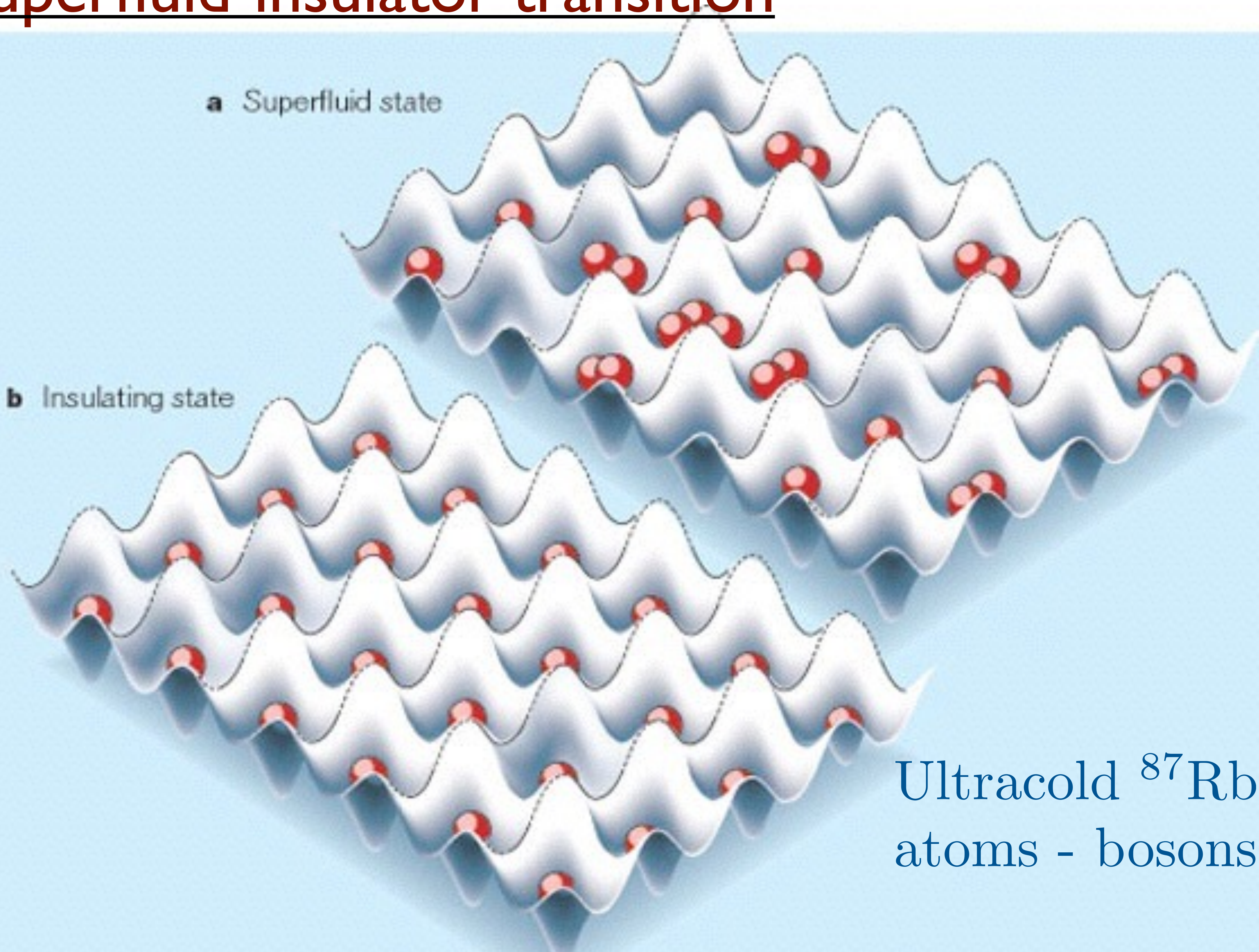
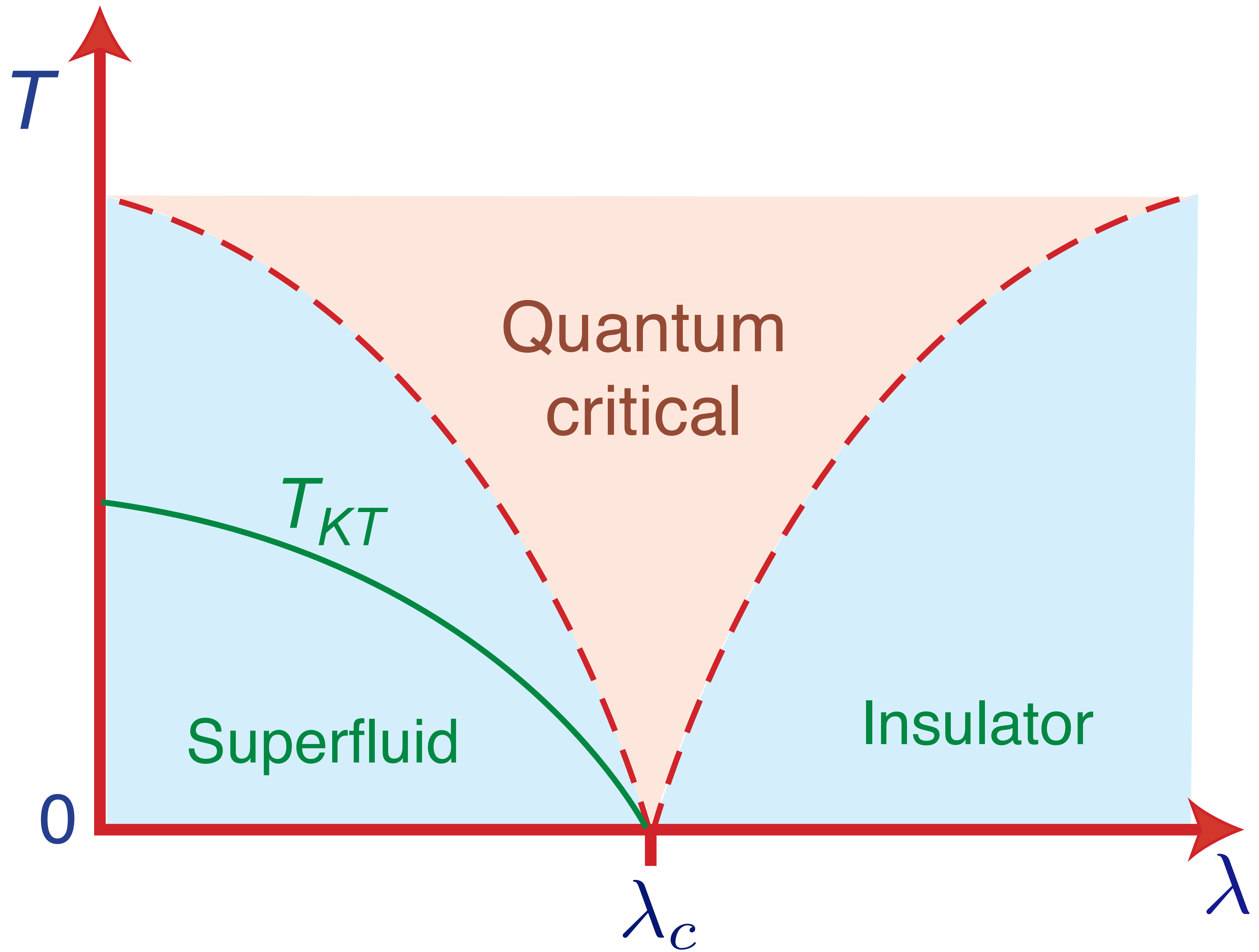
Superfluid-insulator transition

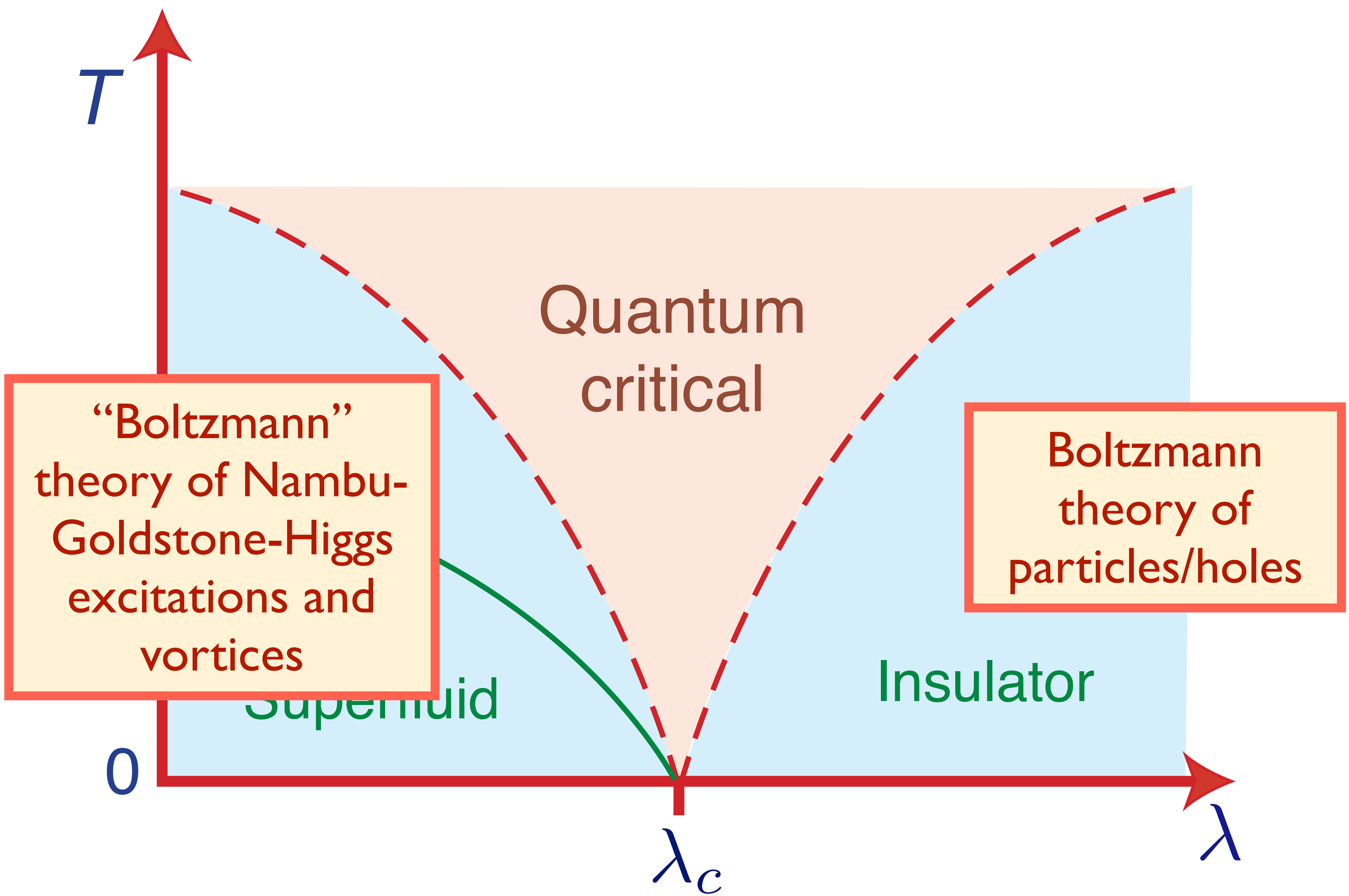


Ultracold ^{87}Rb atoms - bosons

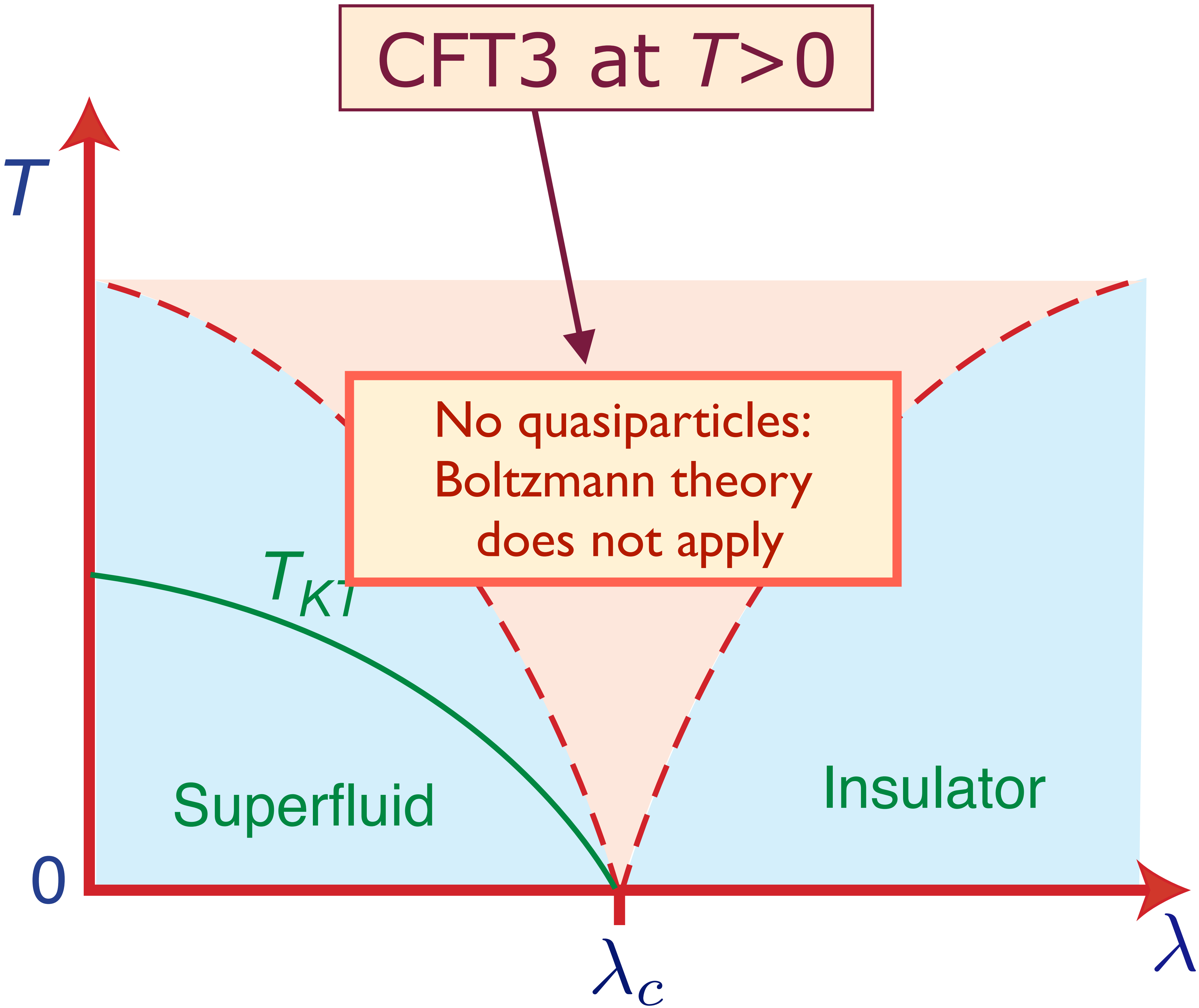
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).



