The phase diagram of the cuprates and the quantum phase transitions of metals in two dimensions

Talk online: sachdev.physics.harvard.edu

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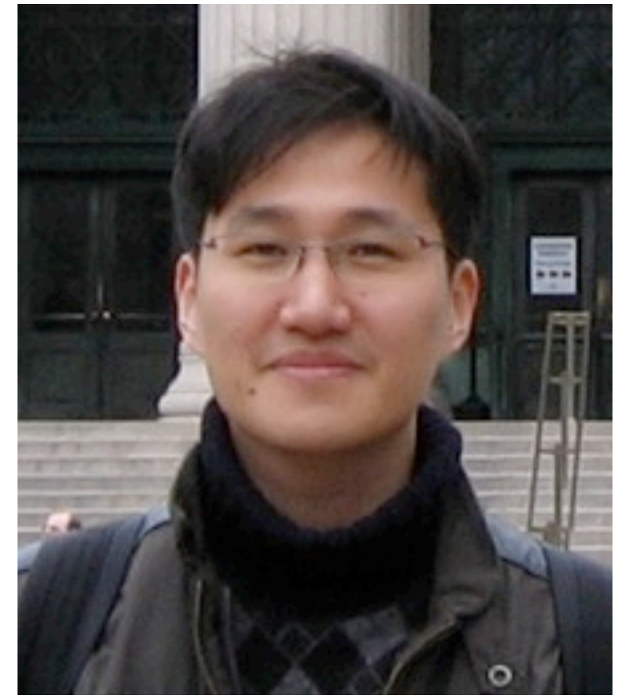
Max Metlitski, Harvard

arXiv:1001.1153



Eun Gook Moon, Harvard

Phys. Rev. B **80**, 035117 (2009)

PHYSICS

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Outline

1. Phase diagram of the cuprates

   Quantum criticality of the competition between antiferromagnetism and superconductivity

2. Influence of an applied magnetic field

   Theoretical predictions and experimental tests

3. Theory of spin density wave ordering in a metal

   Order parameter at zero wavevector

4. Theory of Ising-nematic ordering in a metal

   Order parameter at zero wavevector

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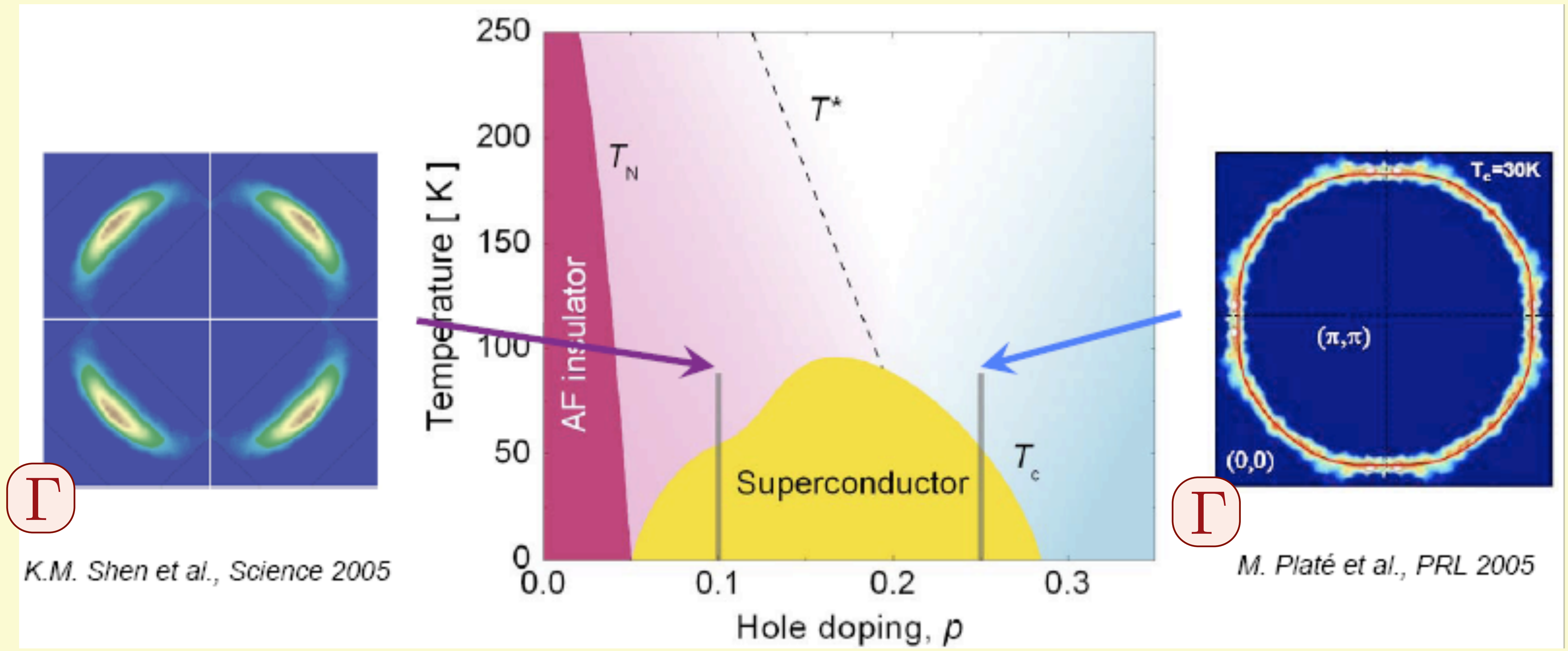
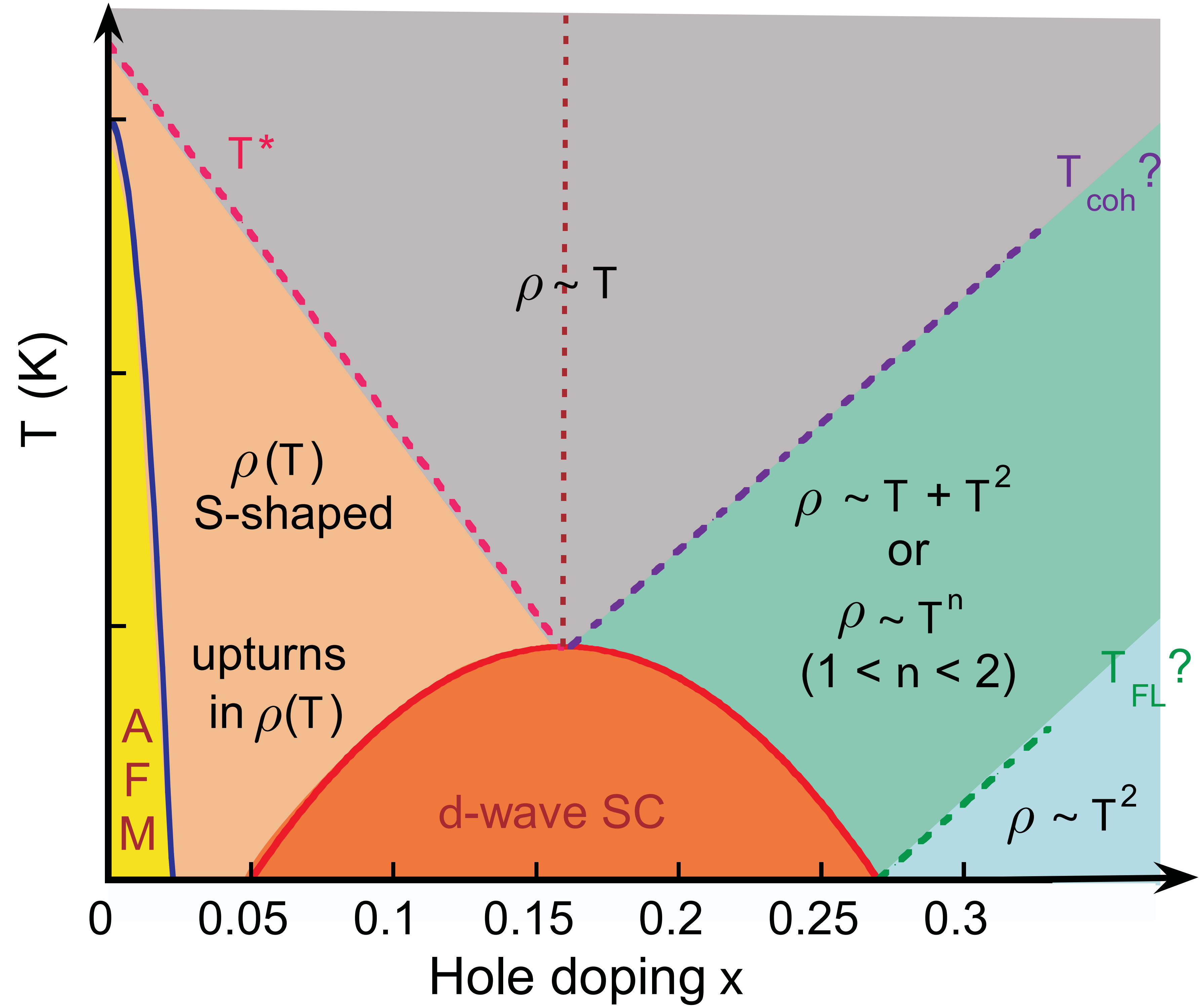
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



Smaller hole Fermi-pockets

Large hole Fermi surface

Crossovers in transport properties of hole-doped cuprates



N. E. Hussey, *J. Phys: Condens. Matter* **20**, 123201 (2008)

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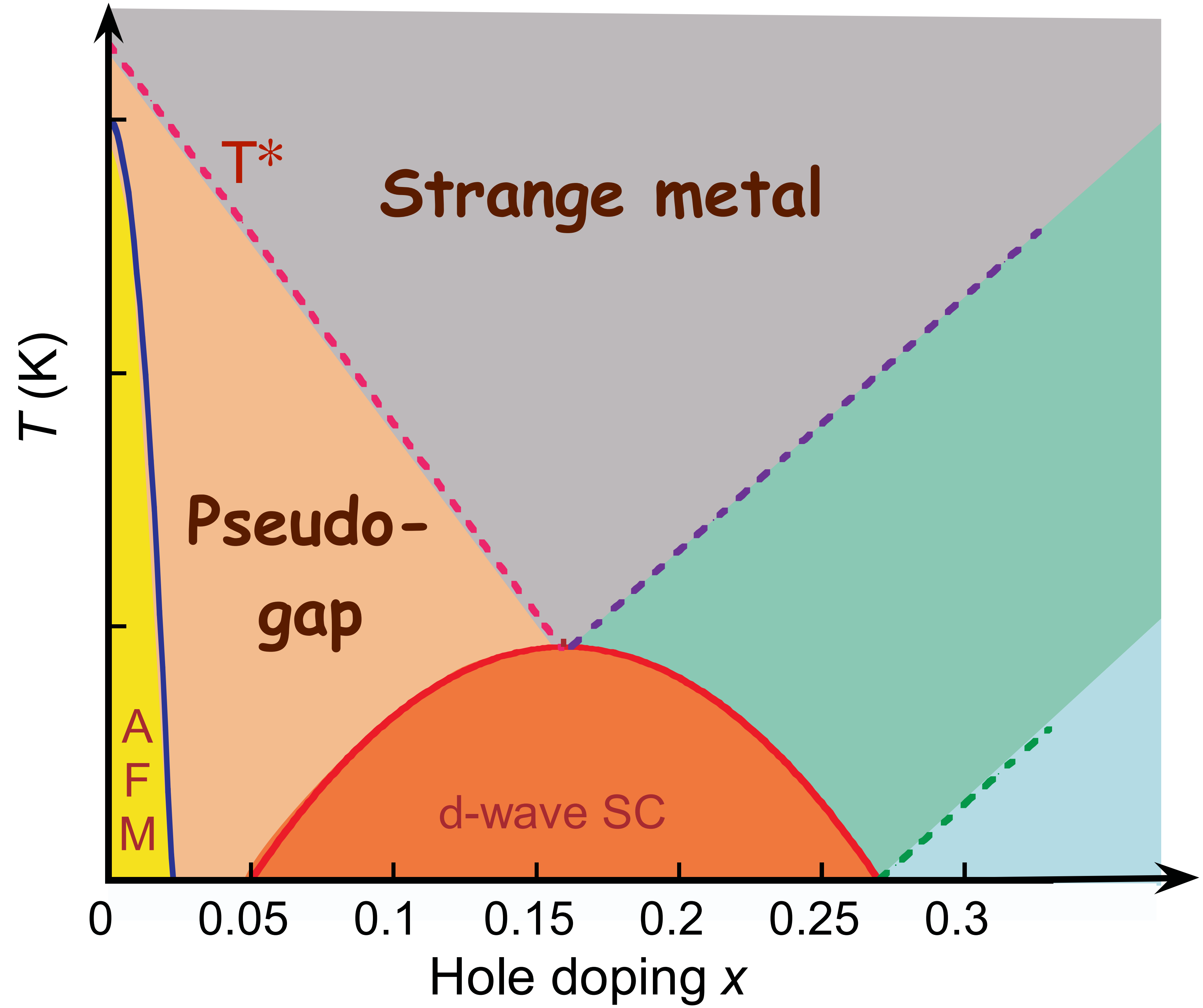

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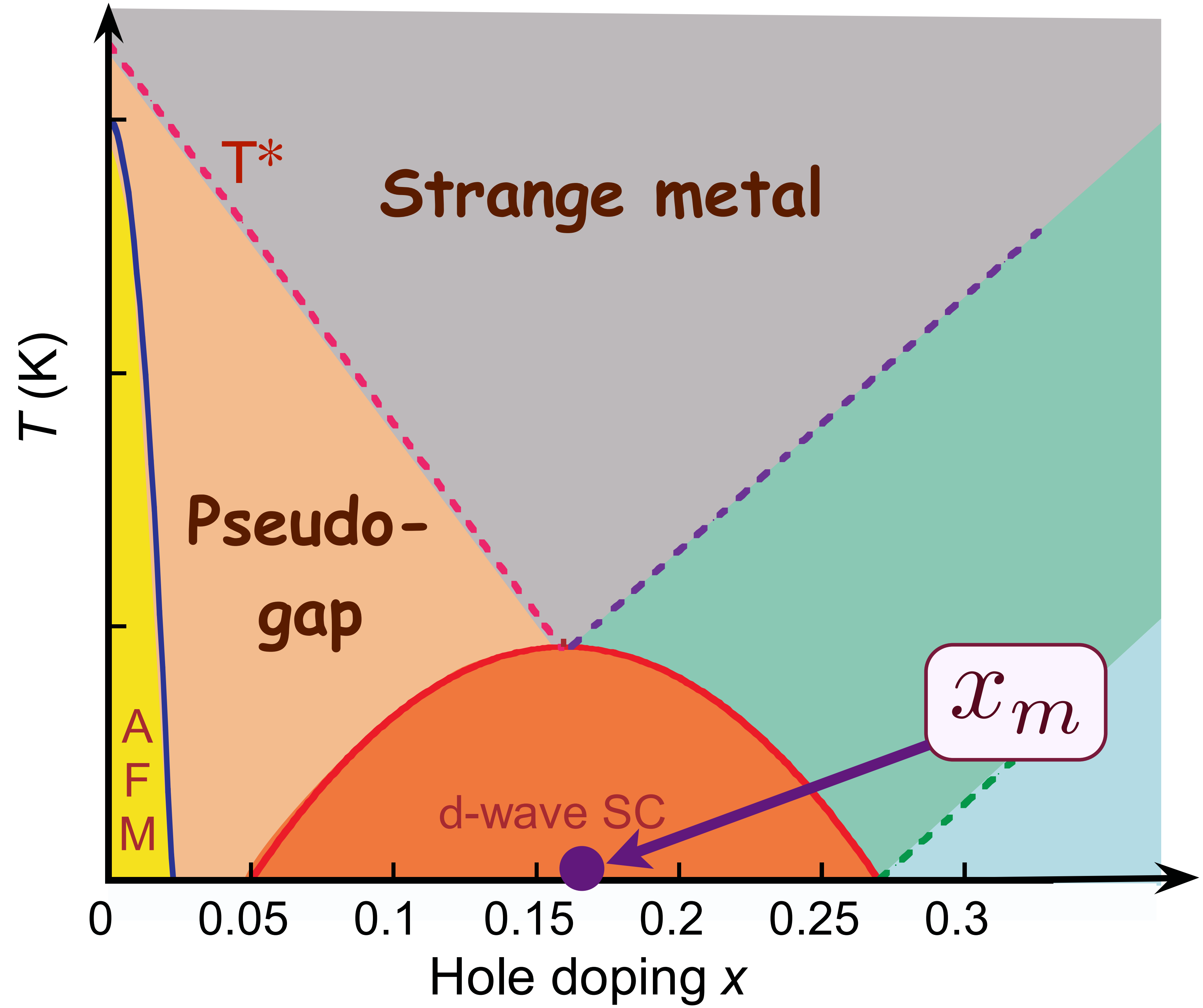
Crossovers in transport properties of hole-doped cuprates



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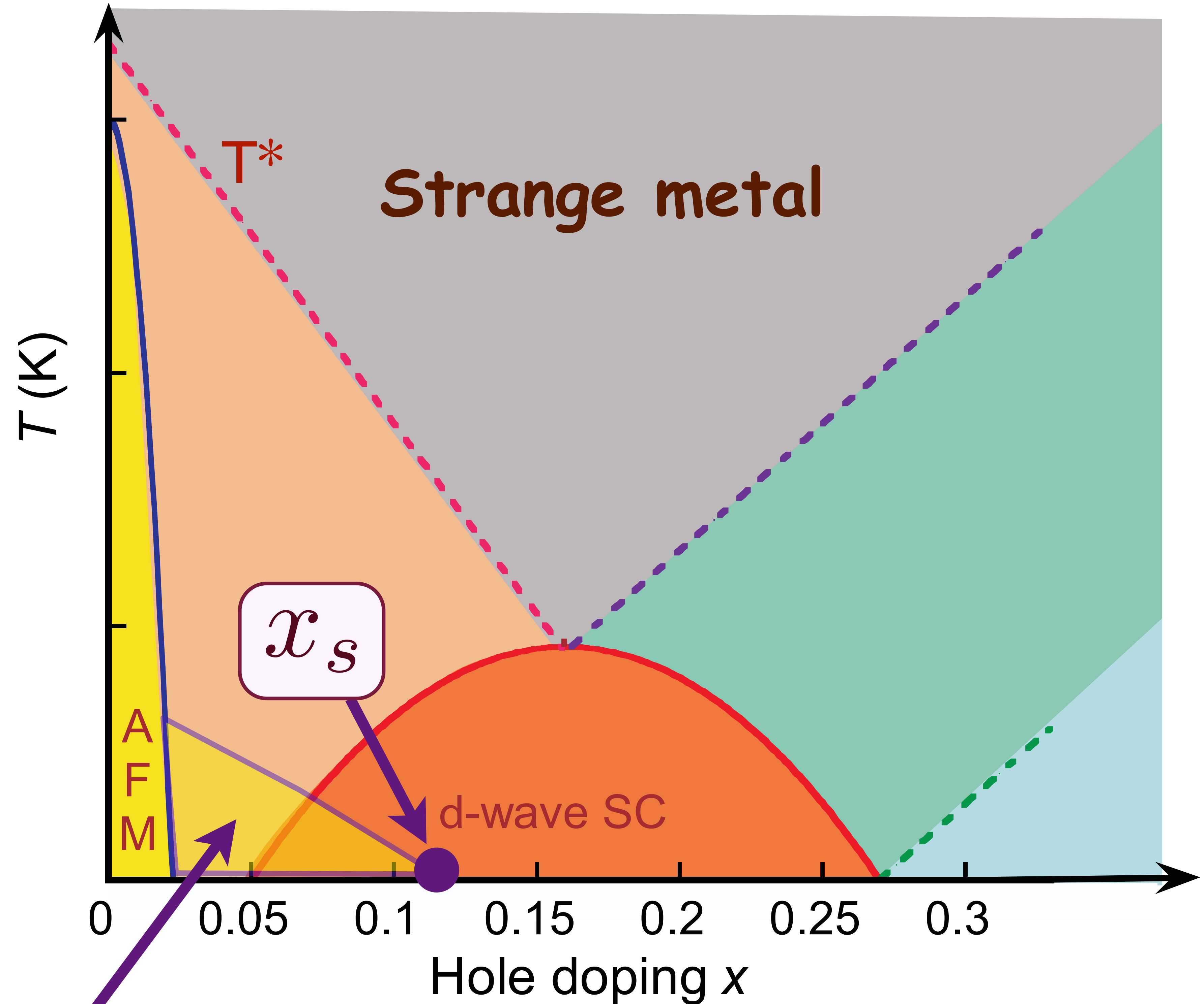
A. J. Millis, *Phys. Rev.* B **48**, 7183 (1993).

C. M. Varma, *Phys. Rev. Lett.* **83**, 3538 (1999).

Strange metal: quantum criticality of optimal doping critical point at $x = x_m$?

Only candidate quantum critical point observed at low T

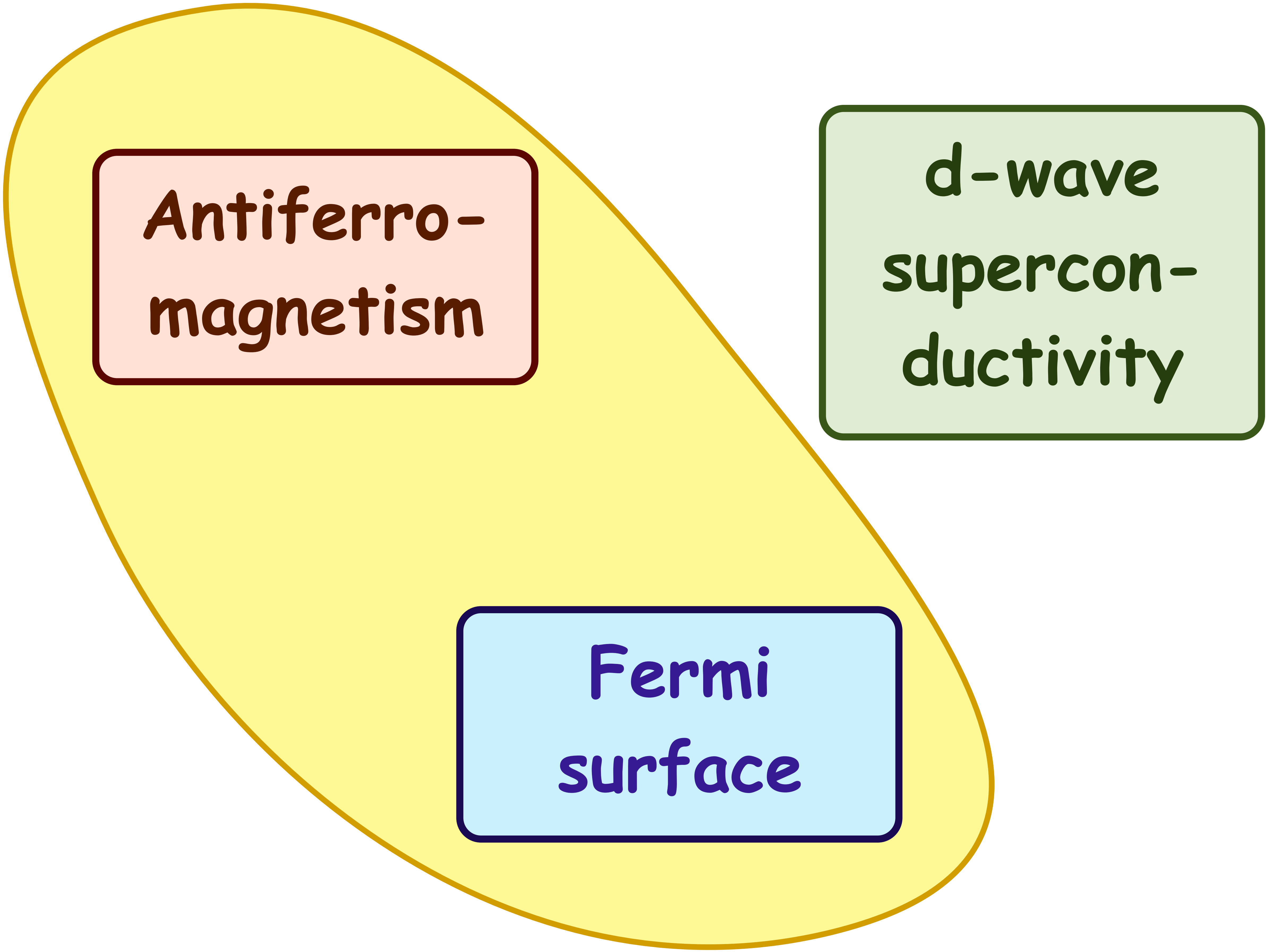


Spin density wave order present below a quantum critical point at $x = x_s$ with $x_s \approx 0.12$ in the La series of cuprates

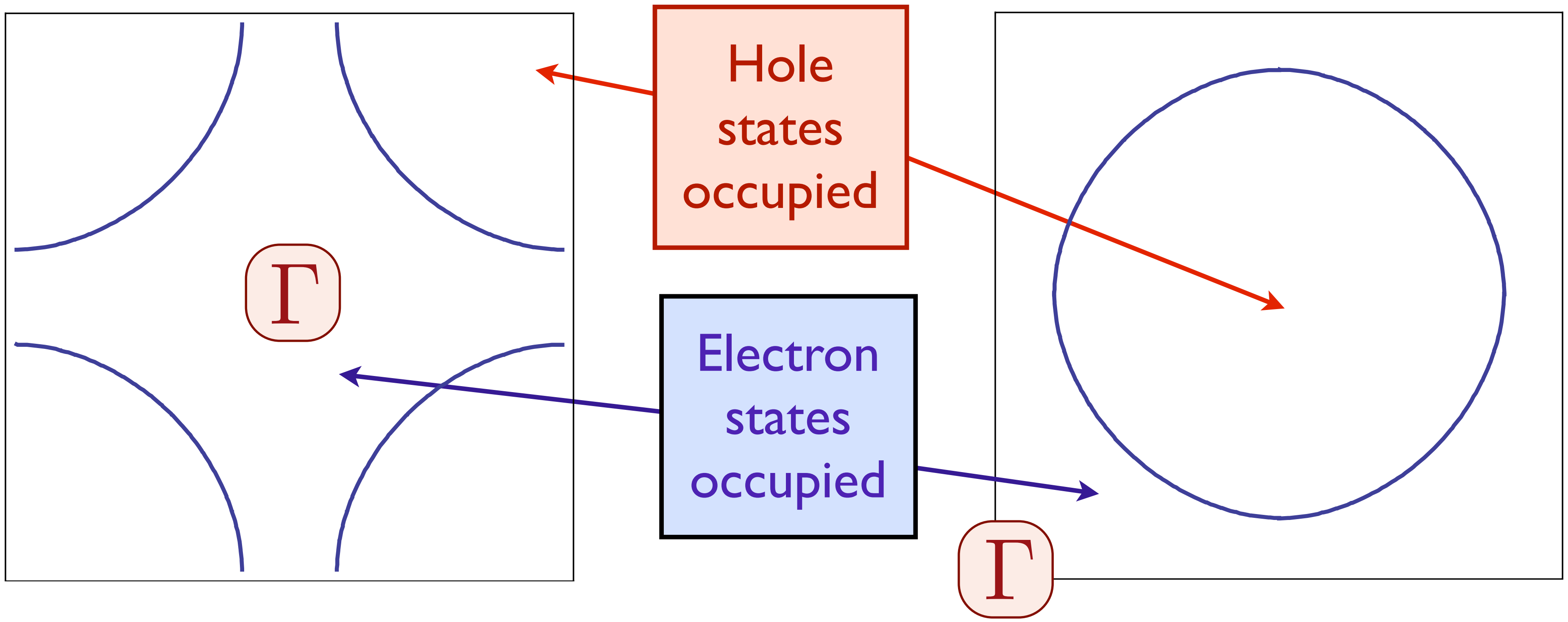
Antiferro-
magnetism

d-wave
supercon-
ductivity

Fermi
surface


Antiferro-
magnetism
d-wave
supercon-
ductivity
Fermi
surface

Fermi surface+antiferromagnetism



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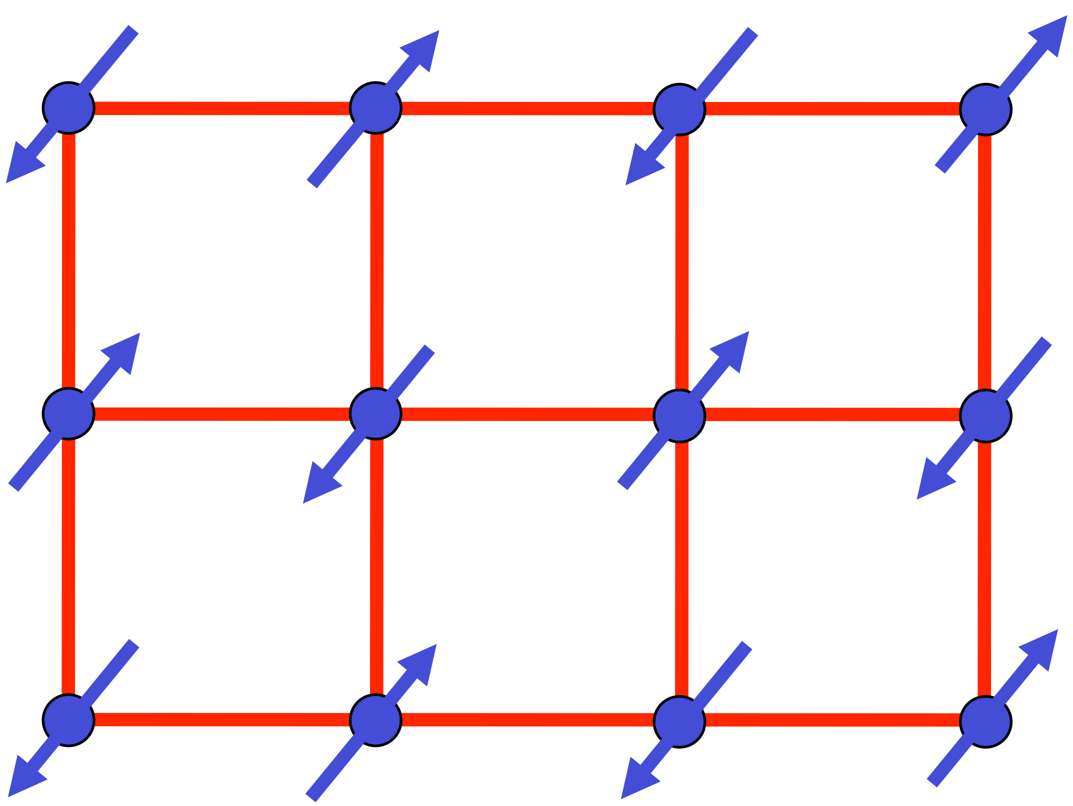
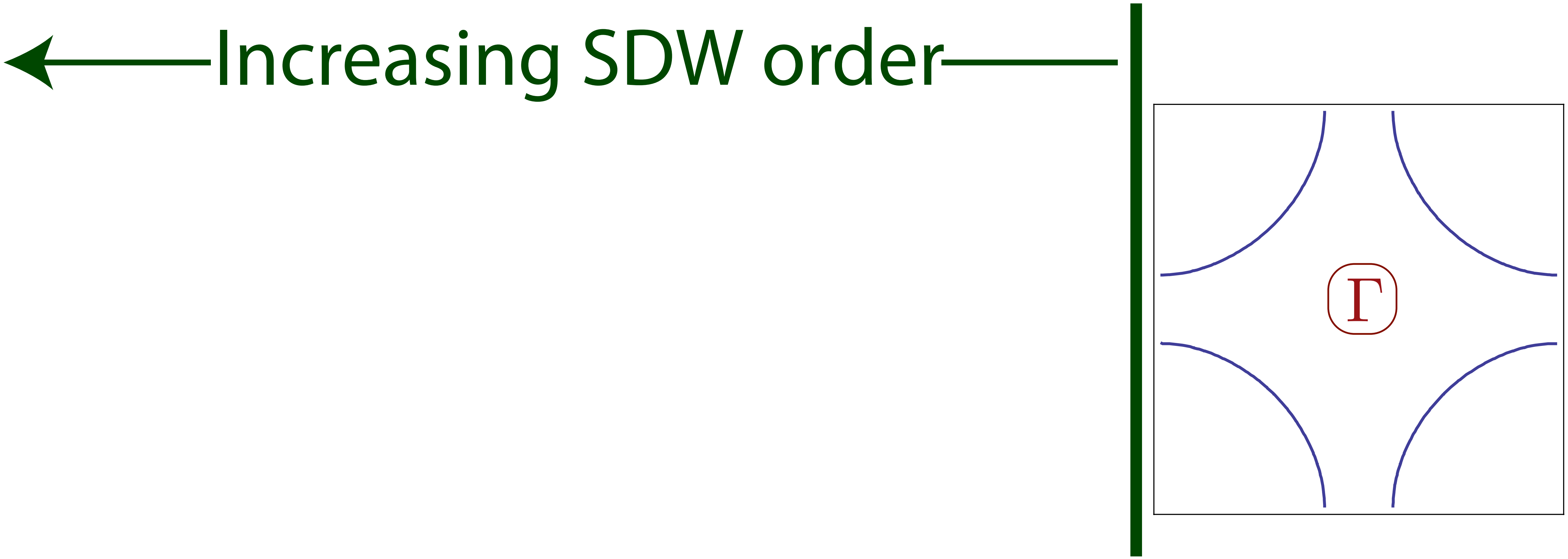


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Hole-doped cuprates



S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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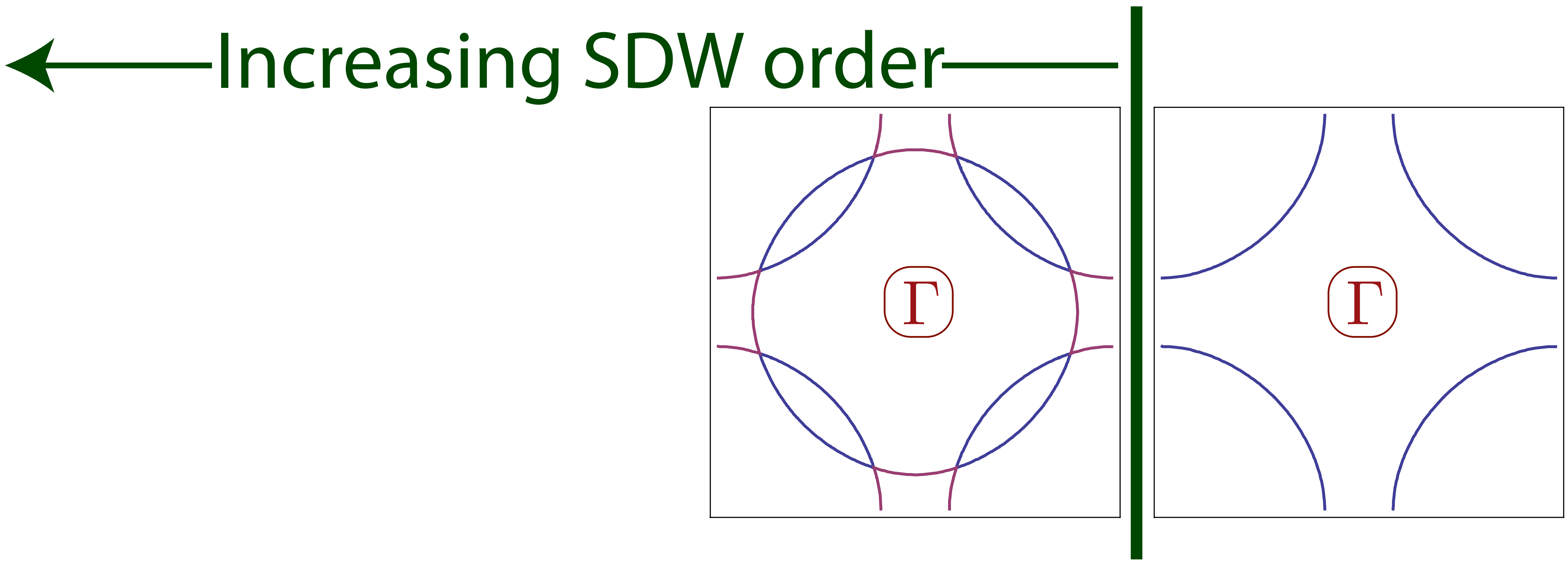

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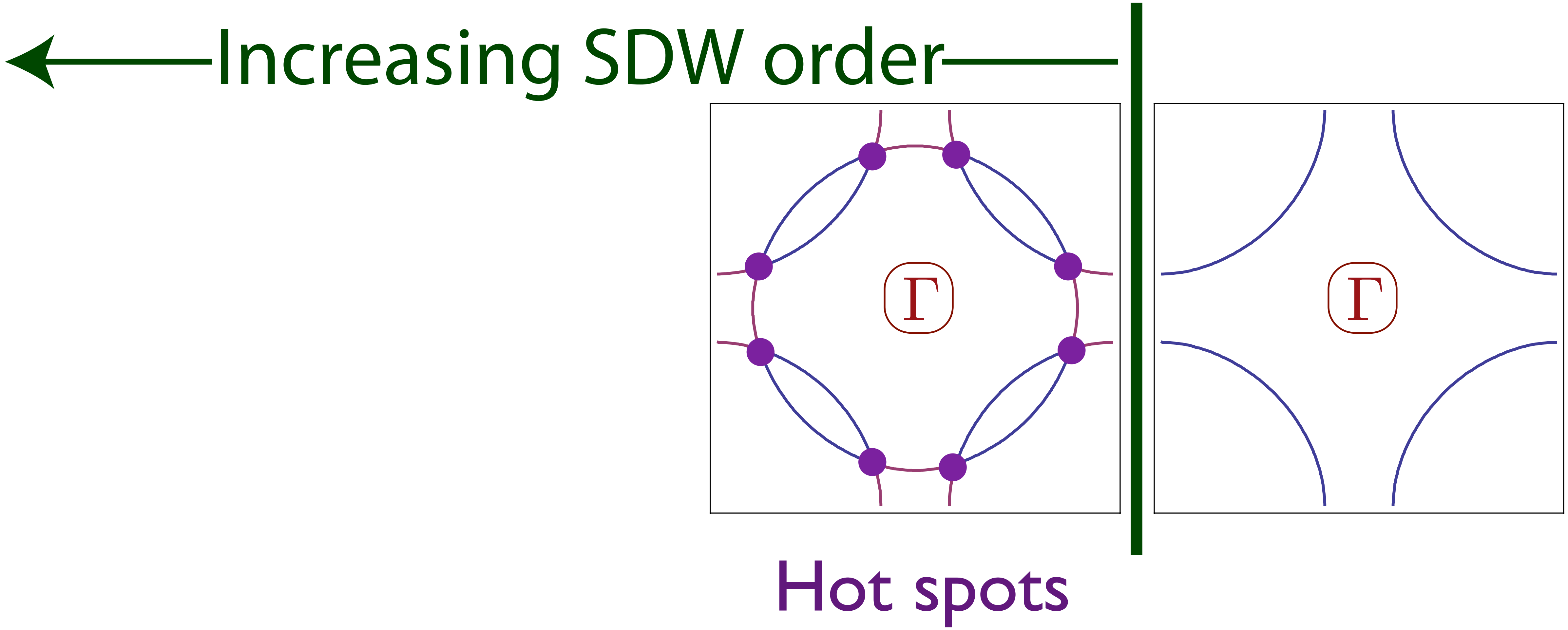

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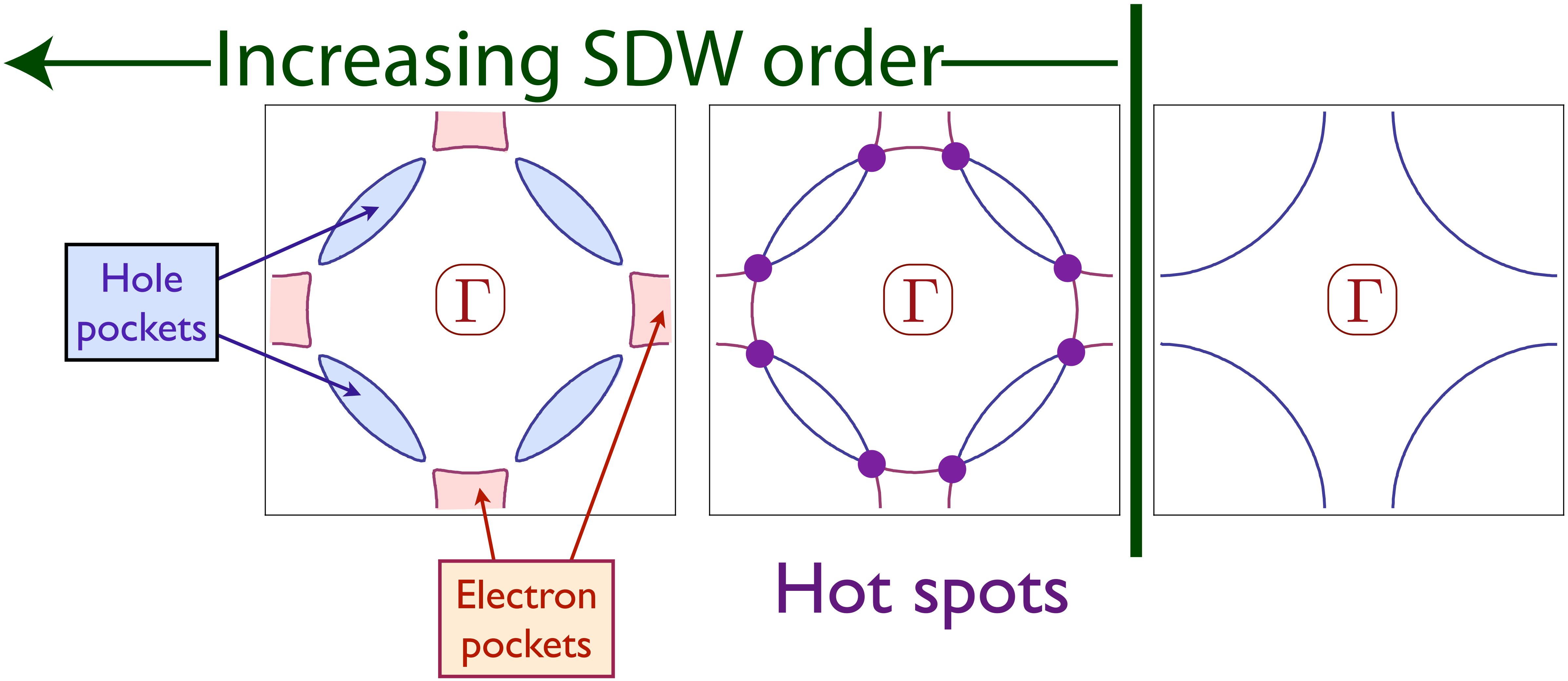

Fermi surface breaks up at hot spots
into electron and hole "pockets"

