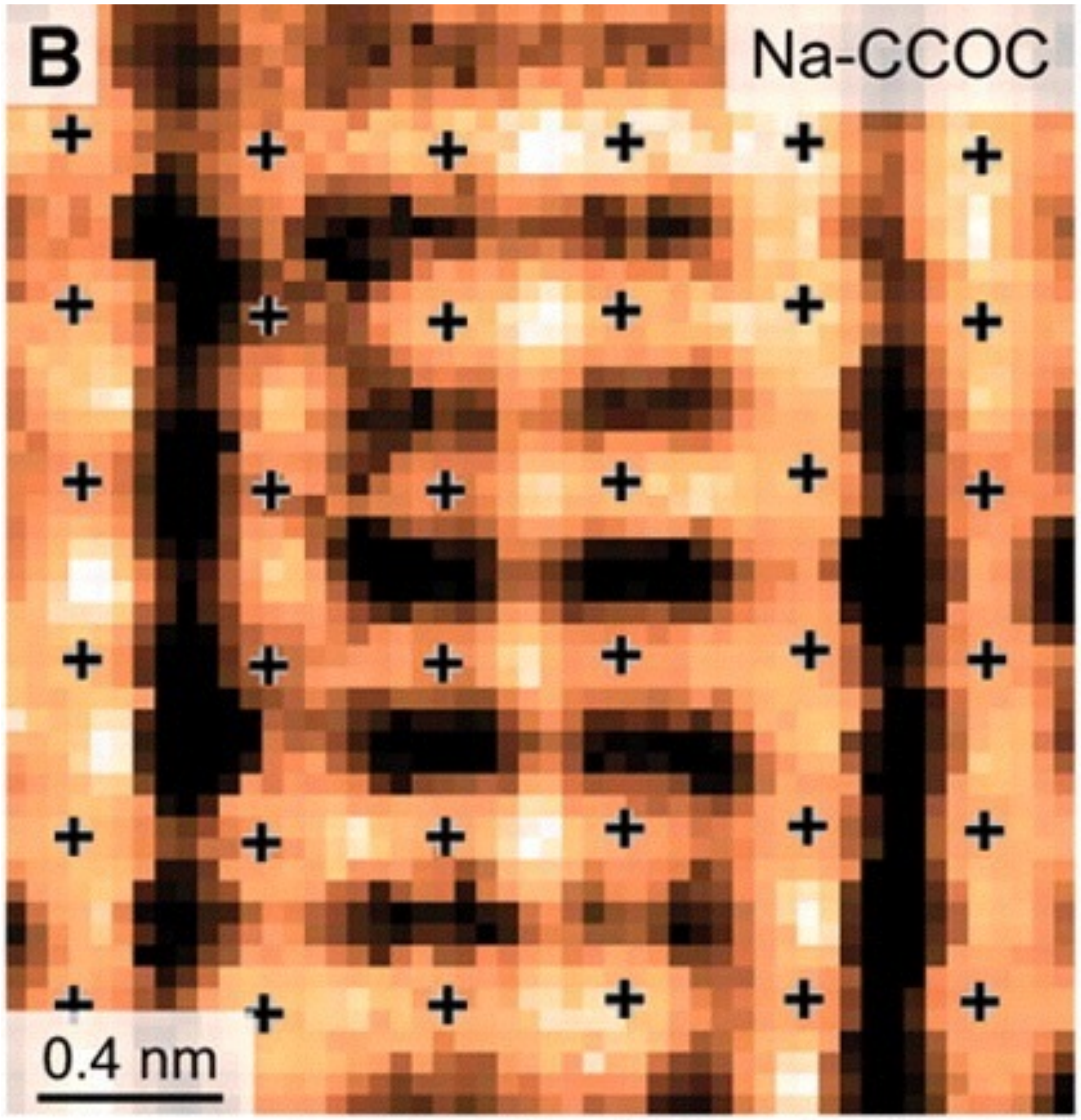
Outline

1. Antiferromagnetism and quantum criticality in insulators

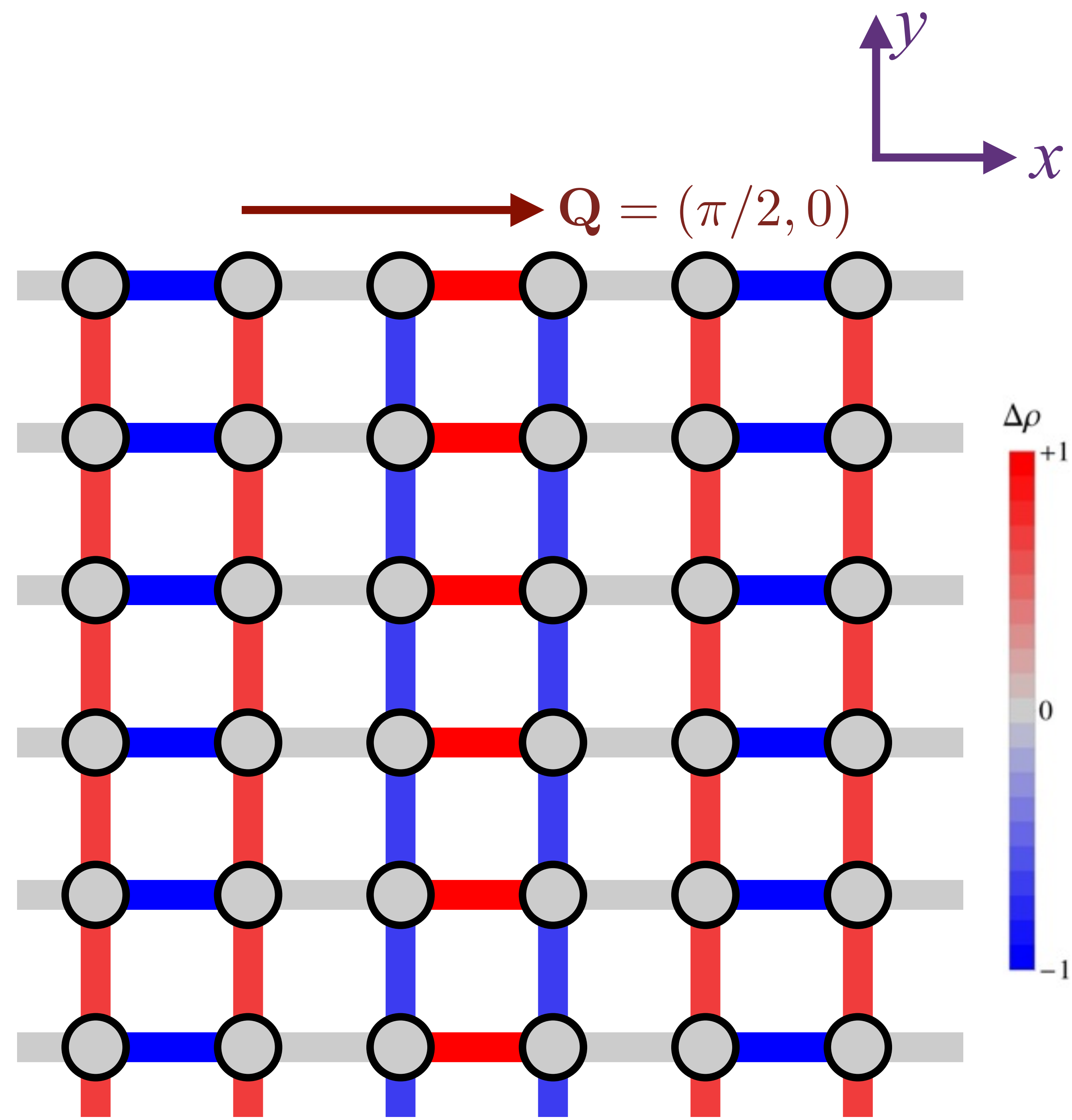
2. Onset of antiferromagnetism in metals, and d-wave superconductivity

3. Competing density wave order, and the pseudogap of the cuprate superconductors

4. Non-Fermi liquids

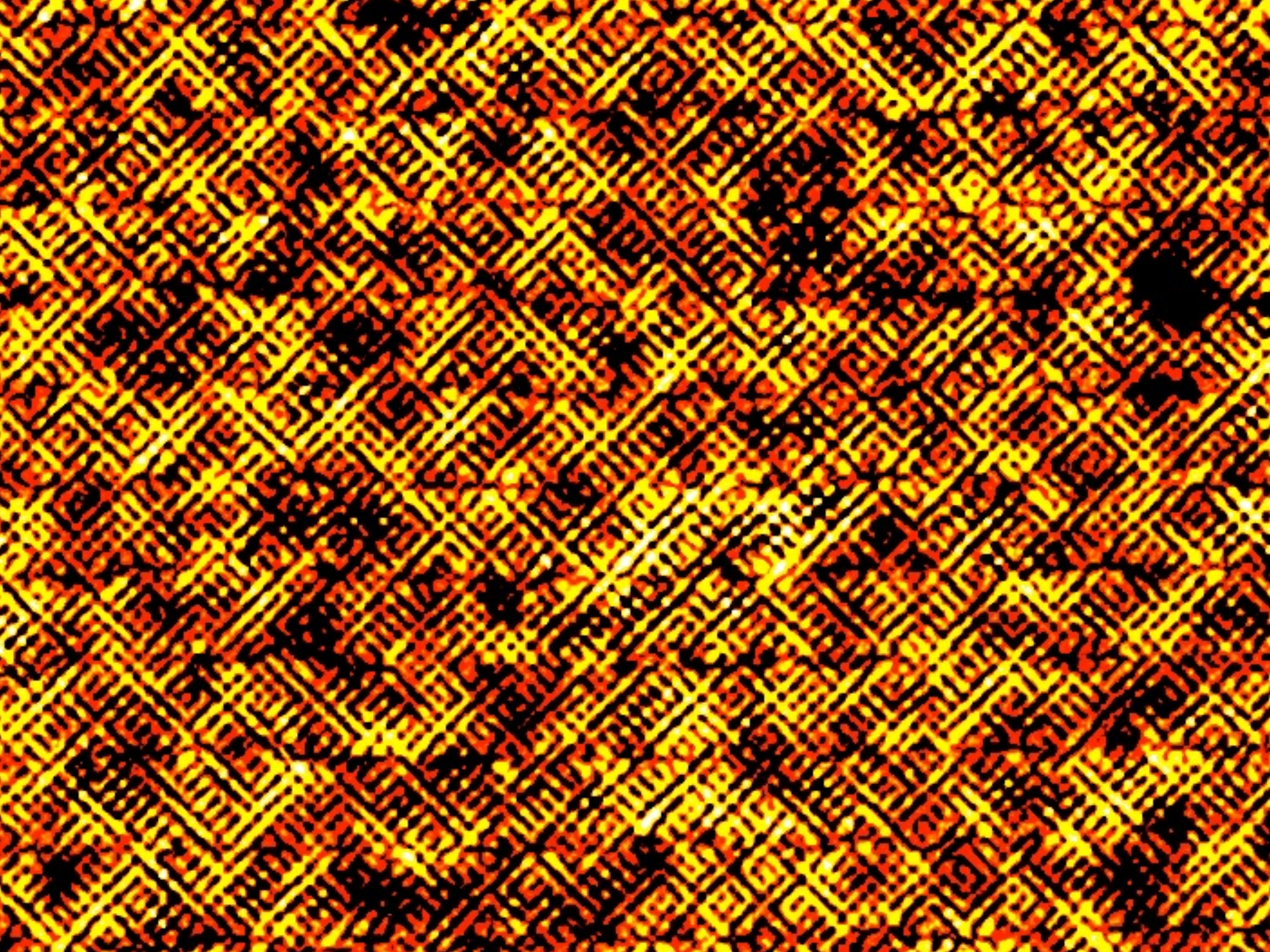


Y. Kohsaka *et al.*, Science **315**, 1380 (2007)

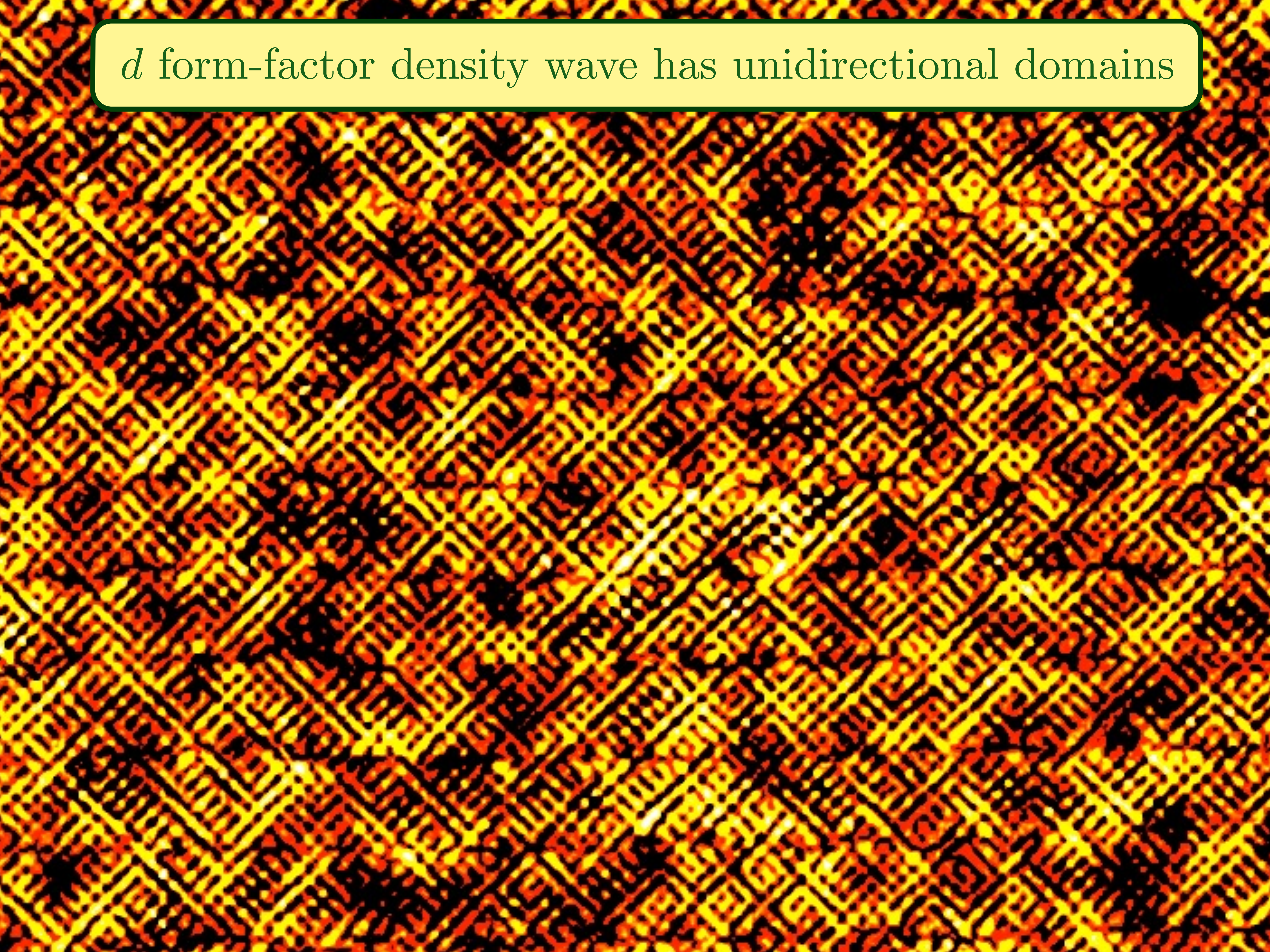


d-form factor density wave order

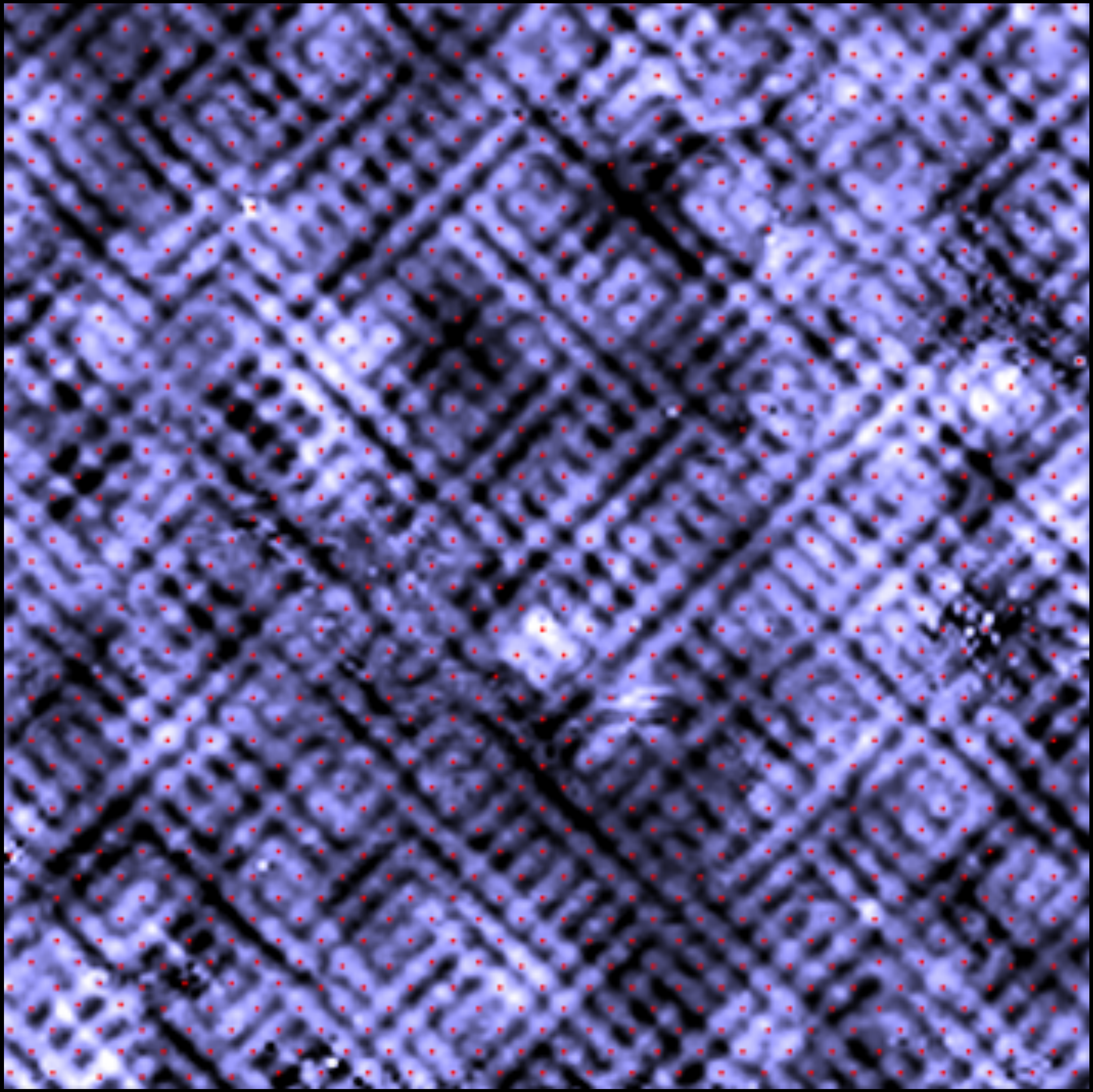
This specific *d*-form factor density wave order (with $\mathbf{Q}$ along the axes) was first predicted in S. Sachdev and R. LaPlaca, Phys. Rev. Lett. **111**, 027202 (2013).



d form-factor density wave has unidirectional domains

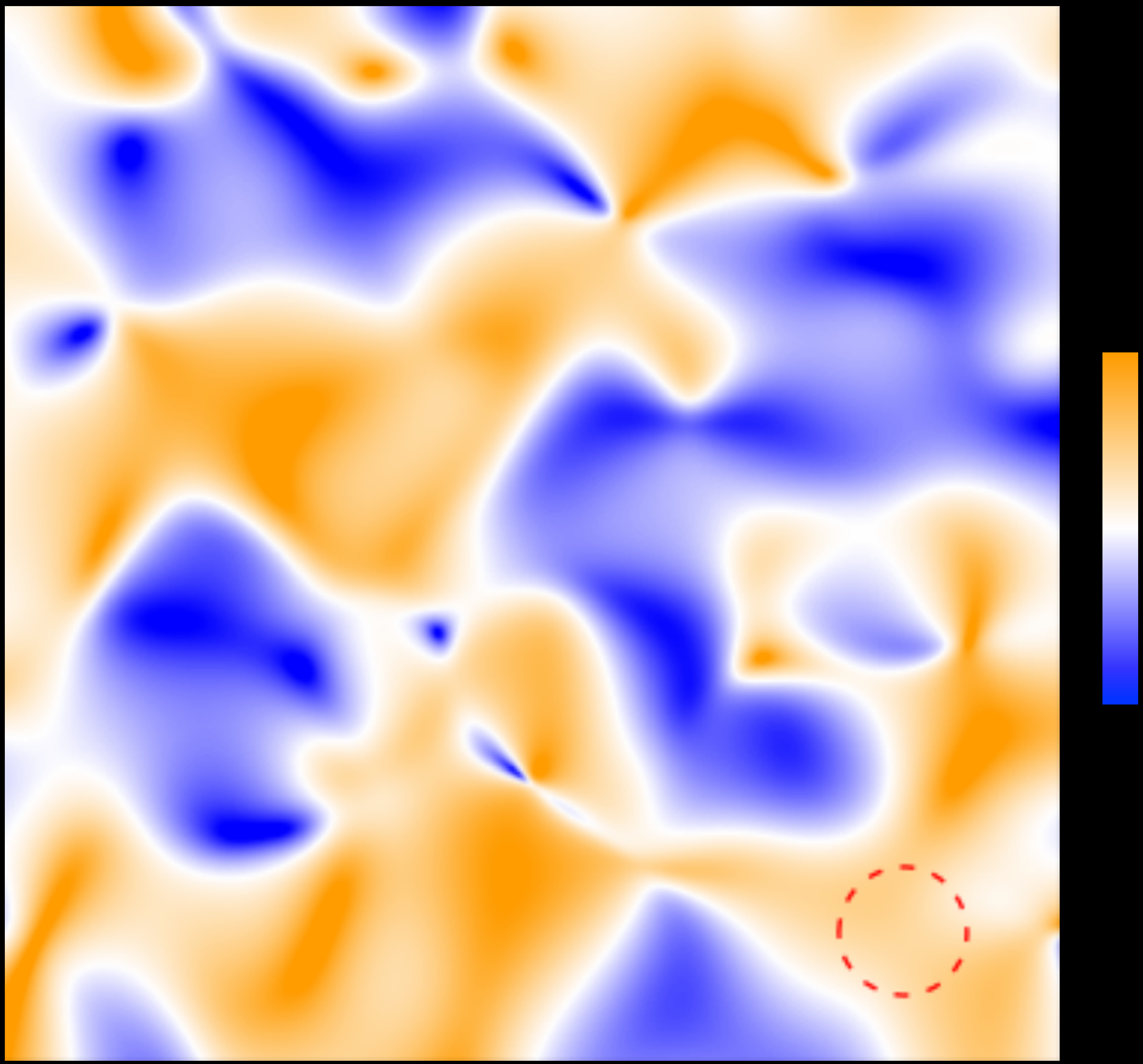


dFF-DW Unidirectional Domains



Z(r,150mV)

