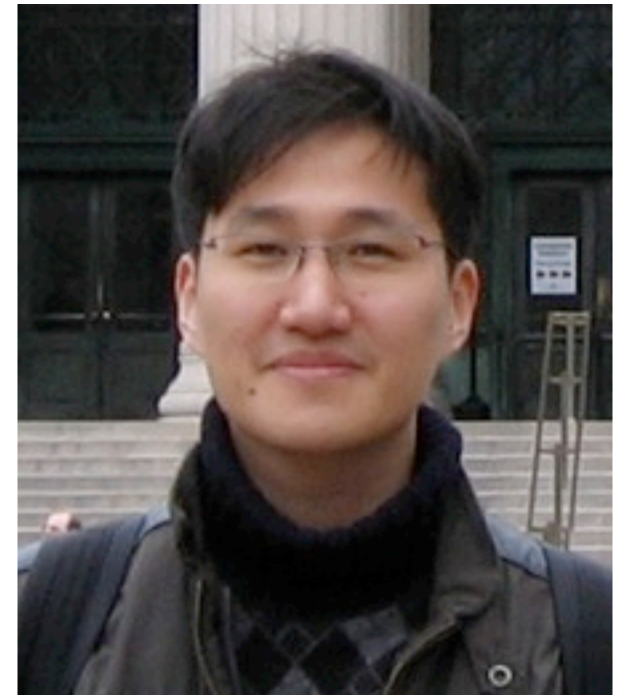
The phase diagram of the cuprates and the quantum phase transitions of metals in two dimensions

Niels Bohr Institute, Copenhagen, May 6, 2010

Talk online: sachdev.physics.harvard.edu

PHYSICS
VE RI TAS
HARVARD



Max Metlitski, Harvard

Eun Gook Moon, Harvard

arXiv:1001.1153

Phys. Rev. B **80**, 035117 (2009)

PHYSICS

VE RI TAS

HARVARD

Outline

1. Phase diagram of the cuprates

   Quantum criticality of the competition between antiferromagnetism and superconductivity

2. Theory of spin density wave ordering in a metal

   Strong-coupling in d=2

3. Instabilities near SDW critical point

   d-wave pairing and bond density wave

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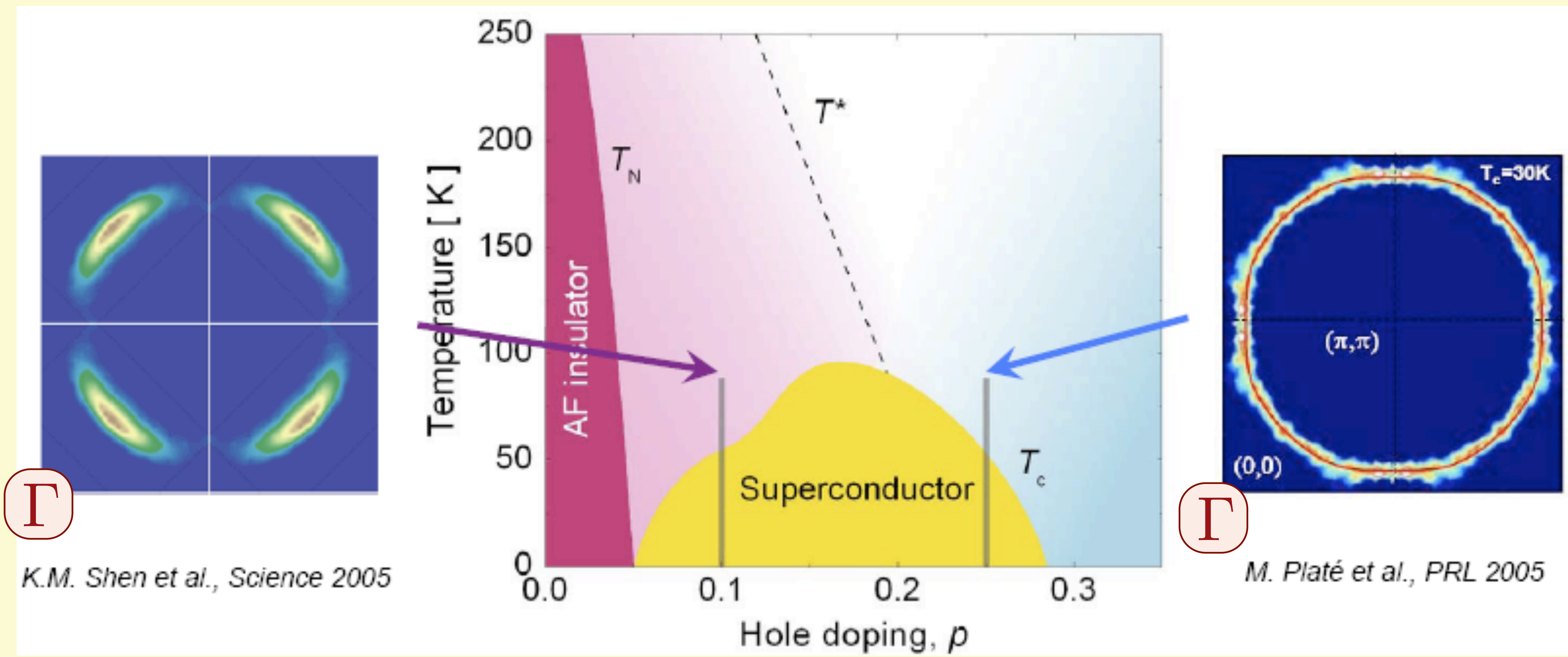
Central ingredients in cuprate phase diagram: antiferromagnetism, superconductivity, and change in Fermi surface



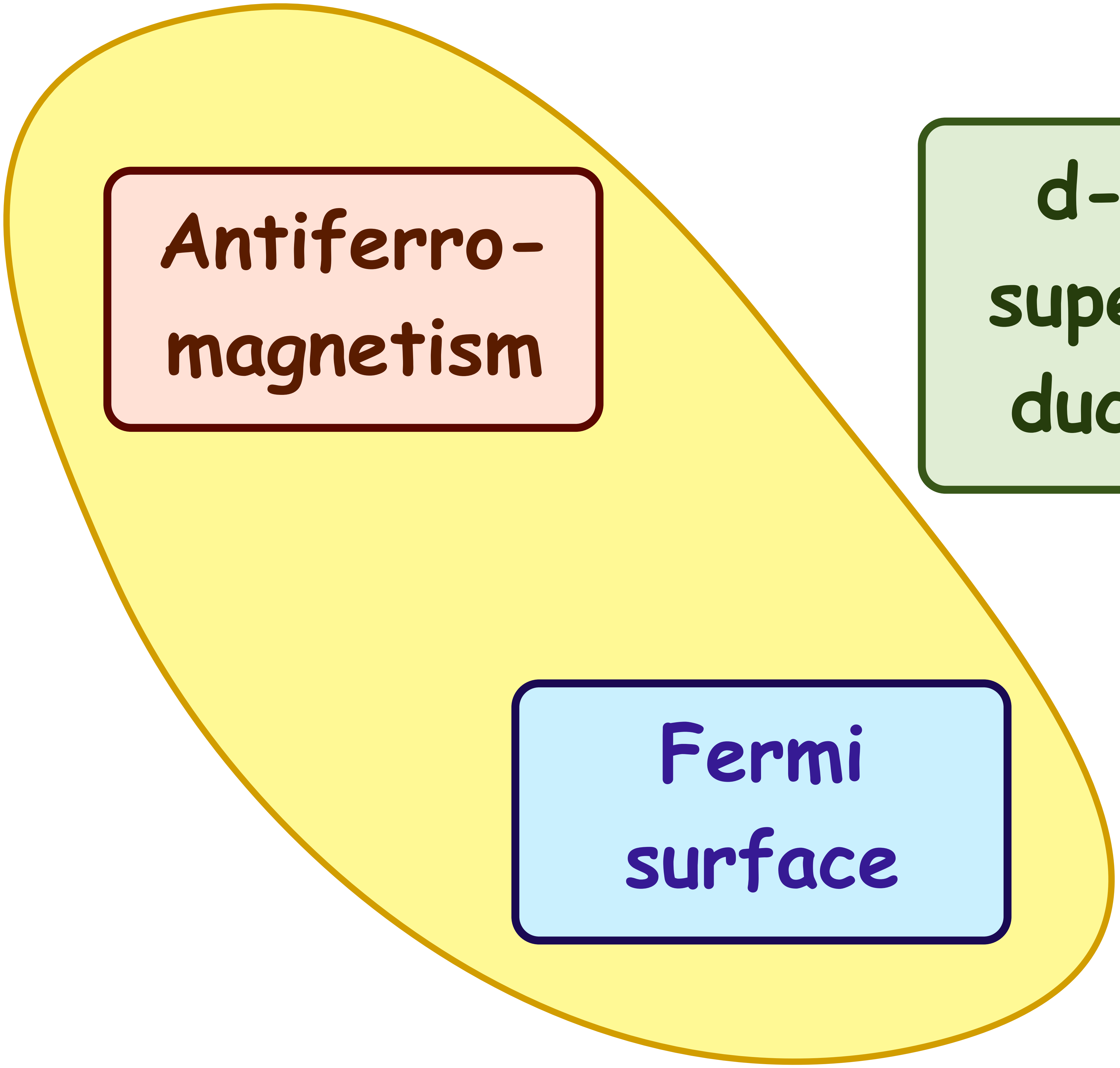
Smaller hole Fermi-pockets

Large hole Fermi surface

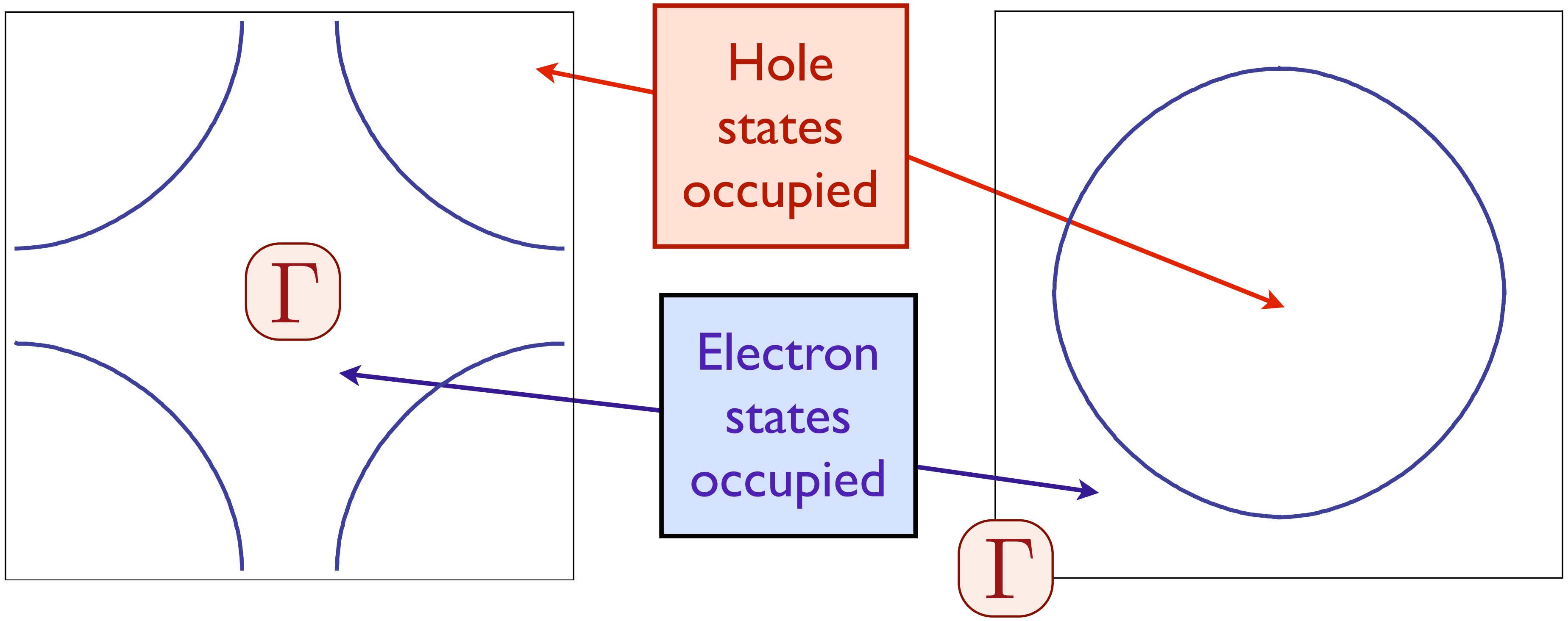
Antiferro-
magnetism

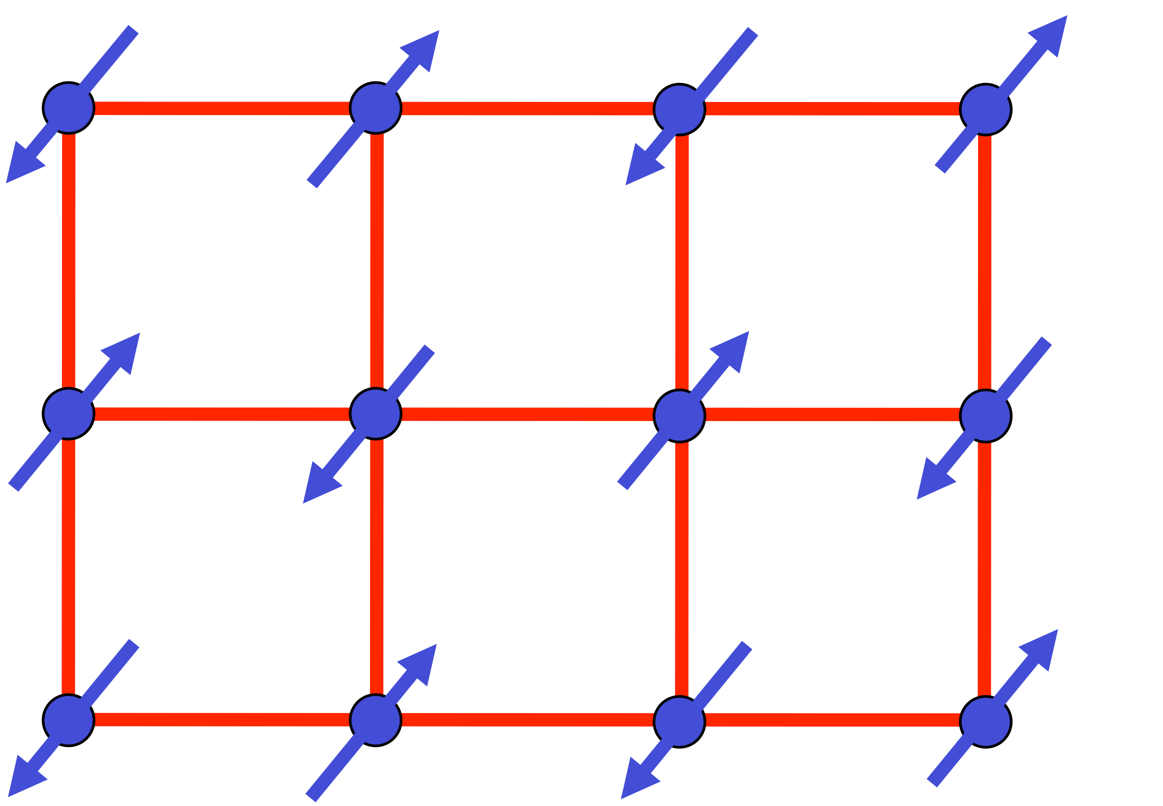
d-wave
supercon-
ductivity

Fermi
surface


Antiferro-
magnetism
d-wave
supercon-
ductivity
Fermi
surface

Fermi surface+antiferromagnetism



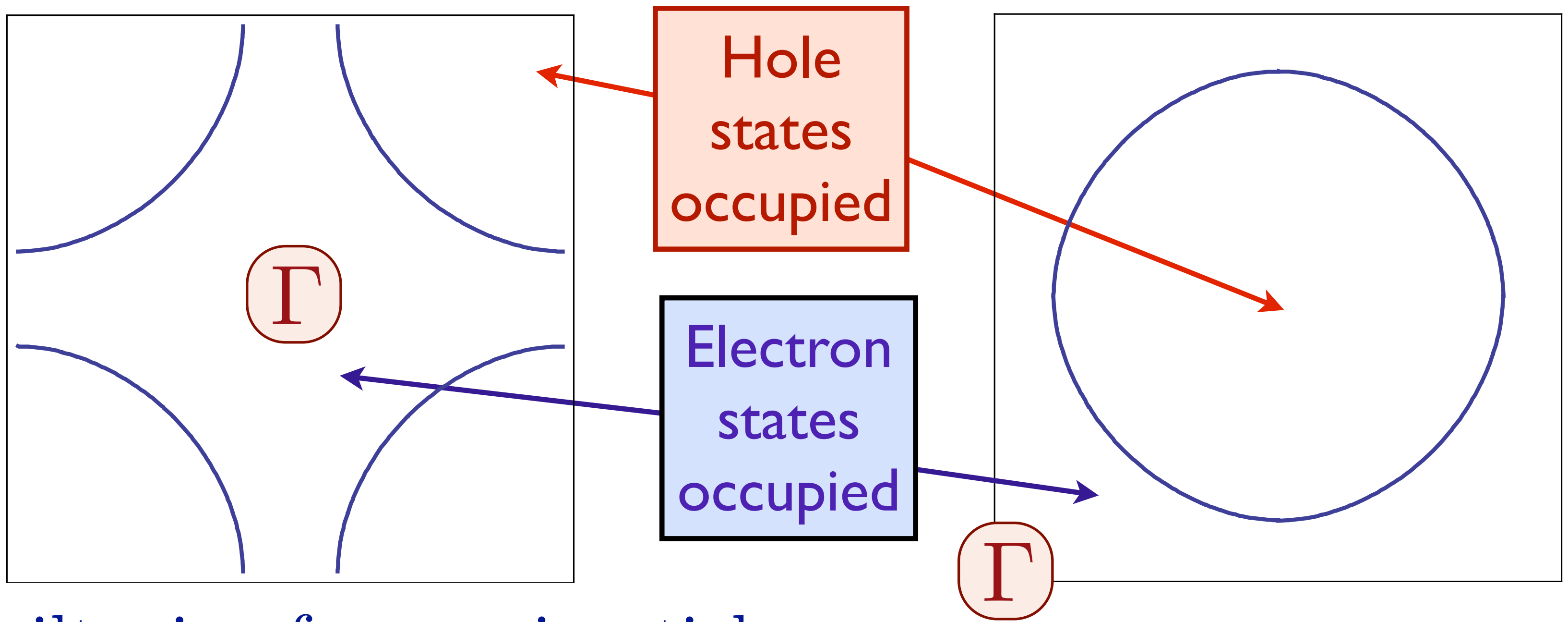


The electron spin polarization obeys

$$\left\langle \vec{S}(\mathbf{r},\tau) \right\rangle = \vec{\varphi}(\mathbf{r},\tau) e^{i\mathbf{K}\cdot\mathbf{r}}$$

where $\mathbf{K}$ is the ordering wavevector.

Fermi surfaces in electron- and hole-doped cuprates



Effective Hamiltonian for quasiparticles:

$$H_0 = -\sum_{i<j} t_{ij} c^\dagger_{i\alpha} c_{i\alpha} \equiv \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

with t_{ij} non-zero for first, second and third neighbor, leads to satisfactory agreement with experiments. The area of the occupied electron states, $\mathcal{A}_e$, from Luttinger's theory is

$$\mathcal{A}_e = \begin{cases} 2\pi^2(1-p) & \text{for hole-doping } p \\ 2\pi^2(1+x) & \text{for electron-doping } x \end{cases}$$

The area of the occupied hole states, $\mathcal{A}_h$, which form a closed Fermi surface and so appear in quantum oscillation experiments is $\mathcal{A}_h = 4\pi^2 - \mathcal{A}_e$.

Spin density wave theory

In the presence of spin density wave order, $\vec{\varphi}$ at wavevector $\mathbf{K} = (\pi, \pi)$, we have an additional term which mixes electron states with momentum separated by $\mathbf{K}$

$$H_{\mathrm{sdw}} = \vec{\varphi} \cdot \sum_{\mathbf{k},\alpha,\beta} c_{\mathbf{k},\alpha} \vec{\sigma}_{\alpha\beta} c_{\mathbf{k}+\mathbf{K},\beta}$$

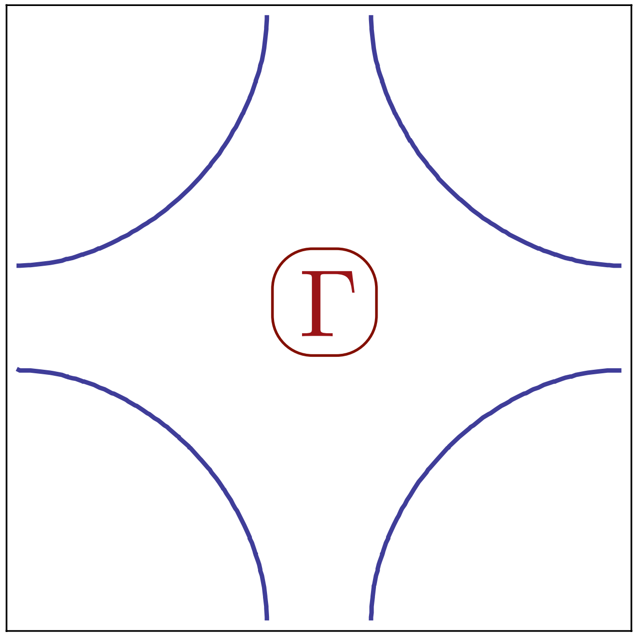
where $\vec{\sigma}$ are the Pauli matrices. The electron dispersions obtained by diagonalizing $H_0 + H_{\mathrm{sdw}}$ for $\vec{\varphi} \propto (0,0,1)$ are

$$E_{\mathbf{k}\pm} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{K}}}{2} \pm \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{K}}}{2}\right) + \varphi^2}$$

This leads to the Fermi surfaces shown in the following slides for electron and hole doping.

Hole-doped cuprates

← Increasing SDW order



S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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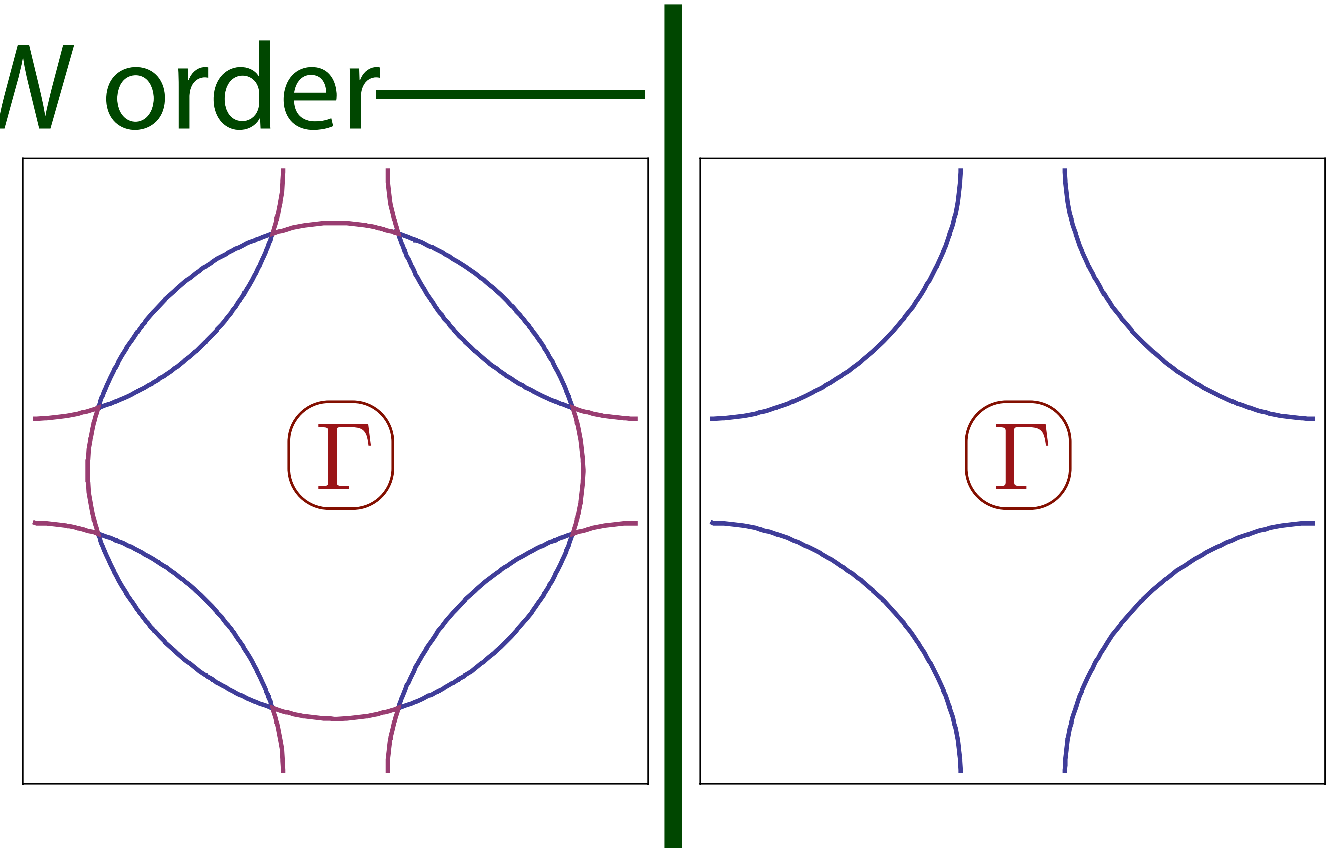
Hole-doped cuprates