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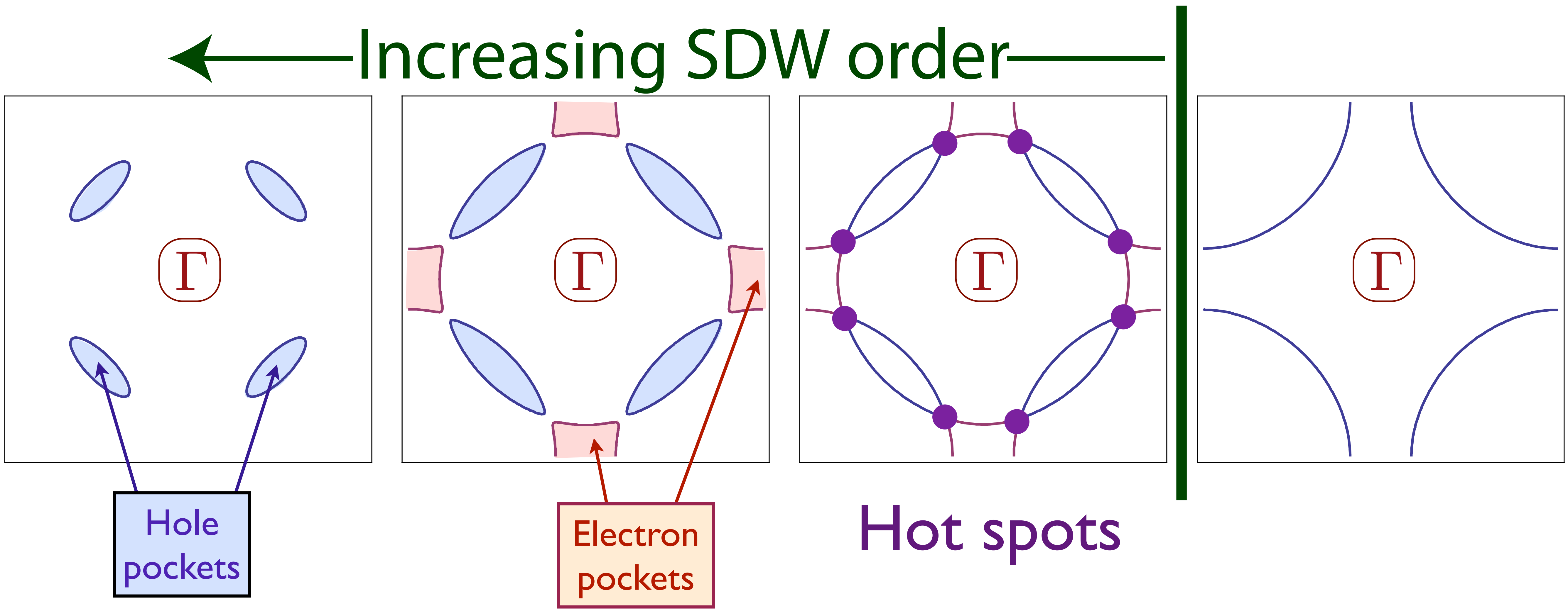

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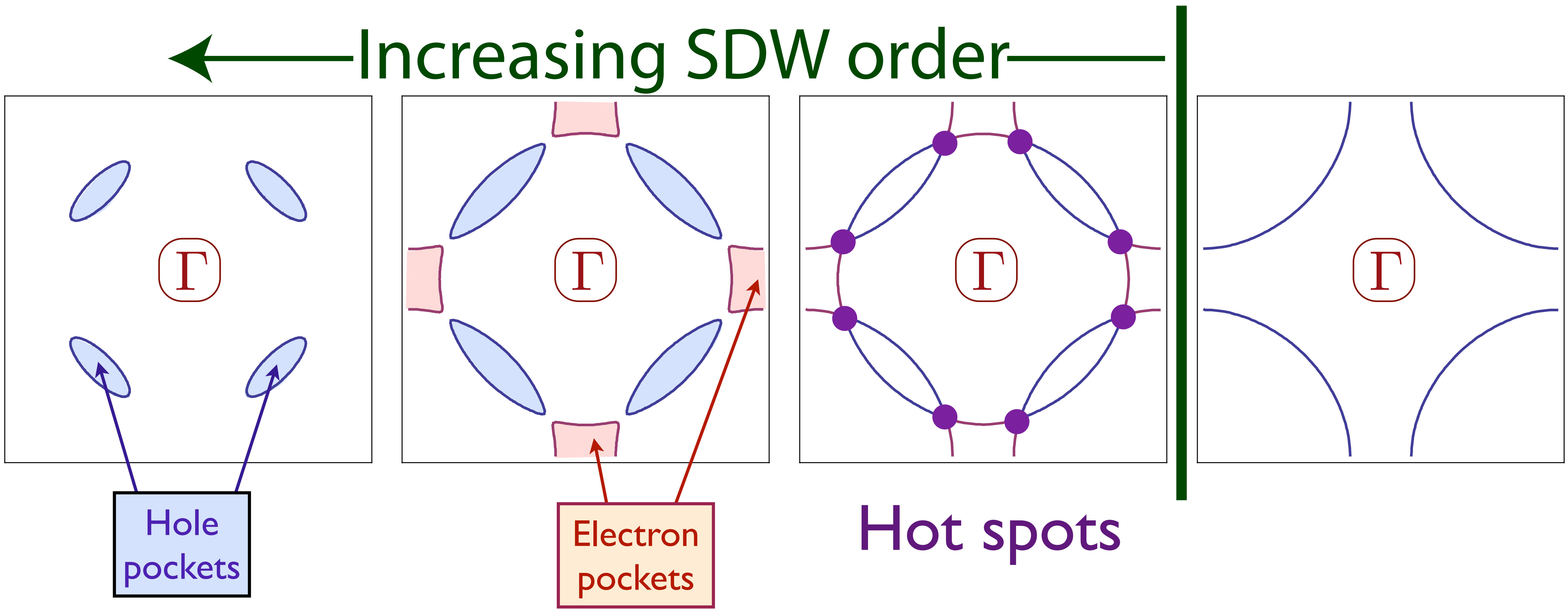

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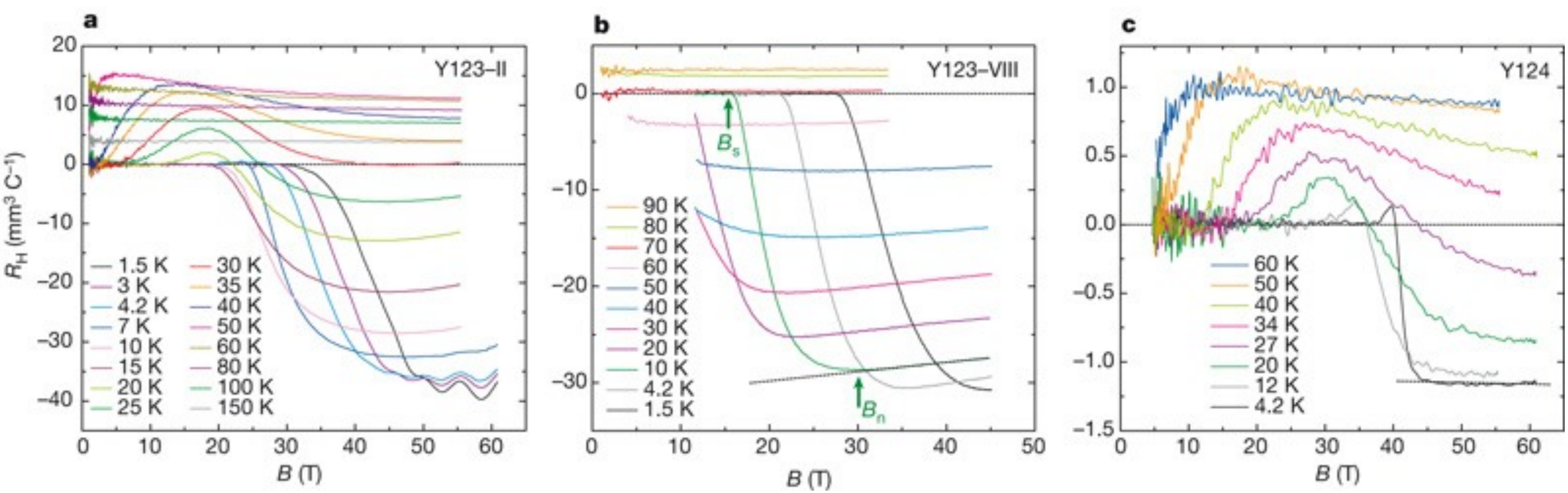
Quantum oscillations

Electron pockets in the Fermi surface of hole-doped high-T_c superconductors

David LeBoeuf[1], Nicolas Doiron-Leyraud[1], Julien Levallois[2], R. Daou[1], J.-B. Bonnemaison[1], N. E. Hussey[3], L. Balicas[4], B. J. Ramshaw[5], Ruixing Liang[5,6], D. A. Bonn[5,6], W. N. Hardy[5,6], S. Adachi[7], Cyril Proust[2] & Louis Taillefer[1,6]

Nature **450**, 533 (2007)



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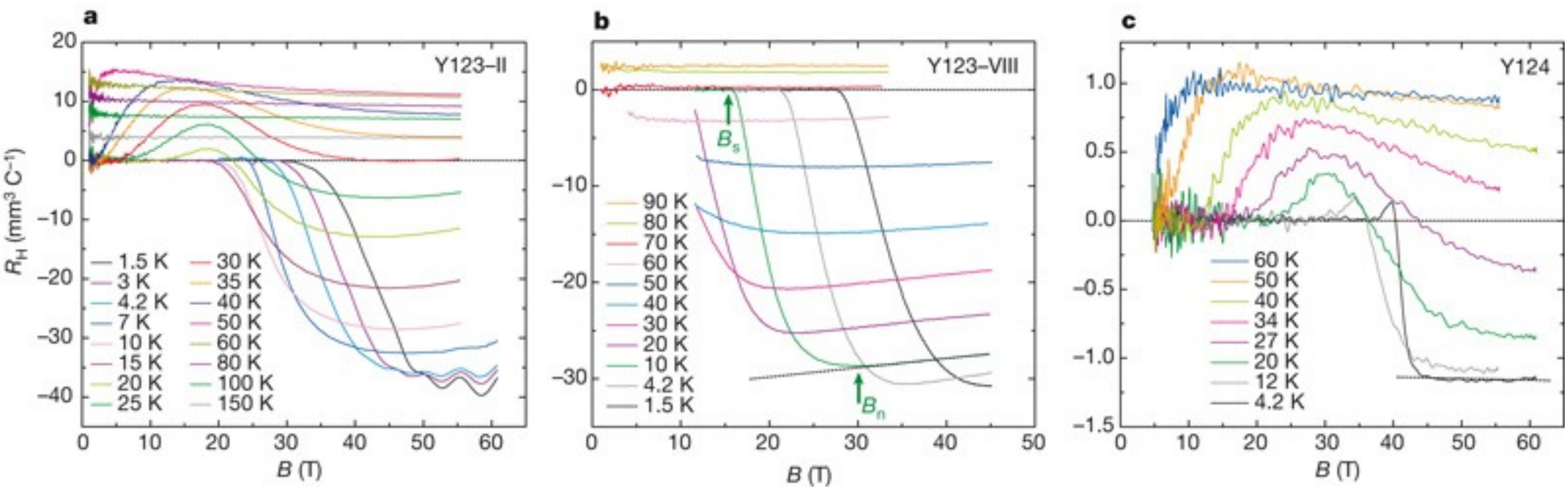


Evidence for small Fermi pockets

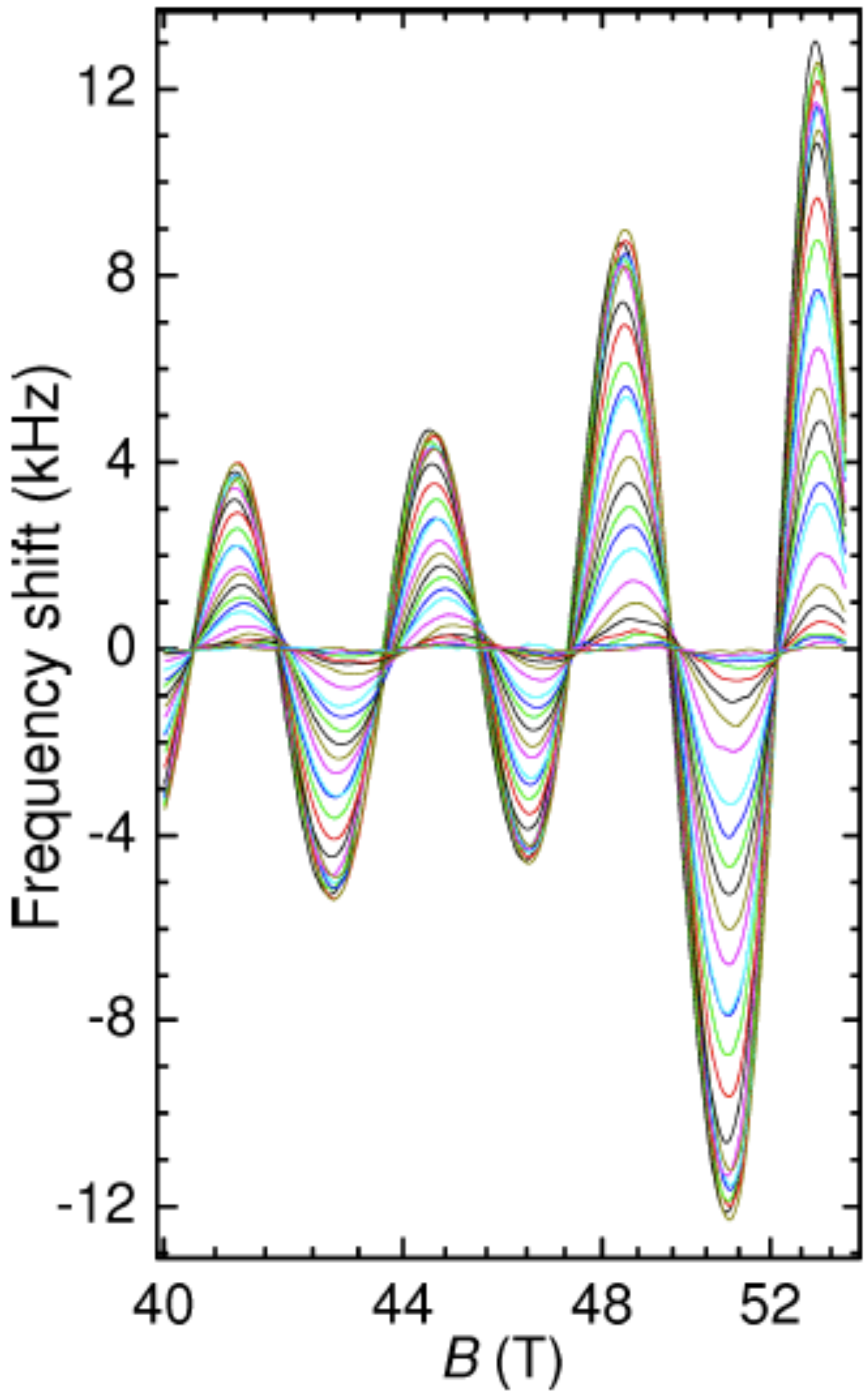


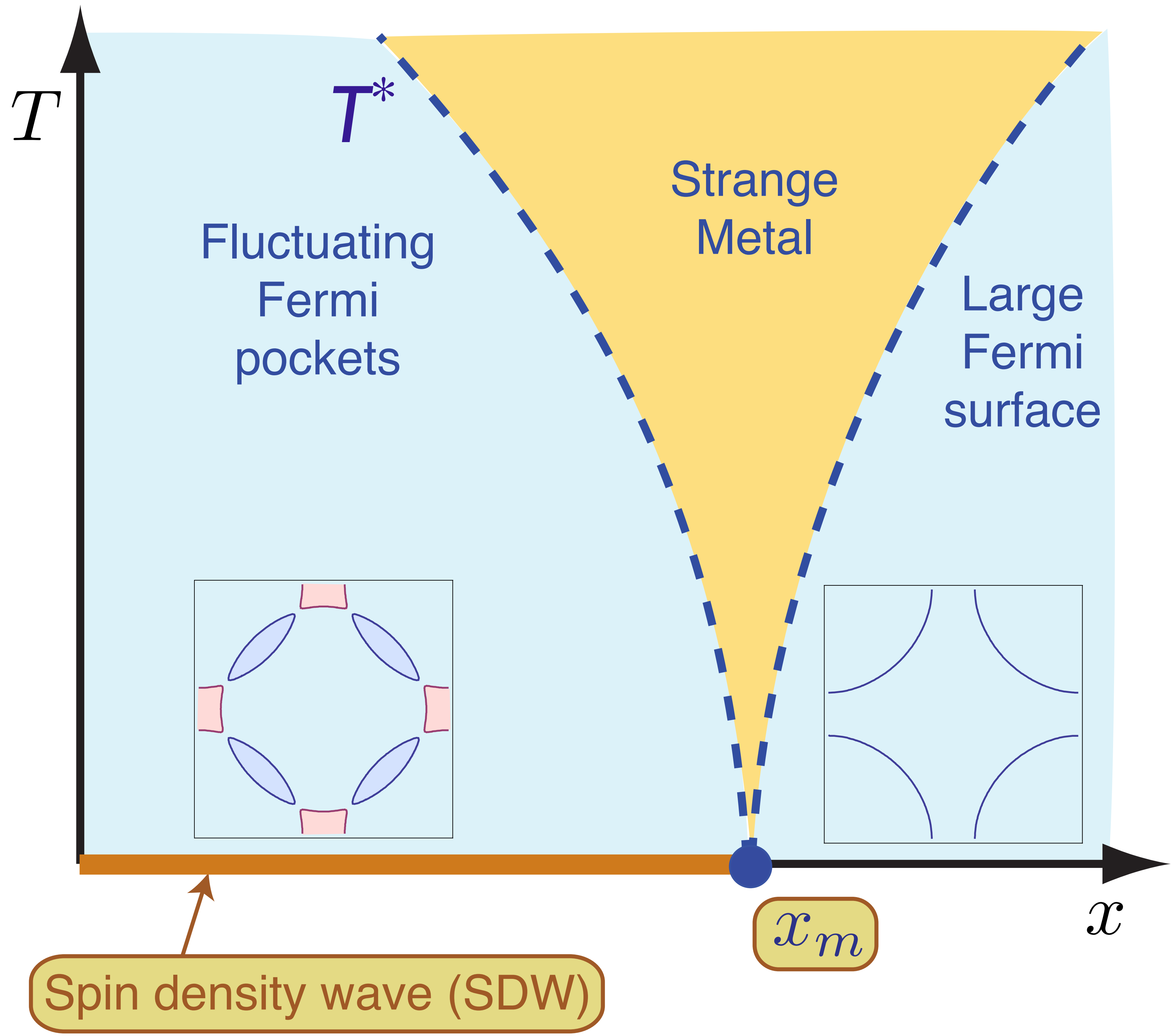
FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of $\sim$ 50 dB between T = 1 and 18 K. The actual T values are provided in Fig. 3.

Fermi liquid behaviour in an underdoped high Tc superconductor

Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich

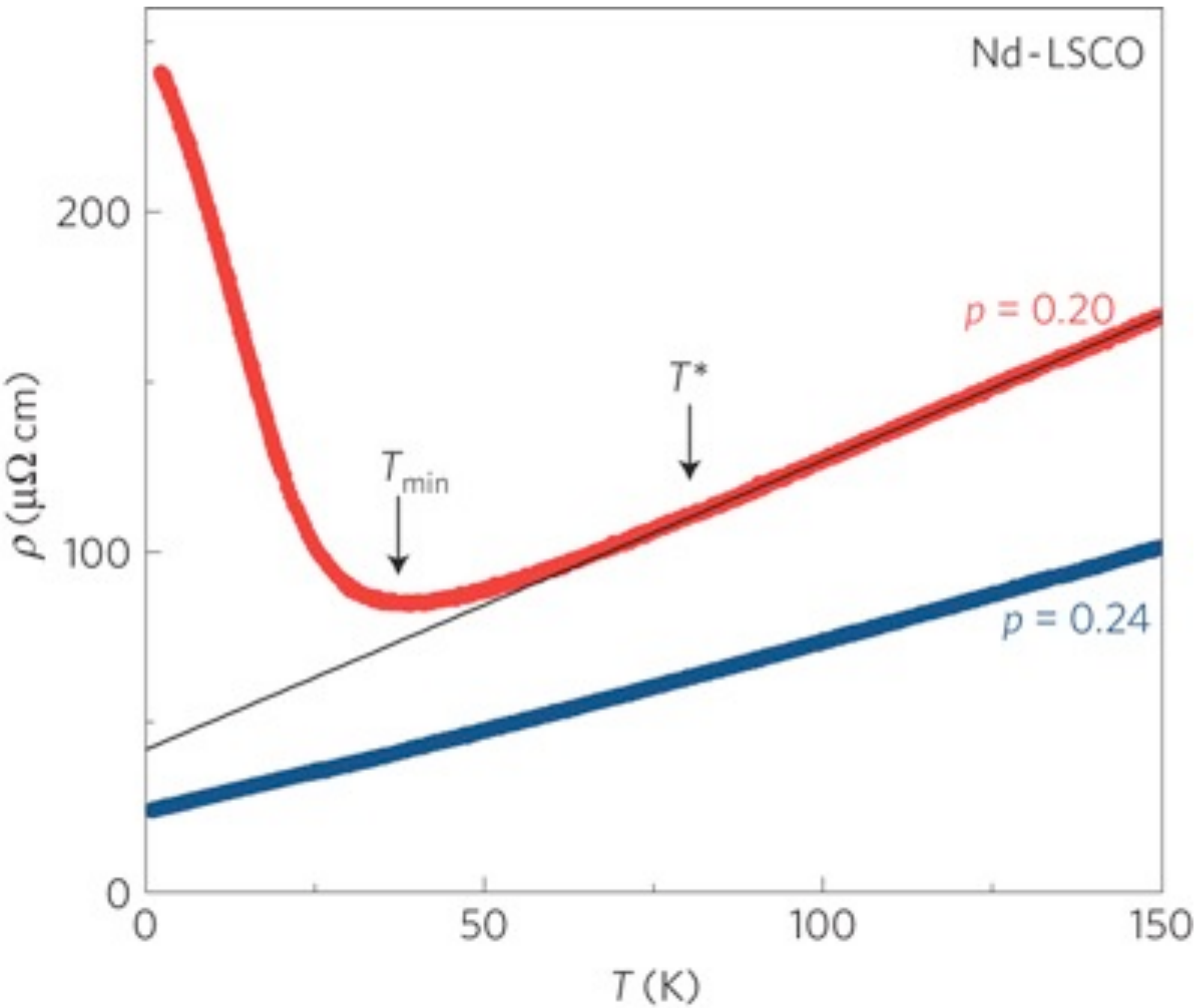
arXiv:0912.3022

Theory of quantum criticality in the cuprates



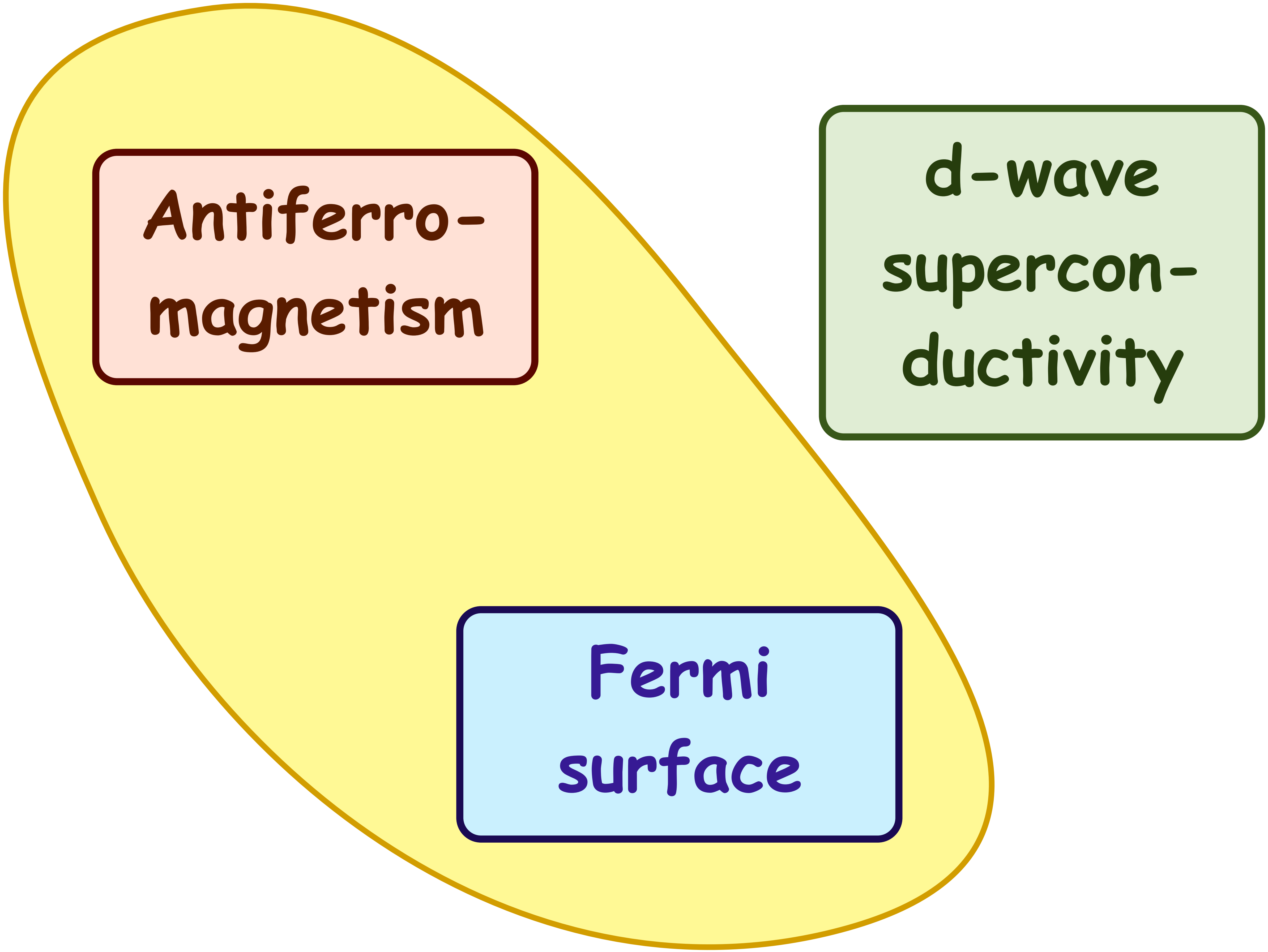
Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

Evidence for connection between linear resistivity and stripe-ordering in a cuprate with a low T_c



- Magnetic field of upto 35 T used to suppress superconductivity
- Identifies $x_m \approx 0.24$

Linear temperature dependence of resistivity and change in the Fermi surface at the pseudogap critical point of a high-T_c superconductor
R. Daou, Nicolas Doiron-Leyraud, David LeBoeuf, S. Y. Li, Francis Laliberté, Olivier Cyr-Choinière, Y. J. Jo, L. Balicas, J.-Q. Yan, J.-S. Zhou, J. B. Goodenough & Louis Taillefer, *Nature Physics* **5**, 31 - 34 (2009)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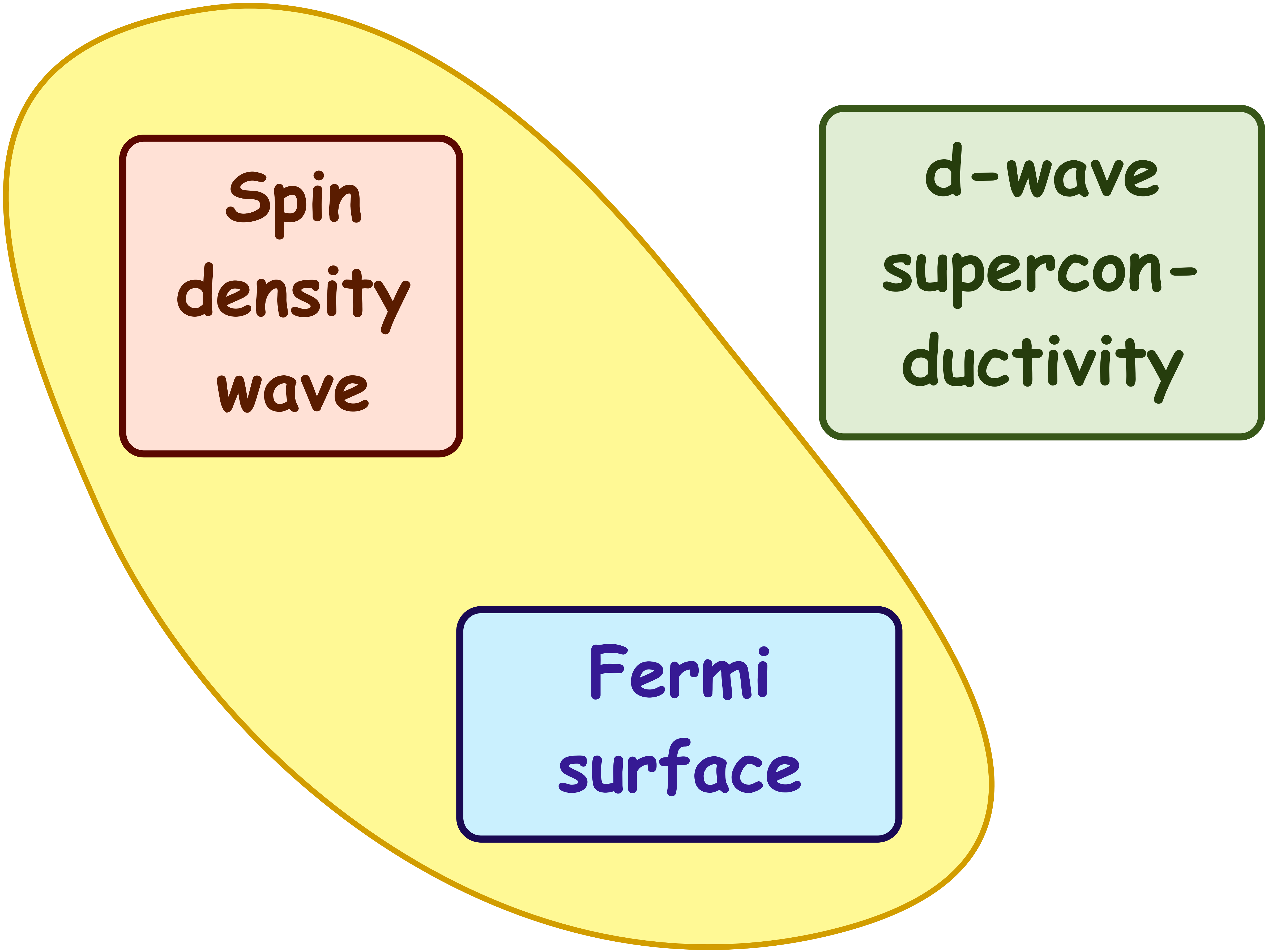
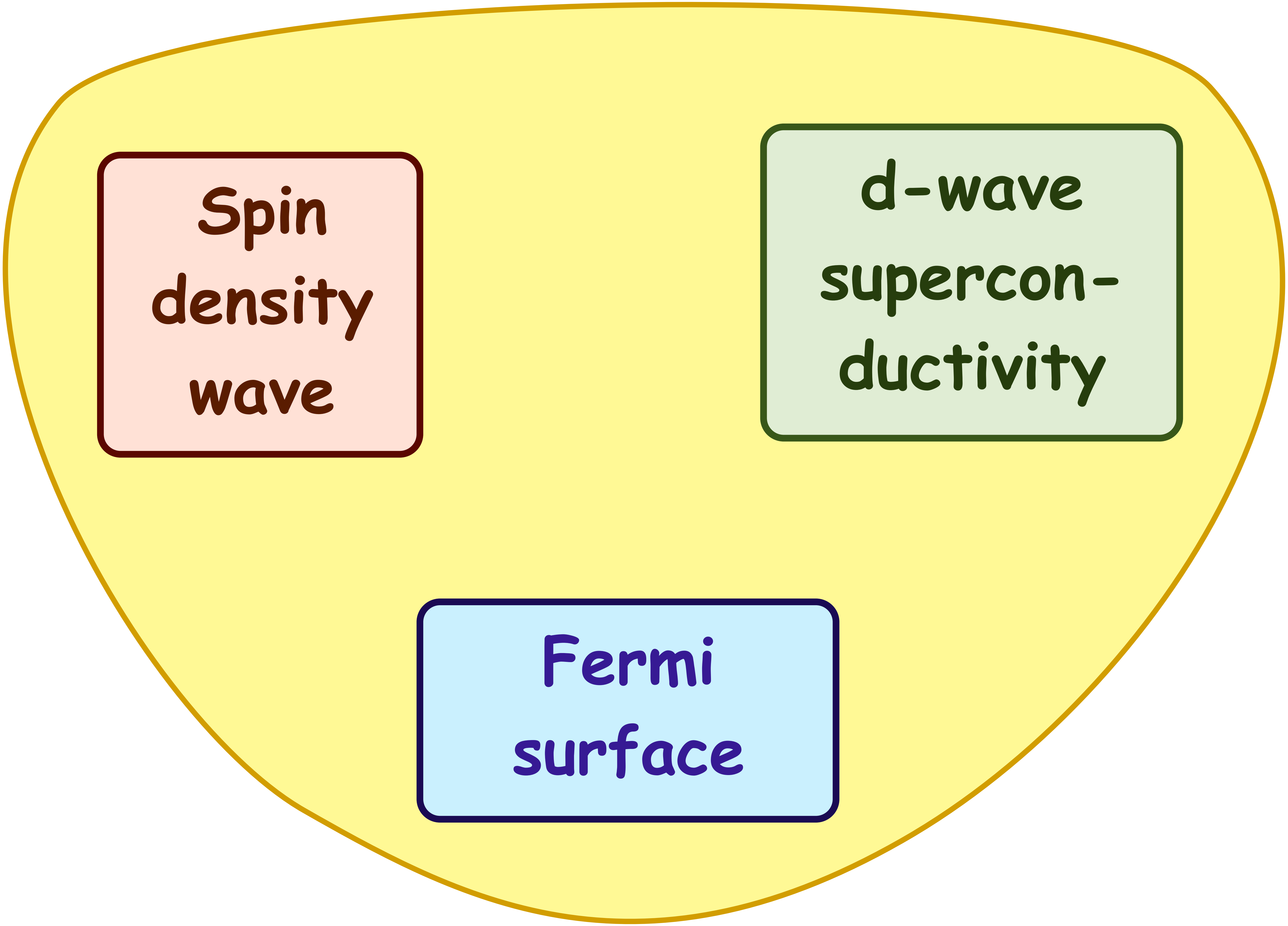

Spin density wave
d-wave supercon-ductivity
Fermi surface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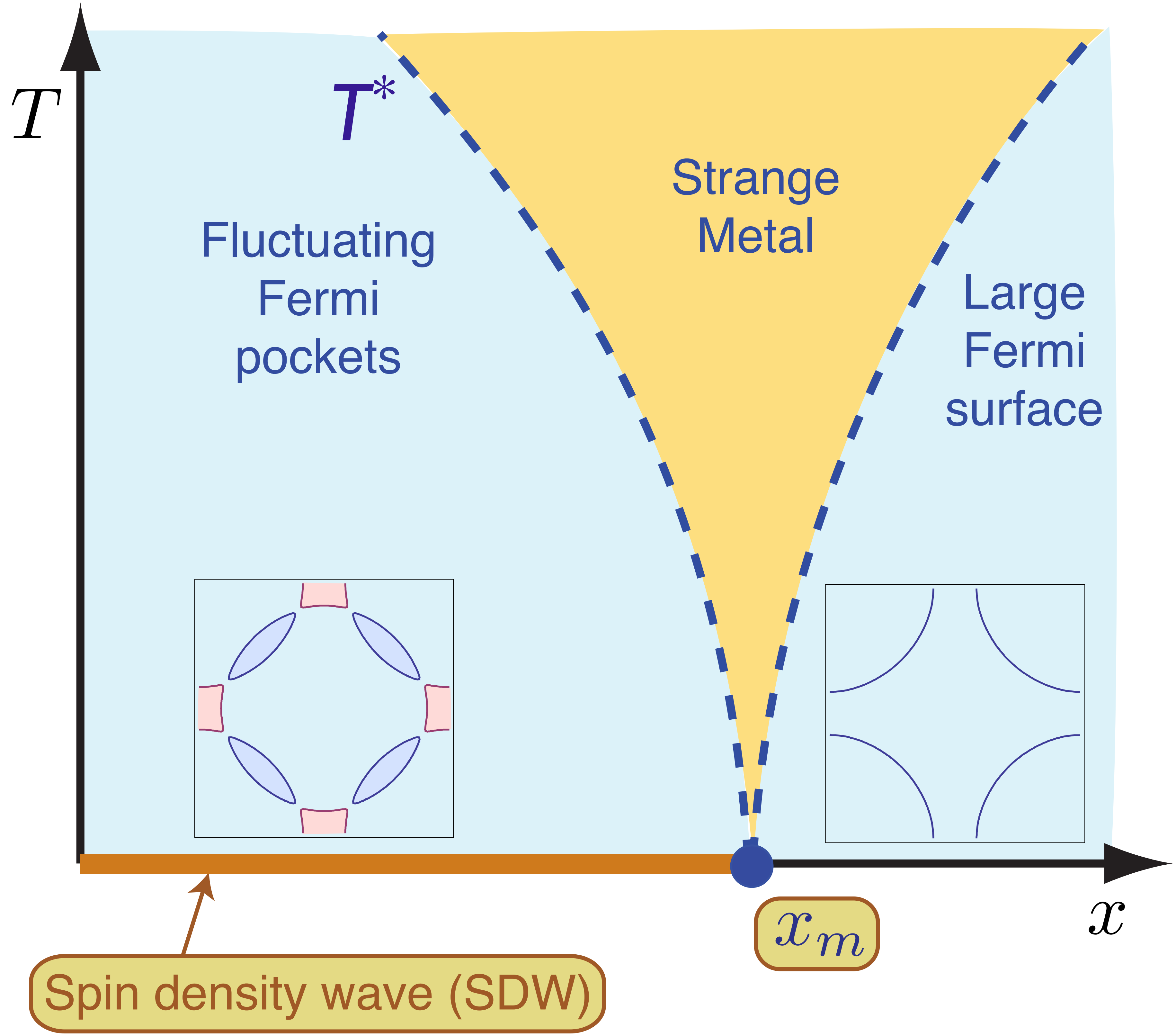

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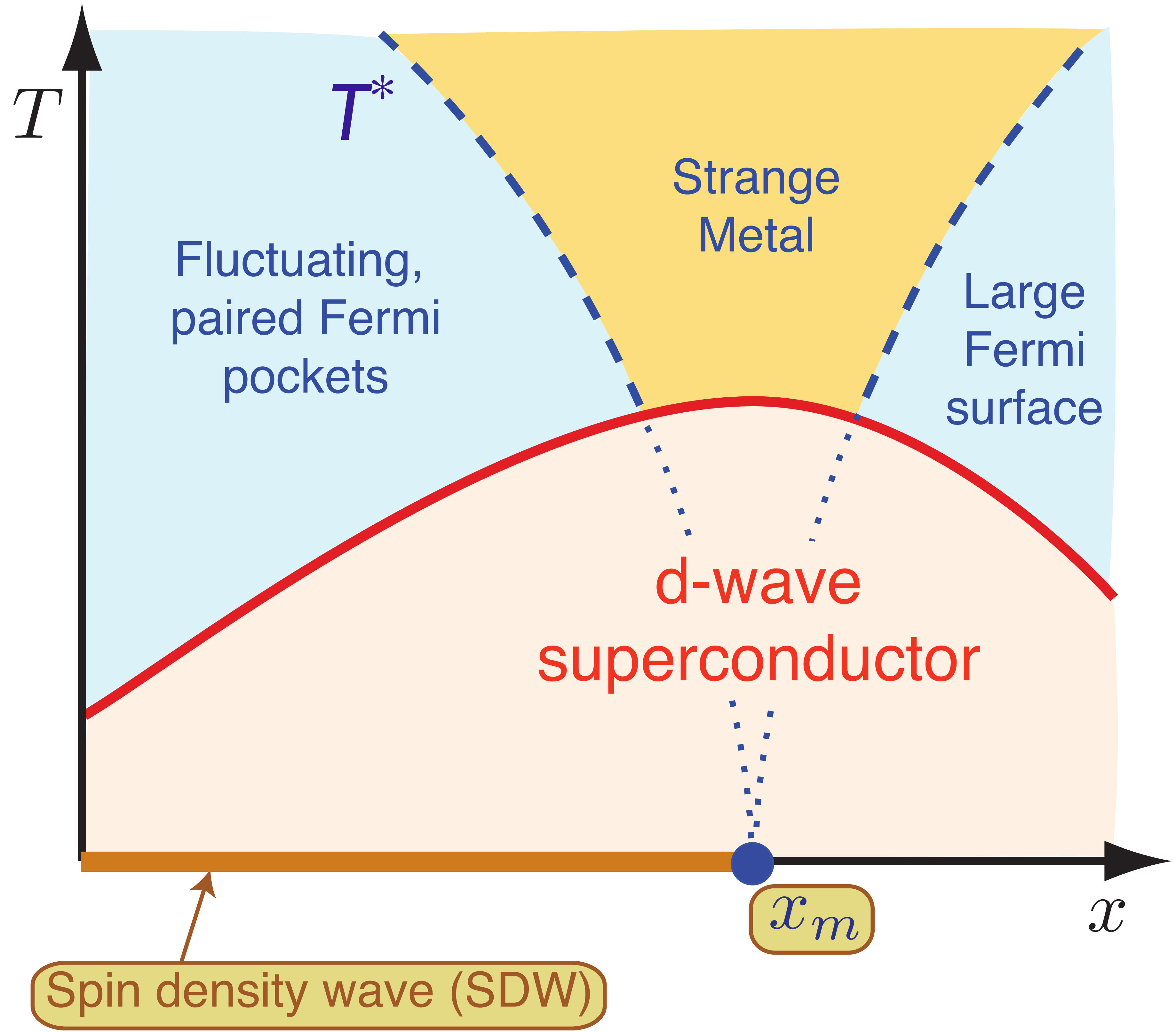
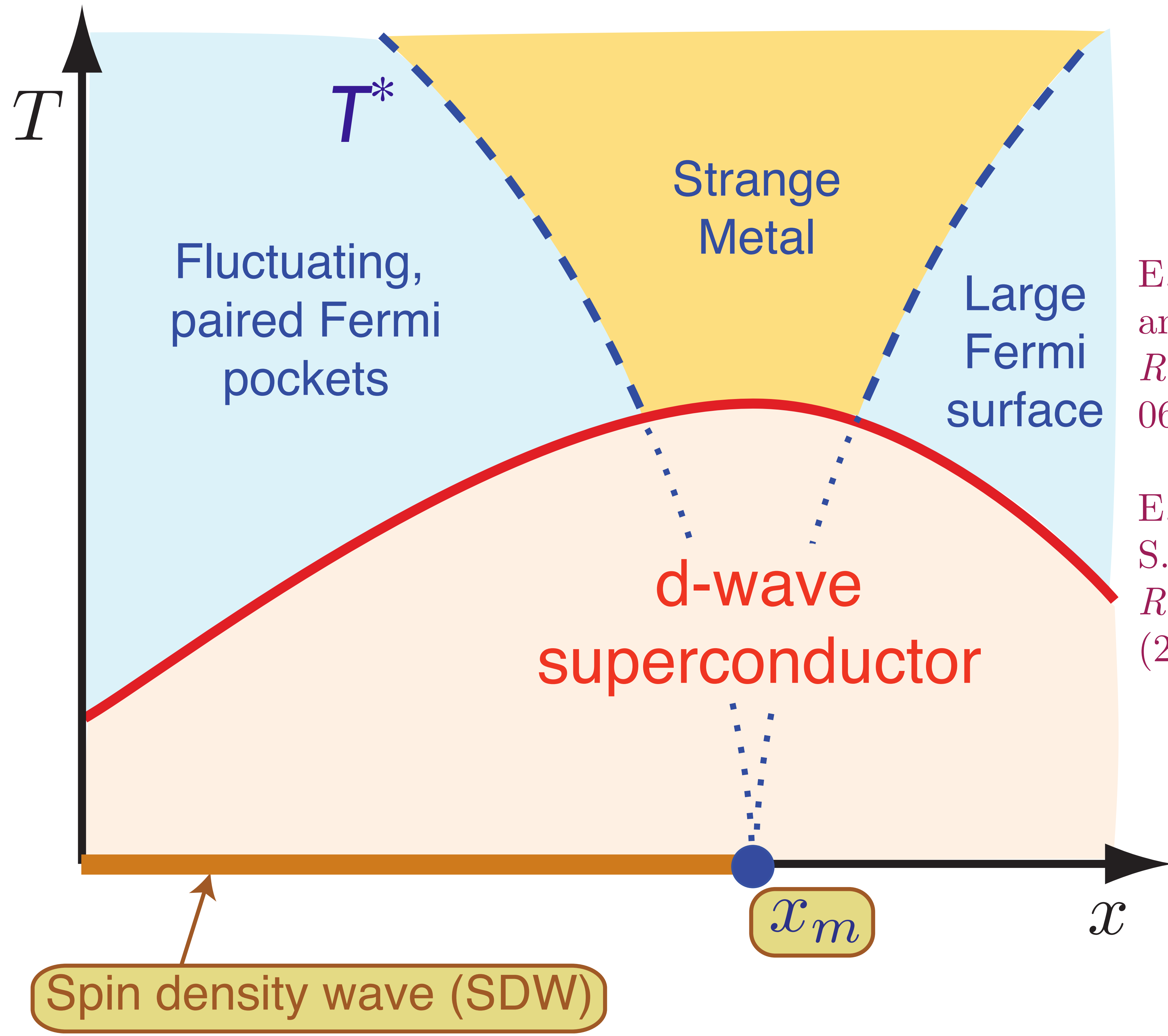


Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates



E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

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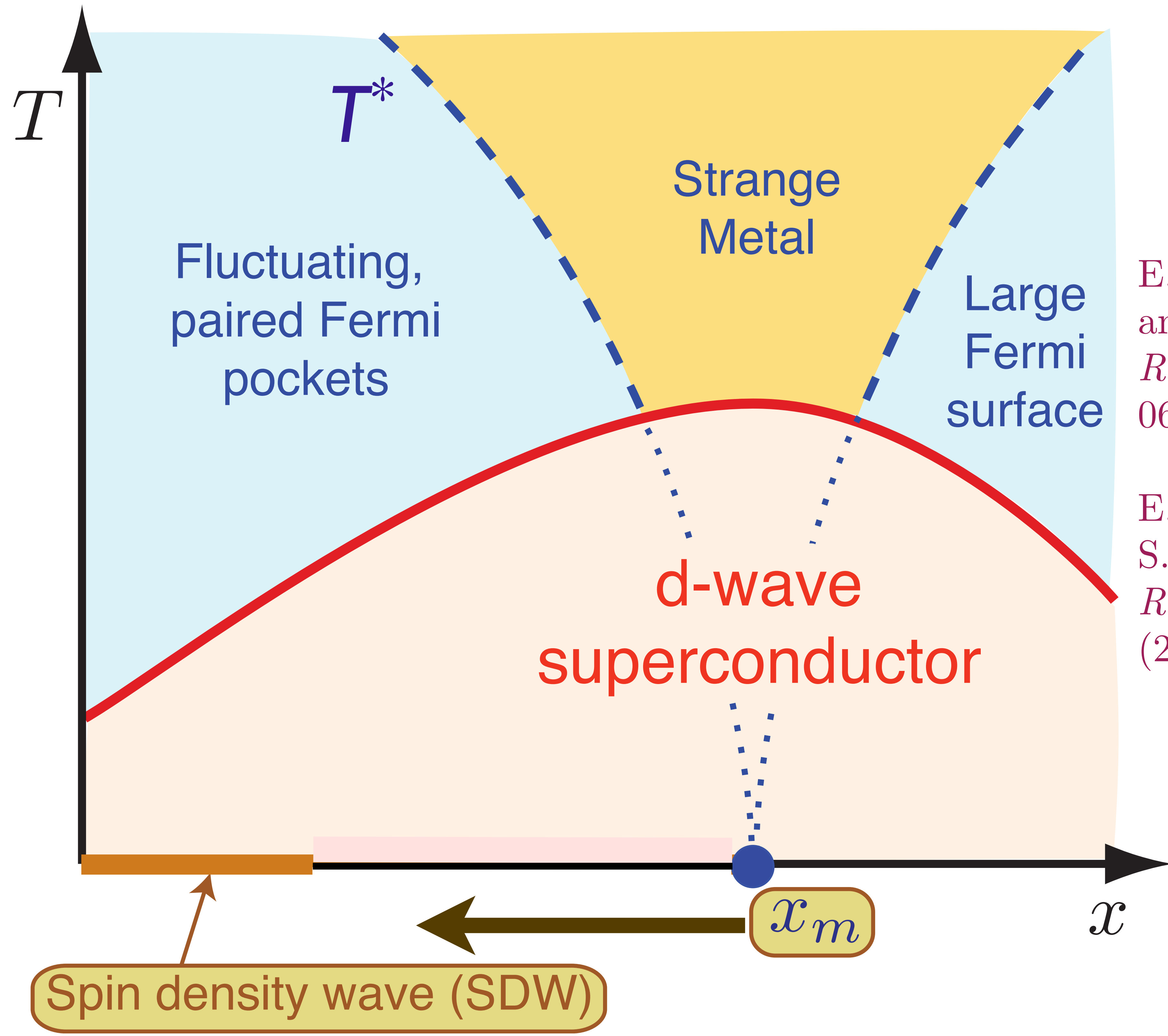

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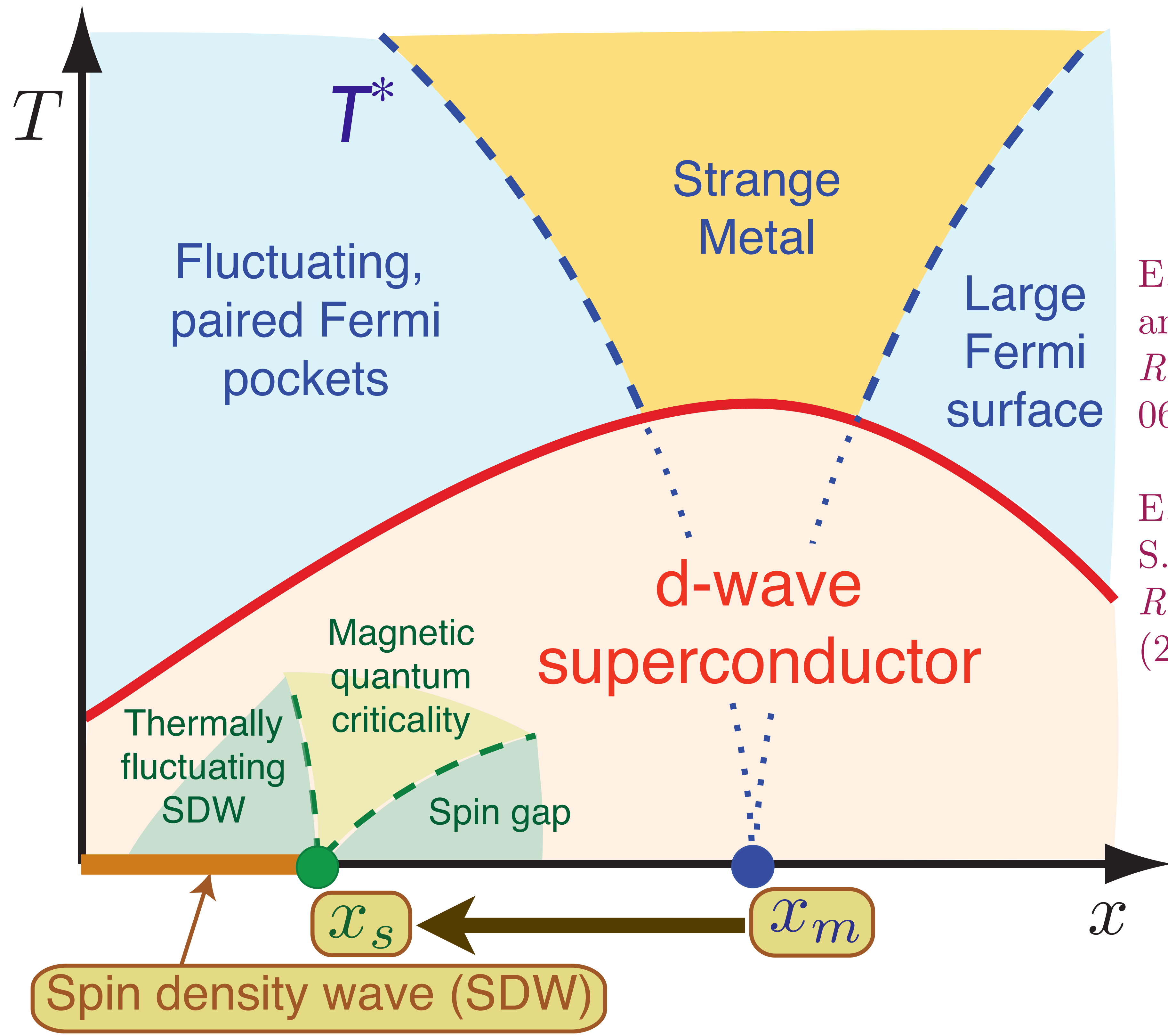

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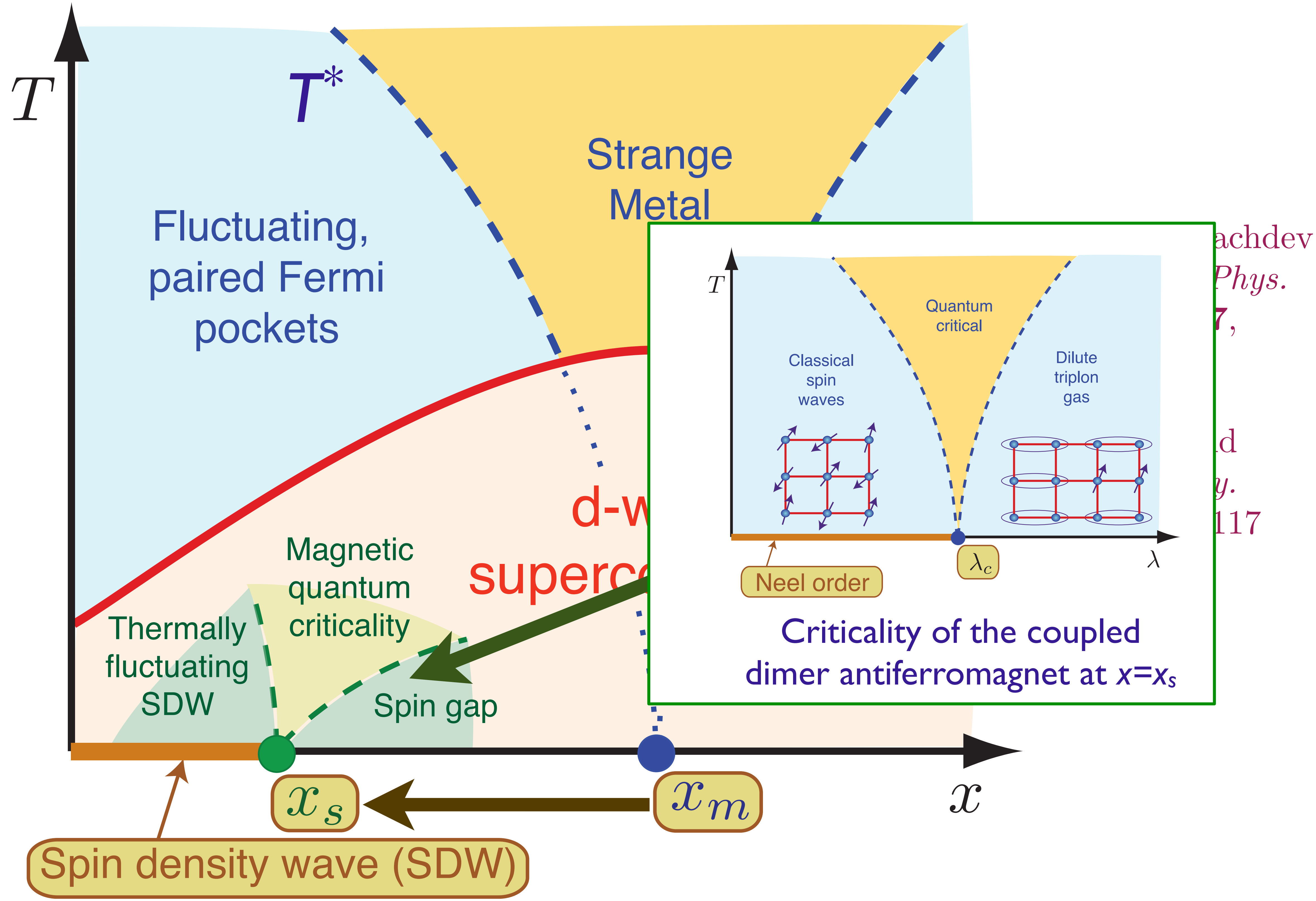

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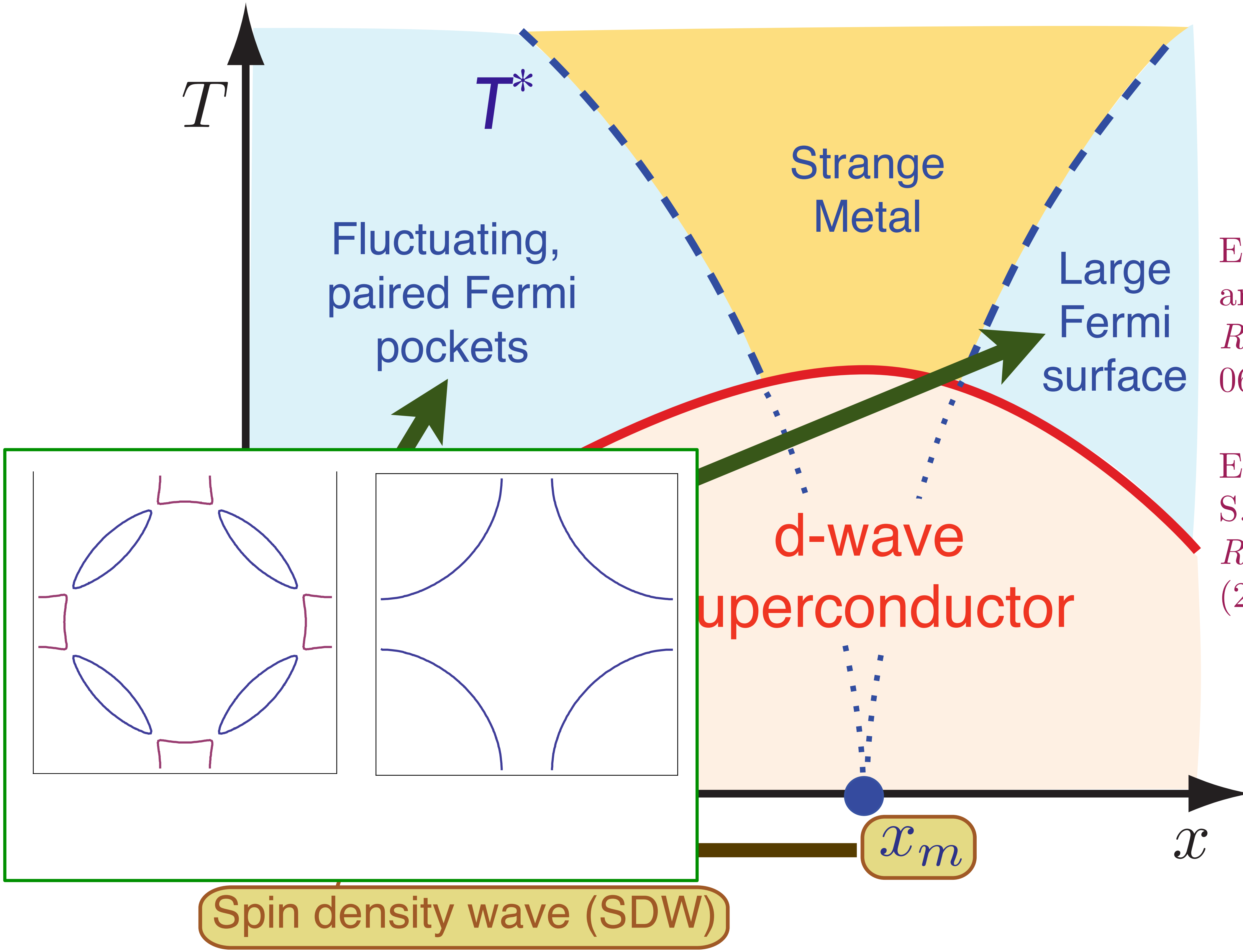

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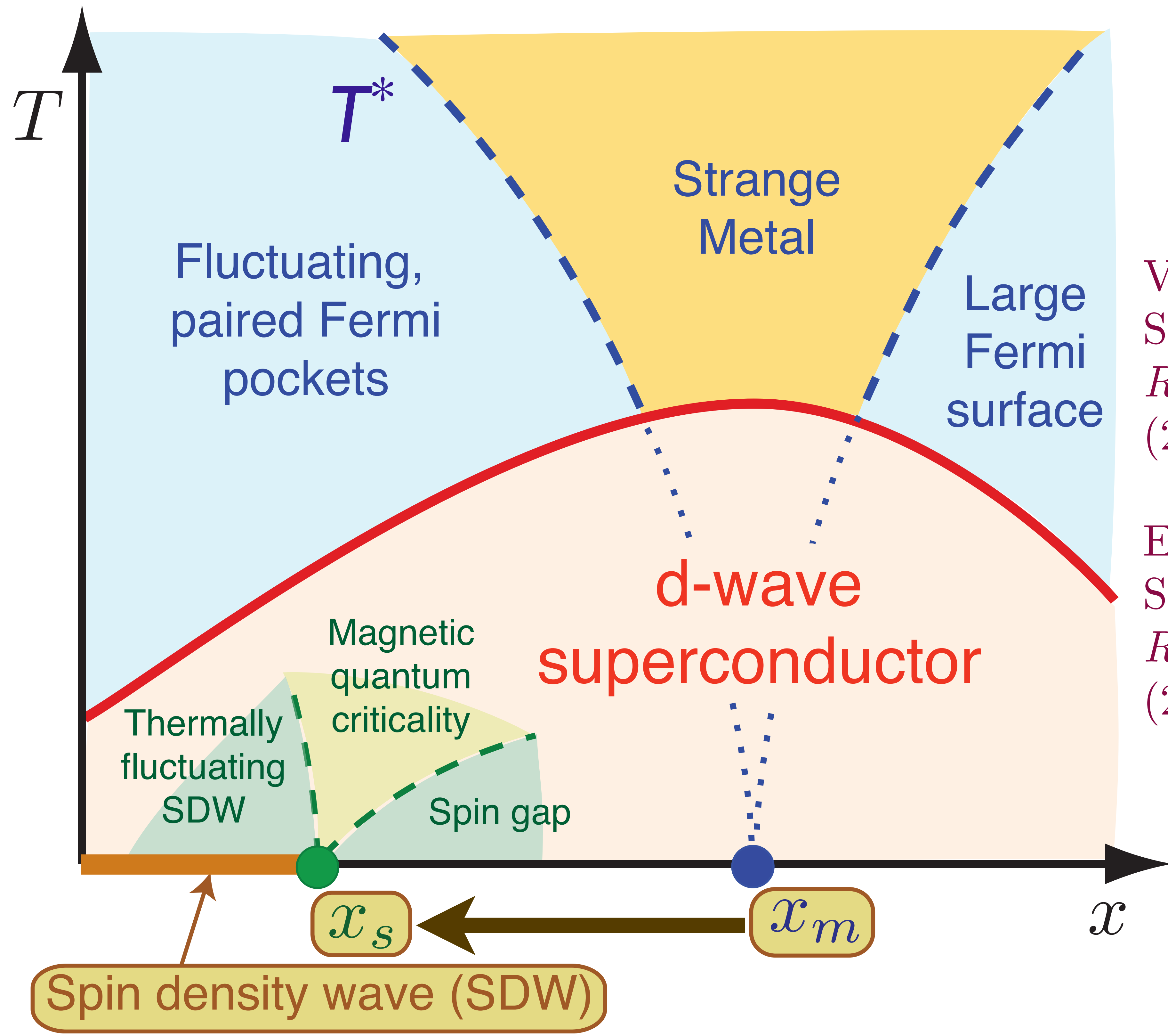

V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Physics of competition: *d*-wave SC and SDW "eat up" same pieces of the large Fermi surface.

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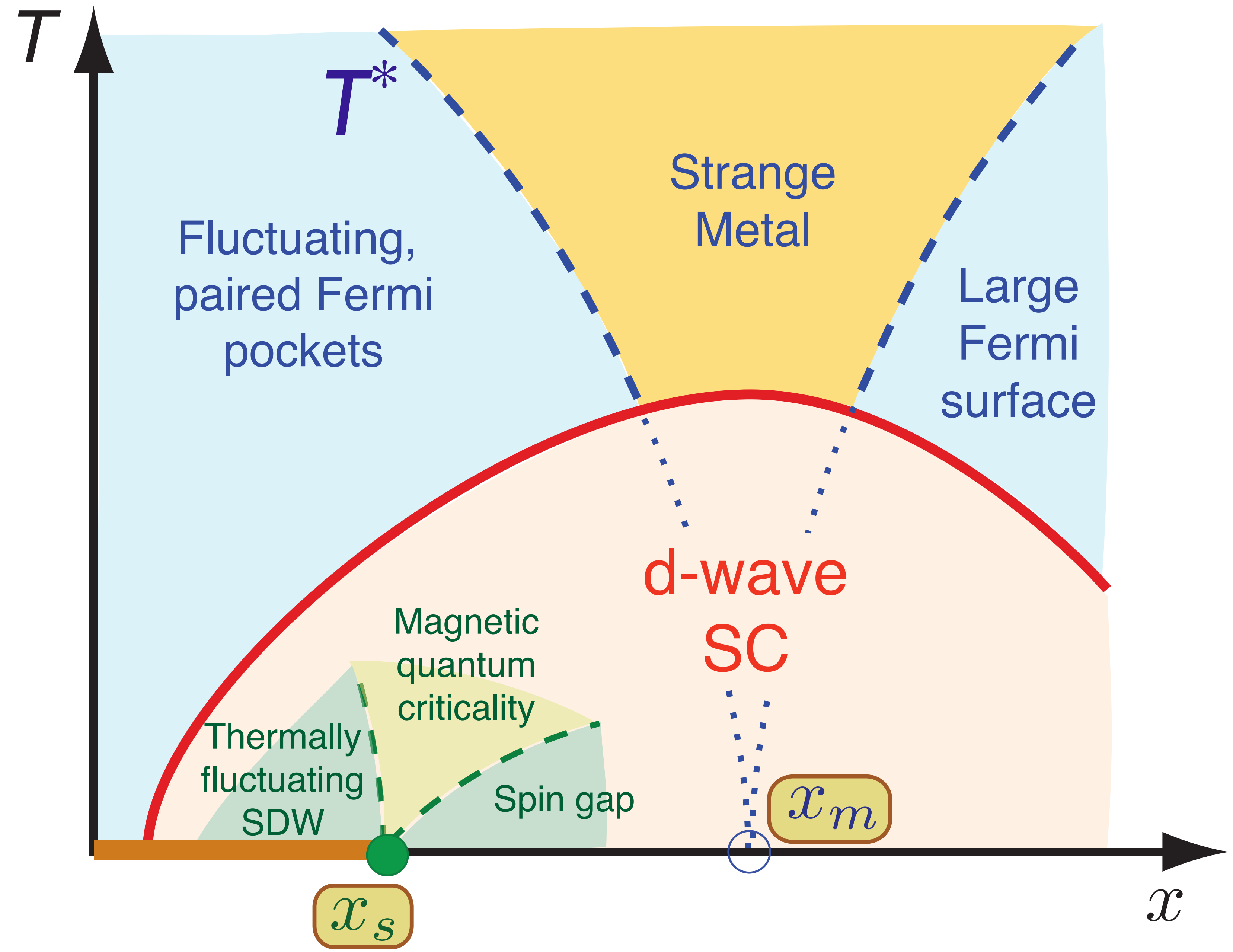
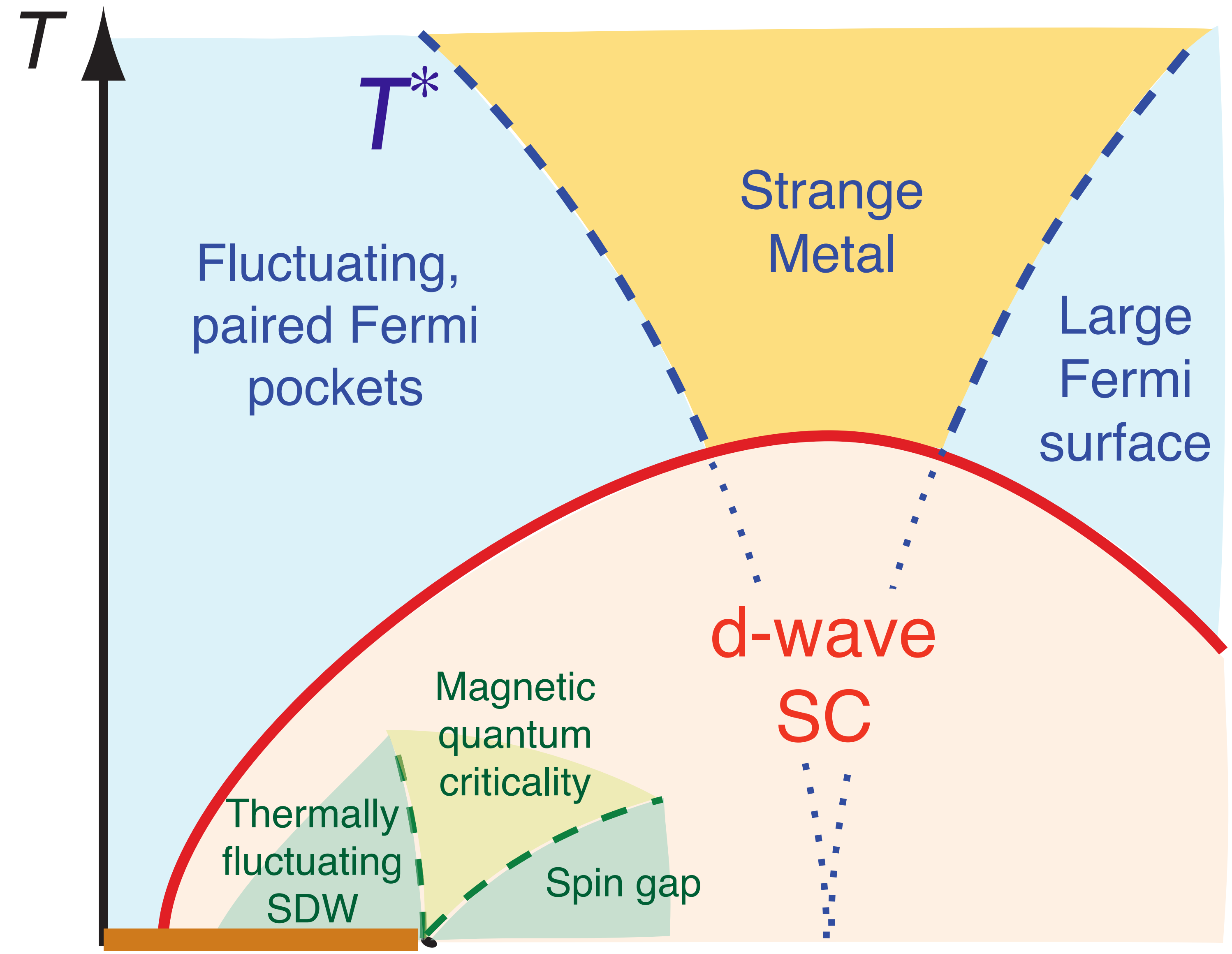


T
T^*
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x



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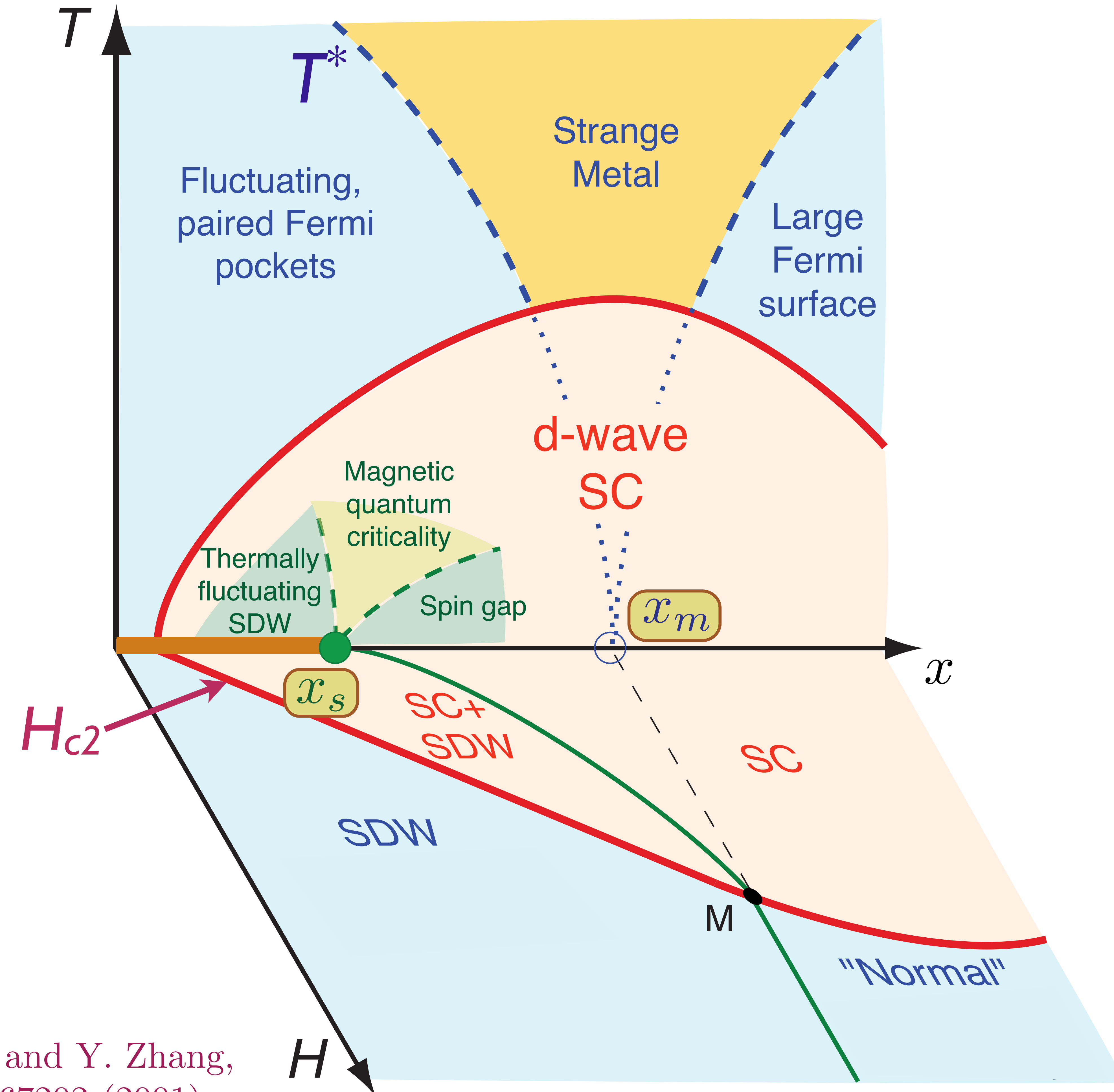

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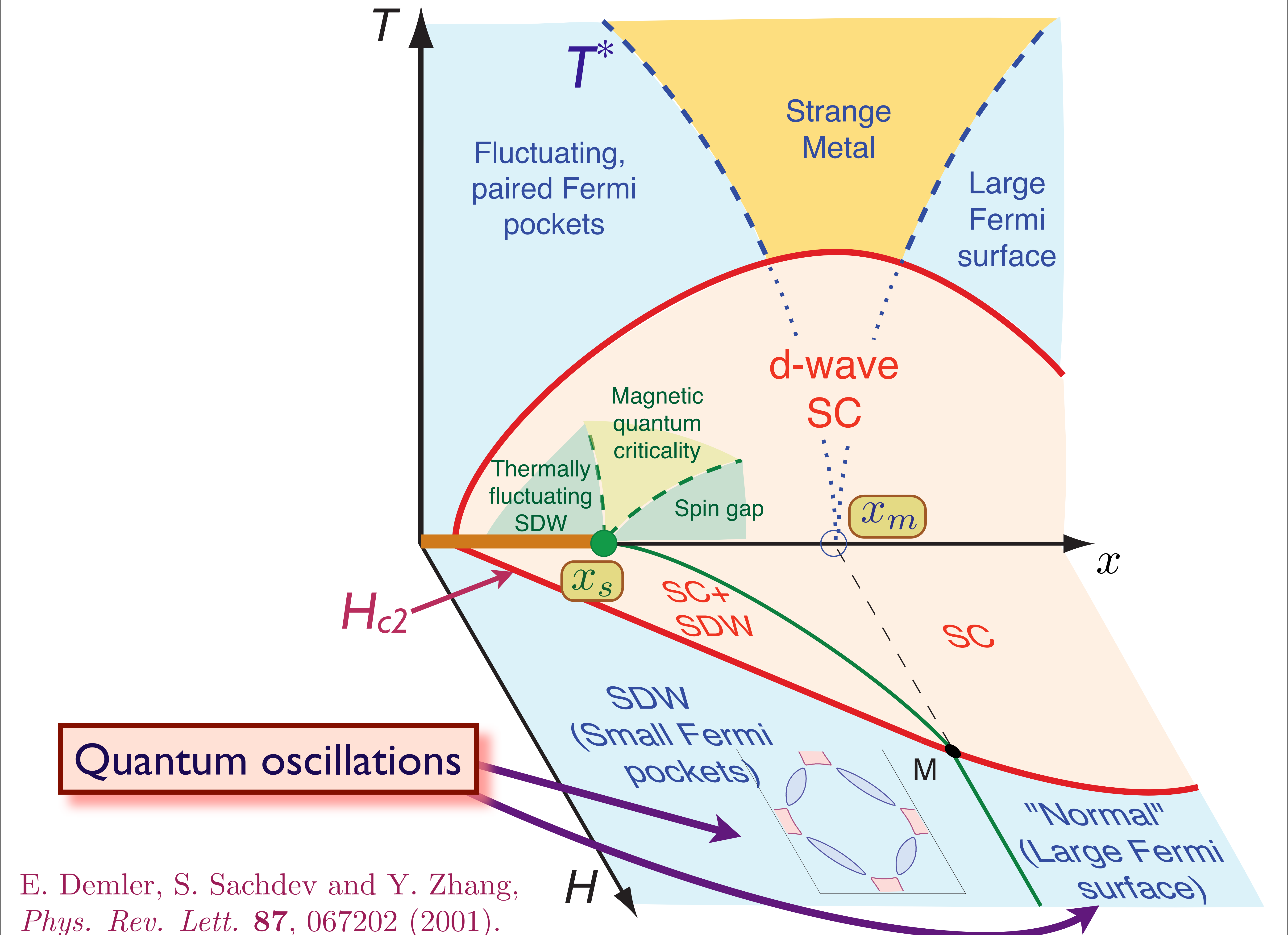

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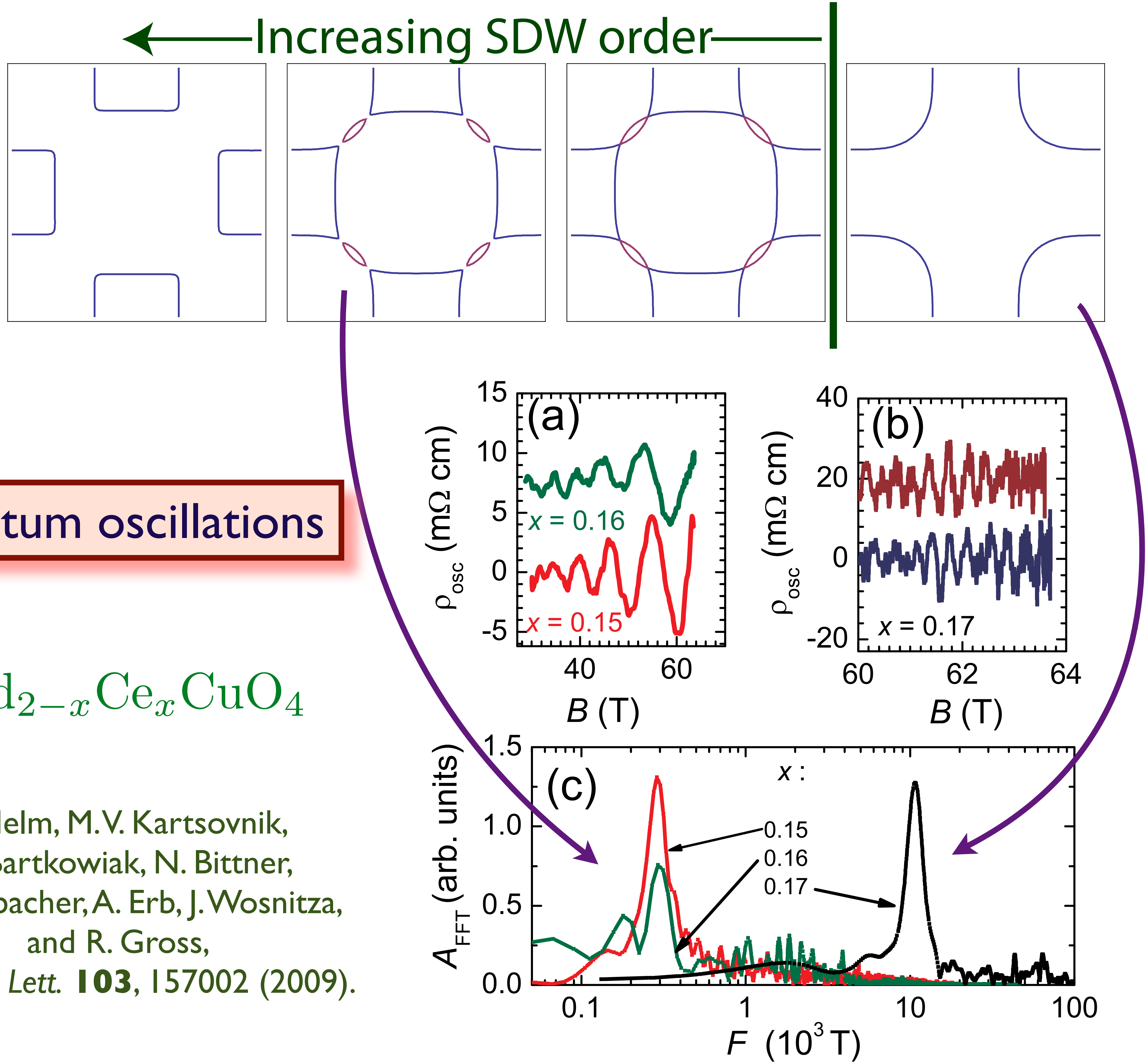
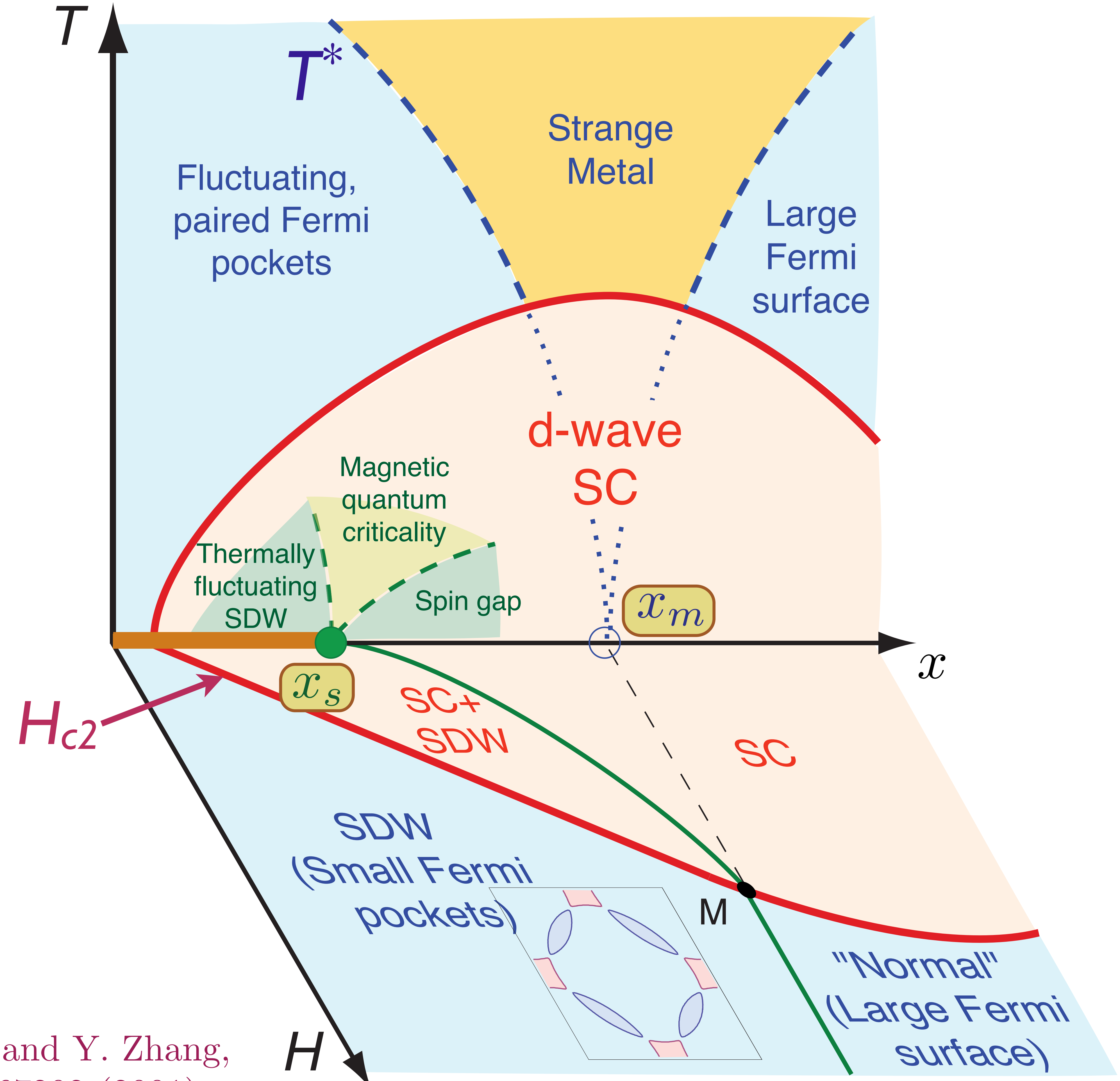


Quantum oscillations

$Nd_{2-x}Ce_xCuO_4$

T. Helm, M.V. Kartsovnik, M. Bartkowiak, N. Bittner, M. Lambacher, A. Erb, J. Wosnitza, and R. Gross, *Phys. Rev. Lett.* **103**, 157002 (2009).



