Many body quantum entanglement: from organic insulators to ultracold atoms

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Outline

1. Organic insulators:
antiferromagnets on the triangular lattice

2. Ultracold atoms:
bosons in tilted Mott insulators

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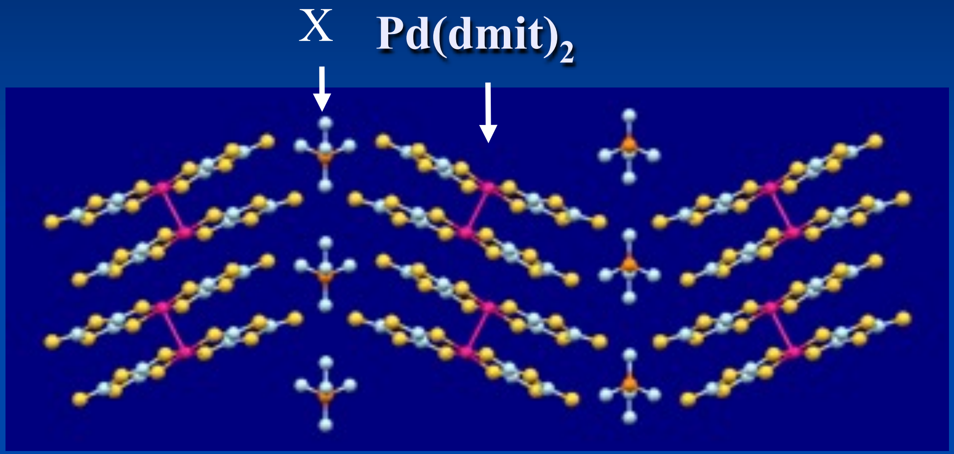
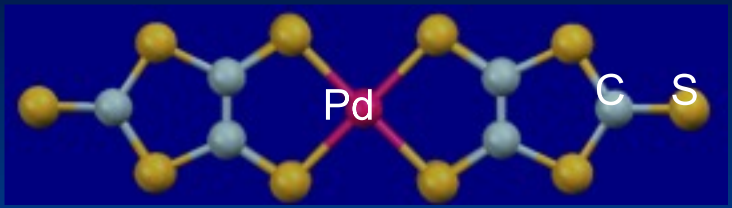
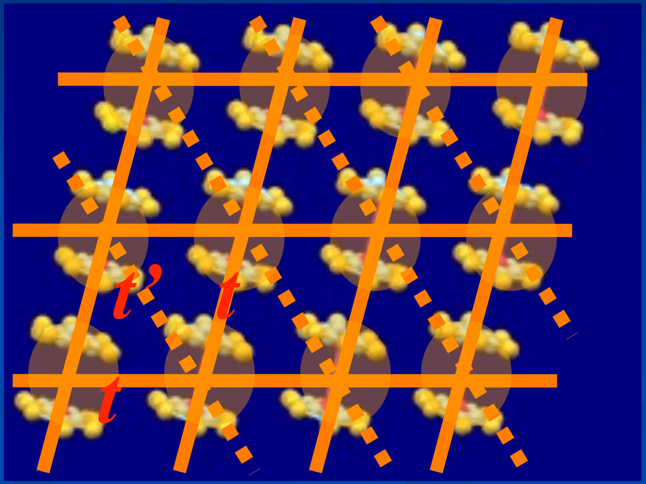
bosons in tilted Mott insulators

X[Pd(dmit)$_2$]$_2$







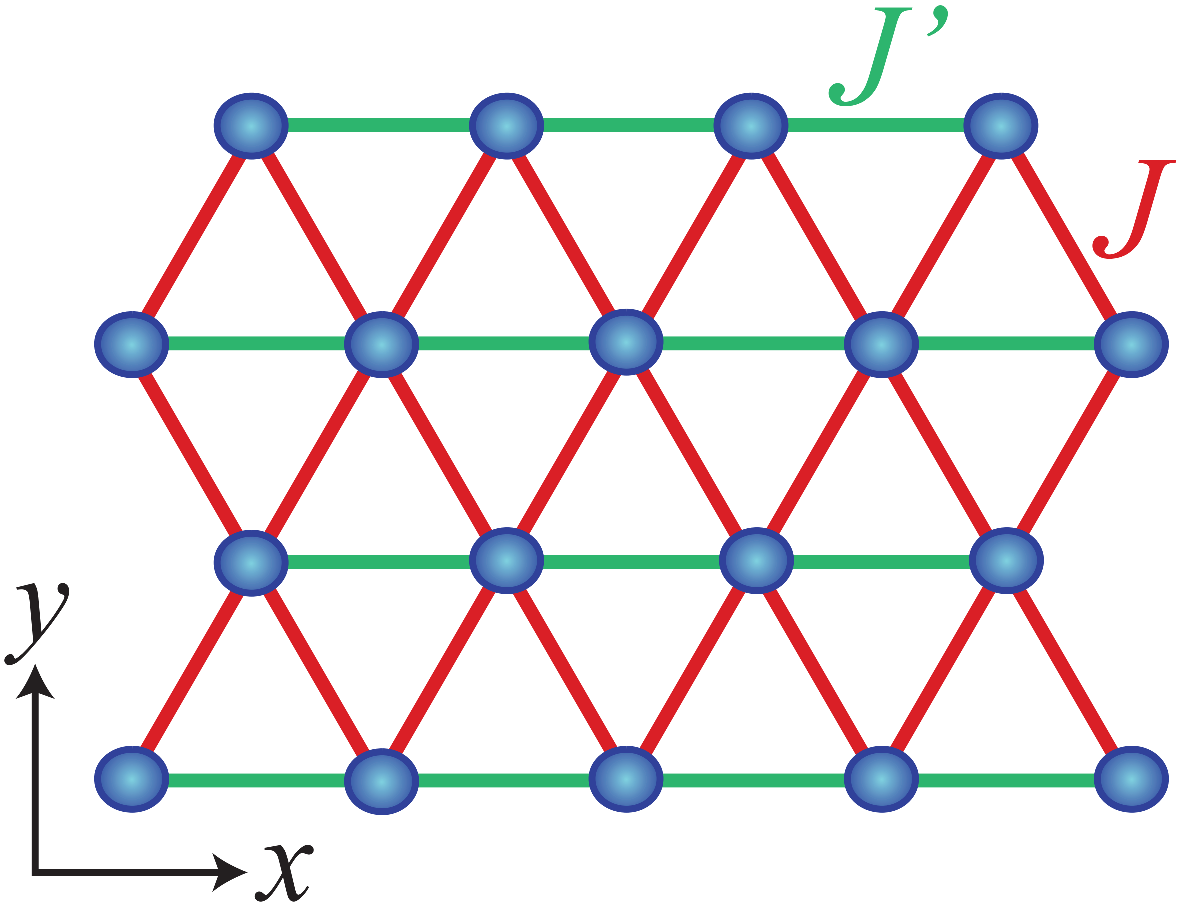
Half-filled band → Mott insulator with spin S = 1/2

Triangular lattice of [Pd(dmit)$_2$]$_2$

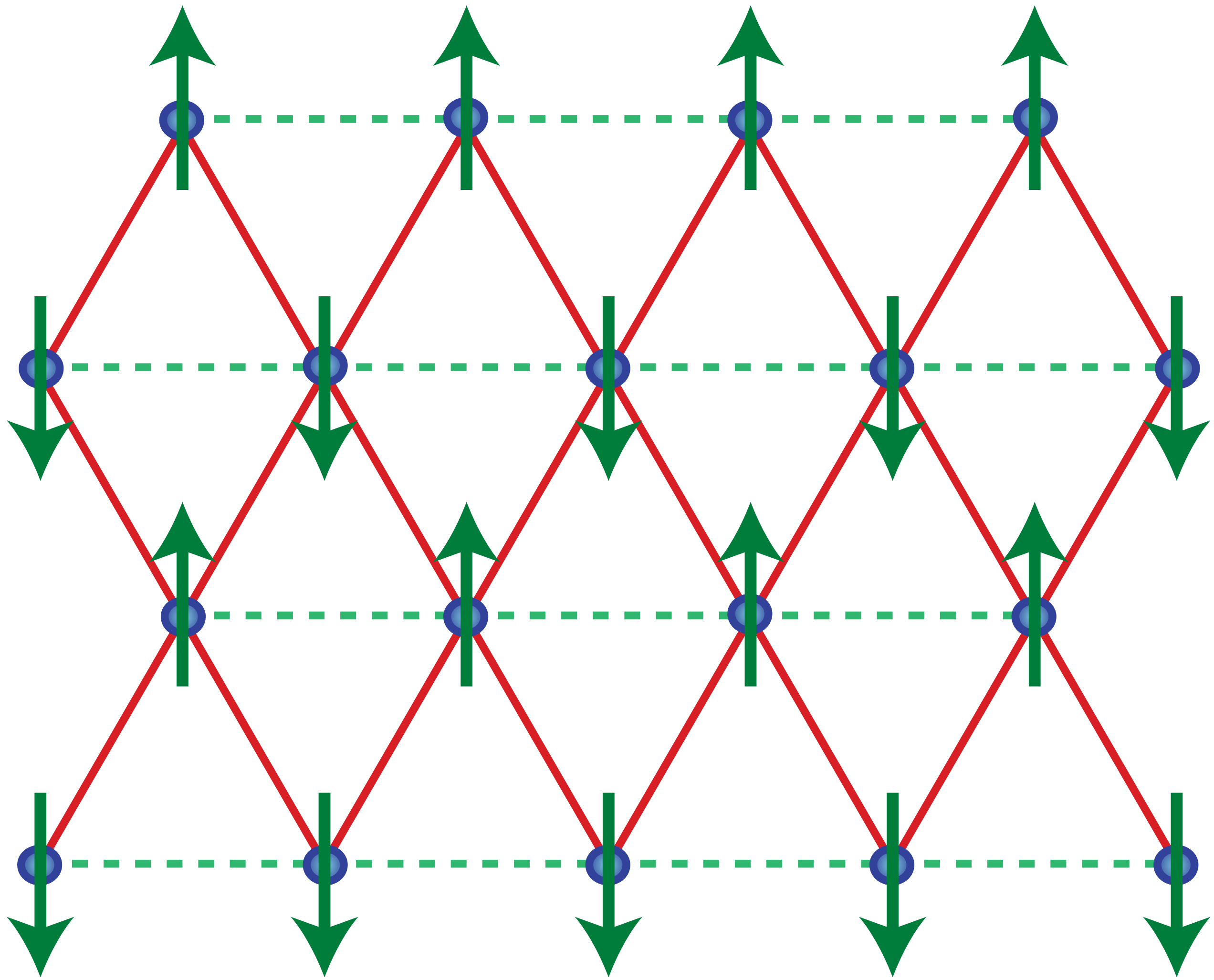
→ frustrated quantum spin system

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$\vec{S}_i \Rightarrow$ spin operator with $S = 1/2$



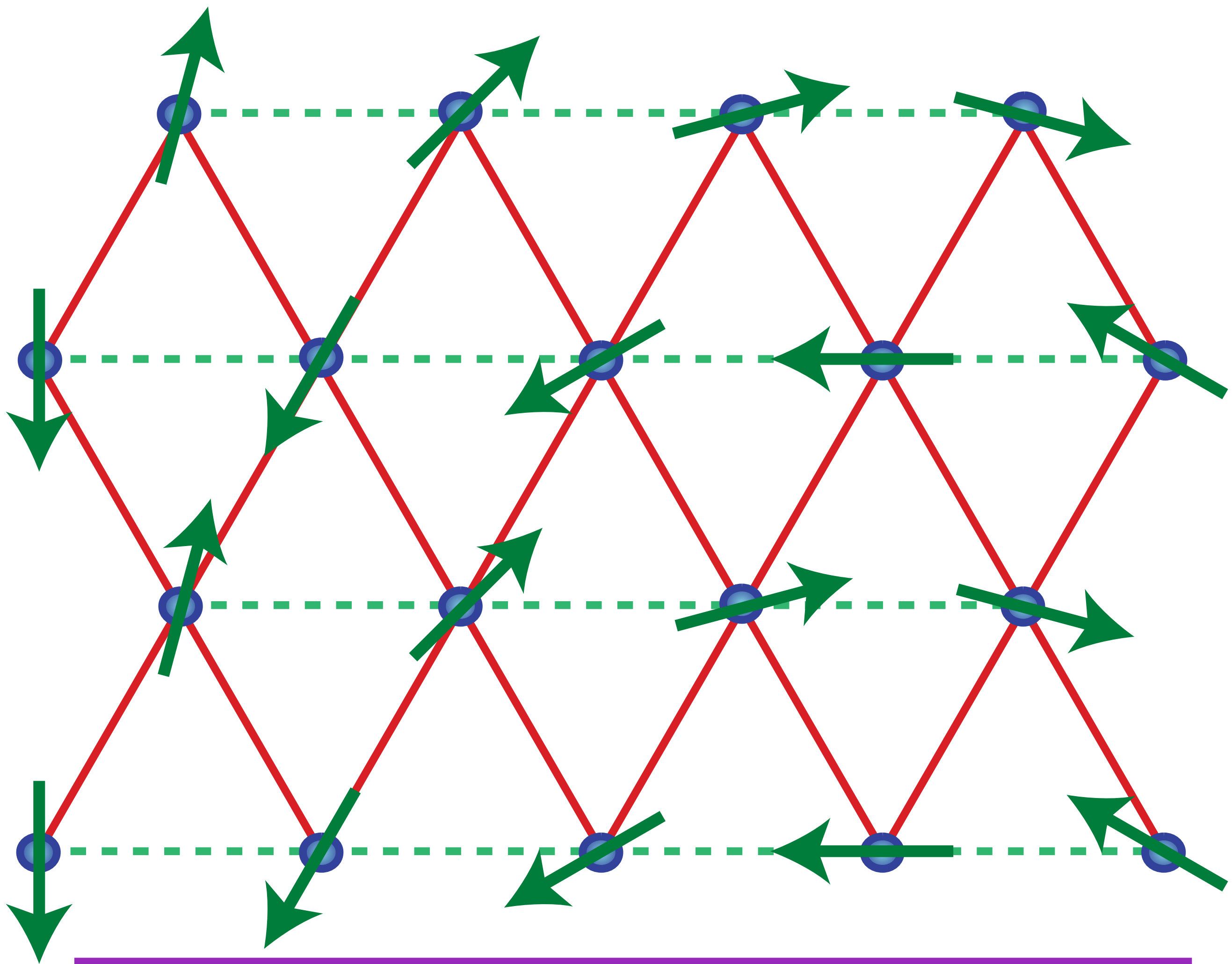
Anisotropic triangular lattice antiferromagnet



Classical ground state for small J'/J

Found in κ-$(ET)_2Cu[N(CN)_2]Cl$

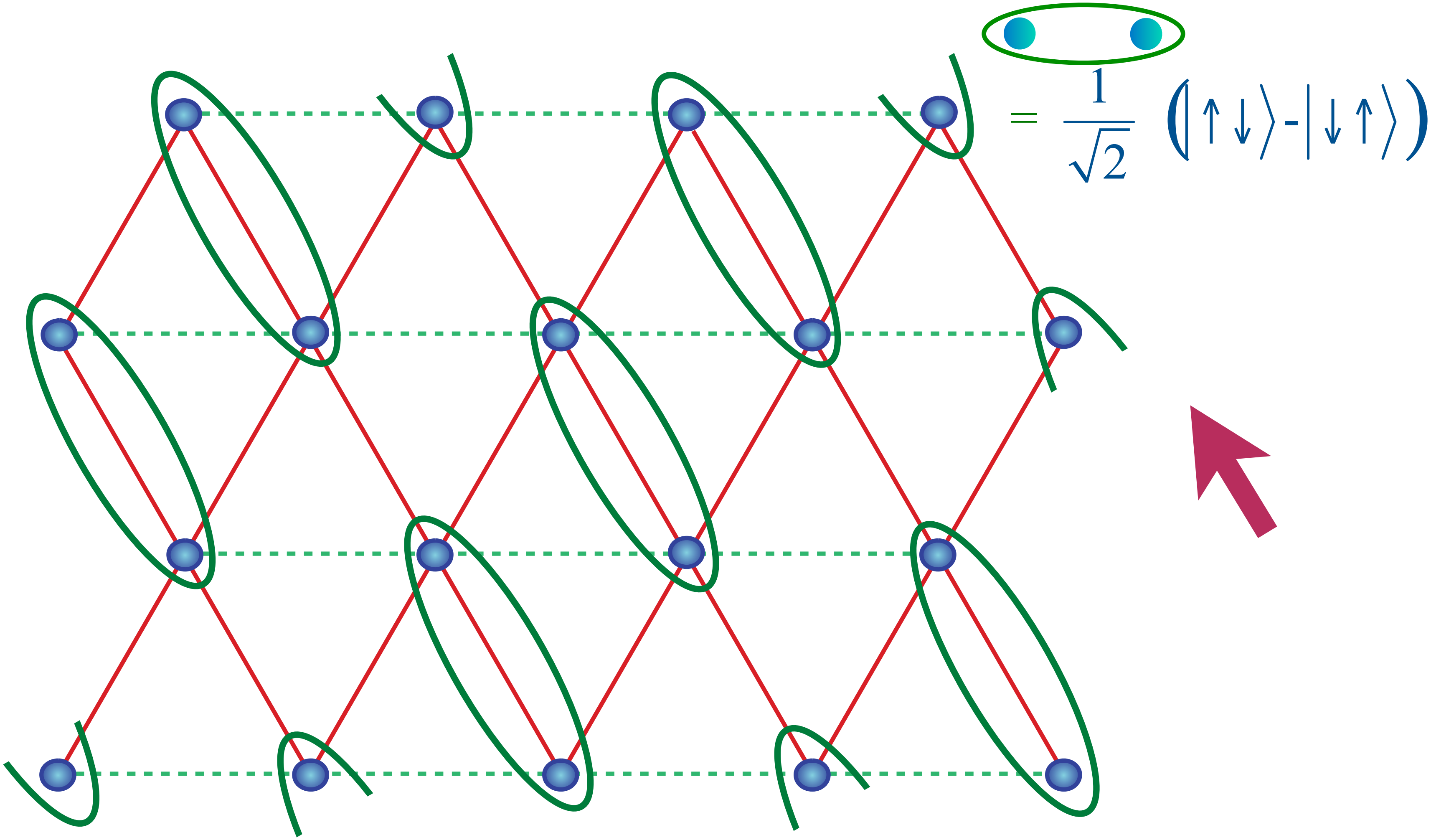
Anisotropic triangular lattice antiferromagnet



Classical ground state for large J'/J

Found in $ETMe_3P[Pd(dmit)_2]_2$ and Cs_2CuCl_4

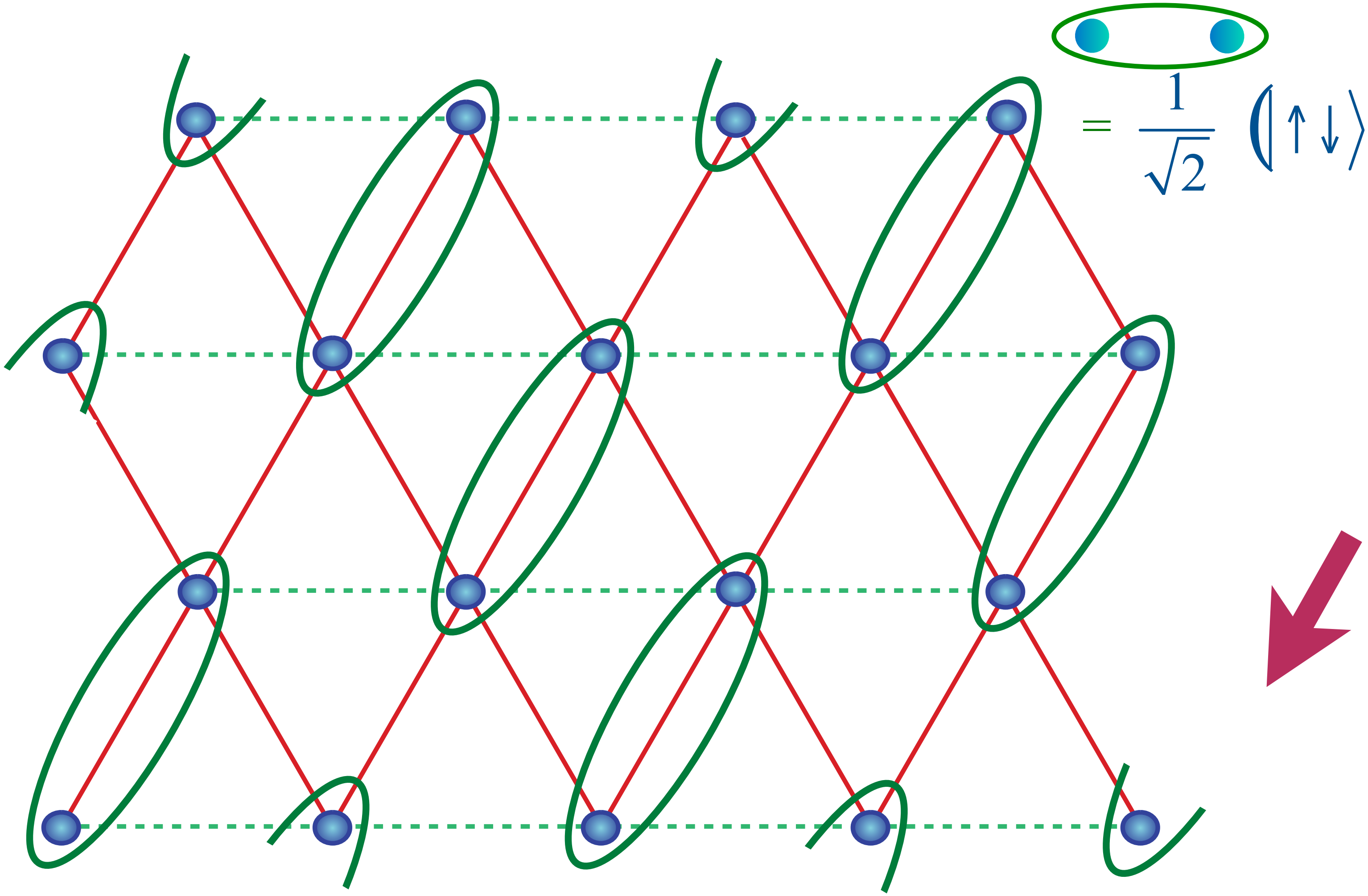
Anisotropic triangular lattice antiferromagnet



Valence bond solid

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)

Anisotropic triangular lattice antiferromagnet



$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle\right)$$

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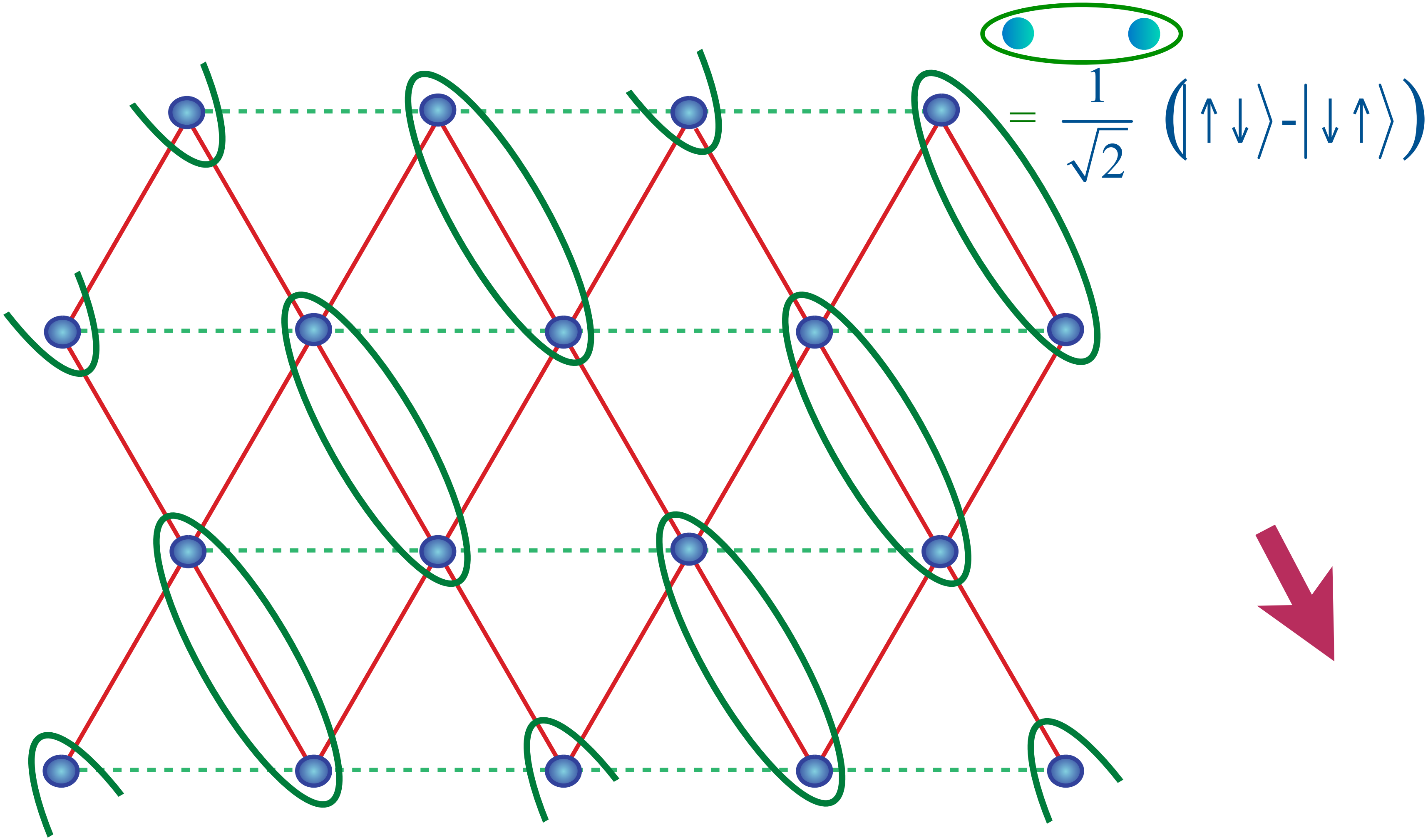

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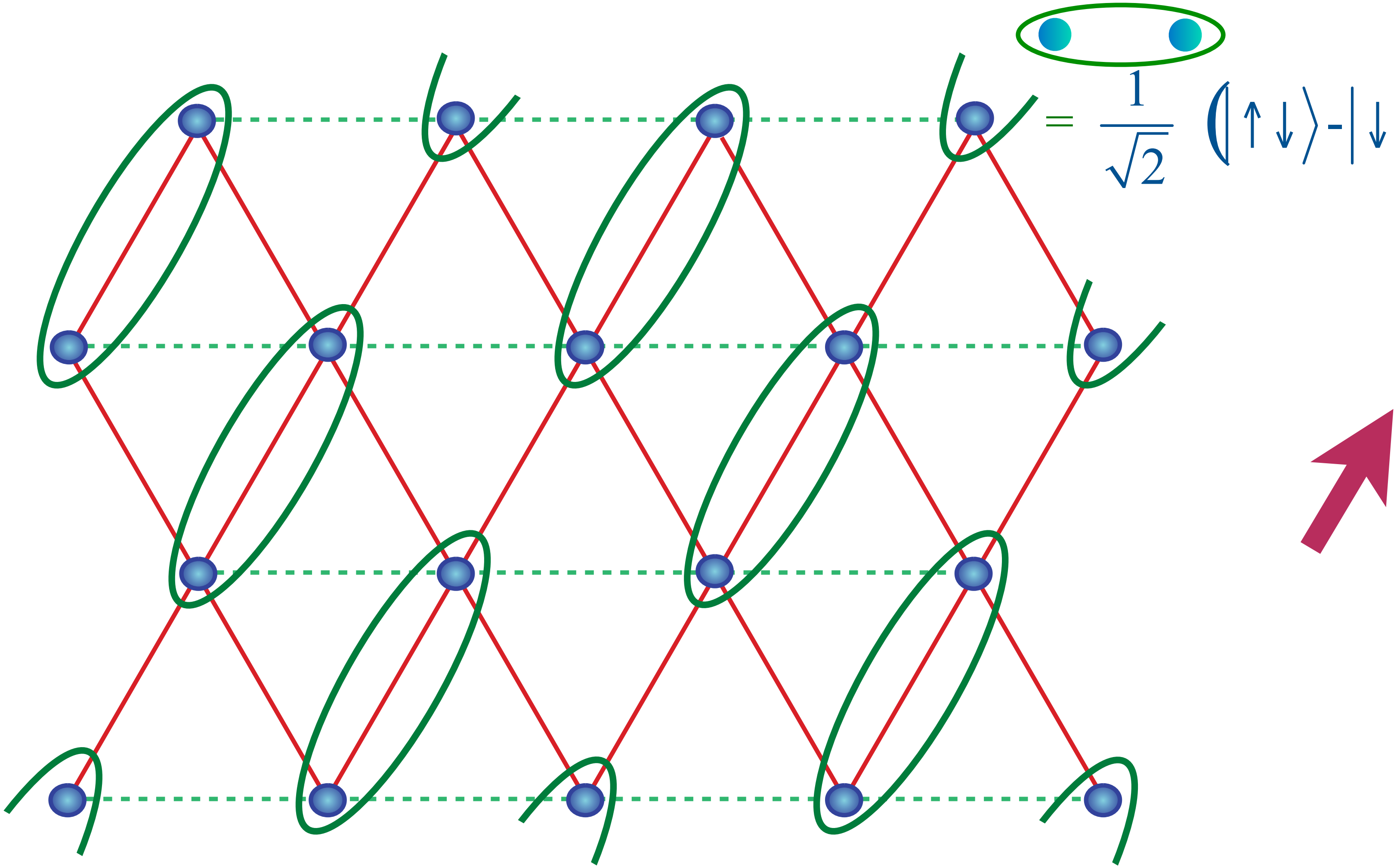

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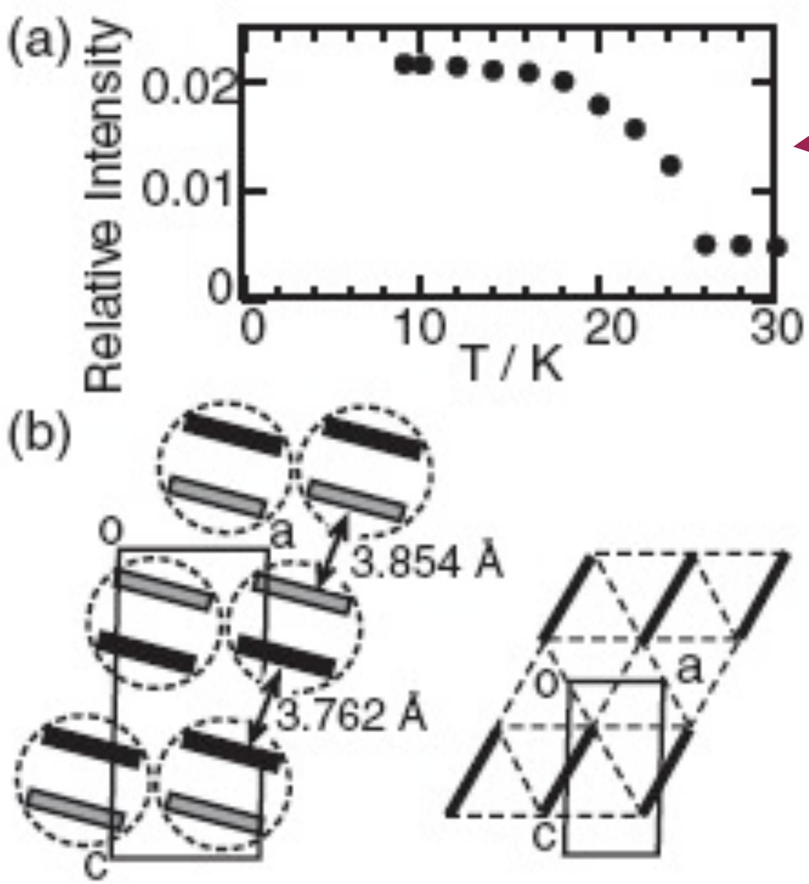
N. Read and S. Sachdev, *Phys. Rev. Lett*. **62**, 1694 (1989)

Observation of a valence bond solid (VBS) in $ETMe_3P[Pd(dmit)_2]_2$

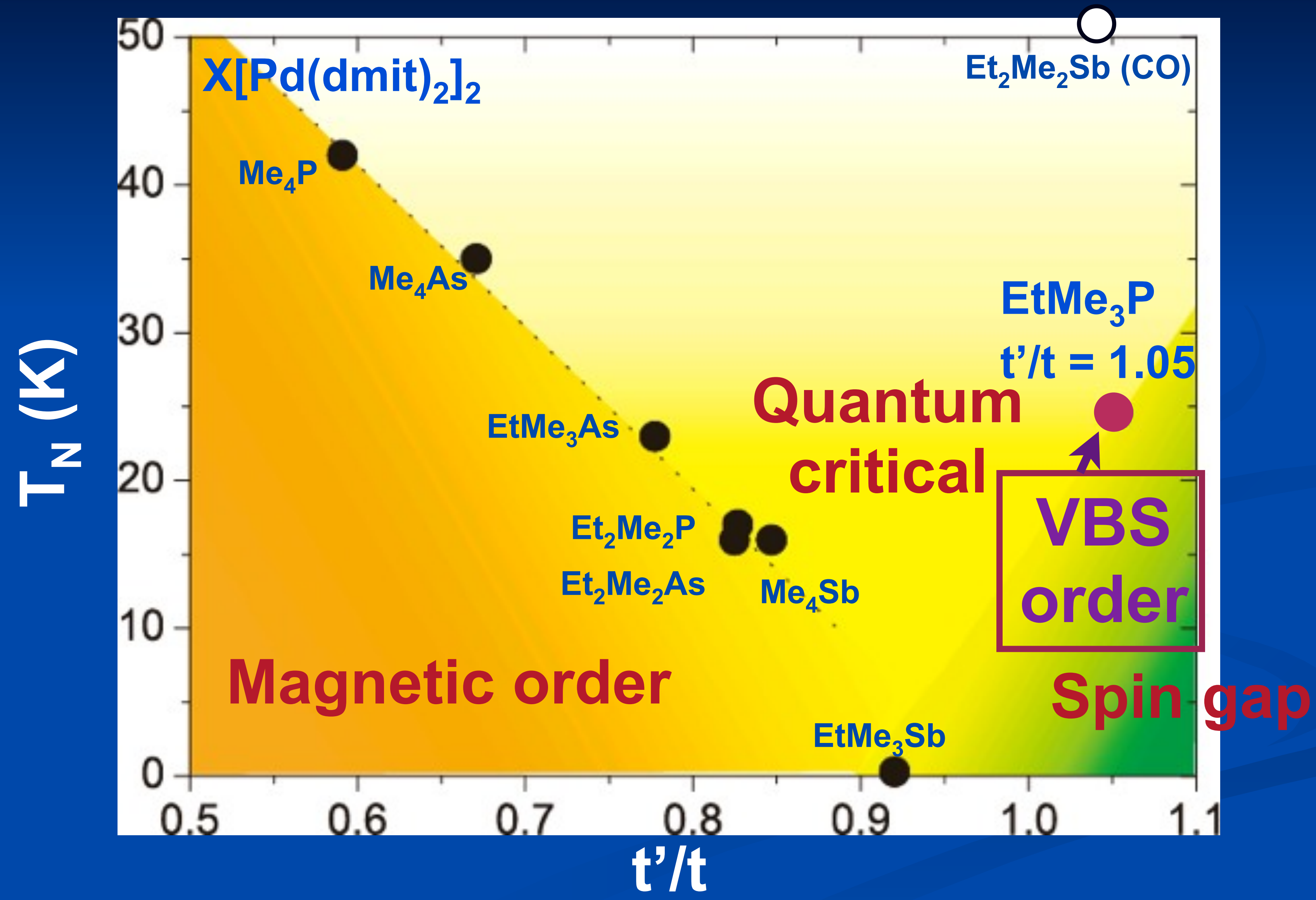


X-ray scattering

Spin gap ~ 40 K
J ~ 250 K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

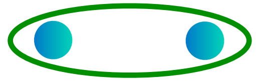
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Triangular lattice antiferromagnet

Z_2 spin liquid



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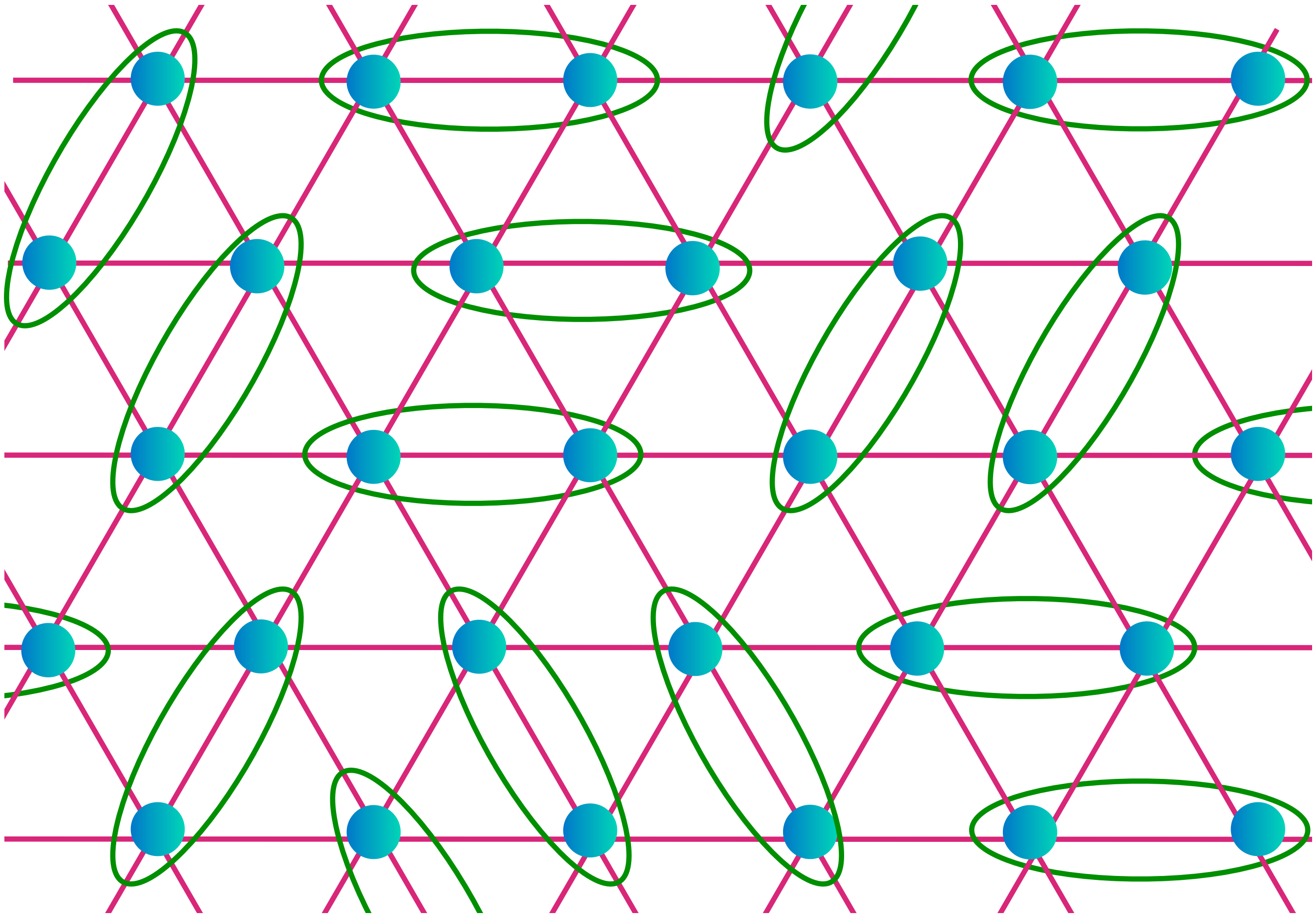