Increasing SDW order



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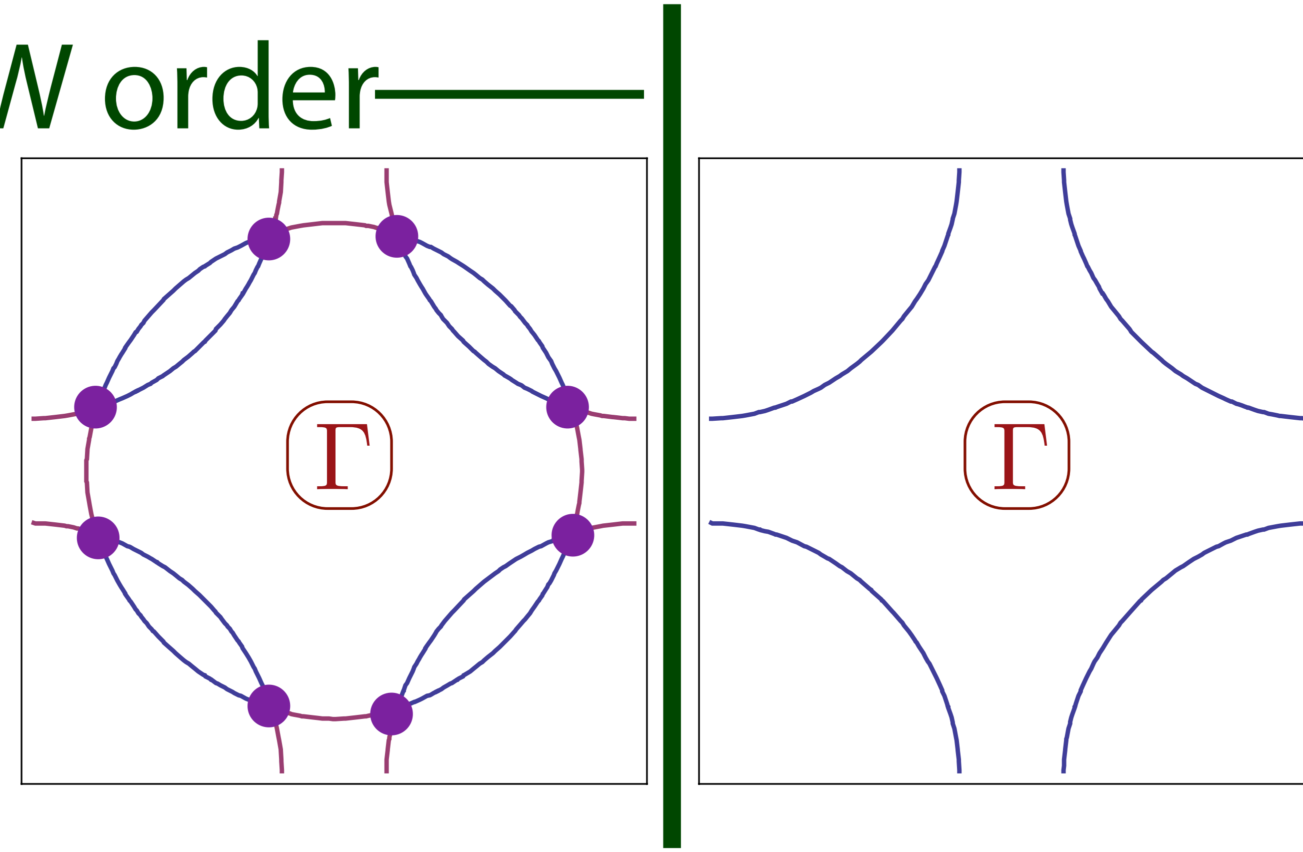
Hole-doped cuprates

Increasing SDW order



Hot spots

S. Sachdev, A. V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
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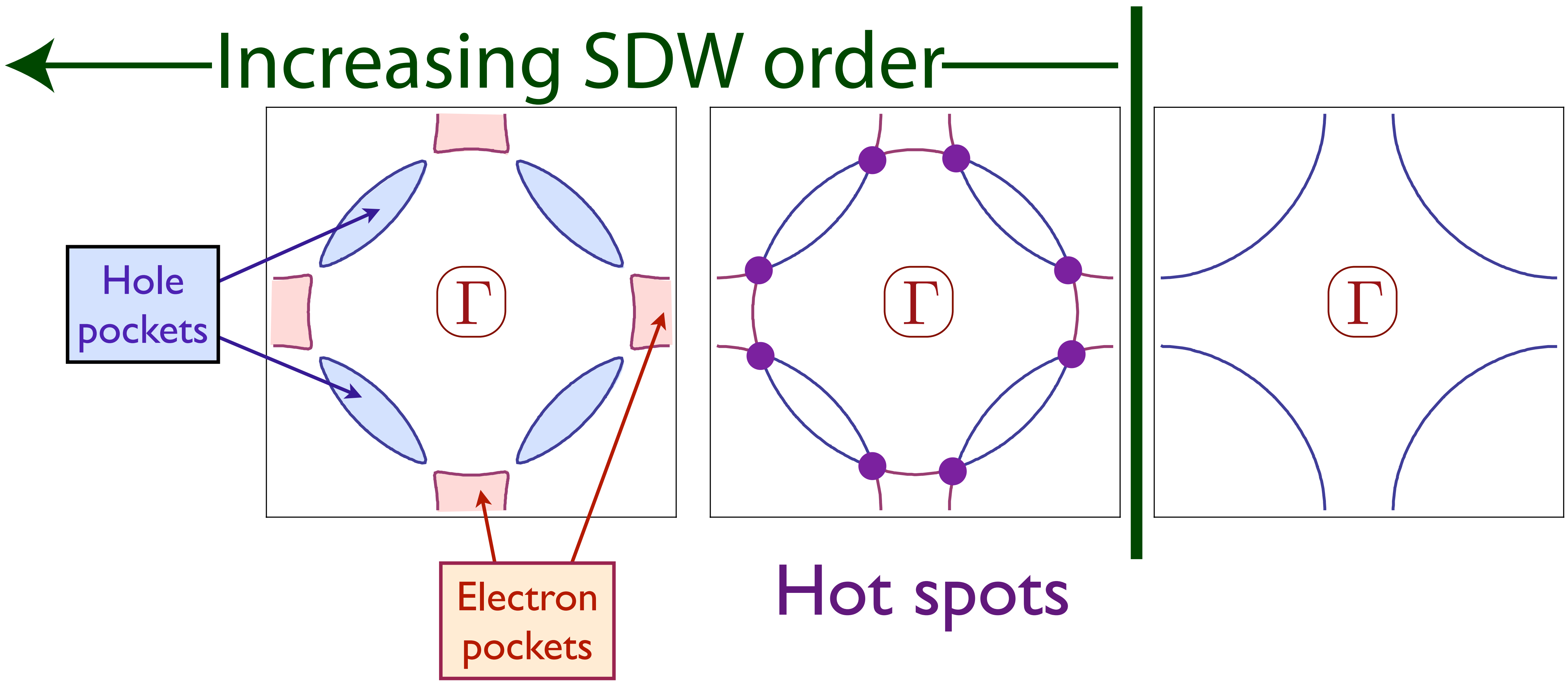
Hole-doped cuprates



Fermi surface breaks up at hot spots into electron and hole "pockets"

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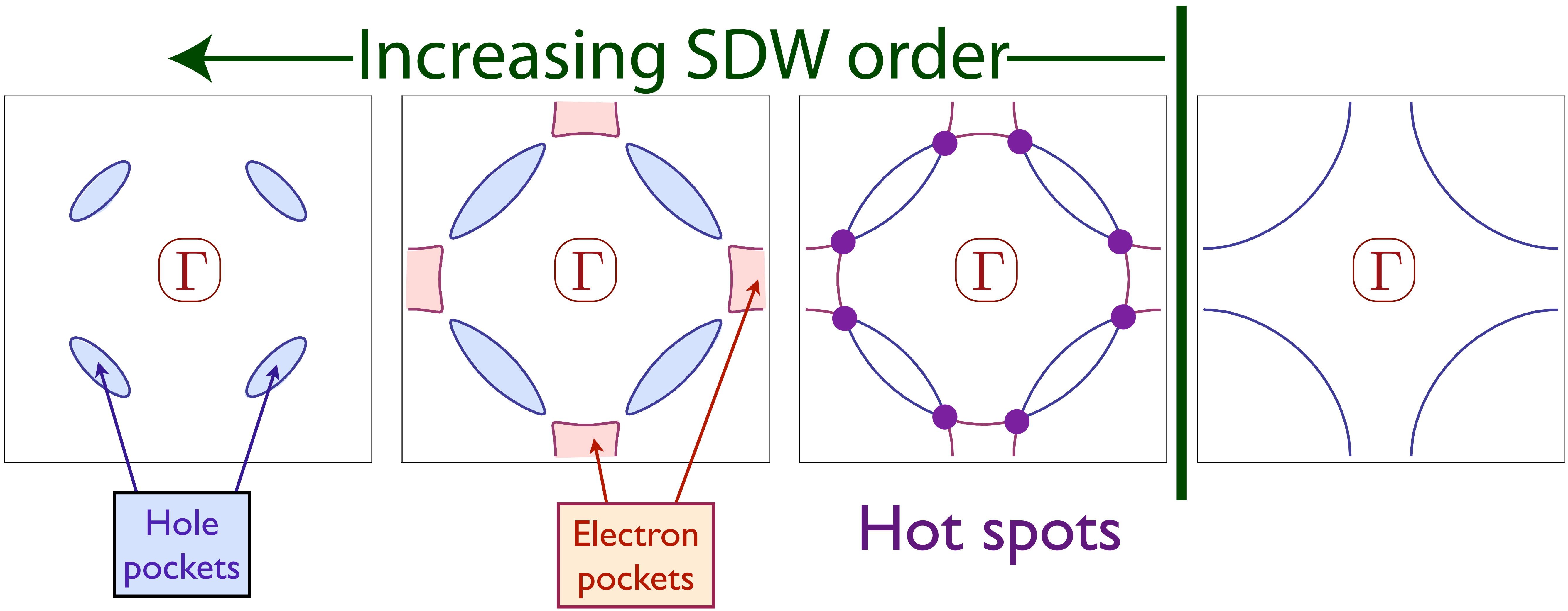
Hole-doped cuprates



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Evidence for small Fermi pockets

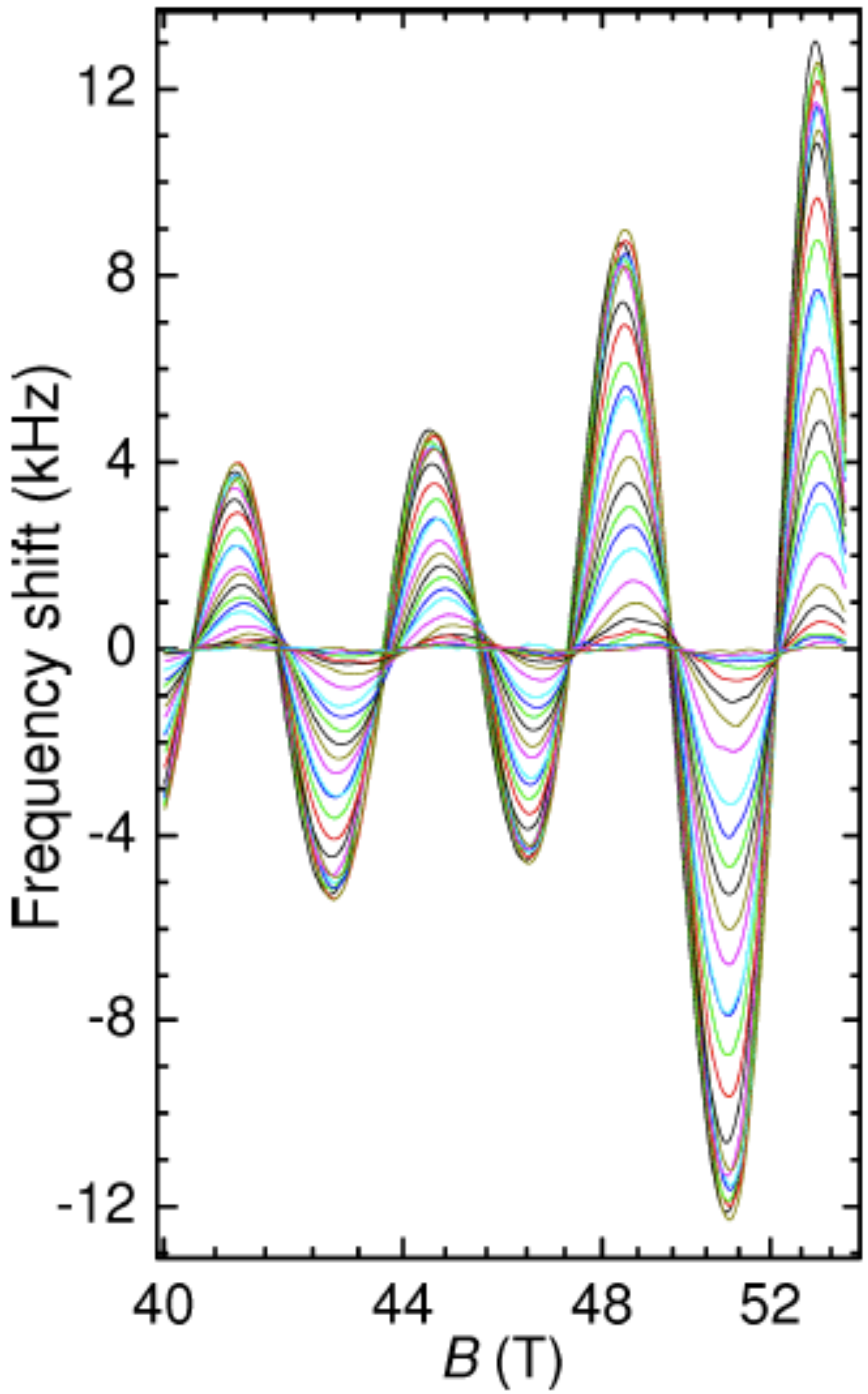


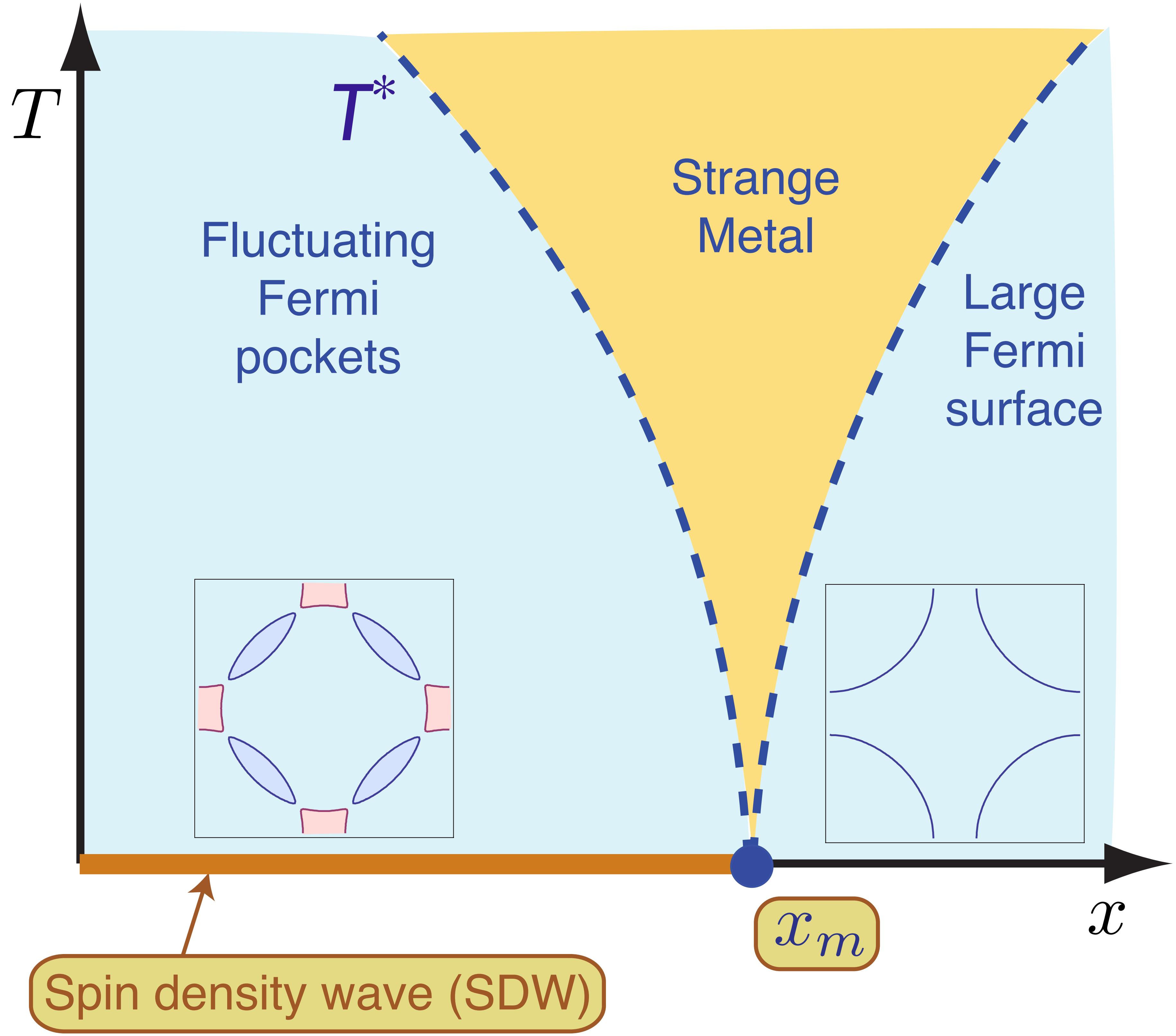
FIG. 2: Magnetic quantum oscillations measured in $YBa_2Cu_3O_{6+x}$ with $x \approx 0.56$ (after background polynomial subtraction). This restricted interval in $B = |\mathbf{B}|$ furnishes a dynamic range of $\sim$ 50 dB between T = 1 and 18 K. The actual T values are provided in Fig. 3.

Fermi liquid behaviour in an underdoped high Tc superconductor

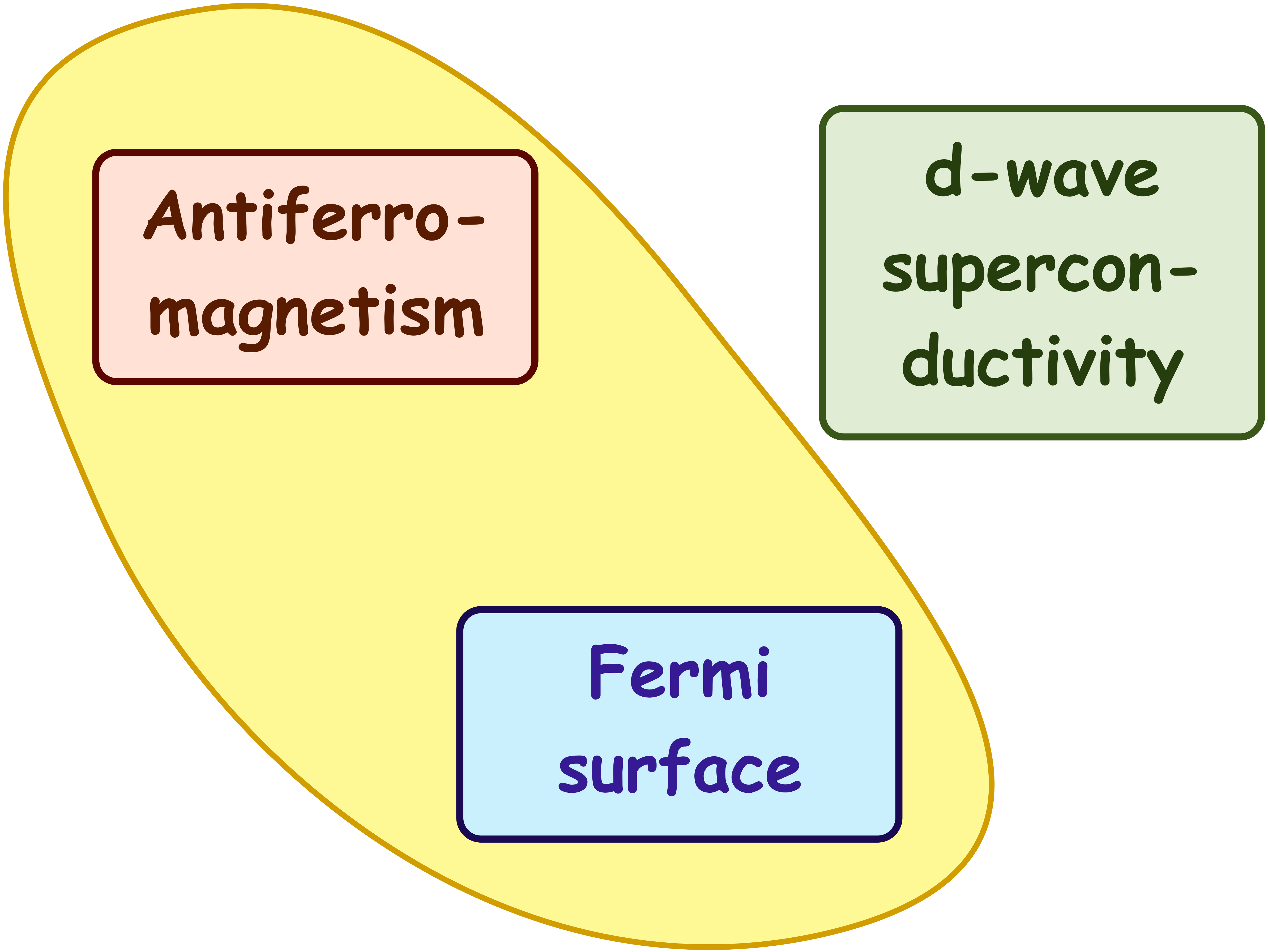
Suchitra E. Sebastian, N. Harrison, M. M. Altarawneh, Ruixing Liang, D. A. Bonn, W. N. Hardy, and G. G. Lonzarich

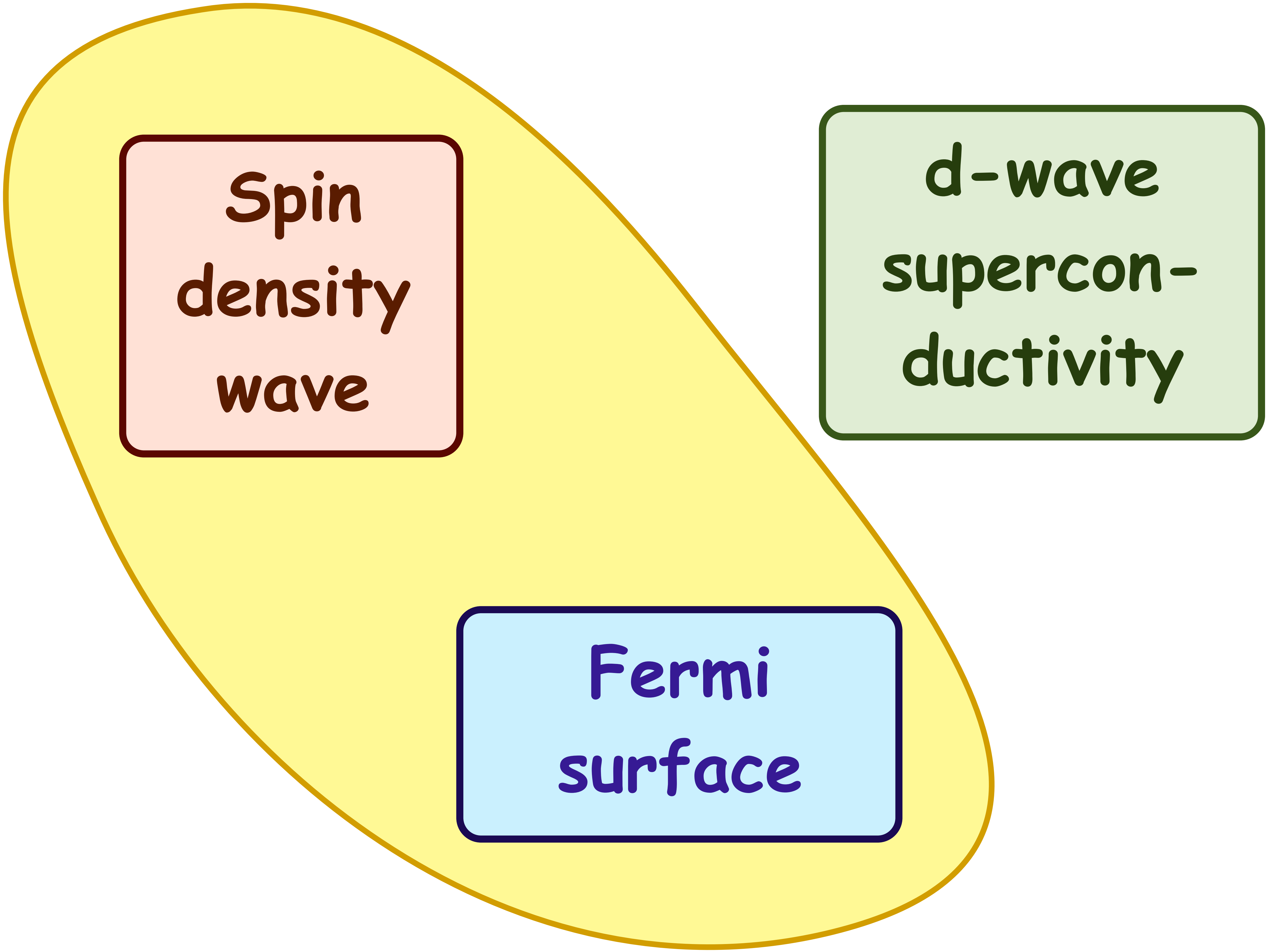
arXiv:0912.3022

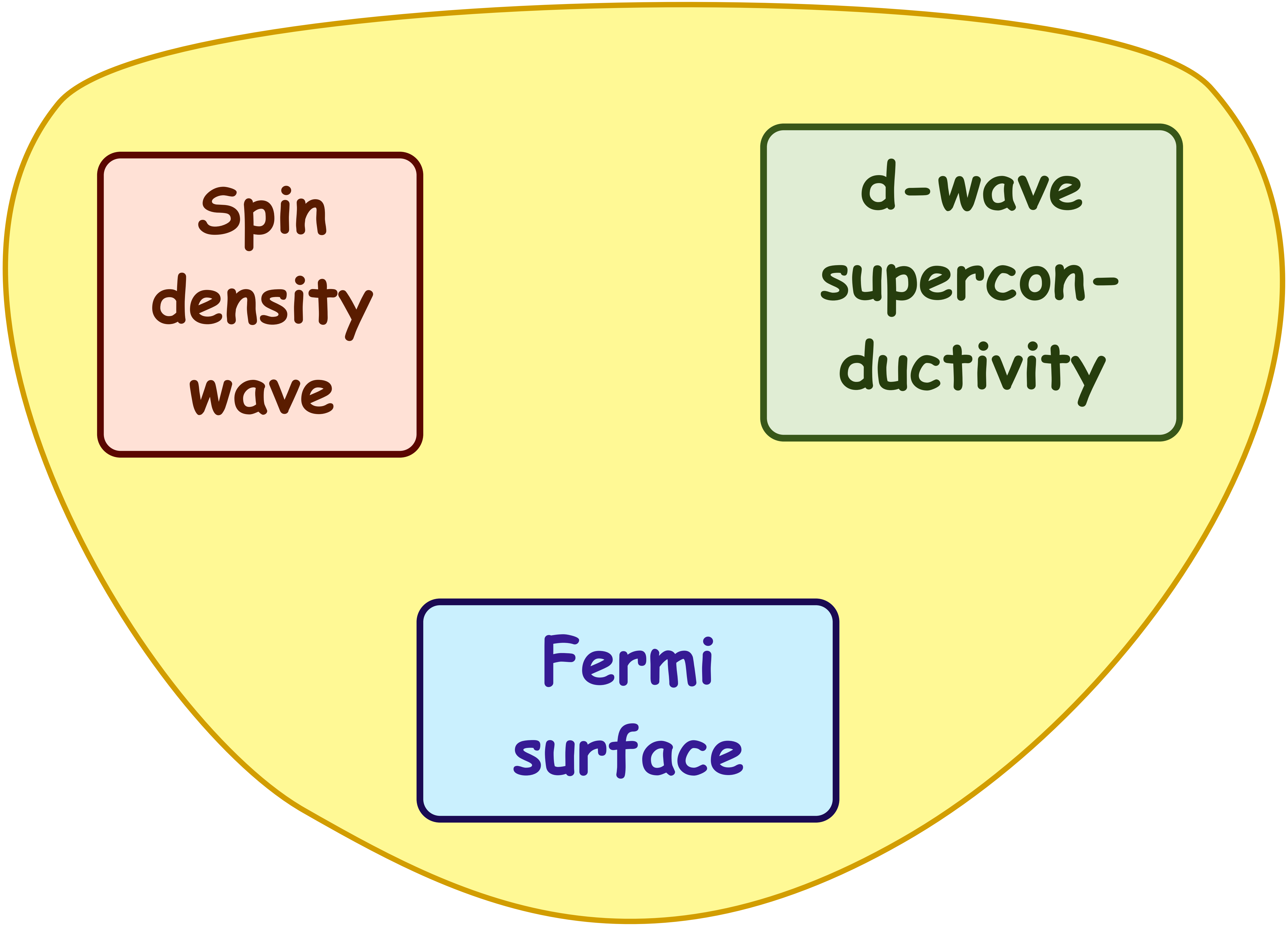
Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$