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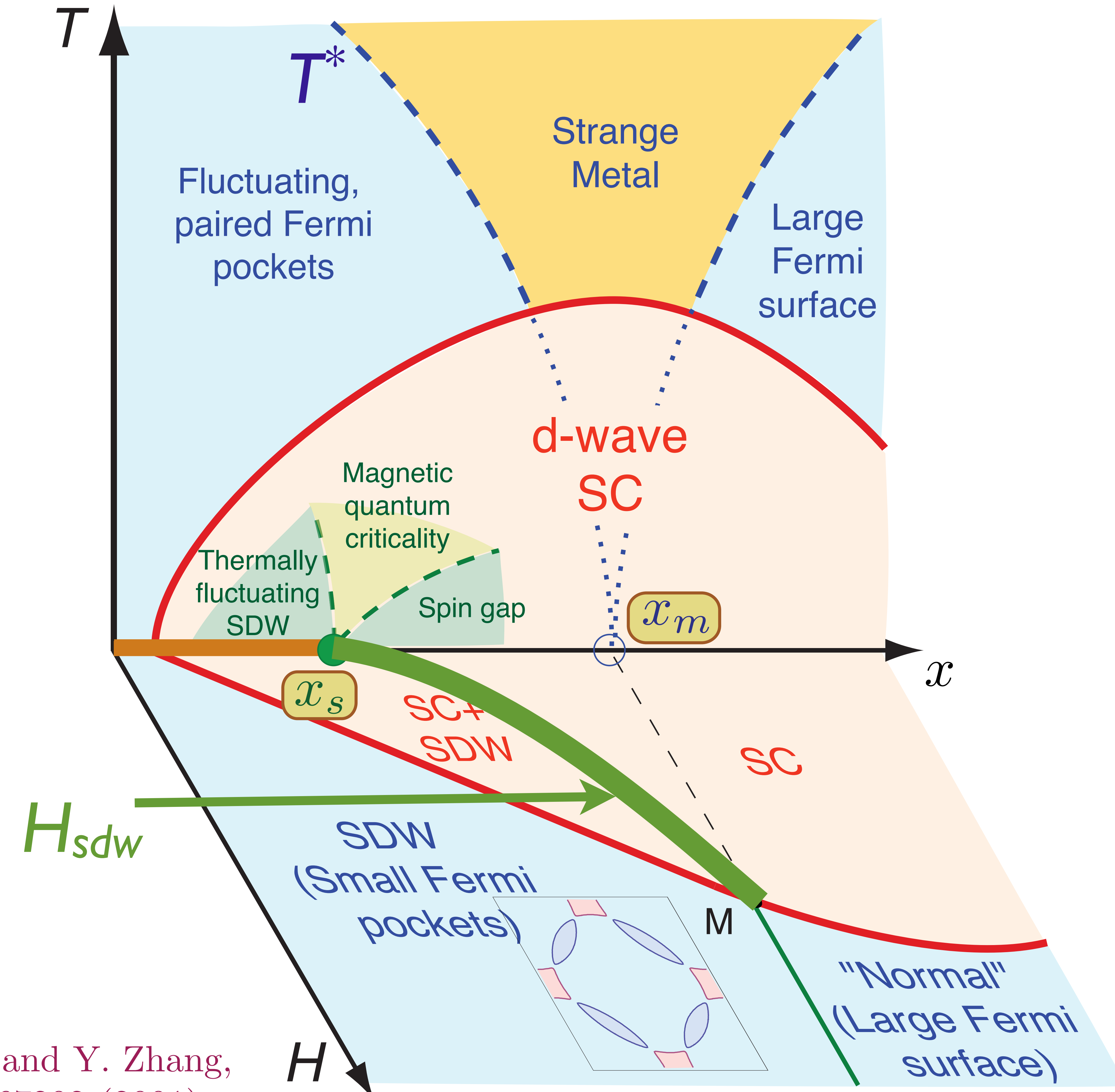

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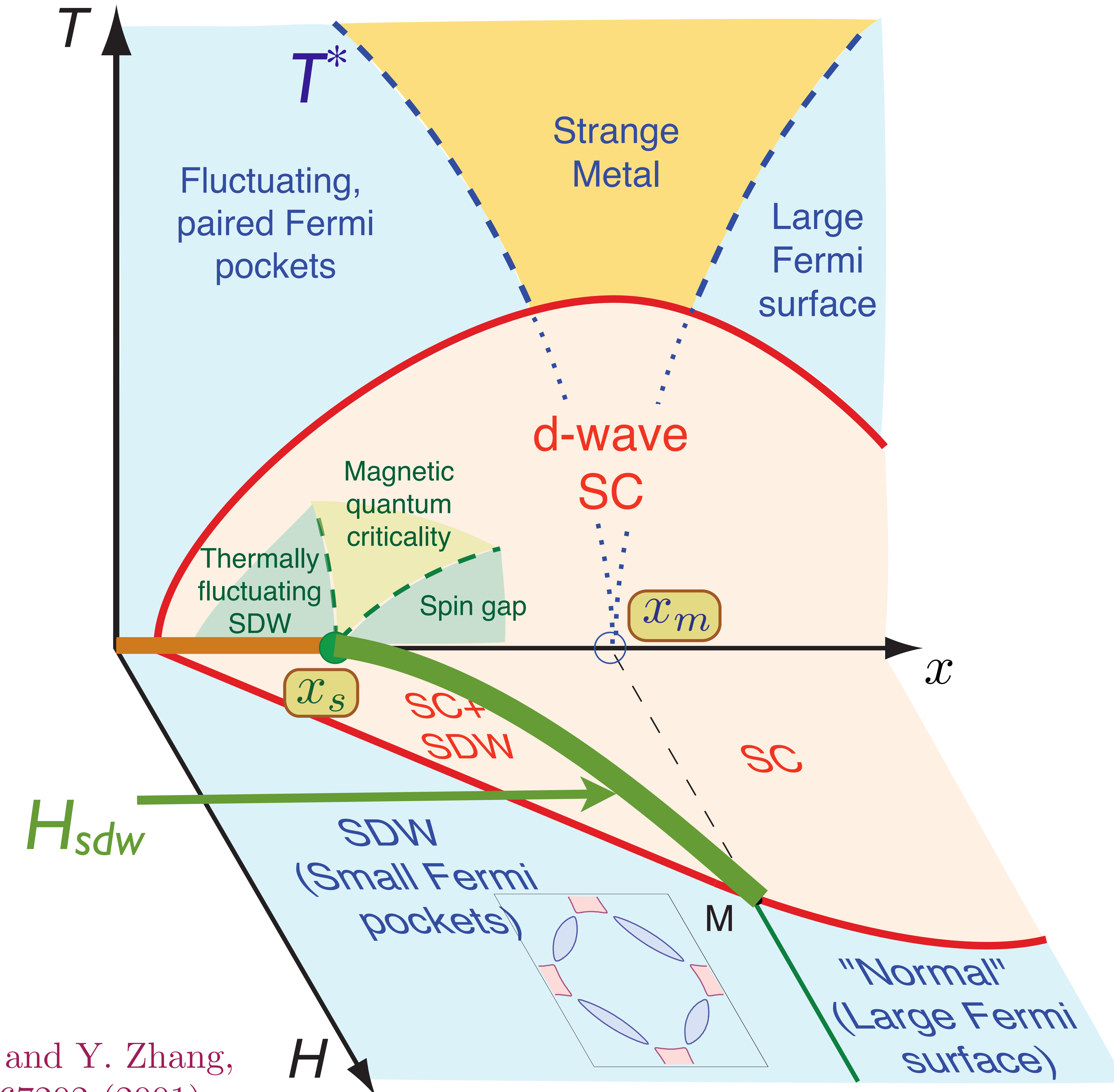

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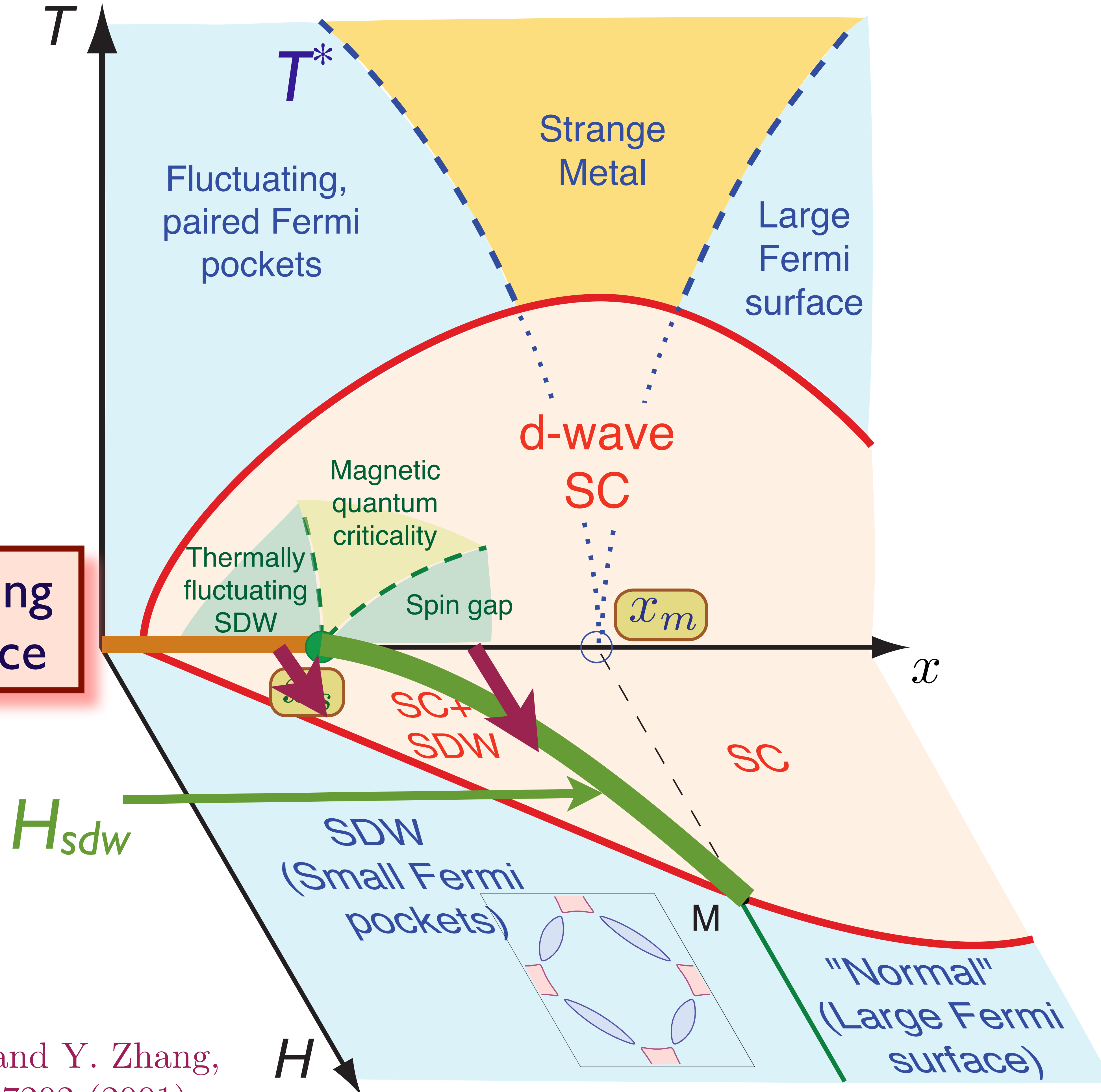
Neutron scattering & muon resonance



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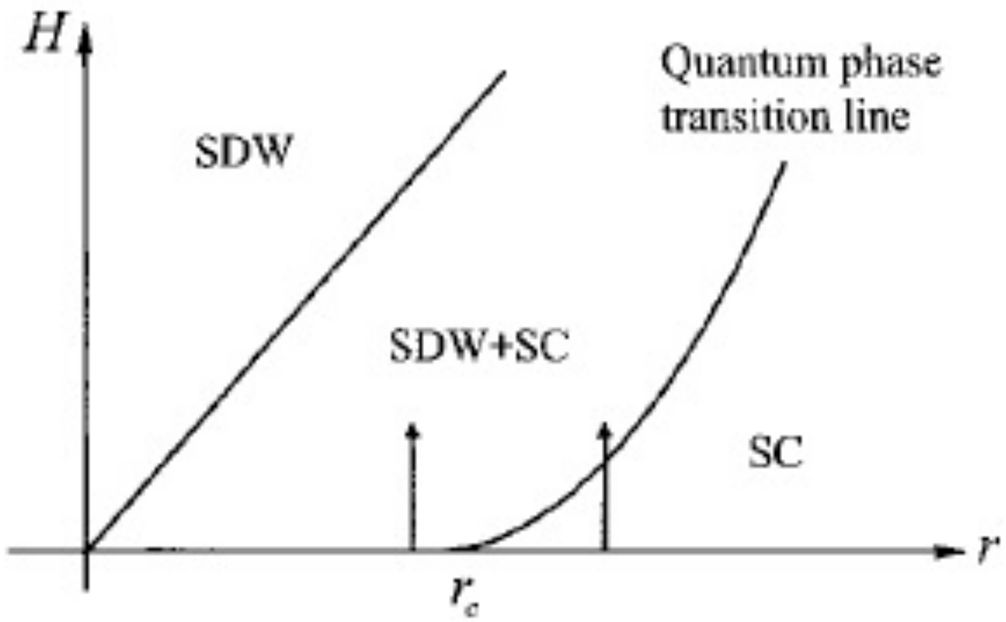
PHYSICAL REVIEW B **71**, 220508(R) (2005)

Field-induced transition between magnetically disordered and ordered phases in underdoped $La_{2-x}Sr_xCuO_4$

B. Khaykovich,[1] S. Wakimoto,[2] R. J. Birgeneau,[3] M. A. Kastner,[1] Y. S. Lee,[1] P. Smeibidl,[4] P. Vorderwisch,[4] and K. Yamada[5]



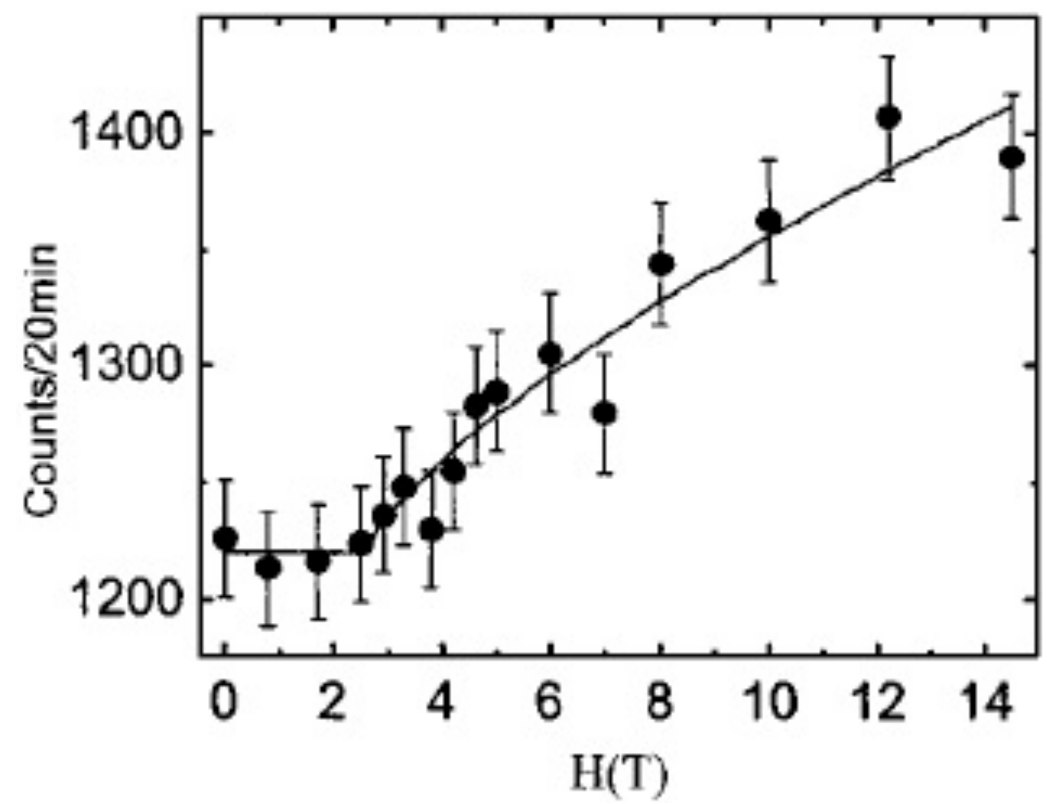
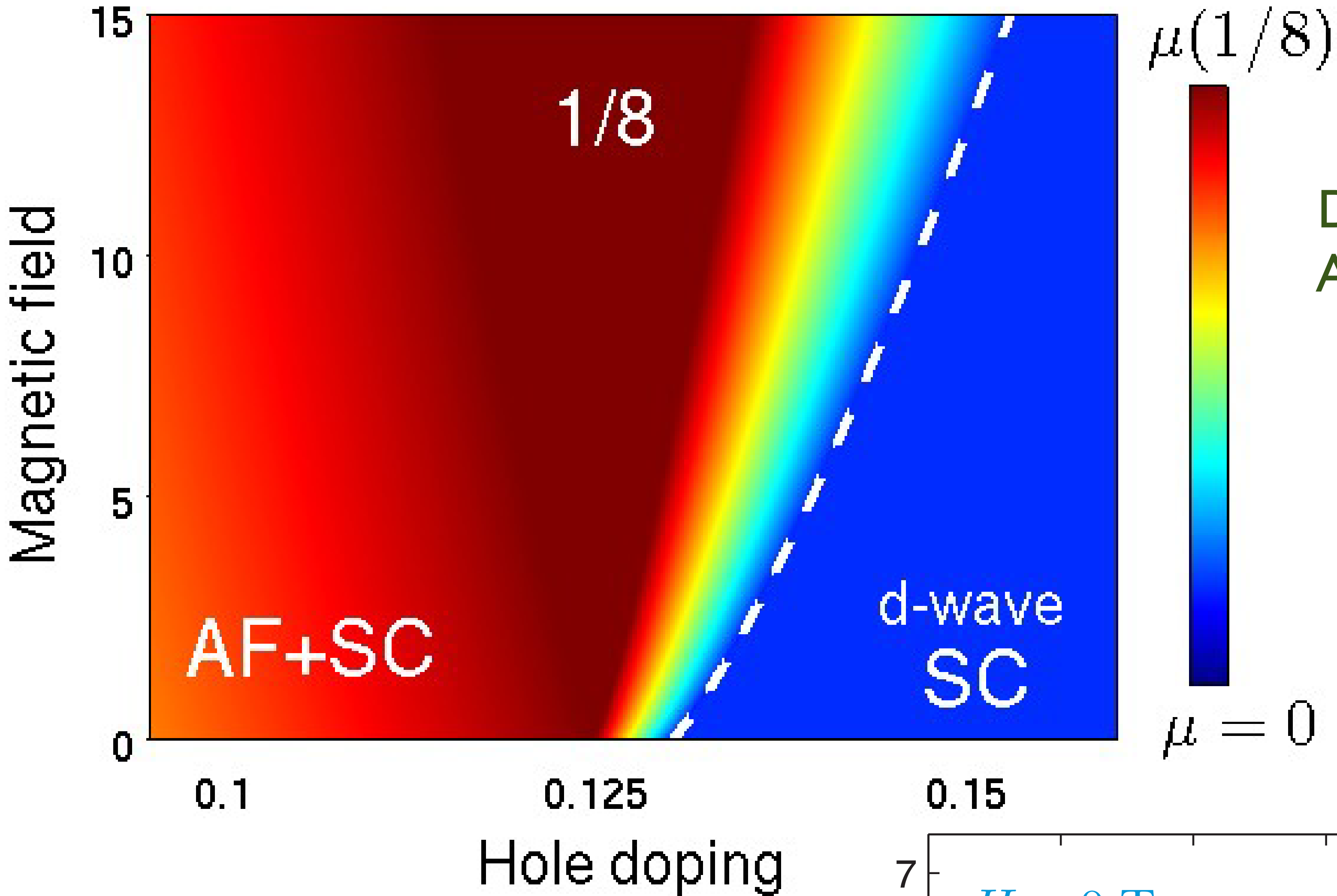
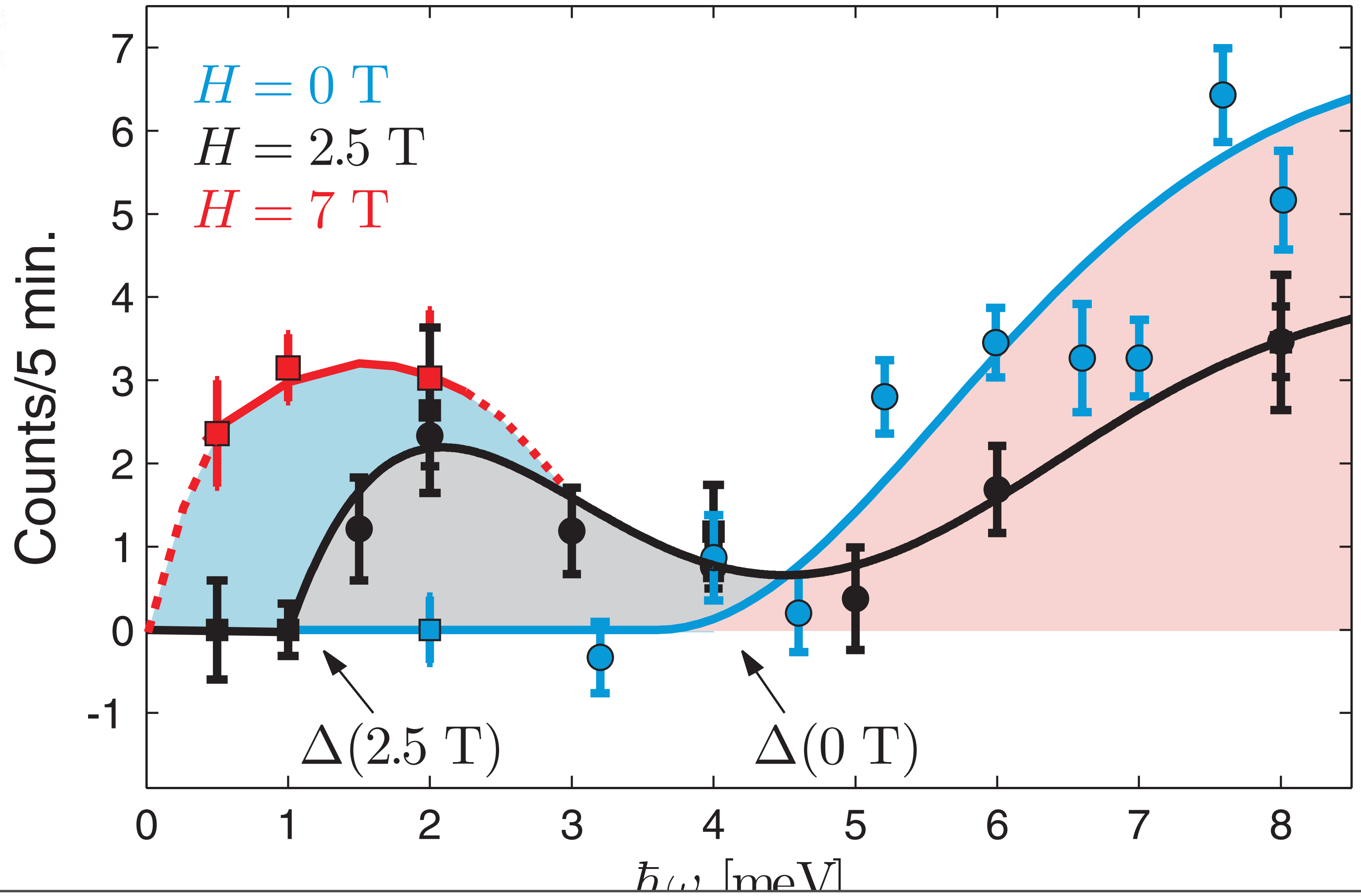


FIG. 1. (a) A fragment of the theoretical phase diagram, adopted from Refs. 4 and 20. The vertical axis is the magnetic field and the horizontal axis is the coupling strength between superconductivity and magnetic order. (b) Field dependence of the magnetic Bragg peak corresponding to the incommensurate SDW peak at $Q=(1.125,0.125,0)$. Every point is measured after field cooling at $T=1.5$ K. The data are fitted to $I=I_0+A|H-H_c|^{2\beta}$ above H_c as explained in the text. Spectrometer configuration: 45-60-Be—S—Be-60-open; cold Be filters were used before and after the sample to eliminate contamination from high-energy neutrons; $E=4$ meV.

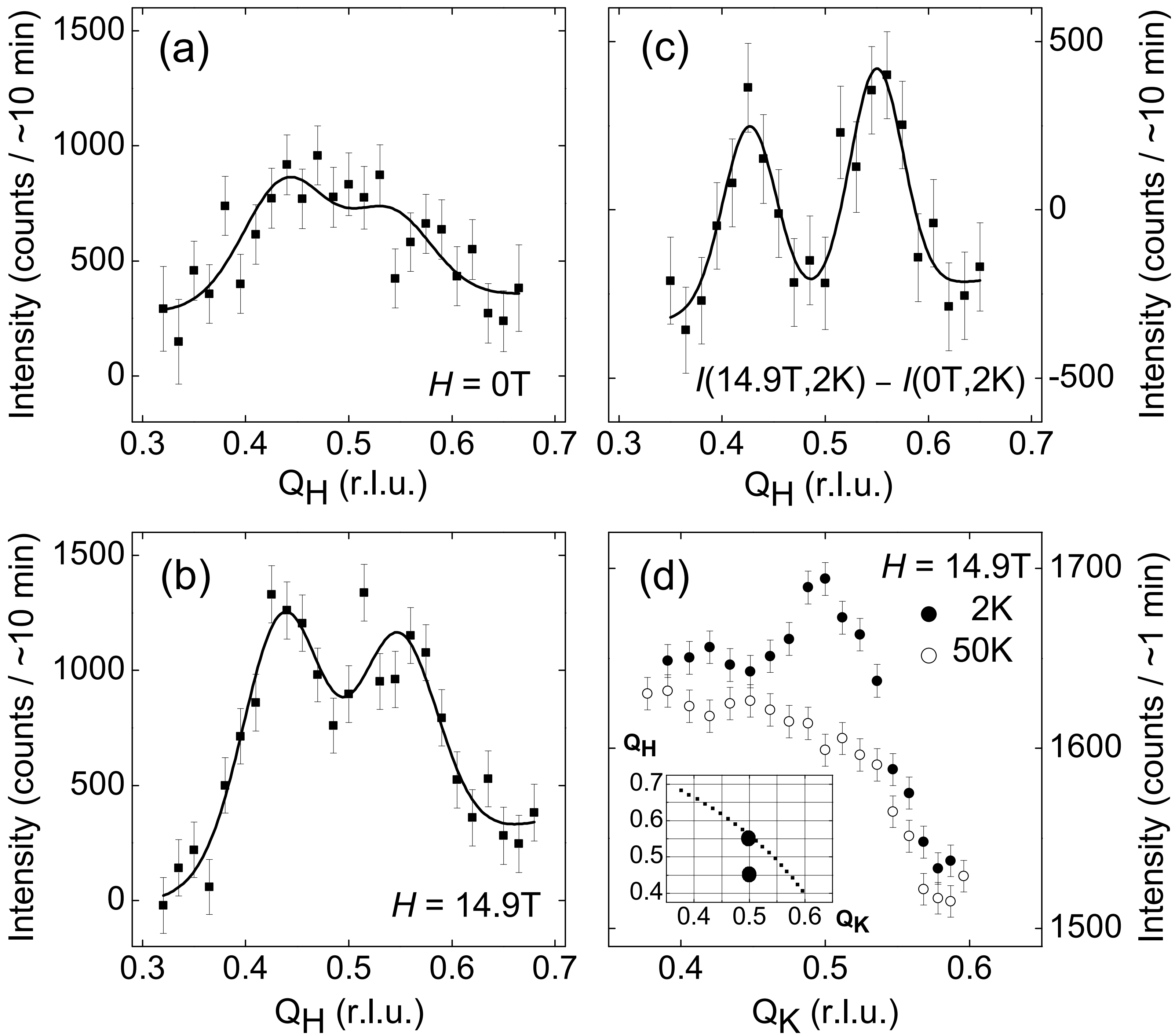
Thursday, February 25, 2010

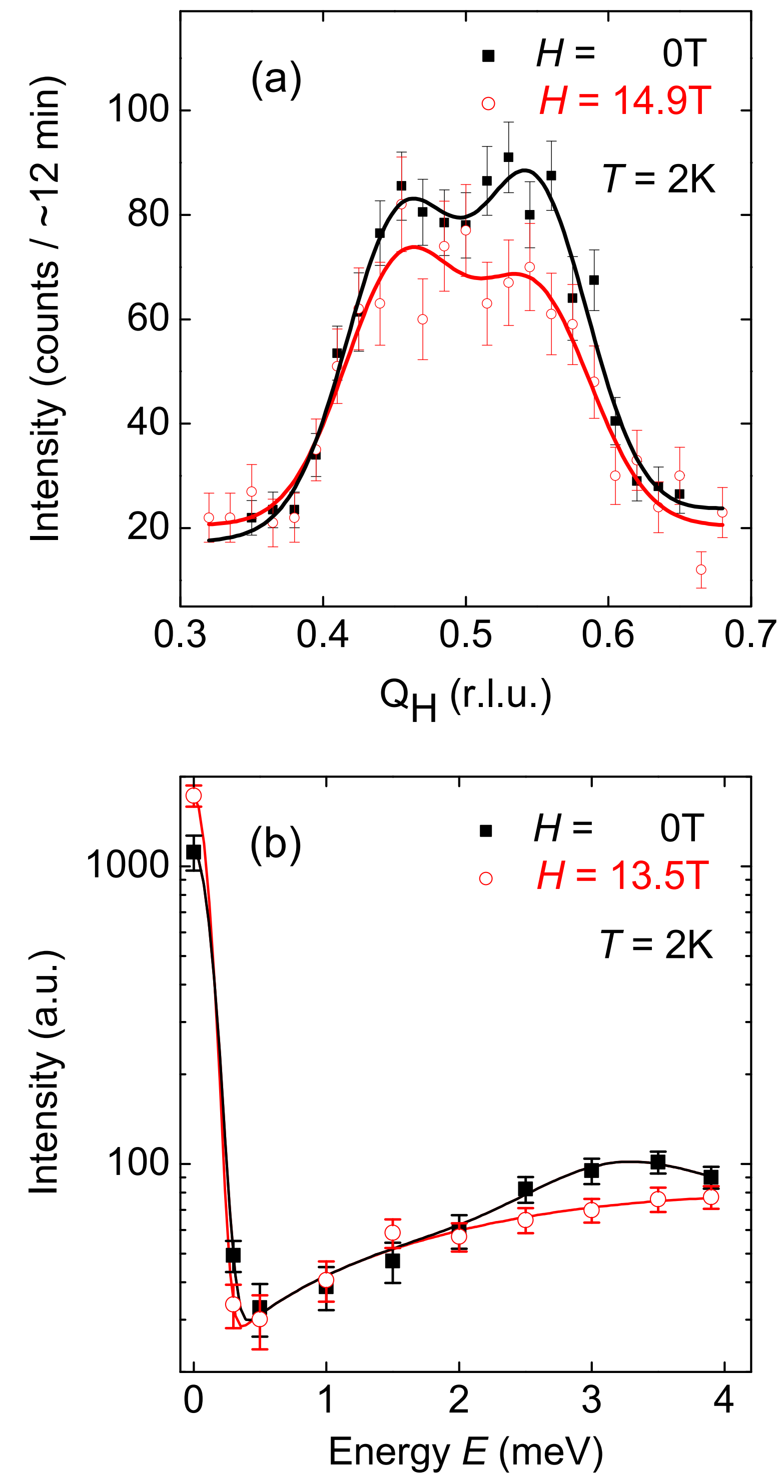


J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).

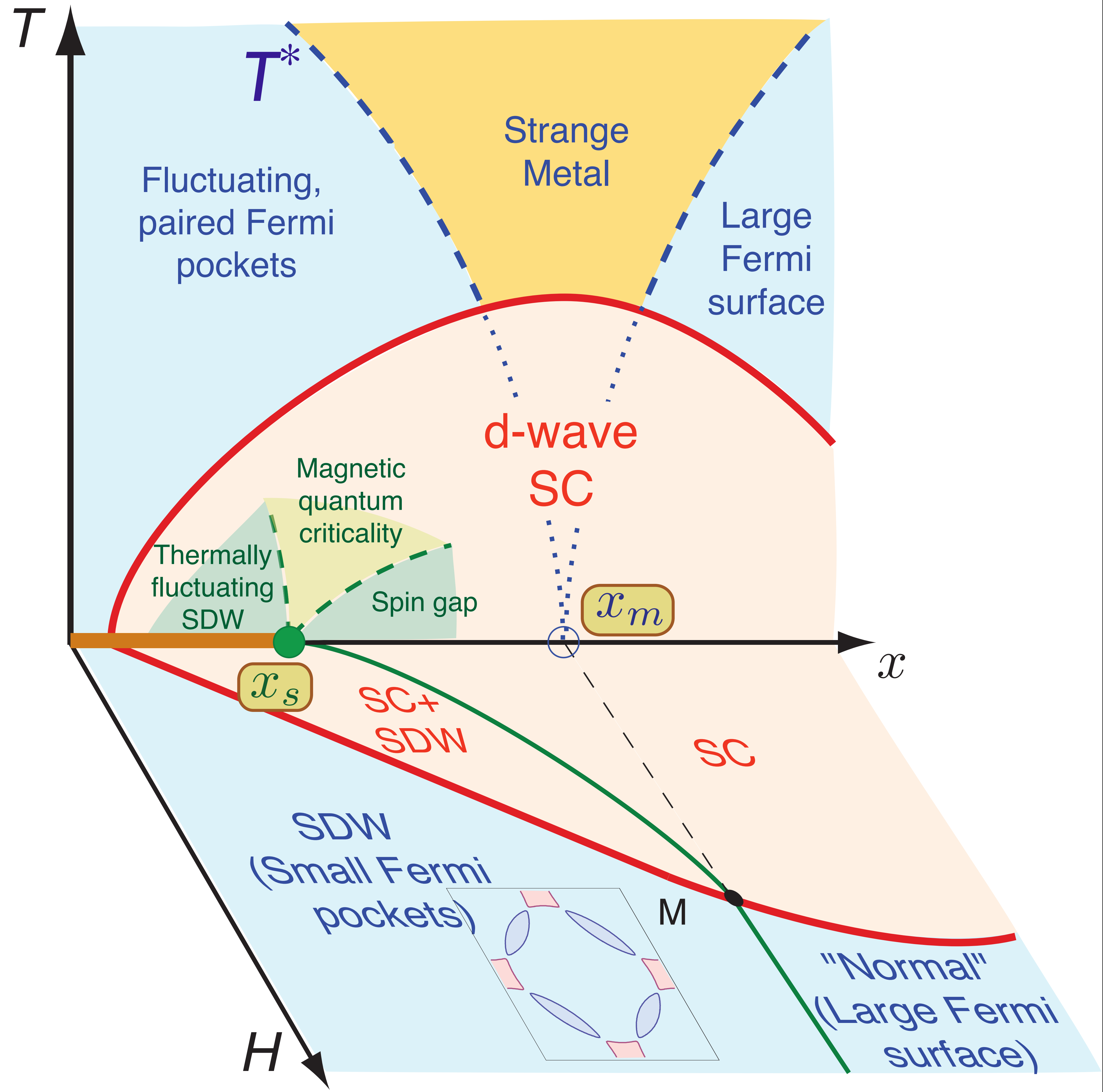


J. Chang, N. B. Christensen, Ch. Niedermayer, K. Lefmann, H. M. Roennow, D. F. McMorrow, A. Schneidewind, P. Link, A. Hiess, M. Boehm, R. Mottl, S. Pailhes, N. Momono, M. Oda, M. Ido, and J. Mesot, *Phys. Rev. Lett.* **102**, 177006 (2009).

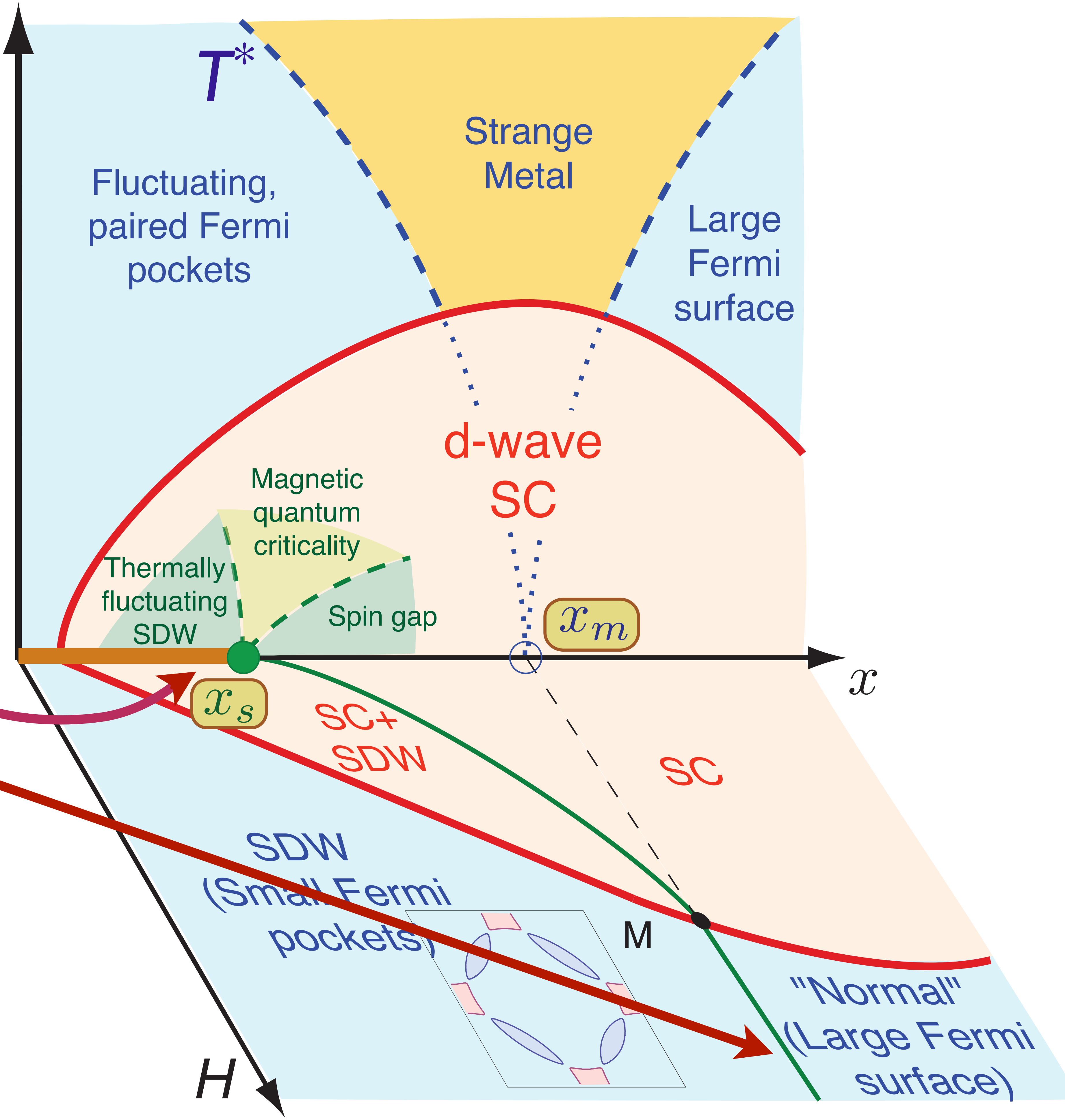




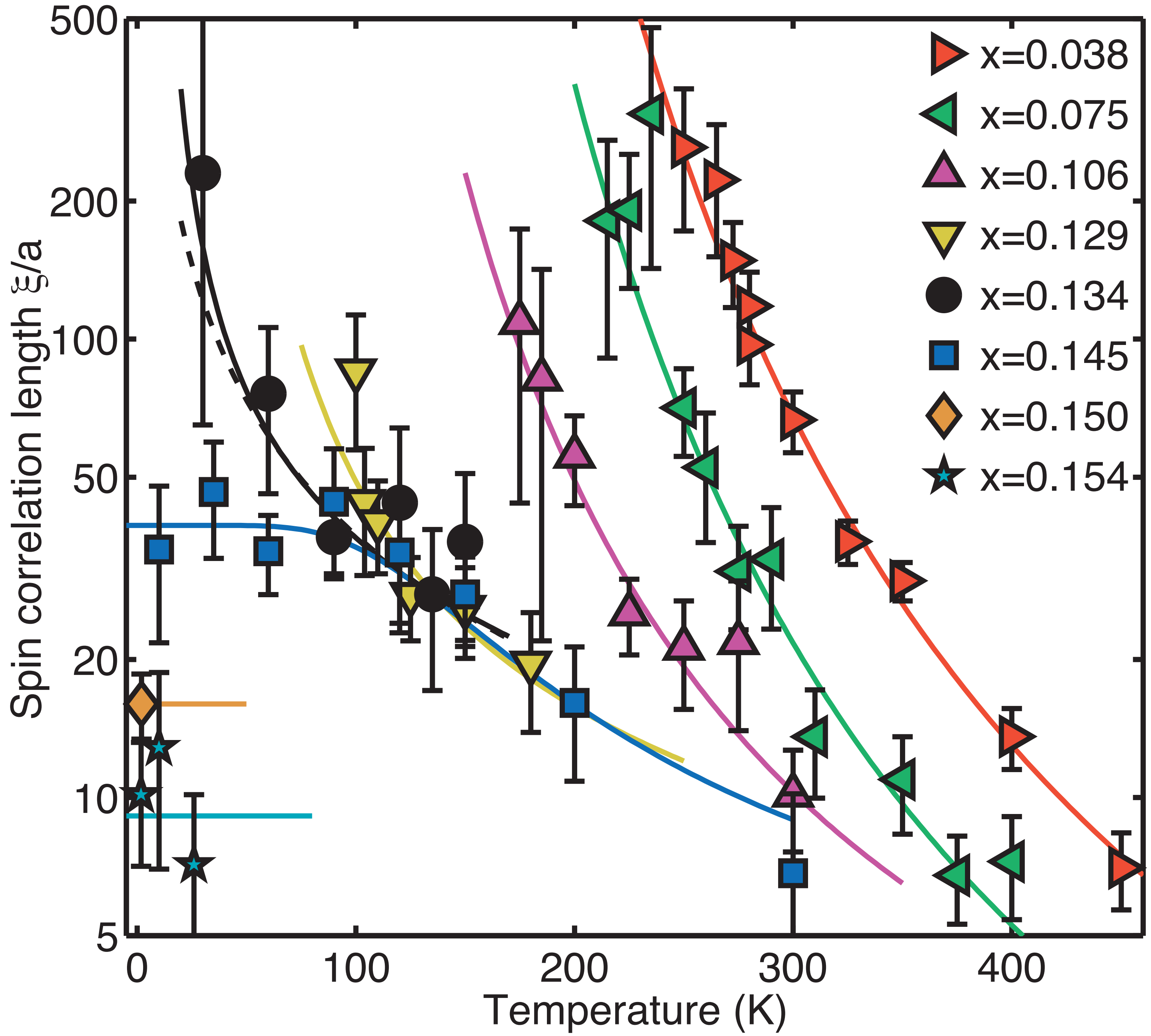
D. Haug, V. Hinkov, A. Suchaneck, D. S. Inosov, N. B. Christensen, Ch. Niedermayer, P. Bourges, Y. Sidis, J. T. Park, A. Ivanov, C. T. Lin, J. Mesot, and B. Keimer, *Phys. Rev. Lett.* **103**, 017001 (2009)


T
T^*
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
Magnetic quantum criticality
Thermally fluctuating SDW
Spin gap
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

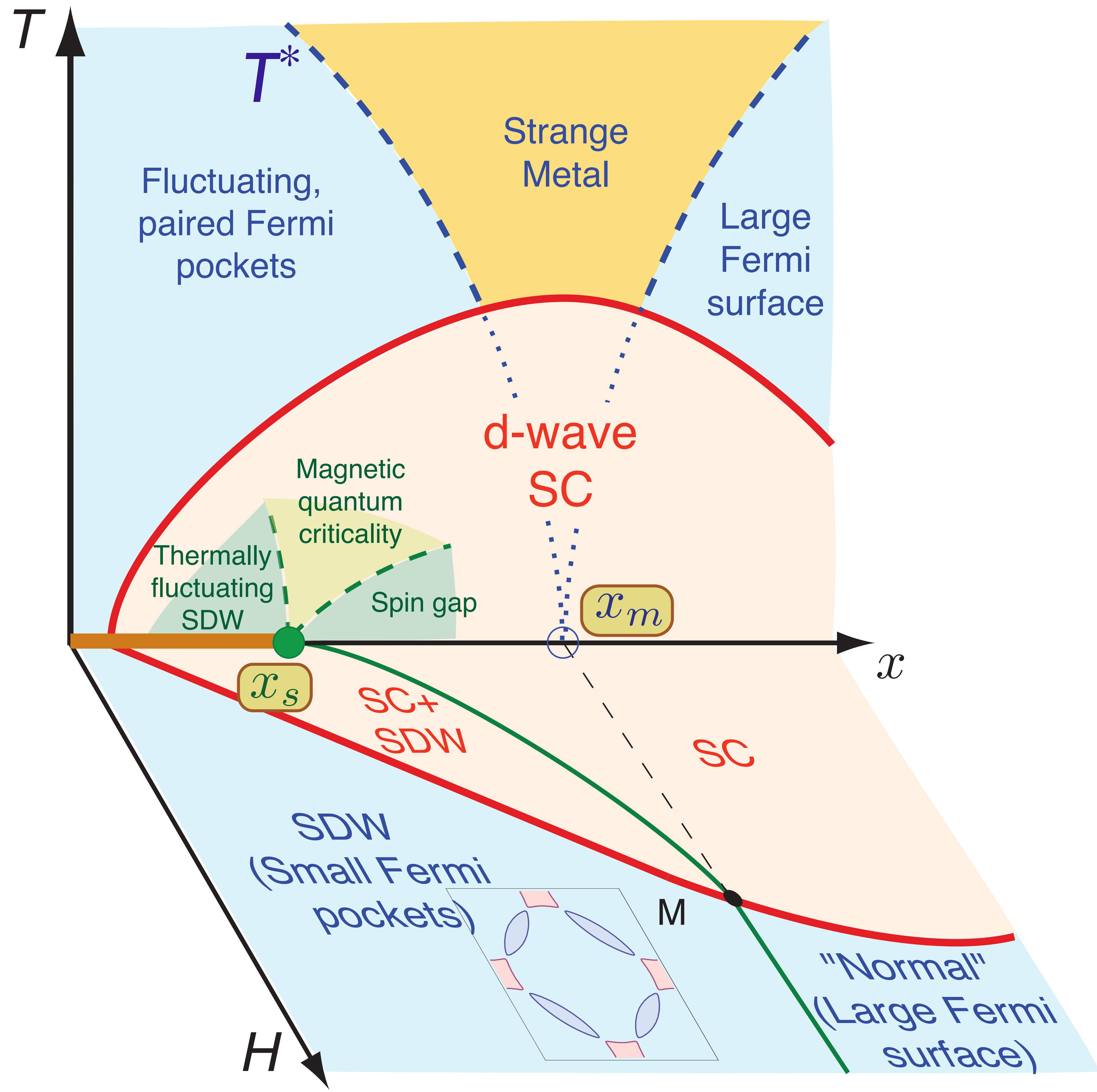
Neutron scattering experiments on $Nd_{2-x}Ce_xCuO_4$ show that at low fields $x_s = 0.14$, while quantum oscillations at high fields show that $x_m = 0.165$.