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).
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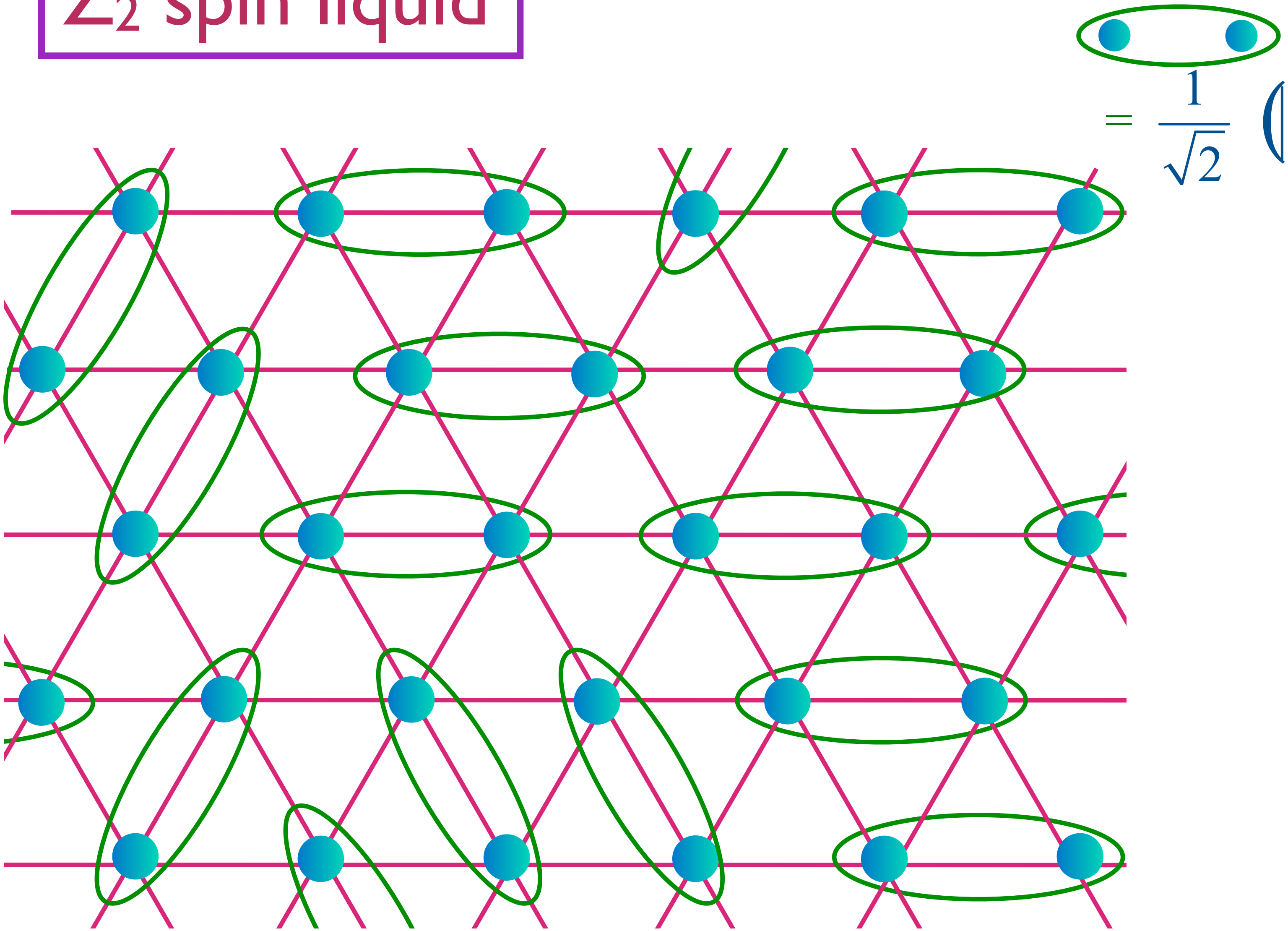

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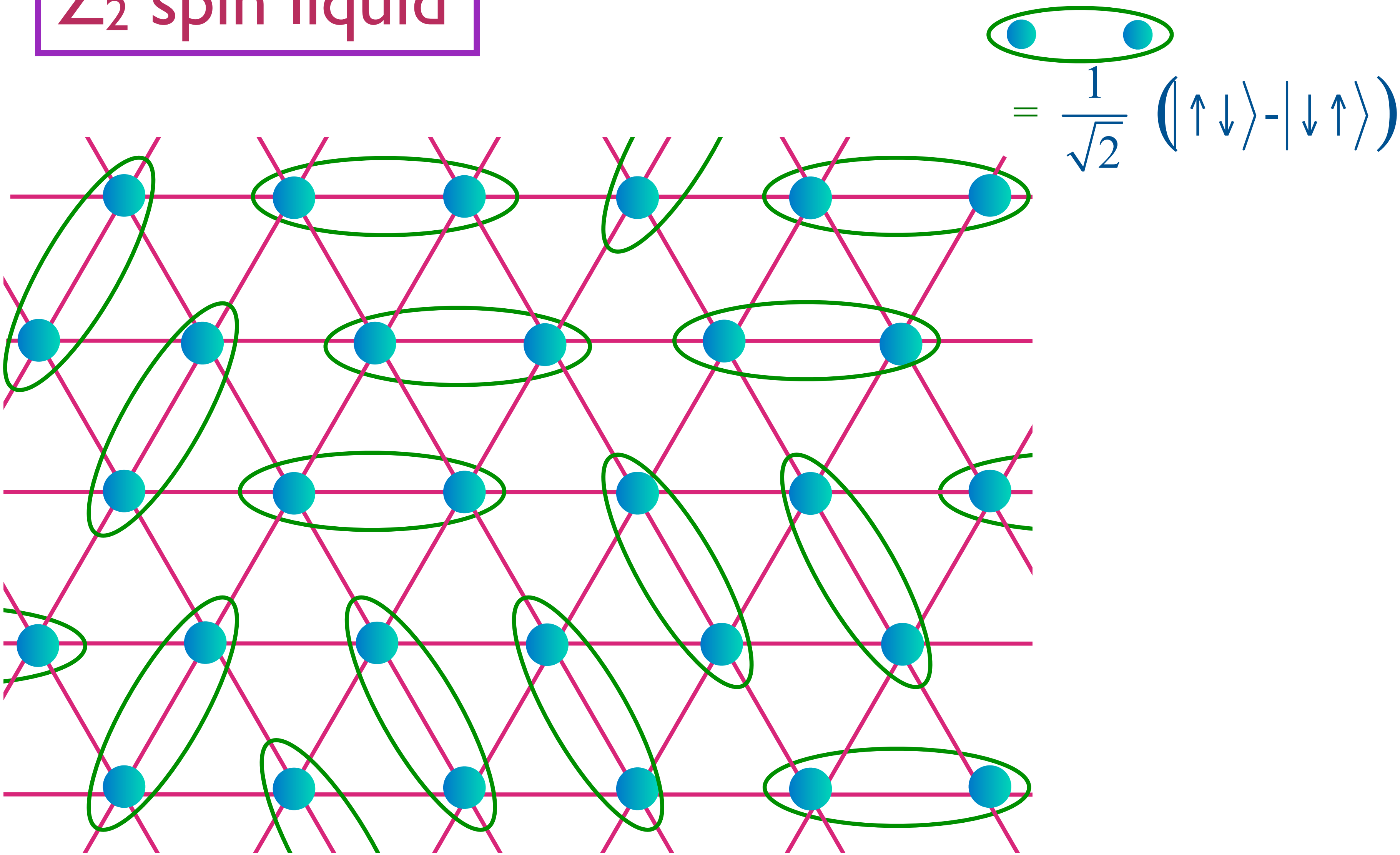

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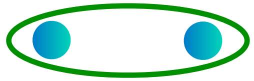

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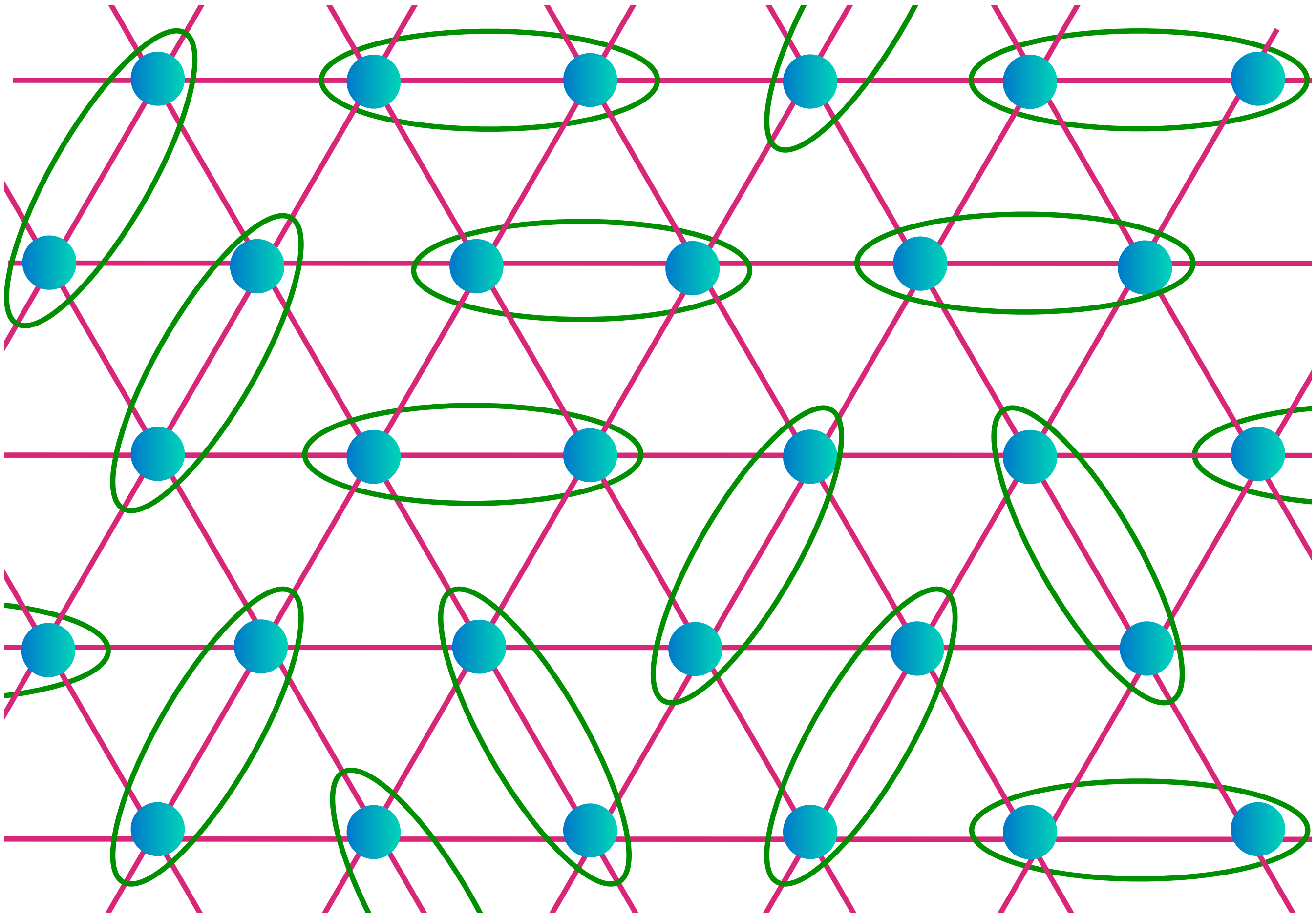

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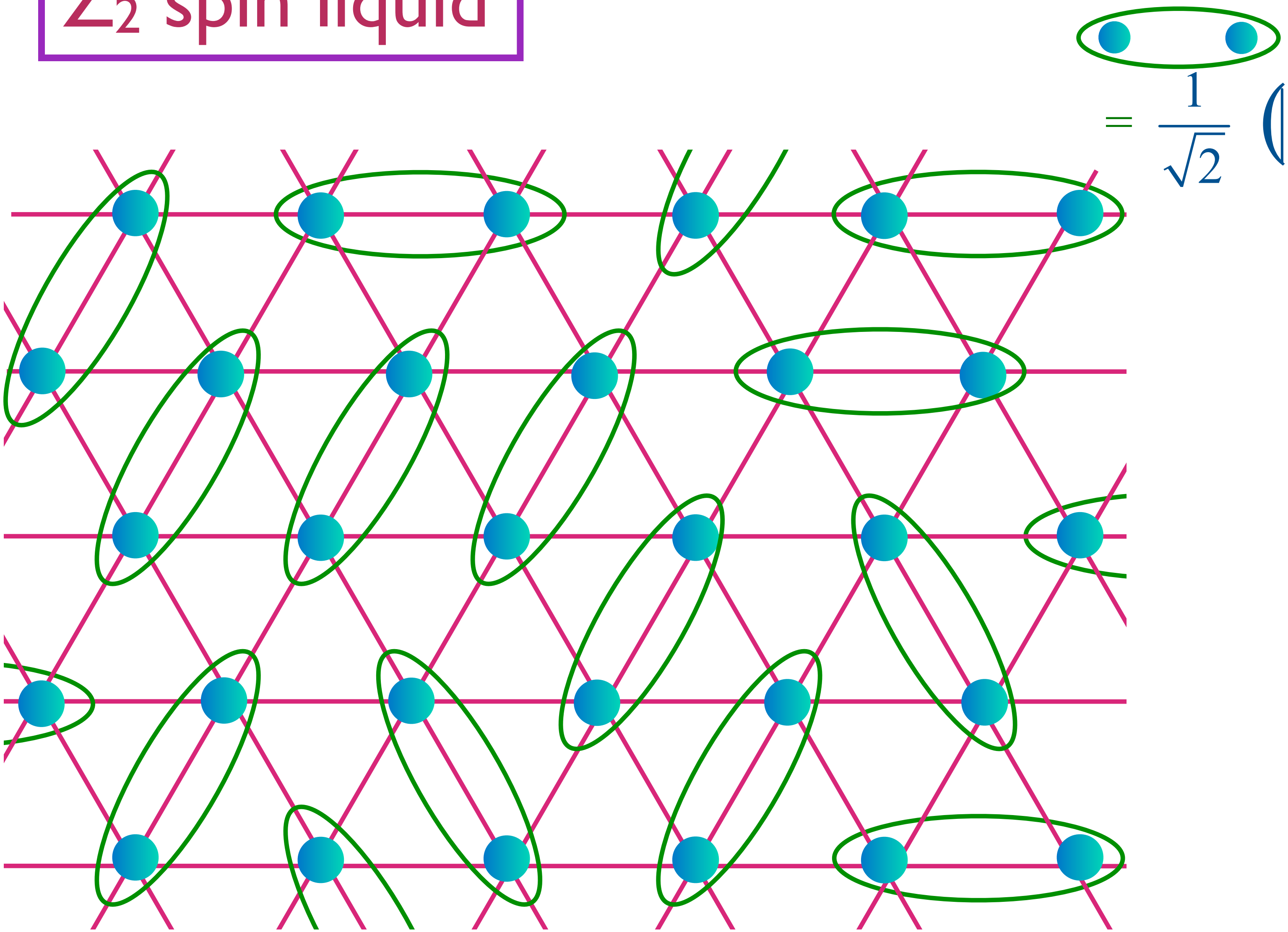

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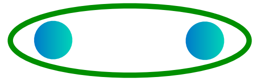

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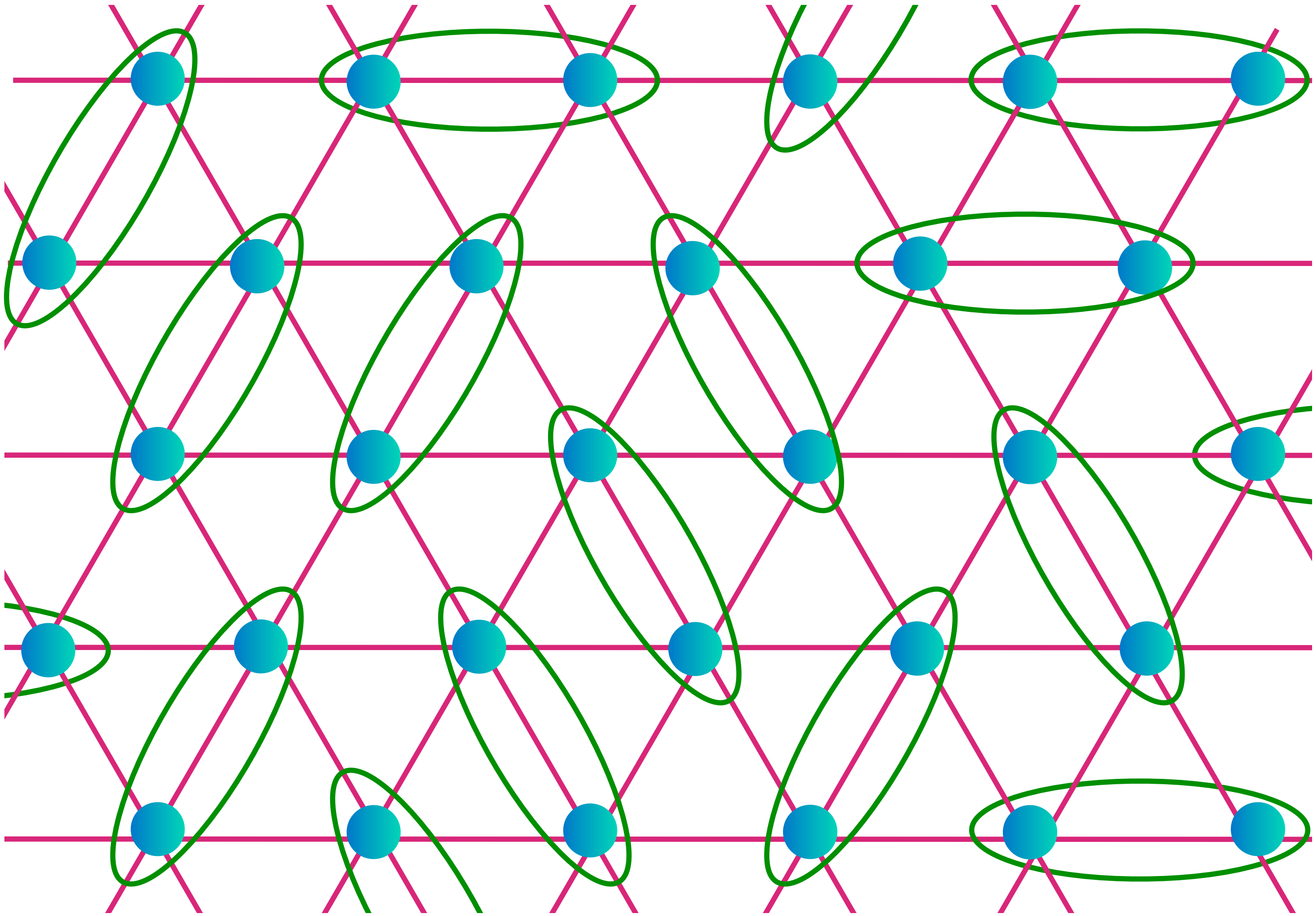

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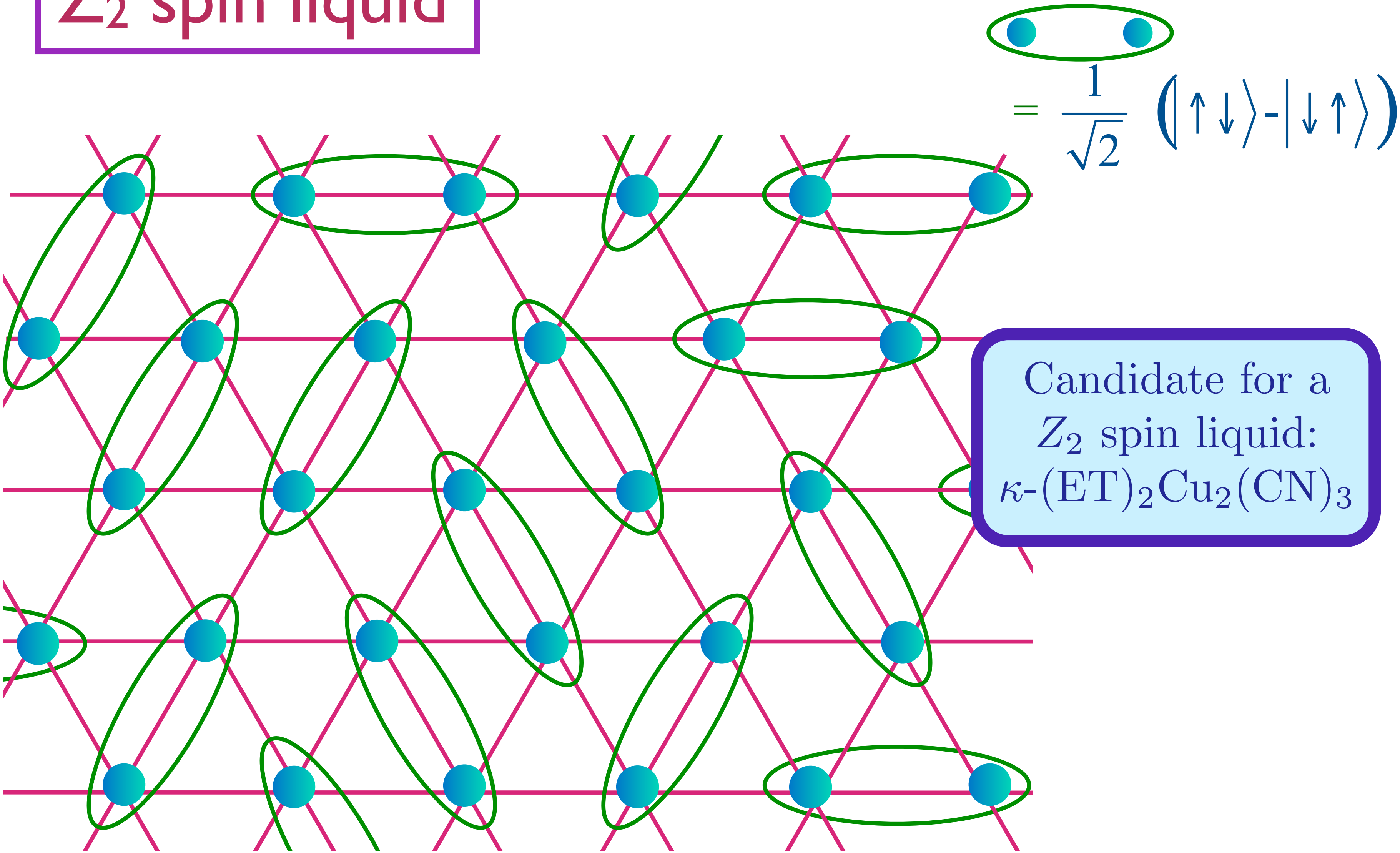
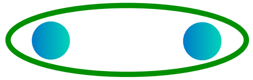


Candidate for a Z_2 spin liquid: κ-$(ET)_2Cu_2(CN)_3$

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Excitations of the Z_2 Spin liquid

A vison



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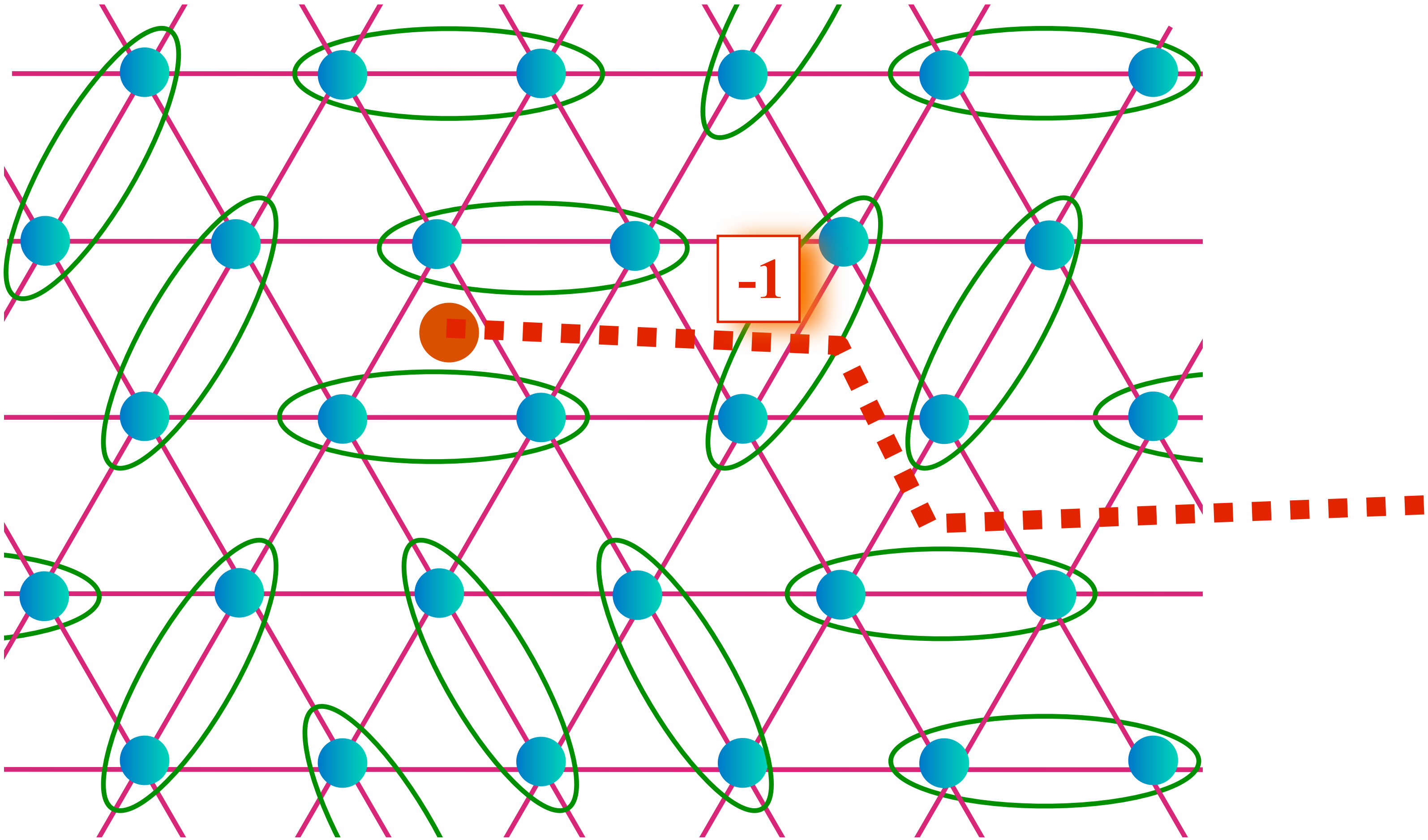

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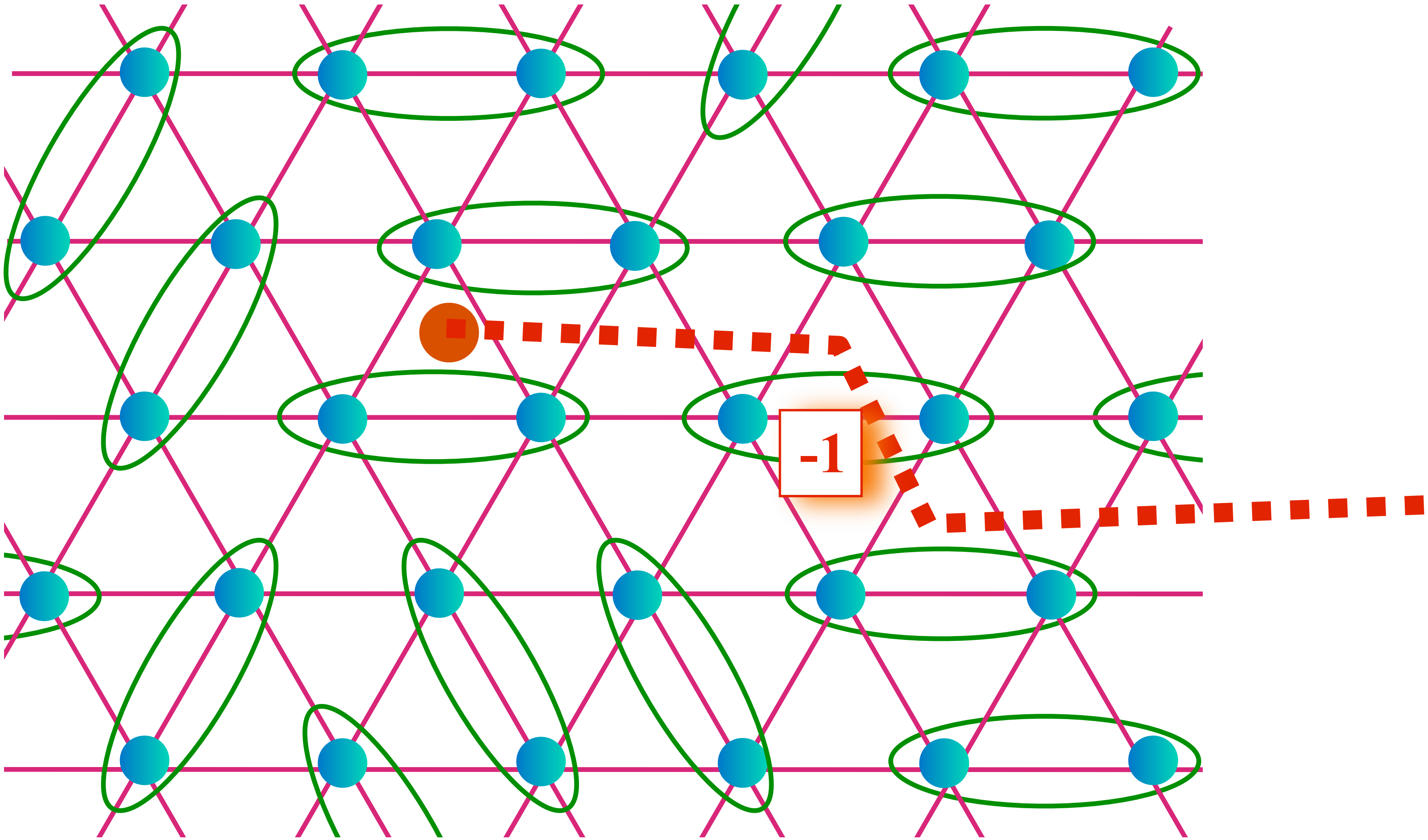

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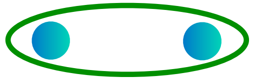

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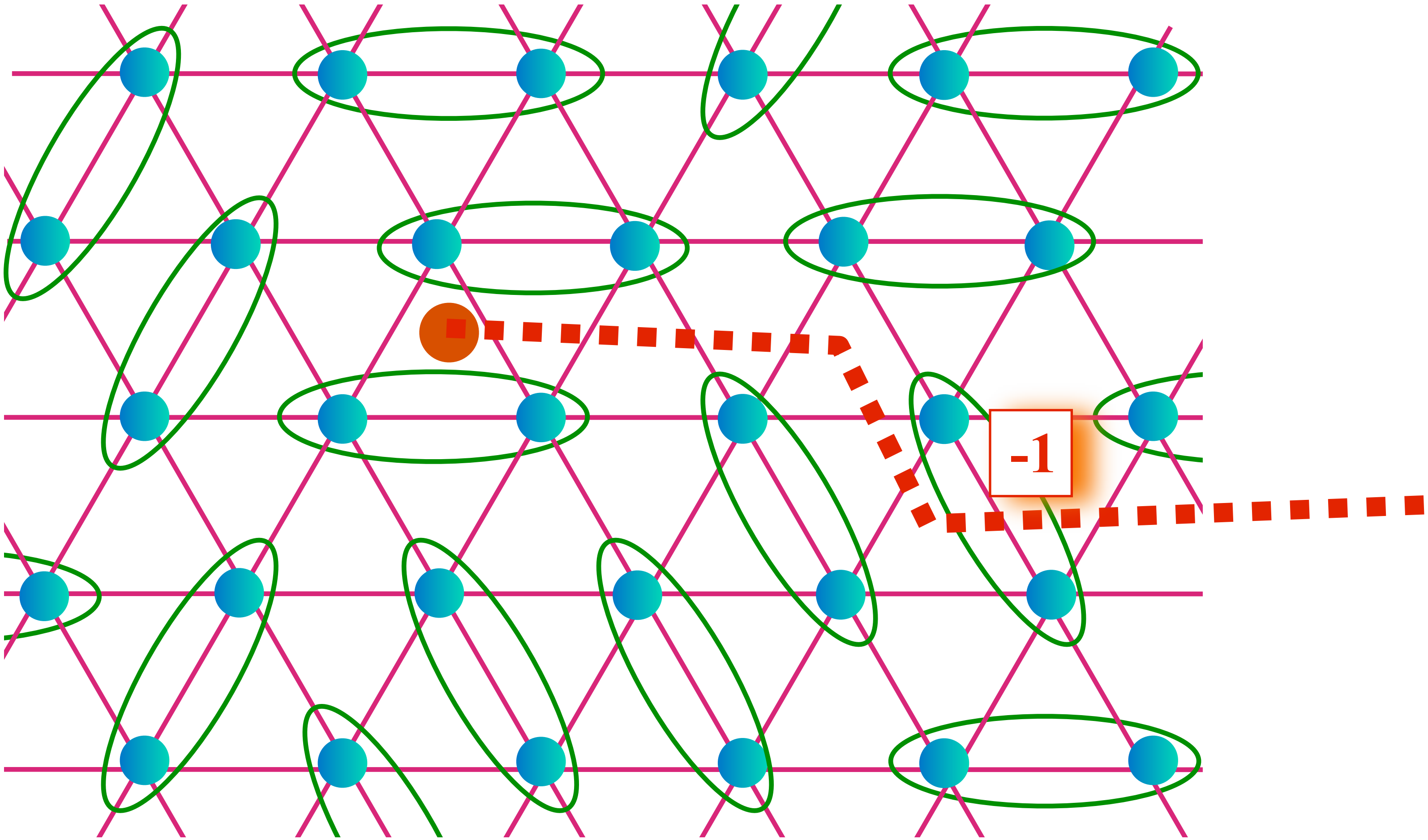

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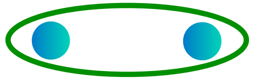

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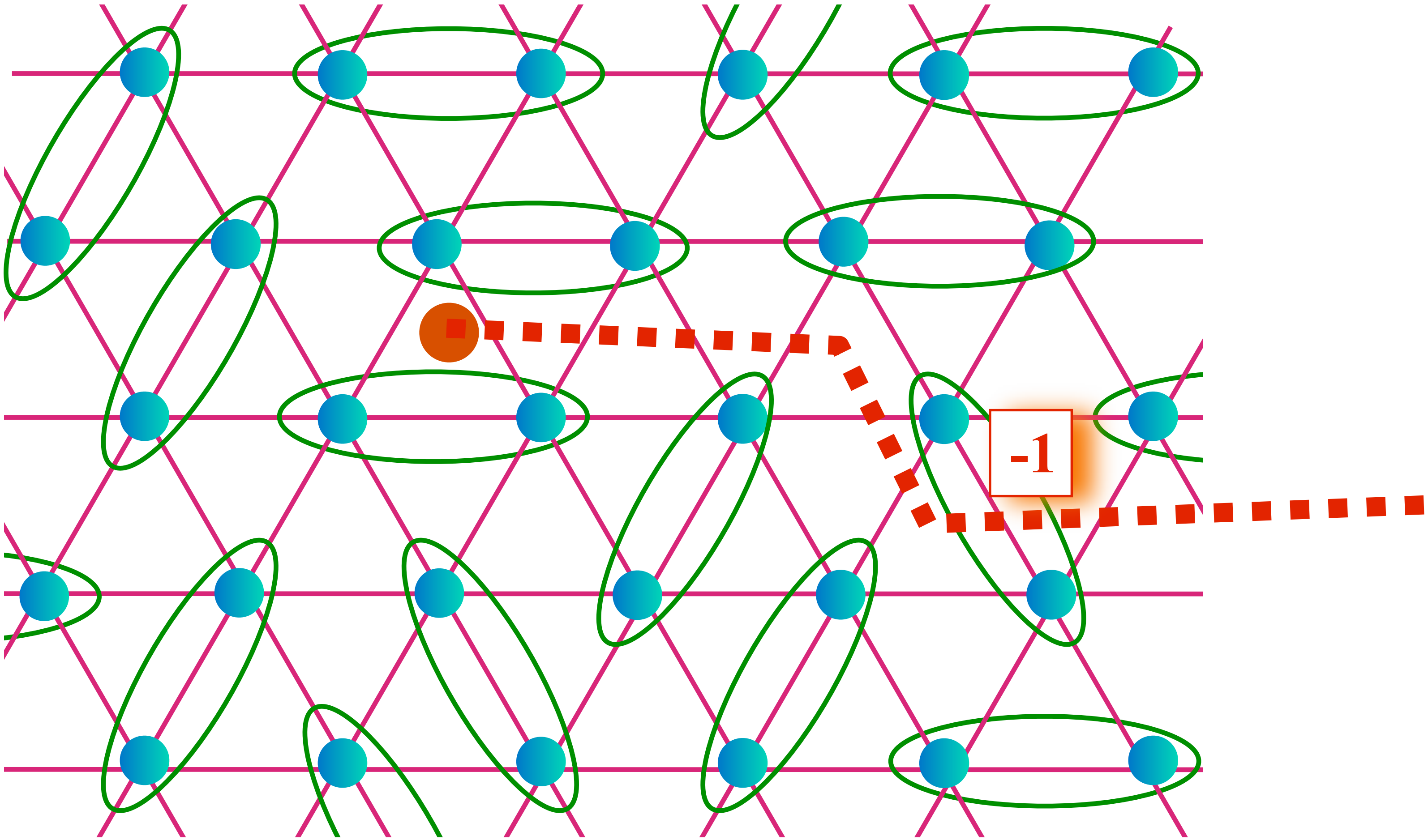

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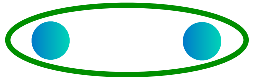

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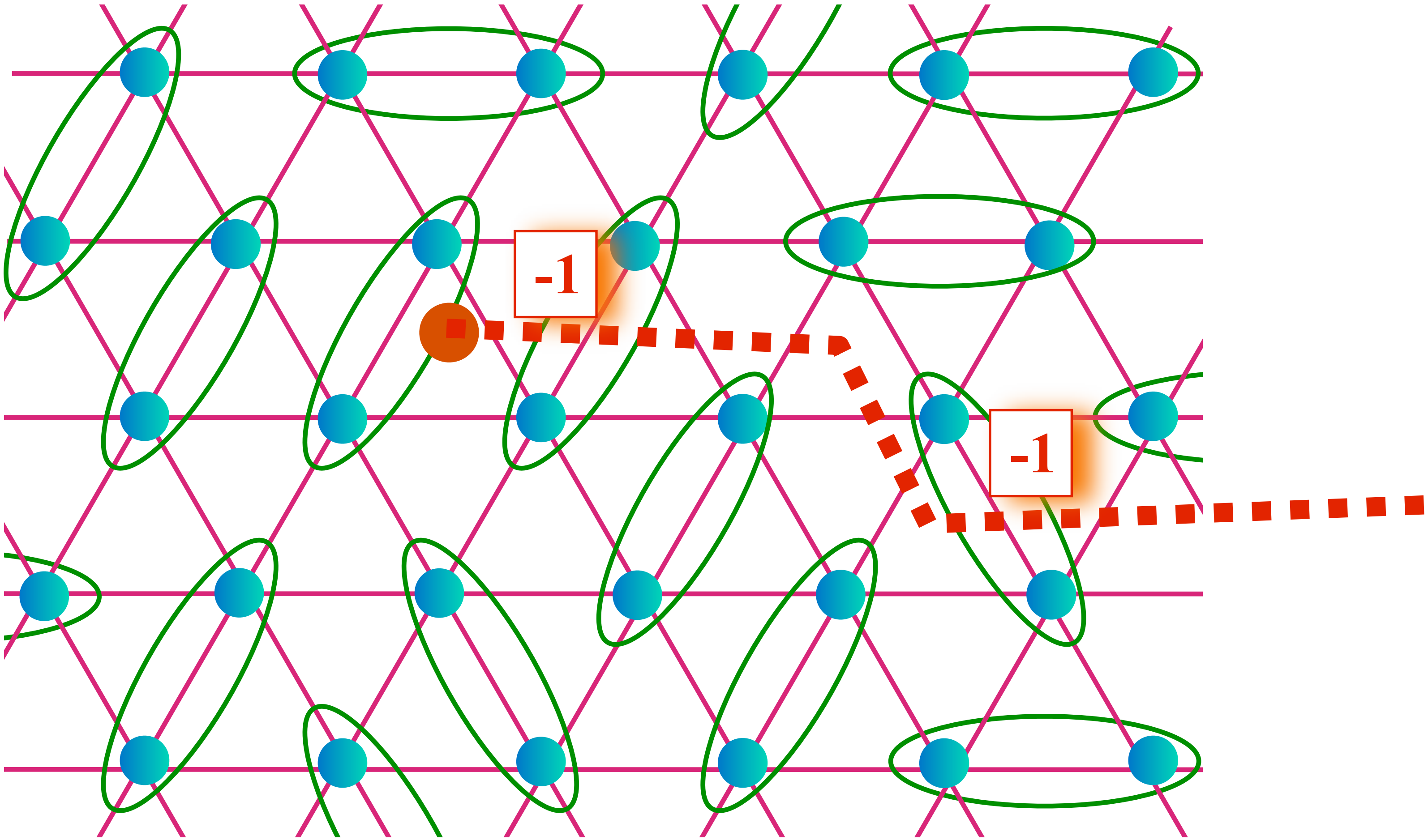