Antiferro-
magnetism
d-wave
supercon-
ductivity
Fermi
surface


Spin density wave
d-wave supercon-ductivity
Fermi surface


Spin density wave
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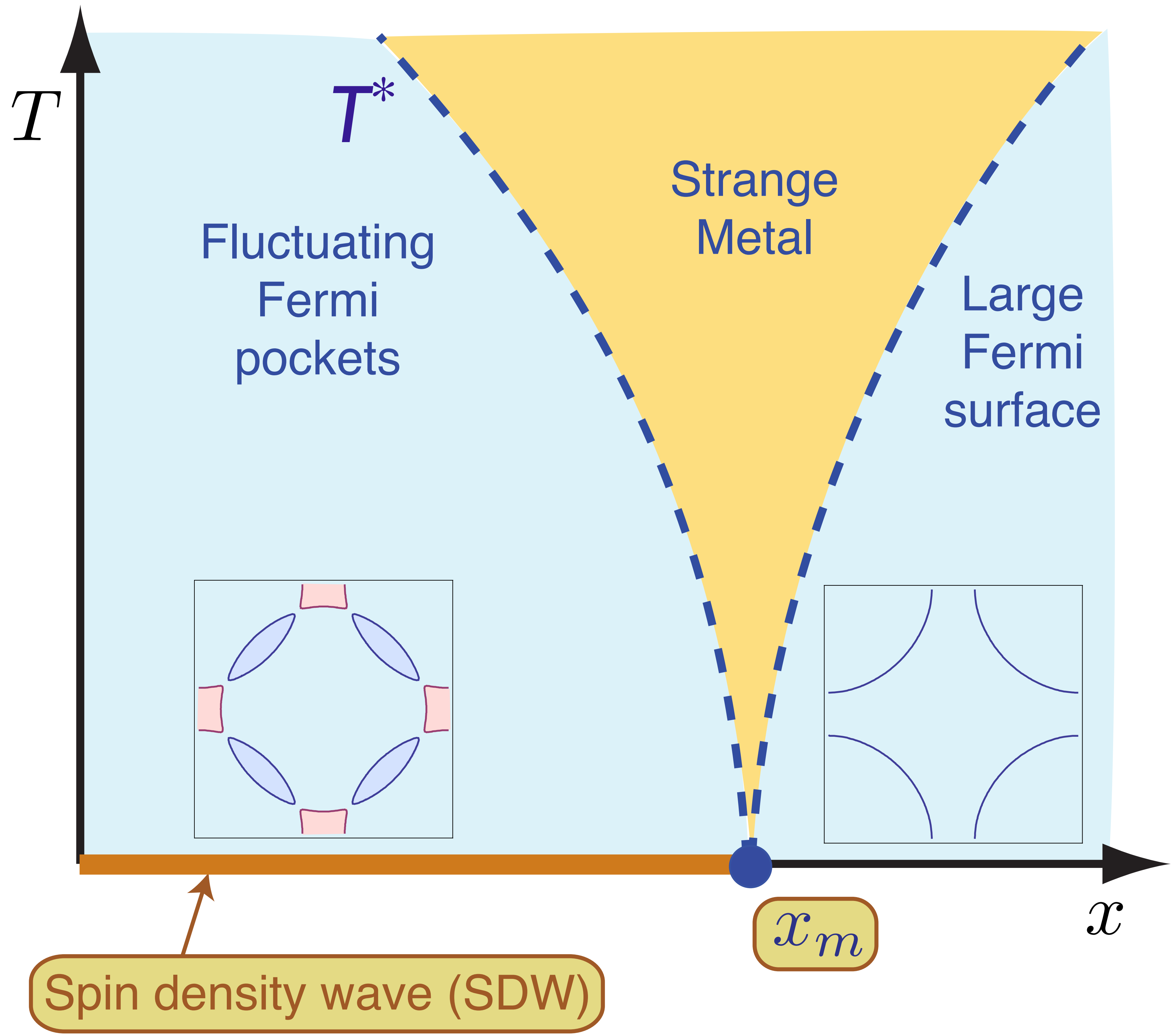

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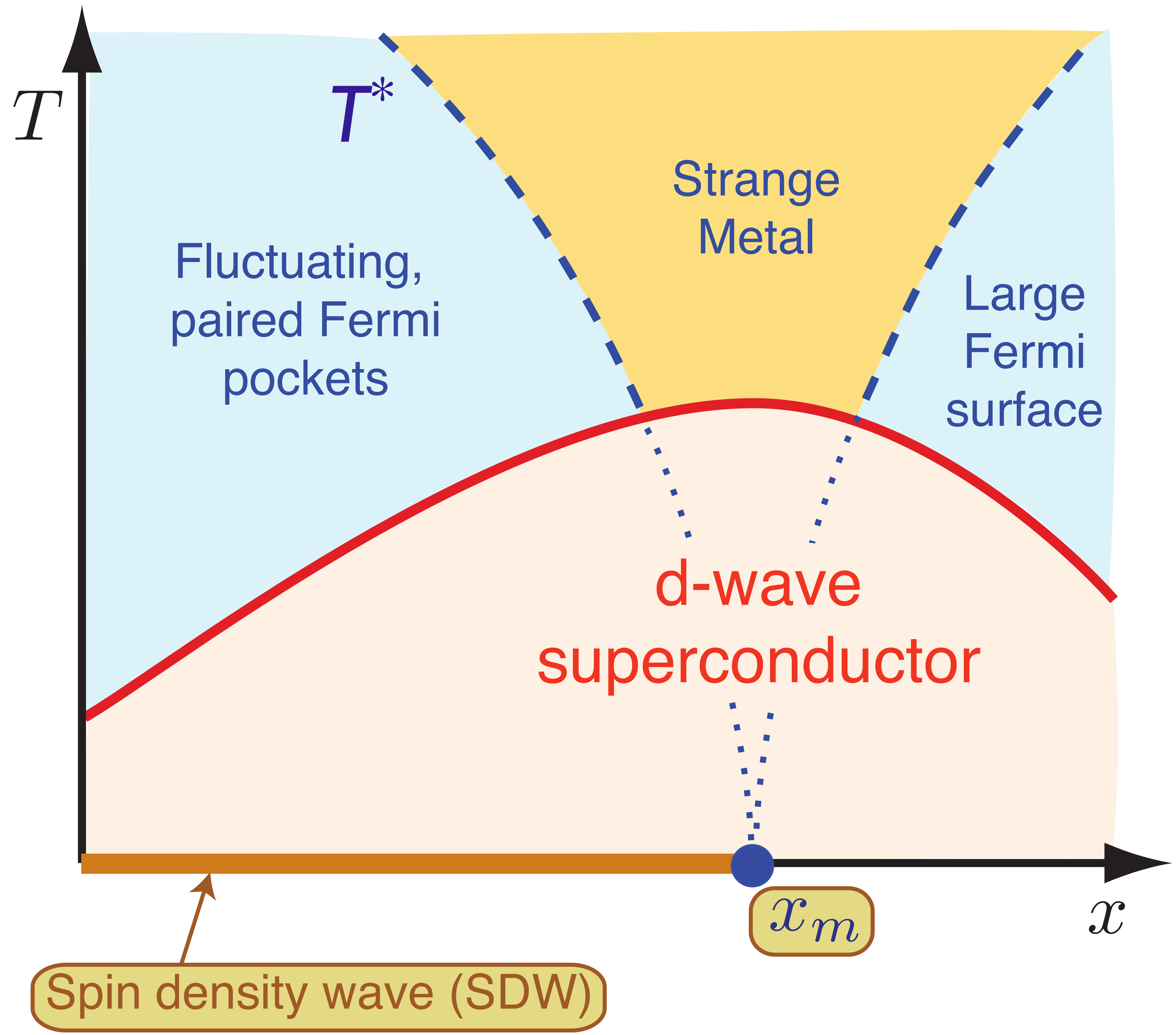
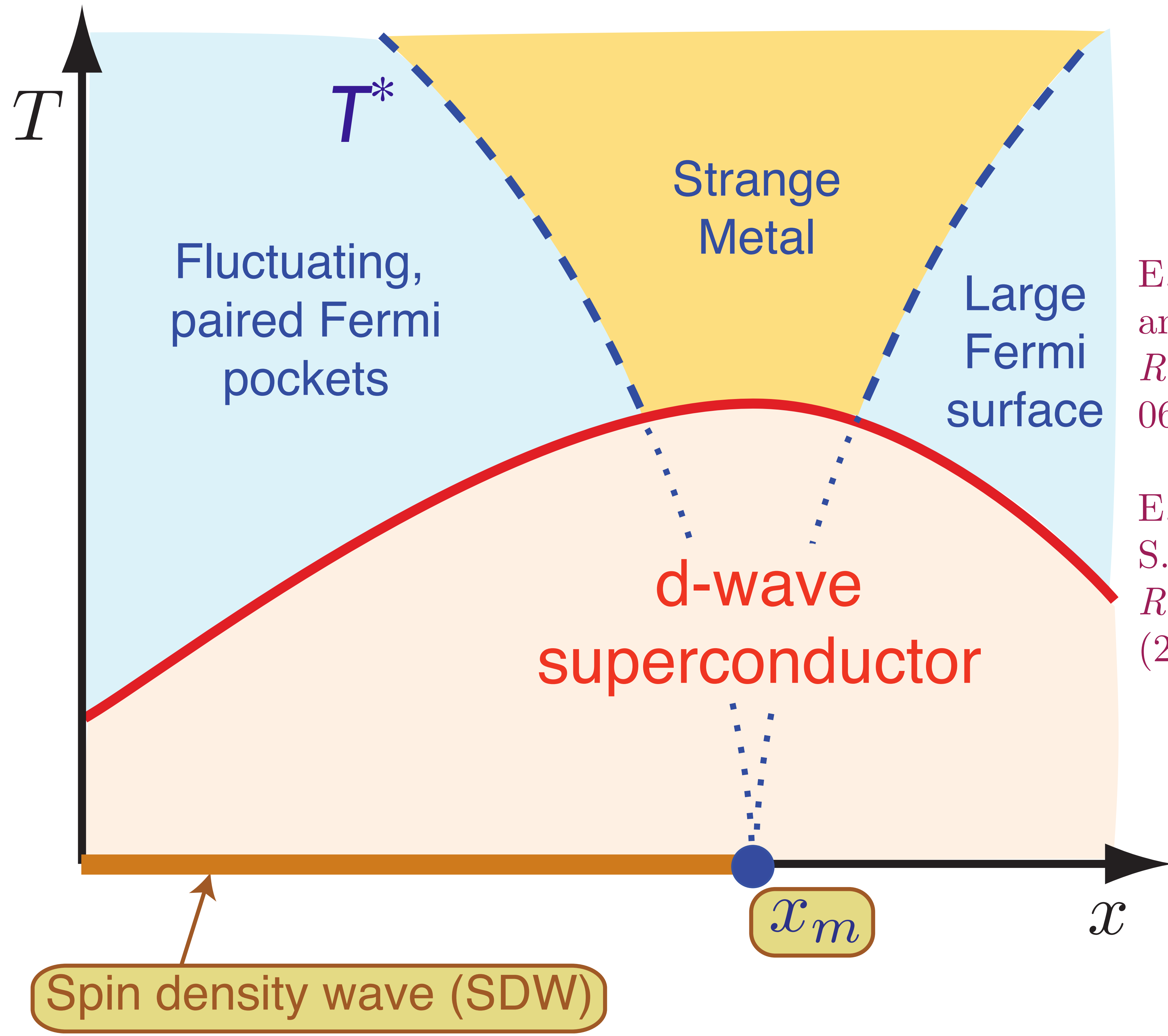


Onset of d-wave superconductivity hides the critical point $x = x_m$

Theory of quantum criticality in the cuprates

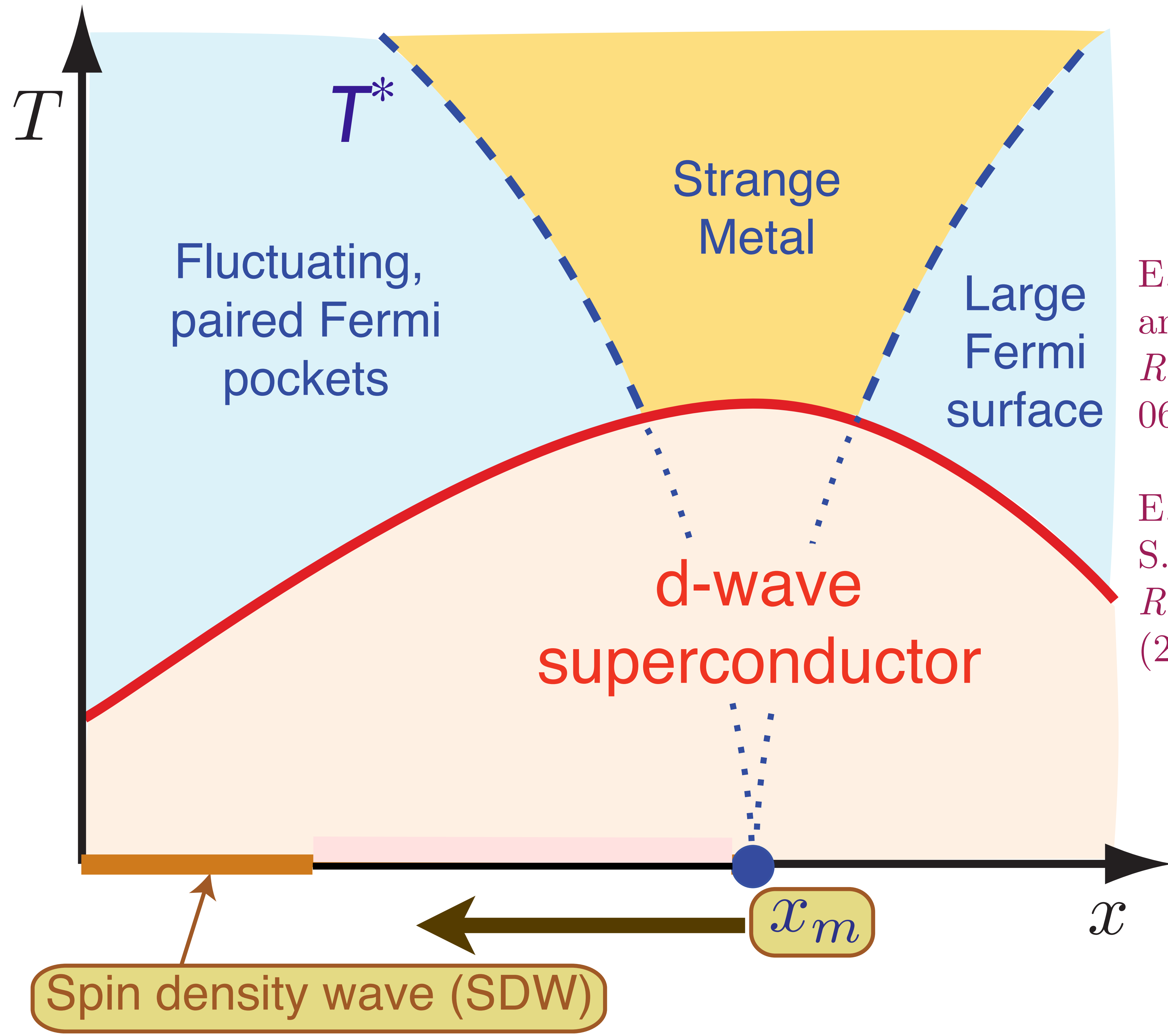


E. Demler, S. Sachdev and Y. Zhang, *Phys. Rev. Lett.* **87**, 067202 (2001).

E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

Theory of quantum criticality in the cuprates



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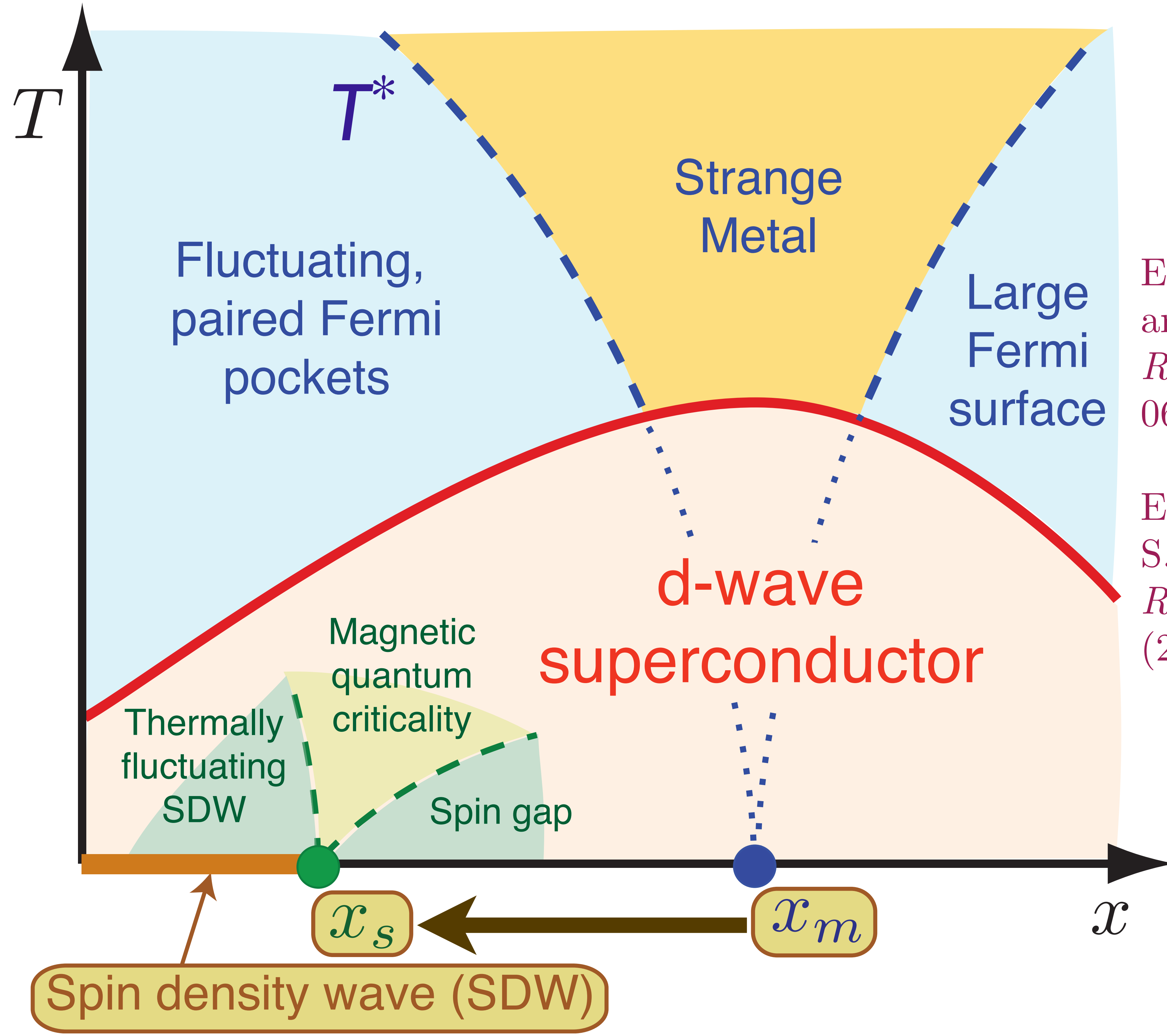

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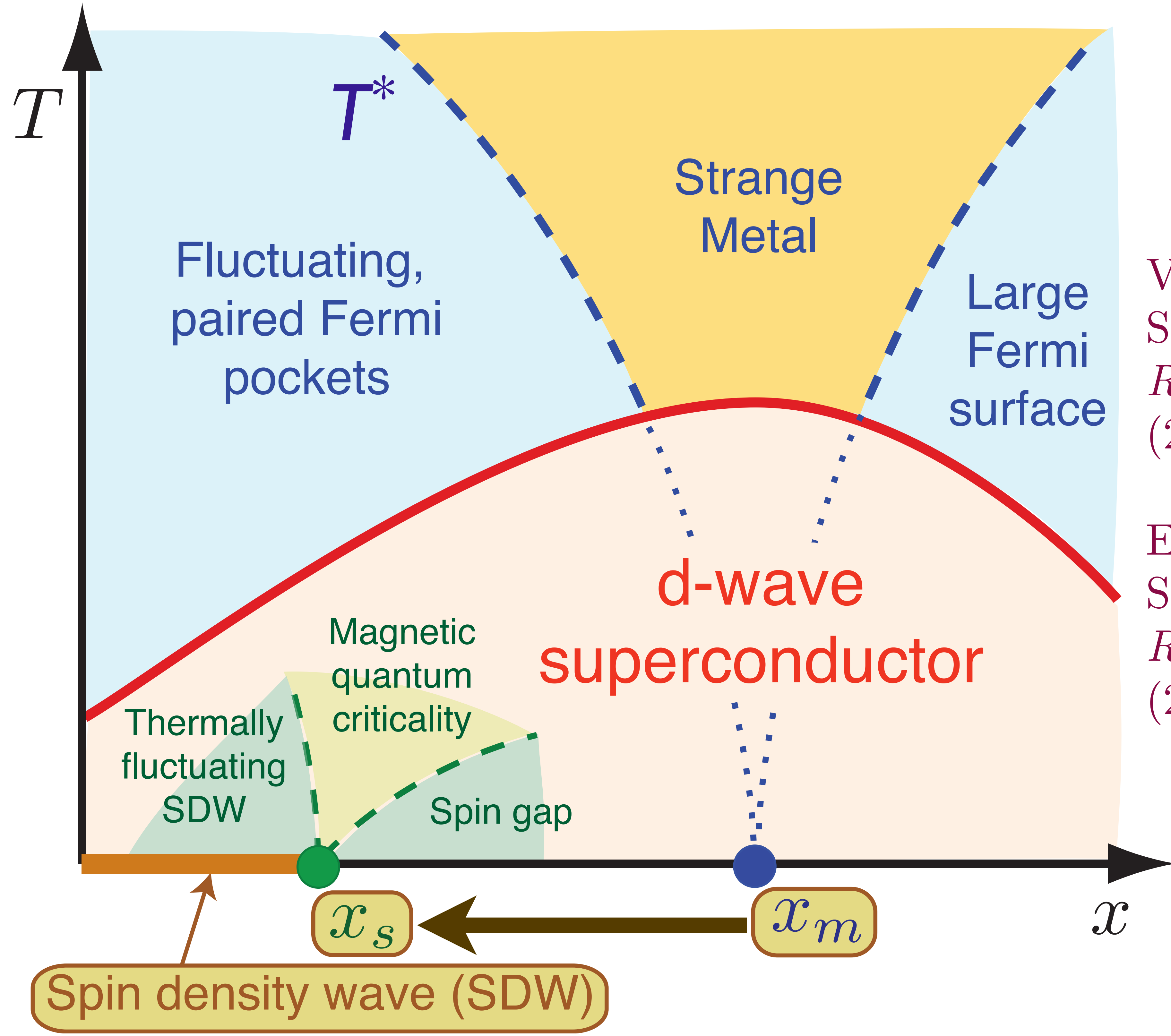
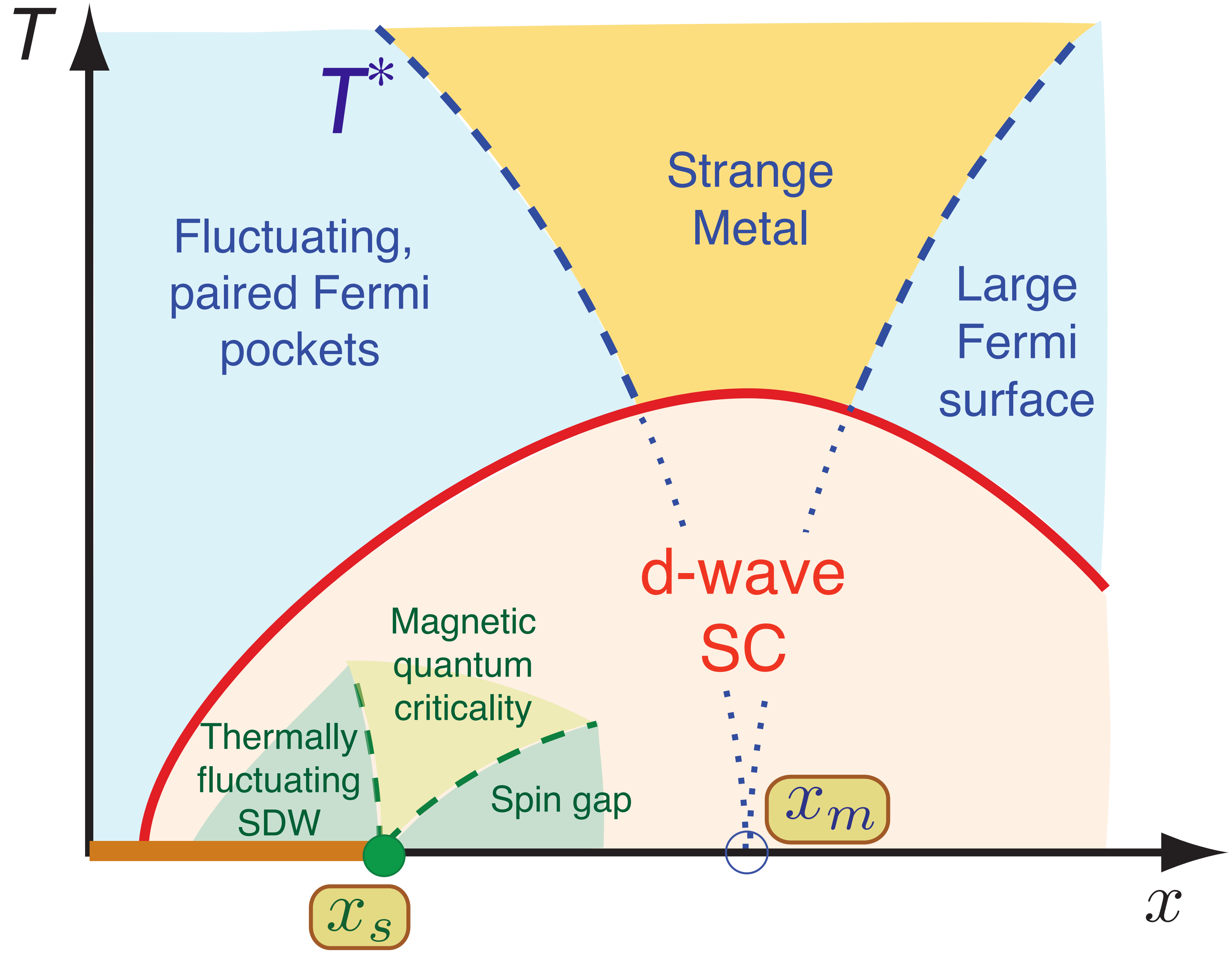
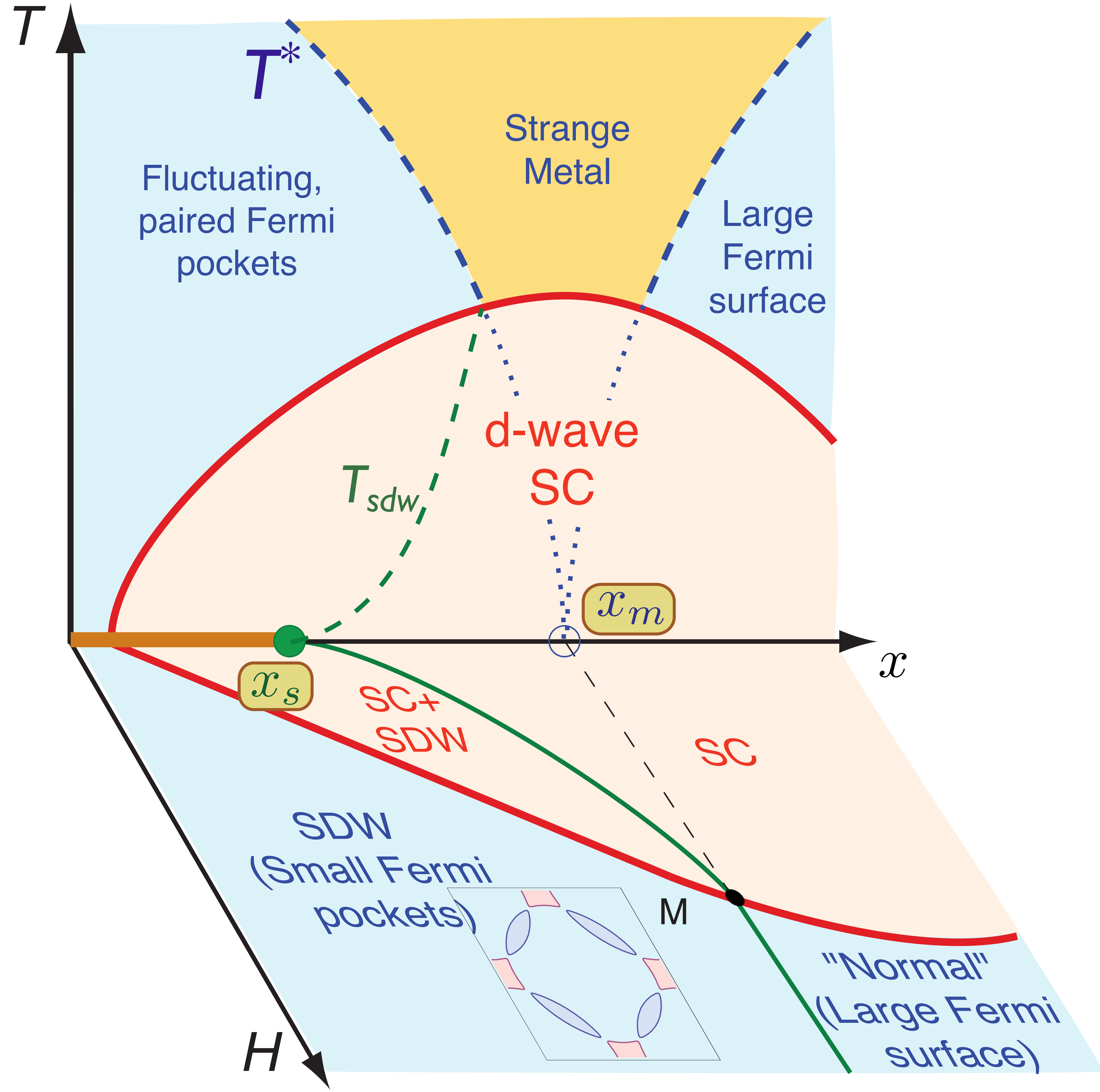