$Nd_{2-x}Ce_xCuO_4$



E. M. Motoyama, G. Yu, I. M. Vishik, O. P. Vajk, P. K. Mang, and M. Greven, *Nature* **445**, 186 (2007).



T
T^*
Fluctuating, paired Fermi pockets
Strange Metal
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Magnetic quantum criticality
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Spin gap
x_m
x_s
x
SC+ SDW
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SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
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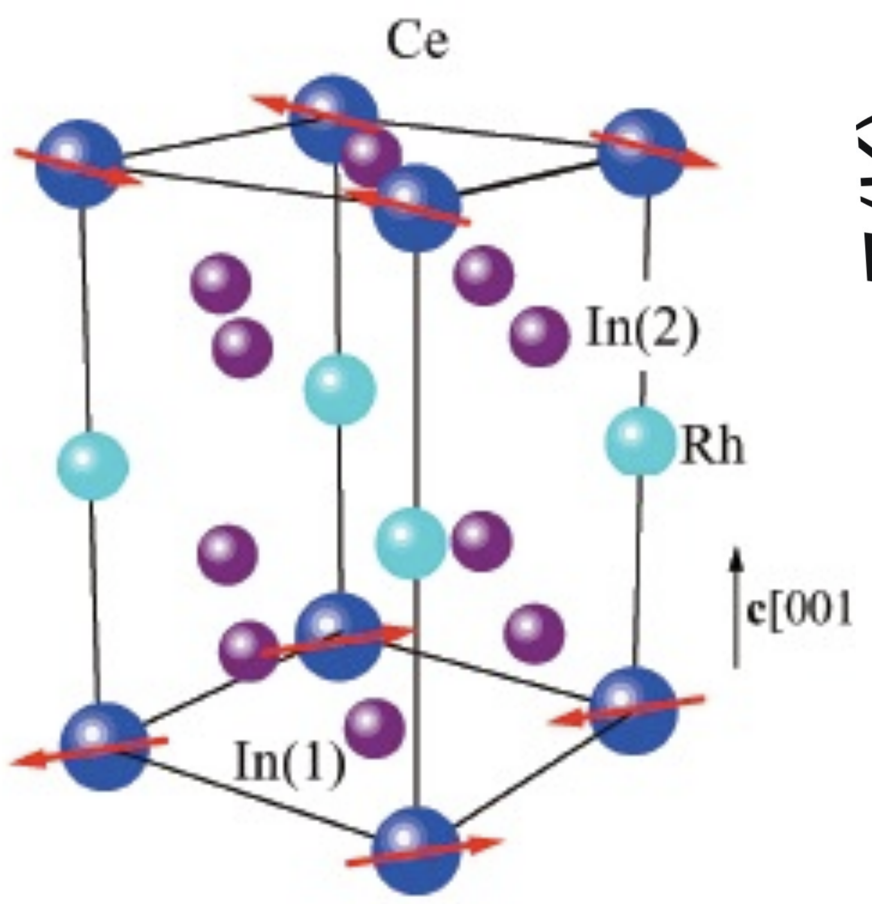
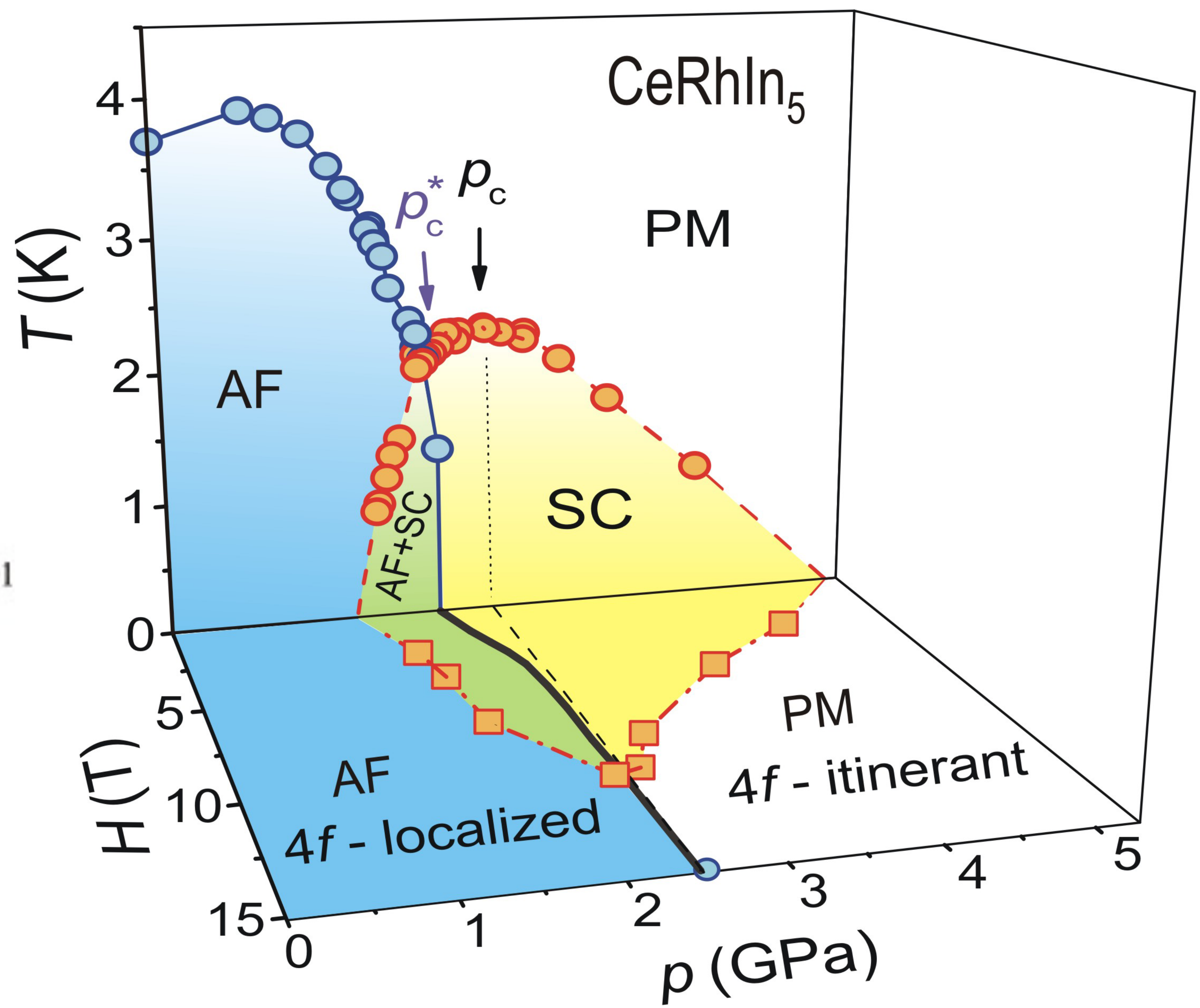
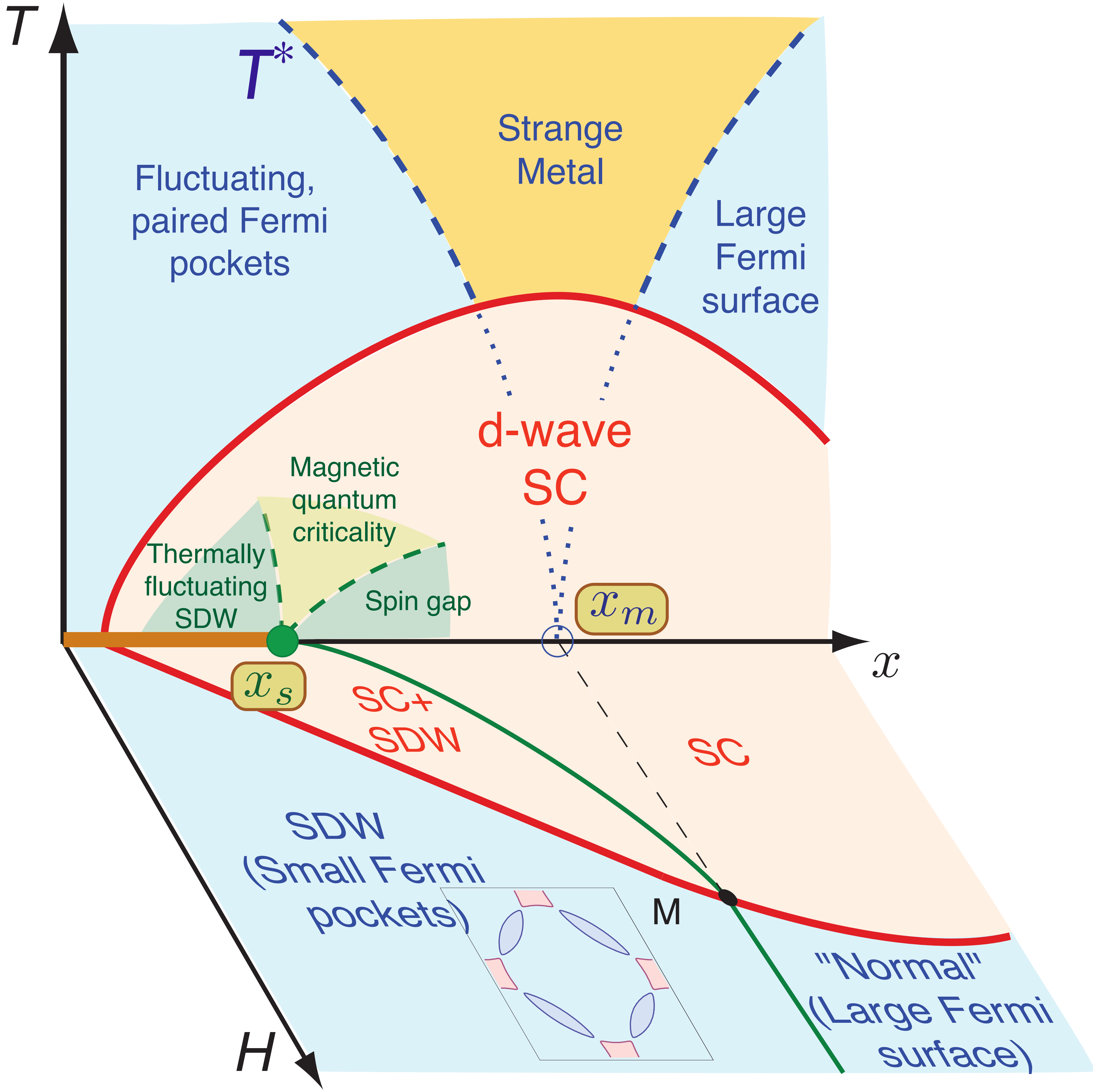
Similar phase diagram for $CeRhIn_5$





G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223



T
T*
Fluctuating, paired Fermi pockets
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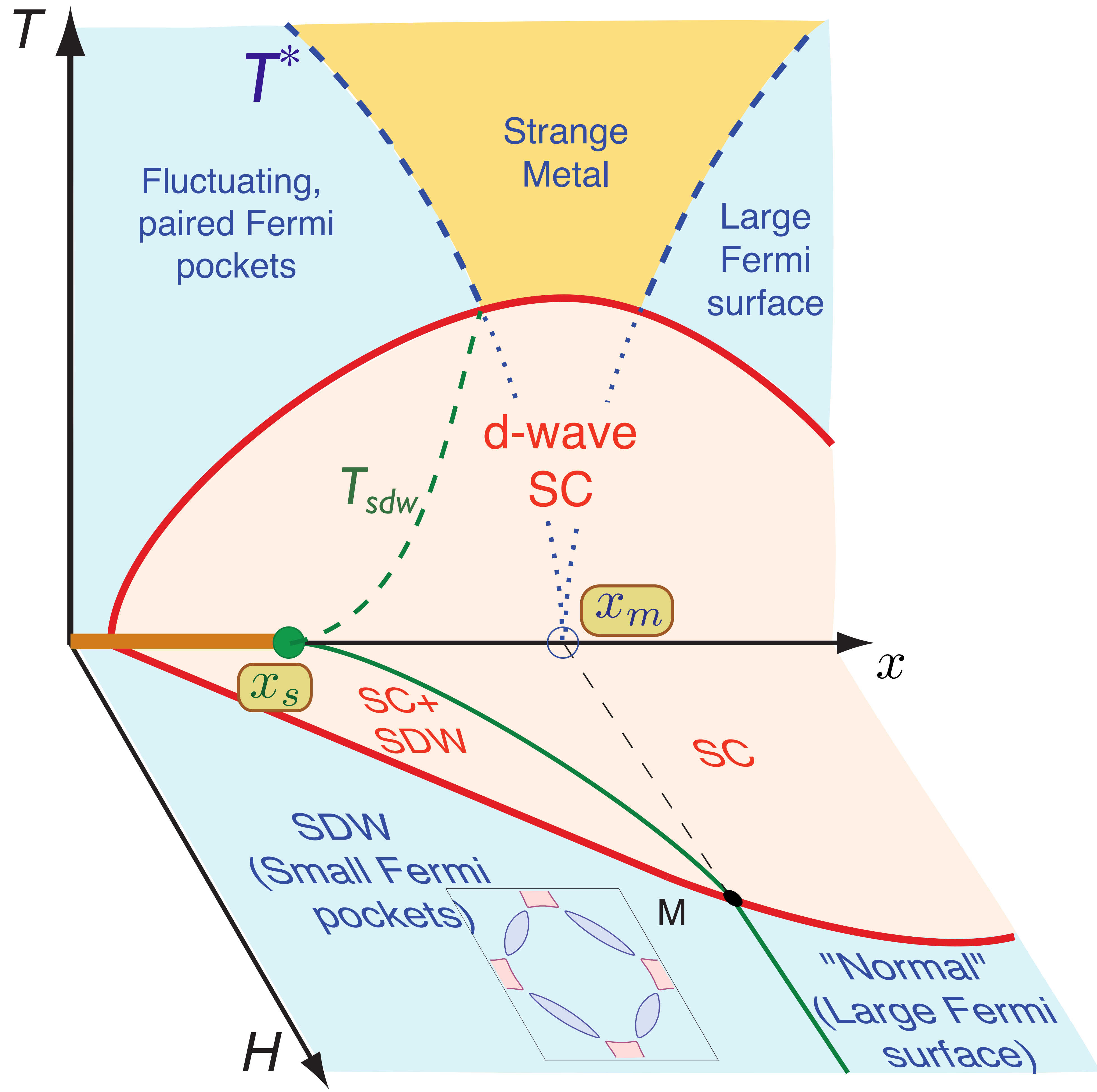
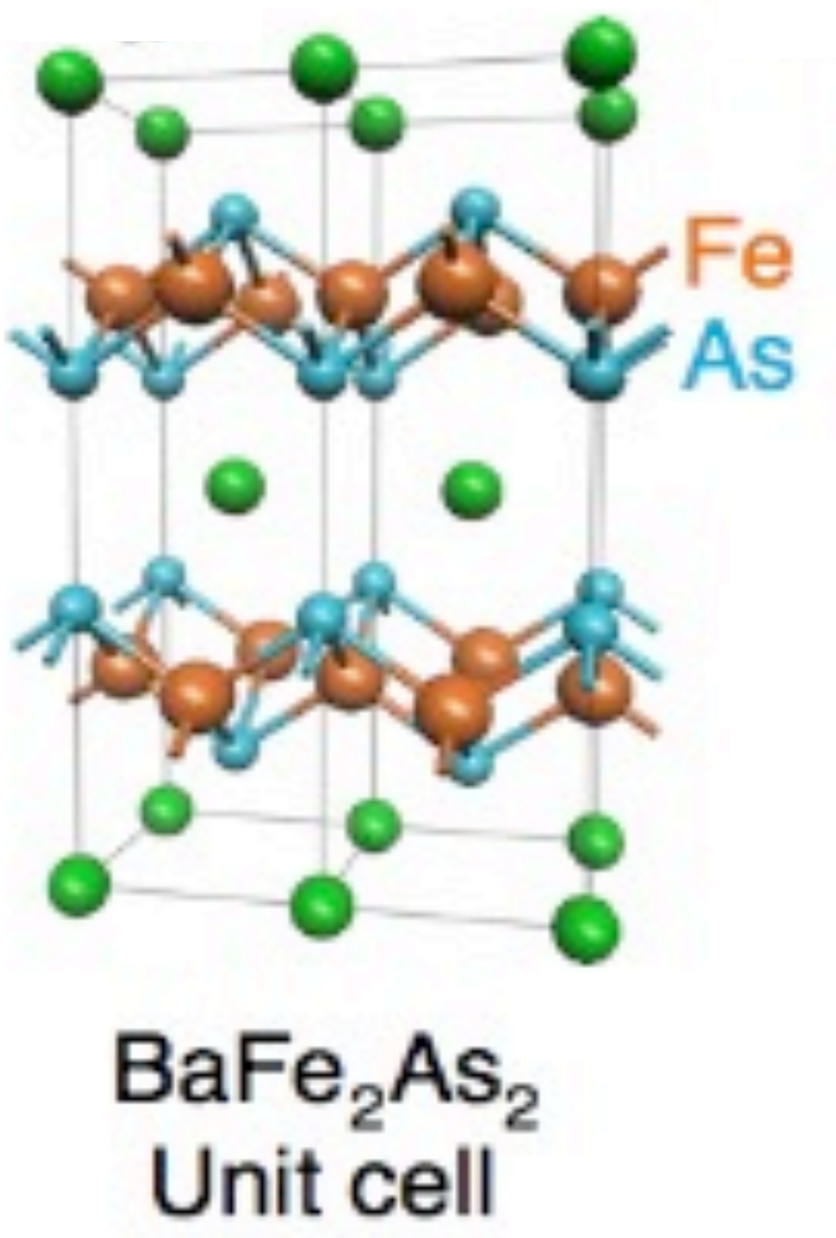
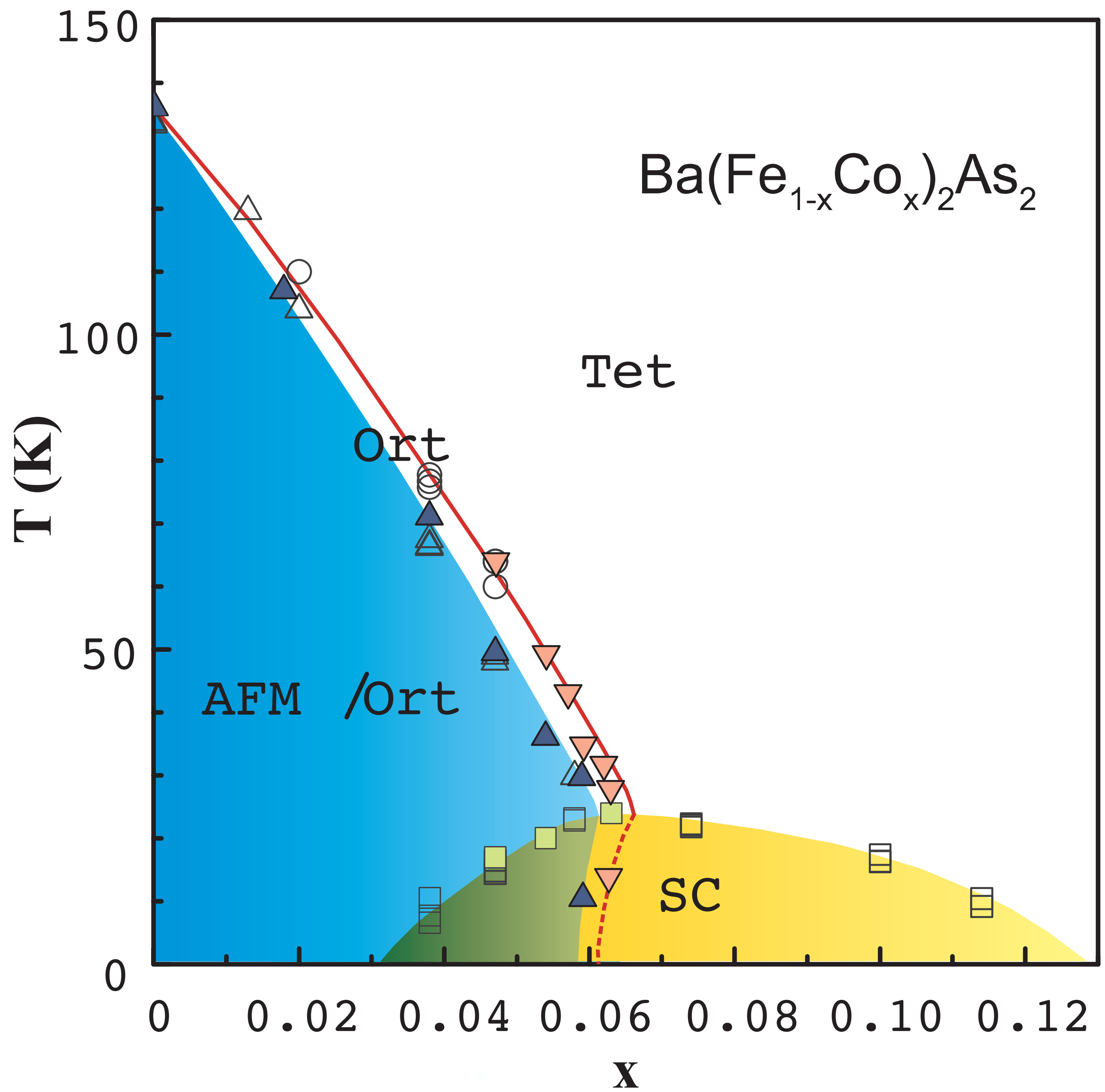
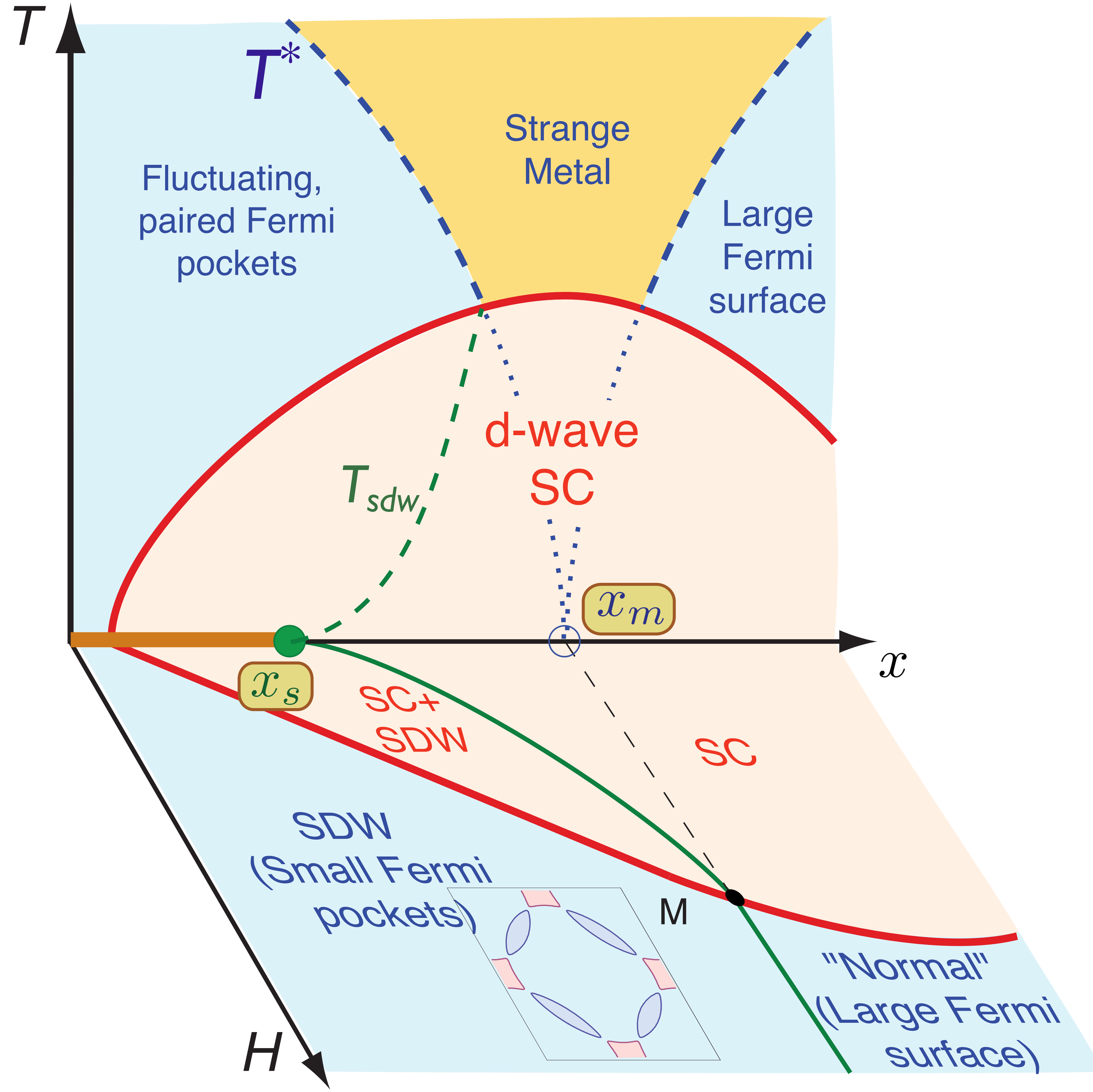

T
T*
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
d-wave
SC
Tsdw
xm
x
xs
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

Similar phase diagram for the pnictides





S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, arXiv:0911.3136.



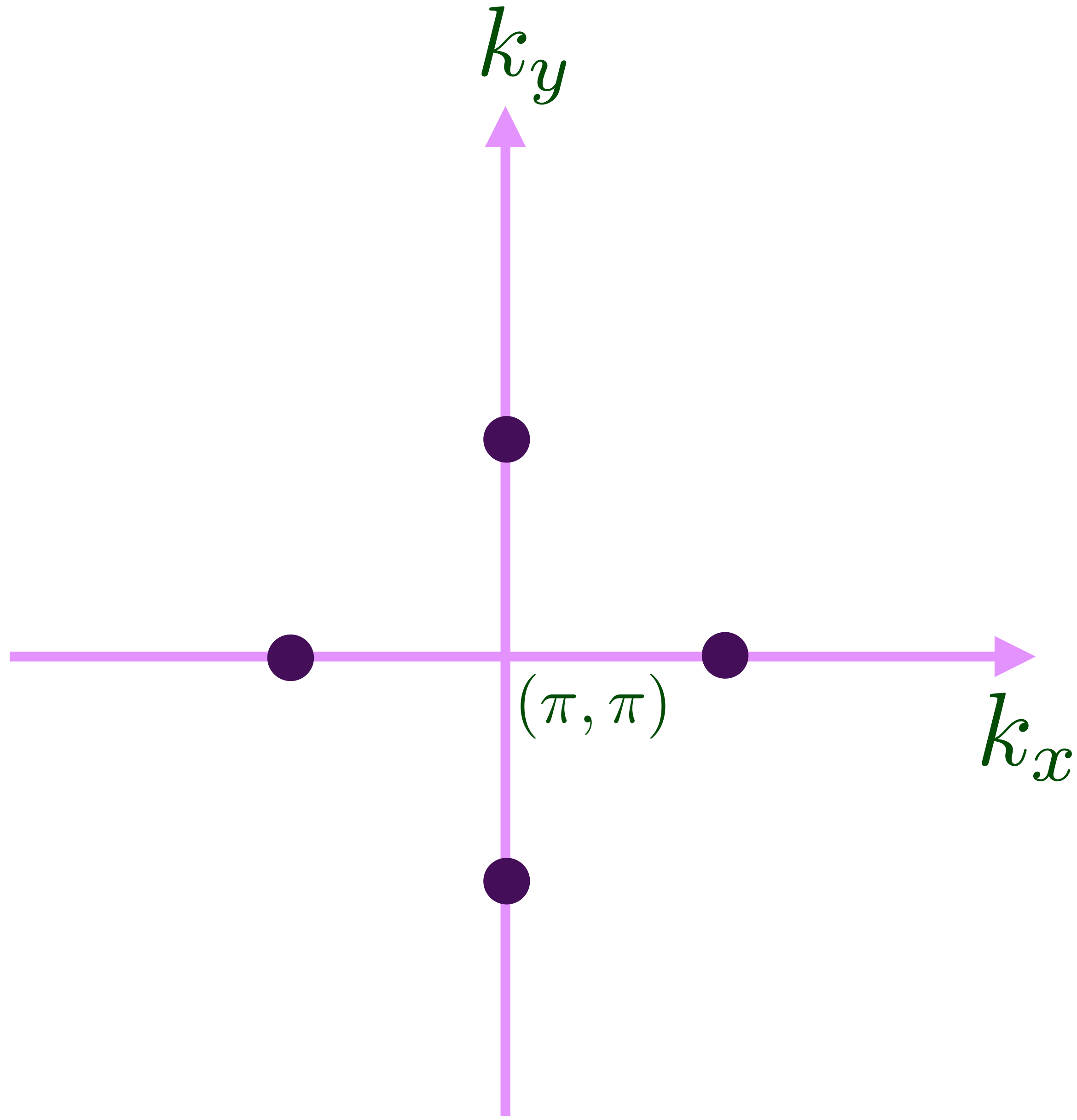
T
T*
Fluctuating, paired Fermi pockets
Strange Metal
Large Fermi surface
d-wave SC
T_sdw
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Remnants of SDW order for $x_s < x < x_m$

For incommensurate ordering, the SDW order parameter consists of 2 complex 3-component vectors $\vec{\Phi}_x$, $\vec{\Phi}_y$:

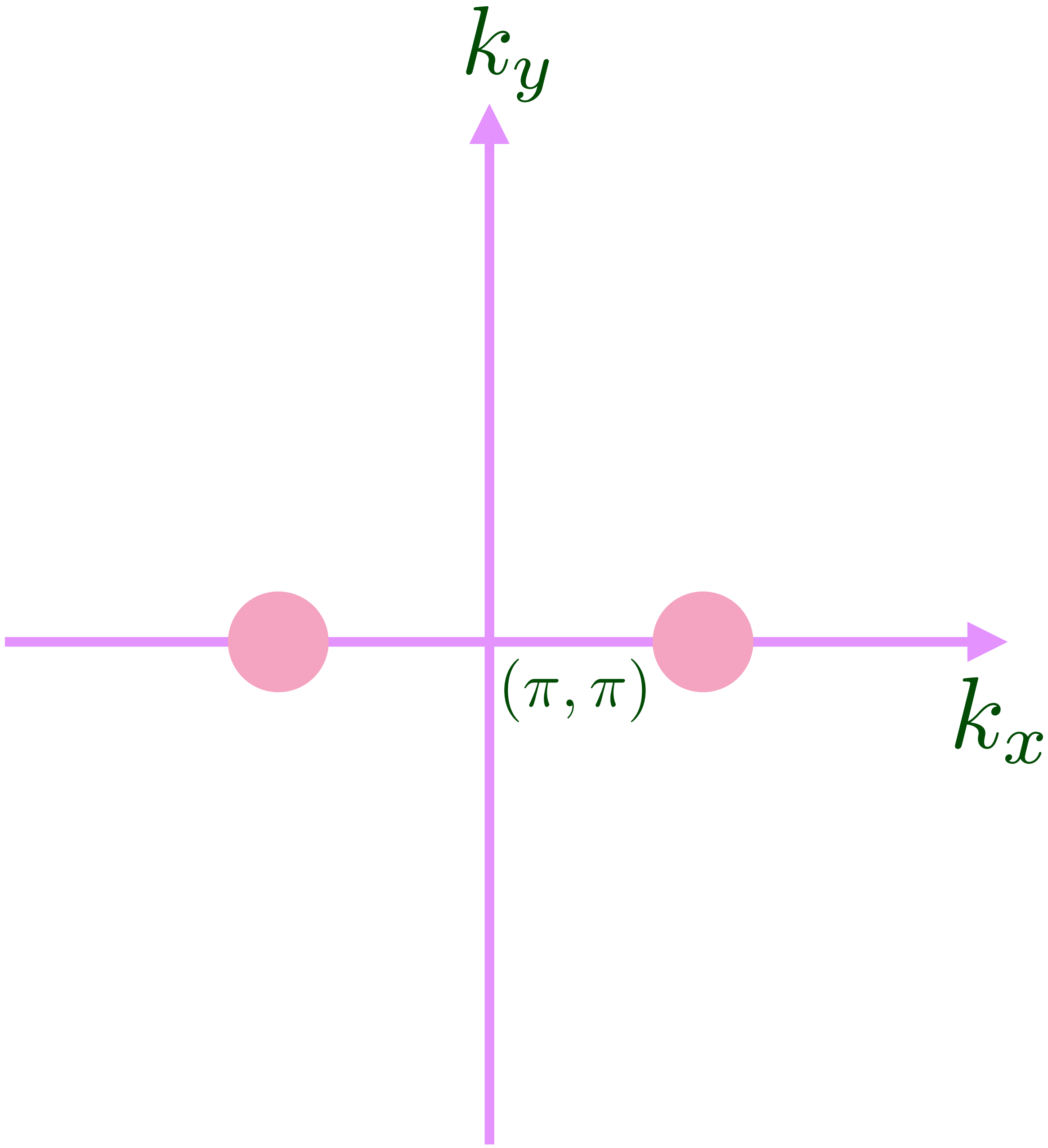
$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\Phi}_x(\mathbf{r},\tau) e^{i\mathbf{K}_x \cdot \mathbf{r}} + \vec{\Phi}_y(\mathbf{r},\tau) e^{i\mathbf{K}_y \cdot \mathbf{r}} + \text{c.c.}$$

where $\mathbf{K}_x = (\pi(1-\vartheta), \pi)$ and $\mathbf{K}_y = (\pi, \pi(1-\vartheta))$, with $\vartheta = 1/4$ near 1/8 doping.



Remnants of SDW order for $x_s < x < x_m$

SDW correlations also Ising nematic order $\phi \propto |\Phi_x|^2 - |\Phi_y|^2$, which can be long-ranged, with SDW and VBS/CDW order all short ranged. This implies of preferential enhancement of electronic exchange/pairing energies along the x or y directions.



S.A. Kivelson, E. Fradkin, and V. J. Emery, *Nature* **393**, 550 (1998).
R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Phys. Rev. B* **78**, 045110 (2008).