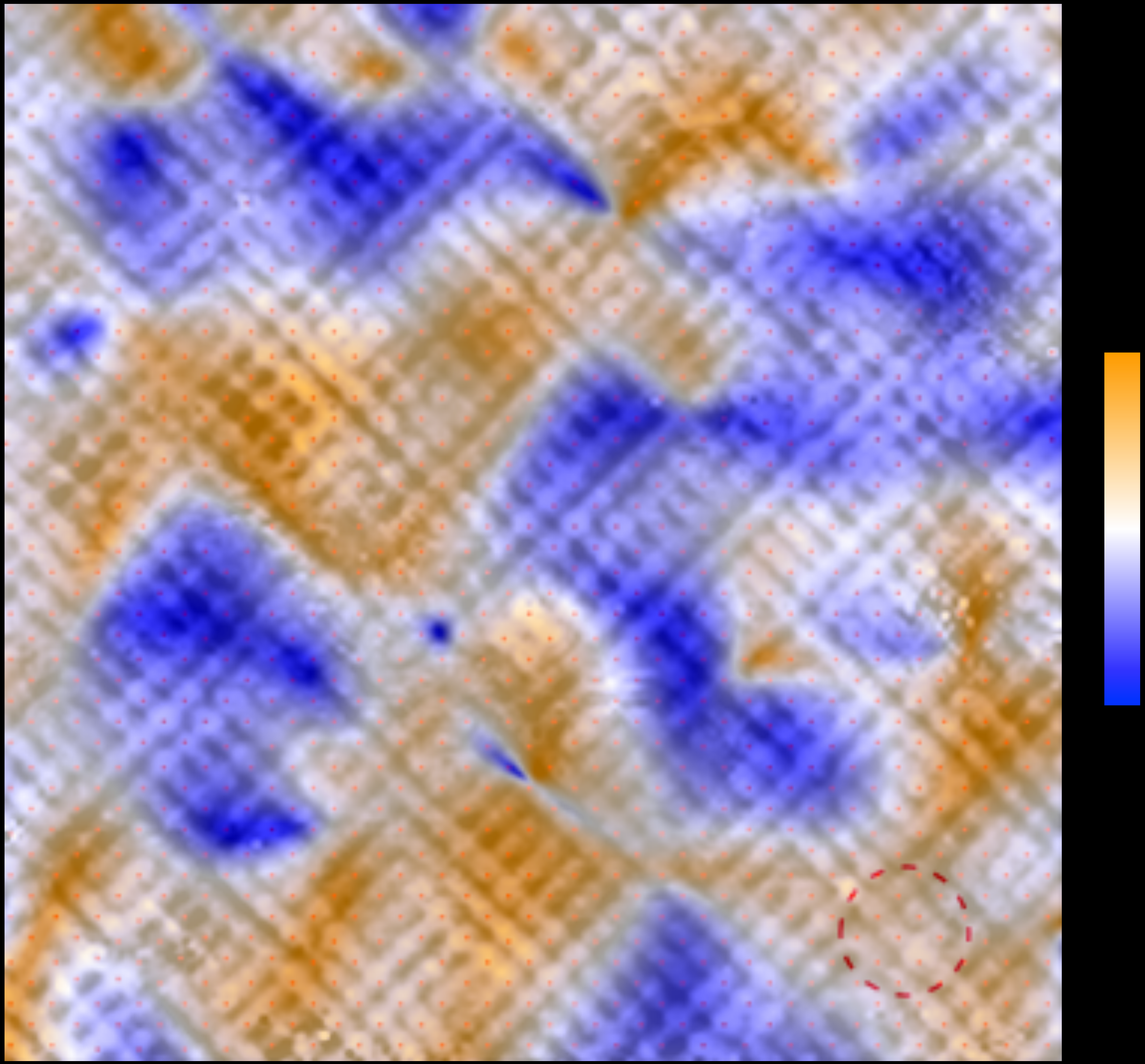
dFF-DW Unidirectional Domains



$(|O_y(r,q=Q_x)|-|O_x(r,q=Q_y)|)/(|O_y(r,q=Q_x)|+|O_x(r,q=Q_y)|)$

Primary DW direction Orange : // (1,0), Blue : //(0,1)

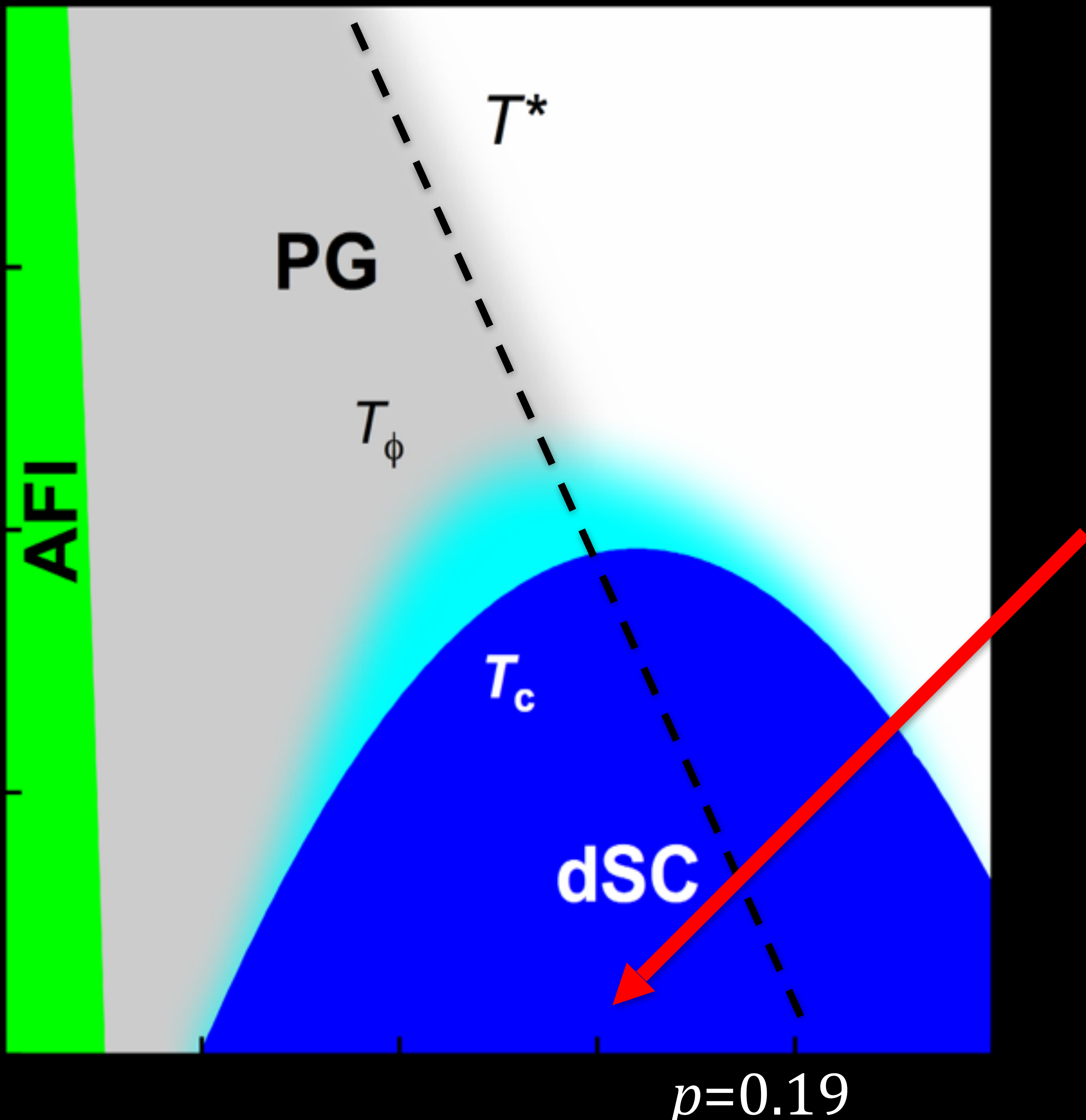
dFF-DW Unidirectional Domains

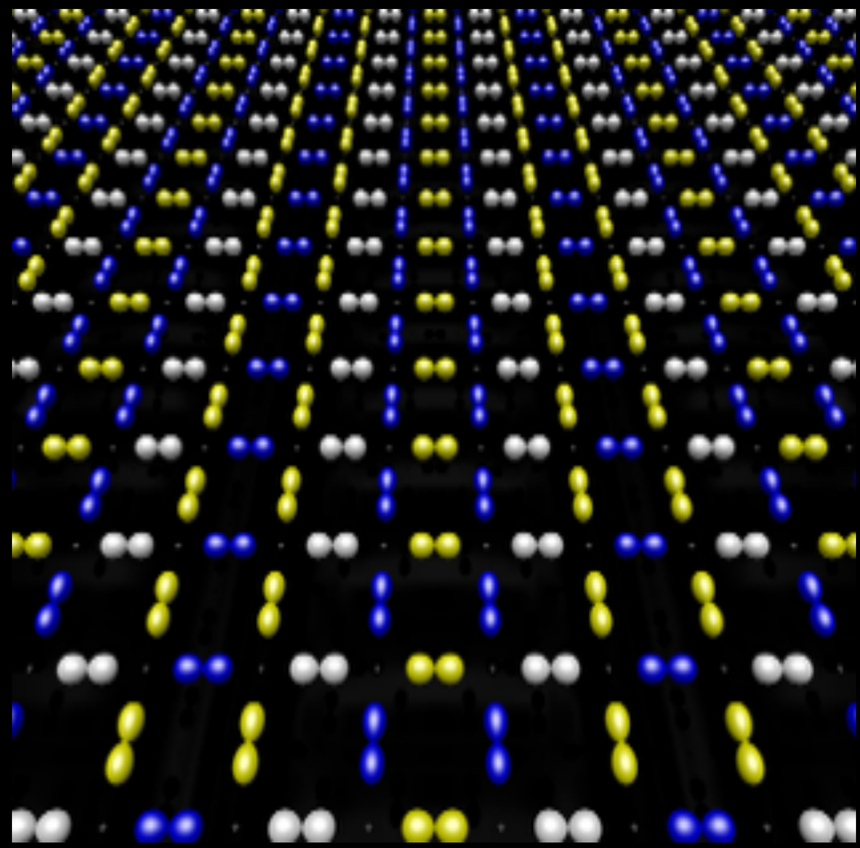


$(|O_y(r,q=Q_x)|-|O_x(r,q=Q_y)|)/(|O_y(r,q=Q_x)|+|O_x(r,q=Q_y)|)$

Primary DW direction Orange : // (1,0), Blue : //(0,1)

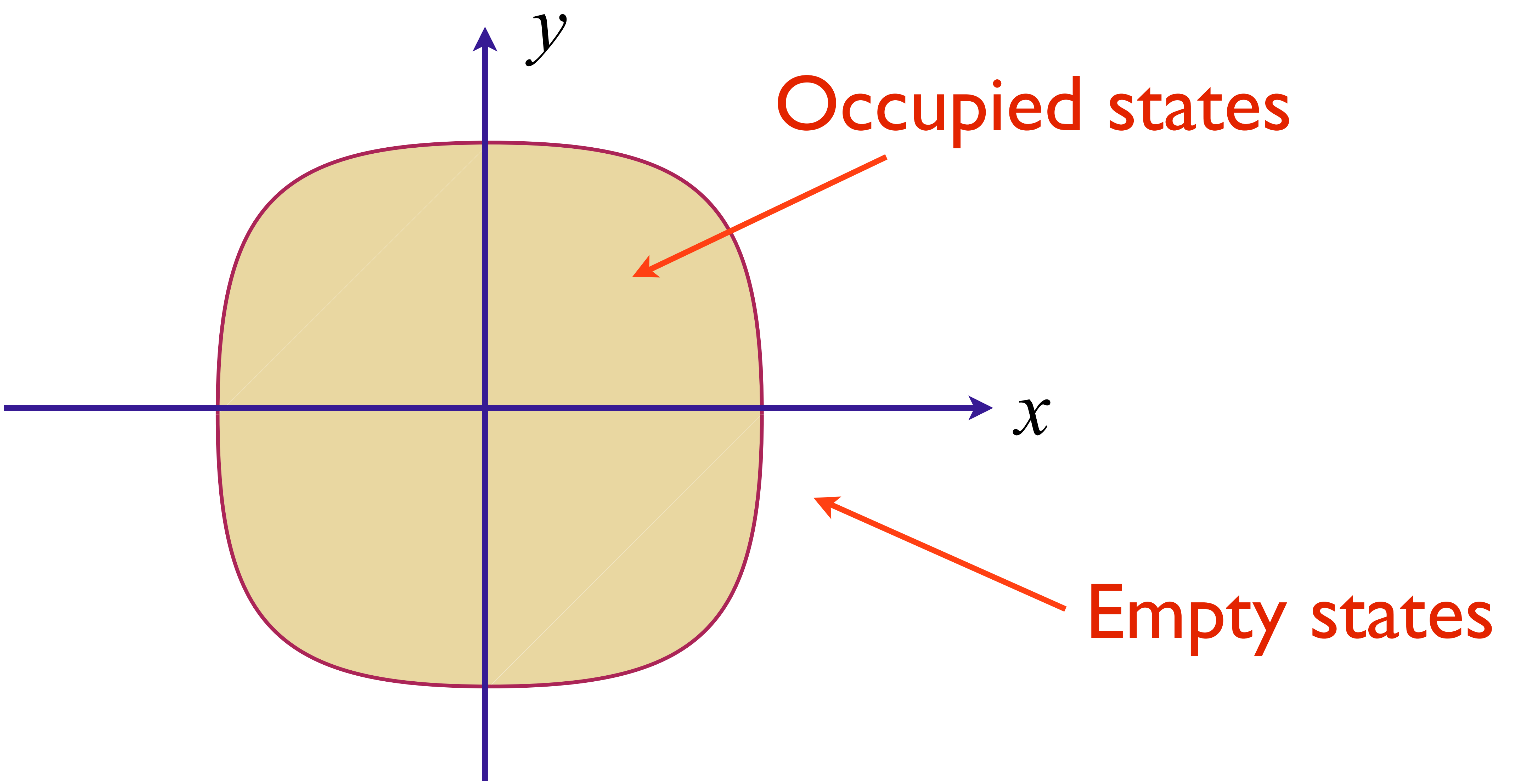
Phase-resolved Visualization of *d*-form factor DW in Cuprates





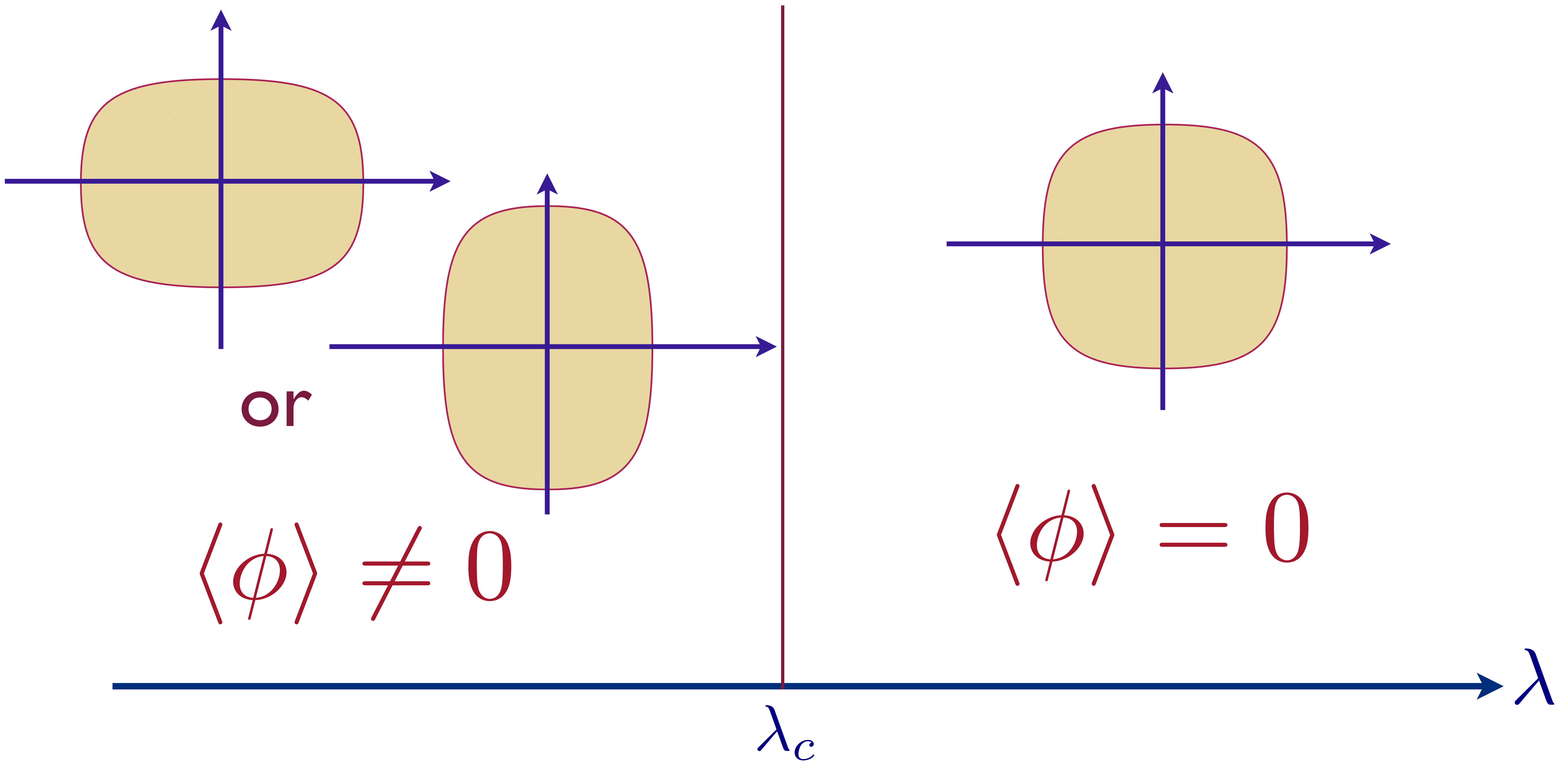
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS to appear, arXiv:1404.0362

Quantum criticality of Ising-nematic ordering in a metal



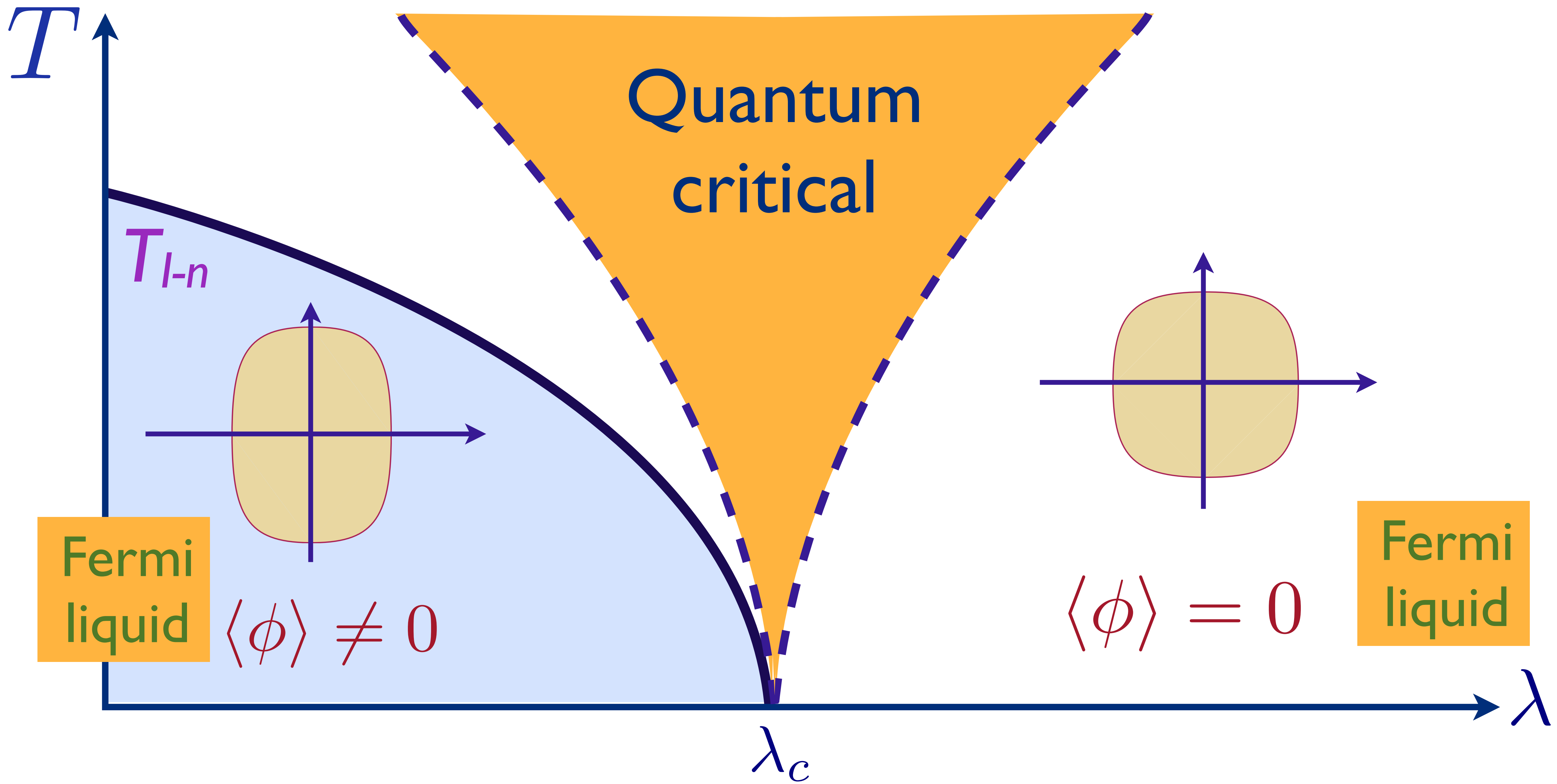
A metal with a Fermi surface
with full square lattice symmetry