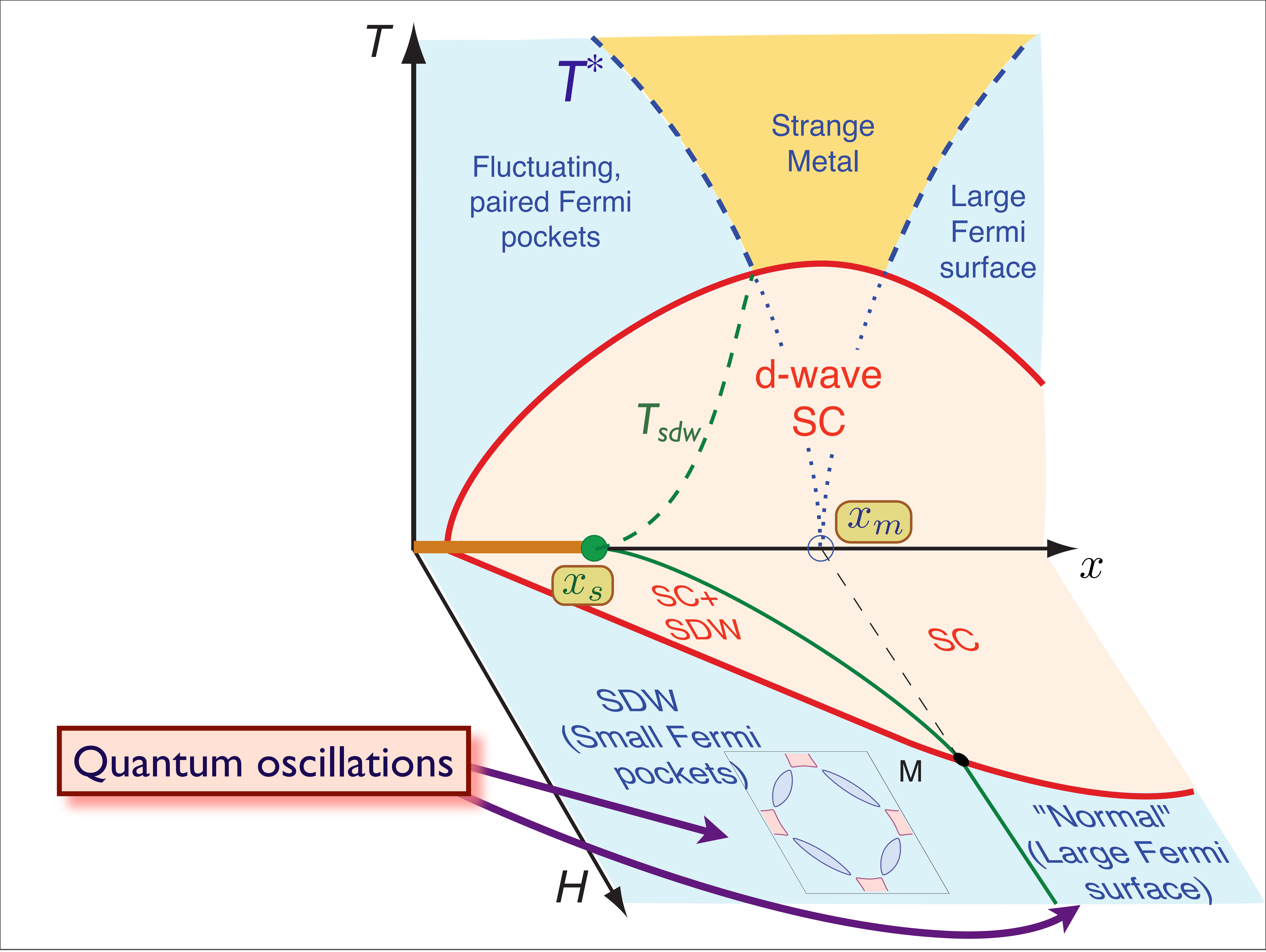
V. Galitski and S. Sachdev, *Phys. Rev.* B **79**, 134512 (2009).

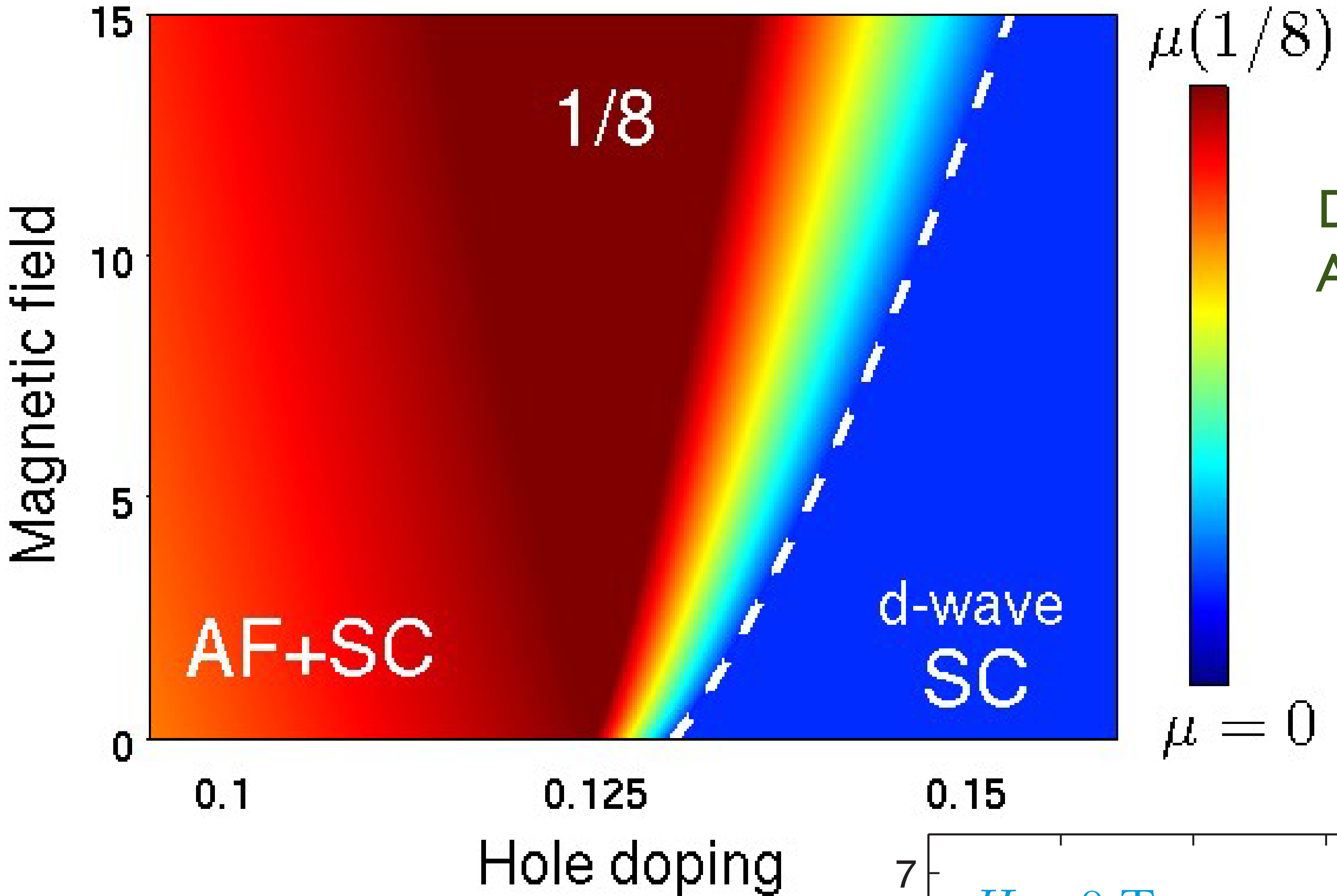
E. G. Moon and S. Sachdev, *Phys. Rev. B* **80**, 035117 (2009)

Physics of competition: *d*-wave SC and SDW "eat up" same pieces of the large Fermi surface.

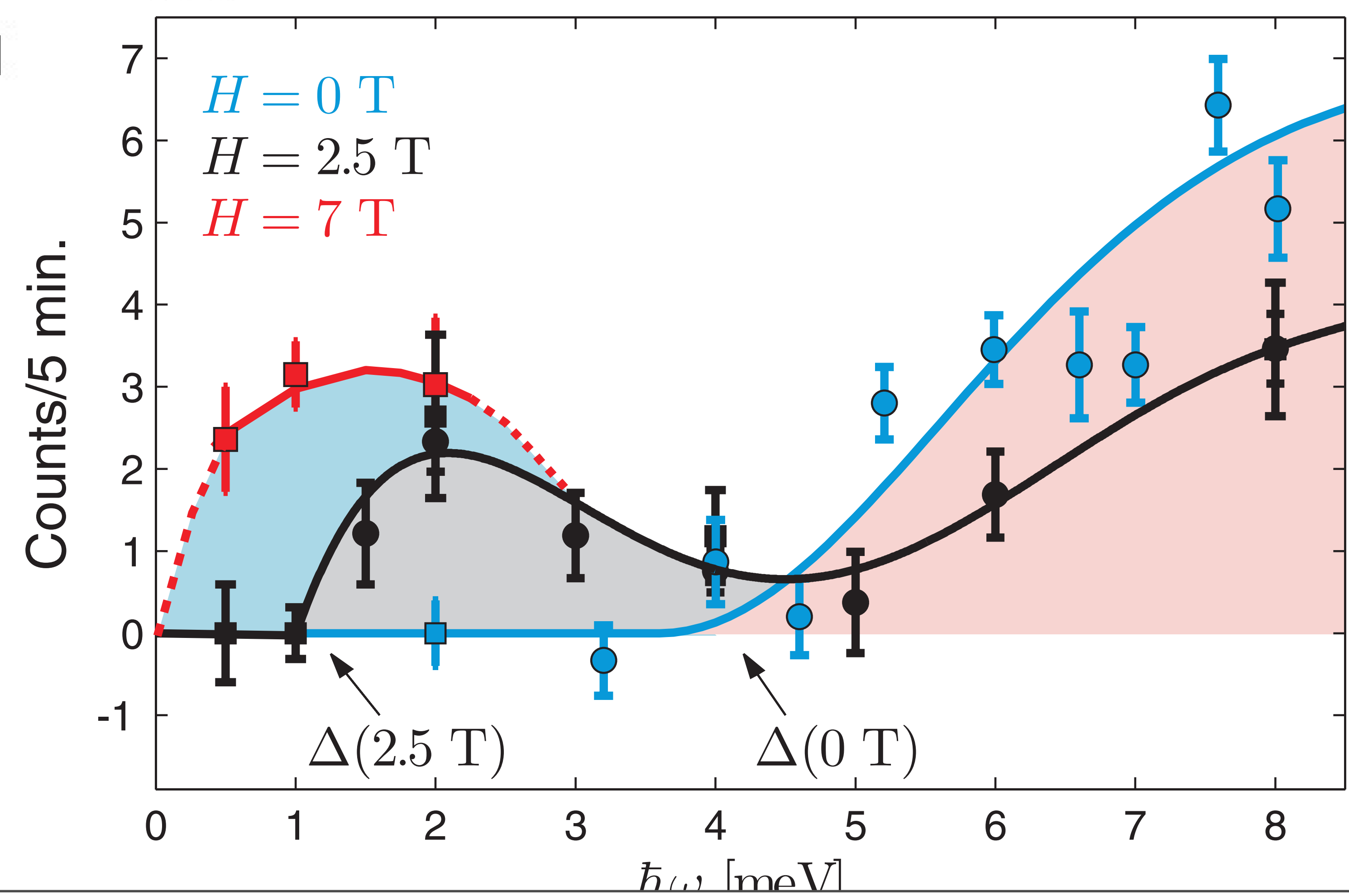

T
T^*
Fluctuating,
paired Fermi
pockets
Strange
Metal
Large
Fermi
surface
d-wave
SC
Magnetic
quantum
criticality
Thermally
fluctuating
SDW
Spin gap
x_m
x_s
x


T
T*
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
d-wave
SC
T_sdw
x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H


T
T*
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
d-wave
SC
T_sdw
x_m
x
x_s
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H
Quantum oscillations



J. Chang, Ch. Niedermayer, R. Gilardi, N.B. Christensen, H.M. Ronnow, D.F. McMorrow, M. Ay, J. Stahn, O. Sobolev, A. Hiess, S. Pailhes, C. Baines, N. Momono, M. Oda, M. Ido, and J. Mesot, *Physical Review B* **78**, 104525 (2008).



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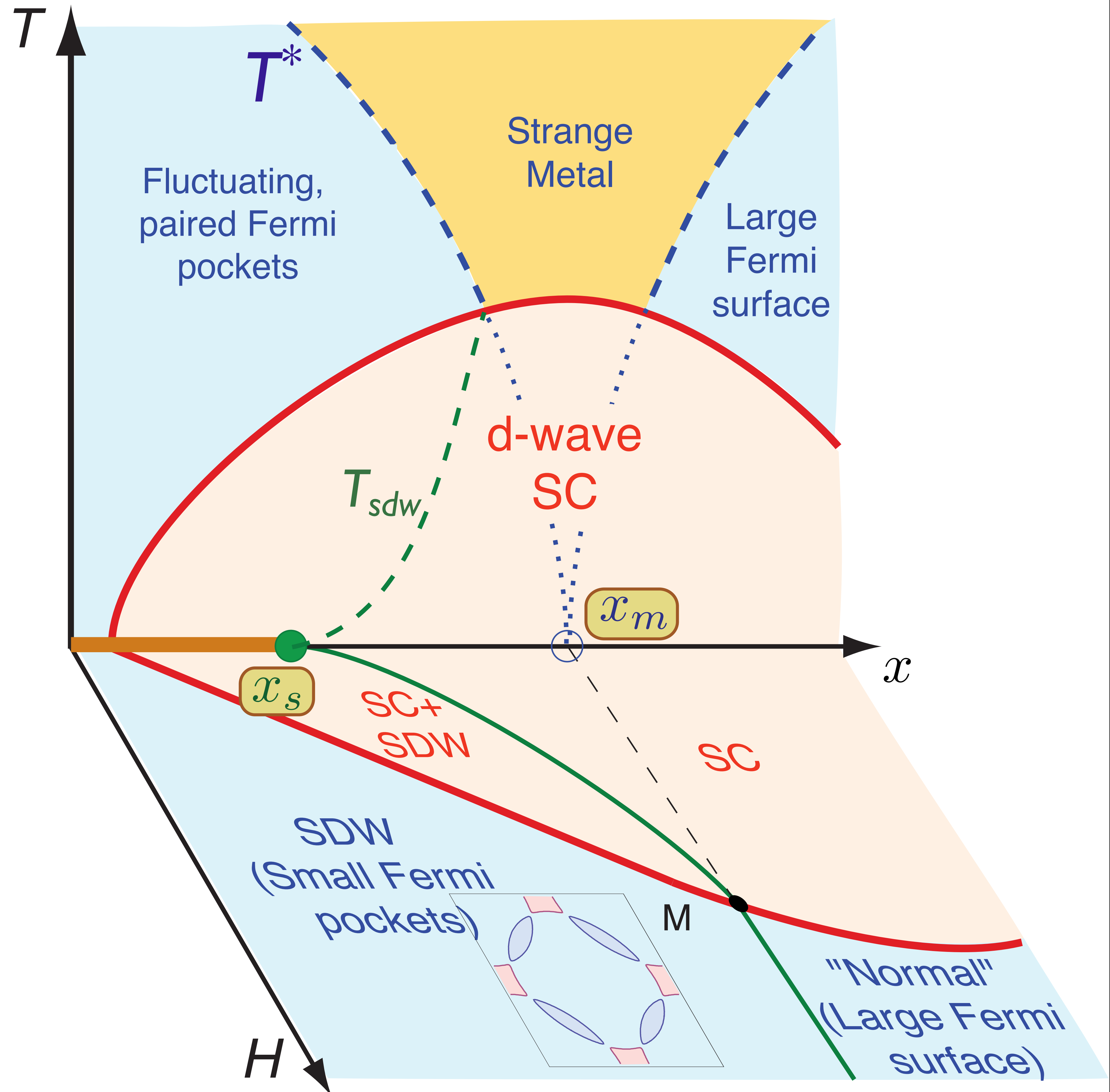
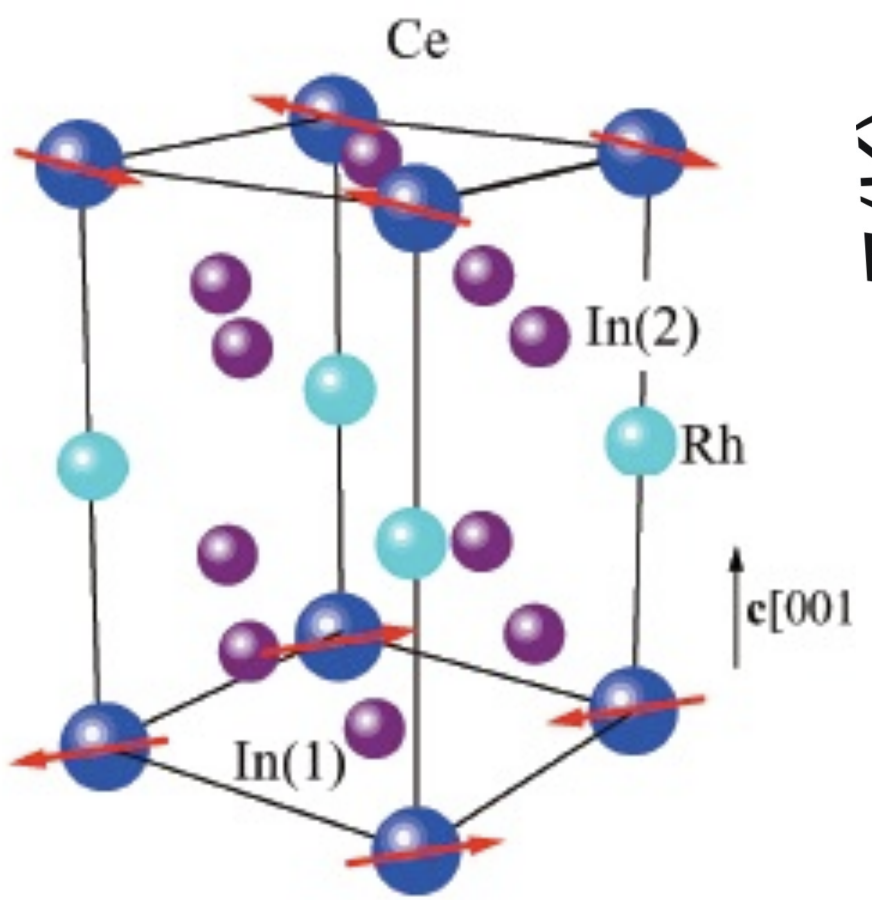
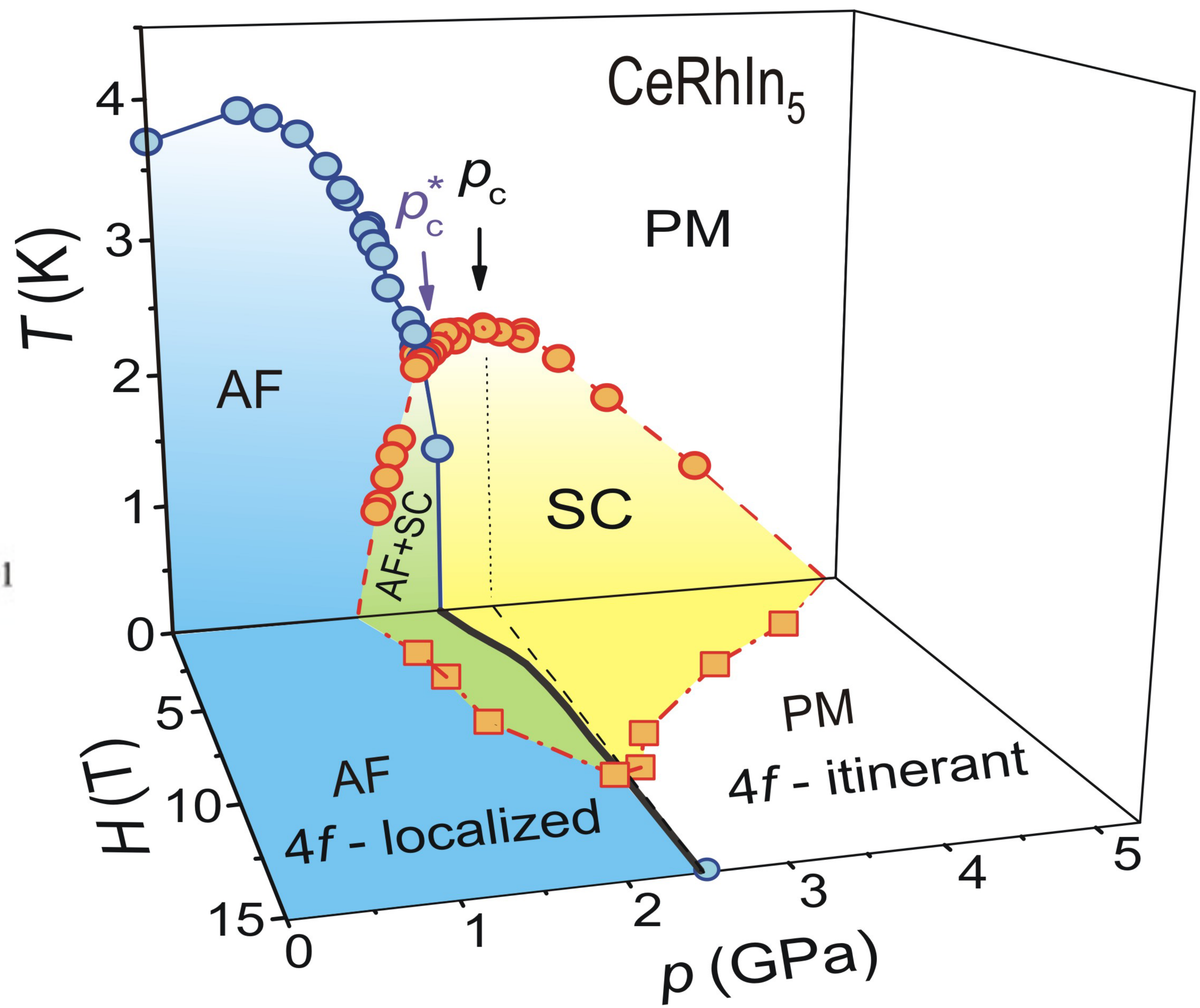


