Compressible quantum liquids: Field theory vs. holography

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PHYSICS

HARVARD

Liza Huijse

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Compressible quantum matter

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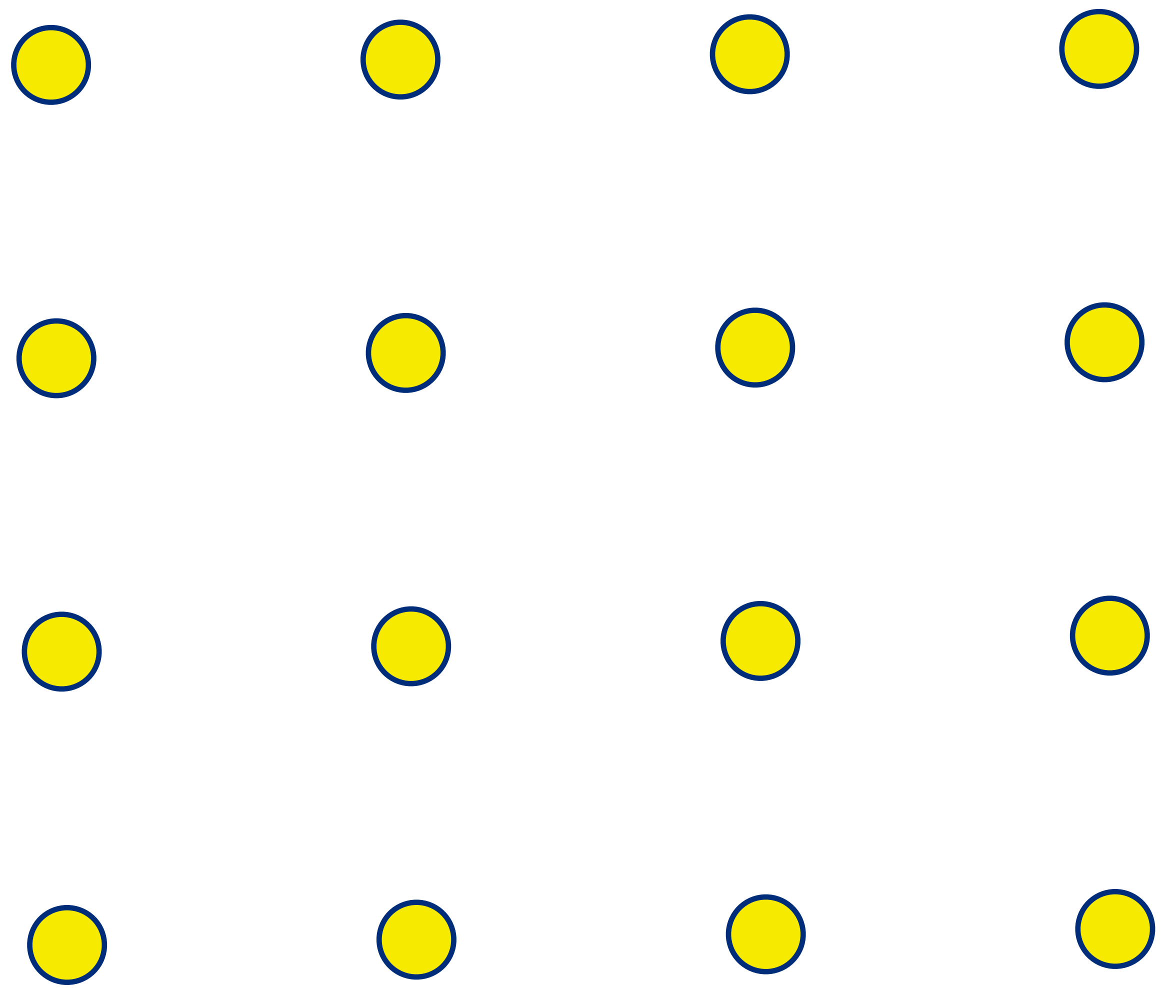
- "Relativistic" quantum critical systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

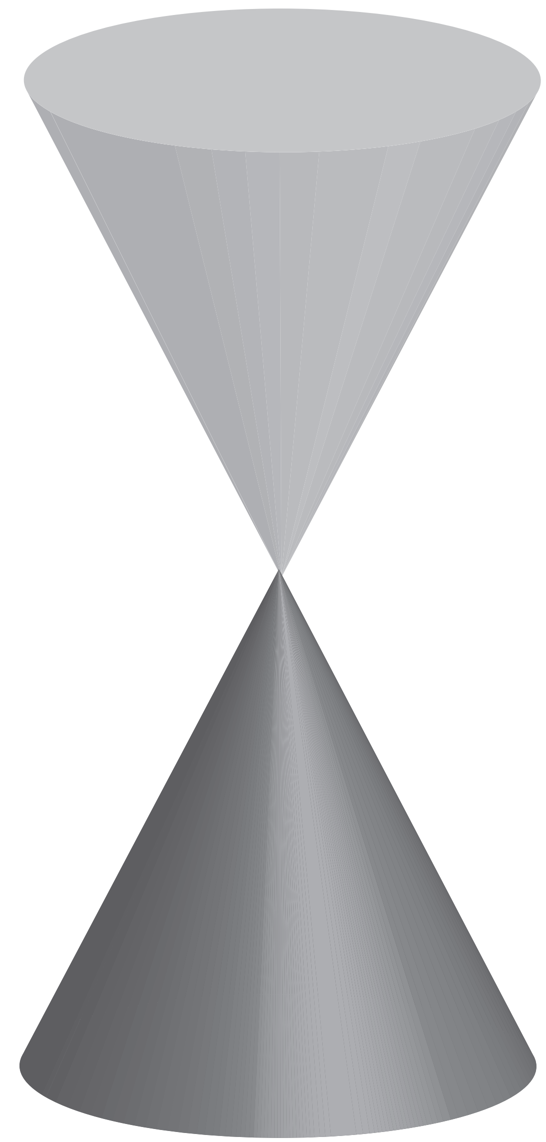
One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

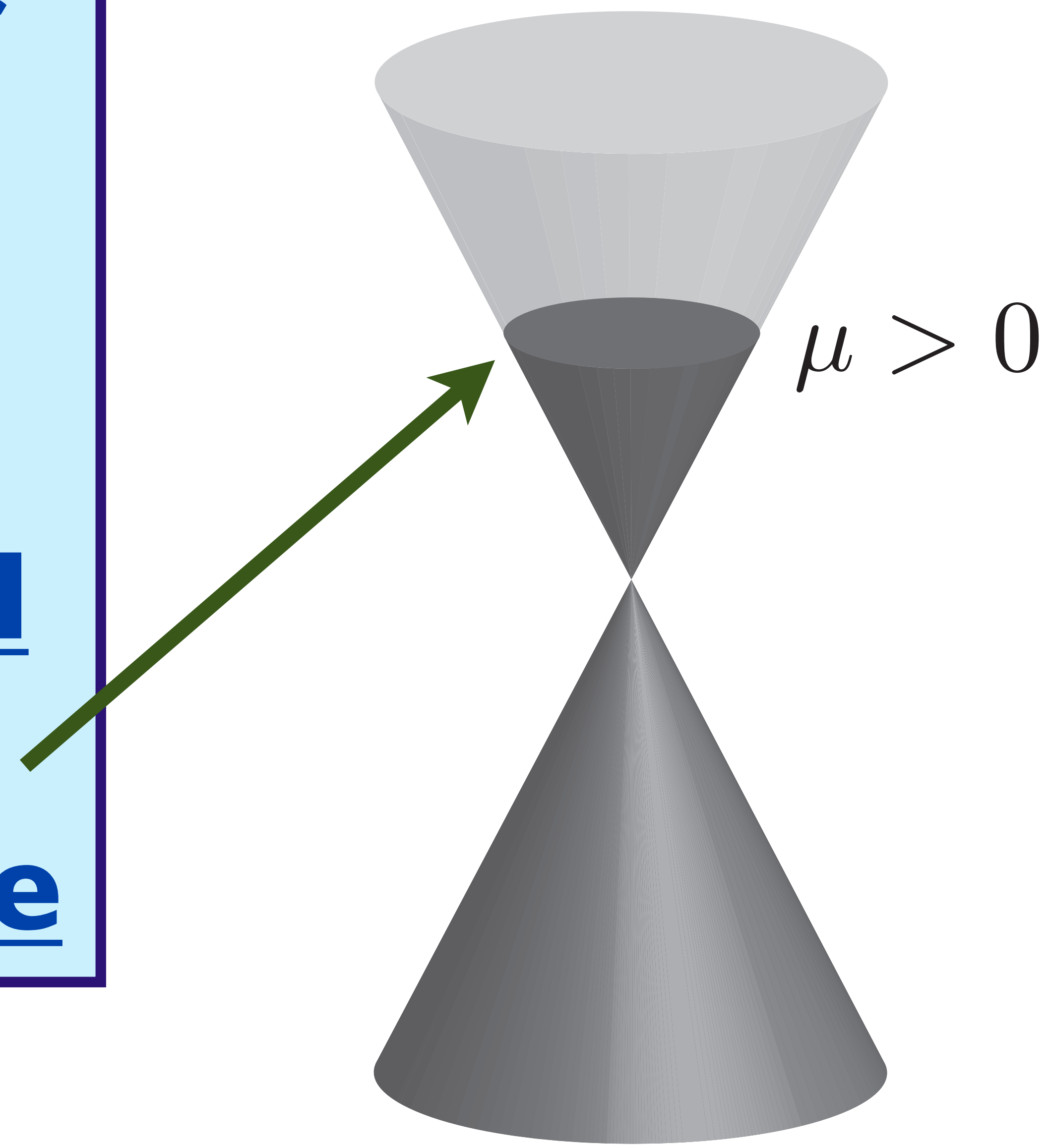
Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs



Graphene

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**



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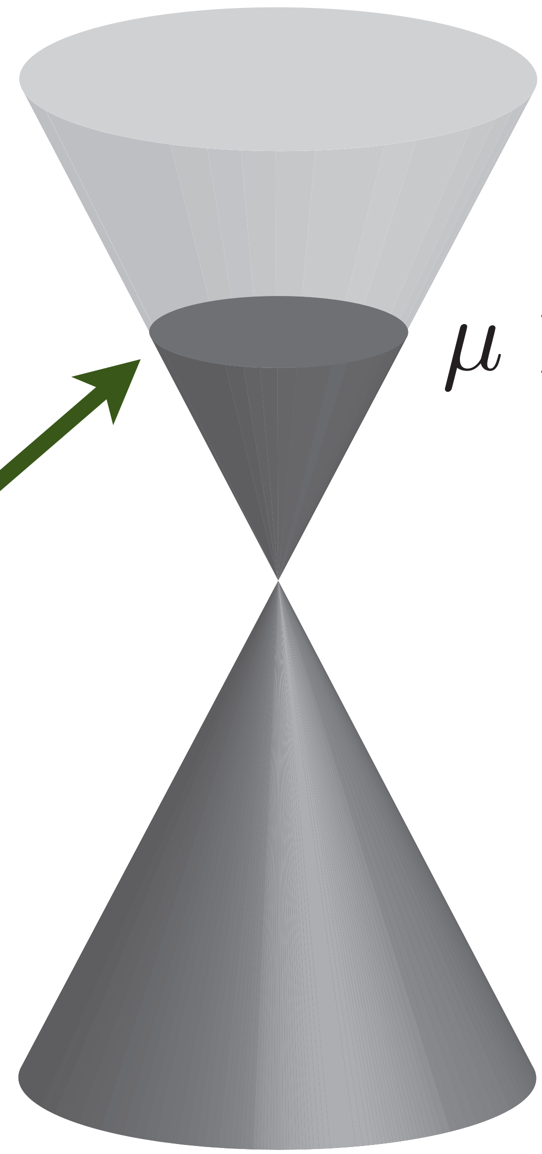

- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

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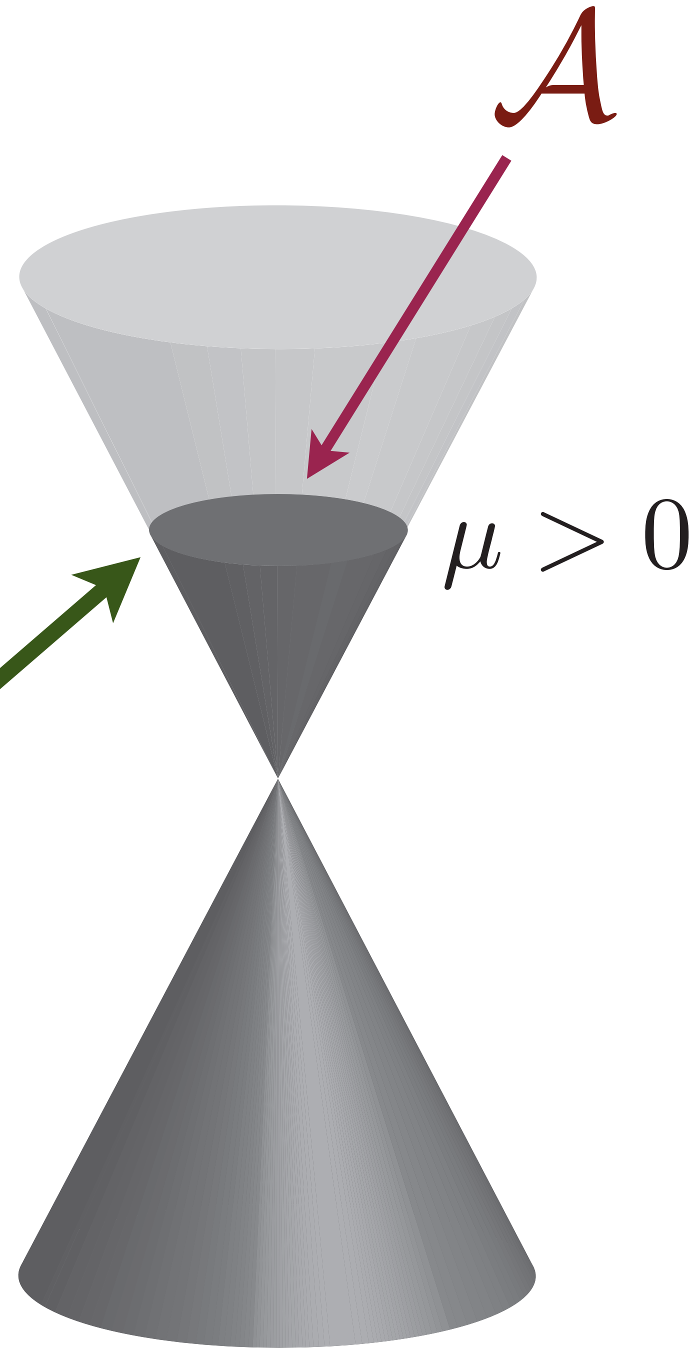

- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

Exotic phases of compressible quantum matter

I. Field theory

II. Holography

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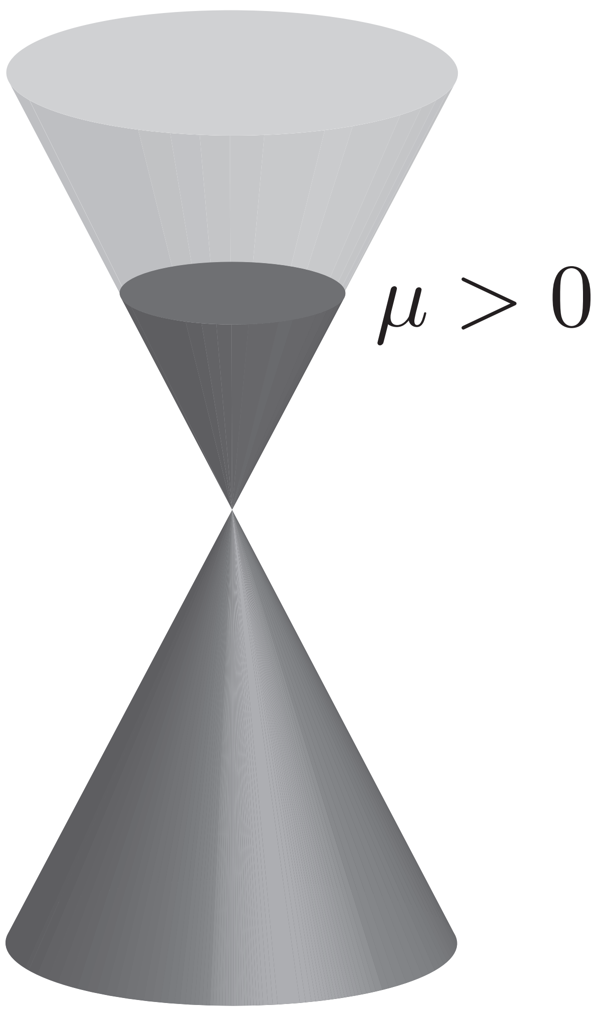
The Fermi Liquid (FL)

Most common example: electrons with short-range interactions (or screened long-range interactions), which are adiabatically connected to the non-interacting limit. The electron Green's function G_f has a pole which crosses zero energy at $k = k_F$, and the Fermi surface has the same area as the non-interacting case.



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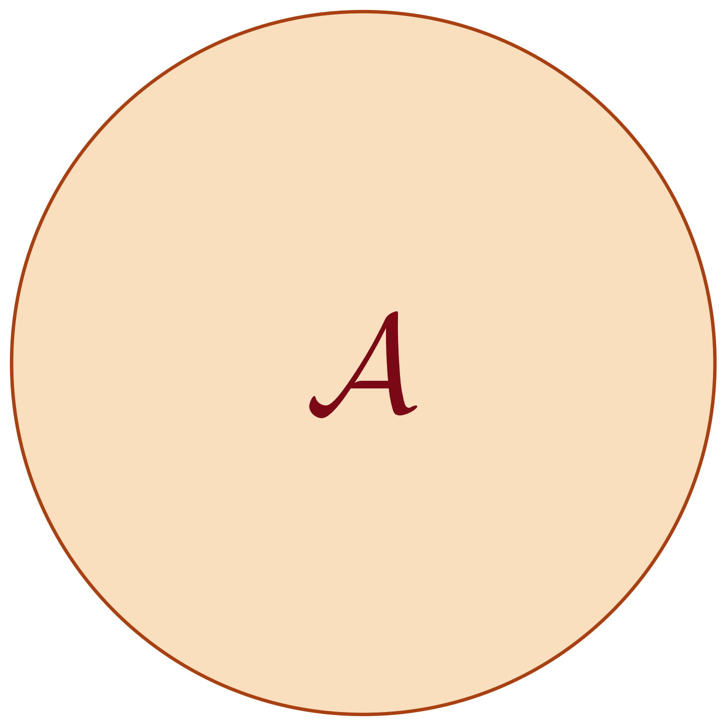
$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma + 4 \text{ Fermi terms}$$

$$\mathcal{A} = \langle f_\sigma^\dagger f_\sigma \rangle = \langle \mathcal{Q}_\sigma \rangle$$

$$G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2}$$



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- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions*.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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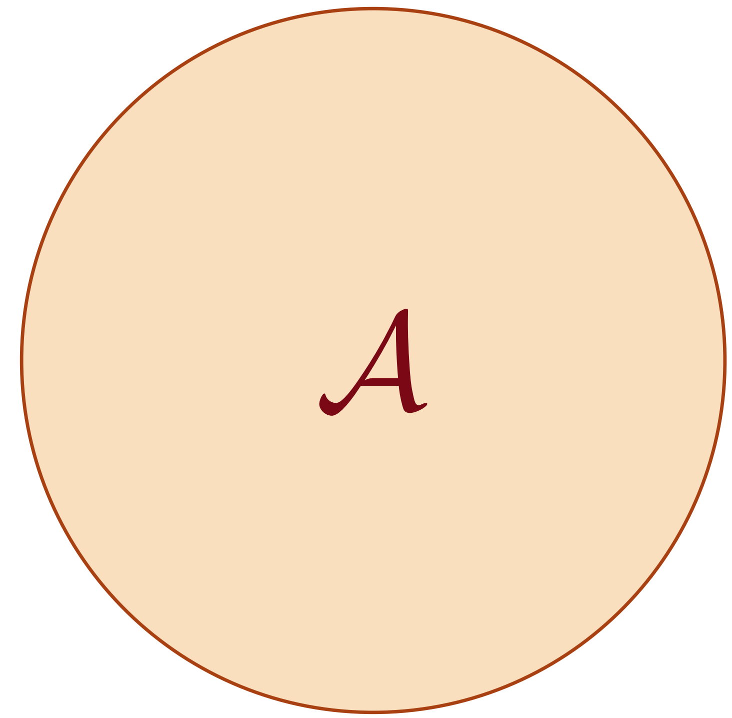

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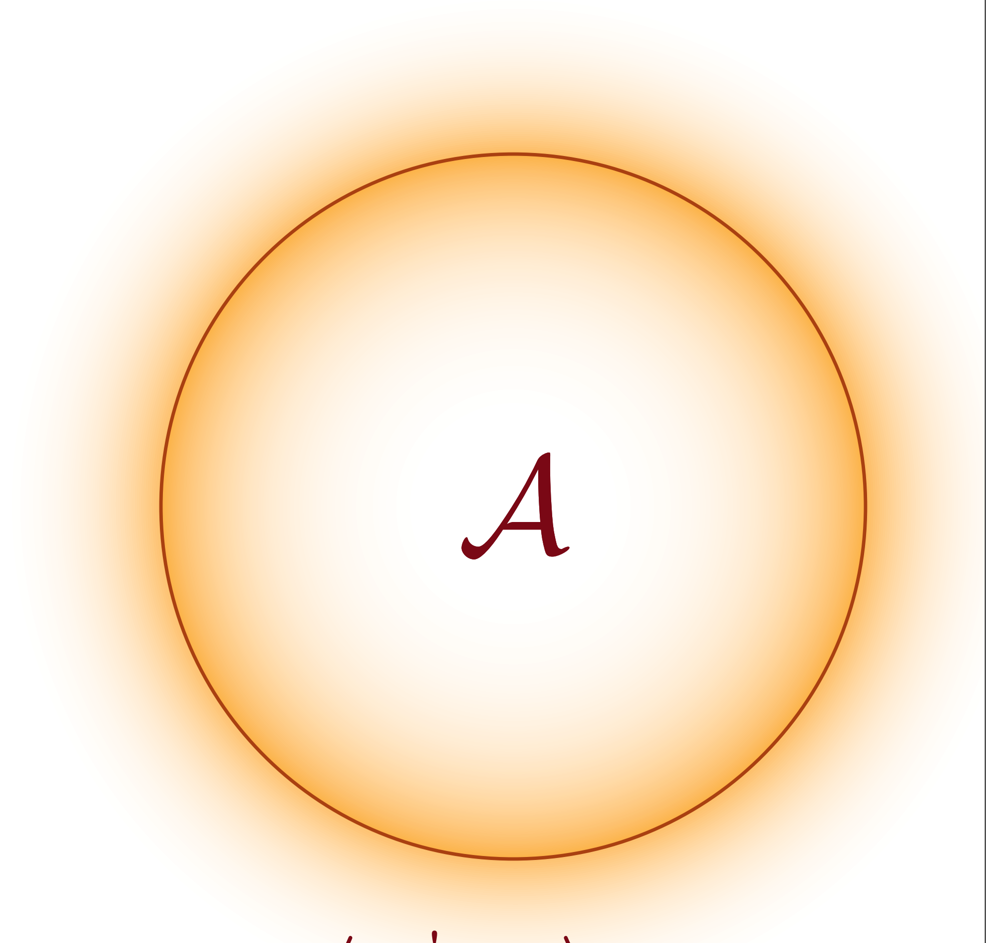
- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*



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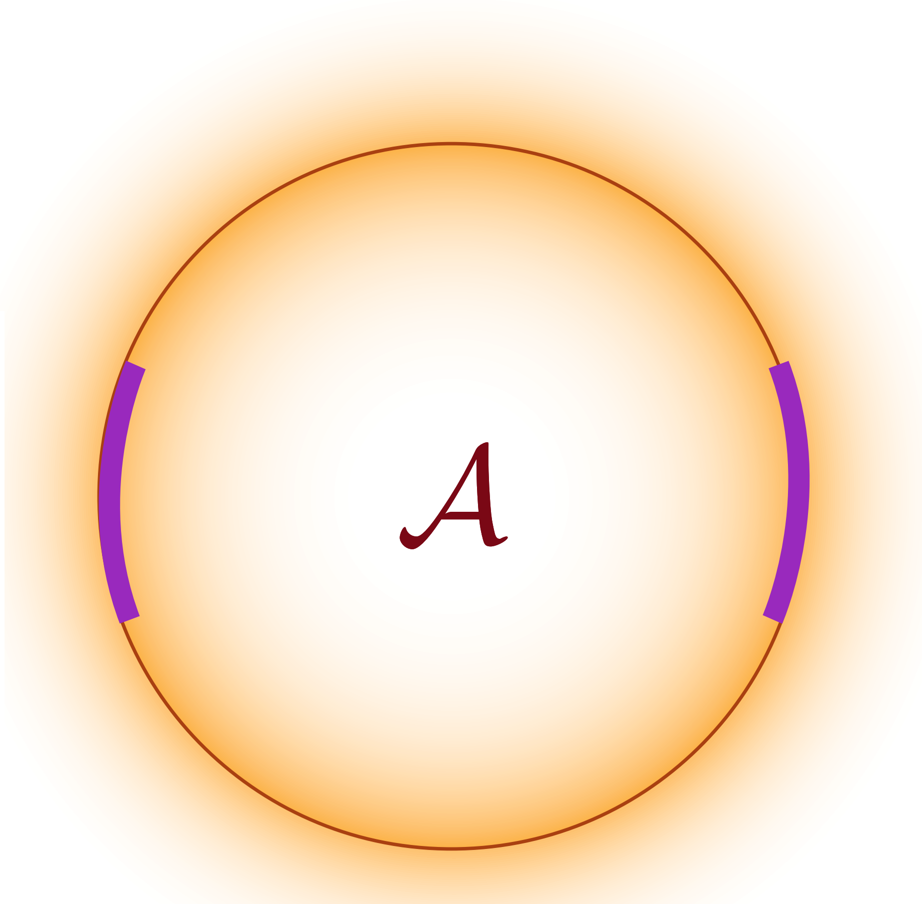

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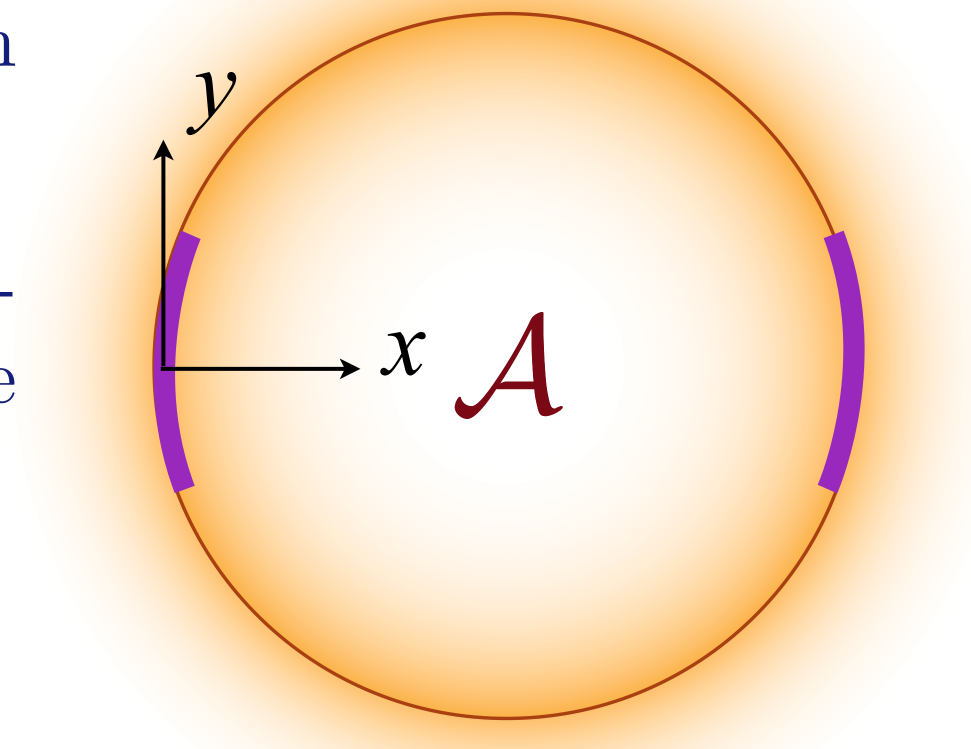
$$G_f^{-1} = q^{1-\eta} F(\omega/q^z)$$

where $q_x = k_x - k_F$, $q_y = k_y$, and $q = q_x + q_y^2/(2k_F)$, and η and z are anomalous exponents. To three-loop order, we find $\eta \neq 0$ and $z = 3/2$.