Quantum criticality of Ising-nematic ordering in a metal



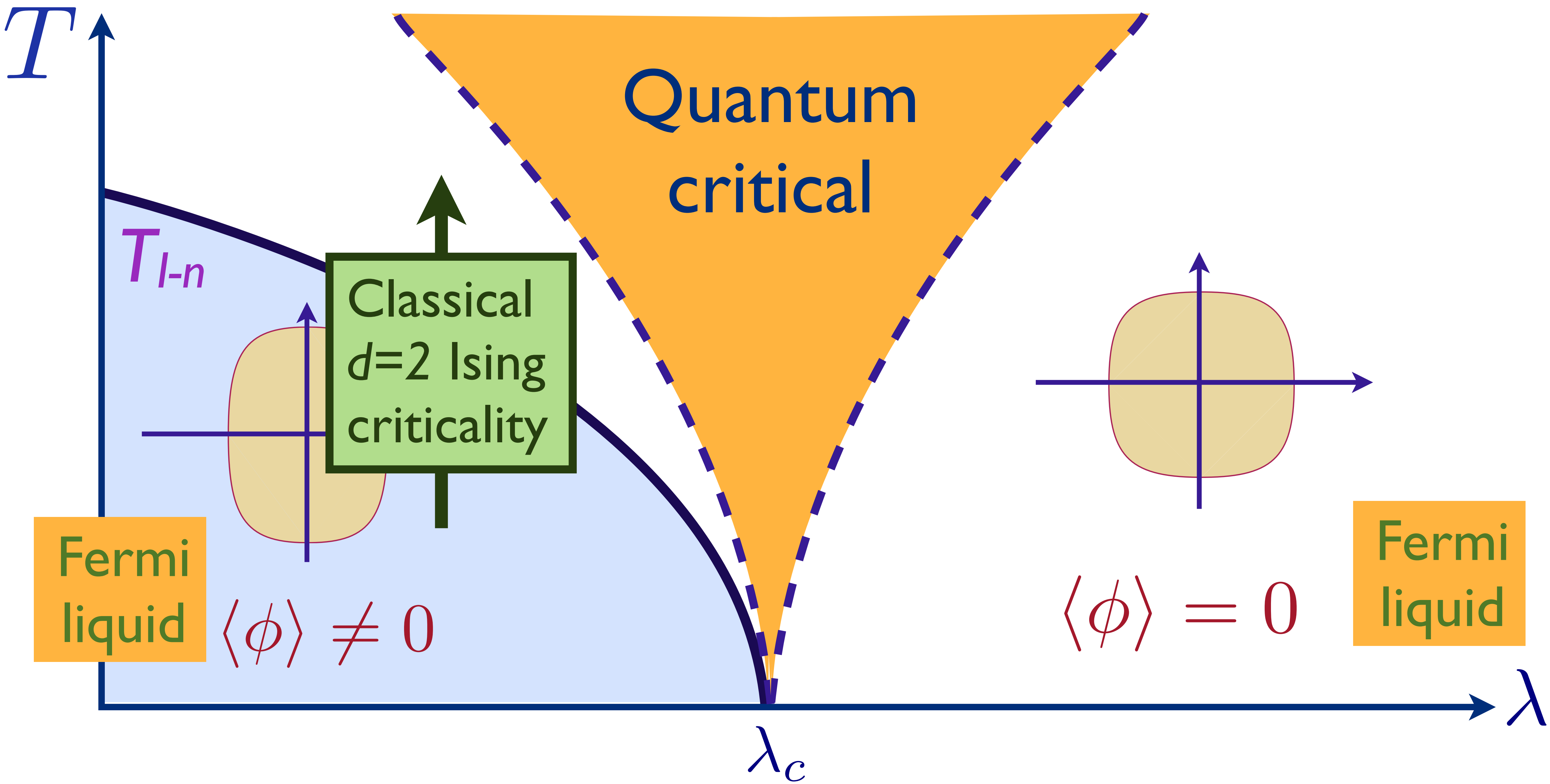
Pomeranchuk instability as a function of coupling λ

Quantum criticality of Ising-nematic ordering in a metal



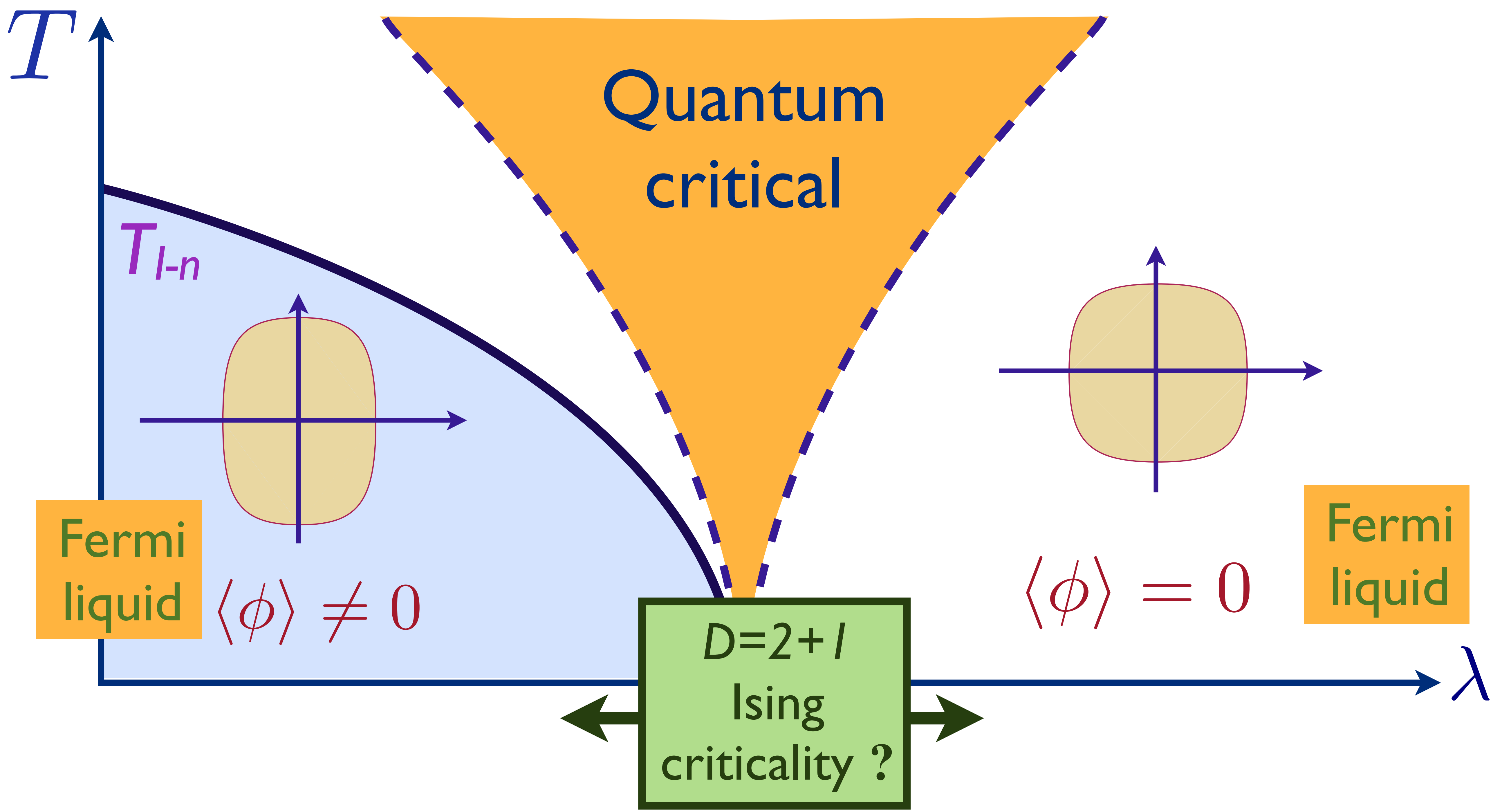
Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

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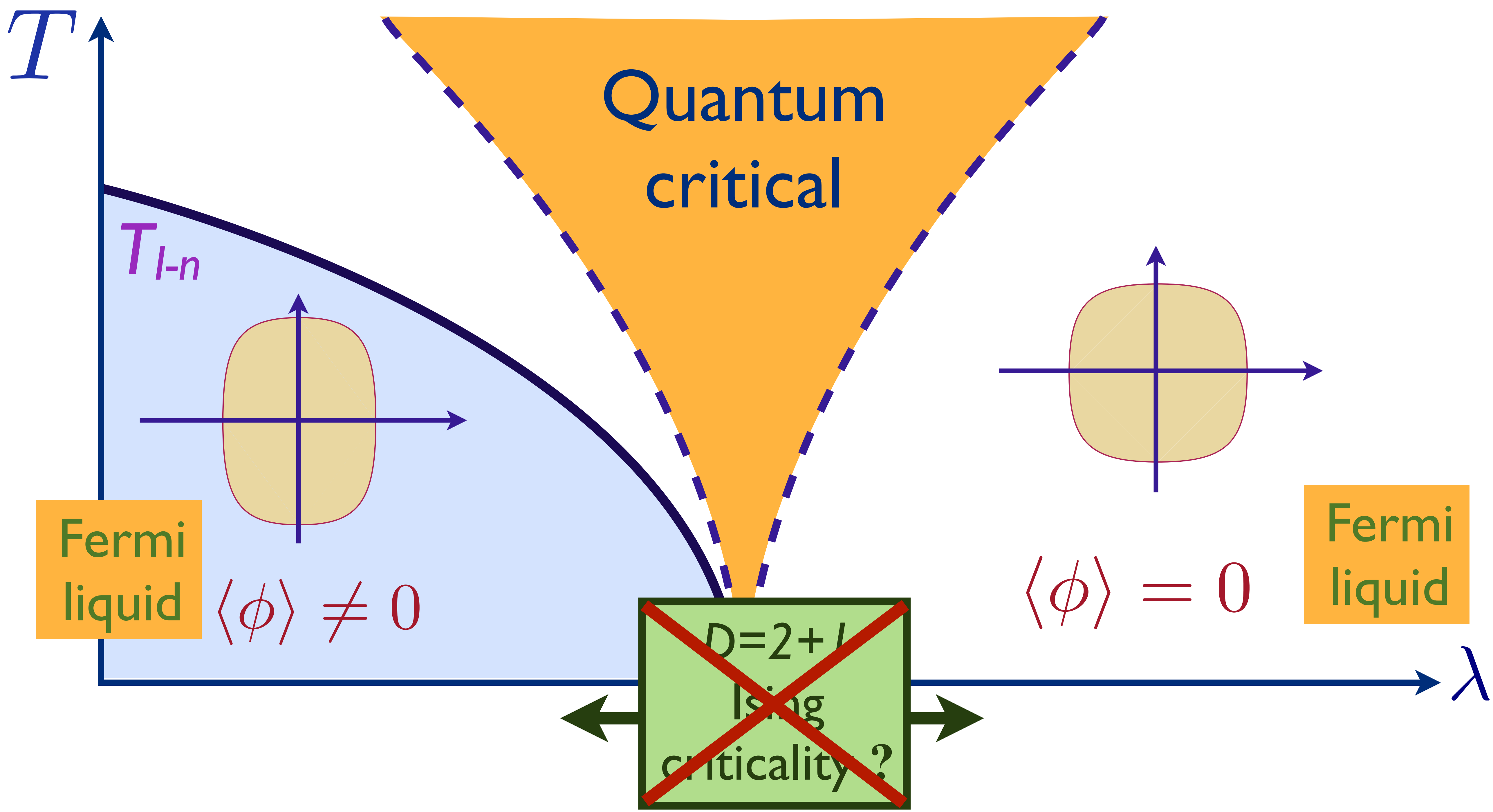

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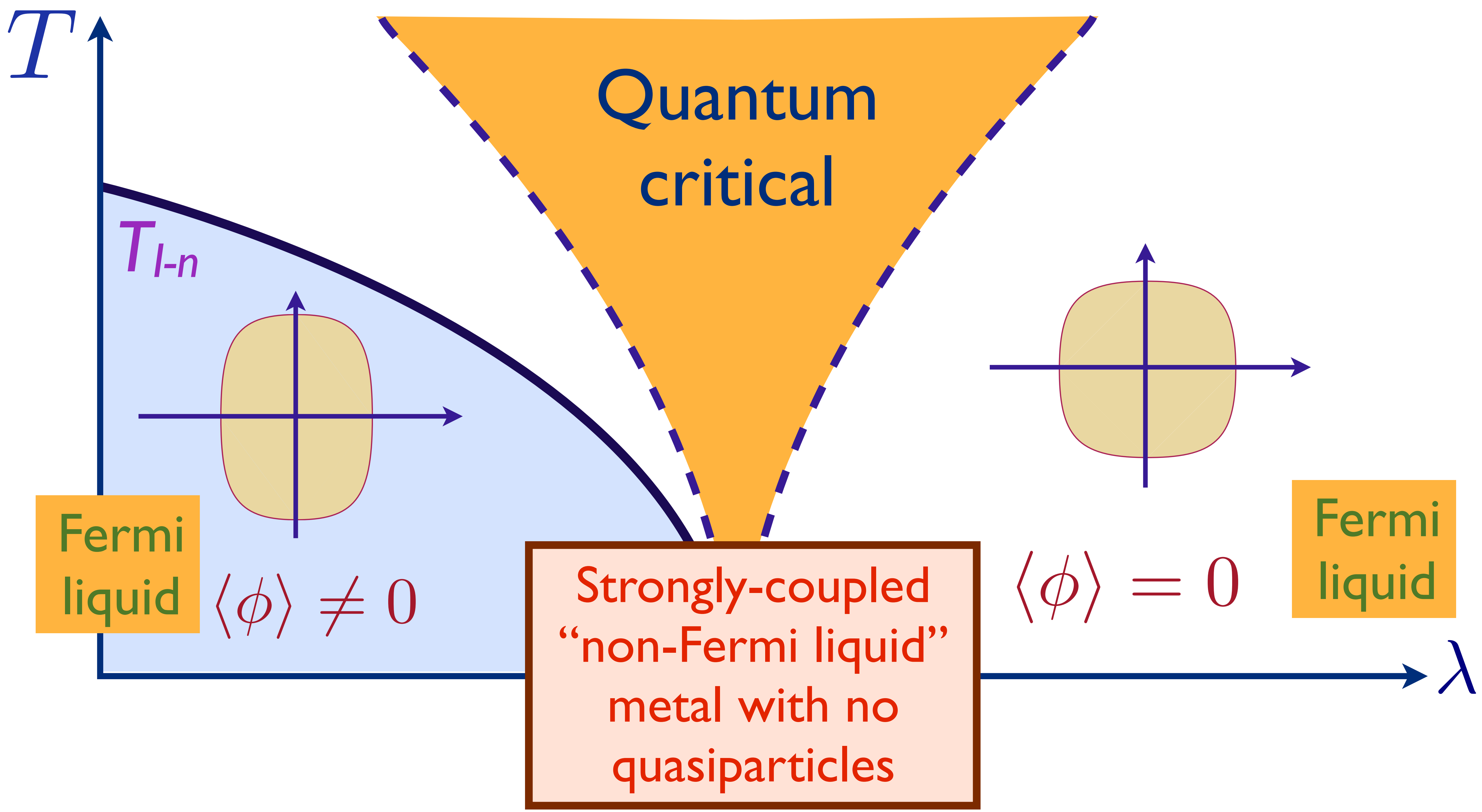

Phase diagram as a function of T and λ

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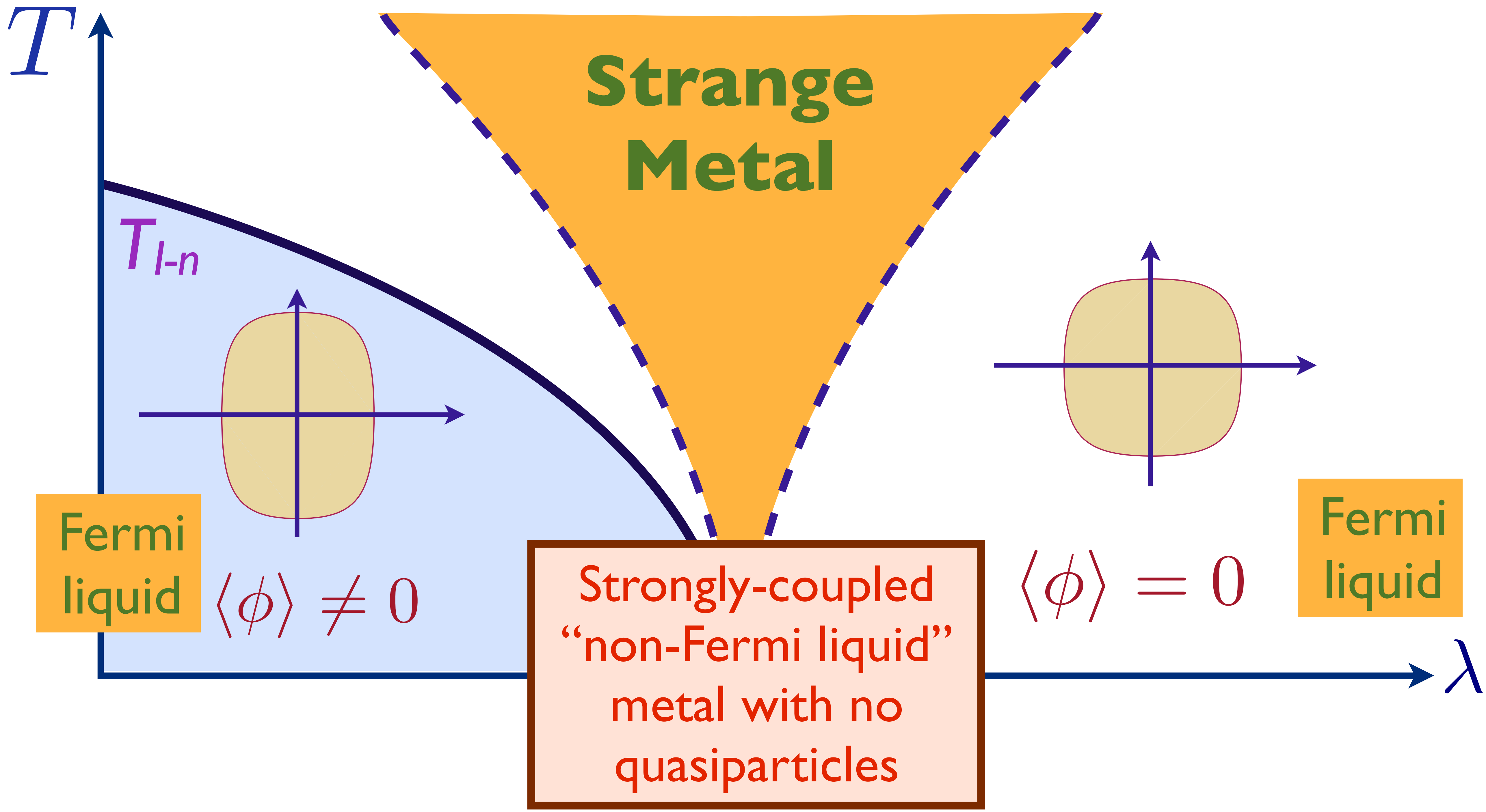


Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

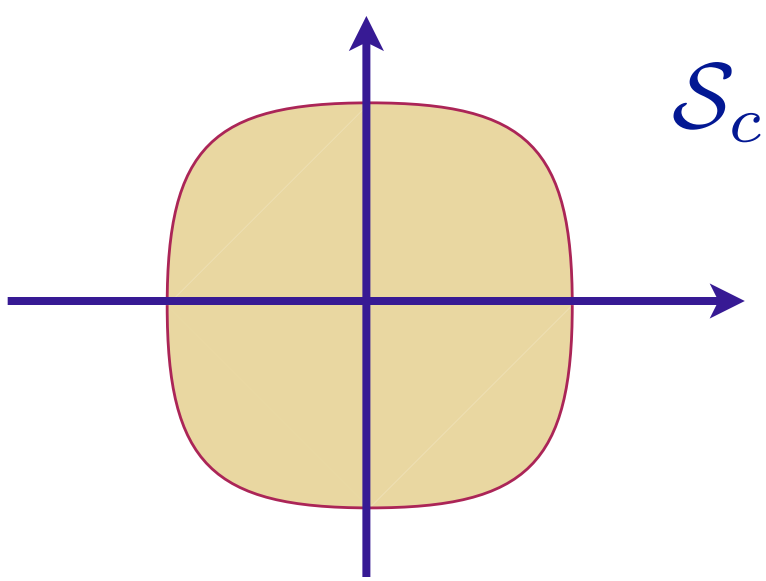
$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Ising-nematic ordering in a metal