T
Quantum critical
T_{KT}
Superfluid
Insulator
0
λ_c
λ



T
Quantum
critical
"Boltzmann"
theory of Nambu-
Goldstone-Higgs
excitations and
vortices
Boltzmann
theory of
particles/holes
Insulator
0
λ_c
λ



CFT3 at $T>0$
T
No quasiparticles:
Boltzmann theory
does not apply
T_{KT}
Superfluid
Insulator
0
λ_c
λ

Title: Conformal field theories at non-zero temperature: operator product expansions, Monte Carlo, and holography

Authors: Emanuel Katz, Subir Sachdev, Erik S. Sorensen, William Witczak-Krempa
(Submitted on 12 Sep 2014 (v1), last revised 2 Dec 2014 (this version, v2))

Abstract: We compute the non-zero temperature conductivity of conserved flavor currents in conformal field theories (CFTs) in 2+1 spacetime dimensions. At frequencies much greater than the temperature, $\hbar\omega >> k_B T$, the ω dependence can be computed from the operator product expansion (OPE) between the currents and operators which acquire a non-zero expectation value at T > 0. Such results are found to be in excellent agreement with quantum Monte Carlo studies of the O(2) Wilson-Fisher CFT. Results for the conductivity and other observables are also obtained in vector 1/N expansions. We match these large ω results to the corresponding correlators of holographic representations of the CFT: the holographic approach then allows us to extrapolate to small $\hbar\omega/(k_B T)$. Other holographic studies implicitly only used the OPE between the currents and the energy-momentum tensor, and this yields the correct leading large ω behavior for a large class of CFTs. However, for the Wilson-Fisher CFT a relevant "thermal" operator must also be considered, and then consistency with the Monte Carlo results is obtained without a previously needed ad hoc rescaling of the T value. We also establish sum rules obeyed by the conductivity of a wide class of CFTs.

Comments:	25+16 pages; 5+2 figures. Single column. v2: Added new appendix about numerical methods, expanded discussion, typos corrected
Subjects:	Strongly Correlated Electrons (cond-mat.str-el); General Relativity and Quantum Cosmology (gr-qc); High Energy Physics - Lattice (hep-lat); High Energy Physics - Theory (hep-th)
Journal reference:	Phys. Rev. B 90, 245109 (2014)

Recent extension away from the critical point

Quantum critical fluid in graphene

ARO MURI meeting, Berkeley
January 11, 2016

Subir Sachdev

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Talk online: sachdev.physics.harvard.edu

PHYSICS
VE RI TAS
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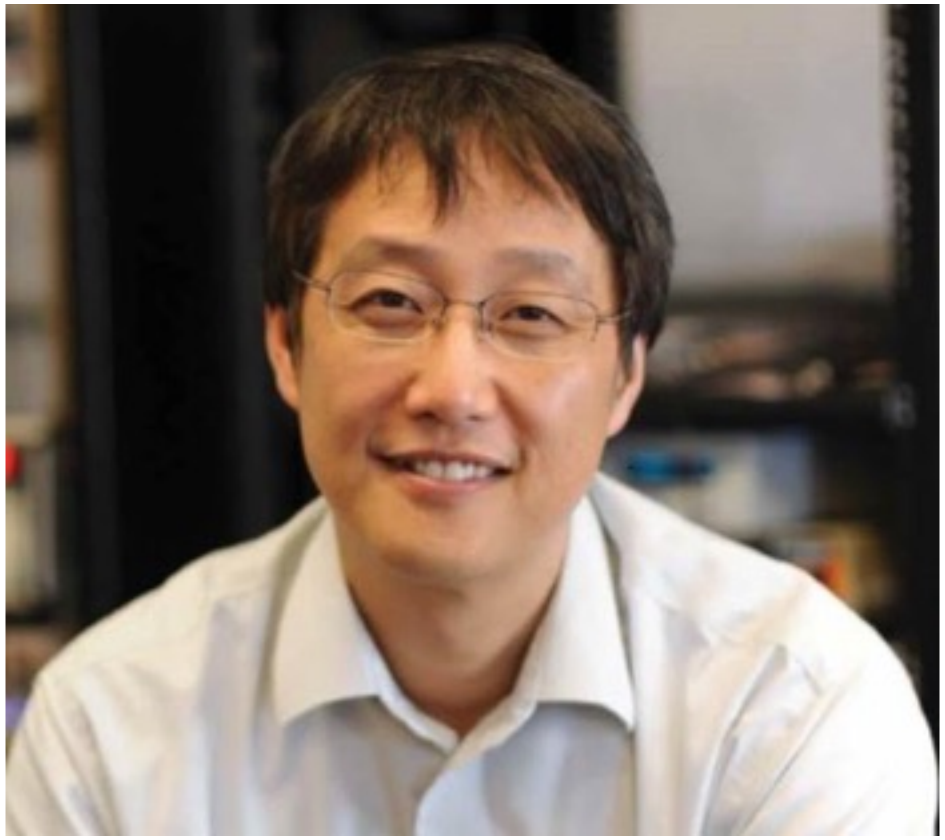
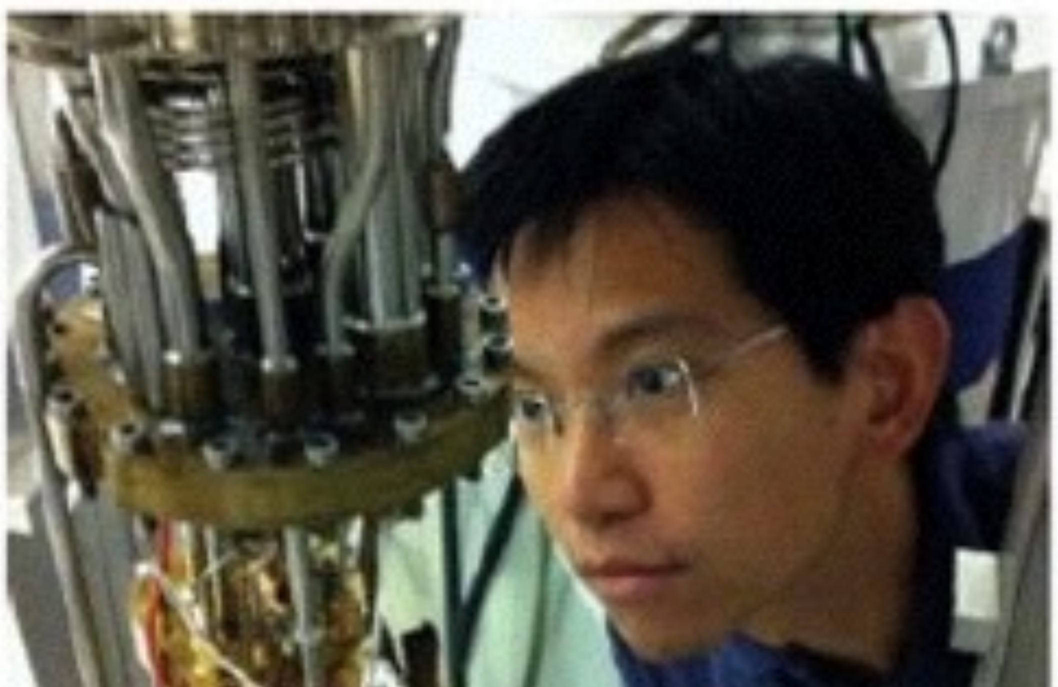
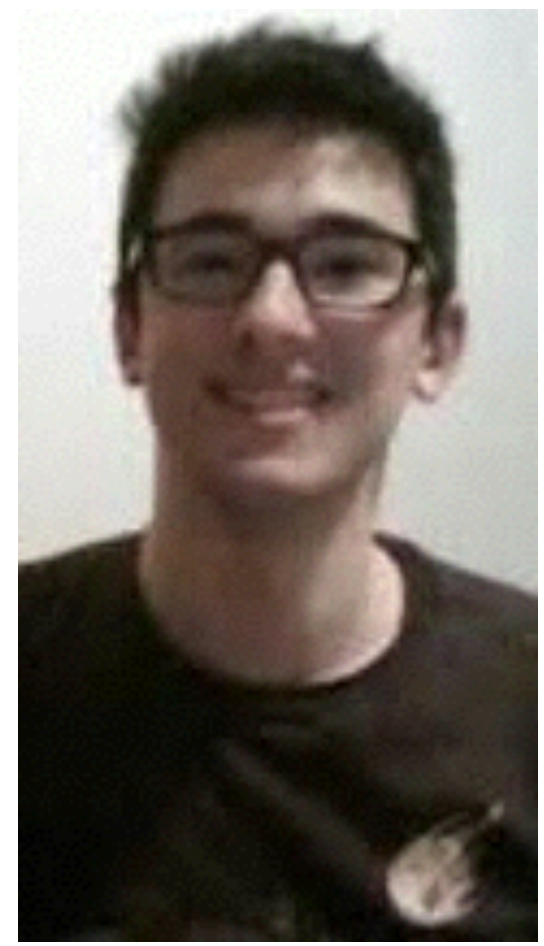


