Building strange metals from SYK models

Subir Sachdev

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Workshop on Advances in Non-Fermi Liquids

University of California, Berkeley

PHYSICS

HARVARD

Daniel Arovas
UCSD

John McGreevy
UCSD

Aavishkar Patel
Harvard

arXiv:1712.05026

and

arXiv:1807.04754

What are quasiparticles ?

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy ε_α

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

In a lattice system of N sites, this parameterizes the energy of $\sim e^{\alpha N}$ states in terms of poly(N) numbers.

What are quasiparticles ?

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time diverges as

$$\tau_{\text{eq}} \sim \frac{\hbar E_F}{(k_B T)^2} \quad , \quad \text{as } T \to 0,$$

where E_F is the Fermi energy.

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- This time is much longer than the 'Planckian time' $\hbar/(k_B T)$, which we will find in systems without quasiparticle excitations.

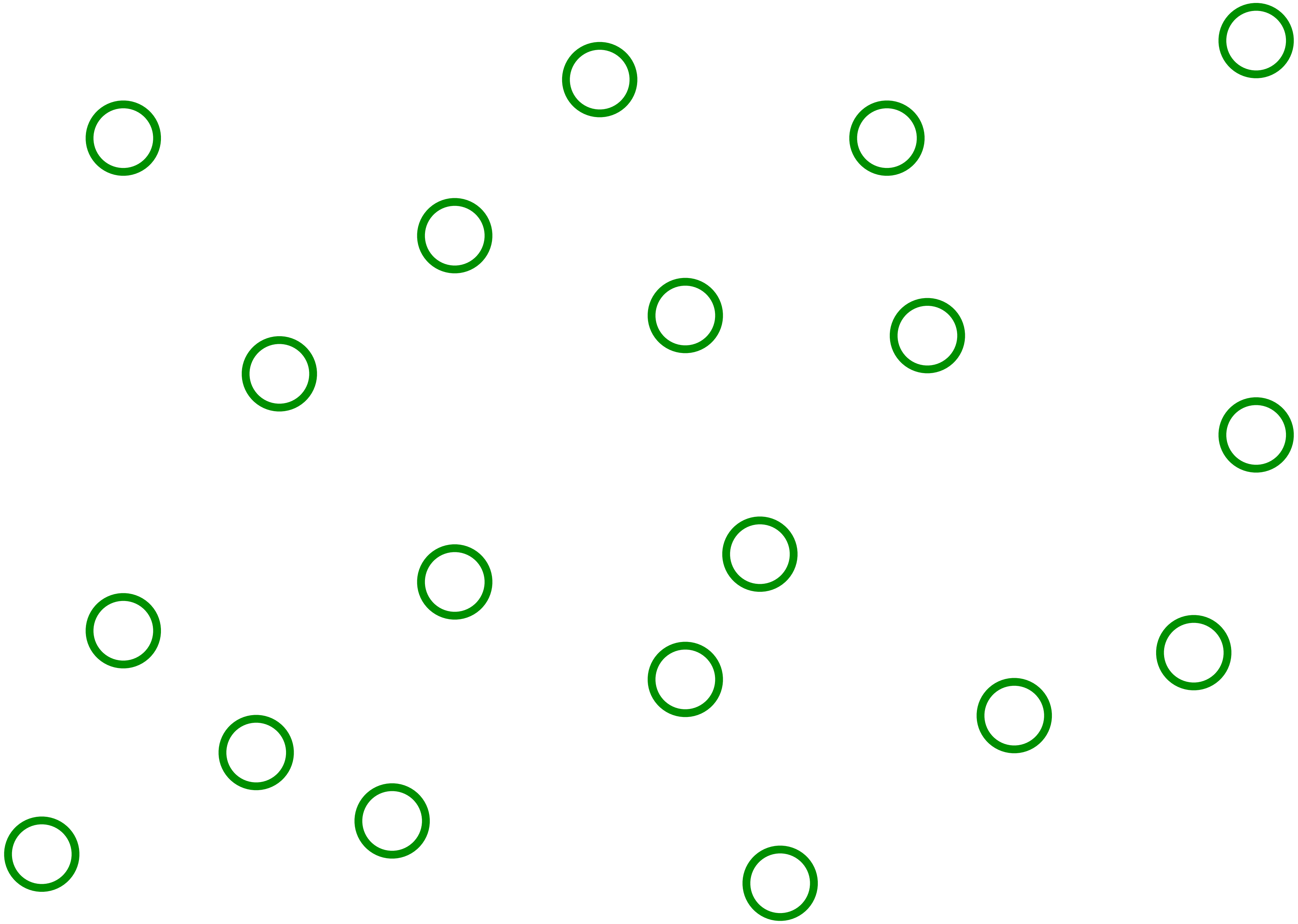
$$\tau_{\rm eq} \gg \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

1. Solvable model without quasiparticles
SYK model of a `quantum island'

2. Lattice models of SYK islands
Theories of strange metals

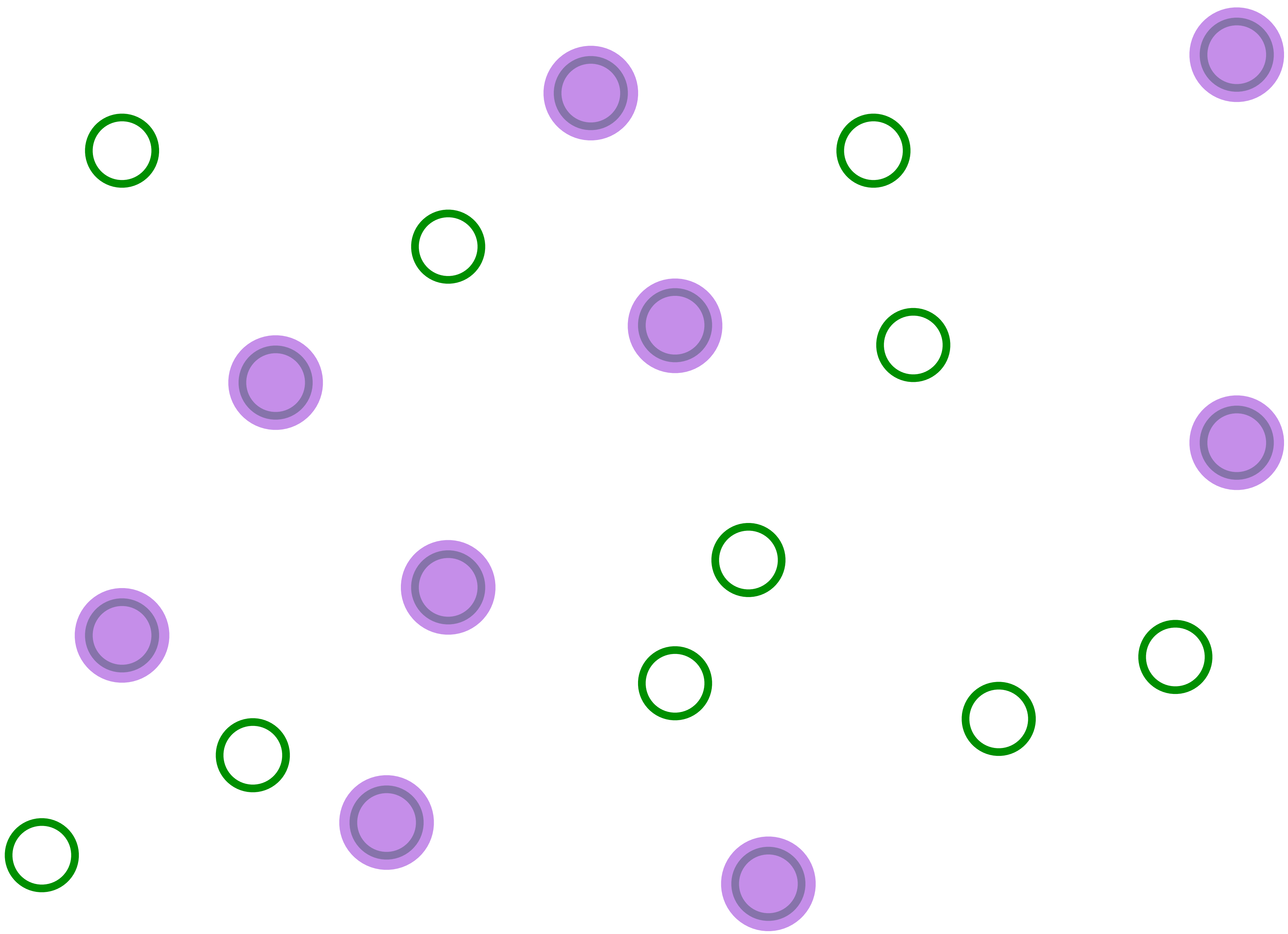
3. SYK U(1) gauge theory
Solvable model with finite density of fermions, emergent gauge fields, and disorder

The Sachdev-Ye-Kitaev (SYK) model



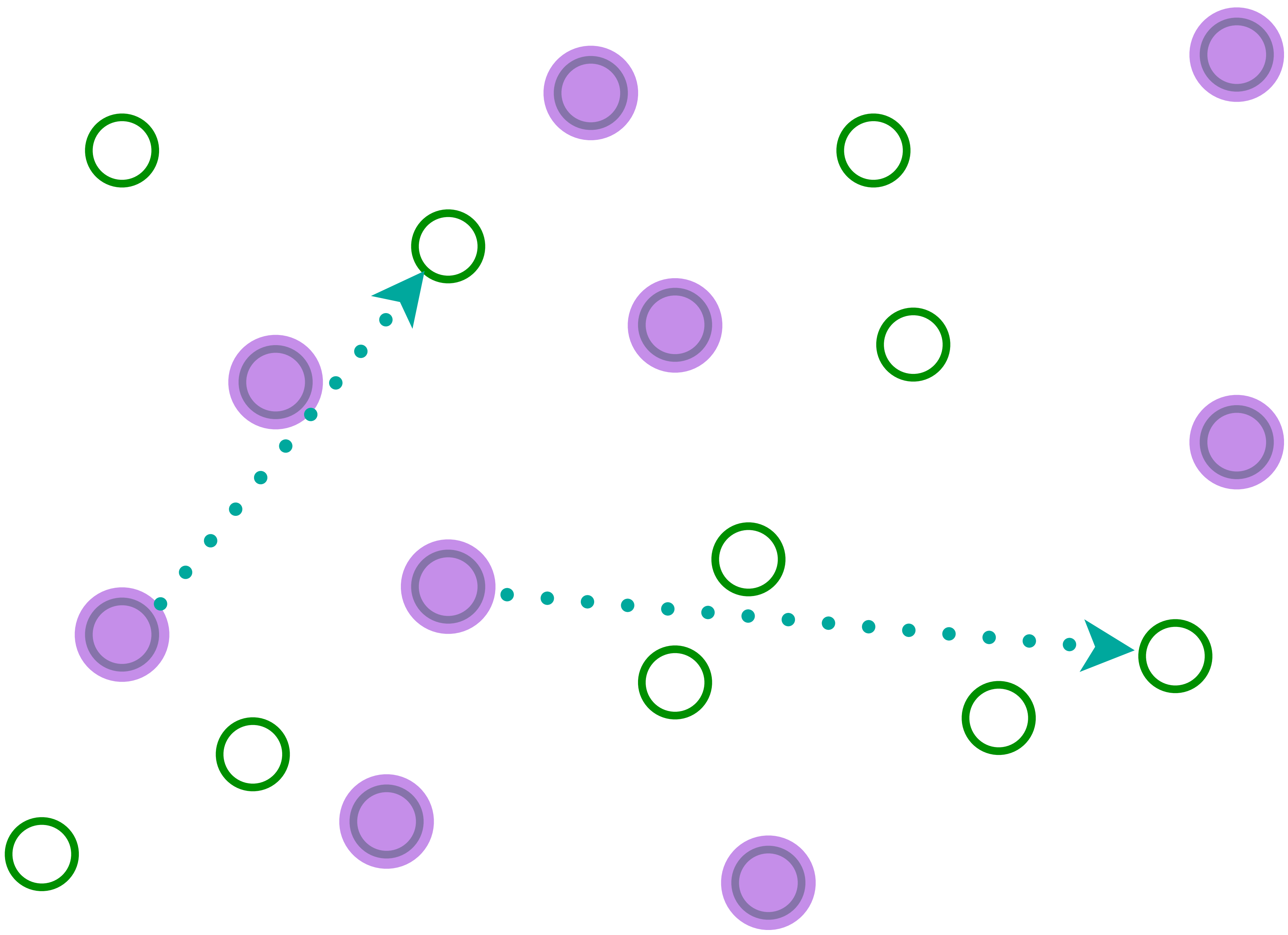
Pick a set of random positions

The SYK model



Place electrons randomly on some sites

The SYK model



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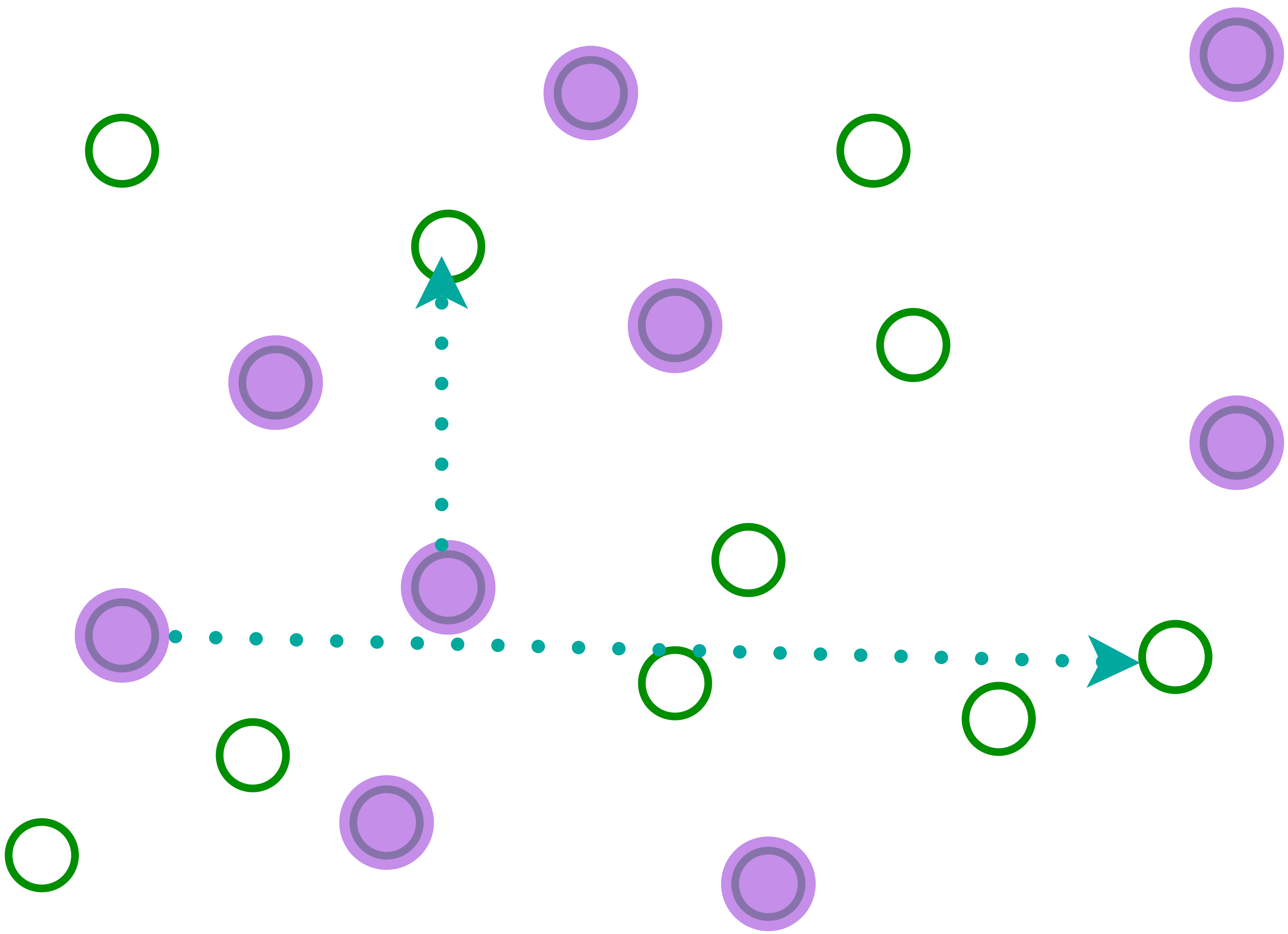

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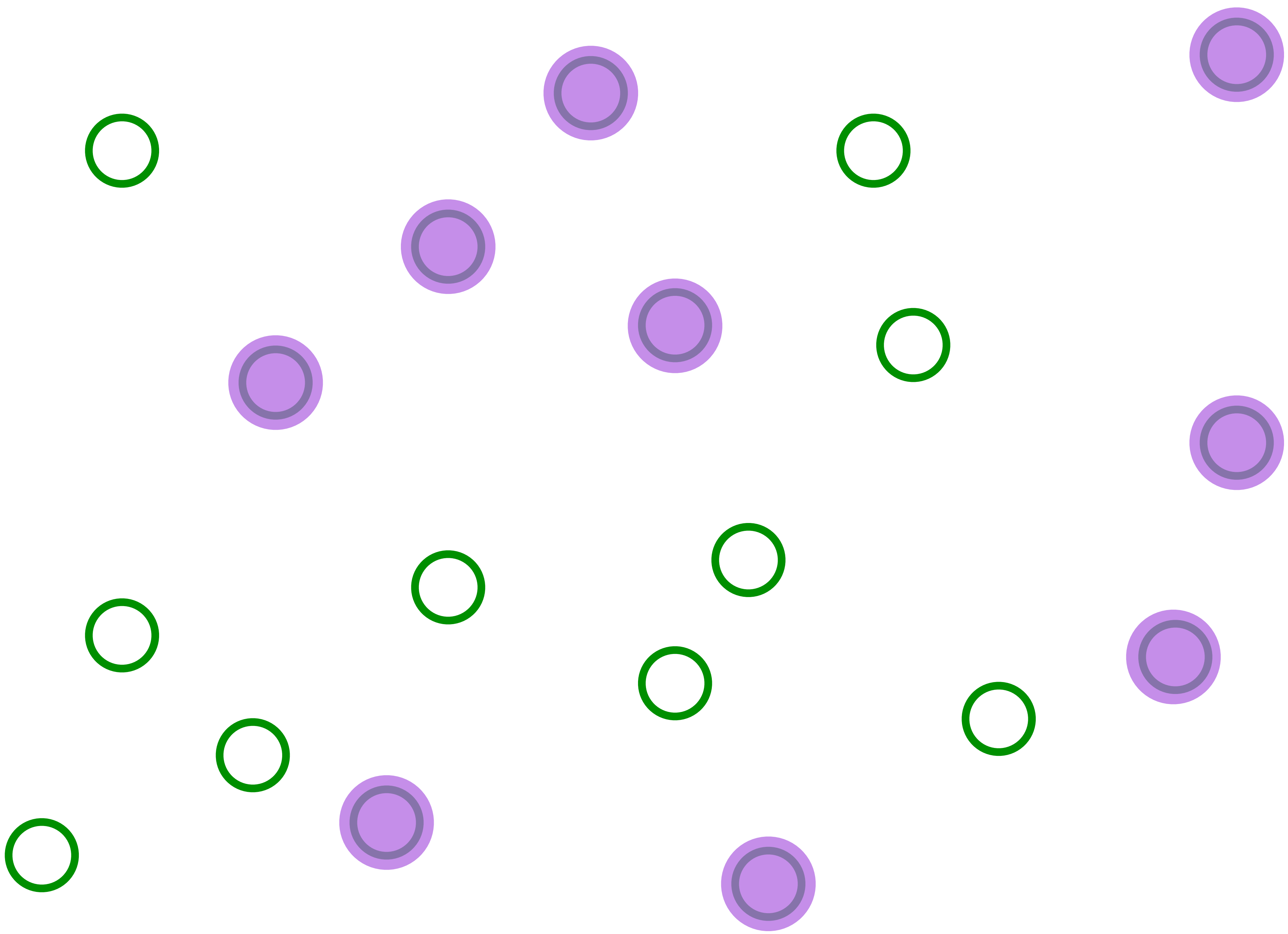
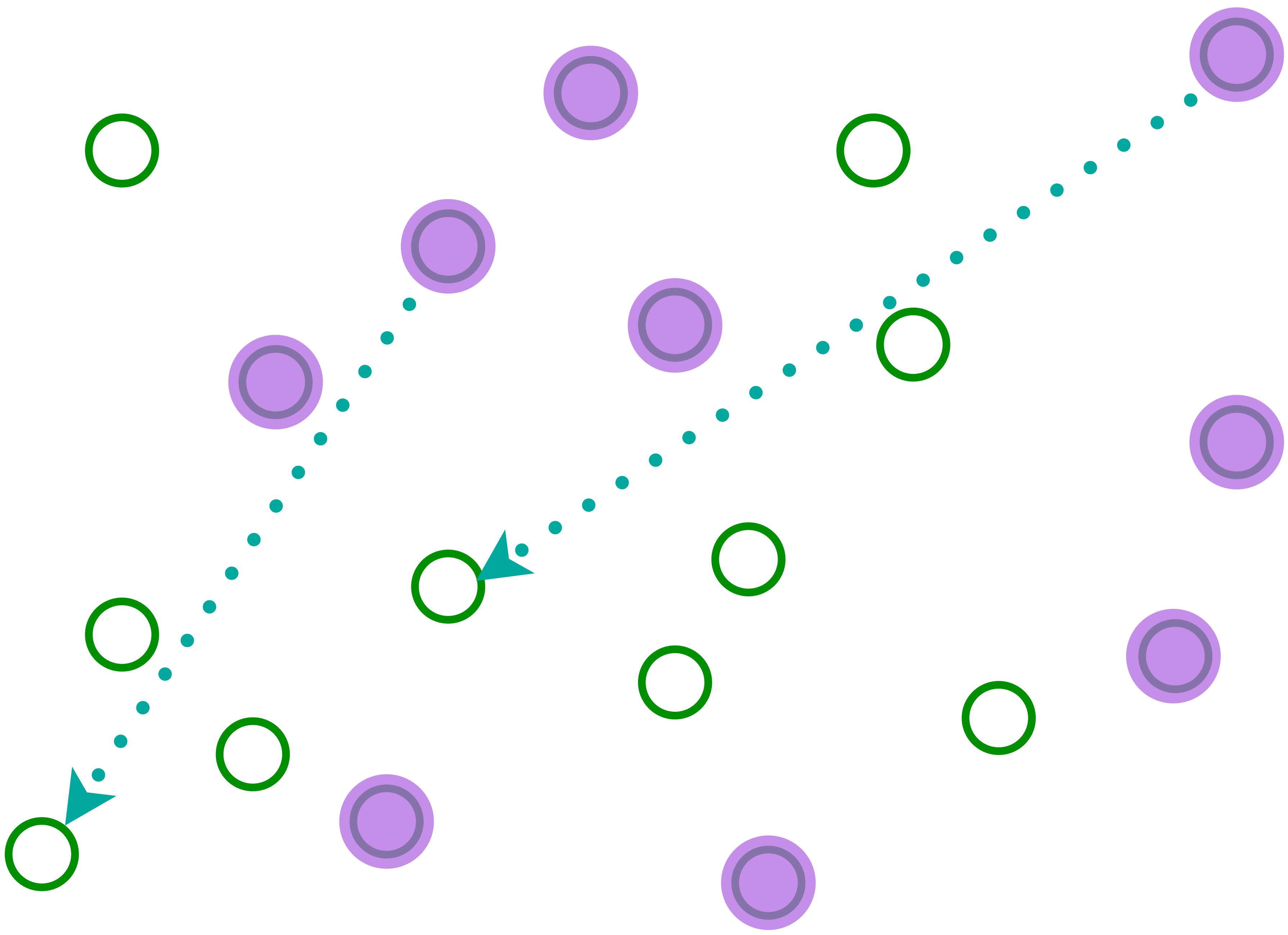


