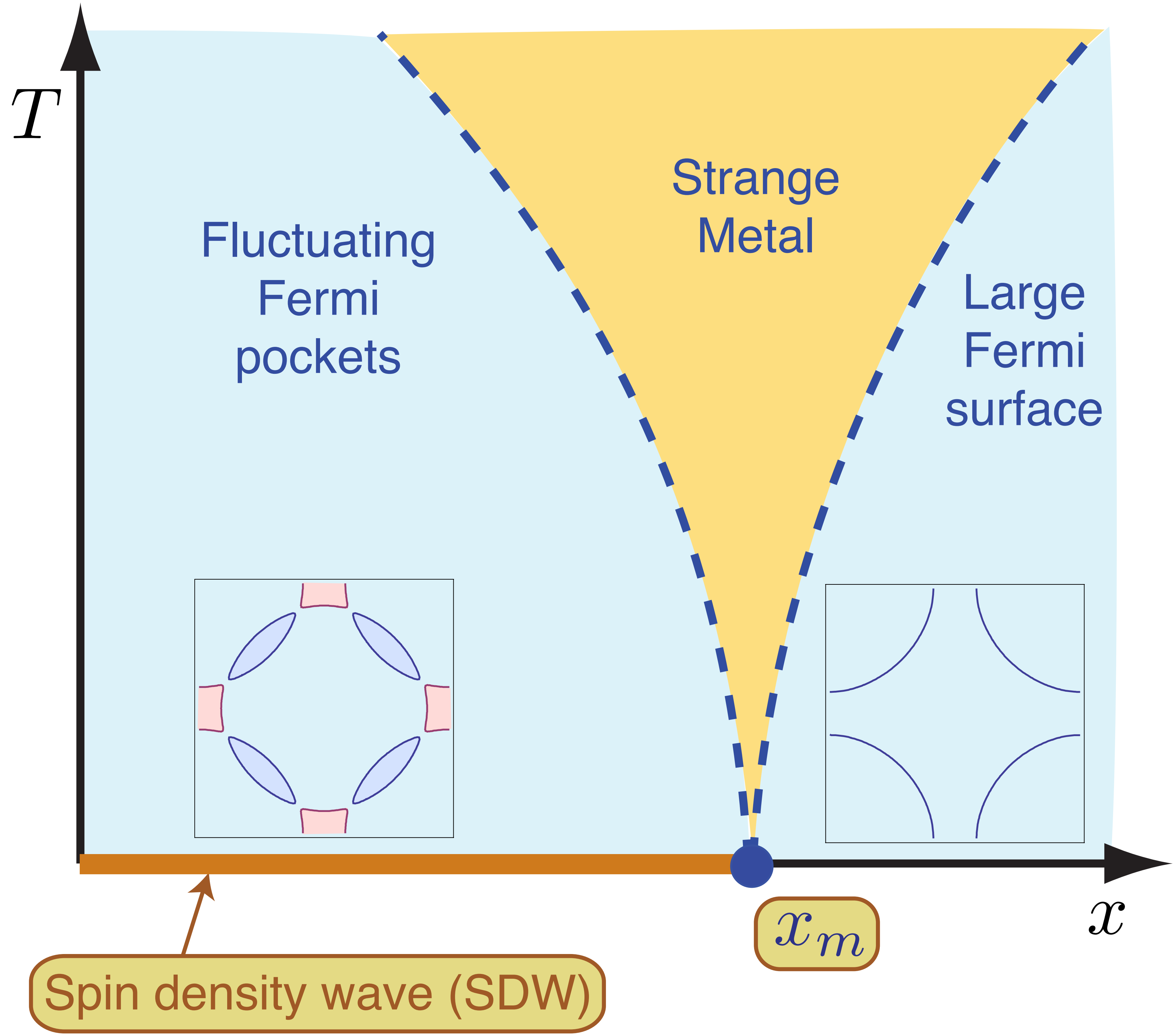
Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c



- Magnetic field of upto 35 T used to suppress superconductivity
- Identifies $x_m \approx 0.24$

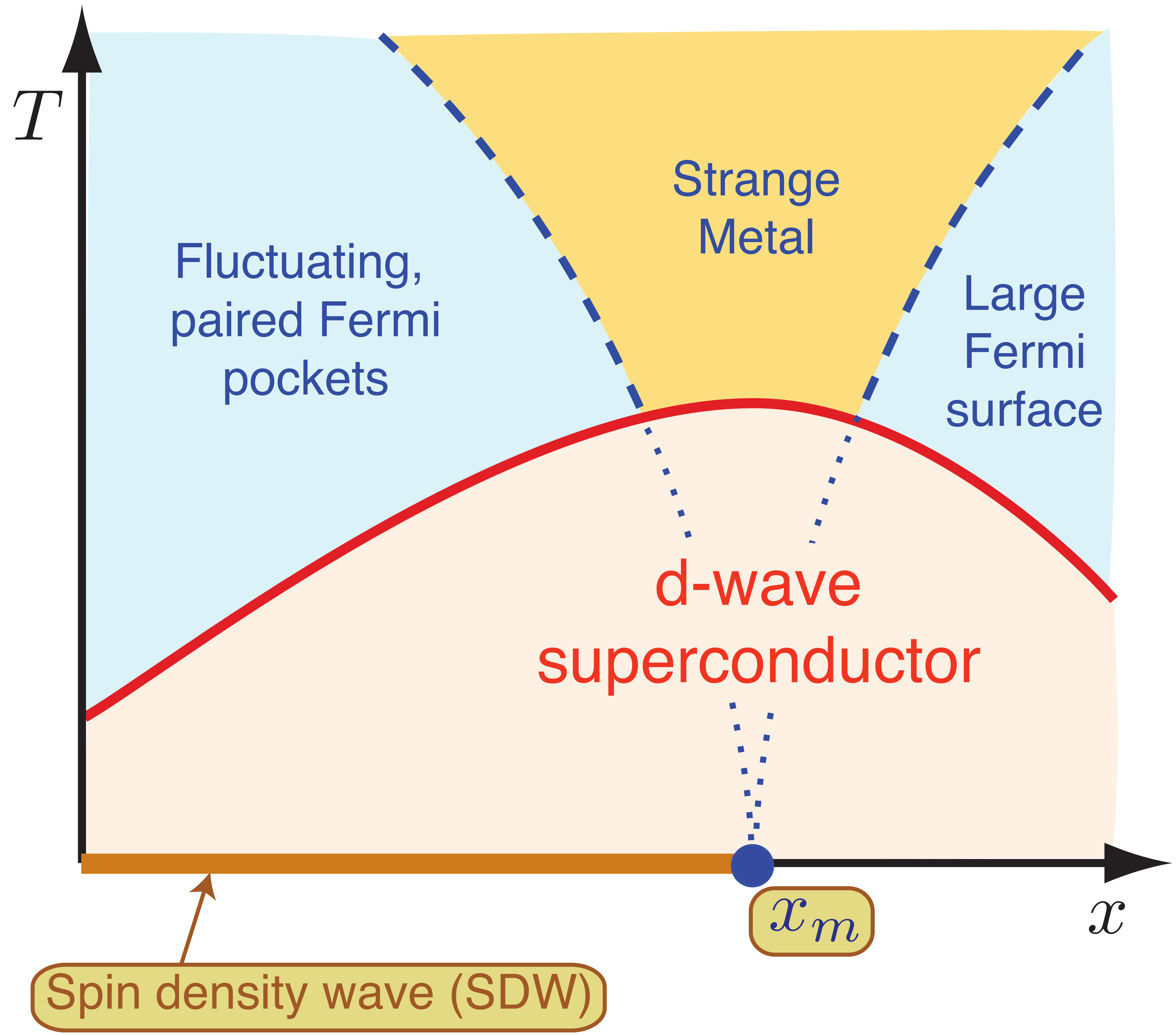
Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-T_c superconductor
R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)

Theory of quantum criticality in the cuprates



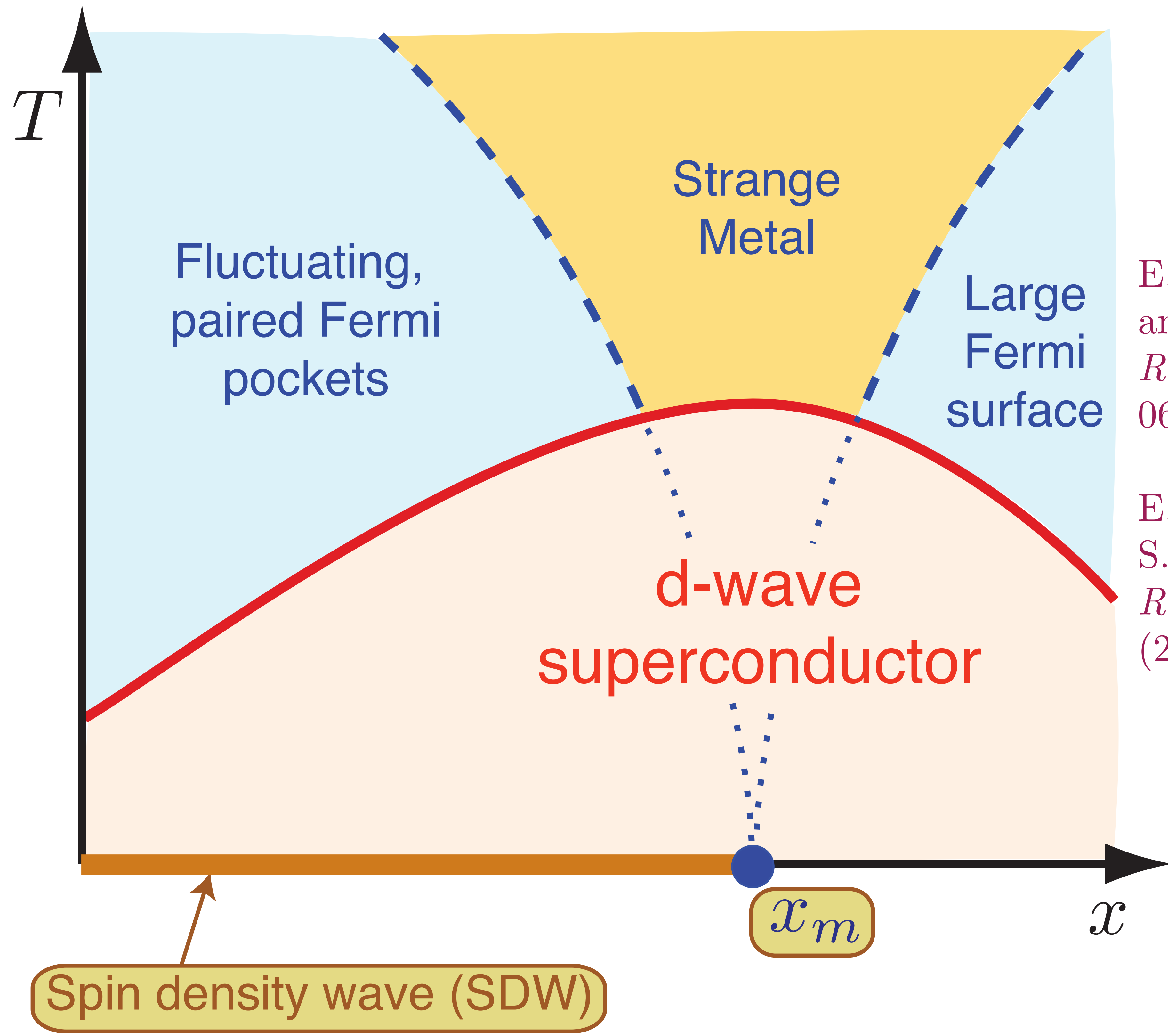
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Theory of quantum criticality in the cuprates



Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates

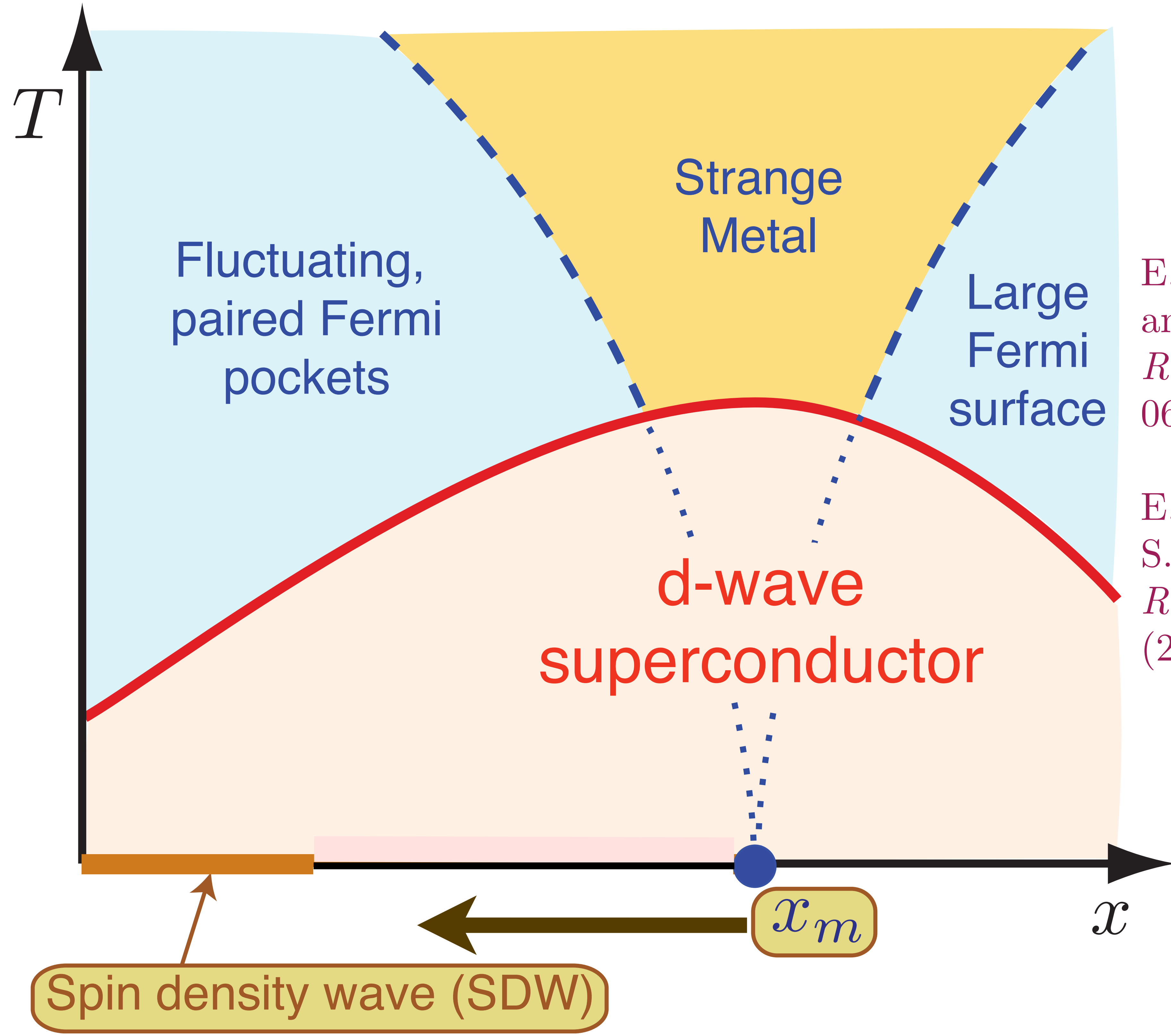


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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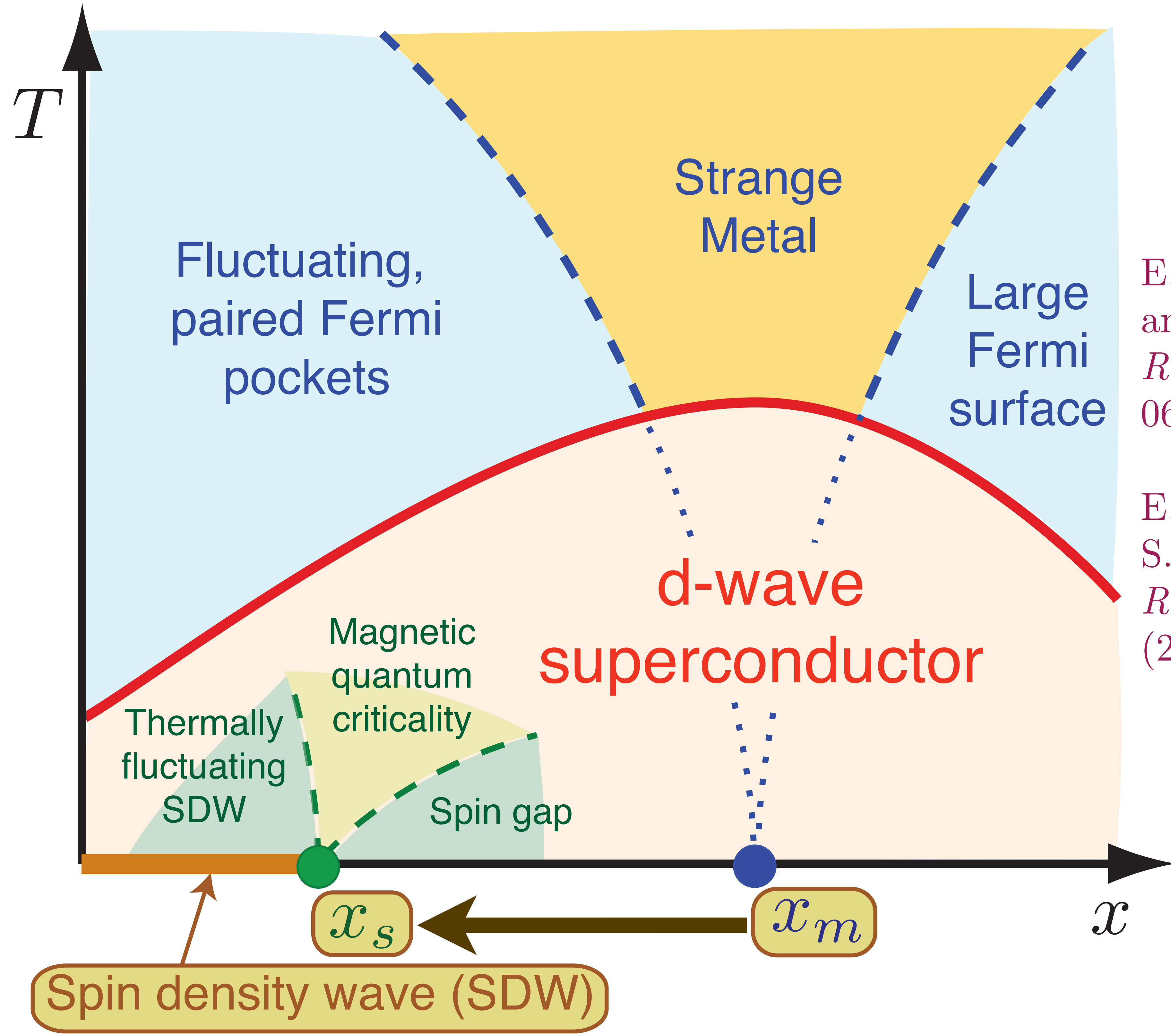