T
T^*
Strange
Metal
Fluctuating,
paired Fermi
pockets
Large
Fermi
surface
d-wave
SC
T_{sdw}
x_m
x_s
x
SC+
SDW
SC
SDW
(Small Fermi
pockets)
M
"Normal"
(Large Fermi
surface)
H

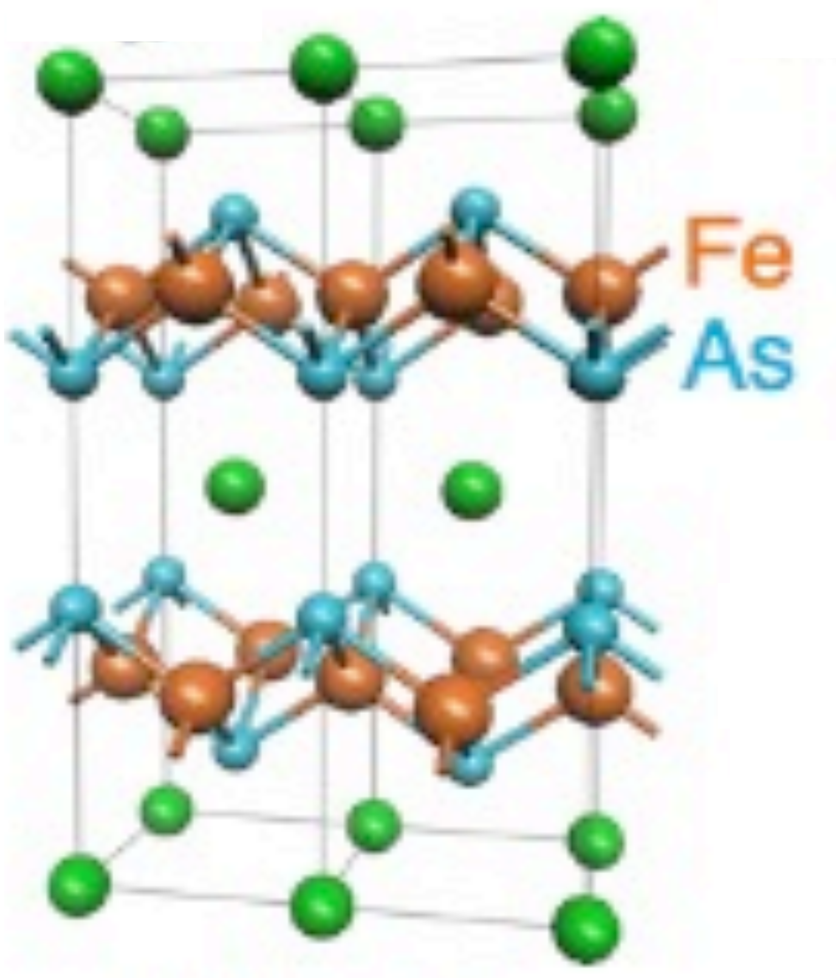
Similar phase diagram for $CeRhIn_5$



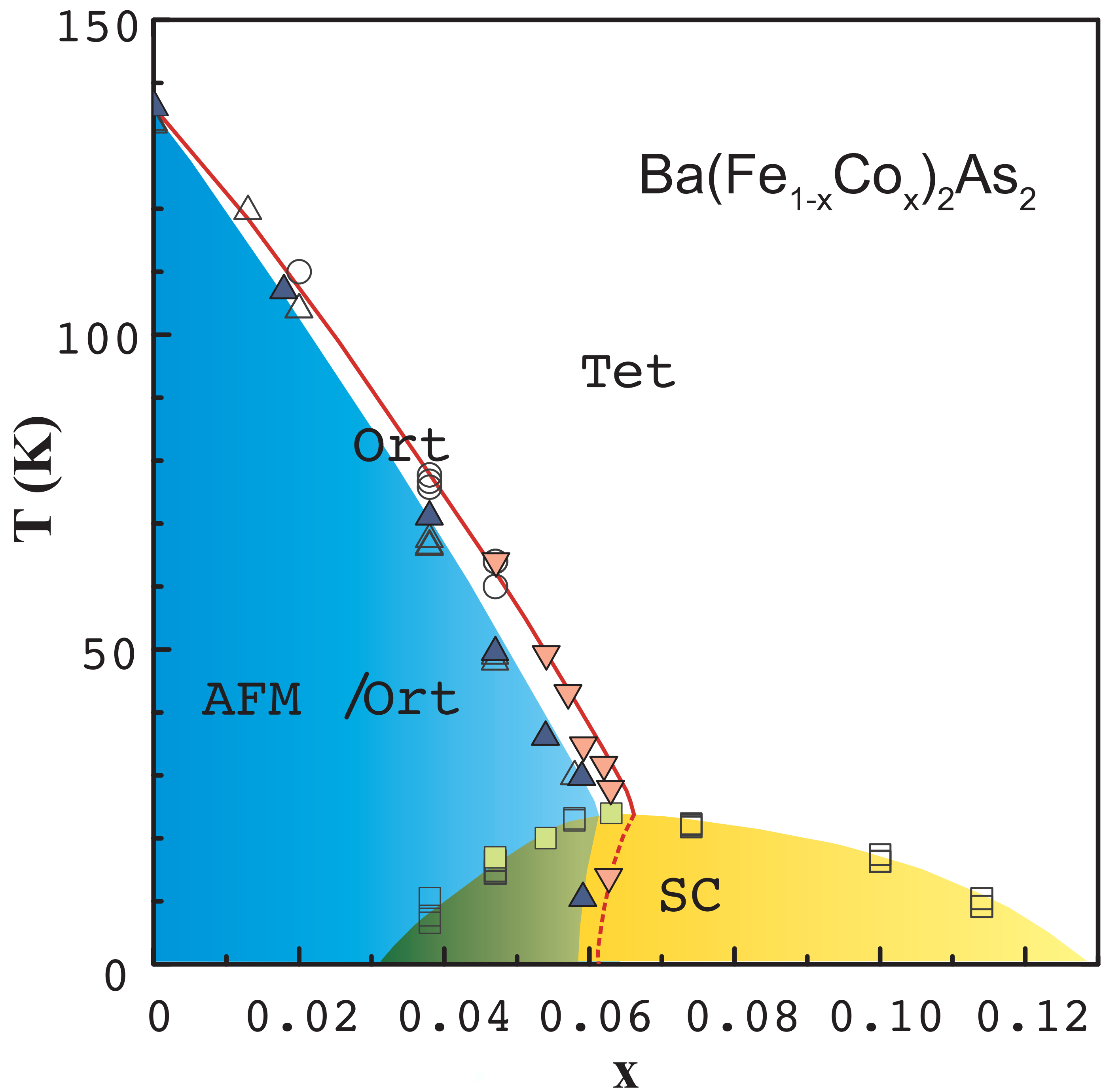


G. Knebel, D. Aoki, and J. Flouquet, arXiv:0911.5223

Similar phase diagram for the pnictides



BaFe$_2$As$_2$
Unit cell



S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, arXiv:0911.3136.

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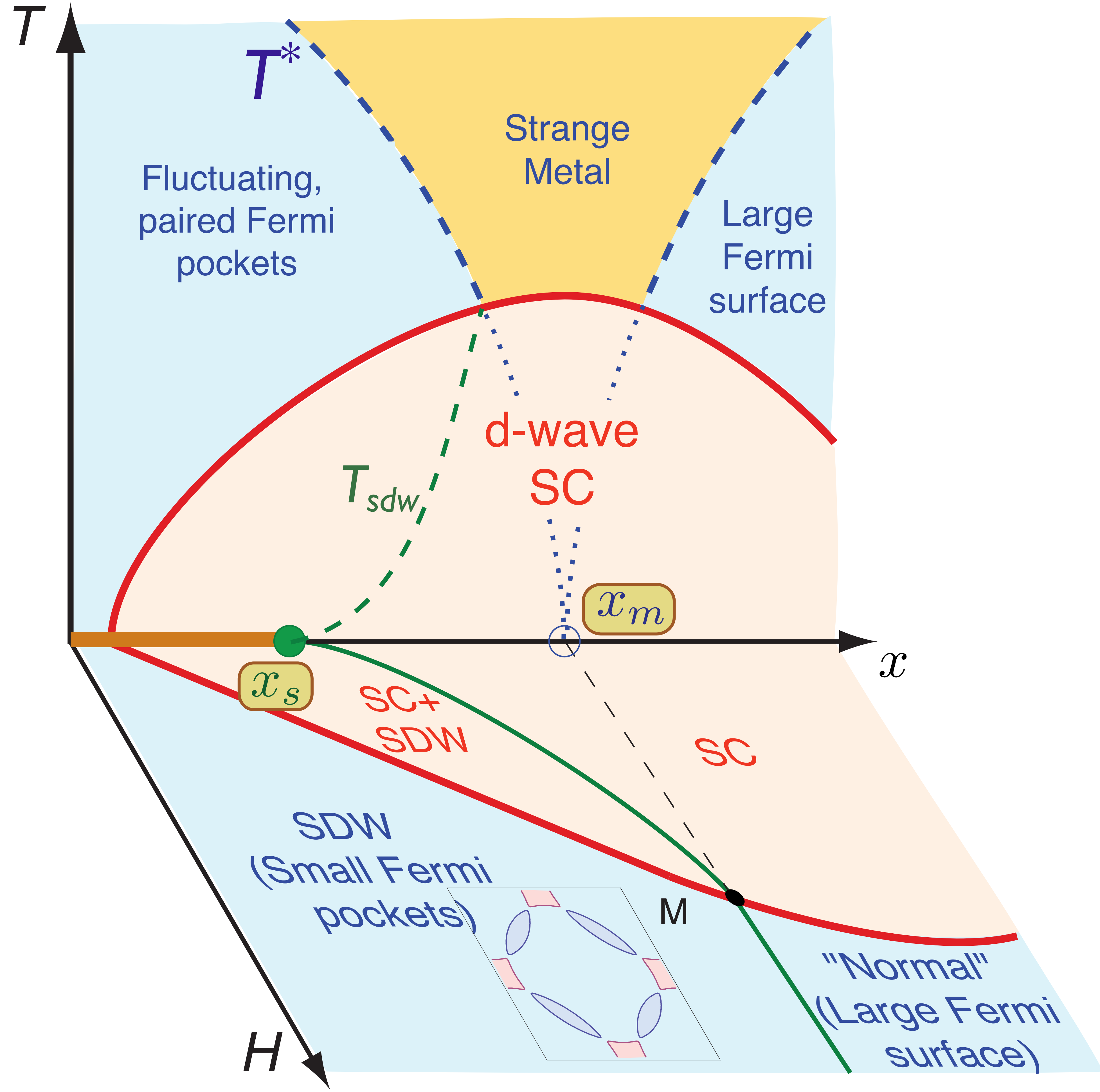
T
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Strange Metal
Fluctuating, paired Fermi pockets
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d-wave SC
T_sdw
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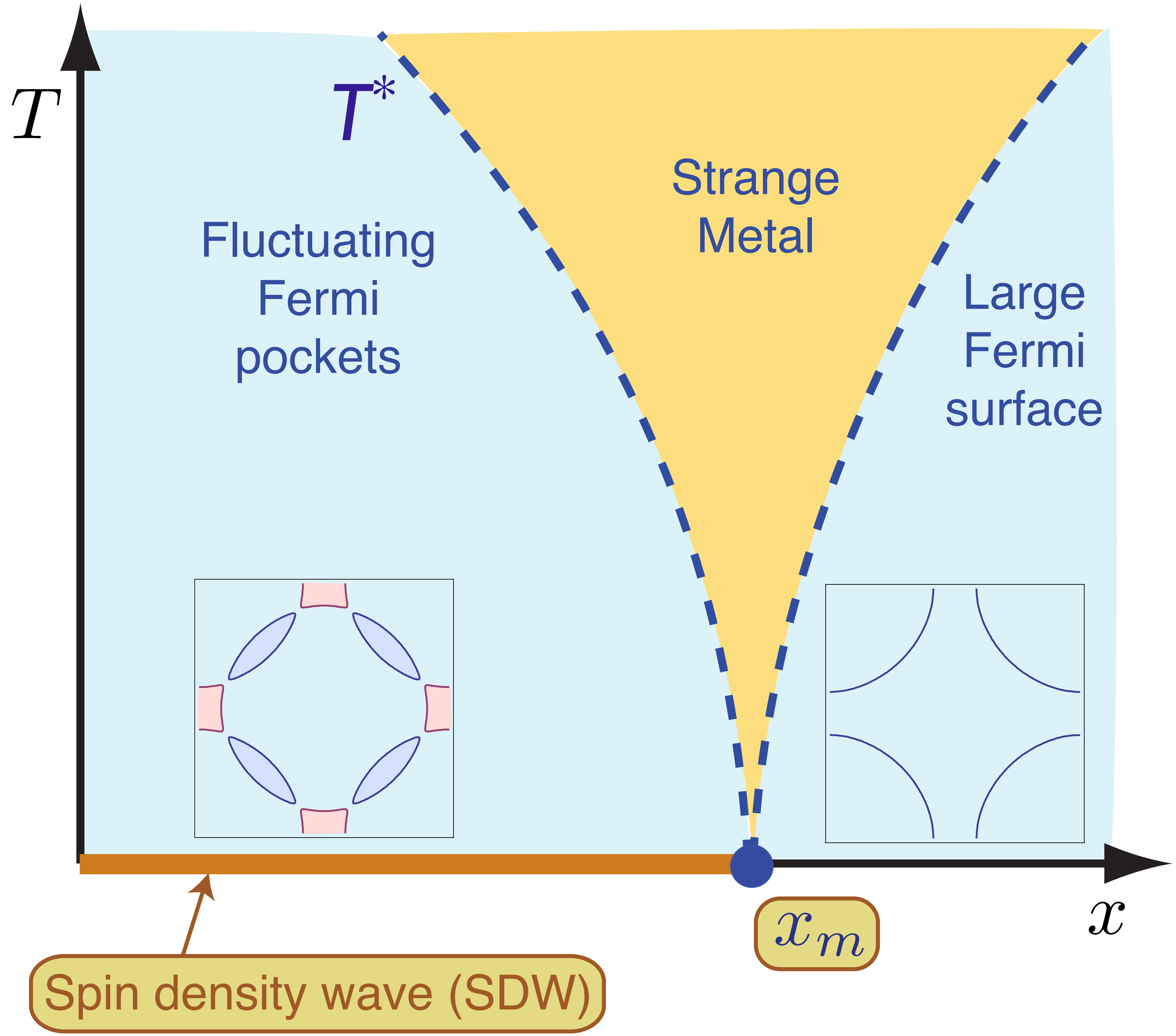
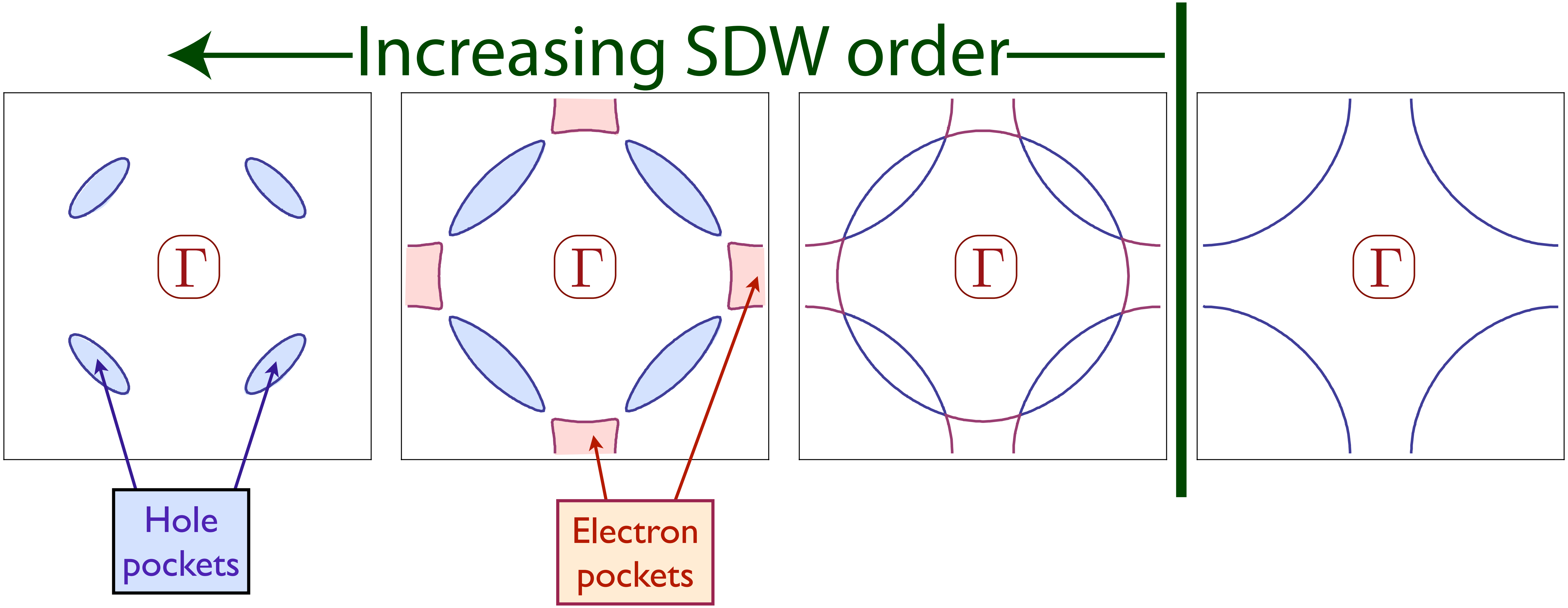
SC+ SDW
SC
SDW (Small Fermi pockets)
M
"Normal" (Large Fermi surface)
H

Theory of quantum criticality in the cuprates



Underlying SDW ordering quantum critical point in **metal** at $x = x_m$

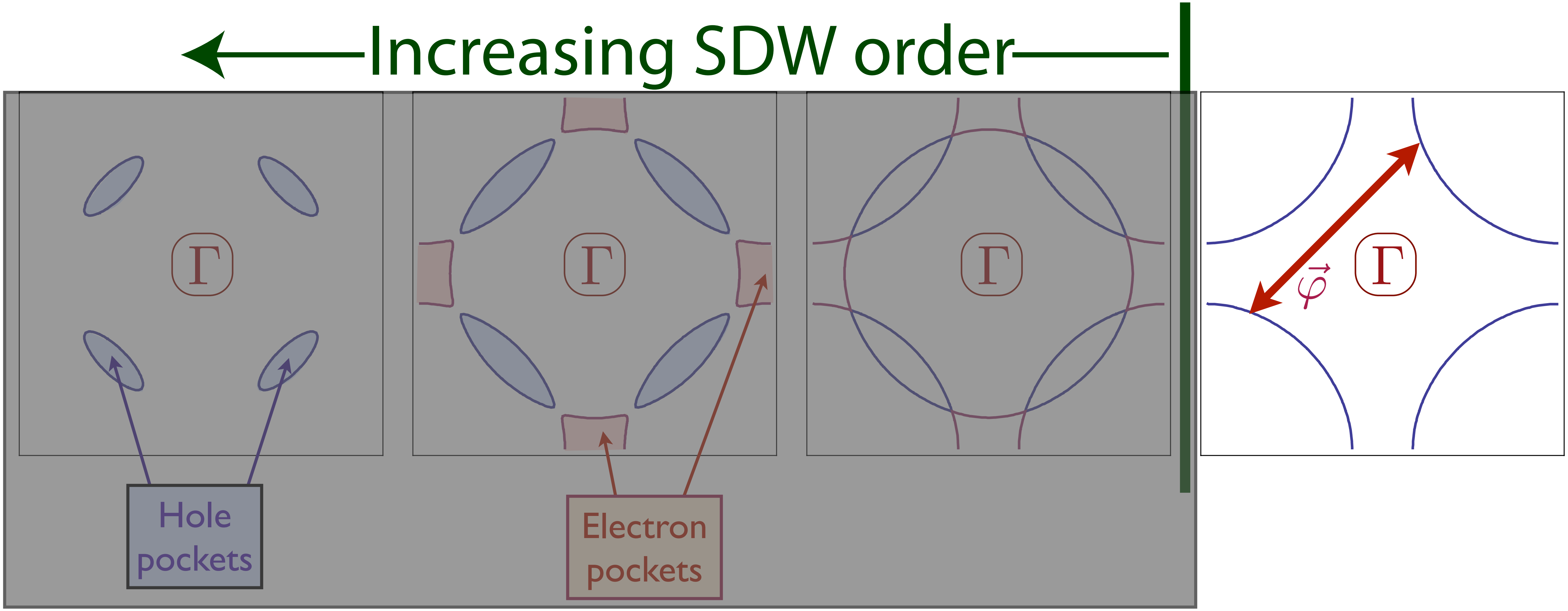
Hole-doped cuprates



Large Fermi surface breaks up into electron and hole pockets

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

Hole-doped cuprates



$\vec{\varphi}$ fluctuations act on the large Fermi surface

S. Sachdev, A.V. Chubukov, and A. Sokol, *Phys. Rev. B* **51**, 14874 (1995).
A.V. Chubukov and D. K. Morr, *Physics Reports* **288**, 355 (1997).

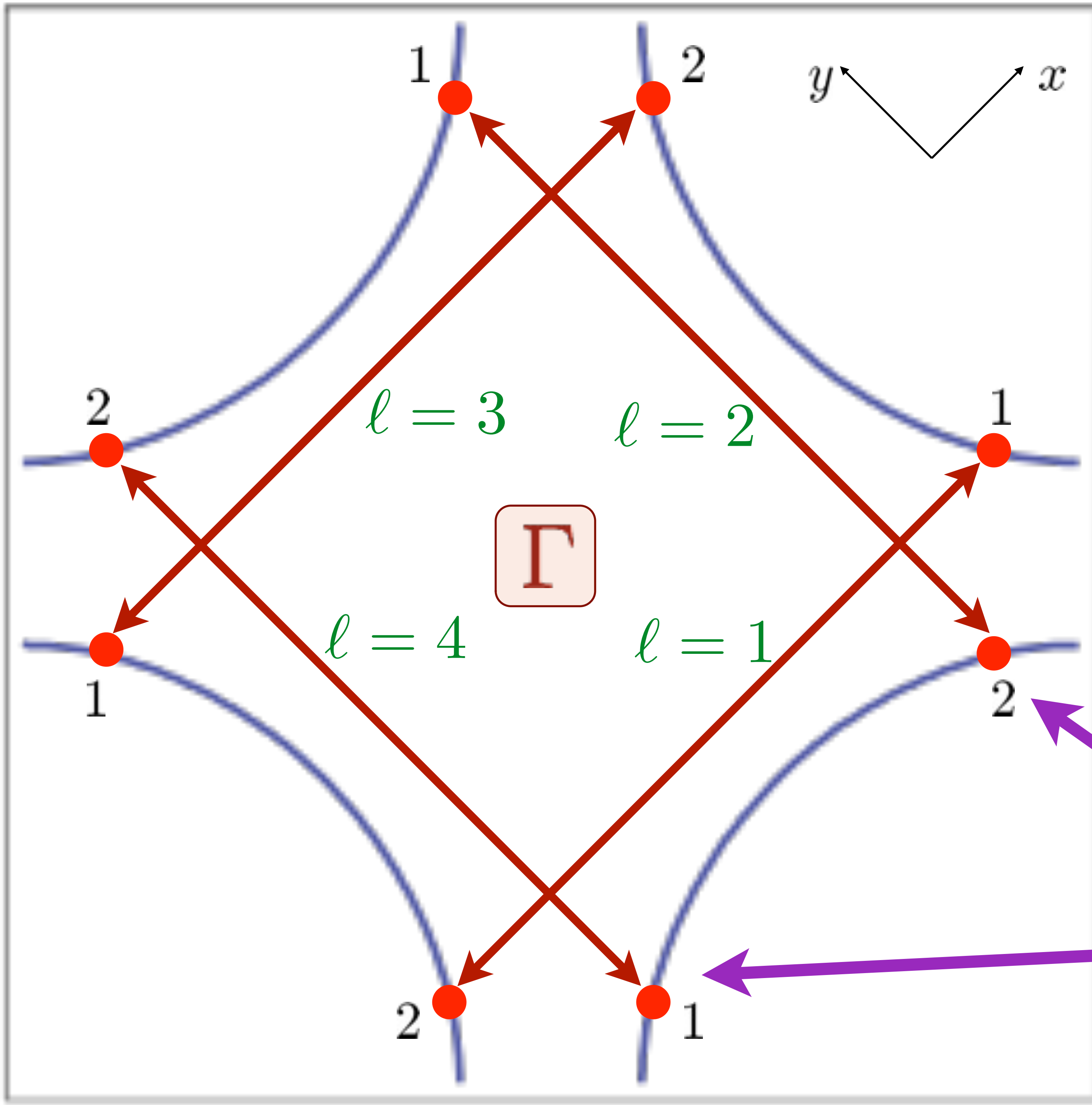
Start from the "spin-fermion" model

$$\mathcal{Z} = \int \mathcal{D}c_\alpha \mathcal{D}\vec{\varphi} \exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \int d\tau \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}\alpha} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

$$- \lambda \int d\tau \sum_i c^\dagger_{i\alpha} \vec{\varphi}_i \cdot \vec{\sigma}_{\alpha\beta} c_{i\beta} e^{i\mathbf{K}\cdot\mathbf{r}_i}$$

$$+ \int d\tau d^2 r \left[\frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4 \right]$$