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2 r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

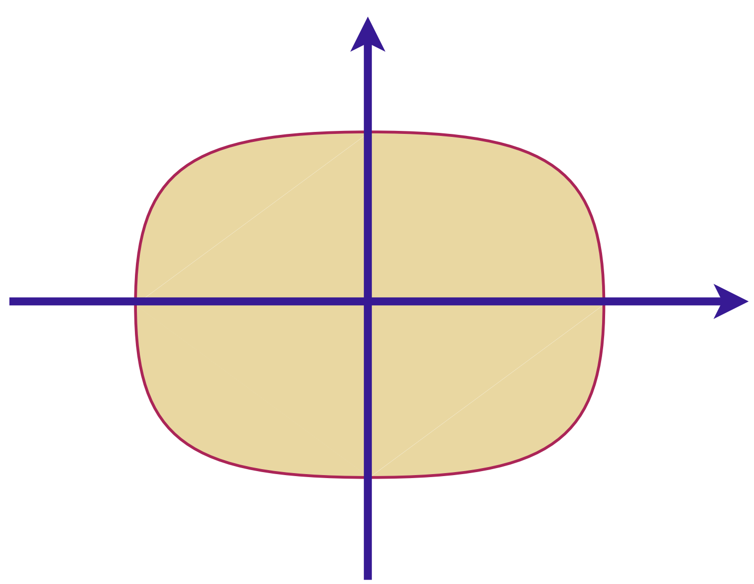
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal

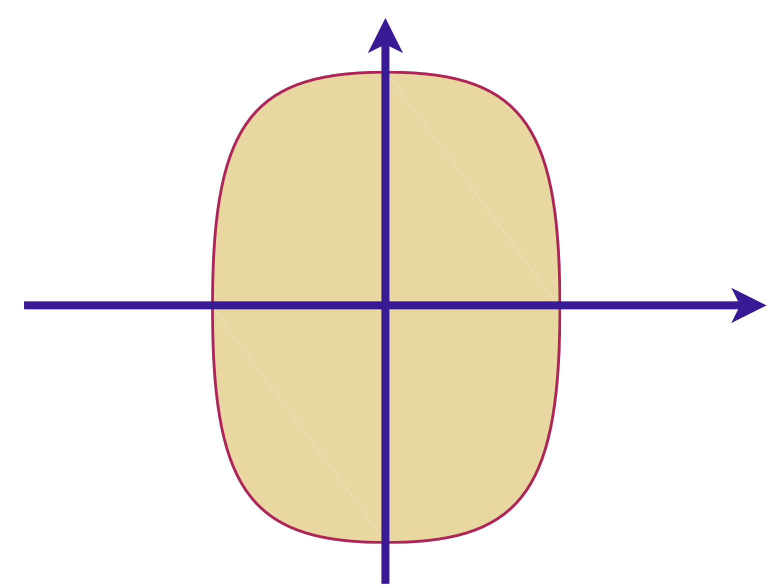
"Yukawa" coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^\dagger_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$$\langle \phi \rangle > 0$$



$$\langle \phi \rangle < 0$$

Quantum criticality of Ising-nematic ordering in a metal

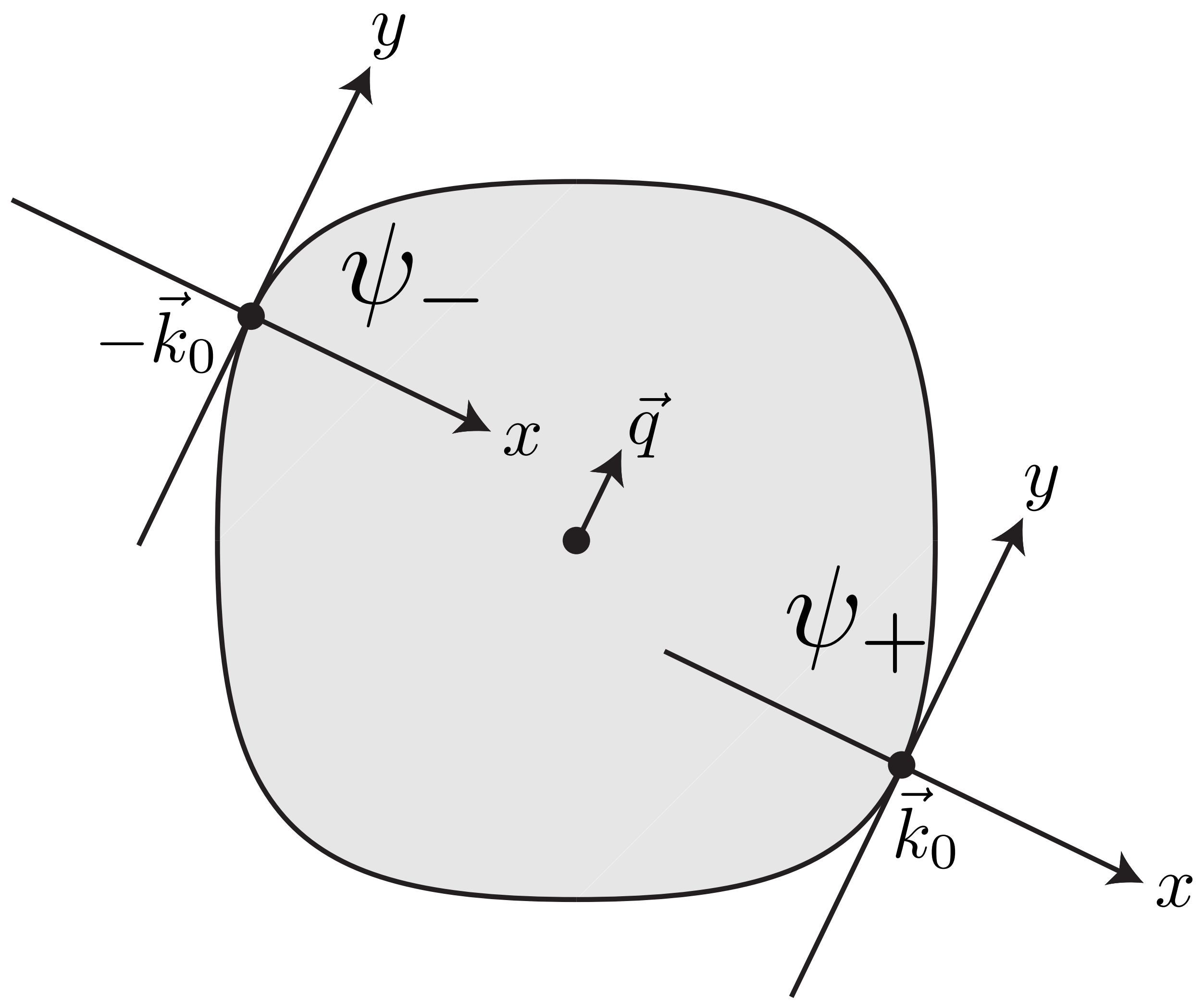
The "standard model":

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

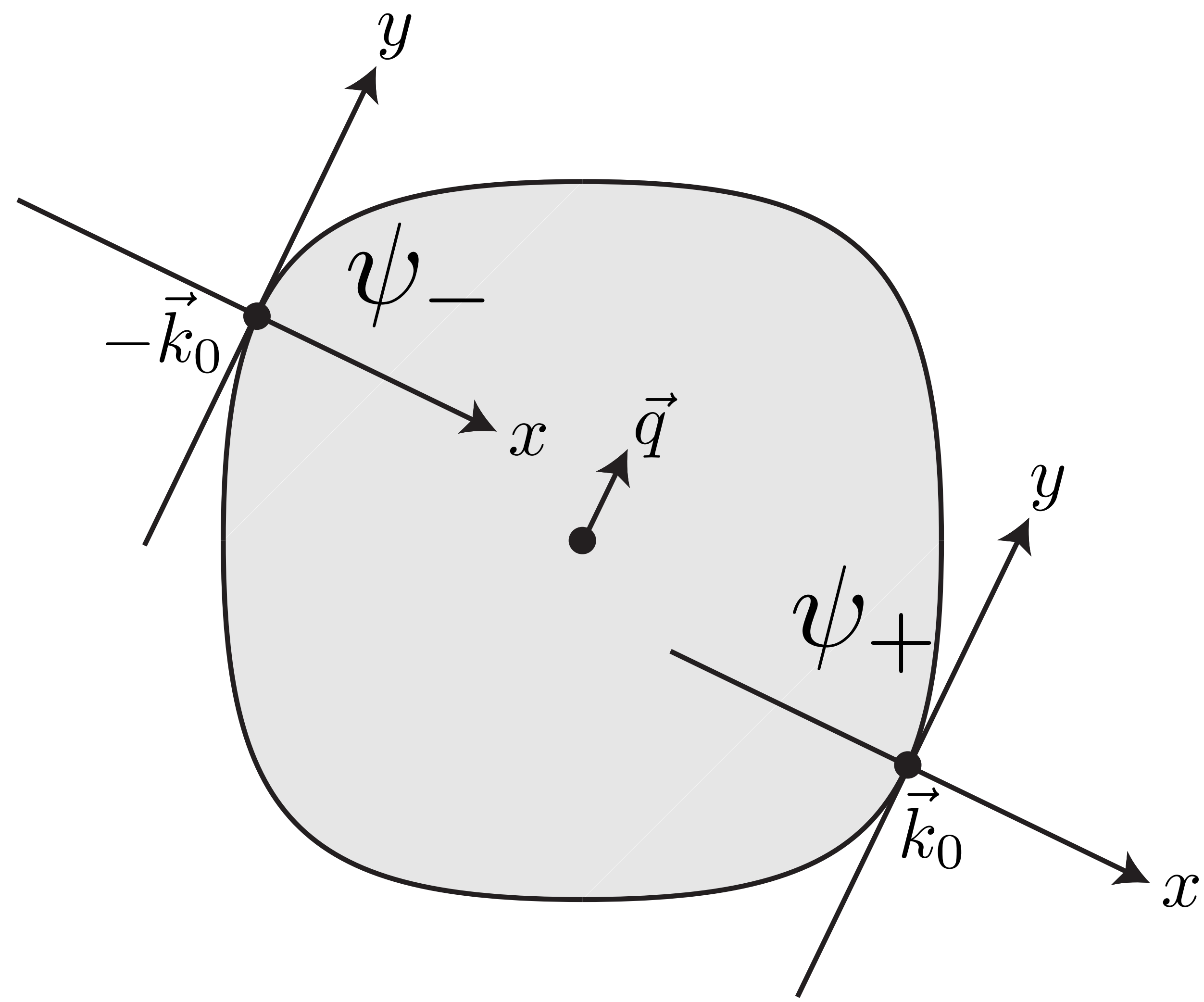
$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

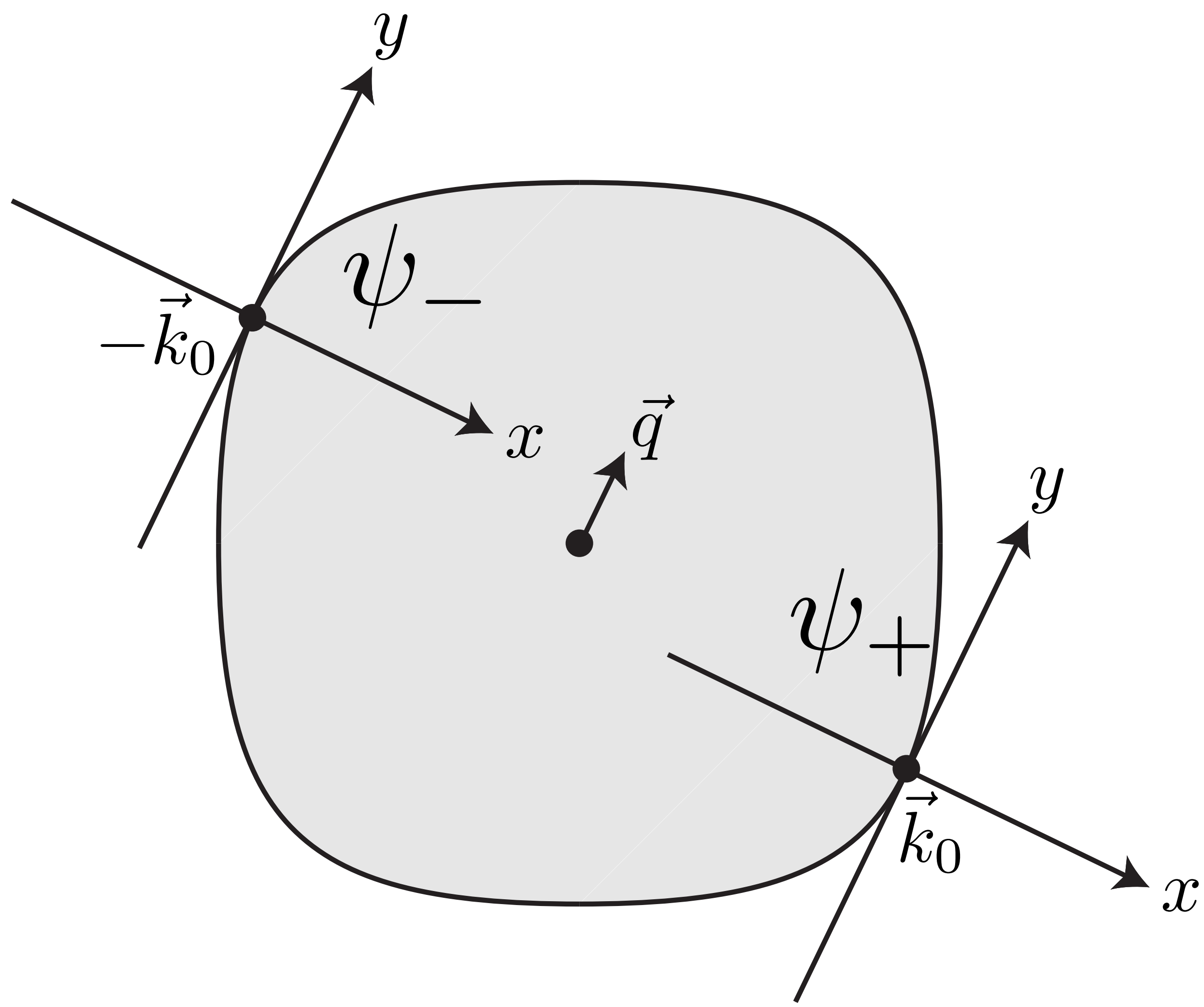
Quantum criticality of Ising-nematic ordering in a metal



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering in a metal

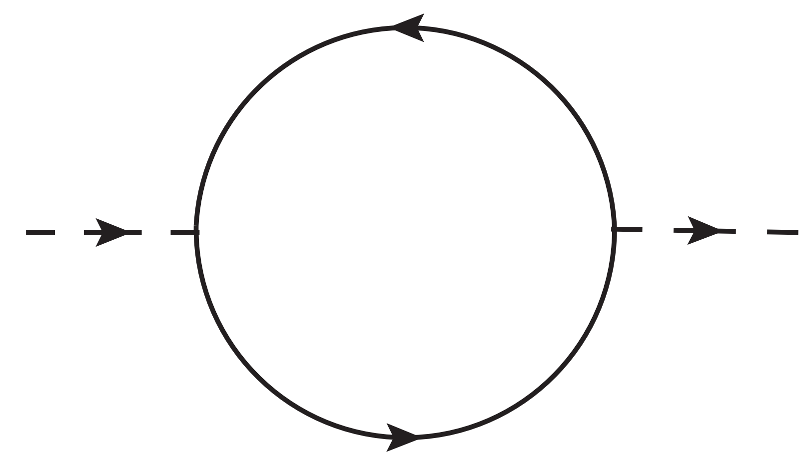


$$\mathcal{L}[\psi_\pm, \phi] =$$
$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$
$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$



One loop ϕ self-energy with N_f fermion flavors:

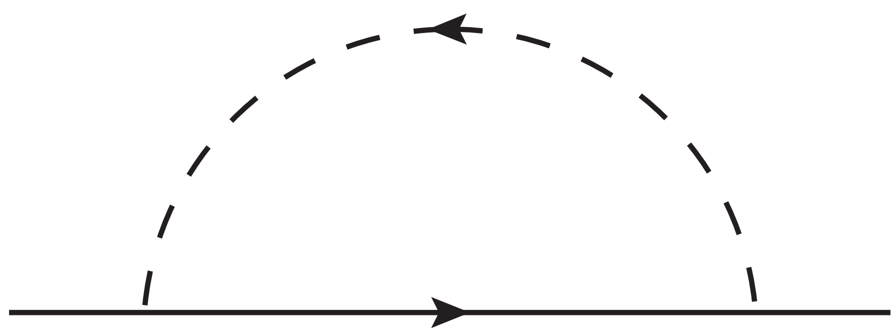
$$\Sigma_\phi(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi}\frac{1}{\left[-i(\Omega+\omega)+k_x+q_x+(k_y+q_y)^2\right]\left[-i\Omega-k_x+k_y^2\right]}$$

$$= \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int\frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\text{sgn}(\Omega)|\Omega|^{2/3}$$

$\sim |\Omega|^{d/3}$ in dimension d.

Quantum criticality of Ising-nematic ordering in a metal

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- \\ &\quad - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2\end{aligned}$$

Schematic form of ϕ and fermion Green's functions in d dimensions

$$D(\vec{q},\omega) = \frac{1/N_f}{q_{\|}^2 + \dfrac{|\omega|}{|q_{\|}|}} \quad , \quad G_f(\vec{q},\omega) = \frac{1}{q_x + q_{\|}^2 - i\,\mathrm{sgn}(\omega)|\omega|^{d/3}/N_f}$$

In the boson case, $q_{\|}^2 \sim \omega^{1/z_b}$ with $z_b = 3/2$.
In the fermion case, $q_x \sim q_{\|}^2 \sim \omega^{1/z_f}$ with $z_f = 3/d$.

Note $z_f < z_b$ for $d > 2 \Rightarrow$ Fermions have *higher* energy than bosons, and perturbation theory in g is OK.

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

Schematic form of ϕ and fermion Green's functions in $d = 2$

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \dfrac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i\text{sgn}(\omega)|\omega|^{2/3}/N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering in a metal

$$\mathcal{L} = \psi_+^\dagger \left(\cancel{\partial_\tau} - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\cancel{\partial_\tau} + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$

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Quantum criticality of Ising-nematic ordering in a metal

$$
\begin{aligned}
\mathcal{L} &= \psi_+^\dagger \left(\cancel{\partial_\tau} - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\cancel{\partial_\tau} + i\partial_x - \partial_y^2\right)\psi_- \\
&\quad - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2
\end{aligned}
$$

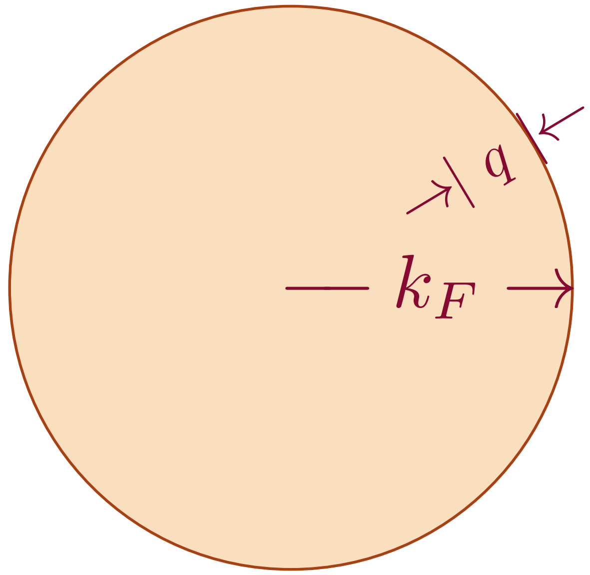
Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$
\begin{aligned}
\phi &\to \phi\, s \\
\psi &\to \psi\, s^{(2z+1)/4} \\
g &\to g\, s^{(3-2z)/4}
\end{aligned}
$$

So the action is invariant provided $z = 3/2$.

FL Fermi liquid



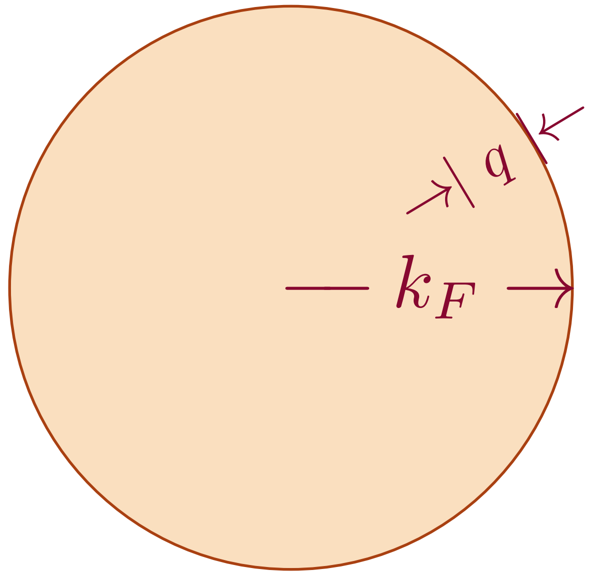
- $k_F^d \sim \mathcal{Q}$, the fermion density

- Sharp fermionic excitations near Fermi surface with $\omega \sim |q|^z$, and $z = 1$.

- Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.

- Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.

FL Fermi liquid



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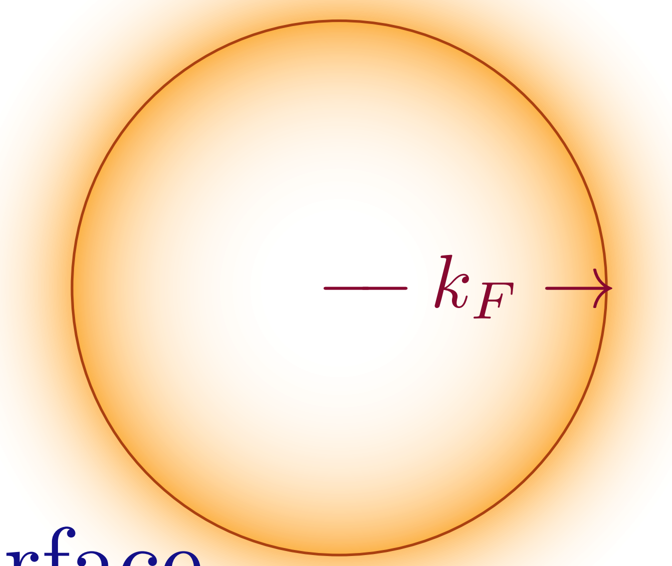
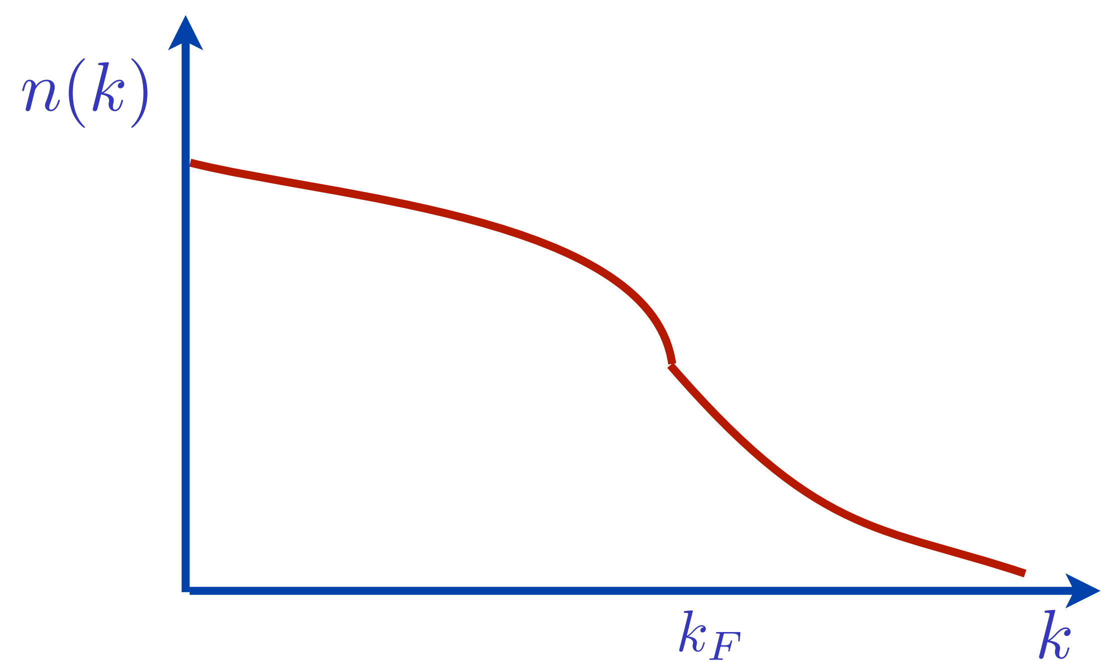
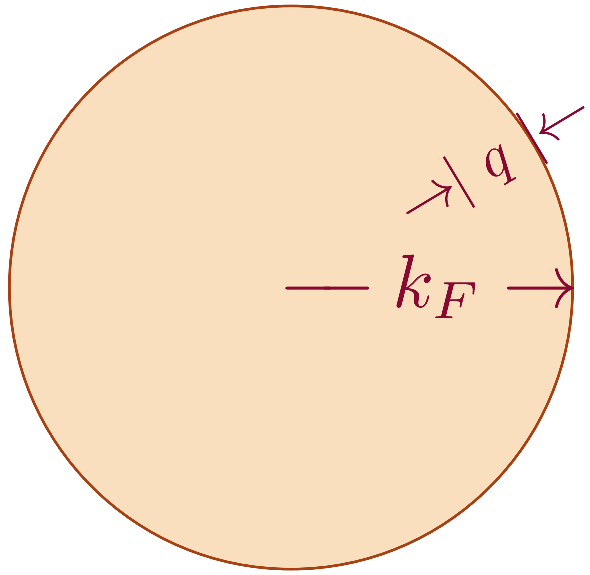
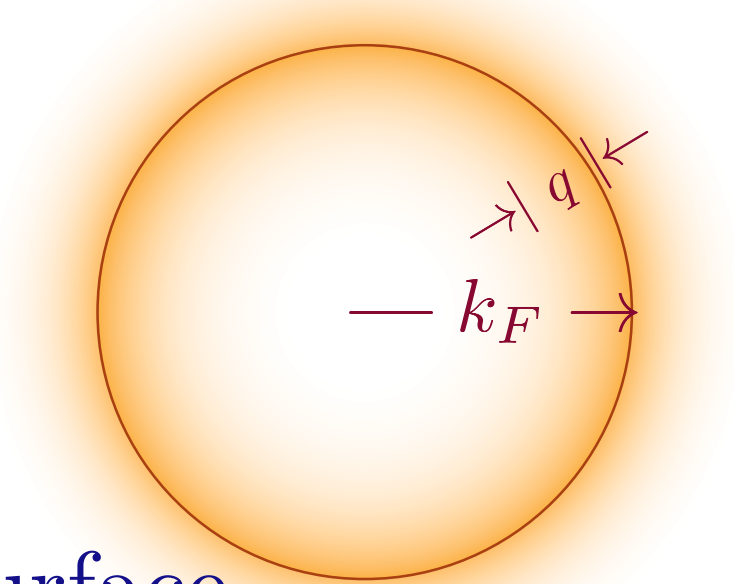
NFL Nematic QCP

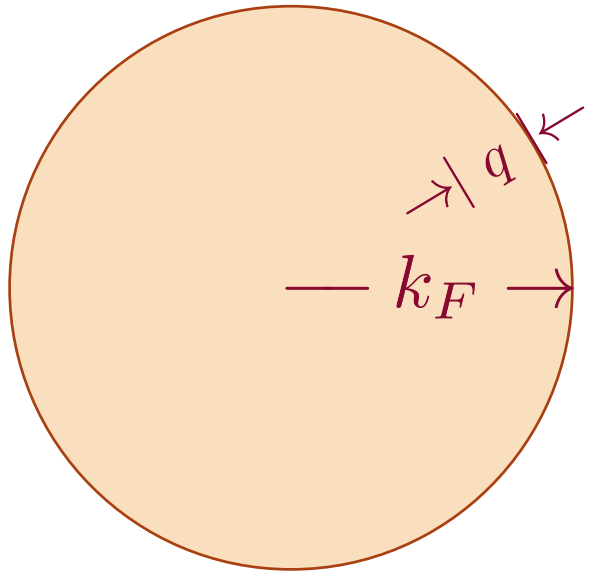


- Fermi surface with $k_F^d \sim \mathcal{Q}$.

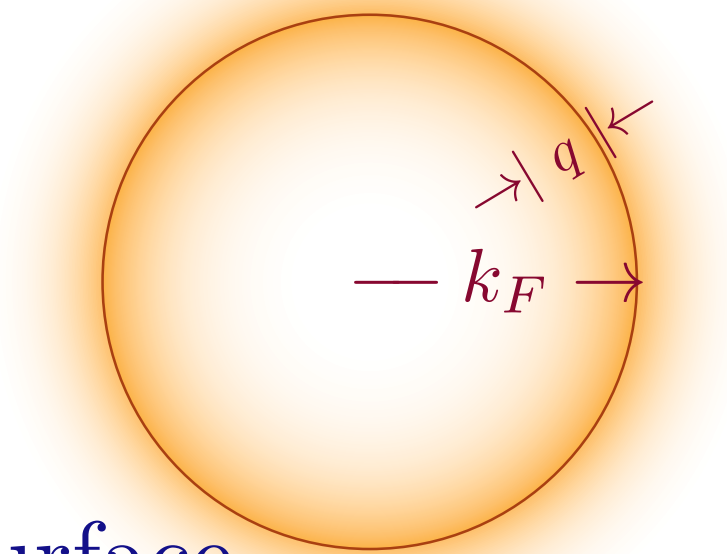


FL Fermi liquid	**NFL Nematic QCP**
	
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Fermi surface with $k_F^d \sim \mathcal{Q}$.
• Sharp fermionic excitations near Fermi surface with $\omega \sim \lvert q \rvert^z$, and $z = 1$.	• Diffuse fermionic excitations with $z = 3/2$ to three loops.
• Entropy density $S \sim T^{(d-\theta)/z}$ with violation of hyperscaling exponent $\theta = d - 1$.	
• Entanglement entropy $S_E \sim k_F^{d-1} P \ln P$.	

FL Fermi liquid



NFL Nematic QCP



FL Fermi liquid	NFL Nematic QCP
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Fermi surface with $k_F^d \sim \mathcal{Q}$.
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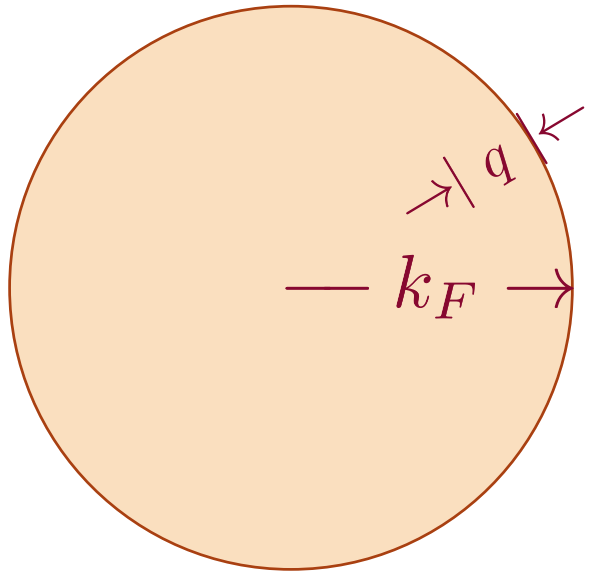	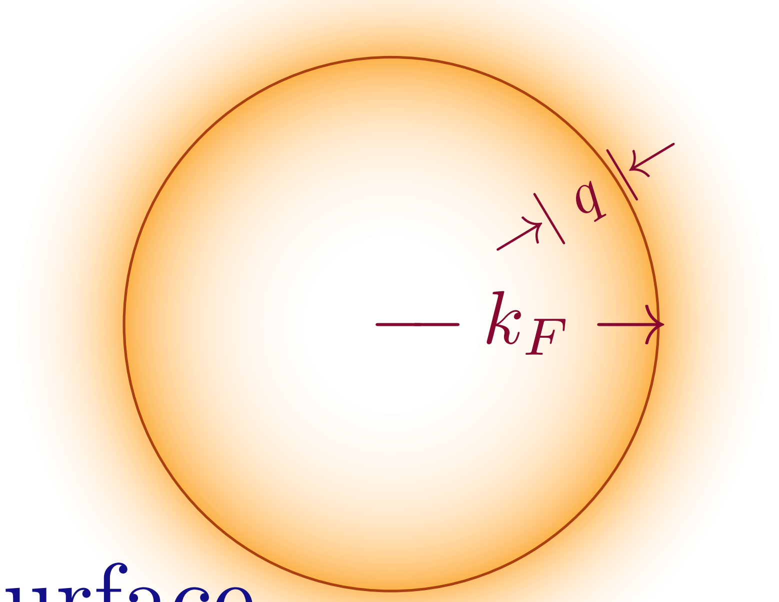
• $k_F^d \sim \mathcal{Q}$, the fermion density	• Fermi surface with $k_F^d \sim \mathcal{Q}$.
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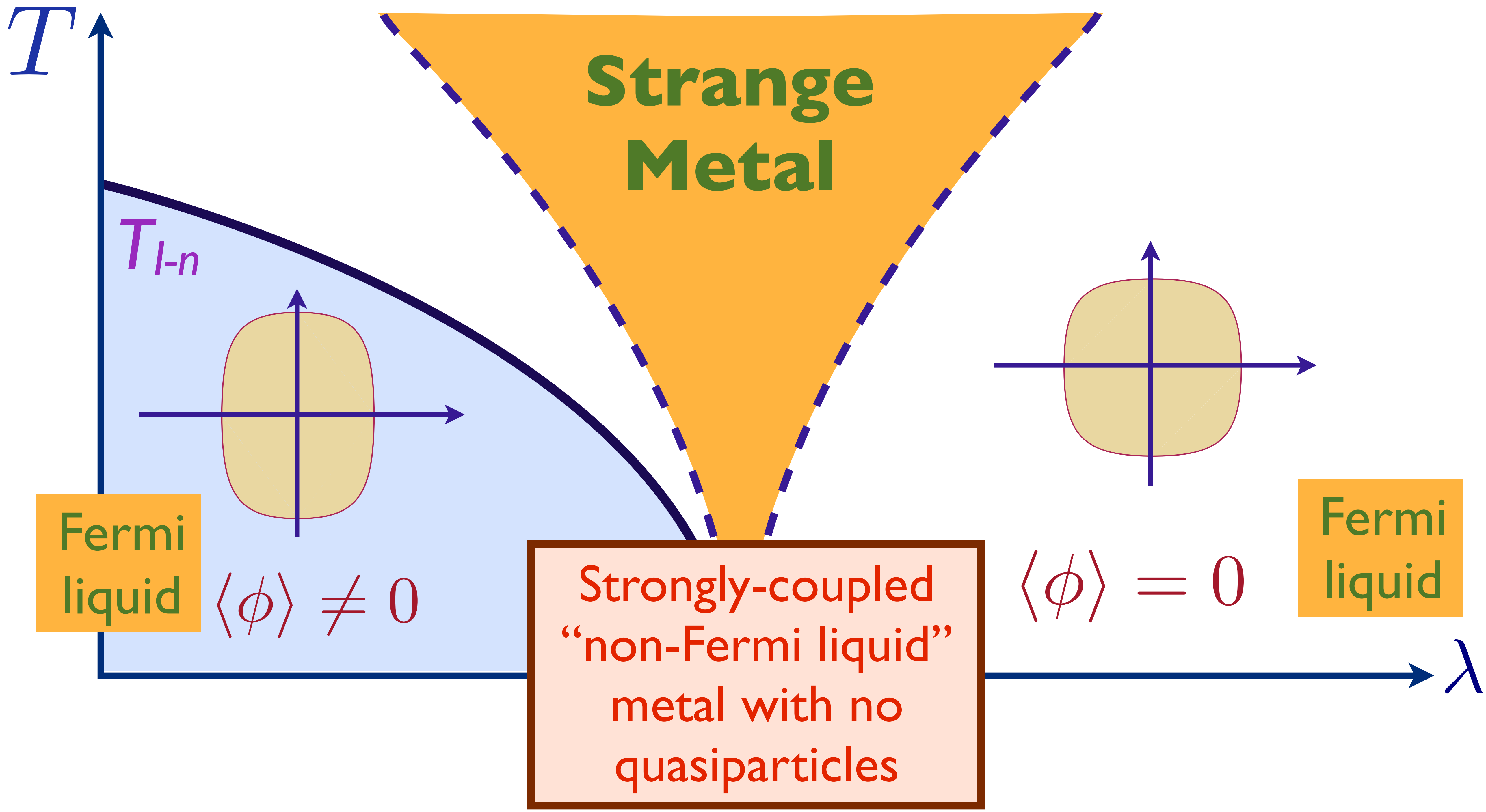
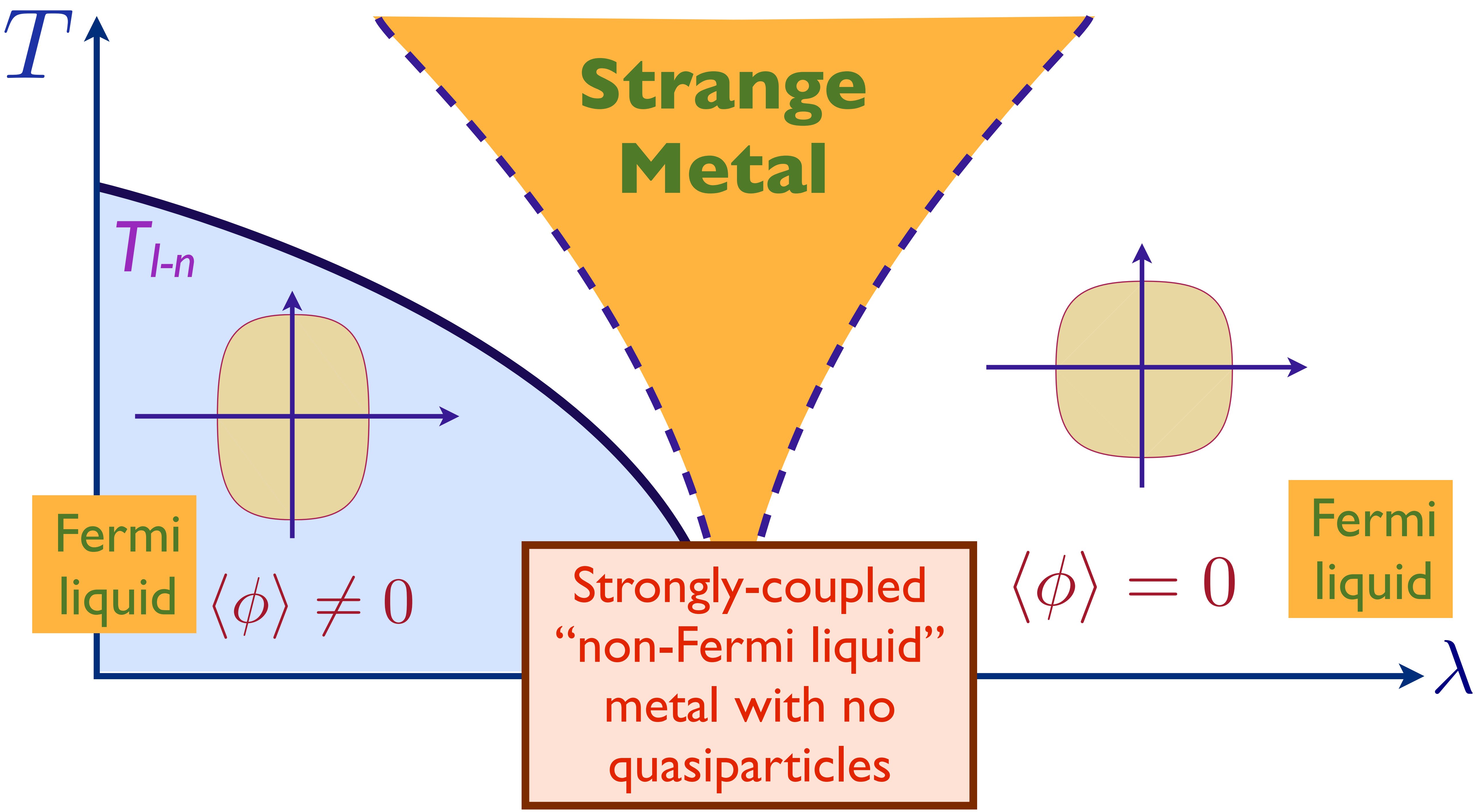
Quantum criticality of Ising-nematic ordering in a metal



Phase diagram as a function of T and λ

Quantum criticality of Ising-nematic ordering in a metal



Common theoretical belief from an analysis of scattering of charged electronic quasiparticles off bosonic ϕ fluctuations: resistivity of strange metal $\rho(T) \sim T^{4/3}$.