Entangle electrons pairwise randomly

The SYK model



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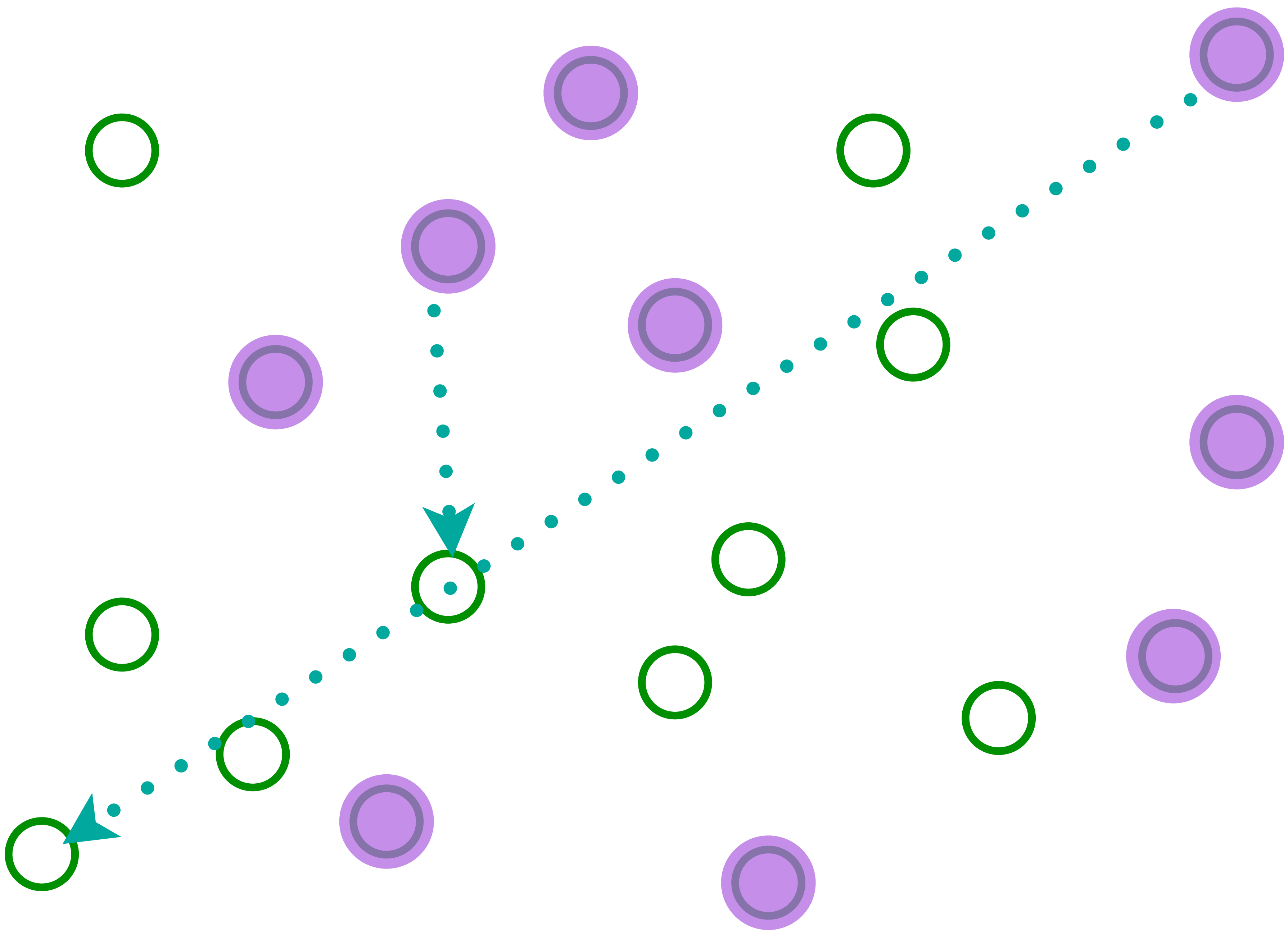

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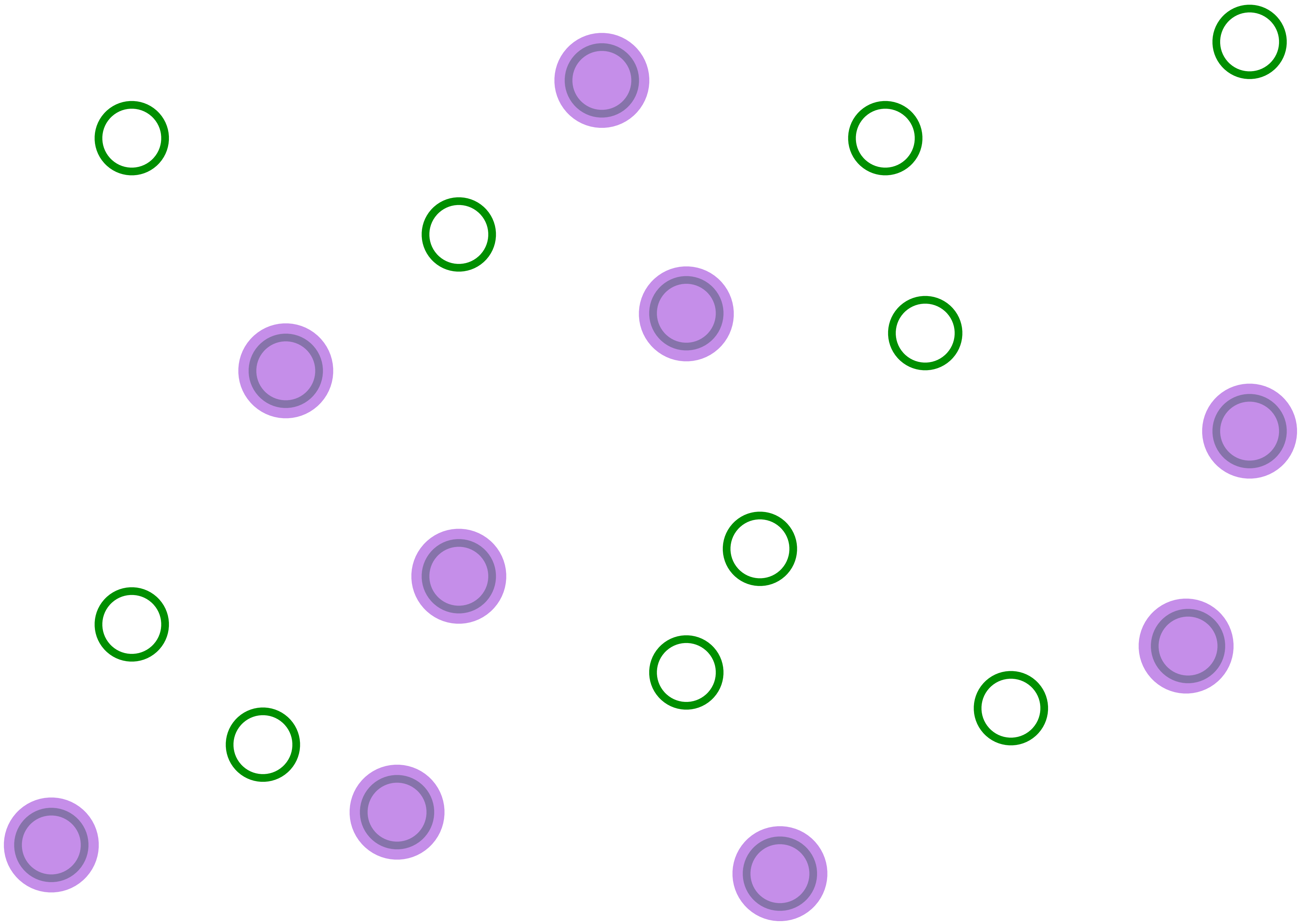

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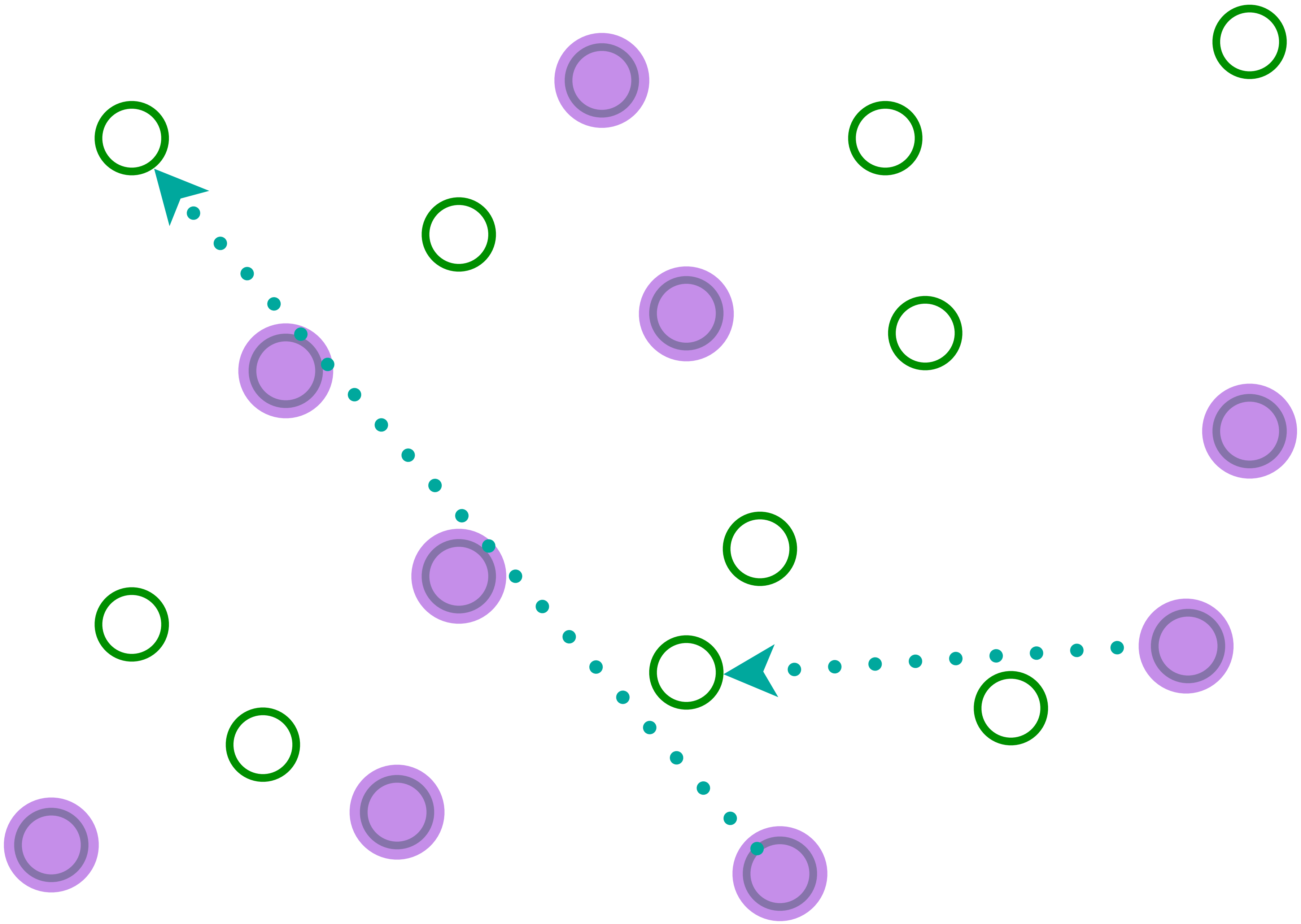

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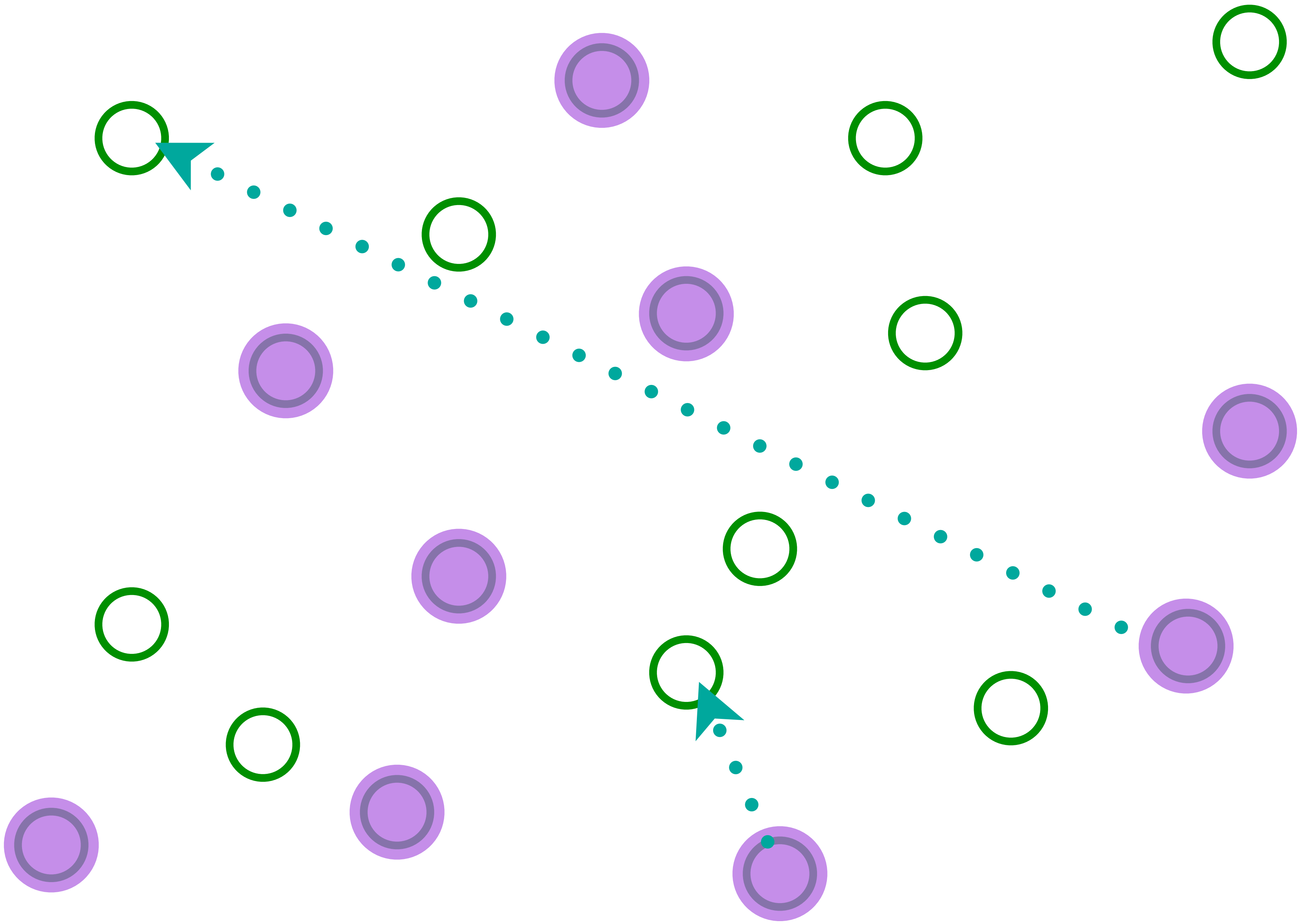

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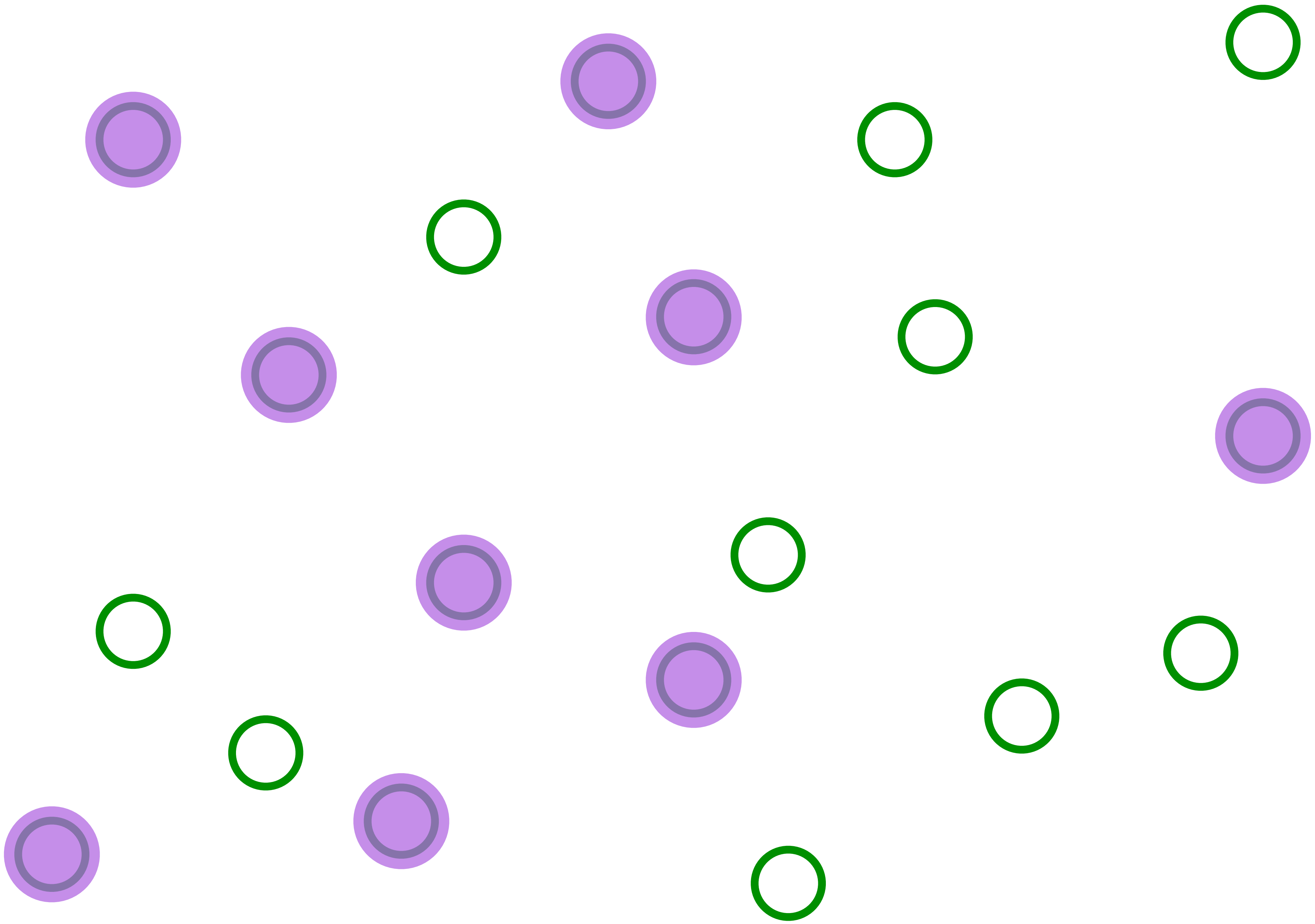

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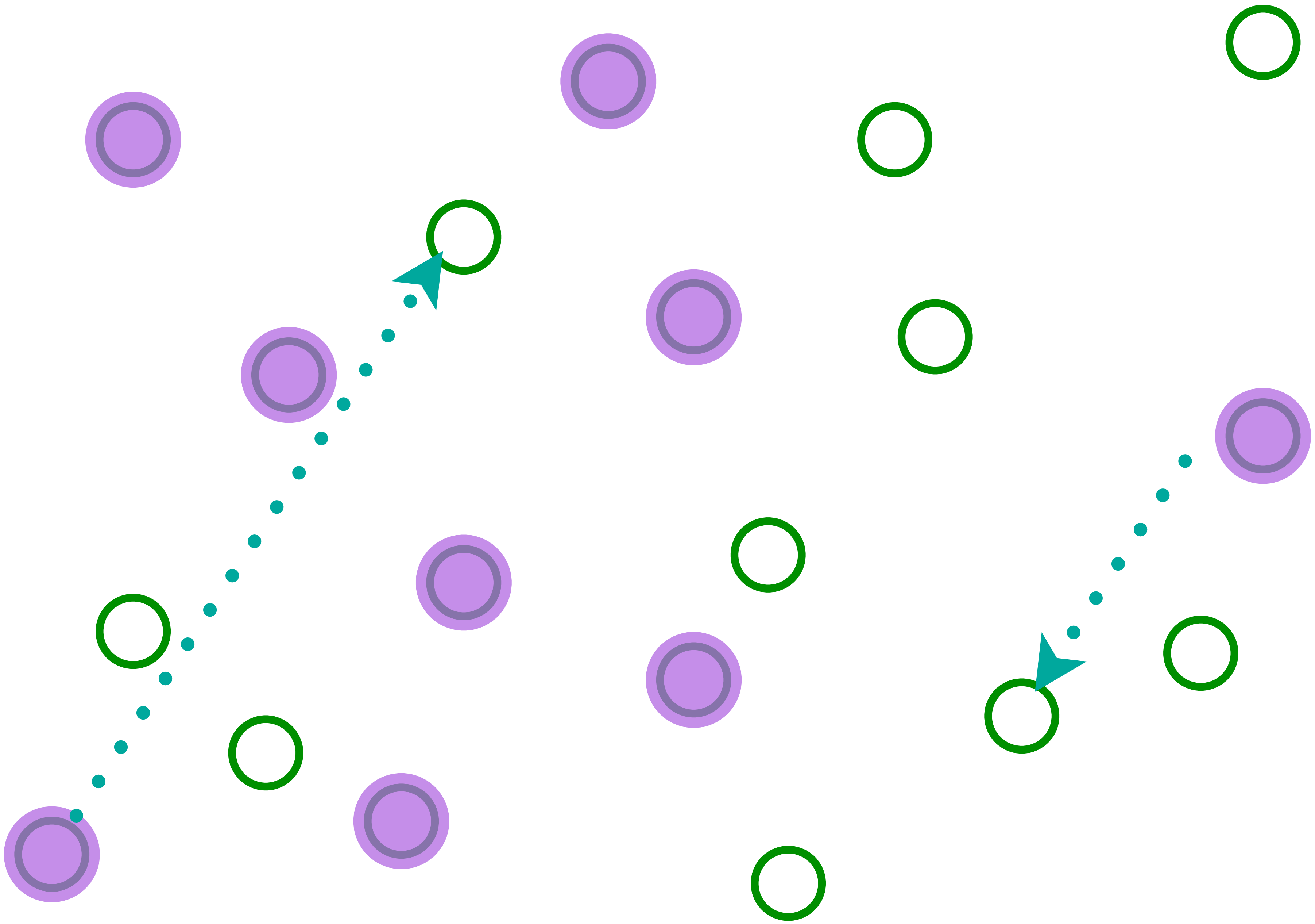