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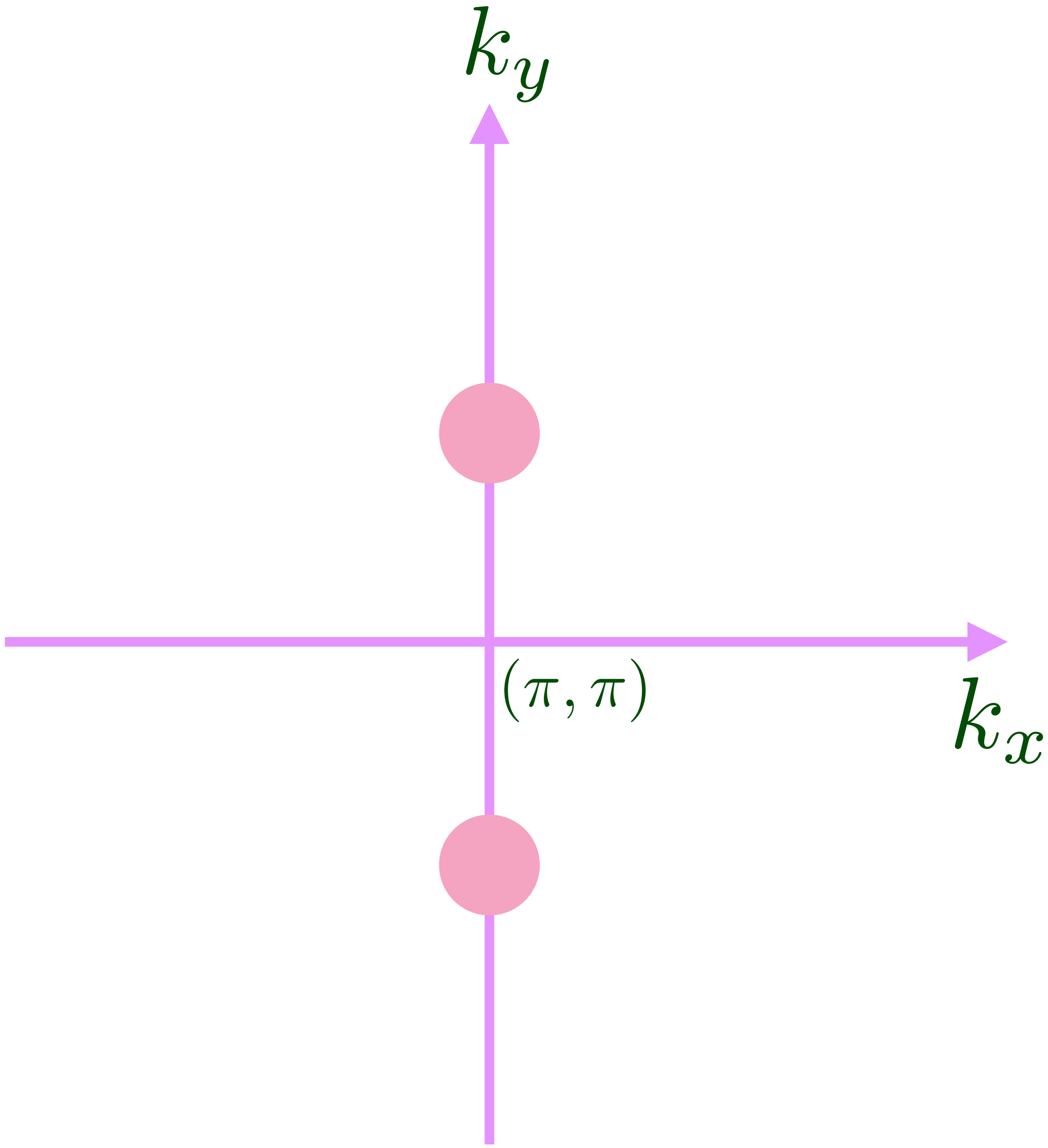

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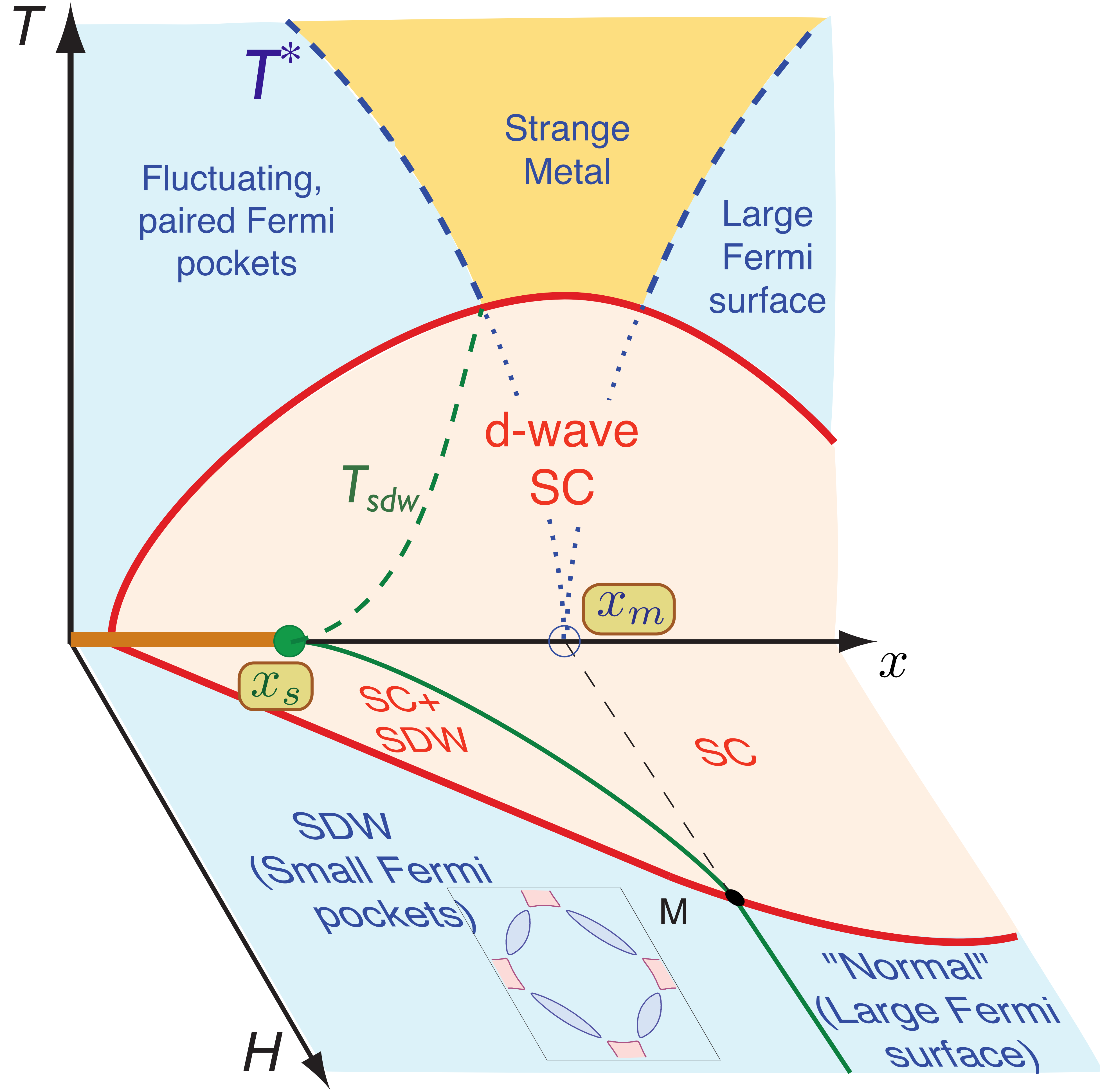
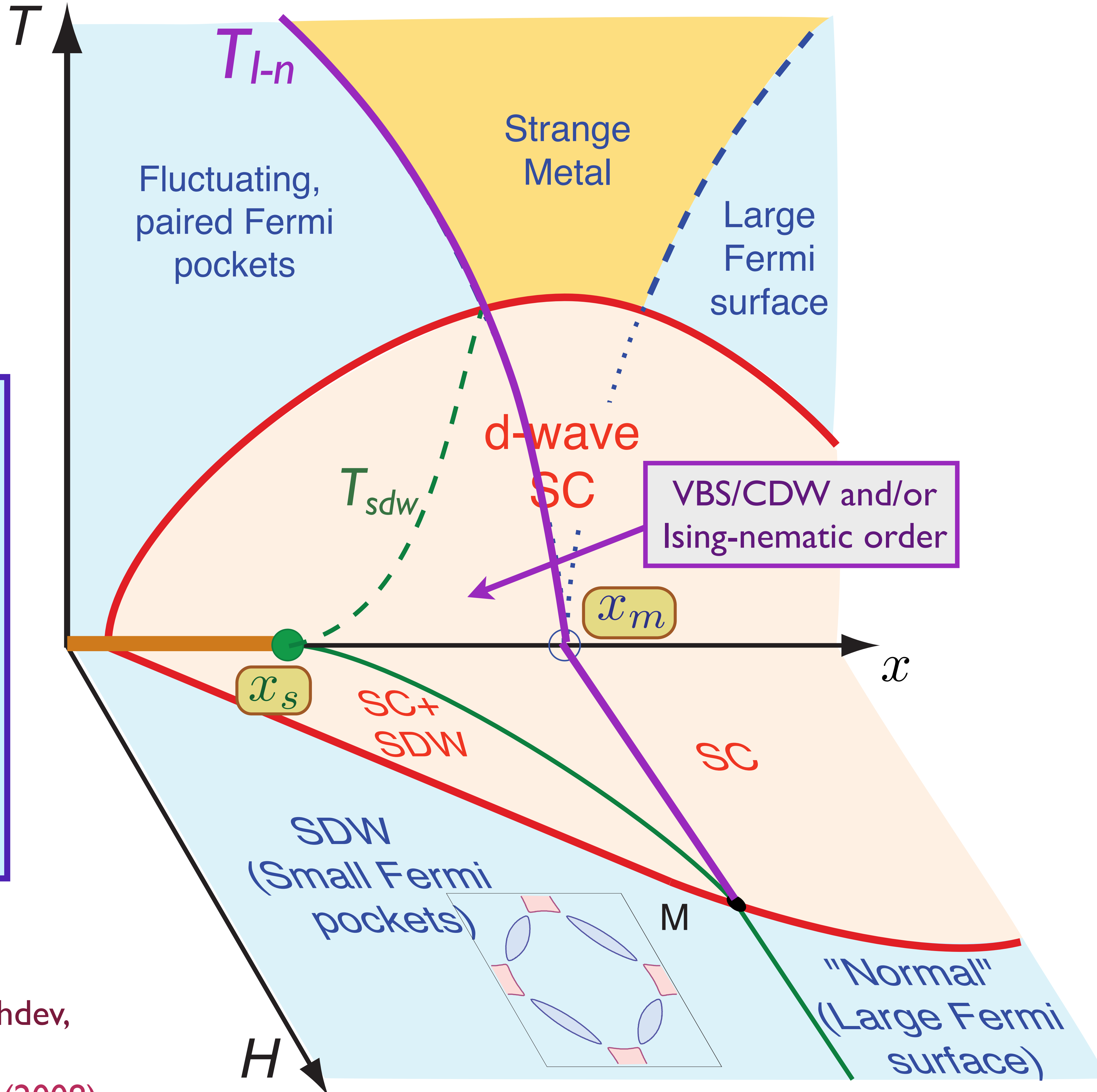


T
T*
Strange Metal
Fluctuating, paired Fermi pockets
Large Fermi surface
d-wave SC
T_sdw
x_m
x_s
x
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Onset of superconductivity disrupts SDW order, but VBS/CDW/ Ising-nematic ordering can survive



R. K. Kaul, M. Metlitksi, S. Sachdev, and Cenke Xu, *Physical Review B* **78**, 045110 (2008).

Outline

1. Phase diagram of the cuprates

   Quantum criticality of the competition between antiferromagnetism and superconductivity

2. Influence of an applied magnetic field

   Theoretical predictions and experimental tests

3. Theory of spin density wave ordering in a metal

   Order parameter at zero wavevector

4. Theory of Ising-nematic ordering in a metal

   Order parameter at zero wavevector

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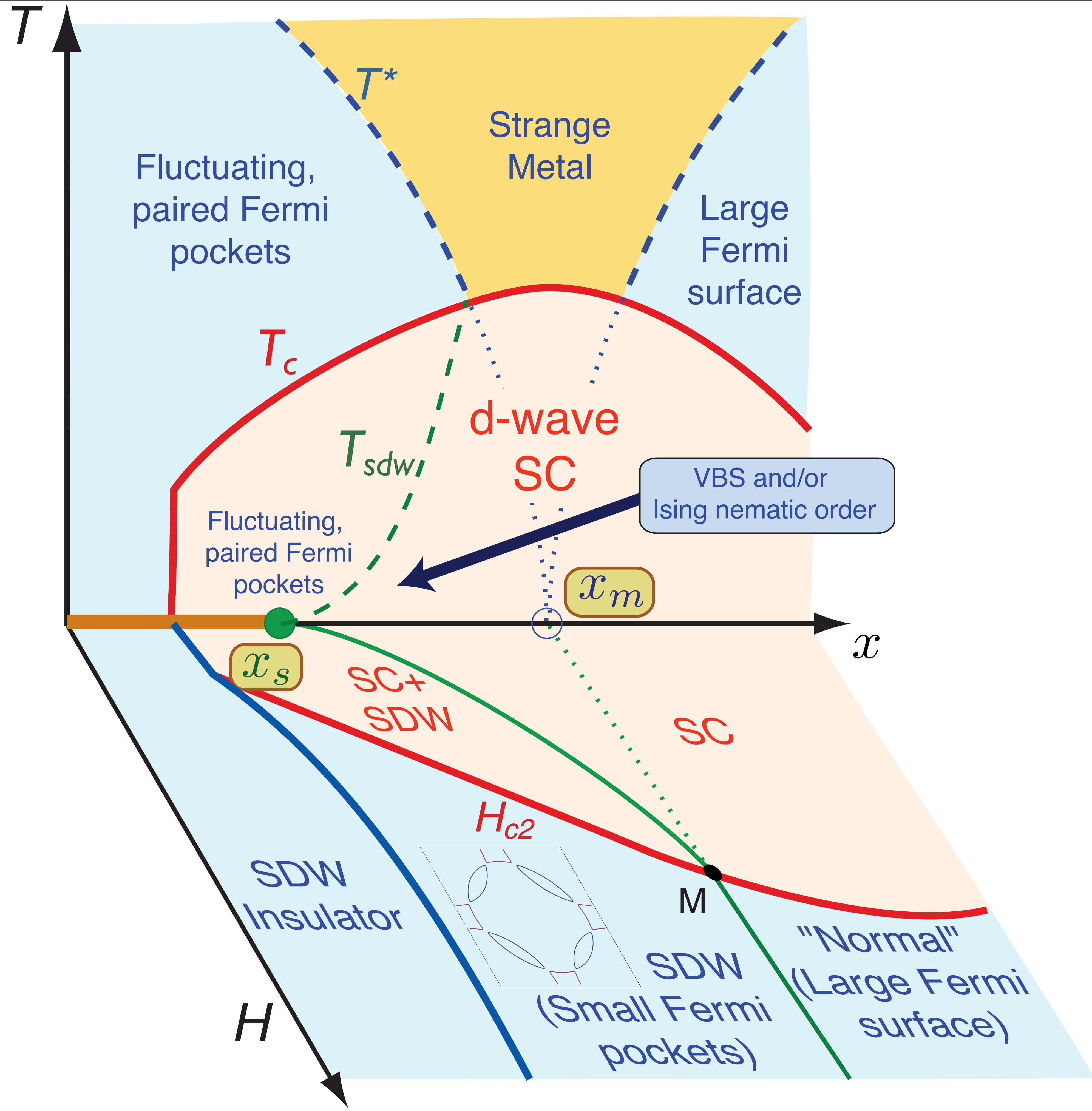
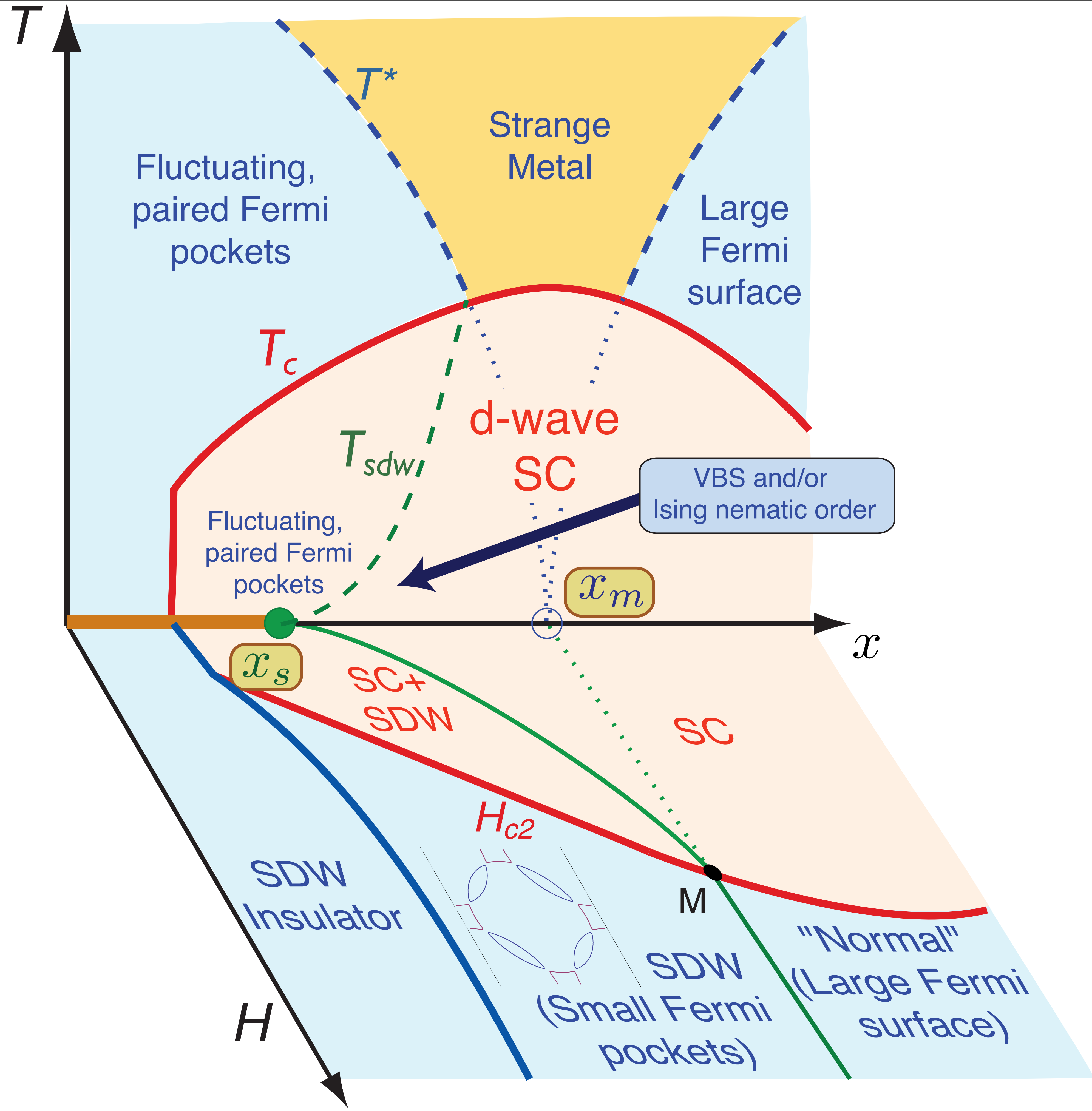


T
T*
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
Tc
d-wave
SC
Tsdw
VBS and/or
Ising nematic order
Fluctuating,
paired Fermi
pockets
xm
x
xs
SC+
SDW
SC
Hc2
SDW
Insulator
M
SDW
(Small Fermi
pockets)
"Normal"
(Large Fermi
surface)
H


T
T*
Strange Metal
Fluctuating, paired Fermi pockets
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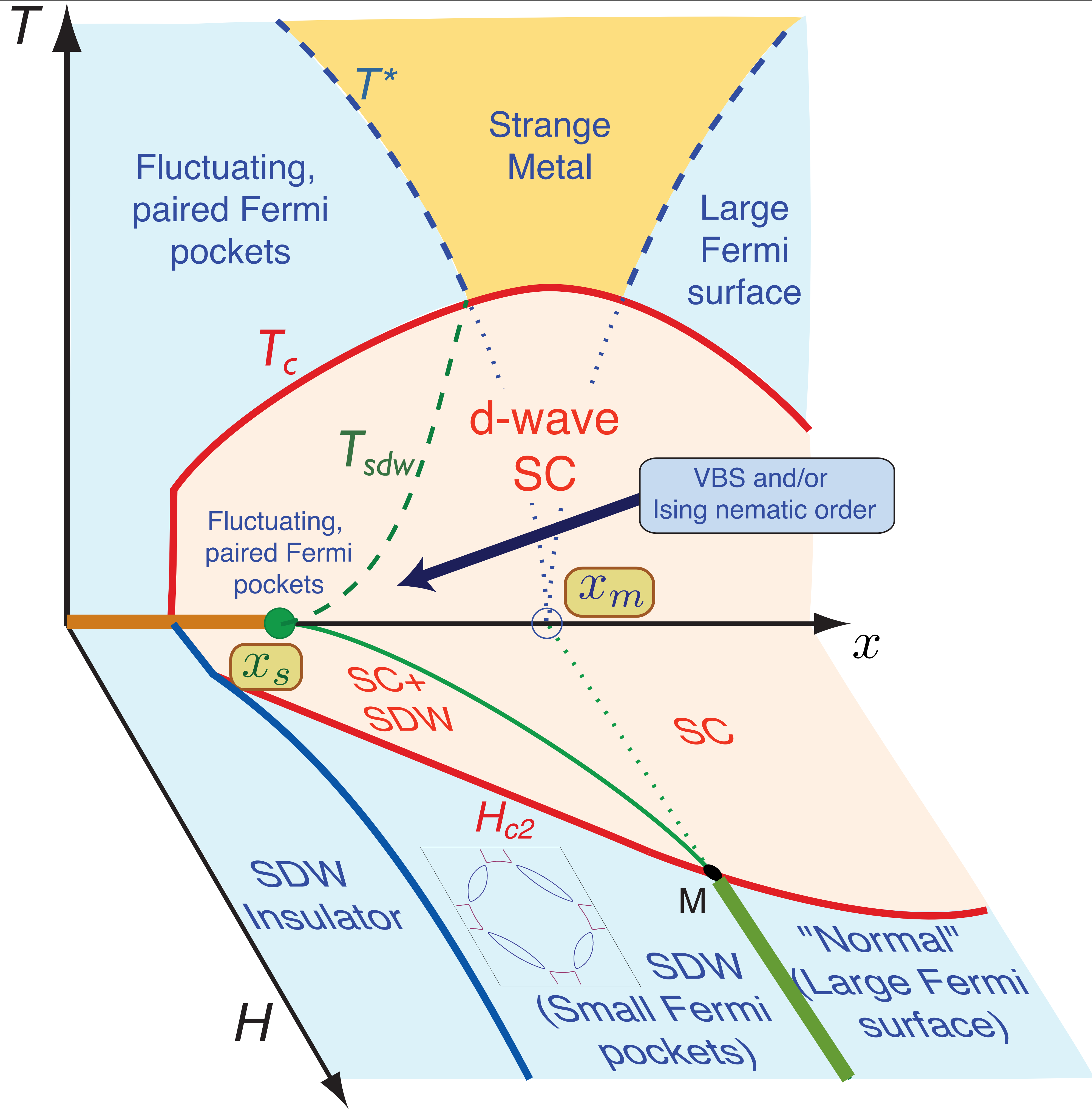
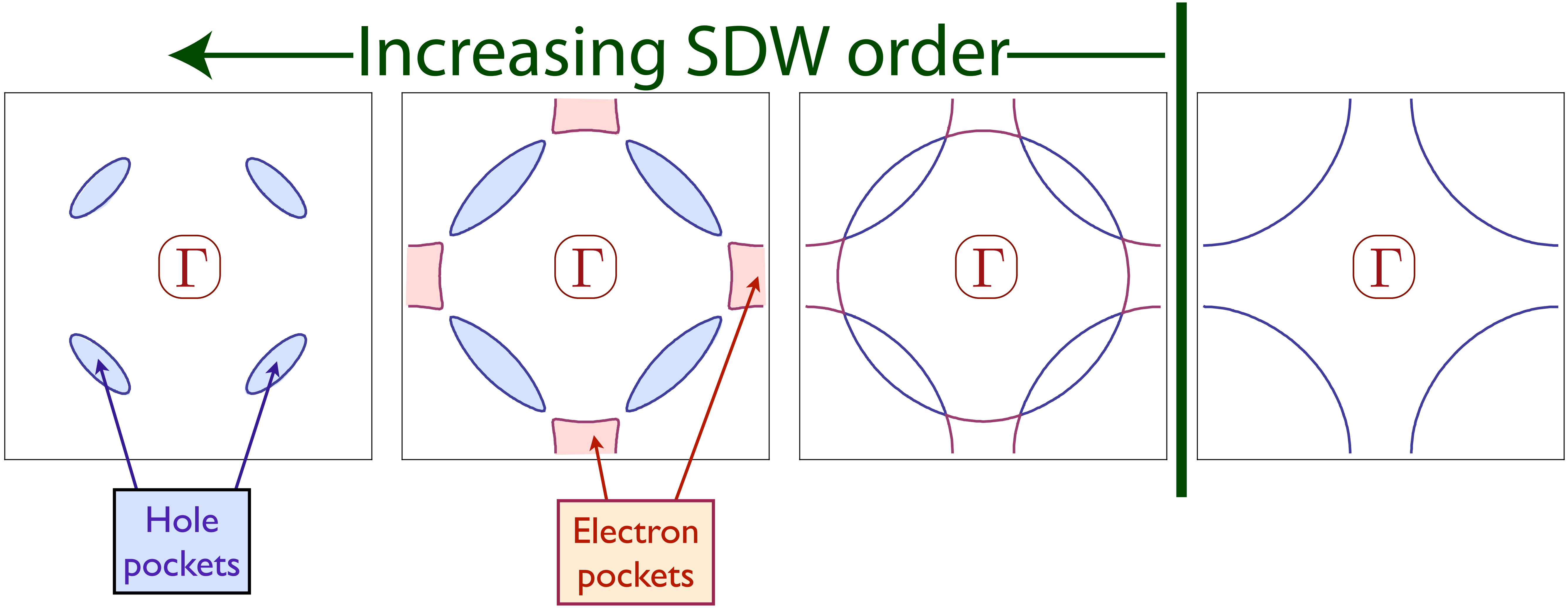
T_c
d-wave SC
T_{sdw}
VBS and/or Ising nematic order
Fluctuating, paired Fermi pockets
x_m
x
x_s
SC+ SDW
SC
H_{c2}
SDW Insulator
M
SDW (Small Fermi pockets)
"Normal" (Large Fermi surface)
H



Thursday, February 25, 2010

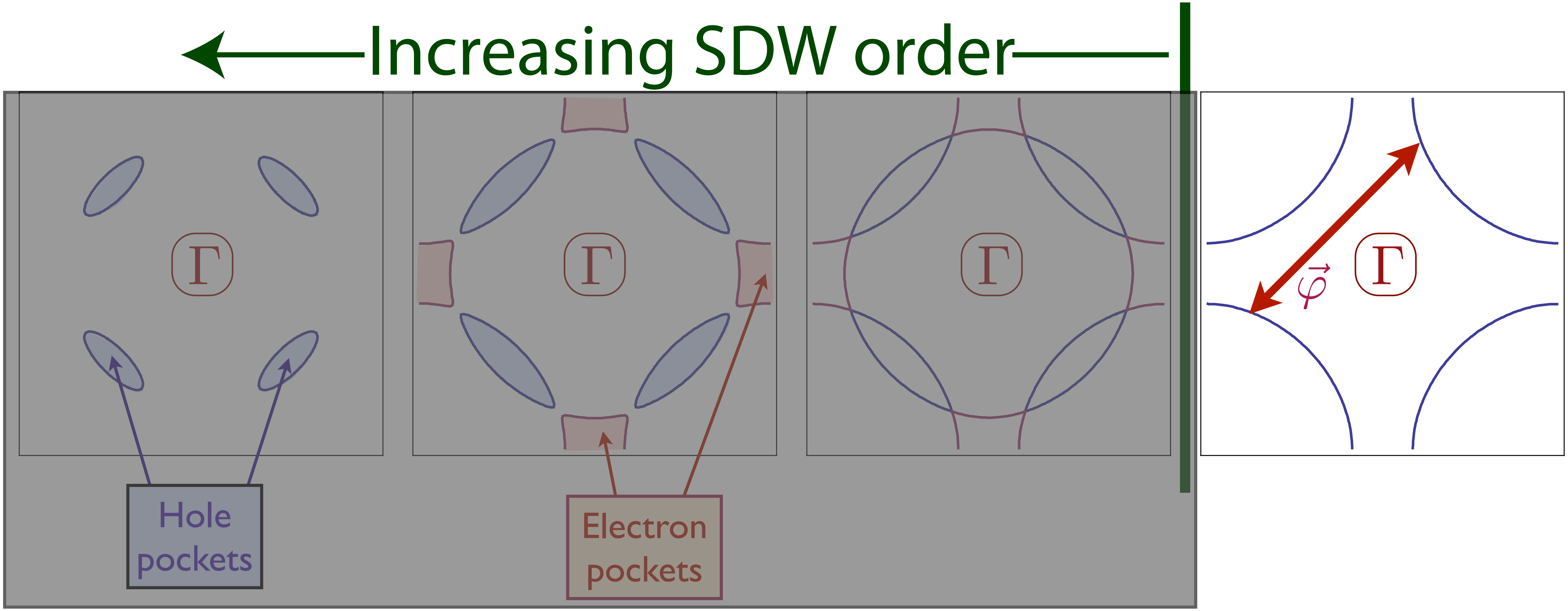
Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



$\vec{\varphi}$ fluctuations act on the large Fermi surface

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

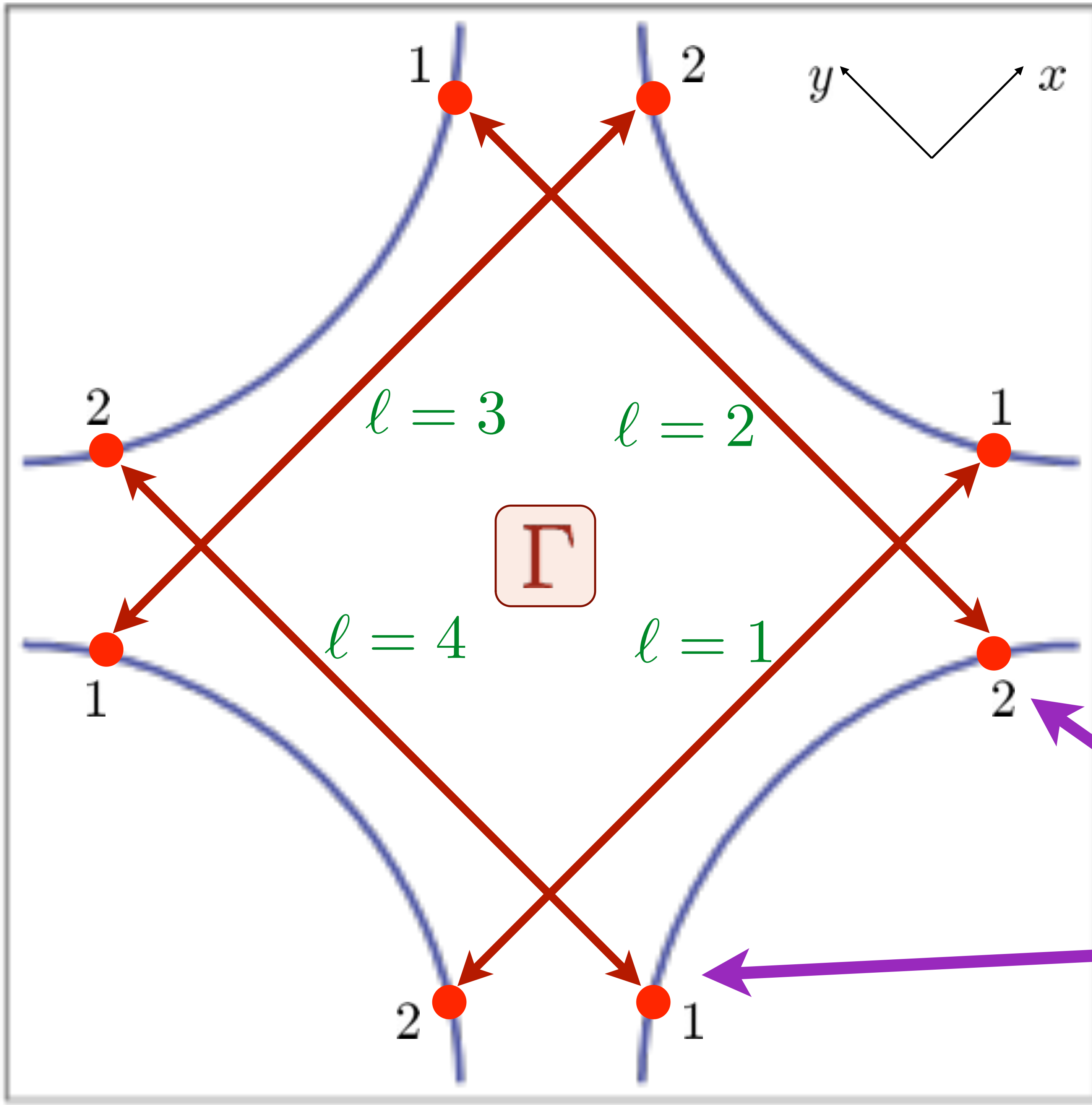
Start from the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c_{\mathbf{k}\alpha}^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c_{i\alpha}^\dagger \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$

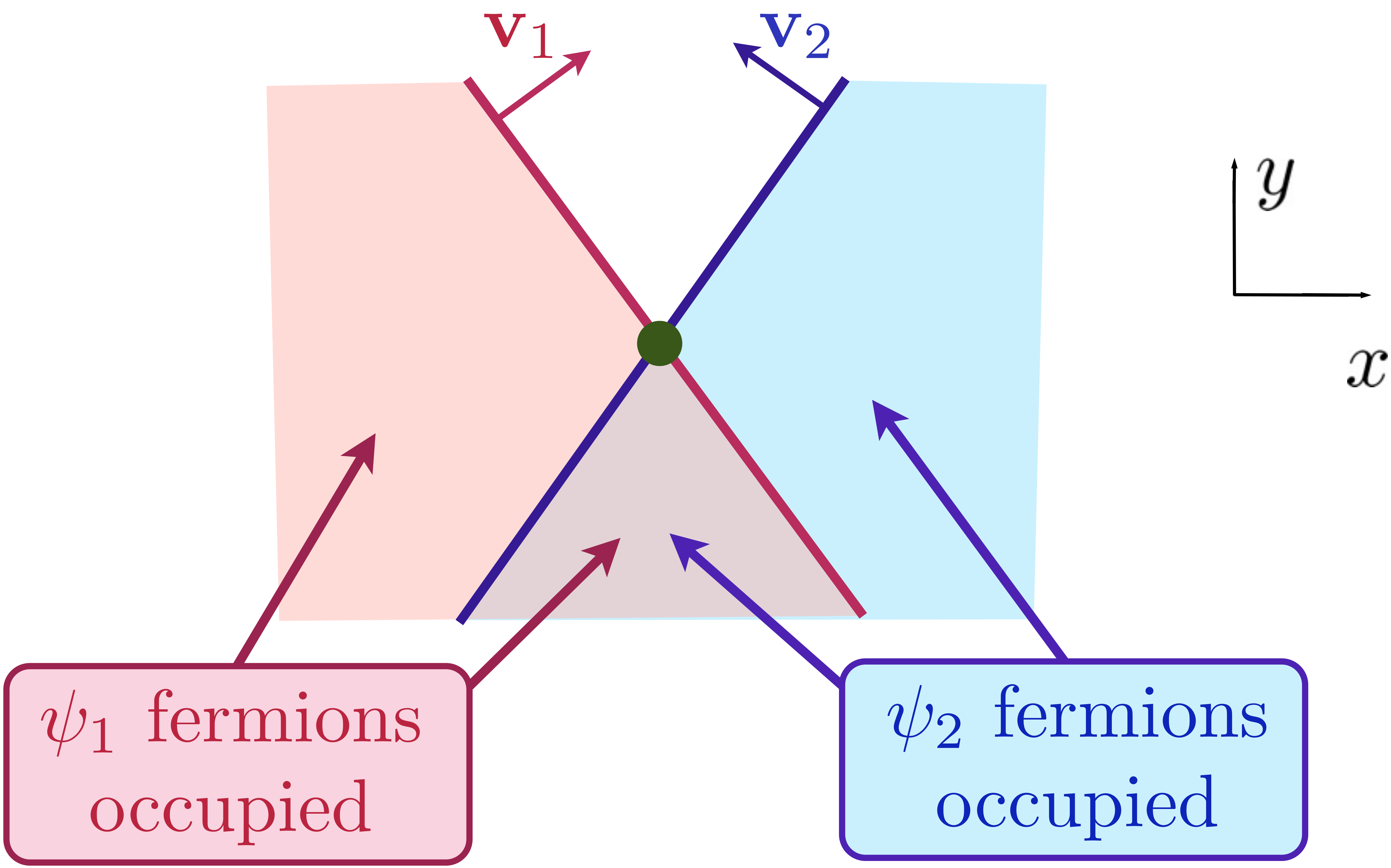


Low energy fermions
$\psi^{\ell}_{1\alpha}$, $\psi^{\ell}_{2\alpha}$
$\ell = 1, \ldots, 4$

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha} \left(\zeta \partial_\tau - i \mathbf{v}^\ell_1 \cdot \boldsymbol{\nabla}_r\right) \psi^{\ell}_{1\alpha} + \psi^{\ell\dagger}_{2\alpha} \left(\zeta \partial_\tau - i \mathbf{v}^\ell_2 \cdot \boldsymbol{\nabla}_r\right) \psi^{\ell}_{2\alpha}$$

$$\mathbf{v}_1^{\ell=1} = (v_x, v_y), \ \mathbf{v}_2^{\ell=1} = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \mathbf{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \mathbf{\nabla}_r\right)\psi_{2\alpha}^\ell$$



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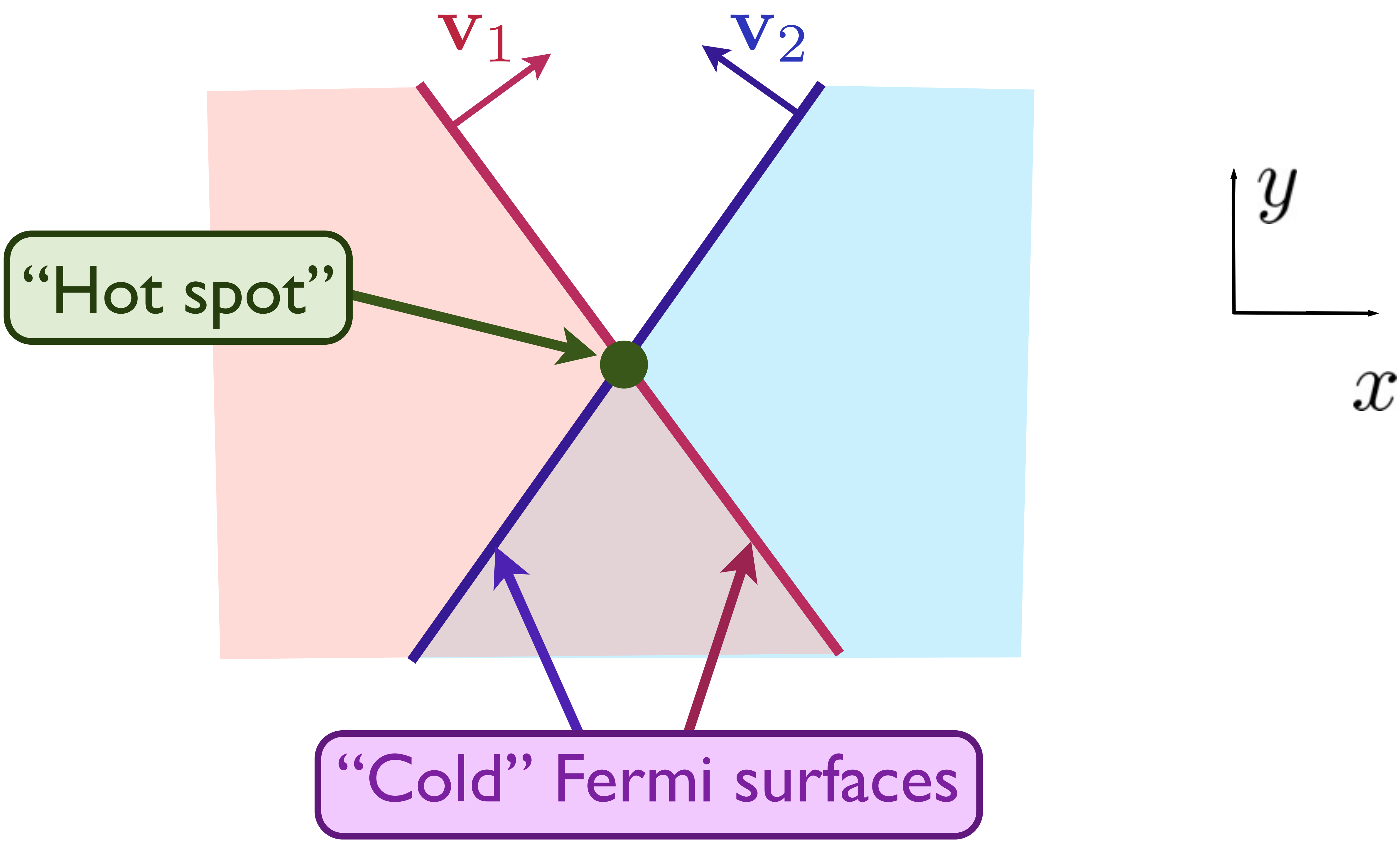

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

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"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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Hertz-Moriya-Millis (HMM) theory

Integrate out fermions and obtain non-local corrections to $\mathcal{L}_\varphi$

$$\mathcal{L}_\varphi = \frac{1}{2}\vec{\varphi}^2\left[\mathbf{q}^2 + \gamma|\omega|\right]/2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

"Yukawa" coupling: $\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$

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Exponent $z = 2$ and mean-field criticality (upto logarithms)

But, higher order terms contain an infinite number of marginal couplings ......

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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Perform RG on both fermions and $\vec{\varphi}$, using a *local* field theory.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$$

Under the rescaling $x' = xe^{-\ell}$, $\tau' = \tau e^{-z\ell}$, the spatial gradients are fixed if the fields transform as

$$\vec{\varphi}' = e^{(d+z-2)\ell/2}\vec{\varphi} \quad ;\text{``}\quad \psi' = e^{(d+z-1)\ell/2}\psi.$$

Then the Yukawa coupling transforms as

$$\lambda' = e^{(4-d-z)\ell/2}\lambda$$

For $d=2$, with $z=2$ the Yukawa coupling is invariant, and the bare time-derivative terms ζ, $\widetilde{\zeta}$ are irrelevant.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

"Yukawa" coupling: $\mathcal{L}_c = -\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$

With $z = 2$ scaling, ζ is irrelevant.
So we take $\zeta \to 0$
(watch for dangerous irrelevancy).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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Set $\vec{\varphi}$ wavefunction renormalization by keeping co-efficient of $(\boldsymbol{\nabla}_r\vec{\varphi})^2$ fixed (as usual).

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha} \left(\zeta\partial_\tau - i\mathbf{v}^\ell_1 \cdot \boldsymbol{\nabla}_r\right) \psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha} \left(\zeta\partial_\tau - i\mathbf{v}^\ell_2 \cdot \boldsymbol{\nabla}_r\right) \psi^\ell_{2\alpha}$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r \vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau \vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

"Yukawa" coupling: $$\mathcal{L}_c = -\vec{\varphi} \cdot \left(\psi^{\ell\dagger}_{1\alpha} \vec{\sigma}_{\alpha\beta} \psi^\ell_{2\beta} + \psi^{\ell\dagger}_{2\alpha} \vec{\sigma}_{\alpha\beta} \psi^\ell_{1\beta}\right)$$

Set fermion wavefunction renormalization by keeping Yukawa coupling fixed.

Y. Huh and S. Sachdev, *Phys. Rev. B* **78**, 064512 (2008).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

"Yukawa" coupling: $\mathcal{L}_c = -\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$

We find consistent two-loop RG factors, as $\zeta \to 0$, for the velocities v_x, v_y, and the wavefunction renormalizations.

Consistency check: the expression for the boson damping constant, $\gamma = \dfrac{2}{\pi v_x v_y}$, is preserved under RG.