Philip Kim

Jesse Crossno

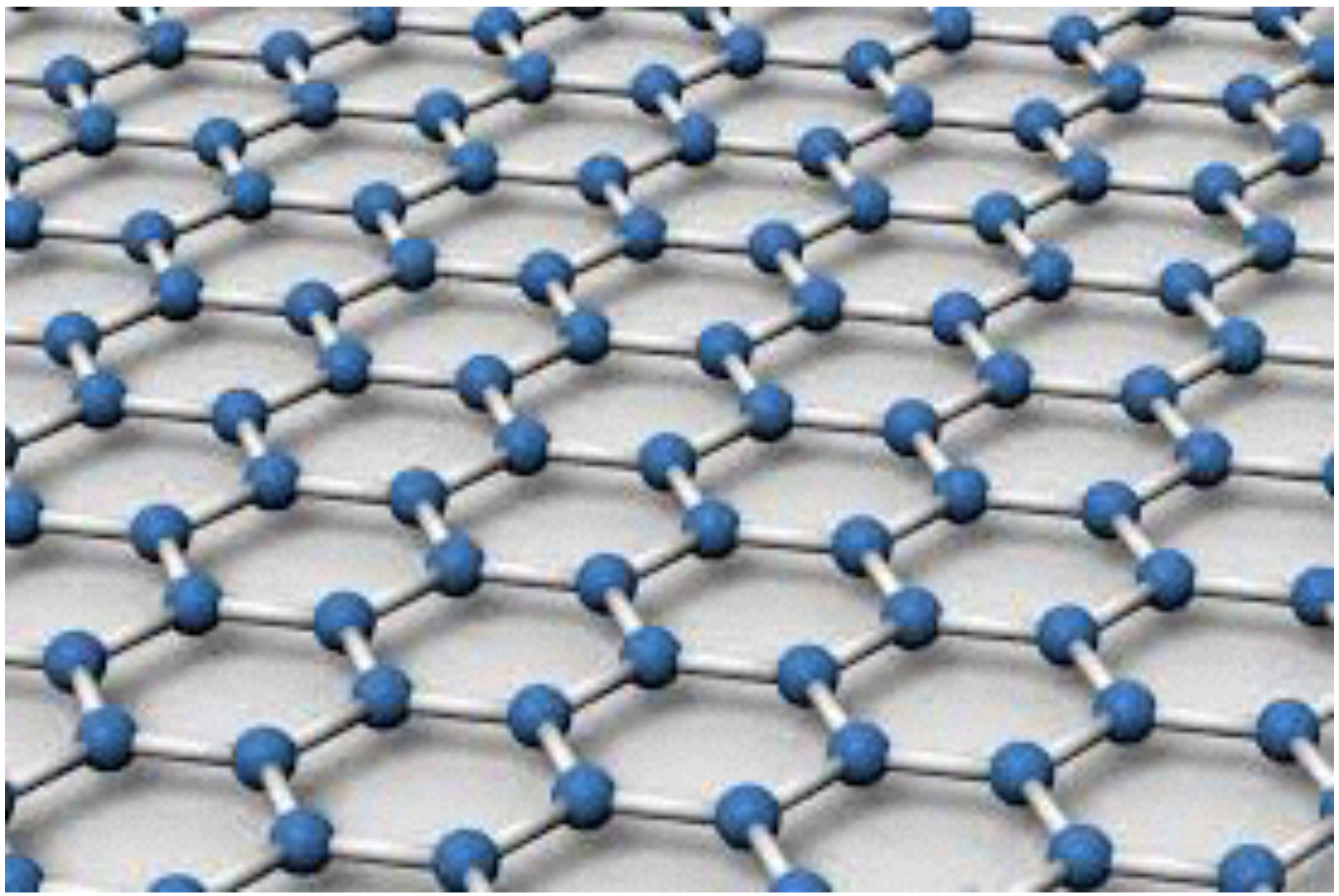


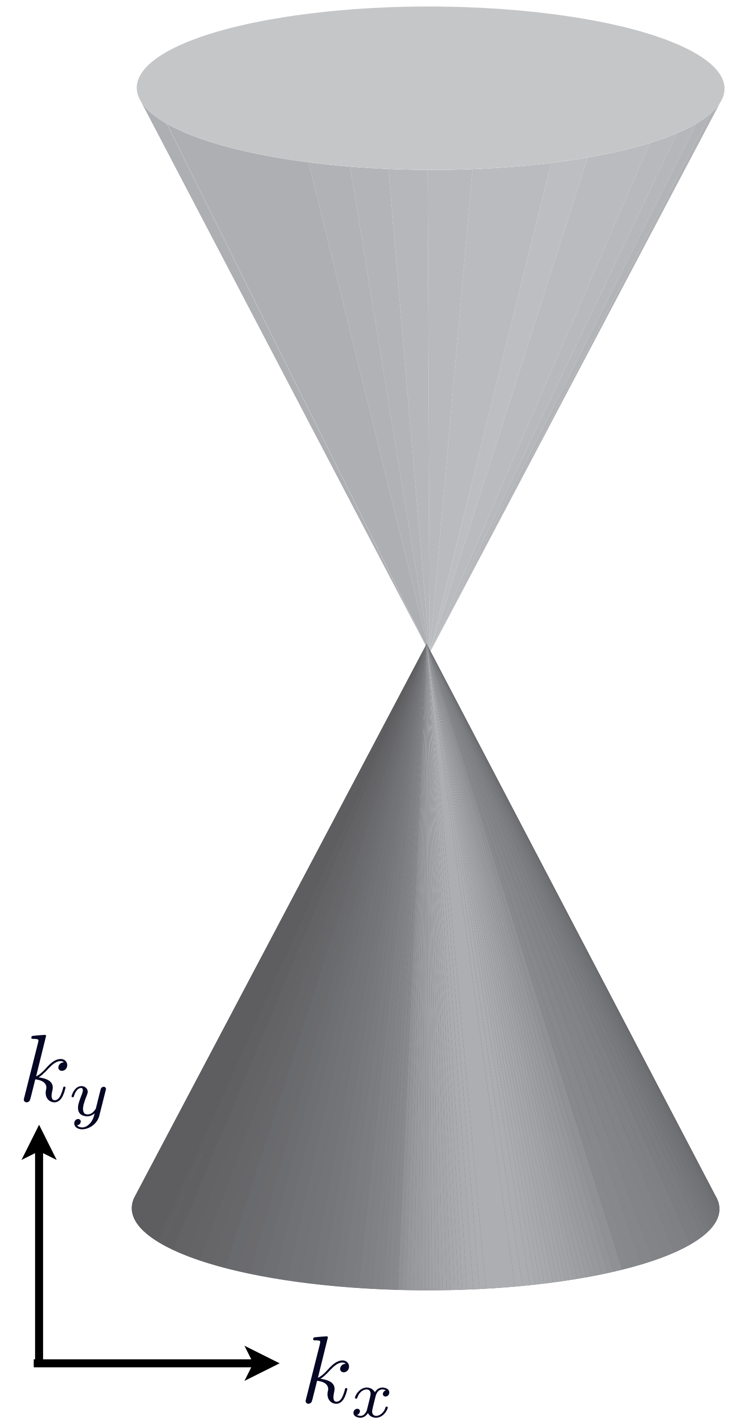
Kin Chung Fong



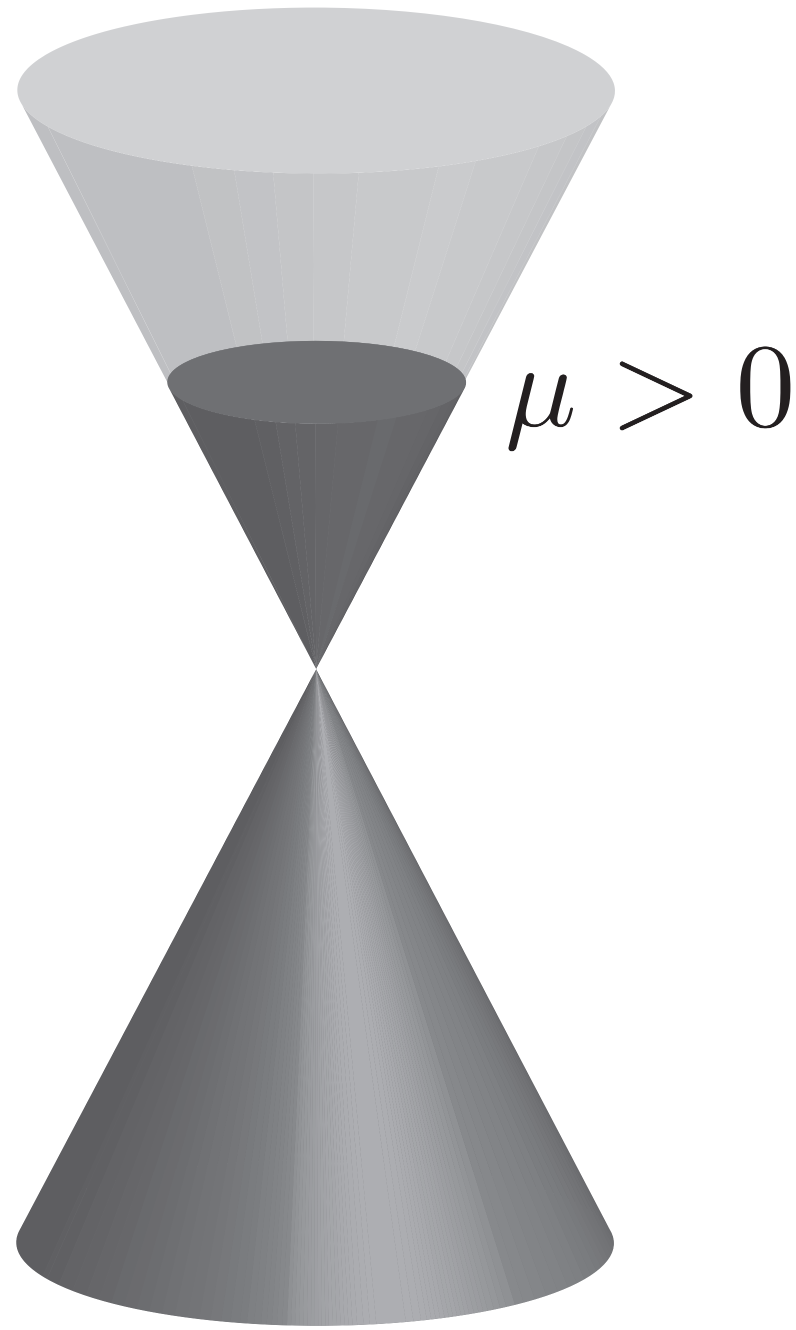
Andrew Lucas

Graphene



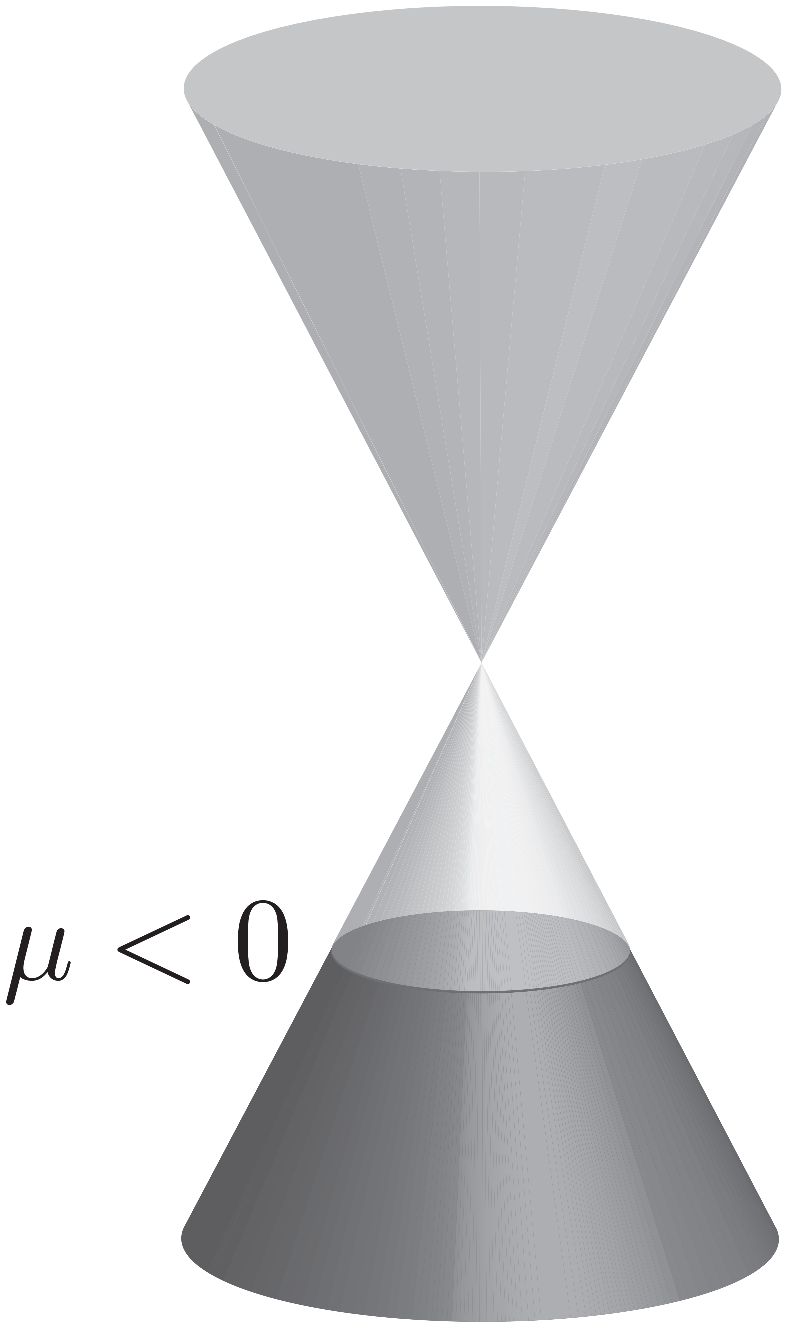


Graphene

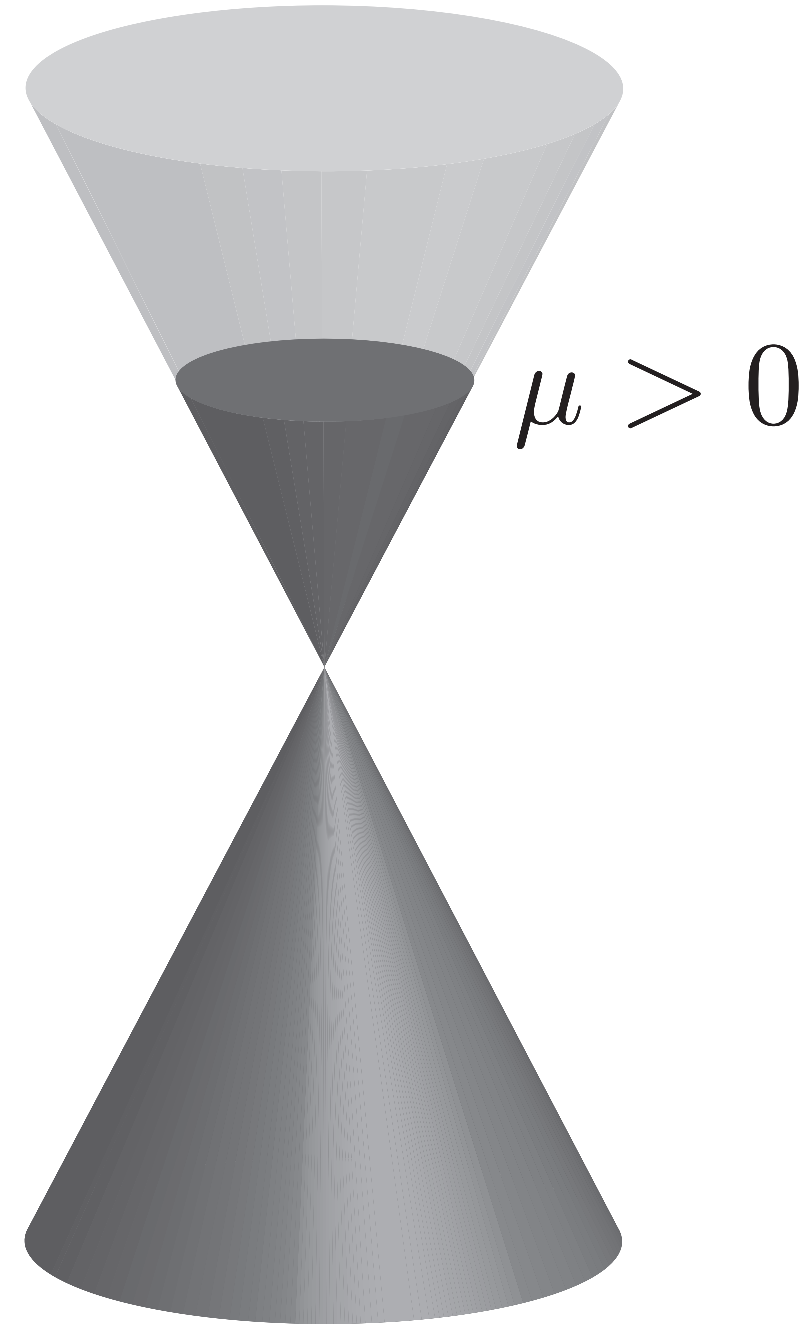


Electron