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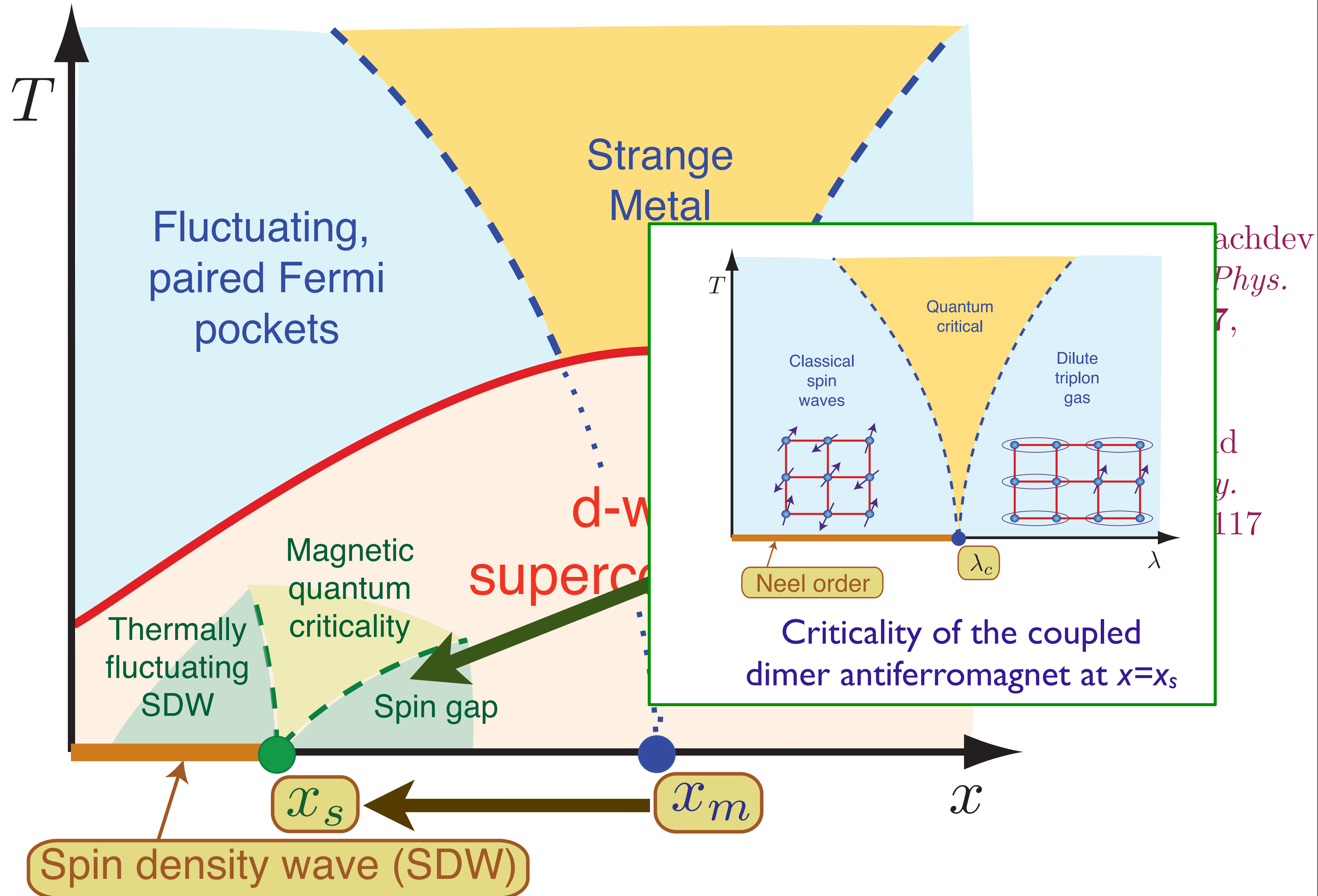

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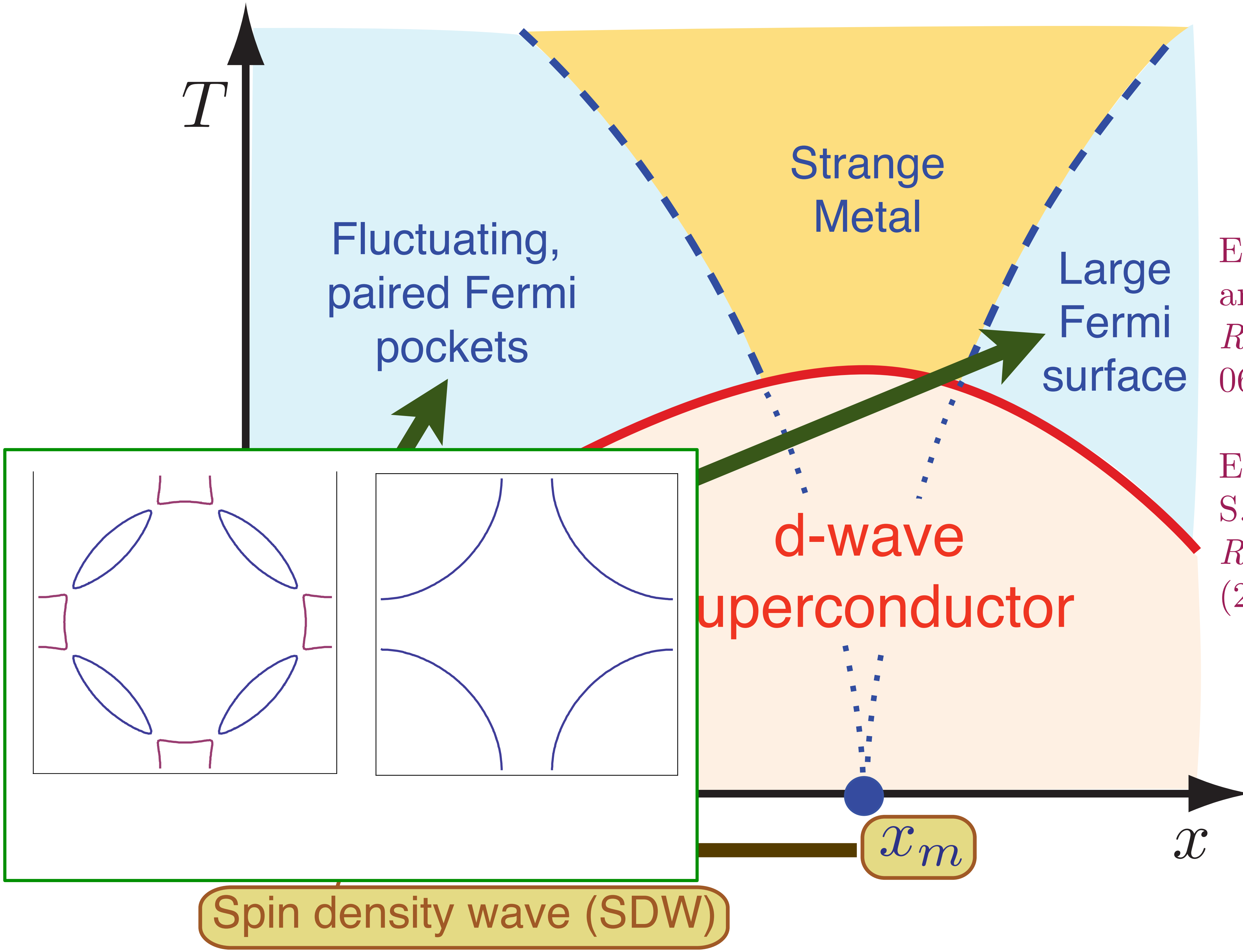

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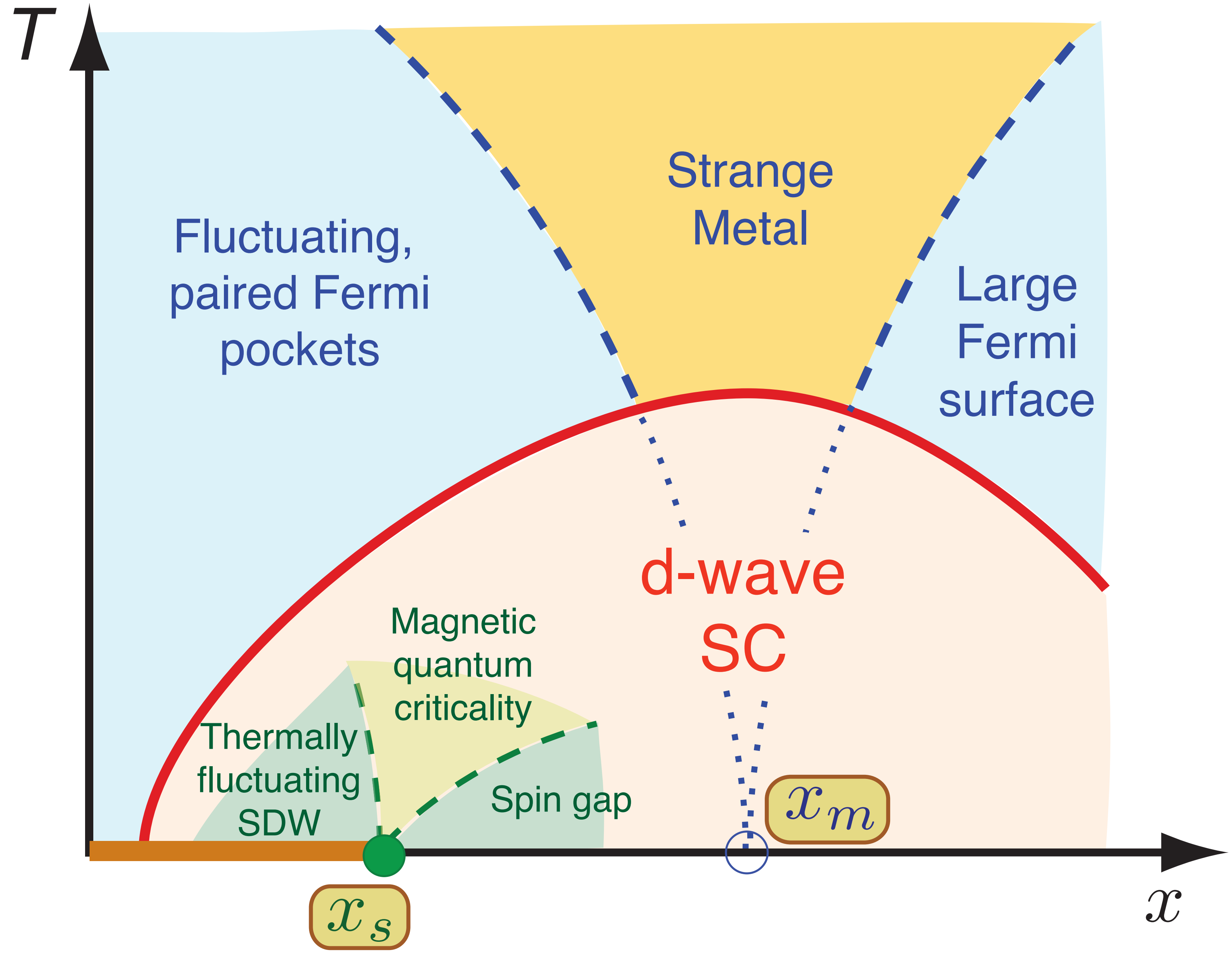
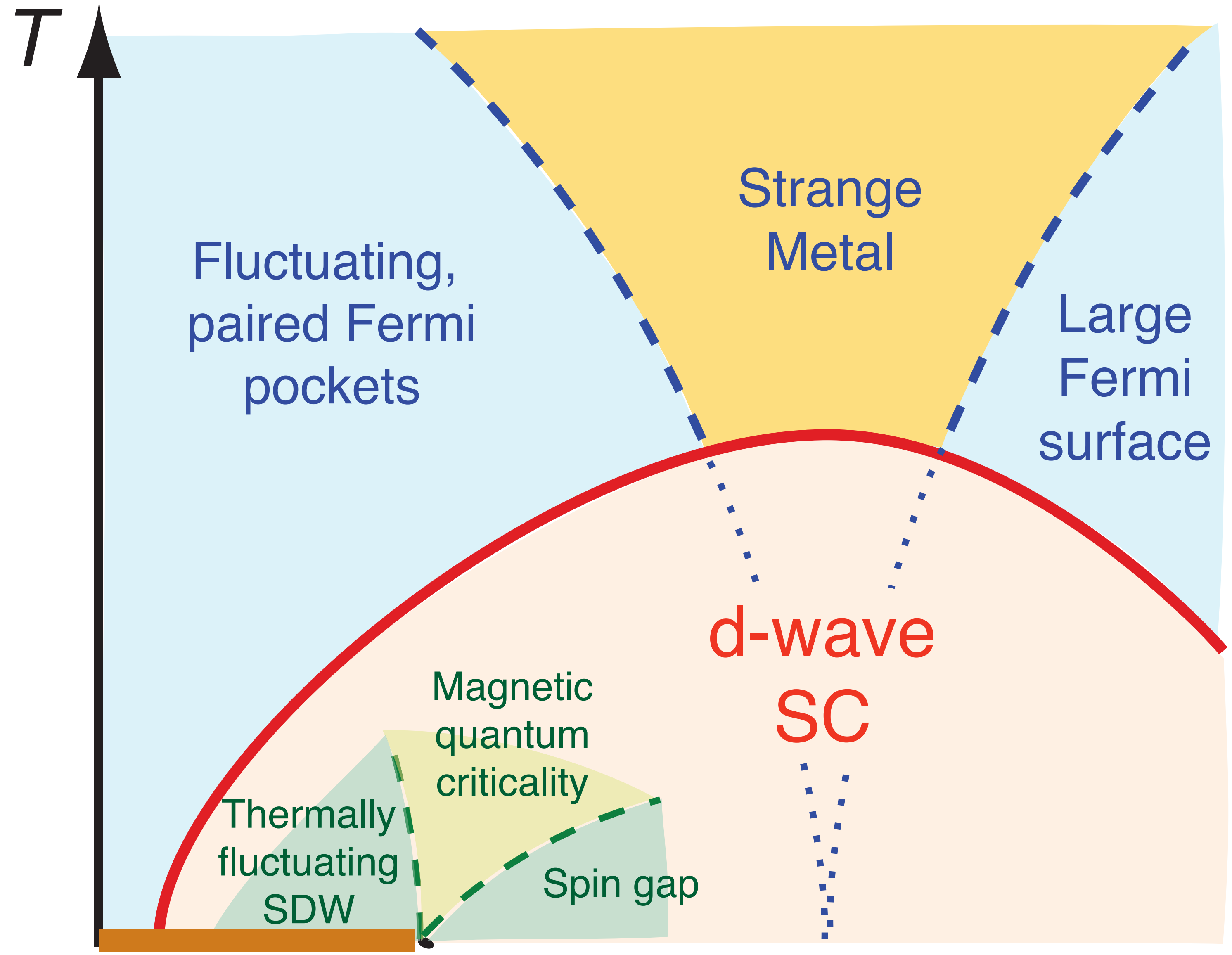


T
Fluctuating,
paired Fermi
pockets
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x_s
x



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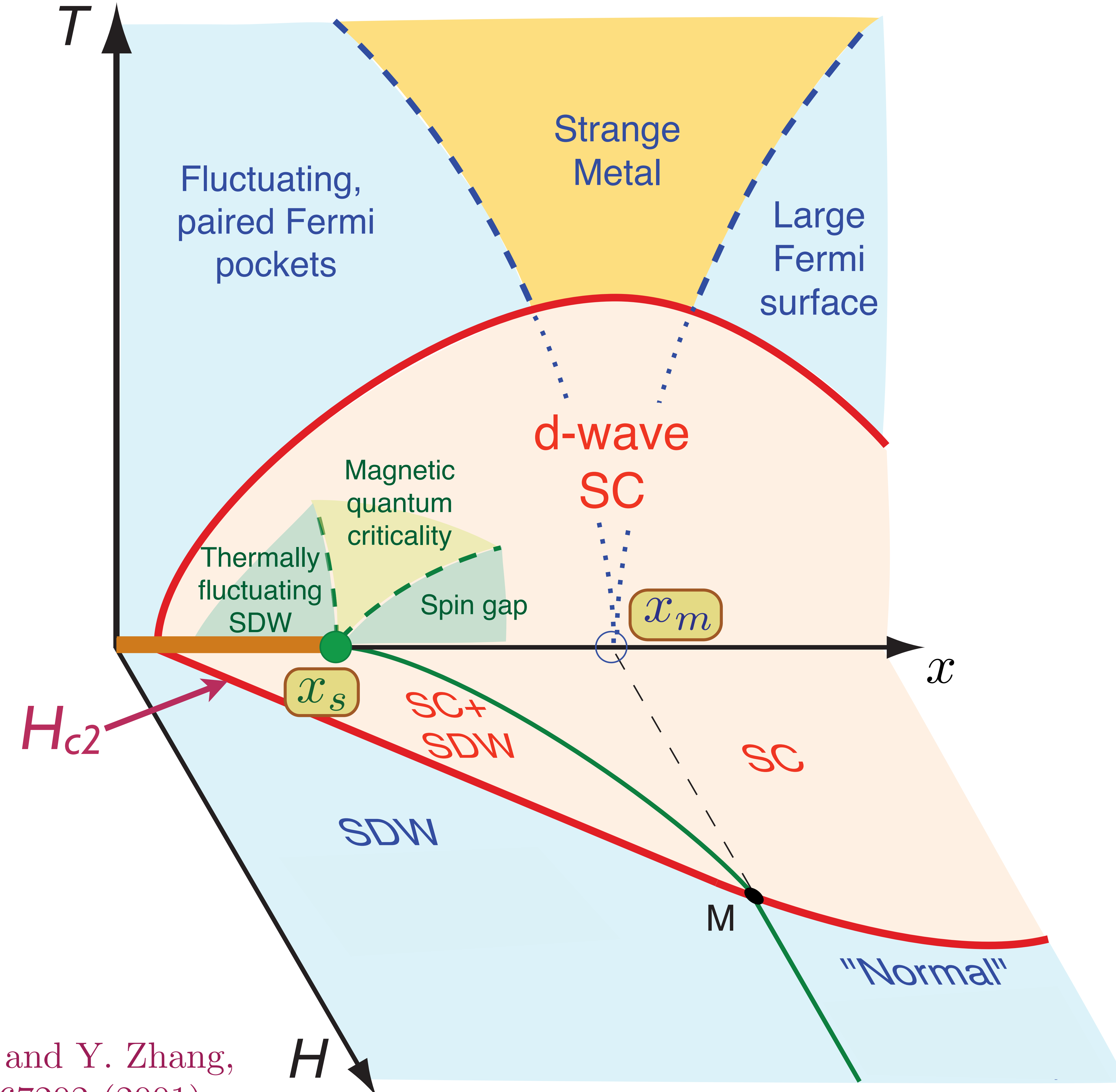