RG-improved Migdal-Eliashberg theory

RG flow can be computed a $1/N$ expansion (with N fermion species) in terms of a single dimensionless coupling $\alpha = v_y/v_x$ whose flow obeys

$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}$$

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$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}$$

The velocities flow as

$$\frac{1}{v_x}\frac{dv_x}{d\ell} = \frac{\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2} \quad ; \quad \frac{1}{v_y}\frac{dv_y}{d\ell} = \frac{-\mathcal{A}(\alpha)+\mathcal{B}(\alpha)}{2}$$

$$\mathcal{A}(\alpha) \equiv \frac{3}{\pi N}\frac{\alpha}{1+\alpha^2}$$

$$\mathcal{B}(\alpha) \equiv \frac{1}{2\pi N}\left(\frac{1}{\alpha}-\alpha\right)\left(1+\left(\frac{1}{\alpha}-\alpha\right)\tan^{-1}\frac{1}{\alpha}\right)$$

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$$\frac{d\alpha}{d\ell} = -\frac{3}{\pi N}\frac{\alpha^2}{1+\alpha^2}$$

The anomalous dimensions of $\vec{\varphi}$ and ψ are

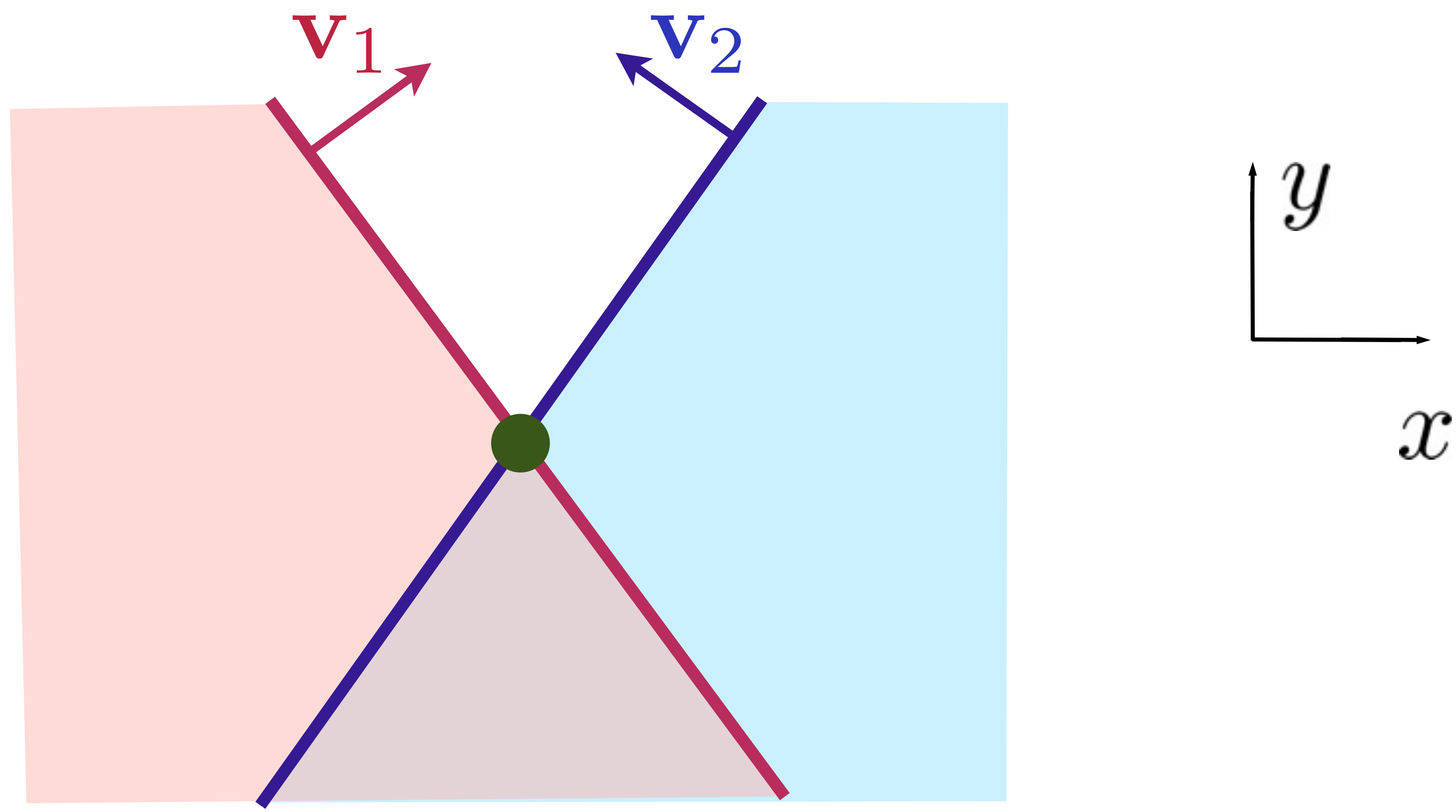
$$\eta_\varphi = \frac{1}{2\pi N}\left(\frac{1}{\alpha} - \alpha + \left(\frac{1}{\alpha^2} + \alpha^2\right)\tan^{-1}\frac{1}{\alpha}\right)$$

$$\eta_\psi = -\frac{1}{4\pi N}\left(\frac{1}{\alpha} - \alpha\right)\left(1 + \left(\frac{1}{\alpha} - \alpha\right)\tan^{-1}\frac{1}{\alpha}\right)$$

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting



Bare Fermi surface

RG-improved Migdal-Eliashberg theory

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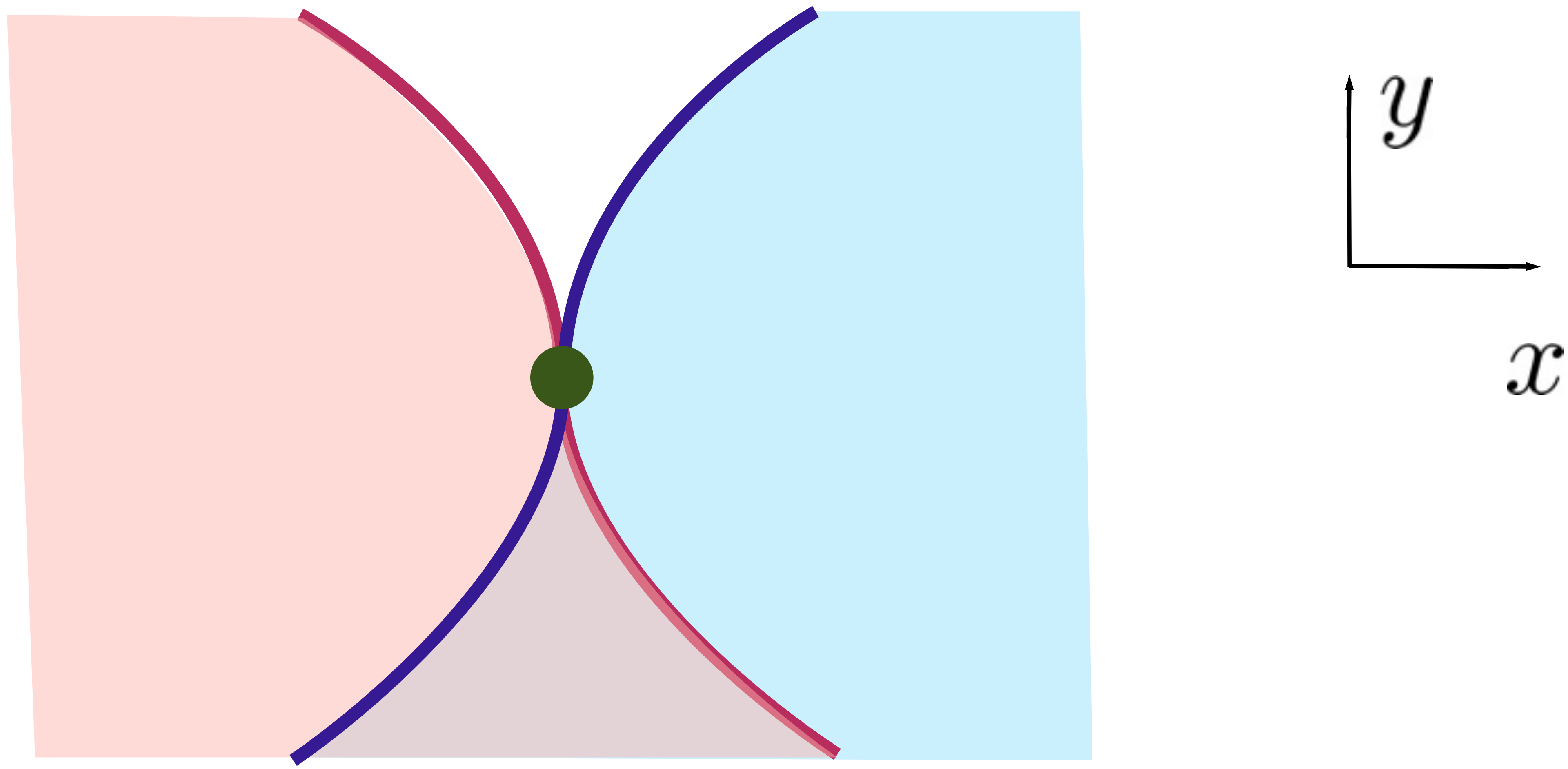
Dynamical Nesting

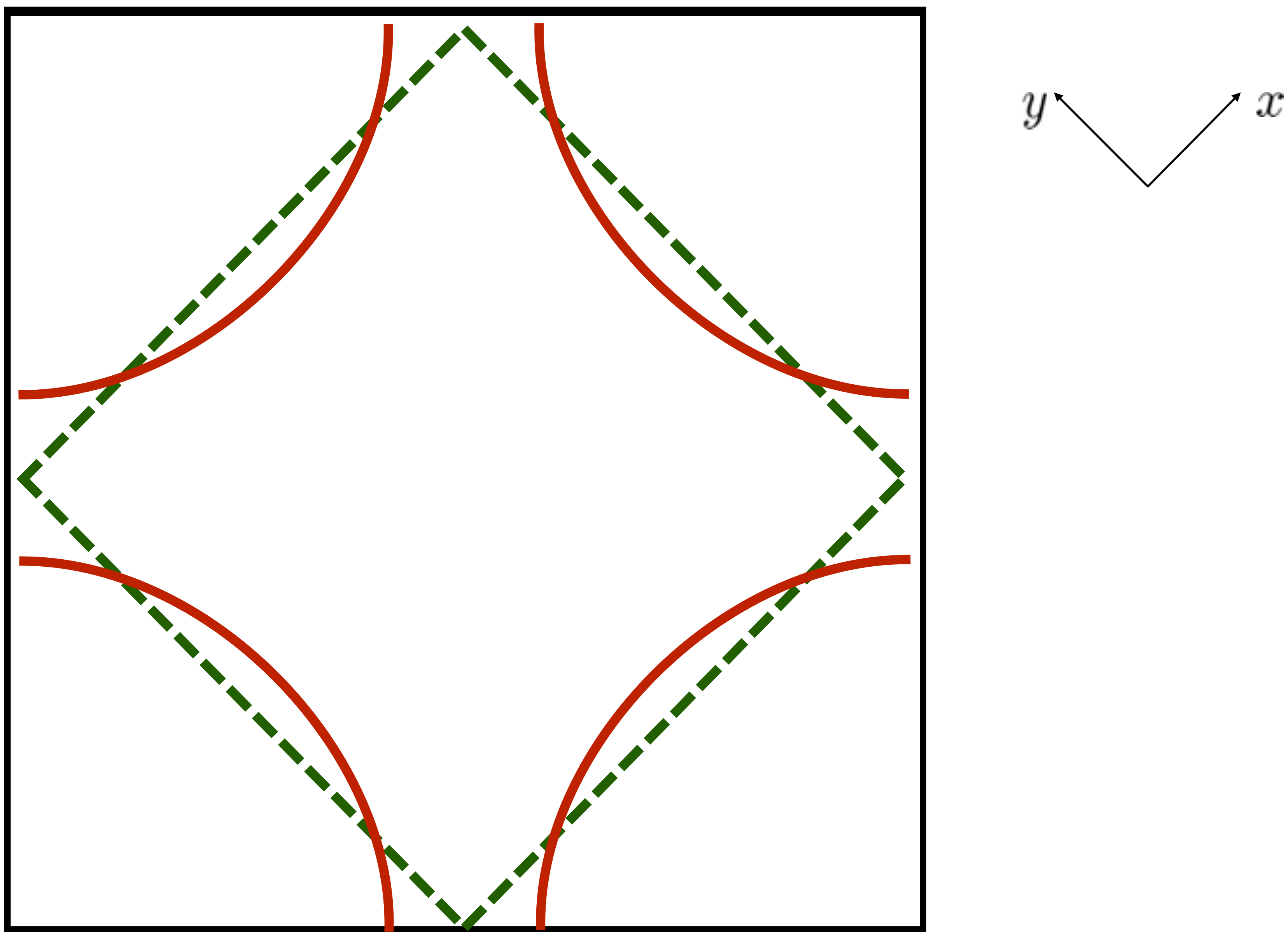


Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting



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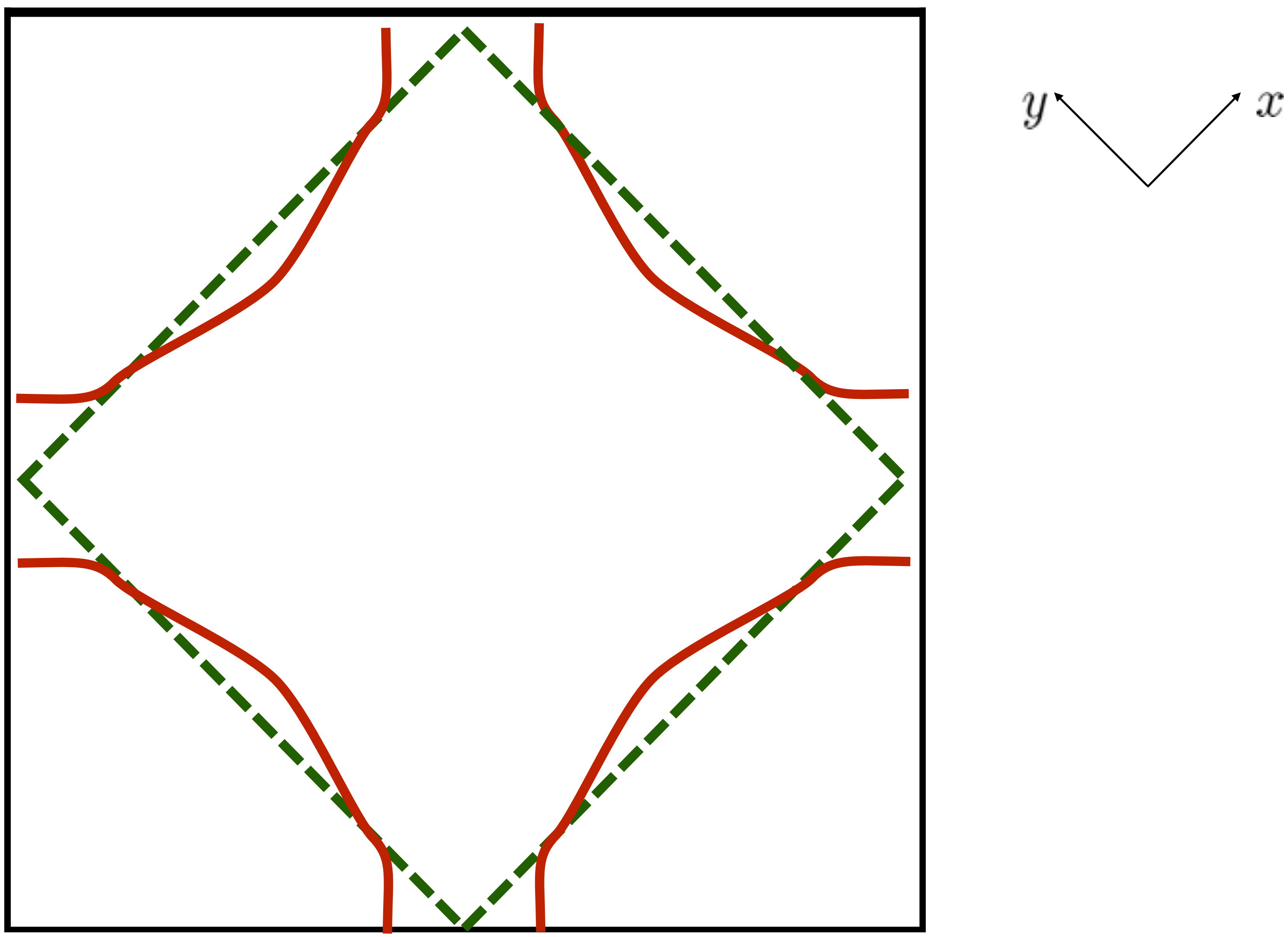
Dynamical Nesting



Dressed Fermi surface

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

In $\vec{\varphi}$ SDW fluctuations, characteristic q and ω scale as

$$q \sim \omega^{1/2} \exp\left(-\frac{3}{64\pi^2}\left(\frac{\ln(1/\omega)}{N}\right)^3\right).$$

However, $1/N$ expansion cannot be trusted in the asymptotic regime.

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

$\vec{\varphi}$ propagator

$$\frac{1}{N} \frac{1}{(q^2 + \gamma|\omega|)}$$

fermion propagator

$$\frac{1}{\mathbf{v} \cdot \mathbf{q} + i\zeta\omega + i\frac{1}{N\sqrt{\gamma}v}\sqrt{\omega}F\left(\frac{v^2q^2}{\omega}\right)}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

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$$\frac{1}{N}\frac{1}{(q^2+\gamma|\omega|)}$$

fermion propagator

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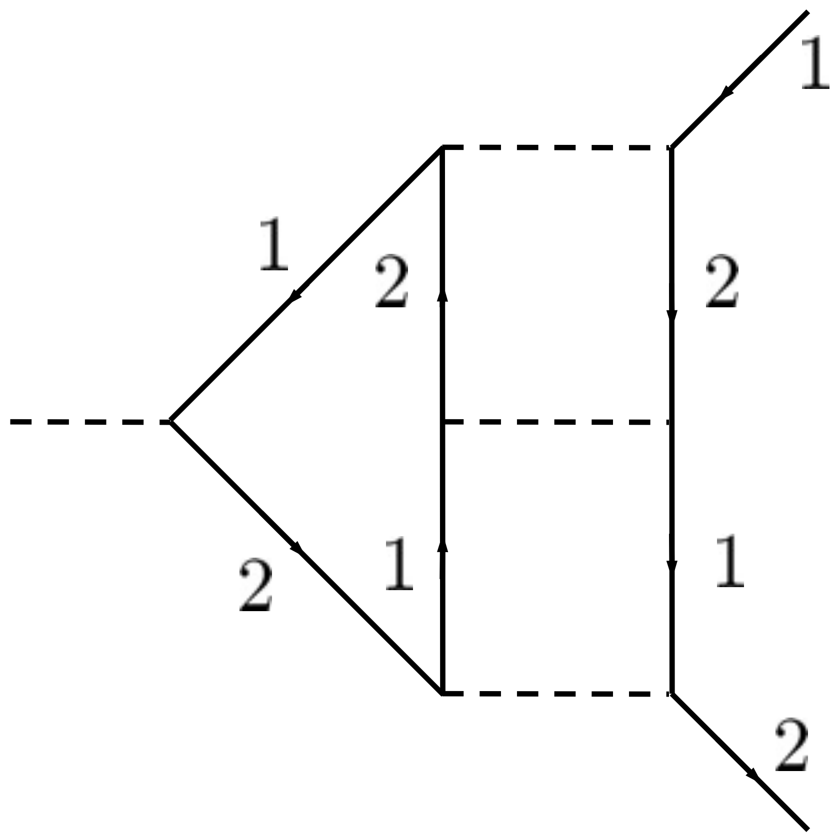
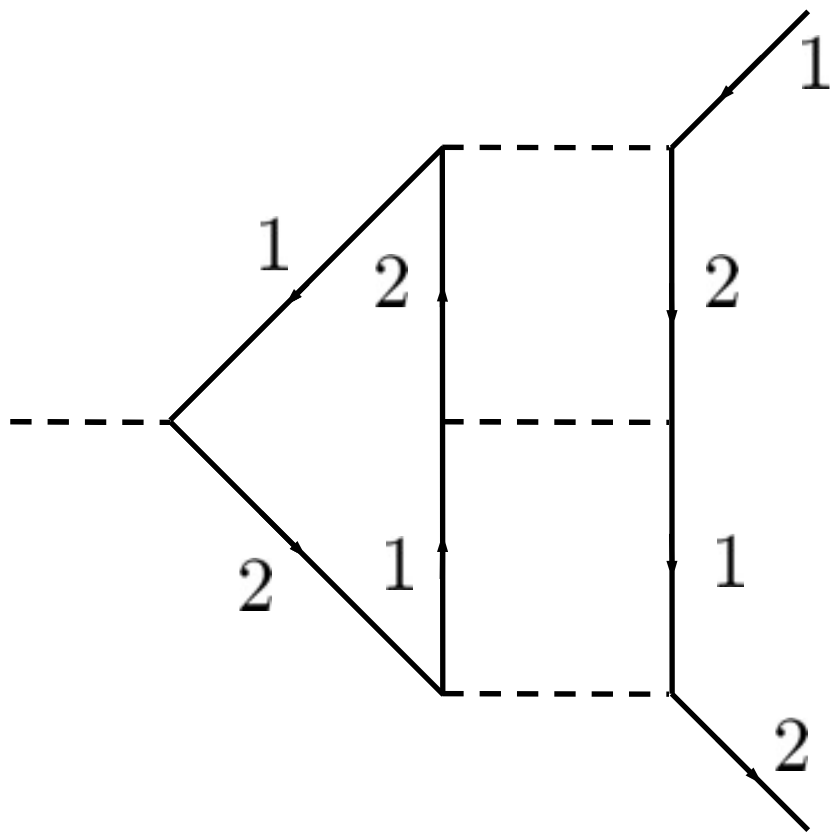
⚠ Dangerous

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \dfrac{1}{N^2} \times \dfrac{1}{\zeta^2} \times \dfrac{1}{\omega}$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Ignoring fermion self energy: $\sim \frac{1}{N^2} \times \frac{1}{\zeta^2} \times \frac{1}{\omega}$

Actual order $\sim \frac{1}{N^0}$

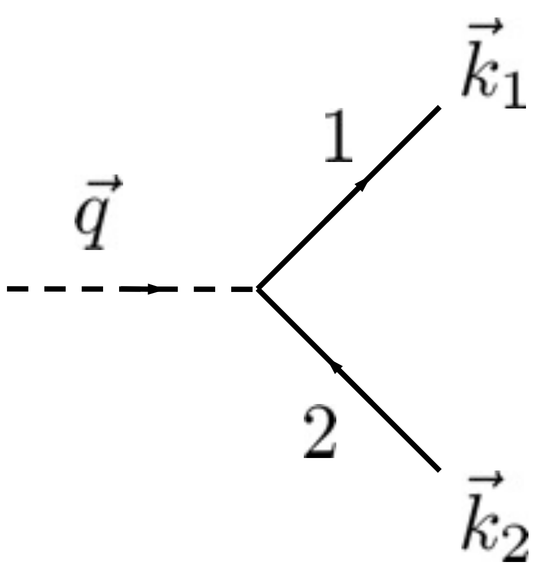
Double line representation

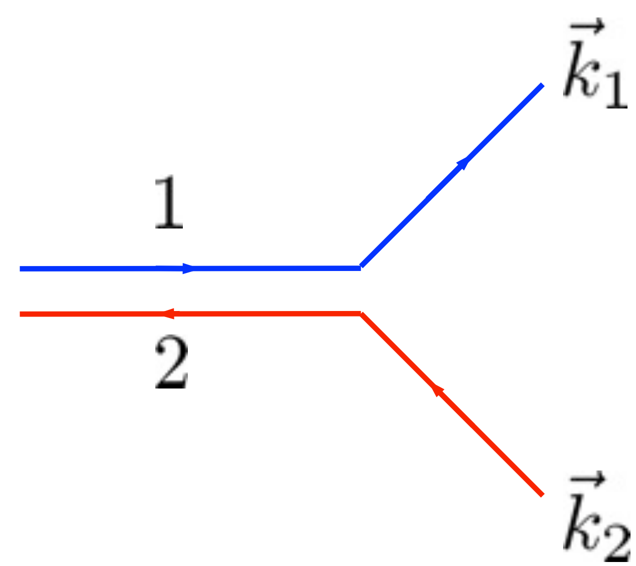
- A way to compute the order of a diagram.
- Extra powers of N come from the Fermi-surface

$$G(\omega, \vec{k}) = \frac{1}{-\Sigma_1(\omega, \vec{k}) - \vec{v} \cdot \vec{k}} \qquad \Sigma_1 \sim \frac{1}{N}$$

- What are the conditions for all propagators to be on the Fermi surface?
- Concentrate on diagrams involving a single pair of hot-spots
- Any bosonic momentum may be (uniquely) written as

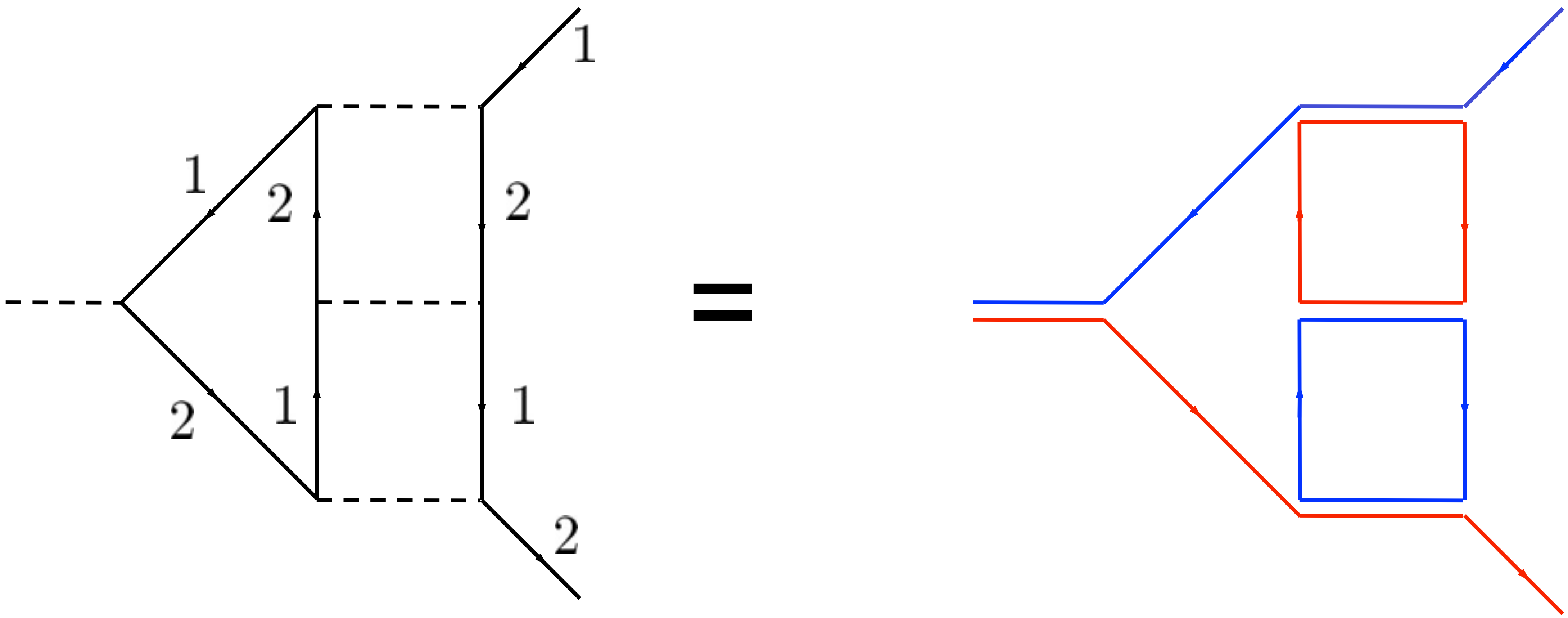
$$\vec{q} = \vec{k}_1 - \vec{k}_2 \qquad \vec{k}_1 \in \mathrm{FS\ of}\ \psi_1 \qquad \vec{k}_2 \in \mathrm{FS\ of}\ \psi_2$$





R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
S. W. Tsai, A. H. Castro Neto, R. Shankar, and D. K. Campbell, Phys. Rev. B **72**, 054531 (2005).

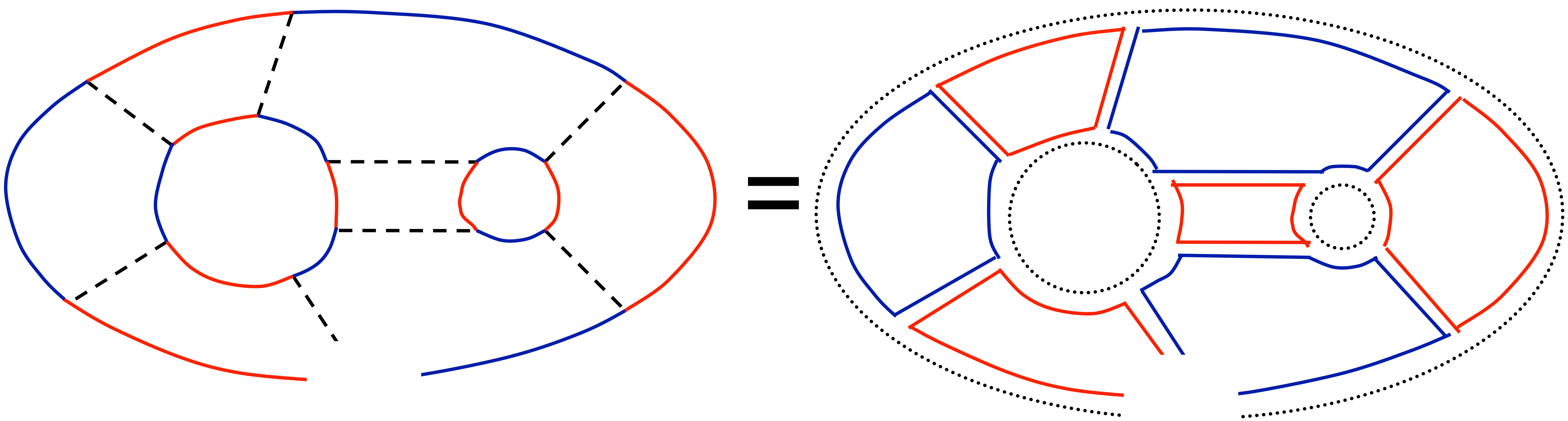
New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



Singularities as $\zeta \to 0$ appear when fermions in closed blue and red line loops are exactly on the Fermi surface

$$\text{Actual order} \sim \frac{1}{N^0}$$

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)

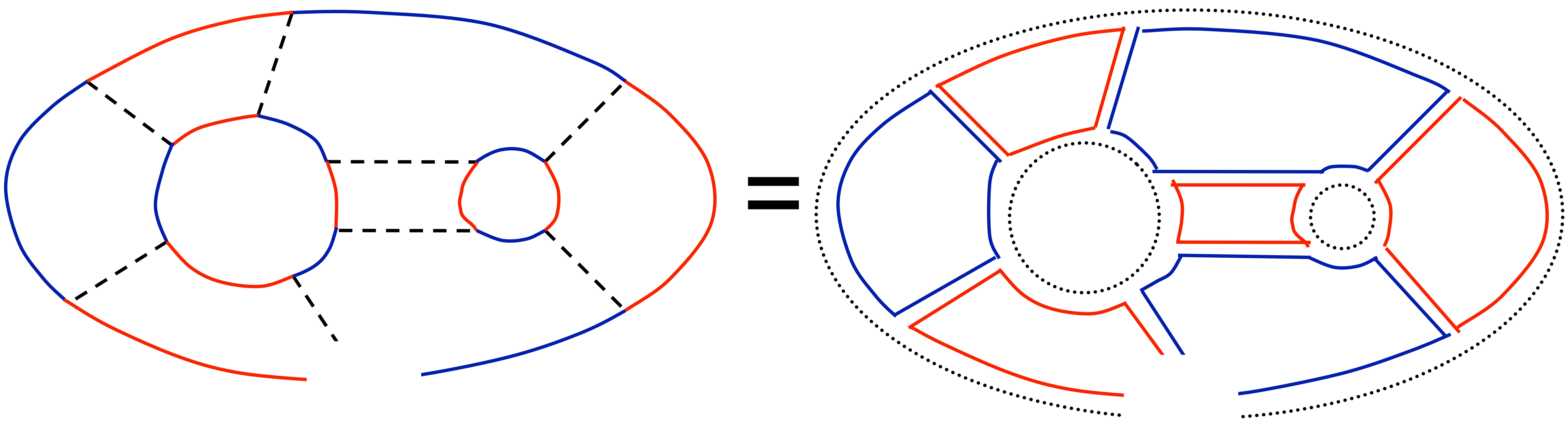


$$\text{Actual order} \sim \frac{1}{N^0}$$

Graph is **planar** after turning fermion propagators also into double lines by drawing additional dotted single line loops for each fermion loop

Sung-Sik Lee, arXiv:0905.4532

New infra-red singularities as $\zeta \to 0$ at higher loops (Breakdown of Migdal-Eliashberg)



$$\text{Actual order} \sim \frac{1}{N^0}$$

A consistent analysis requires resummation of all planar graphs

Outline

1. Phase diagram of the cuprates

Quantum criticality of the competition between antiferromagnetism and superconductivity

2. Influence of an applied magnetic field

Theoretical predictions and experimental tests

3. Theory of spin density wave ordering in a metal

Order parameter at zero wavevector

4. Theory of Ising-nematic ordering in a metal

Order parameter at zero wavevector

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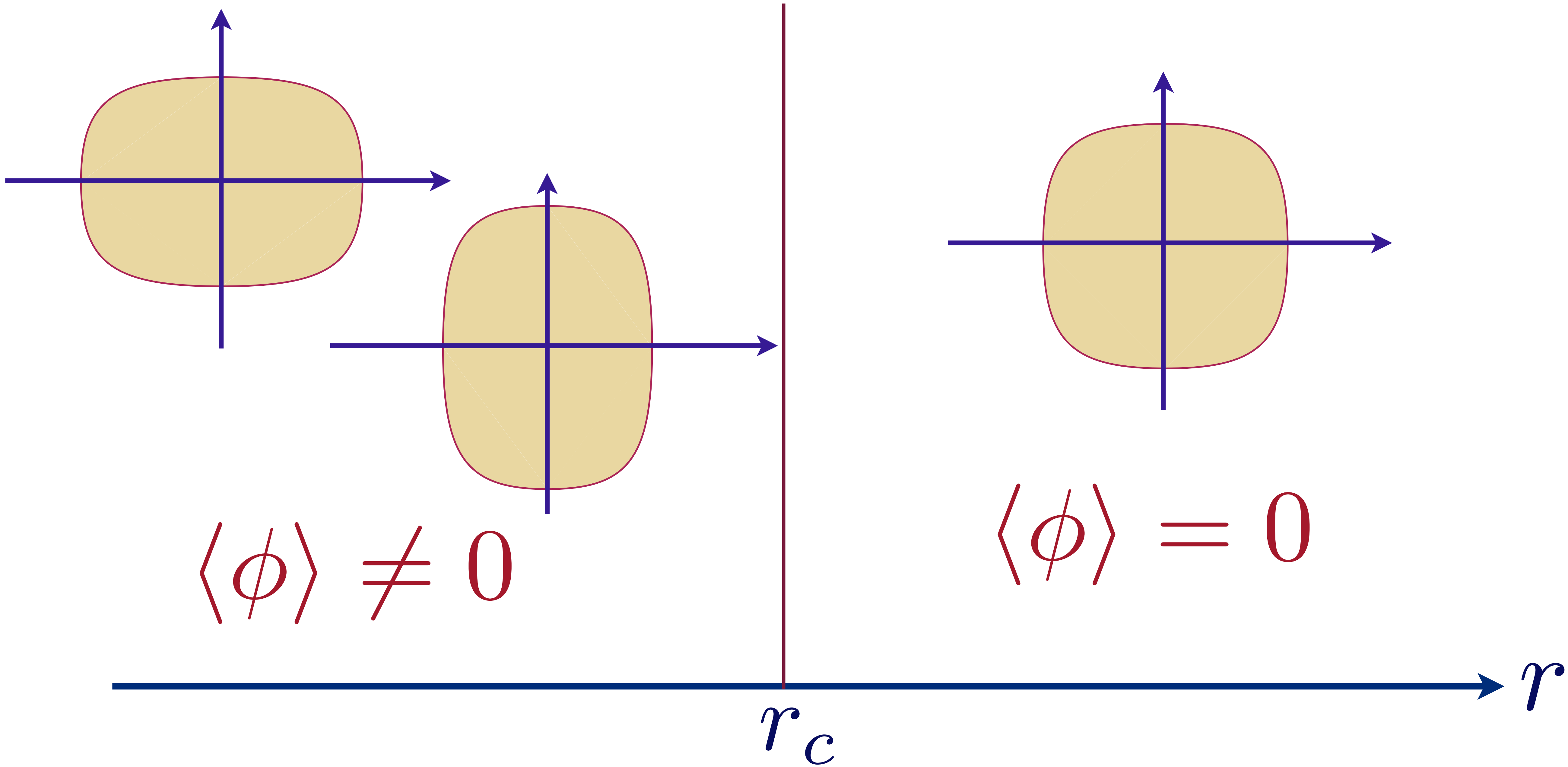
Quantum criticality of Pomeranchuk instability



Pomeranchuk instability as a function of coupling r

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

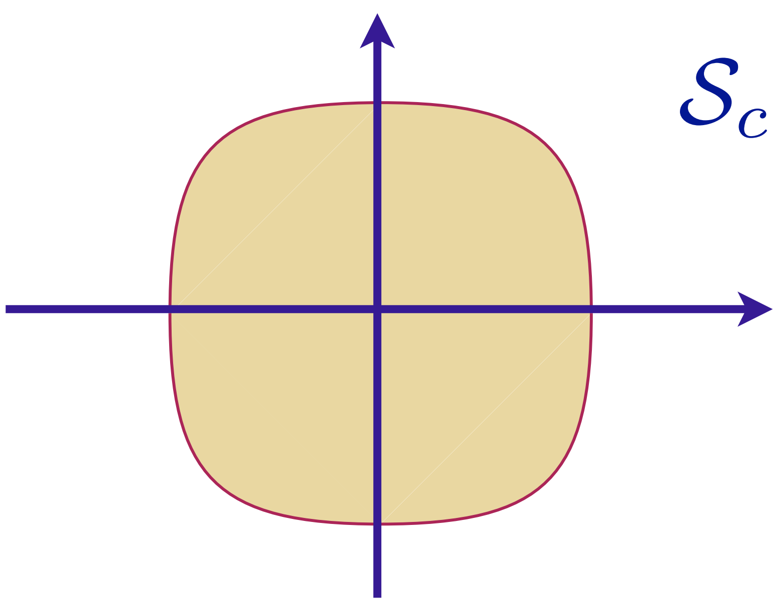
$$\mathcal{S}_\phi = \int d^2x d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (r - r_c) \phi^2 + u \phi^4 \right]$$

Quantum criticality of Pomeranchuk instability

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2x d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (r - r_c)\phi^2 + u\phi^4 \right]$$

Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

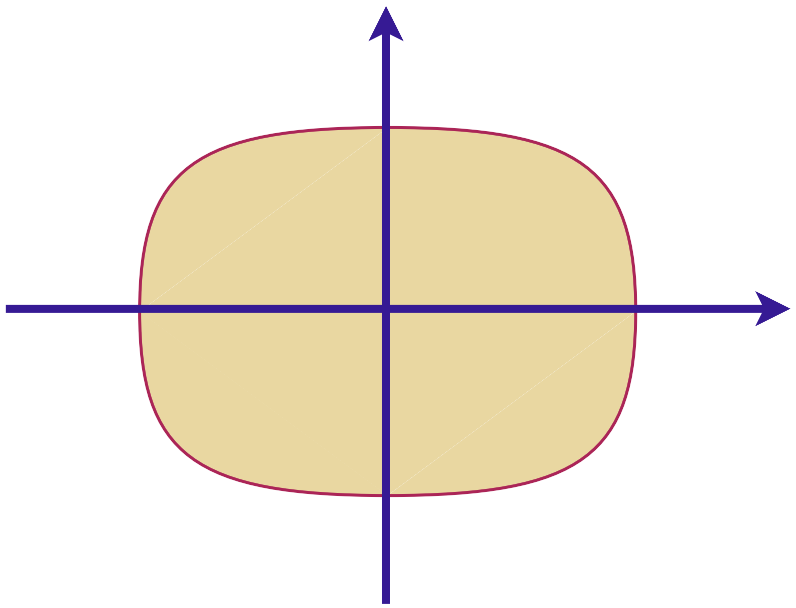
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Pomeranchuk instability

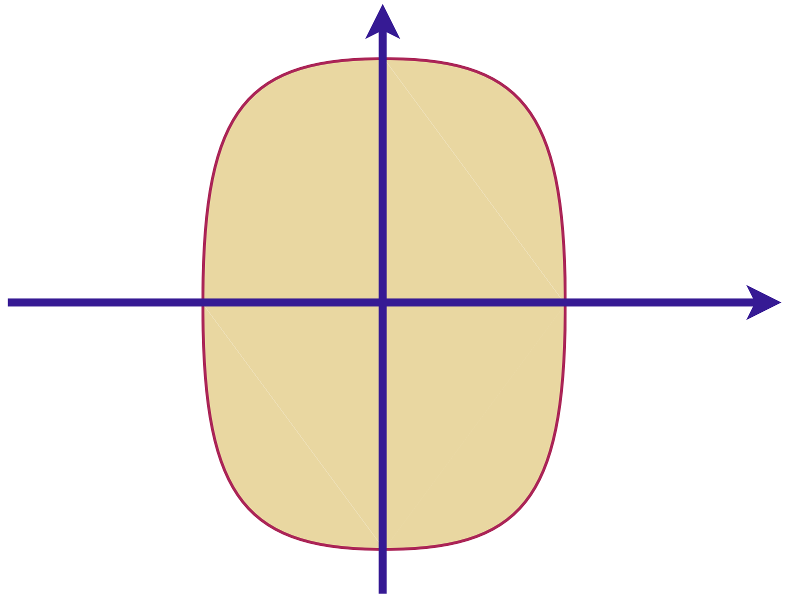
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau\, \phi \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

for spatially independent ϕ



$$\langle \phi \rangle > 0$$



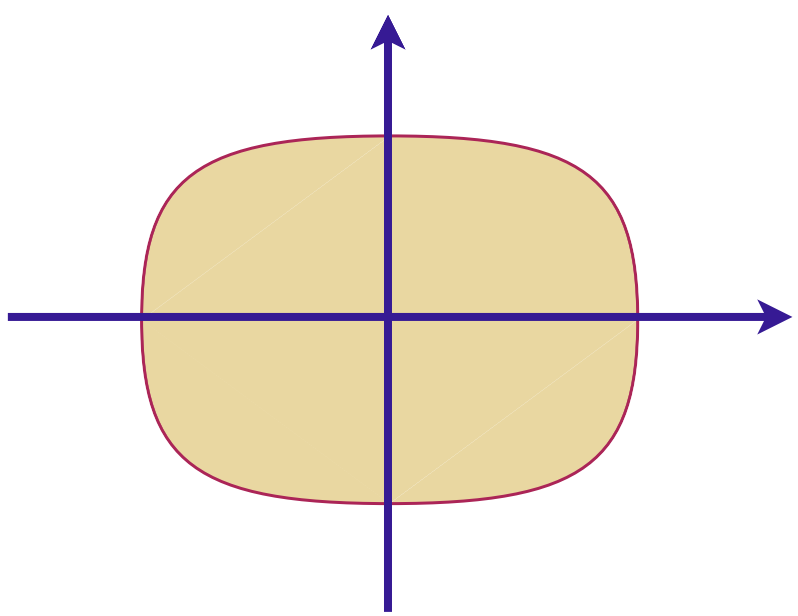
$$\langle \phi \rangle < 0$$

Quantum criticality of Pomeranchuk instability

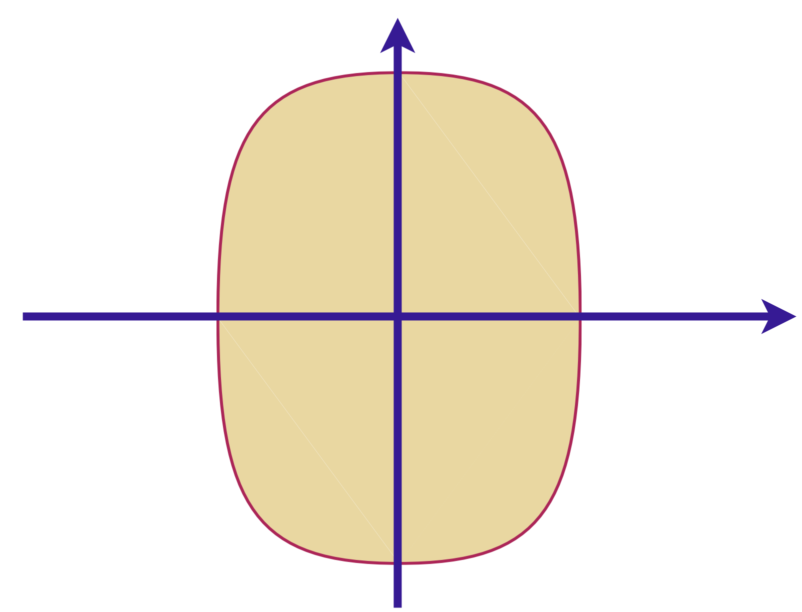
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$\langle \phi \rangle > 0$



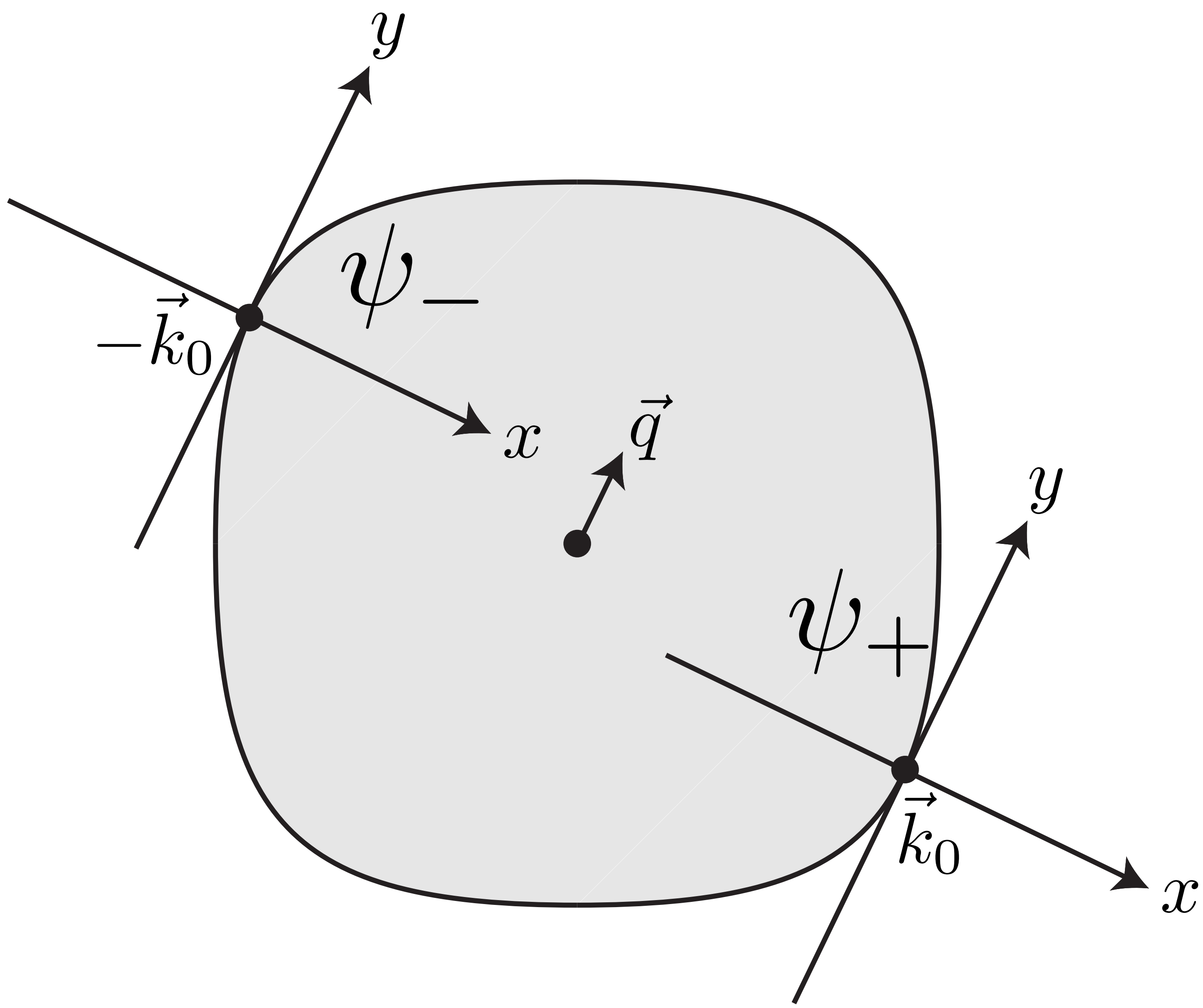
$\langle \phi \rangle < 0$

Quantum criticality of Pomeranchuk instability

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (r - r_c)\phi^2 + u\phi^4 \right]$$