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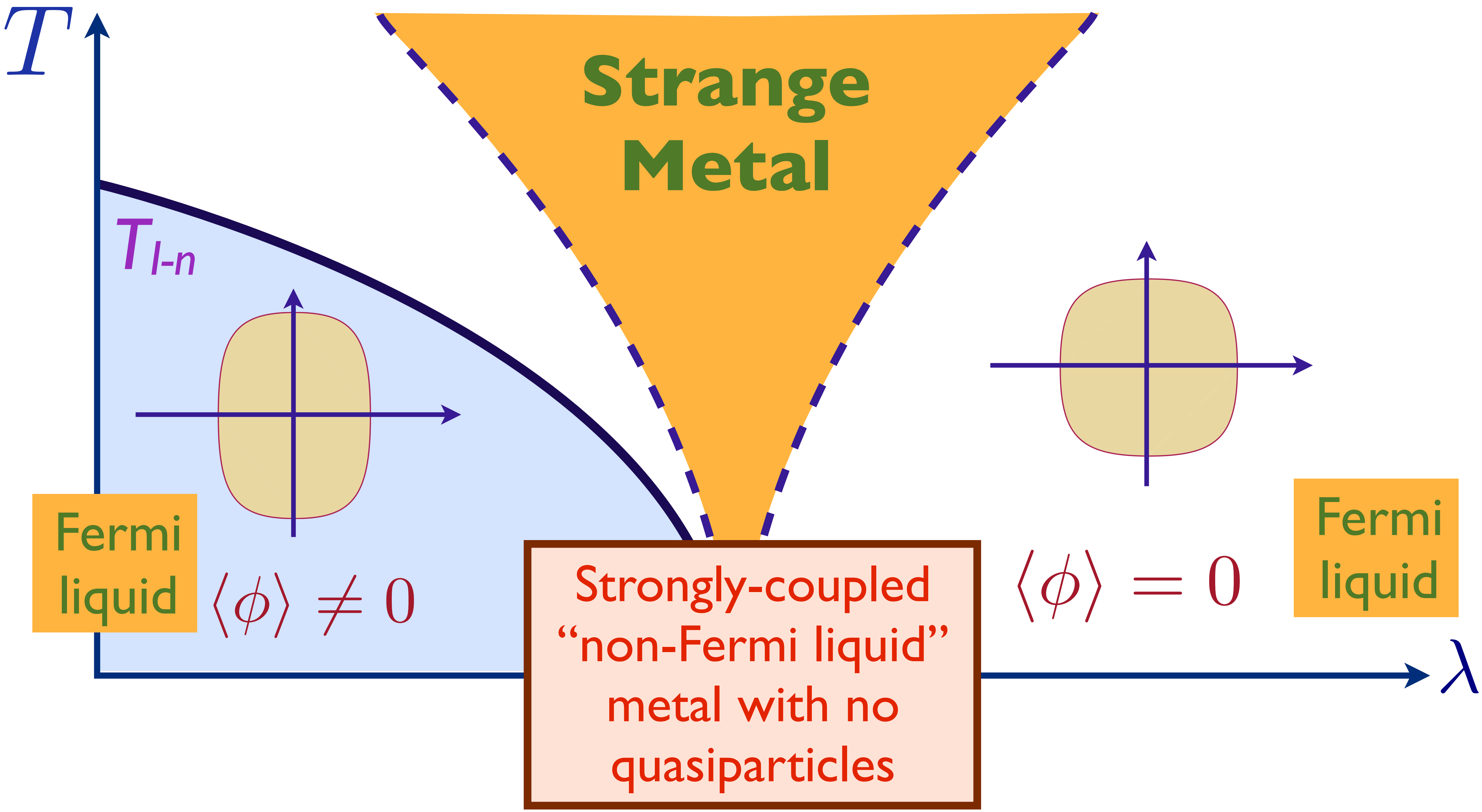

This ignores constraints arising from conservation of total momentum.

D. L. Maslov, V. I. Yudson, and A. V. Chubukov, Phys. Rev. Lett. **106**, 106403 (2011).

H. K. Pal, V. I. Yudson, and D. L. Maslov, Lith. J. Phys. **52**, 142 (2012).

Quantum criticality of Ising-nematic ordering in a metal

Boltzmann view of electrical transport:

- Identify charge carriers: electrons near the Fermi surface. Compute the scattering rate of these charged excitations off the bosonic ϕ fluctuations.

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- "Bloch's law" for the Ising-nematic critical point yields $\rho(T) \sim T^{4/3}$.

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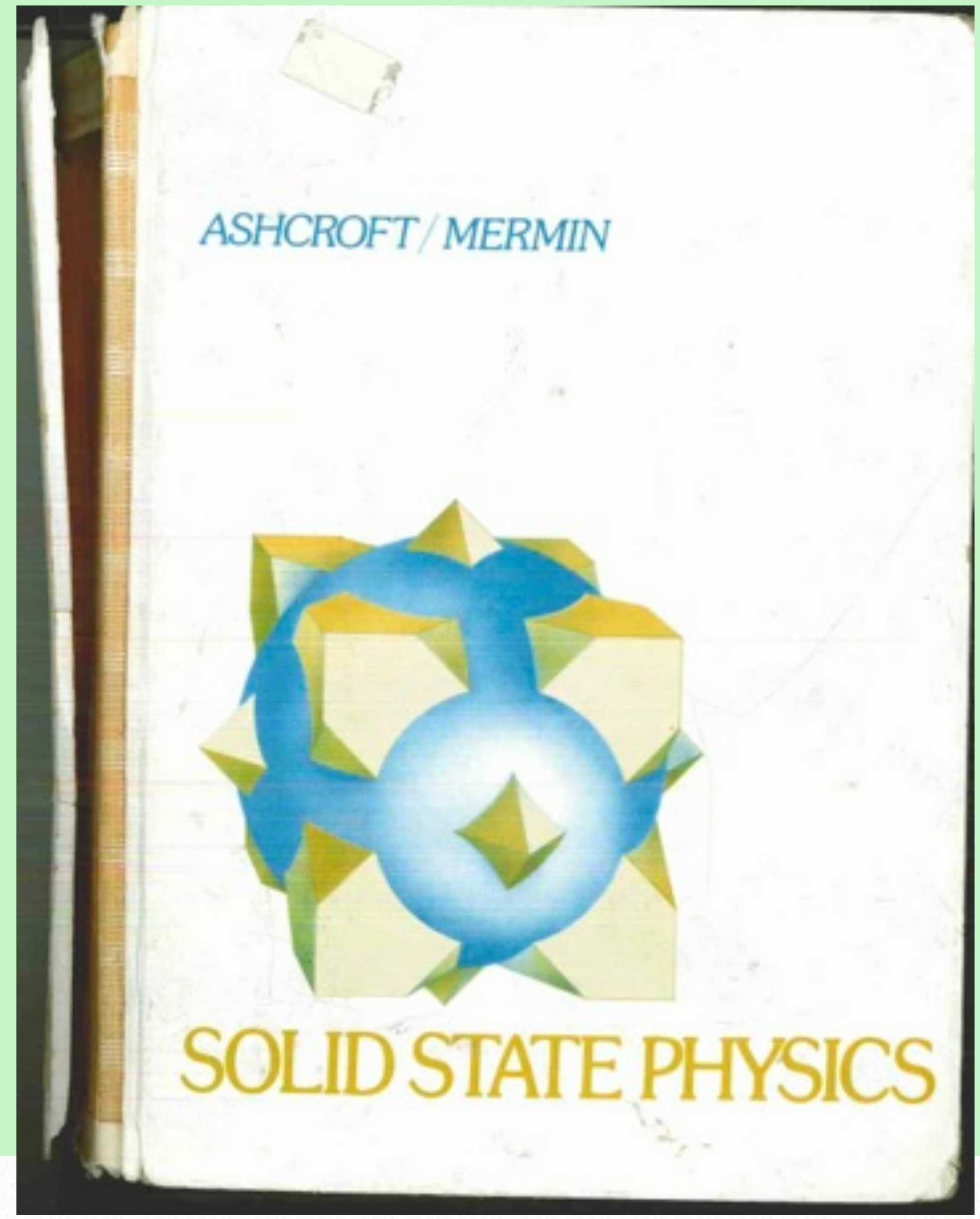

However, this ignores "phonon drag"

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Peierls[28] pointed out a way in which the low temperature resistivity might decline more rapidly than T^5.

[28] R. E. Peierls, *Ann. Phys.* (5) **12**, 154 (1932).

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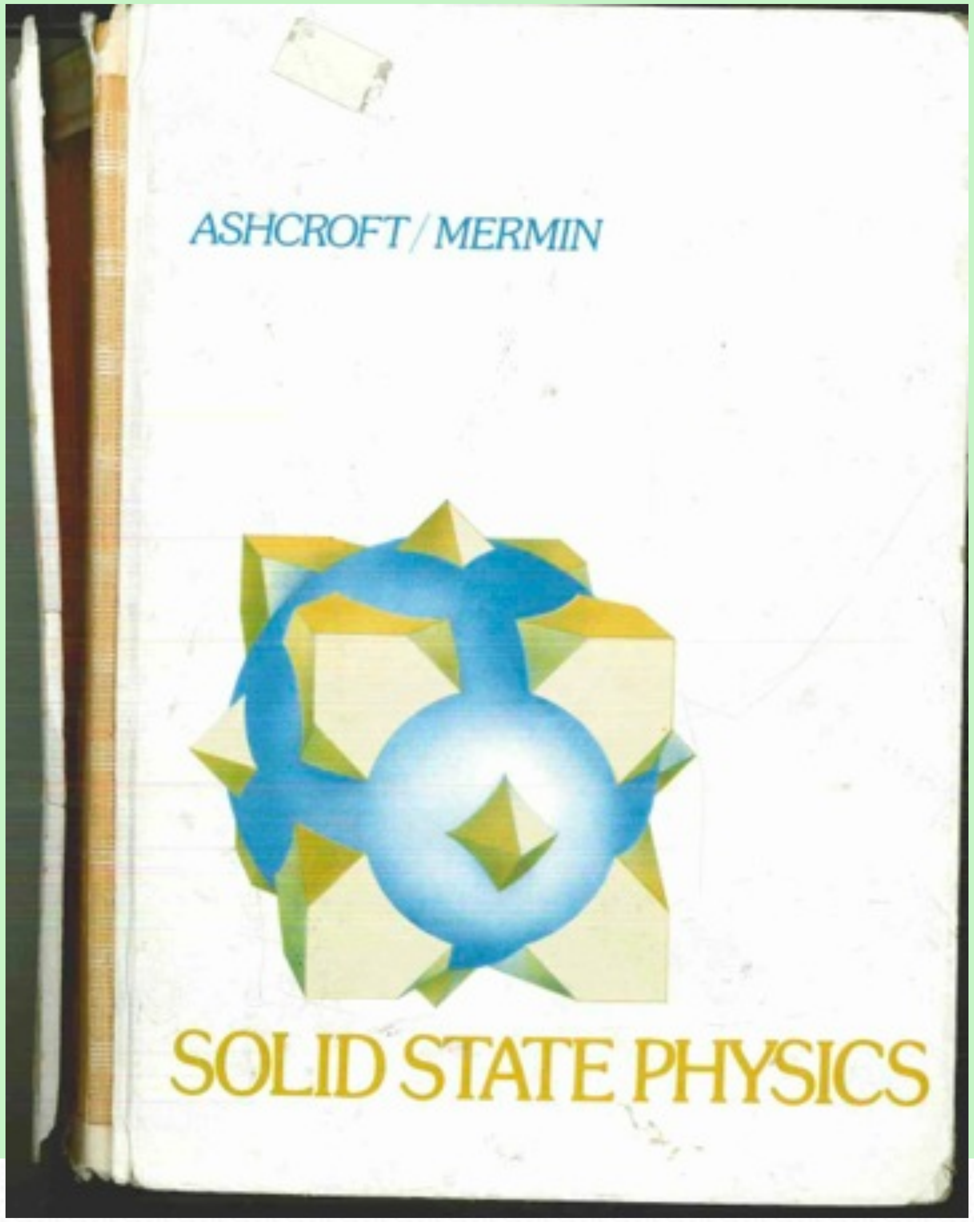

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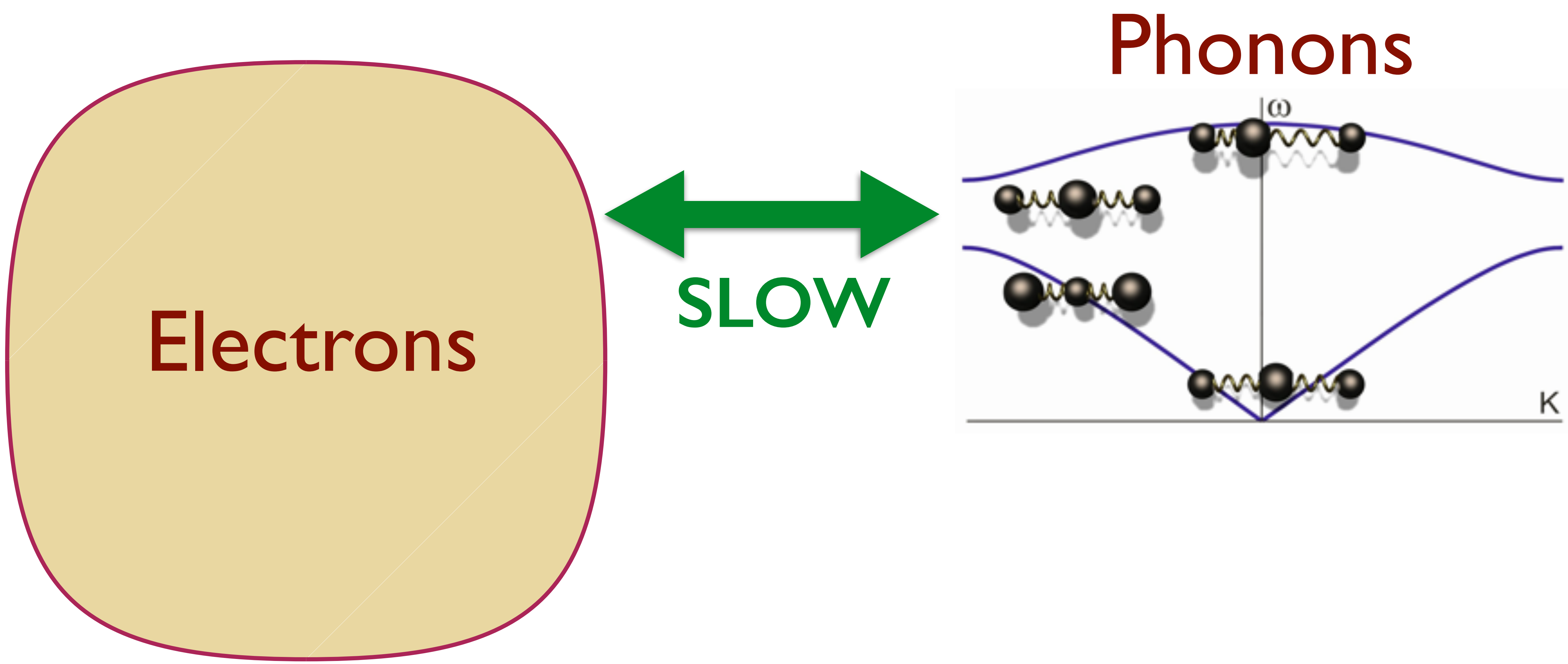

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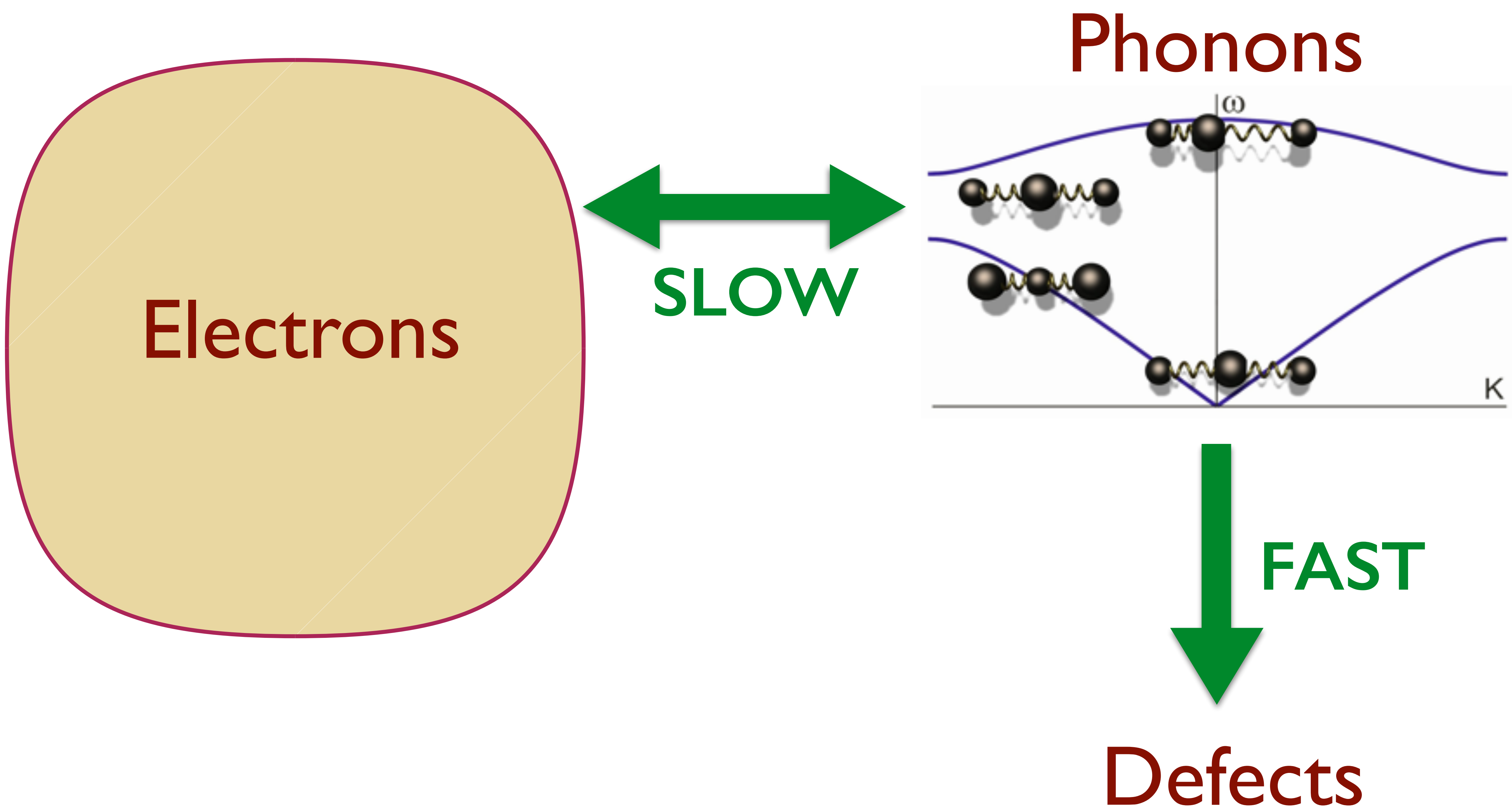

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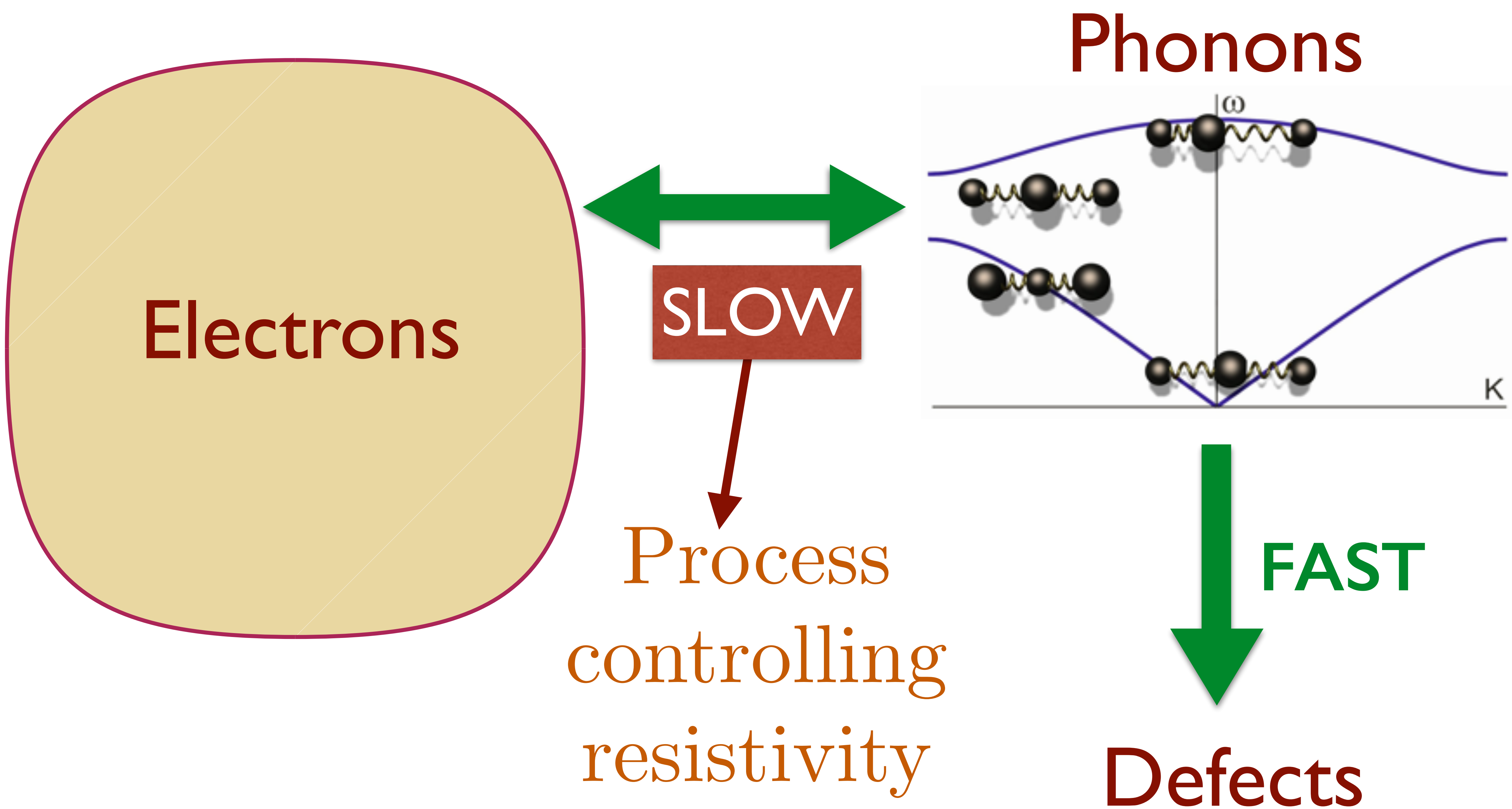