Fermi surface

Graphene

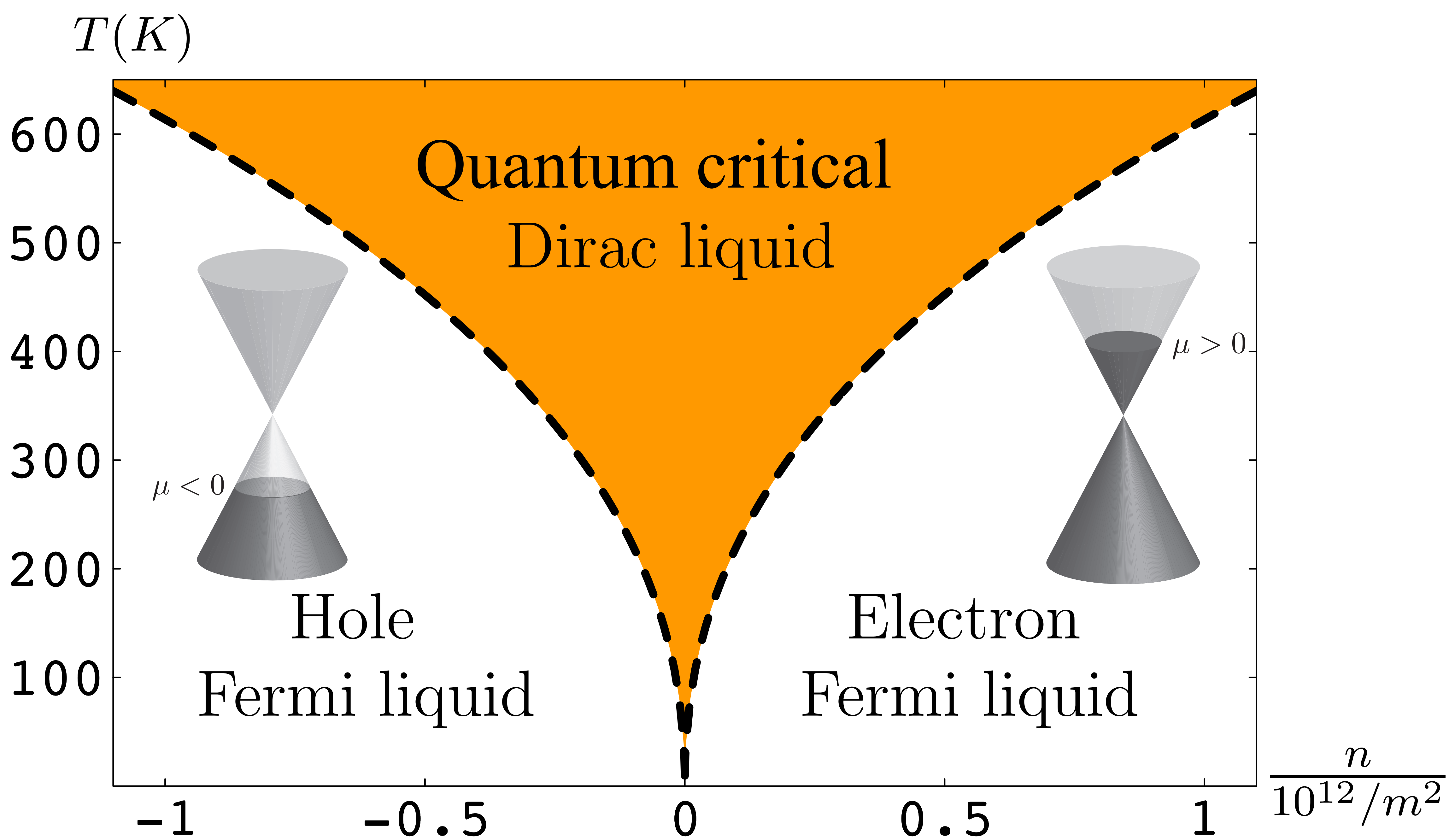


Hole
Fermi surface



Electron
Fermi surface

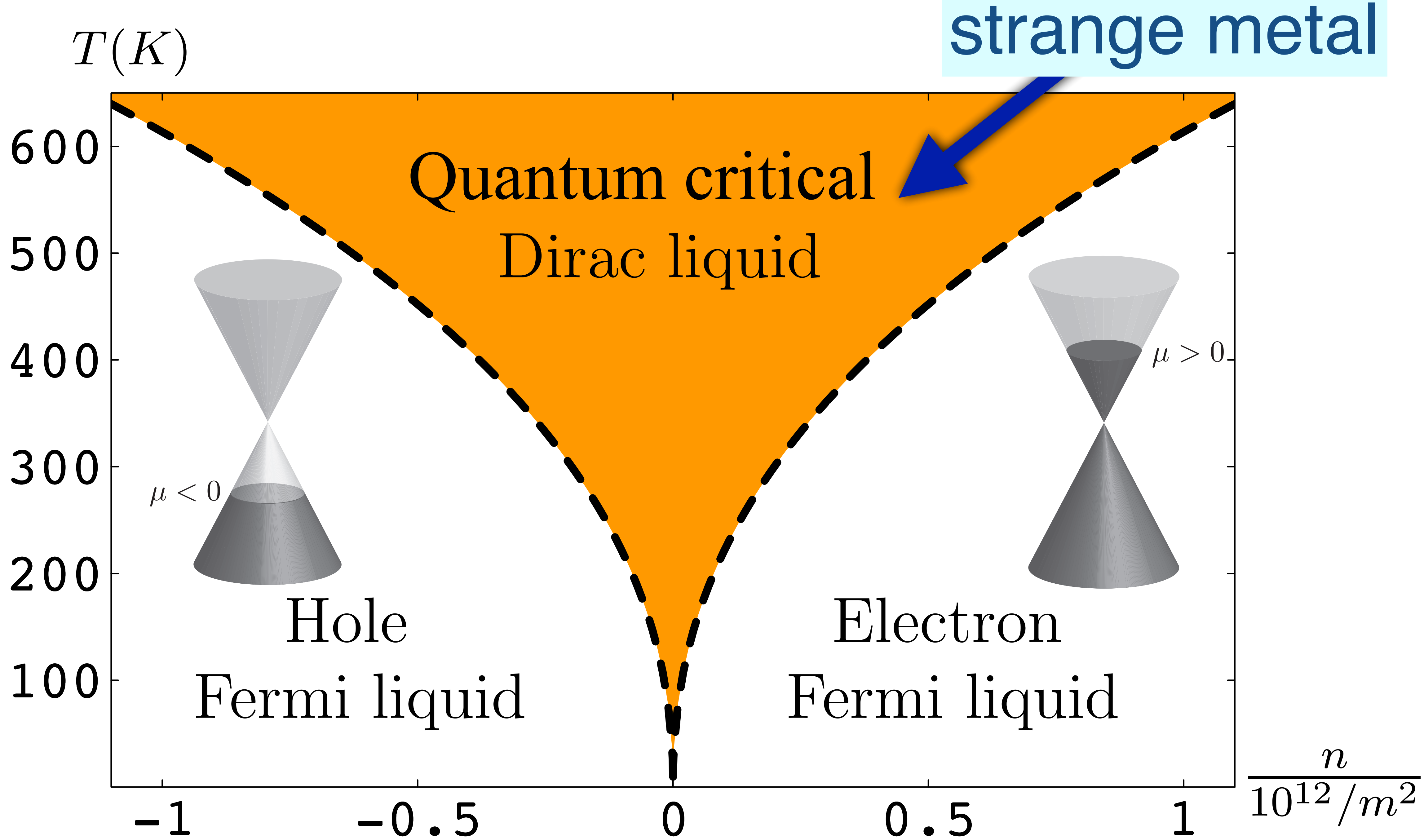
Graphene



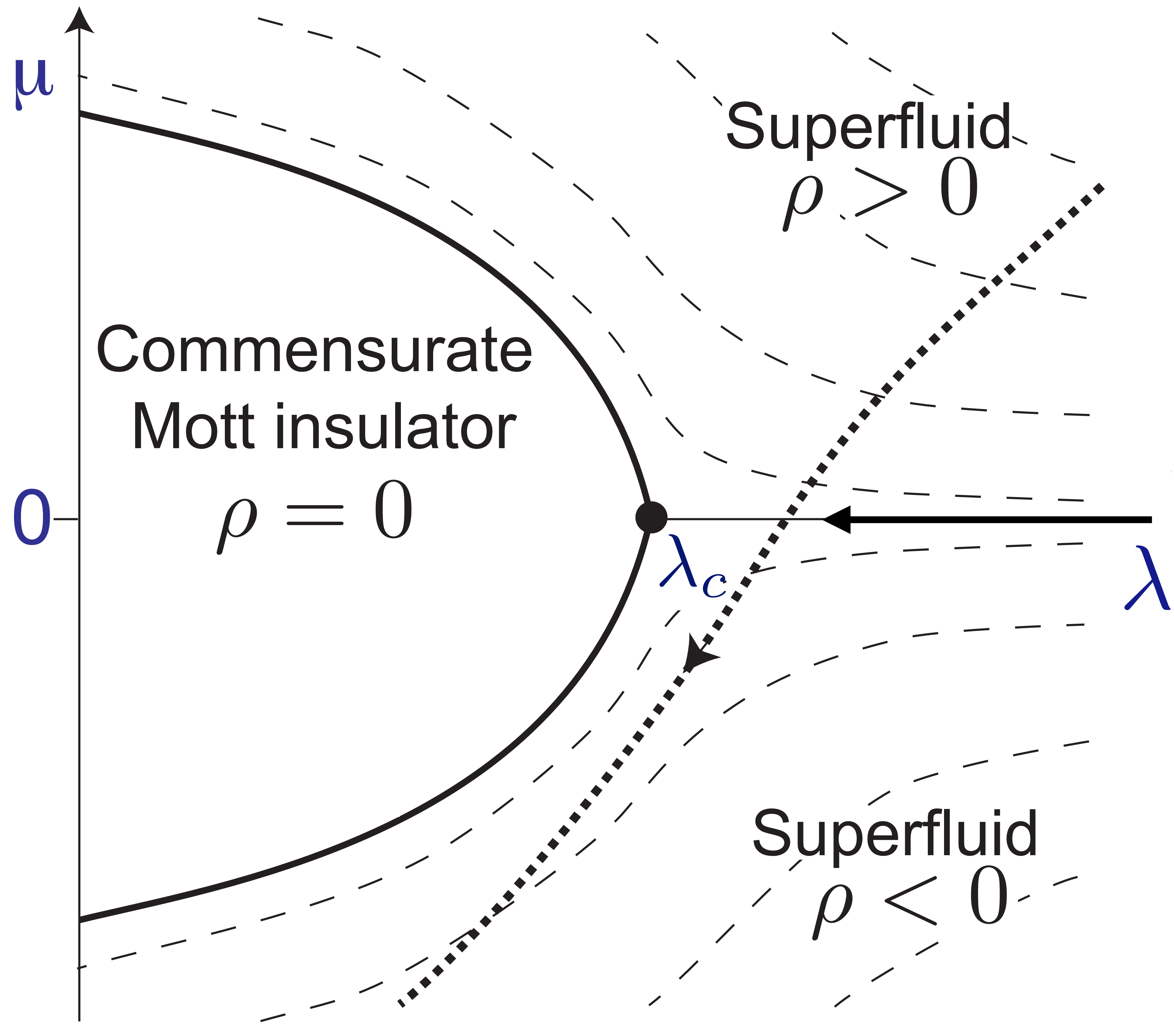
D. E. Sheehy and J. Schmalian, PRL **99**, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)