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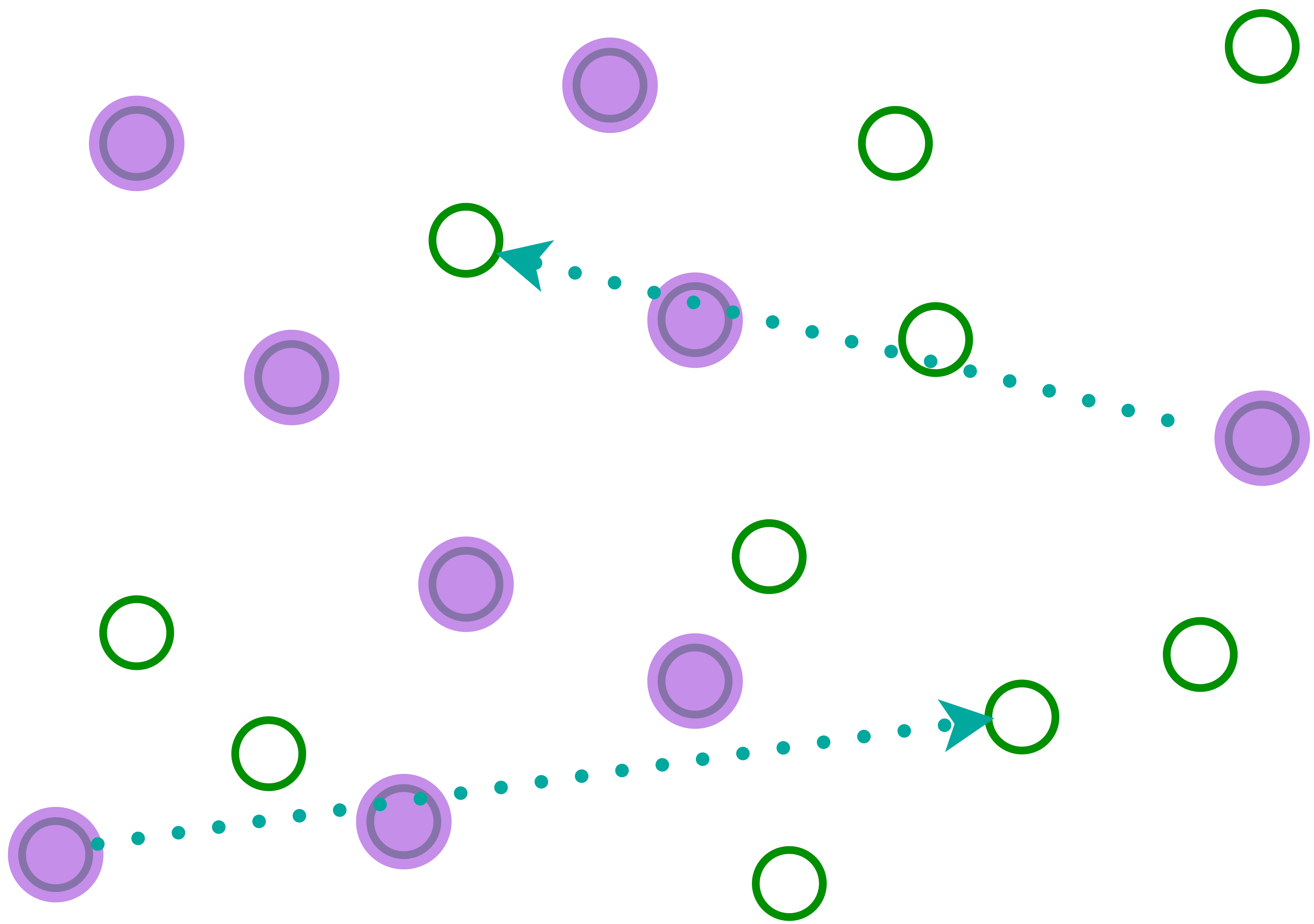

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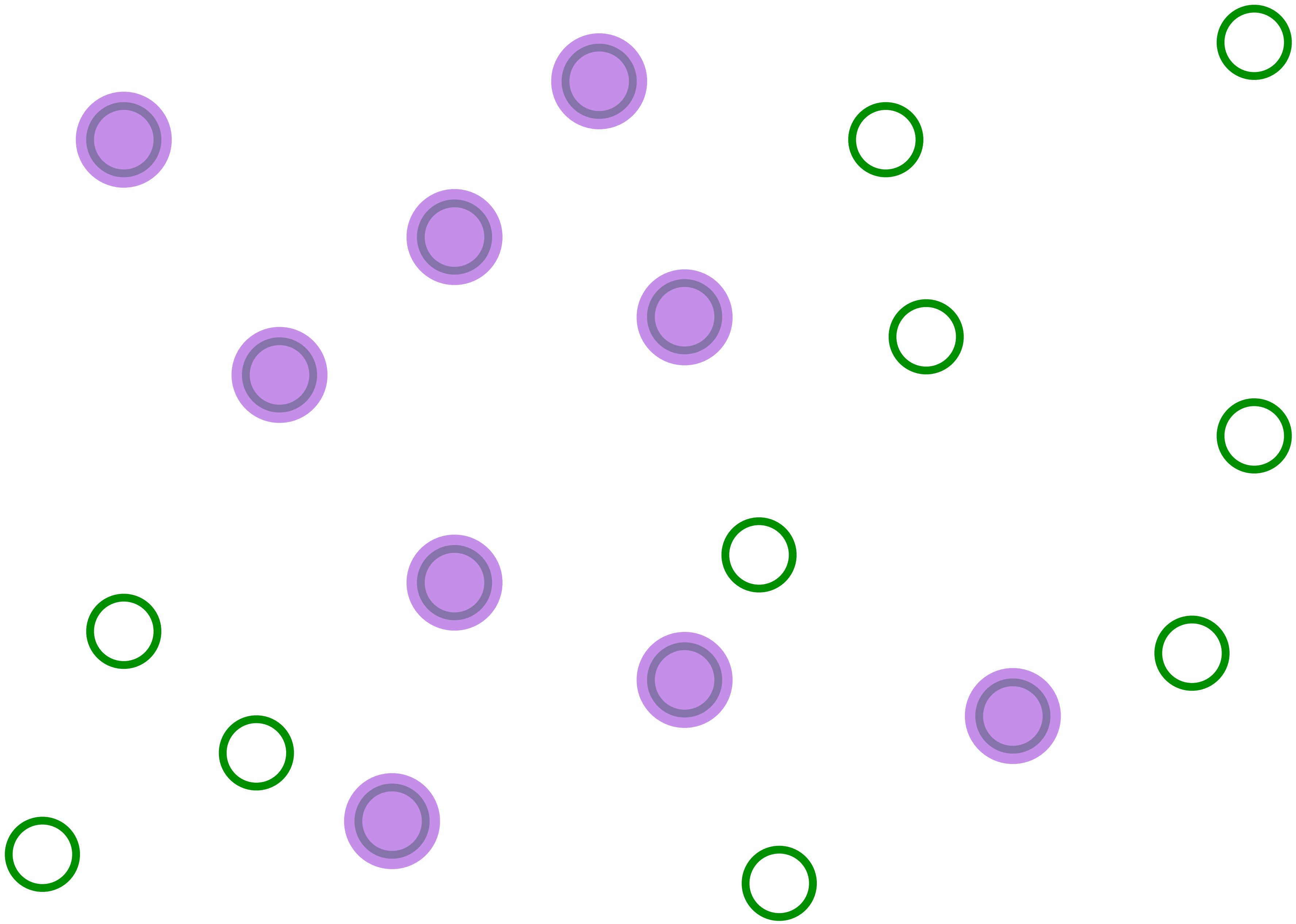

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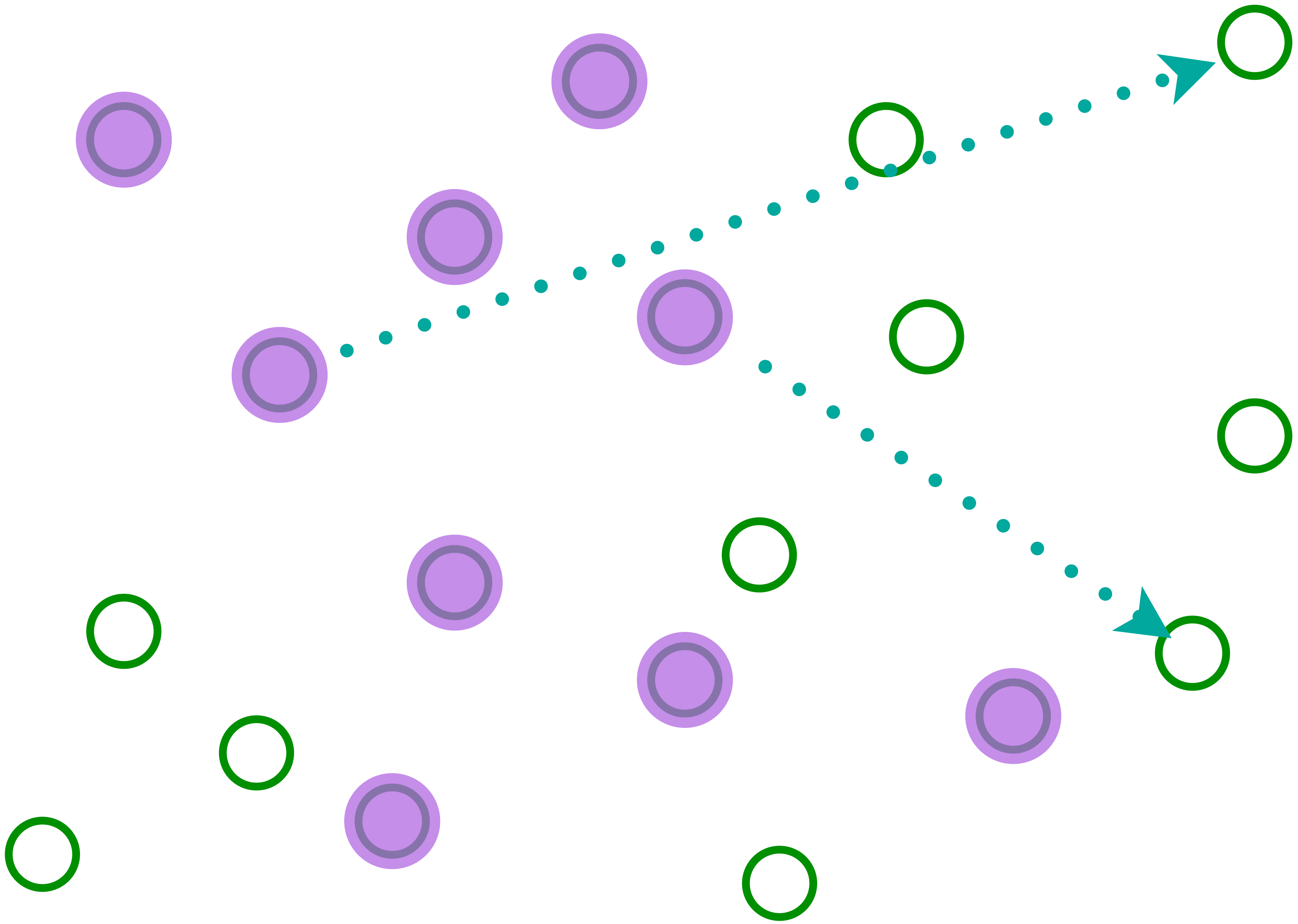

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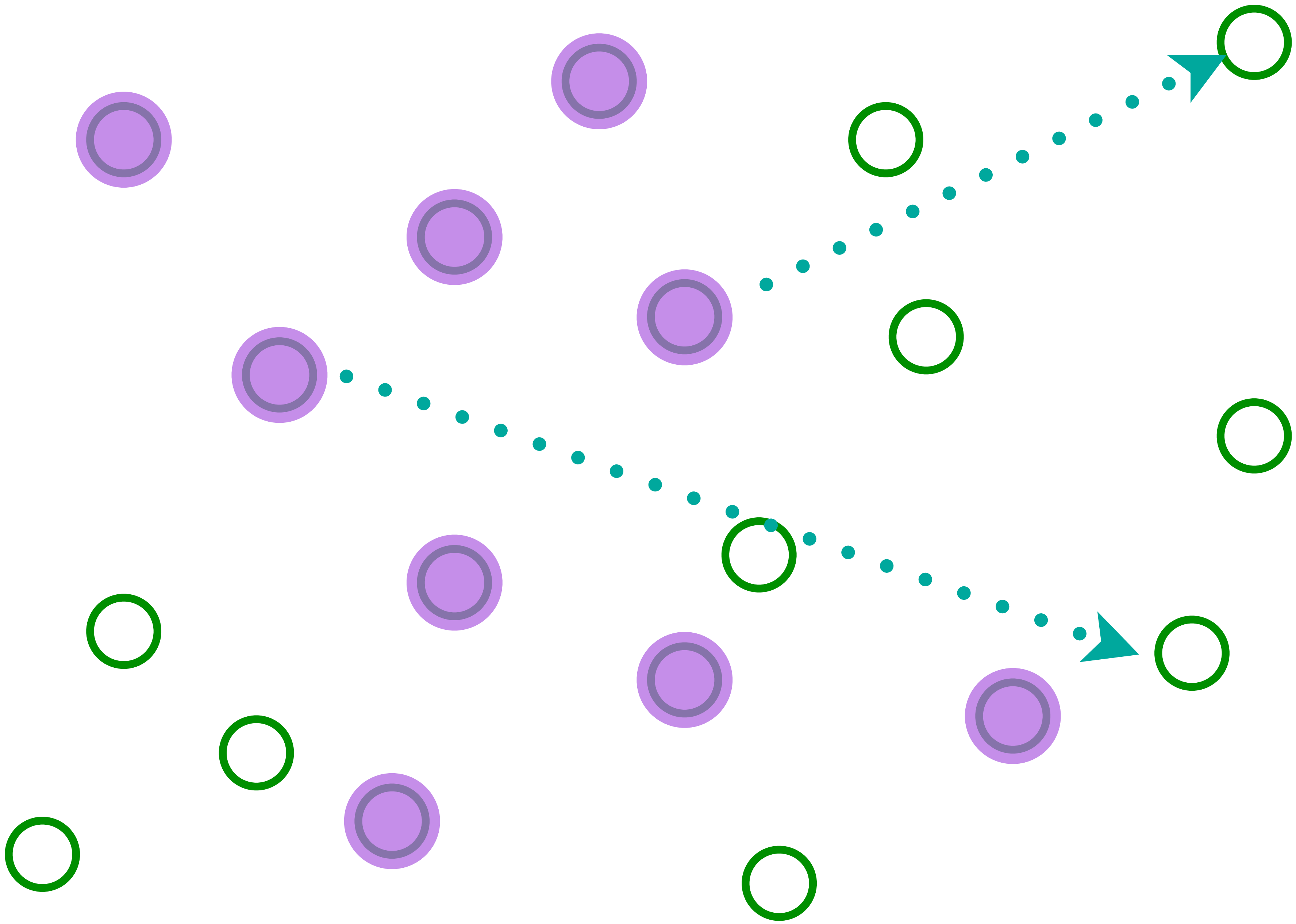

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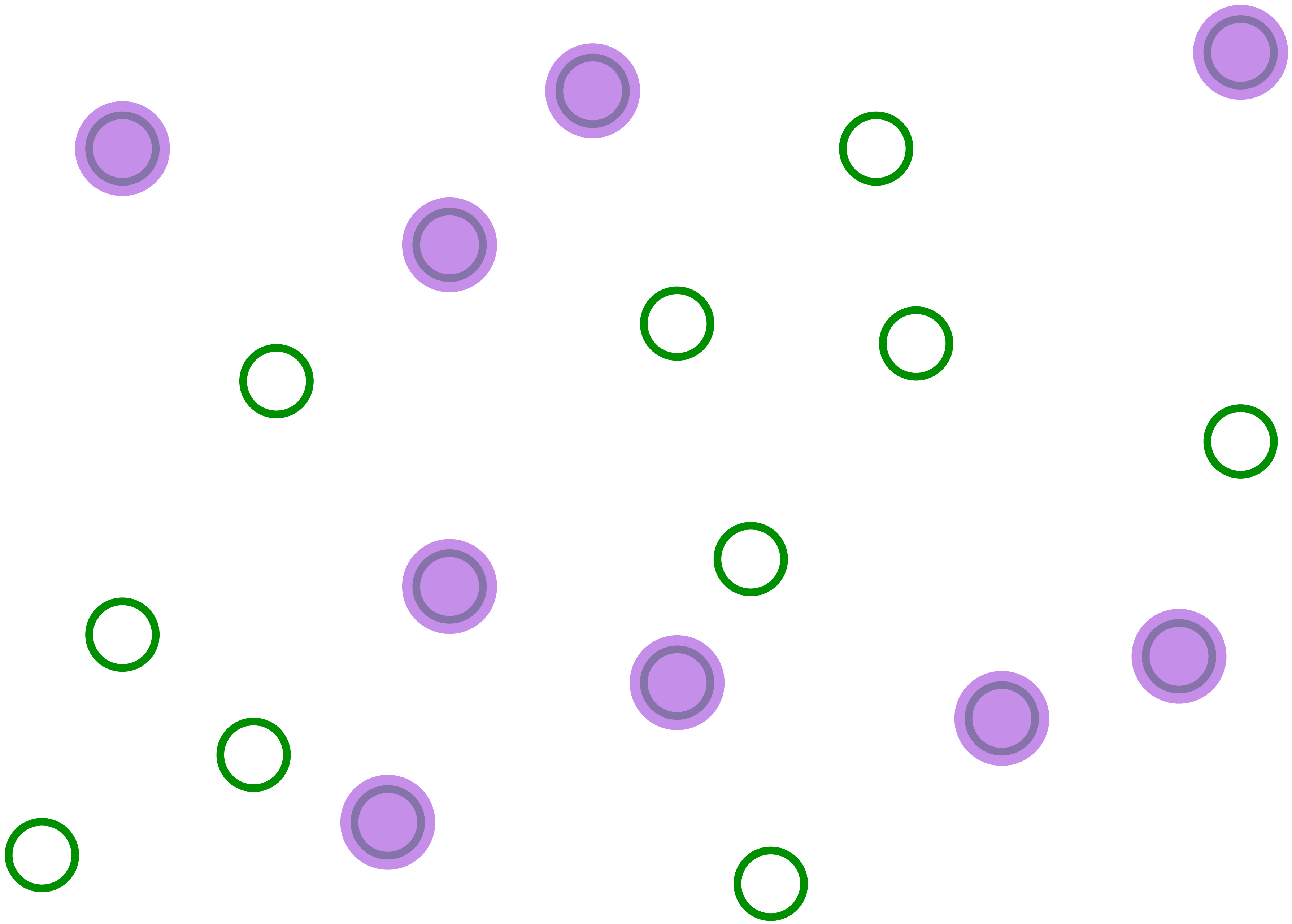

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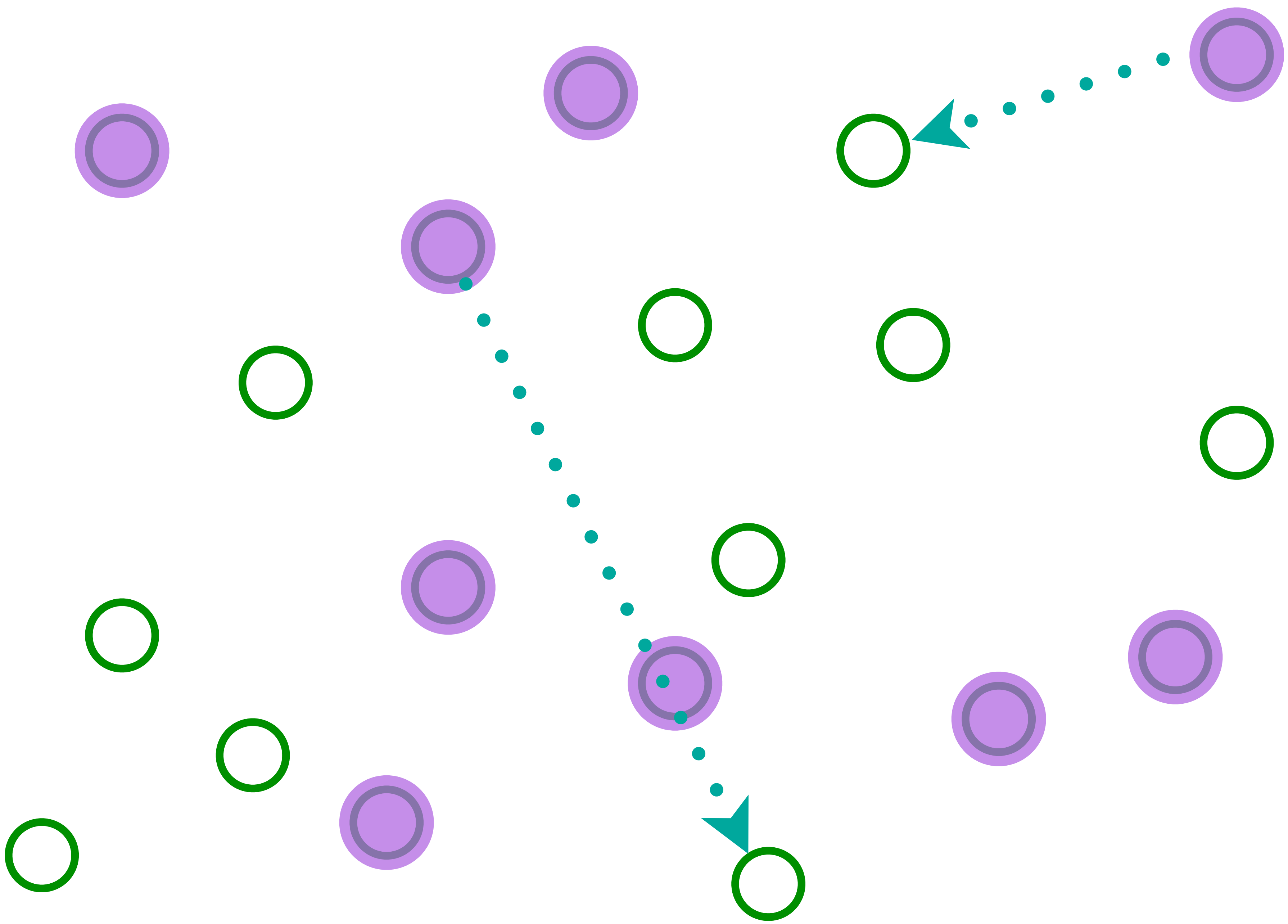

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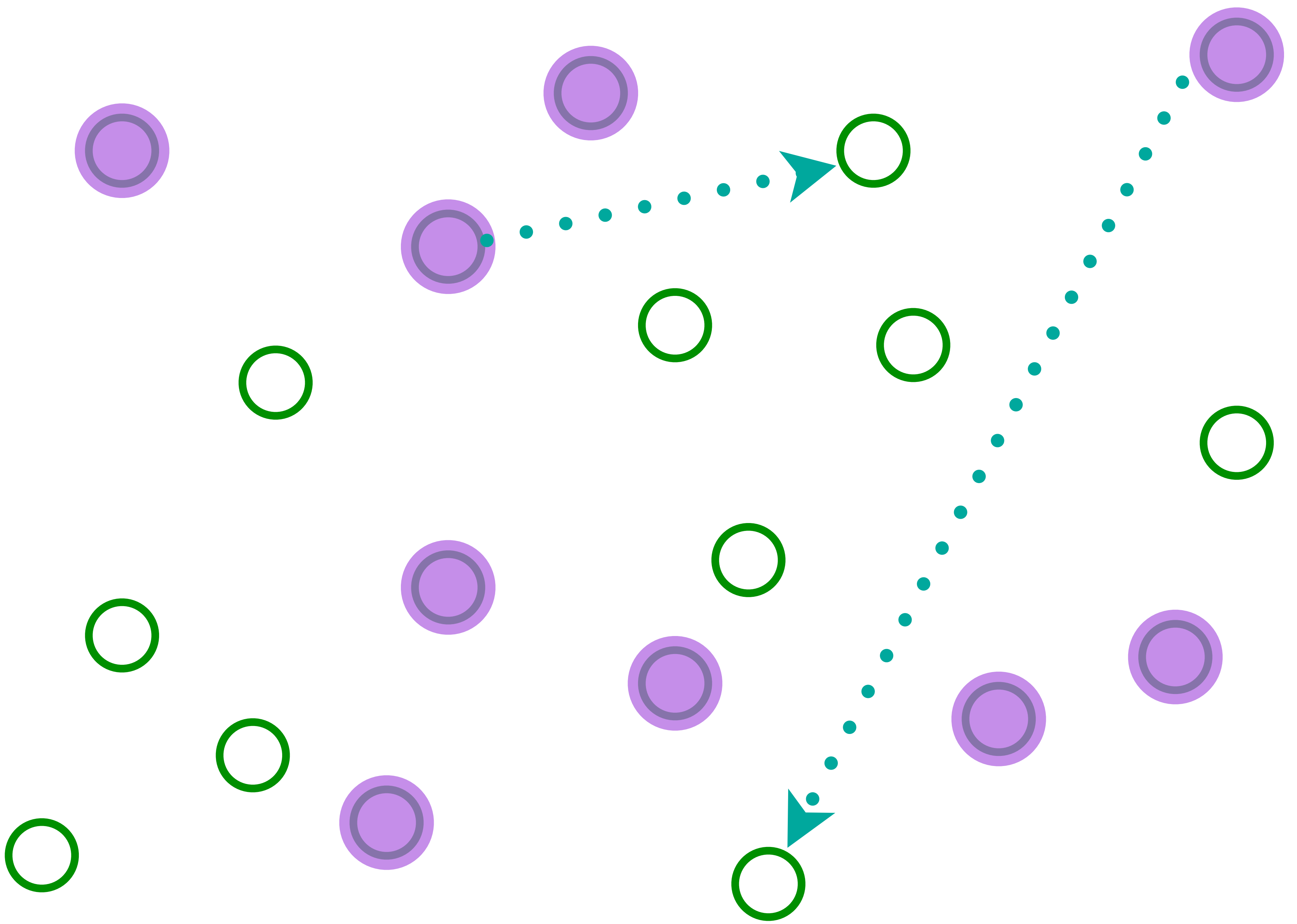