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One-loop order: $G_f^{-1} \sim v_F q + i\omega^{2/3}$

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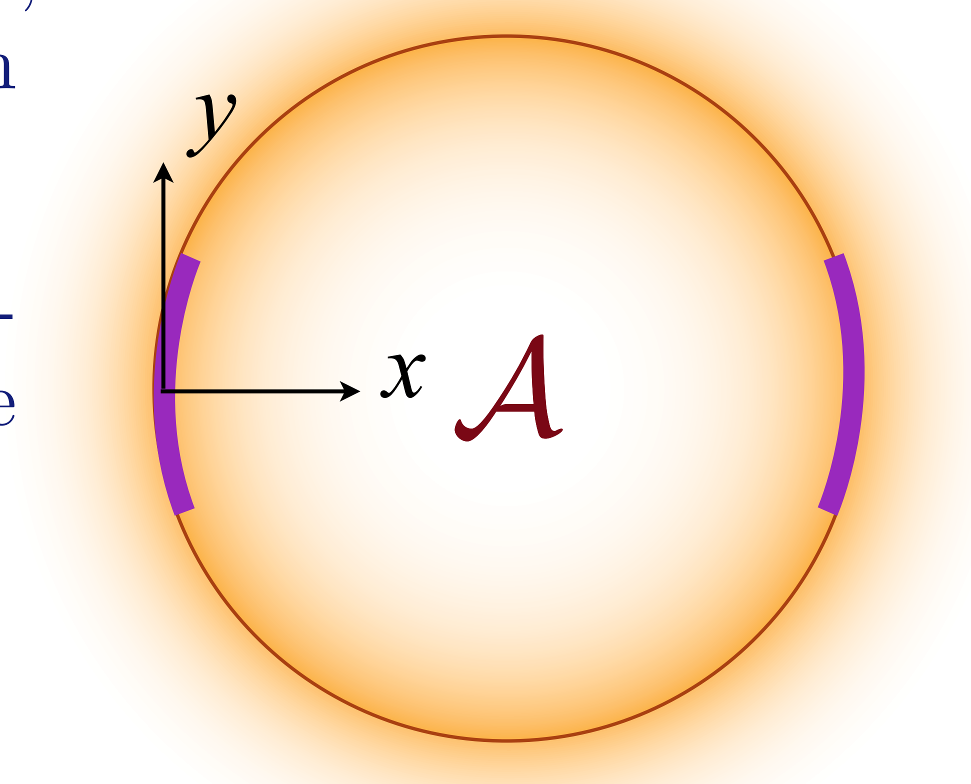

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Key question:

How do we detect the
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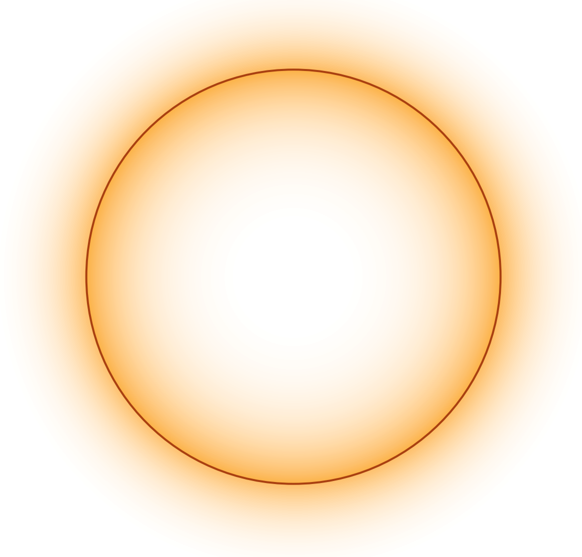
of fermions with gauge charges
in the non-Fermi liquid phases ?



These are not directly visible in the
gauge-invariant fermion correlations
computable via holography

One promising answer:

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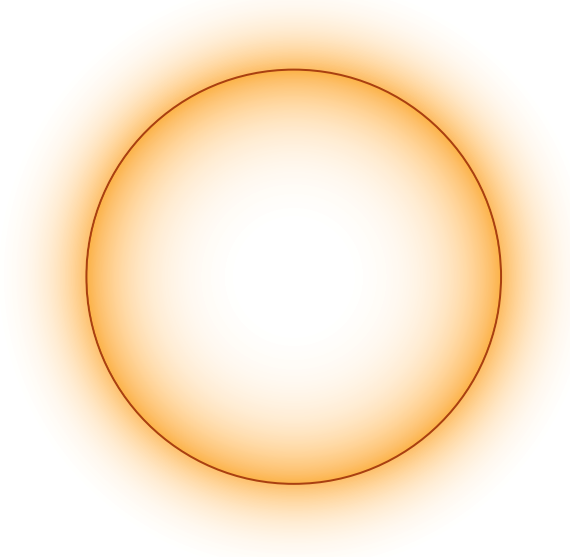
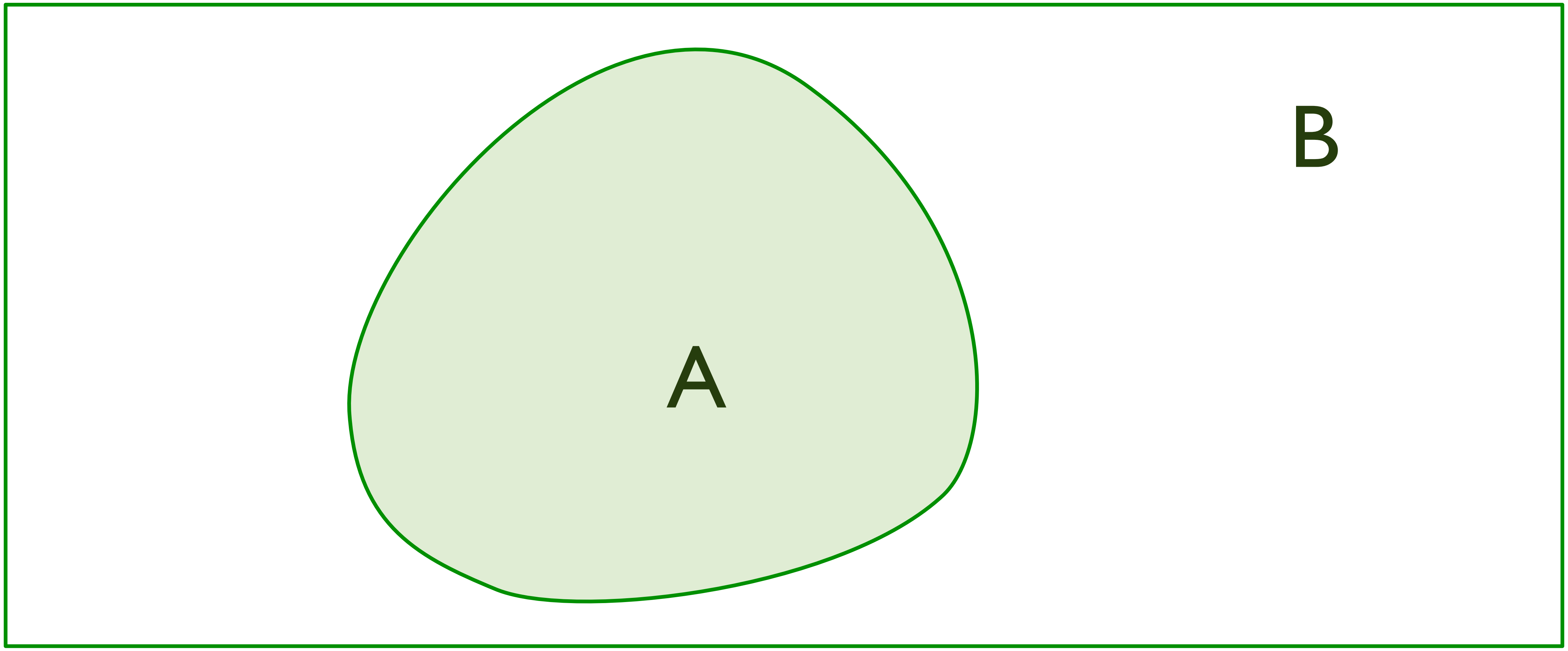


Compute
entanglement entropy

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, B. Swingle, and S. Sachdev arXiv:1112.0573

Entanglement entropy of Fermi surfaces



$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

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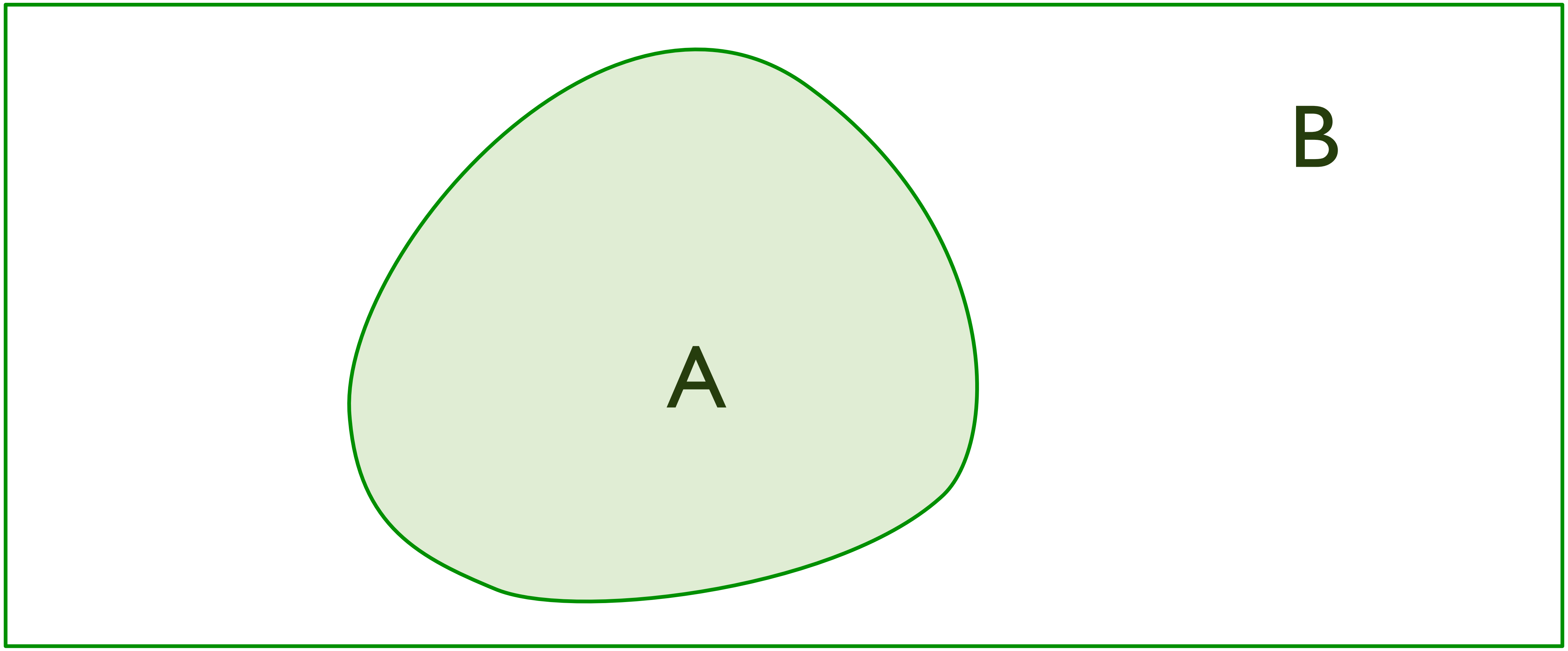

Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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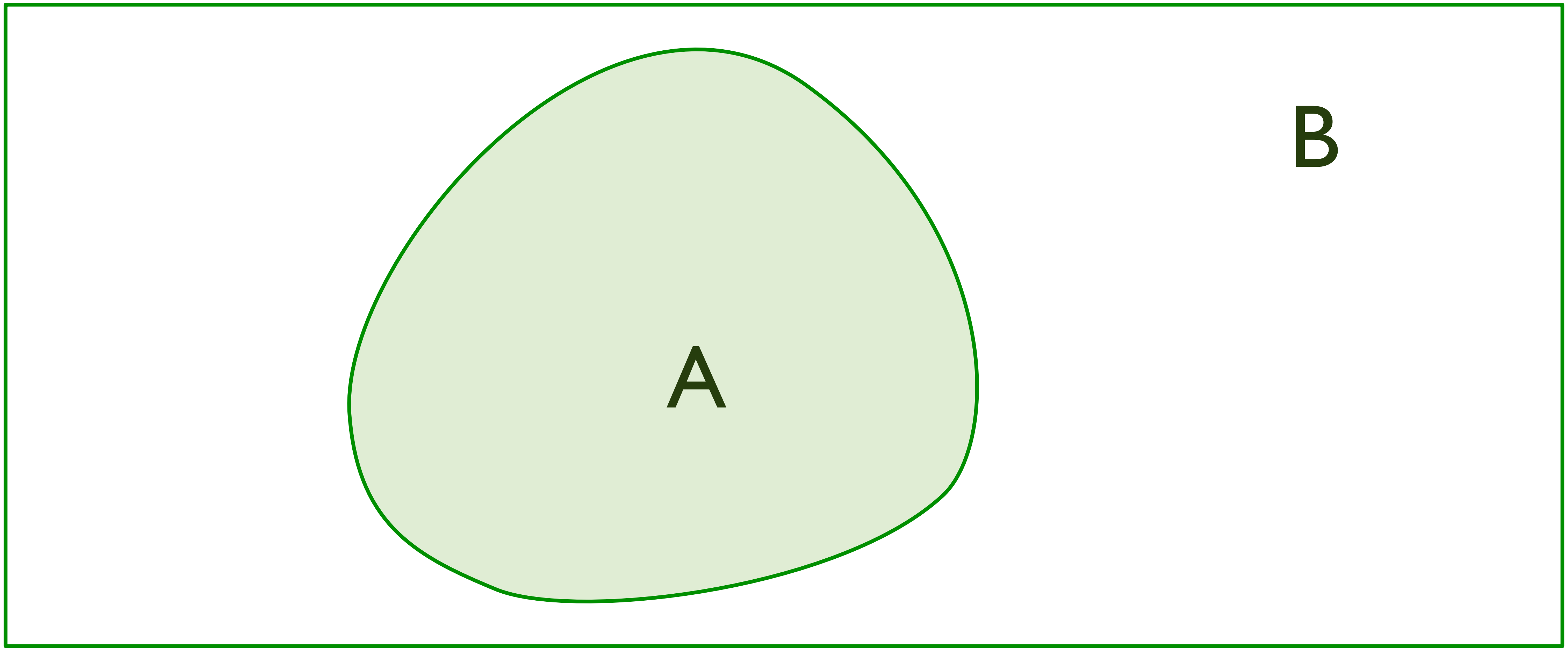

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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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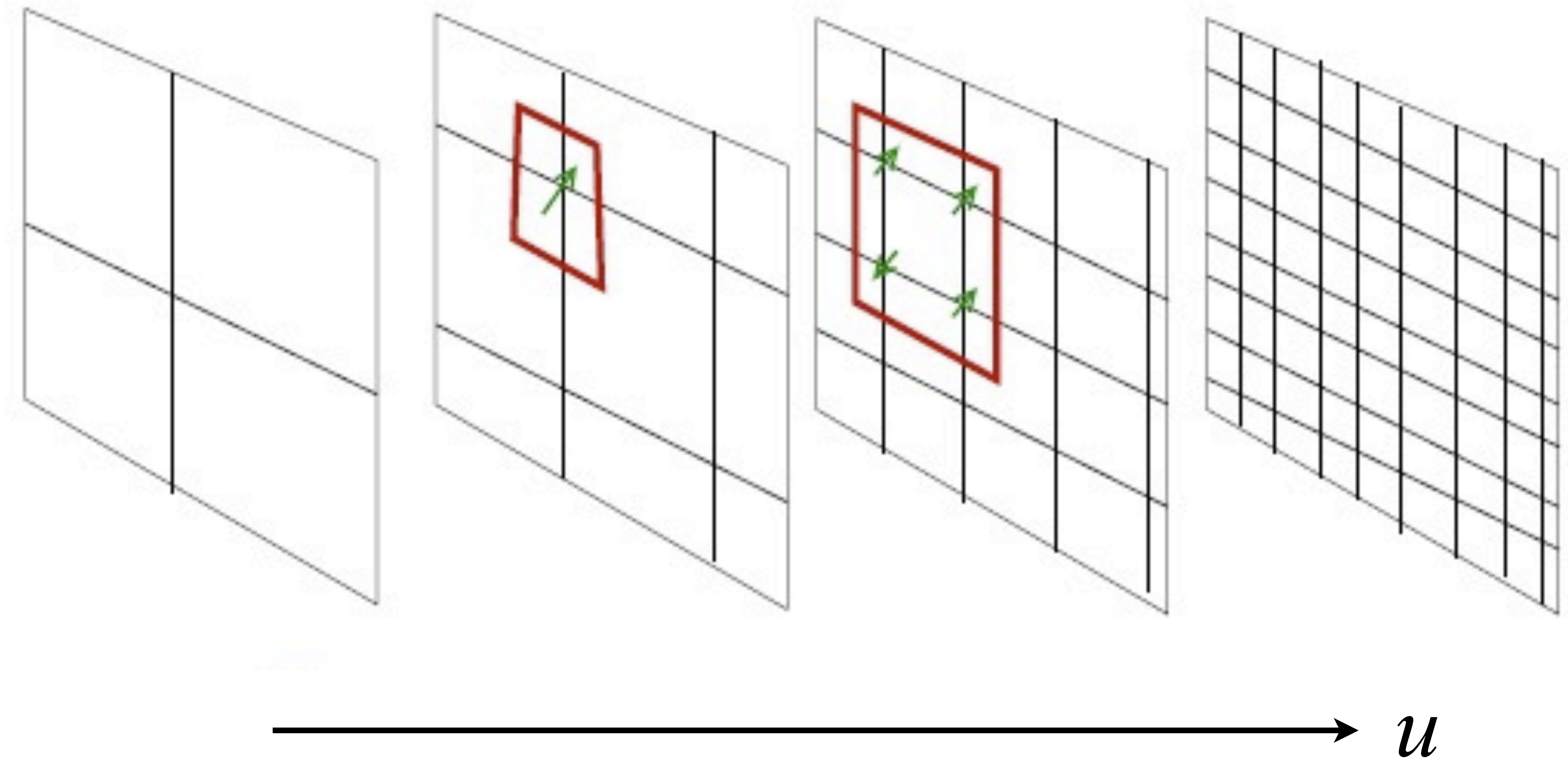
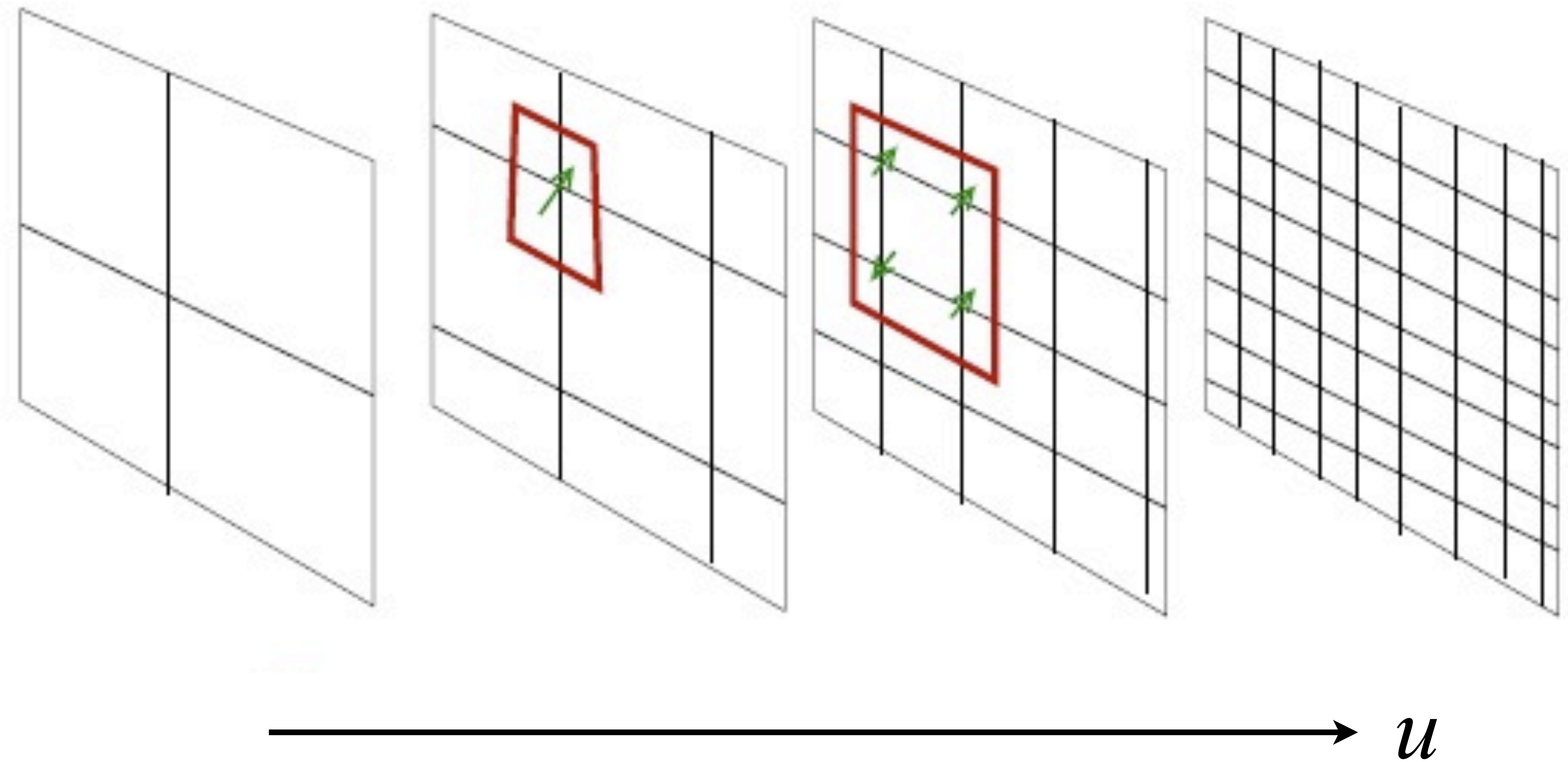
Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.


u



Key idea: $\Rightarrow$ Implement u as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

At the RG fixed point, $\beta(g) = 0$, the $(d+1)$-dimensional "relativistic" field theory is invariant under the scale transformation $(i = 1 \ldots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad u \to u/\zeta$$

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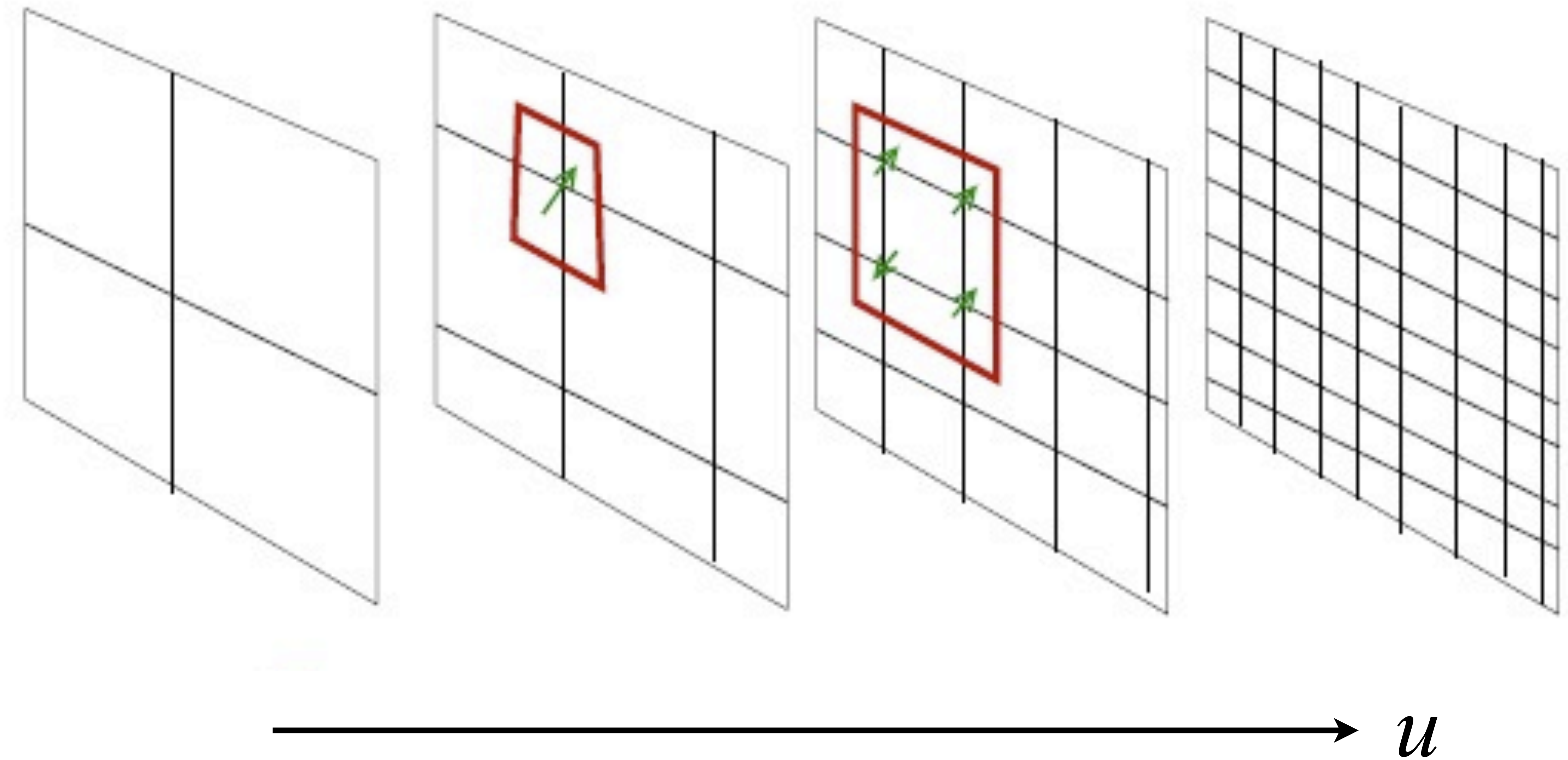
This is assumed to be an invariance of the *metric* of the theory in $d+2$ dimensions. The unique solution is

$$ds^2 = \left(\frac{u}{L}\right)^2 \left(-dt^2 + dx_i^2\right) + L^2 \frac{du^2}{u^2}.$$

Or, using the length scale $r = L^2/u$

$$ds^2 = L^2 \frac{\left(-dt^2 + dx_i^2 + dr^2\right)}{r^2}.$$

This is the space AdS_{d+2}, and L is the AdS radius.