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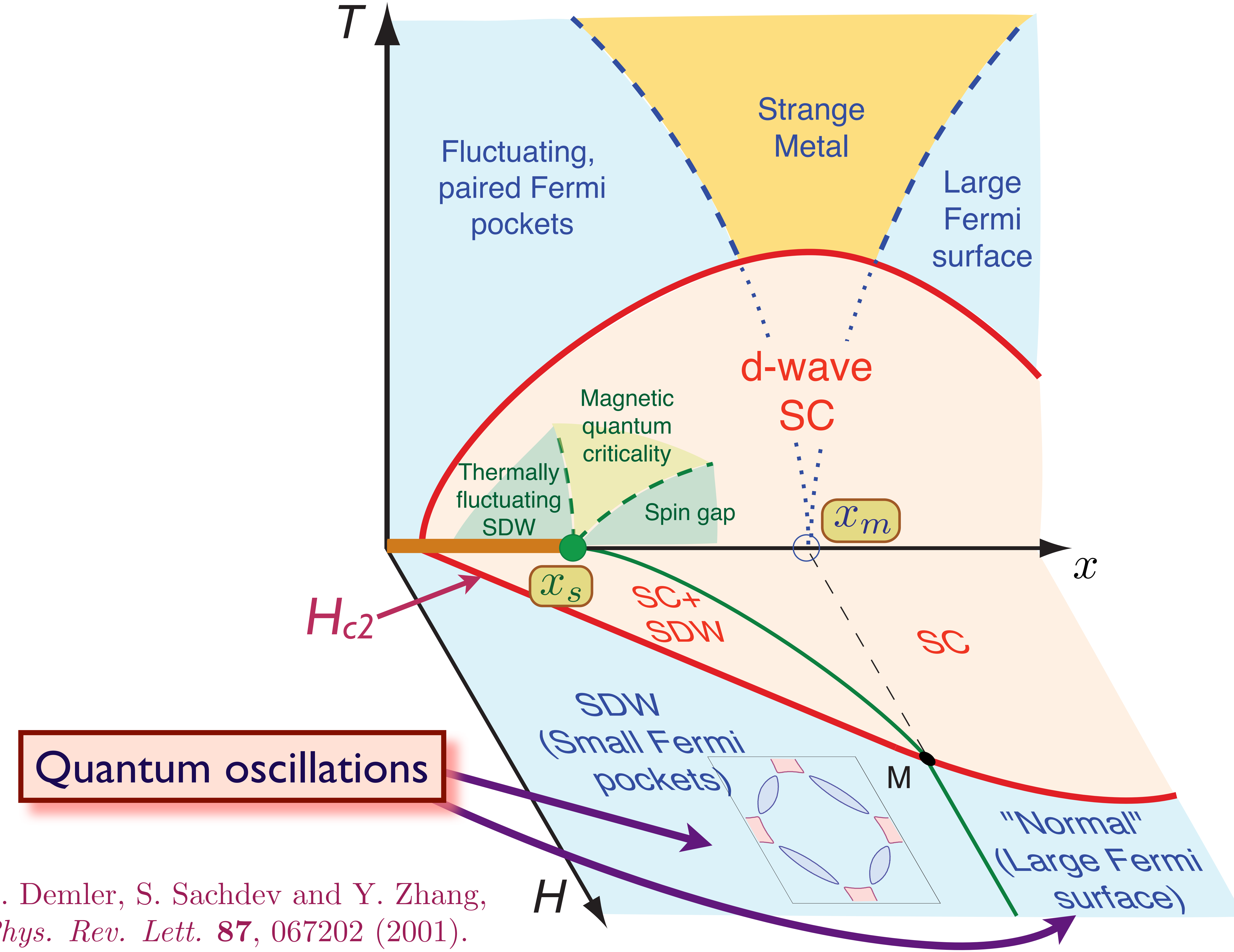
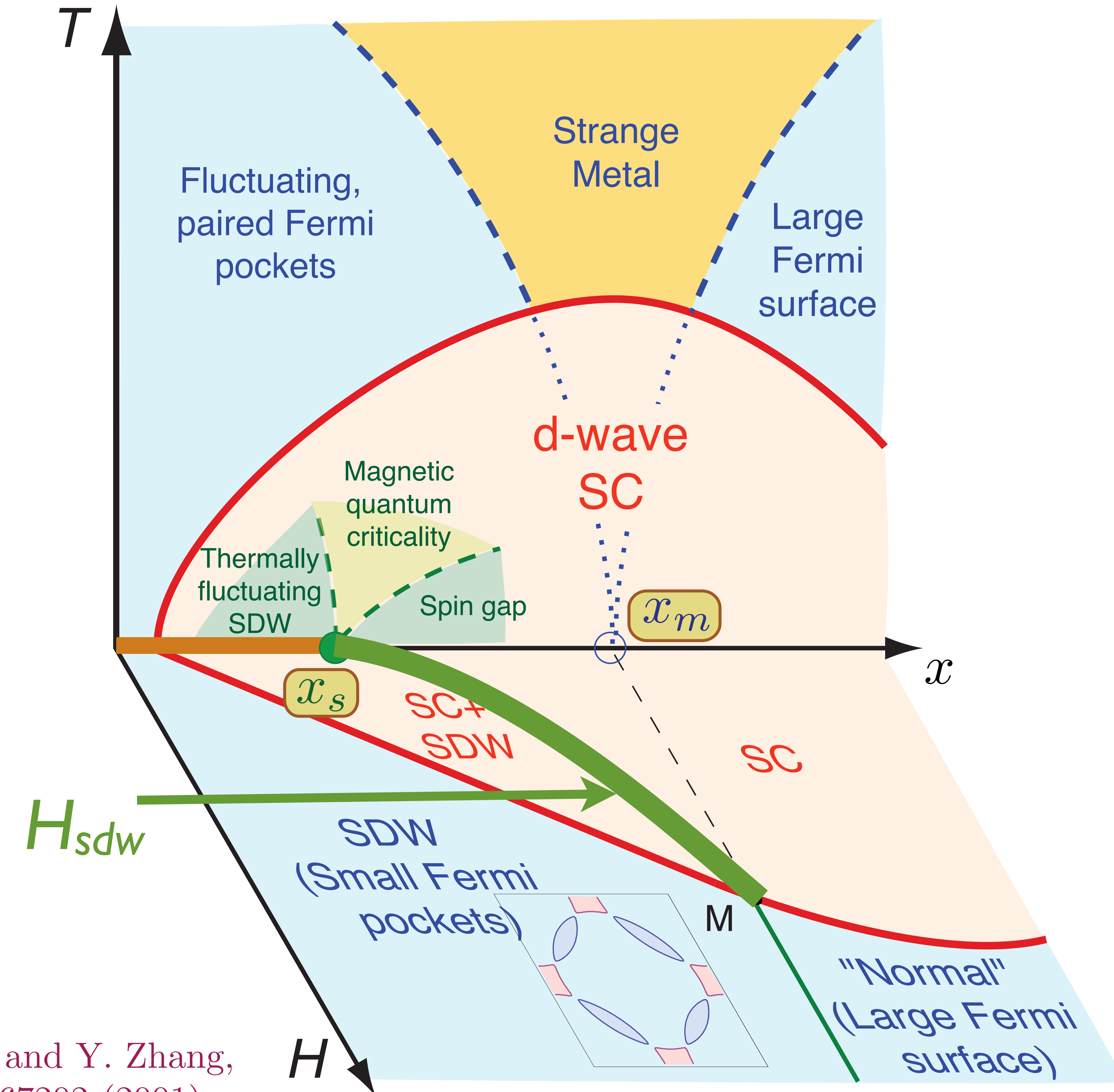


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Thursday, December 3, 2009



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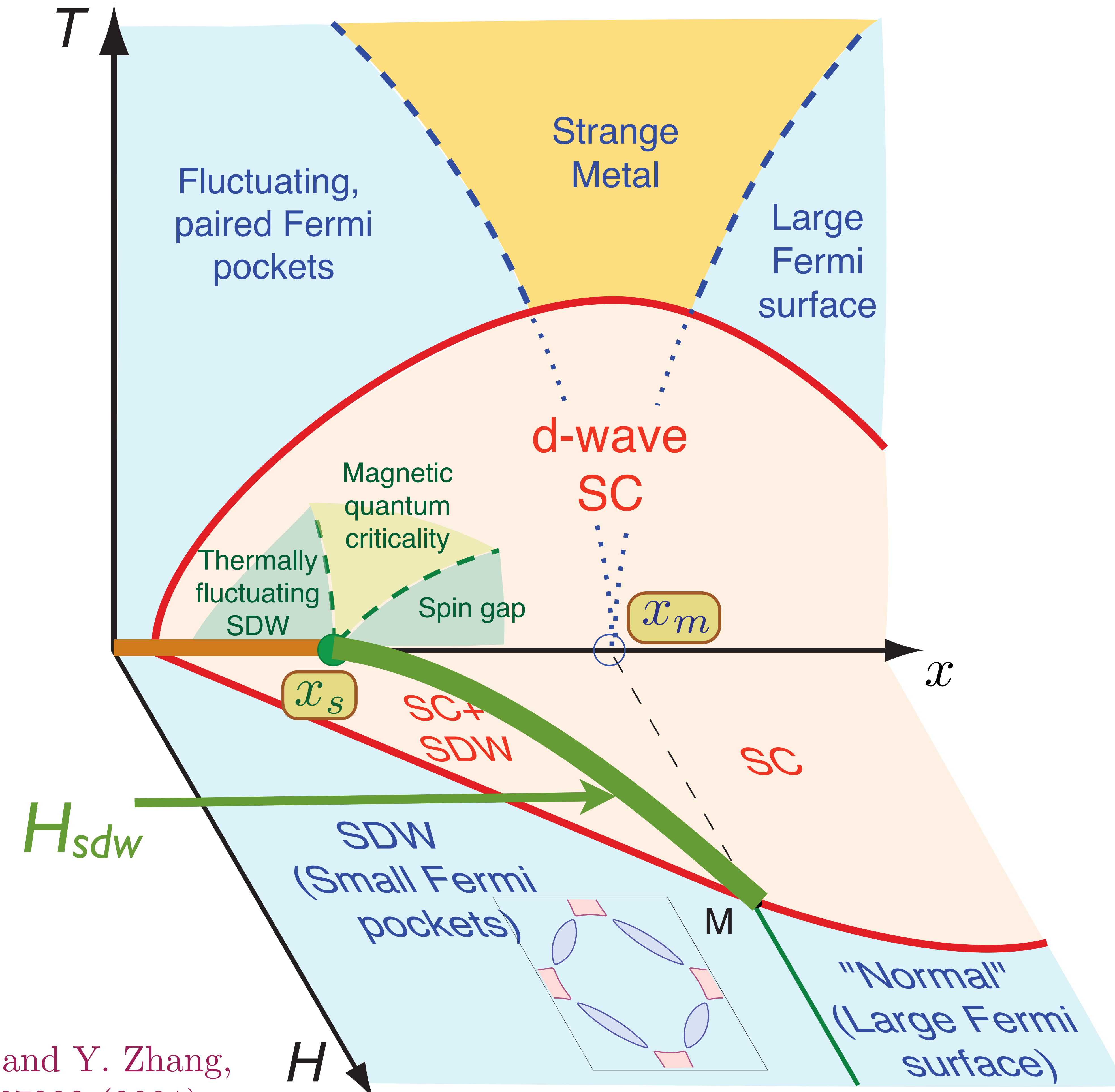
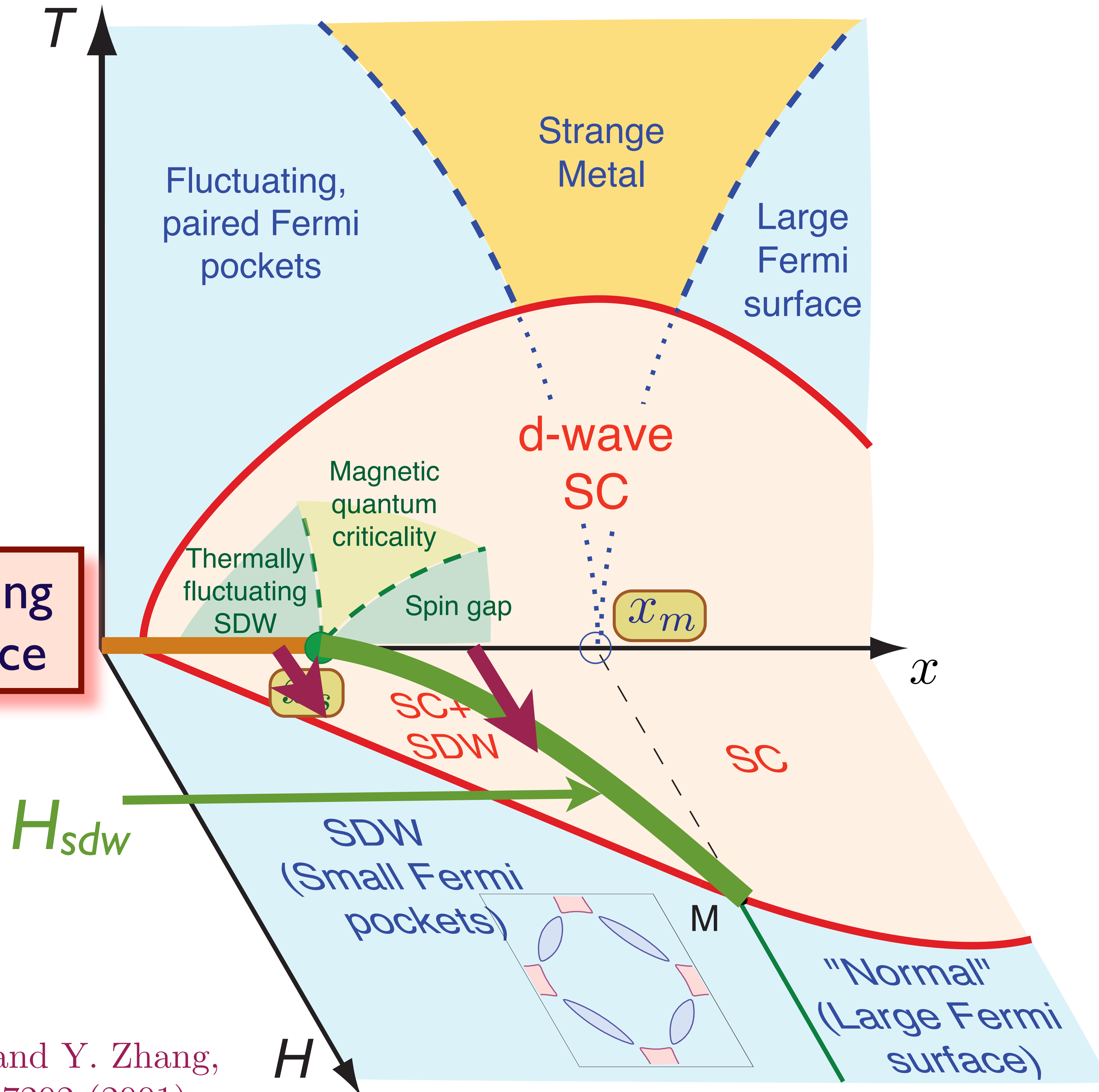


