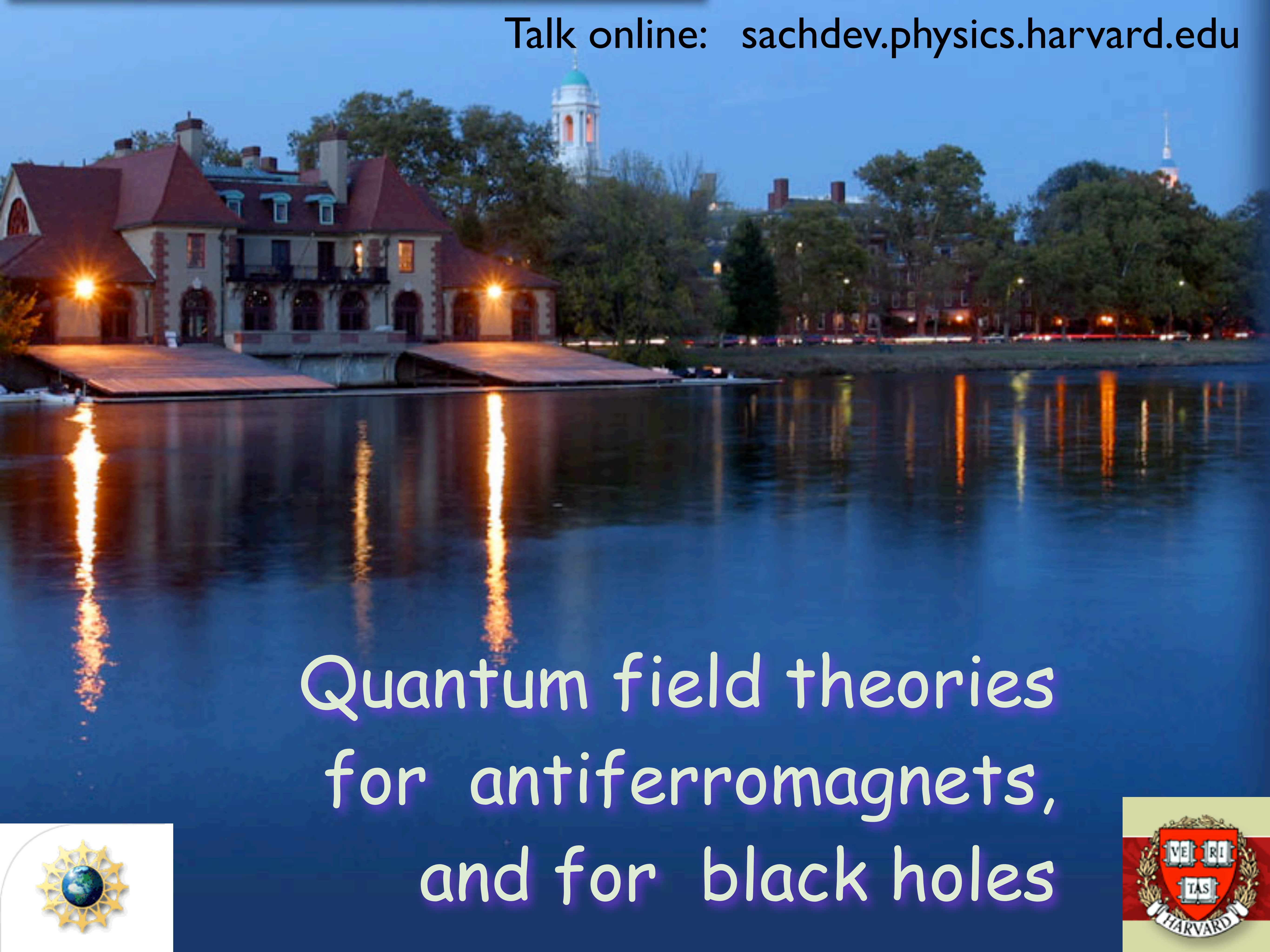
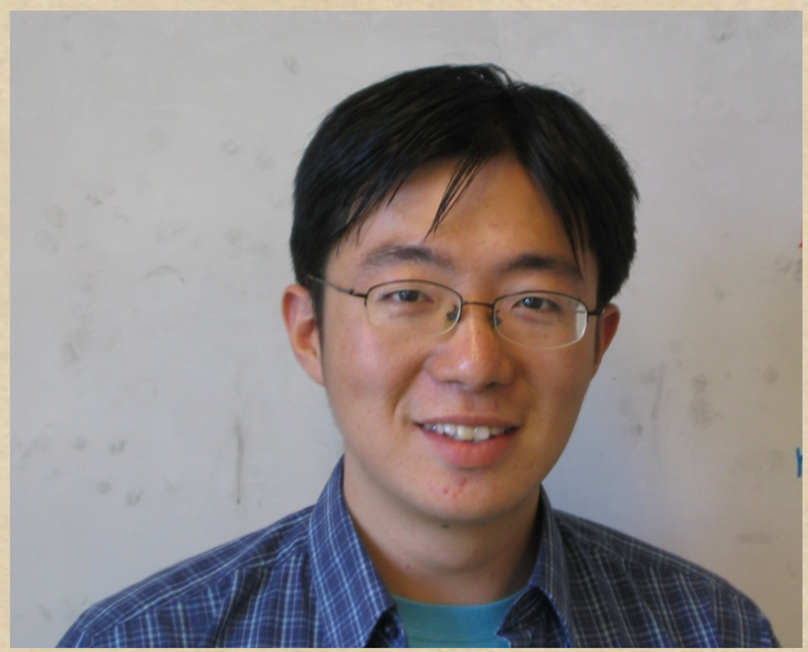
Talk online: sachdev.physics.harvard.edu



Quantum field theories for antiferromagnets, and for black holes

Condensed matter theorists

Markus Mueller
Geneva



Cenke Xu
Harvard

Yang Qi
Harvard

Outline

1. Landau-Ginzburg criticality
   Coupled-dimer antiferromagnets

2. Ground states of the triangular lattice antiferromagnet
   Experiments on $X[Pd(dmit)_2]_2$

3. Spinons, visons, and Berry phases
   Quantum field theories for two-dimensional antiferromagnets

4. Quantum criticality and black holes
   The AdS/CFT correspondence

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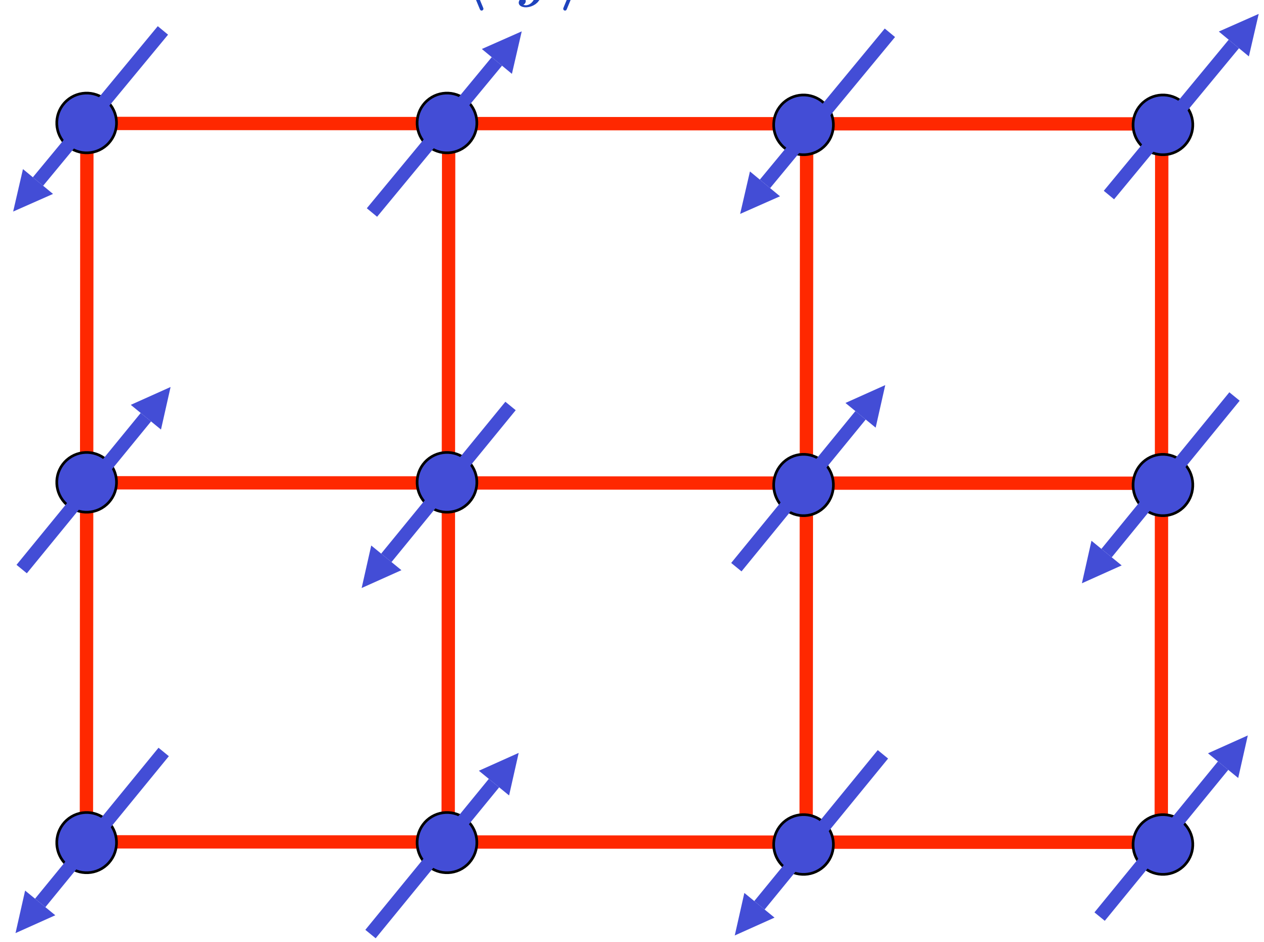
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

Square lattice antiferromagnet

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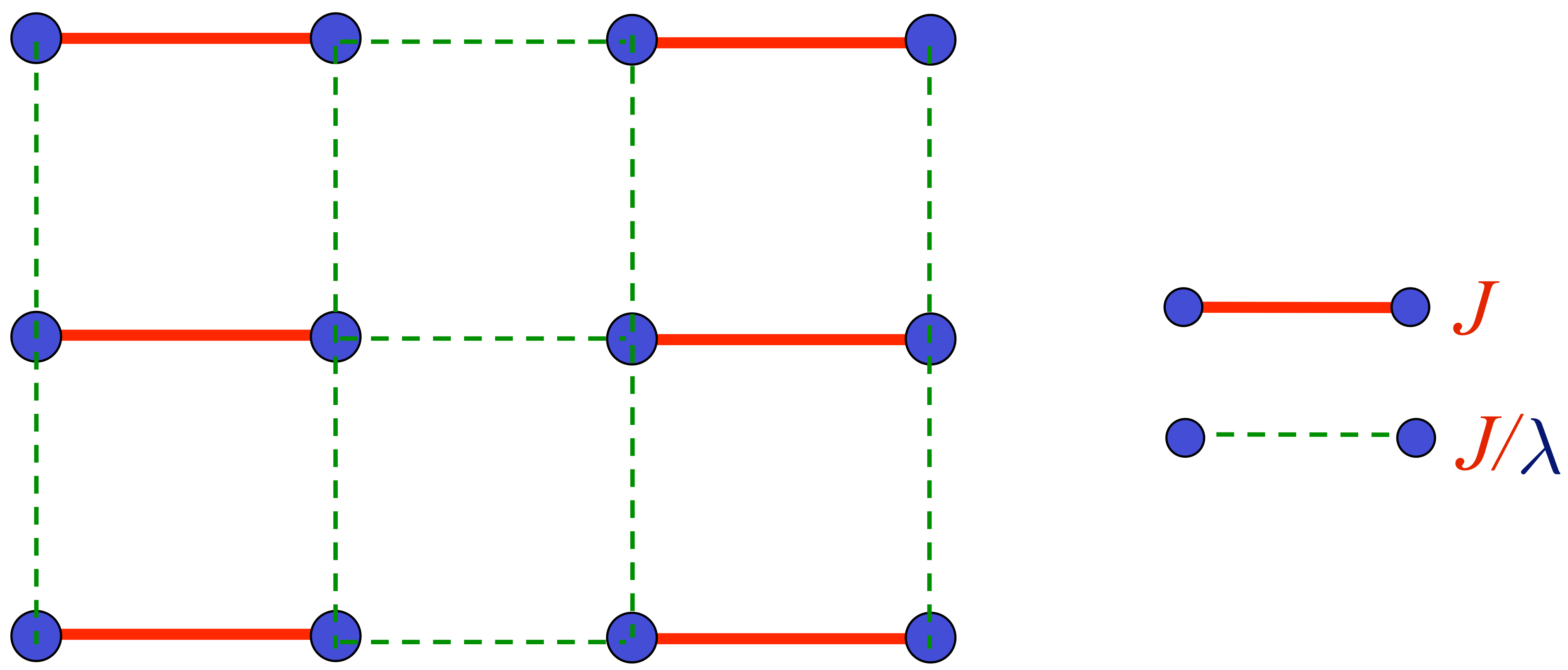
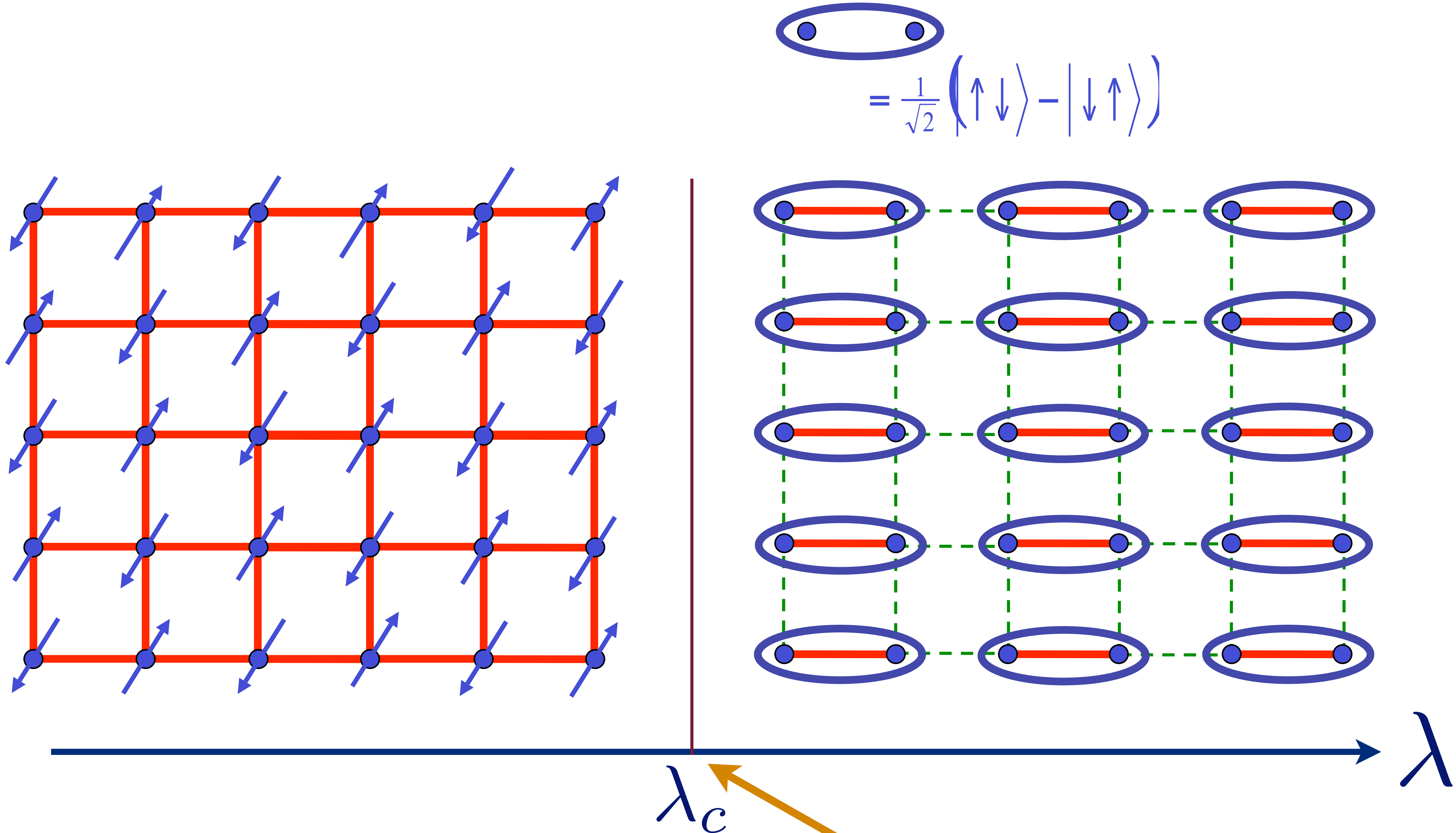
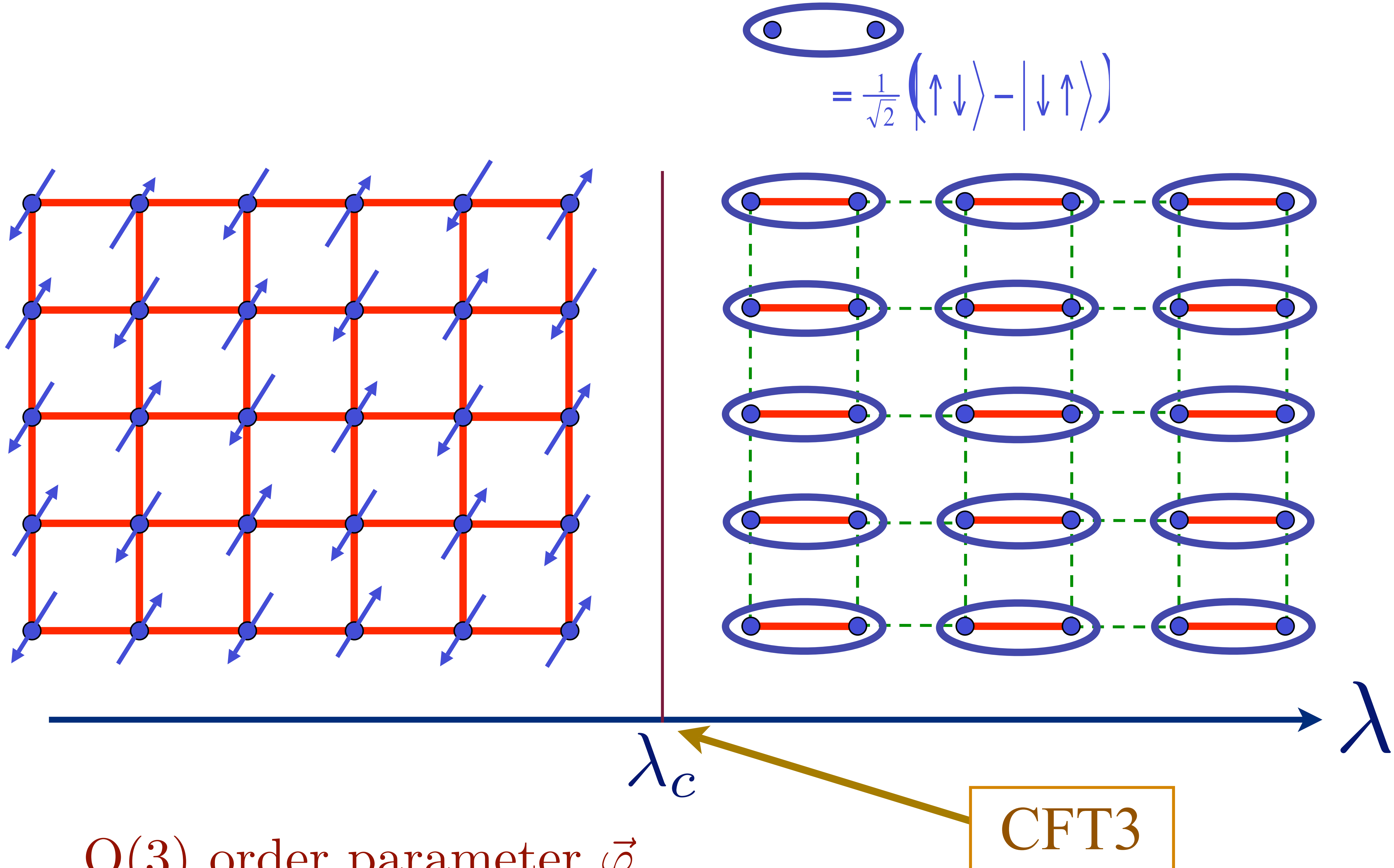


Weaken some bonds to induce spin entanglement in a new quantum phase



Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.* B **65**, 014407 (2002).



O(3) order parameter $\vec{\varphi}$

$$\mathcal{S} = \int d^2 r d\tau \left[(\partial_\tau \varphi)^2 + c^2 (\nabla_r \vec{\varphi})^2 + s\vec{\varphi}^2 + u\left(\vec{\varphi}^2\right)^2 \right]$$

Quantum Monte Carlo - critical exponents

Table IV: Fit results for the critical exponents ν, β/ν, and η. We summarize results including a variation of the critical point within its error bar. For the ladder model (top group of values) fit results and quality of fits are also given at the previous best estimate of α_c. The bottom group are results for the plaquette model. Numbers in [. . .] brackets denote the χ^2/d.o.f. For comparison relevant reference values for the 3D $O(3)$ universality class are given in the last line.

α_c	ν^a	β/ν^b	η^c
$1.9096-\sigma$	0.712(4) [1.8]	0.516(2) [0.5]	0.026(2) [0.2]
1.9096	0.711(4) [1.8]	0.518(2) [1.1]	0.029(5) [0.8]
$1.9096+\sigma$	0.710(4) [1.8]	0.519(3) [2.5]	0.032(7) [1.4]
1.9107^d	0.709(3) [1.7]	0.525(8) [15.3]	0.051(10) [12]
$1.8230-\sigma$	0.708(4) [0.99]	0.515(2) [0.84]	0.025(4) [0.15]
1.8230	0.706(4) [1.04]	0.516(2) [0.40]	0.028(3) [0.31]
$1.8230+\sigma$	0.706(4) [1.10]	0.517(2) [1.6]	0.031(5) [0.80]
Ref. 49	0.7112(5)	0.518(1)	0.0375(5)

[a] $L > 12$.
[b] $L > 16$.
[c] $L > 20$.
[d] Previous best estimate of Ref. 19.