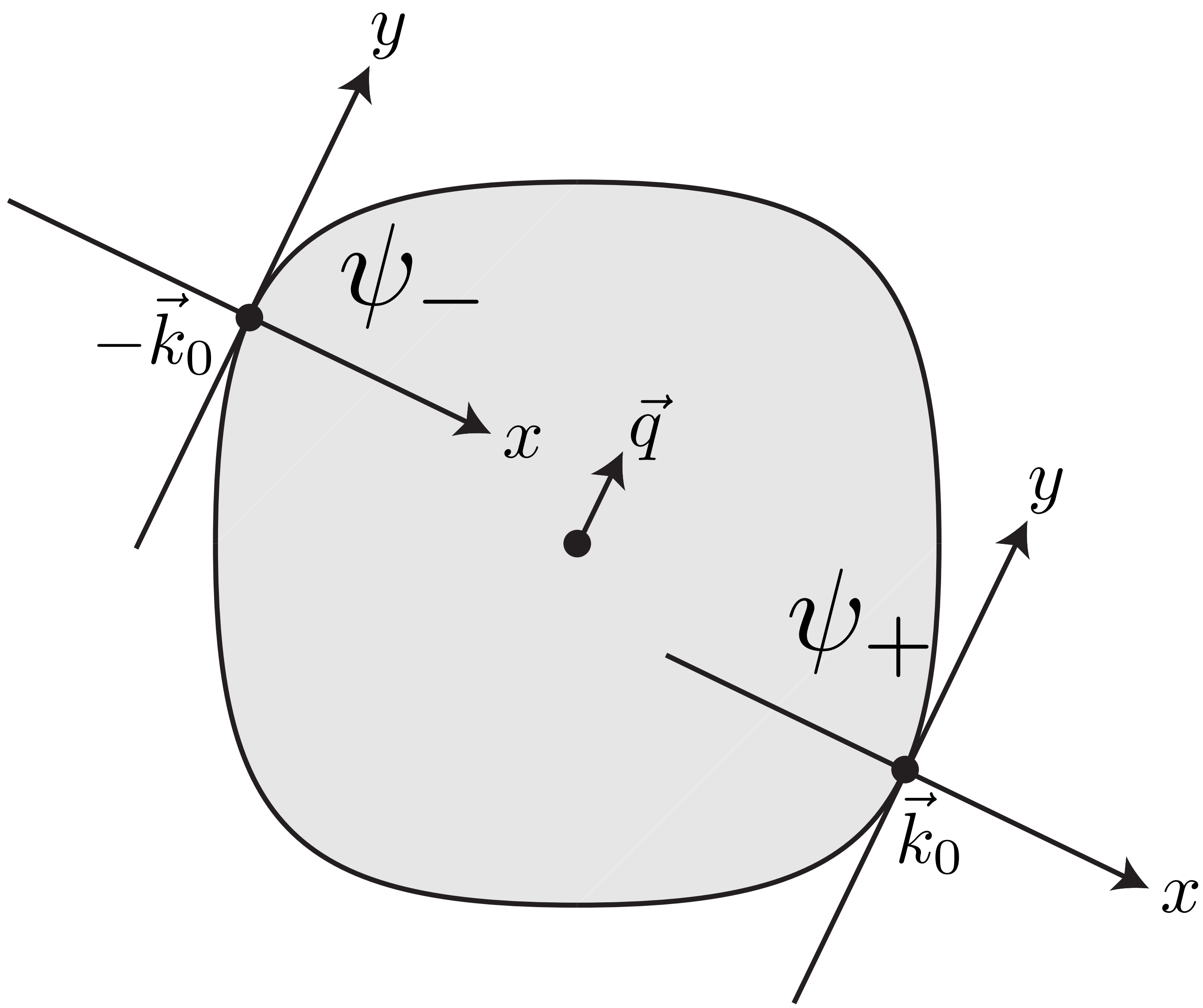
$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c^\dagger_{\mathbf{k}\alpha} \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$



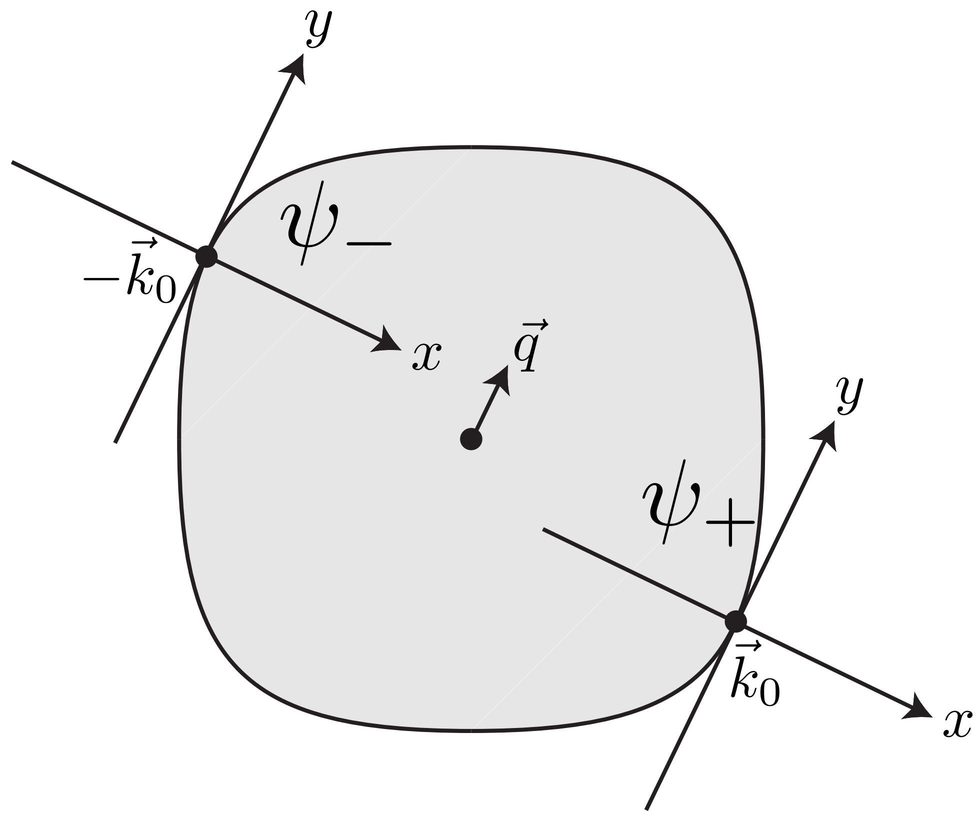
A ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

Expand fermion kinetic energy at wavevectors about $\vec{k}_0$



$$
\begin{aligned}
\mathcal{L} \;=\; & \psi_+^\dagger \left(\zeta \partial_\tau - i \partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\zeta \partial_\tau + i \partial_x - \partial_y^2\right) \psi_- \\
& - \lambda \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_- \right) + \frac{1}{2g} \left(\partial_y \phi\right)^2 + \frac{r}{2} \phi^2
\end{aligned}
$$

Theory of Ising-nematic transition

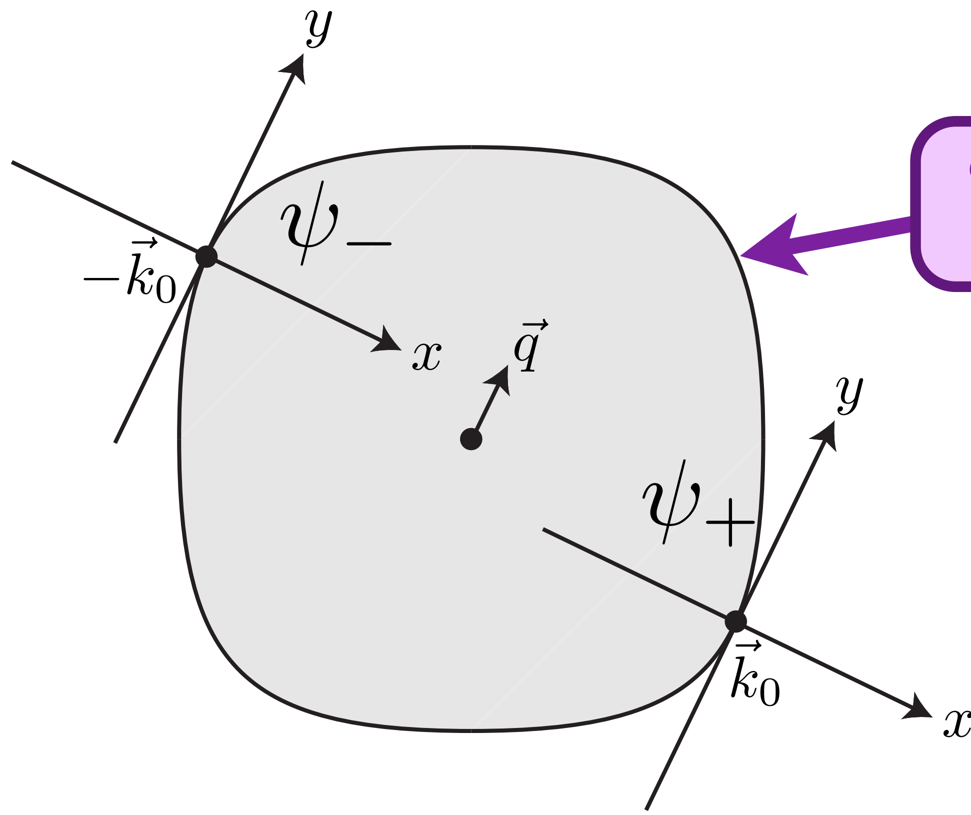


Emergent "Galilean invariance" at low energy ($s = \pm$):

$$\phi(x, y) \to \phi(x, y + \theta x), \quad \psi_s(x, y) \to e^{-is\left(\frac{\theta}{2} y + \frac{\theta^2}{4} x\right)} \psi_s(x, y + \theta x)$$

which implies for the fermion Green's function

$$G(q_x, q_y) = G(s q_x + q_y^2).$$



"Hot" Fermi surfaces

Emergent "Galilean invariance" at low energy ($s = \pm$):

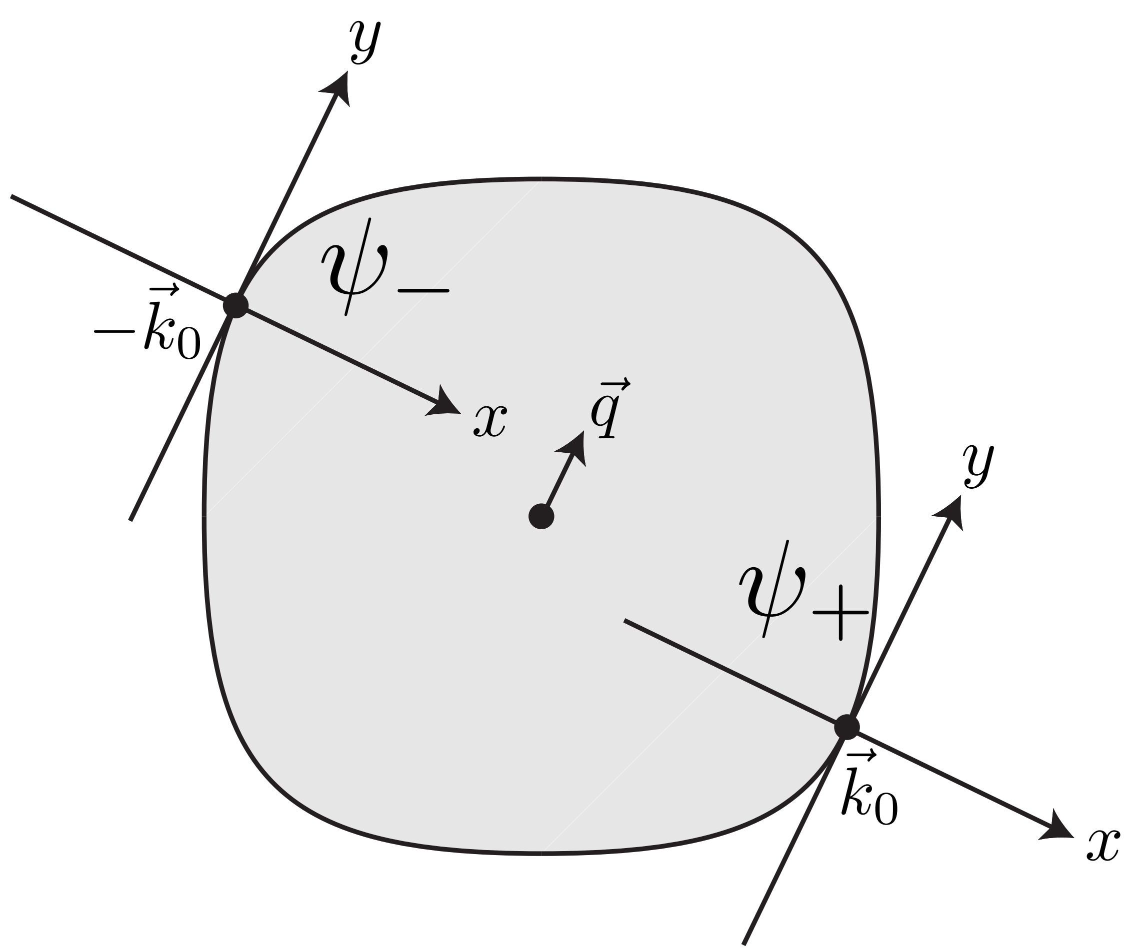
$$\phi(x, y) \to \phi(x, y + \theta x), \quad \psi_s(x, y) \to e^{-is(\frac{\theta}{2} y + \frac{\theta^2}{4} x)} \psi_s(x, y + \theta x)$$

which implies for the fermion Green's function

$$G(q_x, q_y) = G(s q_x + q_y^2).$$

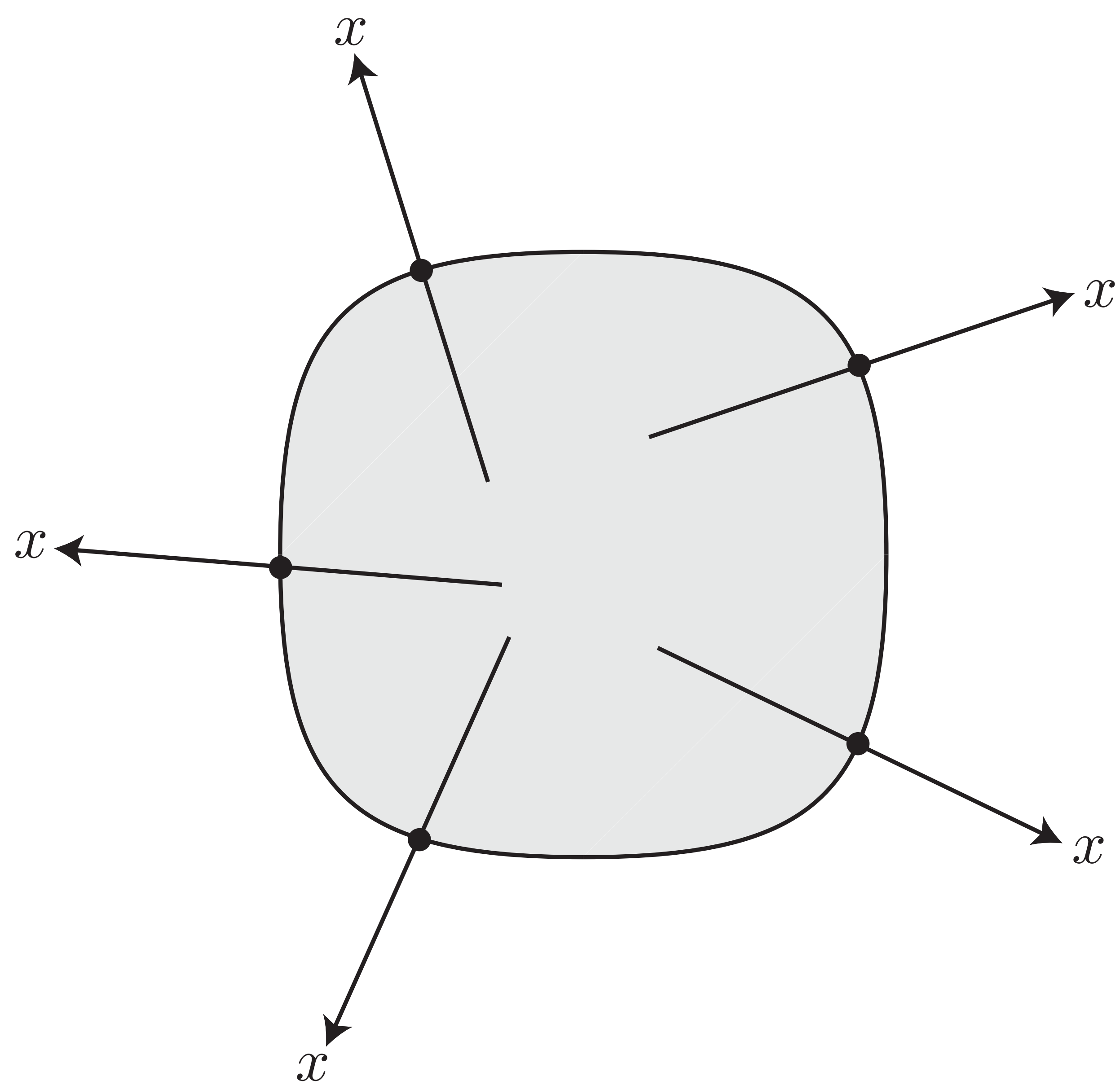
Line of singularities in momentum space on the "hot" Fermi surface $s q_x + q_y^2 = 0$.

- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.



- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.

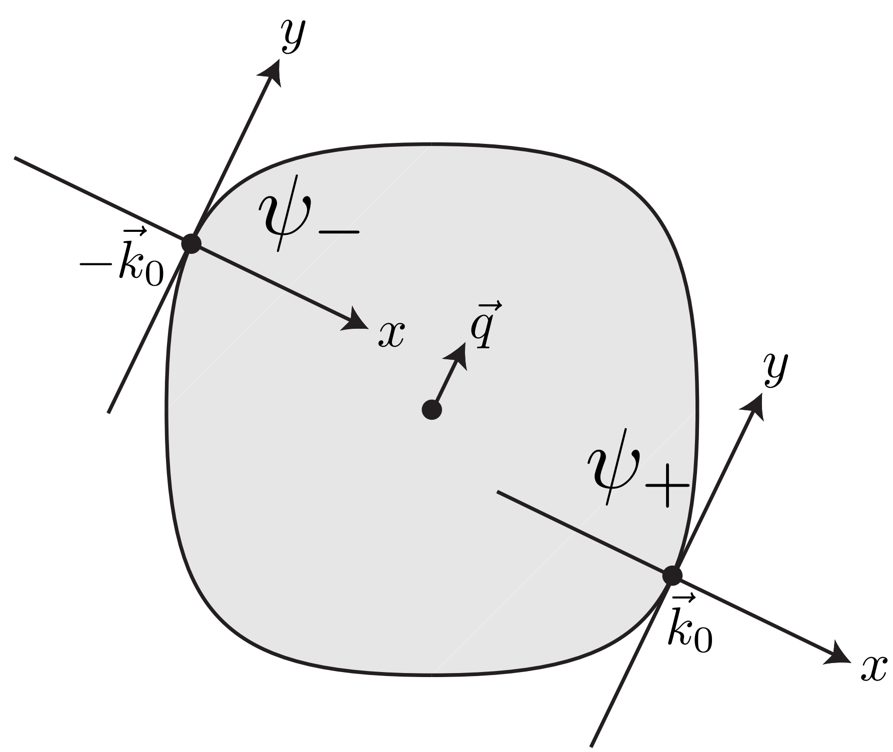
- Contrast with "Fermi surface bosonization" methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.

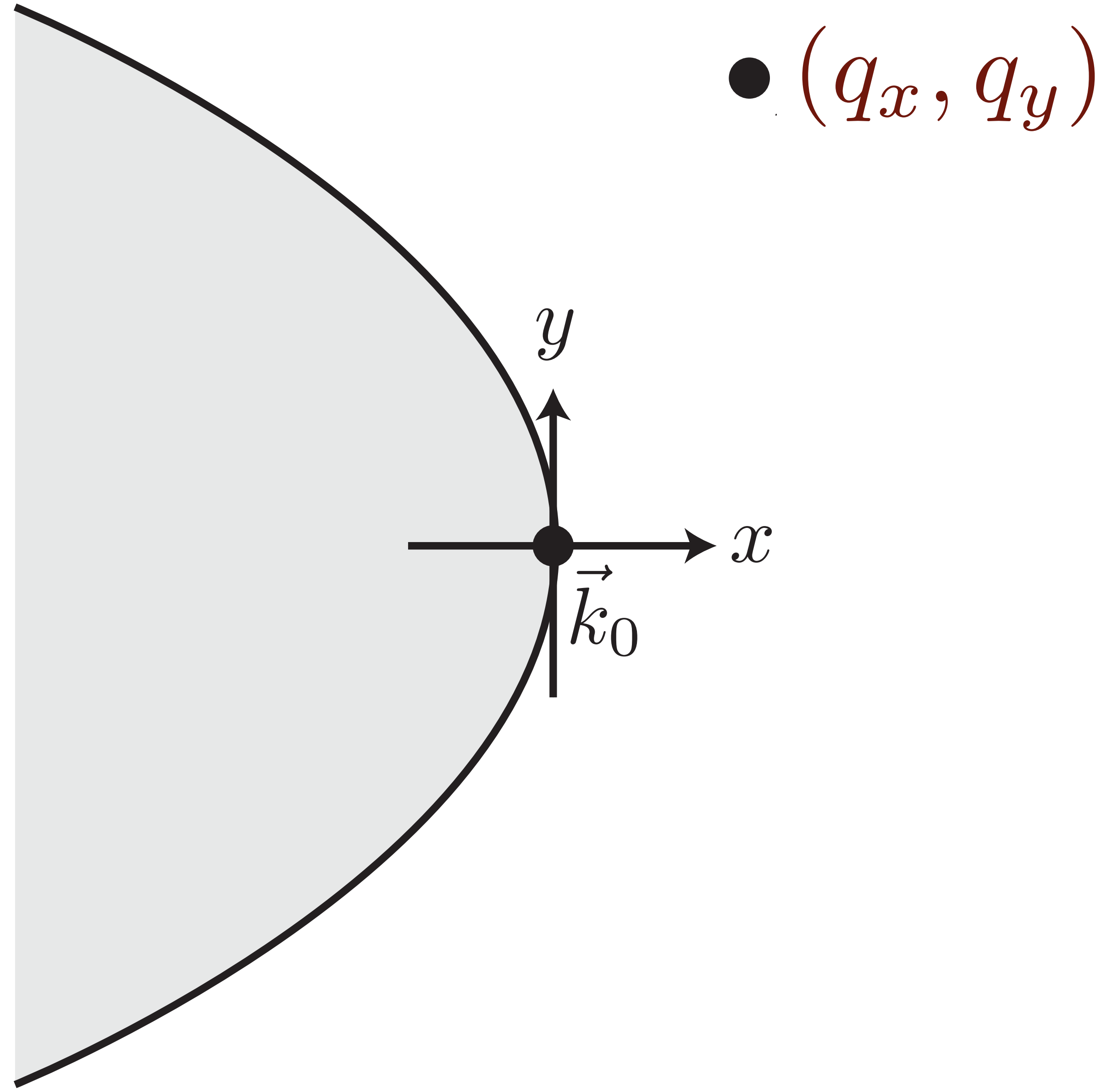


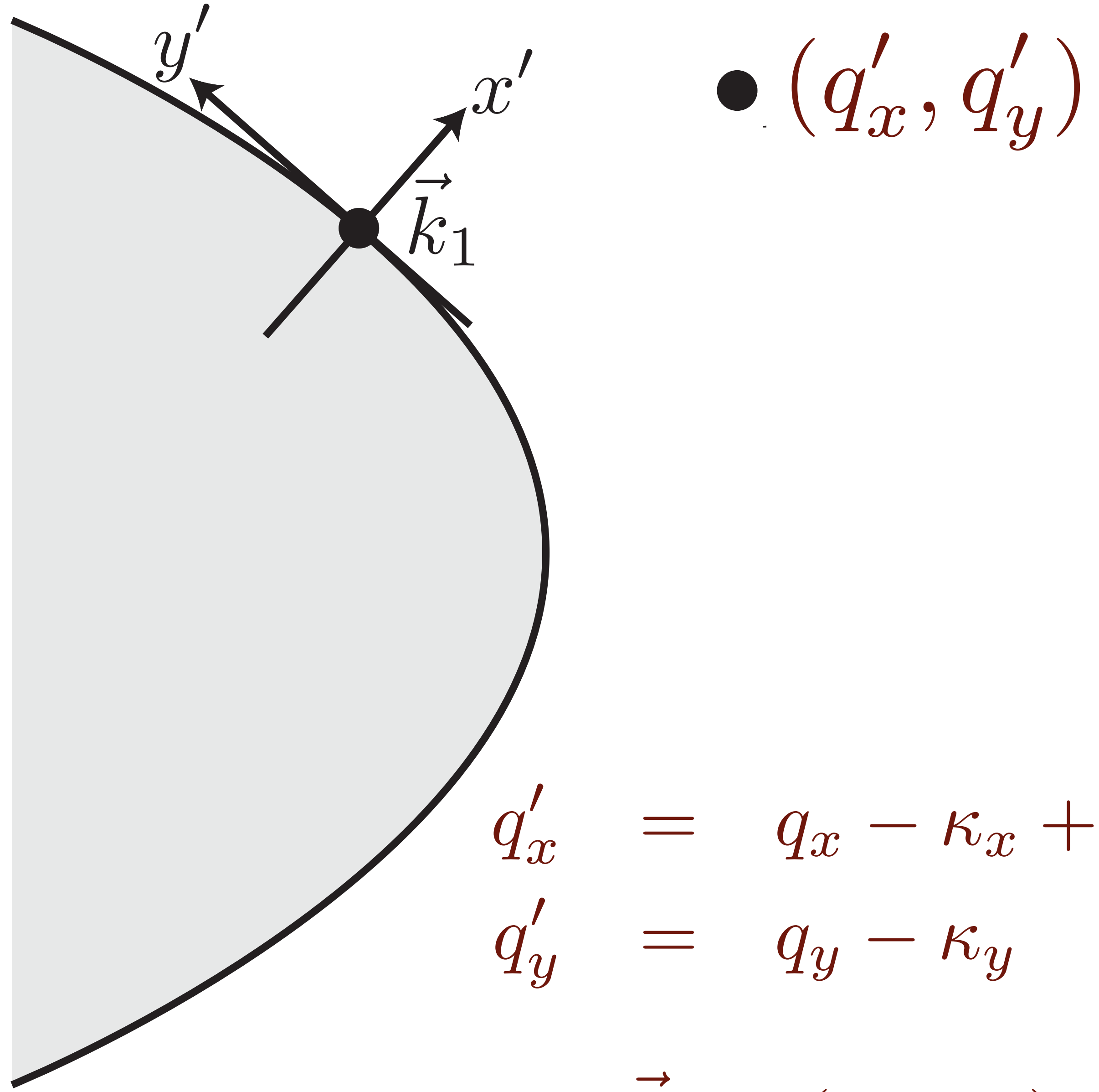
- Critical point is described by an *infinite* set of 2+1 dimensional field theories, one for each direction $\hat{q}$.

- Contrast with "Fermi surface bosonization" methods where there are an infinite set of 1+1 dimensional field theories, one for each direction $\hat{q}$.

- Our approach leads to a redundant description of underlying degrees of freedom. The "Galilean symmetry" ensures consistency of redundant description.

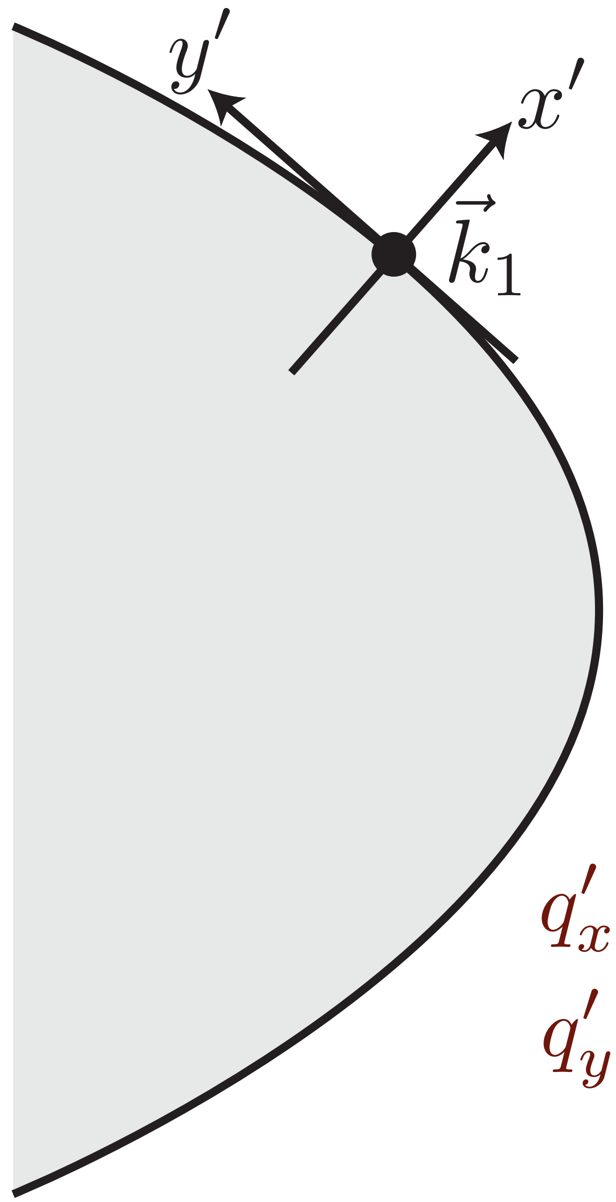



(q_x, q_y)
y
x
$\vec{k}_0$



$$q'_x = q_x - \kappa_x + 2\kappa_y(q_y - \kappa_y)$$
$$q'_y = q_y - \kappa_y \quad ,$$

where $\vec{k}_1 = (\kappa_x, \kappa_y)$ and $\kappa_x + \kappa_y^2 = 0$.



$$
\begin{aligned}
q'_x &= q_x - \kappa_x + 2\kappa_y(q_y - \kappa_y) \\
q'_y &= q_y - \kappa_y \qquad ,
\end{aligned}
$$

where $\vec{k}_1 = (\kappa_x, \kappa_y)$ and $\kappa_x + \kappa_y^2 = 0$.

Note $q'_x + q'^2_y = q_x + q_y^2$: ensures compatibility of redundant 2+1 dimensional field theories.

$$
\begin{aligned}
\mathcal{L} \;=\;& \psi_+^\dagger \left(\zeta\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\zeta\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\
& - \lambda\phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g}\left(\partial_y\phi\right)^2 + \frac{r}{2}\phi^2
\end{aligned}
$$

After tuning the single parameter $r \sim \lambda - \lambda_c$, and sending $\zeta \to 0$, $\mathcal{L}$ describes a critical theory with no coupling constants. There is a separate copy of this critical theory for each direction $\hat{q}$. This theory has 2 independent exponents z and η, and the correlation length and susceptibility exponents are given by

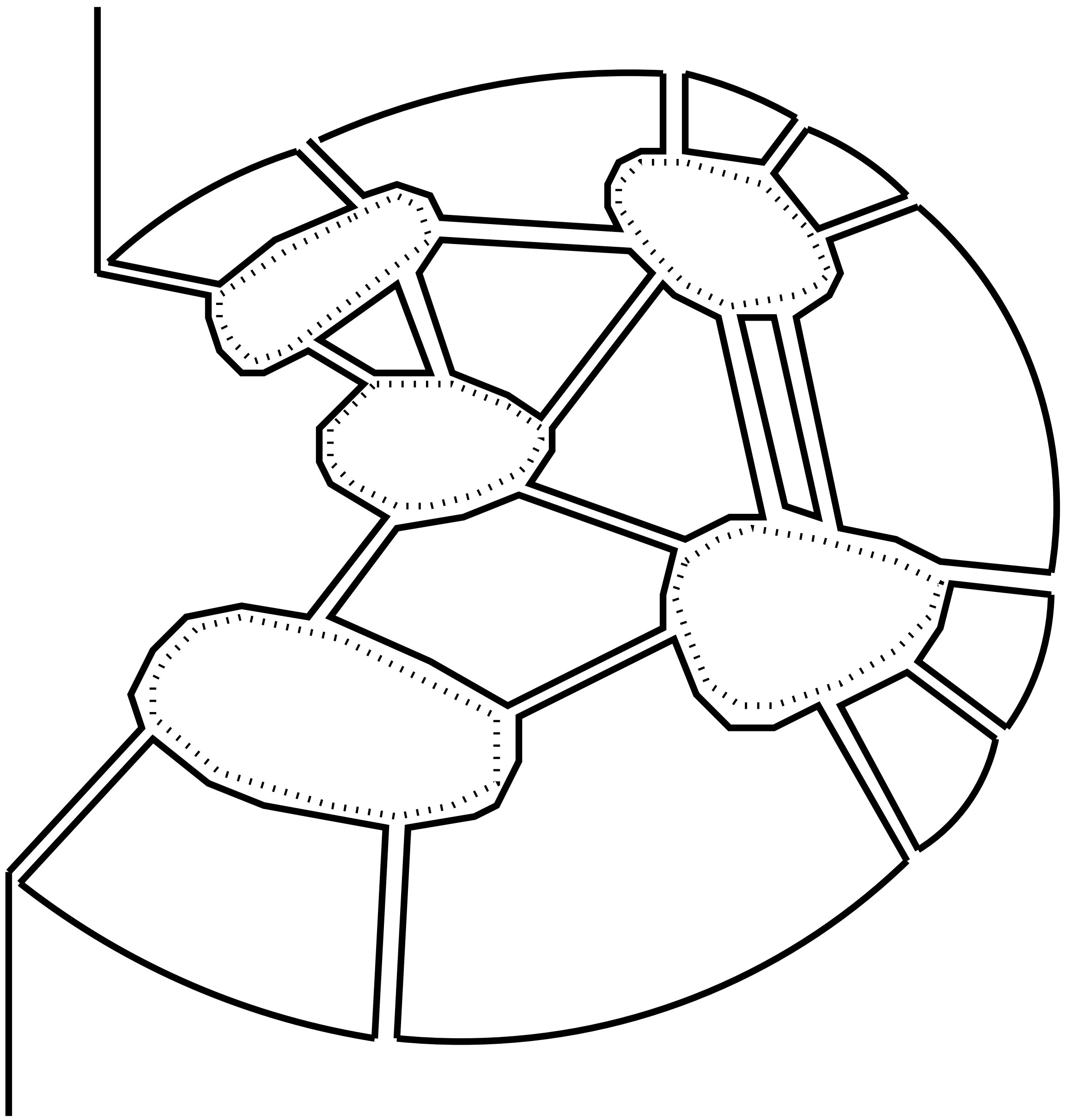
$$
\nu = \frac{1}{z-1} \quad ; \quad \gamma = 1
$$

The fermion and order parameter Green's functions obey the scaling forms

$$
G(\vec{q},\omega) = \xi^{2-\eta}\Phi_\psi\Big((q_x+q_y^2)\xi^2, \omega\xi^z\Big) \;\; ; \;\; D(\vec{q},\omega) = \xi^{z-1}\Phi_\phi\Big(q_y\xi, \omega\xi^z\Big)
$$

We have computed the exponents to three loops, and find $z = 3$ and $\eta = 0.06824$ at this order.

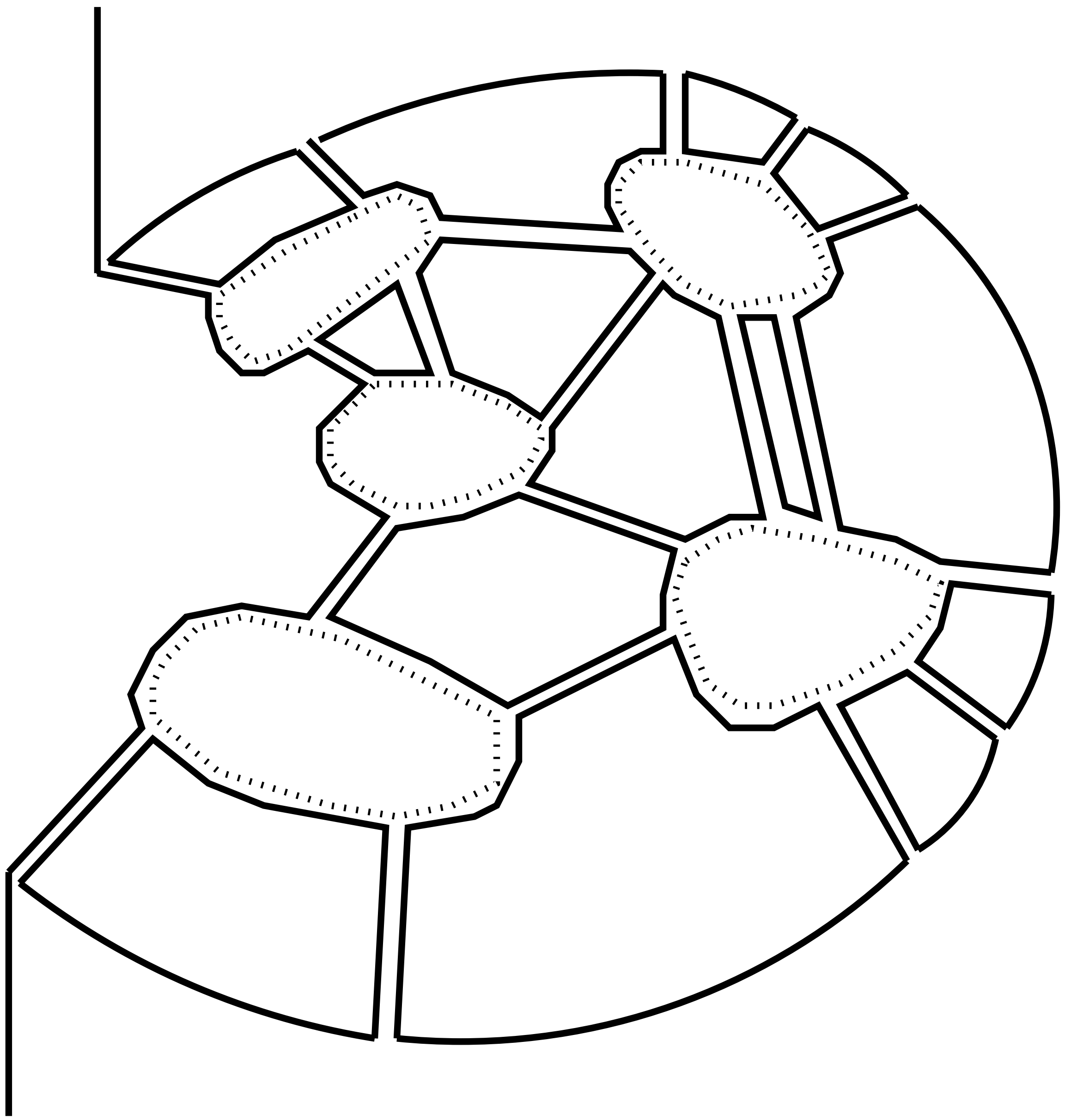
Computations in the 1/N expansion



All planar graphs of ψ_+ alone are as important as the leading term

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Computations in the $1/N$ expansion



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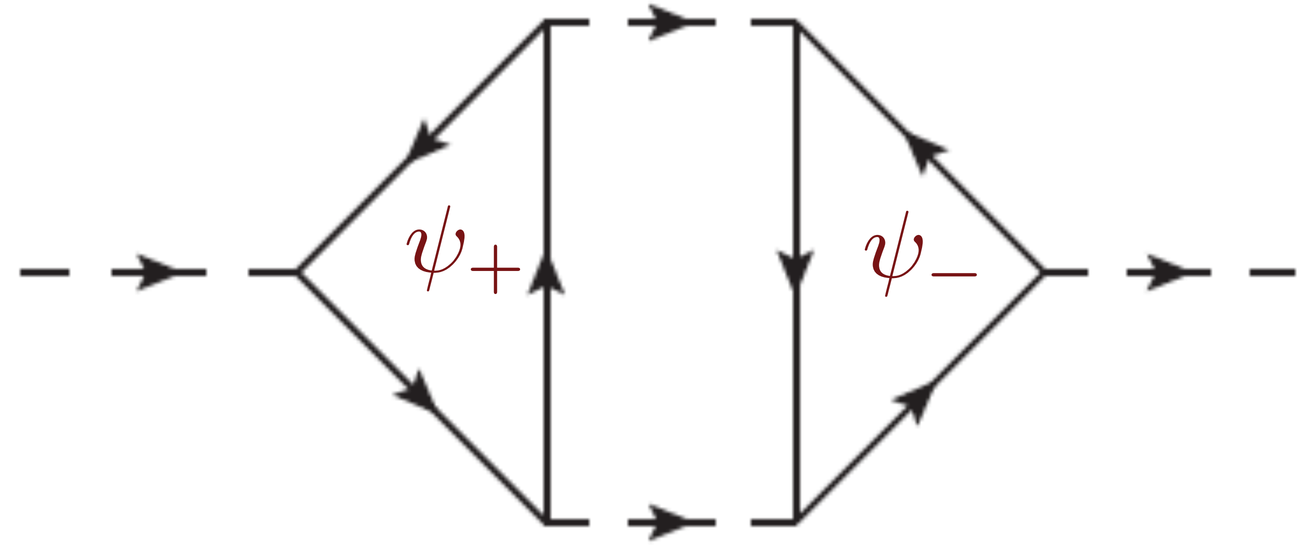

Graph mixing ψ_+ and ψ_- is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity

Conclusions

Theories for the onset of spin density wave and Ising-nematic order in metals are *strongly* coupled in two dimensions