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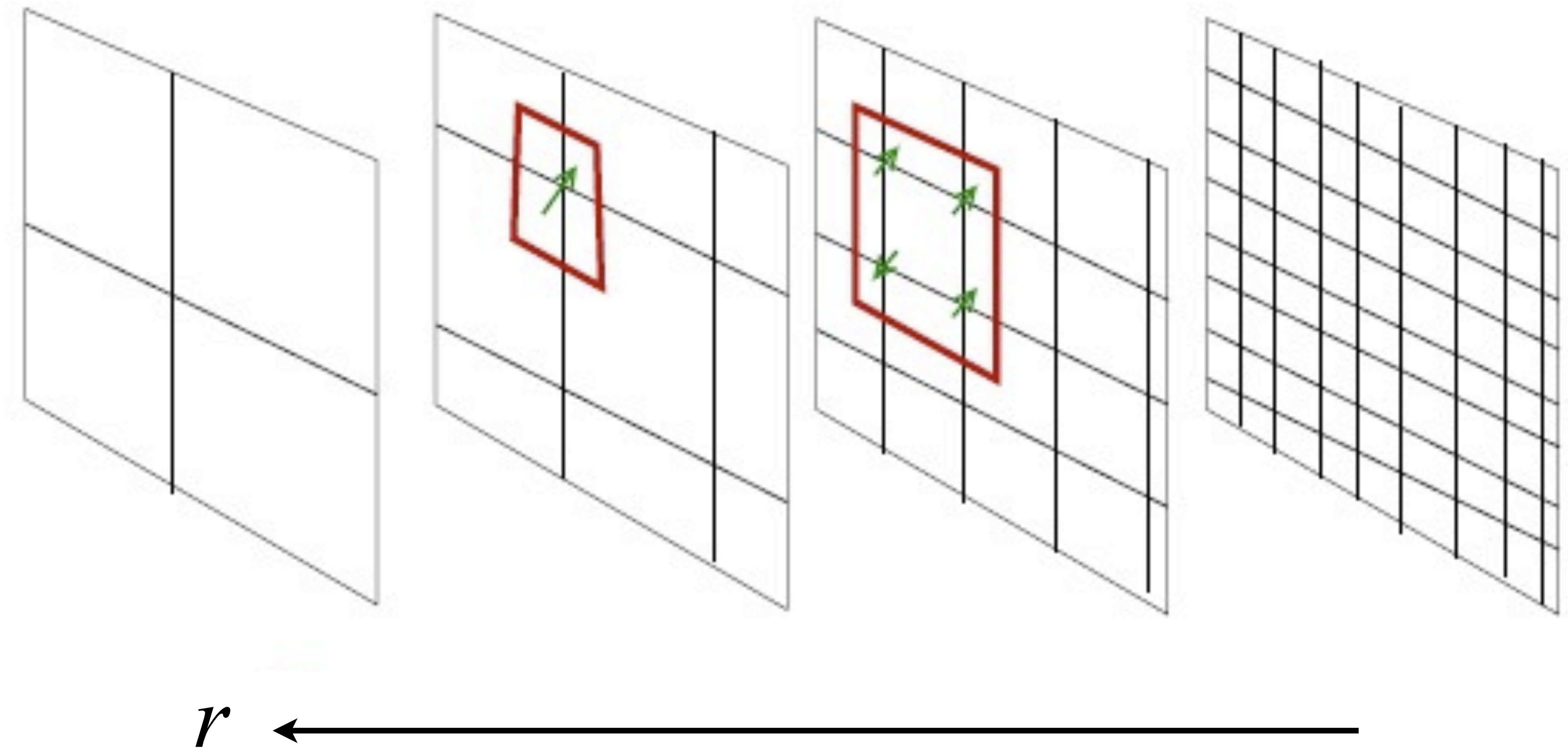
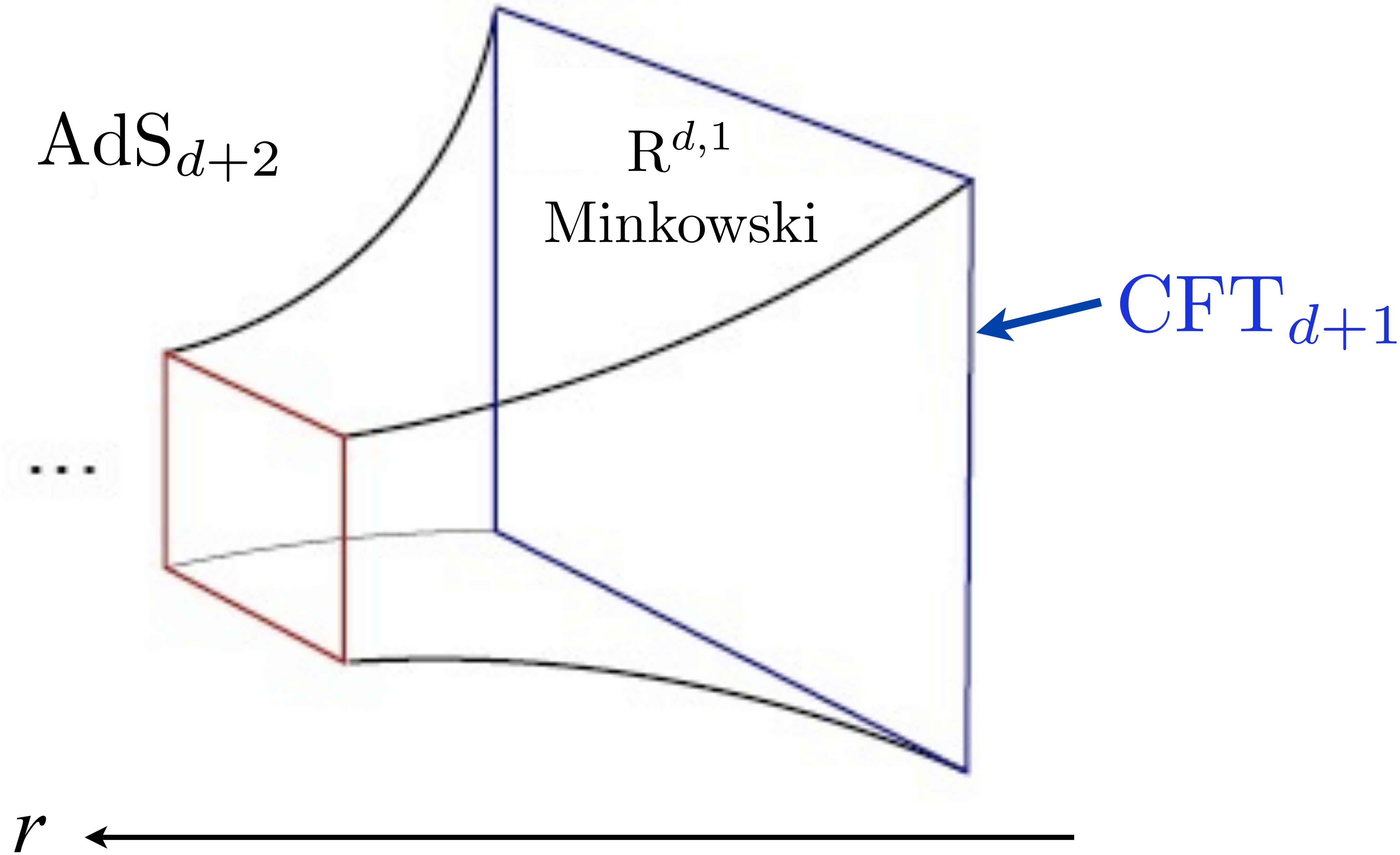


J. McGreevy, arXiv0909.0518



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In general, such scaling arguments lead to the most general metric

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

in d spatial dimensions, $i = 1 \ldots d$. Reparametrization invariance in r has been used to set the prefactor dx_i^2 to equal $1/r^2$.

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$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

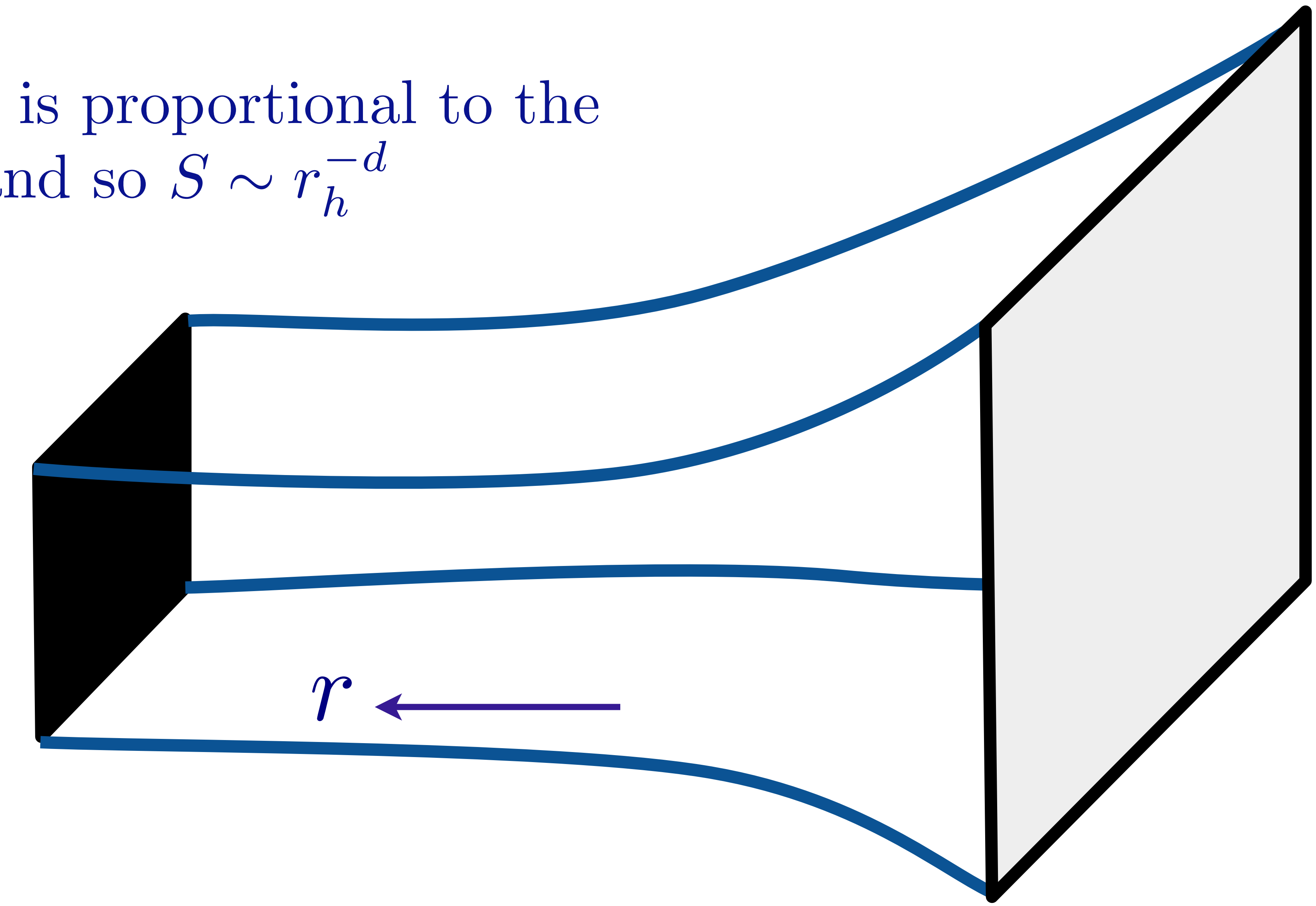
This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

What is θ ? ($\theta = 0$ for "relativistic" quantum critical points).