S. Wenzel and W. Janke, arXiv:0808.1418
M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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Field-theoretic RG of CFT3
E. Vicari *et al.*

S. Wenzel and W. Janke, arXiv:0808.1418
M. Troyer, M. Imada, and K. Ueda, *J. Phys. Soc. Japan* (1997)

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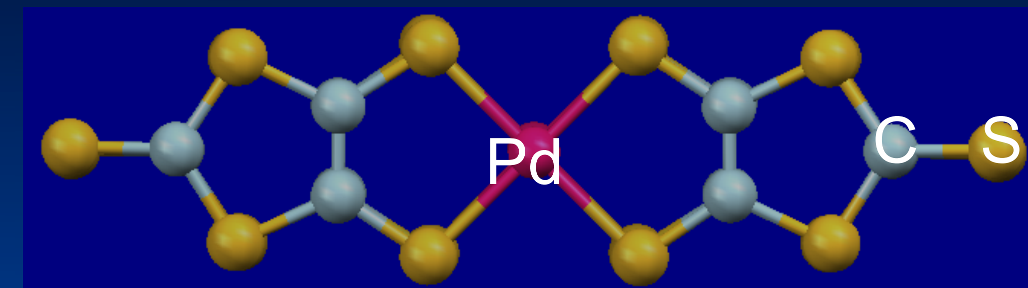
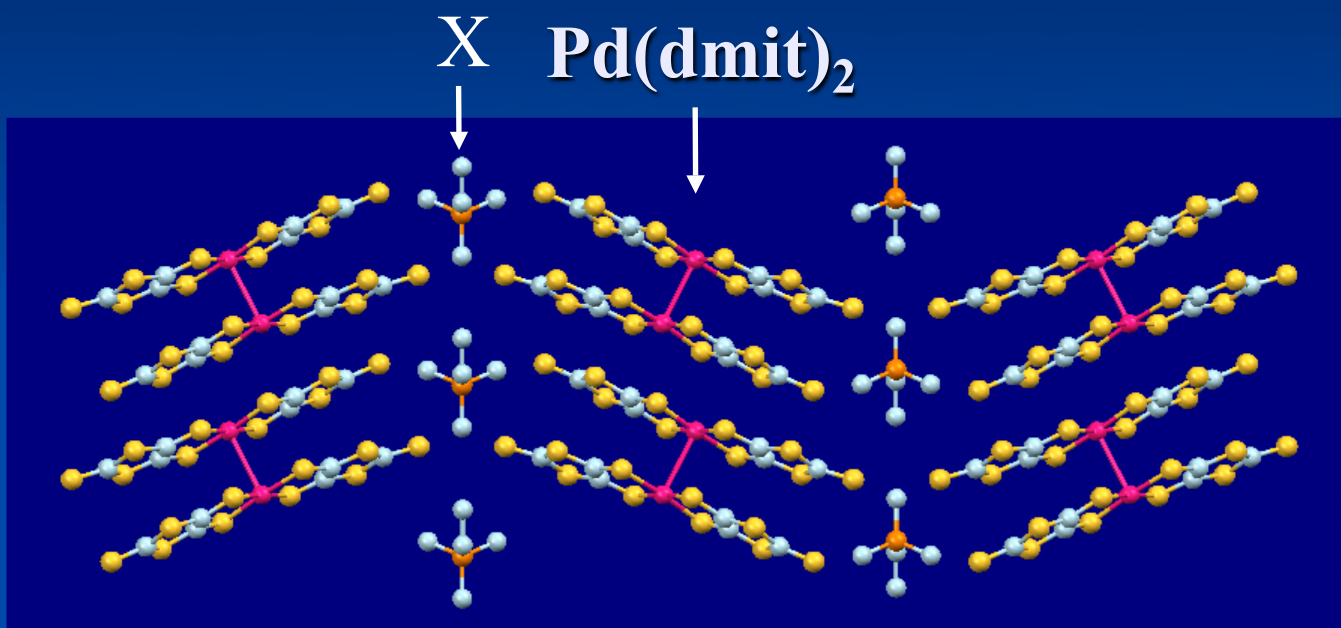
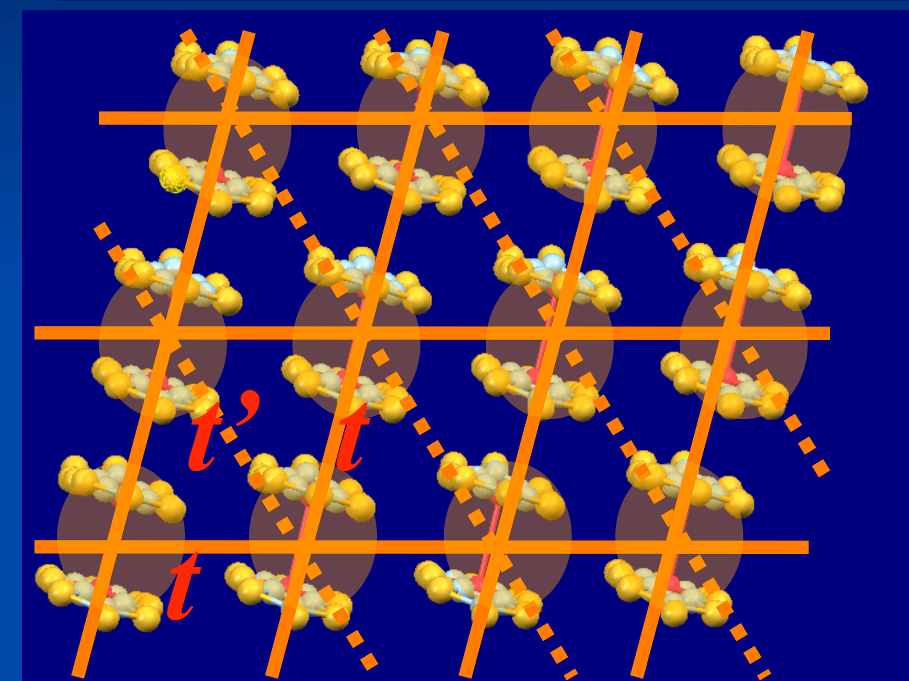
X[Pd(dmit)$_2$]$_2$







Half-filled band → Mott insulator with spin S = 1/2

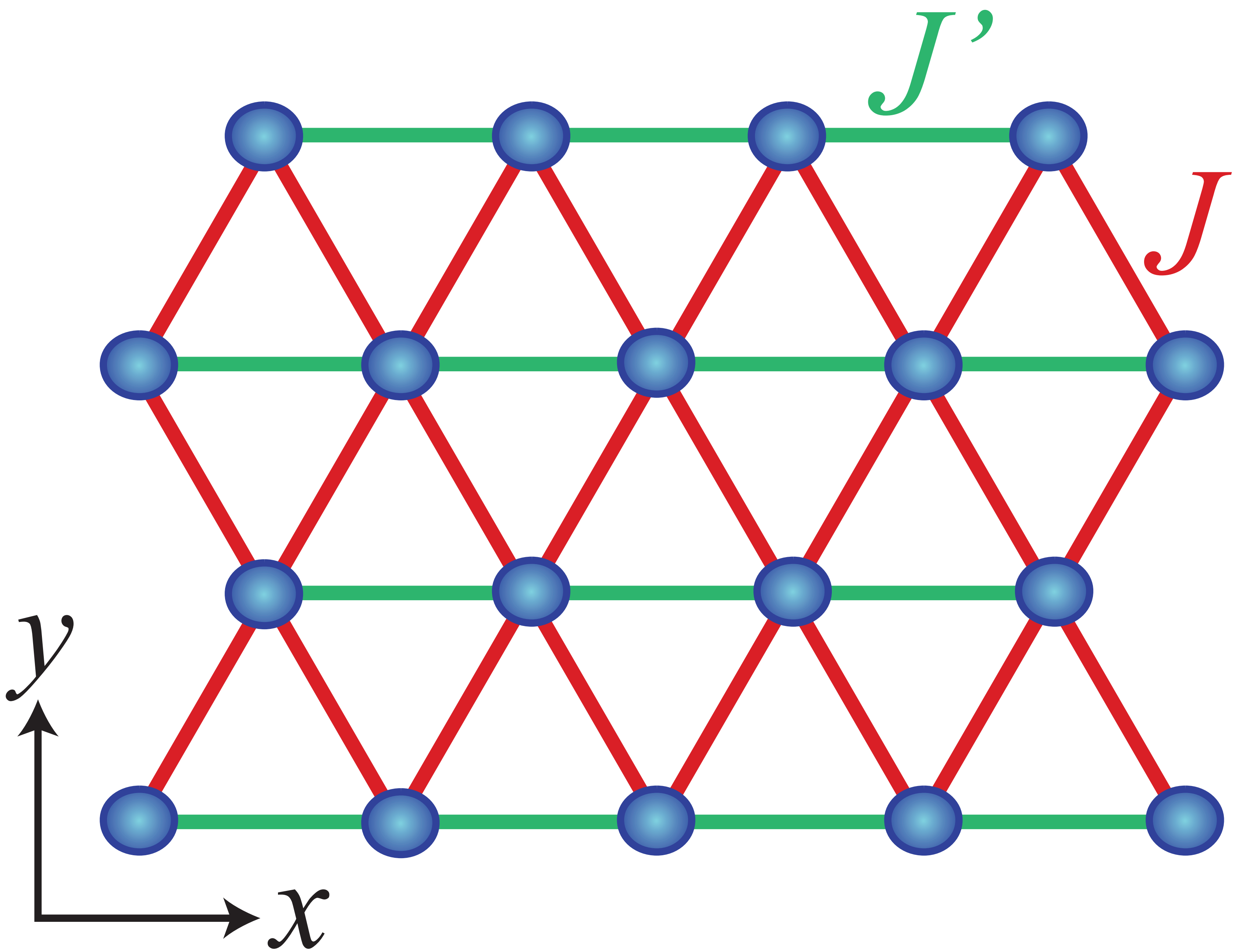
Triangular lattice of [Pd(dmit)$_2$]$_2$

→ frustrated quantum spin system

Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$$

$\vec{S}_i \;\Rightarrow\;$ spin operator with $S = 1/2$



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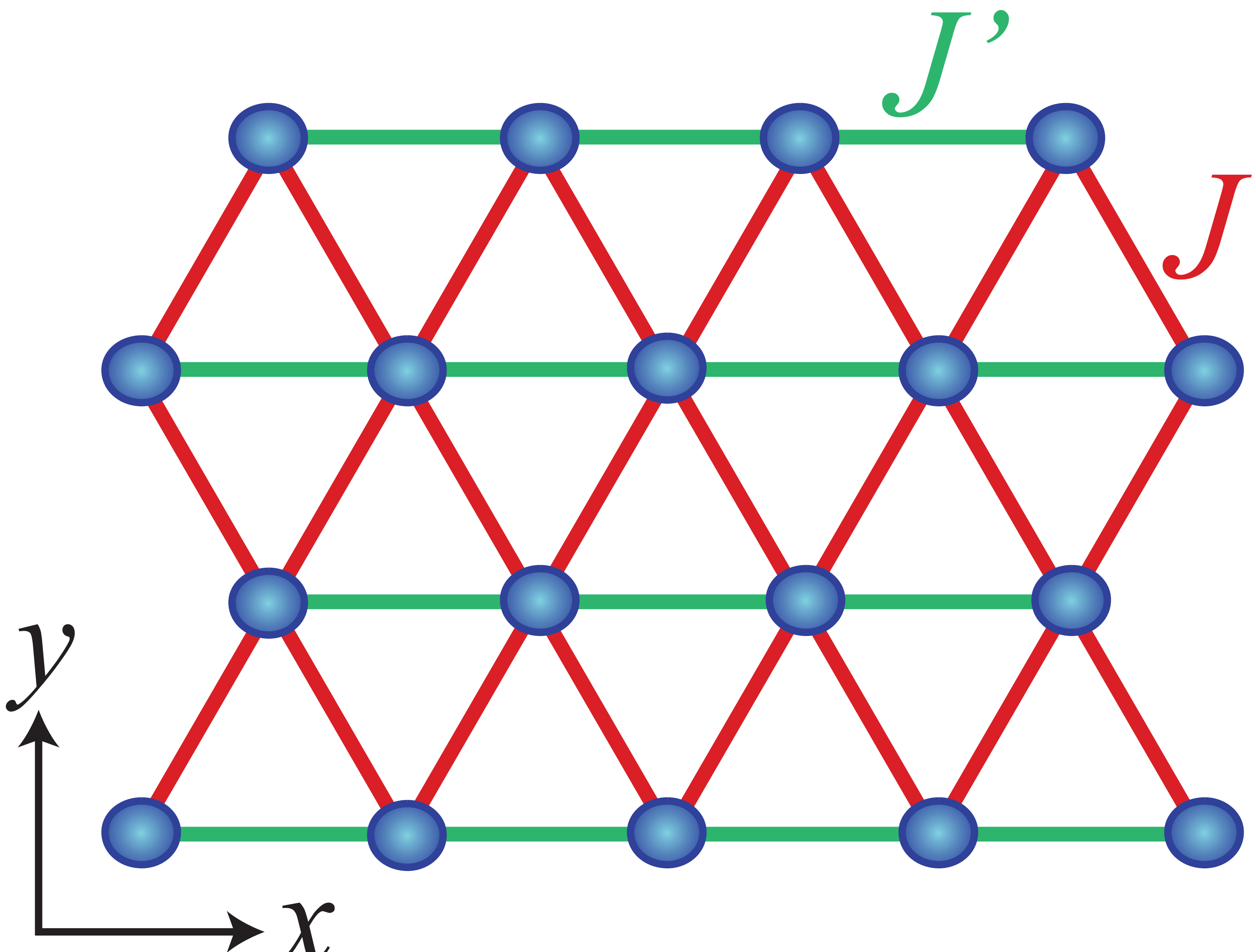
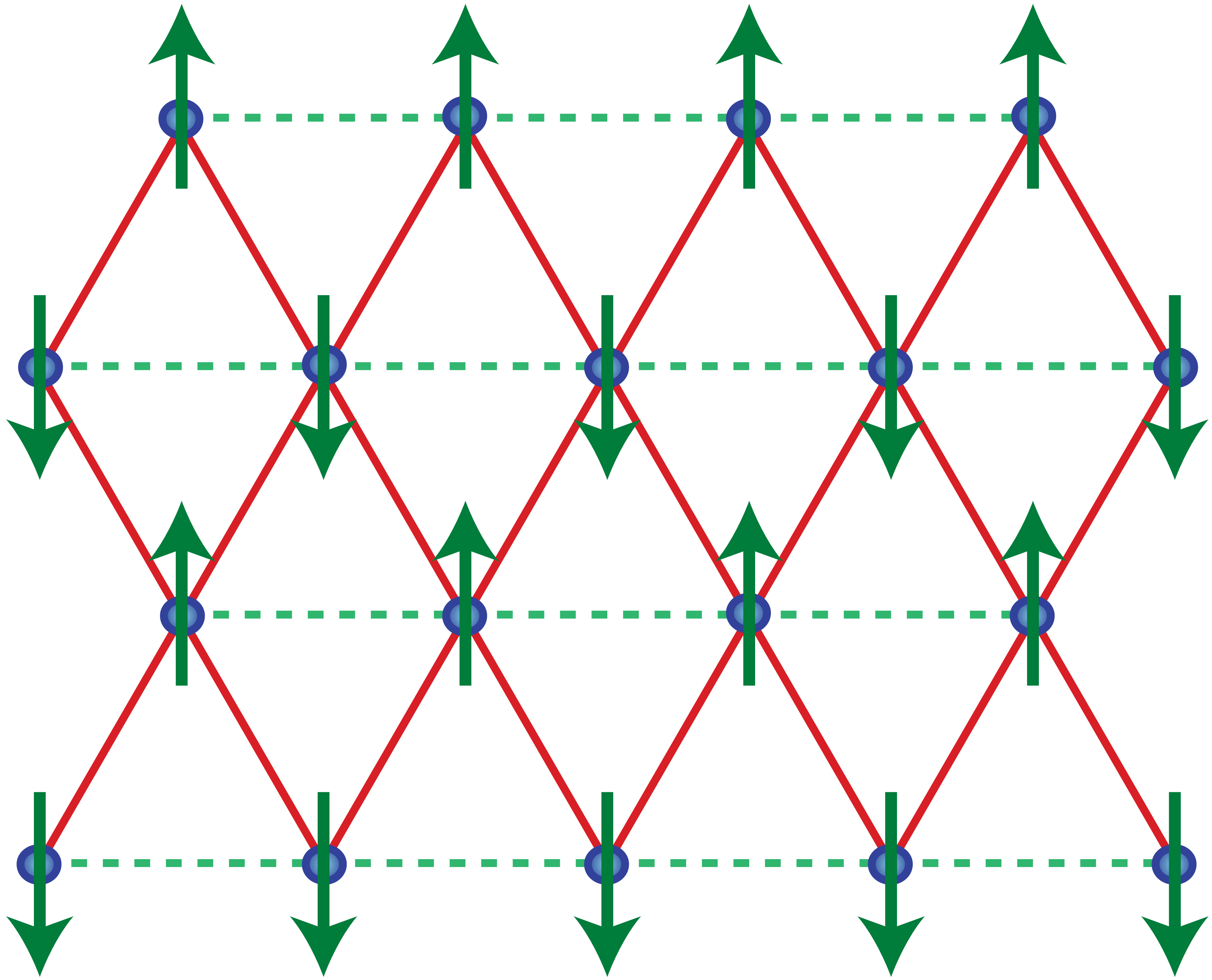


What is the ground state as a function of J'/J ?

Anisotropic triangular lattice antiferromagnet

Broken spin rotation symmetry



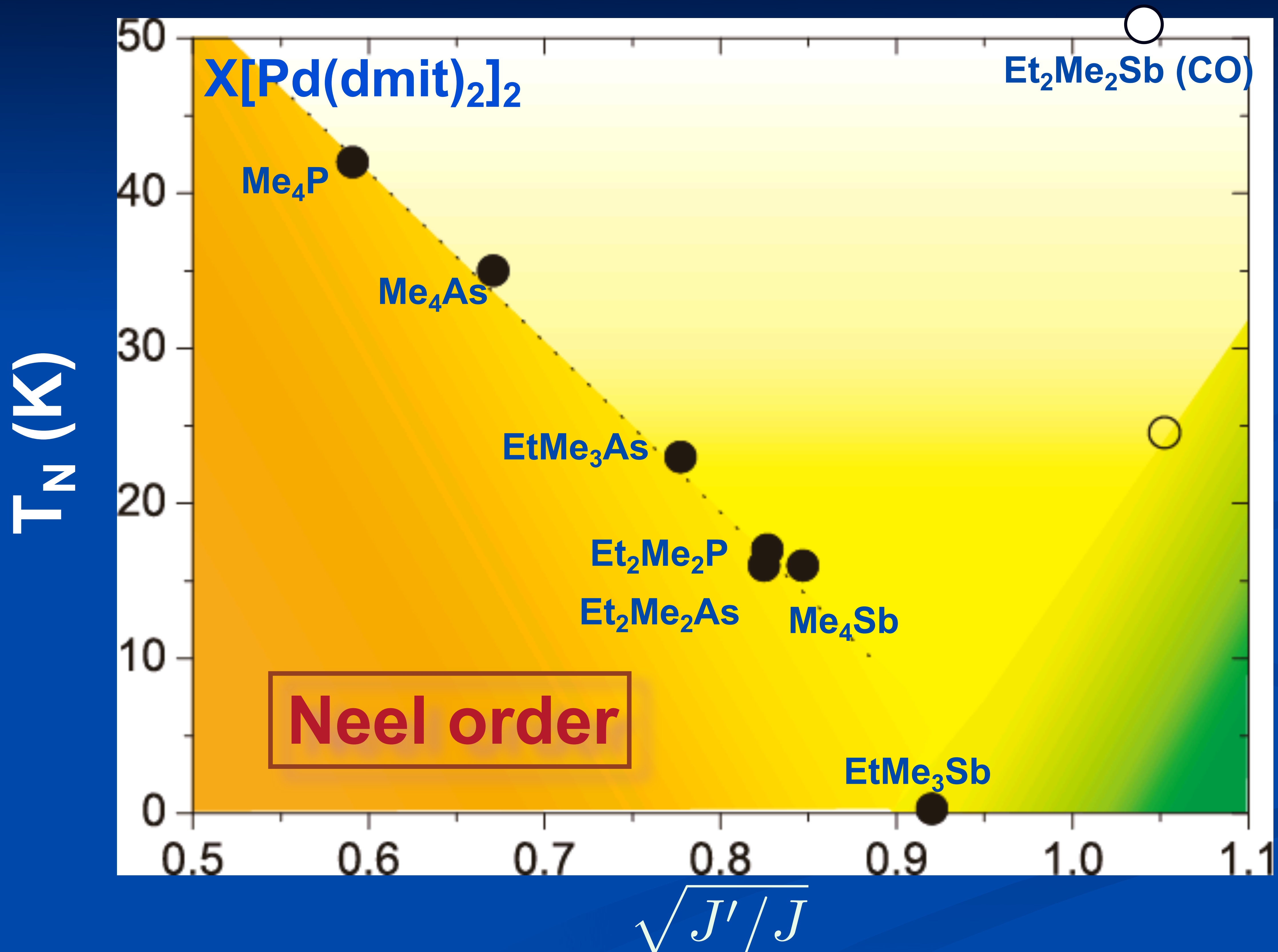
Neel ground state for small J'/J

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

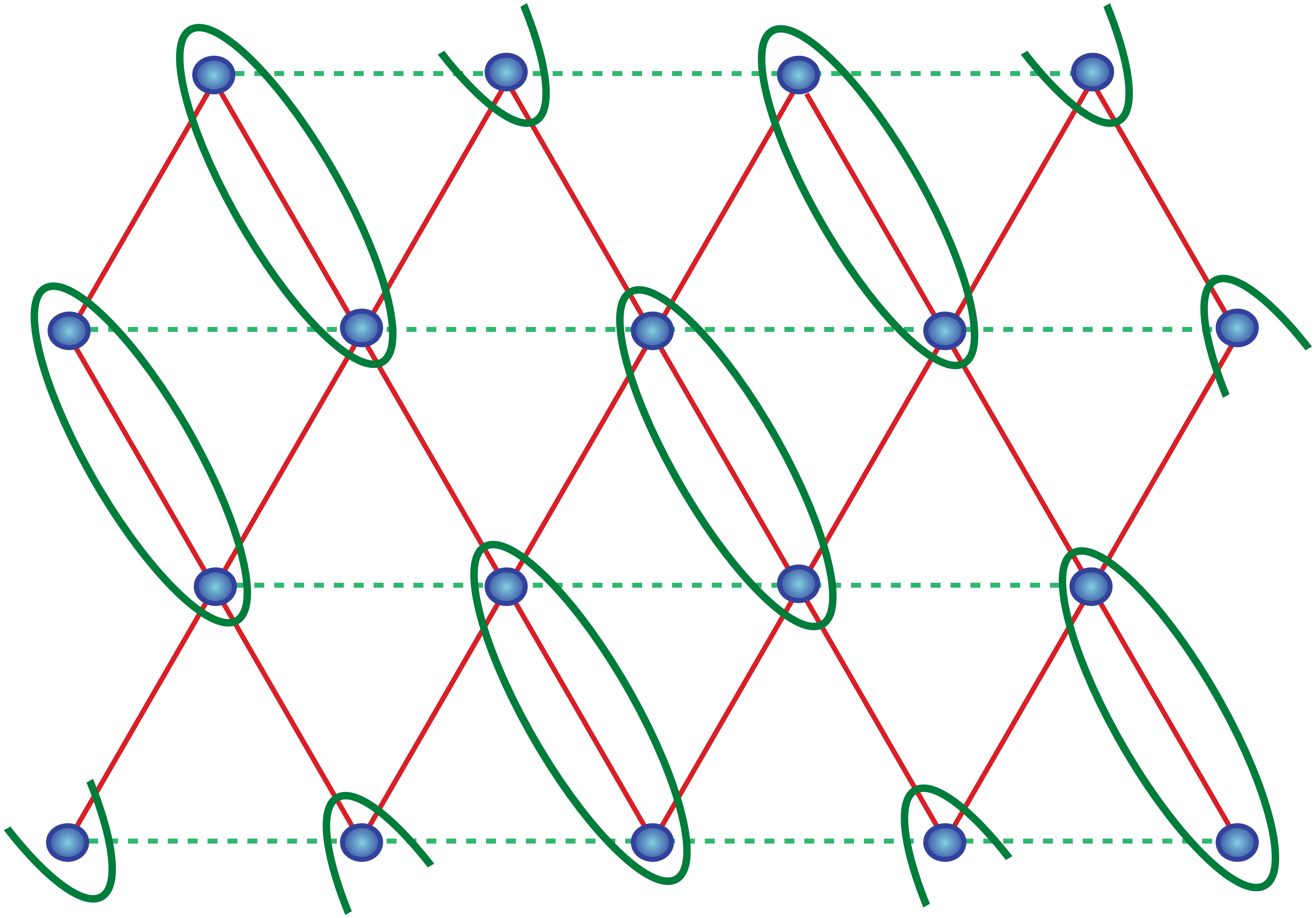
- Néel antiferromagnetic LRO

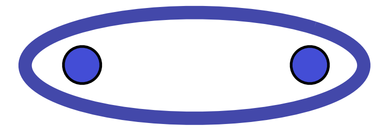
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Anisotropic triangular lattice antiferromagnet





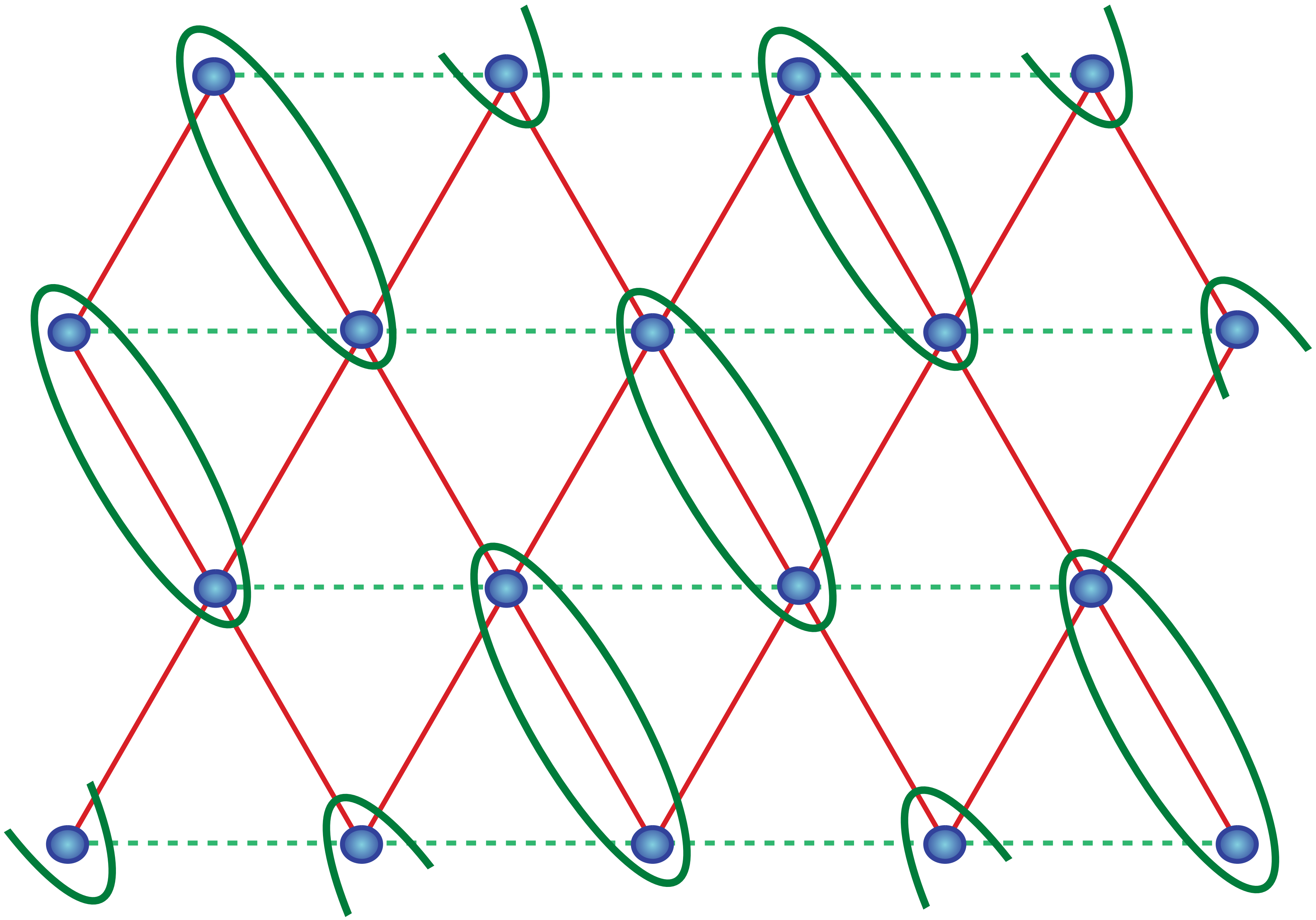
$$= \frac{(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}{\sqrt{2}}$$

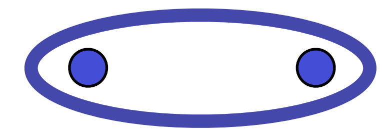
Possible ground state for intermediate J'/J