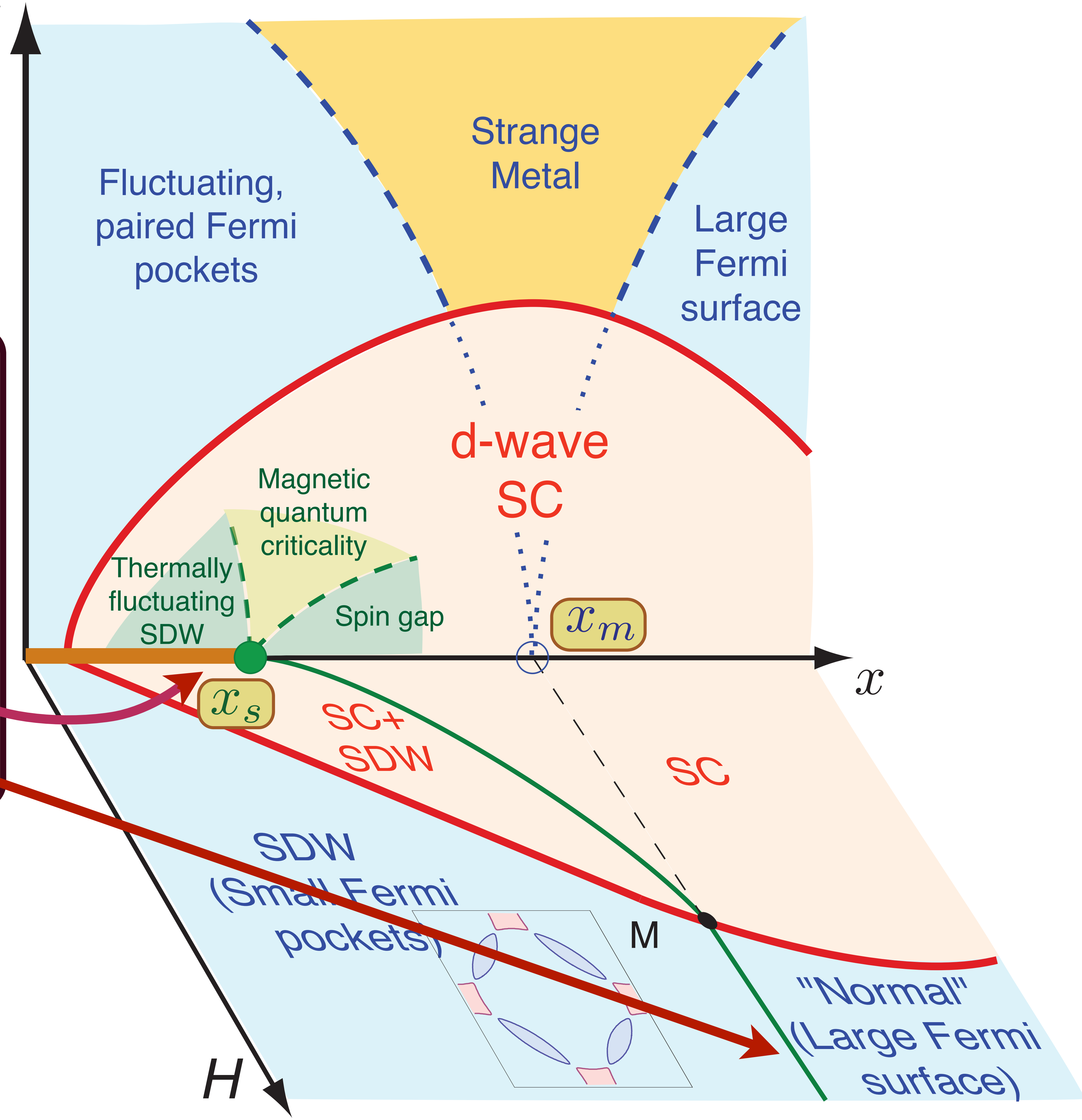
E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering & muon resonance

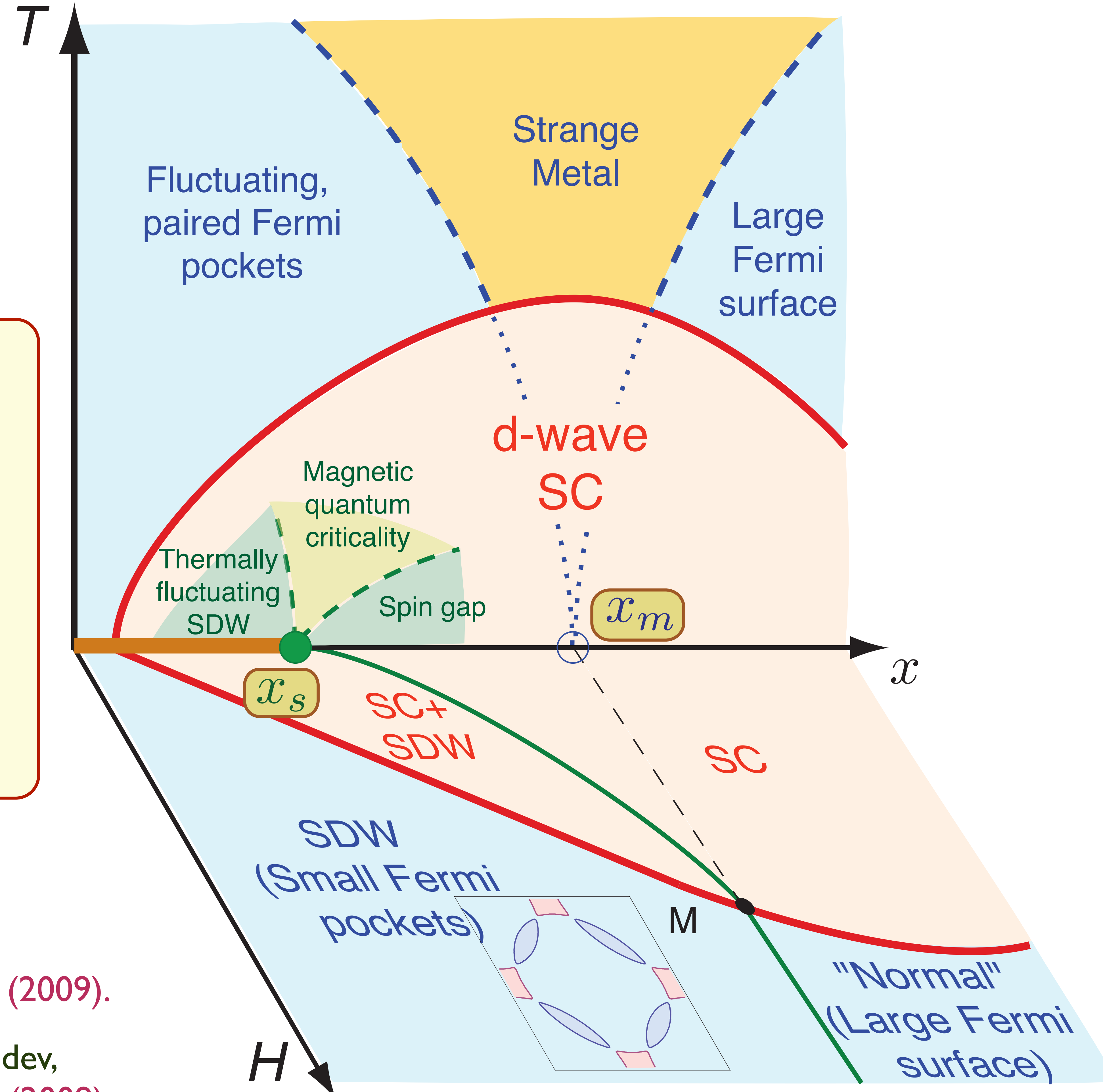


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $\underline{x_s = 0.14}$, while quantum oscillations at high fields show that $\underline{x_m = 0.165}$.



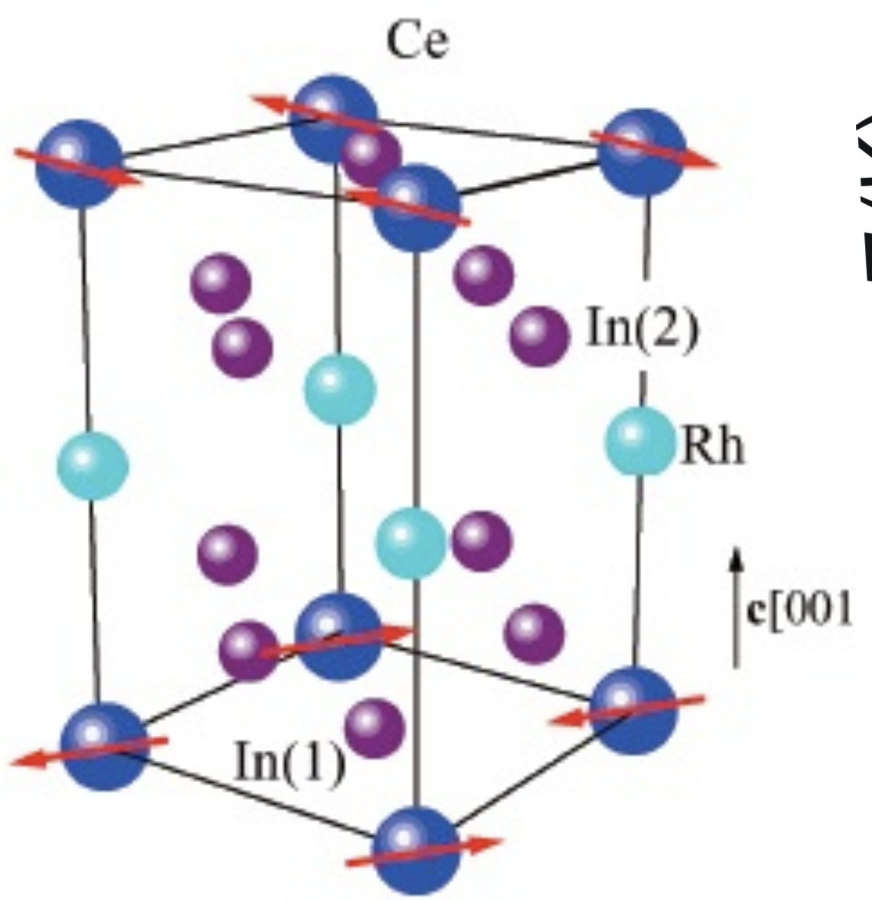
Physics of competition: d-wave SC and SDW "eat up" same pieces of the large Fermi surface.

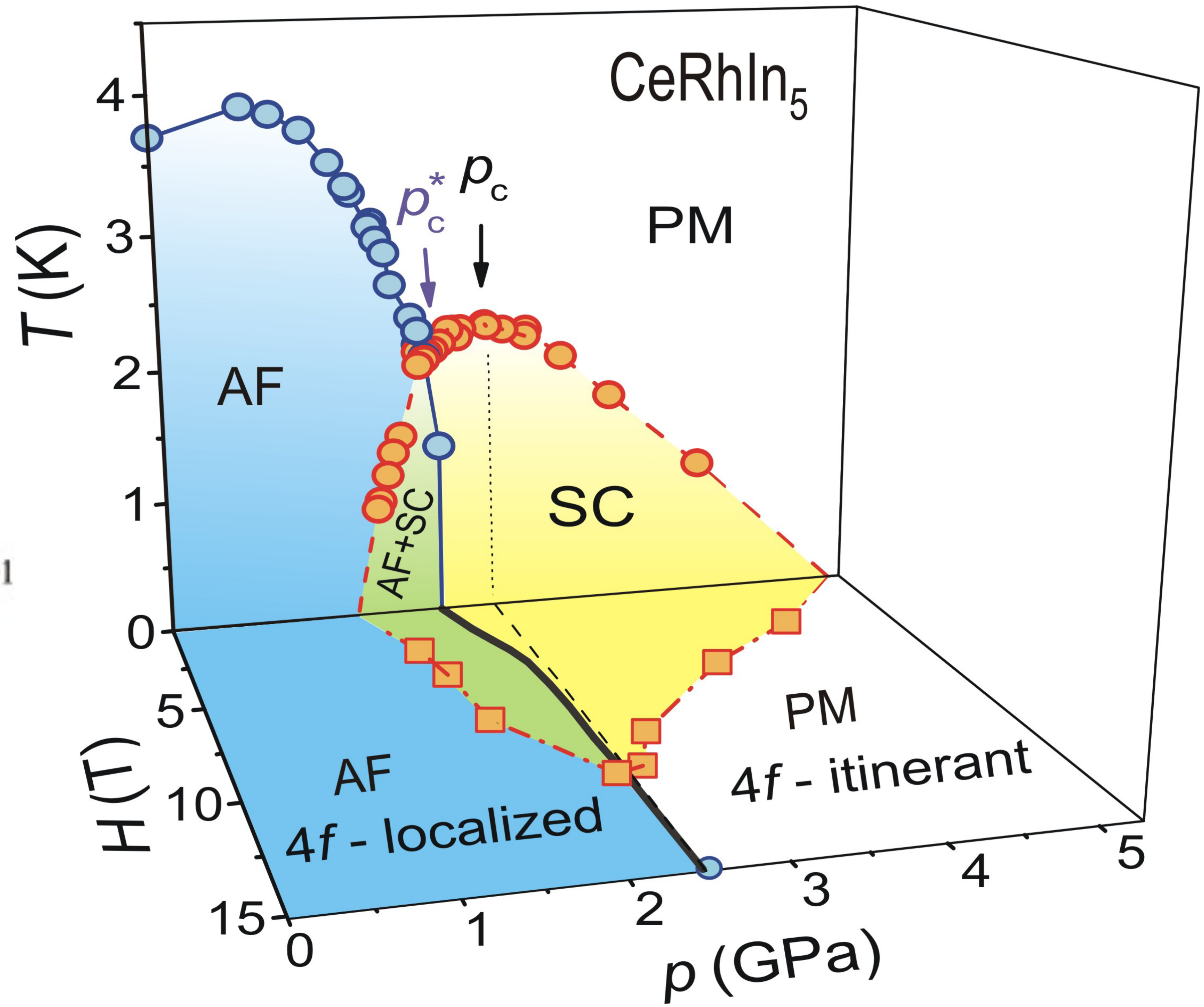


V. Galitski and S. Sachdev, *Physical Review B* **79**, 134512 (2009).

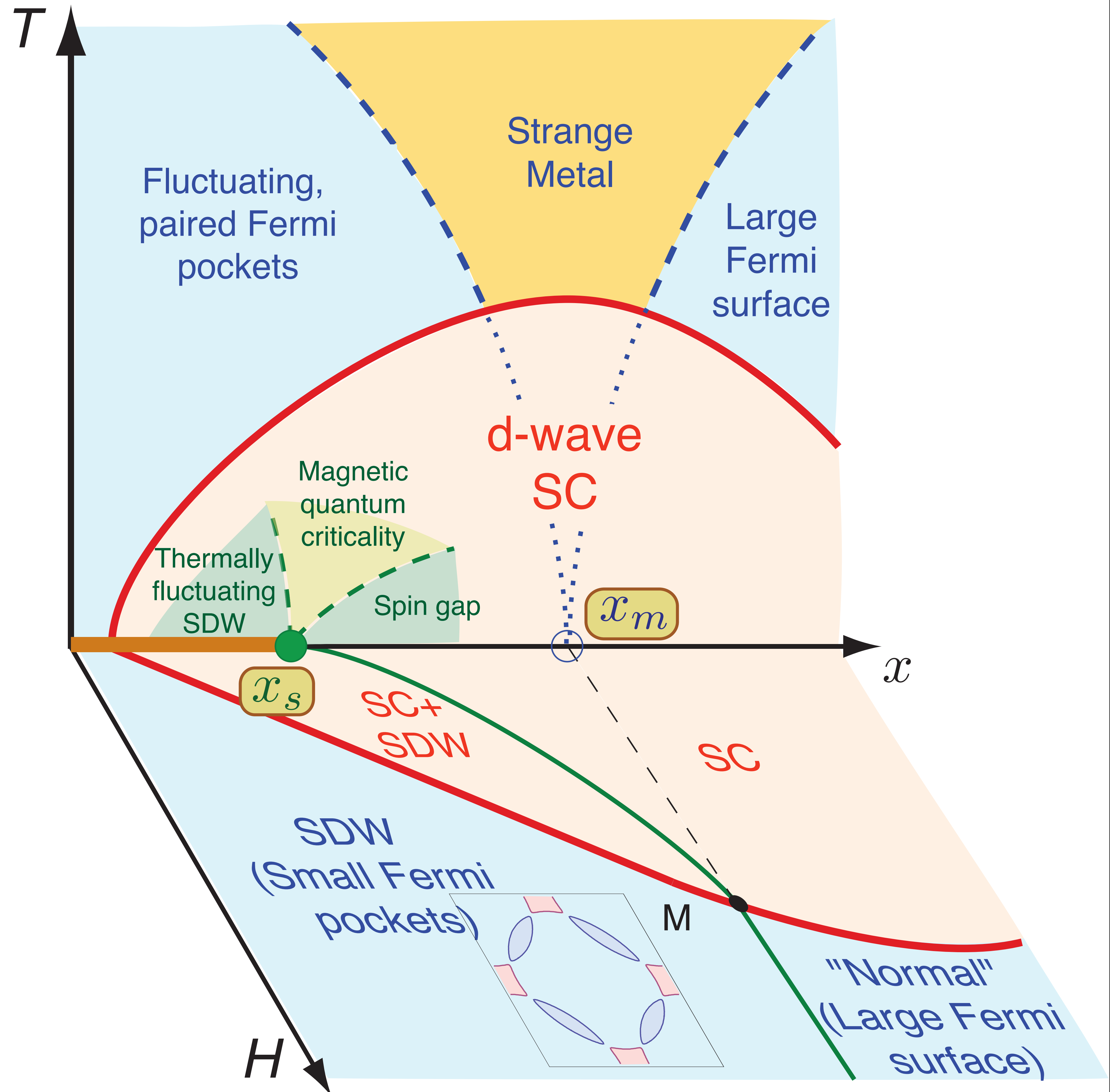
Eun Gook Moon and S. Sachdev, *Physical Review B* **80**, 035117 (2009).