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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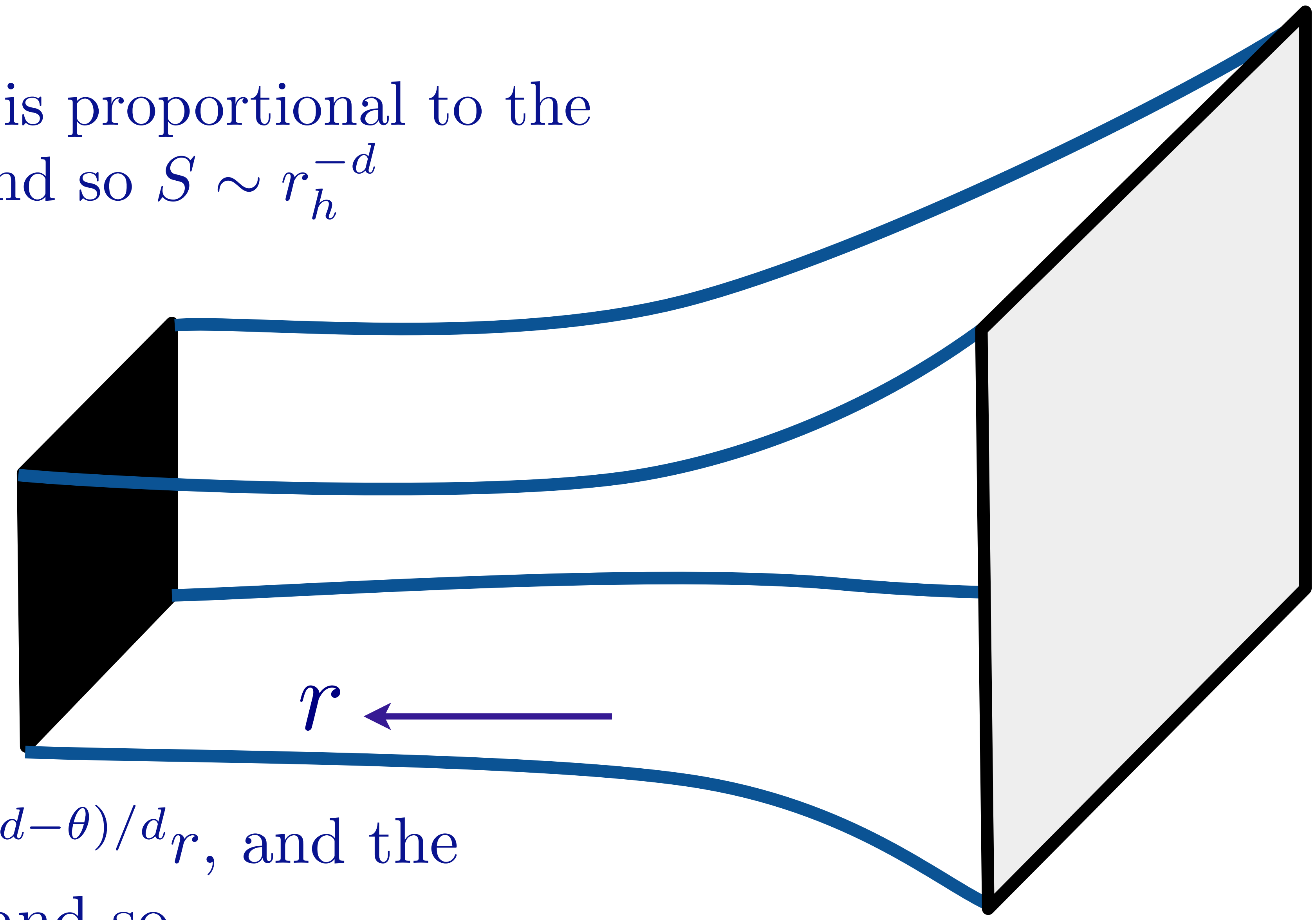


Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

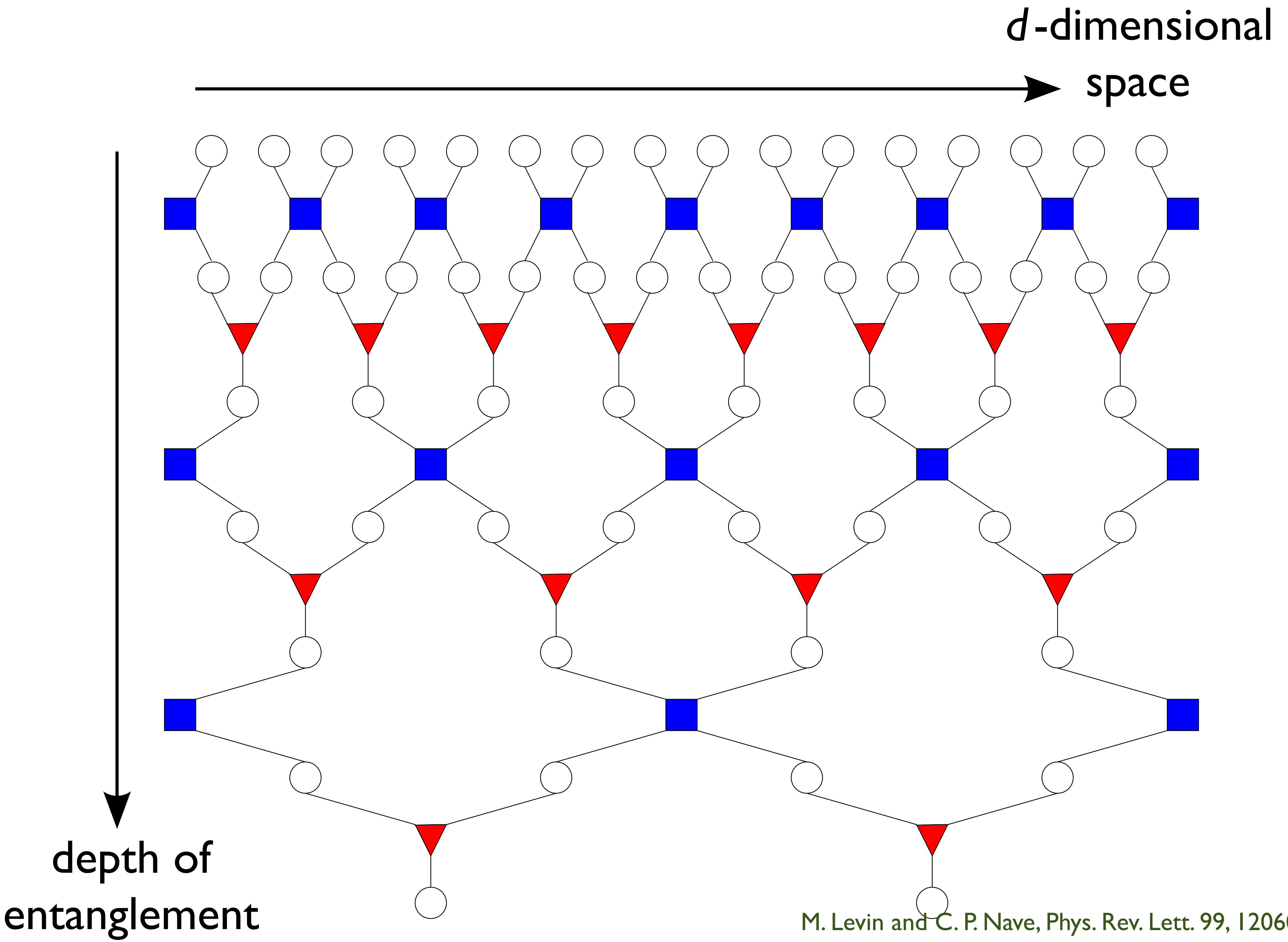
$$S \sim T^{(d-\theta)/z}$$

So θ is the "violation of hyperscaling" exponent.

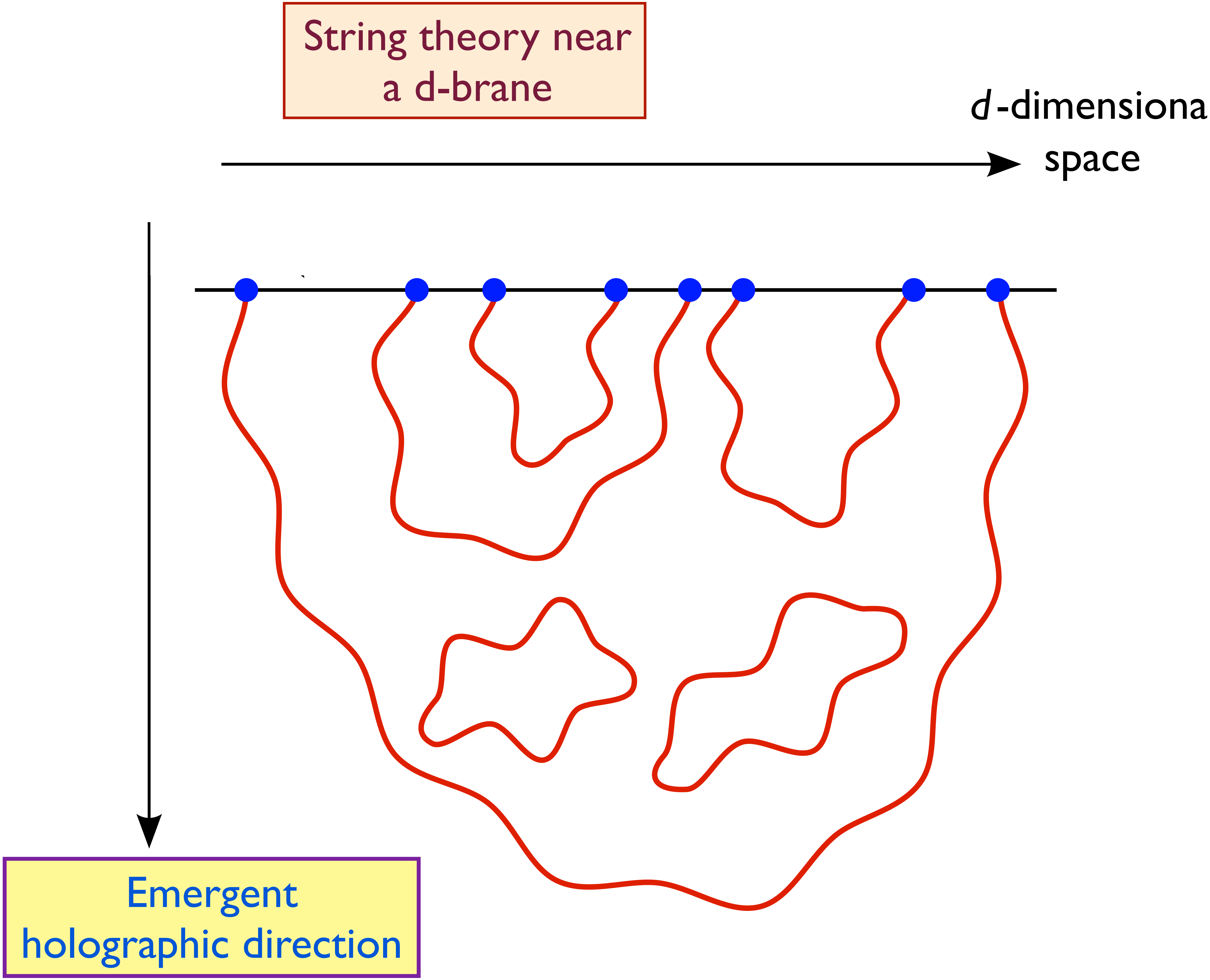
A non-Fermi liquid has gapless fermionic excitations on the Fermi surface, which disperse in the single transverse direction with dynamic critical exponent z. So we expect compressible quantum states to have an effective dimension $d - \theta$ with

$$\theta = d - 1$$

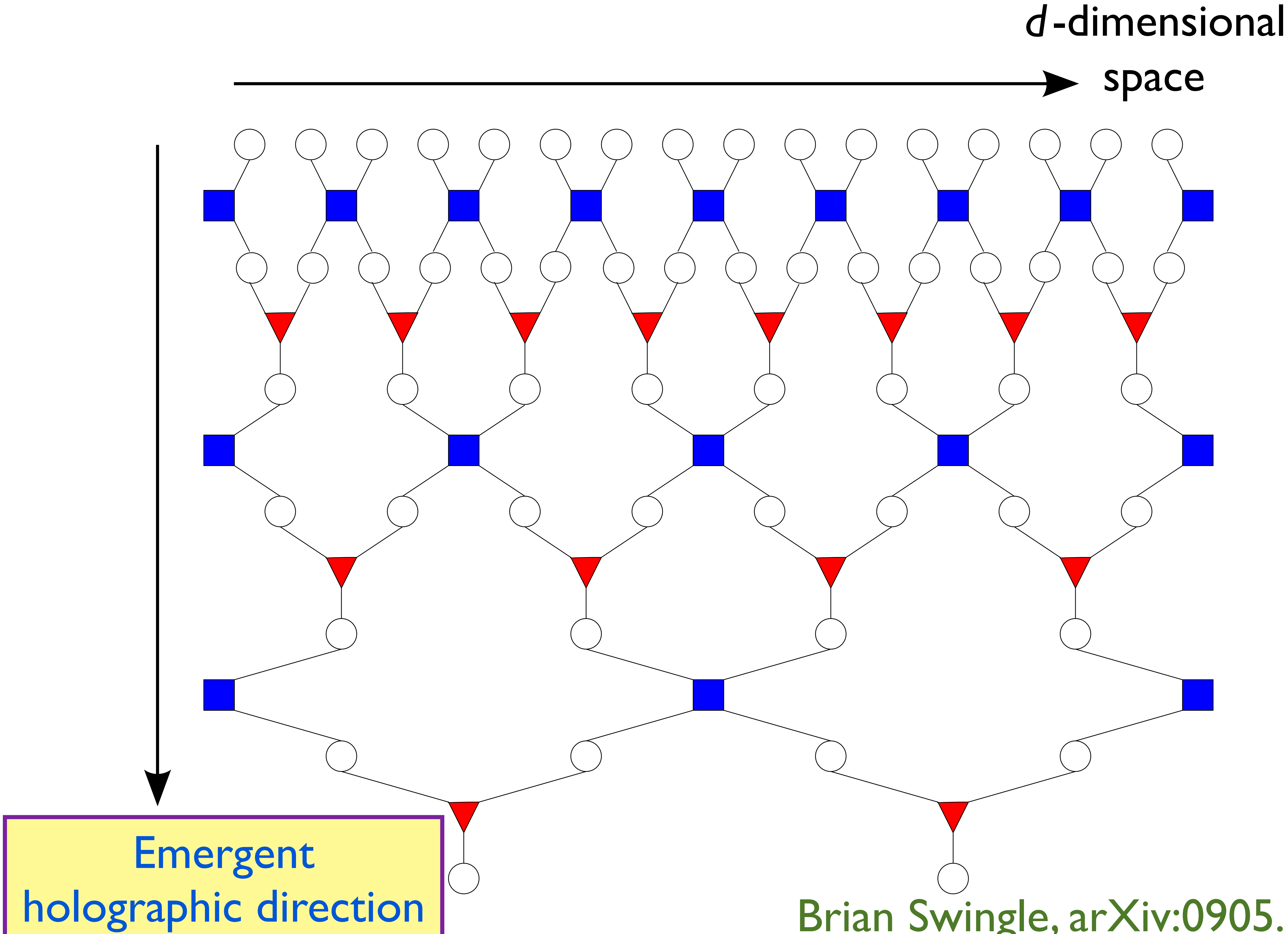
Tensor network representation of entanglement