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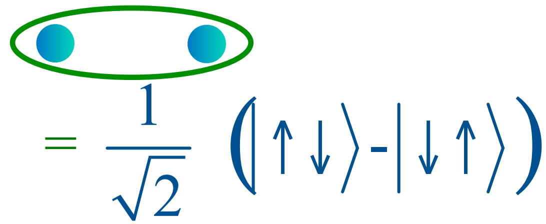
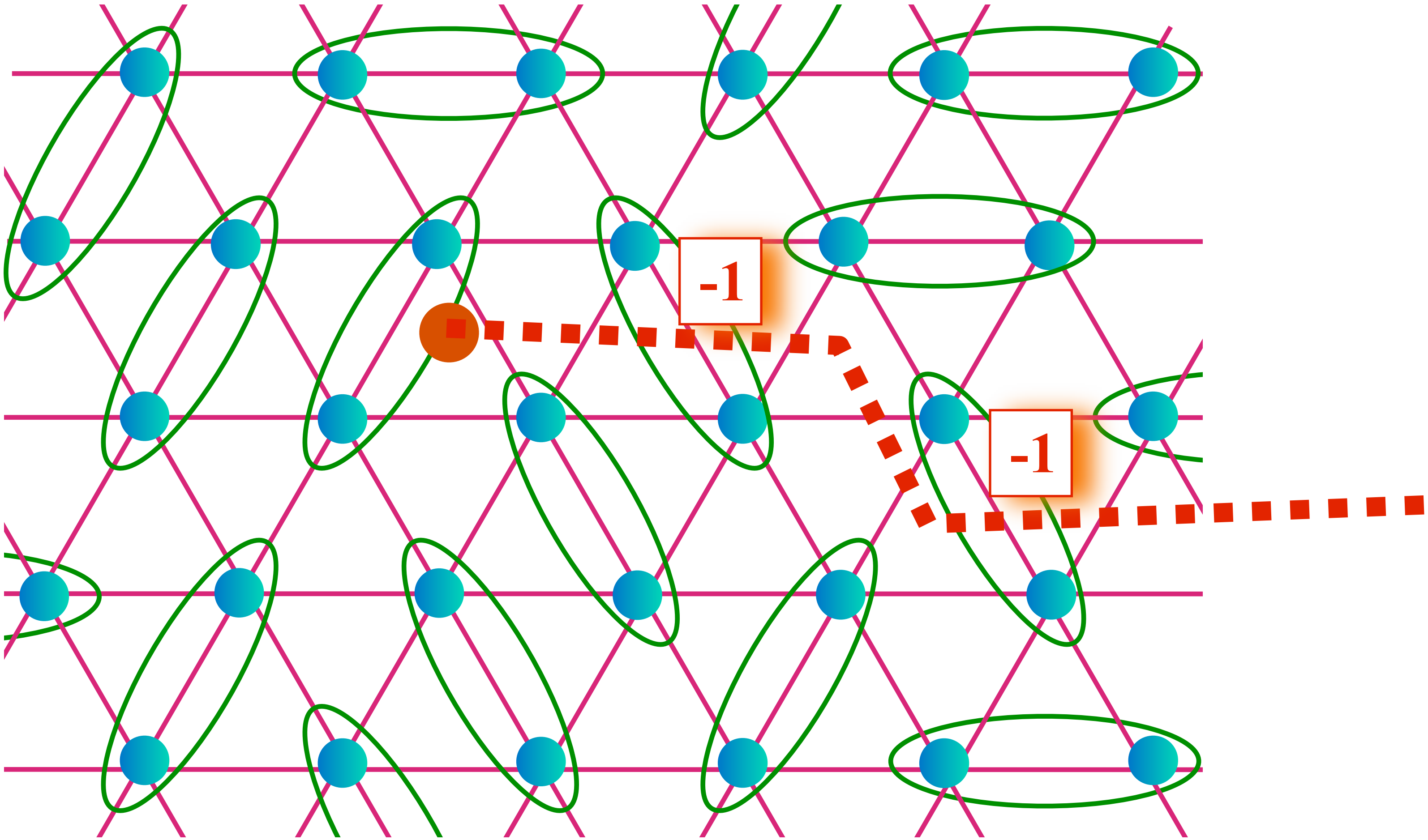




N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

Excitations of the Z_2 Spin liquid

A vison

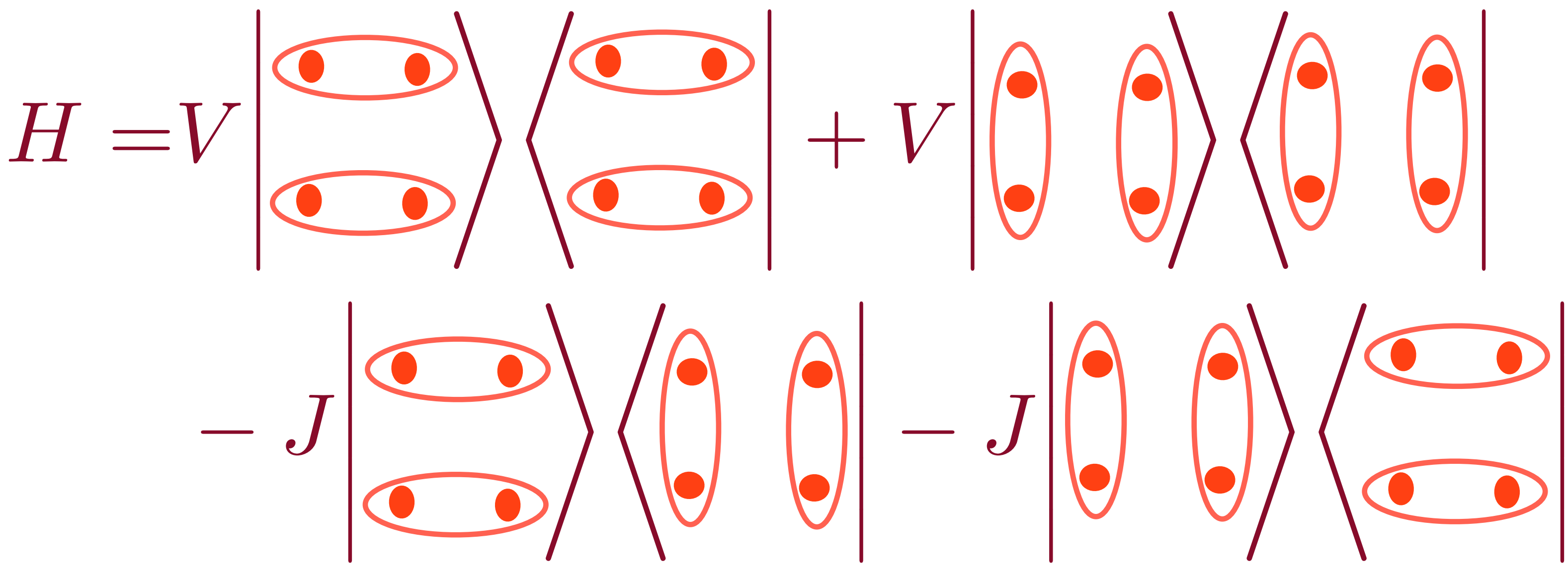
- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate
- A vison is an Abrikosov vortex in the pair condensate of spinons
- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)

Effective description of Z_2 spin liquids, their visons and valence bond solids

Quantum dimer model:

Hilbert space - set of dimer coverings of triangular/square lattice



D. Rokhsar and S.A. Kivelson, *Phys. Rev. Lett.* **61**, 2376 (1988)
R. Moessner and S. L. Sondhi, *Phys. Rev. Lett.* **86**, 1881 (2001)

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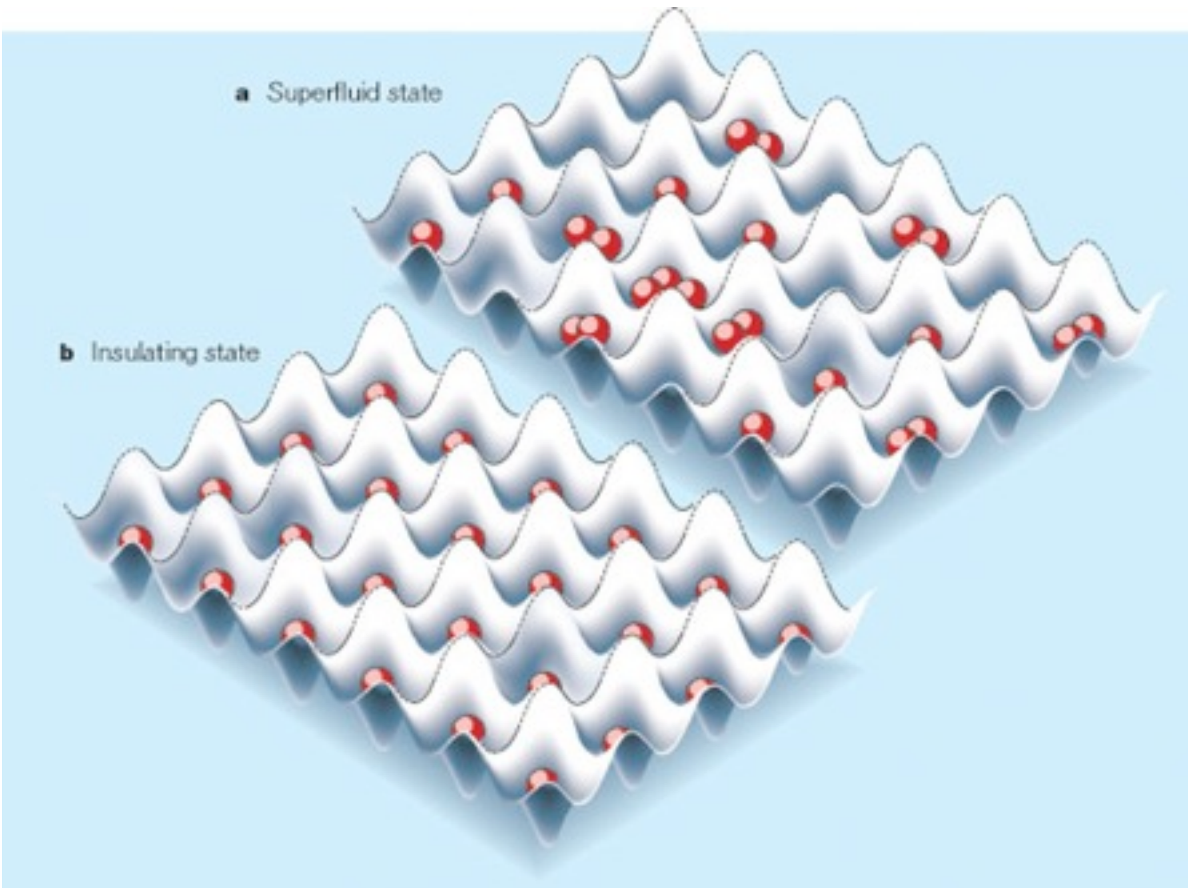
Erez
Berg

S. Sachdev, K. Sengupta, and S.M. Girvin, Phys. Rev. B 66, 075128 (2002)
S. Pielawa, T. Kitagawa, E. Berg, S. Sachdev, arXiv:1101.2897

Superfluid-insulator transition of ^{87}Rb atoms in a magnetic trap and an optical lattice potential



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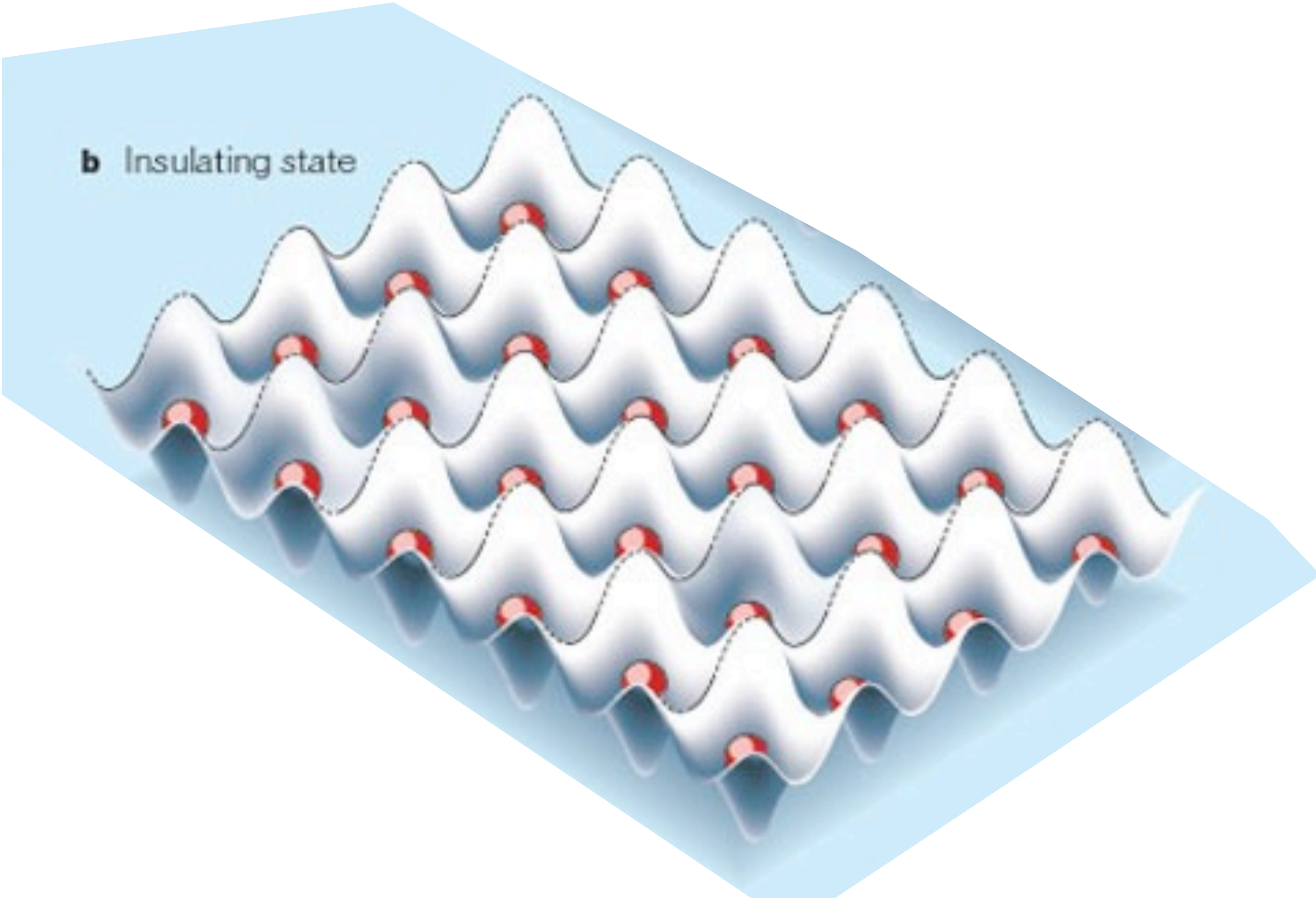

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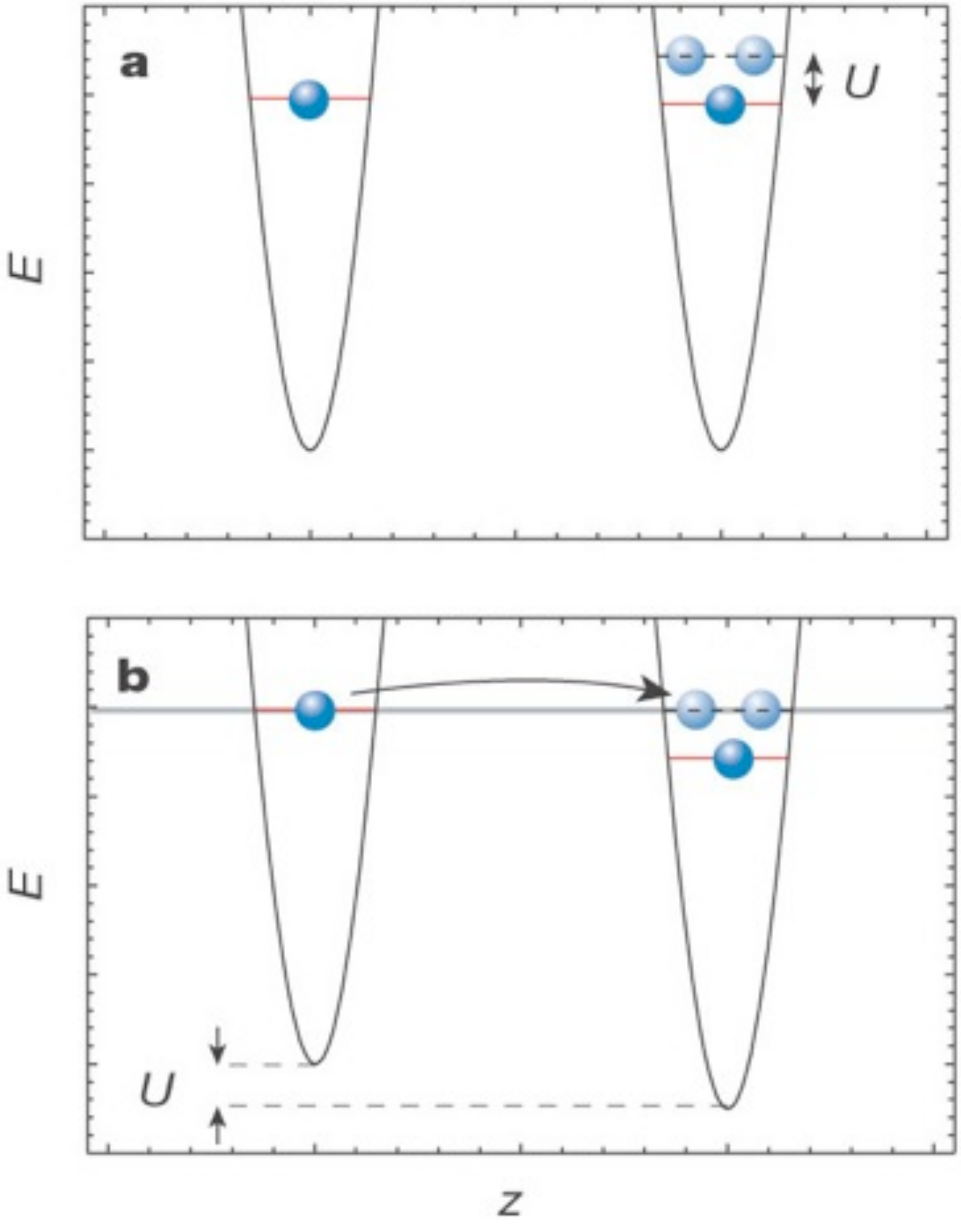
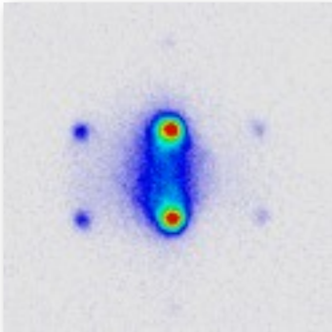
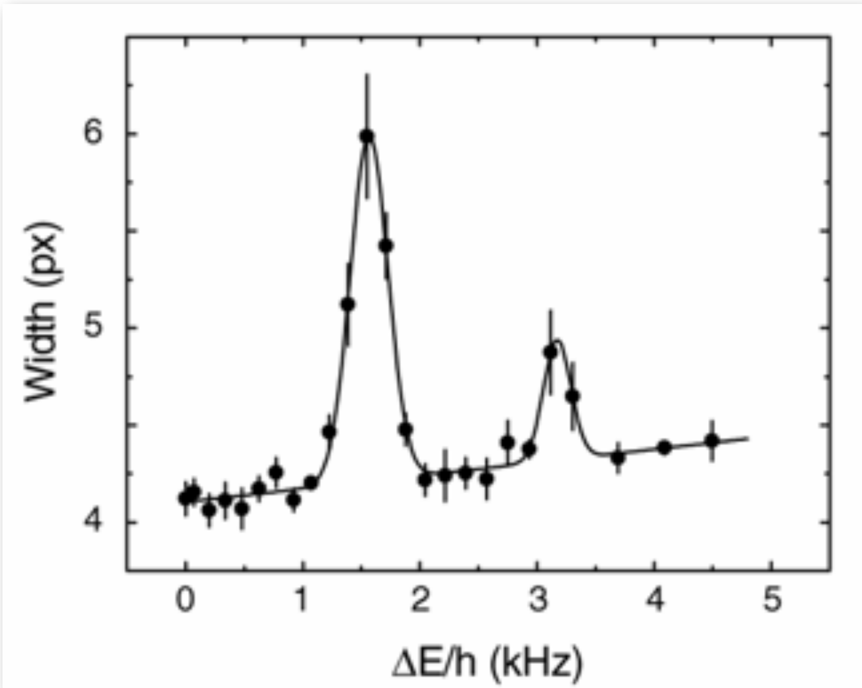
Applying an “electric” field to the Mott insulator







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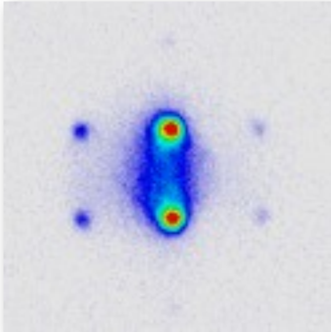
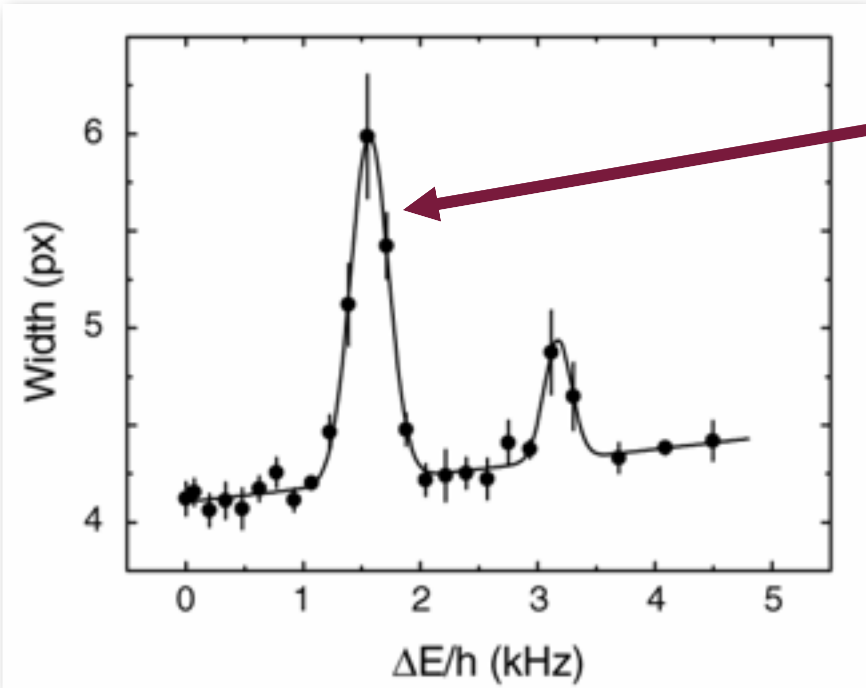
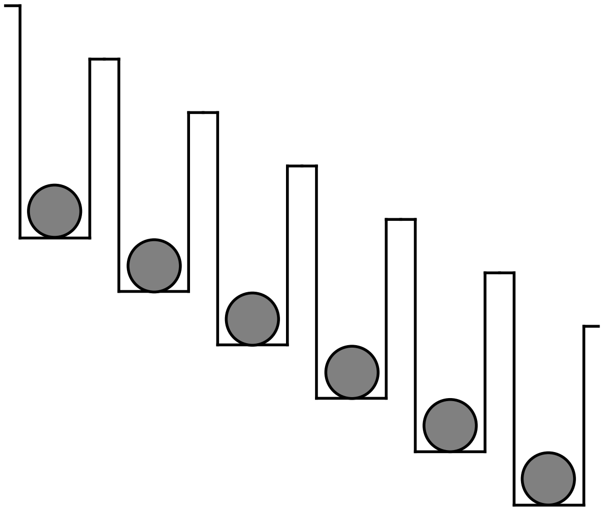
ΔE/h (kHz)





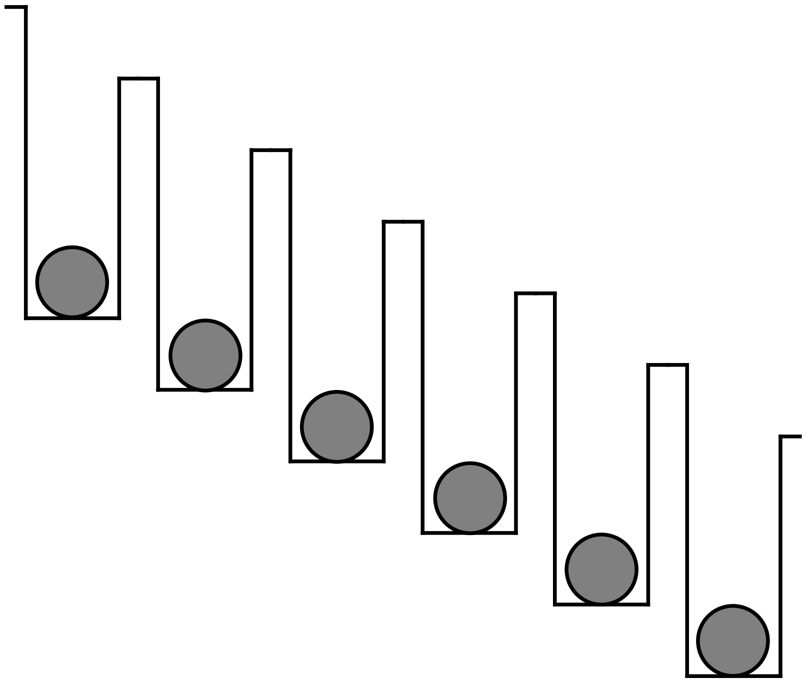
Why is there a peak (and not a threshold) when E = U ?

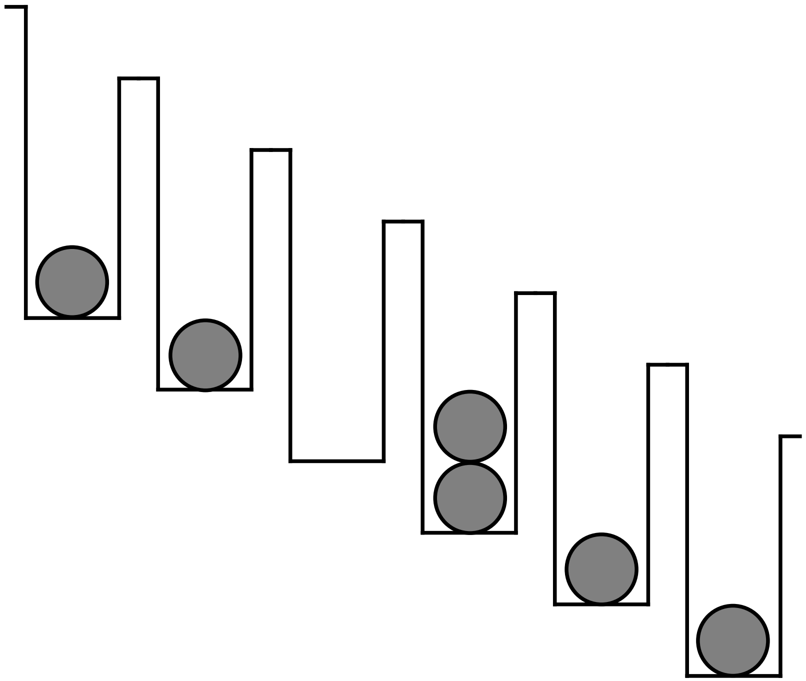


$$H = -t\sum_{\langle ij\rangle}\left(b_i^\dagger b_j + b_j^\dagger b_i\right) + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) - \sum_i \boldsymbol{E}\cdot\boldsymbol{r}_i n_i$$

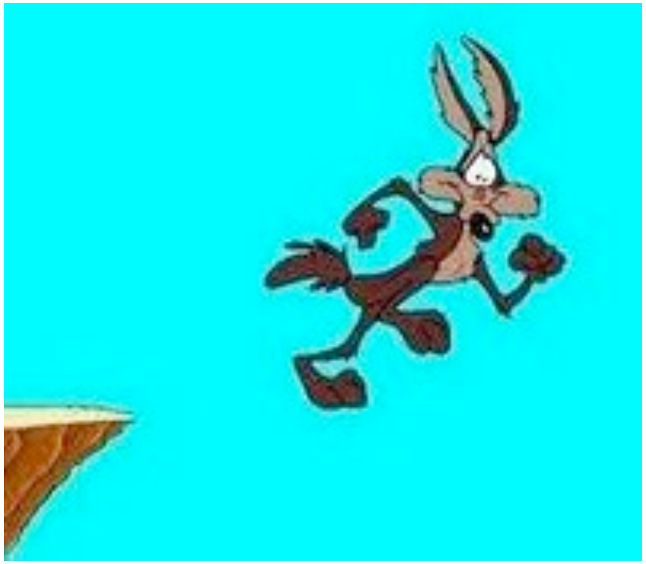
$$n_i = b_i^\dagger b_i$$

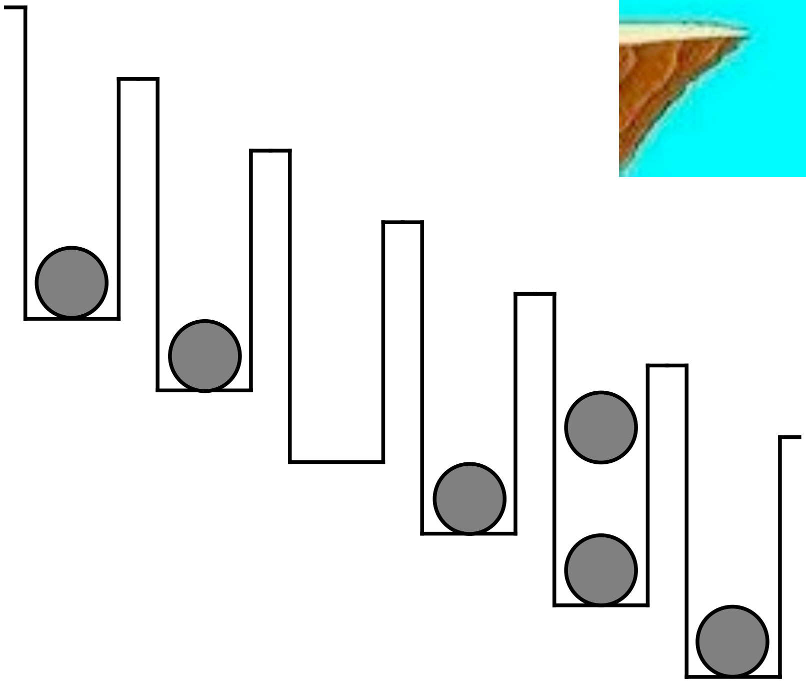
$$\left|U - E\right|, t \ll E, U$$



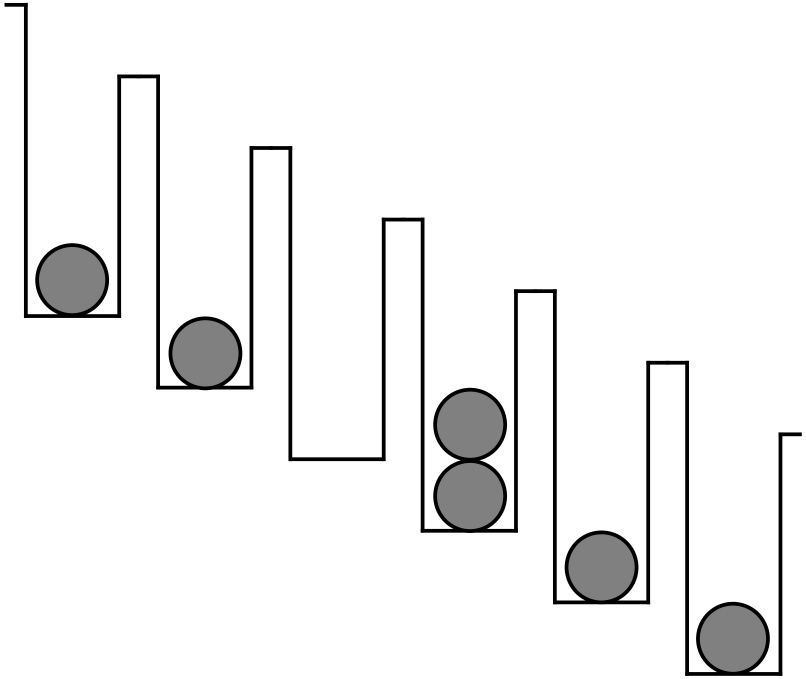


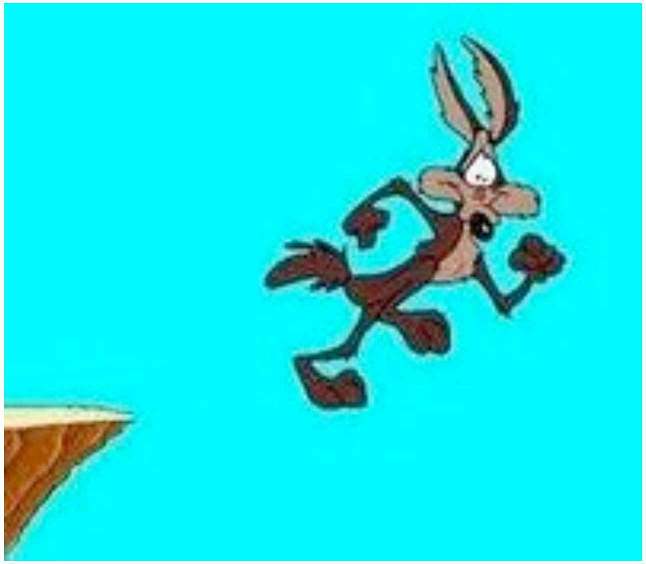
Resonant transition when $E \approx U$

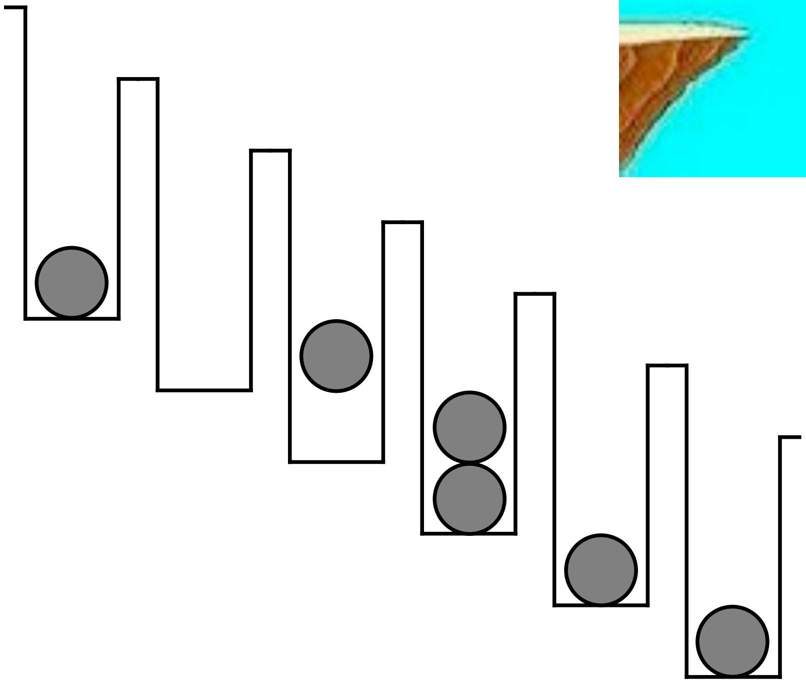




Virtual state







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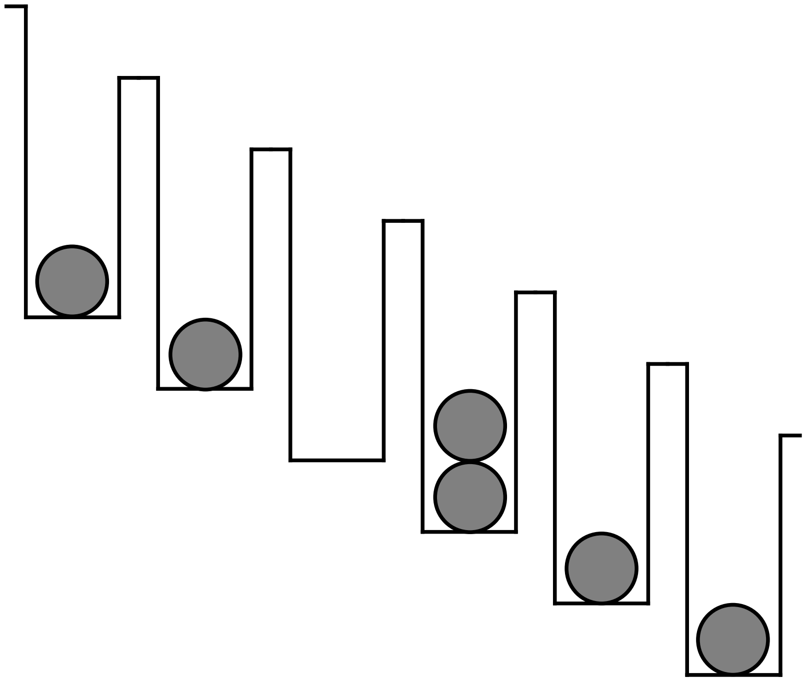