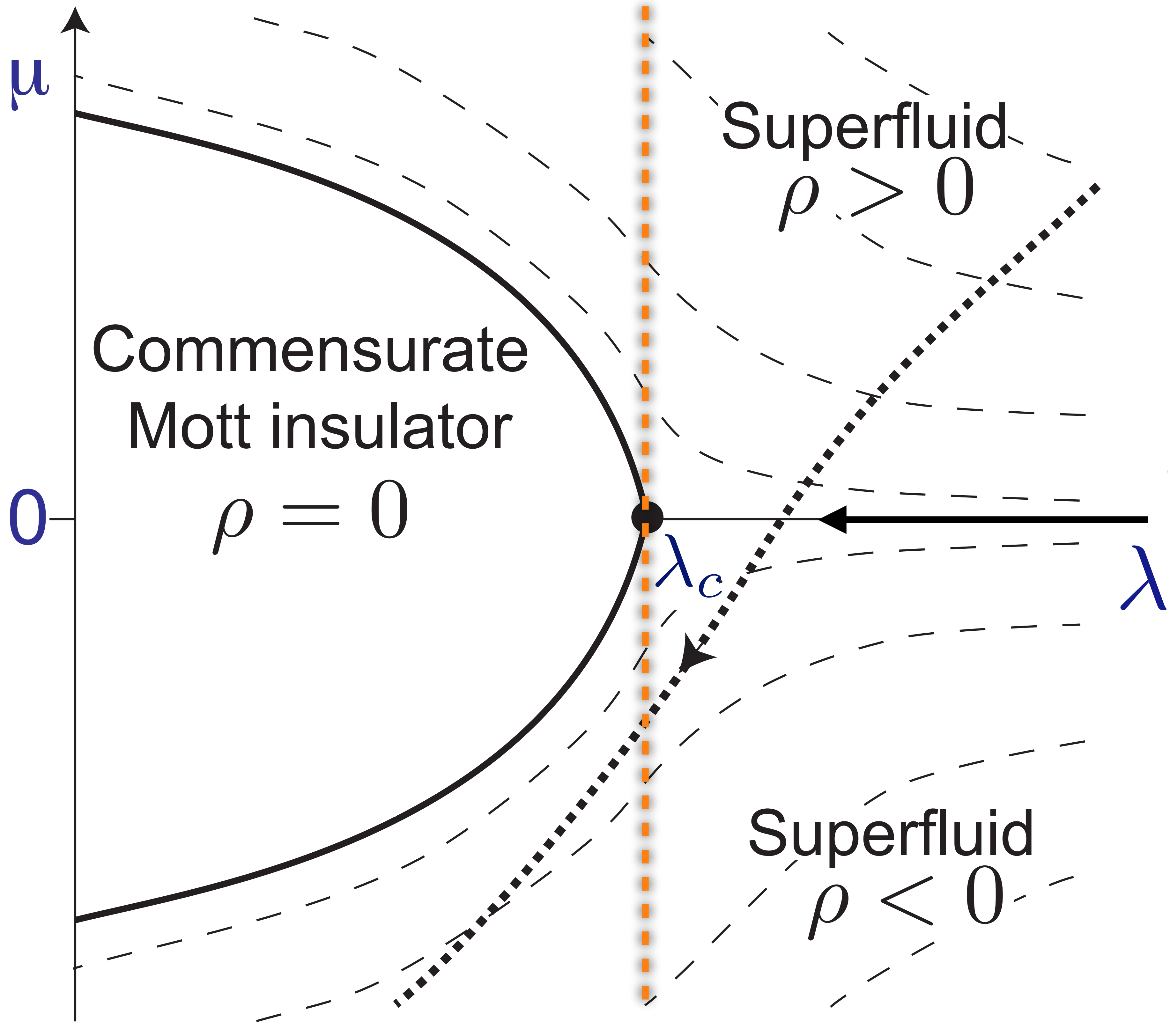
Graphene

Predicted strange metal

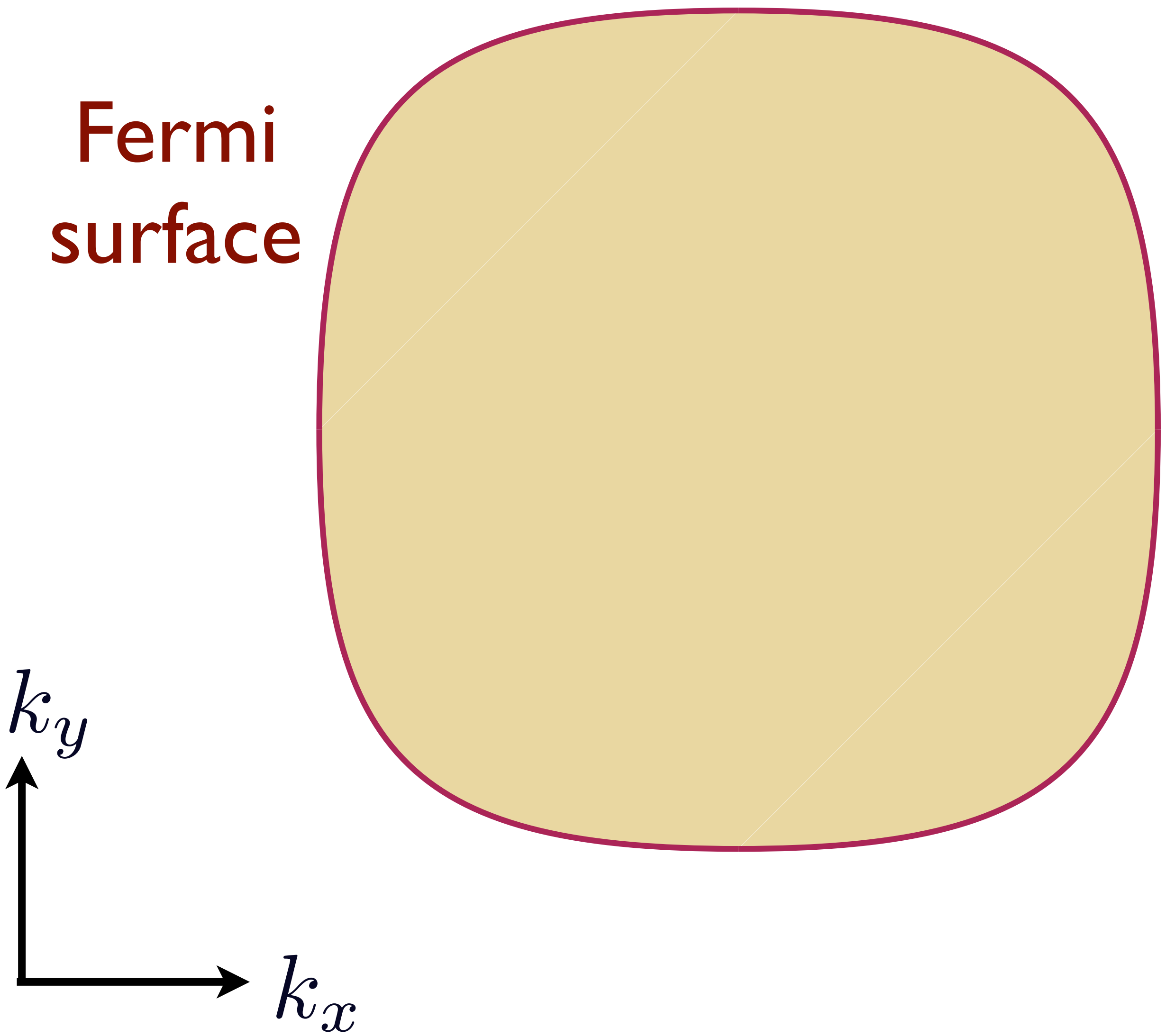


M. Müller, L. Fritz, and S. Sachdev, PRB **78**, 115406 (2008)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)


μ
0
Commensurate
Mott insulator
$\rho = 0$
λ_c
λ
Superfluid
$\rho > 0$
Superfluid
$\rho < 0$


μ
0
Commensurate
Mott insulator
$\rho = 0$
Superfluid
$\rho > 0$
λ_c
λ
Superfluid
$\rho < 0$

Ordinary metals: the Fermi liquid

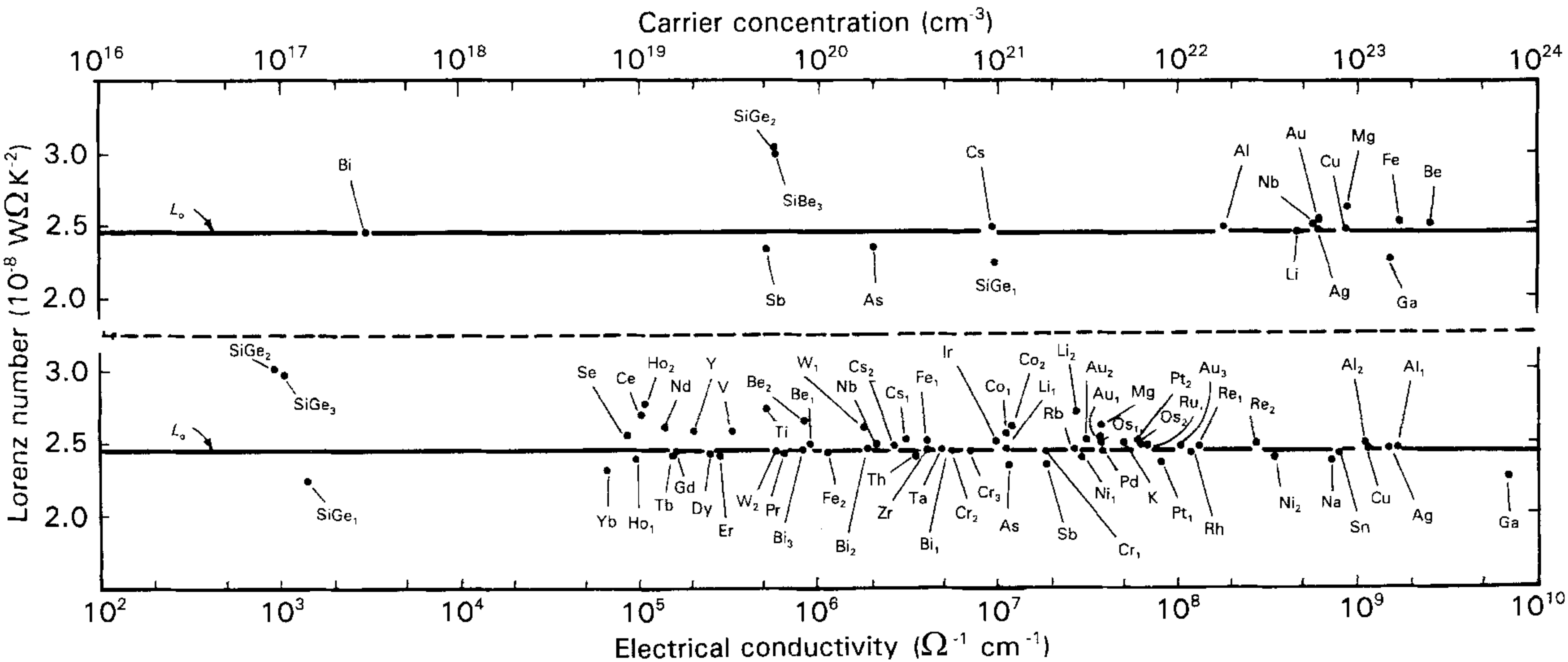


- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = $\mathcal{Q}$. Momenta of low energy excitations fixed by density of *all* electrons.
- Long-lived electron-like quasiparticle excitations near the Fermi surface: lifetime of quasiparticles $\sim 1/T^2$.

- $$\frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}} = \frac{\pi^2 k_B^2}{3e^2} \equiv L_0$$

- Wiedemann-Franz law in a Fermi liquid:

$$\frac{\kappa}{\sigma T} \approx \frac{\pi^2 k_B^2}{3e^2} \approx 2.45 \times 10^{-8}\, \frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}.$$

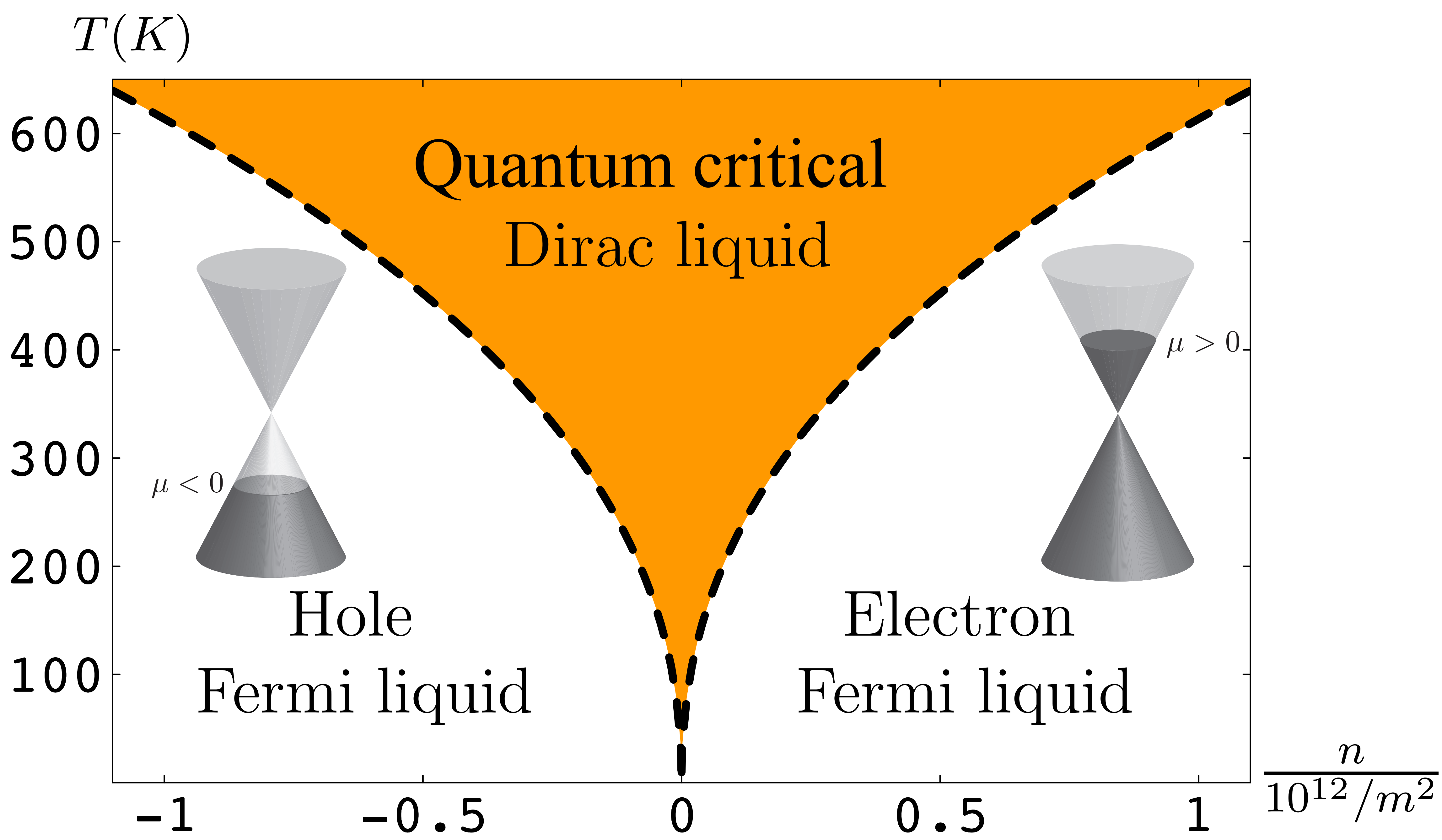


G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. **28**, 4261 (1993)

Key properties of a strange metal

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$ (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)