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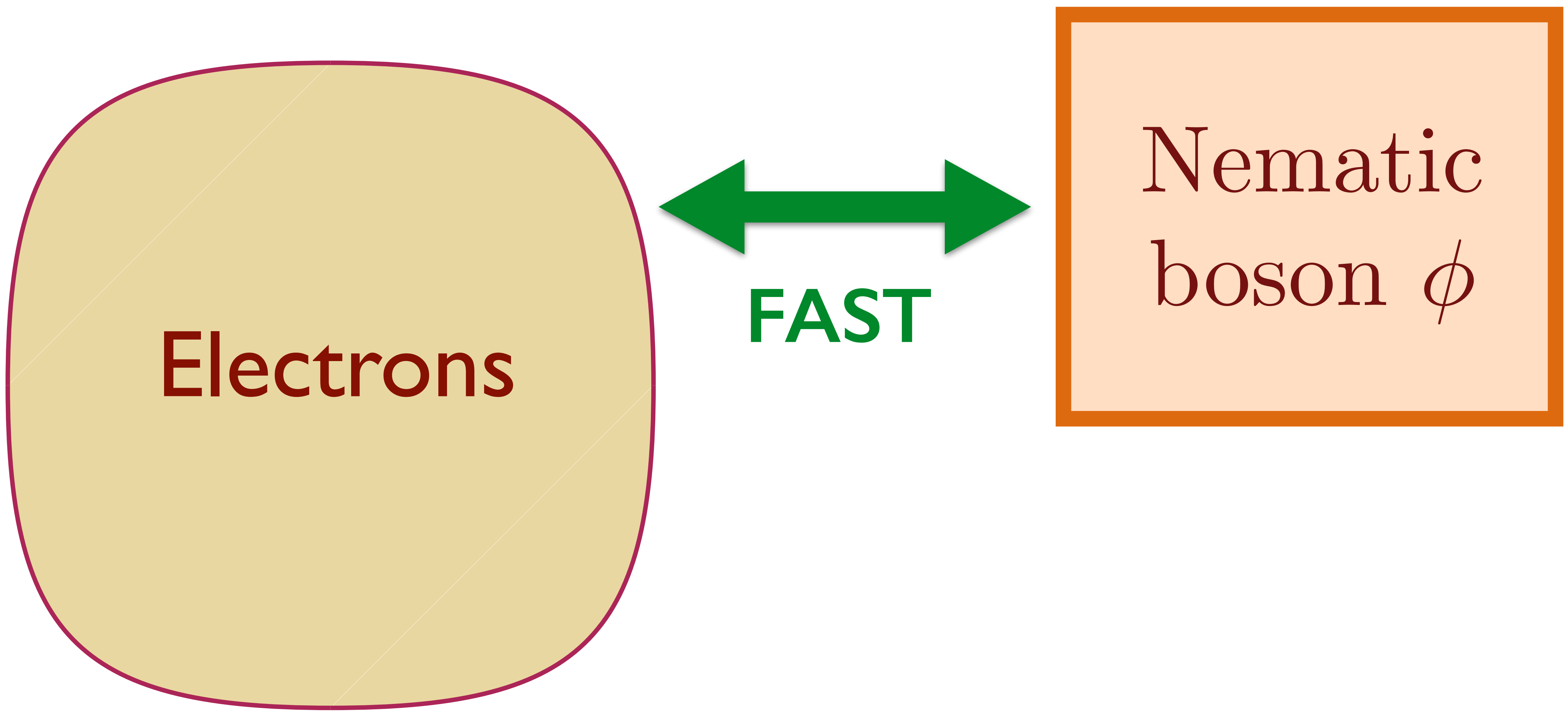

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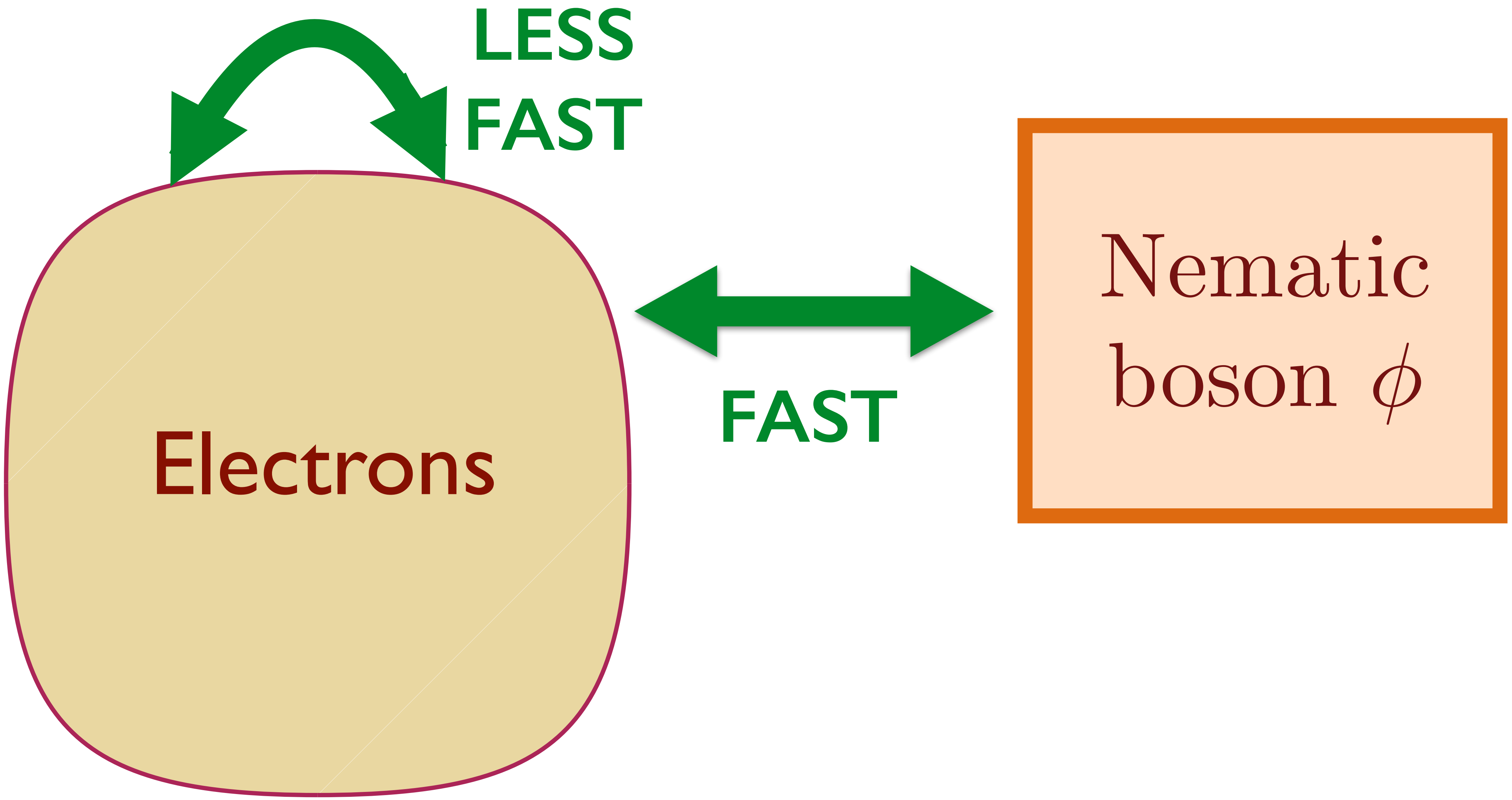

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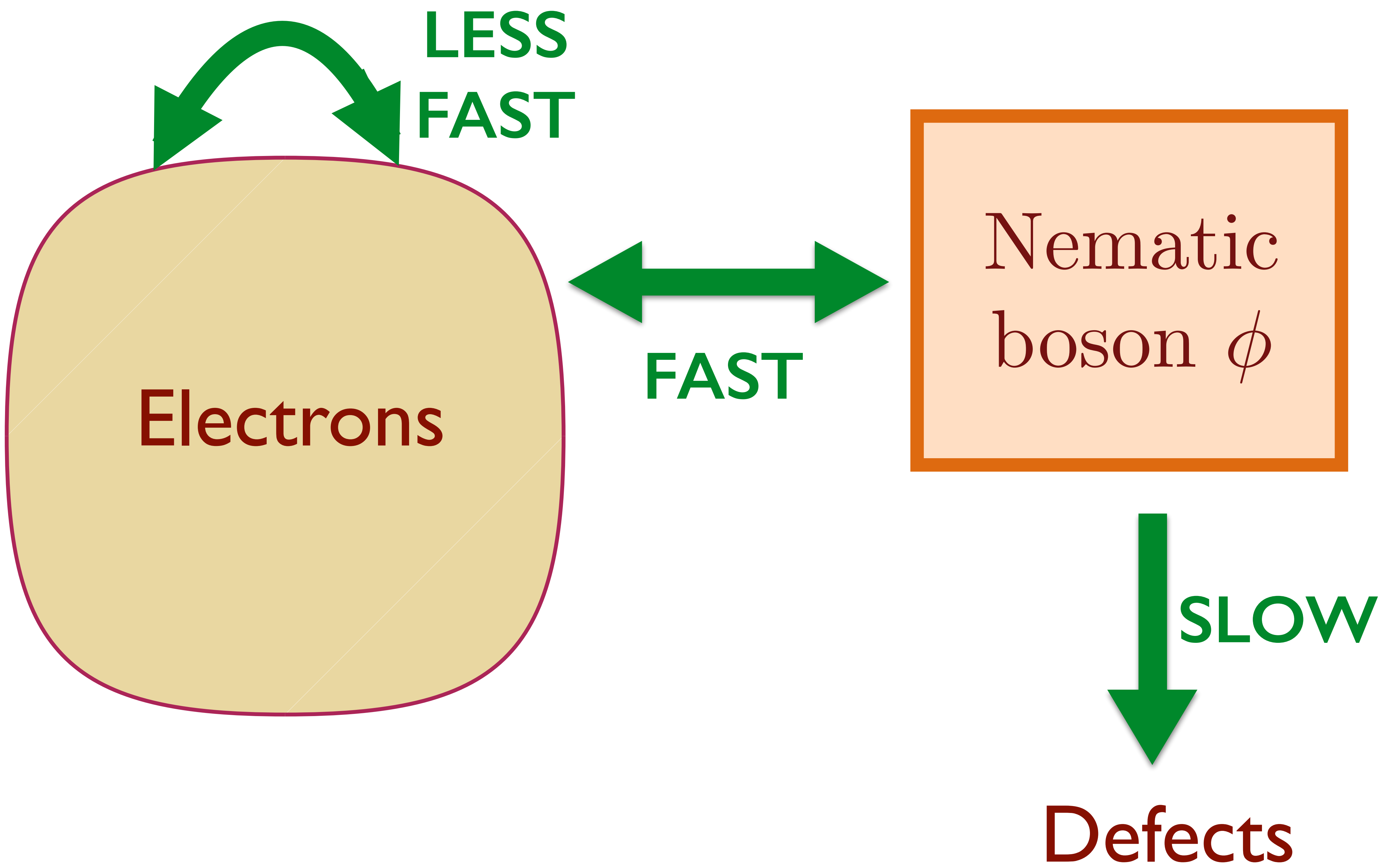

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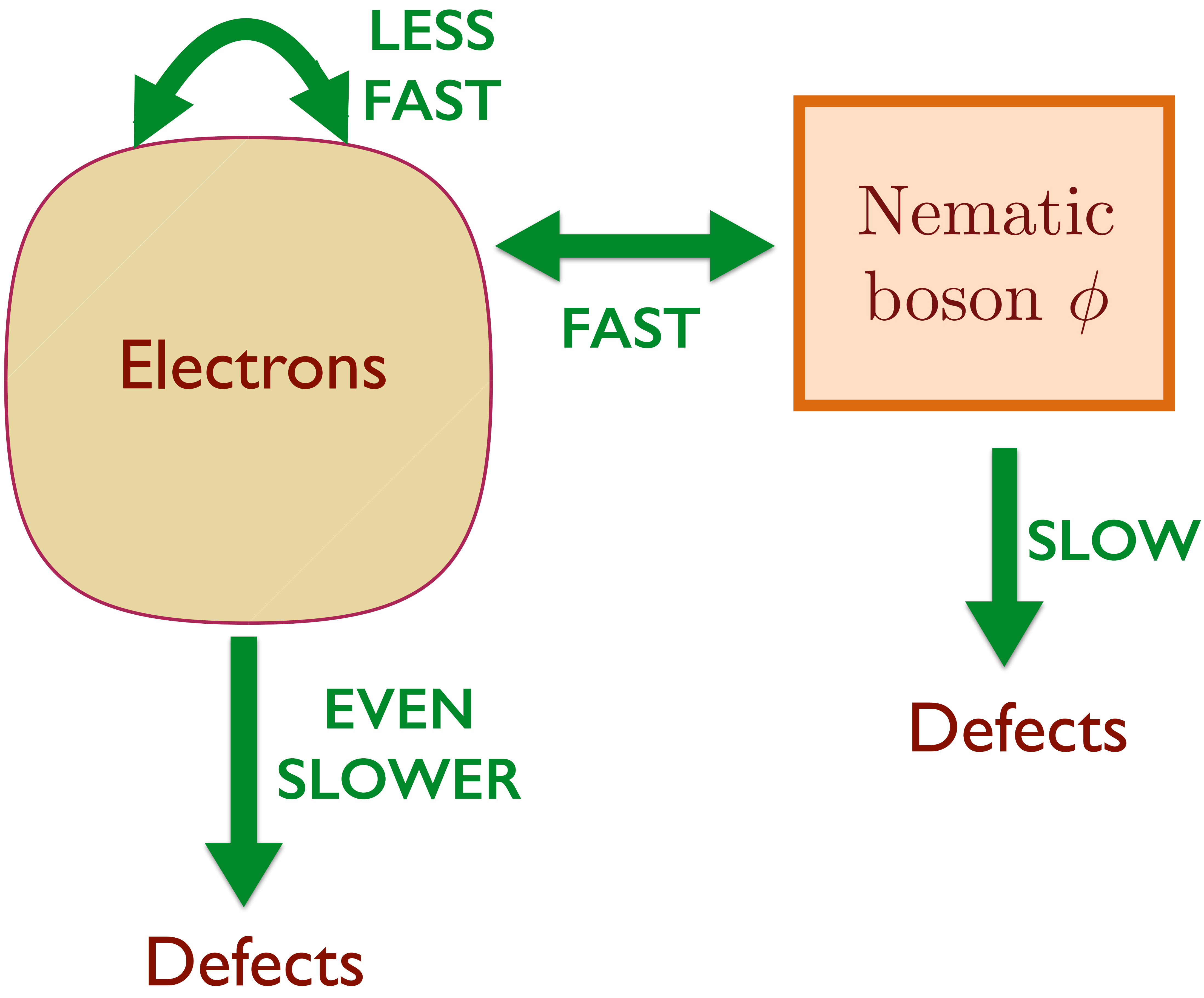

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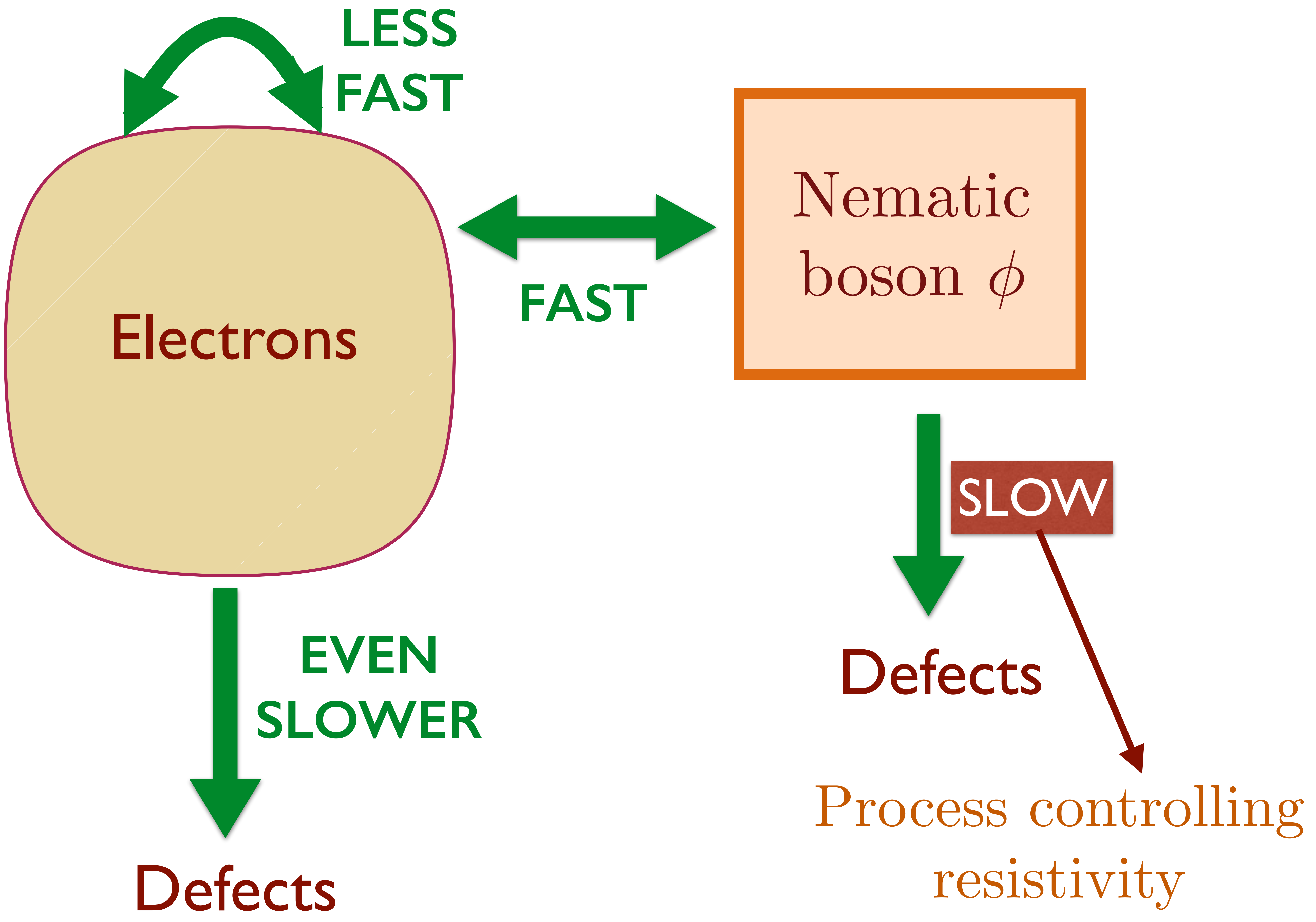

Quantum criticality of Ising-nematic ordering in a metal

The "standard model":

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$\mathcal{S}_{\phi c} = -g \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

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$$\mathcal{S}_{\phi c} = -g \int d^2 r d\tau \sum_{\alpha=1}^{N_f} \phi \left[c_\alpha^\dagger \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha \right\} + \left\{ (\partial_x^2 - \partial_y^2 + \ldots) c_\alpha^\dagger \right\} c_\alpha \right]$$

This continuum theory has has strong electron−ϕ scattering, and no quasi-particle excitations. But it has a conserved momentum $\mathbf{P}$, and $\chi_{\mathbf{J},\mathbf{P}} \neq 0$ ("phonon drag"), and so the resistivity $\rho(T) = 0$.

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

- Focus on the interplay between J_μ and $T_{\mu\nu}$!

The most-probable state with a non-zero current $\mathbf{J}$ has a non-zero momentum $\mathbf{P}$ (and vice versa). At non-zero density, $\mathbf{J}$ "drags" $\mathbf{P}$.

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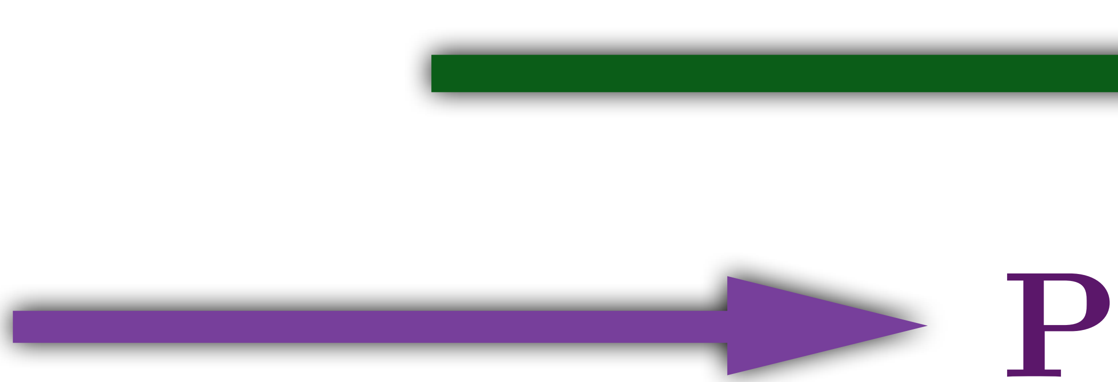

The most-probable state with a non-zero current **J** has a non-zero momentum **P** (and vice versa). At non-zero density, **J** "drags" **P**.