M. Levin and C. P. Nave, Phys. Rev. Lett. 99, 120601 (2007)
F. Verstraete, M. M. Wolf, D. Perez-Garcia, and J. I. Cirac, Phys. Rev. Lett. 96, 220601 (2006)

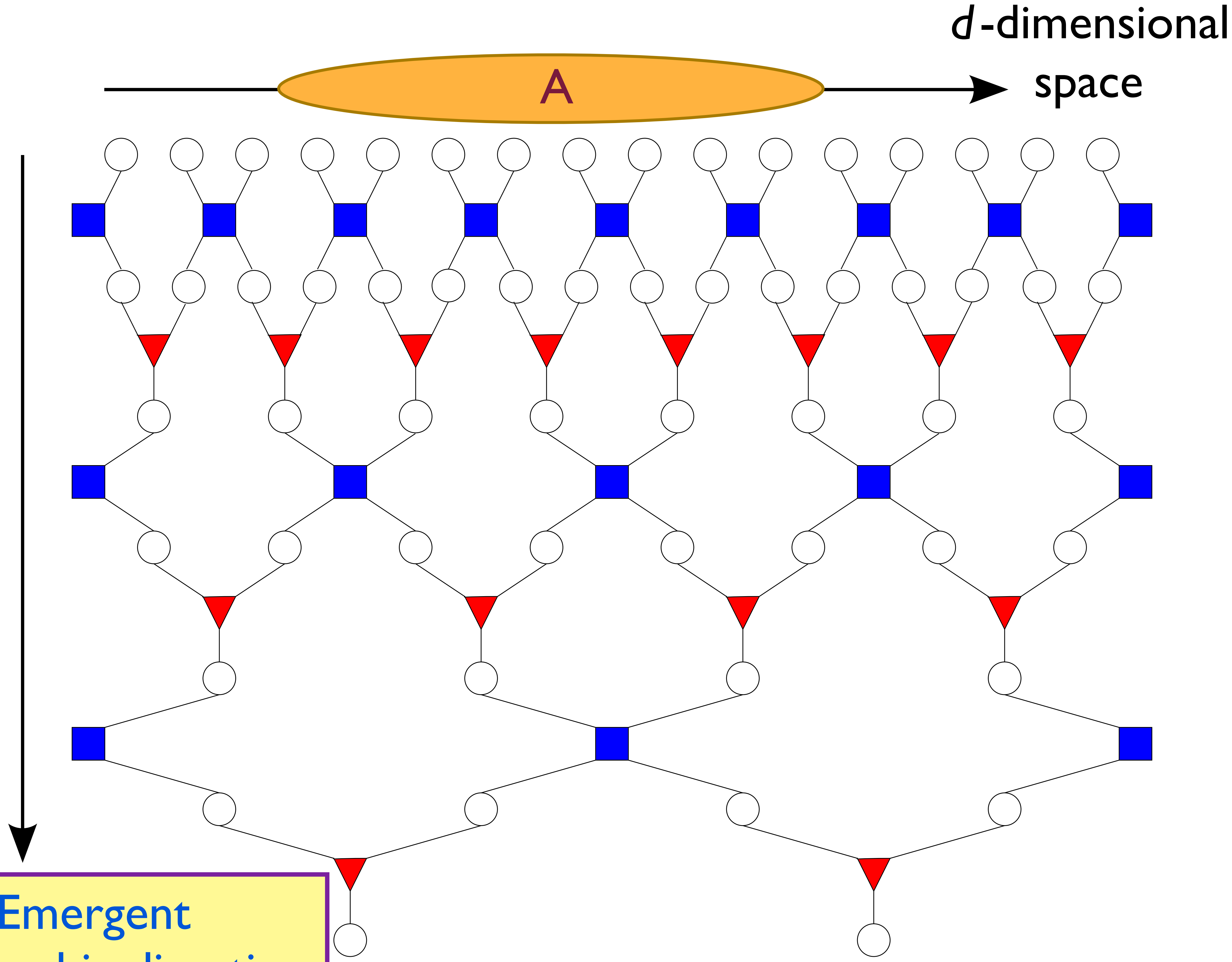

String theory near a d-brane
d-dimensional space
Emergent holographic direction

Tensor network representation of entanglement



Brian Swingle, arXiv:0905.1317

Entanglement entropy



Emergent holographic direction

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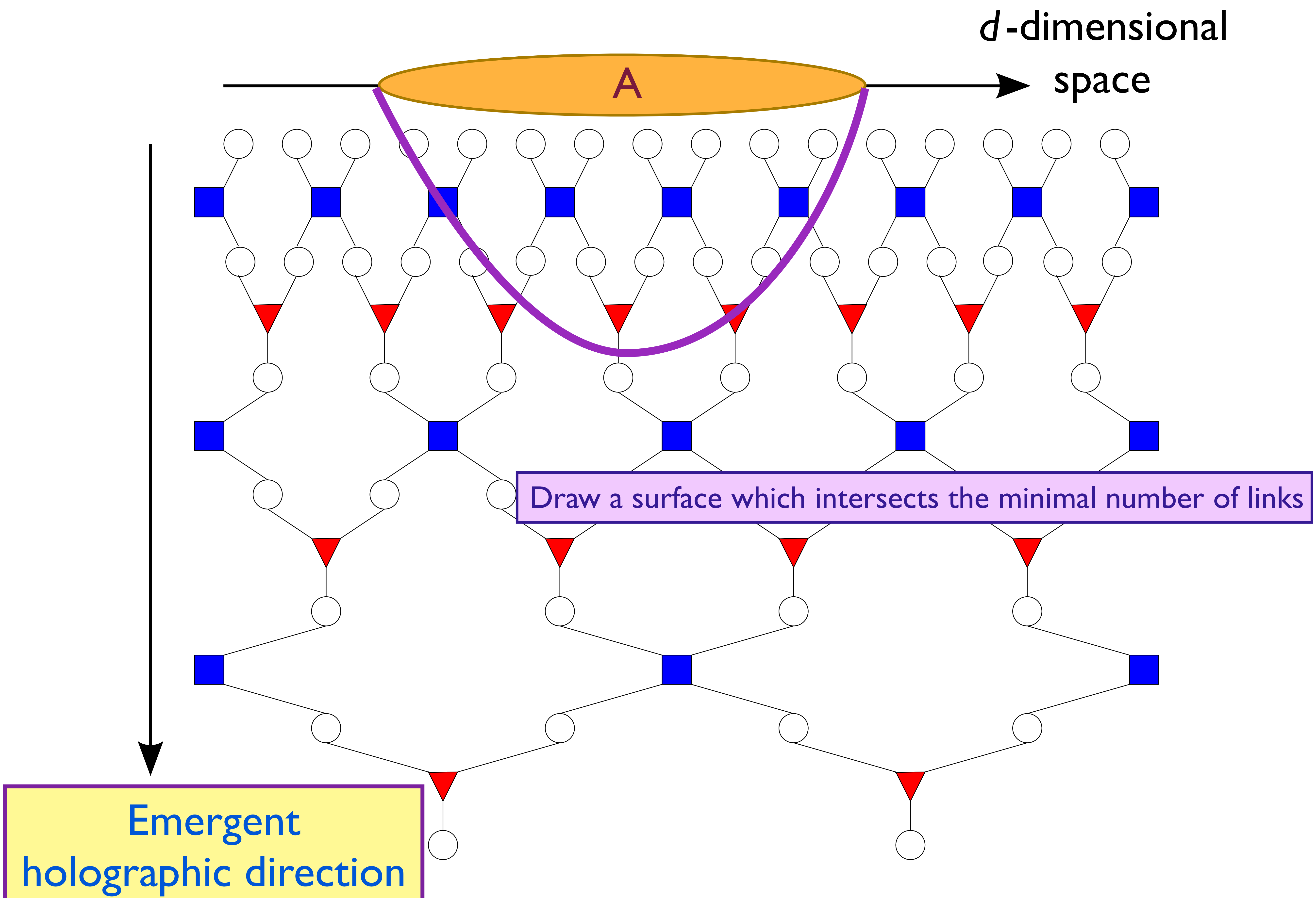

Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

This can be seen in both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317

Entanglement entropy of the metric

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The area law is obeyed for

$$\theta \leq d - 1$$

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For $\theta = d - 1$, the value expected for compressible quantum states, the entanglement entropy has log-violation of the area law

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N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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$$S_E = \eta\, \mathcal{Q}^{(d-1)/d}\Sigma\, \ln\left(\mathcal{Q}^{(d-1)/d}\Sigma\right).$$

- η is a dimensionless constant which is *independent* of $\mathcal{Q}$ and of any property of the entangling region.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- Σ is the $(d-1)$-dimensional surface area of entangling region (in $d = 2$, $\Sigma = P$ is the perimeter). Note S_E is otherwise independent of the shape of the entangling region, unlike other gapless systems. This is a characteristic property of a Fermi surface

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Entanglement entropy of the metric

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

For $\theta = d-1$, the value expected for compressible quantum states, the entanglement entropy has log-violation of the area law

$$S_E = \eta\, \mathcal{Q}^{(d-1)/d}\Sigma \ln\left(\mathcal{Q}^{(d-1)/d}\Sigma\right).$$

- $\mathcal{Q}$ is the total conserved charge. The metric has a complicated dependence on $\mathcal{Q}$, but S_E is just proportional to $\mathcal{Q}^{(d-1)/d}$. Many UV details are irrelevant, and S_E flows to the universal $\mathcal{Q}$ dependence in the IR. By Luttinger's relation $\mathcal{Q} \sim k_F^{d-1}$, and so prefactor is the area of the Fermi surface, as expected from field theory.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Inequalities

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The area law of entanglement entropy is obeyed for

$$\theta \leq d - 1.$$

The "null energy condition" of the gravity theory yields

$$z \geq 1 + \frac{\theta}{d}.$$

Remarkably, for $d = 2$, $\theta = d - 1$ and $z = 1 + \theta/d$, we obtain $z = 3/2$, the same value associated with the field theory.