Similar phase diagram for $CeRhIn_5$

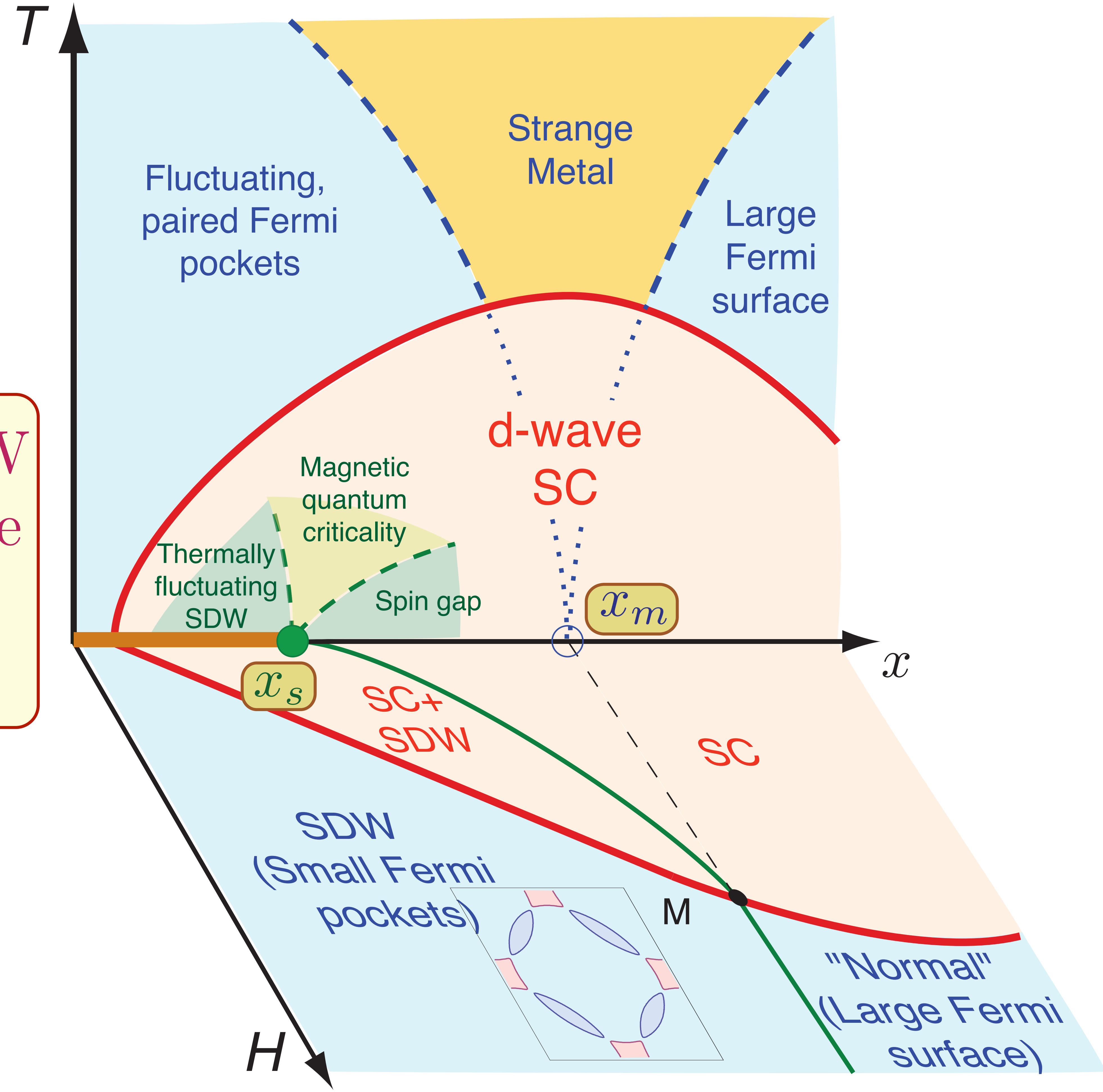




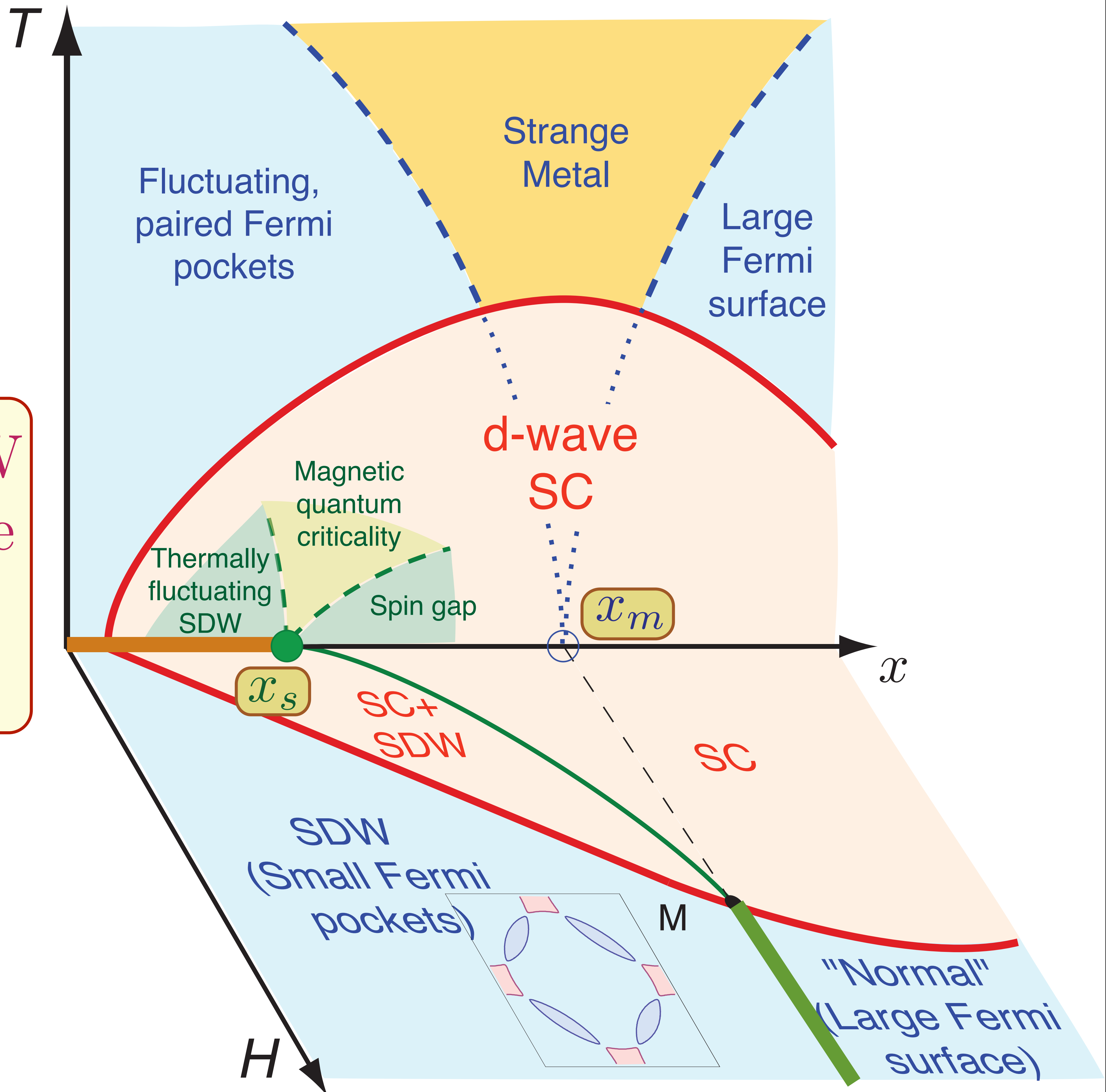
G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223


T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Theory of SDW quantum phase transition in metal



Theory of SDW quantum phase transition in metal



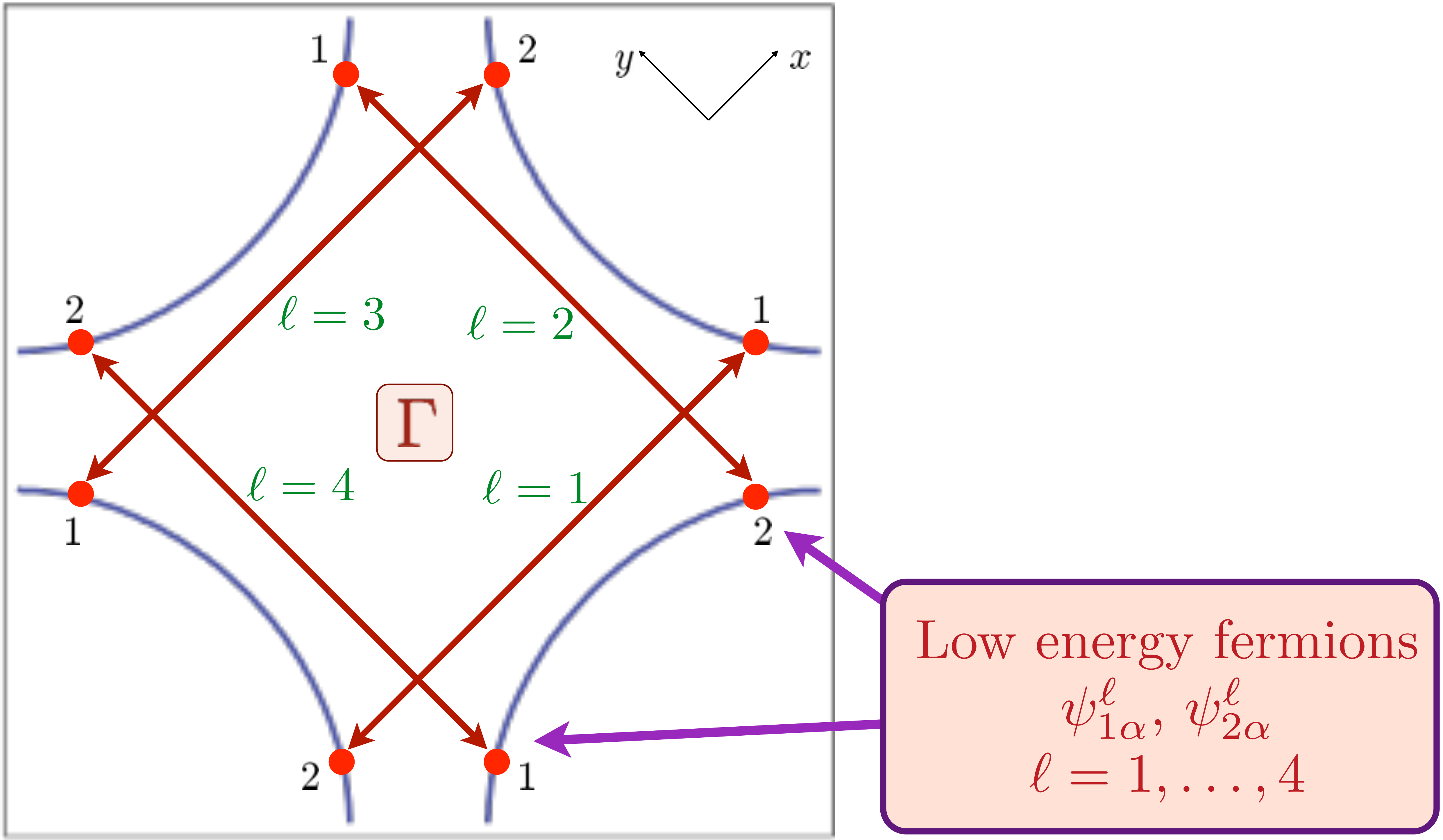
Start from the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial\tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

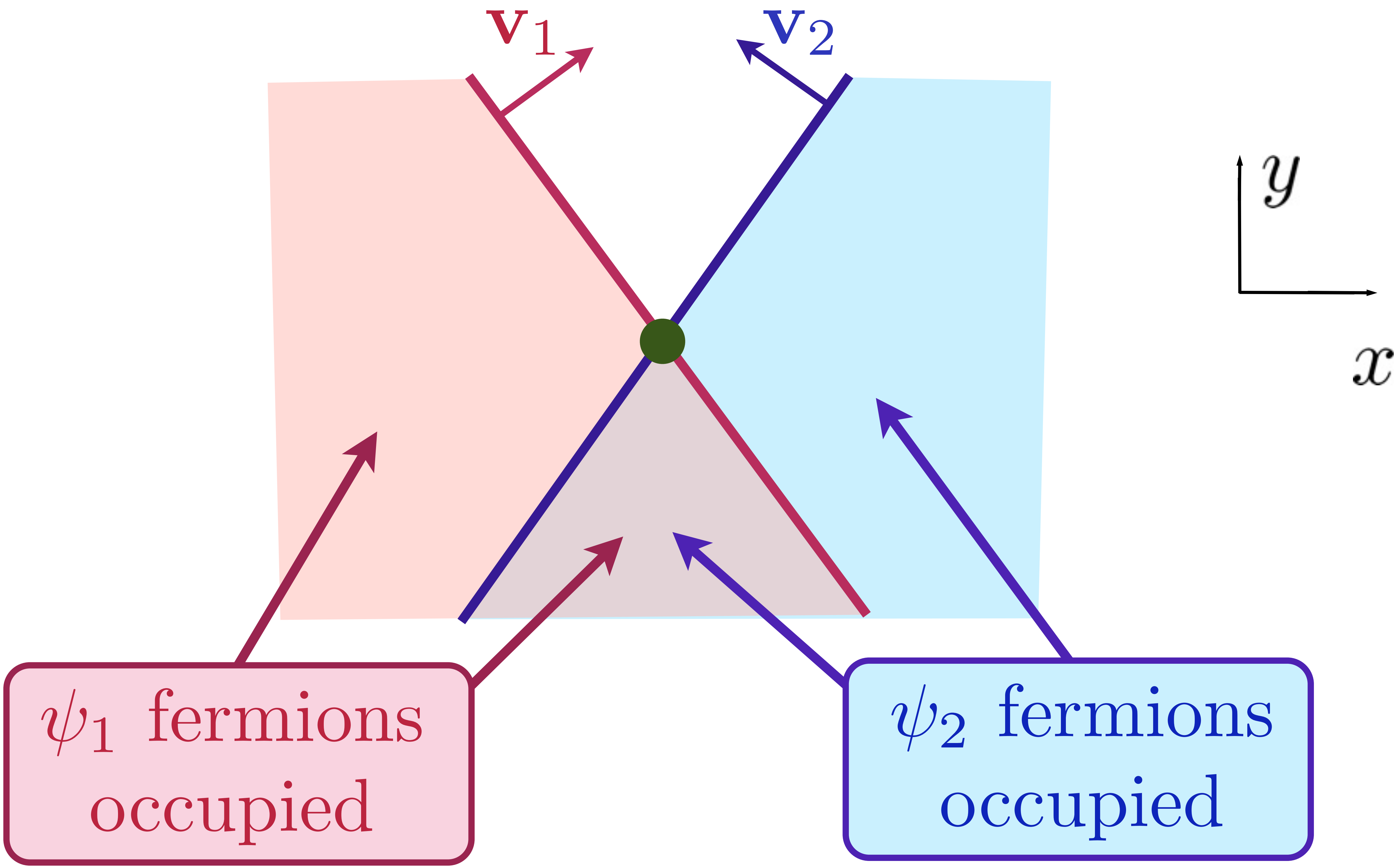
$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$



$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

$$\mathbf{v}^{\ell=1}_1 = (v_x, v_y),\ \mathbf{v}^{\ell=1}_2 = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$



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Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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"Yukawa" coupling: $\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi^{\ell\dagger}_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{2\beta} + \psi^{\ell\dagger}_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{1\beta}\right)$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Hertz-Moriya-Millis (HMM) theory

Integrate out fermions and obtain non-local corrections to $\mathcal{L}_\varphi$

$$\mathcal{L}_\varphi = \frac{1}{2}\vec{\varphi}^2\left[\mathbf{q}^2 + \gamma|\omega|\right]/2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Exponent $z = 2$ and mean-field criticality (upto logarithms)

But, higher order terms contain an infinite number of marginal couplings . . . . . .

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

"Yukawa" coupling: $\mathcal{L}_c = -\lambda \vec{\varphi} \cdot \left(\psi^{\ell\dagger}_{1\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{2\beta} + \psi^{\ell\dagger}_{2\alpha}\vec{\sigma}_{\alpha\beta}\psi^\ell_{1\beta}\right)$

Perform RG on both fermions and $\vec{\varphi}$, using a *local* field theory.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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Under the rescaling $x' = xe^{-\ell}$, $\tau' = \tau e^{-z\ell}$, the spatial gradients are fixed if the fields transform as

$$\vec{\varphi}' = e^{(d+z-2)\ell/2}\vec{\varphi} \quad ;" \quad \psi' = e^{(d+z-1)\ell/2}\psi.$$

Then the Yukawa coupling transforms as

$$\lambda' = e^{(4-d-z)\ell/2}\lambda$$

For $d = 2$, with $z = 2$ the Yukawa coupling is invariant, and the bare time-derivative terms ζ, $\widetilde{\zeta}$ are irrelevant.

Two approaches:

(A) Fix $\lambda = 1$ and perform RG in a $1/N$ expansion, where N is the number of fermion flavors

(B) Make λ part of the bare fermion dispersion by transforming electrons to a 'rotating reference frame' determined by the local orientation of the SDW order $\vec{\varphi}$.

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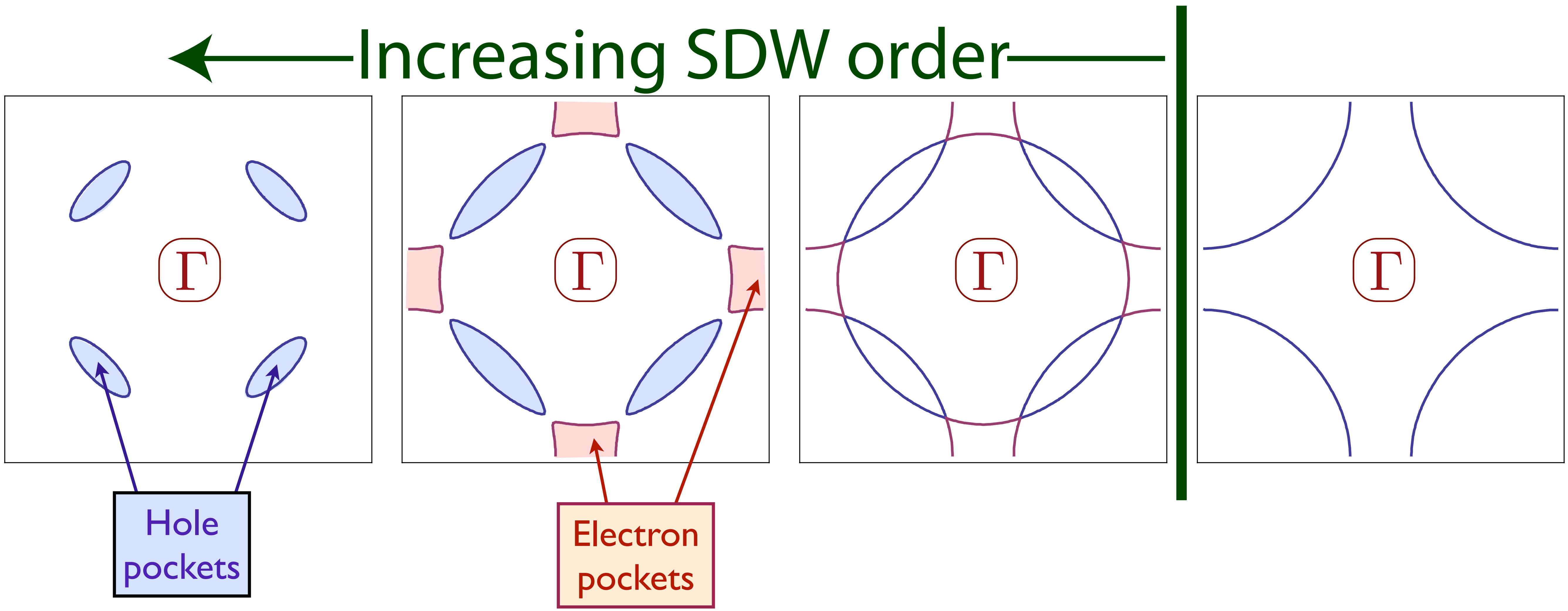
Max Metlitski

M. Metlitski and S. Sachdev, *to appear*

Ar. Abanov, A.V. Chubukov, and J. Schmalian, *Advances in Physics* **52**, 119 (2003)

Sung-Sik Lee, arXiv:0905.4532.