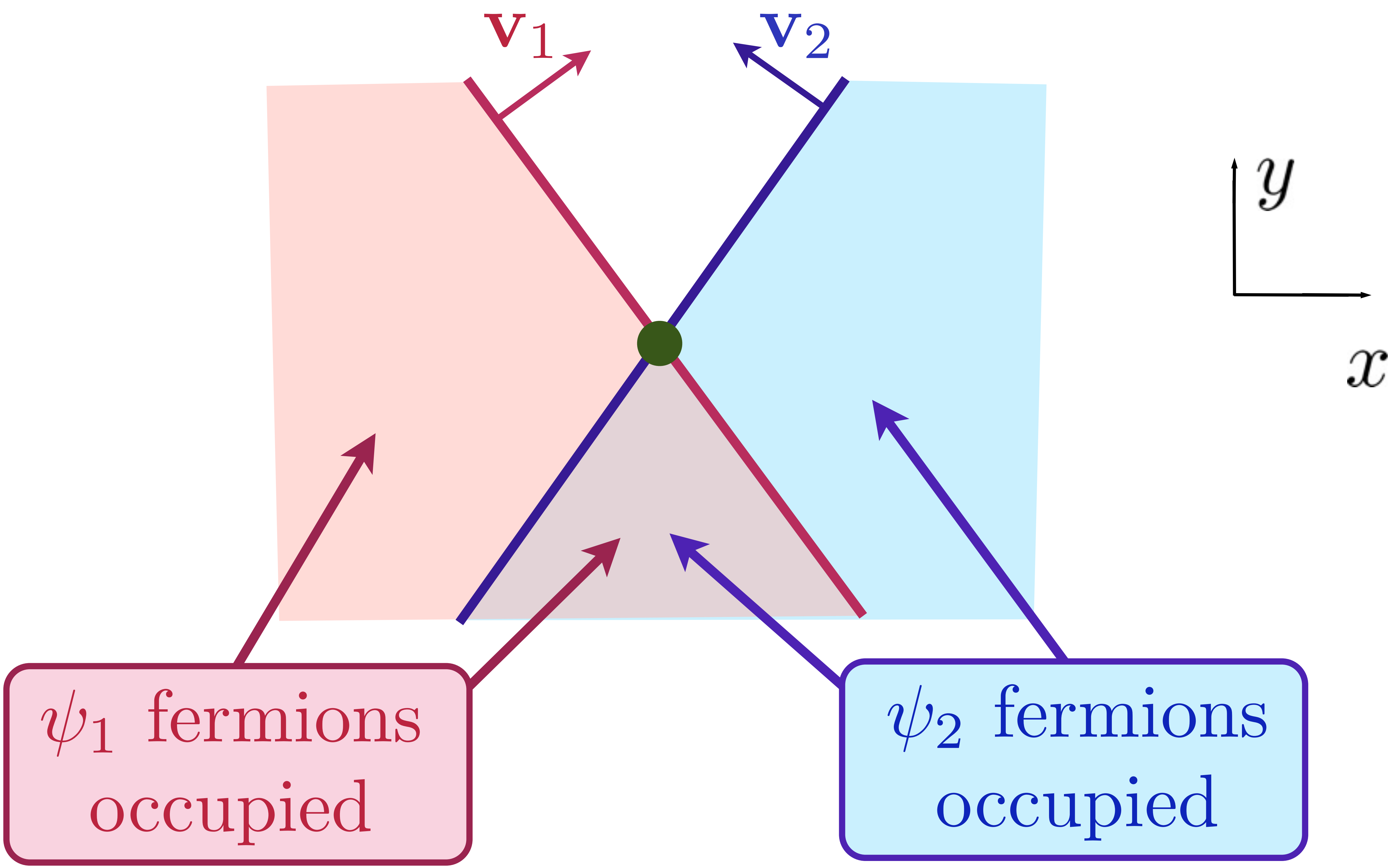


Low energy fermions
$\psi^{\ell}_{1\alpha}$, $\psi^{\ell}_{2\alpha}$
$\ell = 1, \ldots, 4$

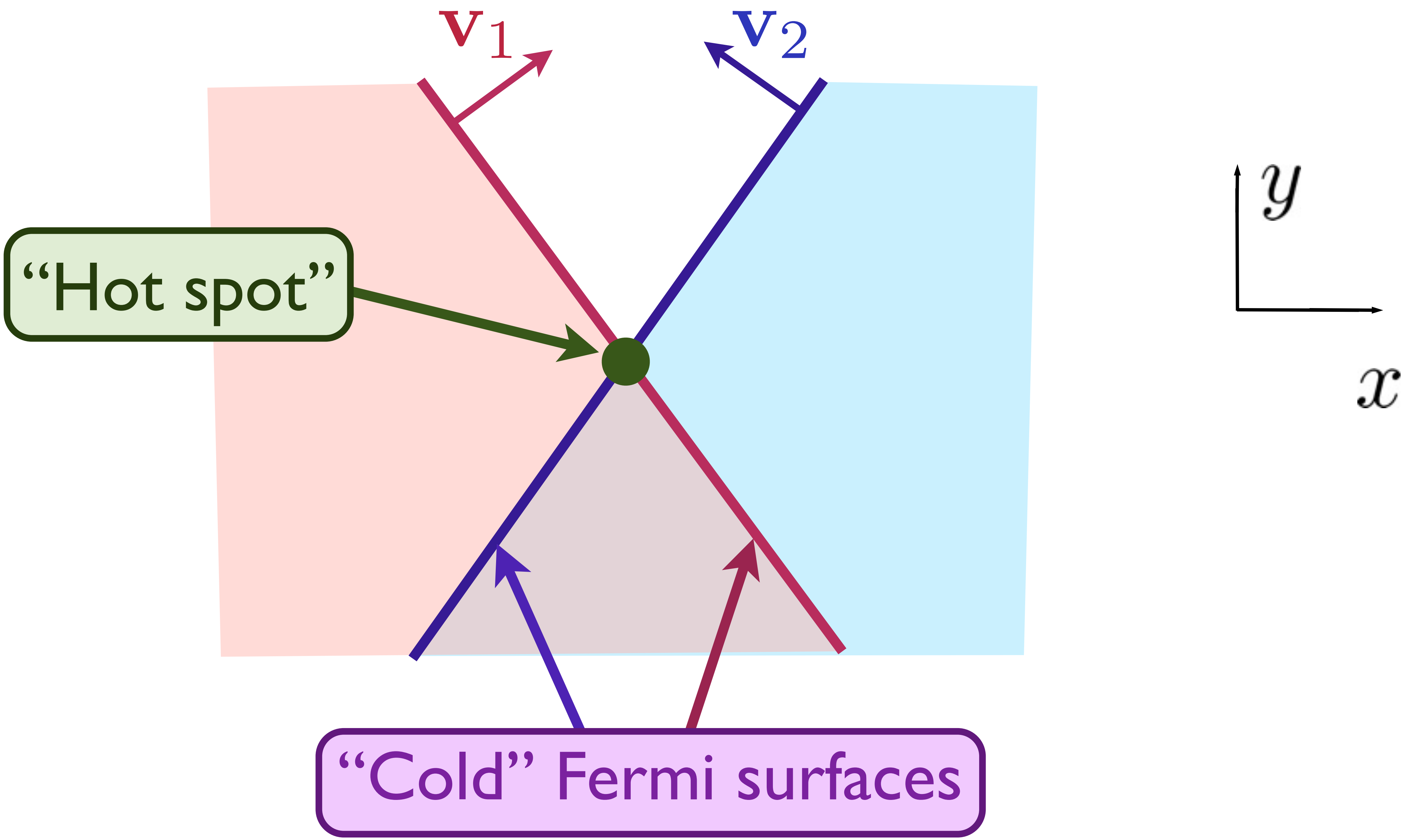
$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_1 \cdot \boldsymbol{\nabla}_r\right)\psi^{\ell}_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_2 \cdot \boldsymbol{\nabla}_r\right)\psi^{\ell}_{2\alpha}$$

$$\mathbf{v}^{\ell=1}_1 = (v_x, v_y),\ \mathbf{v}^{\ell=1}_2 = (-v_x, v_y)$$

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$



$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_1\cdot\boldsymbol{\nabla}_r\right)\psi^{\ell}_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^{\ell}_2\cdot\boldsymbol{\nabla}_r\right)\psi^{\ell}_{2\alpha}$$



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Order parameter: $\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$

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"Yukawa" coupling: $$\mathcal{L}_c = -\lambda\vec{\varphi}\cdot\left(\psi_{1\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{2\beta}^\ell + \psi_{2\alpha}^{\ell\dagger}\vec{\sigma}_{\alpha\beta}\psi_{1\beta}^\ell\right)$$

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Hertz theory

Integrate out fermions and obtain non-local corrections to $\mathcal{L}_\varphi$

$$\mathcal{L}_\varphi = \frac{1}{2}\vec{\varphi}^2\left[\mathbf{q}^2 + \gamma|\omega|\right]/2 \quad ; \quad \gamma = \frac{2}{\pi v_x v_y}$$

Exponent $z = 2$ and mean-field criticality (upto logarithms)

$$\mathcal{L}_f = \psi^{\ell\dagger}_{1\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_1\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{1\alpha} + \psi^{\ell\dagger}_{2\alpha}\left(\zeta\partial_\tau - i\mathbf{v}^\ell_2\cdot\boldsymbol{\nabla}_r\right)\psi^\ell_{2\alpha}$$

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Exponent $z = 2$ and mean-field criticality (upto logarithms)

OK in $d = 3$, but higher order terms contain an infinite number of marginal couplings in $d = 2$

Ar. Abanov and A.V. Chubukov, *Phys. Rev. Lett.* **93**, 255702 (2004).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Perform RG on both fermions and $\vec{\varphi}$, using a *local* field theory.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

Order parameter: $$\mathcal{L}_\varphi = \frac{1}{2}\left(\boldsymbol{\nabla}_r\vec{\varphi}\right)^2 + \frac{\widetilde{\zeta}}{2}\left(\partial_\tau\vec{\varphi}\right)^2 + \frac{s}{2}\vec{\varphi}^2 + \frac{u}{4}\vec{\varphi}^4$$

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Under the rescaling $x' = xe^{-\ell}$, $\tau' = \tau e^{-z\ell}$, the spatial gradients are fixed if the fields transform as

$$\vec{\varphi}' = e^{(d+z-2)\ell/2}\vec{\varphi} \quad ; \text{``} \quad \psi' = e^{(d+z-1)\ell/2}\psi.$$

Then the Yukawa coupling transforms as

$$\lambda' = e^{(4-d-z)\ell/2}\lambda$$

For $d = 2$, with $z = 2$ the Yukawa coupling is invariant, and the bare time-derivative terms ζ, $\widetilde{\zeta}$ are irrelevant.

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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With $z = 2$ scaling, ζ is irrelevant.
So we take $\zeta \to 0$
(watch for dangerous irrelevancy).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Set $\vec{\varphi}$ wavefunction renormalization by keeping co-efficient of $(\boldsymbol{\nabla}_r\vec{\varphi})^2$ fixed (as usual).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_1^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger}\left(\zeta\partial_\tau - i\mathbf{v}_2^\ell\cdot\boldsymbol{\nabla}_r\right)\psi_{2\alpha}^\ell$$

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Set fermion wavefunction renormalization by keeping Yukawa coupling fixed.

Y. Huh and S. Sachdev, *Phys. Rev. B* **78**, 064512 (2008).

$$\mathcal{L}_f = \psi_{1\alpha}^{\ell\dagger} \left(\zeta \partial_\tau - i \mathbf{v}_1^\ell \cdot \boldsymbol{\nabla}_r \right) \psi_{1\alpha}^\ell + \psi_{2\alpha}^{\ell\dagger} \left(\zeta \partial_\tau - i \mathbf{v}_2^\ell \cdot \boldsymbol{\nabla}_r \right) \psi_{2\alpha}^\ell$$

Order parameter: $\mathcal{L}_\varphi = \frac{1}{2} \left(\boldsymbol{\nabla}_r \vec{\varphi} \right)^2 + \frac{\widetilde{\zeta}}{2} \left(\partial_\tau \vec{\varphi} \right)^2 + \frac{s}{2} \vec{\varphi}^2 + \frac{u}{4} \vec{\varphi}^4$

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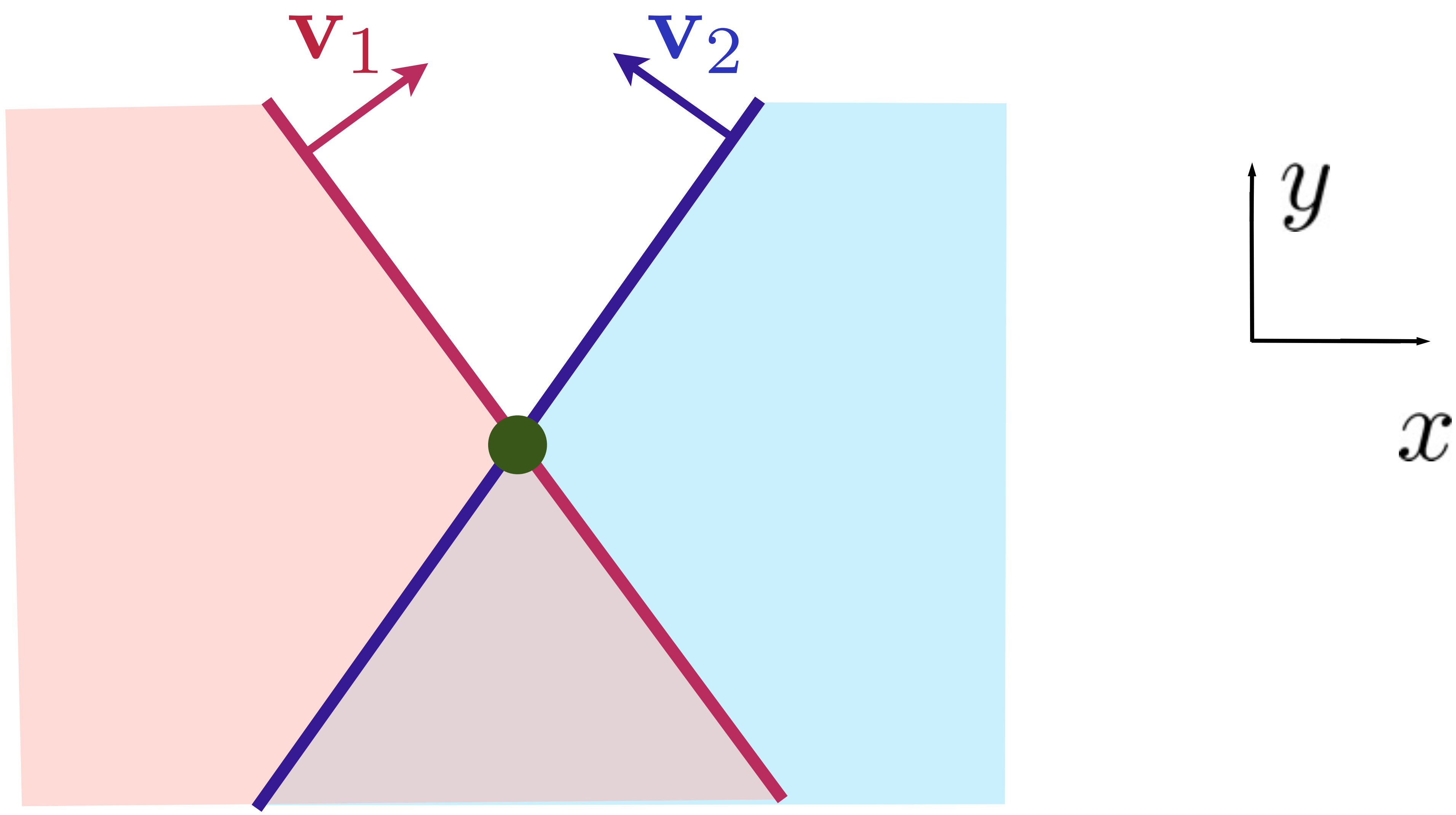
We find consistent two-loop RG factors, as $\zeta \to 0$, for the velocities v_x, v_y, and the wavefunction renormalizations.

Consistency check: the expression for the boson damping constant, $\gamma = \dfrac{2}{\pi v_x v_y}$, is preserved under RG.

RG-improved Migdal-Eliashberg theory

$\alpha = v_y/v_x \to 0$ logarithmically in the infrared.

Dynamical Nesting



Bare Fermi surface

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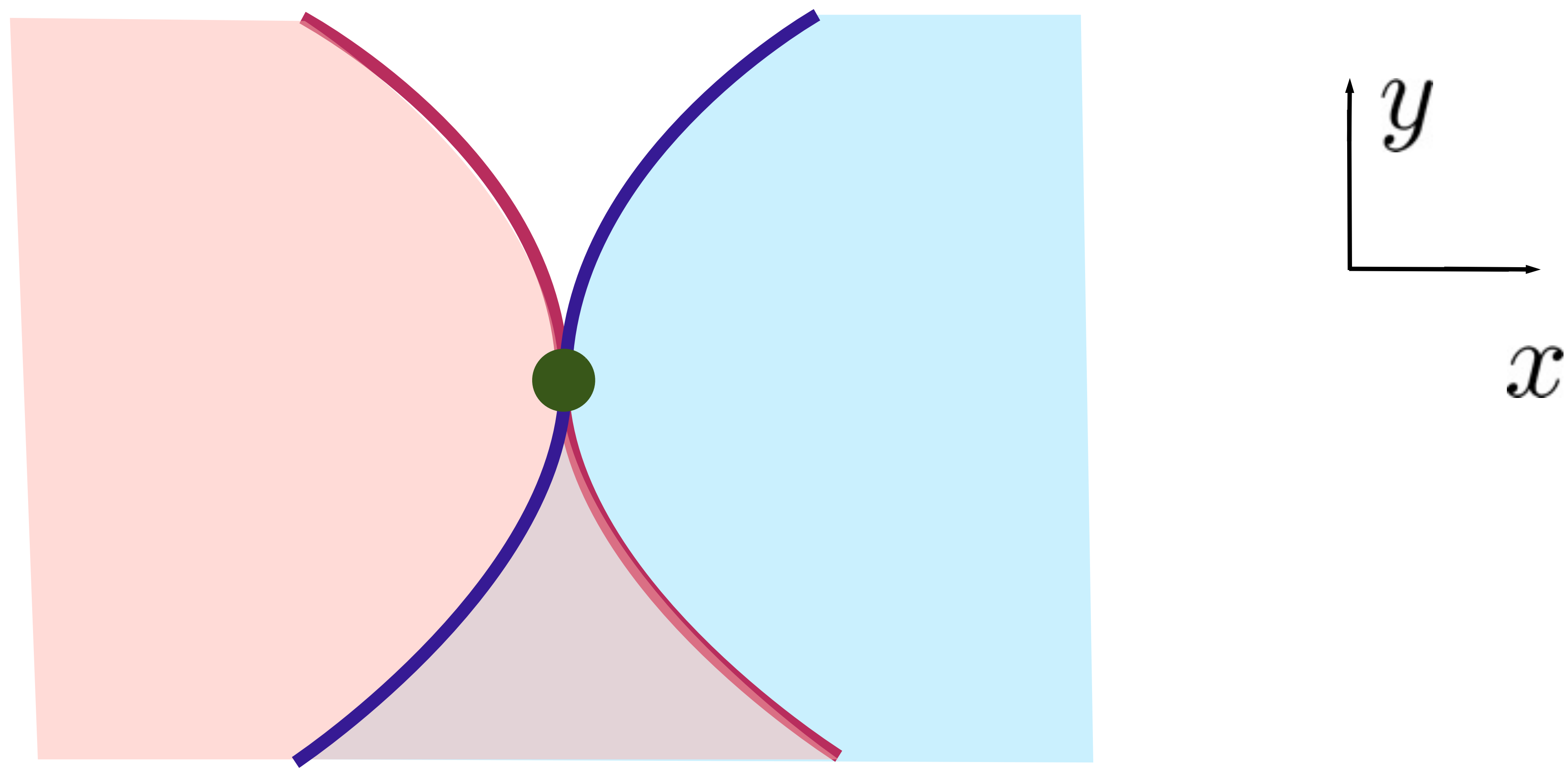

Dressed Fermi surface

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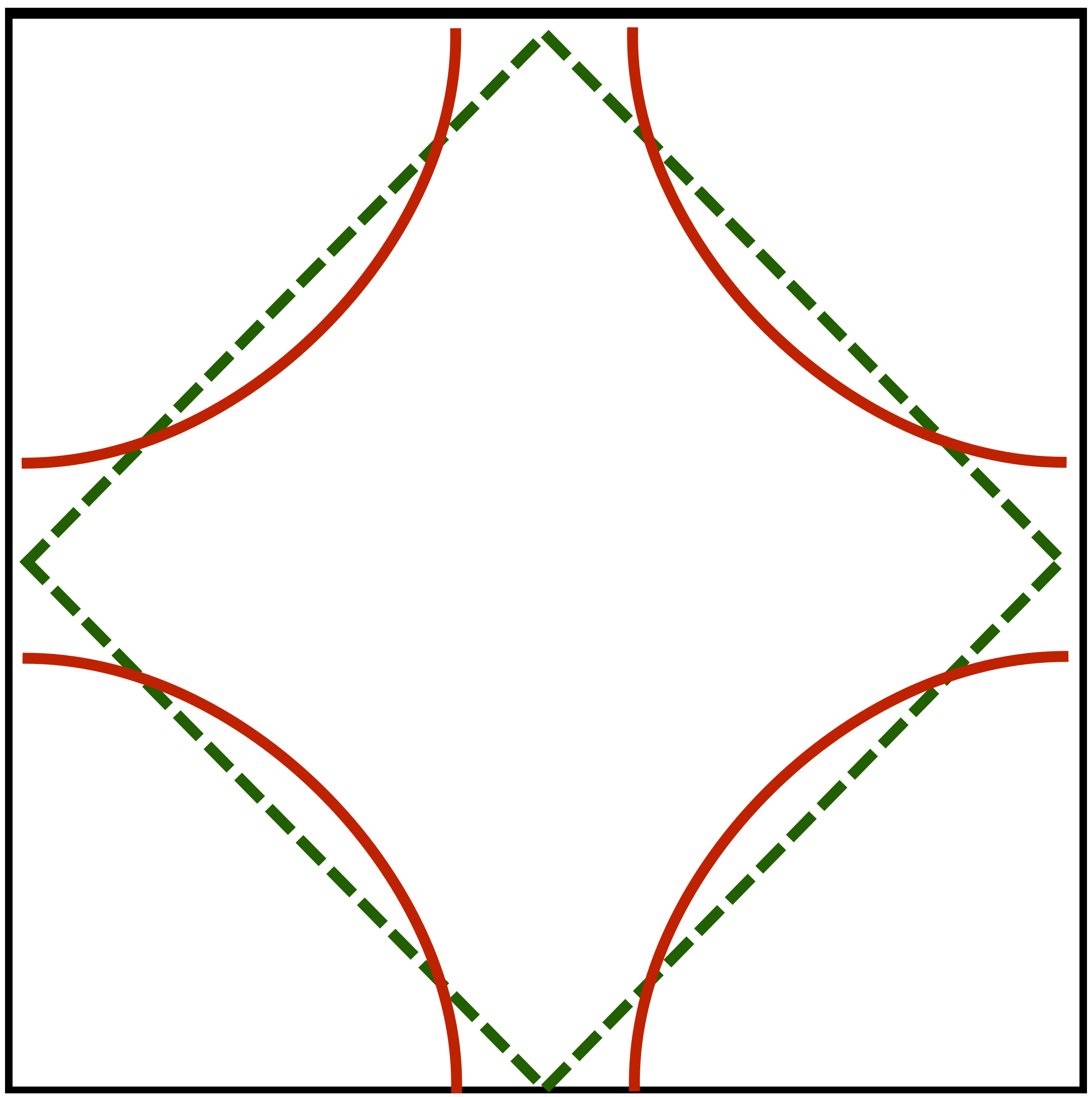
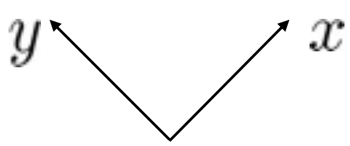
<u>Dynamical Nesting</u>





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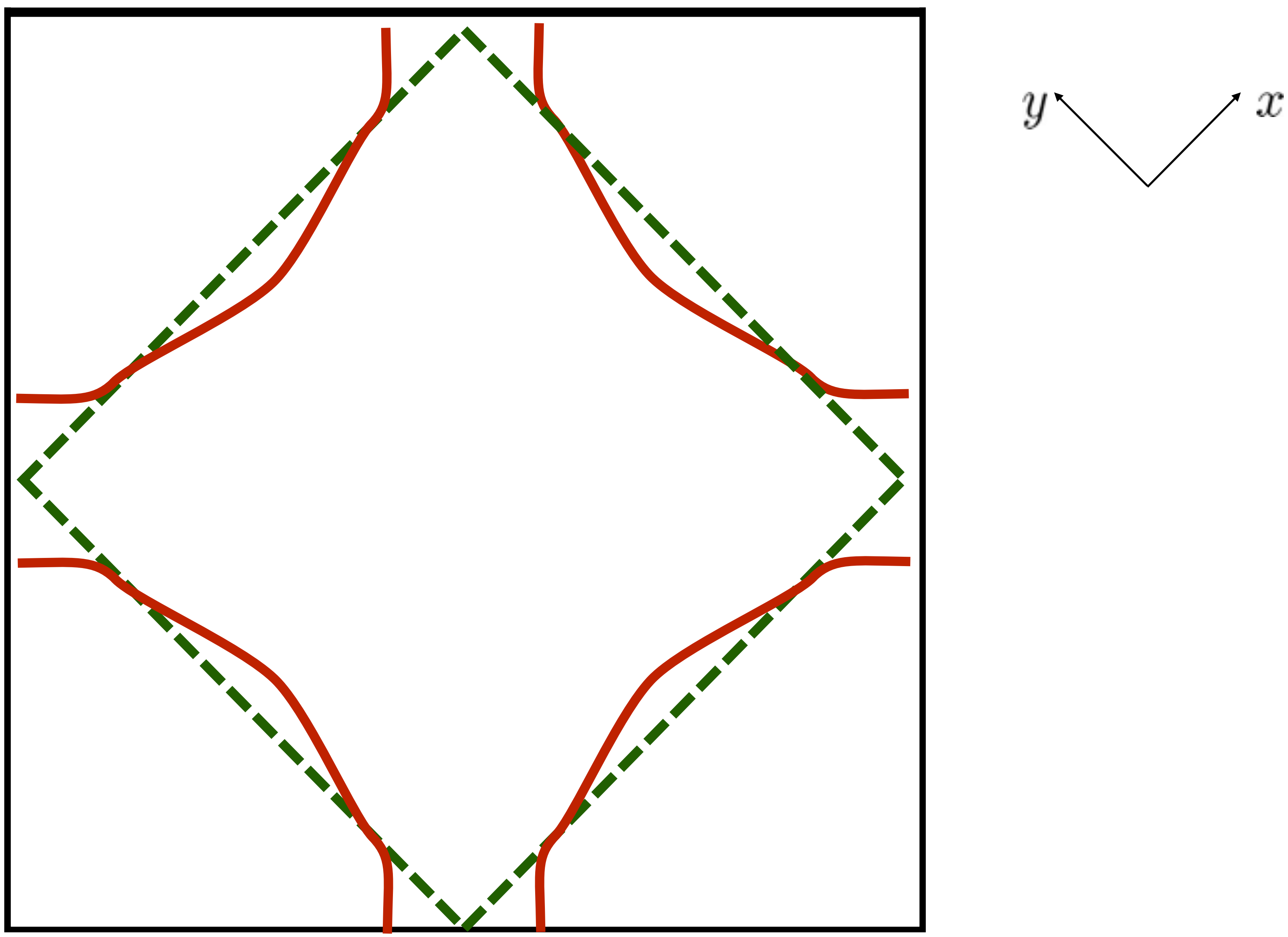

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In $\vec{\varphi}$ SDW fluctuations, characteristic q and ω scale as

$$q \sim \omega^{1/2} \exp\left(-\frac{3}{64\pi^2}\left(\frac{\ln(1/\omega)}{N}\right)^3\right).$$

However, $1/N$ expansion cannot be trusted in the asymptotic regime.