N. Read and S. Sachdev, *Phys. Rev. Lett*. **62,** *1694 (1989)*

Anisotropic triangular lattice antiferromagnet

Broken lattice space group symmetry





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Valence bond solid (VBS)

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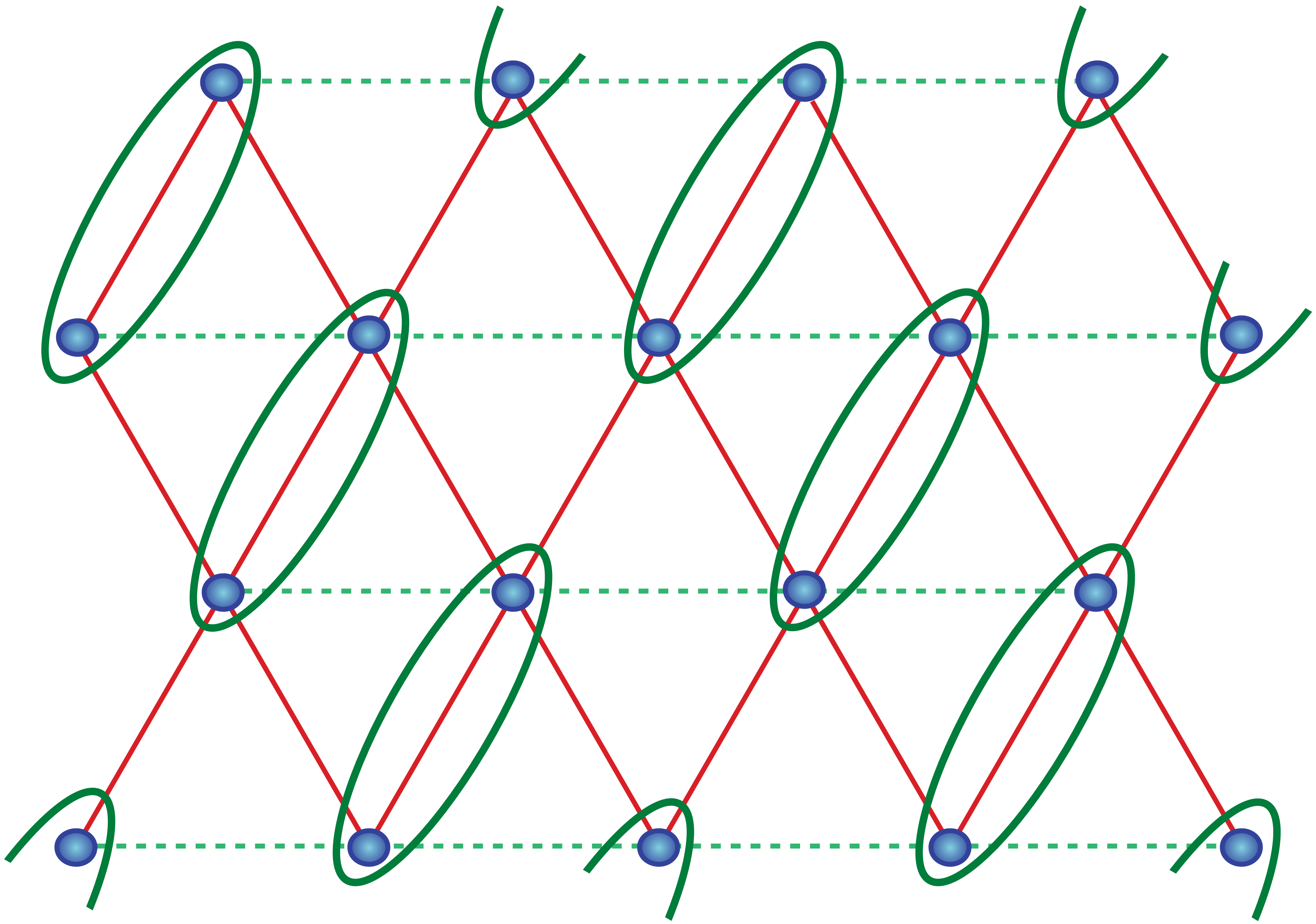

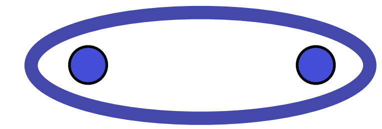

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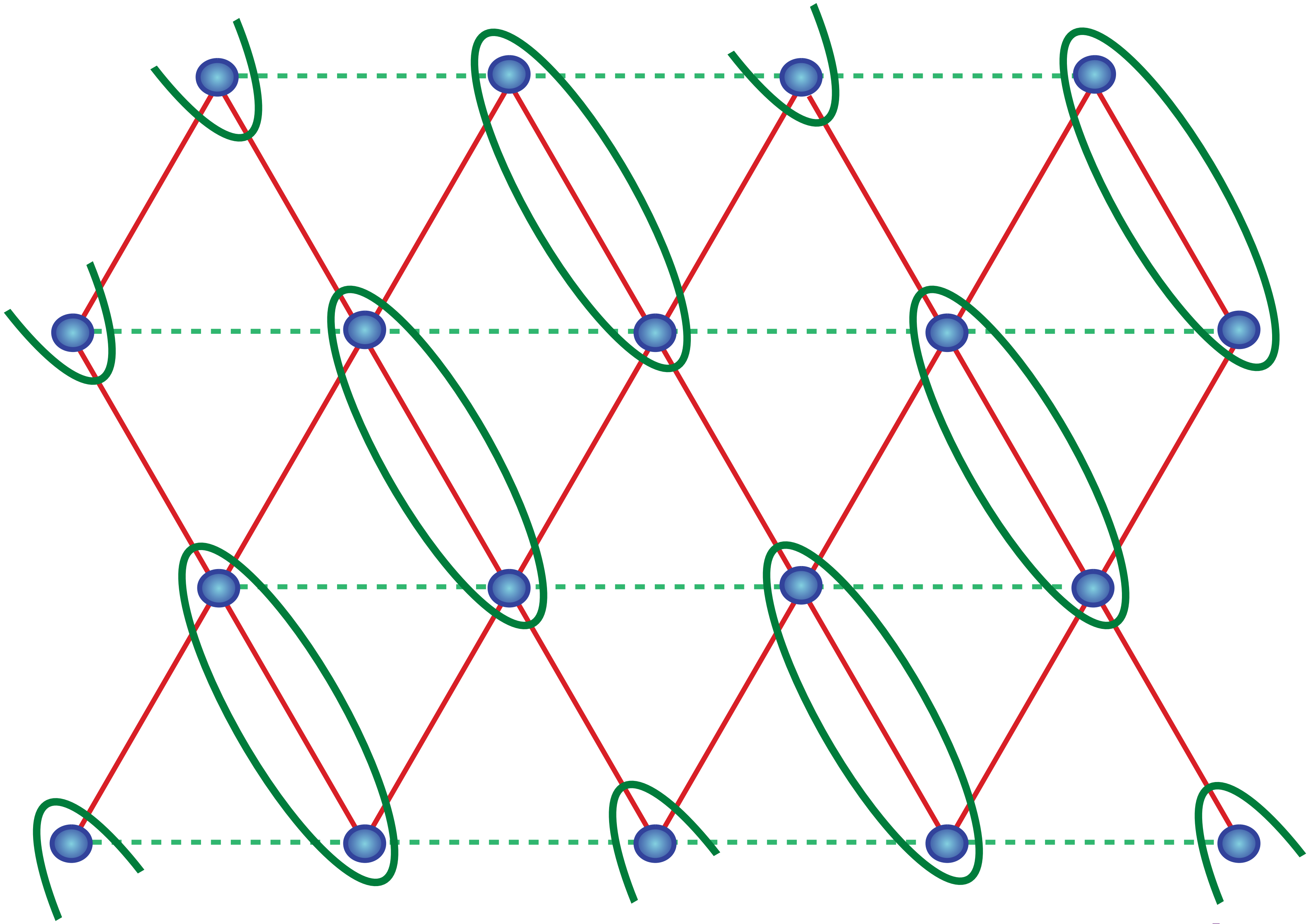

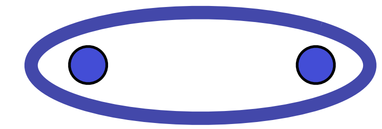

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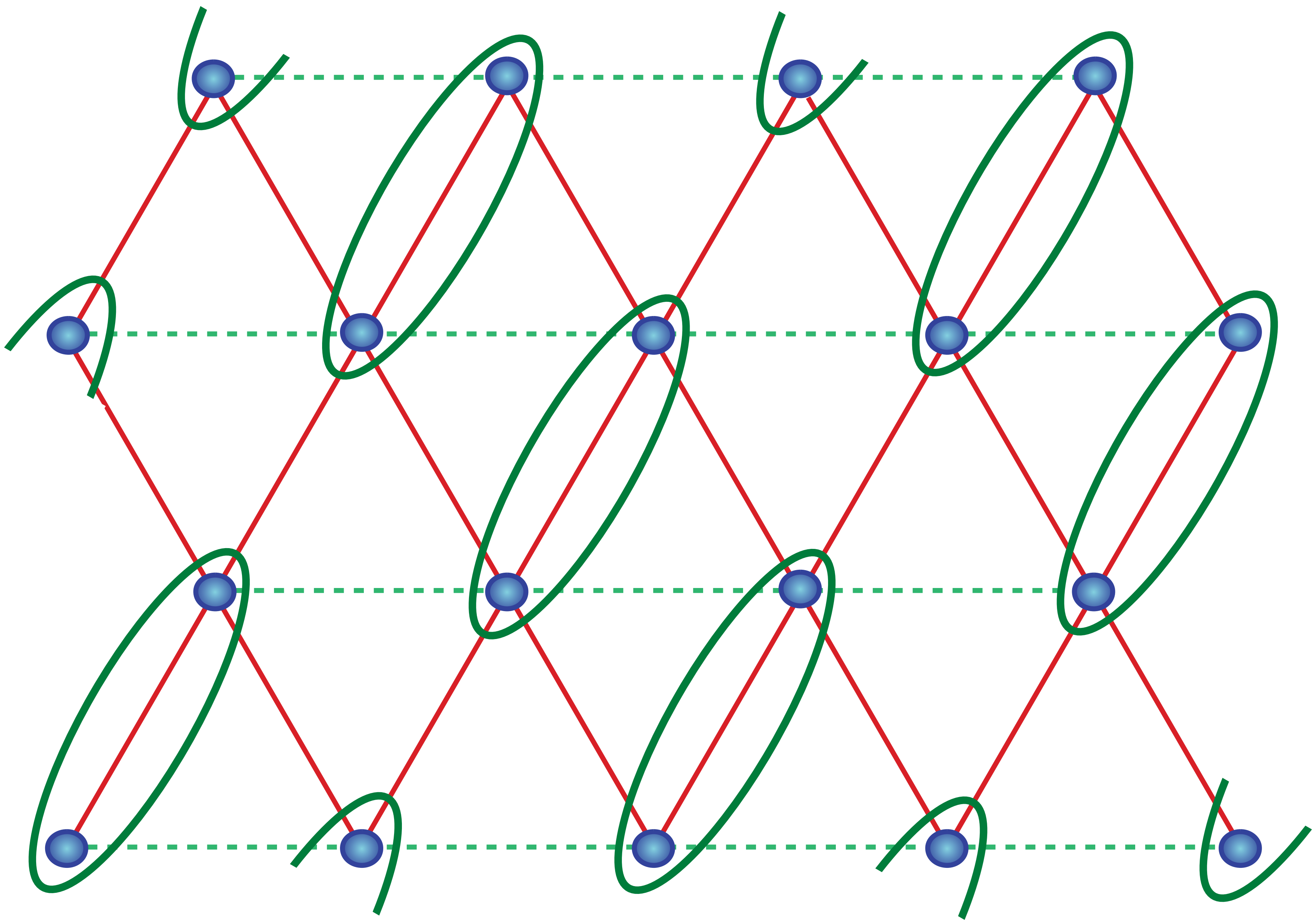

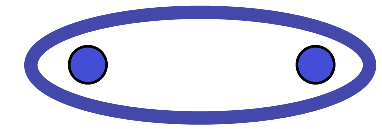

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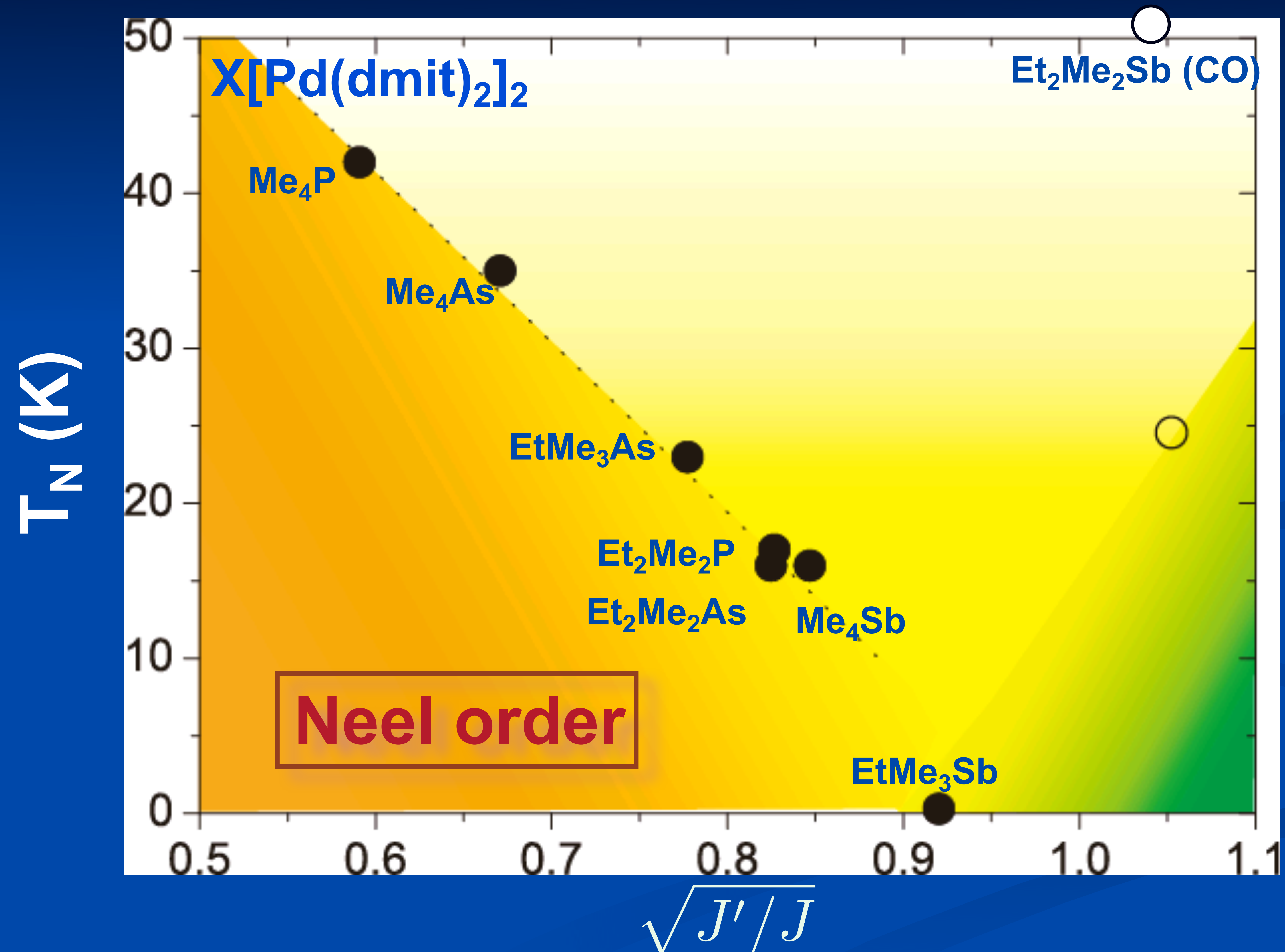
N. Read and S. Sachdev, *Phys. Rev. Lett.* **62,** *1694 (1989)*

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

Magnetic Criticality



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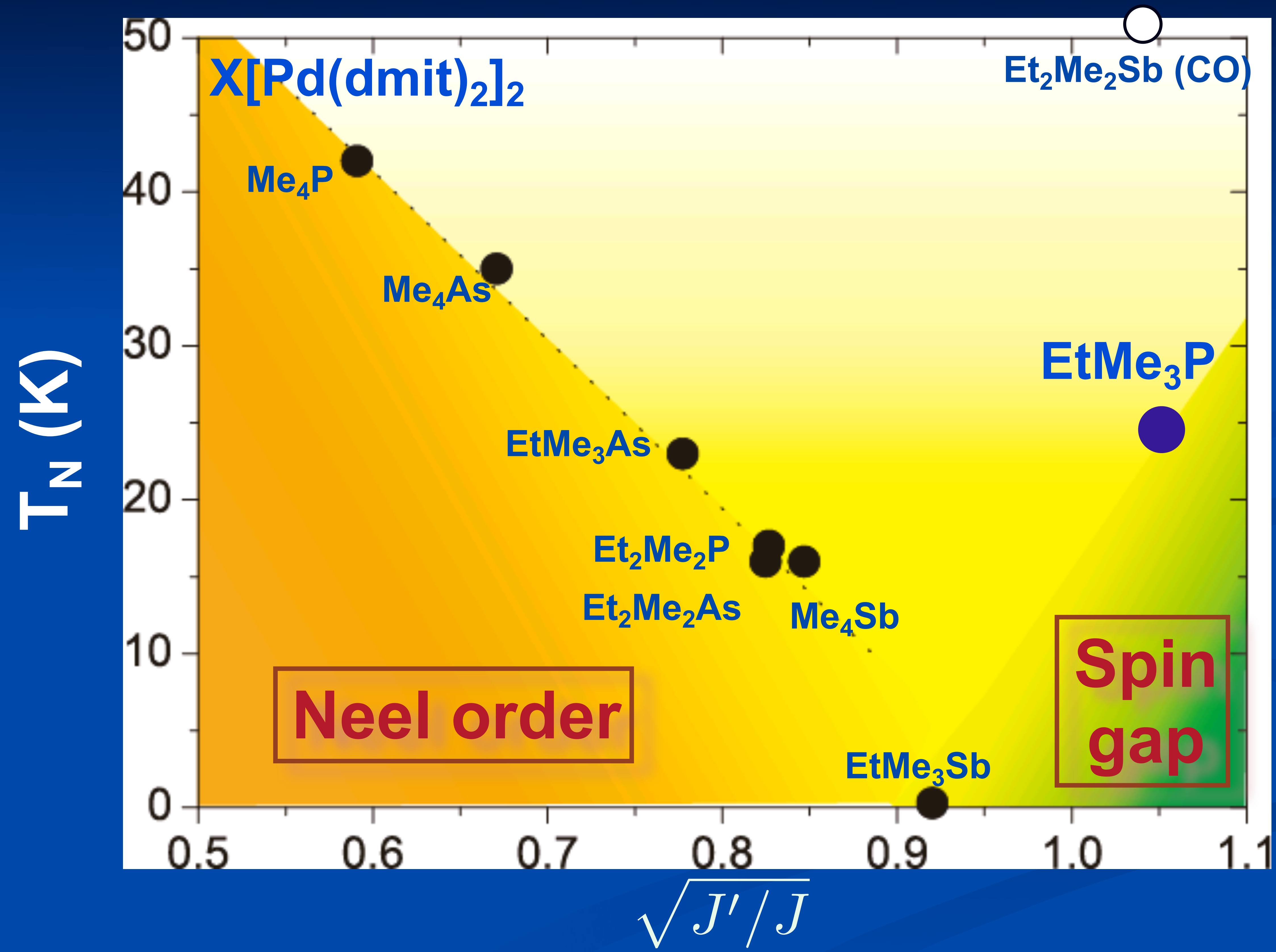

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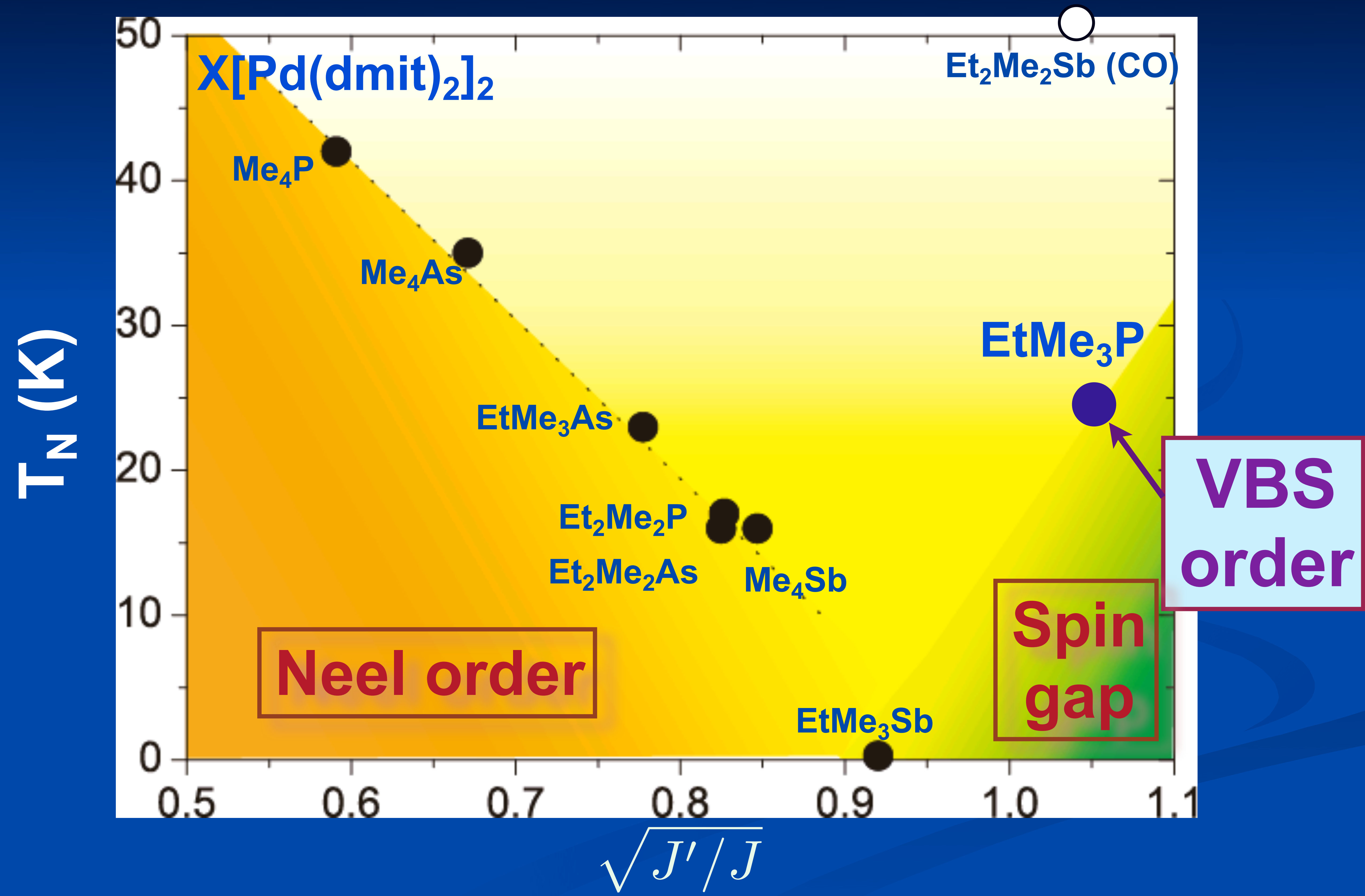
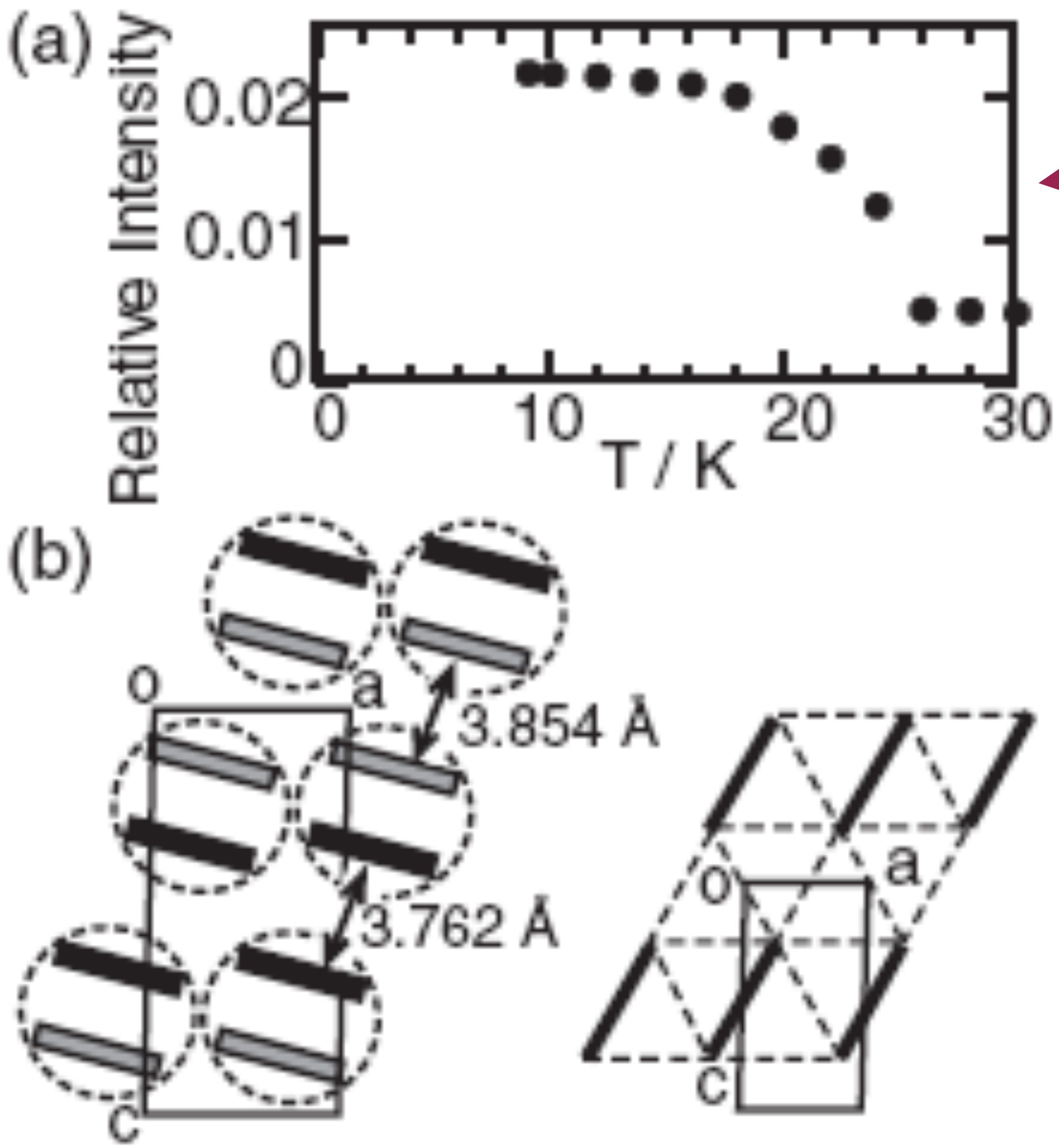


Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Observation of a valence bond solid (VBS) in $ETMe_3P[Pd(dmit)_2]_2$

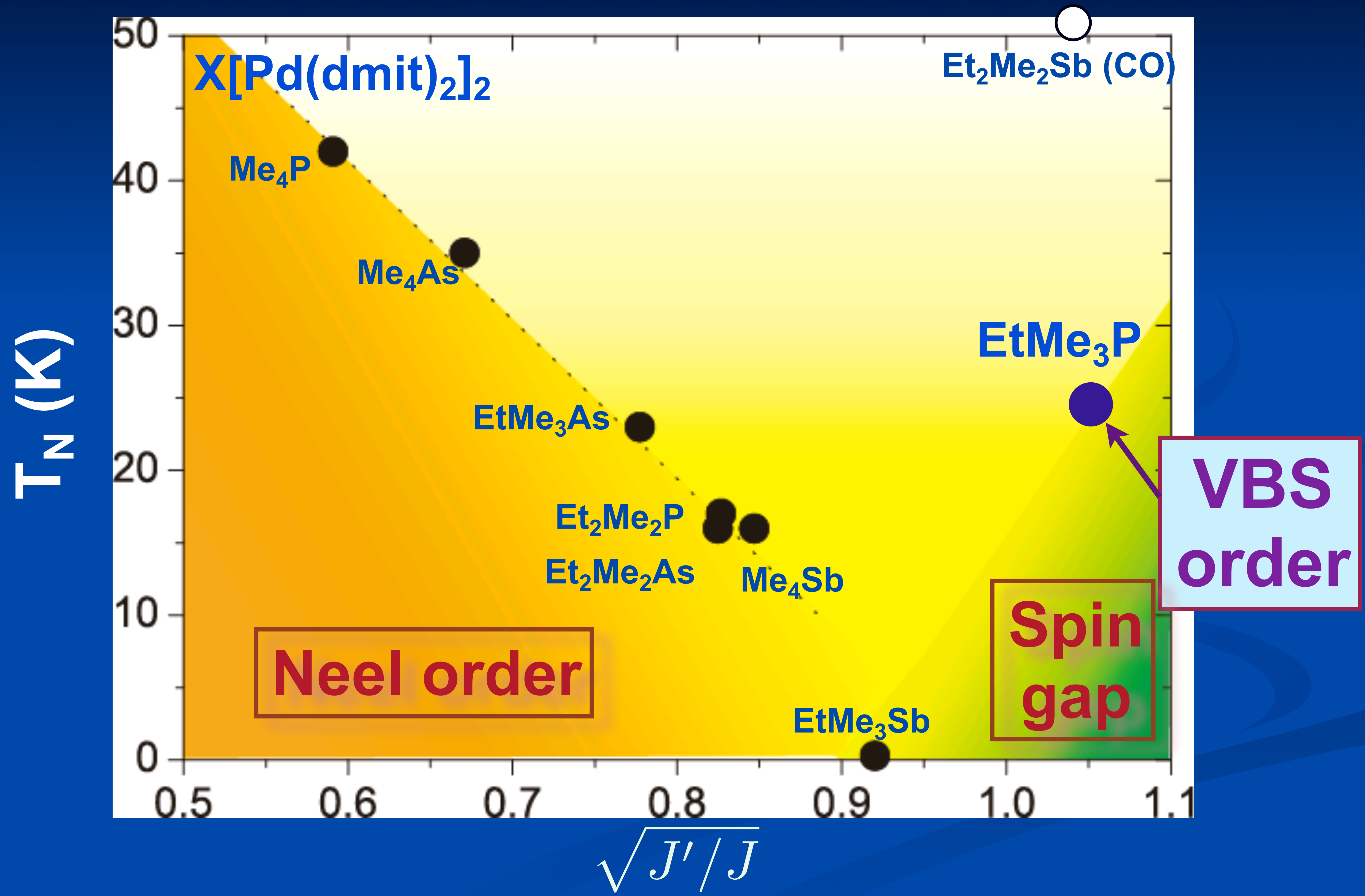


X-ray scattering

Spin gap ~ 40 K
J ~ 250 K

M. Tamura, A. Nakao and R. Kato, *J. Phys. Soc. Japan* **75**, 093701 (2006)
Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *Phys. Rev. Lett.* **99**, 256403 (2007)

Magnetic Criticality



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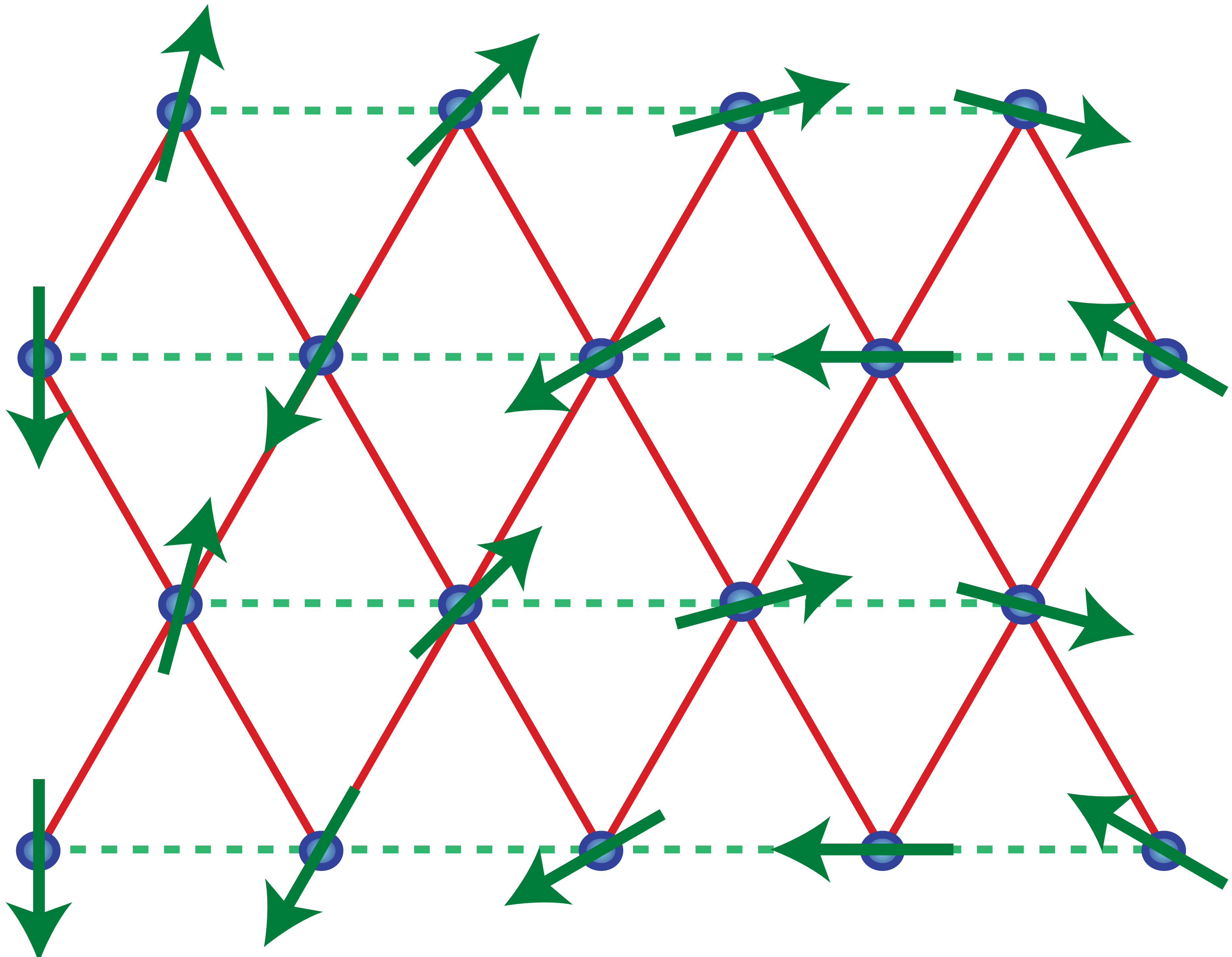
Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid

Anisotropic triangular lattice antiferromagnet



Classical ground state for large J'/J

Found in Cs_2CuCl_4

Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO

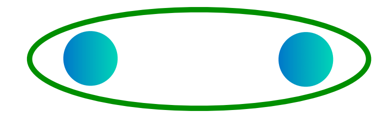
Anisotropic triangular lattice antiferromagnet

Possible ground states as a function of J'/J

- Néel antiferromagnetic LRO
- Valence bond solid
- Spiral LRO
- Z_2 spin liquid: preserves all symmetries of Hamiltonian

Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with S=1/2 per unit cell



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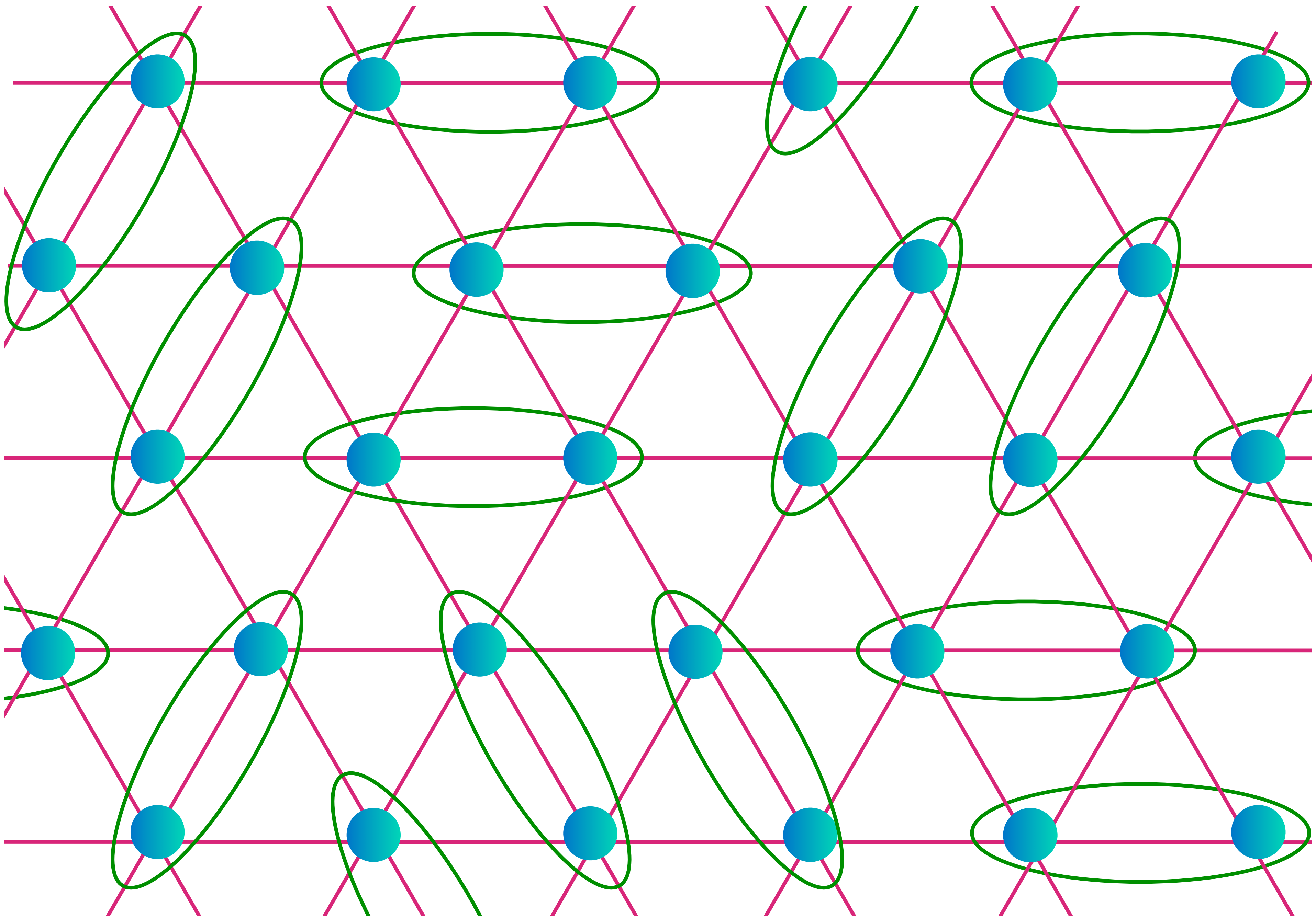

P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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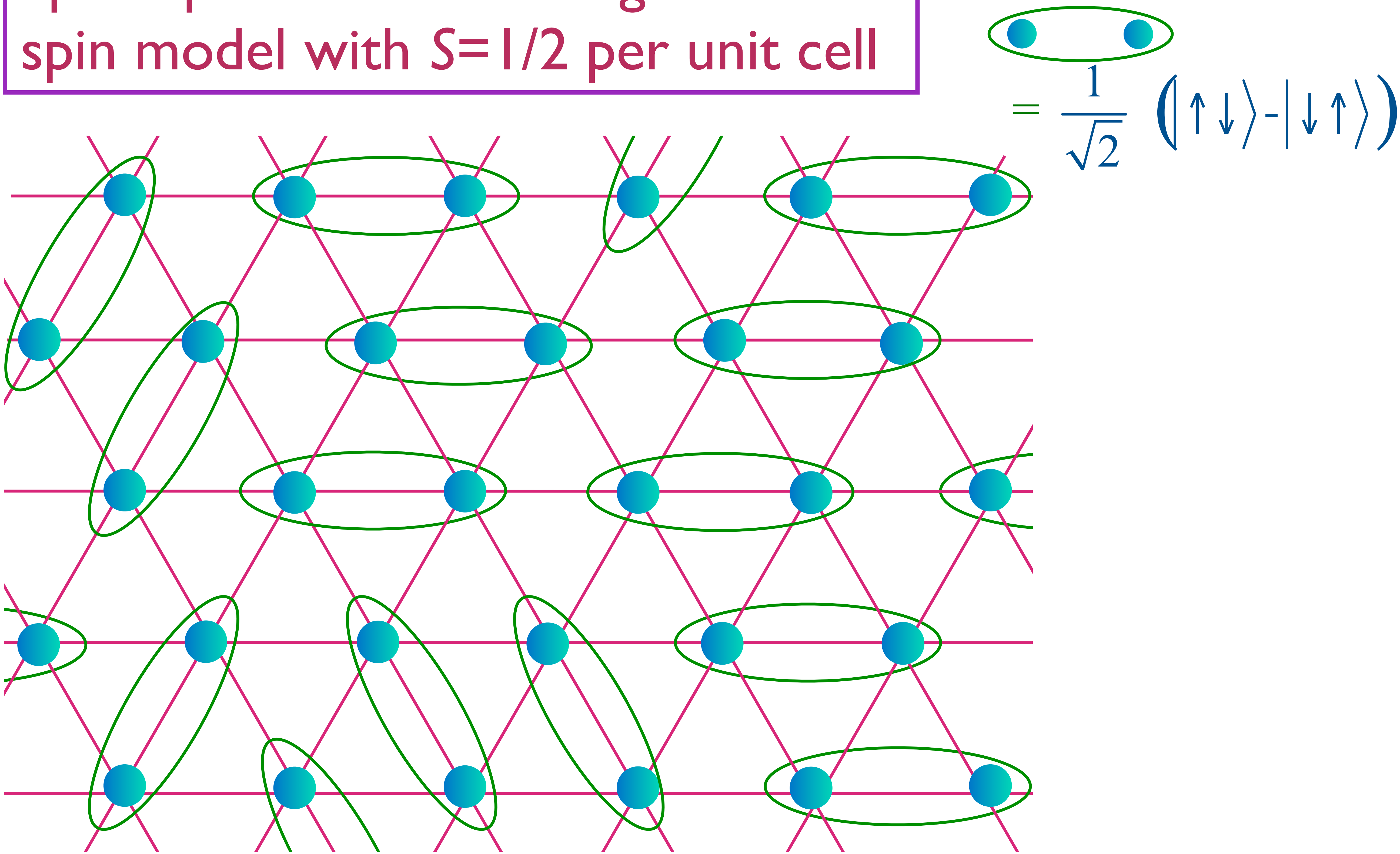

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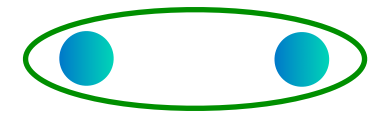

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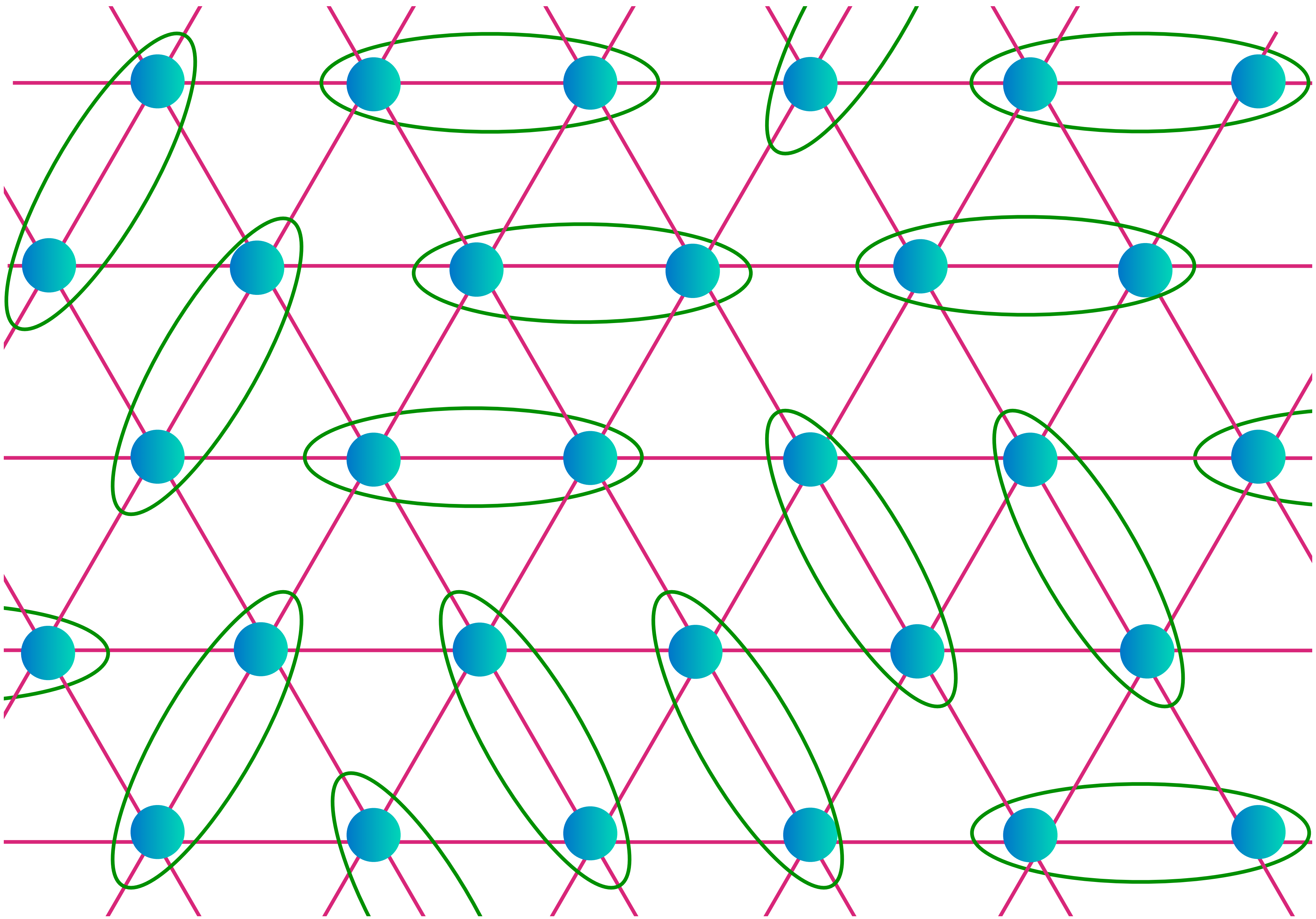

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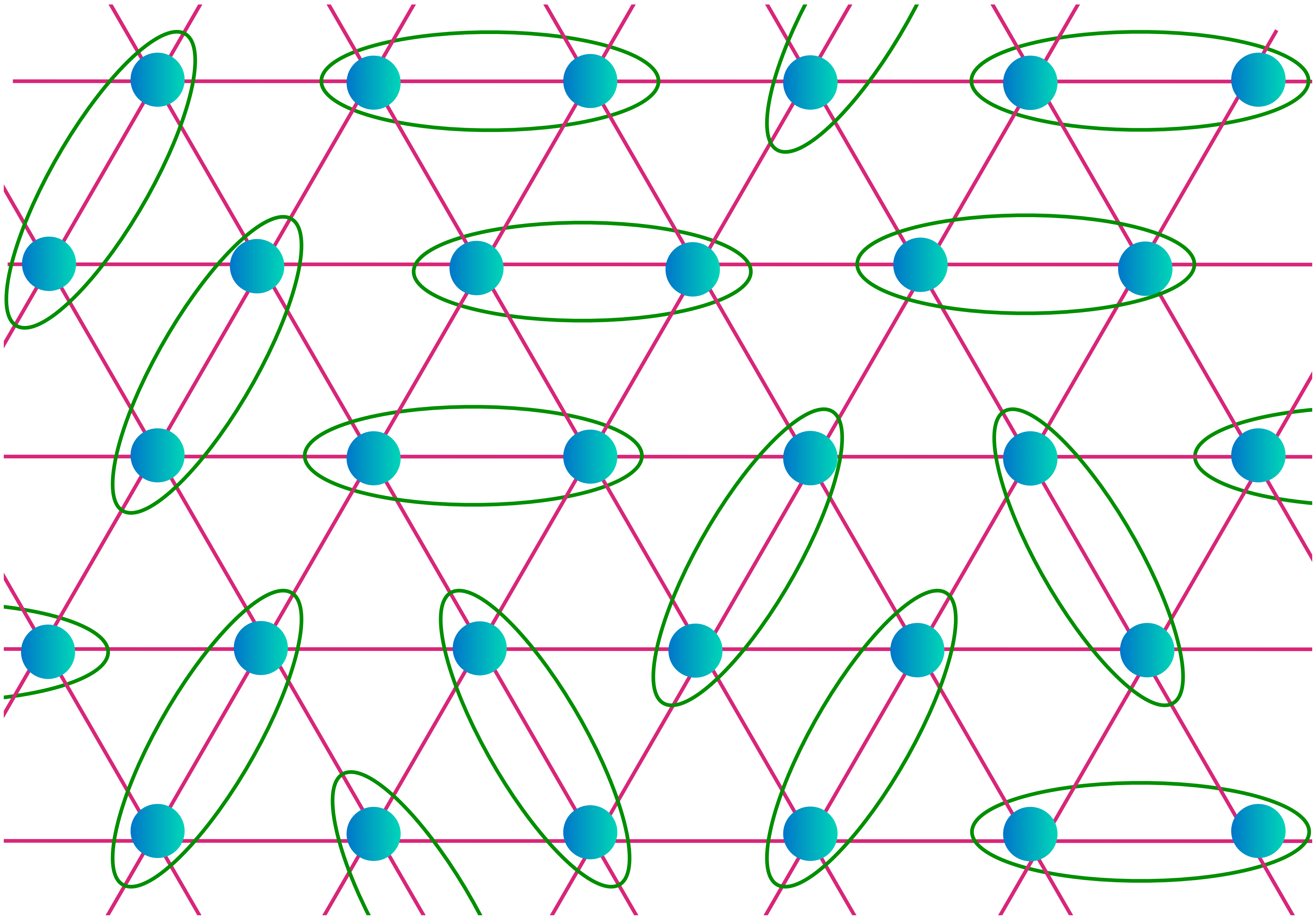

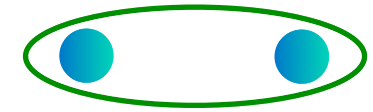

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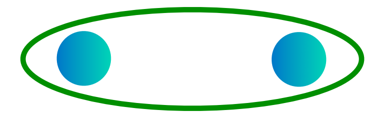