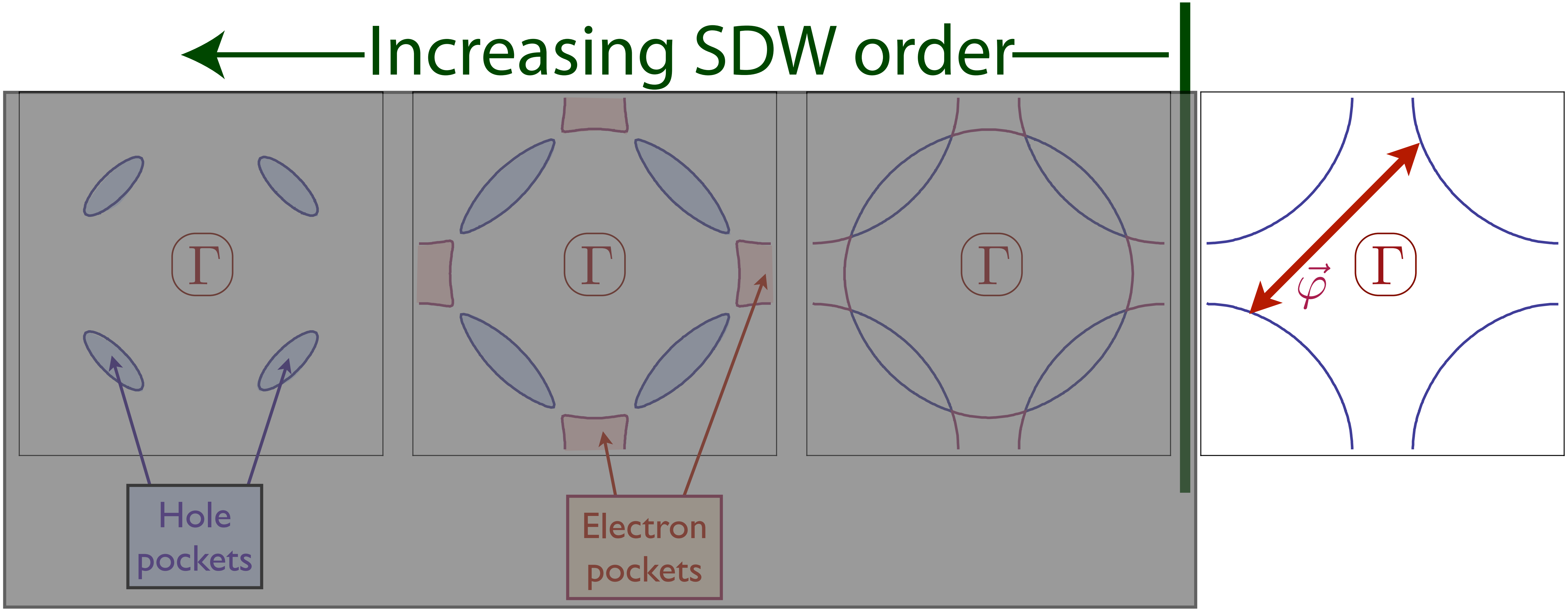
Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



$\vec{\varphi}$ fluctuations act on the large Fermi surface

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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With $z = 2$ scaling, ζ is irrelevant.
So we take $\zeta \to 0$
(watch for dangerous irrelevancy).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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Set $\vec{\varphi}$ wavefunction renormalization by keeping co-efficient of $(\boldsymbol{\nabla}_r\vec{\varphi})^2$ fixed (as usual).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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Set fermion wavefunction renormalization by keeping Yukawa coupling fixed.

Y. Huh and S. Sachdev, *Phys. Rev. B* **78**, 064512 (2008).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2 \cdot \boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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We find consistent two-loop RG factors, as $\zeta \to 0$, for the velocities v_x, v_y, and the wavefunction renormalizations.

Consistency check: the expression for the boson damping constant, $\gamma = \dfrac{2}{\pi v_x v_y}$, is preserved under RG.

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}$$

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}$$

The velocities flow as

$$\frac{1}{v_x}\frac{dv_x}{d\ell} = \frac{\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2} \quad ; \quad \frac{1}{v_y}\frac{dv_y}{d\ell} = \frac{-\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2}$$

$$\mathcal{A}(\alpha) \equiv \frac{3}{\pi N}\frac{\alpha}{1+\alpha^2}$$

$$\mathcal{B}(\alpha) \equiv \frac{1}{2\pi N}\left(\frac{1}{\alpha}-\alpha\right)\left(1+\left(\frac{1}{\alpha}-\alpha\right)\tan^{-1}\frac{1}{\alpha}\right)$$

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The anomalous dimensions of $\vec{\varphi}$ and ψ are

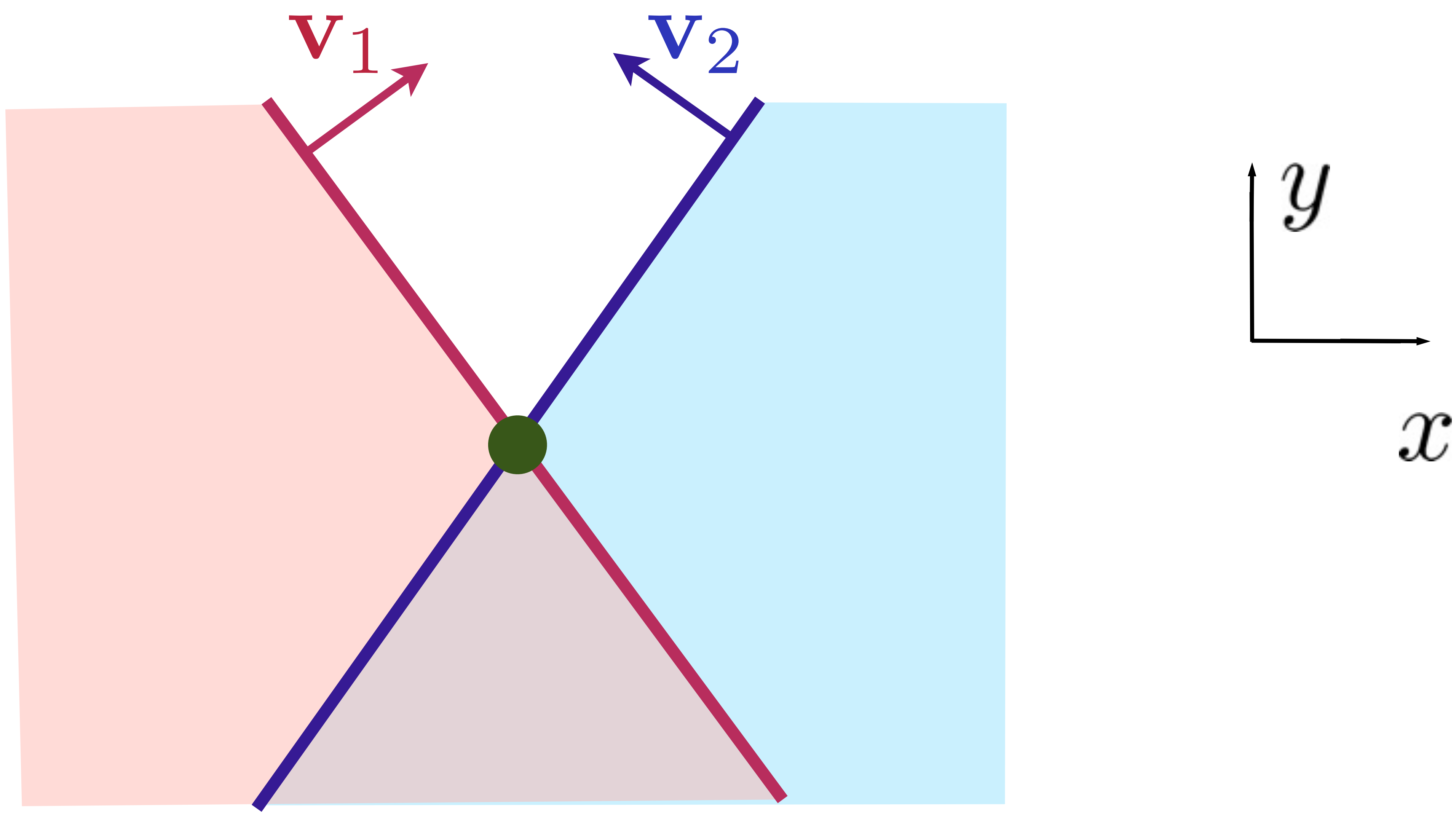
$$\eta_\varphi = \frac{1}{2\pi N}\left(\frac{1}{\alpha} - \alpha + \left(\frac{1}{\alpha^2} + \alpha^2\right)\tan^{-1}\frac{1}{\alpha}\right)$$

$$\eta_\psi = -\frac{1}{4\pi N}\left(\frac{1}{\alpha} - \alpha\right)\left(1 + \left(\frac{1}{\alpha} - \alpha\right)\tan^{-1}\frac{1}{\alpha}\right)$$

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting

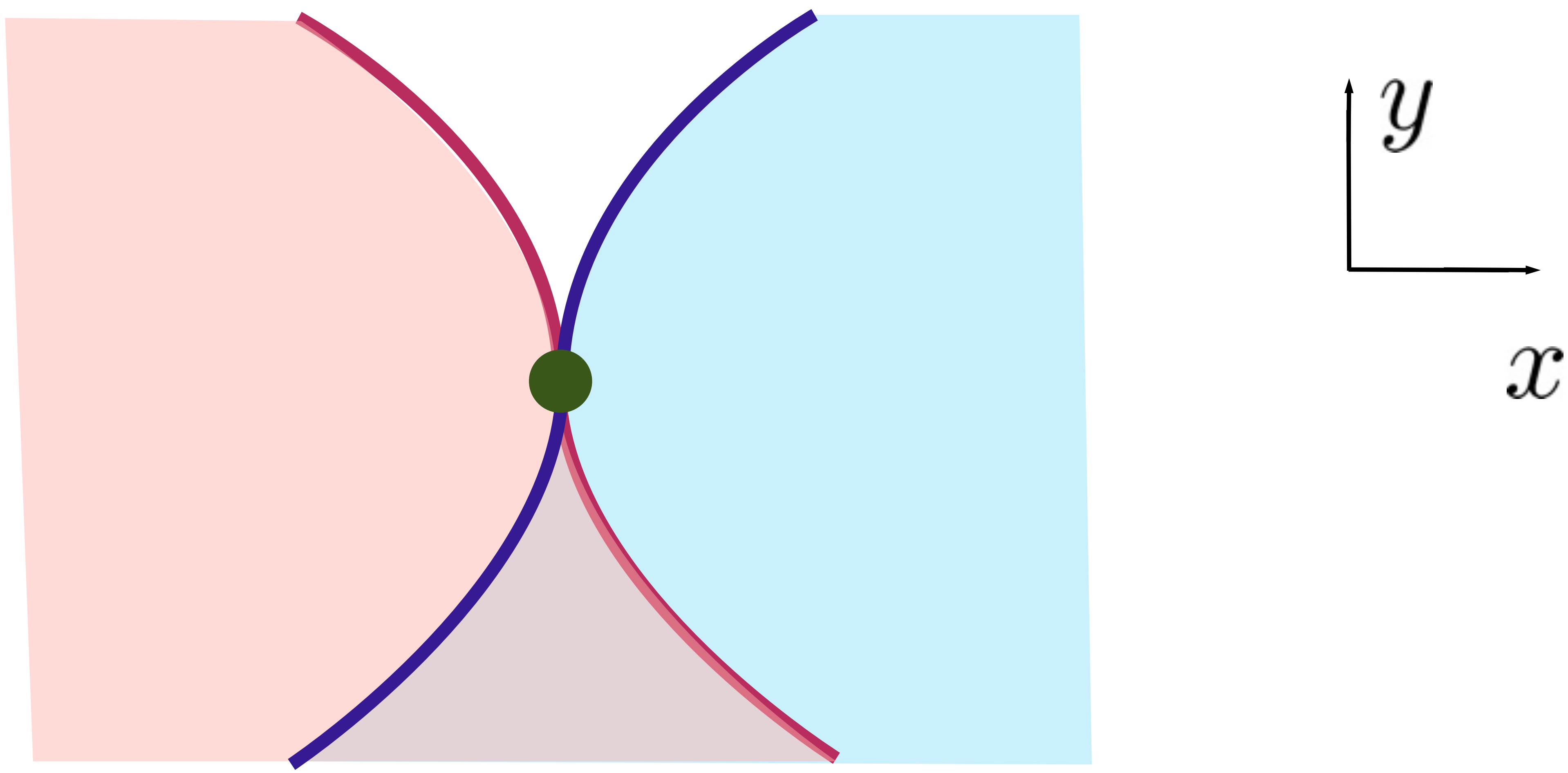


Bare Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y / v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting



Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

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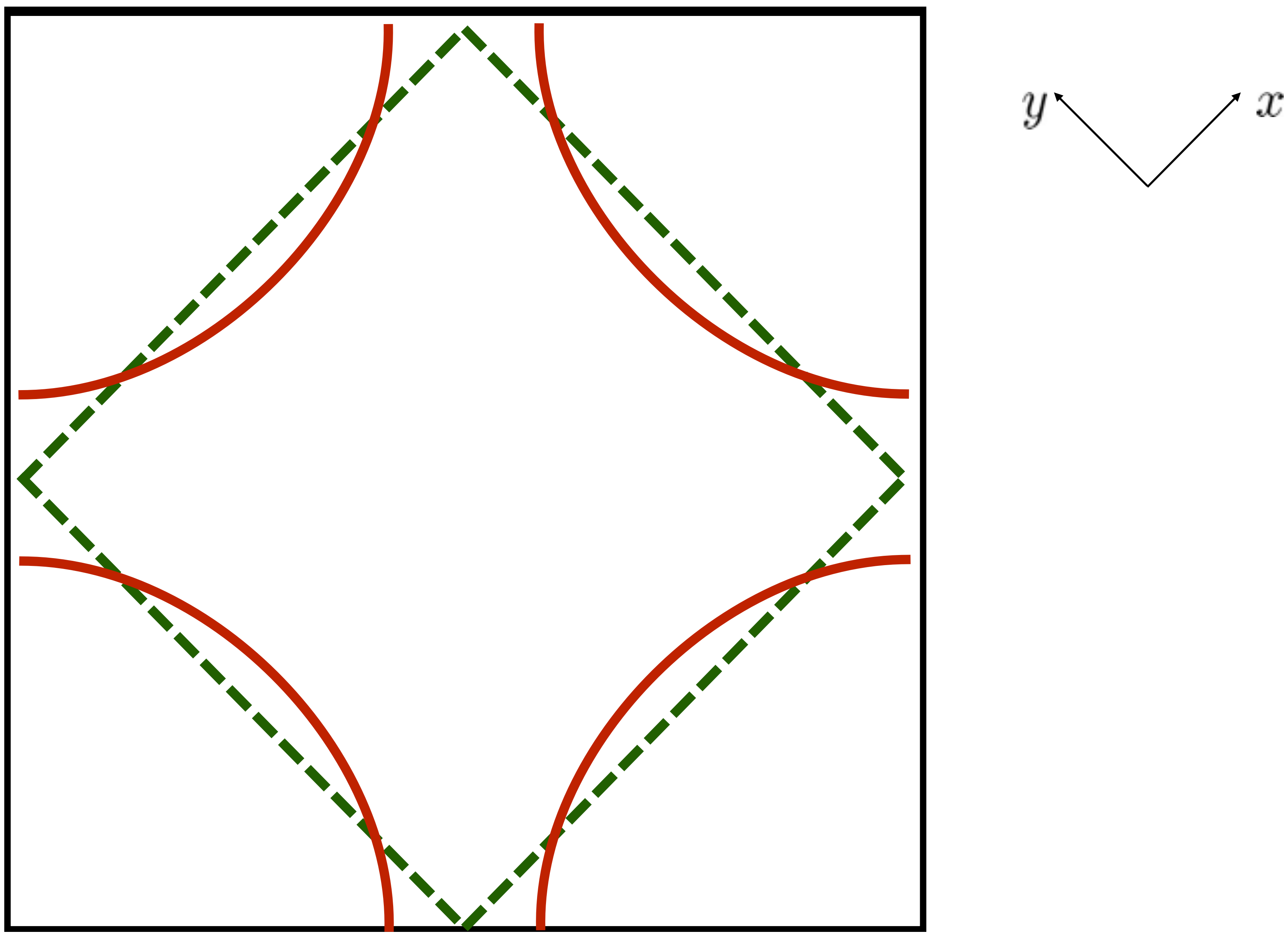
Dynamical Nesting