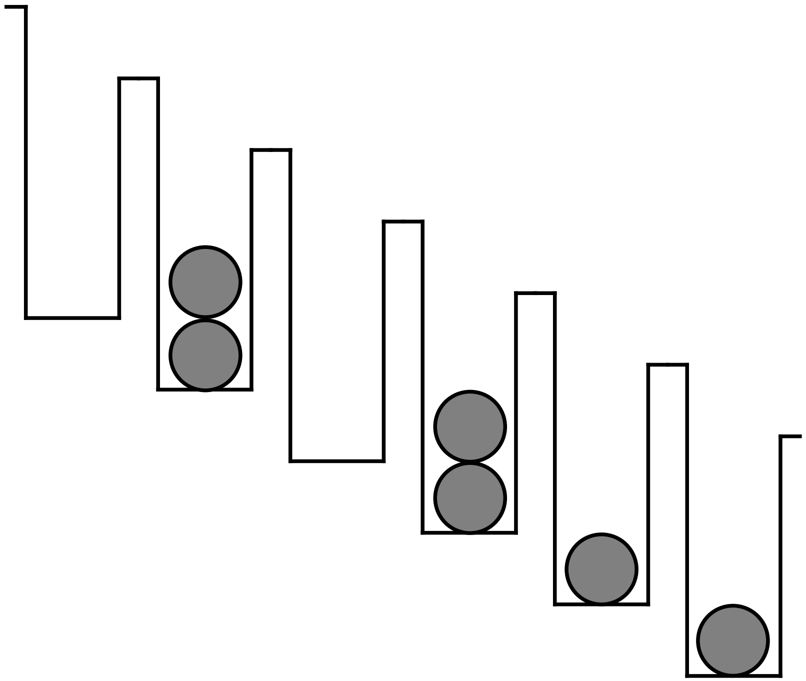

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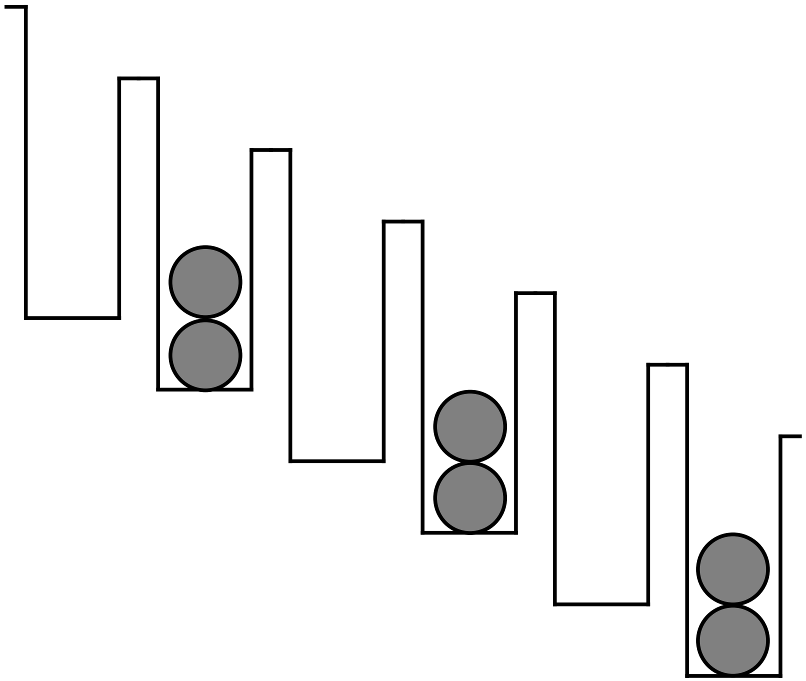
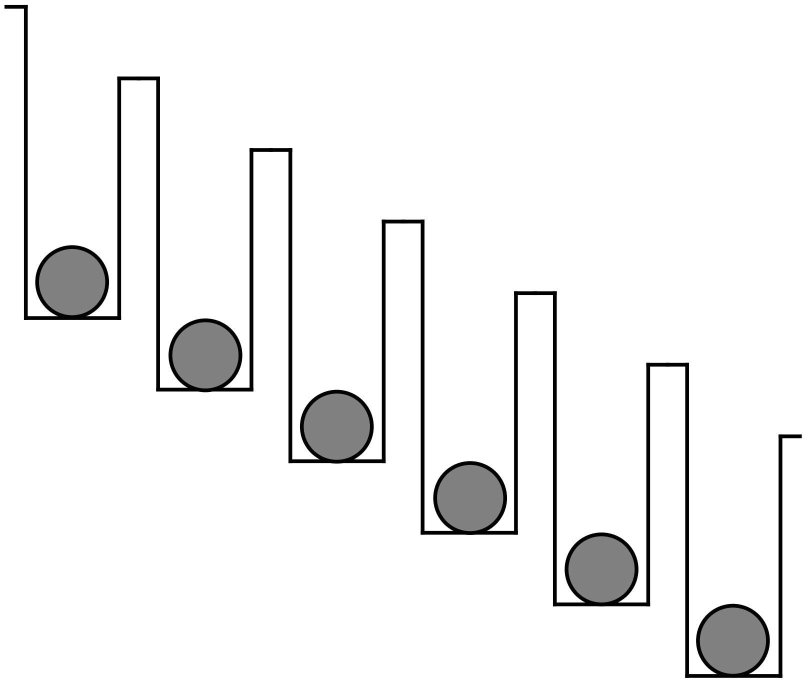
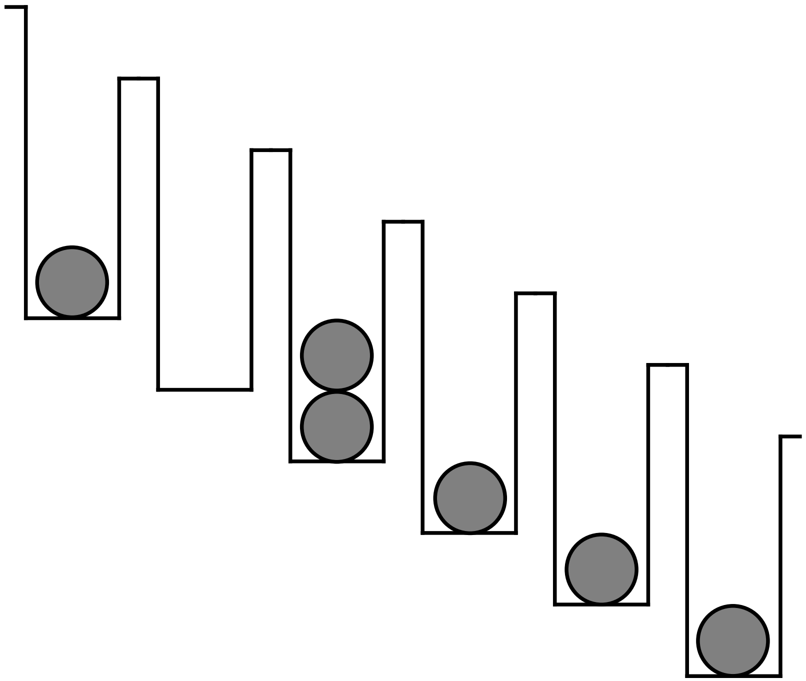
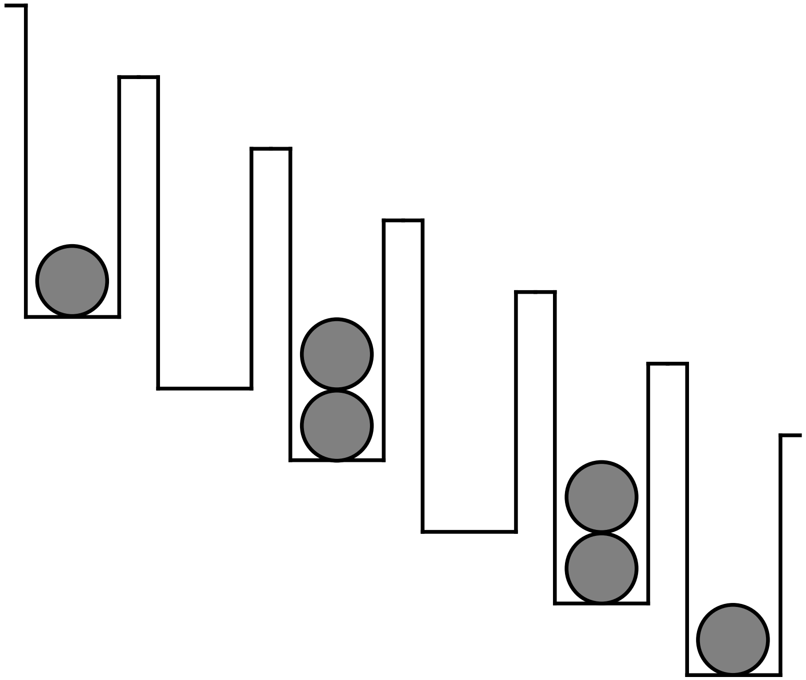
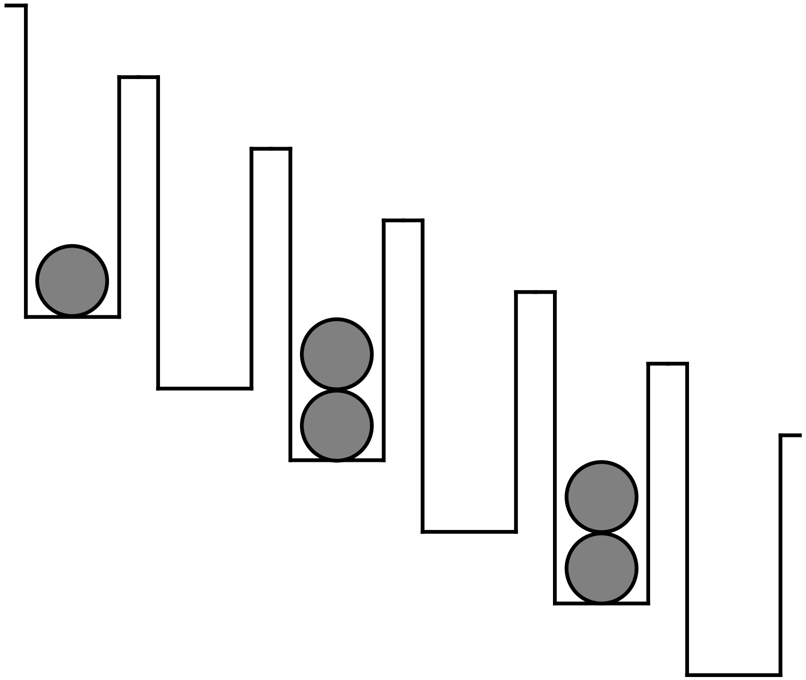


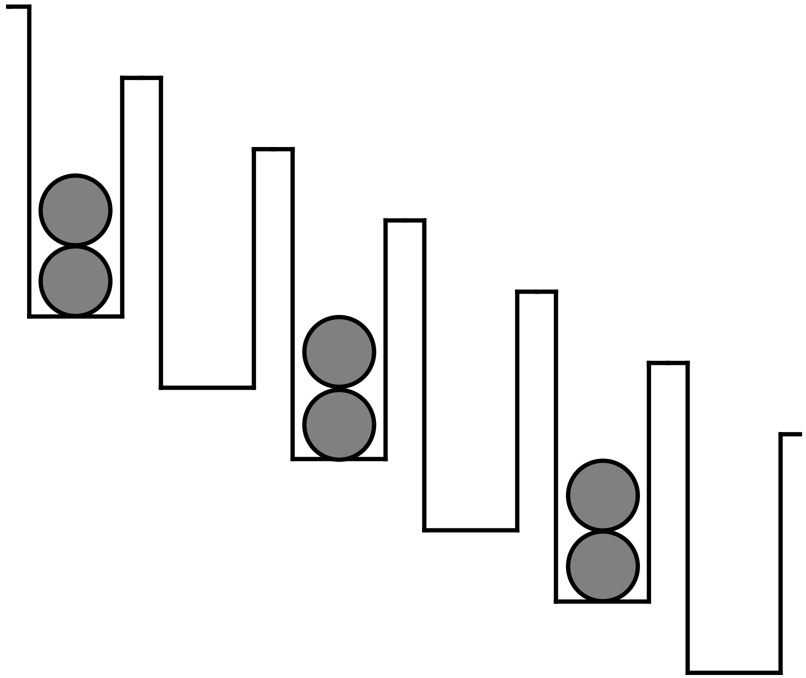
Resonant transition when $E \approx U$



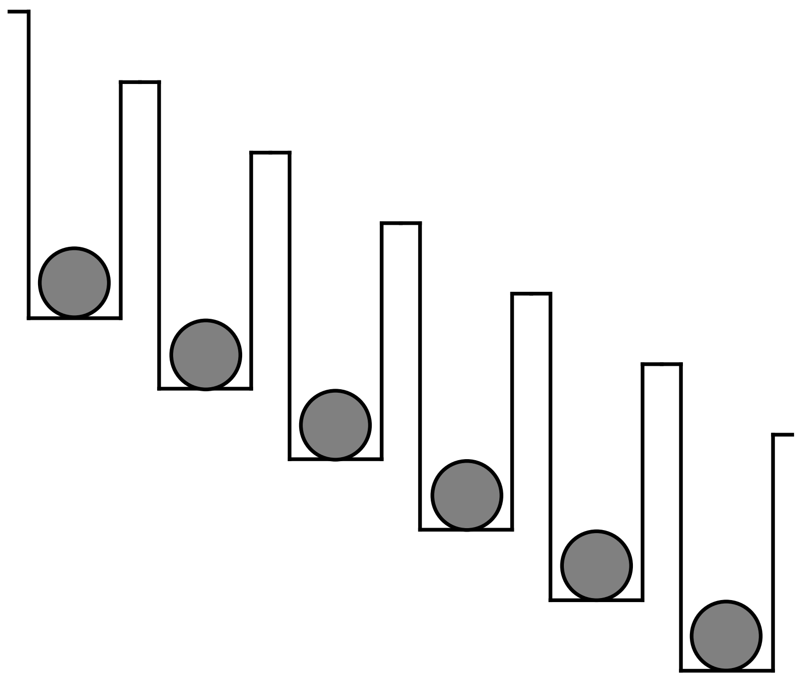








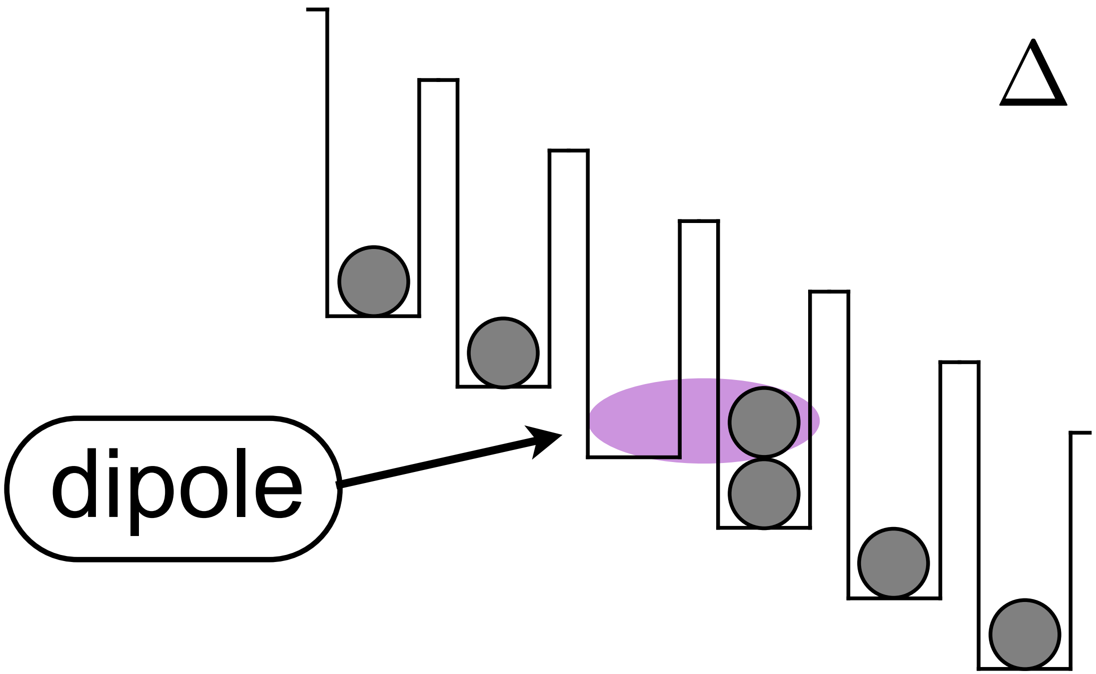
Hamiltonian of resonant subspace



Hamiltonian of resonant subspace

$$\hat{H} = -\sqrt{2}t \sum_i \left(\hat{d}_i^\dagger + \hat{d}_i \right) + \Delta \sum_i \hat{d}_i^\dagger \hat{d}_i$$

$$\Delta = U - E$$



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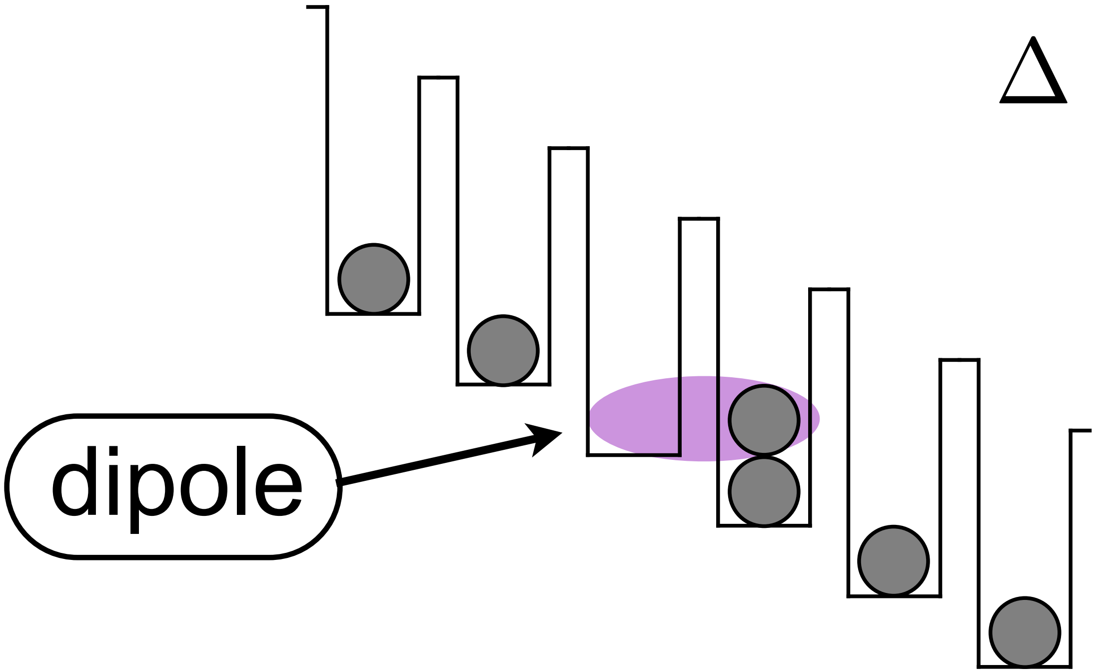


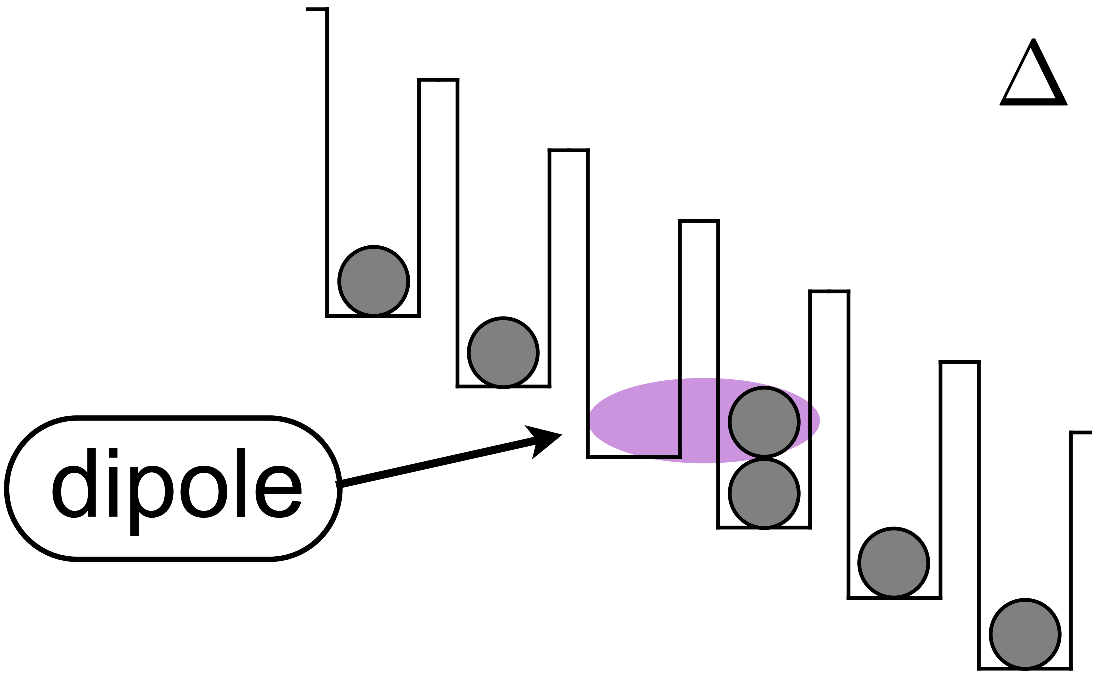
Constraints:

max one dipole per site: $\hat{d}_i^\dagger \hat{d}_i \leq 1$

no neighboring dipoles: $\hat{d}_i^\dagger \hat{d}_i \hat{d}_{i+1}^\dagger \hat{d}_{i+1} = 0$

Hamiltonian of resonant subspace

$$\hat{H} = -\sqrt{2}t \sum_i \left(\hat{d}_i^\dagger + \hat{d}_i \right) + \Delta \sum_i \hat{d}_i^\dagger \hat{d}_i$$



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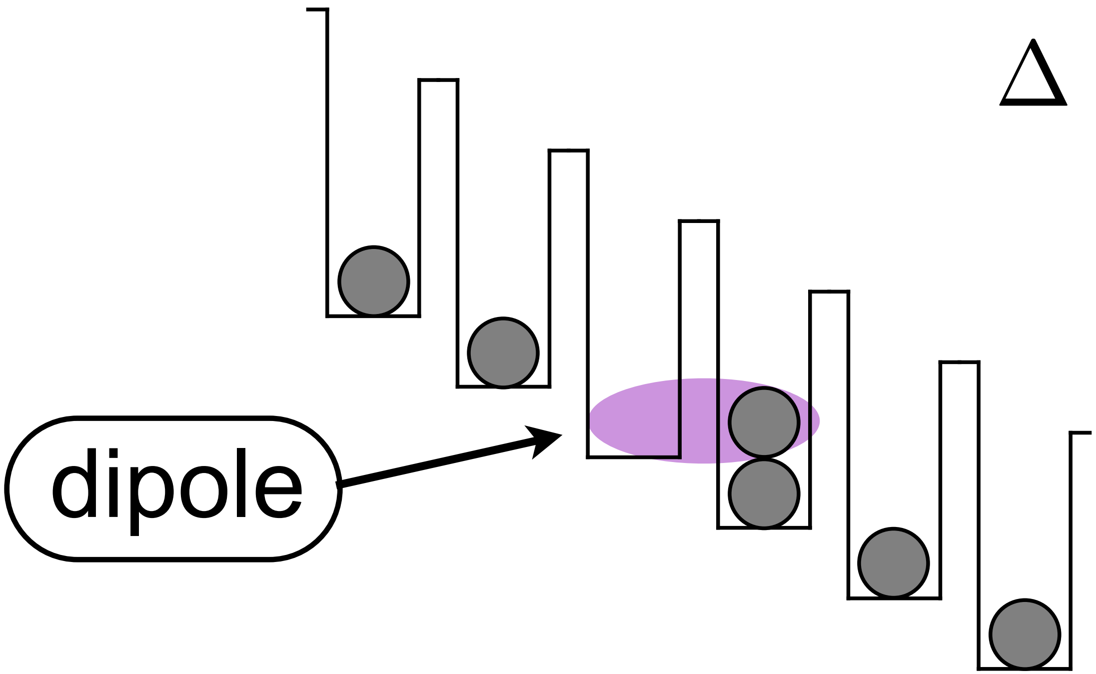


Strong off-site quantum correlations

Constraints:

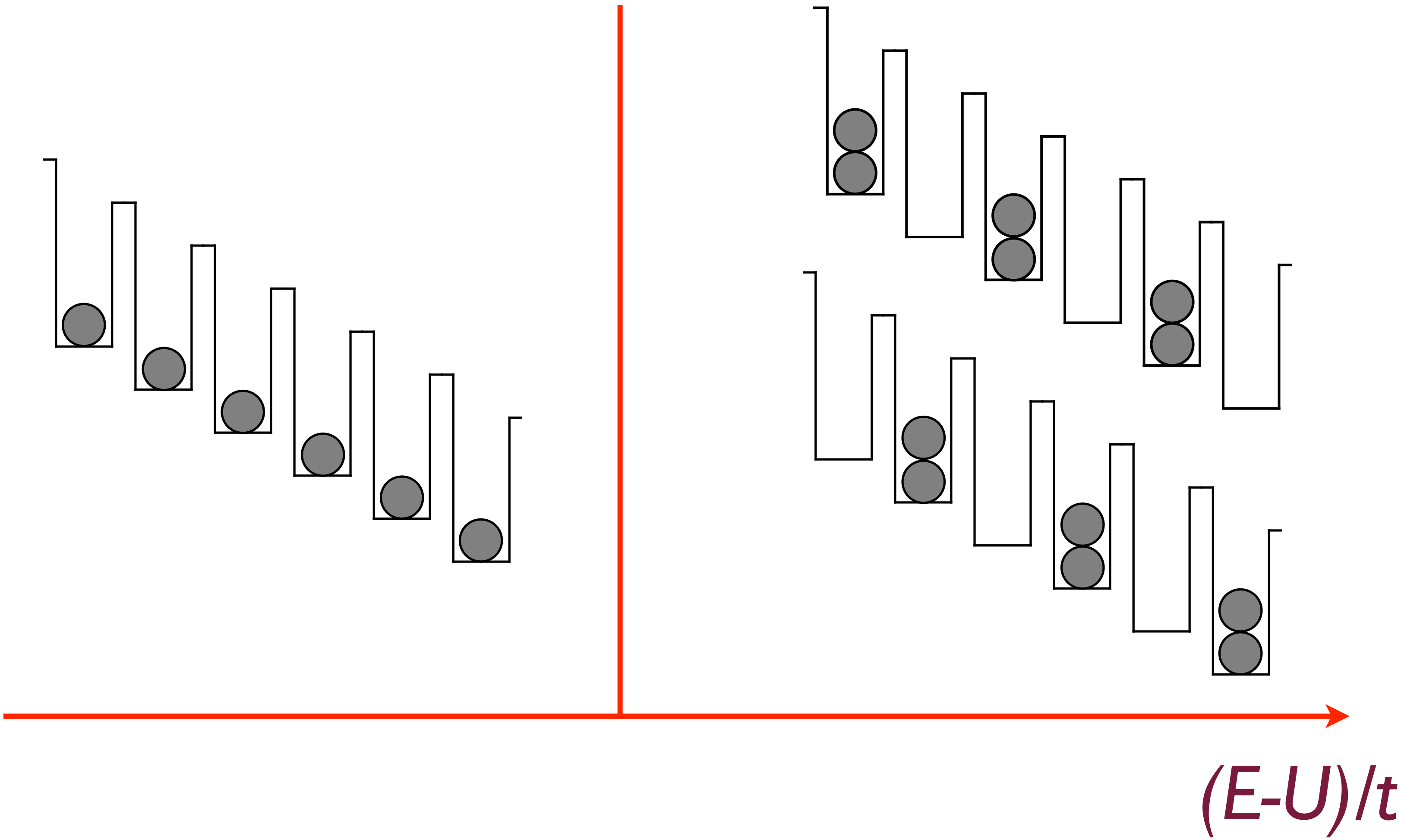
max one dipole per site:

$$\hat{d}_i^\dagger \hat{d}_i \leq 1$$

no neighboring dipoles:

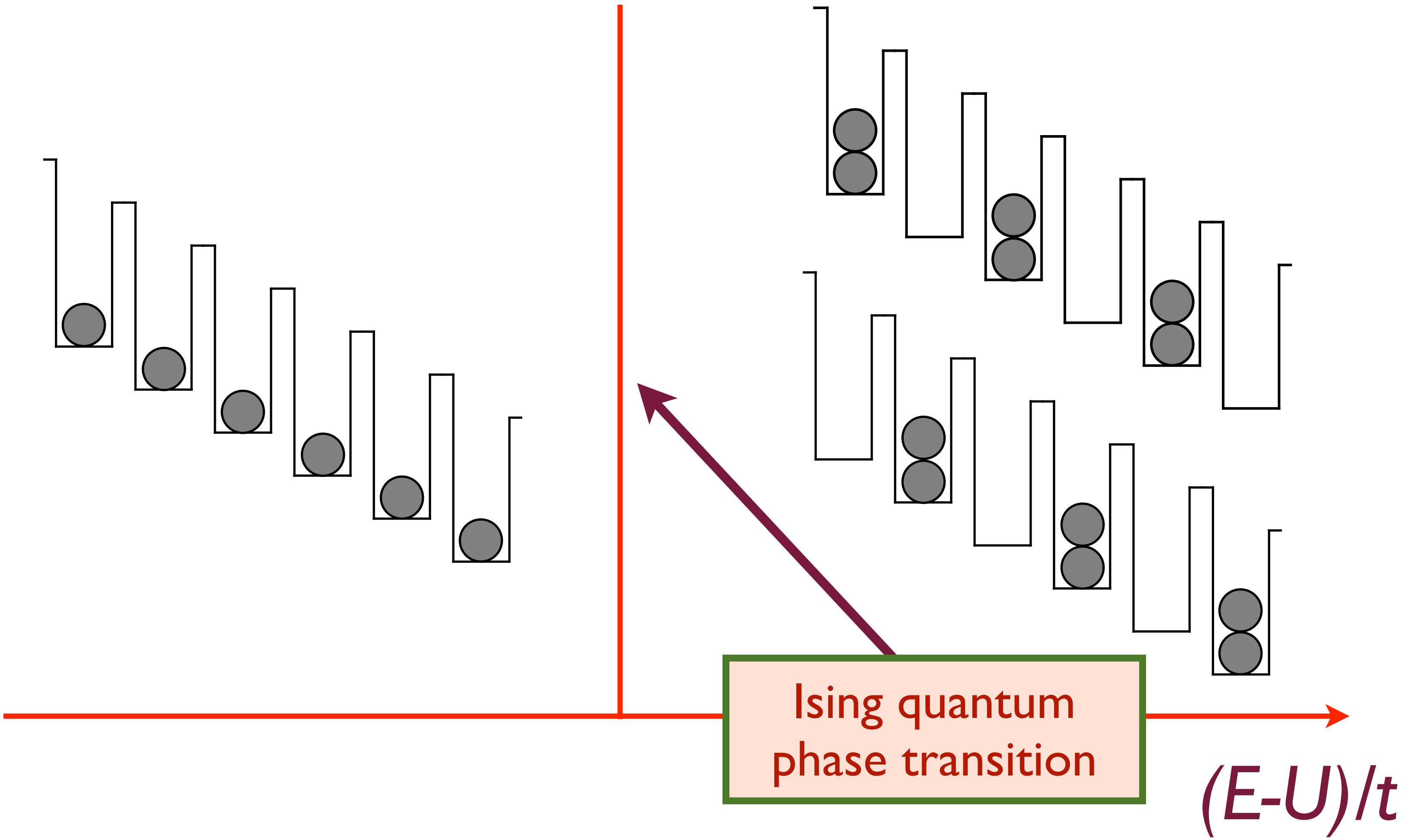
$$\hat{d}_i^\dagger \hat{d}_i \hat{d}_{i+1}^\dagger \hat{d}_{i+1} = 0$$

Phase diagram of dipole model



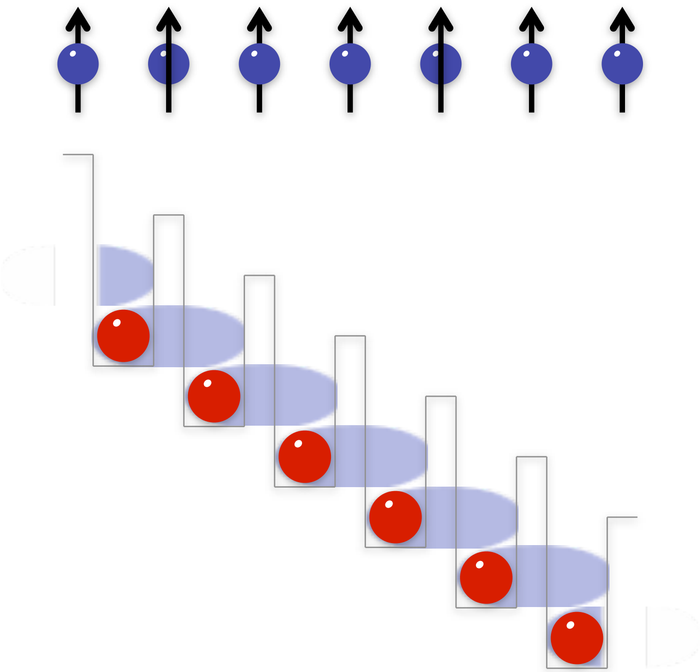
S. Sachdev, K. Sengupta, and S.M. Girvin, Phys. Rev. B 66, 075128 (2002)

Phase diagram of dipole model

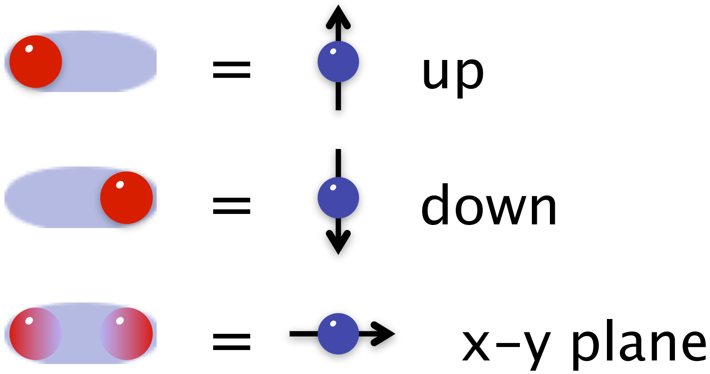


S. Sachdev, K. Sengupta, and S.M. Girvin, Phys. Rev. B 66, 075128 (2002)

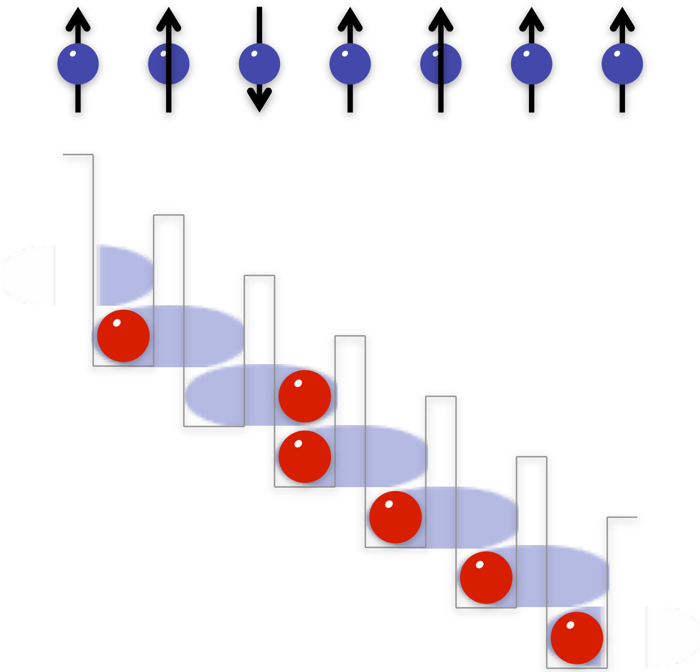
Hamiltonian of resonant subspace



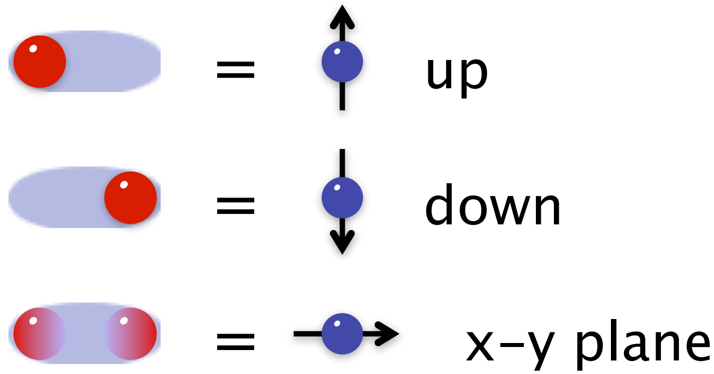
Effective Hamiltonian can be written as spin model



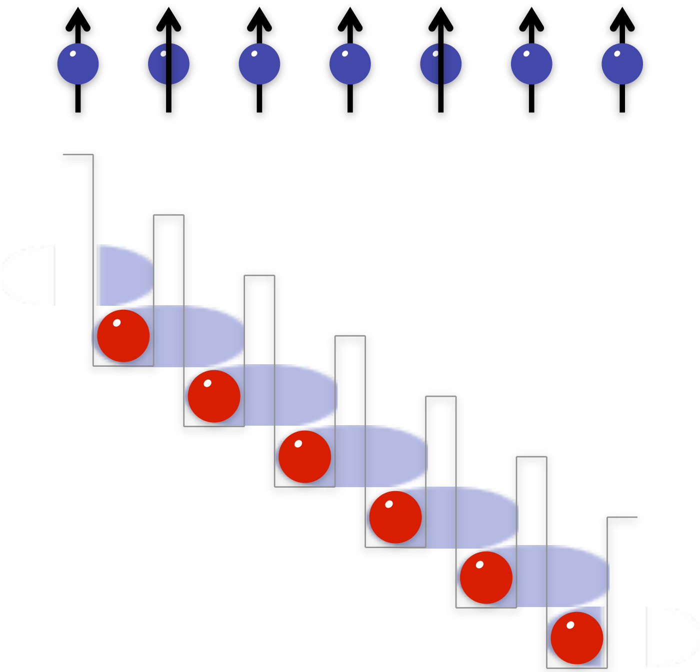
Hamiltonian of resonant subspace



Effective Hamiltonian can be written as spin model



Hamiltonian of resonant subspace



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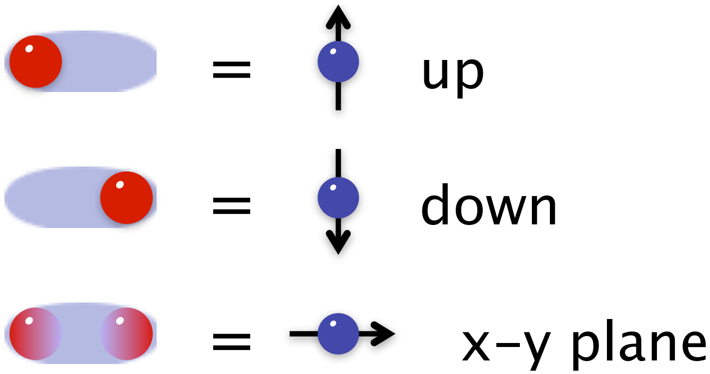
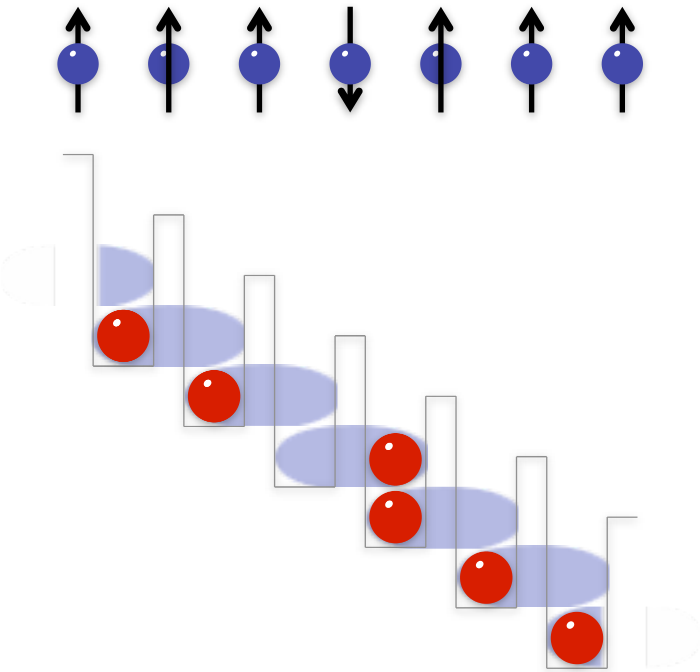


Hamiltonian of resonant subspace



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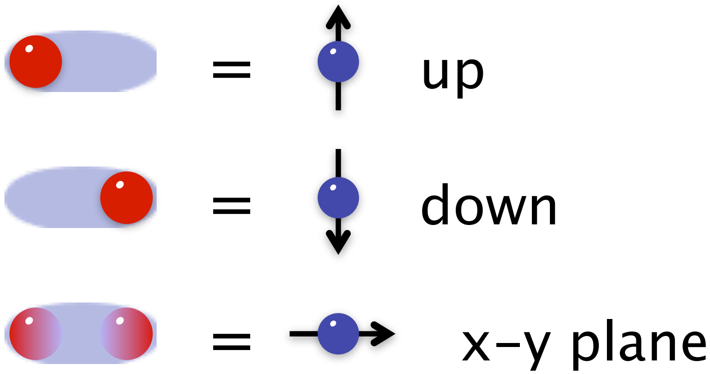