Outline

1. Phase diagram of the cuprates

Quantum criticality of the competition between antiferromagnetism and superconductivity

2. Theory of spin density wave ordering in a metal

Strong-coupling in d=2

3. Instabilities near SDW critical point

d-wave pairing and bond density wave

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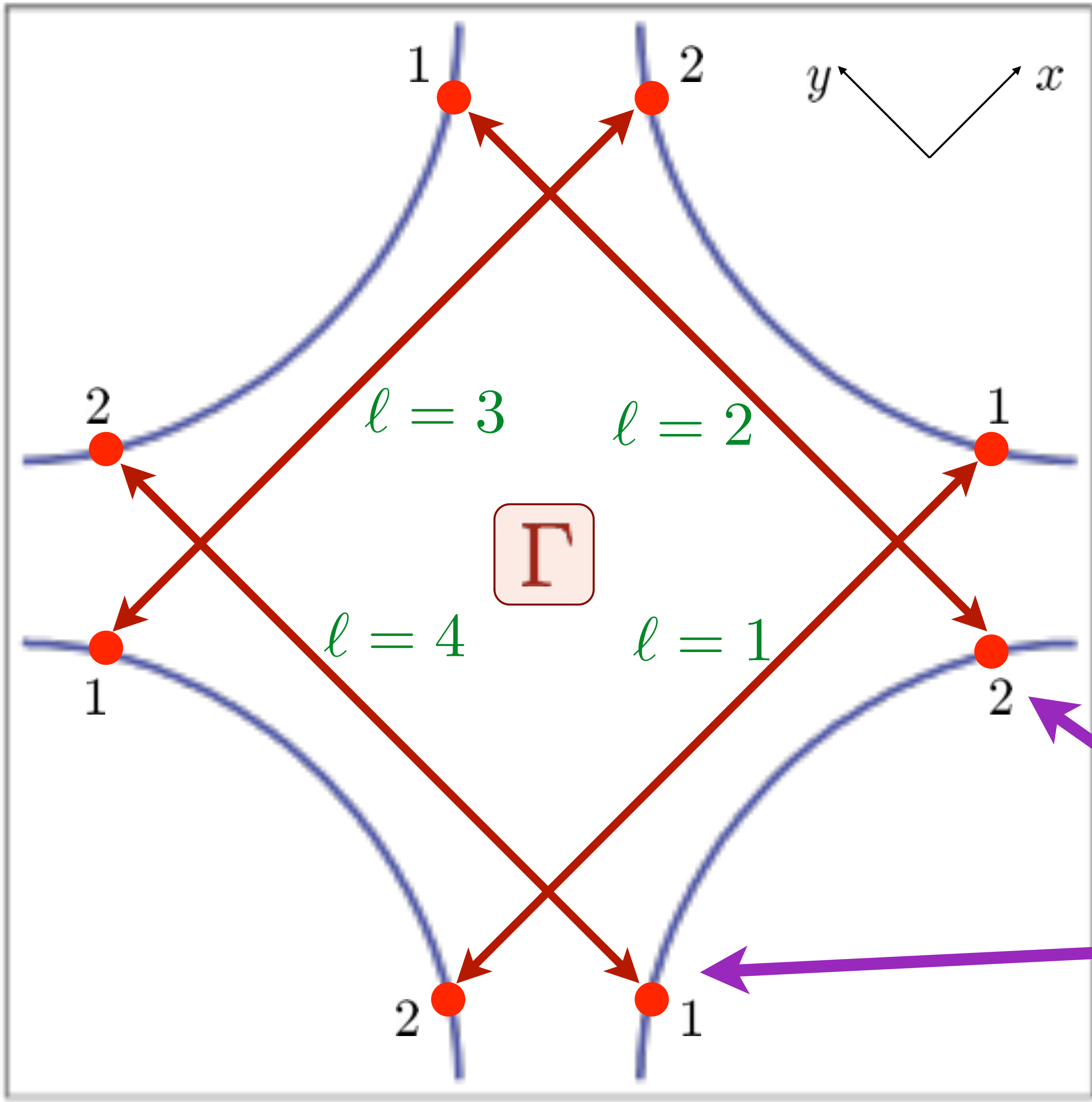
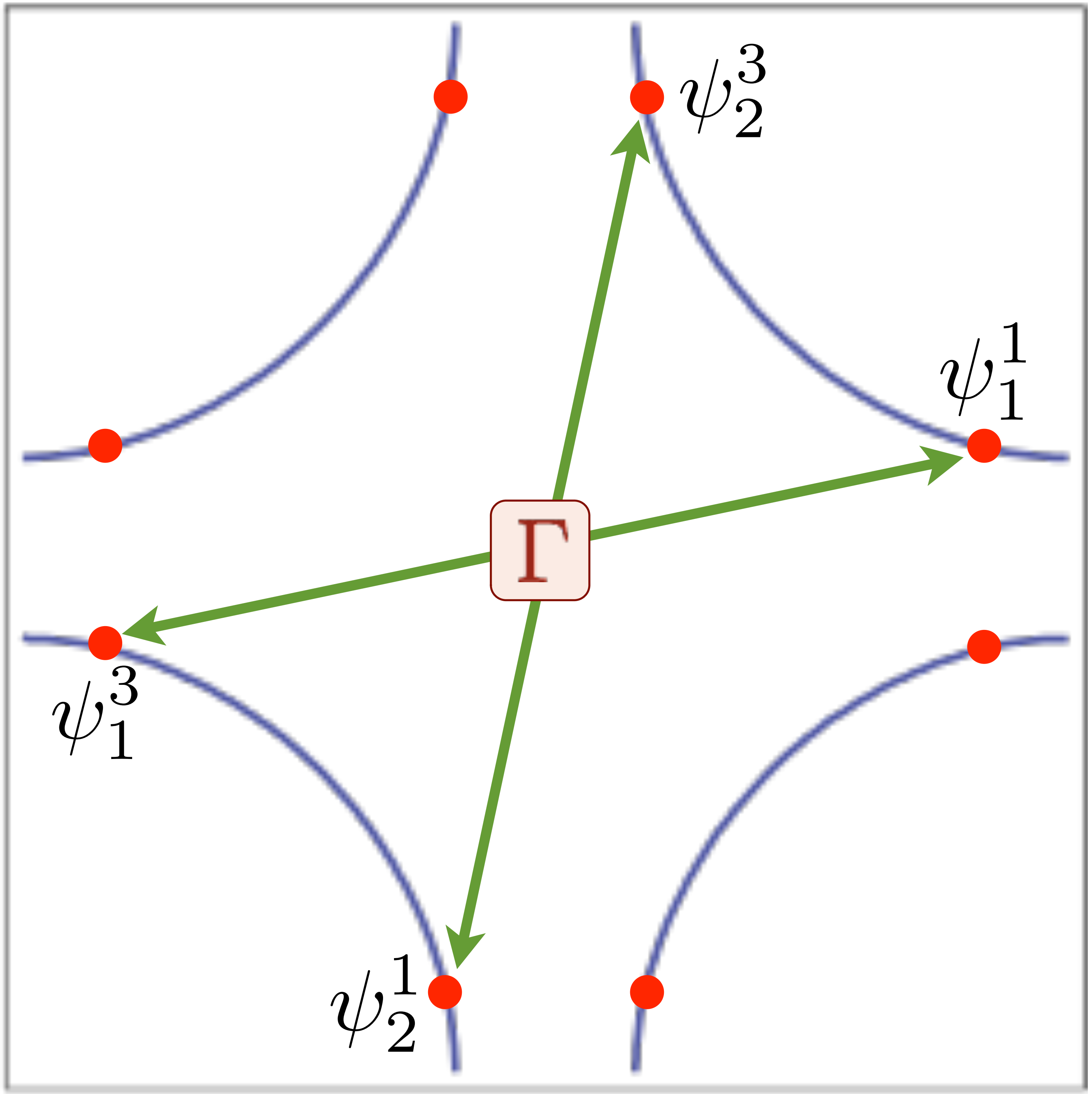


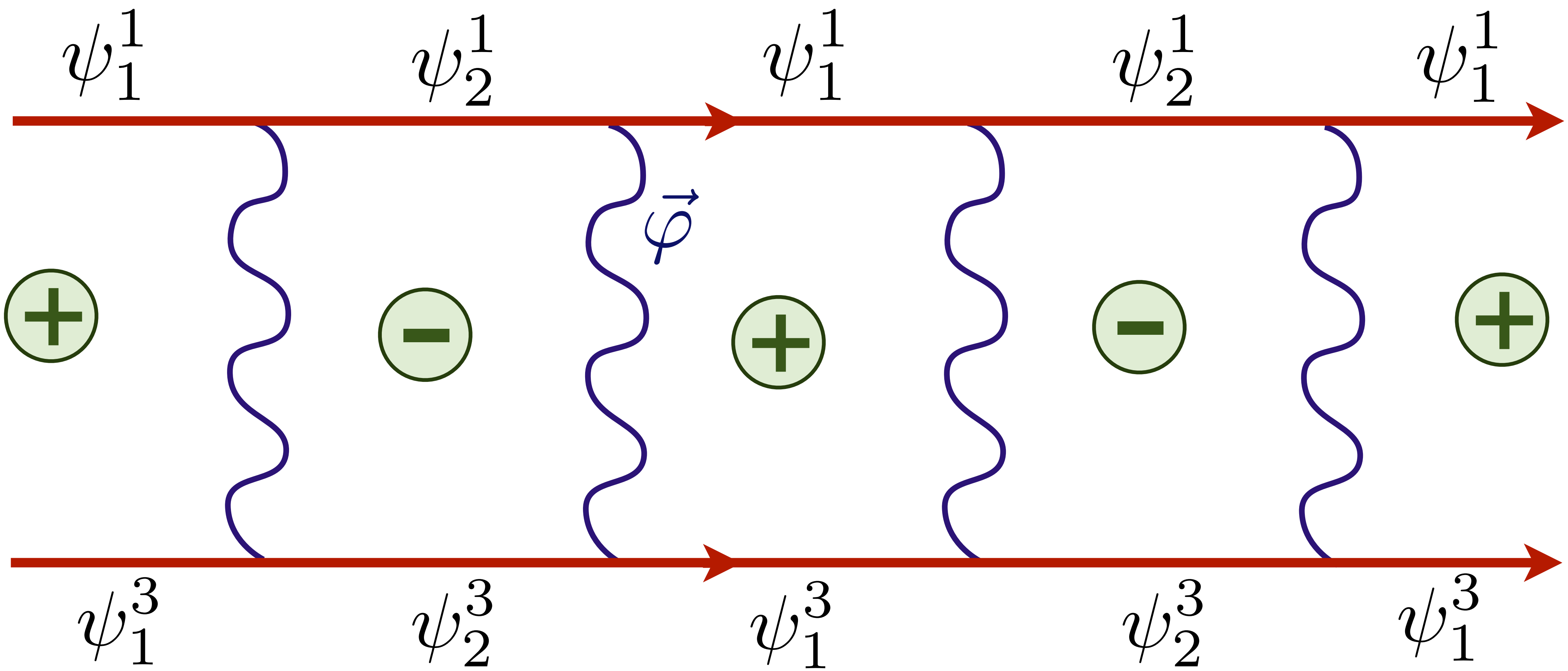
Low energy fermions
$\psi^{\ell}_{1\alpha}$, $\psi^{\ell}_{2\alpha}$
$\ell = 1, \ldots, 4$



Hot spots have strong instability to *d*-wave pairing near SDW critical point. This instability is stronger than the BCS instability of a Fermi liquid.

Pairing order parameter:

$$\varepsilon^{\alpha\beta}\left(\psi^3_{1\alpha}\psi^1_{1\beta} - \psi^3_{2\alpha}\psi^1_{2\beta}\right)$$



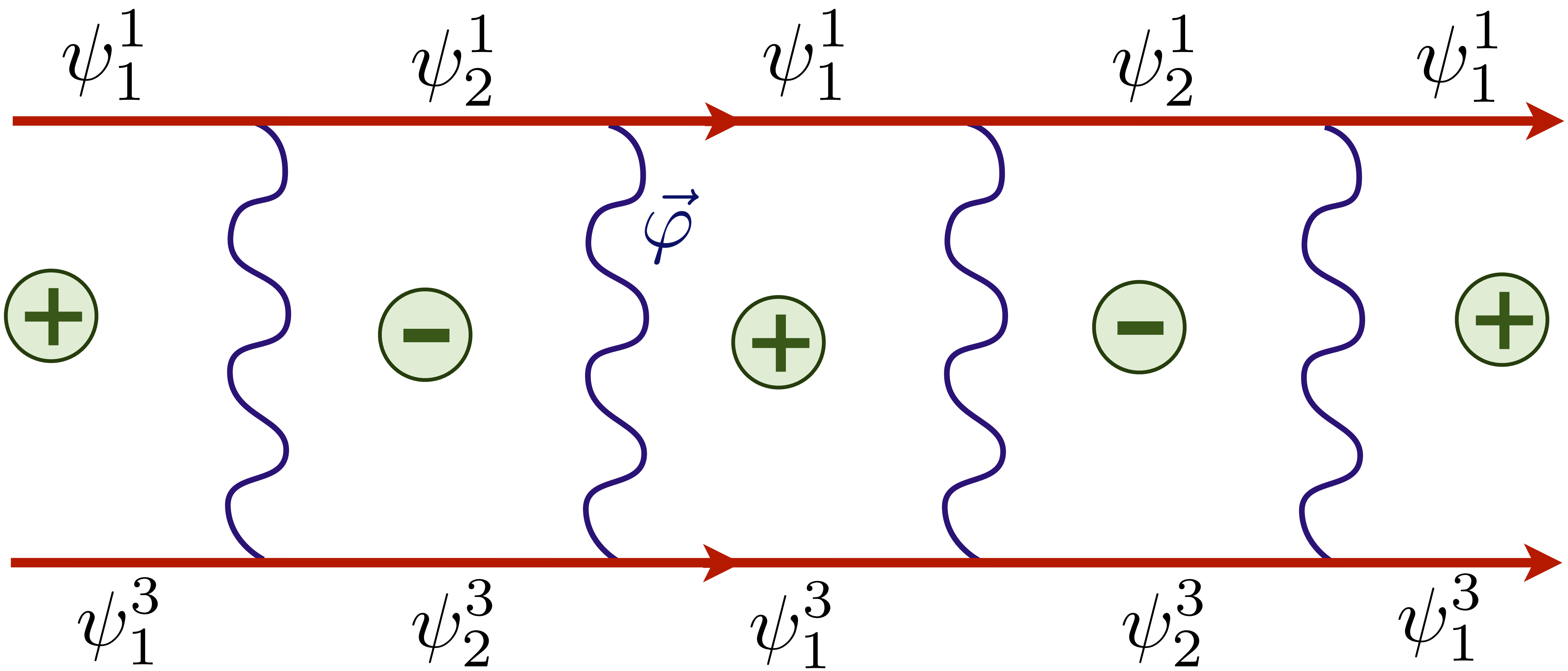
d-wave Cooper pairing instability in particle-particle channel

Emergent Pseudospin symmetry

Continuum theory of hotspots in invariant under:

$$\begin{pmatrix} \psi^{\ell}_{\uparrow} \\ \psi^{\ell\dagger}_{\downarrow} \end{pmatrix} \rightarrow U^{\ell} \begin{pmatrix} \psi^{\ell}_{\uparrow} \\ \psi^{\ell\dagger}_{\downarrow} \end{pmatrix}$$

where U^{ℓ} are arbitrary SU(2) matrices which can be *different* on different hotspots ℓ.



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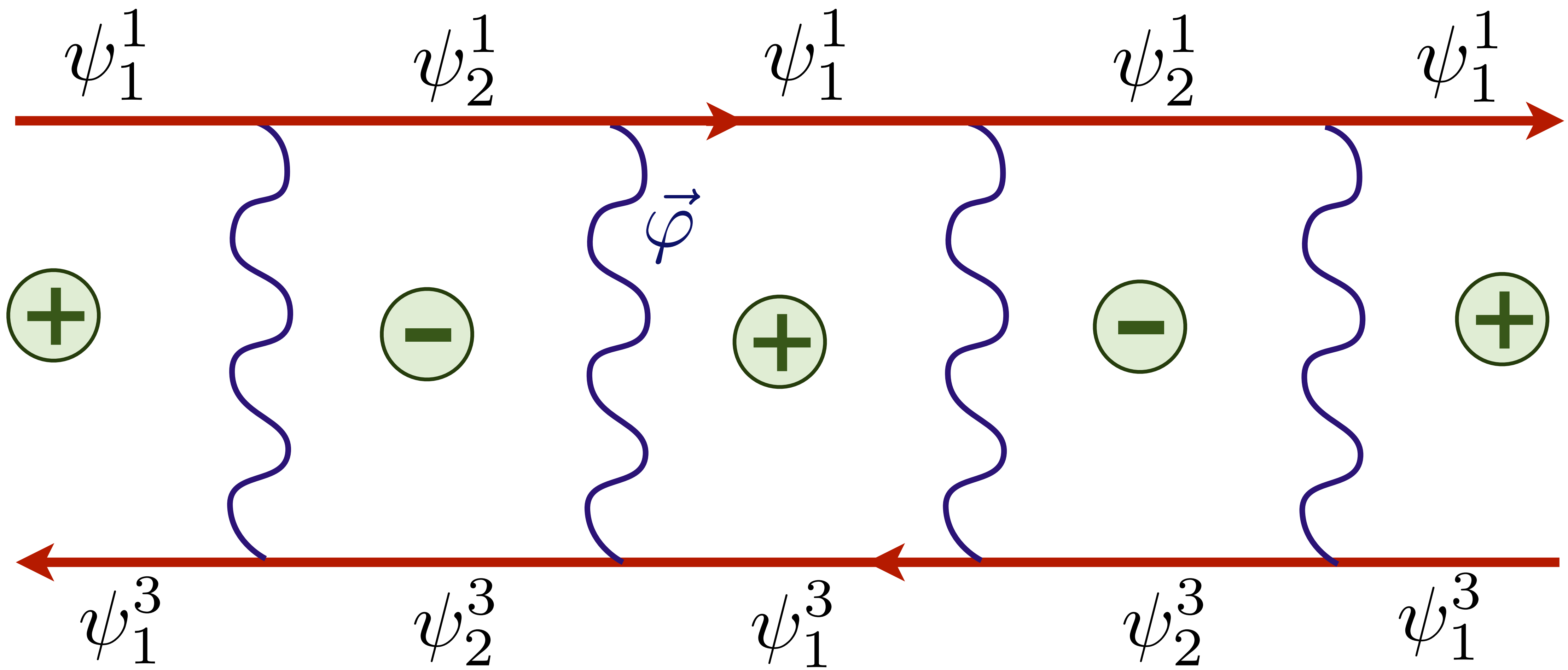
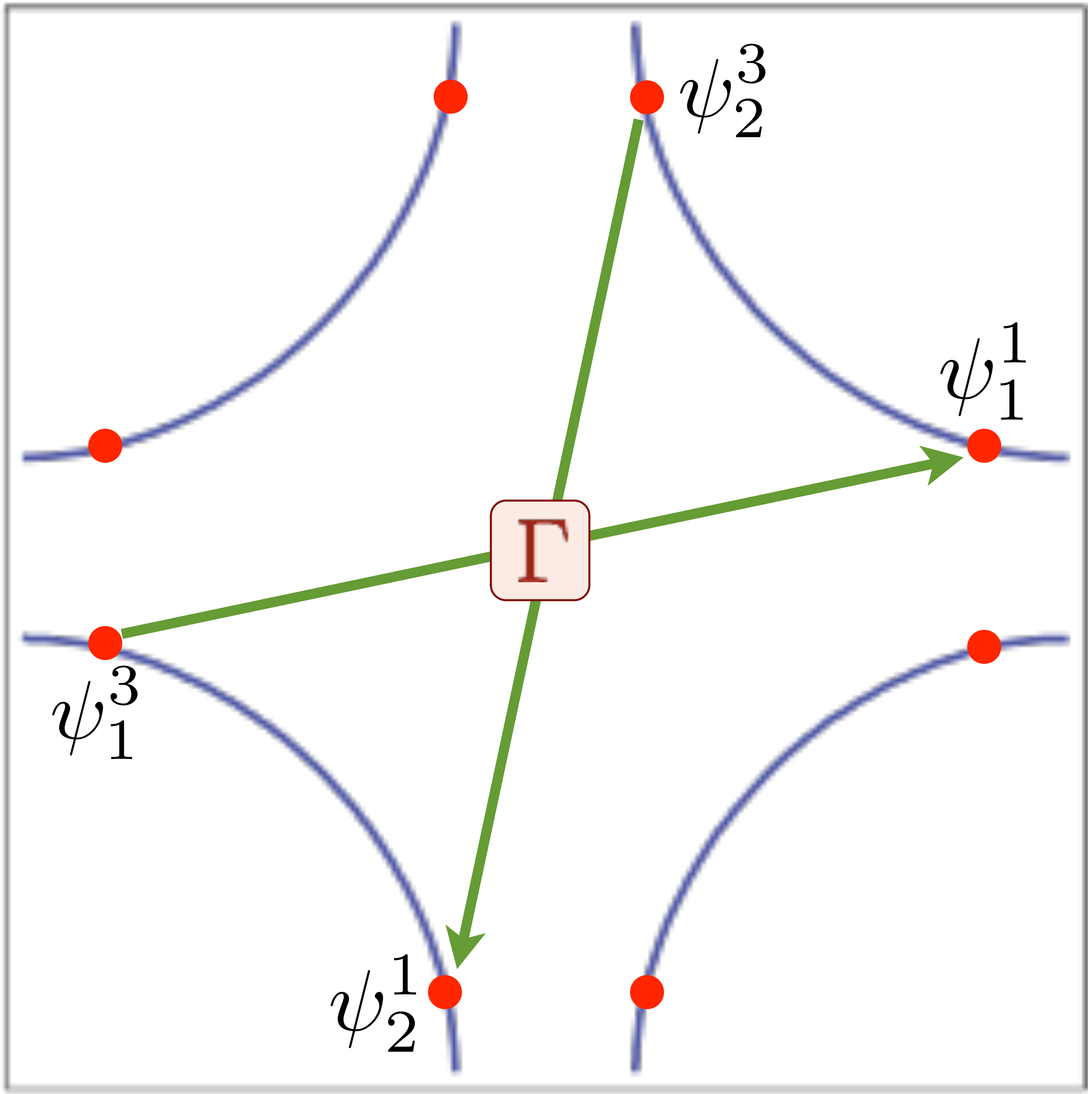


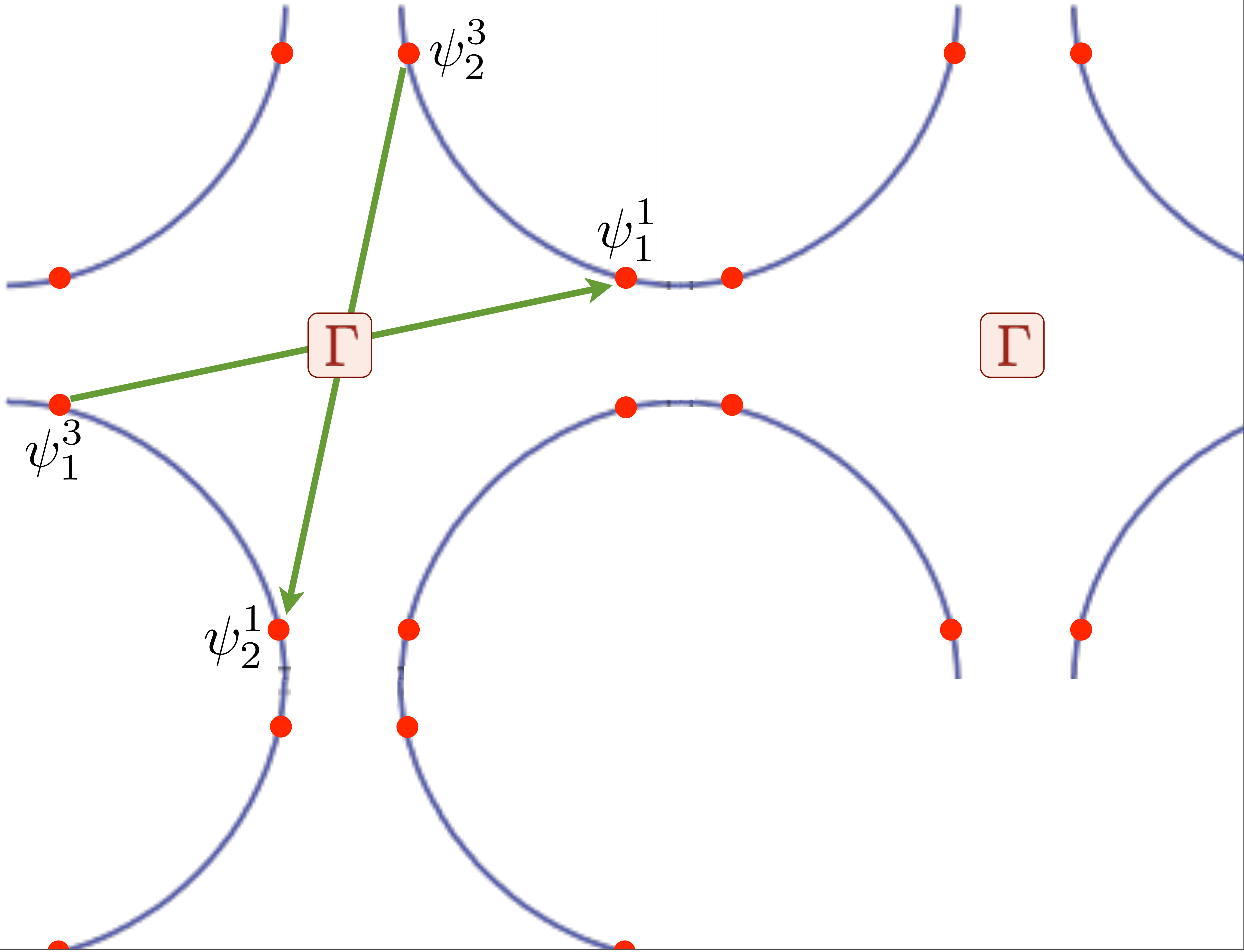
Bond density wave (with local Ising-nematic order) instability in particle-hole channel