Graphene



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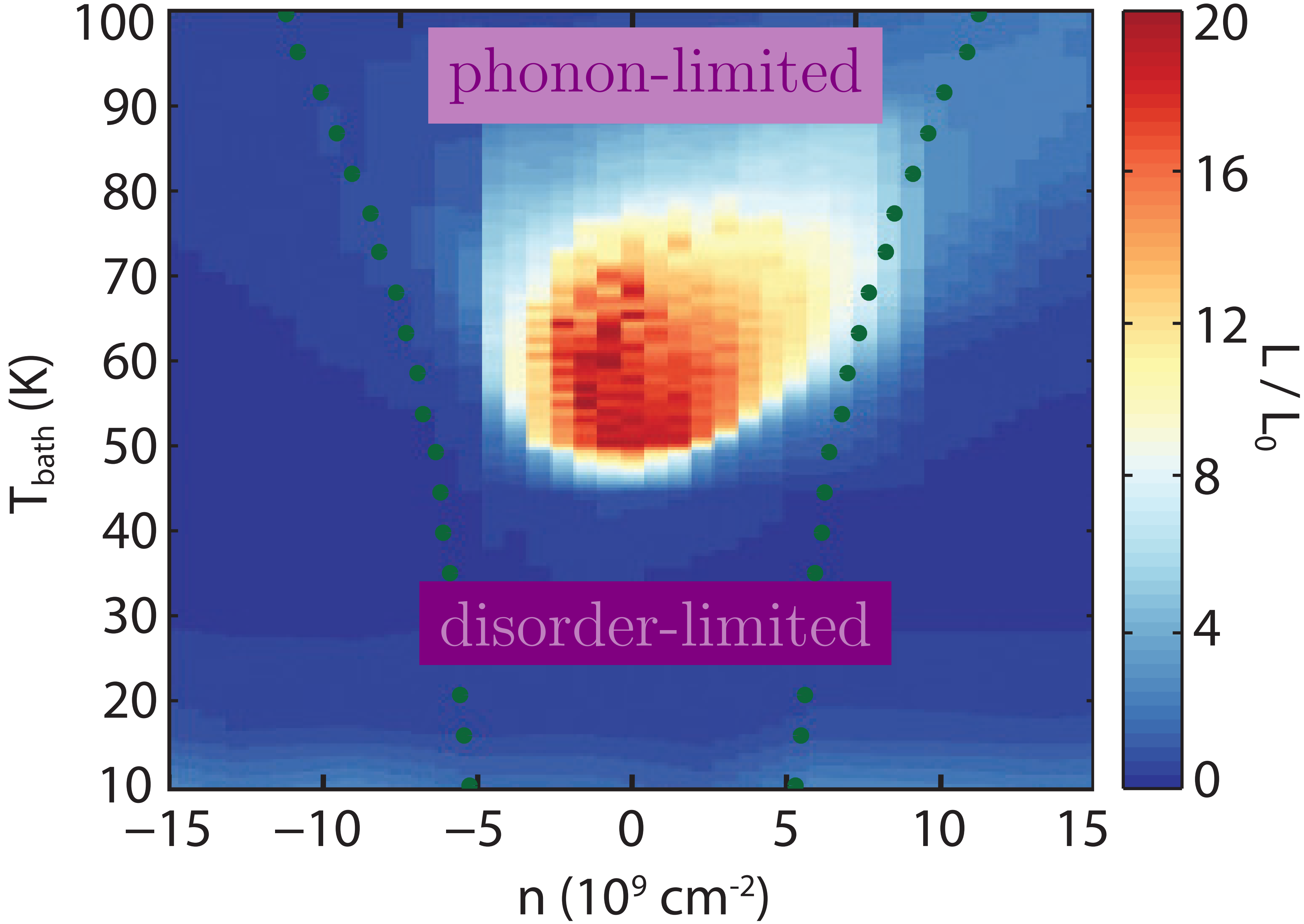
J. Crossno *et al.* arXiv:1509.04713; Science, to appear

Strange metal in graphene



$$L = \frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}}; \quad L_0 \equiv \frac{\pi^2 k_B^2}{3e^2}$$

J. Crossno *et al.* arXiv:1509.04713; Science, to appear

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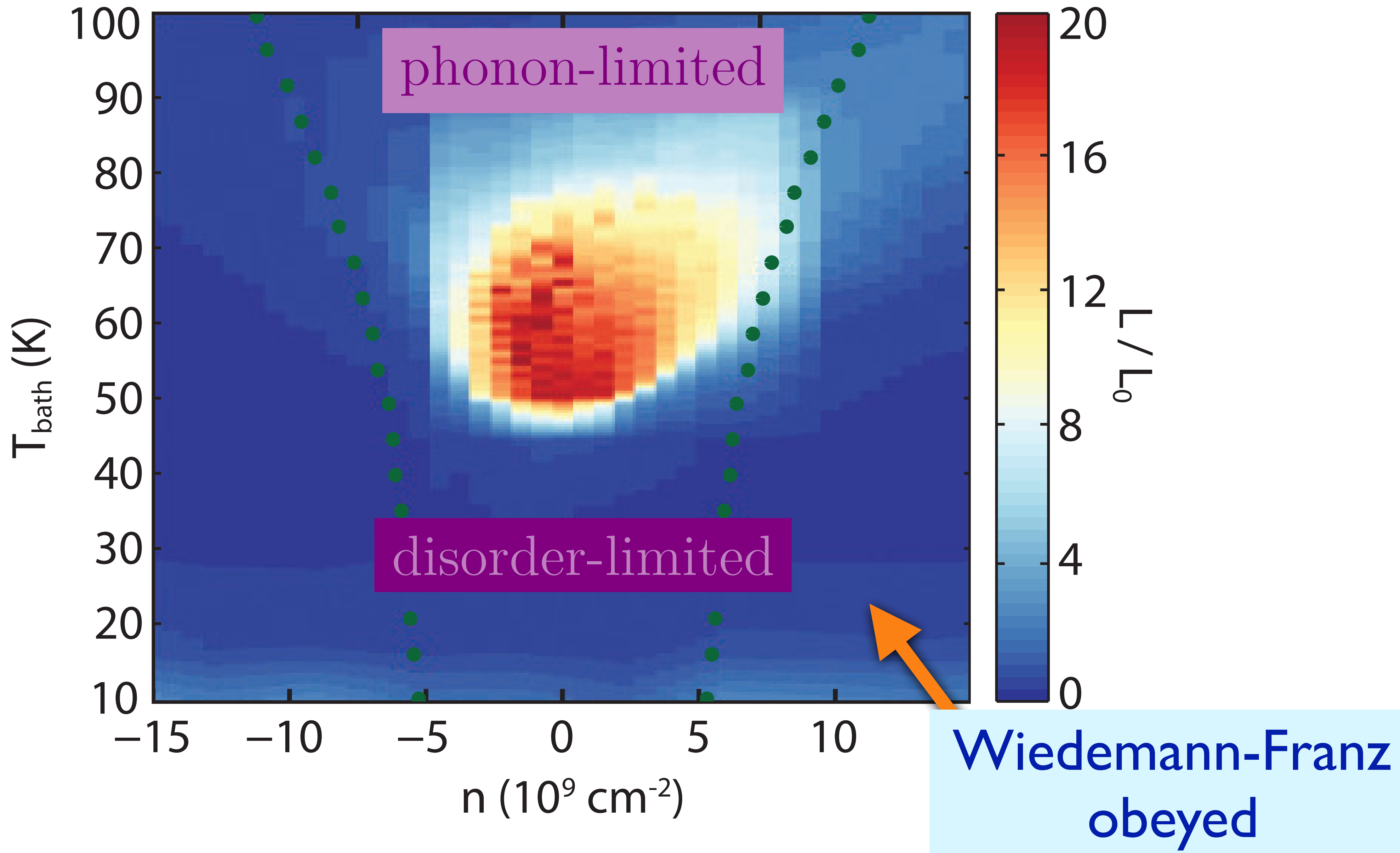

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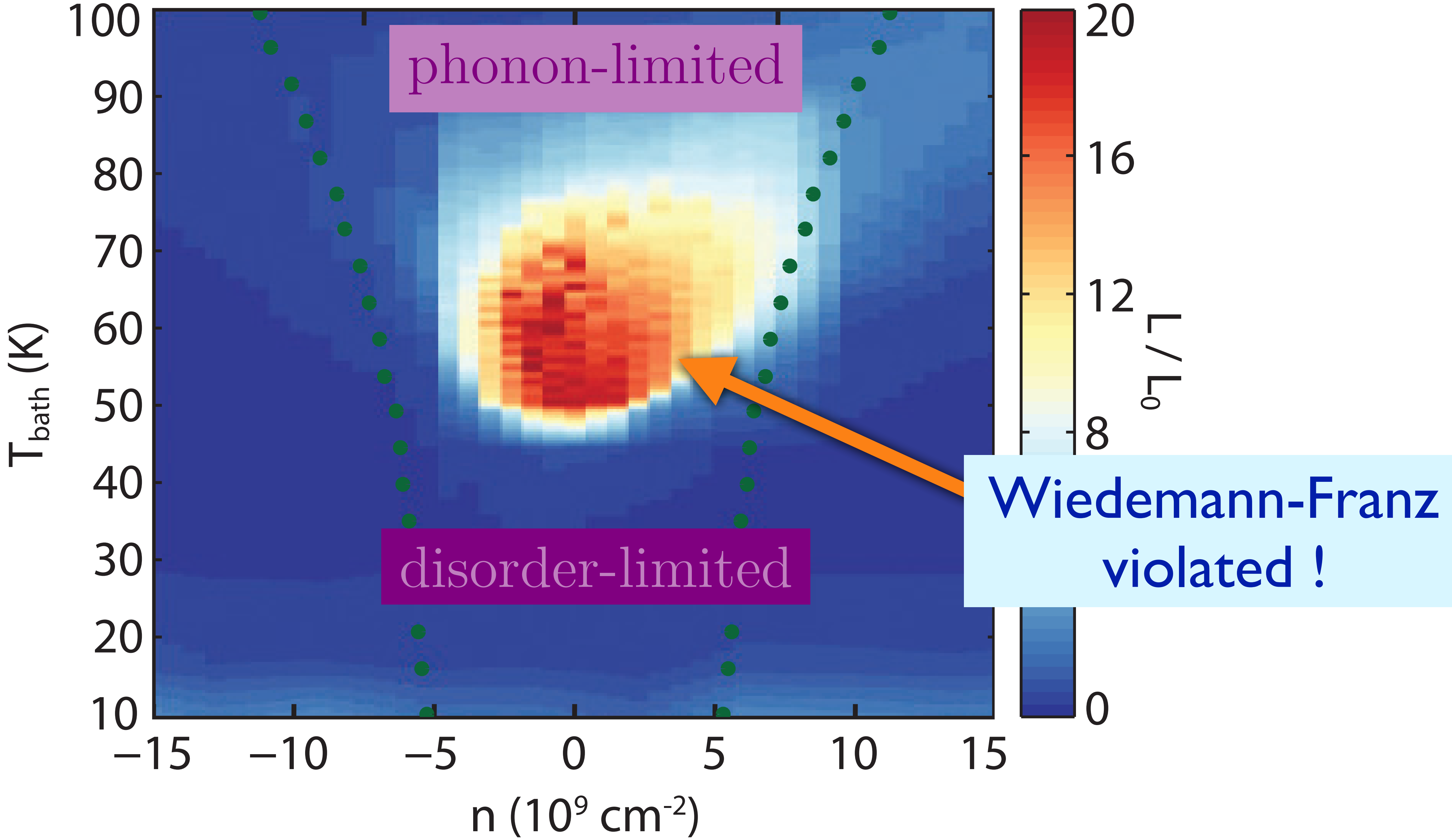
Strange metal in graphene



$$L = \frac{\text{(Thermal conductivity)}}{T\,\text{(Electrical conductivity)}}; \quad L_0 \equiv \frac{\pi^2 k_B^2}{3e^2}$$

Quasiparticle transport in metals:

- Focus on infinite number of (near) conservation laws (momenta of quasiparticles on the Fermi surface) and compute how they are slowly violated by the lattice or impurities

Transport in strange metals

- There are no quasiparticles, and so the Fermi surface is not a central actor in transport (although a Fermi surface can be precisely defined in some cases).

Transport in strange metals

- There are no quasiparticles, and so the Fermi surface is not a central actor in transport (although a Fermi surface can be precisely defined in some cases).

- Focus on relaxation of *total* momentum (including contributions of the Fermi surface (if present) and all critical bosons) by the lattice or impurities

Transport in Strange Metals

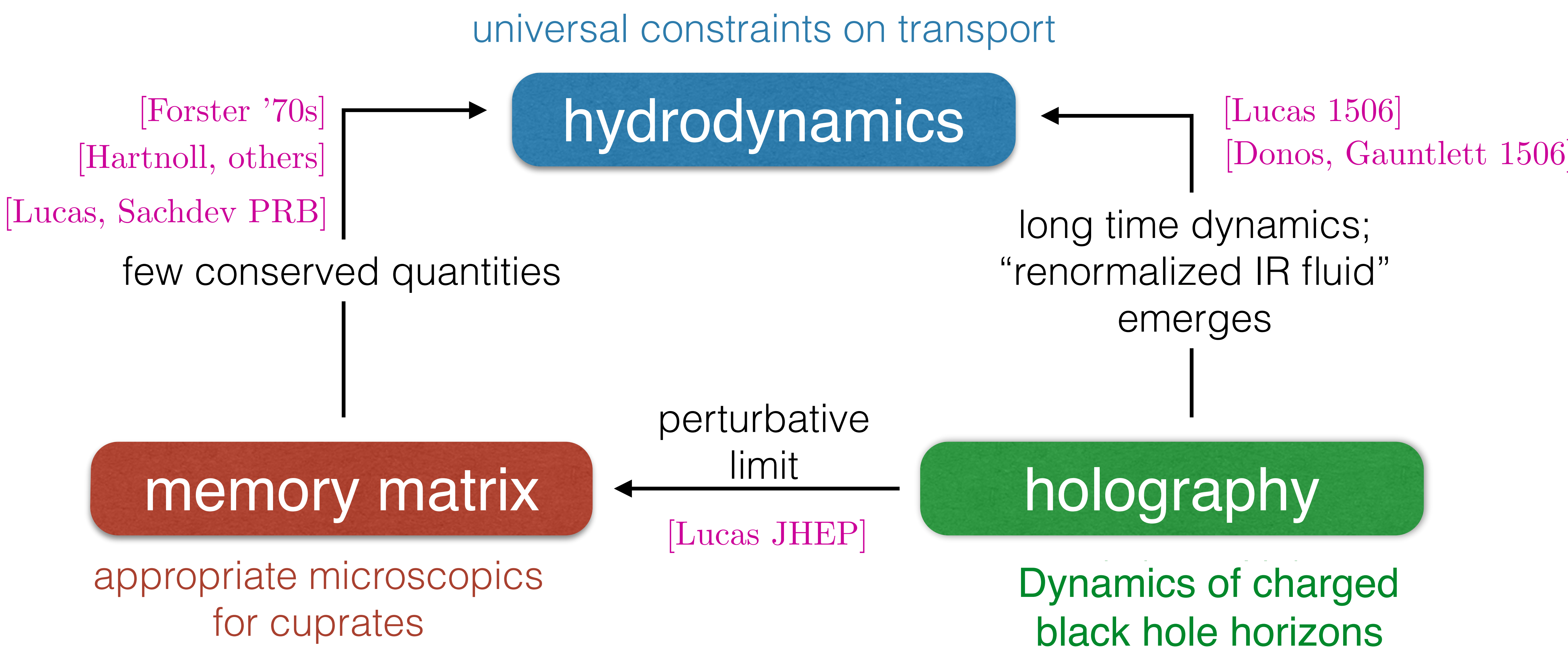


figure from [Lucas, Sachdev, *Physical Review* **B91** 195122 (2015)]

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Prediction for transport in the graphene strange metal

Recall that in a Fermi liquid, the Lorenz ratio $L = \kappa/(T\sigma)$, where κ is the thermal conductivity, and σ is the conductivity, is given by $L = \pi^2 k_B^2/(3e^2)$.