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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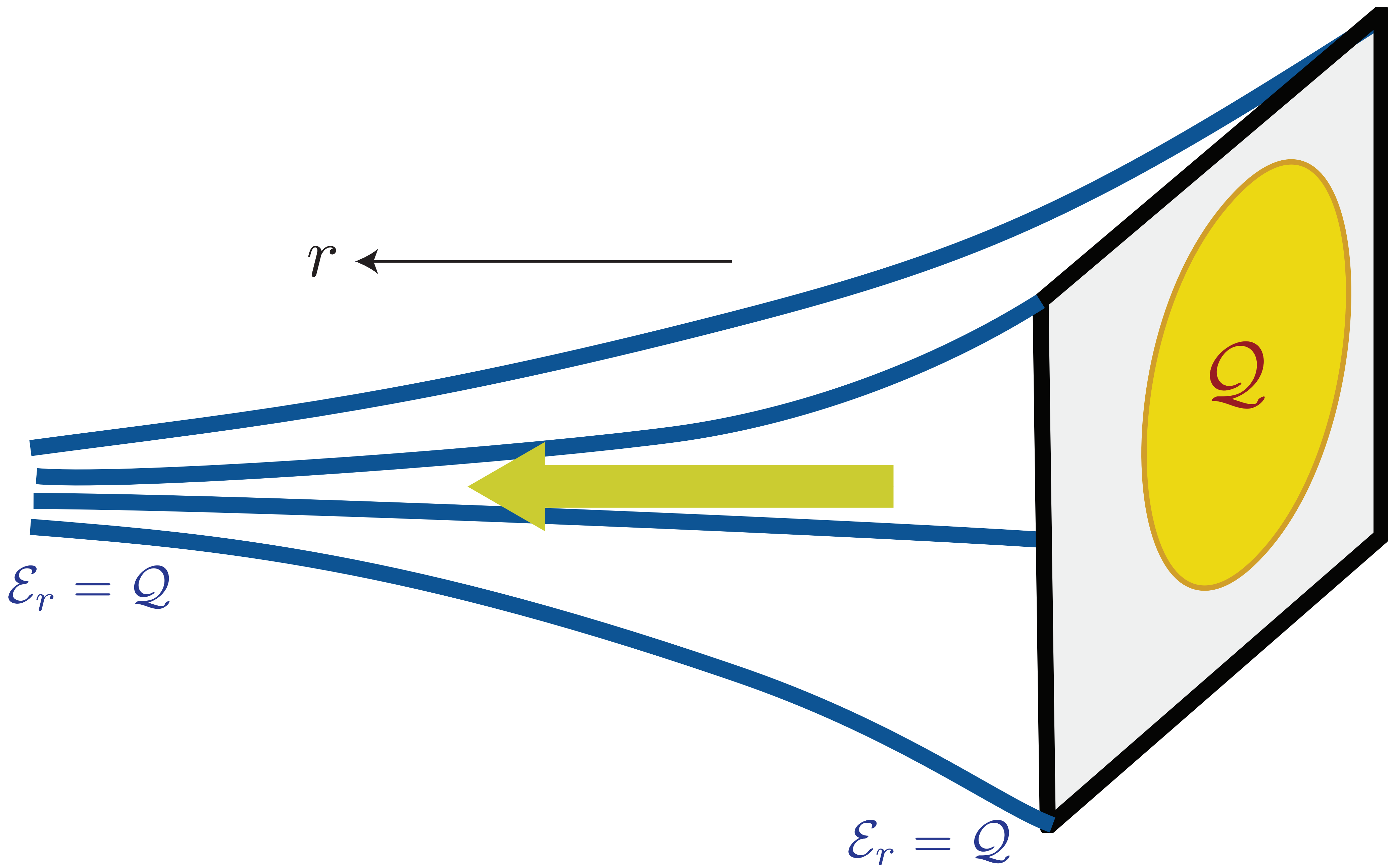
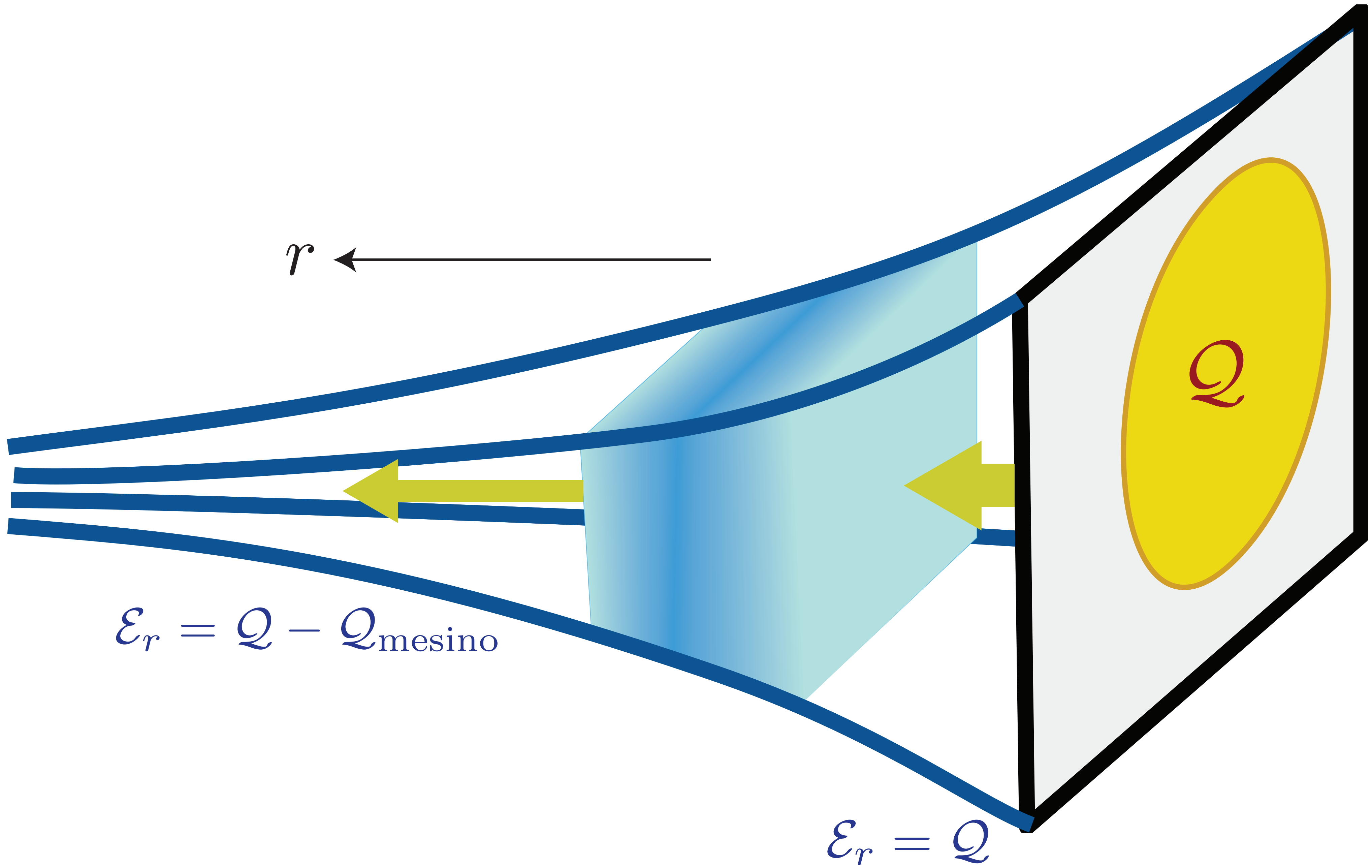
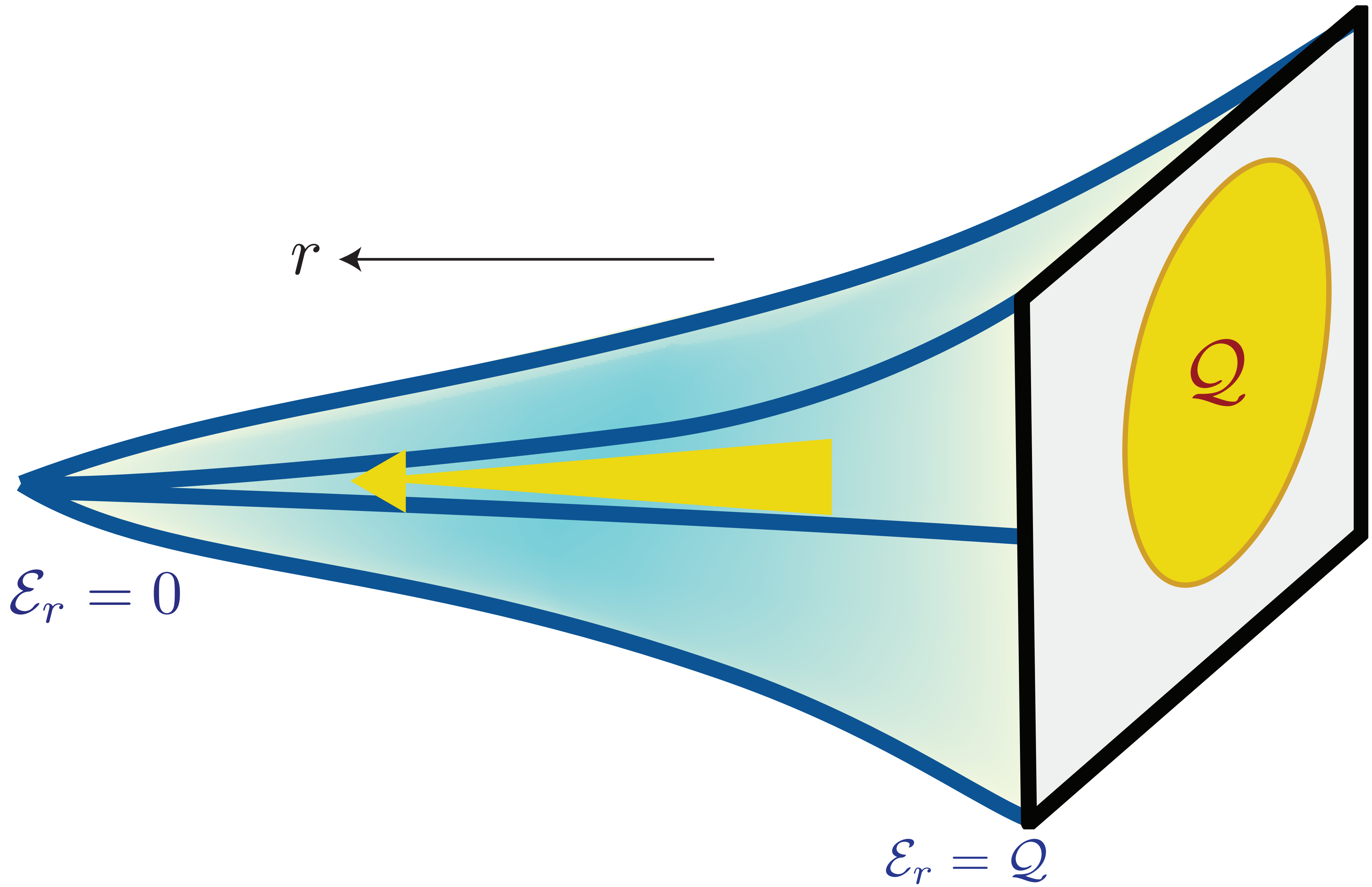
Holographic theory of a non-Fermi liquid (NFL)



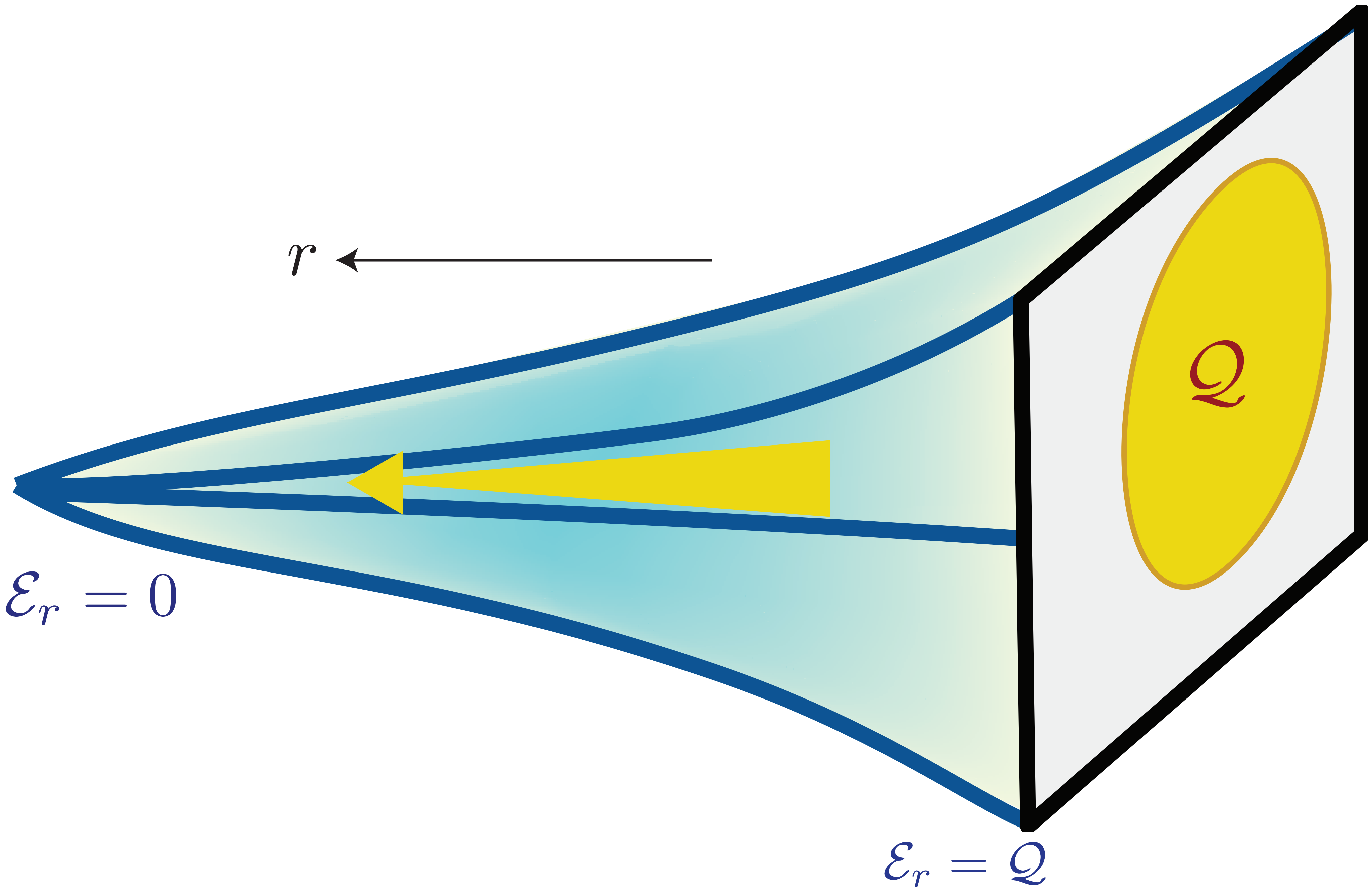
Holographic theory of a fractionalized-Fermi liquid (FL*)



Holographic theory of a Fermi liquid (FL)



Holographic theory of a Fermi liquid (FL)



Gauss Law in the bulk

$\Leftrightarrow$ Luttinger theorem on the boundary

Theory of a non-Fermi liquid (NFL)

Field theory	Holography
A gauge-dependent Fermi surface of overdamped gapless fermions.	Fermi surface is hidden.

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Logarithmic violation of area law of entanglement entropy, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.	Logarithmic violation of area law of entanglement entropy for $\theta = d - 1$, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.

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Gauge neutral 'mesinos' reduce the volume enclosed by Fermi surfaces of gauge-charged fermions to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.	Gauge neutral 'mesinos' reduce the volume enclosed by hidden Fermi surfaces to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.