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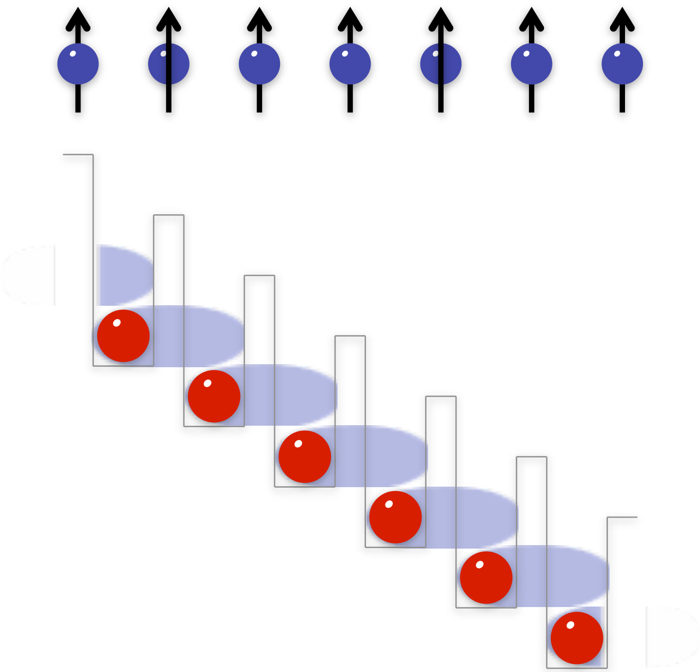

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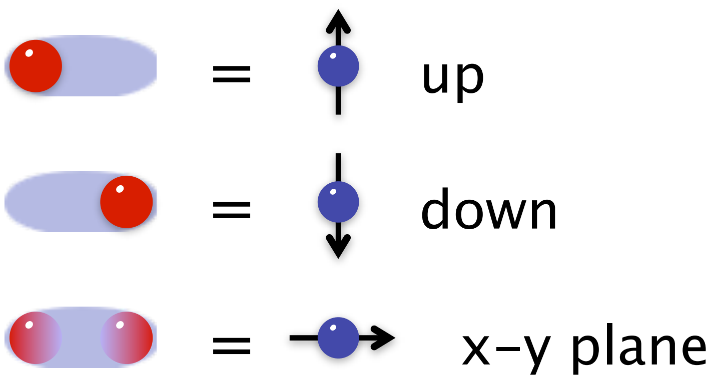
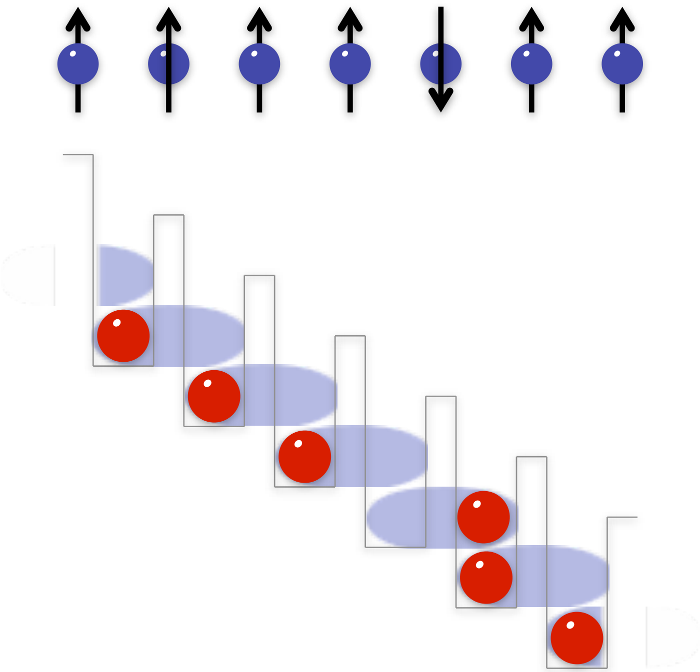


Hamiltonian of resonant subspace



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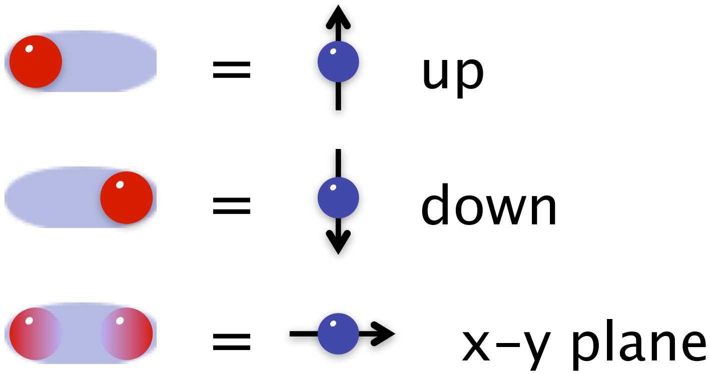
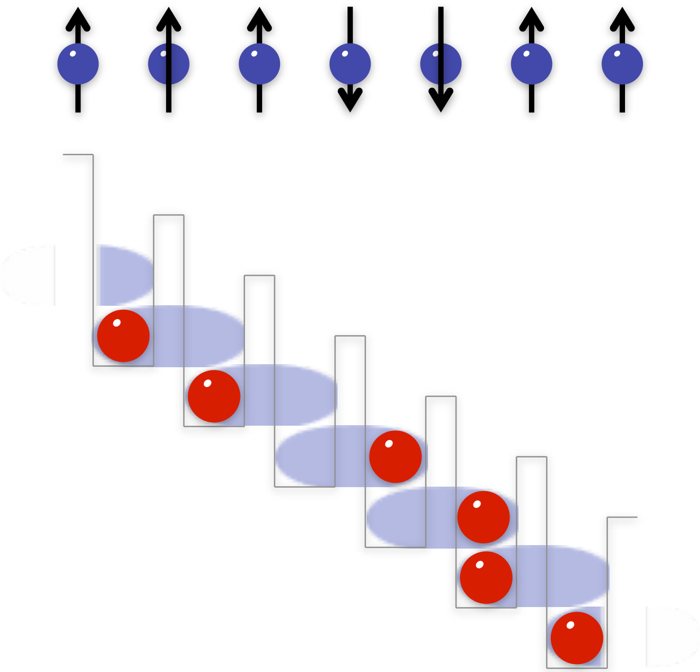
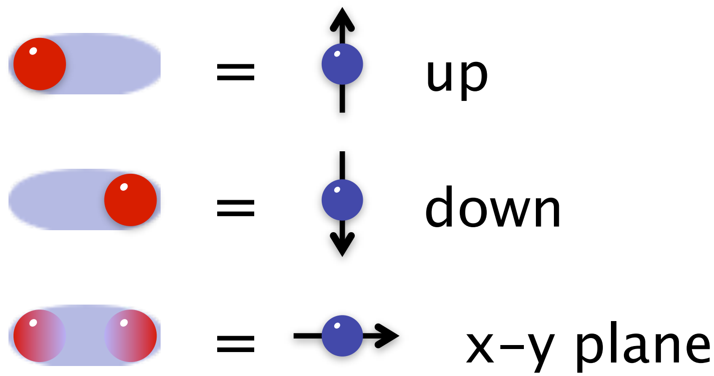


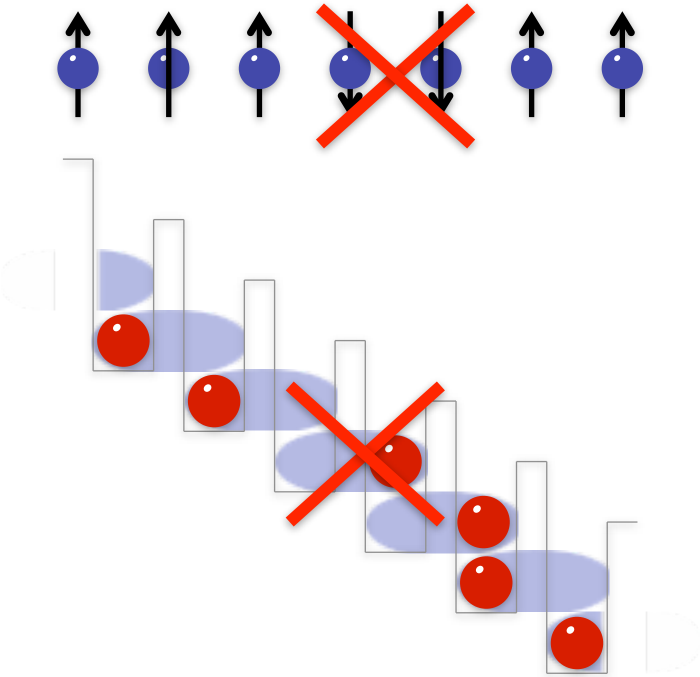
Hamiltonian of resonant subspace



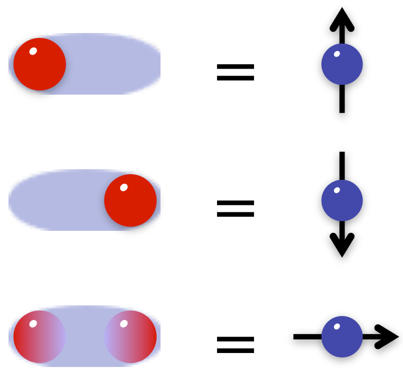
Effective Hamiltonian can be written as spin model



Hamiltonian of resonant subspace



Effective Hamiltonian can be written as spin model

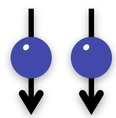


up

down

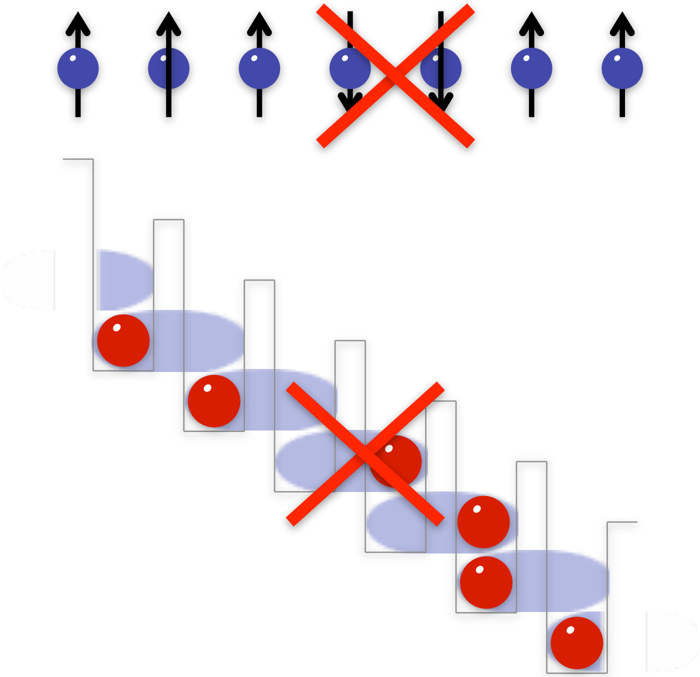
x−y plane

Constraint:

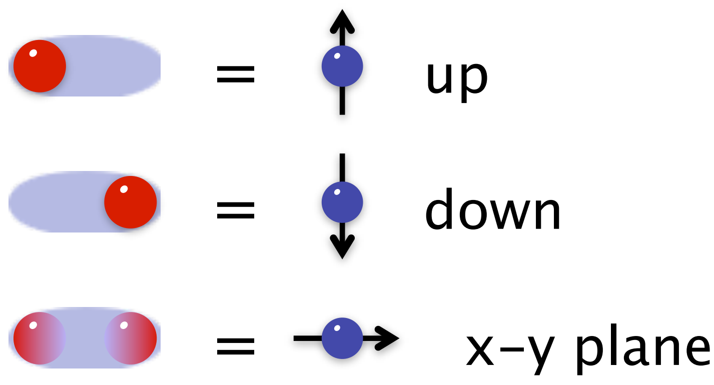


forbidden !

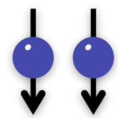
Hamiltonian of resonant subspace



Effective Hamiltonian can be written as spin model



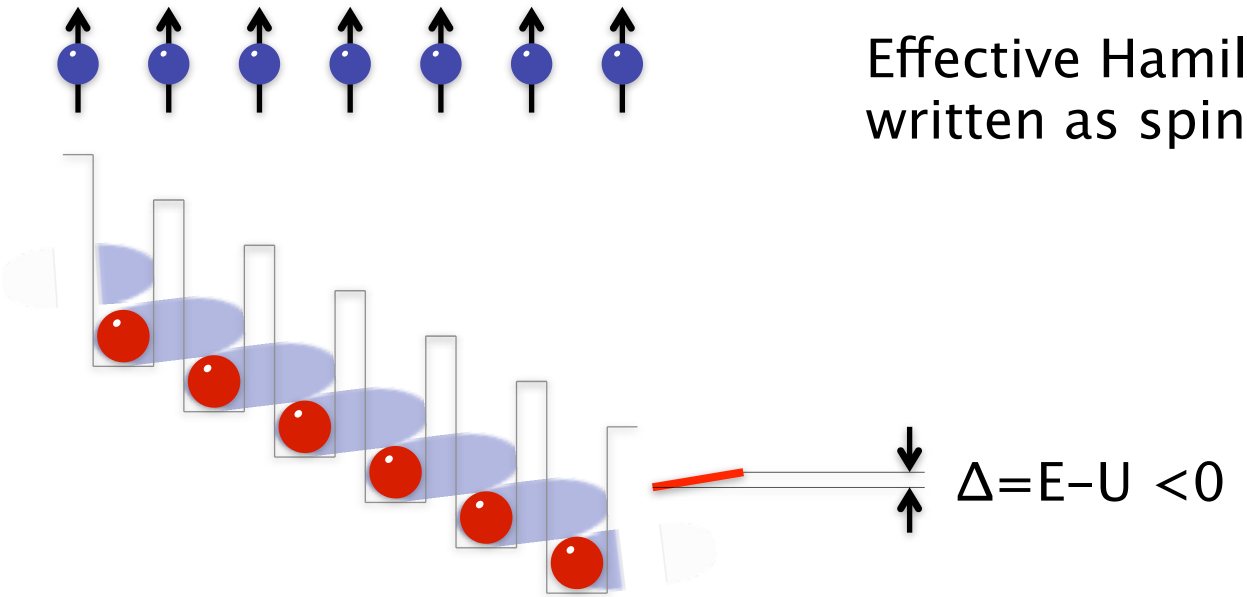
Constraint:



forbidden !

Include a term $J \sum_i \left(S_i^z - \frac{1}{2}\right)\left(S_{i+1}^z - \frac{1}{2}\right)$
and send $J \to \infty$. Infinite exchange interaction !

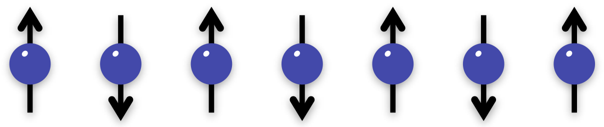
Hamiltonian of resonant subspace



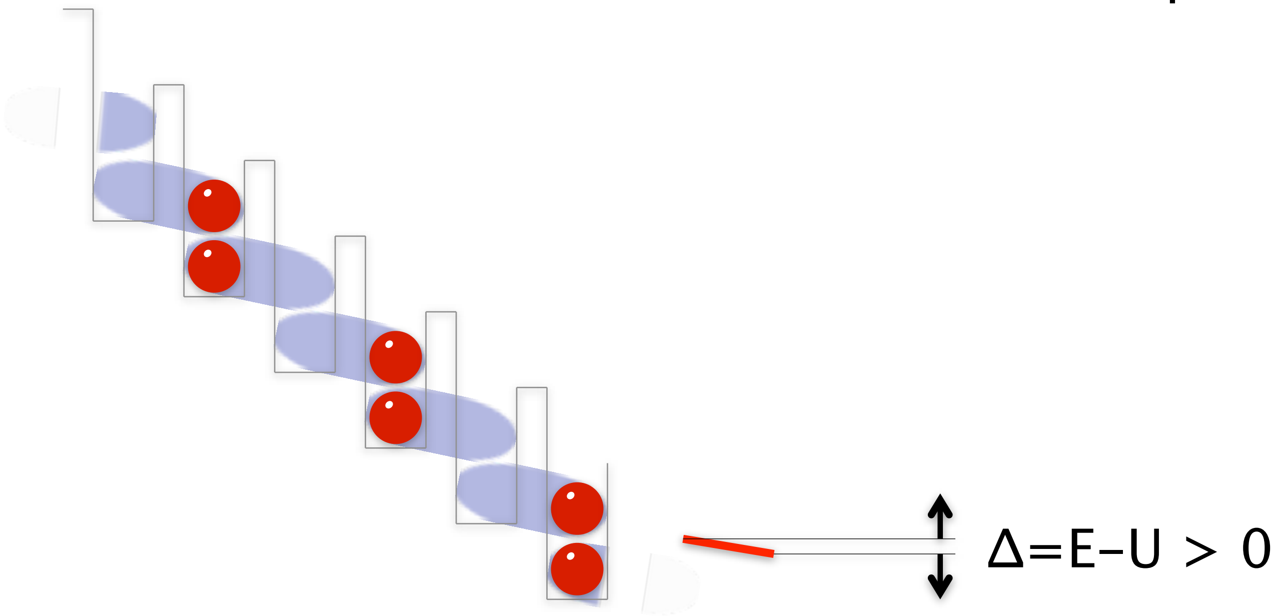
Effective Hamiltonian can be written as spin model

Paramagnetic state

Hamiltonian of resonant subspace

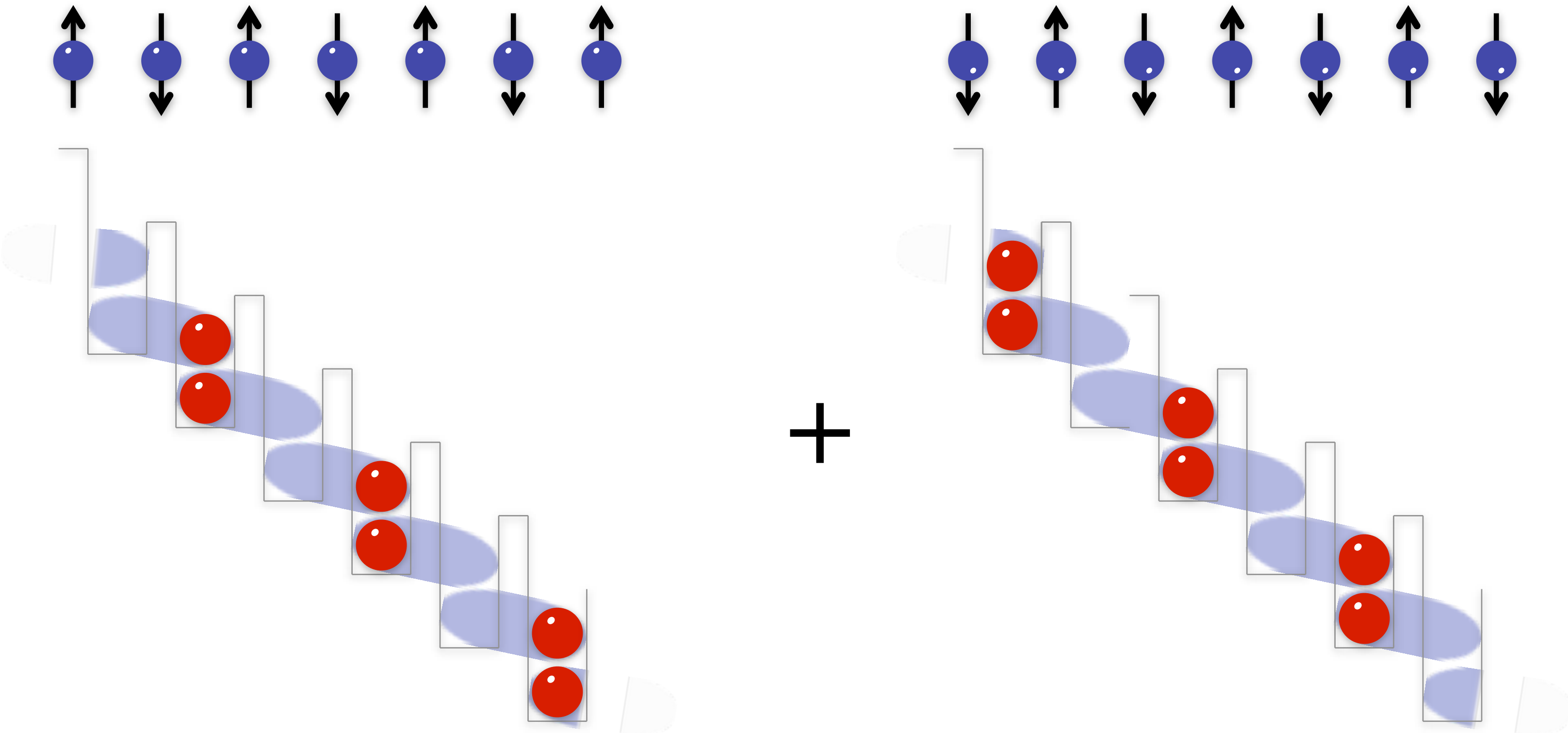


Effective Hamiltonian can be written as spin model



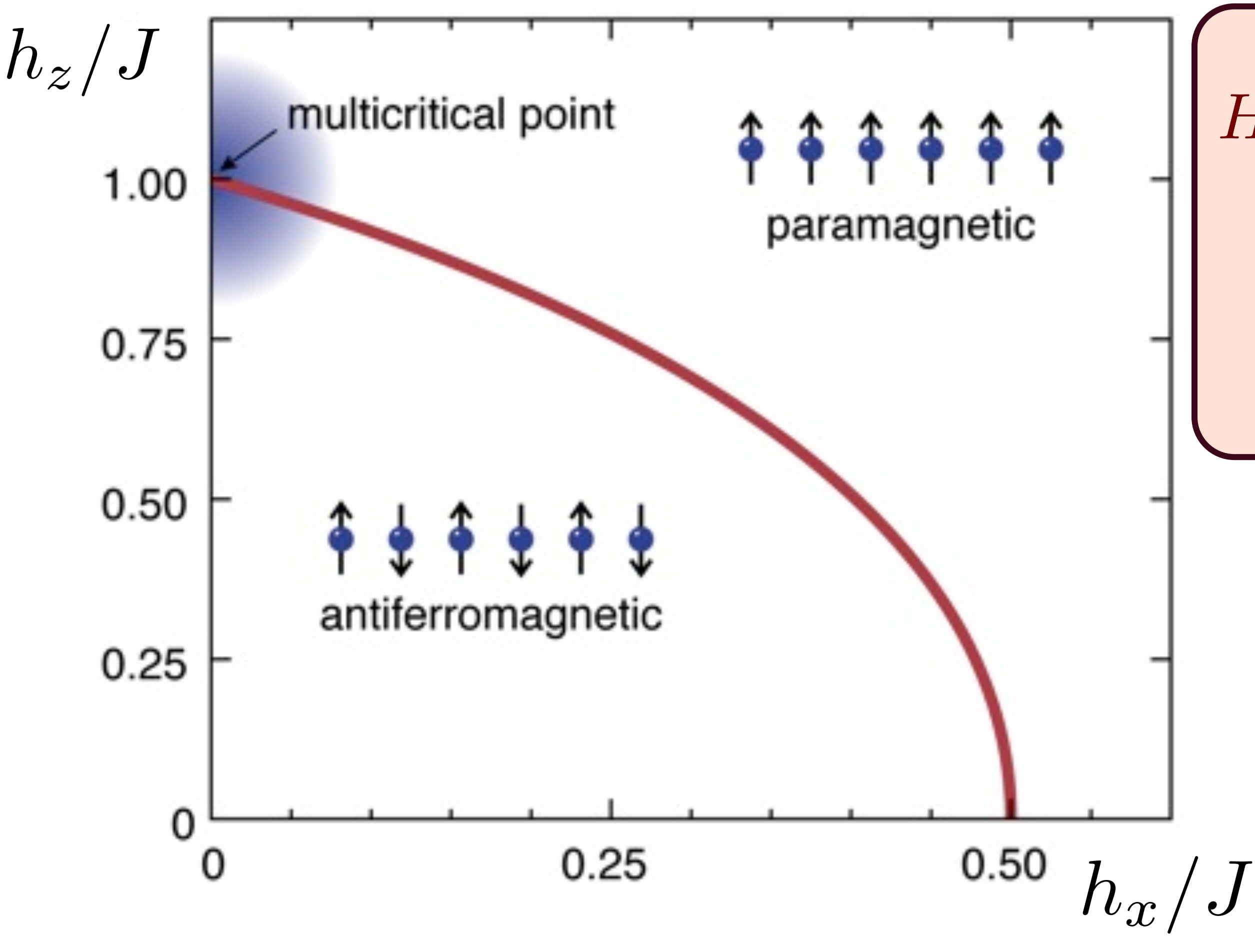
Antiferromagnetic state, two fold degenerate

Hamiltonian of resonant subspace



Antiferromagnetic state, two fold degenerate

Phase diagram of spin model



$$H = \sum_i \Big[J S_i^z S_{i+1}^z - h_z S_i^z - h_x S_i^x \Big]$$

$$J \to \infty,$$

$$h_z = J + (U - E),$$

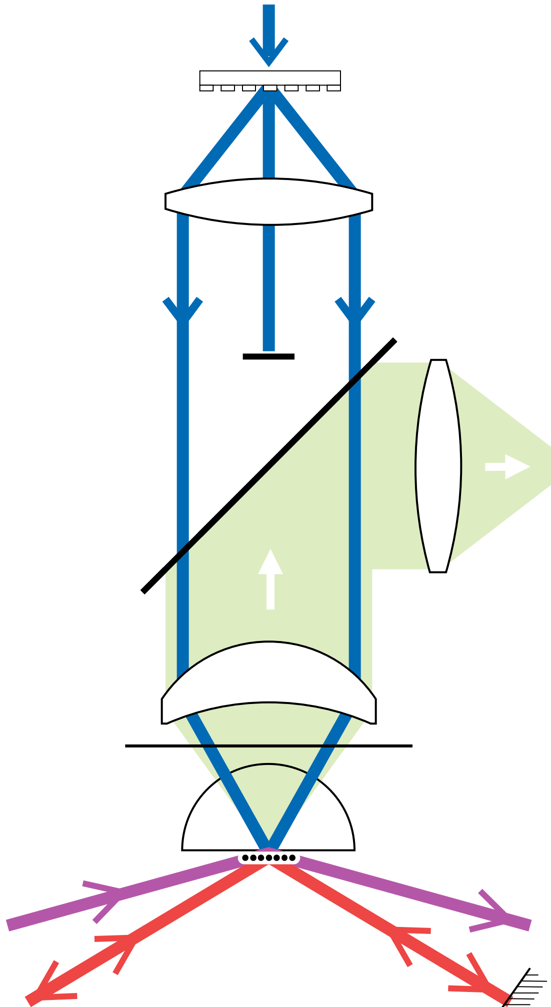
$$h_x = 2\sqrt{2}t$$

hx =0: classical first order phase transition
Finite hx: quantum phase transition, second order

Quantum gas microscope

High fidelity single atom single site imaging

- **First:** **Many-body physics in conservative lattice potential**
- **Then:** **increase lattice depth, fluorescence imaging**



Quantum gas microscope

Bakr *et al.*, Nature 462, 74 (2009)
Bakr *et al.*, Science.1192368 (June 2010)

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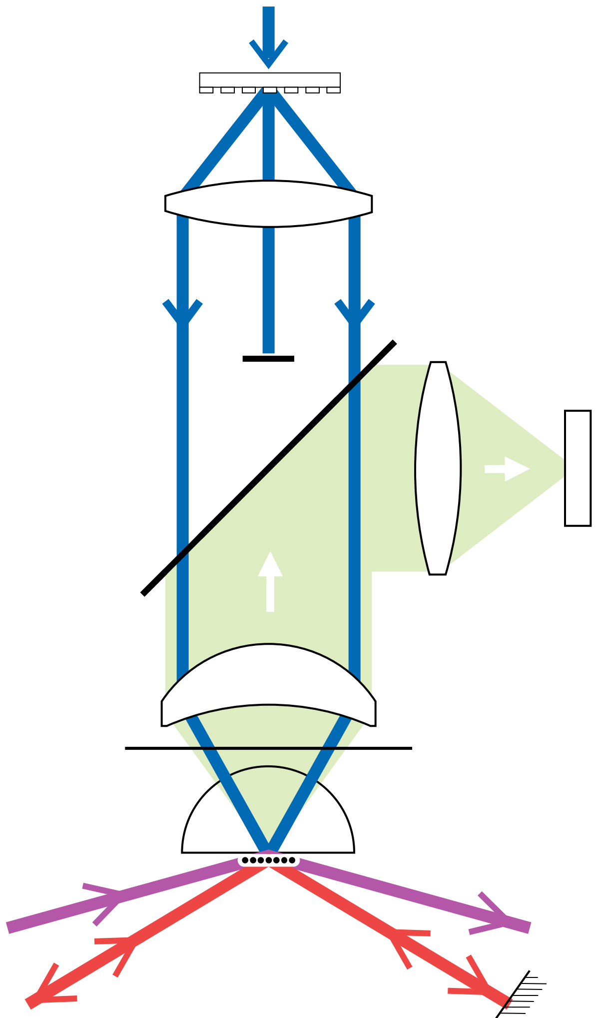


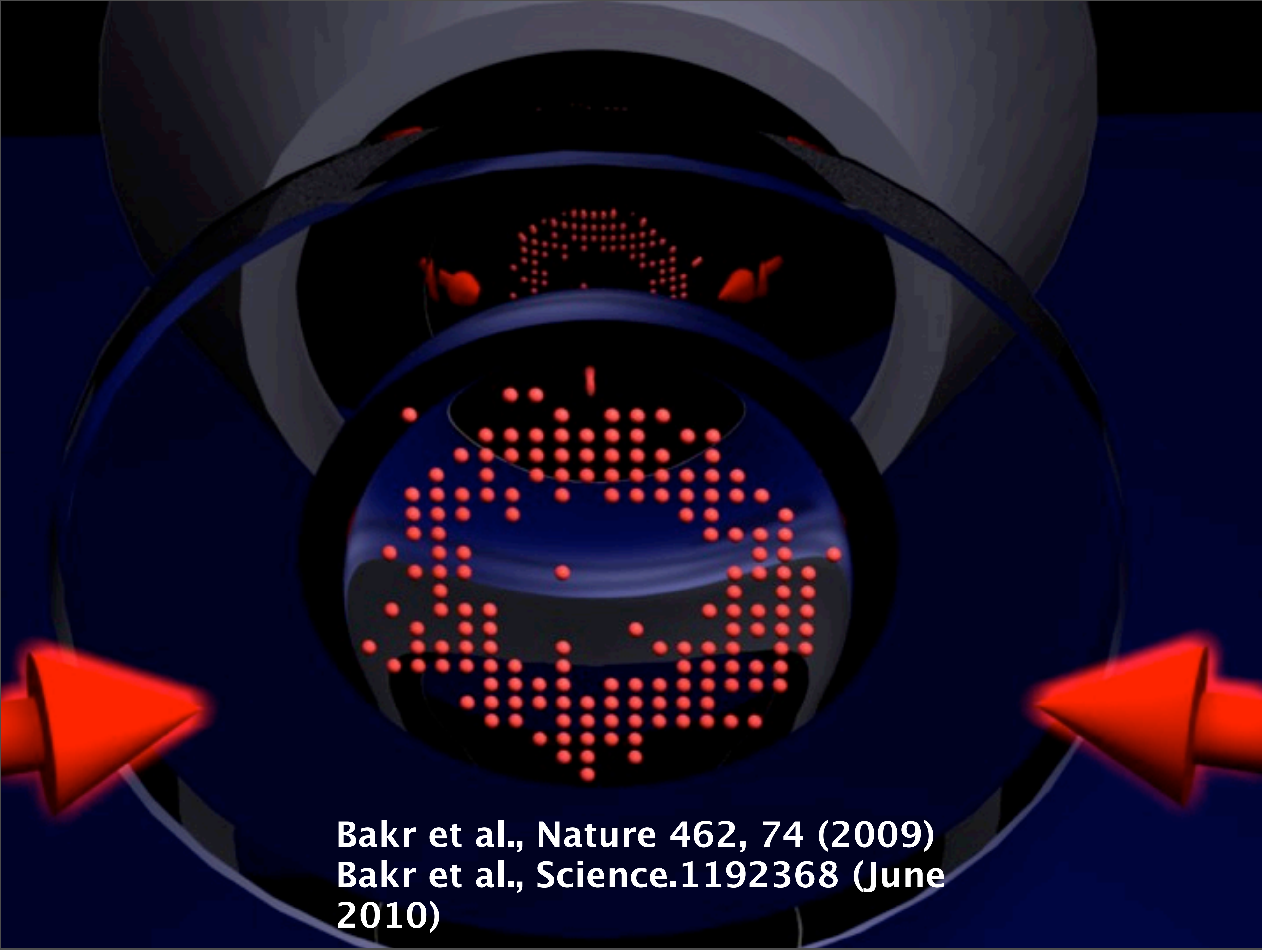
High resolution imaging

High aperture objective NA=0.8

Solid immersion lens in vacuum

Optical Molasses during imaging

2D quantum gas in optical lattice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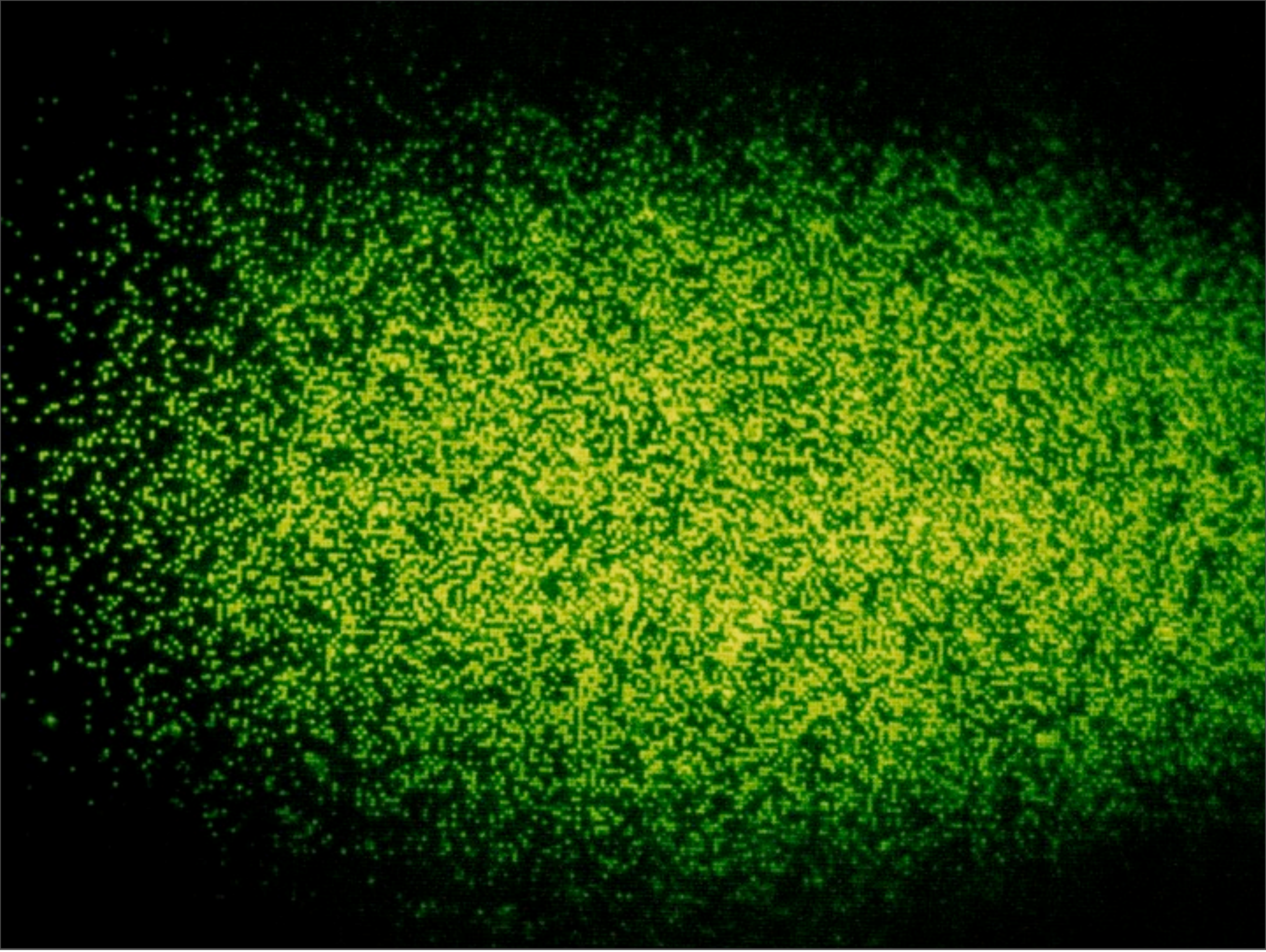
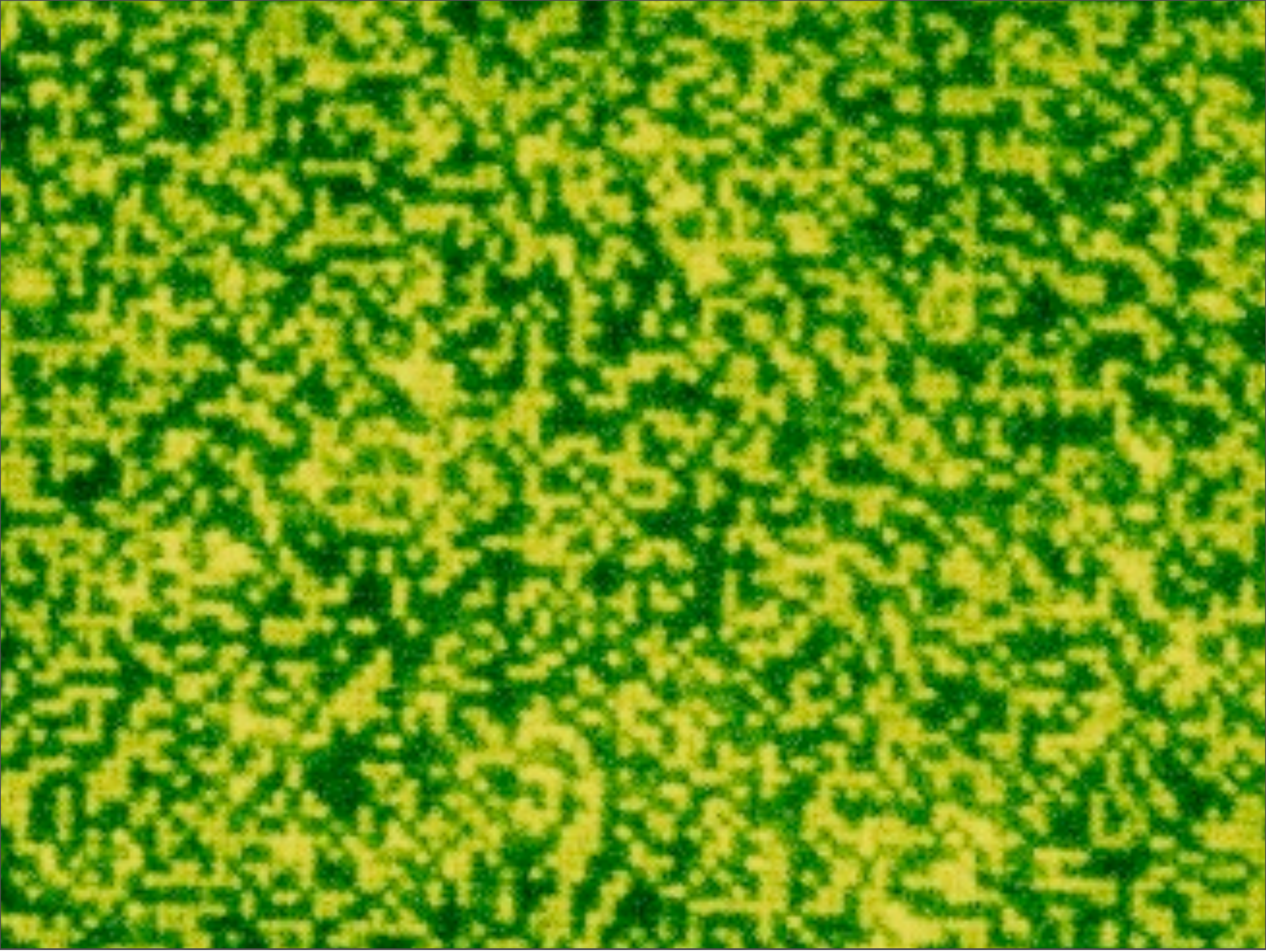
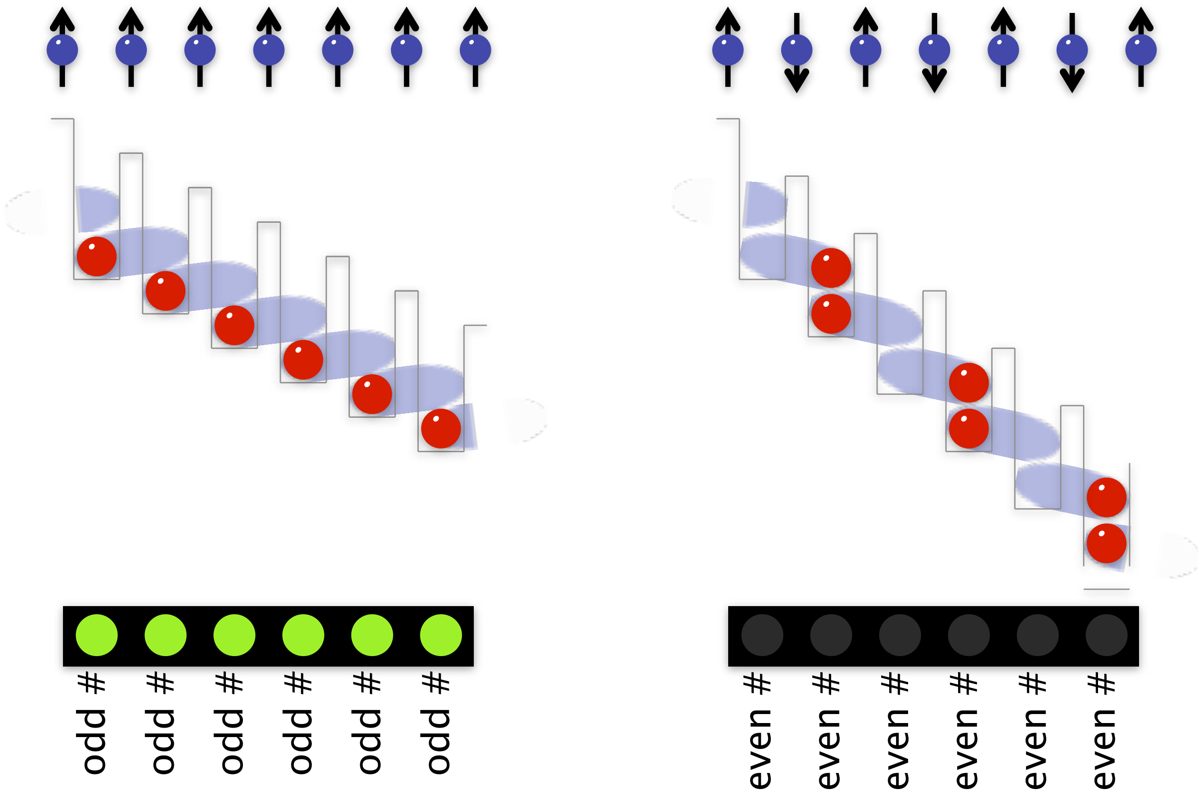
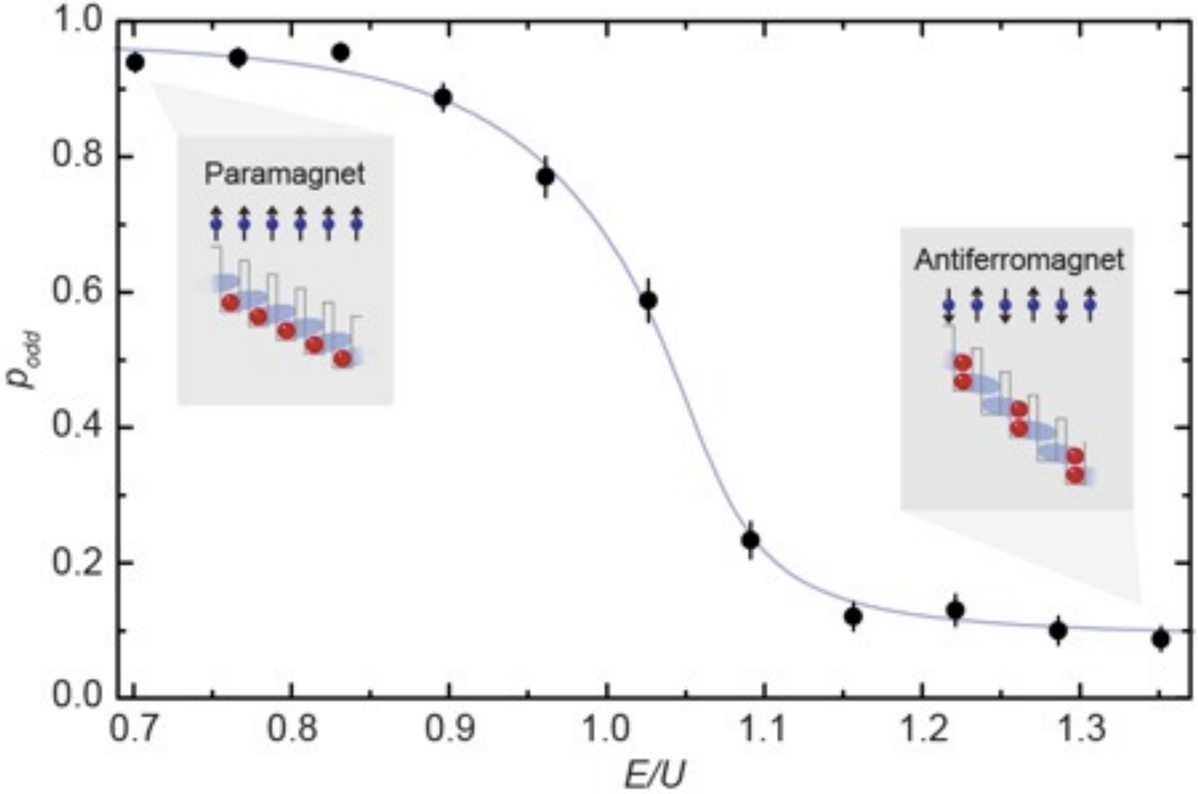