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield

$$\sigma = \sigma_Q \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right) \quad , \quad \kappa = \frac{v_F^2 \mathcal{H}\tau_{\rm imp}}{T}\left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}}{\mathcal{H}\sigma_Q}\right)^{-1}$$

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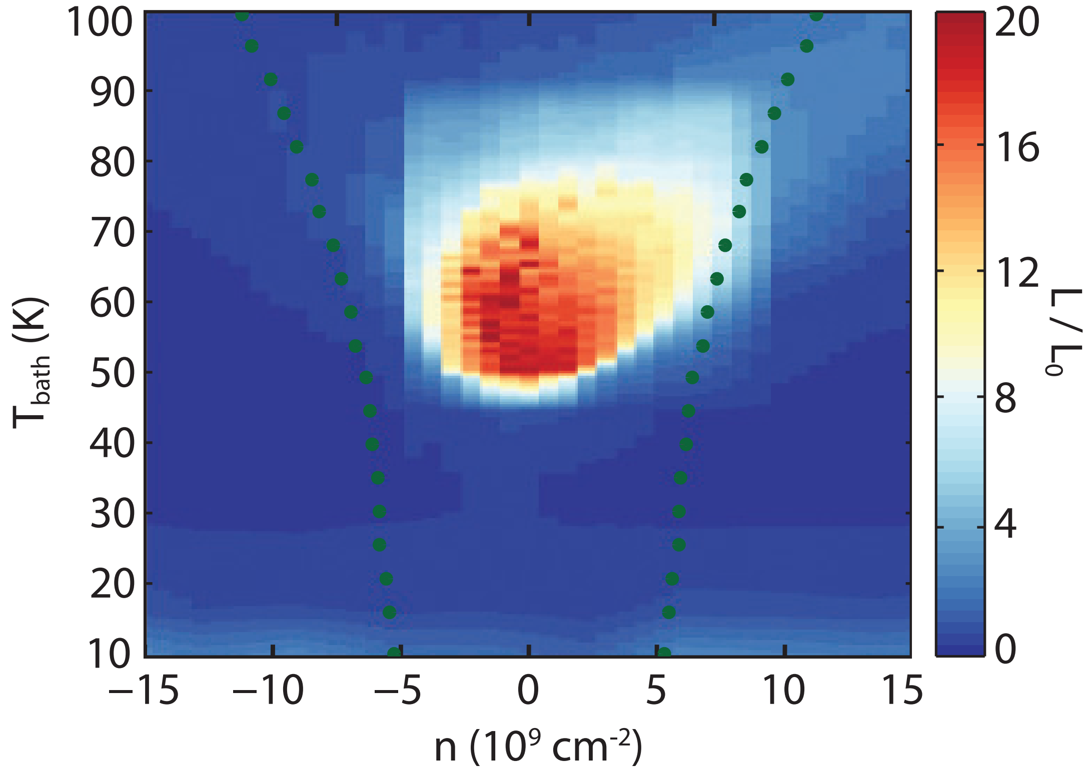
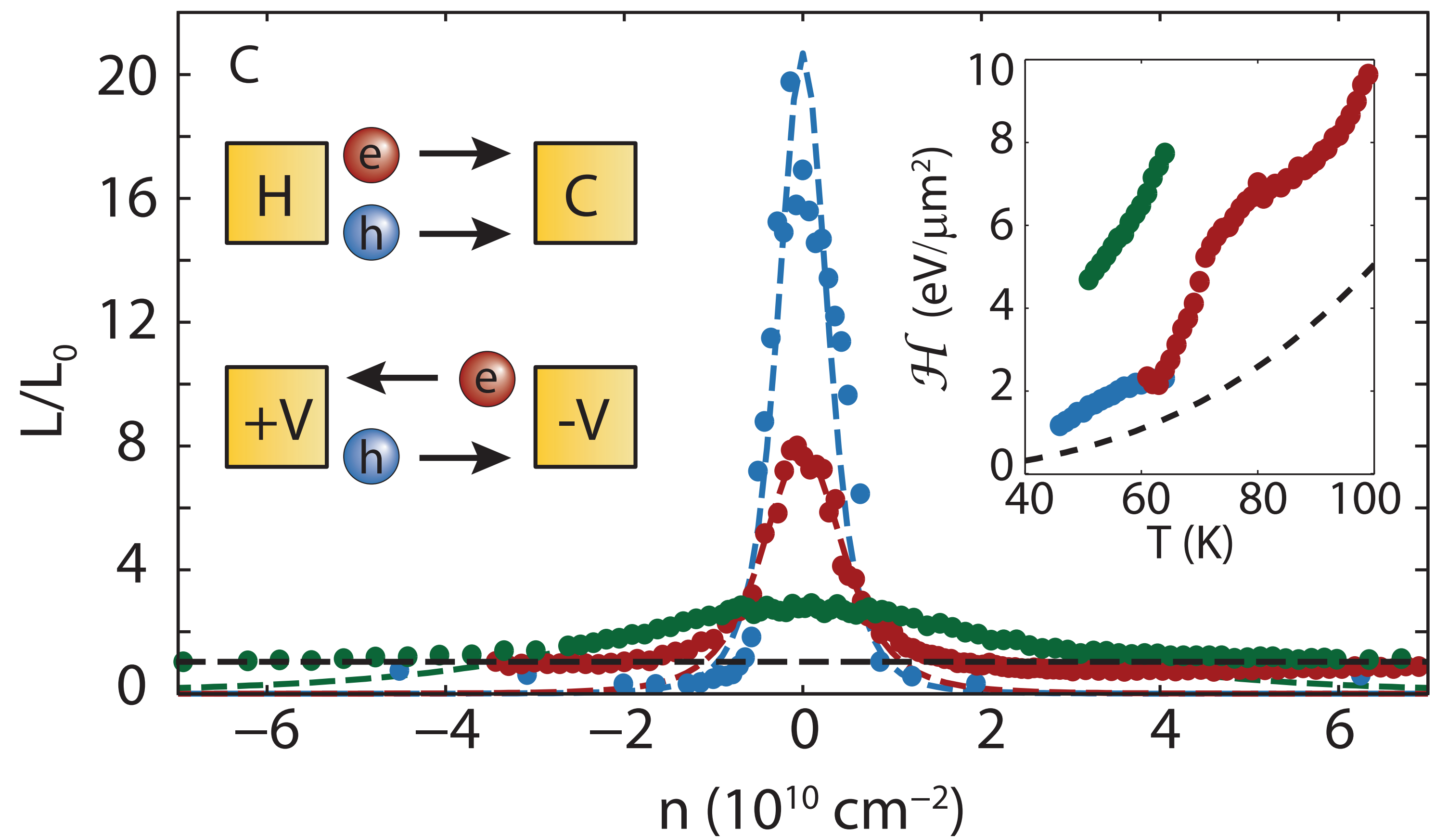
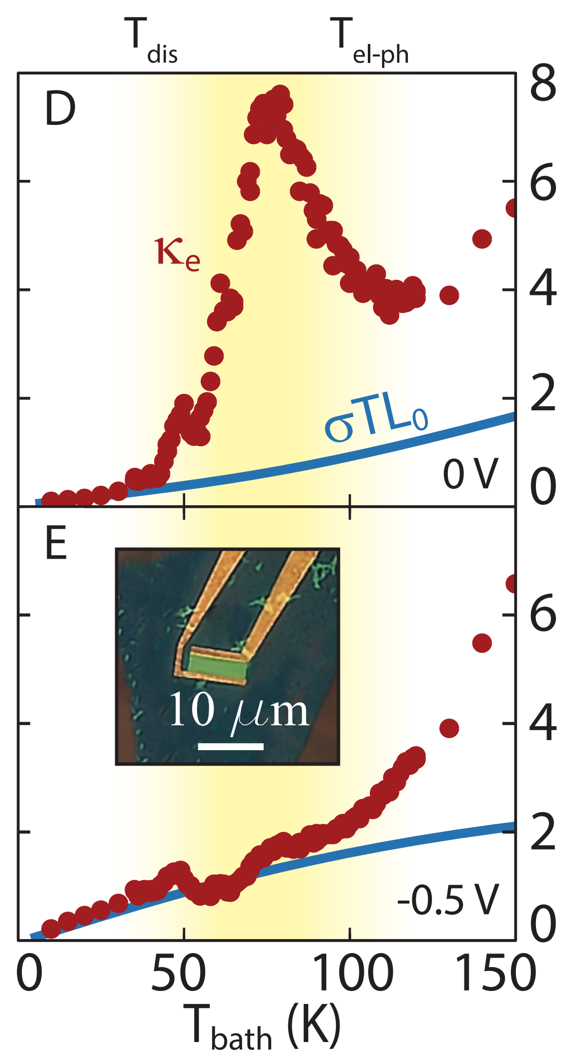
where $\mathcal{H}$ is the enthalpy density, $\tau_{\rm imp}$ is the momentum relaxation time (from impurities), while $\sigma = \sigma_Q$, an intrinsic, finite, "quantum critical" conductivity. Note that the limits $\mathcal{Q} \to 0$ and $\tau_{\rm imp} \to \infty$ do not commute.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)







$$\text{Lorentz ratio } L = \kappa/(T\sigma)$$

$$= \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \frac{1}{\left(1 + e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}} / (\mathcal{H} \sigma_Q)\right)^2}$$

J. Crossno *et al.* arXiv:1509.04713; Science, to appear

Relativistic hydrodynamics

- hydrodynamics when $l \gg l_{\rm ee}$, $t \gg t_{\rm ee}$
- long time dynamics governed by conservation laws:

$$\partial_\nu T^{\mu\nu} = J_\nu \left(F^{\rm ext}\right)^{\mu\nu}, \qquad \partial_\mu J^\mu = 0.$$

dynamics of relaxation to equilibrium

- expand $T^{\mu\nu}$, J^μ in perturbative parameter $l_{\rm ee}\partial_\mu$:

$$T^{\mu\nu} = P\eta^{\mu\nu} + (\epsilon + P)u^\mu u^\nu$$

$$J^\mu = \mathcal{Q}u^\mu - \sigma_{\rm Q}\mathcal{P}^{\mu\rho}\left(\partial_\rho \mu - \frac{\mu}{T}\partial_\rho T - u^\nu F^{\rm ext}_{\rho\nu}\right) + \cdots,$$

$$\mathcal{P}^{\mu\nu} \equiv \eta^{\mu\nu} + u^\mu u^\nu,$$

$$Q^i = T^{ti} - \mu J^i$$

- New (and only) transport co-efficient, σ_Q: "quantum critical" conductivity at $\mathcal{Q} = 0$.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)

Translational symmetry breaking

Momentum relaxation by an external source h coupling to the operator $\mathcal{O}$

$$H = H_0 - \int d^d x \, h(x) \, \mathcal{O}(x).$$

Leads to an additional term in equations of motion:

$$\partial_\mu T^{\mu i} = \ldots - \frac{T^{it}}{\tau_{\text{imp}}} + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

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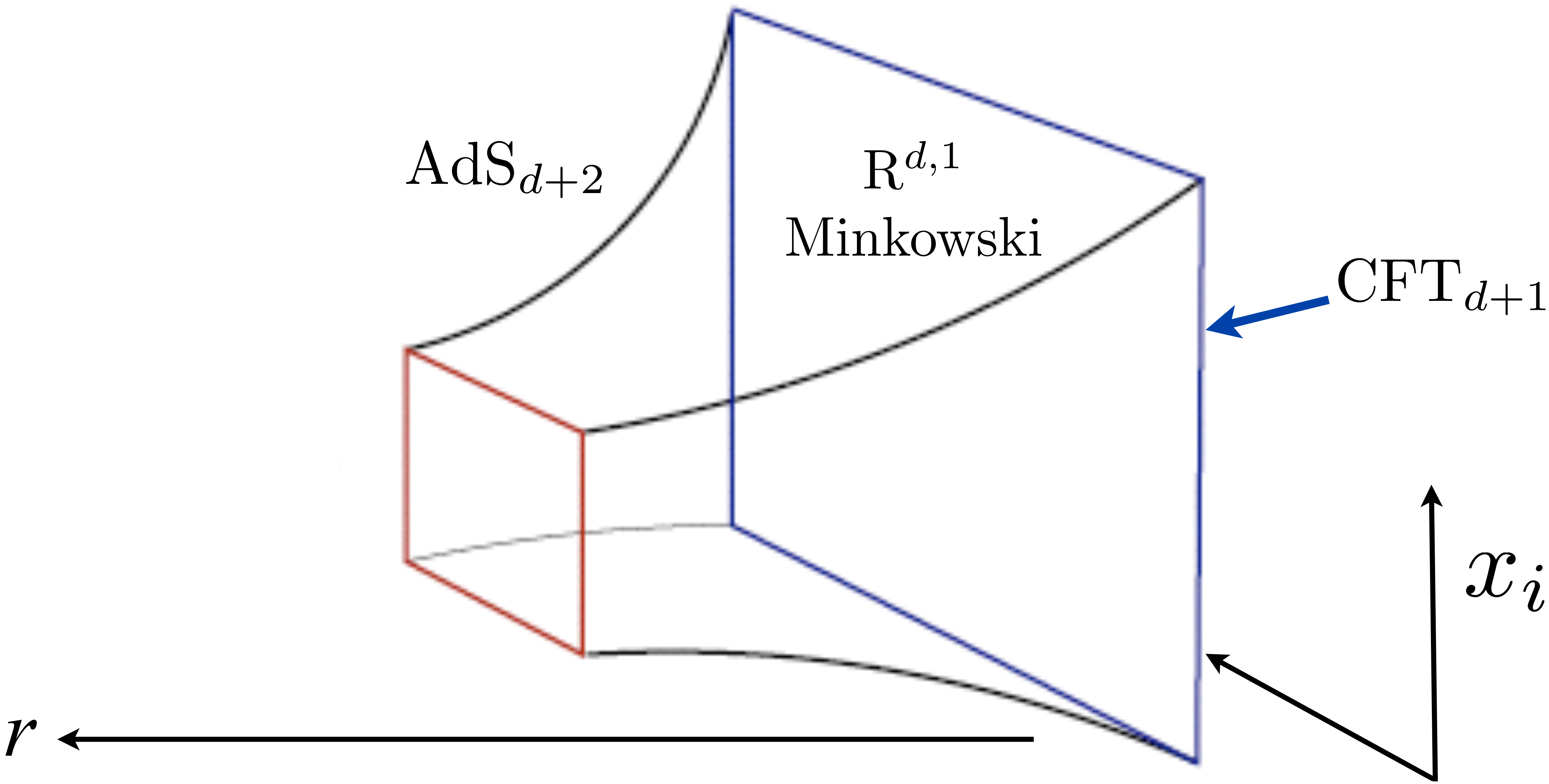
"Memory function" methods yield an explicit expression for τ_{imp}:

$$\frac{\mathcal{M}}{\tau_{\mathrm{imp}}} = \lim_{\omega \to 0} \int d^d q \, |h(q)|^2 q_x^2 \frac{\mathrm{Im} \left(G^{\mathrm{R}}_{\mathcal{O}\mathcal{O}}(q,\omega) \right)_{H_0}}{\omega} + \ldots$$