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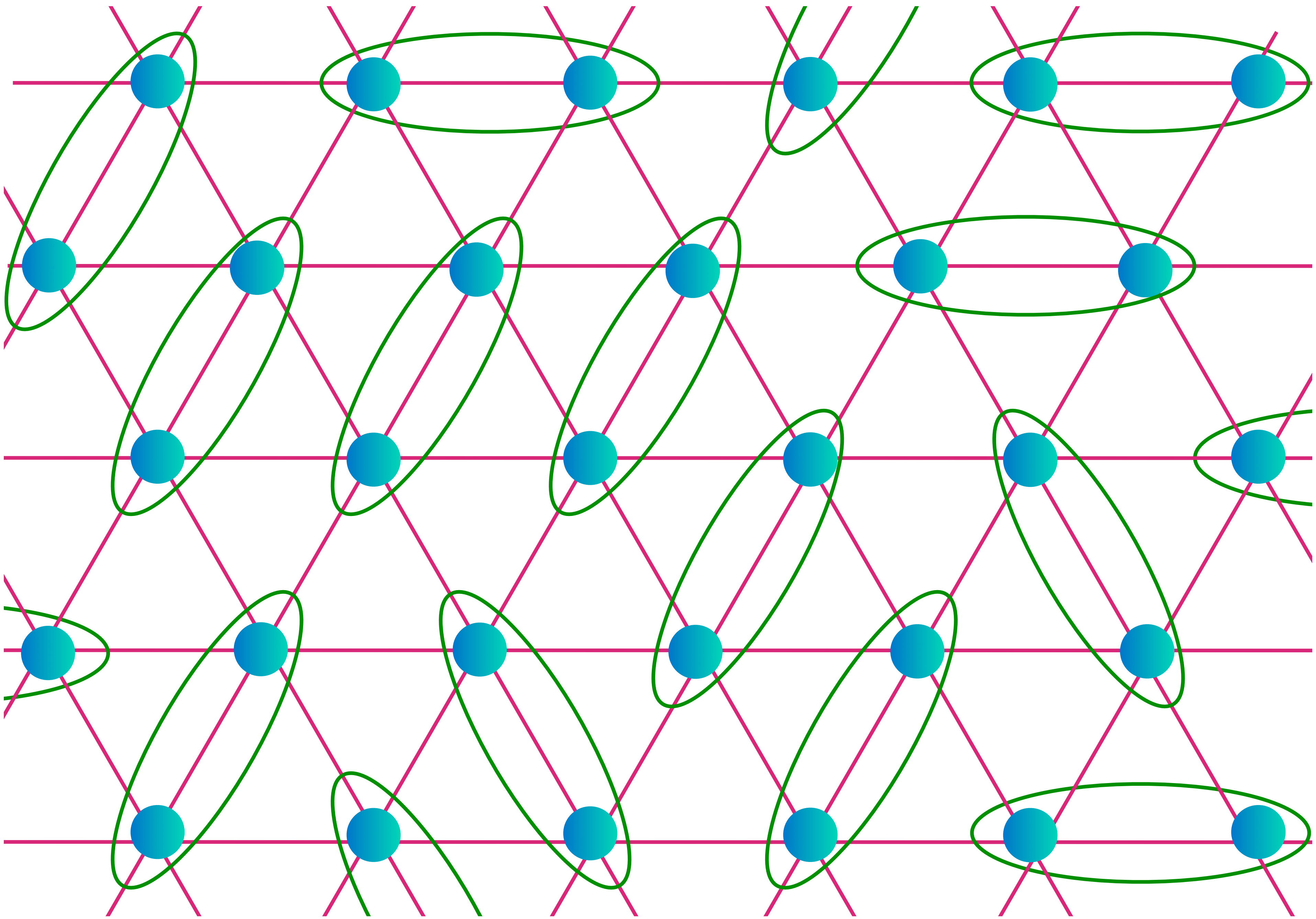

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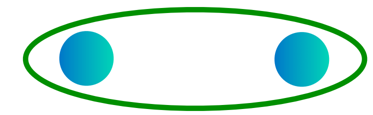

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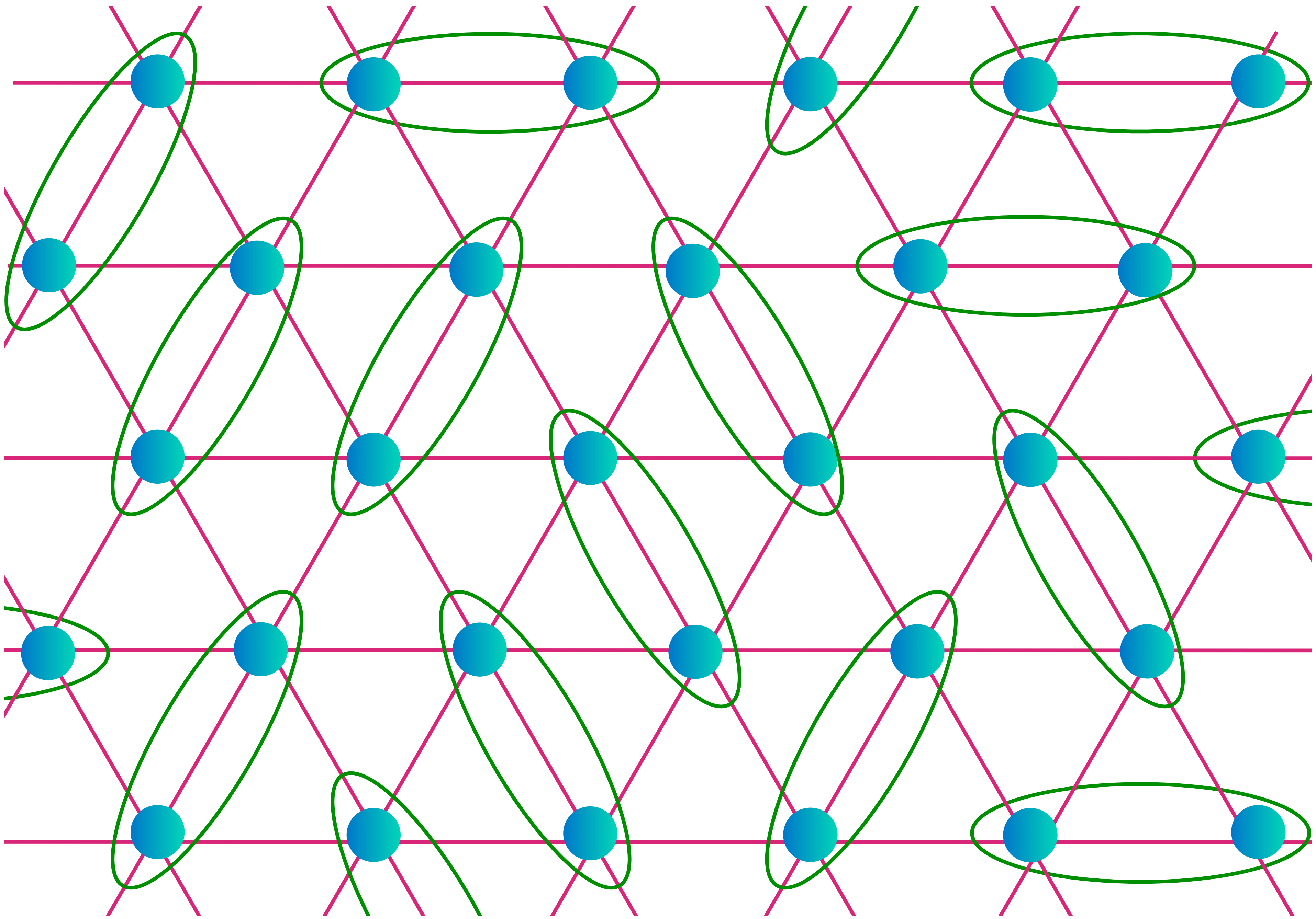

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   Coupled-dimer antiferromagnets

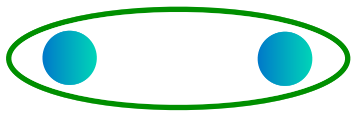
2. Ground states of the triangular lattice antiferromagnet
   Experiments on $X[Pd(dmit)_2]_2$

3. Spinons, visons, and Berry phases
   Quantum field theories for two-dimensional antiferromagnets

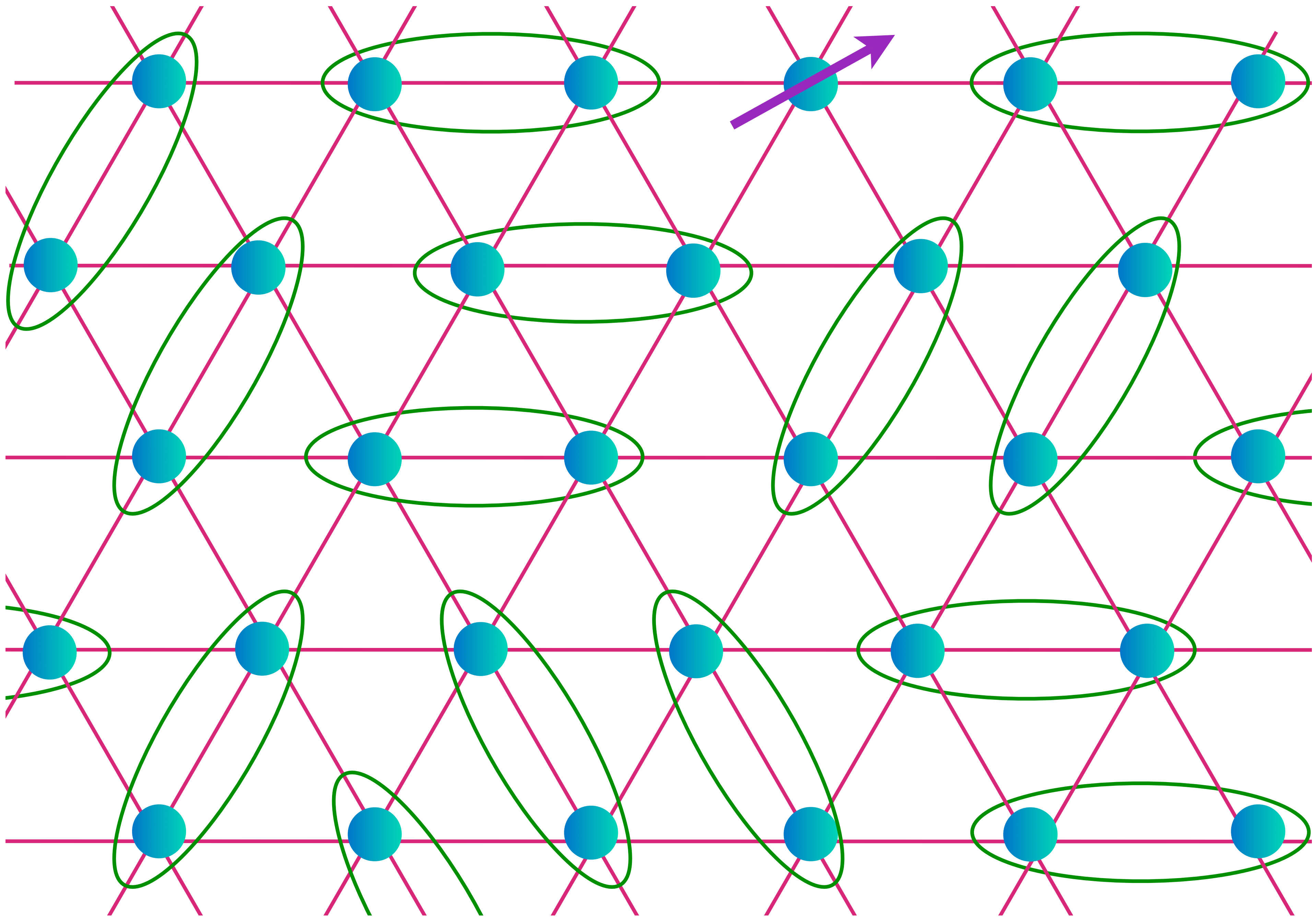
4. Quantum criticality and black holes
   The AdS/CFT correspondence

Excitations of the Z_2 Spin liquid

A spinon



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



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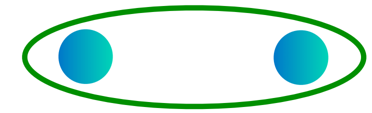

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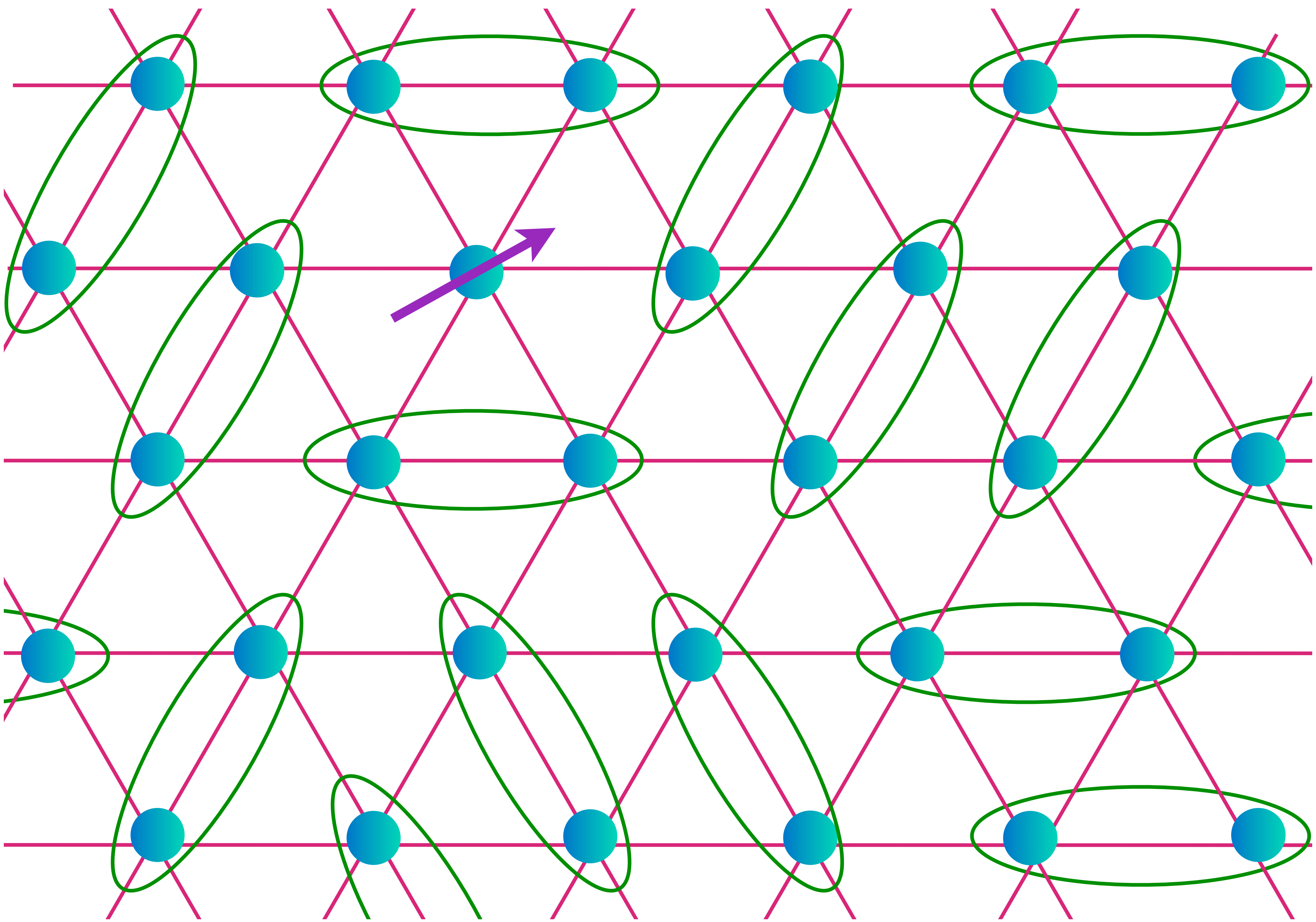

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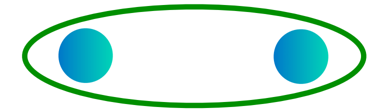

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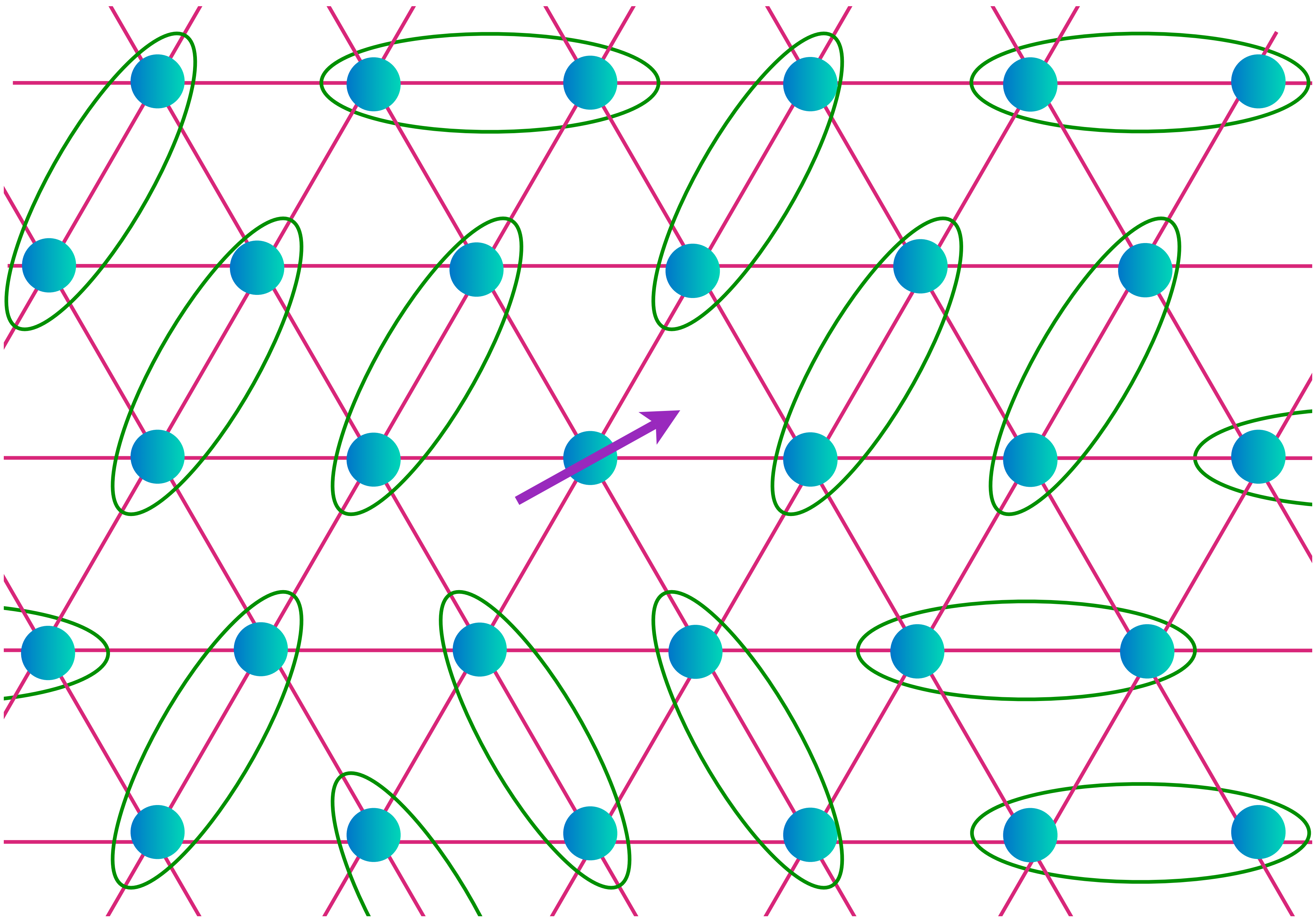

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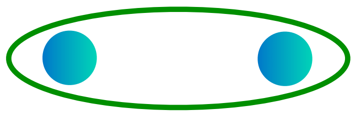

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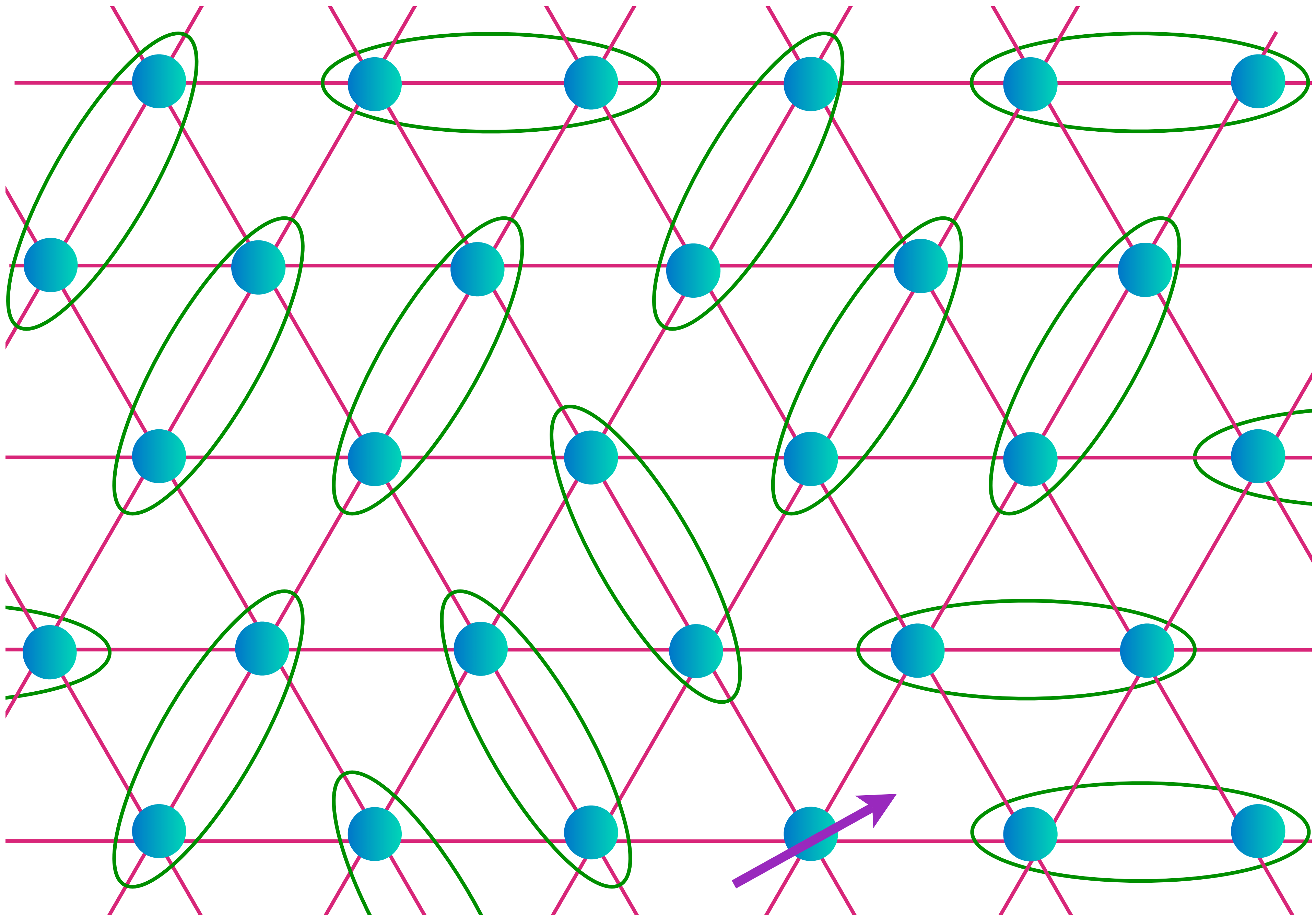

Excitations of the Z_2 Spin liquid

A spinon

The spinon annihilation operator is a spinor z_α, where $\alpha = \uparrow, \downarrow$.

The Néel order parameter, $\vec{\varphi}$ is a composite of the spinons:

$$\vec{\varphi} = z_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} z_{i\beta}$$

where $\vec{\sigma}$ are Pauli matrices

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where $\vec{\sigma}$ are Pauli matrices

The theory for quantum phase transitions is expressed in terms of fluctuations of z_α, and *not* the order parameter $\vec{\varphi}$.