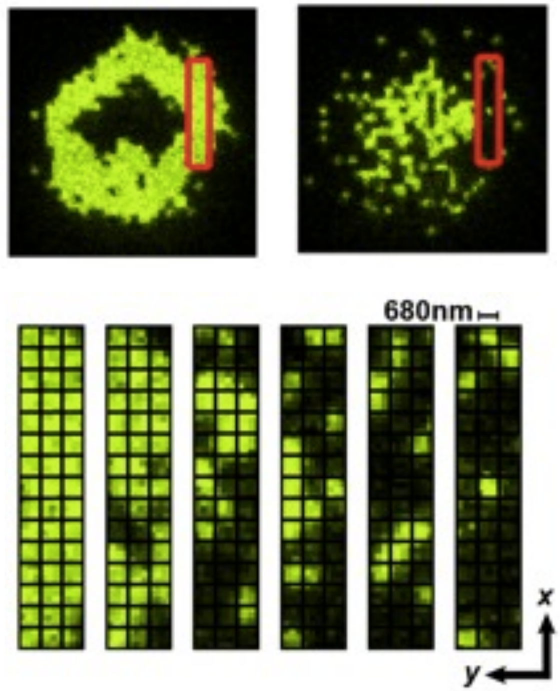


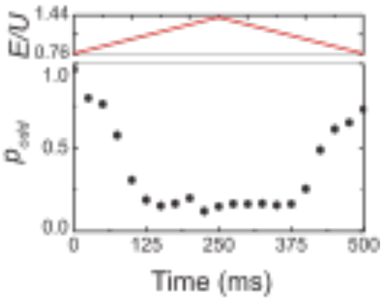
paramagnetic ground state

antiferromagnetic ground state

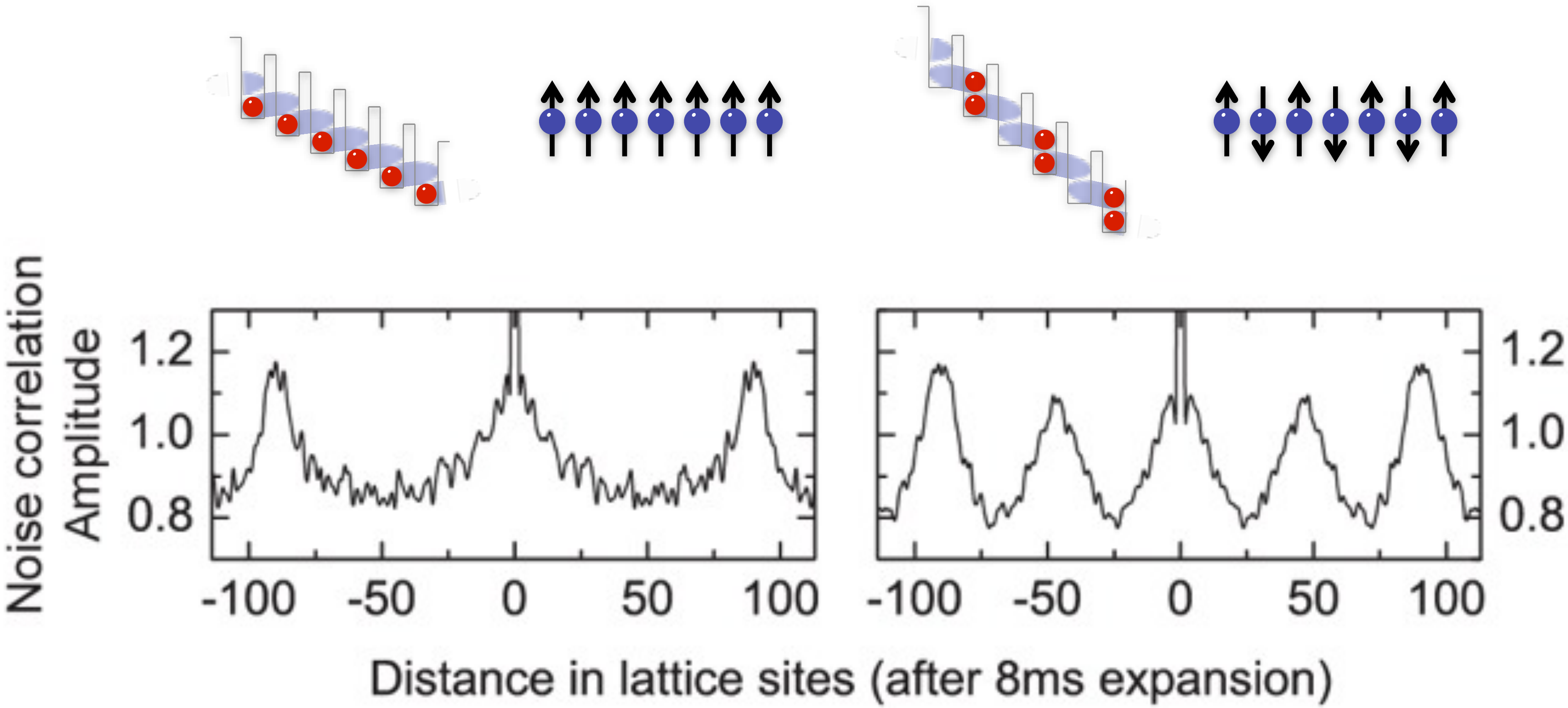






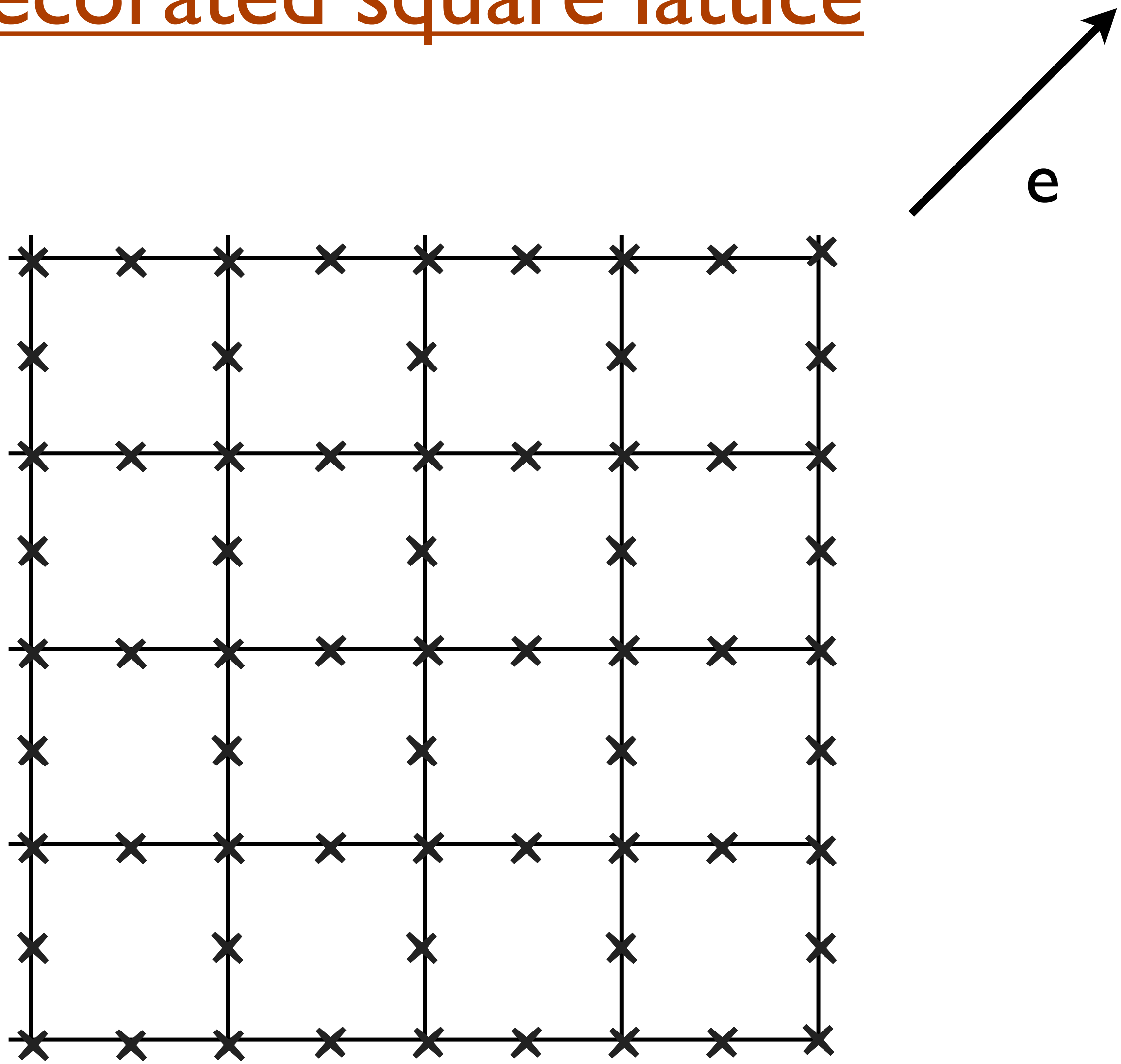


Thursday, March 10, 2011



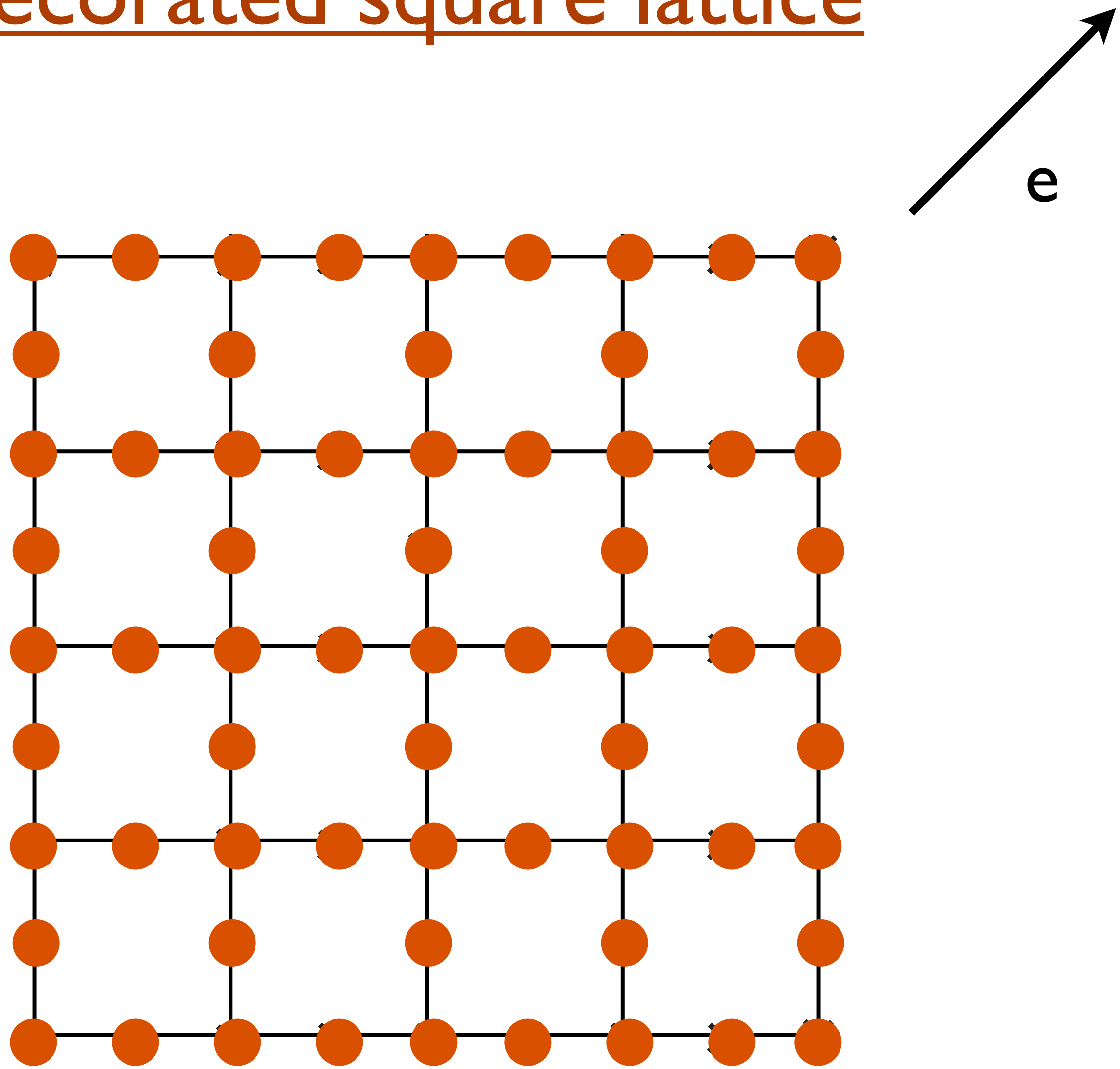
- Expand within each 1D tube, detect individual atoms, and calculate correlation function
- See Foelling et al., Nature 434, 481–484 (2005)

Tilting a decorated square lattice



Susanne Pielawa

Tilting a decorated square lattice



Susanne Pielawa

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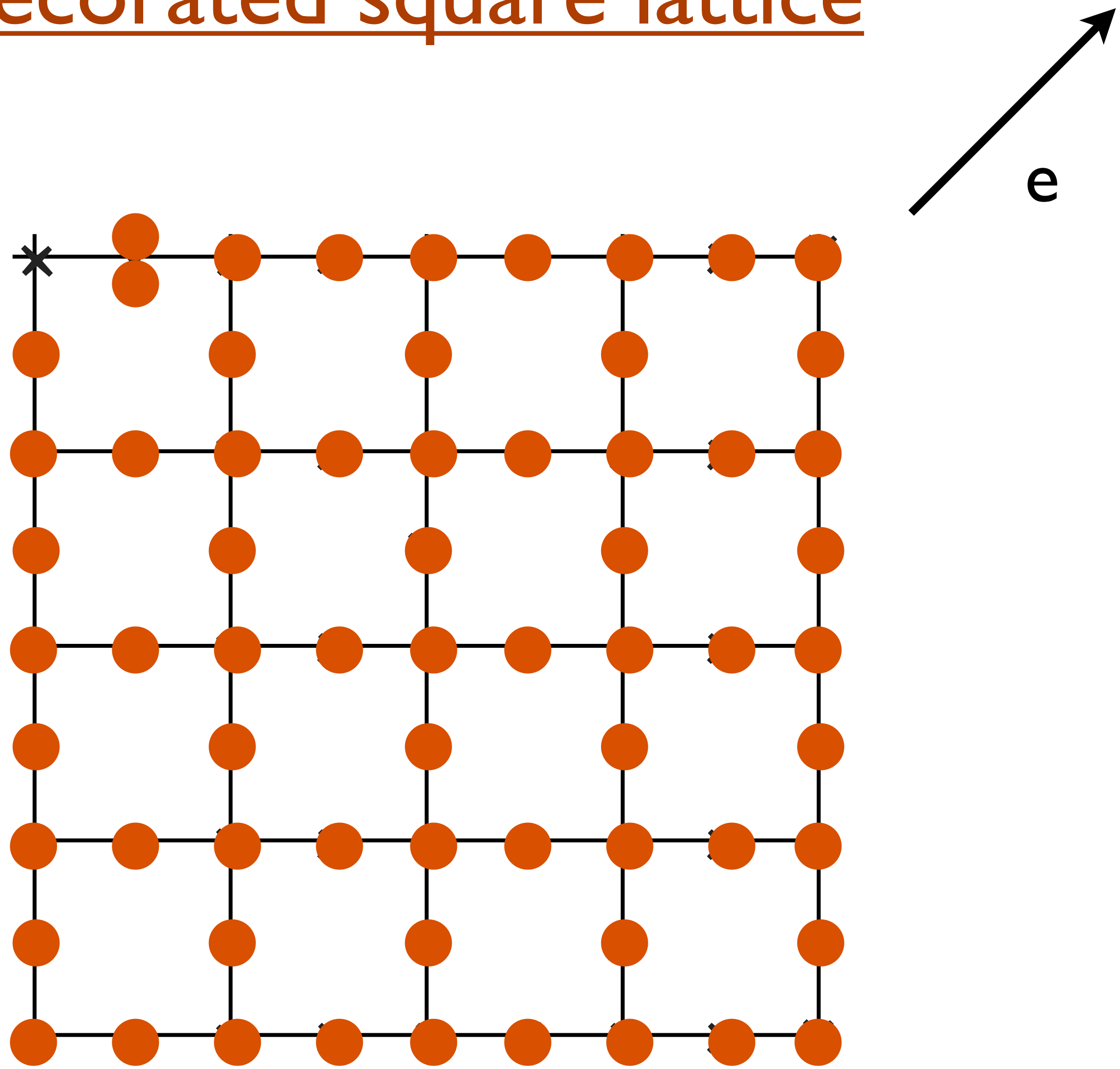

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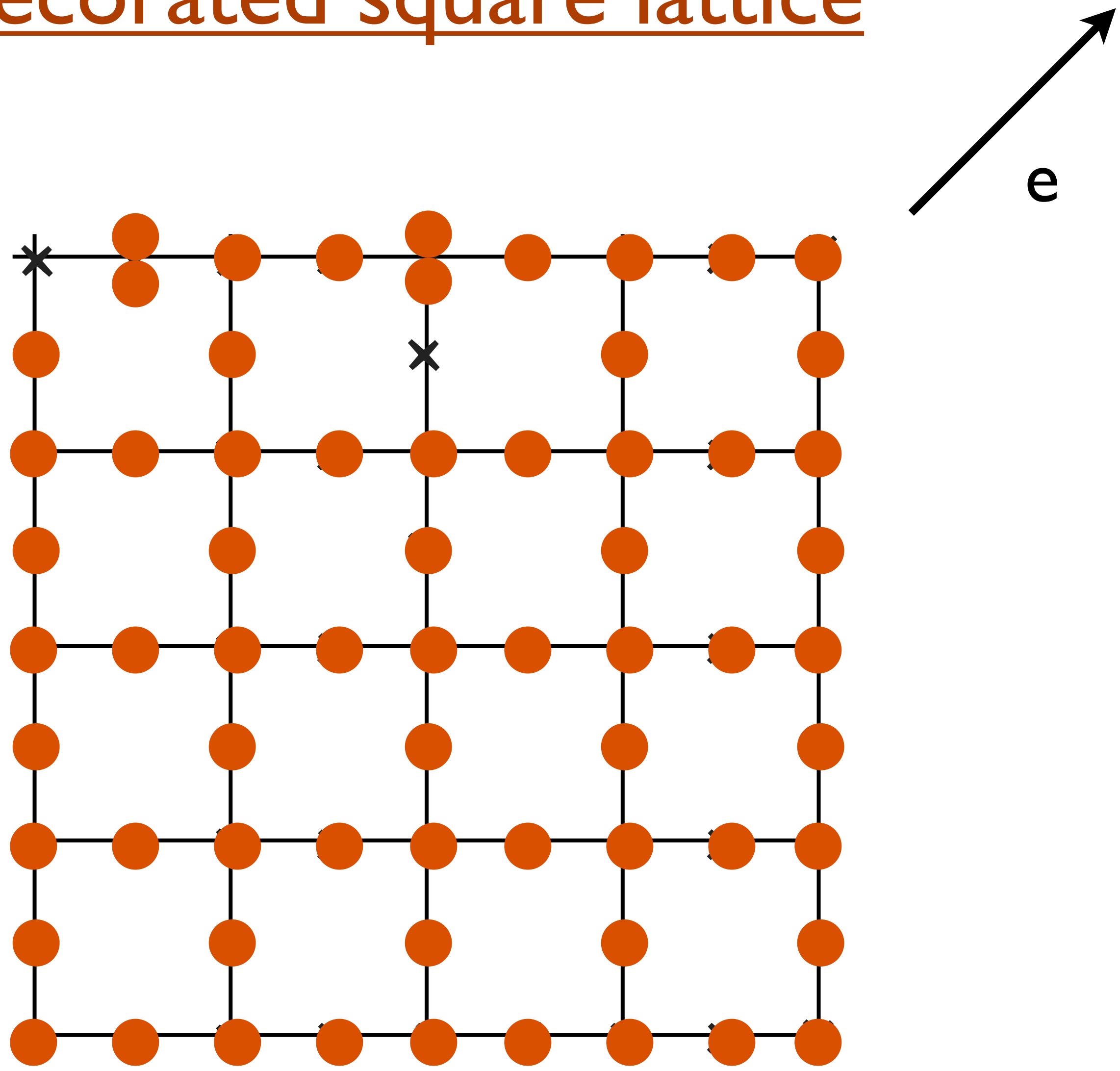
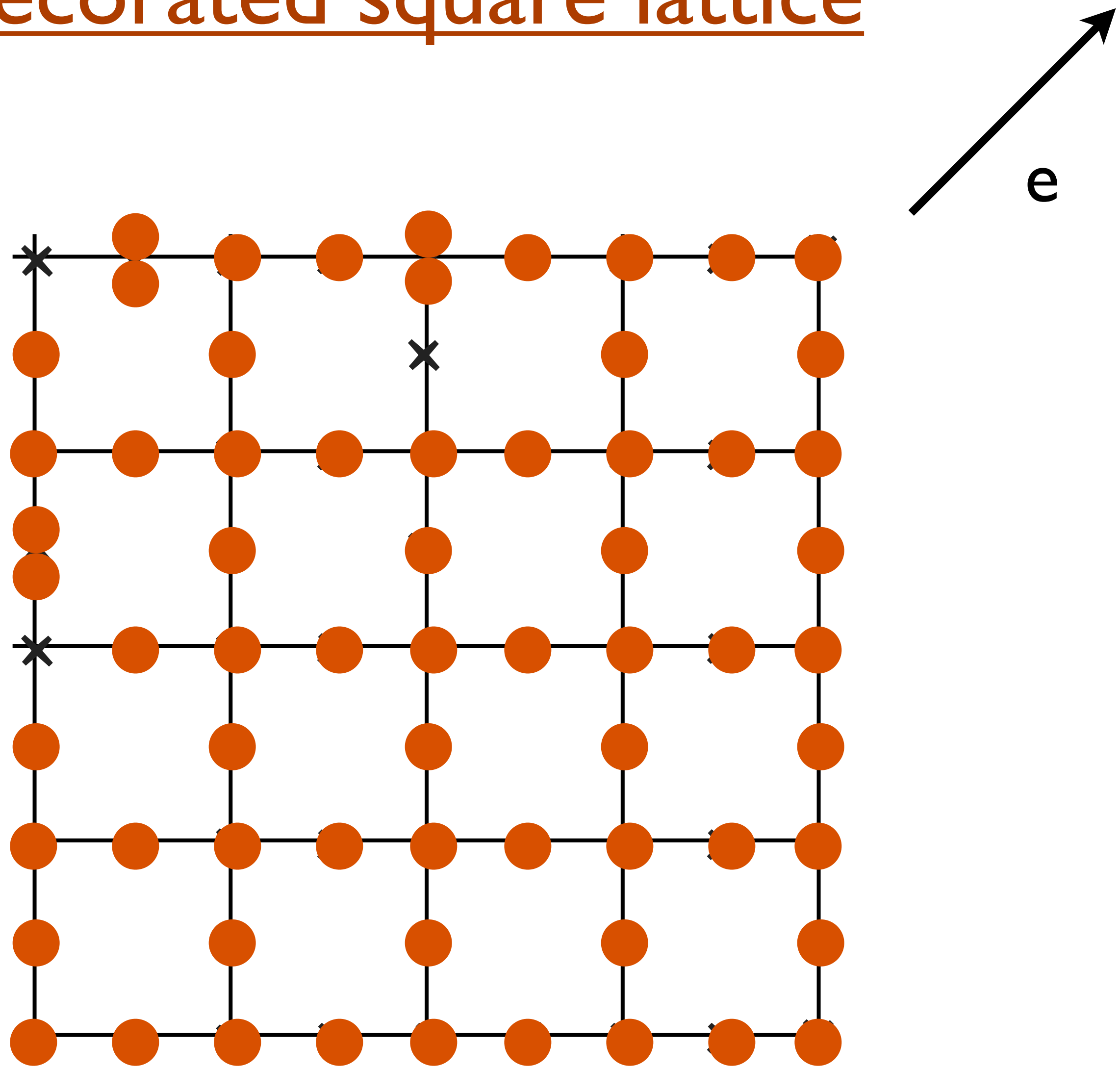


Strong tilt: maximize sites with 2 bosons

Tilting a decorated square lattice



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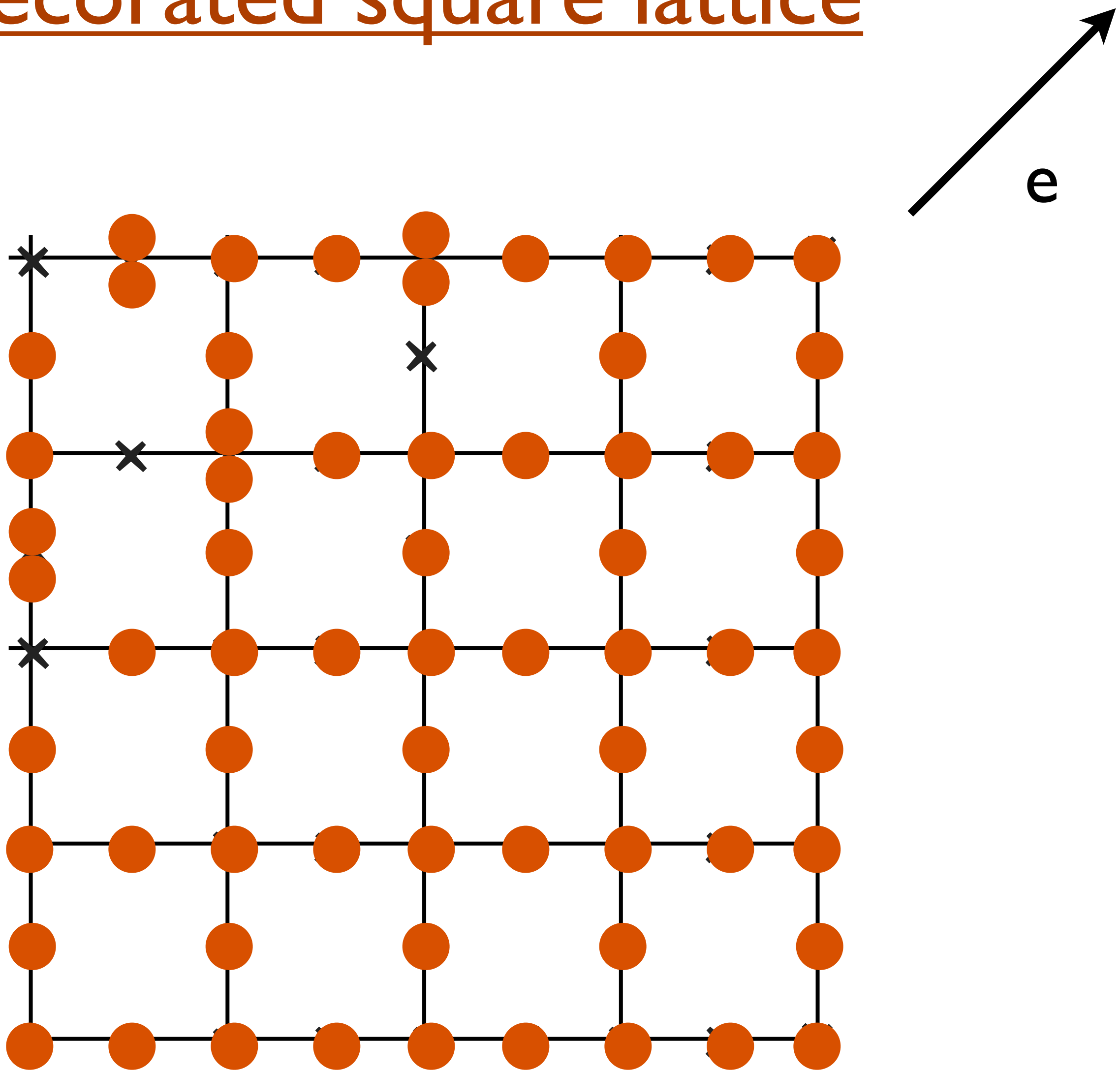

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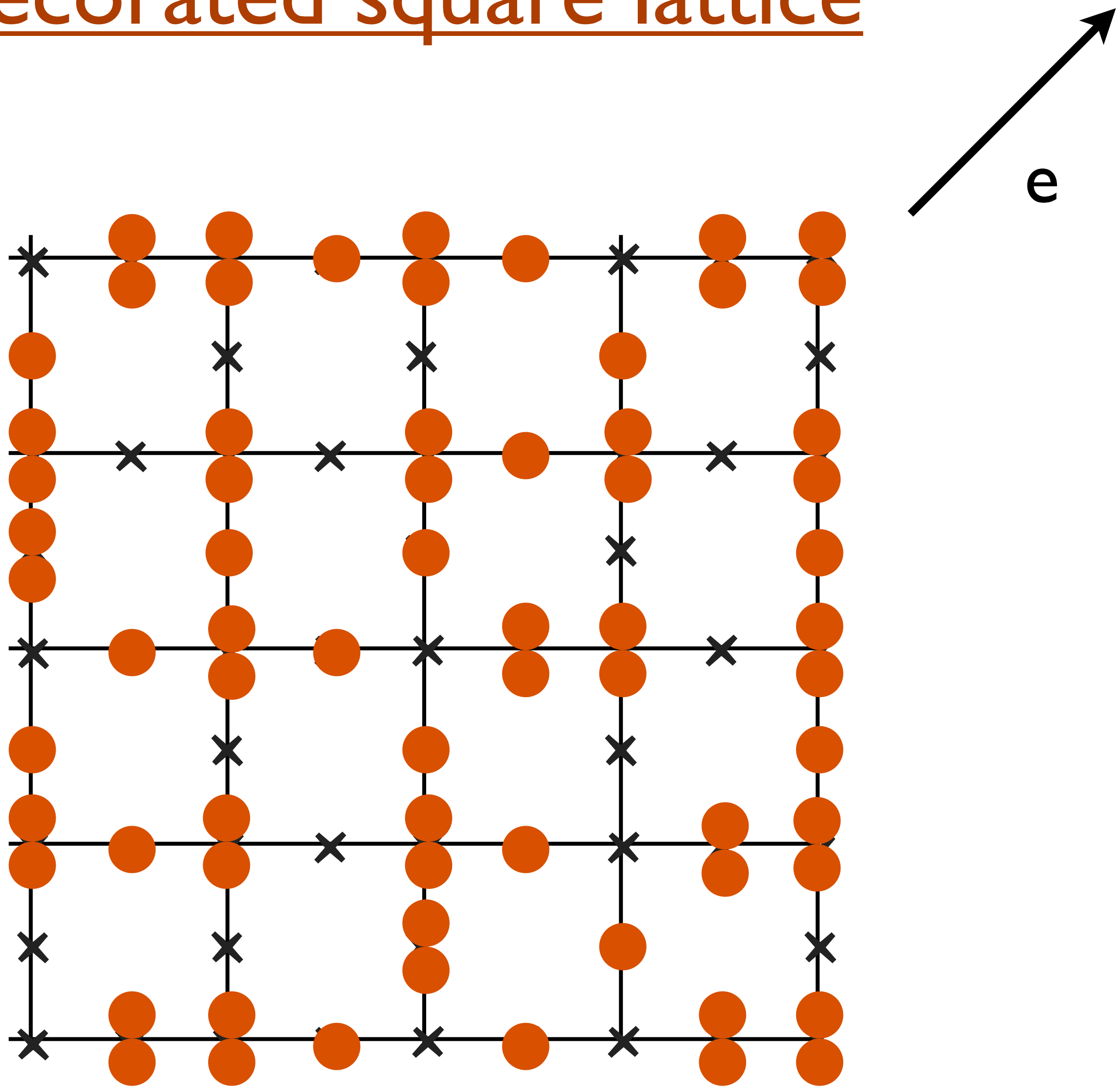
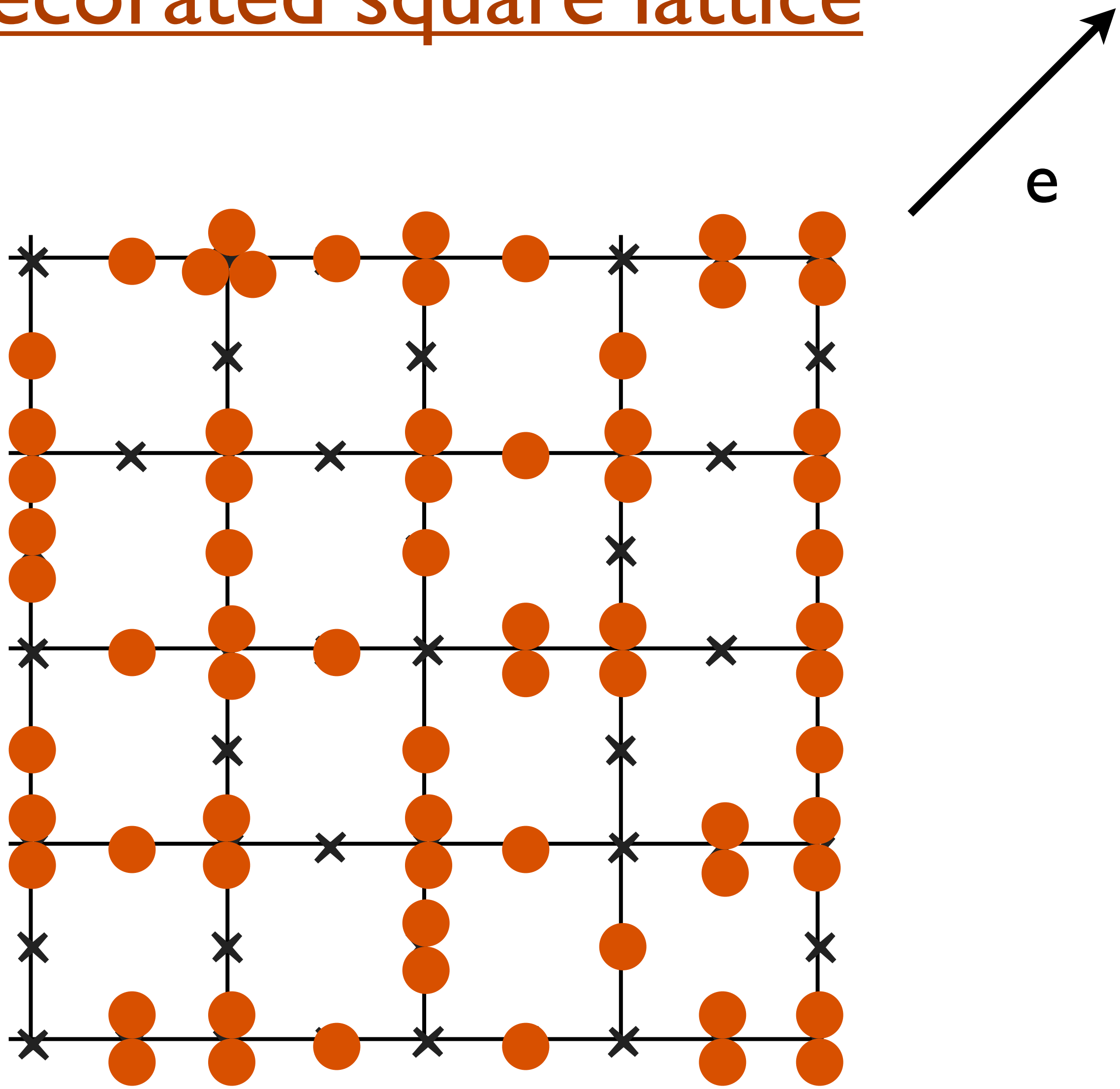


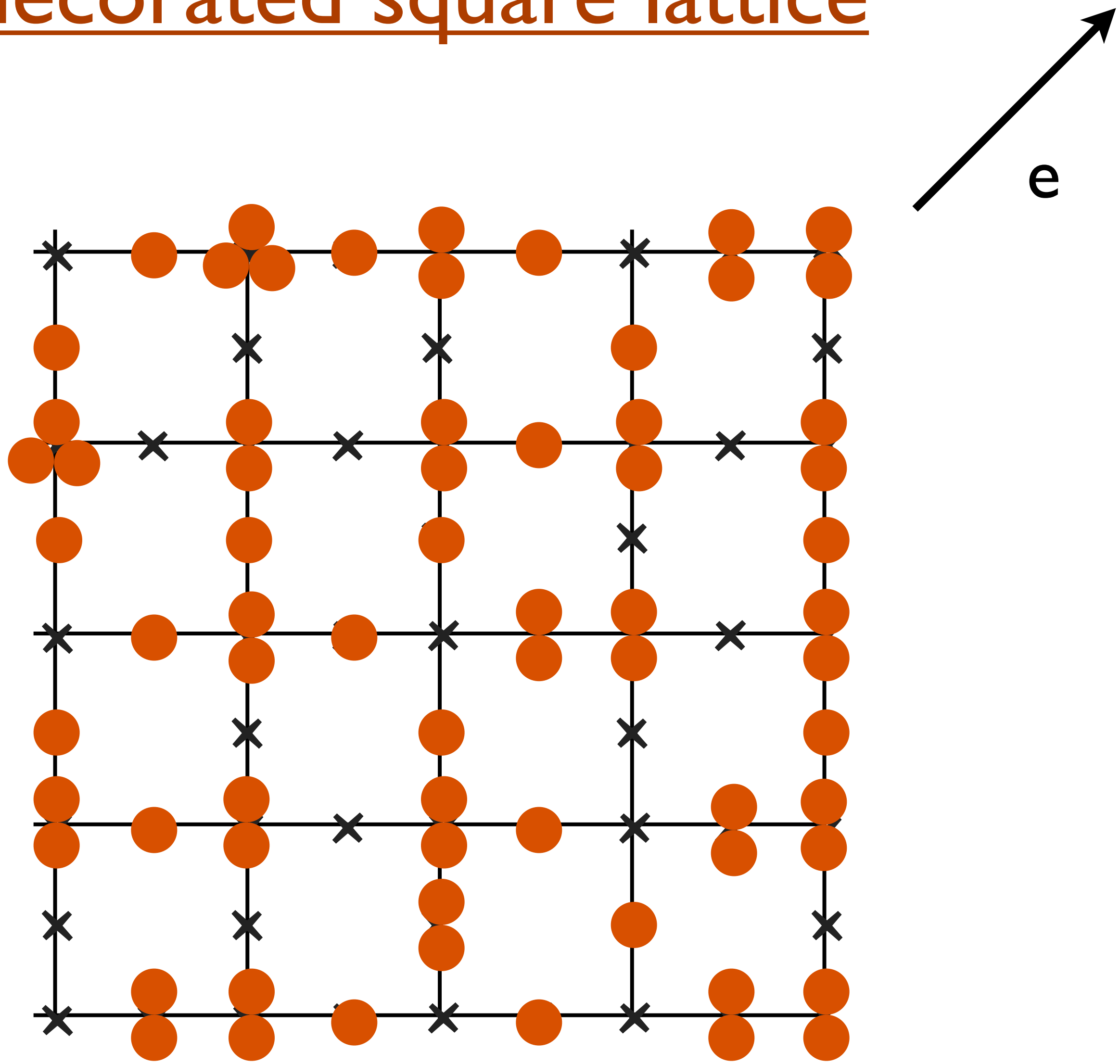
Maximum number of 2's

Tilting a decorated square lattice



Can also get some 3's from neighboring 2's.