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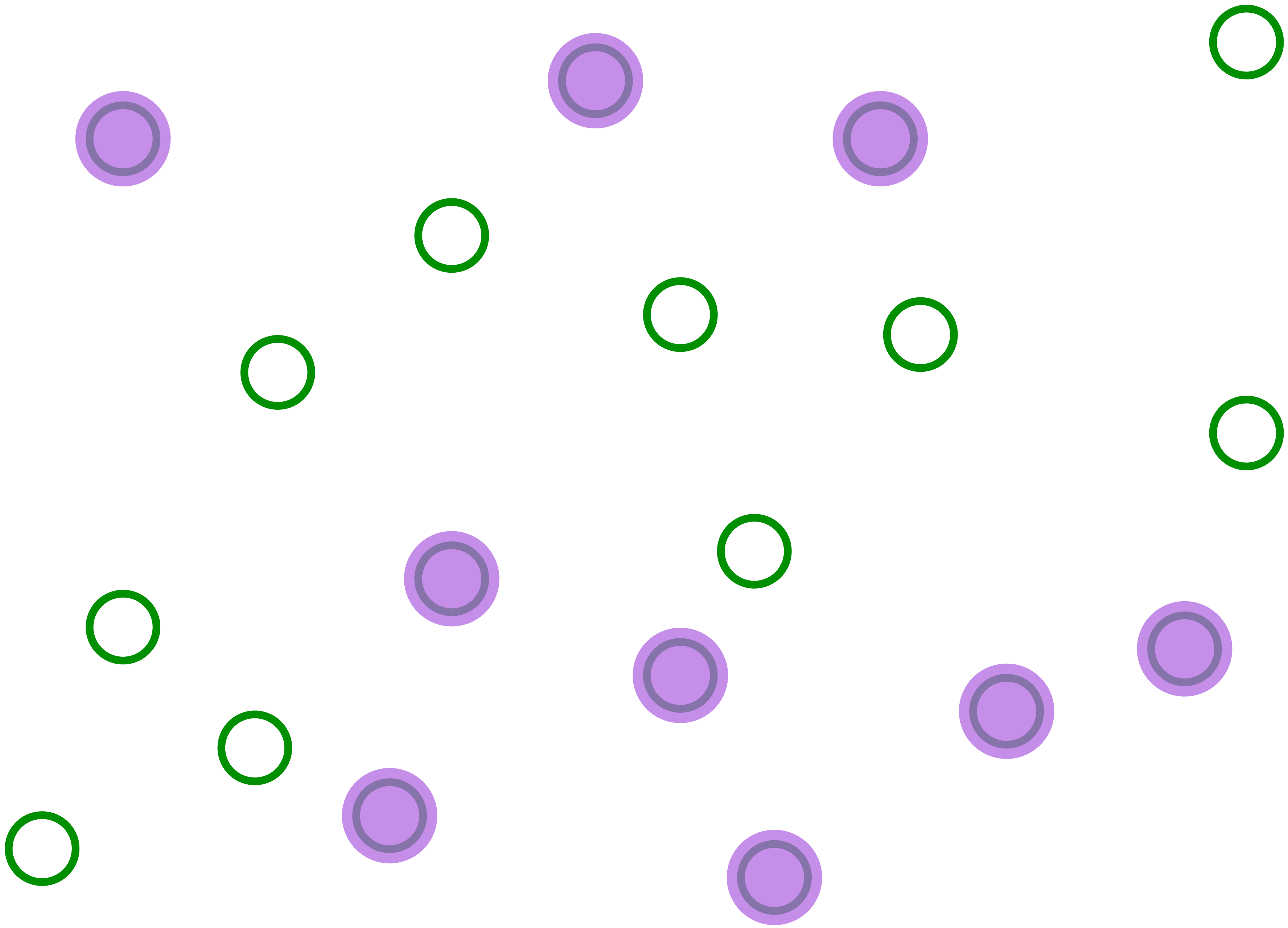

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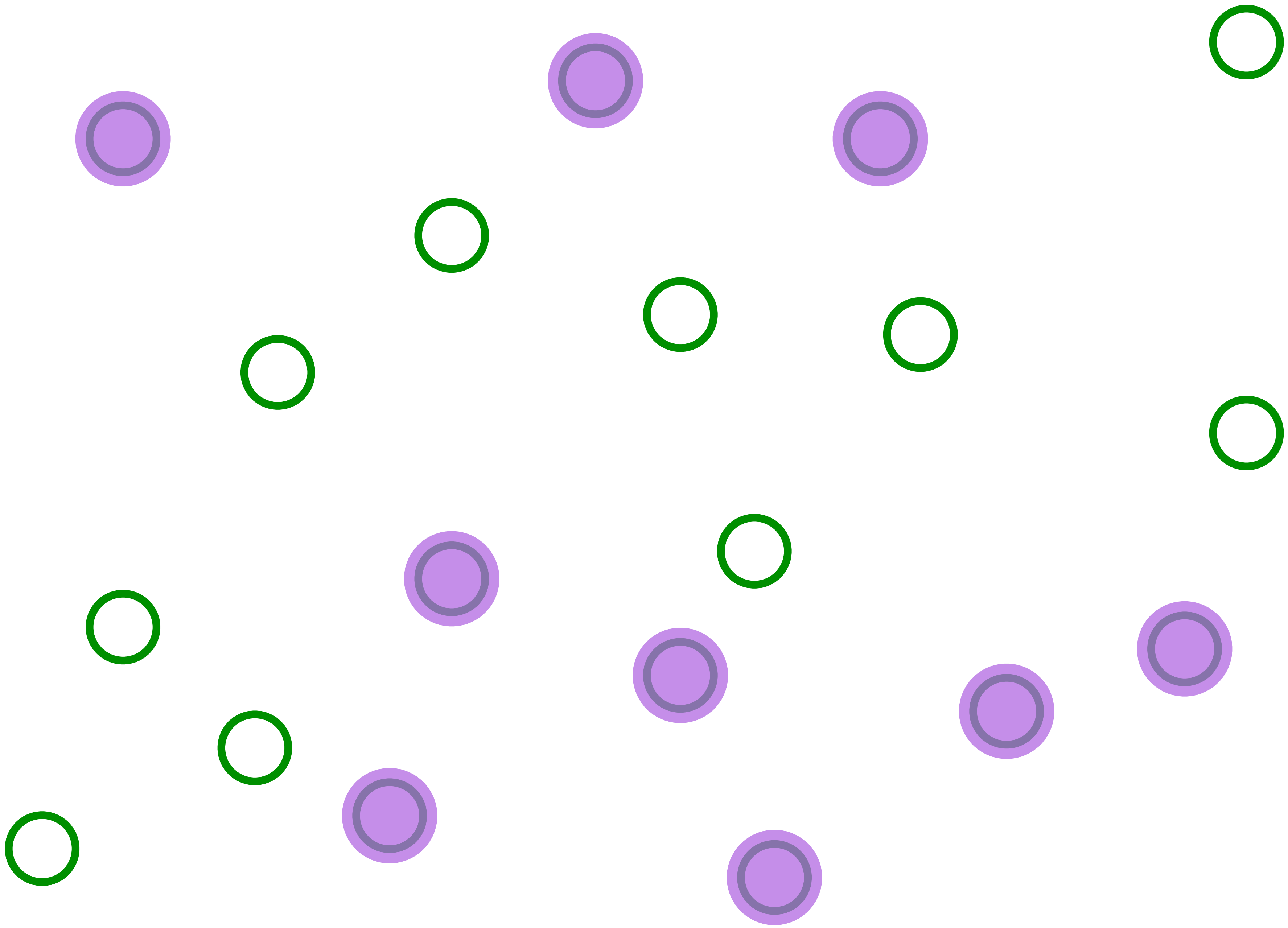


This describes both a strange metal and a black hole!

The SYK model

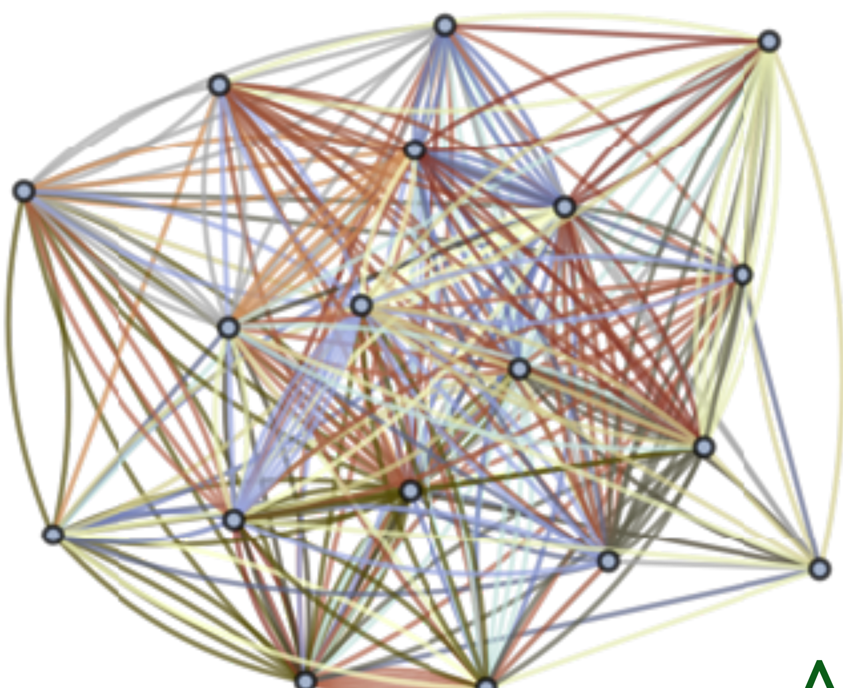
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large N limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



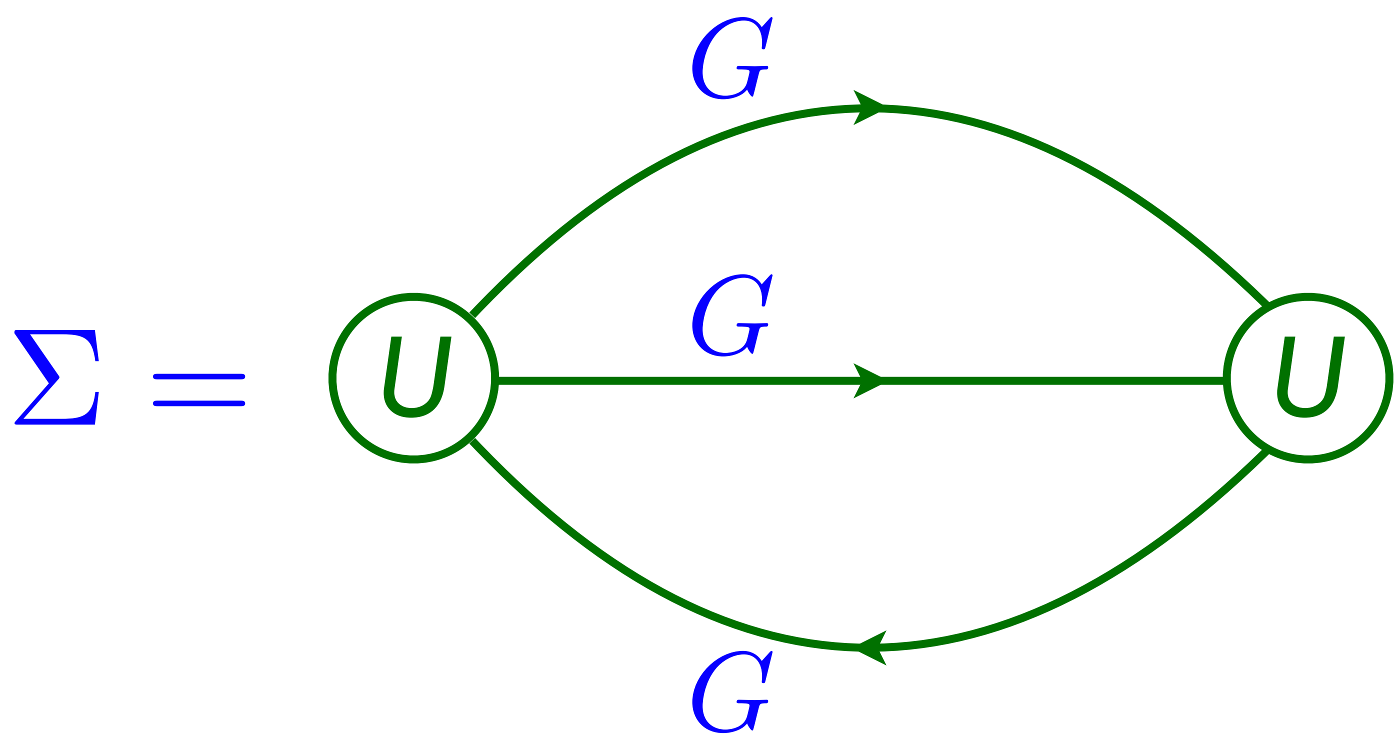
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

The SYK model

Feynman graph expansion in $U_{ijk\ell}$, and graph-by-graph average, yields exact equations in the large N limit:

$$G(i\omega) = \frac{1}{i\omega + \mu - \Sigma(i\omega)} \quad , \quad \Sigma(\tau) = -U^2 G^2(\tau) G(-\tau)$$
$$G(\tau = 0^-) = \mathcal{Q}.$$



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$$G(\tau = 0^-) = \mathcal{Q}.$$

Low frequency analysis shows that the solutions must be gapless and obey

$$\Sigma(z) = \mu - \frac{e^{i(\pi/4+\theta)}}{A}\sqrt{z} + \dots \quad , \quad G(z) = \frac{A e^{-i(\pi/4+\theta)}}{\sqrt{z}}$$

where $A = (\pi/U^2 \cos(2\theta))^{1/4}$. The value of θ is universally related to $\mathcal{Q}$ by a Luttinger-Ward functional analysis similar to that used to establish the Luttinger theorem of Fermi liquid theory:

$$\mathcal{Q} = \frac{1}{2} - \frac{\theta}{\pi} - \frac{\sin(2\theta)}{4}$$

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993)

A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

The SYK model

No quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

A. Georges and O. Parcollet
PRB **59**, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and
J. Steinberg, PRB **96**, 205123 (2017)

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A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

- Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

Absence of quasiparticles $\Leftrightarrow$ Fastest possible thermalization

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   SYK model of a `quantum island'

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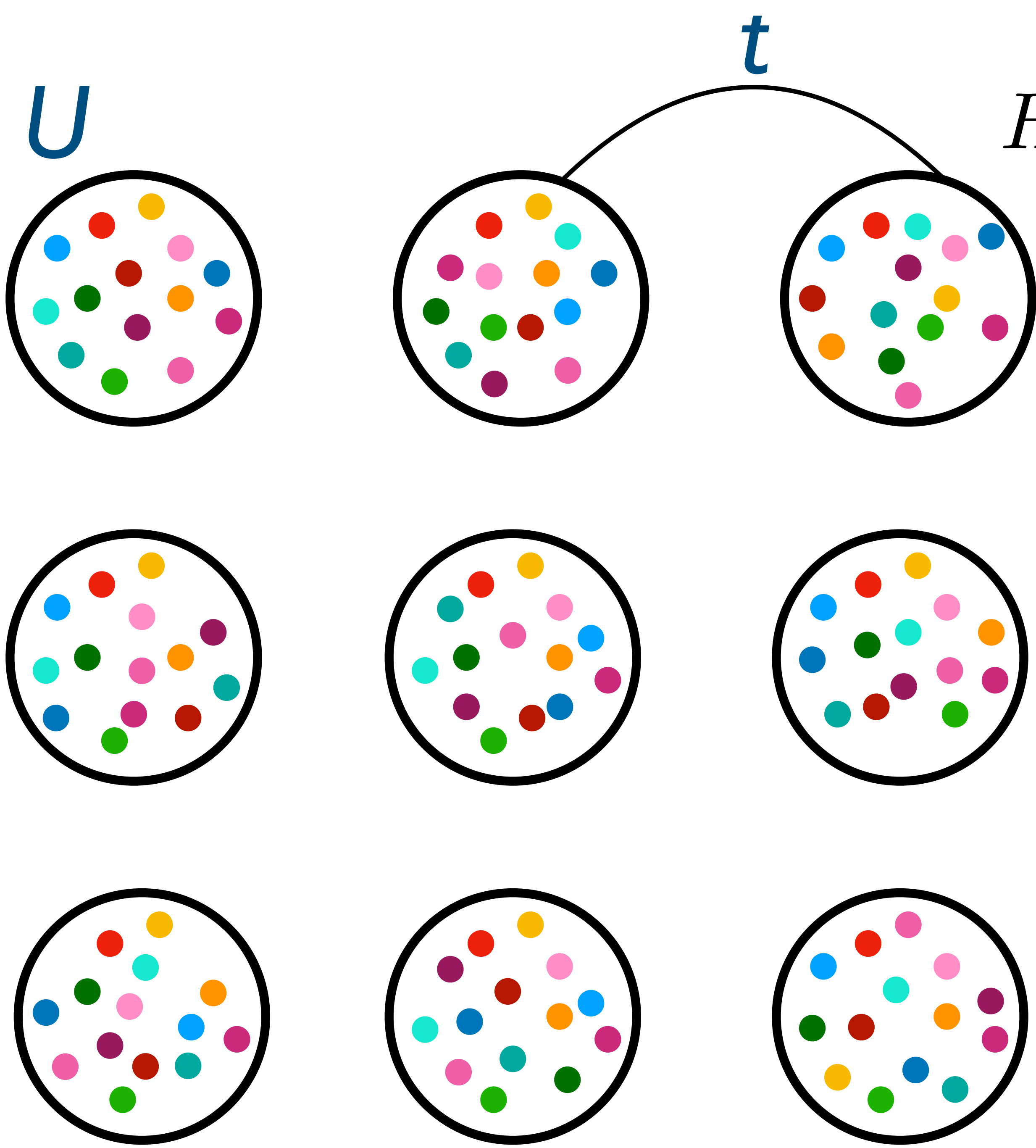
Coupled SYK Islands

SYK quantum islands of electrons with random hopping between them.



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

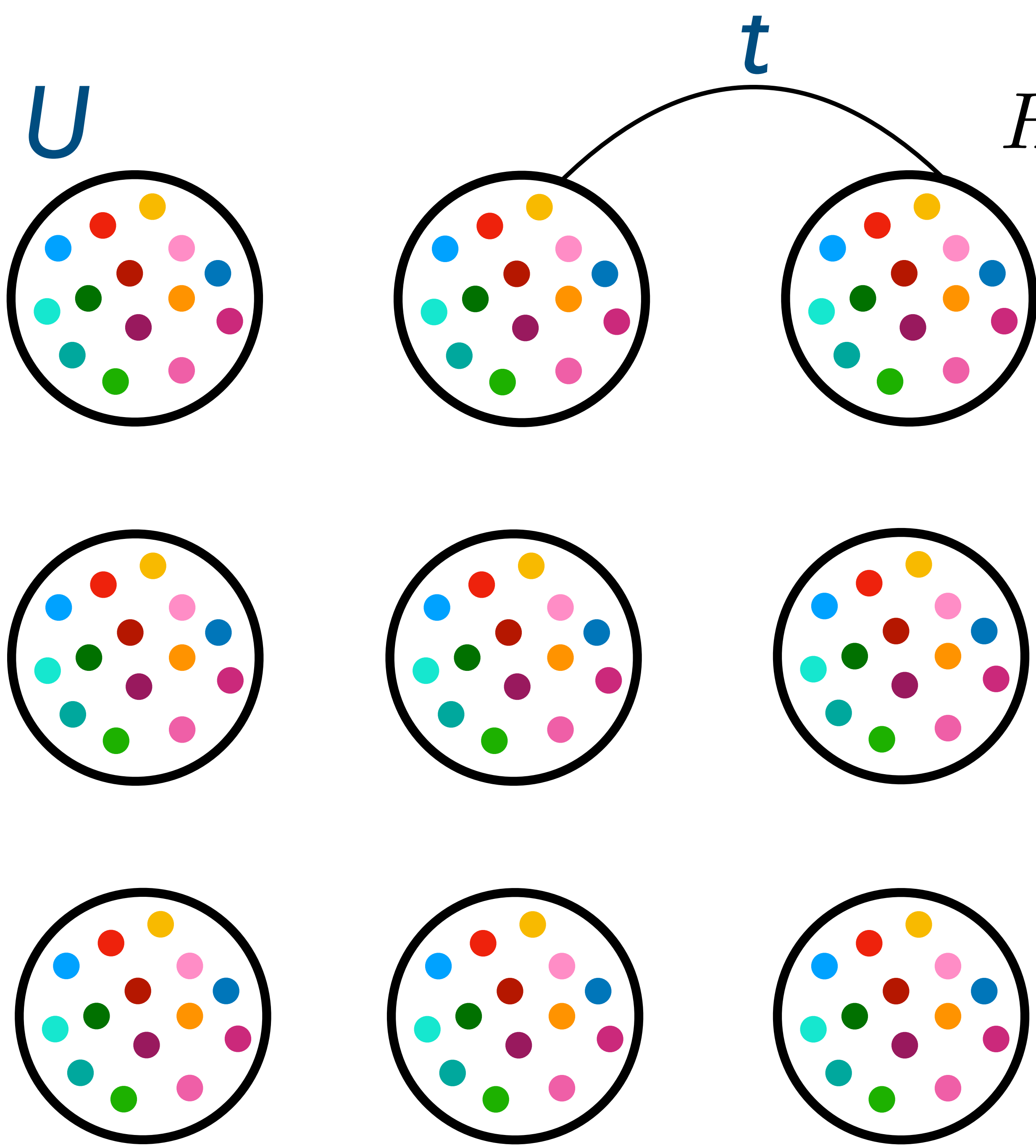
$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Can also use non-random *t*, and the same *U* on all "islands".