Effective theory for z_α must be invariant under the U(1) gauge transformation

$$z_{i\alpha} \rightarrow e^{i\theta} z_{i\alpha}$$

Excitations of the Z_2 Spin liquid

A vison

- A characteristic property of a Z_2 spin liquid is the presence of a spinon pair condensate

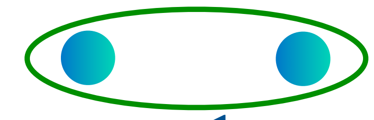
- A vison is an Abrikosov vortex in the pair condensate of spinons

- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)

Excitations of the Z_2 Spin liquid

A vison



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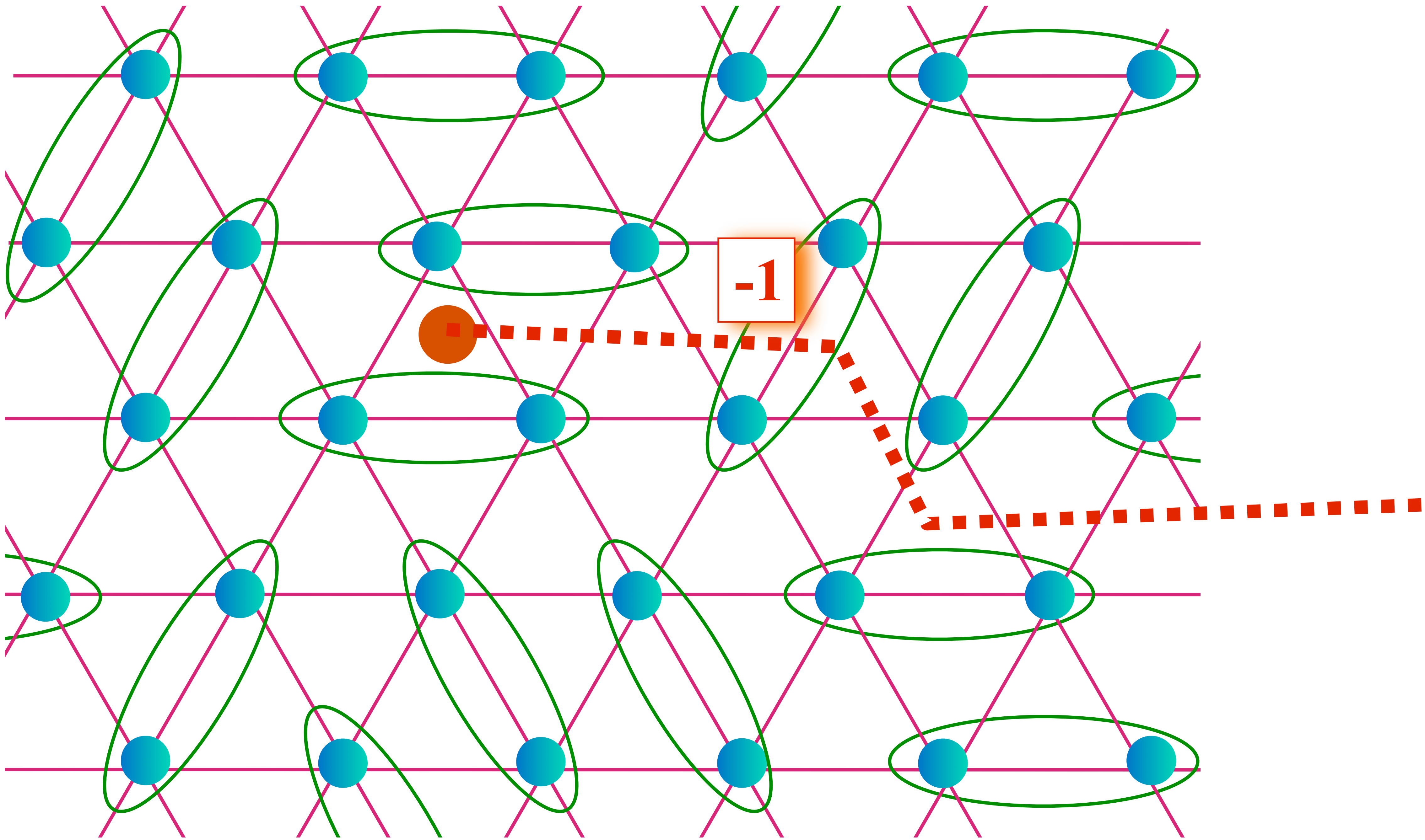

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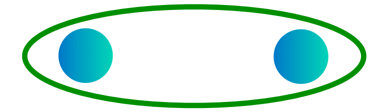

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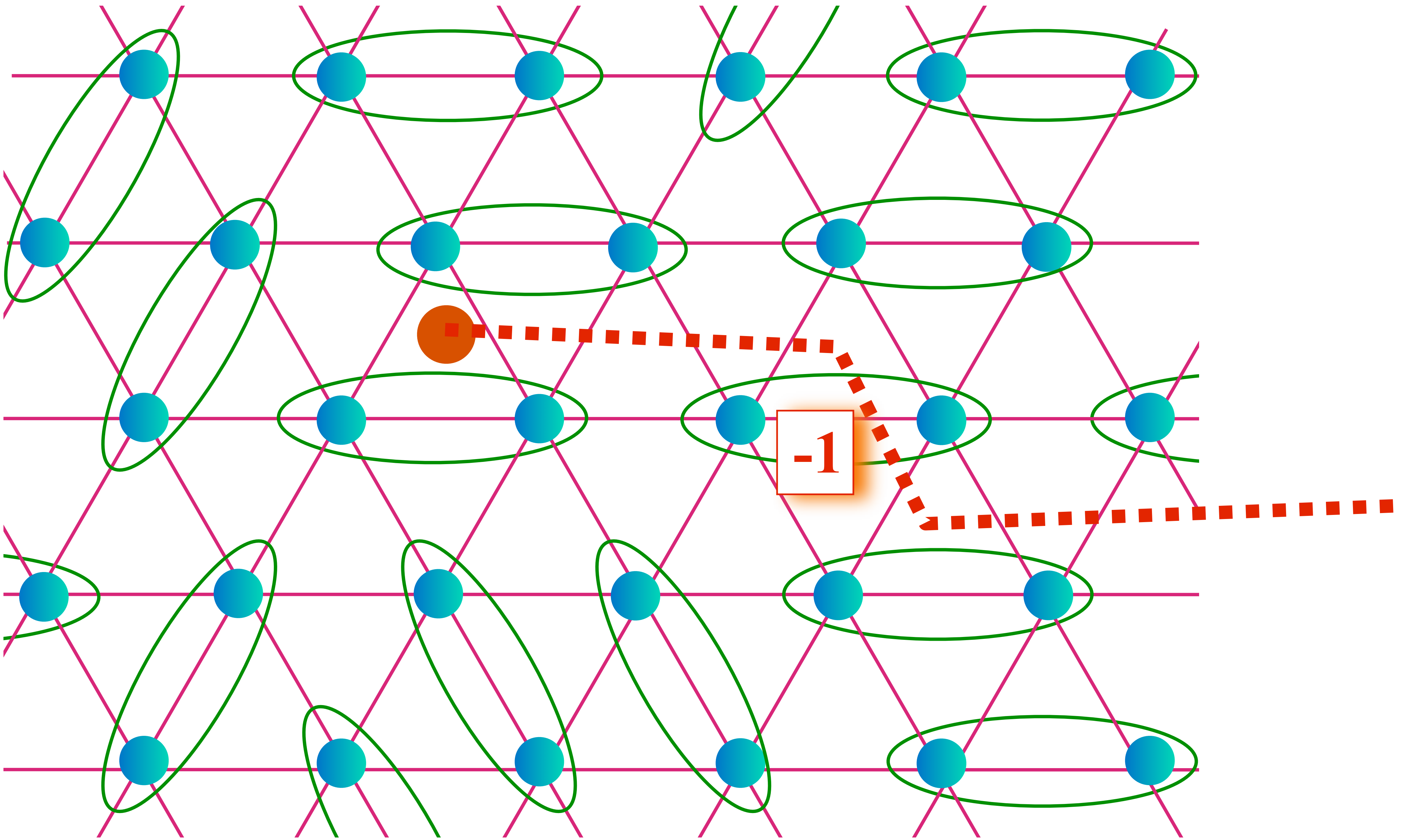

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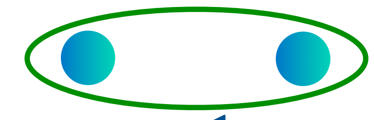

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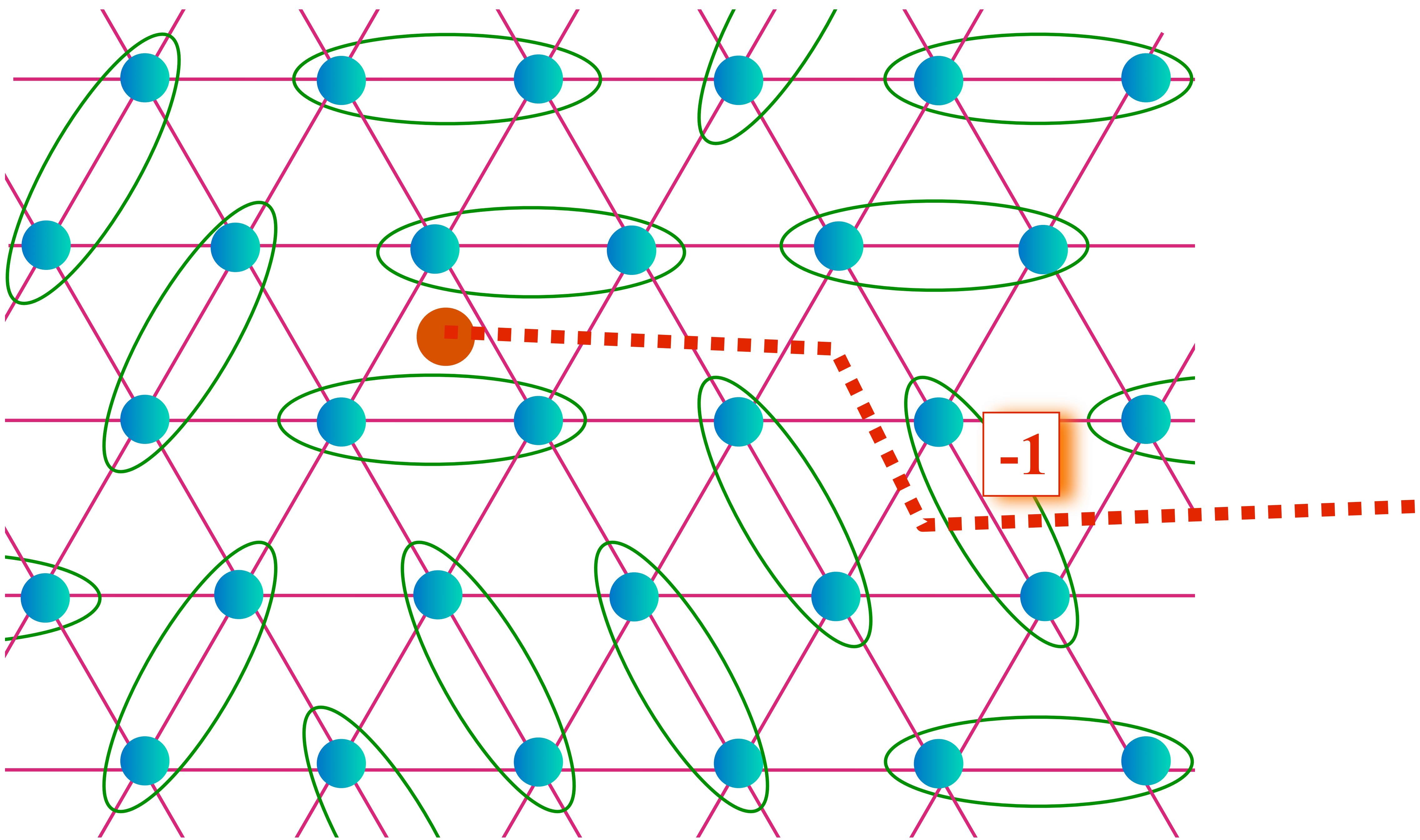

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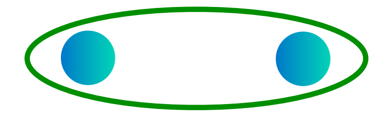

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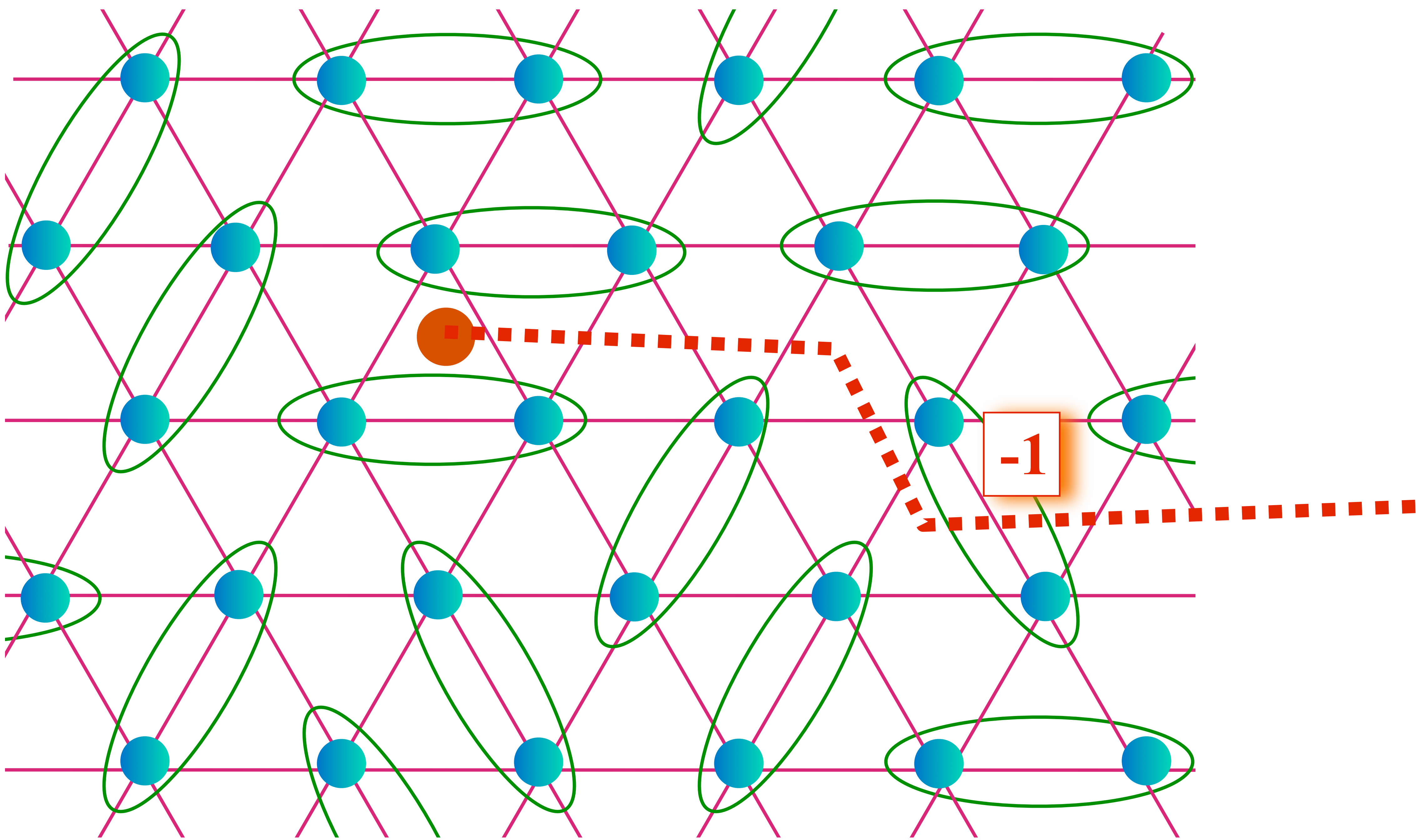

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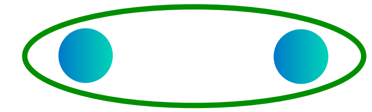

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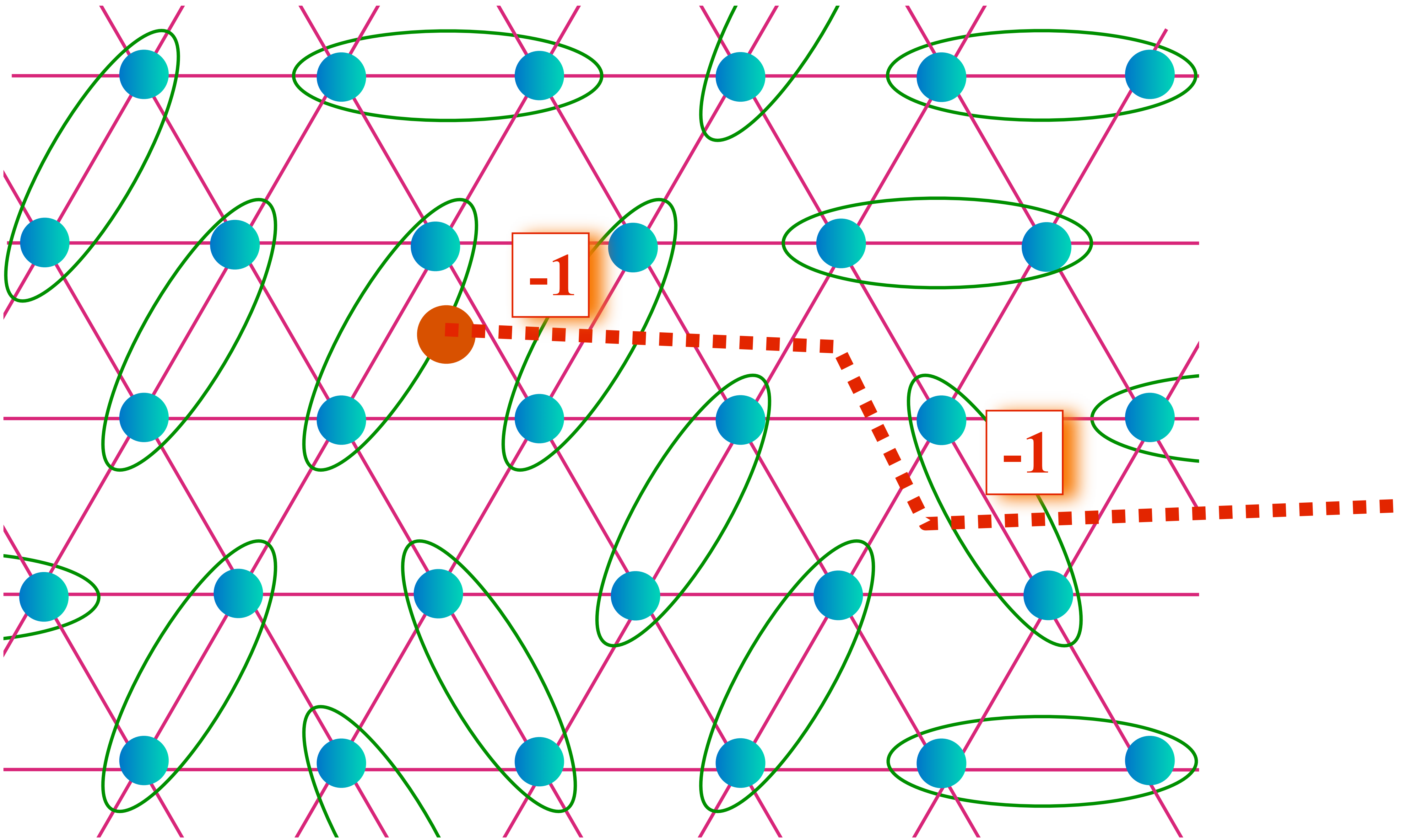

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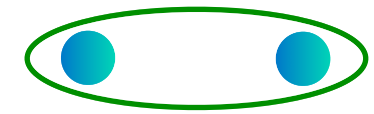

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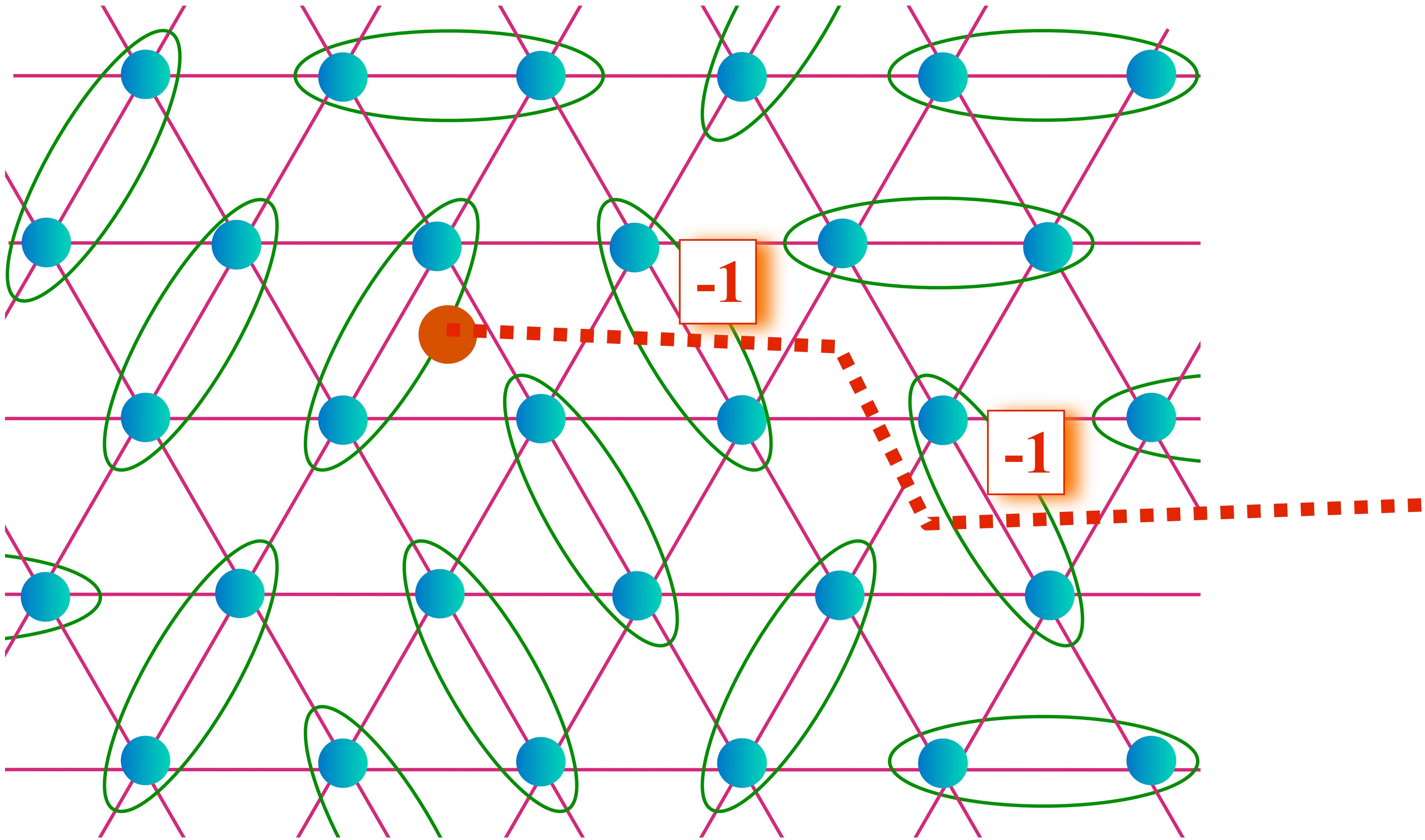

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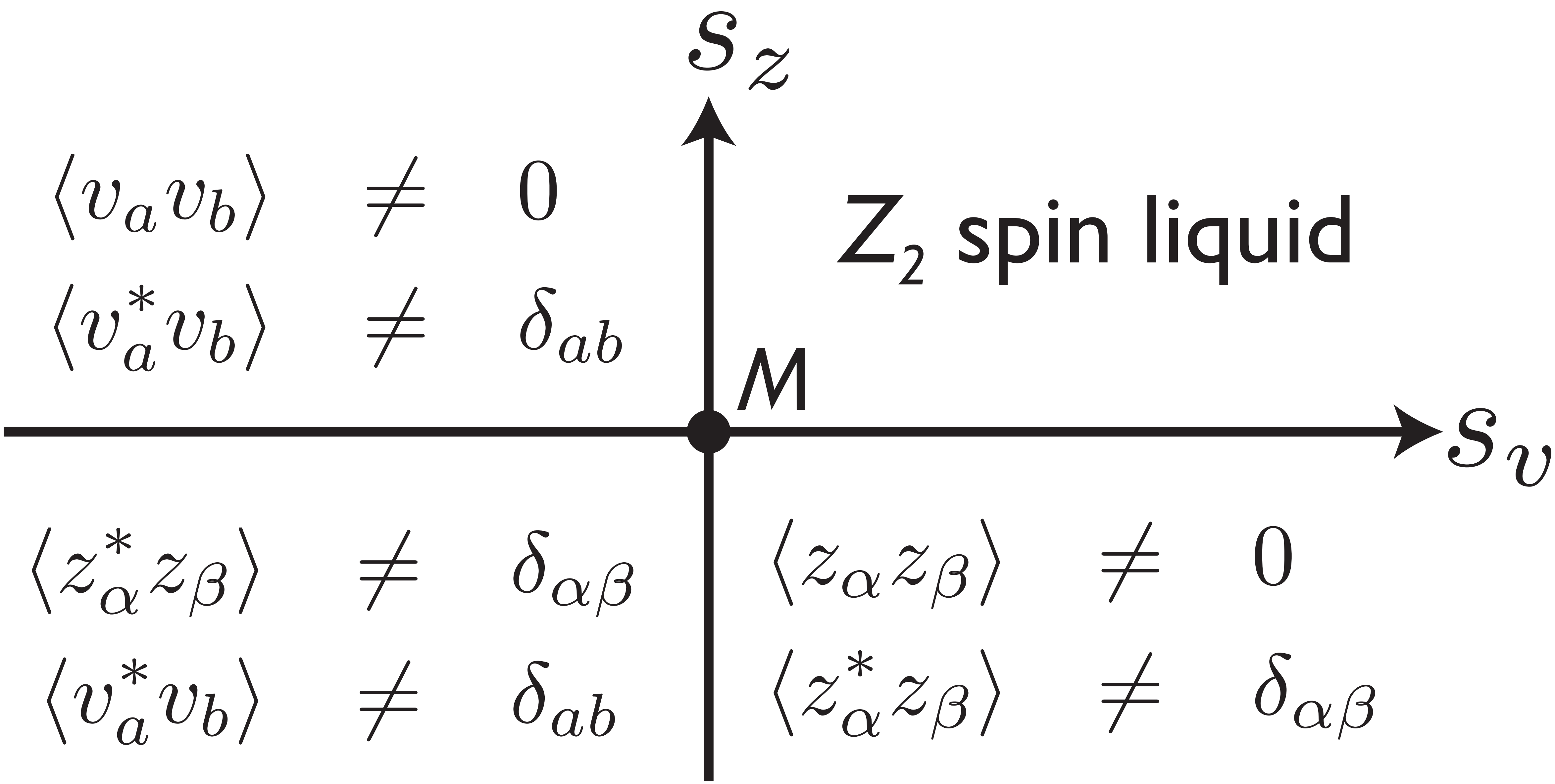
Mutual Chern-Simons Theory

Express theory in terms of the physical excitations of the Z_2 spin liquid: the spinons, z_α, and the visons. After accounting for Berry phase effects, the visons can be described by complex fields v_a, which transforms non-trivially under the square lattice space group operations.