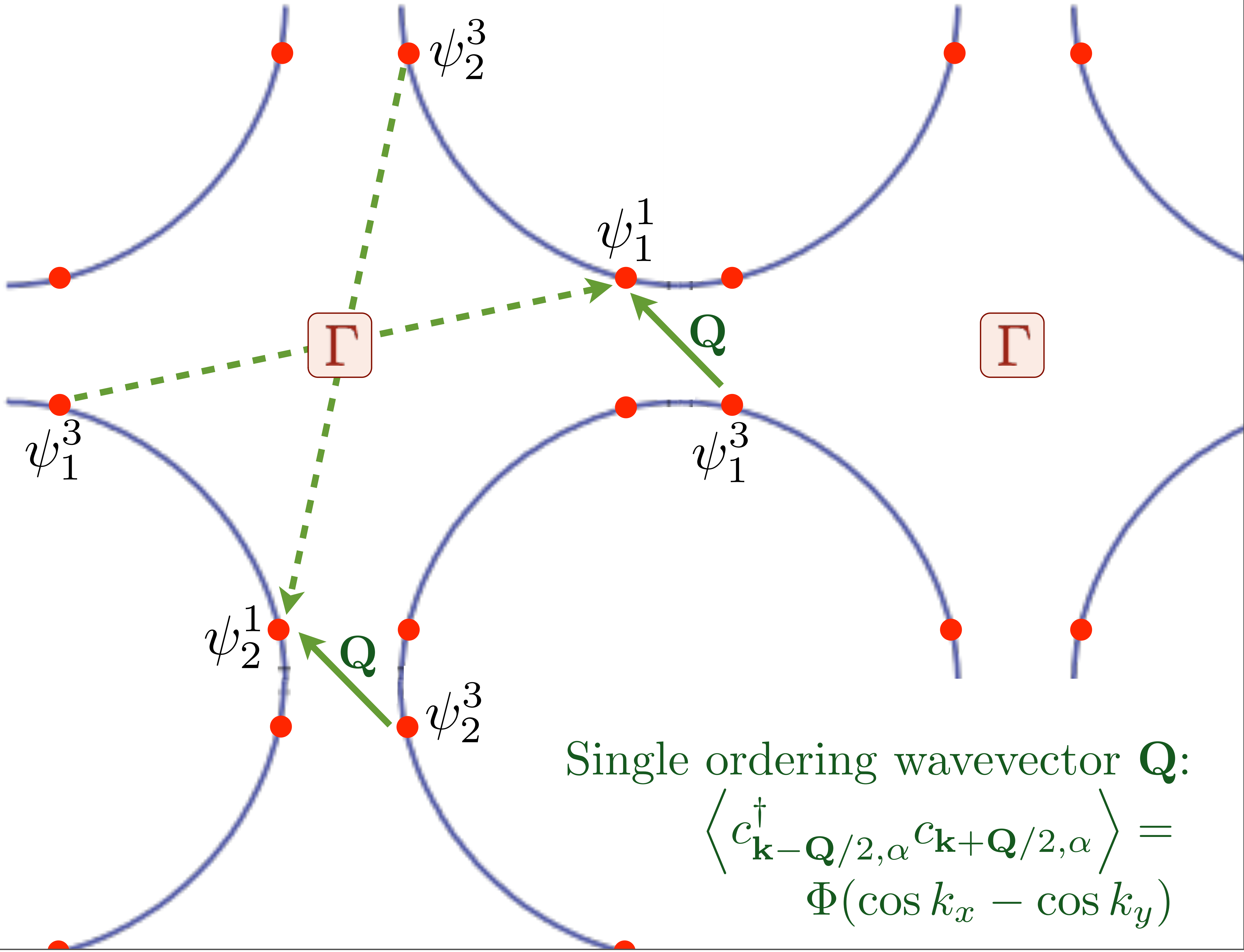


d-wave pairing has a partner instability in the particle-hole channel

Density-wave order parameter:

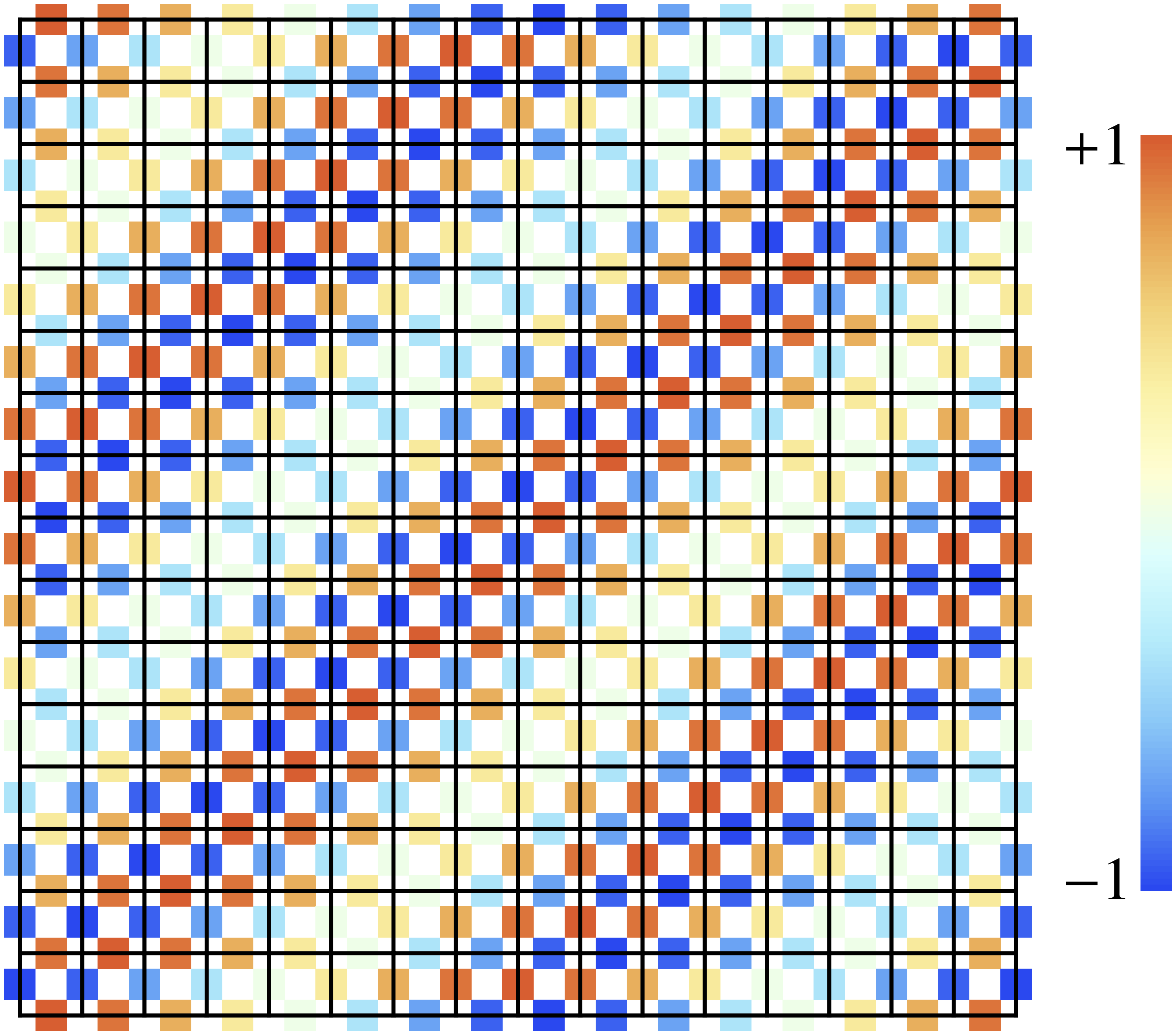
$$\left(\psi_{1\alpha}^{3\dagger}\psi_{1\alpha}^{1} - \psi_{2\alpha}^{3\dagger}\psi_{2\alpha}^{1}\right)$$


ψ_2^3
ψ_1^1
Γ
Γ
ψ_1^3
ψ_2^1



Single ordering wavevector $\mathbf{Q}$:

$$\left\langle c^\dagger_{\mathbf{k}-\mathbf{Q}/2,\alpha} c_{\mathbf{k}+\mathbf{Q}/2,\alpha} \right\rangle = \Phi(\cos k_x - \cos k_y)$$



"Bond density" measures amplitude for electrons to be in spin-singlet valence bond: VBS order

No modulations on sites. Modulated bond-density wave with local Ising-nematic ordering:

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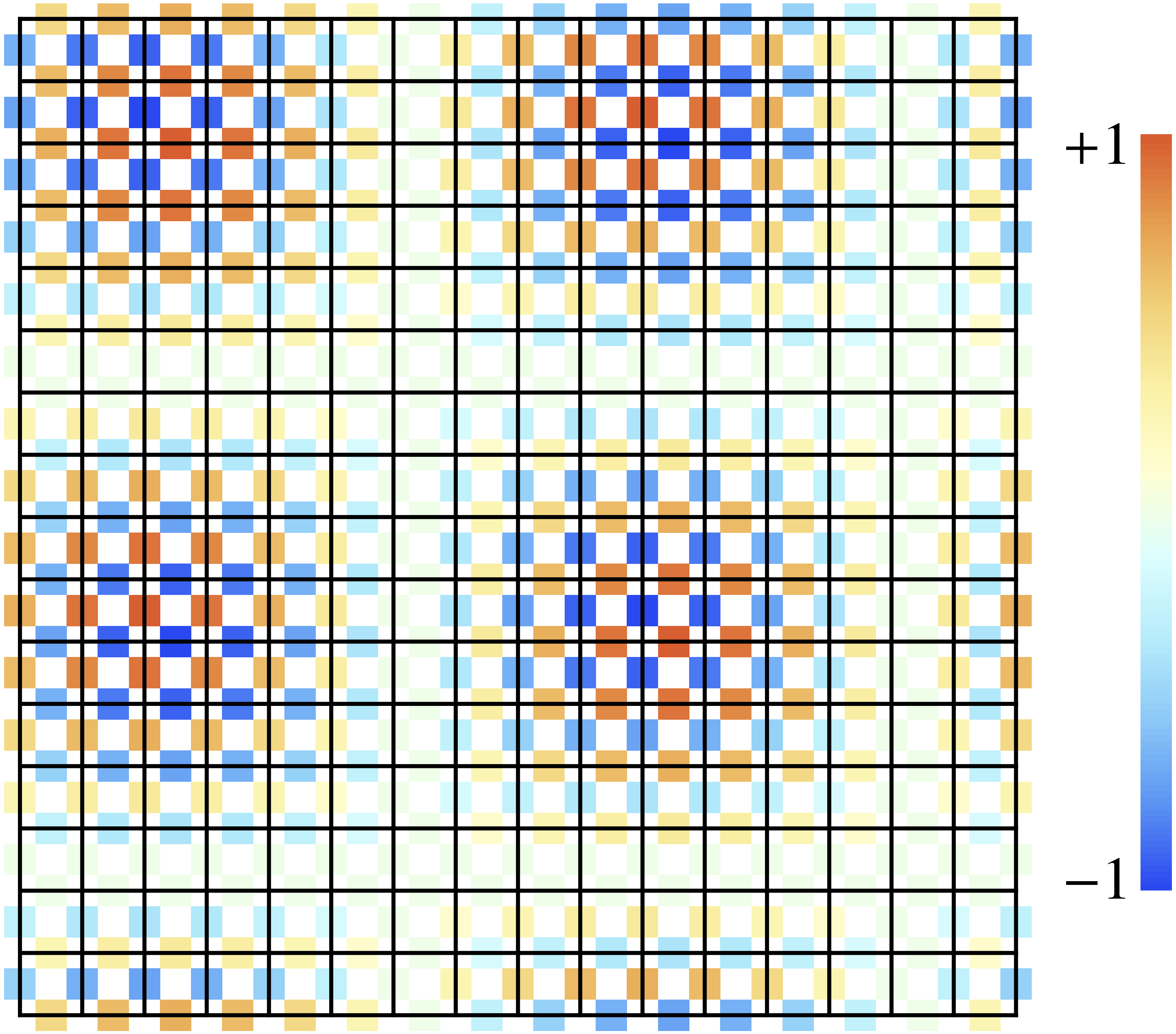

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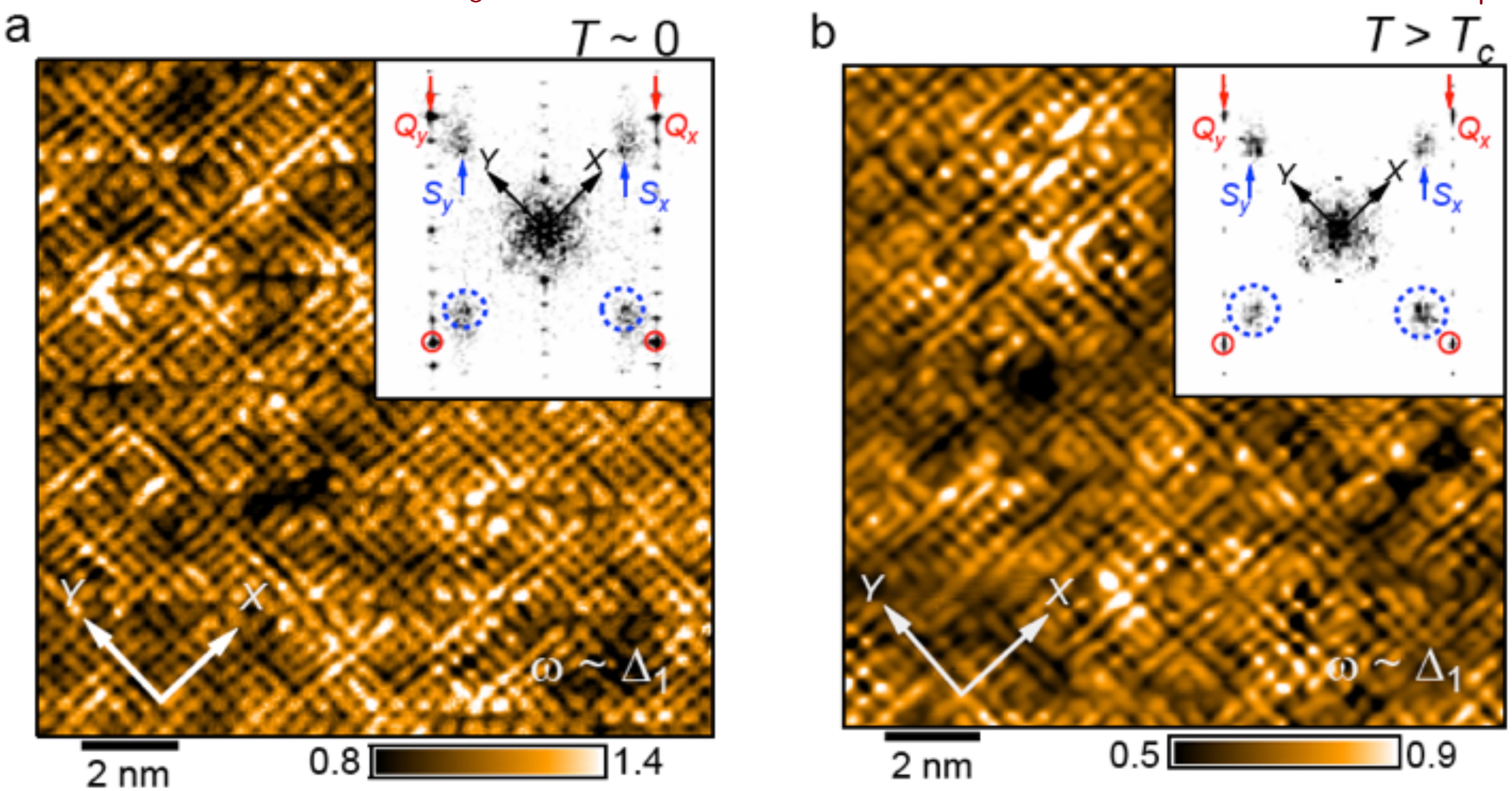
STM measurements of $Z(r)$, the energy asymmetry in density of states in $Bi_2Sr_2CaCu_2O_{8+\delta}$.



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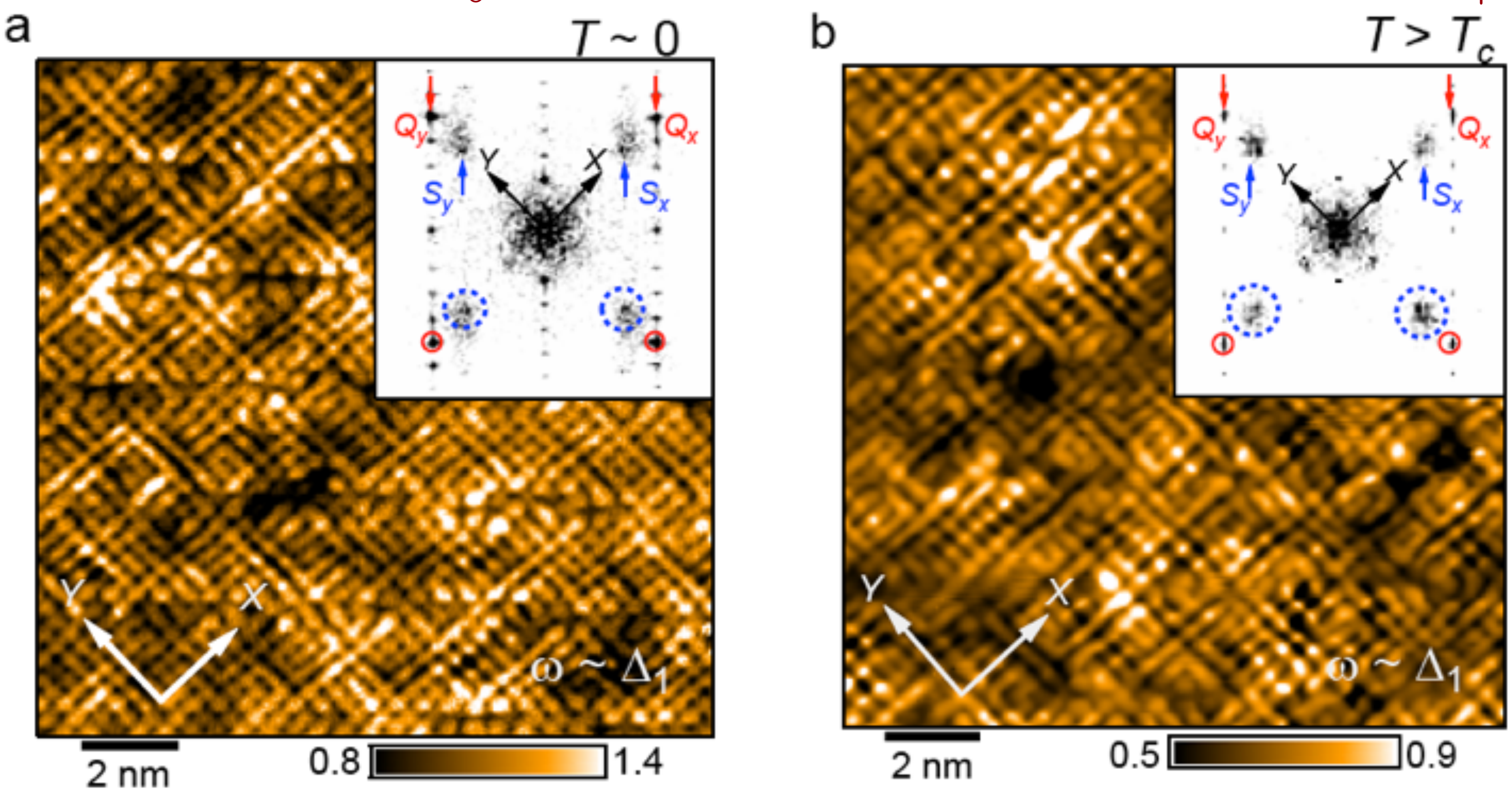

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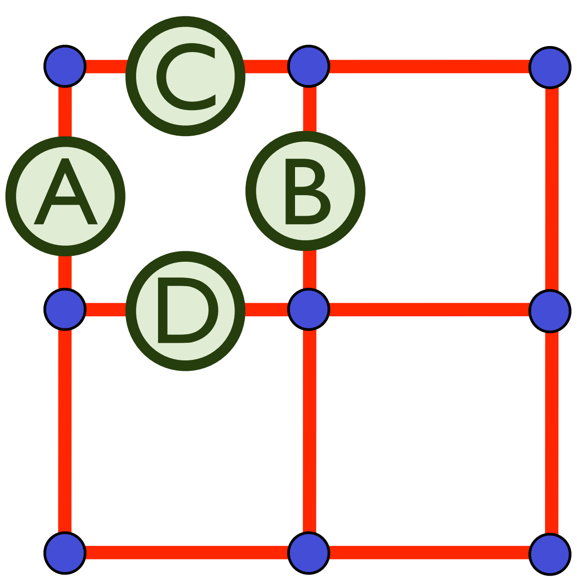

$$O_N = Z_A + Z_B - Z_C - Z_D$$

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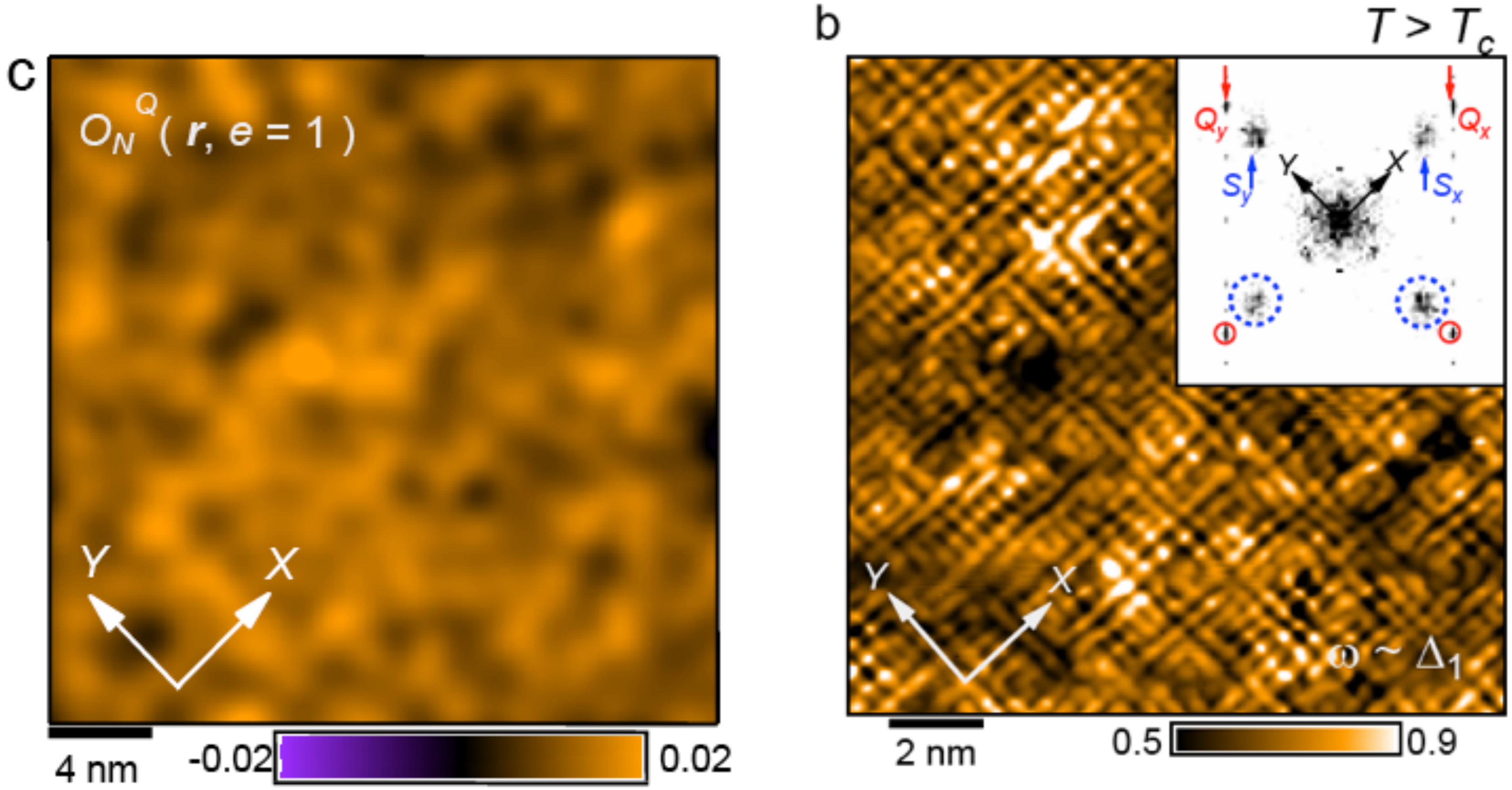

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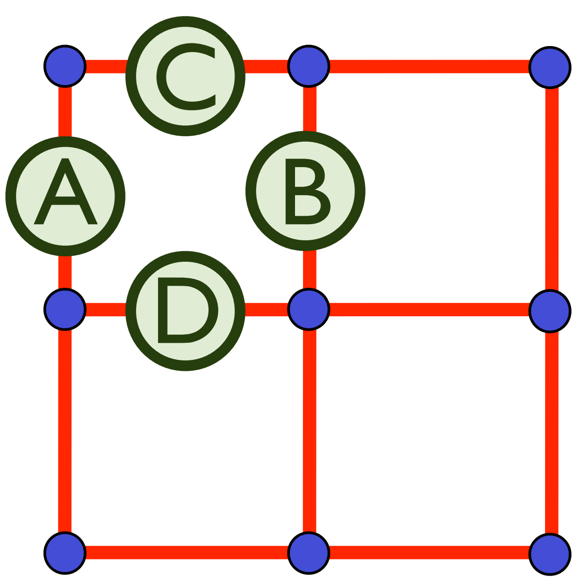

$$O_N = Z_A + Z_B - Z_C - Z_D$$

Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

Conclusions

Identified quantum criticality in cuprate superconductors with a critical point at optimal doping associated with onset of spin density wave order in a metal

Elusive optimal doping quantum critical point has been "hiding in plain sight".

It is shifted to lower doping by the onset of superconductivity

Conclusions

Theory for the onset of spin density wave in metals is *strongly* coupled in two dimensions

For the cuprate Fermi surface, there are strong instabilities near the quantum critical point to
d-wave pairing
and
bond density waves with local Ising-nematic ordering