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl} \quad c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx}$$

$$+ \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij} \qquad c^\dagger_{i,x} c_{j,x'}$$

Pengfei Zhang, PRB **96**, 205138 (2017)

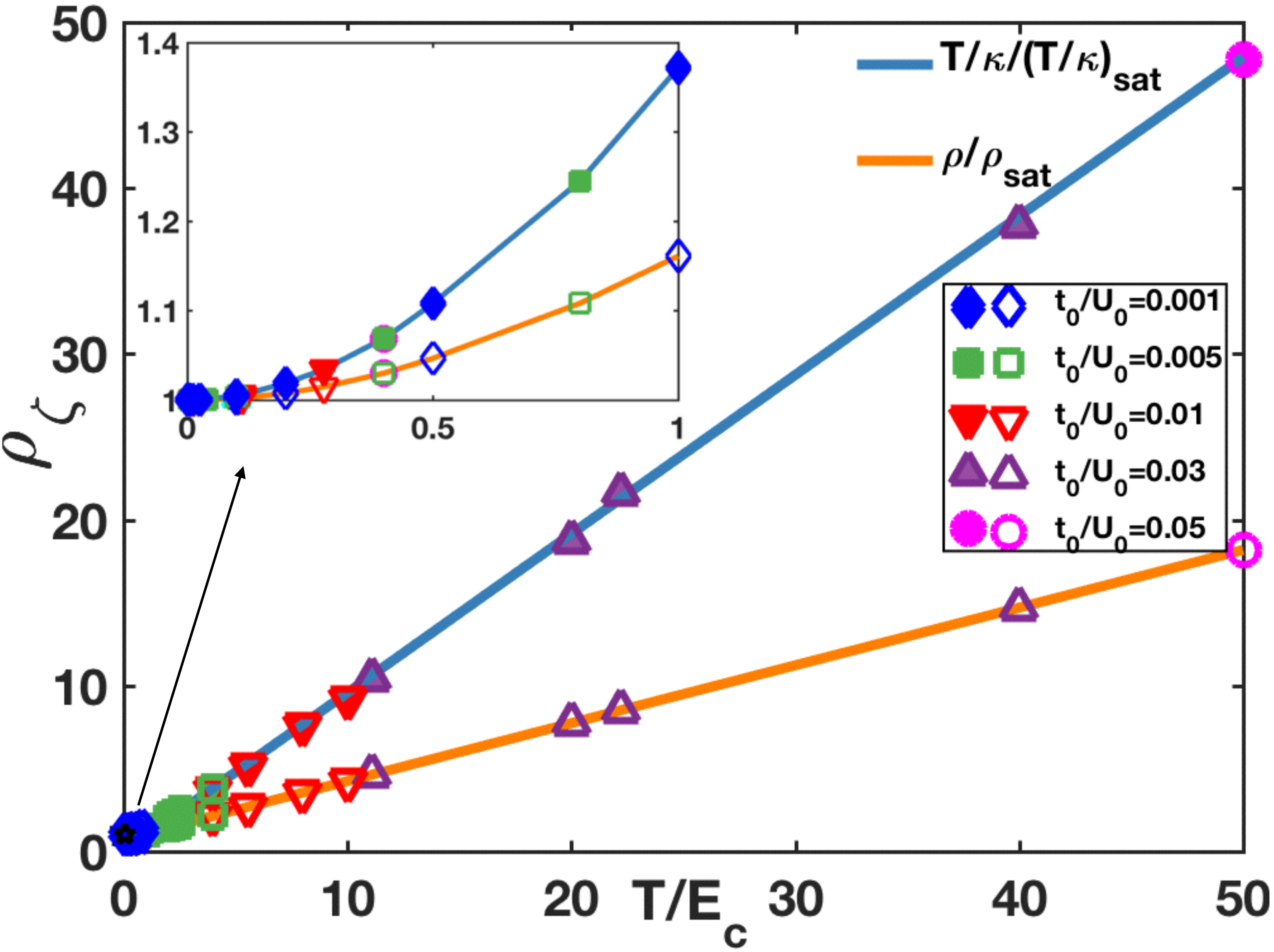
Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 021049 (2018)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

Coupled SYK Islands

Low 'coherence' scale

$$E_c \sim \frac{t_0^2}{U}$$



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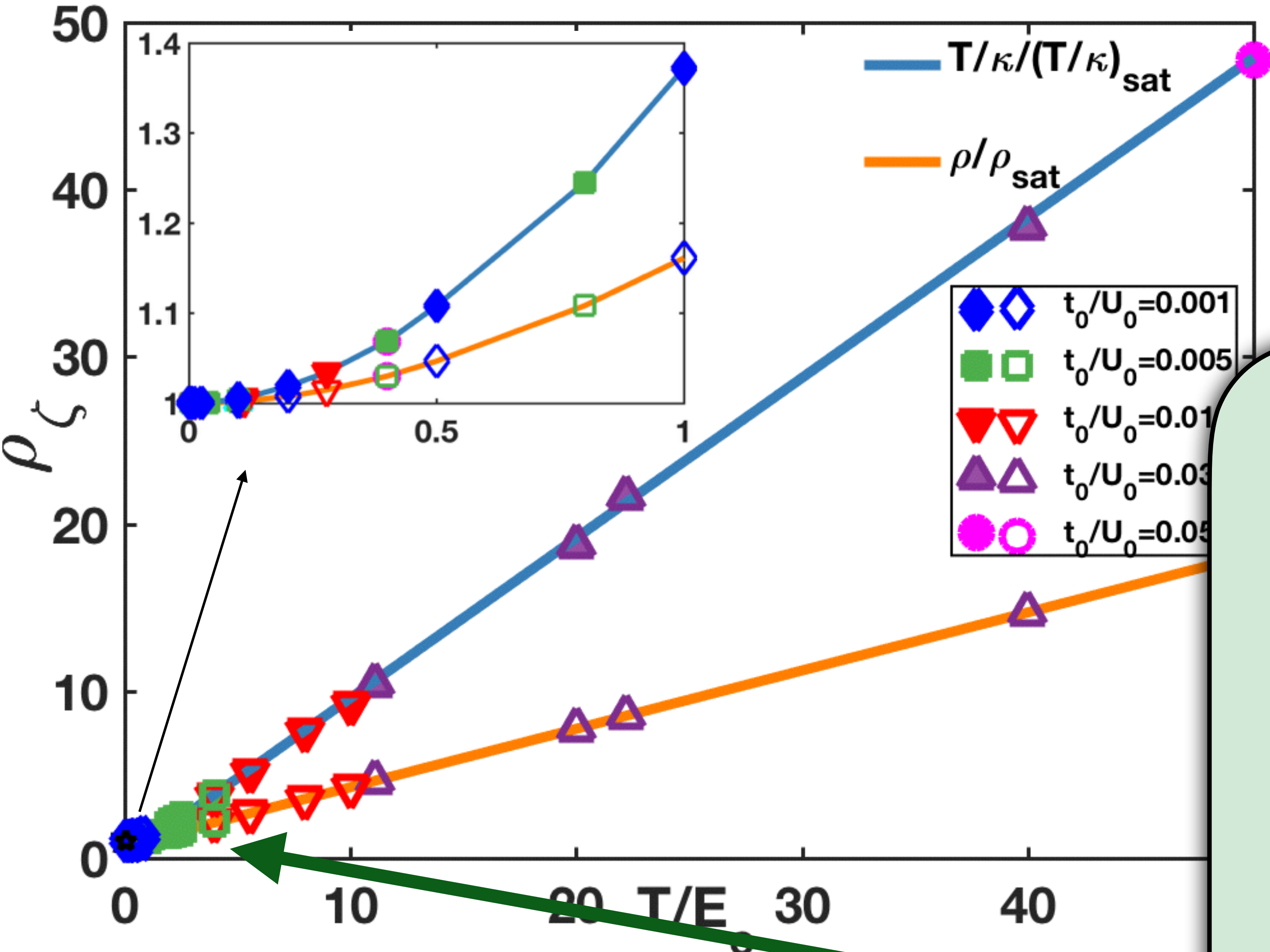

For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

$$s \sim s_0\left(\frac{T}{E_c}\right)$$

Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

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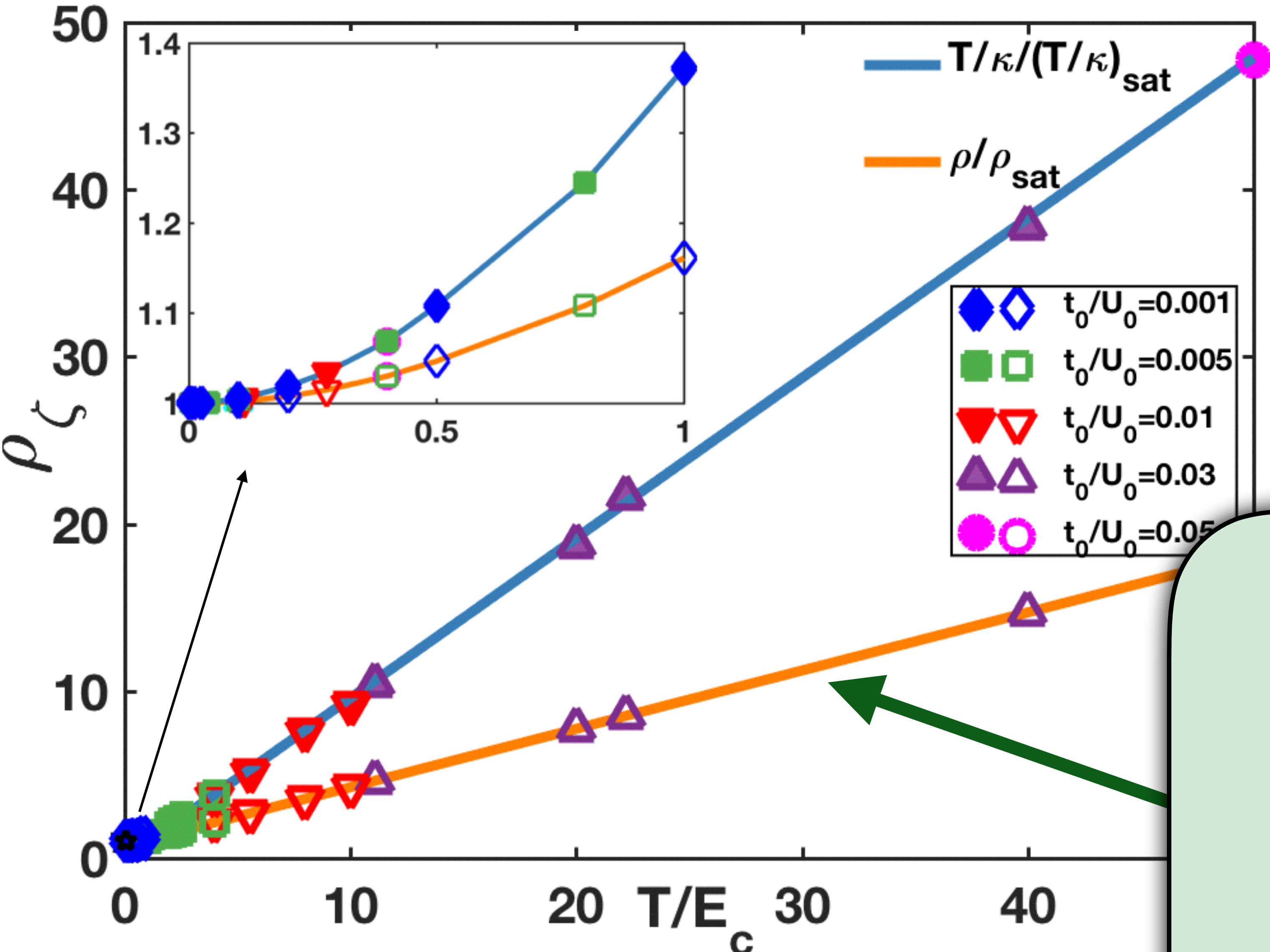
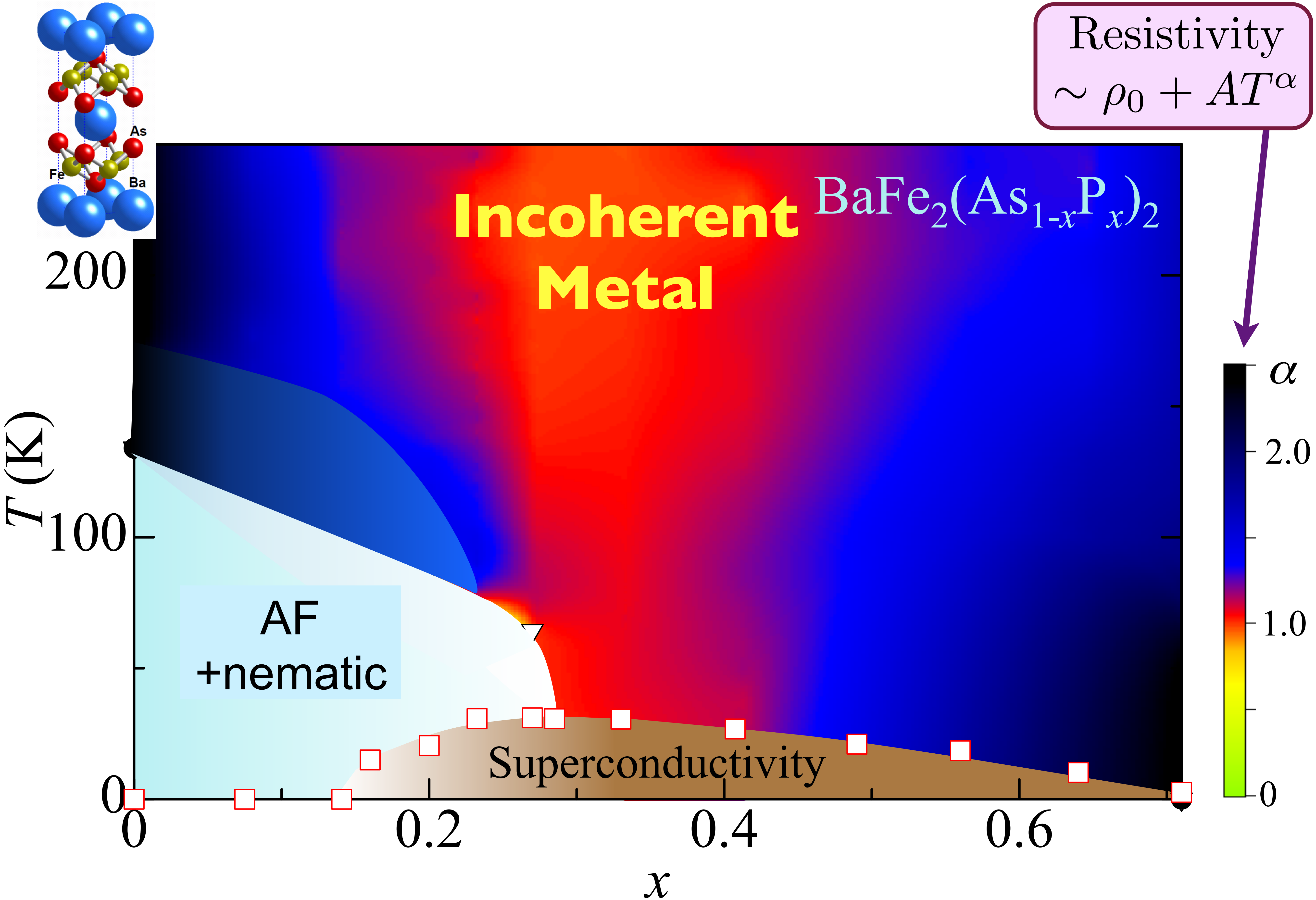


For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

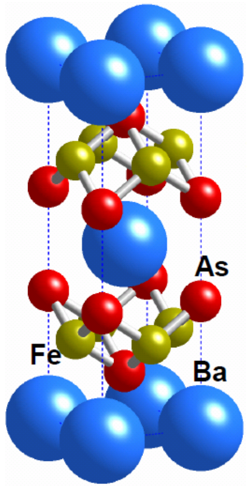
$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right) \quad , \qquad s = s_0$$

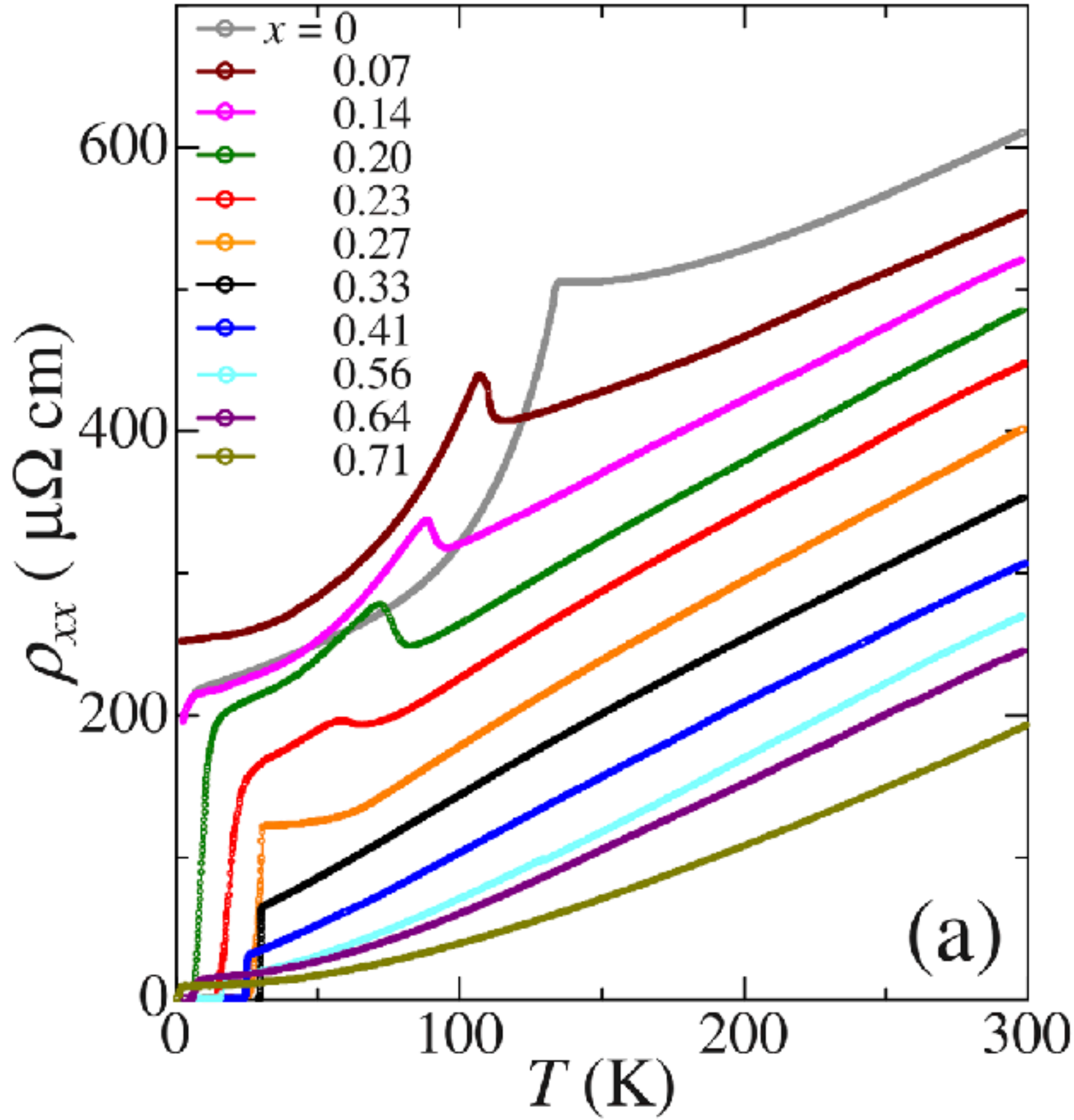
Xue-Yang Song, Chao-Ming Jian, and L. Balents, PRL **119**, 216601 (2017)

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)





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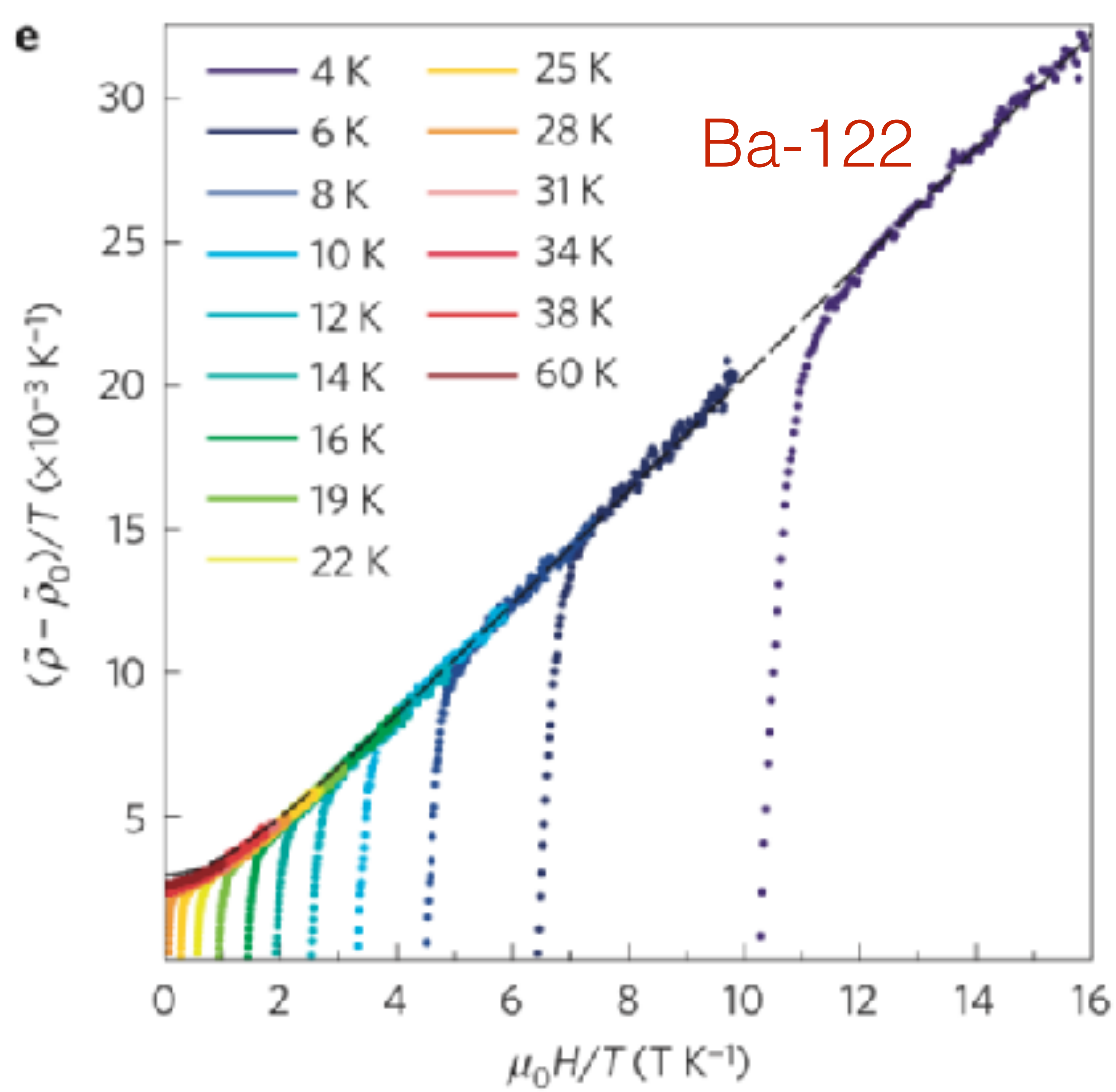
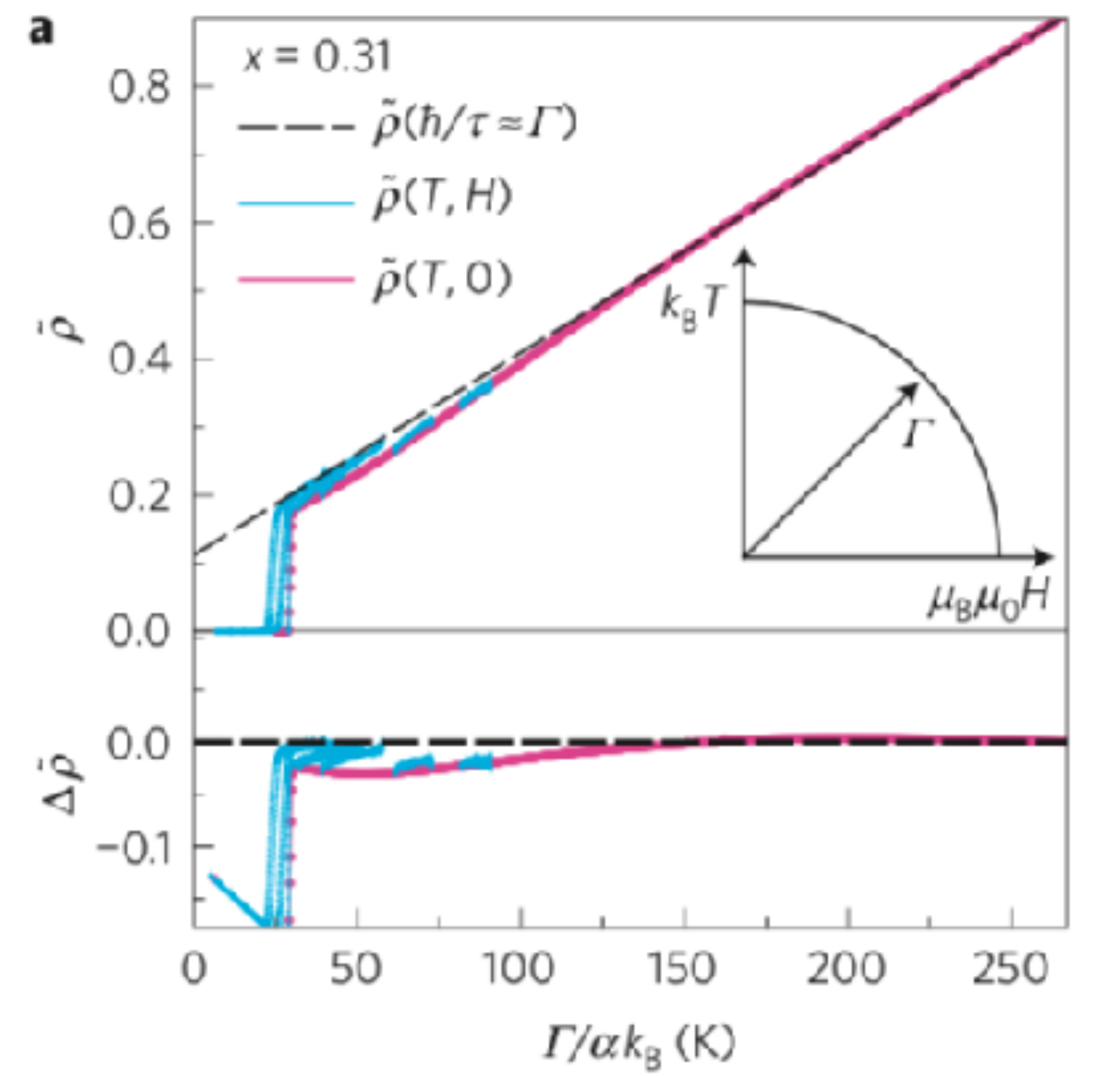
Linear-in-B magnetoresistance with B/T scaling