A related Berry phase is the phase of -1 acquired by a spinon encircling a vortex. This is implemented in the following "mutual Chern-Simons" theory at $k = 2$:

$$
\begin{aligned}
\mathcal{L} &= \sum_{\alpha=1}^{2} \left\{ |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_z |z_\alpha|^2 + u_z (|z_\alpha|^2)^2 \right\} \\
&+ \sum_{a=1}^{N_v} \left\{ |(\partial_\mu - i b_\mu) v_a|^2 + s_v |v_a|^2 + u_v (|v_a|^2)^2 \right\} \\
&+ \frac{ik}{2\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu b_\lambda + \cdots
\end{aligned}
$$

Cenke Xu and S. Sachdev, arXiv:0811.1220



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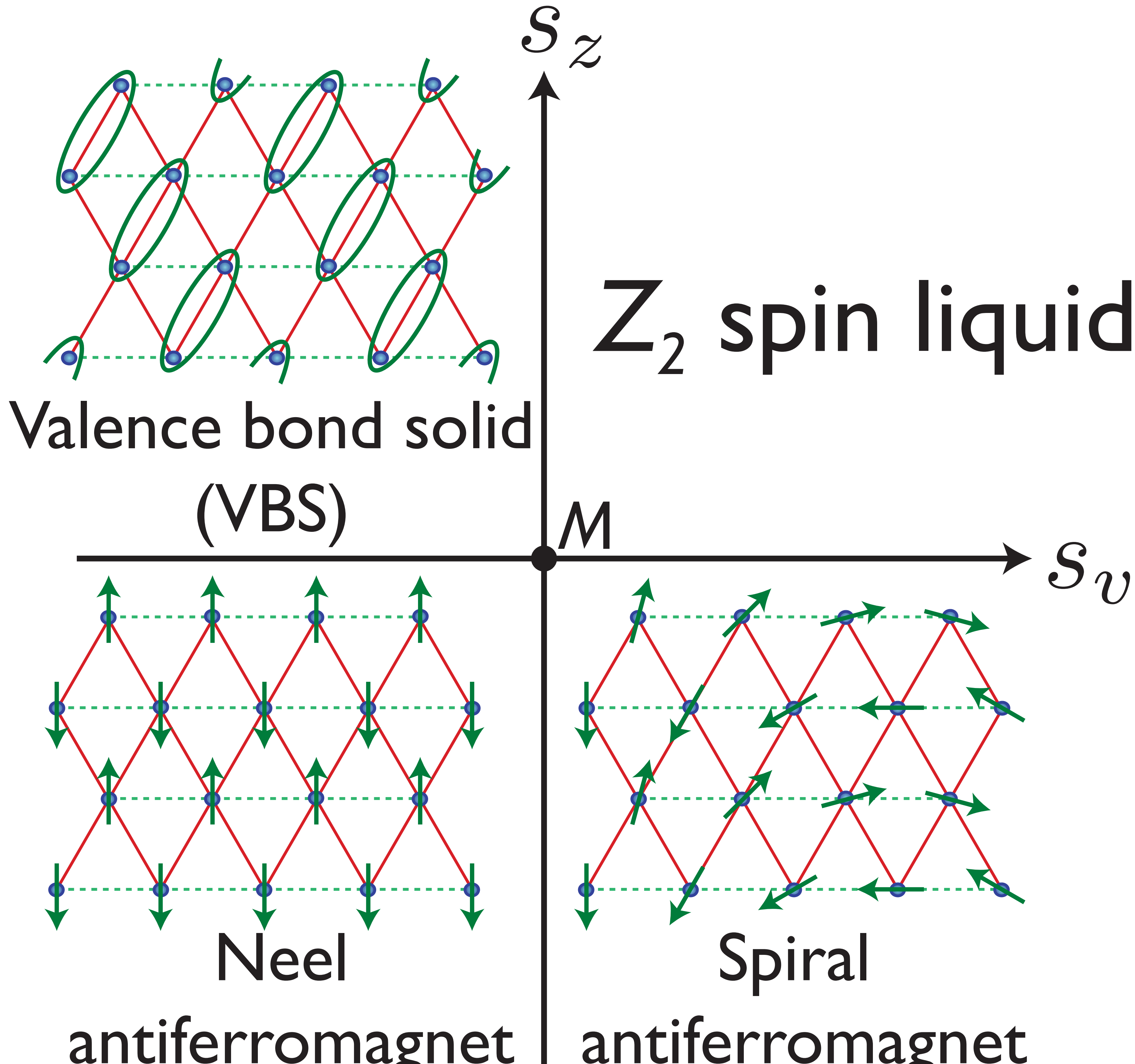
Theoretical global phase diagram



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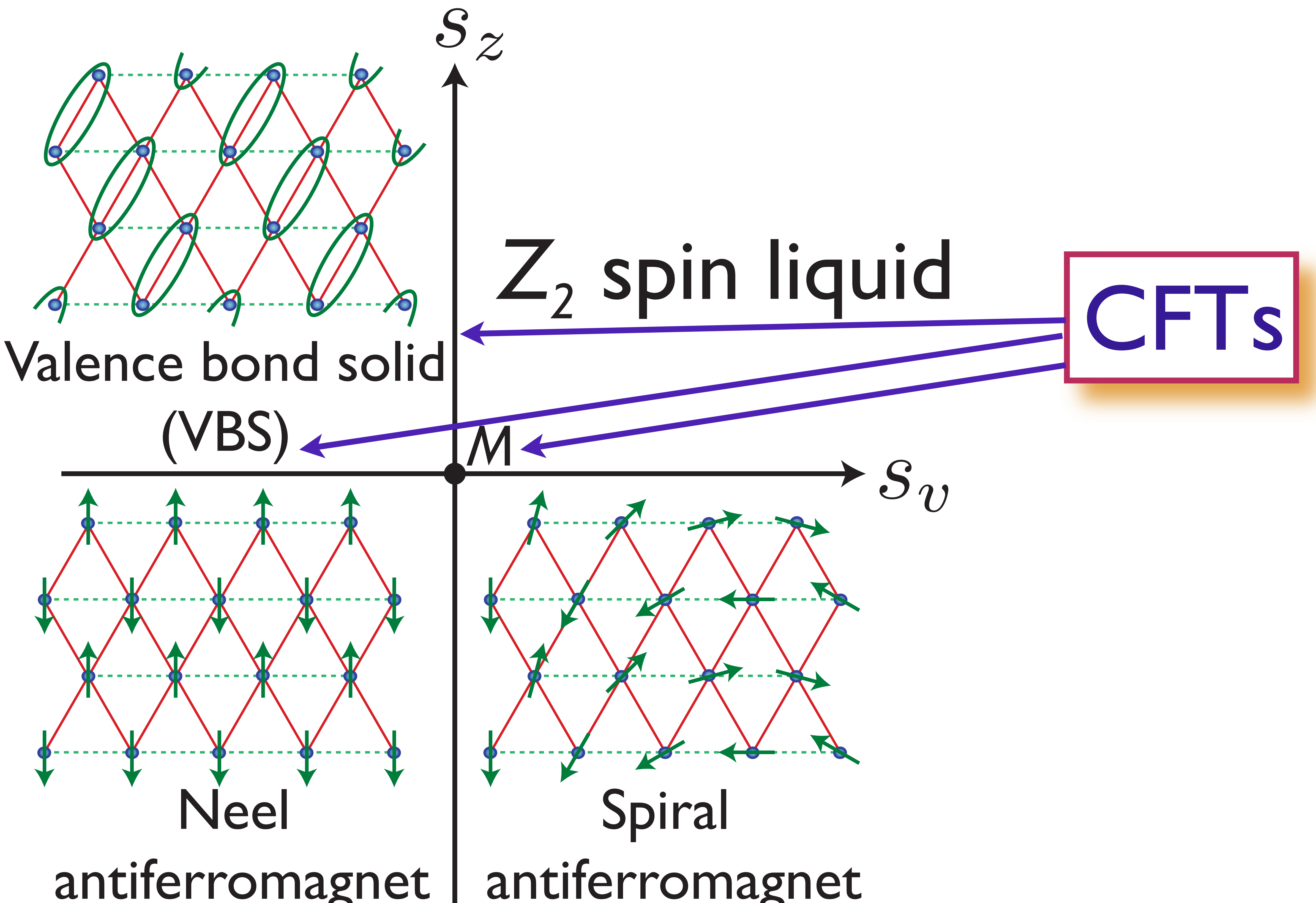

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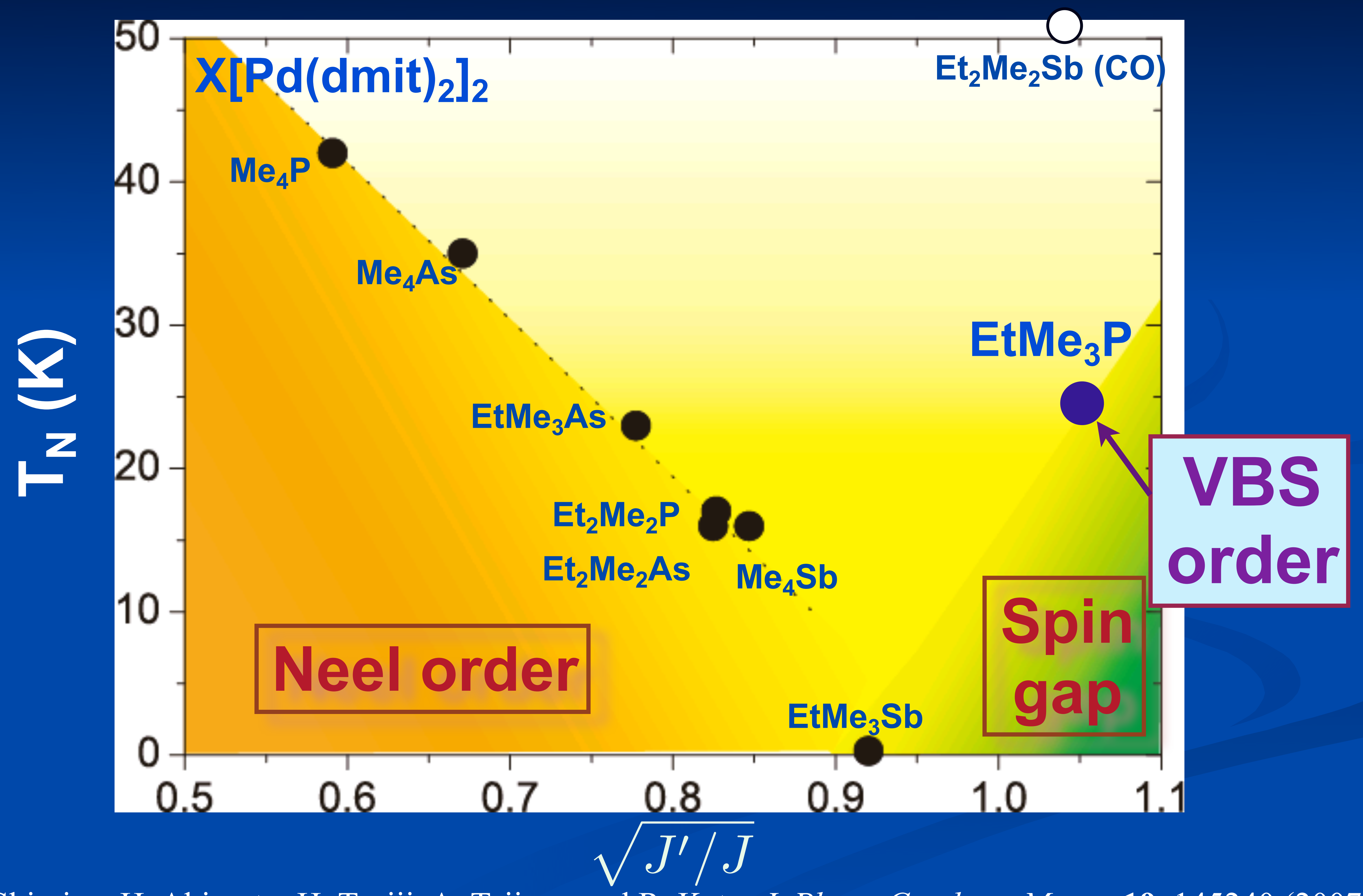
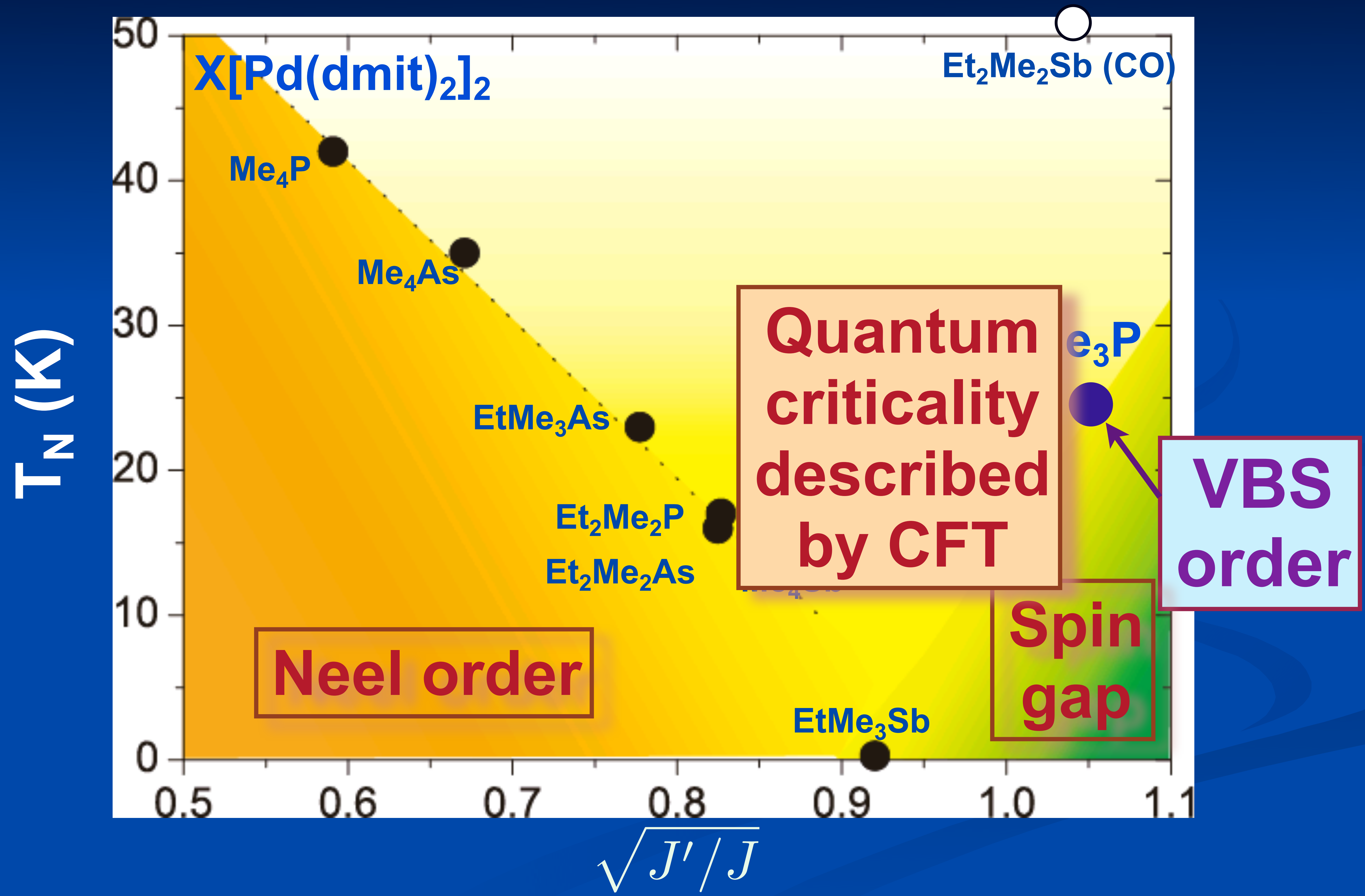
Magnetic Criticality



Y. Shimizu, H. Akimoto, H. Tsujii, A. Tajima, and R. Kato, *J. Phys.: Condens. Matter* **19**, 145240 (2007)

Magnetic Criticality



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From quantum antiferromagnets to string theory

A direct generalization of the CFT of the multicritical point M ($s_z = s_v = 0$) to $\mathcal{N} = 4$ supersymmetry and the U(N) gauge group was shown by O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, JHEP **0810**, 091 (2008) to be dual to a theory of quantum gravity (M theory) on $\text{AdS}_4 \times \text{S}_7/Z_k$.

$$
\begin{aligned}
\mathcal{L} &= \sum_{\alpha=1}^{2} \left\{ |(\partial_\mu - i a_\mu) z_\alpha|^2 + s_z |z_\alpha|^2 + u_z (|z_\alpha|^2)^2 \right\} \\
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   Experiments on $X[Pd(dmit)_2]_2$

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Black Holes

Objects so massive that light is gravitationally bound to them.

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The region inside the black hole horizon is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \dfrac{k_B A}{4\ell_P^2}$

where A is the area of the horizon, and

$\ell_P = \sqrt{\dfrac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$

Black Hole Thermodynamics

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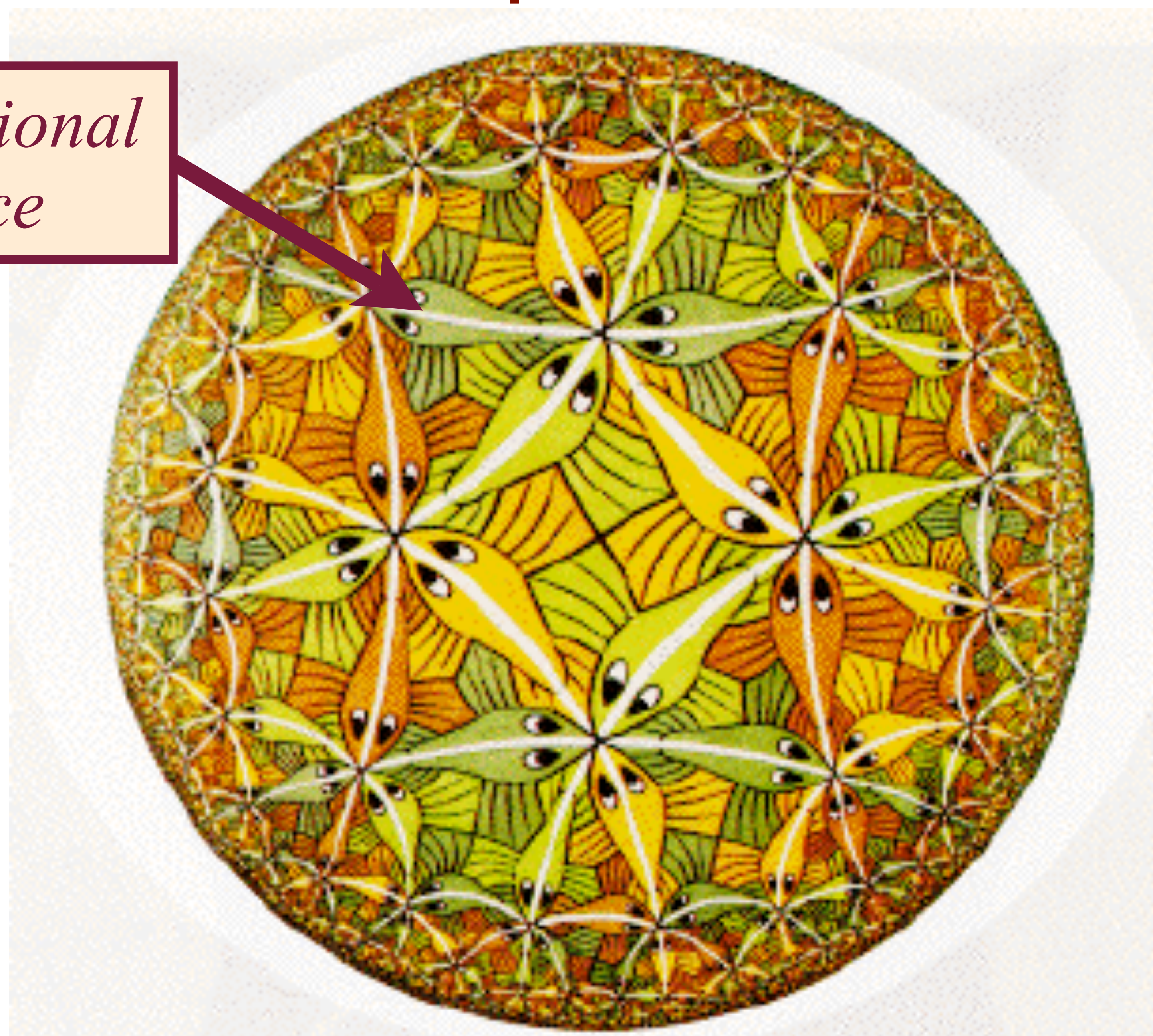
$$\text{Horizon temperature: } 4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$$

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena, Gubser, Klebanov, Polyakov, Witten

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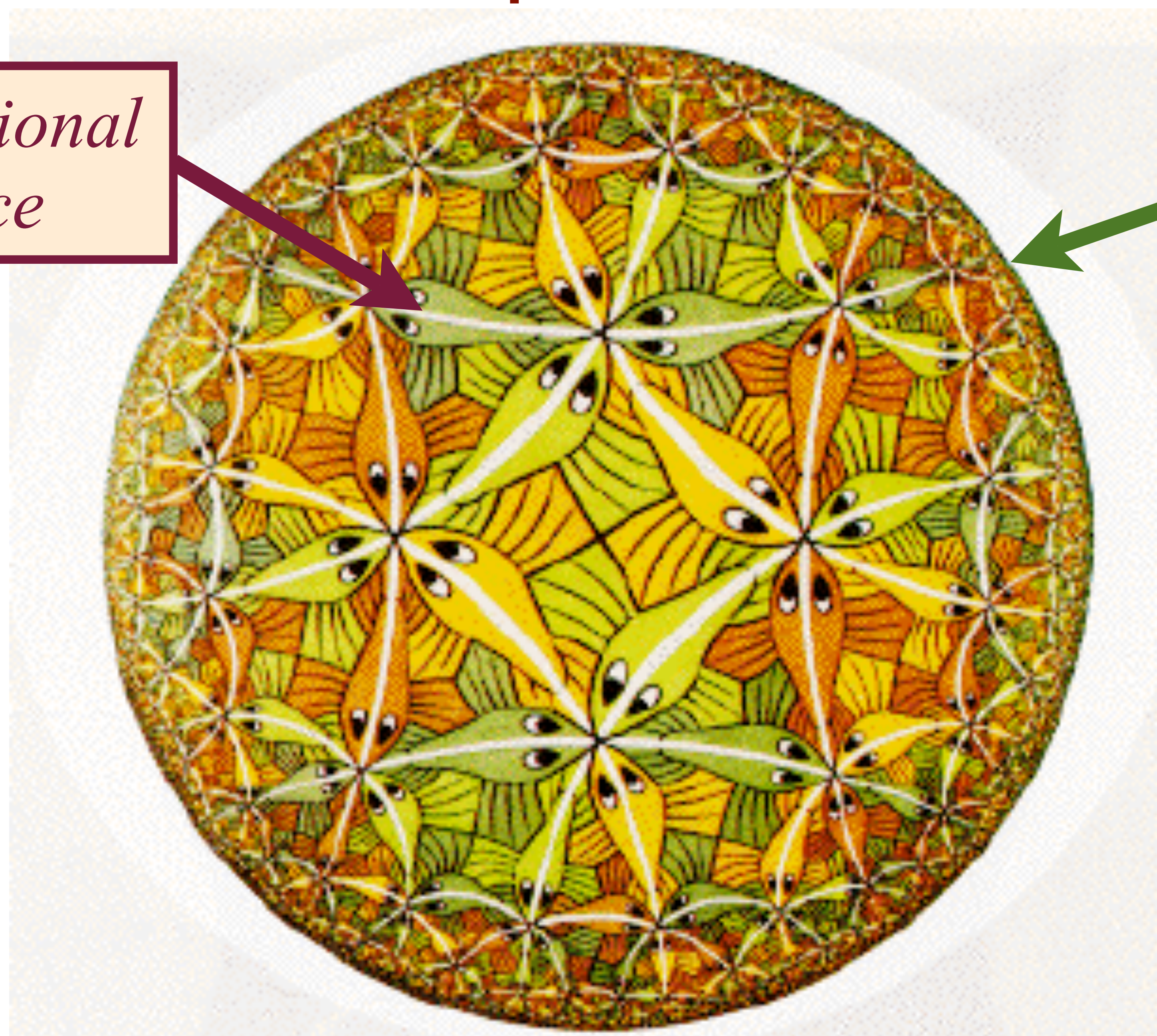
3+1 dimensional AdS space