Tilting a decorated square lattice



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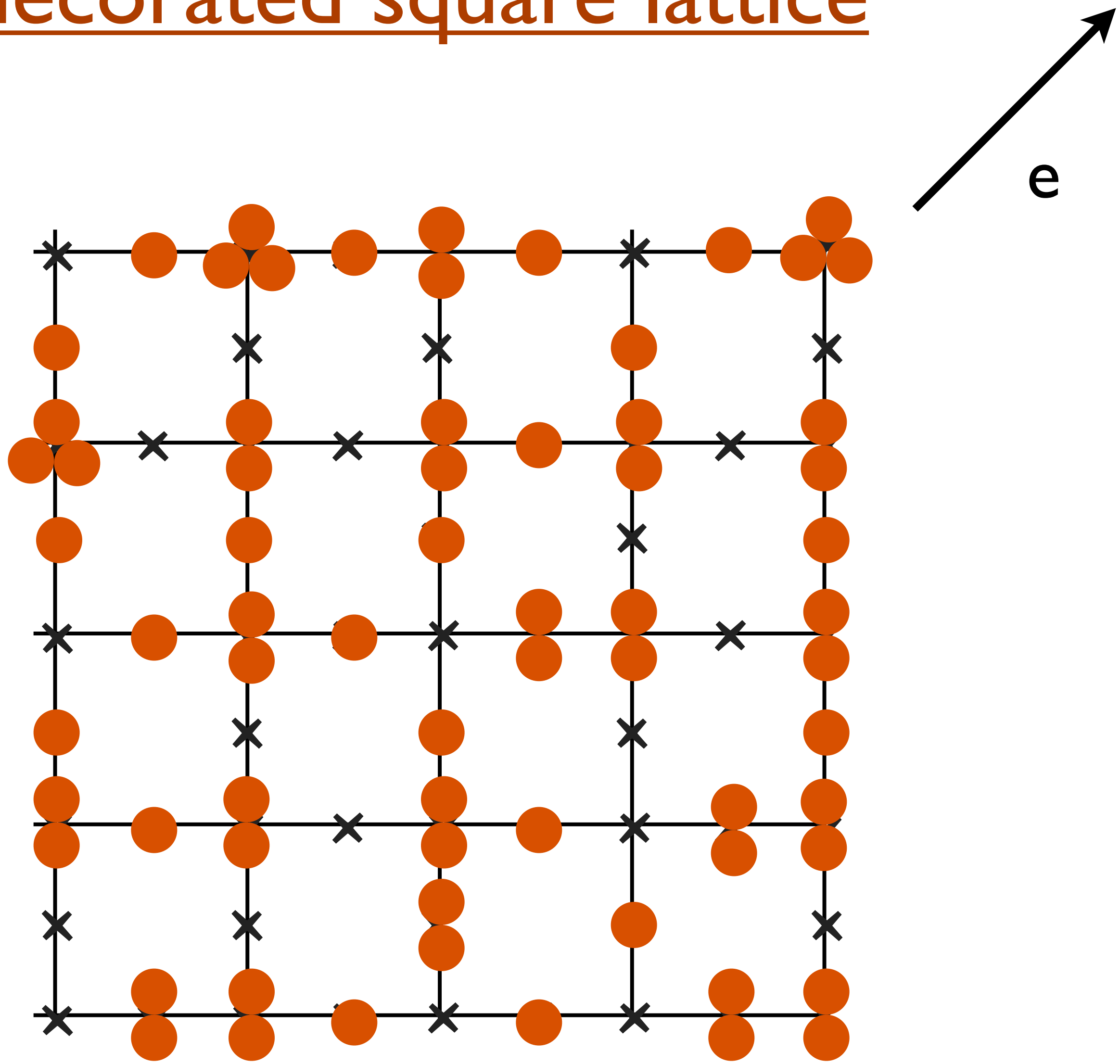

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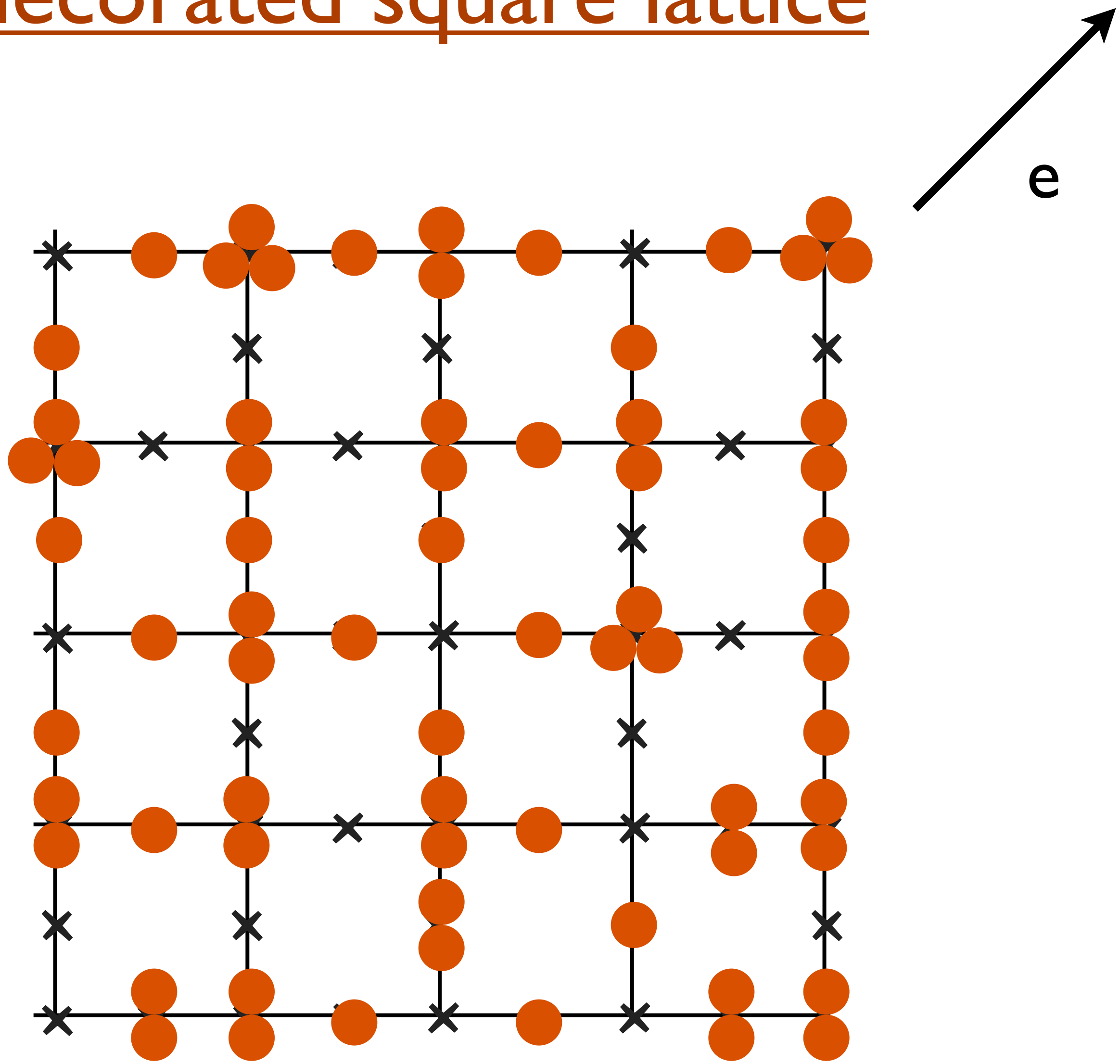

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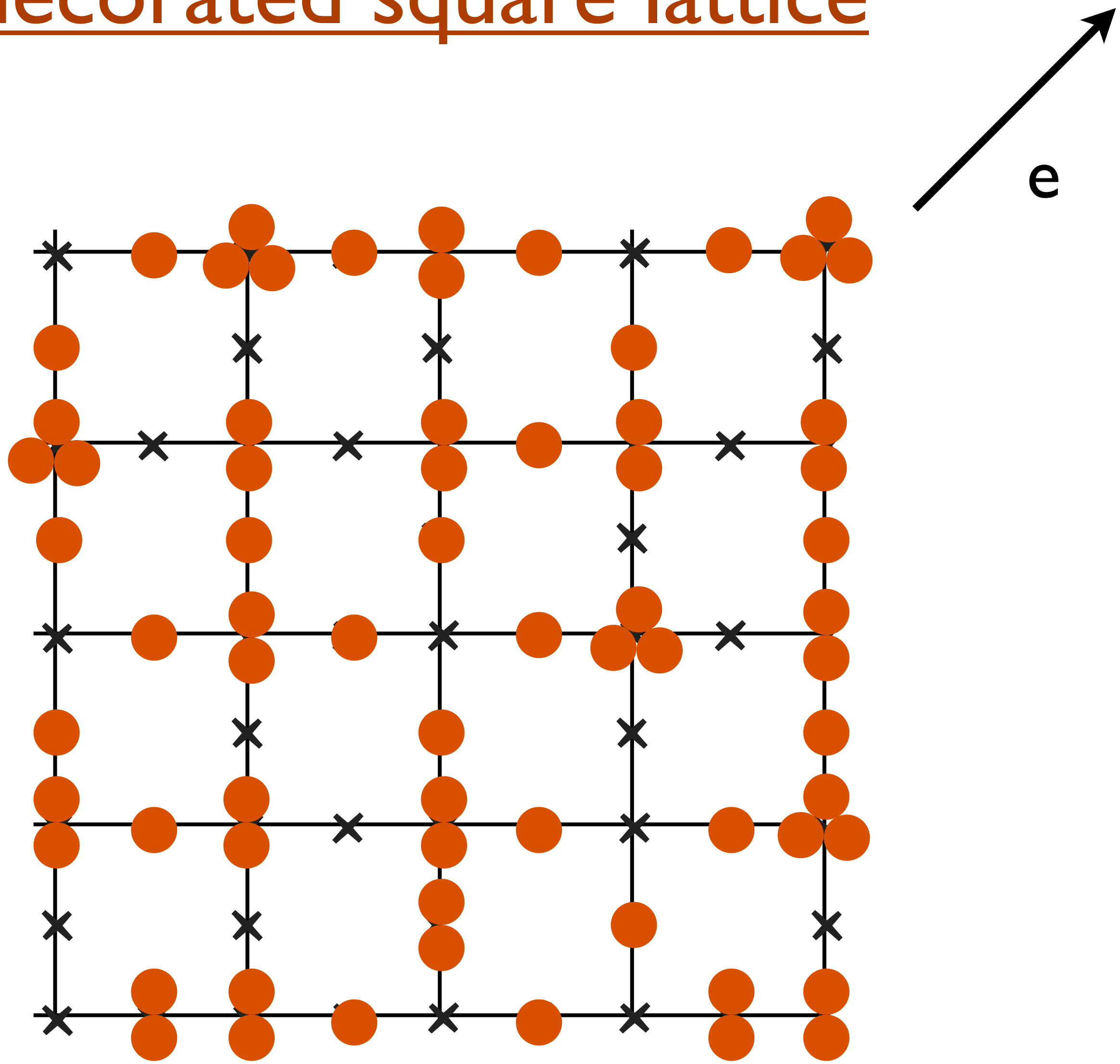

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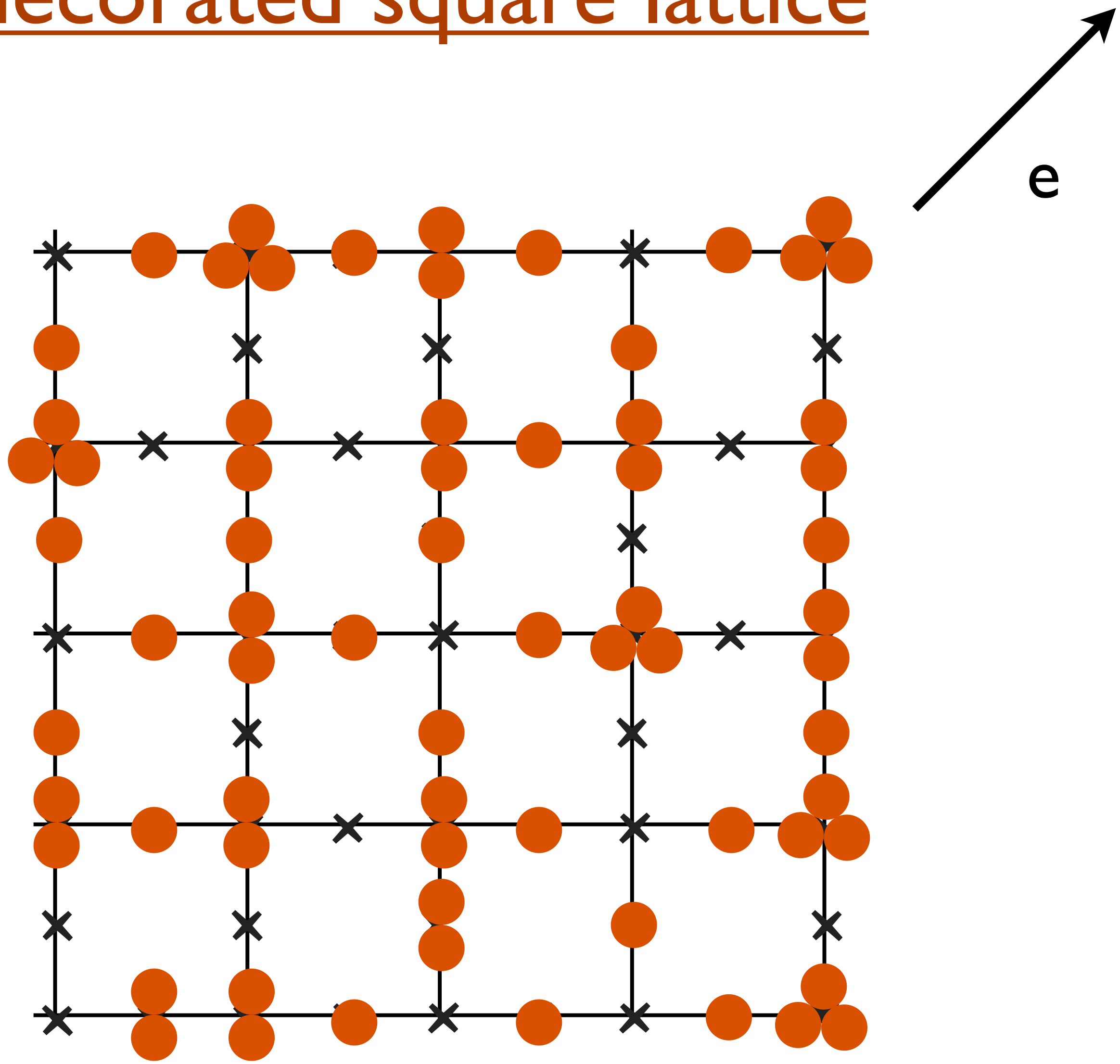

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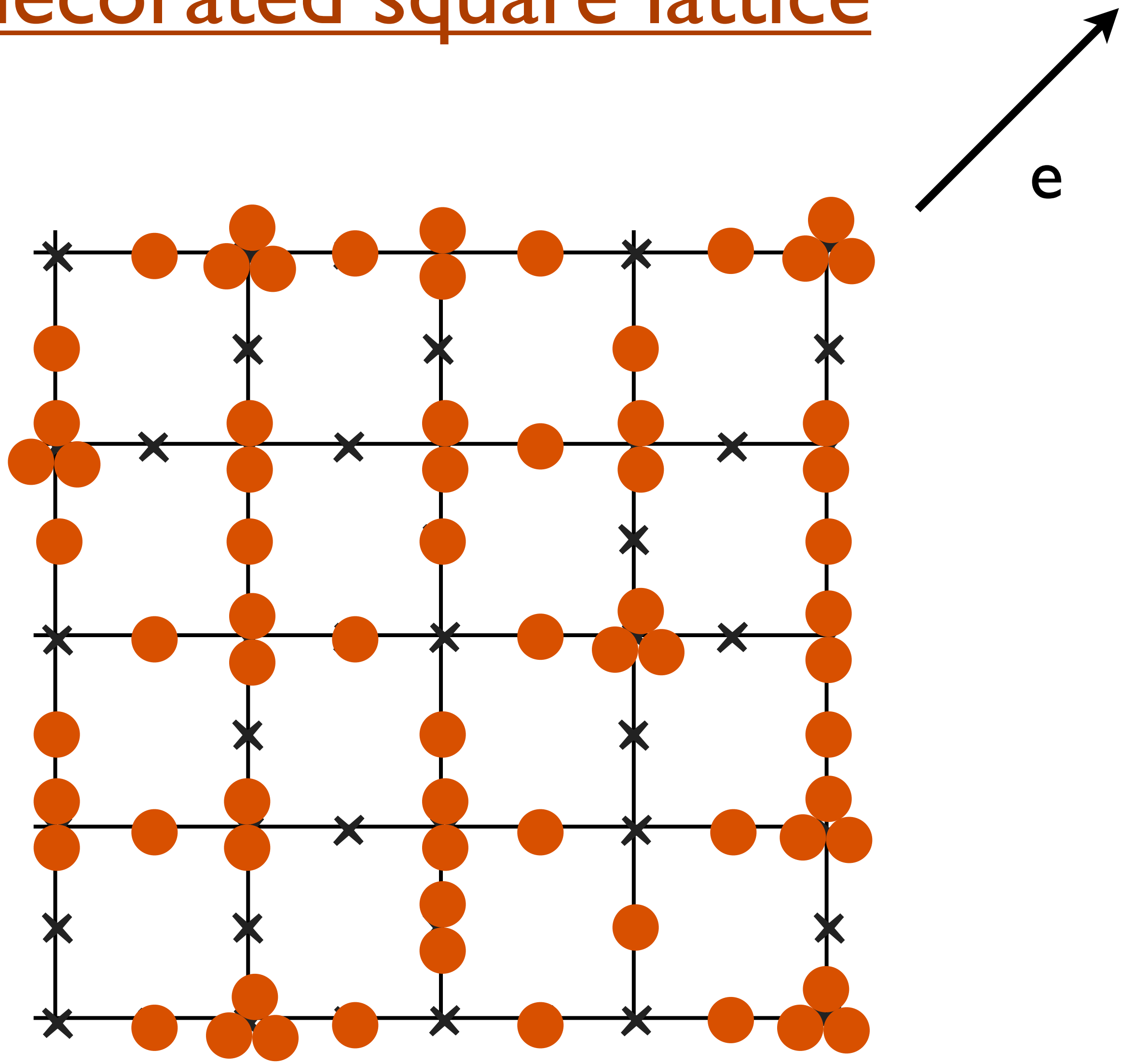

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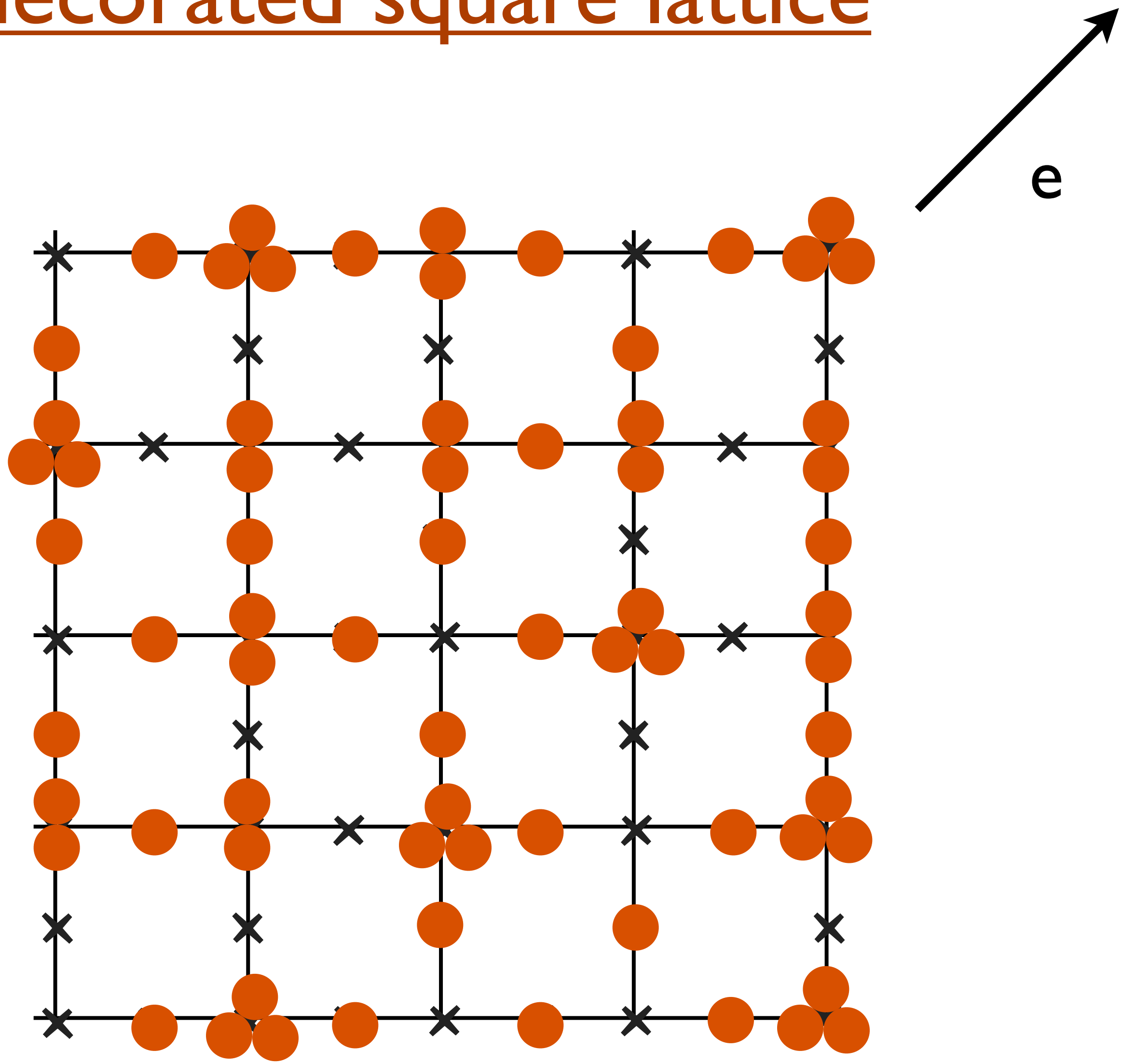
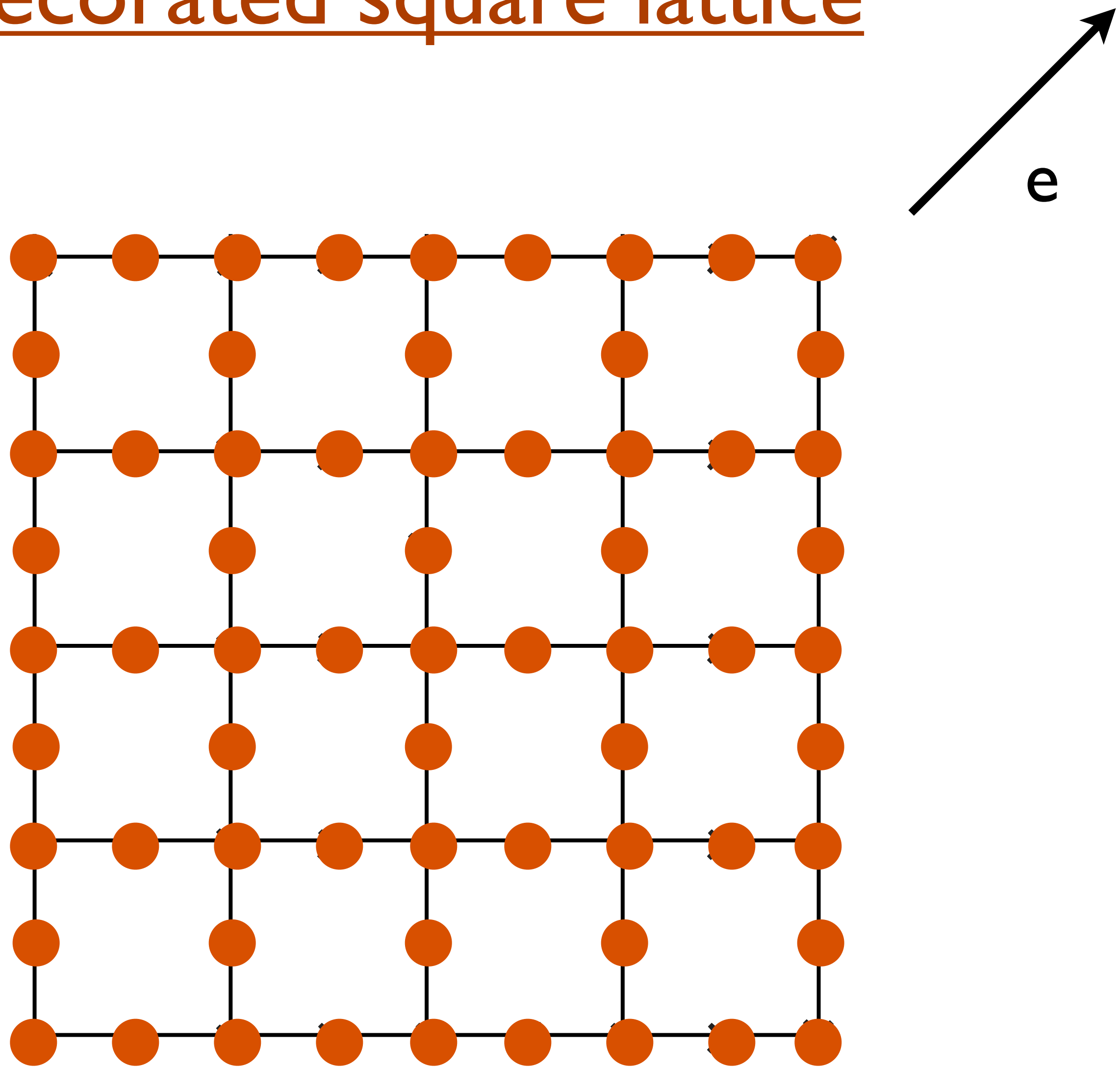


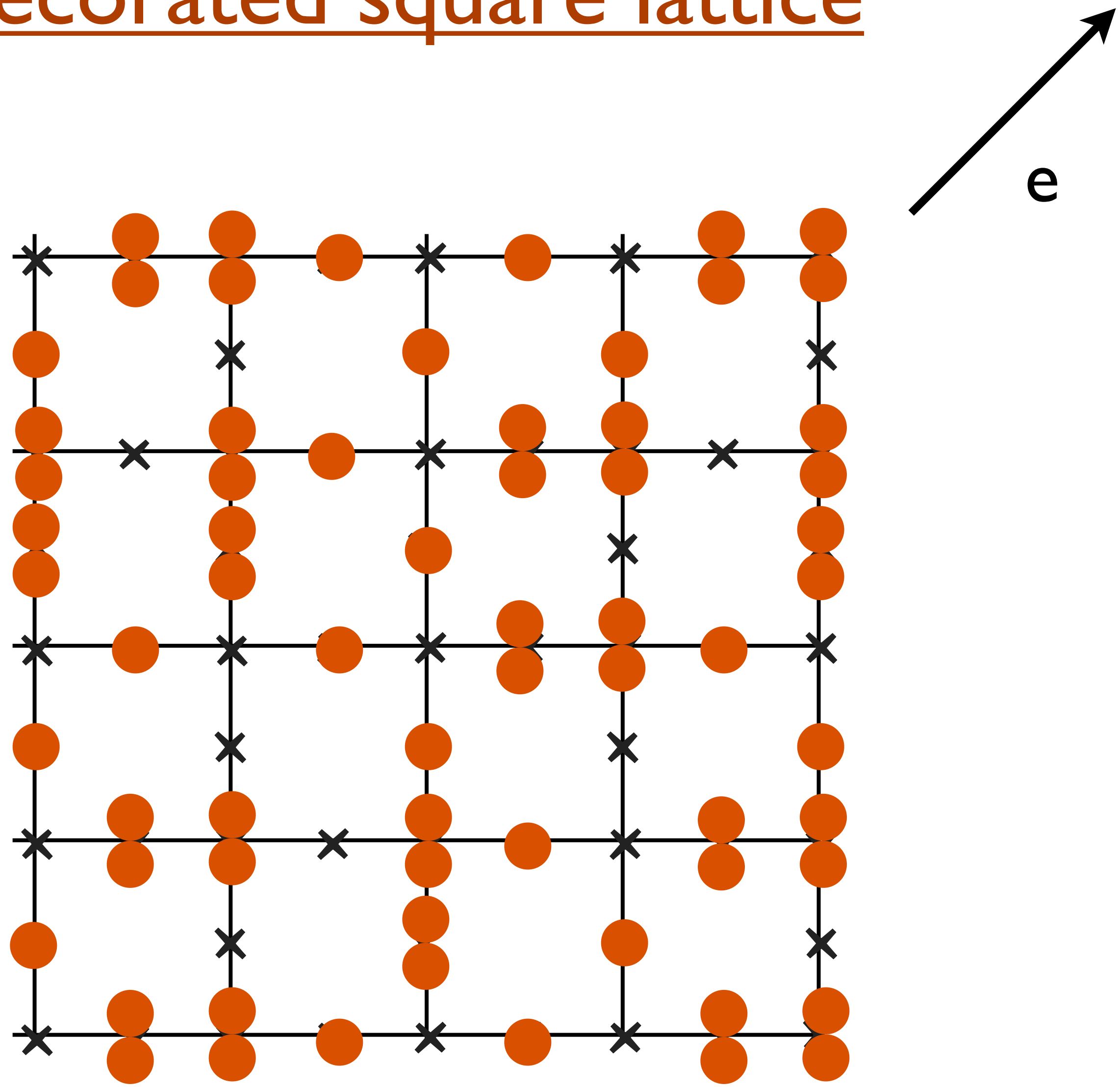
No more 3's are possible, but some 2's are left over