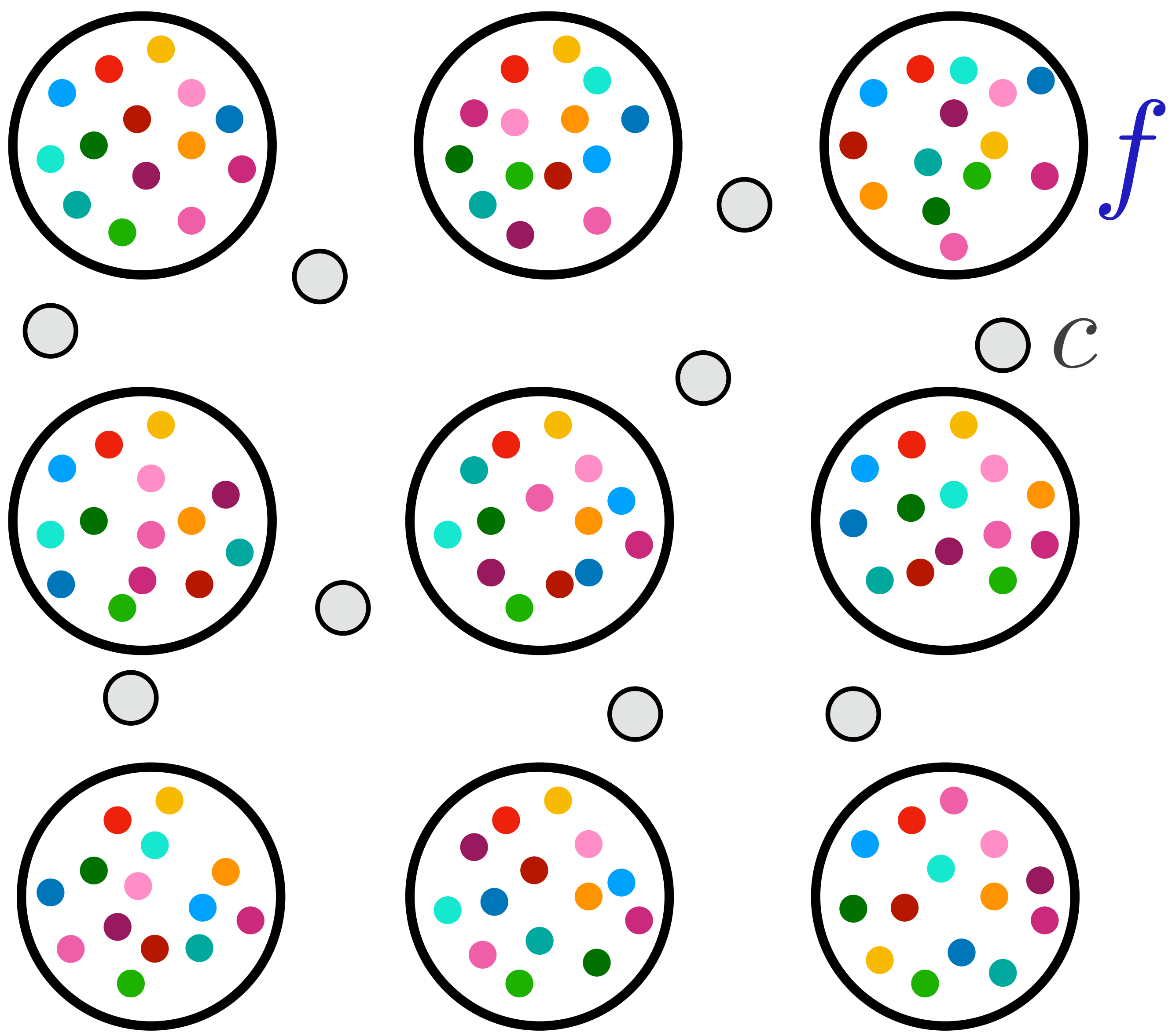


$$\rho(H,T) - \rho(0,0) \propto \sqrt{(\alpha k_B T)^2 + \left(\gamma \mu_B \mu_0 H\right)^2} \equiv \Gamma$$

I. M. Hayes, R. D. McDonald, N. P. Breznay, T. Helm, P. J. W. Moll, M. Wartenbe, A. Shekhter, and J. G. Analytis, Nature Physics 12, 916 (2016)

SYK-Kondo lattice models

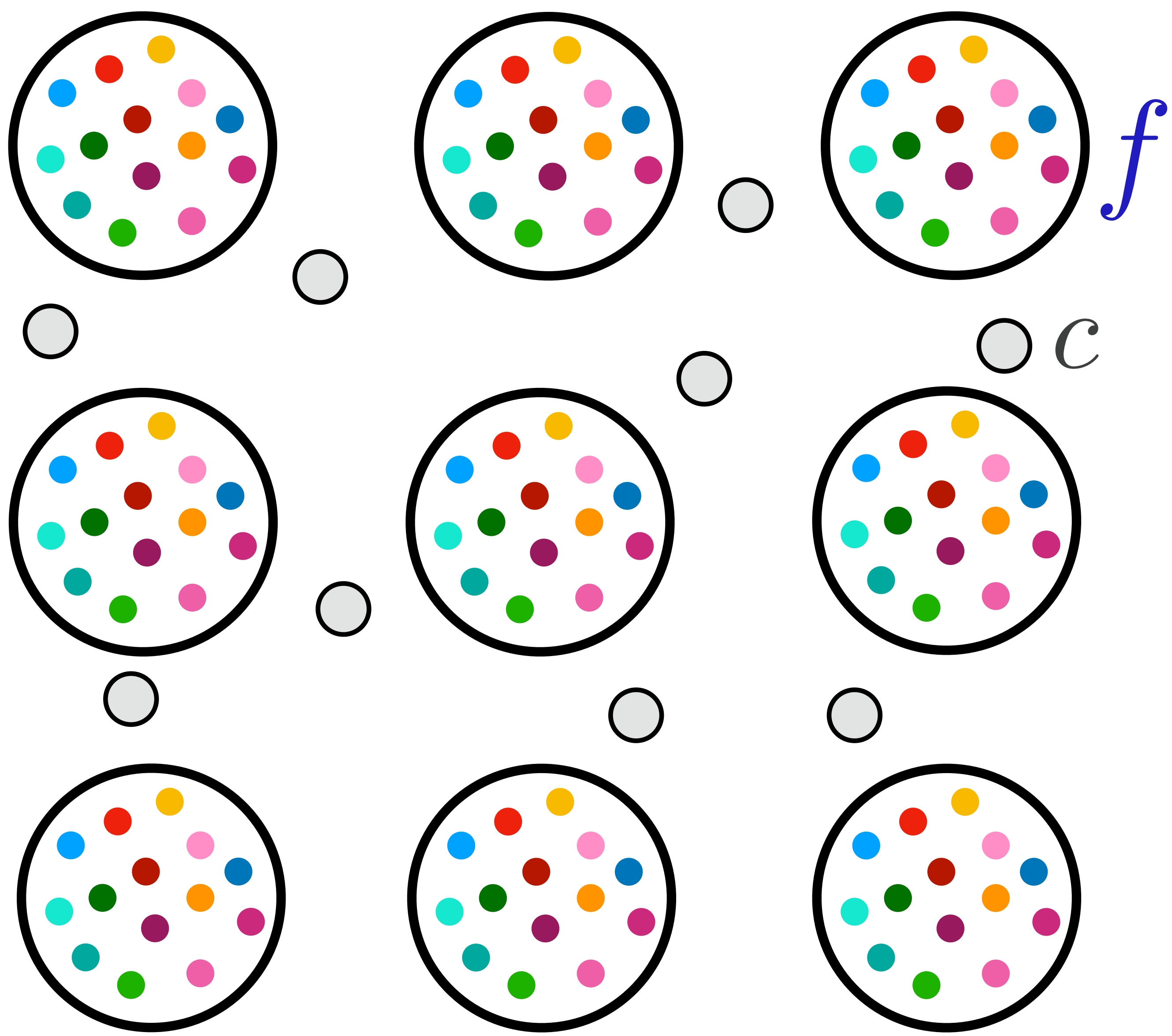
Mobile electrons (c) interacting with SYK quantum islands (f) with random exchange interactions.



Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

SYK-Kondo lattice models

Mobile electrons (c) interacting with SYK quantum islands (f) with non-random exchange interactions.



Debanjan Chowdhury, Yochai Werman, Erez Berg, T. Senthil, PRX **8**, 031024 (2018)

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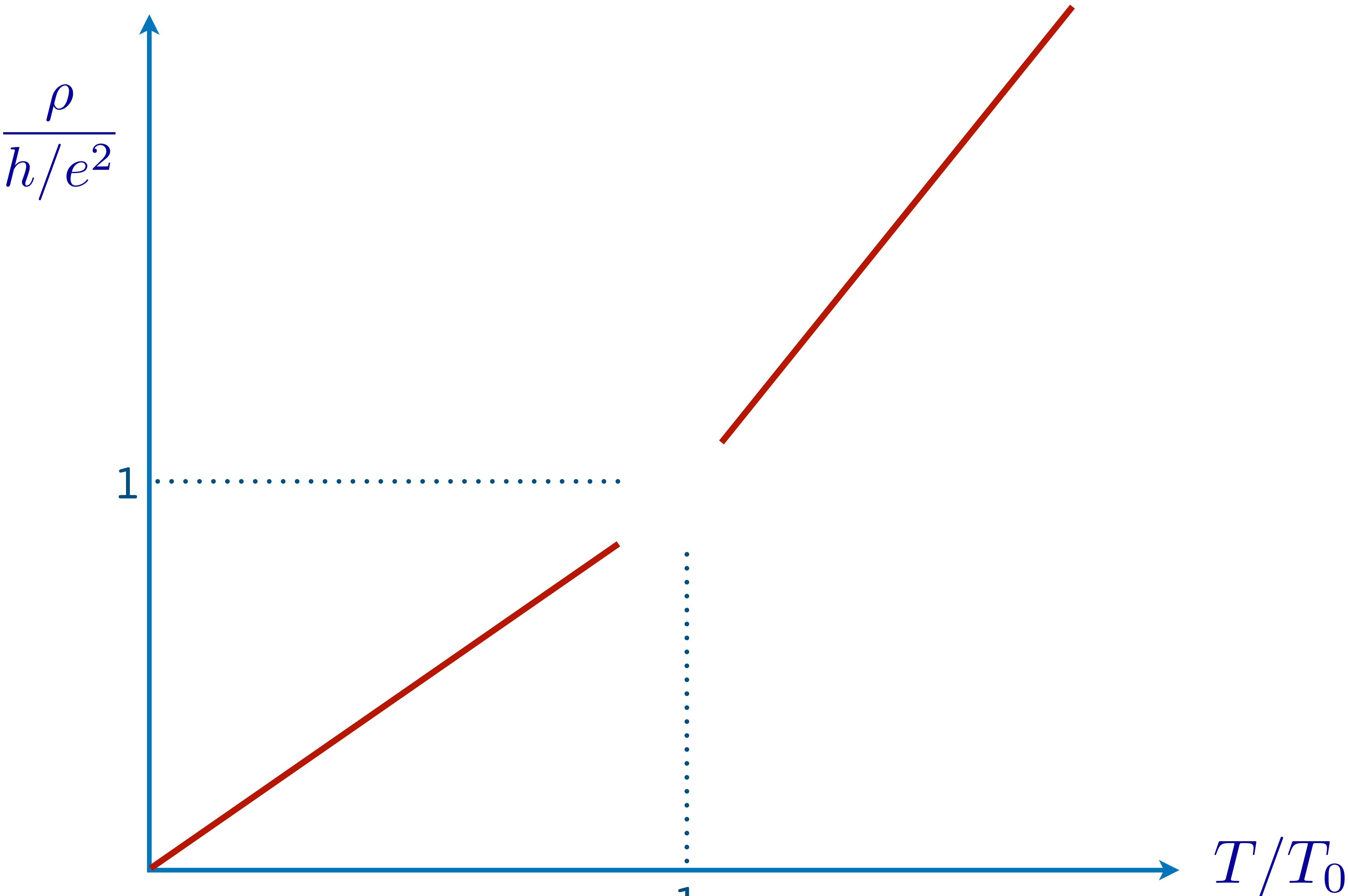

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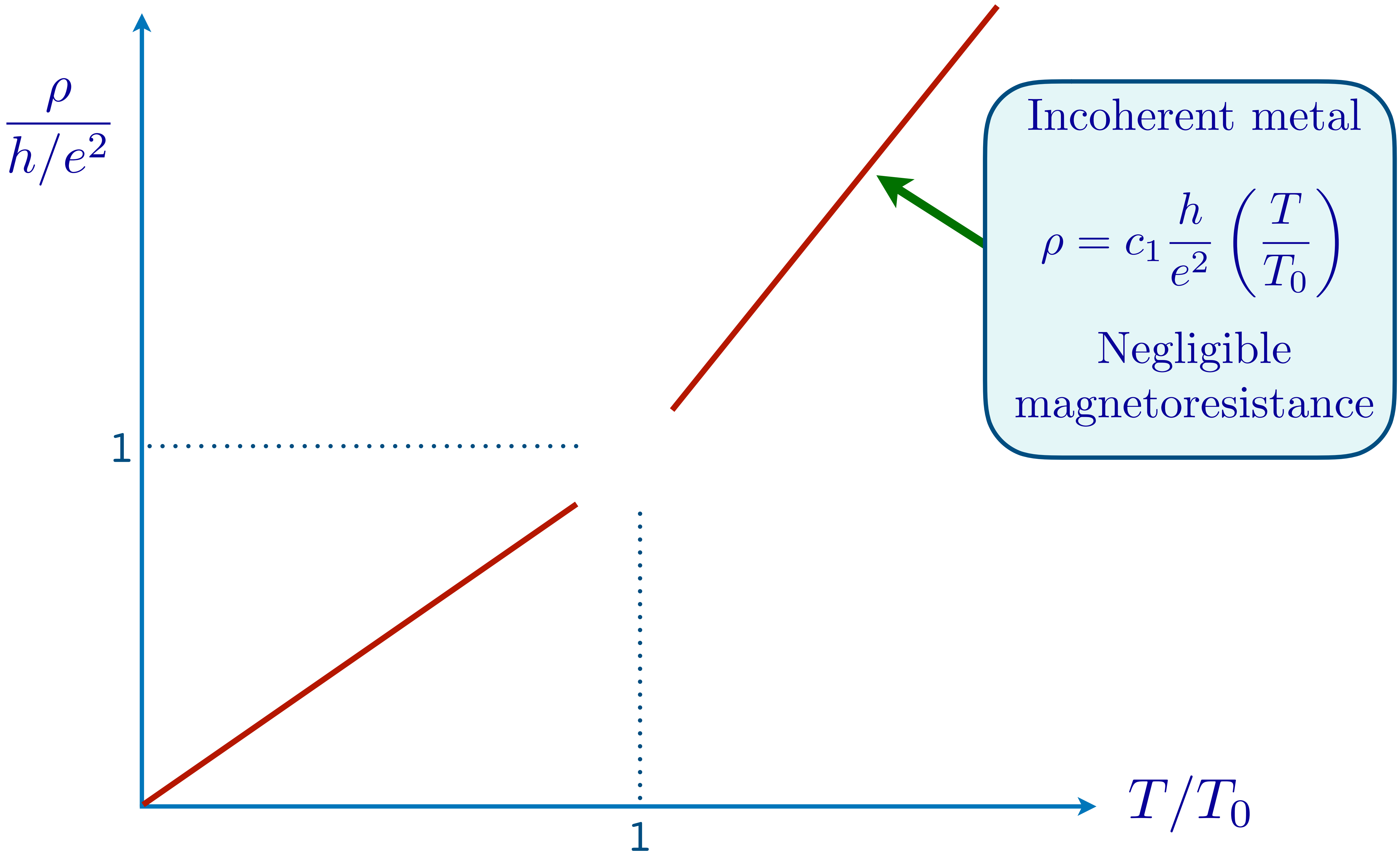

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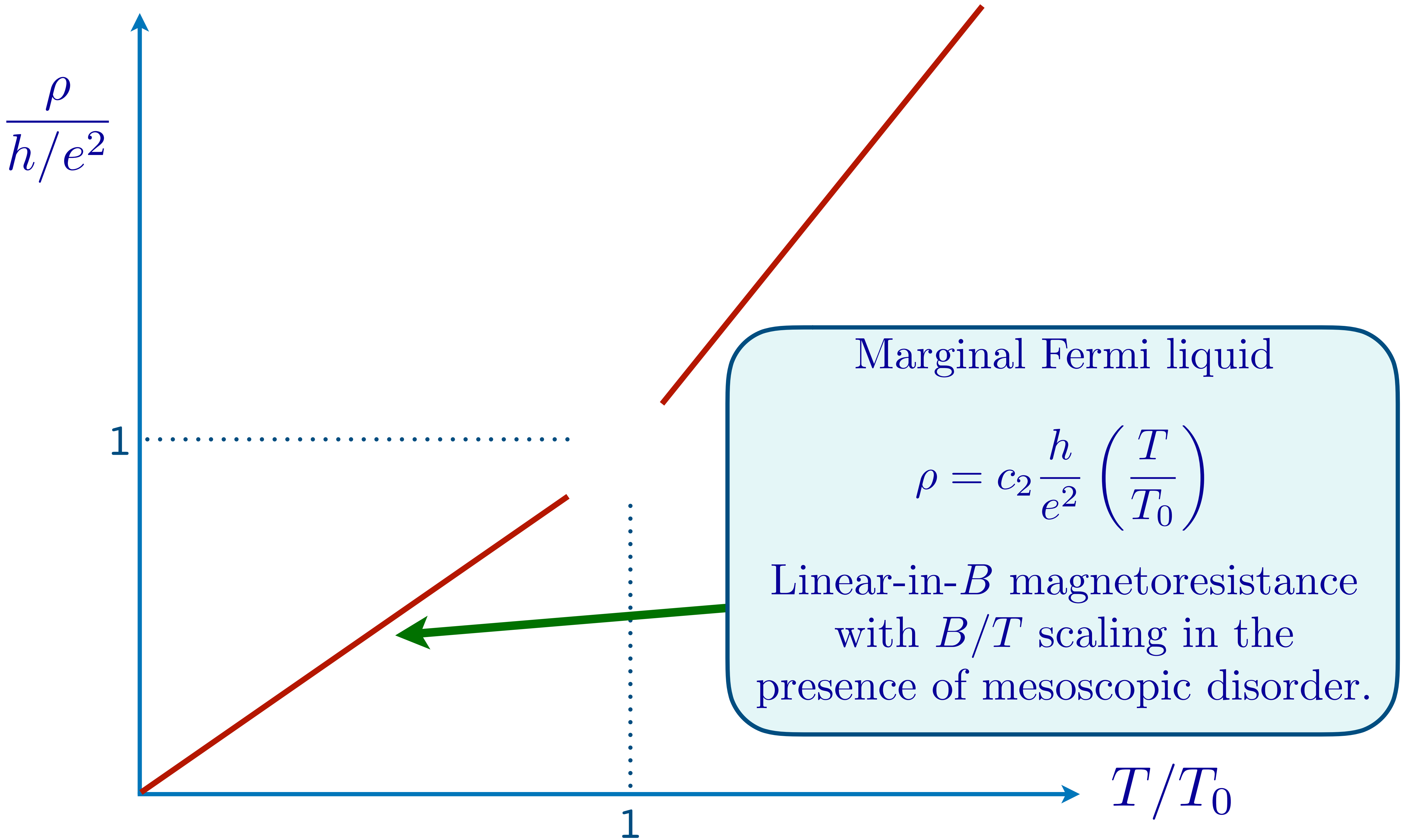