A 2+1 dimensional system at its quantum critical point

Maldacena, Gubser, Klebanov, Polyakov, Witten

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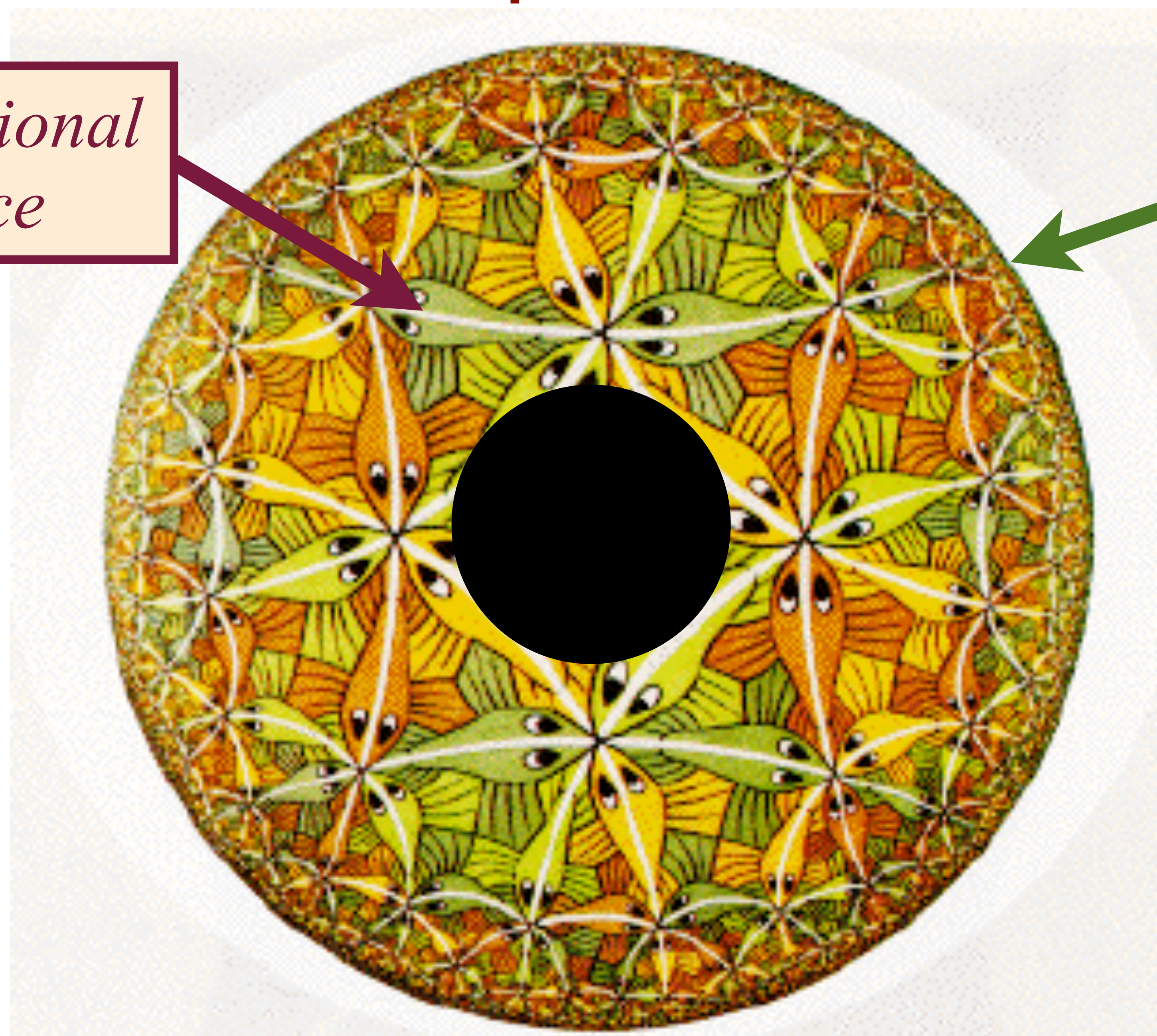

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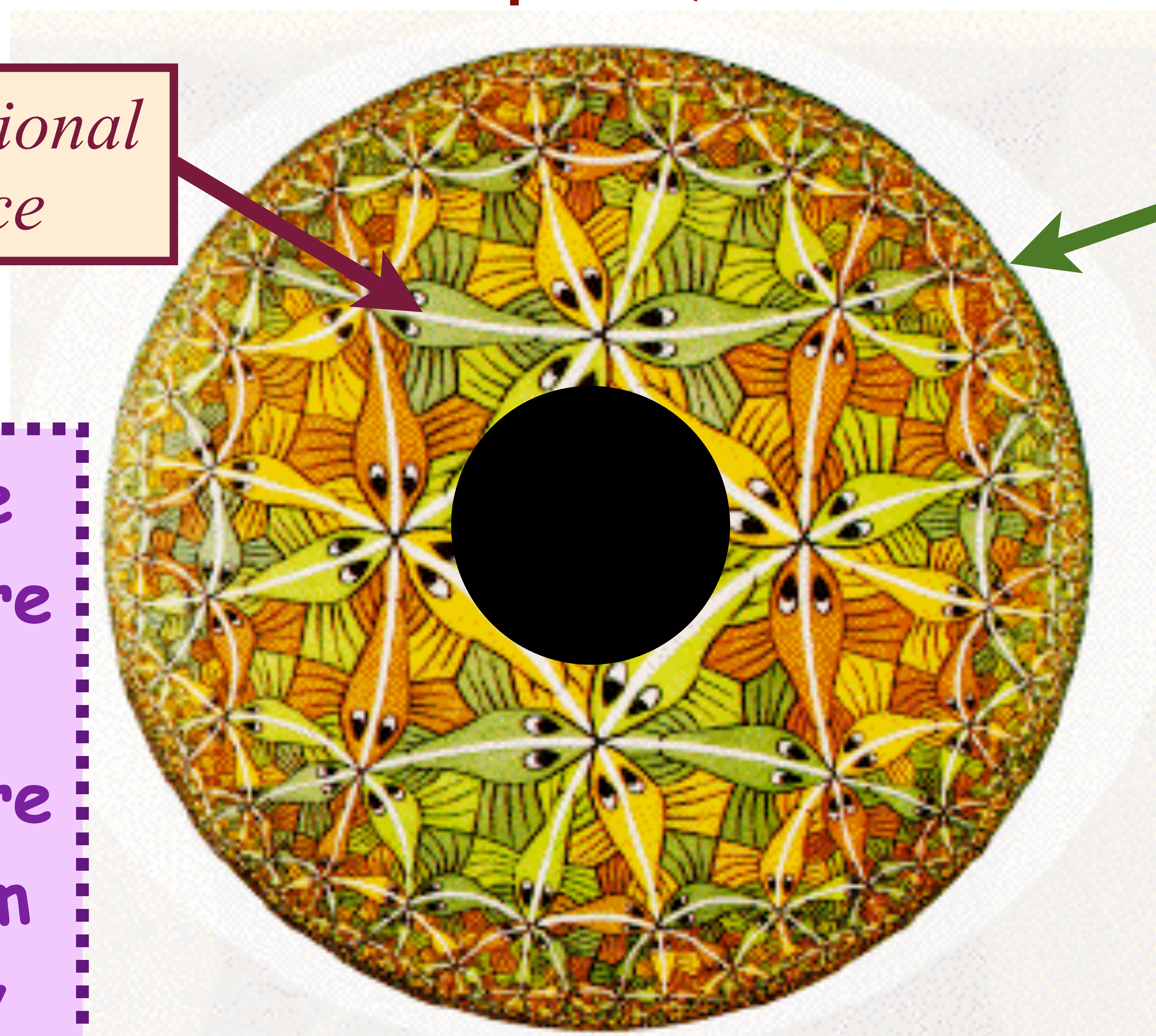
3+1 dimensional AdS space



Quantum criticality in 2+1 dimensions

Black hole temperature = temperature of quantum criticality

Maldacena, Gubser, Klebanov, Polyakov, Witten

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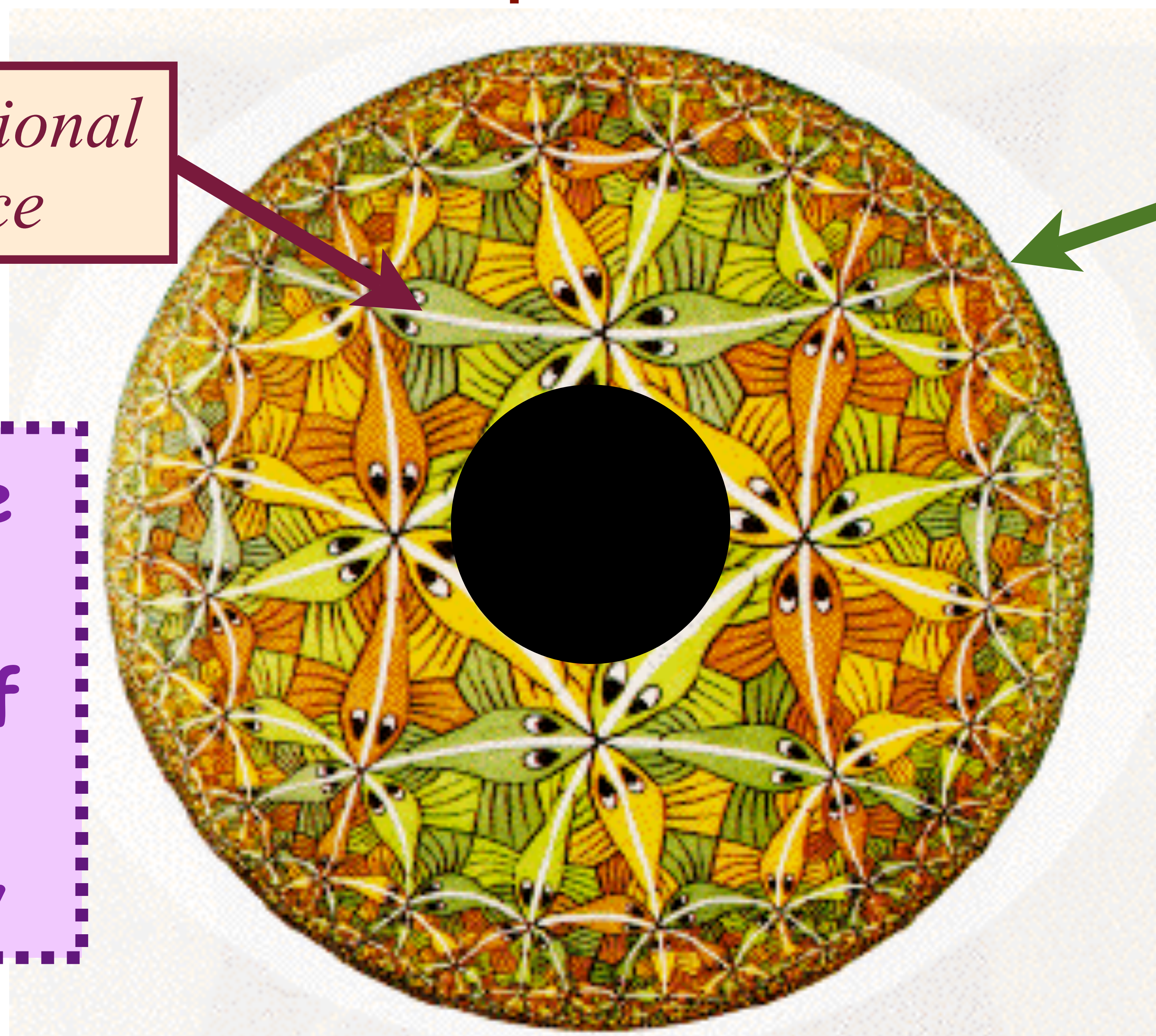

3+1 dimensional AdS space

Quantum criticality in 2+1 dimensions

Black hole entropy = entropy of quantum criticality

Strominger, Vafa

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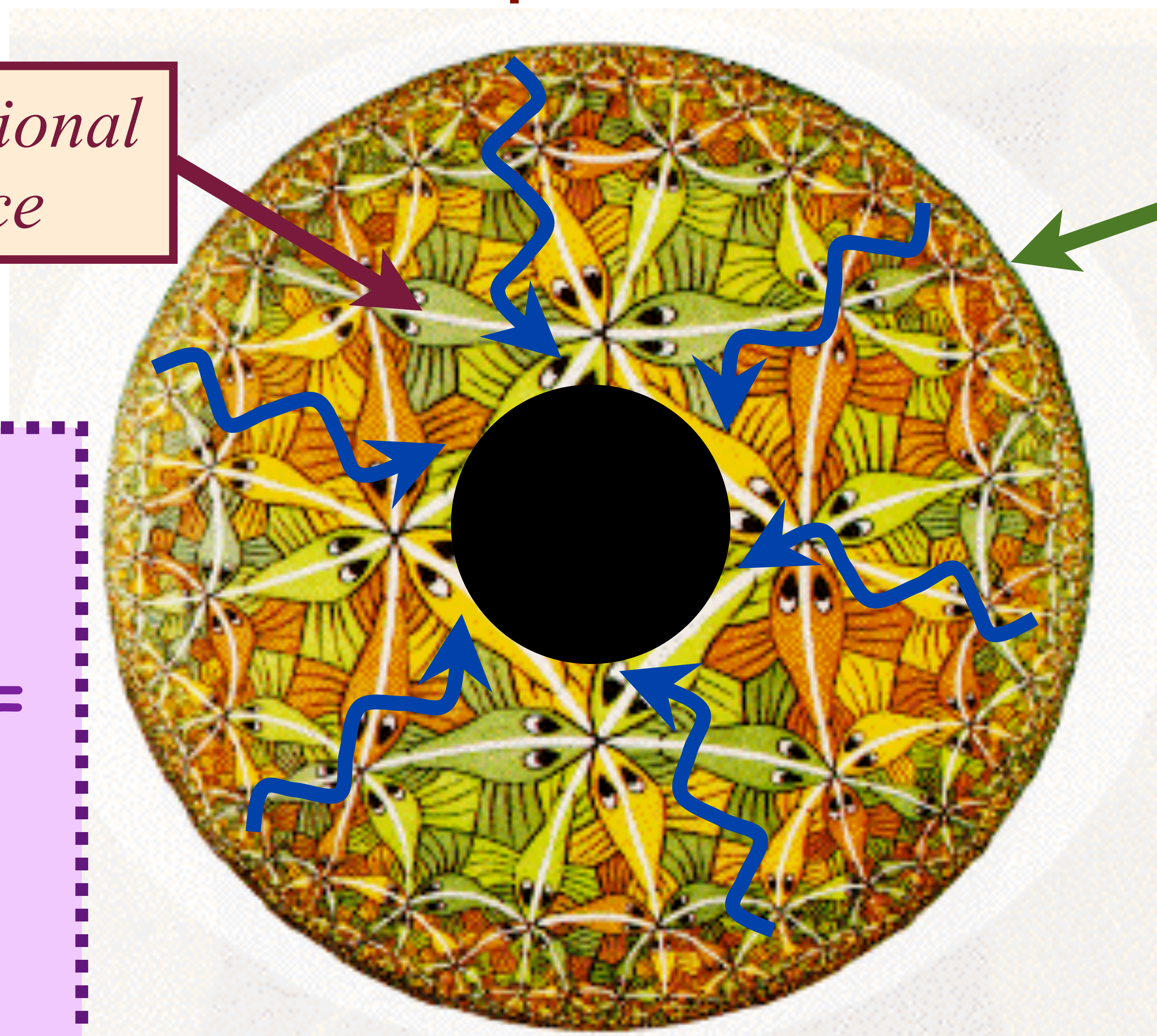

Quantum criticality in 2+1 dimensions

Quantum critical dynamics = waves in curved space

Maldacena, Gubser, Klebanov, Polyakov, Witten

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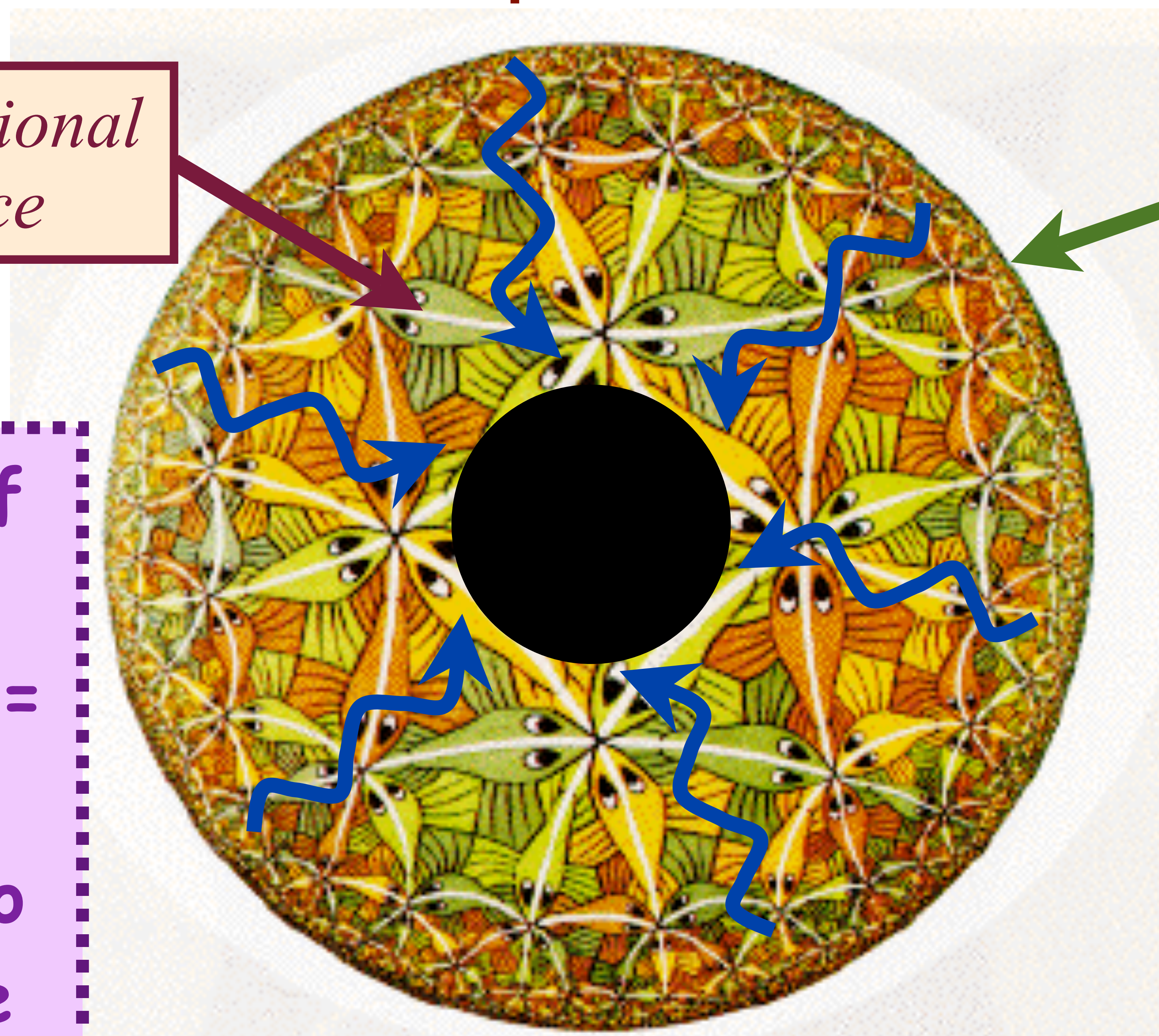
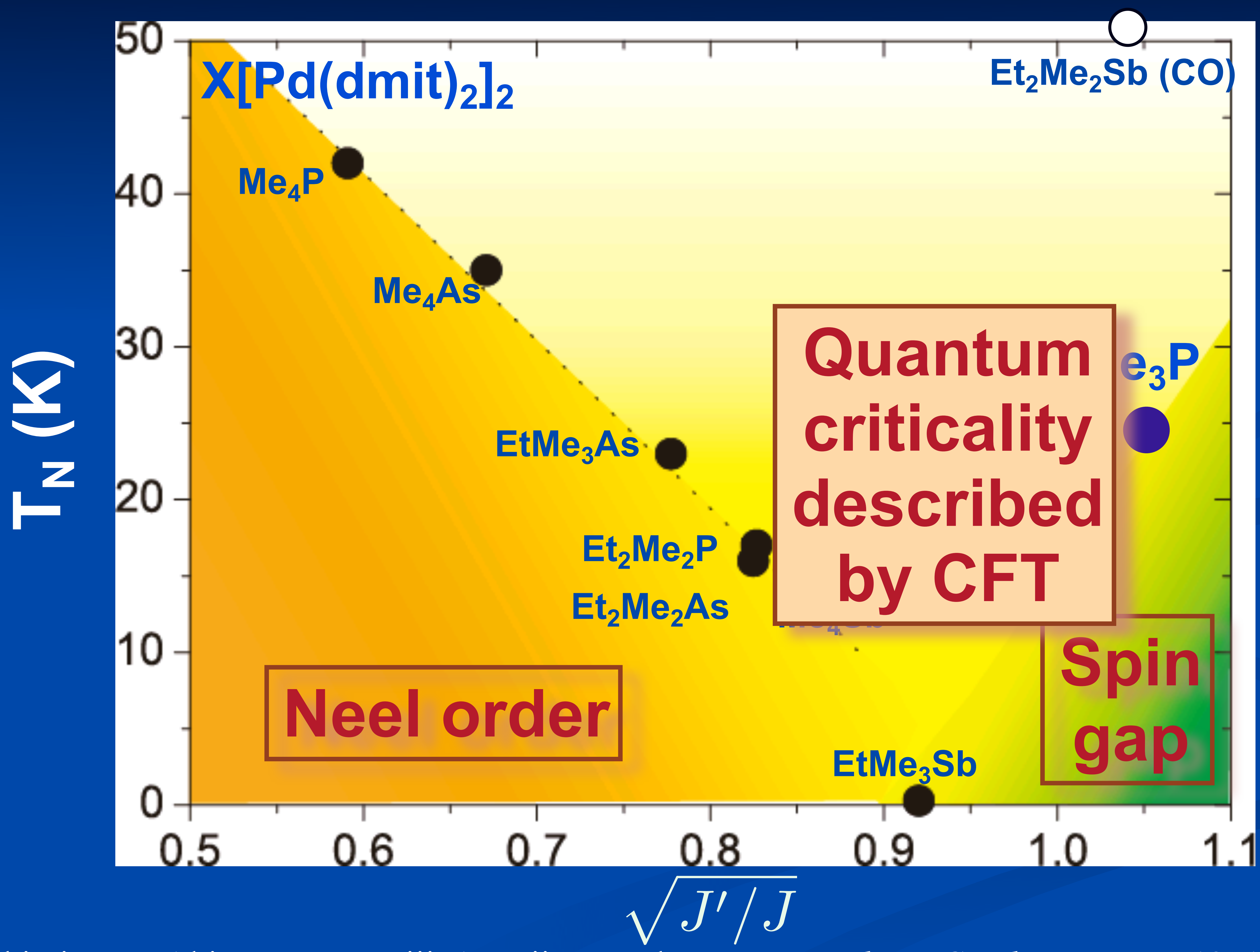


Quantum criticality in 2+1 dimensions

Friction of quantum criticality = waves falling into black hole

Kovtun, Policastro, Son

Magnetic Criticality



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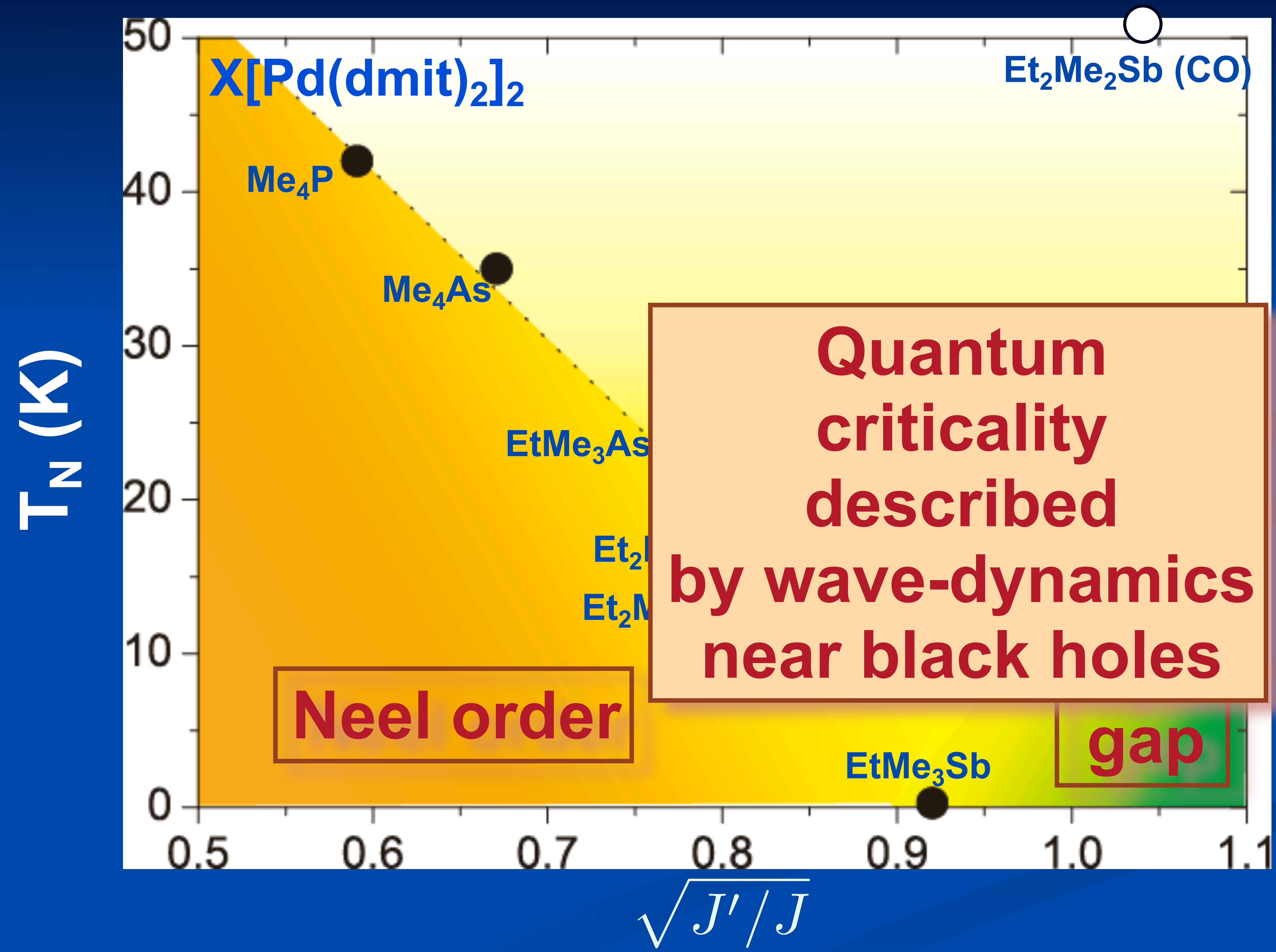

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Conclusions

- Berry phases lead to new field theories for transitions in antiferromagnets which are not part of the Landau-Ginzburg classification
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems in dimension $d > 1$, and were valuable in determining general structure of transport in the quantum critical region