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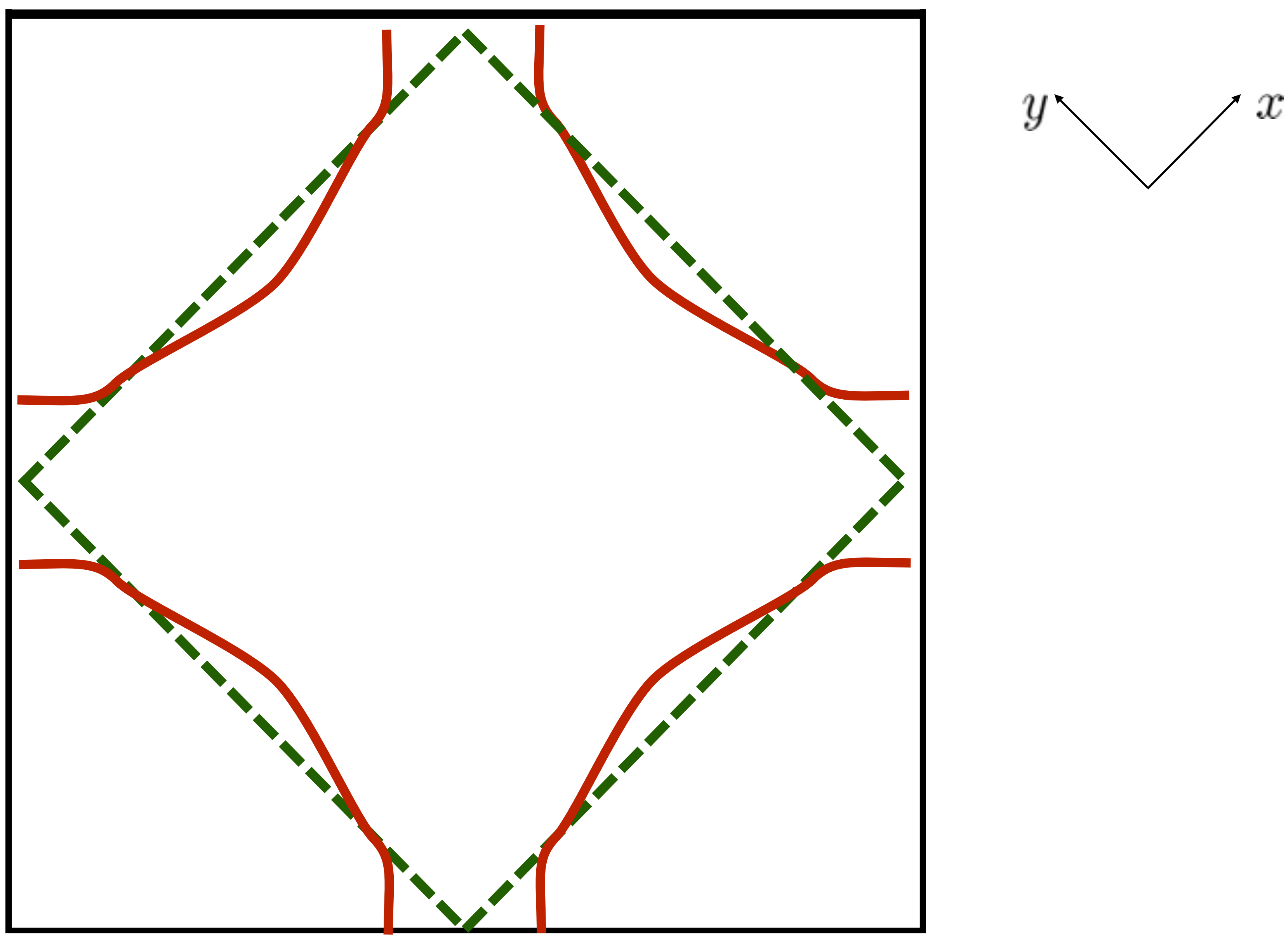
Dynamical Nesting



Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

In $\vec{\varphi}$ SDW fluctuations, characteristic q and ω scale as

$$q \sim \omega^{1/2} \exp\left(-\frac{3}{64\pi^2}\left(\frac{\ln(1/\omega)}{N}\right)^3\right).$$

However, $1/N$ expansion cannot be trusted in the asymptotic regime.

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

$$\frac{1}{N}\frac{1}{(q^2+\gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v}\cdot\mathbf{q}+i\zeta\omega+i\frac{1}{N\sqrt{\gamma}v}\sqrt{\omega}F\left(\frac{v^2q^2}{\omega}\right)}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

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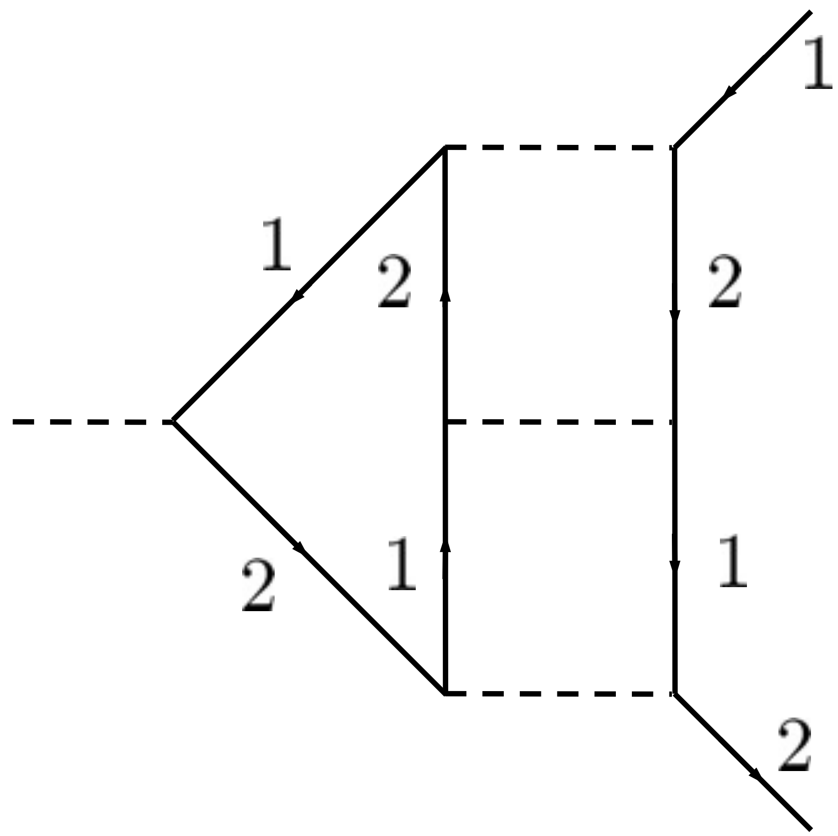
$$\frac{1}{N} \frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v} \cdot \mathbf{q} + i\zeta\omega + i\frac{1}{N\sqrt{\gamma}v}\sqrt{\omega}F\left(\frac{v^2q^2}{\omega}\right)}$$

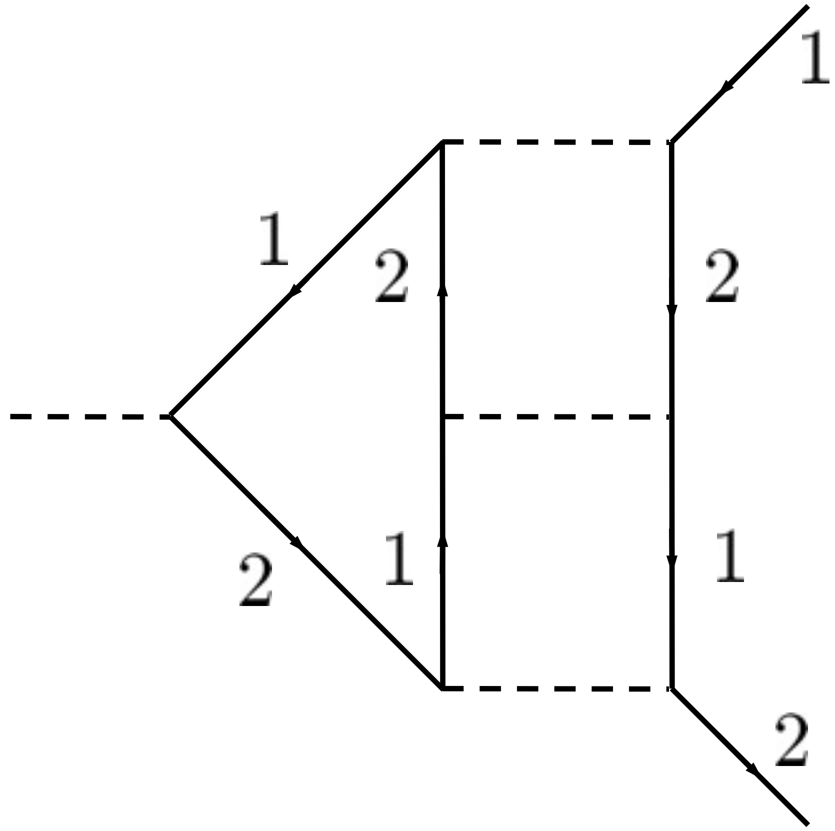
Dangerous

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \dfrac{1}{N^2} \times \dfrac{1}{\zeta^2} \times \dfrac{1}{\omega}$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \dfrac{1}{N^2} \times \dfrac{1}{\zeta^2} \times \dfrac{1}{\omega}$

Actual order $\sim \dfrac{1}{N^0}$

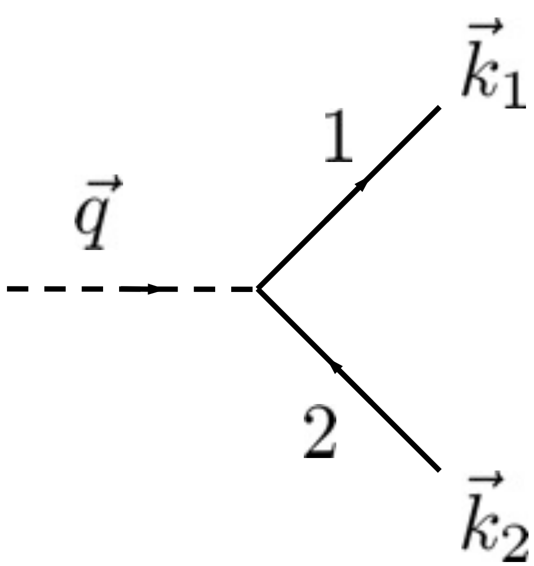
Double line representation

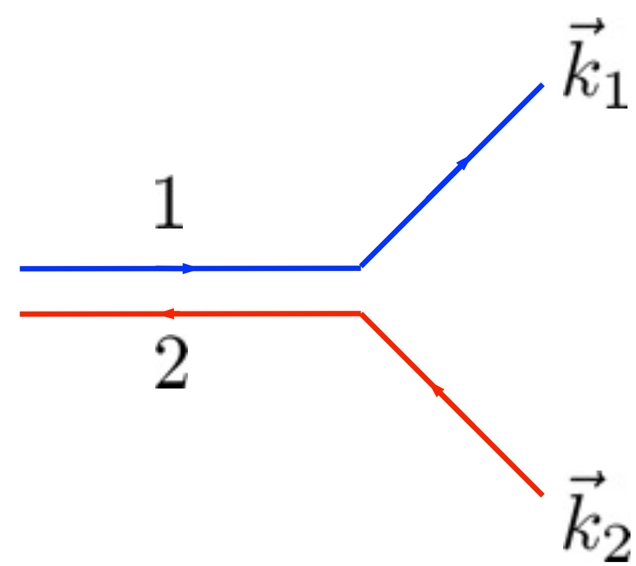
- A way to compute the order of a diagram.
- Extra powers of N come from the Fermi-surface

$$G(\omega, \vec{k}) = \frac{1}{-\Sigma_1(\omega, \vec{k}) - \vec{v} \cdot \vec{k}} \qquad \Sigma_1 \sim \frac{1}{N}$$

- What are the conditions for all propagators to be on the Fermi surface?
- Concentrate on diagrams involving a single pair of hot-spots
- Any bosonic momentum may be (uniquely) written as

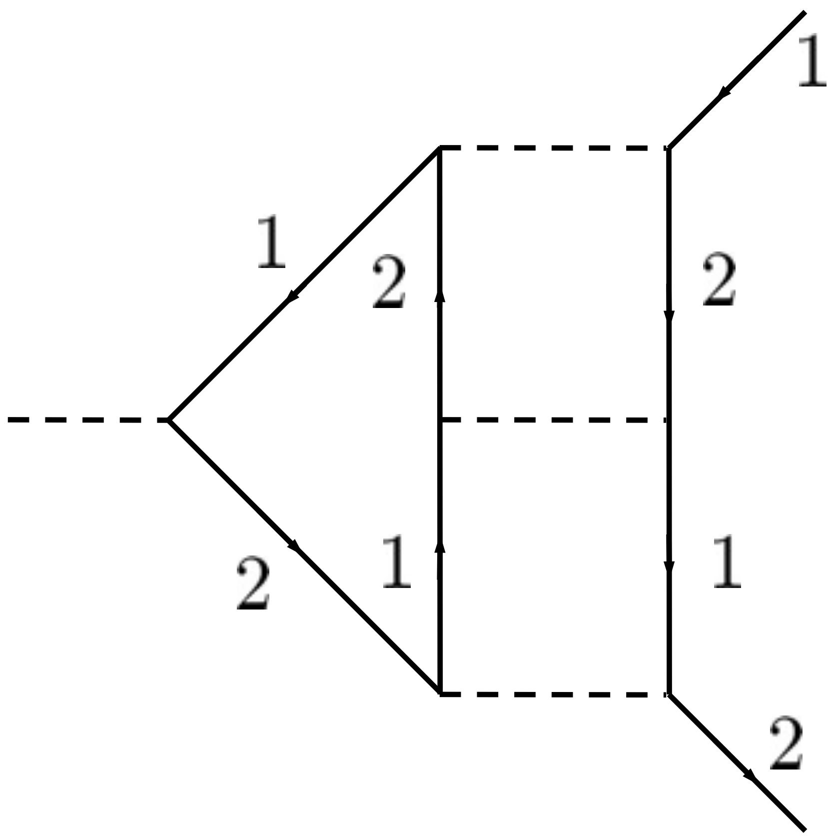
$$\vec{q} = \vec{k}_1 - \vec{k}_2 \qquad \vec{k}_1 \in \mathrm{FS\ of}\ \psi_1 \qquad \vec{k}_2 \in \mathrm{FS\ of}\ \psi_2$$





R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
S. W. Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Phys. Rev. B **72**, 054531 (2005).

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



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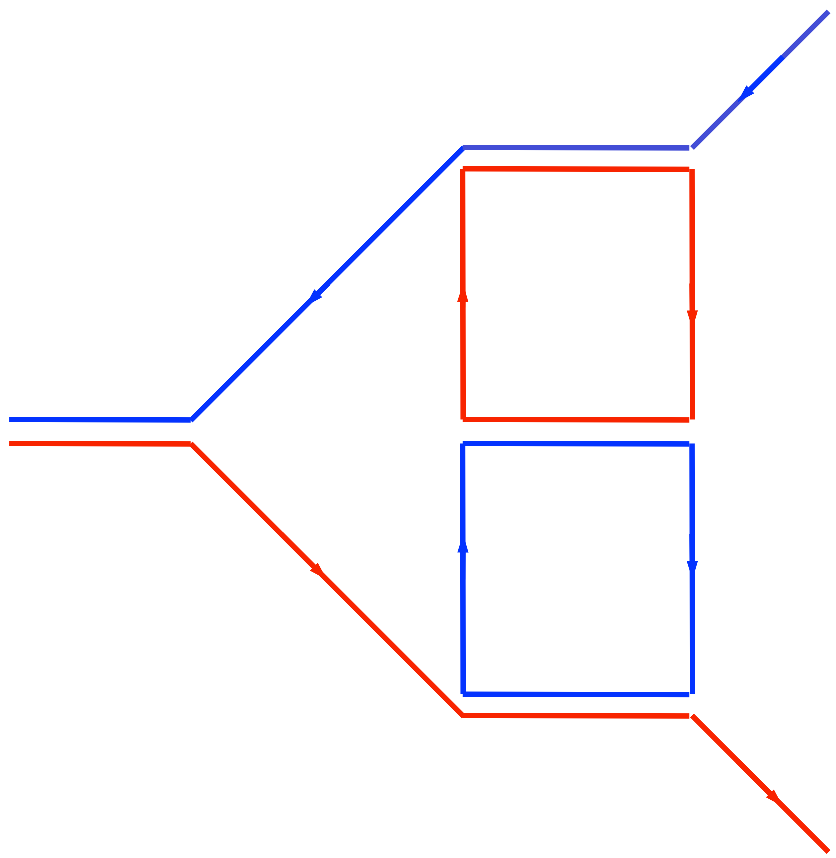
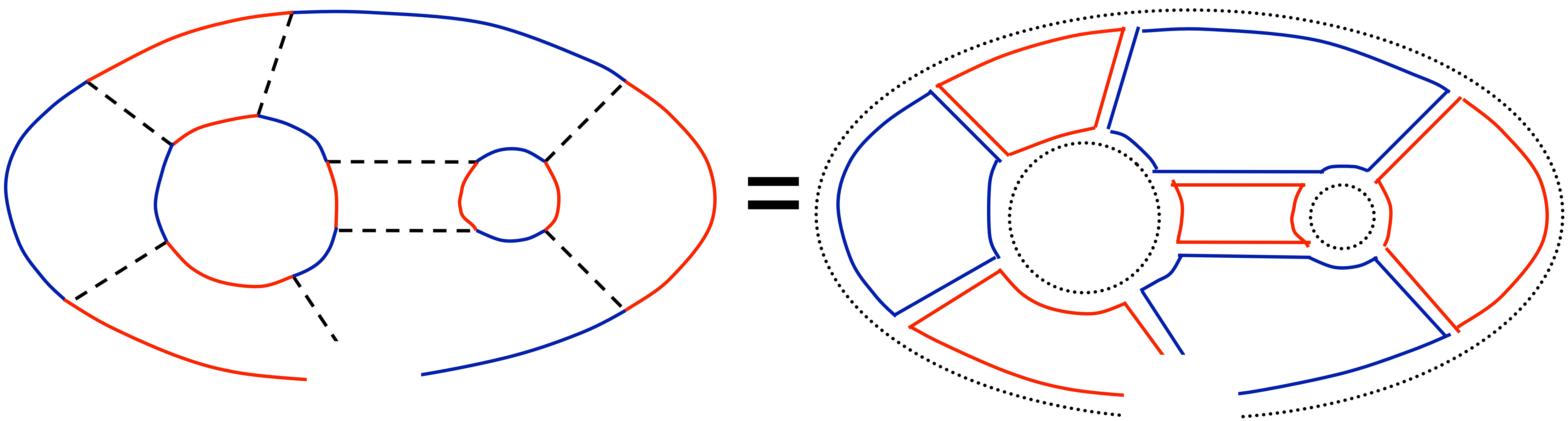