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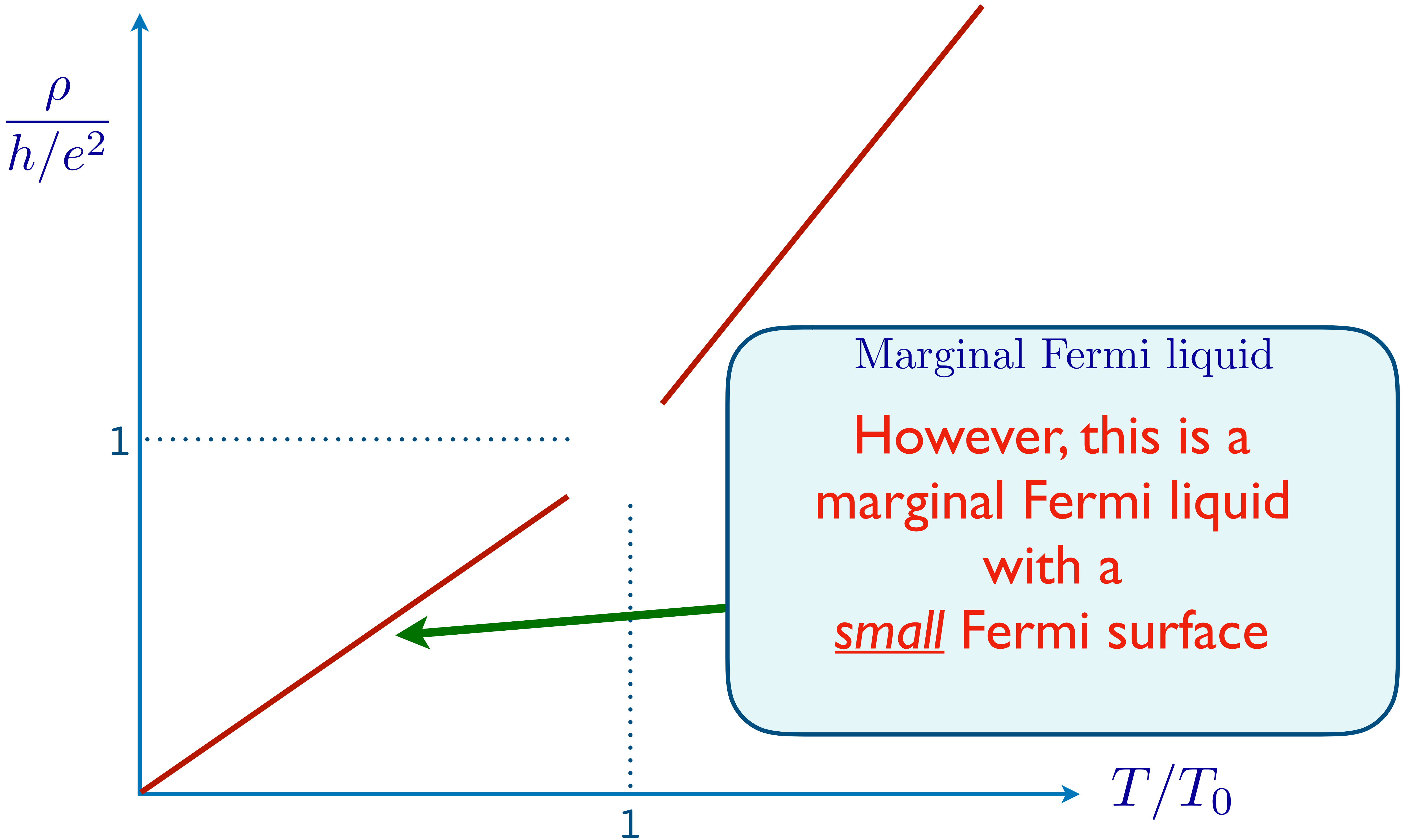

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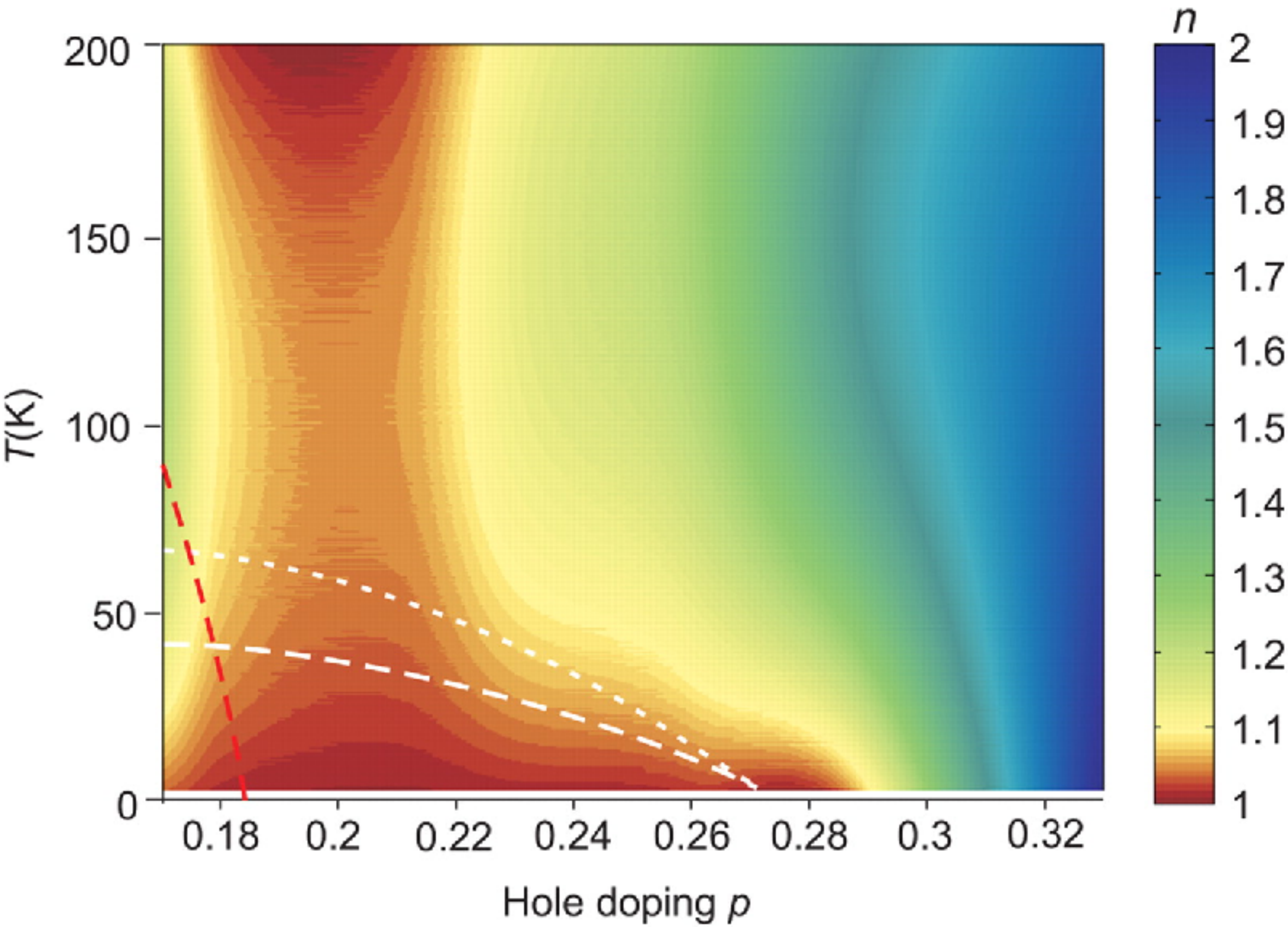
Aavishkar A. Patel, John McGreevy, Daniel P. Arovas, Subir Sachdev, PRX **8**, 021049 (2018)

SCIENCE VOL 323 30 JANUARY 2009
603

Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$

R. A. Cooper,[1] Y. Wang,[1] B. Vignolle,[2] O. J. Lipscombe,[1] S. M. Hayden,[1] Y. Tanabe,[3] T. Adachi,[3] Y. Koike,[3] M. Nohara,[4]* H. Takagi,[4] Cyril Proust,[2] N. E. Hussey[1]†



Universal *T*-linear resistivity and Planckian limit in overdoped cuprates

arXiv:1805.02512

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1],

B. Vignolle[3], D. Vignolles[3], H. Raffy[5], Z. Z. Li[5], P. Auban-Senzier[5],

N. Doiron-Leyraud[1], P. Fournier[1,6], D. Colson[2], L. Taillefer[1,6], and C. Proust[3,6]

From the resistivity, they determined the value of the number α defined by

$$\rho(T) = \rho_0 + \alpha \frac{h}{2e^2} \left(\frac{T}{T_F} \right)$$

where $T_F = (\pi\hbar^2/k_B)(n/m^*)$ and m^* is determined from the specific heat. This expression is obtained from the Drude form $\rho = m^*/(ne^2\tau)$ and $\hbar/\tau = \alpha\, k_B T$.

Universal *T*-linear resistivity and Planckian limit in overdoped cuprates

arXiv:1805.02512

A. Legros[1,2], S. Benhabib[3], W. Tabis[3,4], F. Laliberté[1], M. Dion[1], M. Lizaire[1],

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Material		n (10^{27} m^{-3})	m^* (m_0)	A_1 / d (Ω / K)	$h / (2e^2 T_F)$ (Ω / K)	α
Bi2212	$p = 0.23$	6.8	8.4 ± 1.6	8.0 ± 0.9	7.4 ± 1.4	1.1 ± 0.3
Bi2201	$p \sim 0.4$	3.5	7 ± 1.5	8 ± 2	8 ± 2	1.0 ± 0.4
LSCO	$p = 0.26$	7.8	9.8 ± 1.7	8.2 ± 1.0	8.9 ± 1.8	0.9 ± 0.3
Nd-LSCO	$p = 0.24$	7.9	12 ± 4	7.4 ± 0.8	10.6 ± 3.7	0.7 ± 0.4
PCCO	$x = 0.17$	8.8	2.4 ± 0.1	1.7 ± 0.3	2.1 ± 0.1	0.8 ± 0.2
LCCO	$x = 0.15$	9.0	3.0 ± 0.3	3.0 ± 0.45	2.6 ± 0.3	1.2 ± 0.3
TMTSF	$P = 11$ kbar	1.4	1.15 ± 0.2	2.8 ± 0.3	2.8 ± 0.4	1.0 ± 0.3

Slope of *T*-linear resistivity vs Planckian limit in seven materials.

Assessment of SYK lattice models

- Reasonable model of incoherent metal behavior for $t^2/U < T < t$.

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- What is the origin of the "foot" at overdoping ??

1. Solvable model without quasiparticles
   SYK model of a `quantum island'

2. Lattice models of SYK islands
   Theories of strange metals

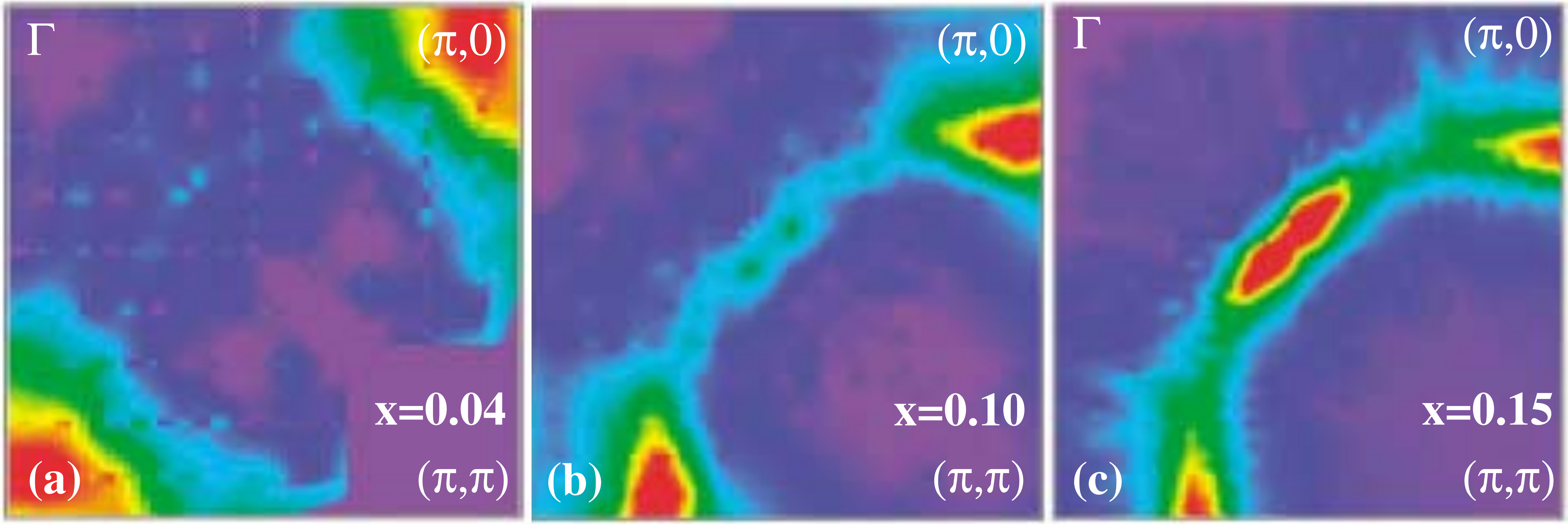
3. SYK U(1) gauge theory
   Solvable model with finite density of fermions, emergent gauge fields, and disorder

Electronic spectrum in pseudogap metal is well described by the Higgs phase of a SU(2) gauge theory

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero,
PRX **8**, 021048 (2018)

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev,
PNAS **115**, E3665 (2018)

$Nd_{2-x}Ce_xCuO_{4\pm\delta}$



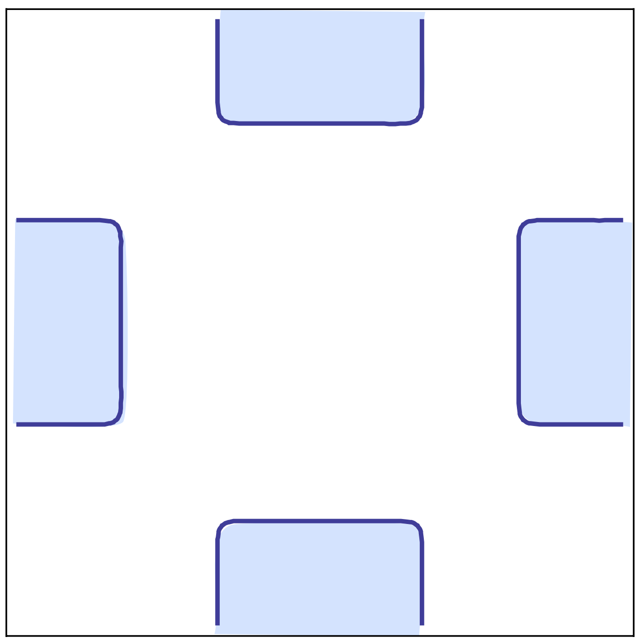
Doping Dependence of an n-Type Cuprate Superconductor Investigated by Angle-Resolved Photoemission Spectroscopy

N. P. Armitage, F. Ronning, D. H. Lu, C. Kim, A. Damascelli, K. M. Shen, D. L. Feng, H. Eisaki, Z.-X. Shen, P. K. Mang, N. Kaneko, M. Greven, Y. Onose, Y. Taguchi, and Y. Tokura

Phys. Rev. Lett. **88**, 257001 (2002)

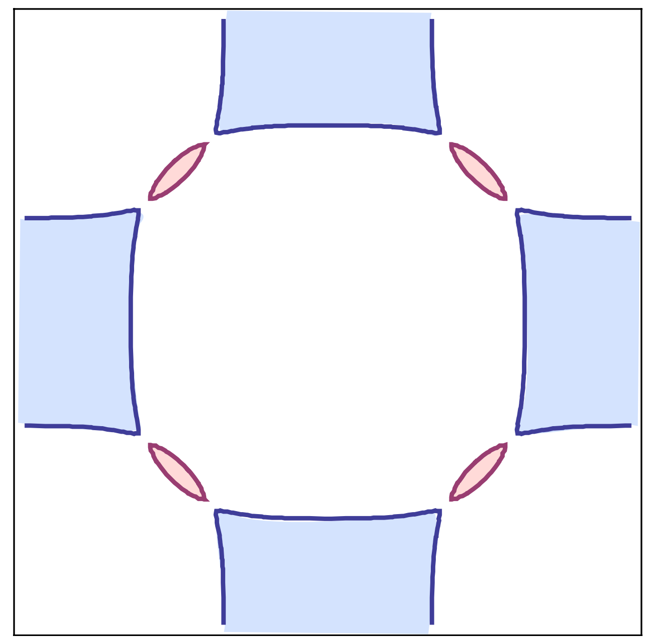
Square lattice Hubbard model with electron doping

$\langle \vec{\Phi} \rangle \neq 0$ and large



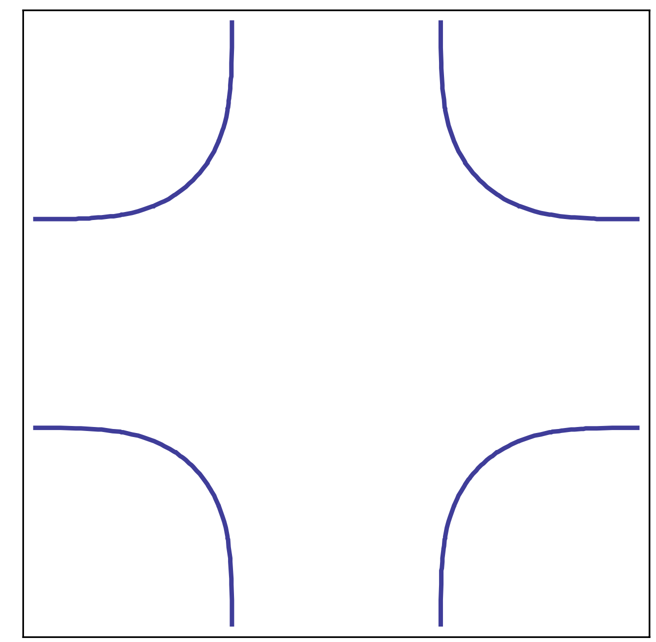
Metal with electron pockets

$\langle \vec{\Phi} \rangle \neq 0$ and small



Metal with electron and hole pockets

$\langle \vec{\Phi} \rangle = 0$



Metal with "large" Fermi surface

s

Fermi surface reconstruction in electron-doped cuprates without antiferromagnetic long-range order

J.-F. He, C. R. Rotundu, M. S. Scheurer, Y. He, M. Hashimoto, K. Xu, Y. Wang, E. W. Huang, T. Jia, S.-D. Chen, B. Moritz, D.-H. Lu, Y. S. Lee, T. P. Devereaux and Z.-X. Shen

Fermi surface (FS) topology is a fundamental property of metals and superconductors. In electron- doped cuprate Nd2-xCexCuO4 (NCCO), an unexpected FS reconstruction has been observed in optimal- and over-doped regime (x=0.15-0.17) by quantum oscillation measurements (QOM). This is all the more puzzling because neutron scattering suggests that the antiferromagnetic (AFM) long-range order, which is believed to reconstruct the FS, vanishes before x=0.14. Then, a widely discussed external magnetic field-induced AFM long-range order in QOM explains the FS reconstruction as an extrinsic property. Here, we report angle-resolved photoemission (ARPES) evidence of FS reconstruction in optimal- and over-doped NCCO.

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The observed FSs are in quantitative agreement with quantum oscillation measurements, suggesting an intrinsic FS reconstruction without (magnetic) field. This reconstructed FS cannot be explained under the traditional scheme (of reconstruction by antiferromagnetism). Furthermore, the energy gap of the reconstruction collapses near x=0.17 like an order parameter, echoing the quantum critical doping in transport. The totality of the data points to a mysterious order between x=0.14 and 0.17, whose appearance favors the FS reconstruction and disappearance defines the quantum critical doping. A recent topological proposal provides an ansatz for its origin.