The resistivity of this metal is *not* determined by the scattering rate of charged excitations near the Fermi surface, but by the dominant rate of momentum loss by *any* excitation, whether neutral or charged, or fermionic or bosonic

Quantum criticality of Ising-nematic ordering in a metal

Transport without quasiparticles:

S. A. Hartnoll, R. Mahajan, M. Punk and S. Sachdev, arXiv:1401.7012.
A. Lucas, S. Sachdev, and K. Schalm, arXiv:1401.7933

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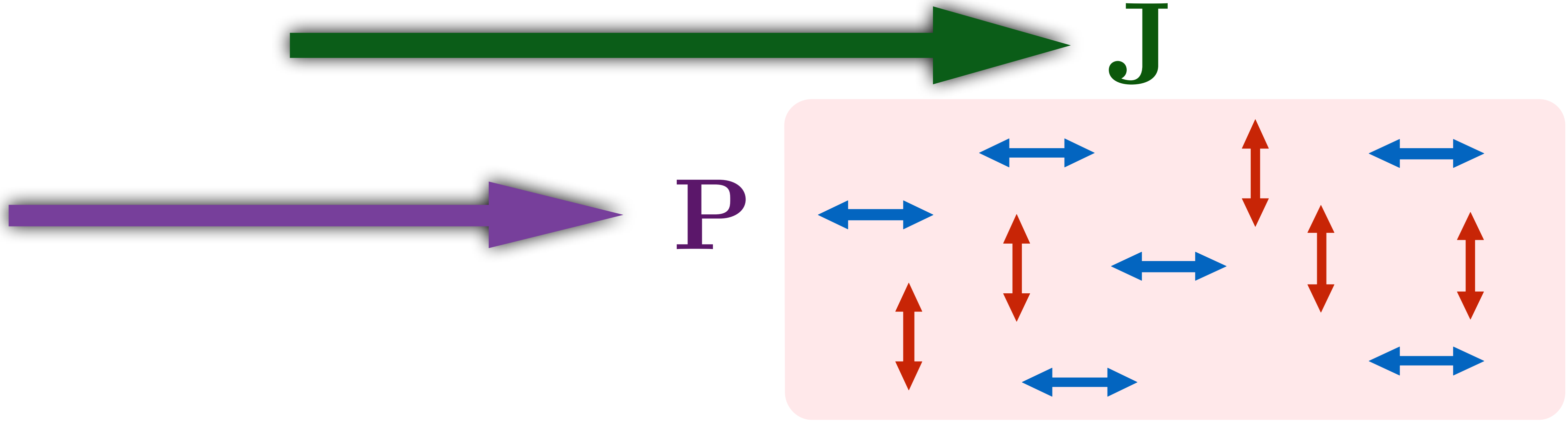
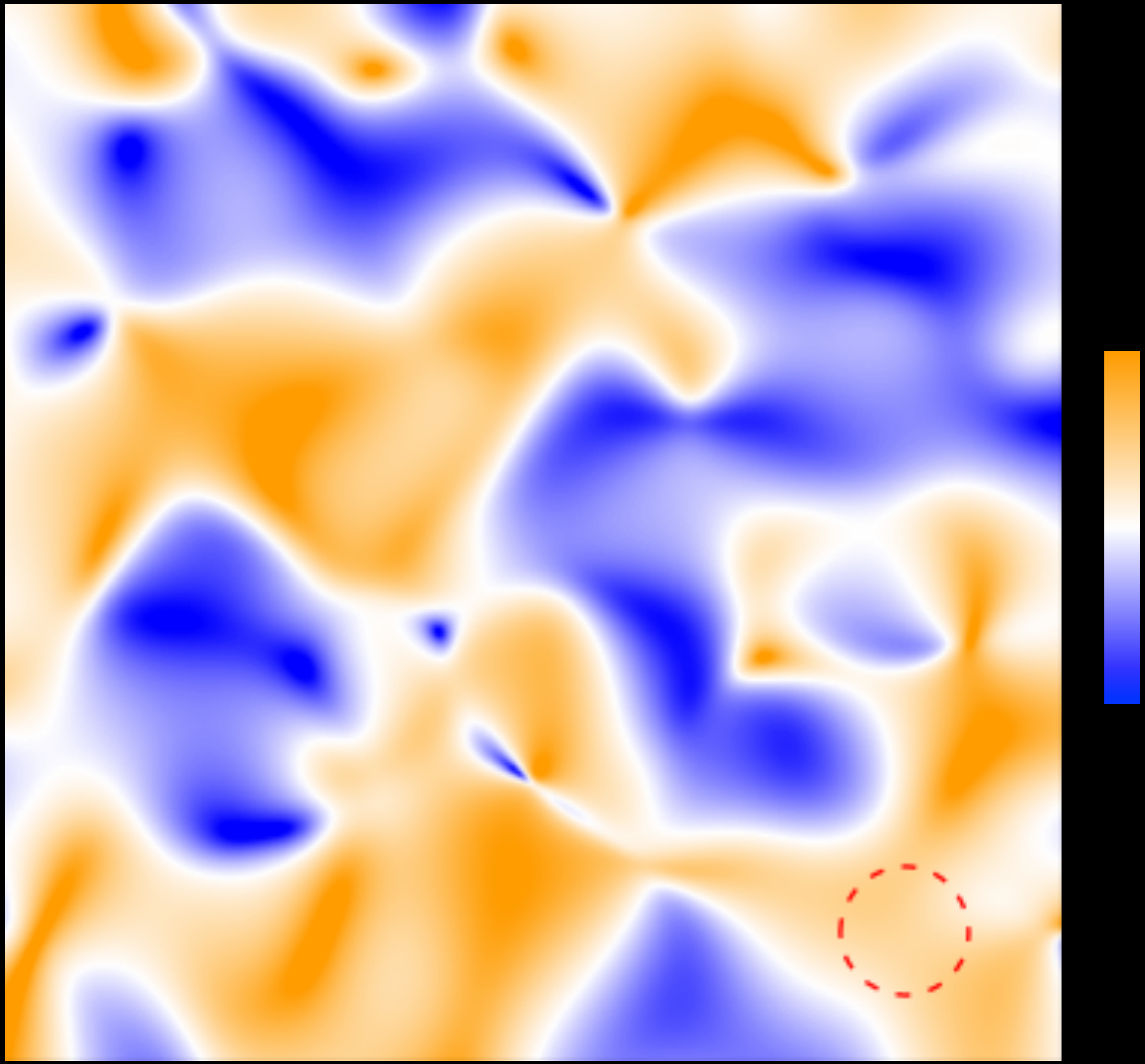


The dominant momentum loss occurs via the scattering of the neutral bosonic ϕ excitations off **random fields**.
This is good news for the AdS/CMT approaches, which do not capture the Fermi surface of most of the charged carriers.

dFF-DW Unidirectional Domains



$(|O_y(r,q=Q_x)|-|O_x(r,q=Q_y)|)/(|O_y(r,q=Q_x)|+|O_x(r,q=Q_y)|)$

Primary DW direction Orange : // (1,0), Blue : //(0,1)

Resistivity of strange metal

In the presence of weak disorder of quenched Gaussian random fields

$$\mathcal{S}_{\rm dis} = \int d^2 r d\tau \left[V(\mathbf{r})\, c^\dagger c + h(\mathbf{r})\, \phi \right] ,$$

$$\overline{V(\mathbf{r})} = 0 \quad ; \quad \overline{V(\mathbf{r})V(\mathbf{r}')} = V_0^2\, \delta(\mathbf{r} - \mathbf{r}') ,$$

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we use the memory-function approach to obtain the *resistivity* for current along angle ϑ

$$\rho(T) = \frac{1}{\chi^2_{\mathbf{J},\mathbf{P}}} \lim_{\omega \to 0} \int \frac{d^2 k}{(2\pi)^2}\, k^2 \cos^2(\theta_{\mathbf{k}} - \vartheta) \left(V_0^2\, \frac{\mathrm{Im}\, \Pi^R_{c^\dagger c}(\omega, \mathbf{k})}{\omega} + h_0^2\, \frac{\mathrm{Im}\, D^R_\phi(\omega, \mathbf{k})}{\omega} \right) .$$

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Fermi surface term: Obtain T-dependent corrections to residual resistivity similar to earlier work

G. Zala, B. N. Narozhny, and I. L. Aleiner, Phys. Rev. B **64**, 214204 (2001)
I. Paul, C. Pépin, B. N. Narozhny, and D. L. Maslov, Phys. Rev. Lett. **95**, 017206 (2005).

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Bosonic term: Dominant contribution:

$$\rho(T) \sim h_0^2\, T^{(d - z + \eta)/z}$$

Crosses over from the "relativistic" form ($z = 1$, $\eta \approx 0$) with $\rho(T) \sim h_0^2\, T$ at higher T, to the "Landau-damped" form ($z = 3$, $\eta = 0$) with $\rho(T) \sim h_0^2\, (T \ln(1/T))^{-1/2}$ at lower T (subtle corrections to scaling specific to this field theory).

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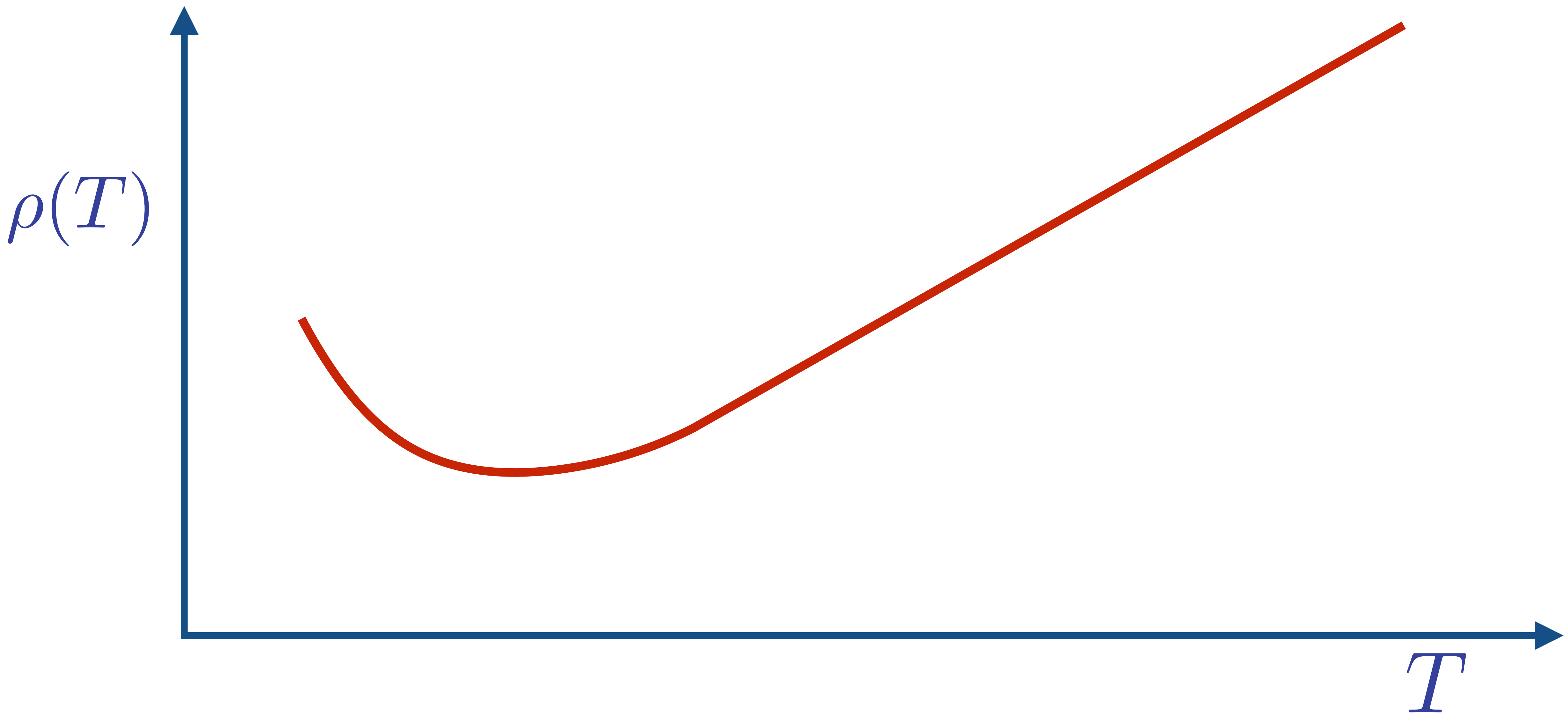
Transport without quasiparticles:

Resistivity from random-field disorder



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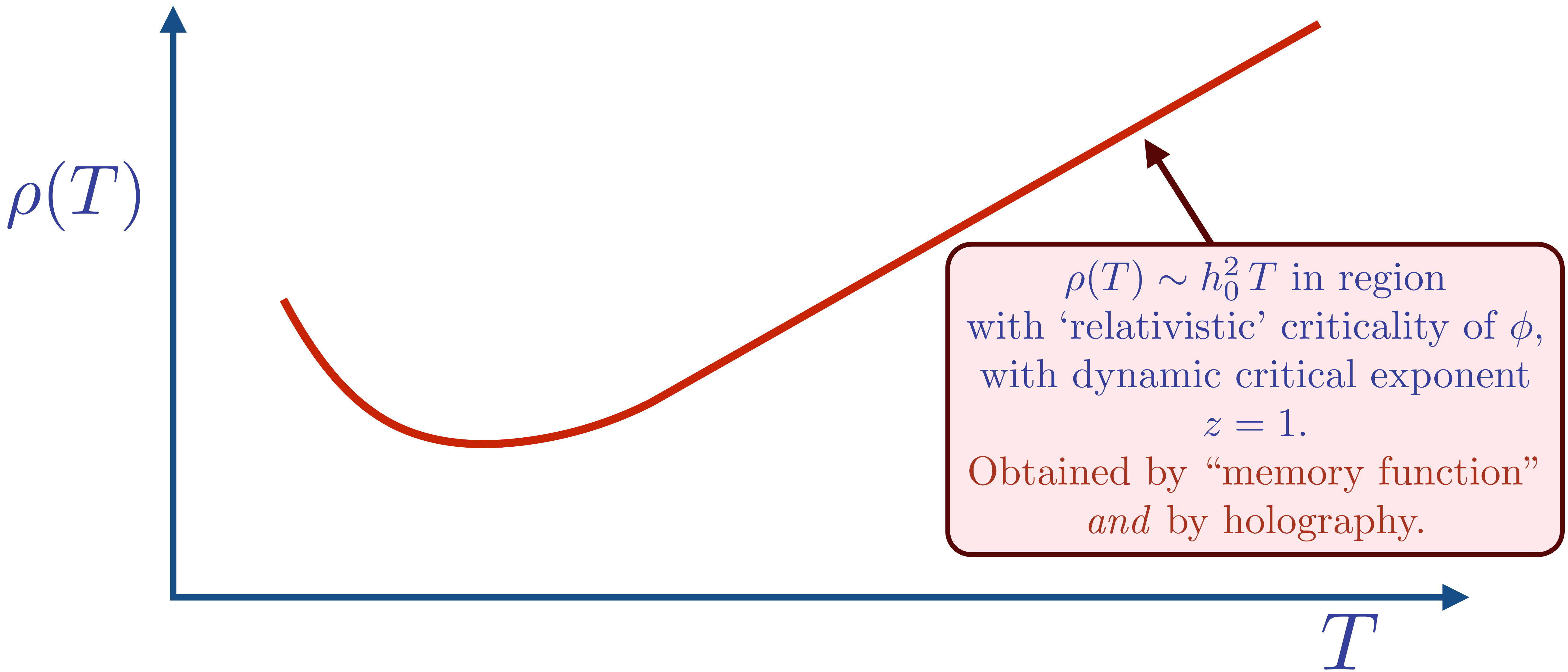

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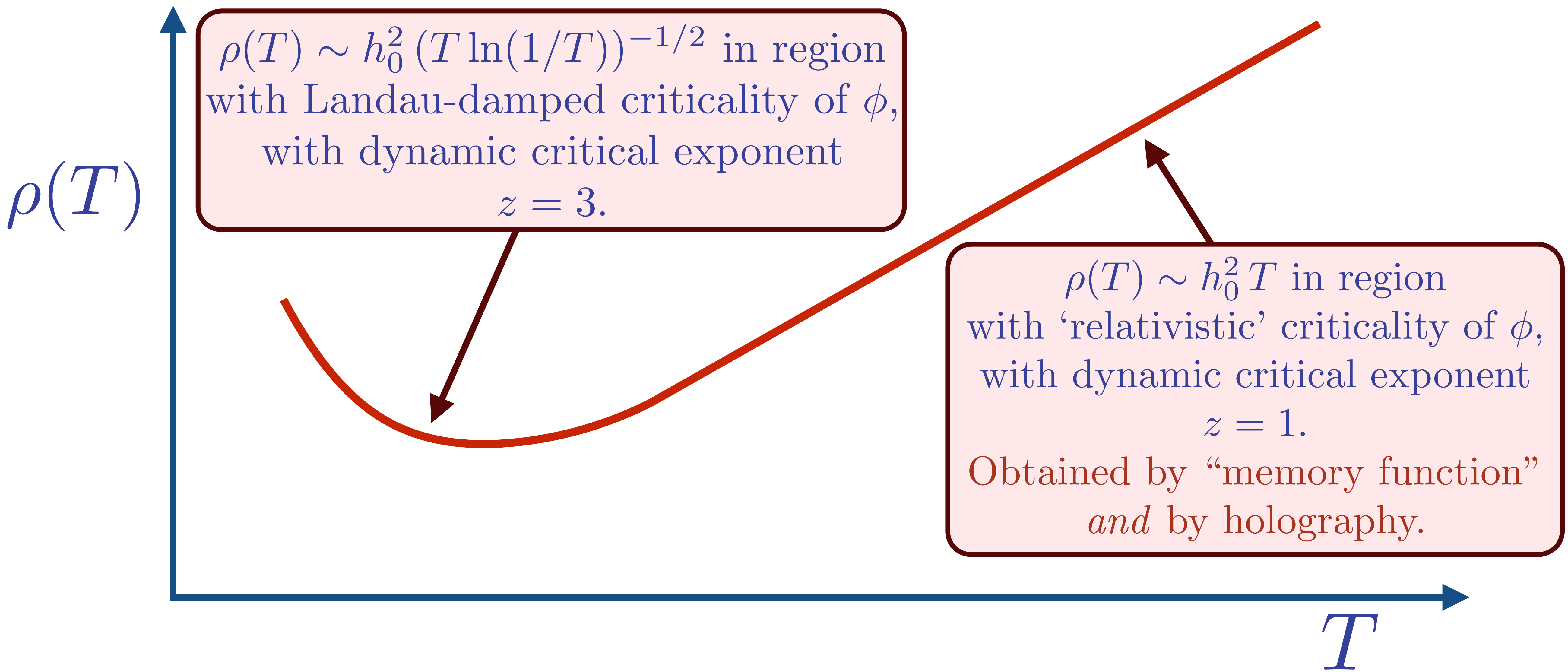

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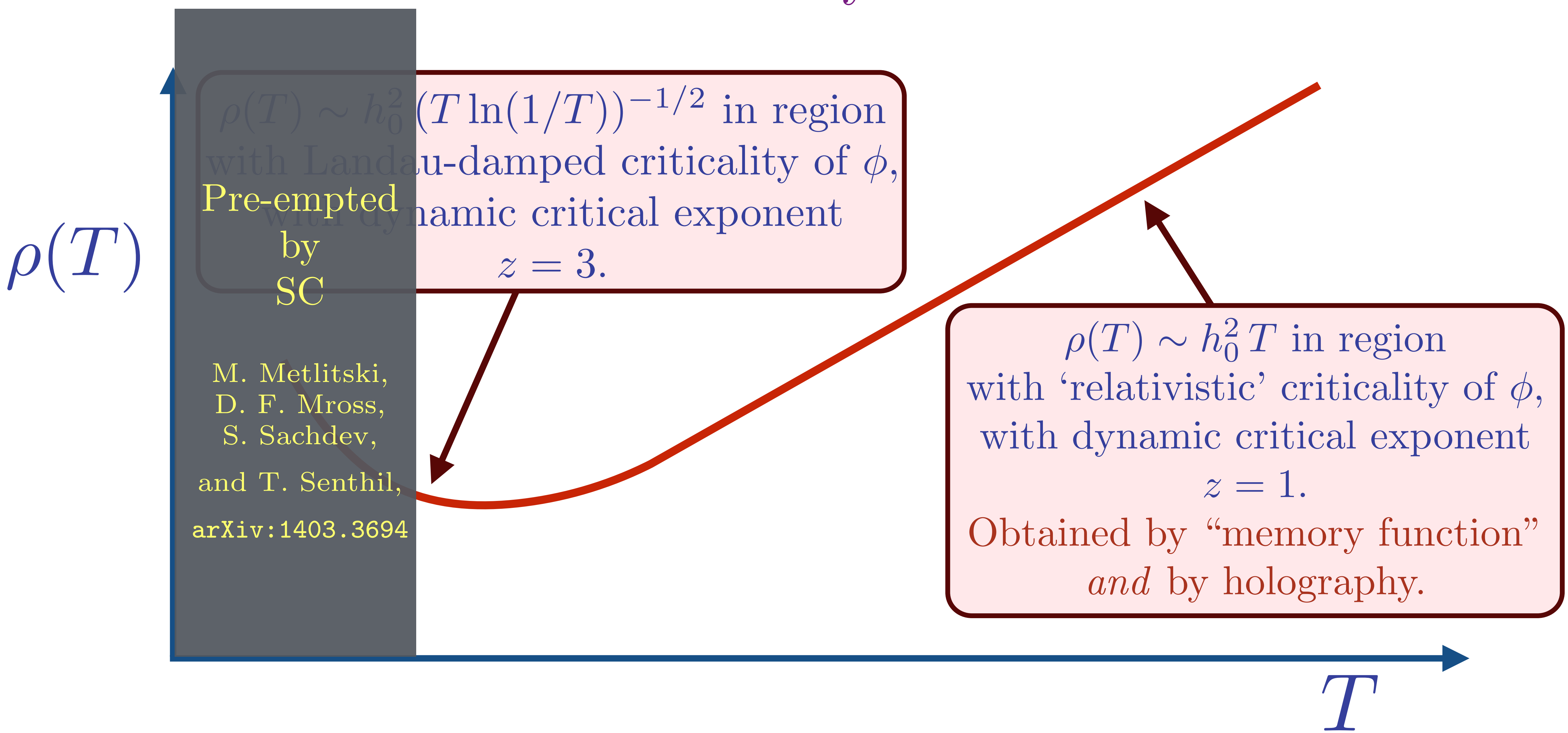

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Conclusions

1. Antiferromagnetism and quantum criticality in insulators: triplons, spin-waves, and “Higgs” in $TlCuCl_3$

2. Onset of antiferromagnetism in metals, and *d*-wave superconductivity

3. Experimental evidence for *d*-form factor density wave order, linked to the pseudogap, in the cuprate superconductors

4. Non-Fermi liquid at the Ising-nematic quantum critical point in a two-dimensional metal, and its transport properties