Tilting a decorated square lattice



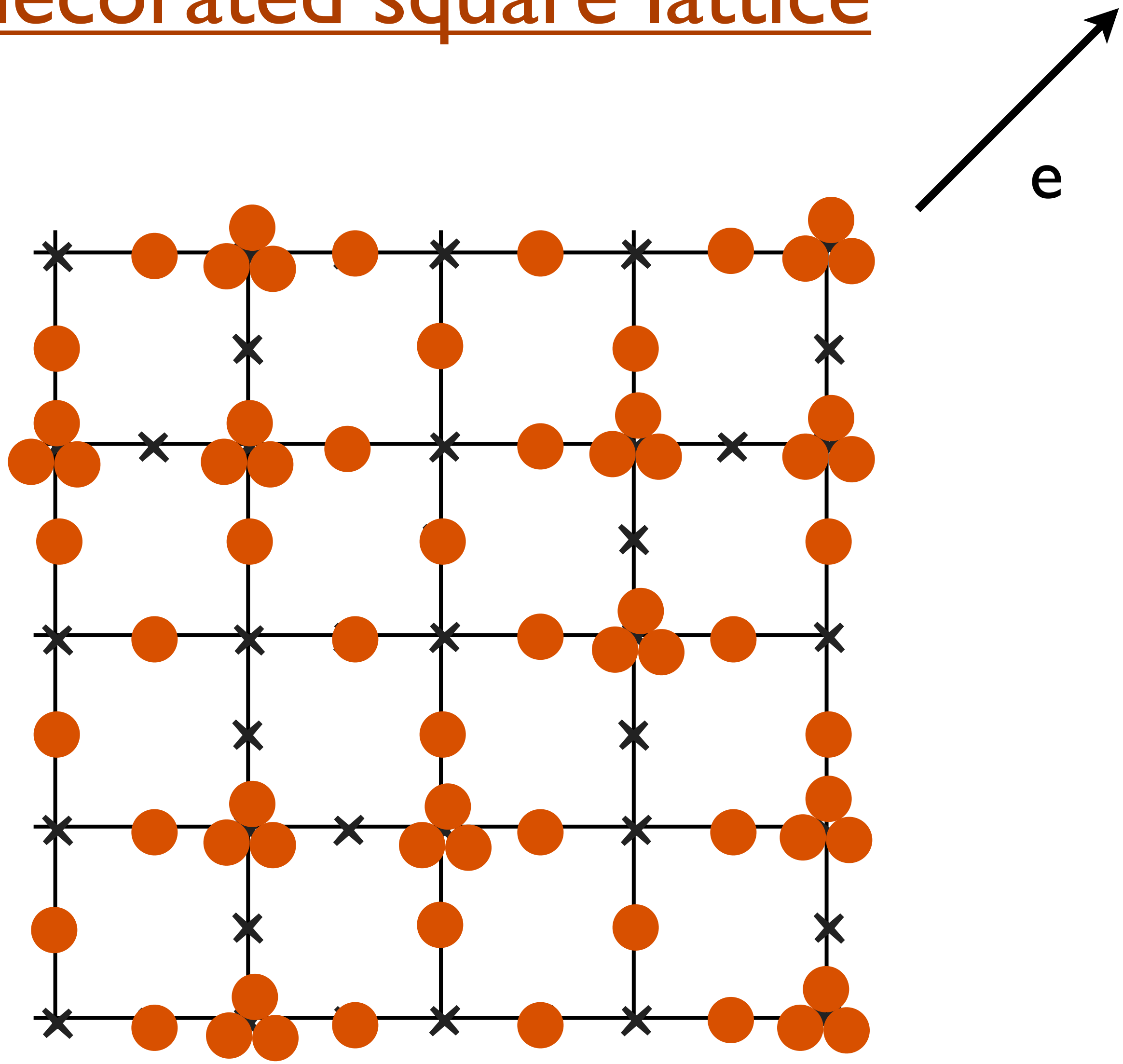
Start again

Tilting a decorated square lattice



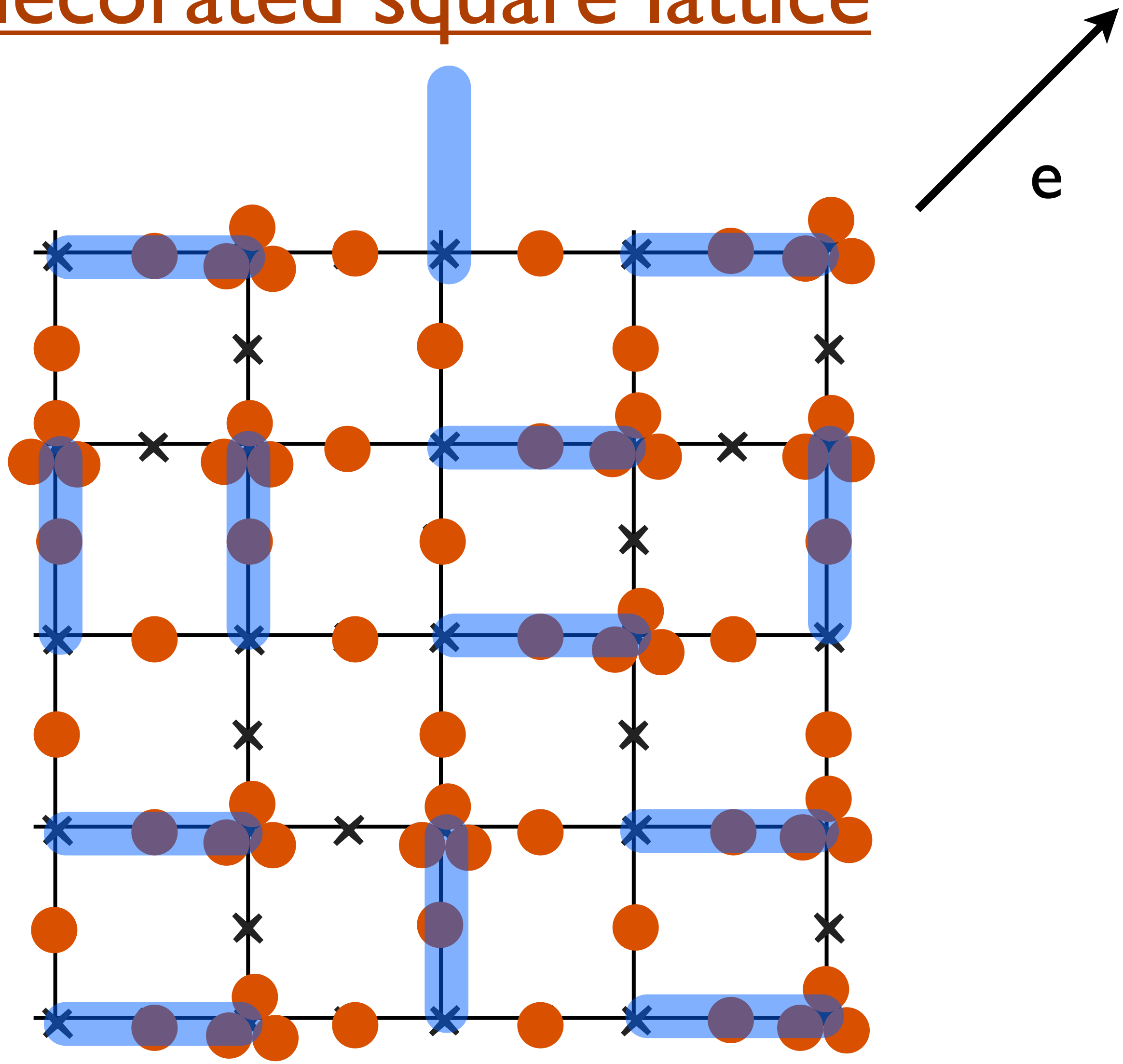
Another maximal set of 2's

Tilting a decorated square lattice



Maximum number of 3's with
no 2's left over

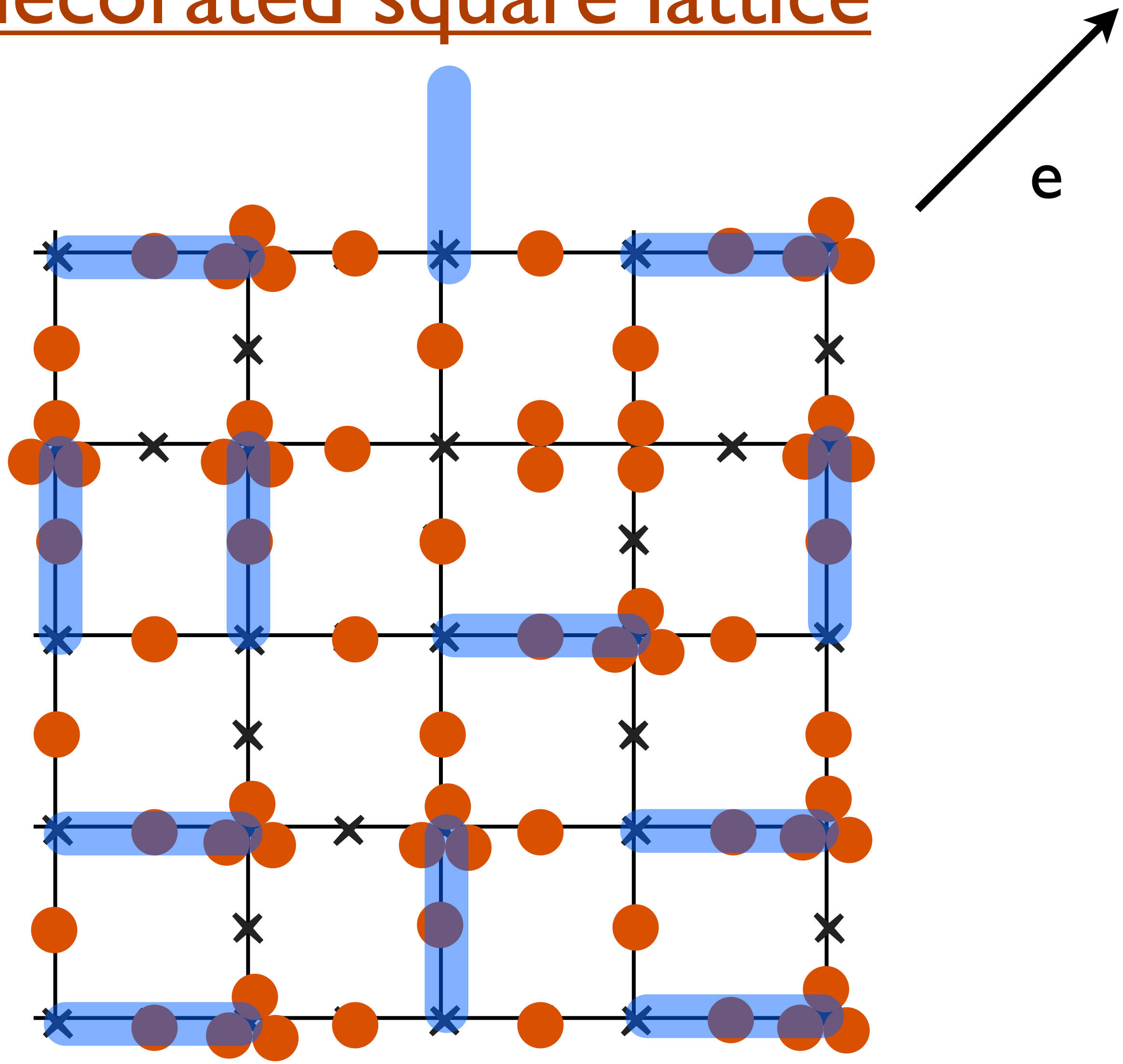
Tilting a decorated square lattice



Susanne Pielawa

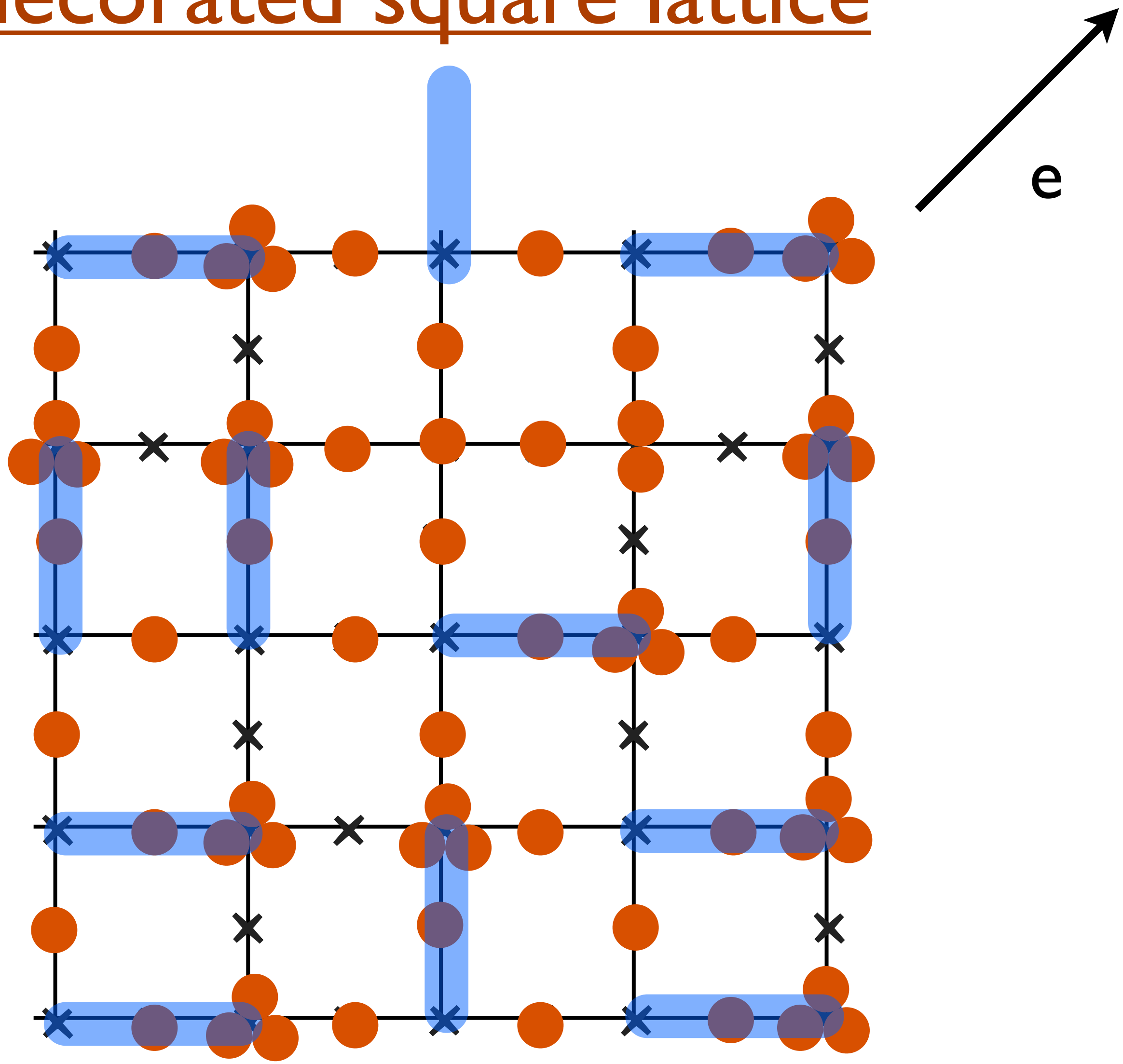
Configurations map onto dimer coverings of the square lattice !

Tilting a decorated square lattice



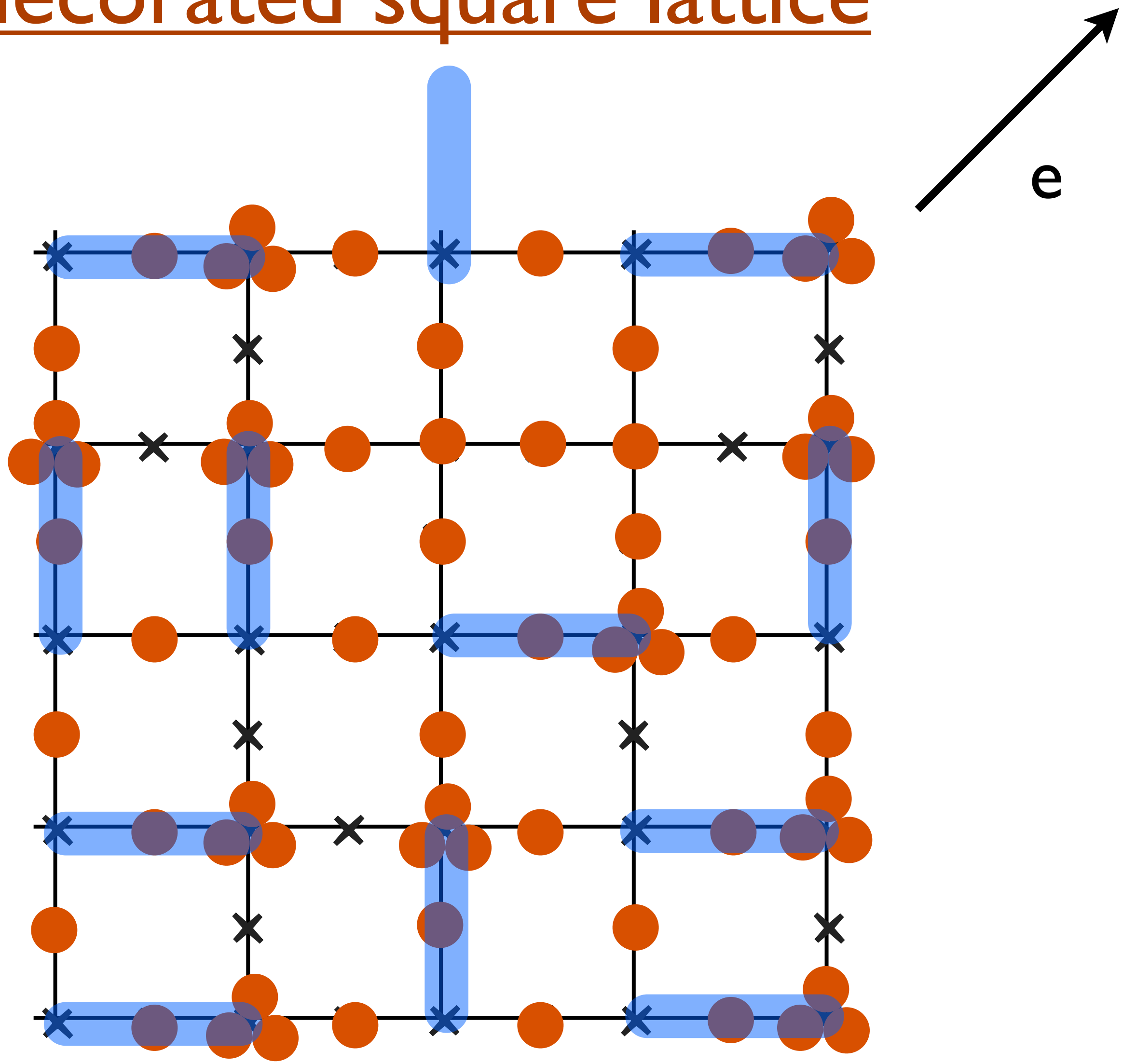
Go backwards around a plaquette

Tilting a decorated square lattice



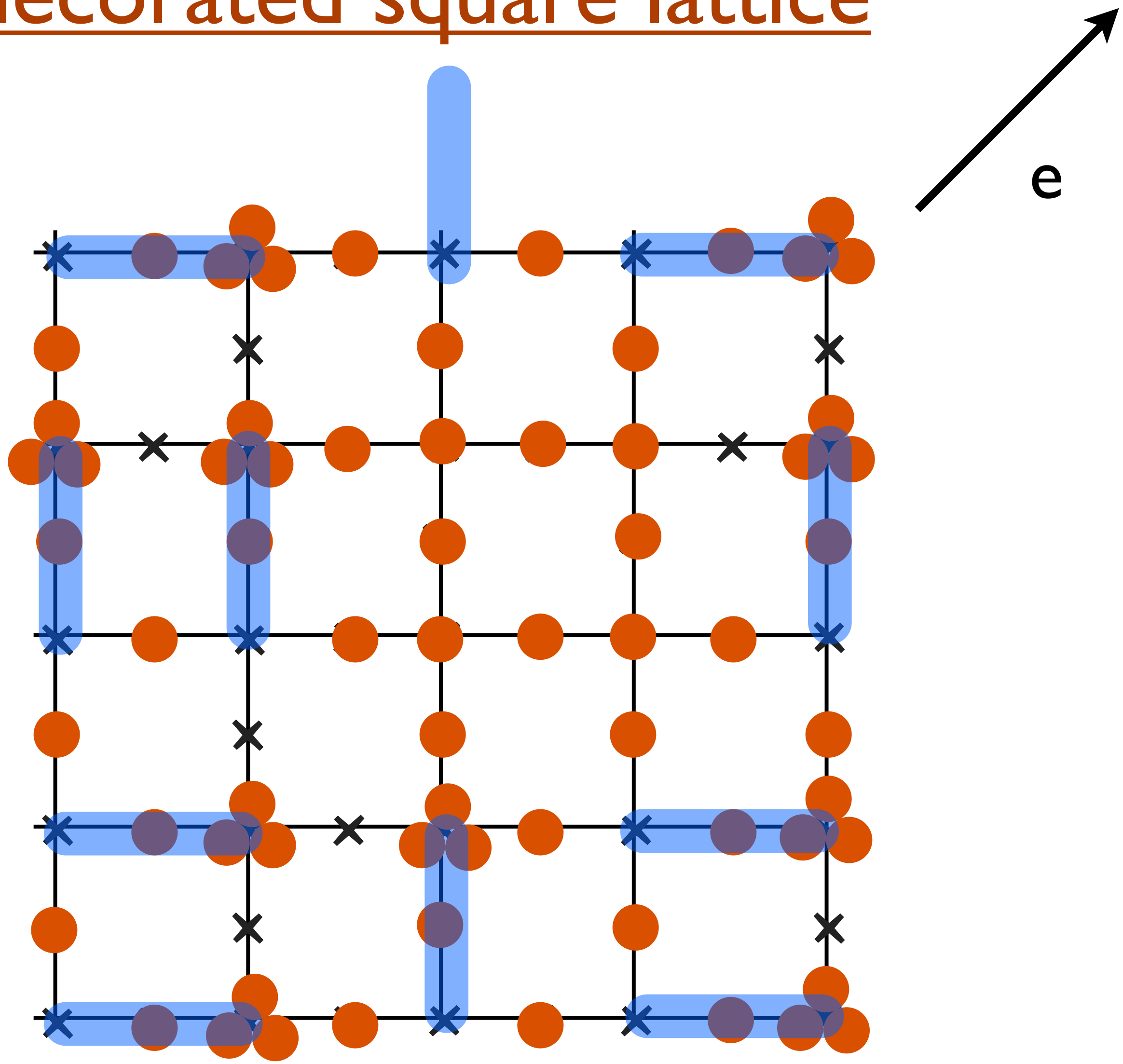
Go backwards around a plaquette

Tilting a decorated square lattice



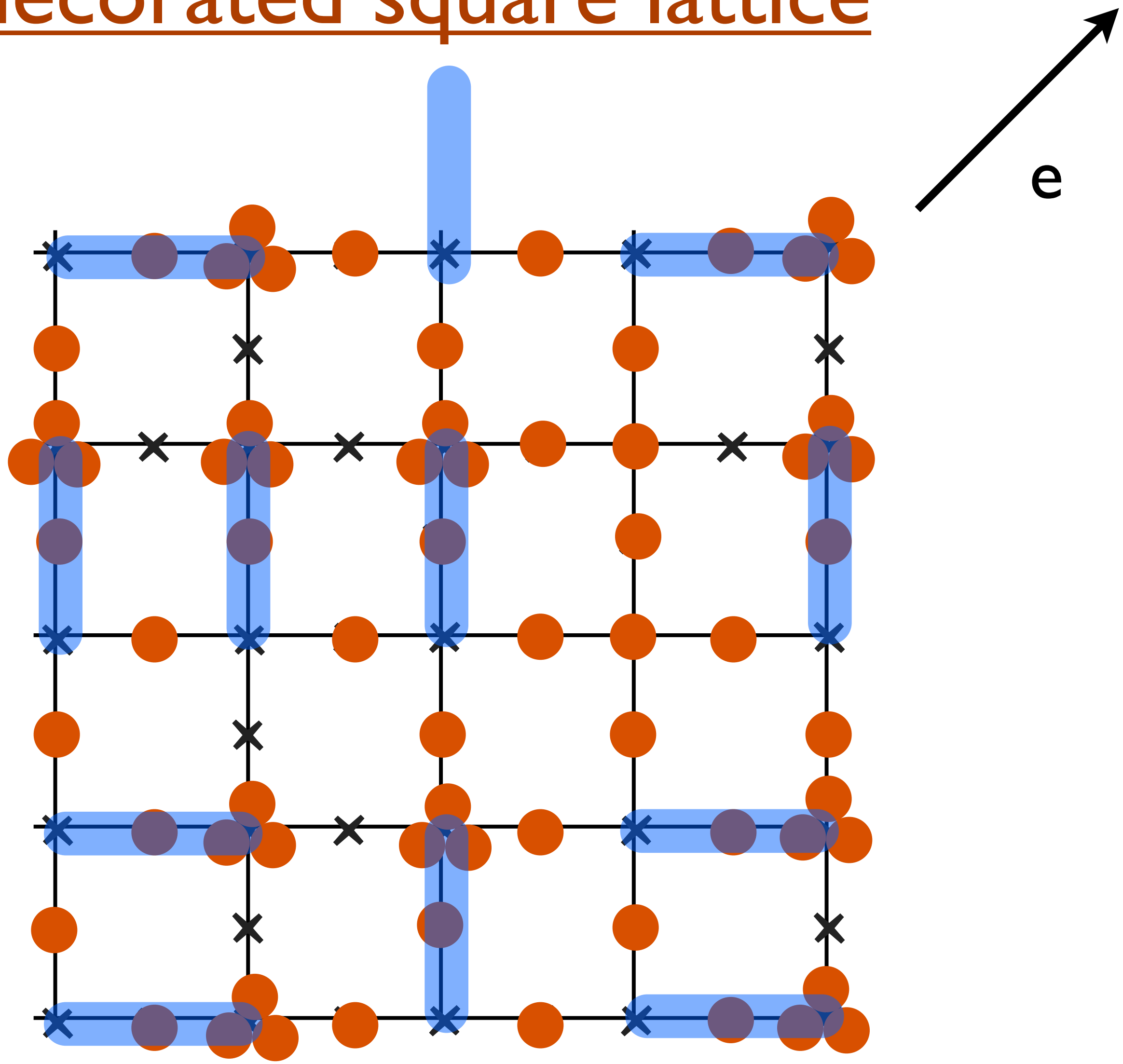
Go backwards around a plaquette

Tilting a decorated square lattice



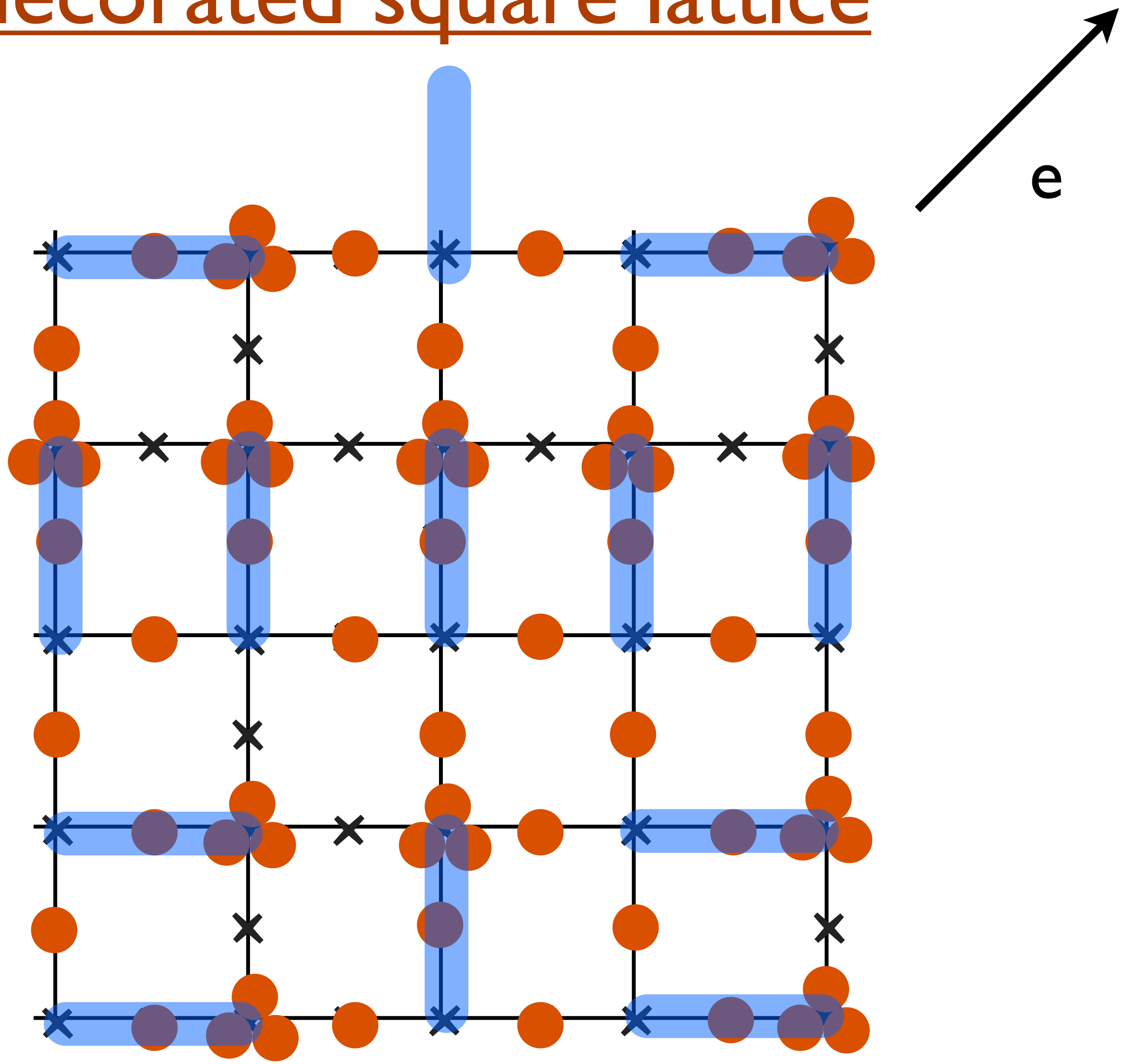
Go backwards around a plaquette

Tilting a decorated square lattice



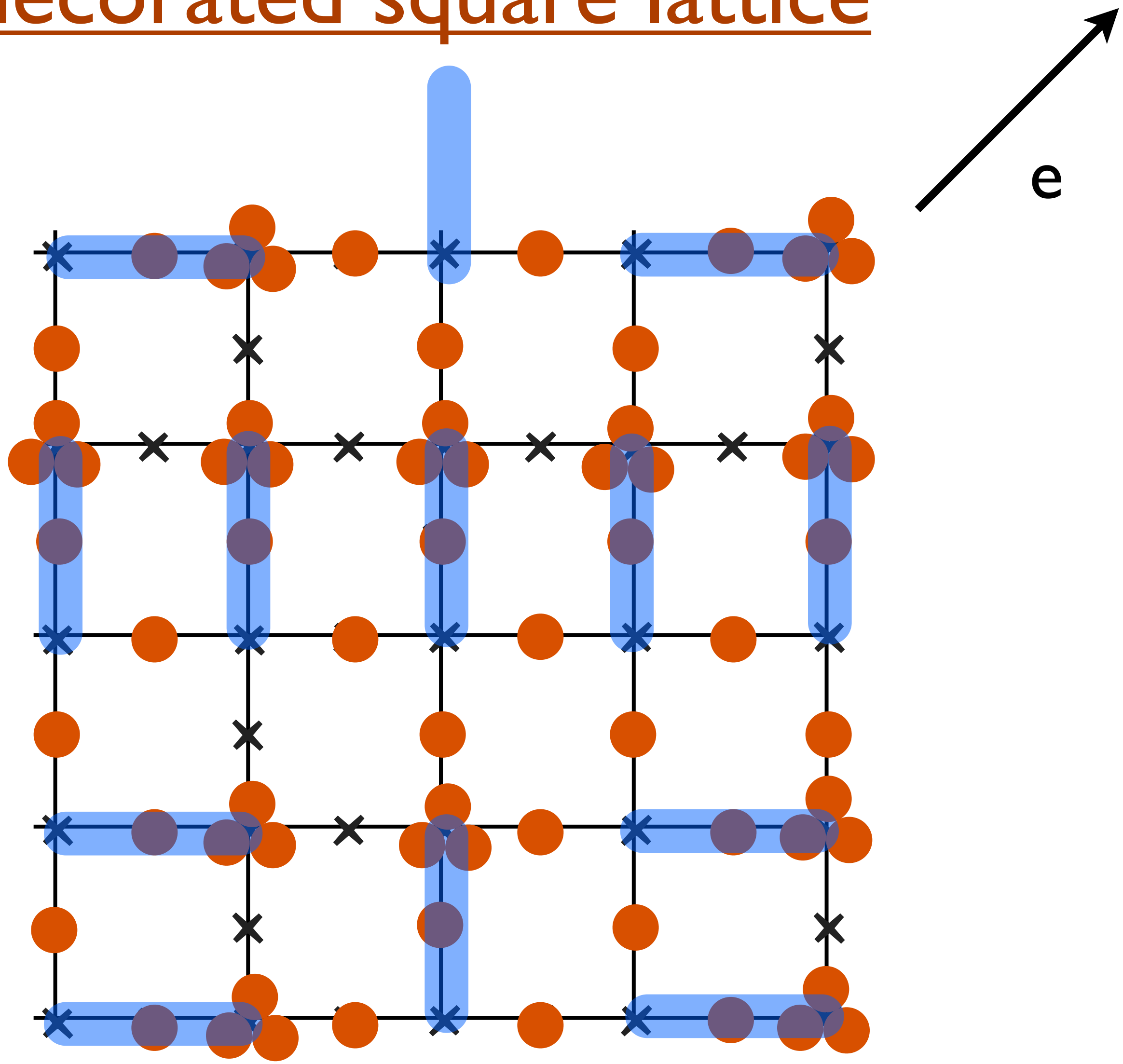
Then create a different set of 3's

Tilting a decorated square lattice



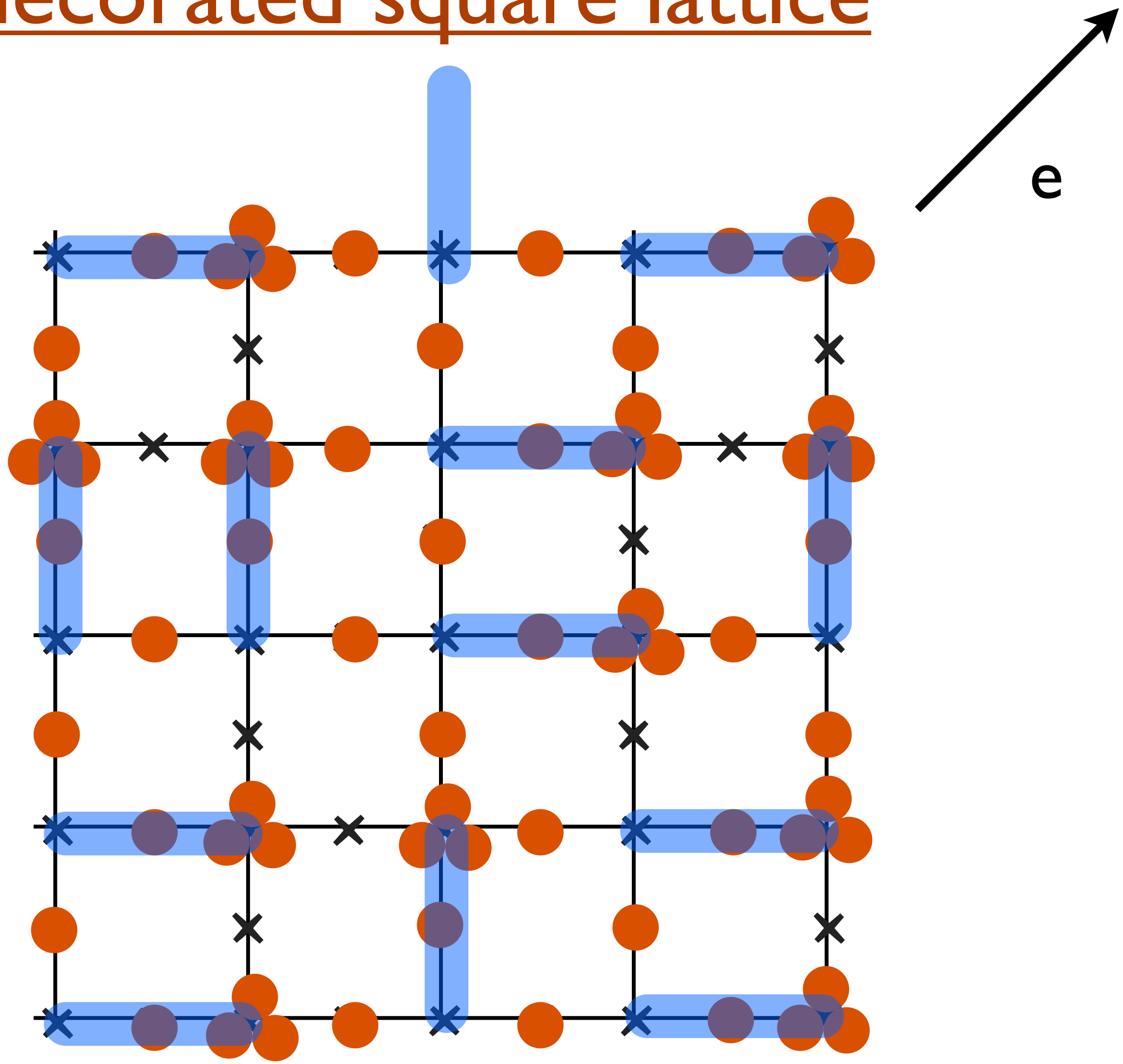
Then create a different set of 3's

Tilting a decorated square lattice



A different dimer covering

Tilting a decorated square lattice



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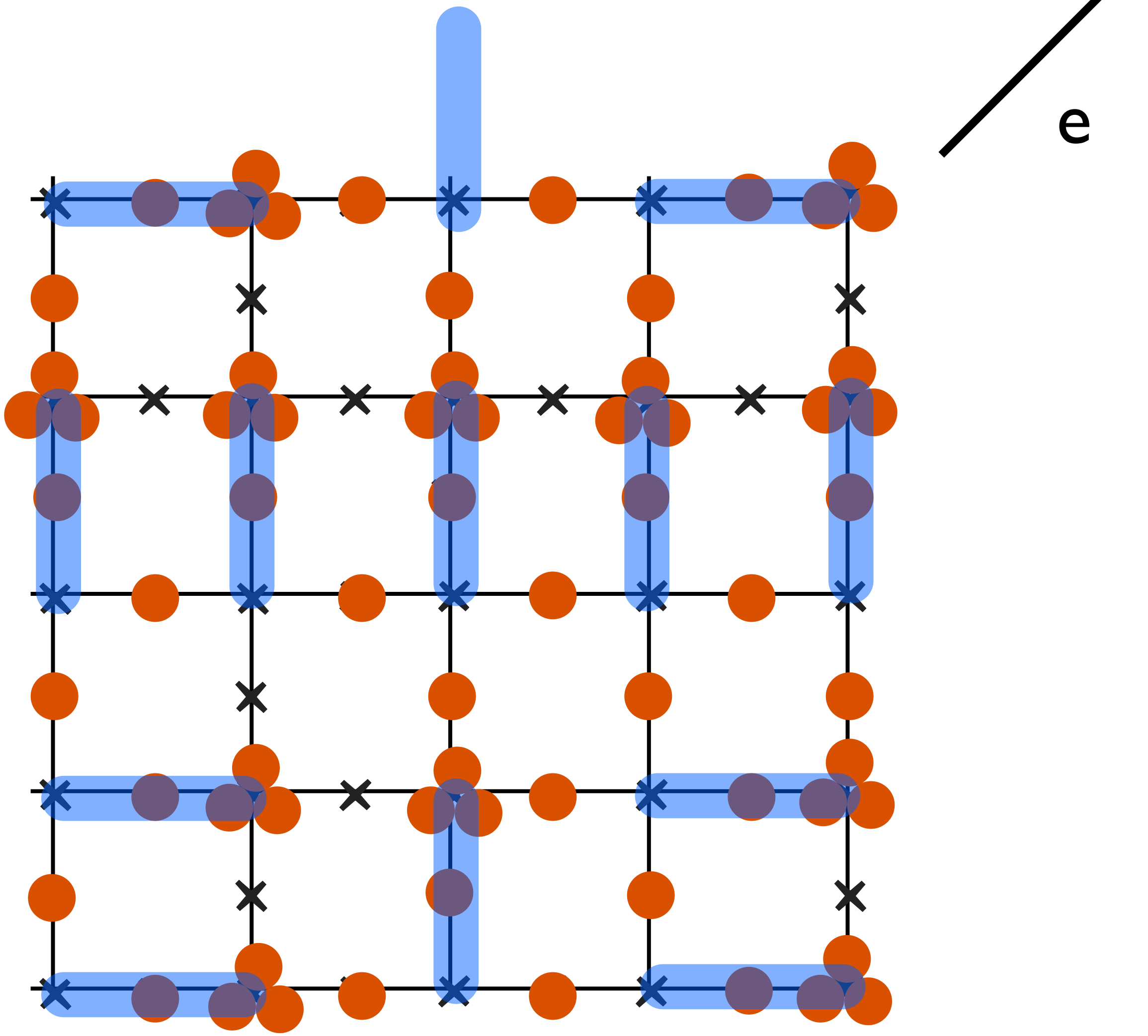
Dimers can resonate around a plaquette

Tilting a decorated square lattice



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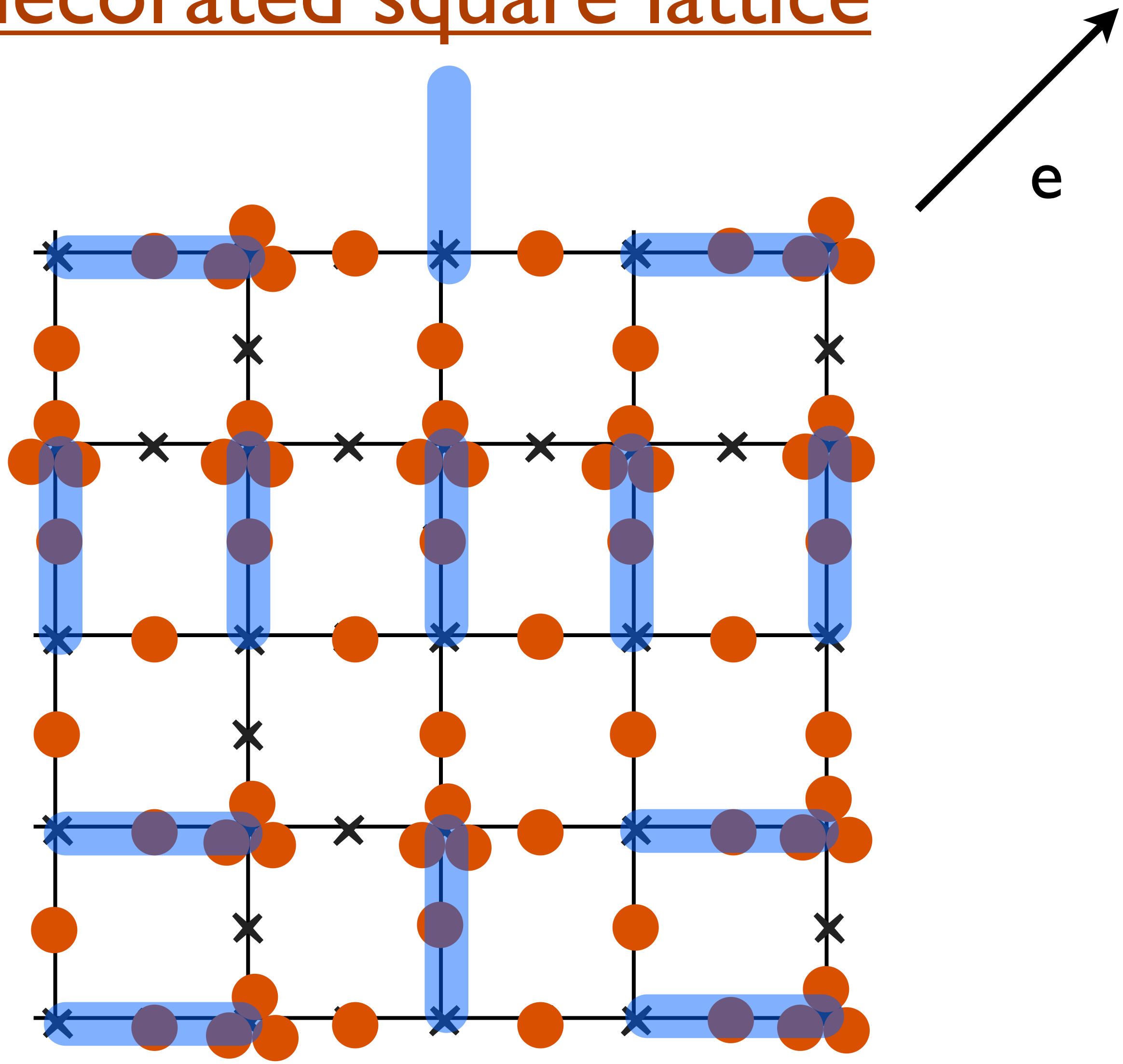
Dimers can resonate around a plaquette

Tilting a decorated square lattice



Susanne Pielawa

Strong tilt:
effective quantum dimer model

Conclusions

- Many common issues on many body quantum correlations in condensed matter and ultracold atoms

- Tilting Mott insulators can generate many interesting states with interesting quantum entanglement