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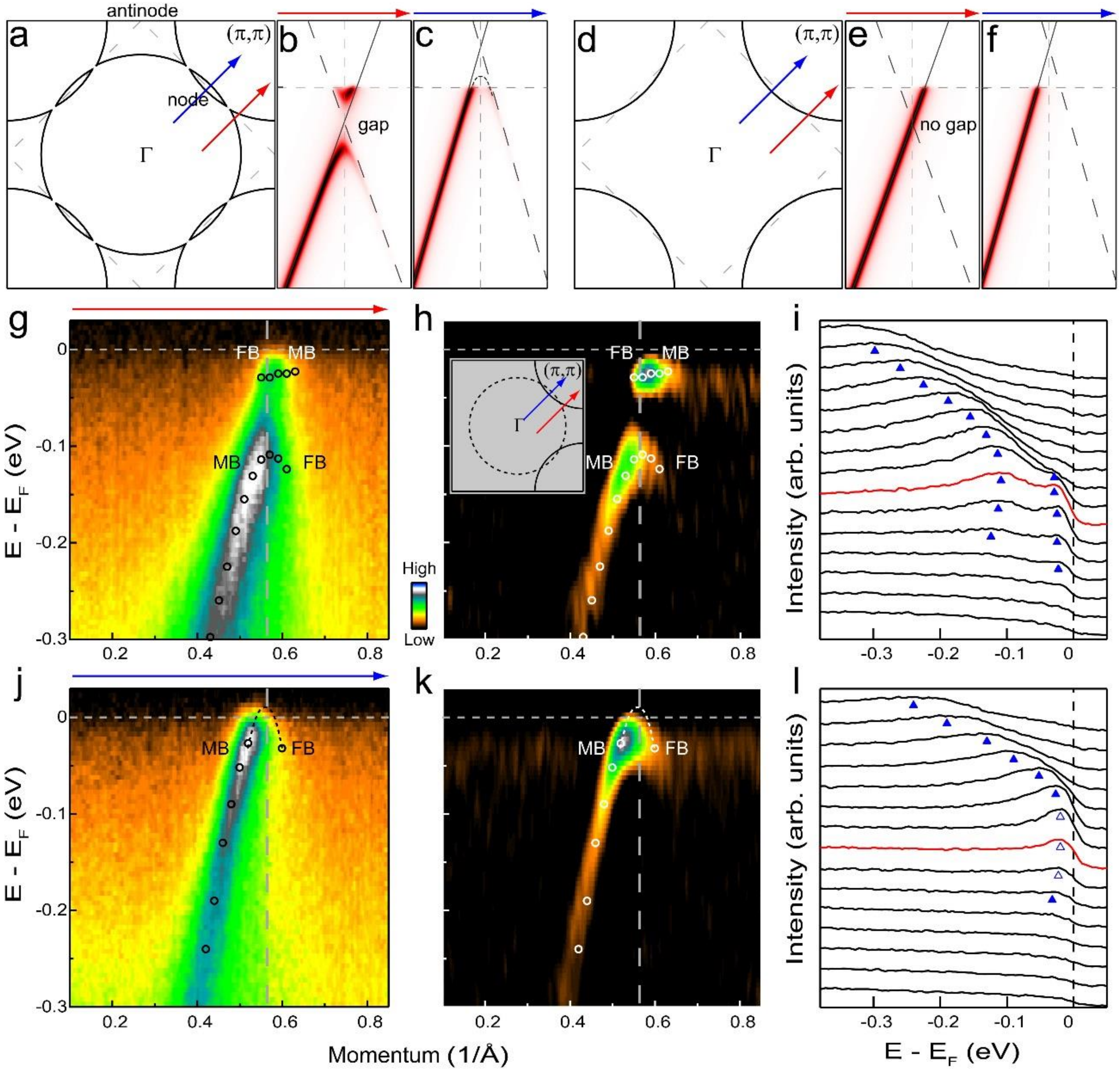
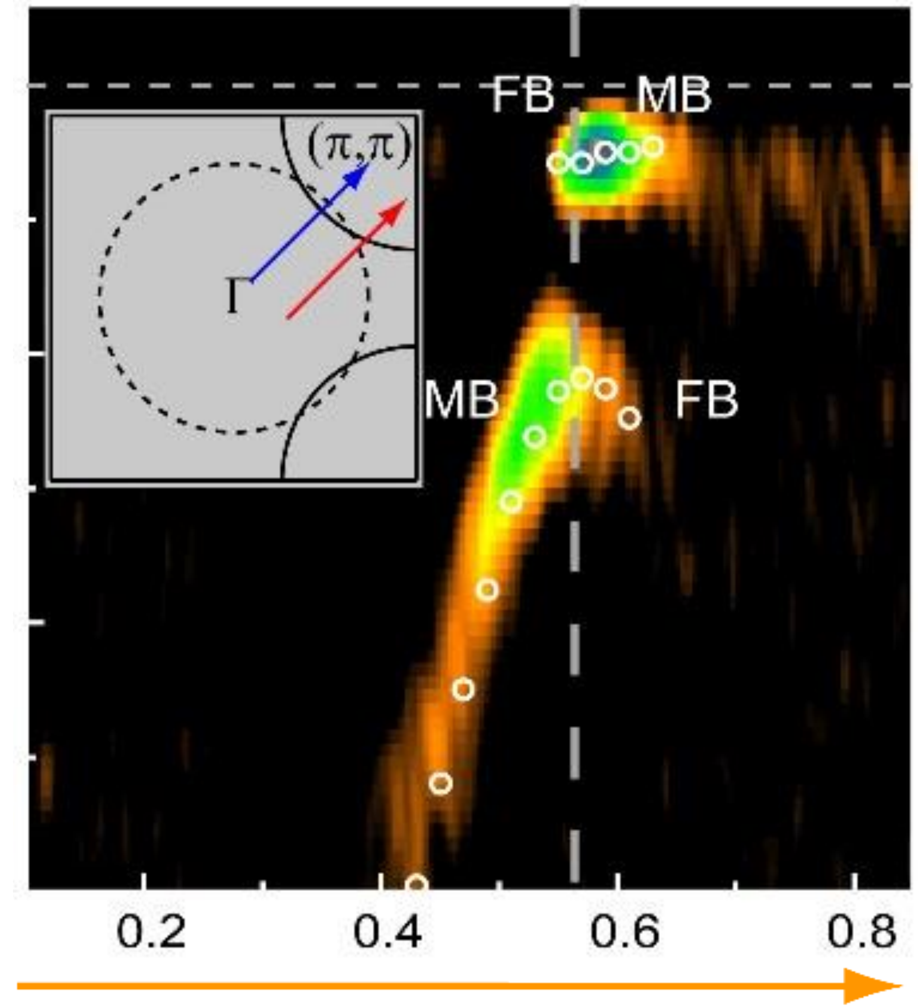
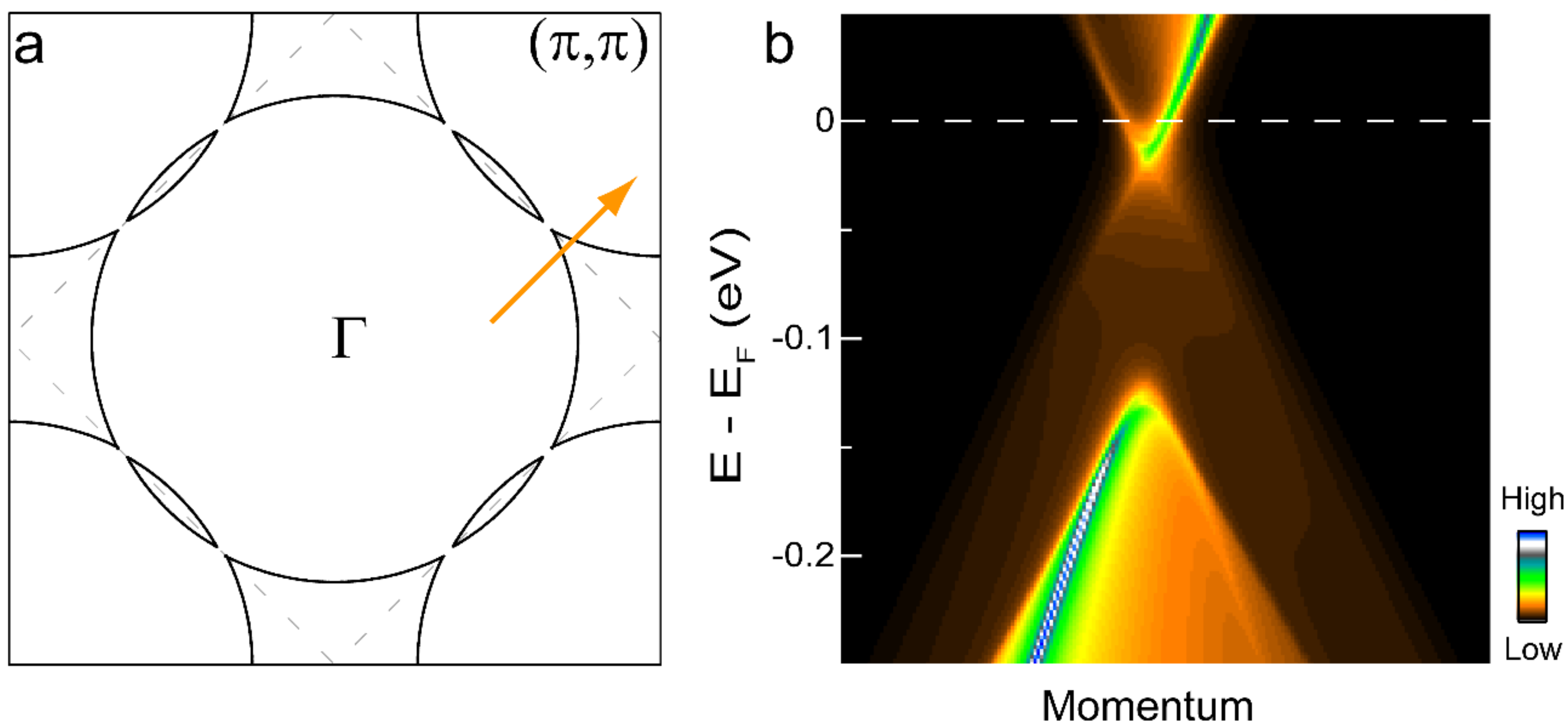


antinode
a
b
c
d
e
f
(π,π)
node
Γ
gap
no gap
g
h
i
j
k
l
FB
MB
E - E_F (eV)
Intensity (arb. units)
High
Low
Momentum (1/Å)
E - E_F (eV)

S. Sachdev, Topological order and Fermi surface reconstruction, arXiv:1801.01125

M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, Proceedings of the National Academy of Sciences **115**, E3665 (2018)





Electronic spectrum in pseudogap metal is well described by the Higgs phase of a SU(2) gauge theory

Wei Wu, M. S. Scheurer, S. Chatterjee, S. Sachdev, A. Georges, and M. Ferrero, PRX **8**, 021048 (2018)

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M. S. Scheurer, S. Chatterjee, Wei Wu, M. Ferrero, A. Georges, and S. Sachdev, PNAS **115**, E3665 (2018)

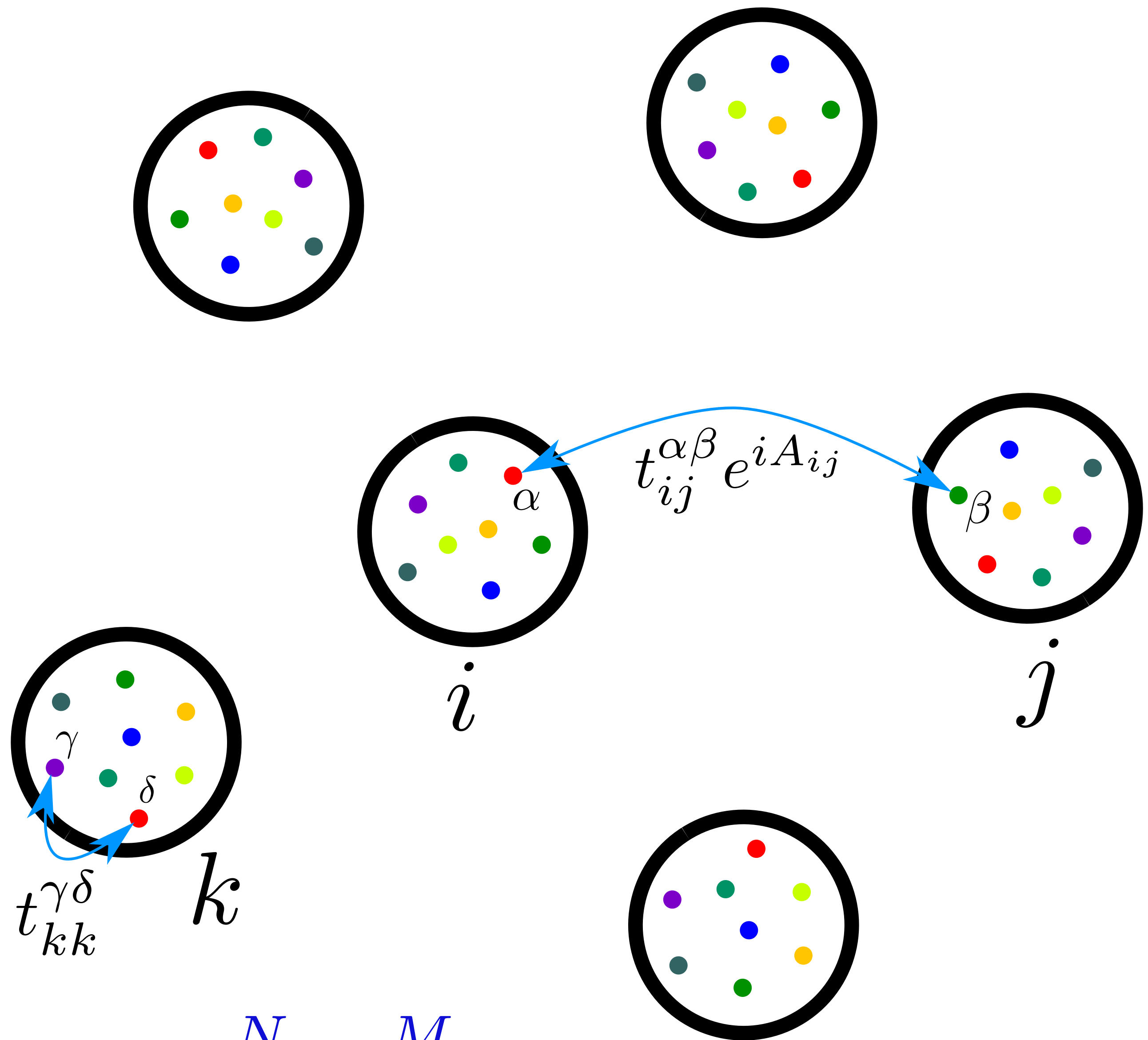
Optimal doping critical point is associated with vanishing of the Higgs condensate. Overdoped regime is described by (a large Fermi surface of) electrically-charged fermions coupled to an emergent SU(2) gauge field in the presence of disorder

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

Fermions with random hopping coupled to a fluctuating U(1) gauge field

Aavishkar Patel



$$H = -\frac{1}{(MN)^{1/2}} \sum_{ij=1}^{N} \sum_{\alpha\beta=1}^{M} \left[t_{ij}^{\alpha\beta} e^{iA_{ij}} f_{i\alpha}^{\dagger} f_{j\beta} + (MN)^{1/2} \mu \delta_{ij}^{\alpha\beta} f_{i\alpha}^{\dagger} f_{i\alpha} \right]$$

$$\ll t_{ij}^{\alpha\beta} t_{ji}^{\beta\alpha} \gg = \ll |t_{ij}^{\alpha\beta}|^2 \gg = t^2, \quad A_{ji} = -A_{ij}.$$

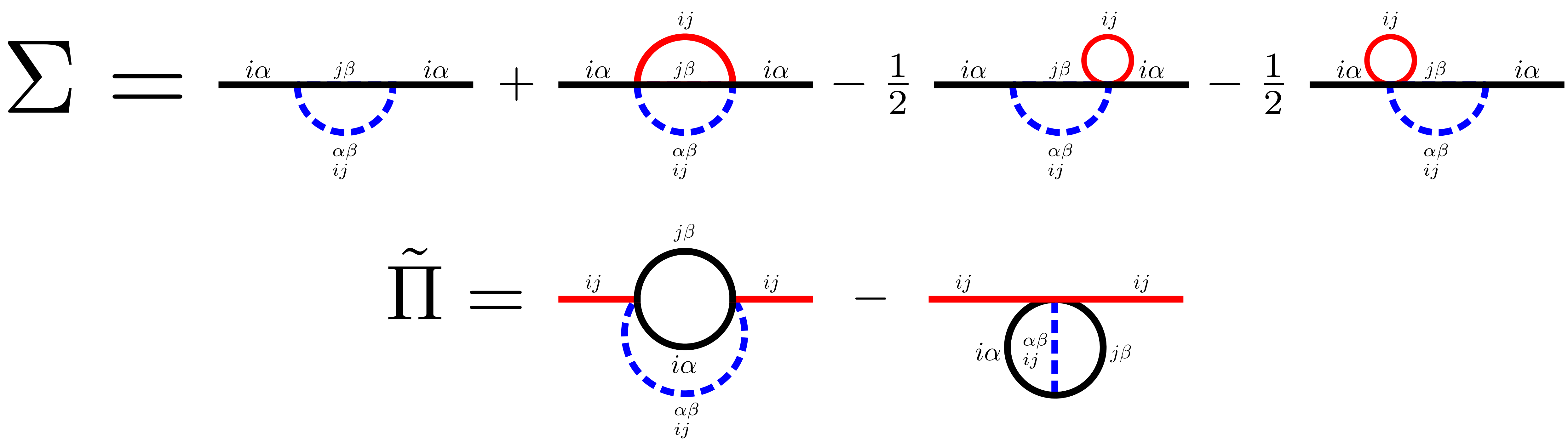
arXiv:1807.04754

Fermions with random hopping coupled to a fluctuating U(1) gauge field


Aavishkar Patel

$$\Sigma(i\omega_n) = t^2 G(i\omega_n) + t^2 T \sum_{\Omega_m \neq 0} \frac{G(i\omega_n + i\Omega_m) - G(i\omega_n)}{\Pi(i\Omega_m) - \Pi(i\Omega_m = 0)},$$

$$\Pi(i\Omega_m) = 2t^2 T \frac{M}{N} \sum_{\omega_n} G(i\omega_n) G(i\omega_n + i\Omega_m), \quad G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)}.$$



arXiv:1807.04754

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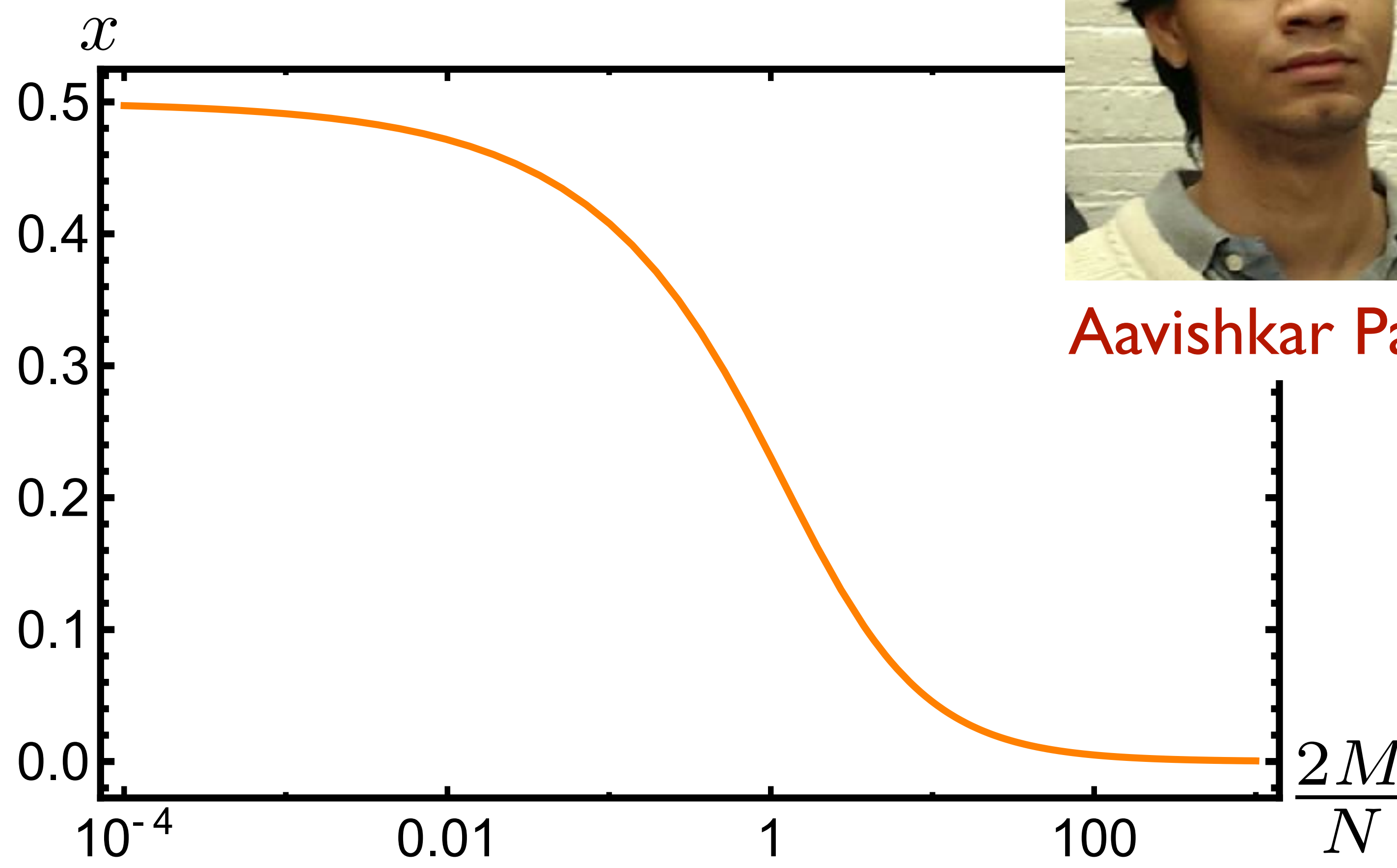


Aavishkar Patel

General low energy solution

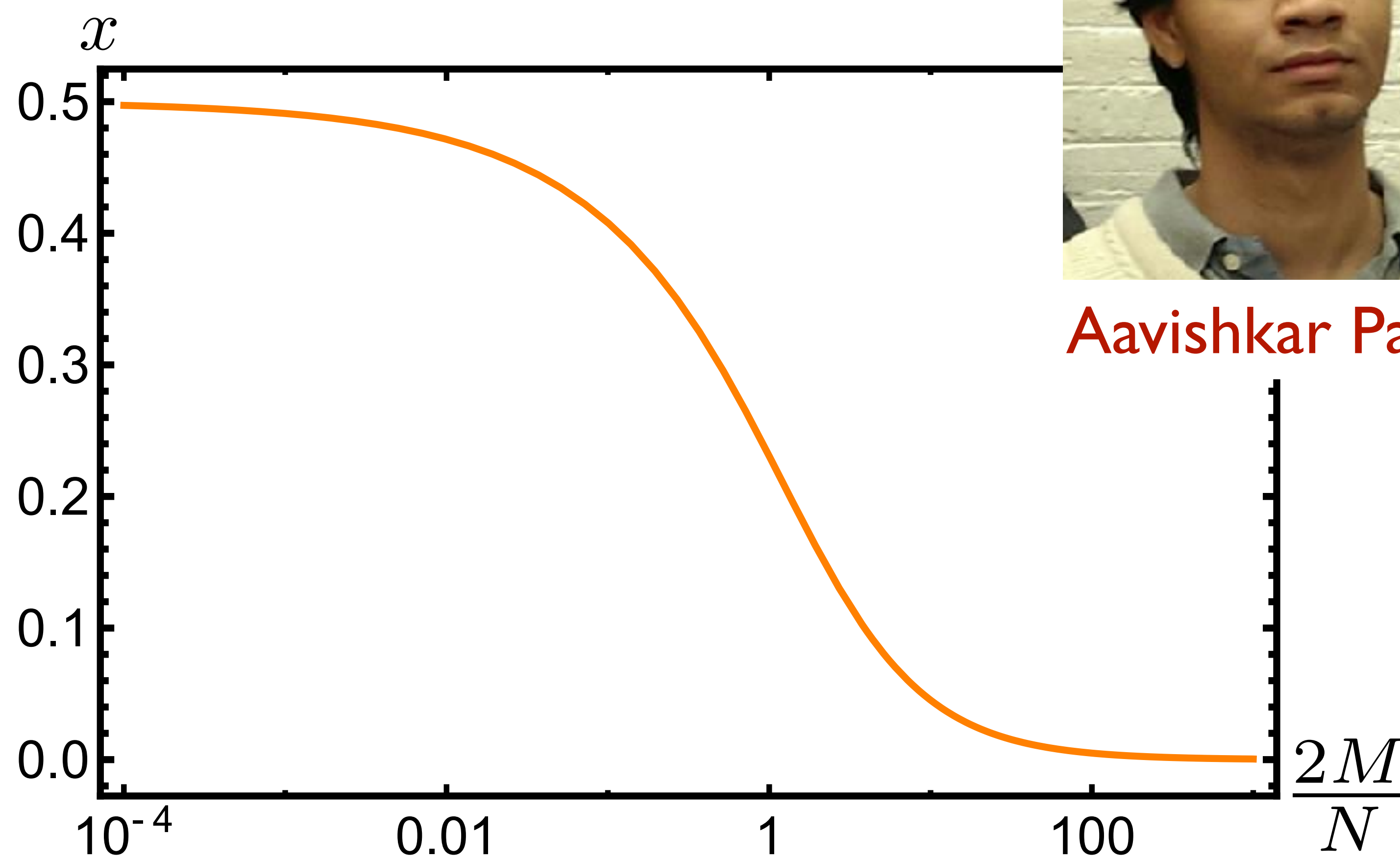
$$G(\tau > 0) = -\frac{C(\mathcal{E})}{t^{1-x}\tau^{1-x}}, \quad G(\tau < 0) = \frac{C(\mathcal{E})e^{-2\pi\mathcal{E}}}{t^{1-x}|\tau|^{1-x}}.$$

where $\mathcal{E}$ is a parameter universally related to the filling fraction ($\mathcal{E} = 0$ at half-filling). The exponent x is the solution to

$$\frac{(1/x - 2)(\cosh(2\pi\mathcal{E}) - \cos(\pi x))}{\tan(\pi x)\sin(\pi x)} = \frac{2M}{N}.$$

arXiv:1807.04754

Fermions with random hopping coupled to a fluctuating U(1) gauge field



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$$\text{Resistivity} \quad \rho \sim \frac{h}{e^2}\left(\frac{T}{t}\right)^{2x}$$

Disordered strange metal as $T \to 0$
with *all* electrons contributing to transport.

arXiv:1807.04754

The SYK model

No quasiparticles

- Rapid local thermal equilibration (of fermion correlators) in a 'Planckian' time

$$\tau_{\rm eq} \sim \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

A. Georges and O. Parcollet PRB **59**, 5341 (1999)
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, PRB **96**, 205123 (2017)

- Presence of quasiparticles should slow down thermalization, so *all* quantum systems obey

$$\tau_{\rm eq} > C \frac{\hbar}{k_B T} \quad , \quad \text{as } T \to 0.$$

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

Absence of quasiparticles $\Leftrightarrow$ Fastest possible thermalization

Conclusions

- Solvable model without quasiparticles: SYK model of a 'quantum island'

- Lattice models of SYK islands: Bad metal behavior with $\rho \sim (T/E_c)(h/e^2)$ for $T > E_c$, and Fermi liquid behavior for $T < E_c$.

- SYK-Kondo lattice models: Bad metal behavior with $\rho \sim (T/T_0)(h/e^2)$ for $T > T_0$, and marginal Fermi liquid (MFL) behavior for $T < T_0$ with $\rho \sim (T/T_0)(h/e^2)$. MFL regime has *small* Fermi surface, and magnetoresistance B/T scaling (with mesoscopic disorder).

- SYK U(1) gauge theory: solvable model with finite density of fermions, emergent gauge fields, and disorder. Strange metal behavior with $\rho \sim (T/t)^{2x}(h/e^2)$ as $T \to 0$, with *all* electrons mobile.