Singularities as $\zeta \to 0$ appear when fermions in closed blue and red line loops are exactly on the Fermi surface

$$\text{Actual order} \sim \frac{1}{N^0}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

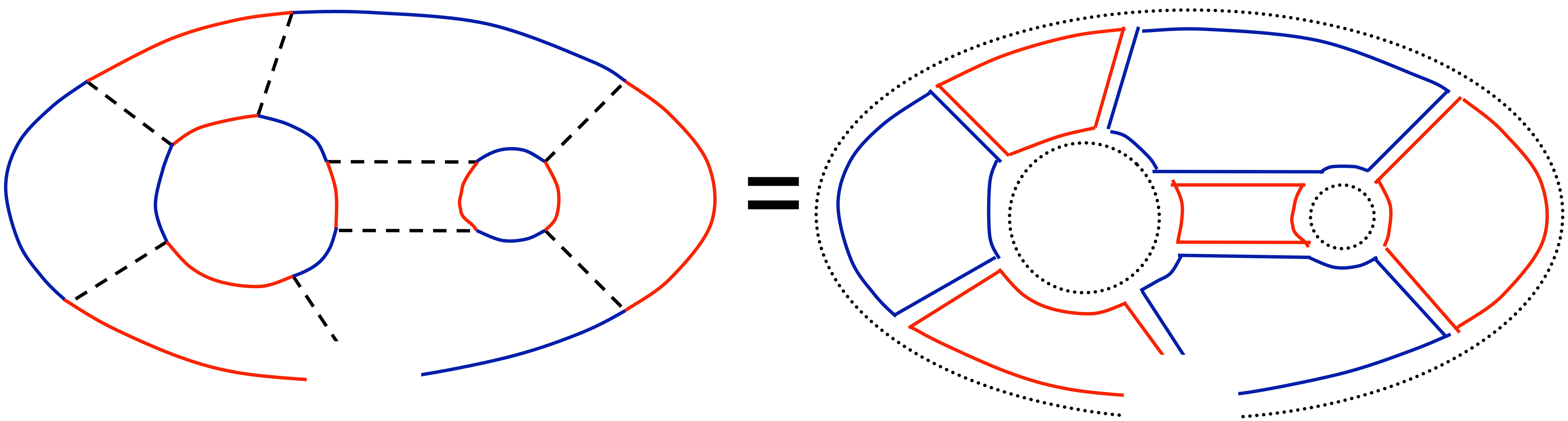


$$\text{Actual order} \sim \frac{1}{N^0}$$

Graph is **planar** after turning fermion propagators also into double lines by drawing additional dotted single line loops for each fermion loop

Sung-Sik Lee, arXiv:0905.4532

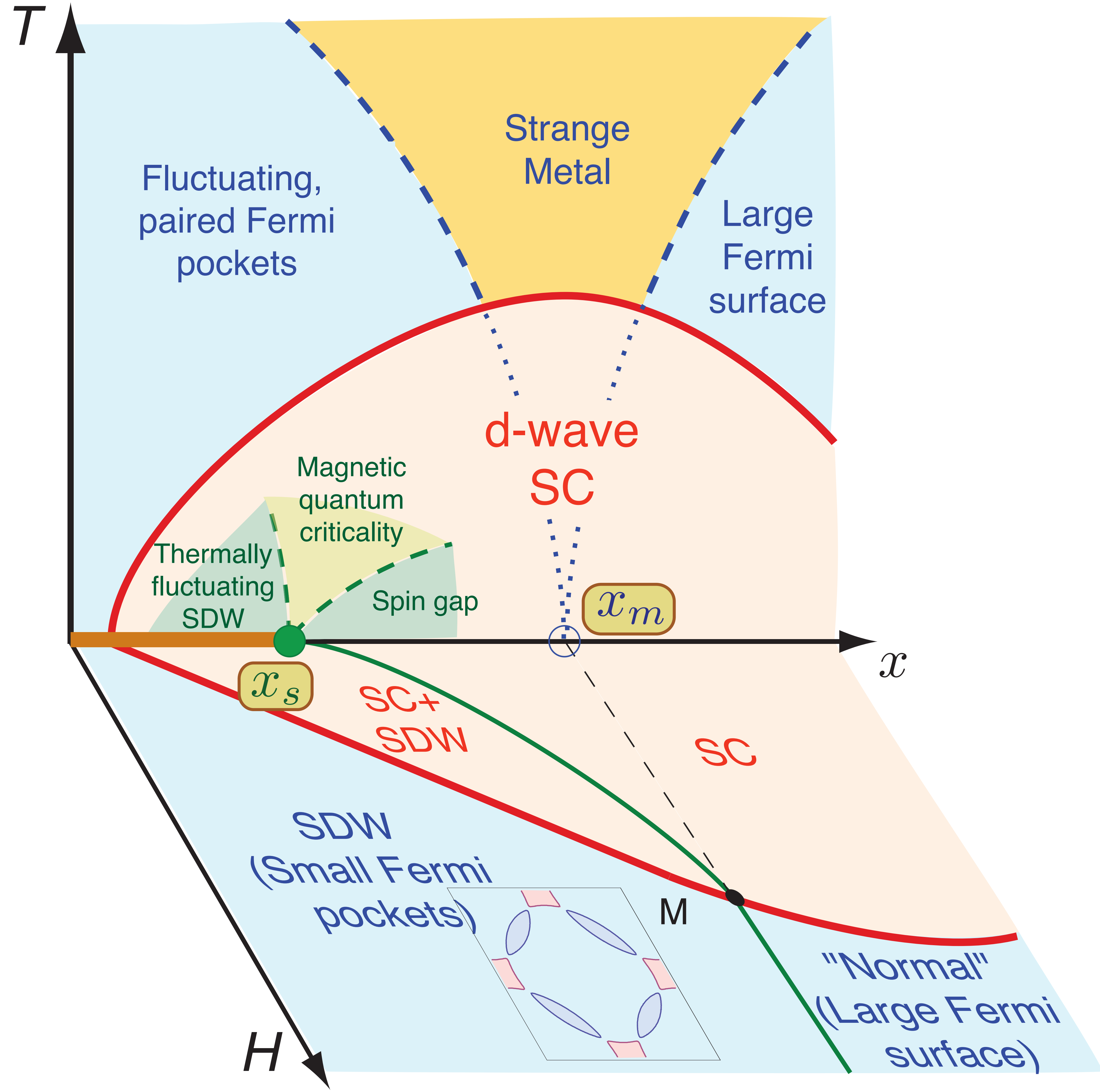
New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



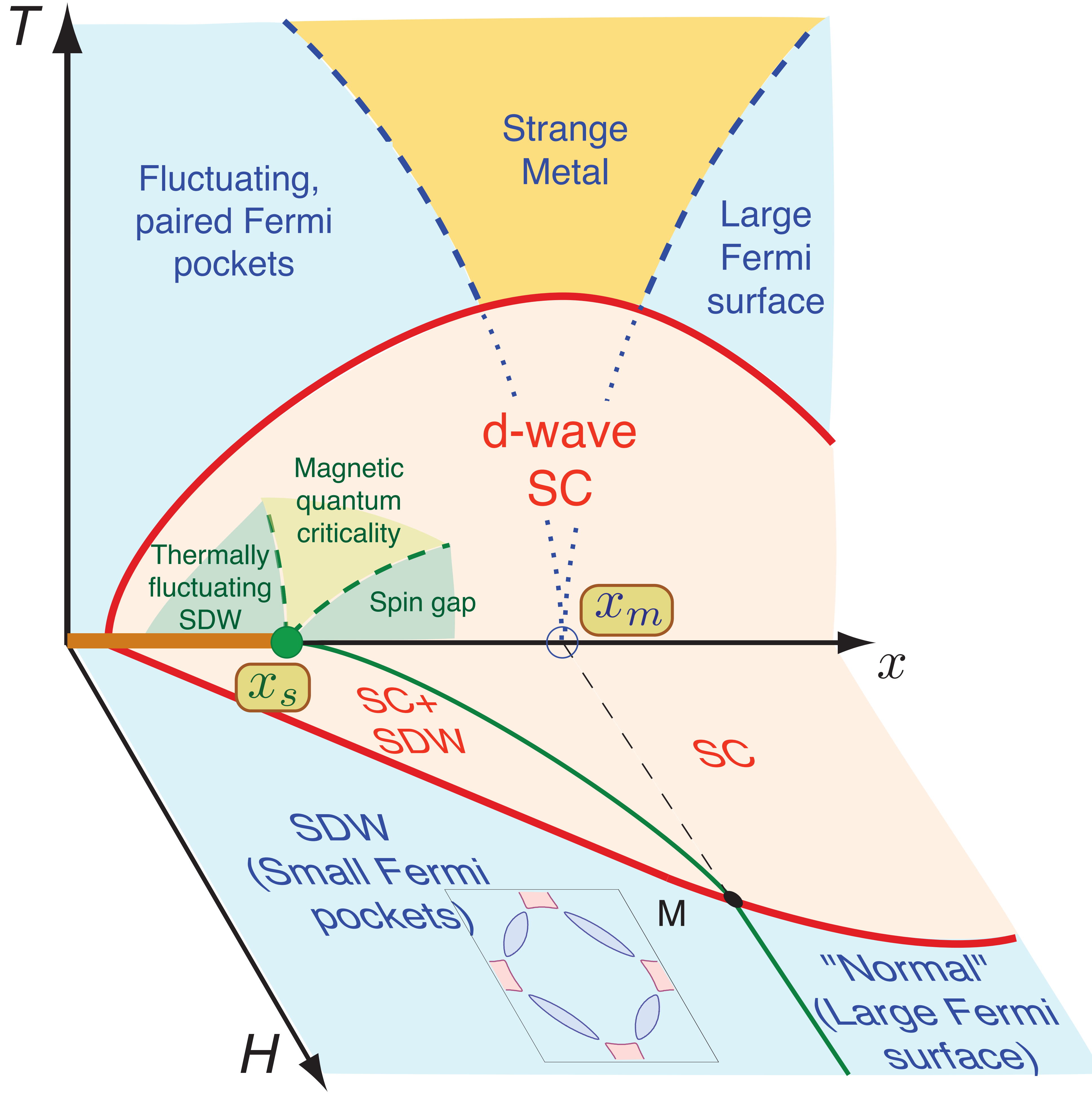
$$\text{Actual order} \sim \frac{1}{N^0}$$

A consistent analysis requires resummation of all planar graphs

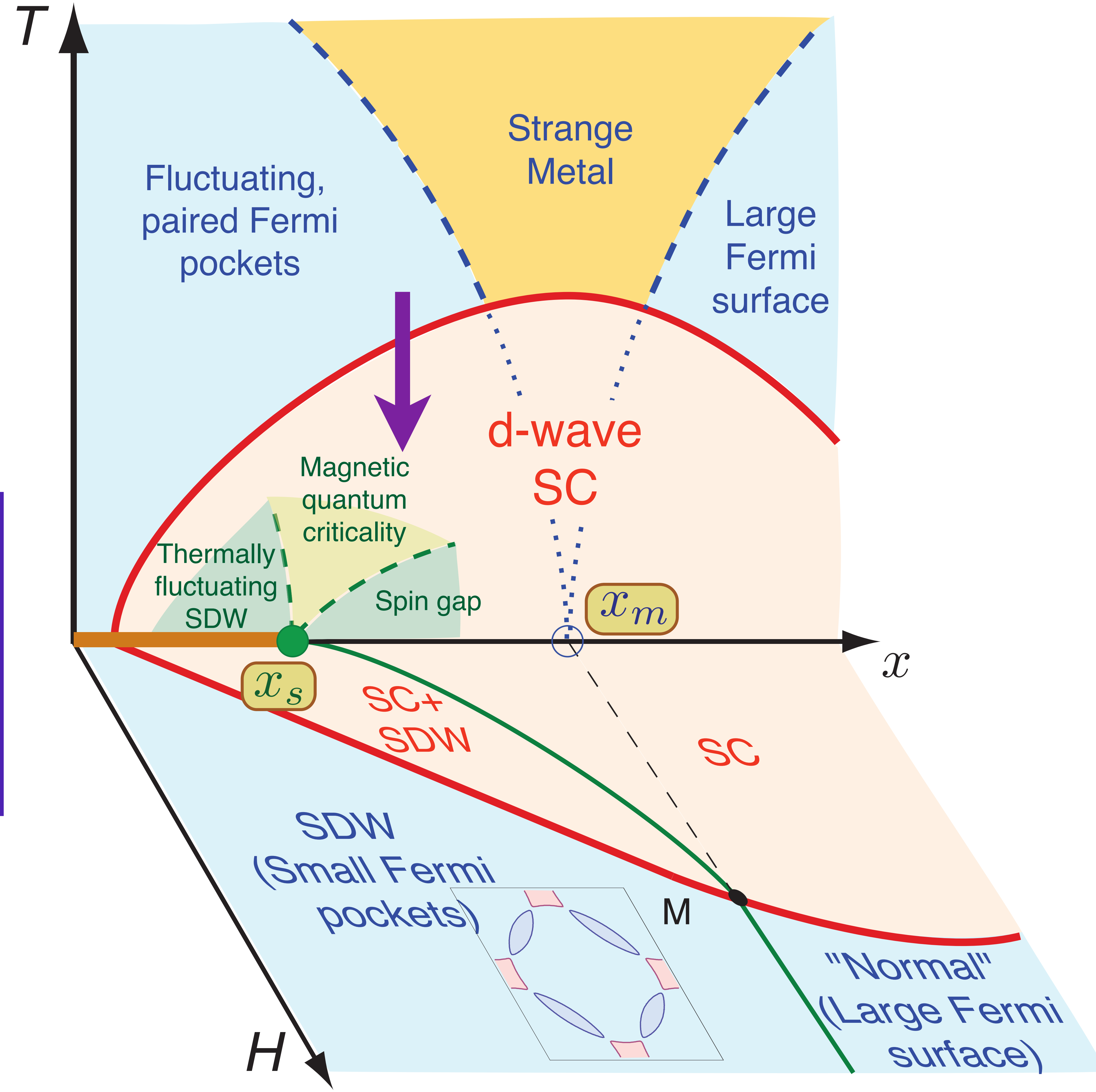
Theory for the onset of spin density wave order in metals is *strongly* coupled in two dimensions

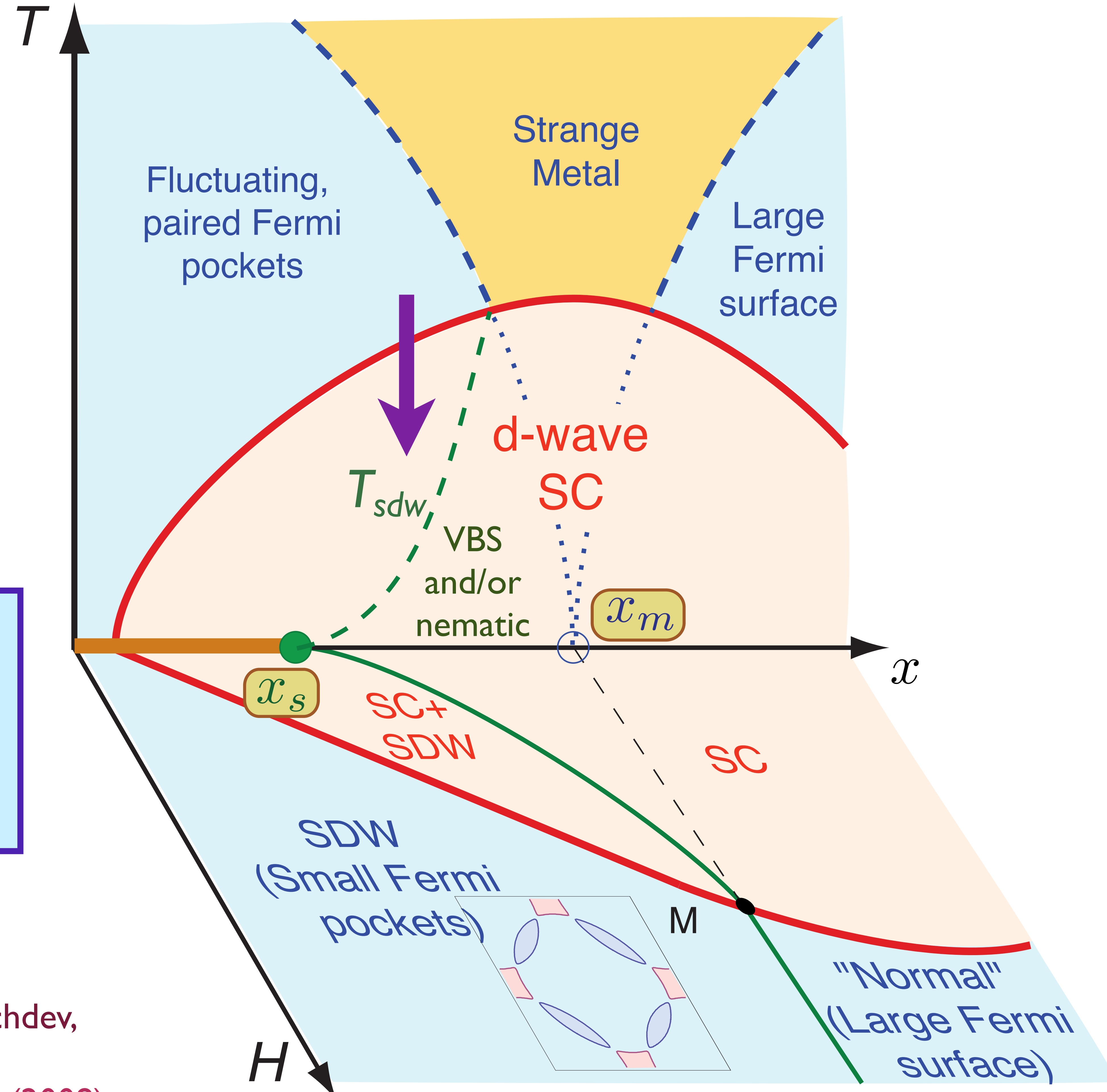

T
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x
x_s
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Naturally formulated in route B: theory of fluctuating Fermi pockets



Naturally formulated in route B: theory of fluctuating Fermi pockets





Onset of superconductivity induces confinement

R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).