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
A. Lucas and S. Sachdev, PRB **91**, 195122 (2015)

AdS/CFT correspondence at zero temperature

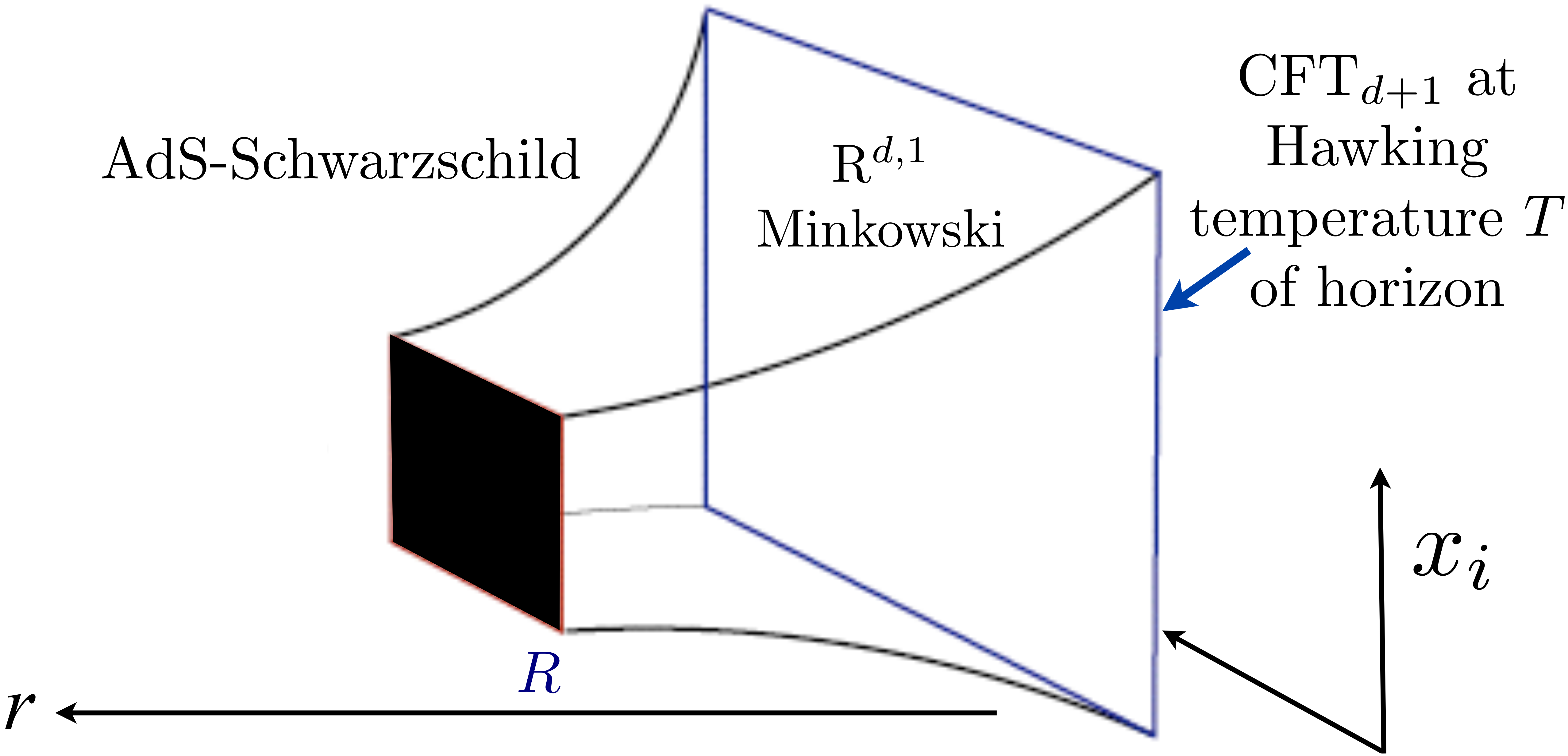
Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[dr^2 - dt^2 + d\vec{x}^2\right]$$

AdS/CFT correspondence at non-zero temperature

Einstein gravity $\mathcal{S}_E = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(\mathcal{R} + \frac{d(d+1)}{L^2}\right)\right]$

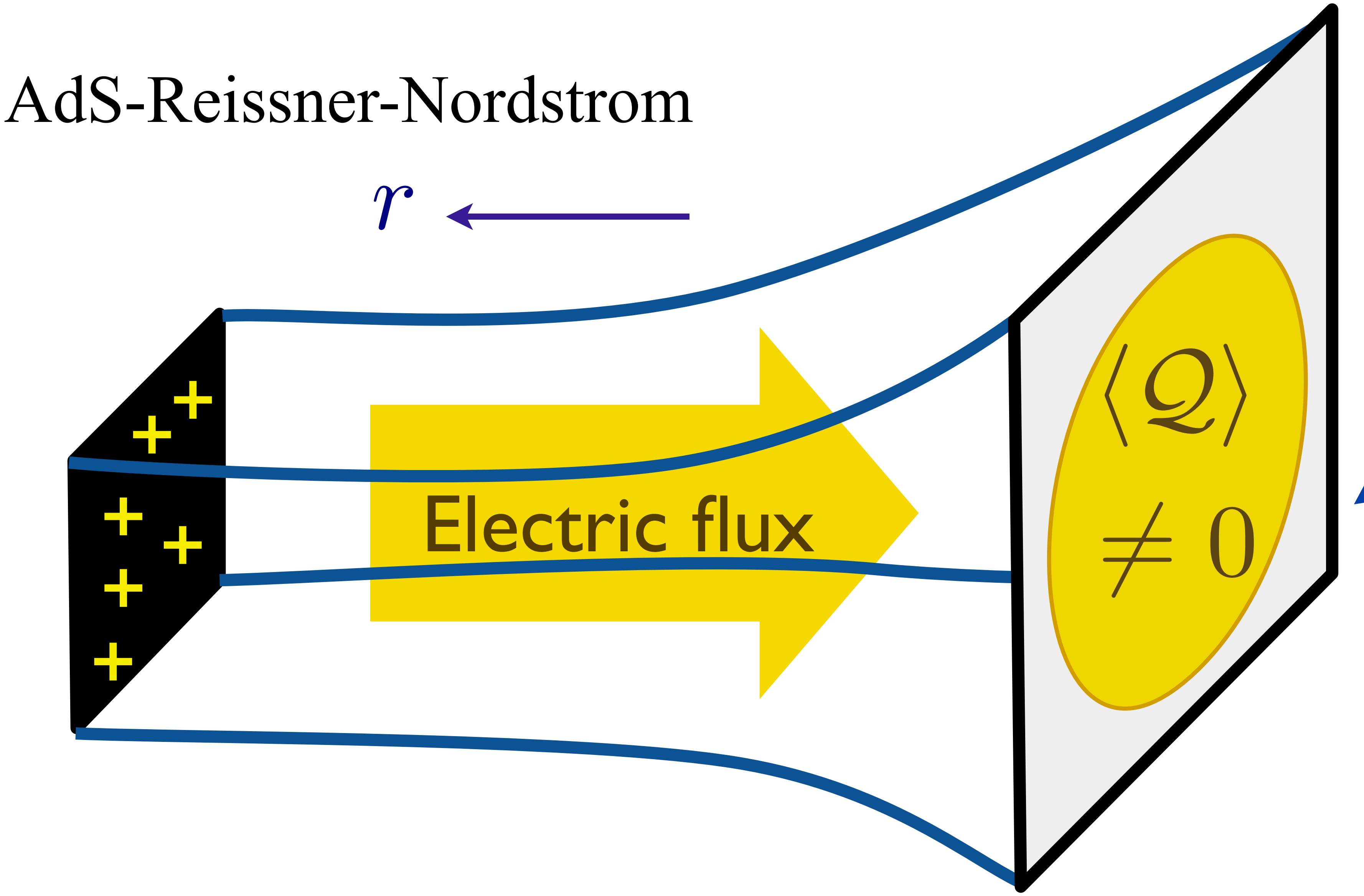


Entropy density of CFT_{d+1}, $\mathcal{S} \sim T^d$

Bekenstein-Hawking entropy density, $\mathcal{S}_{\mathrm{BH}} \sim T^d$

Charged black branes

Einstein-Maxwell theory $\mathcal{S}_{EM} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2} F^2 \right) \right]$

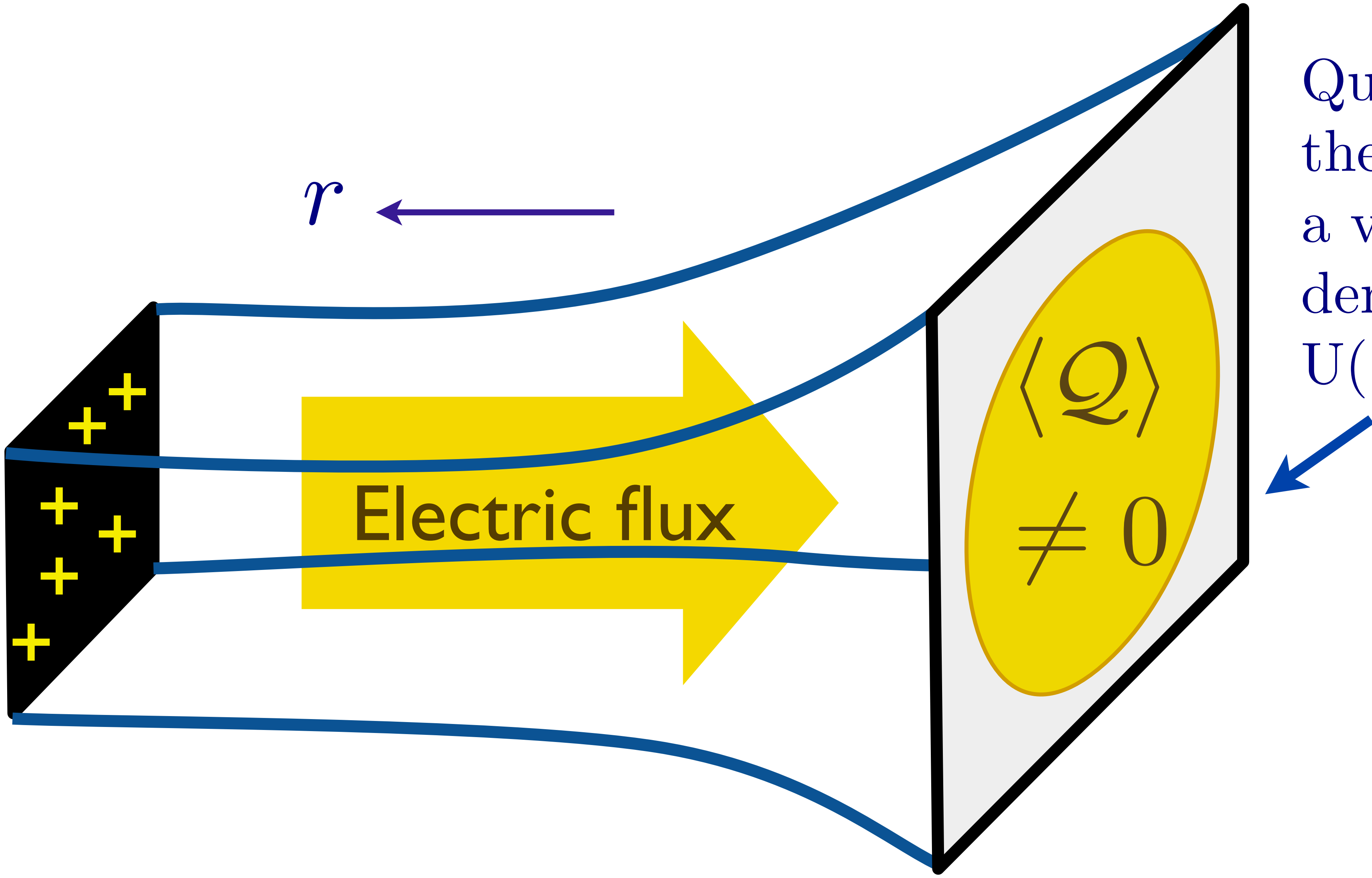


Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, $\mathcal{Q}$, at $T = 0$ which does not have any quasiparticle excitations.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, 99

Charged black branes



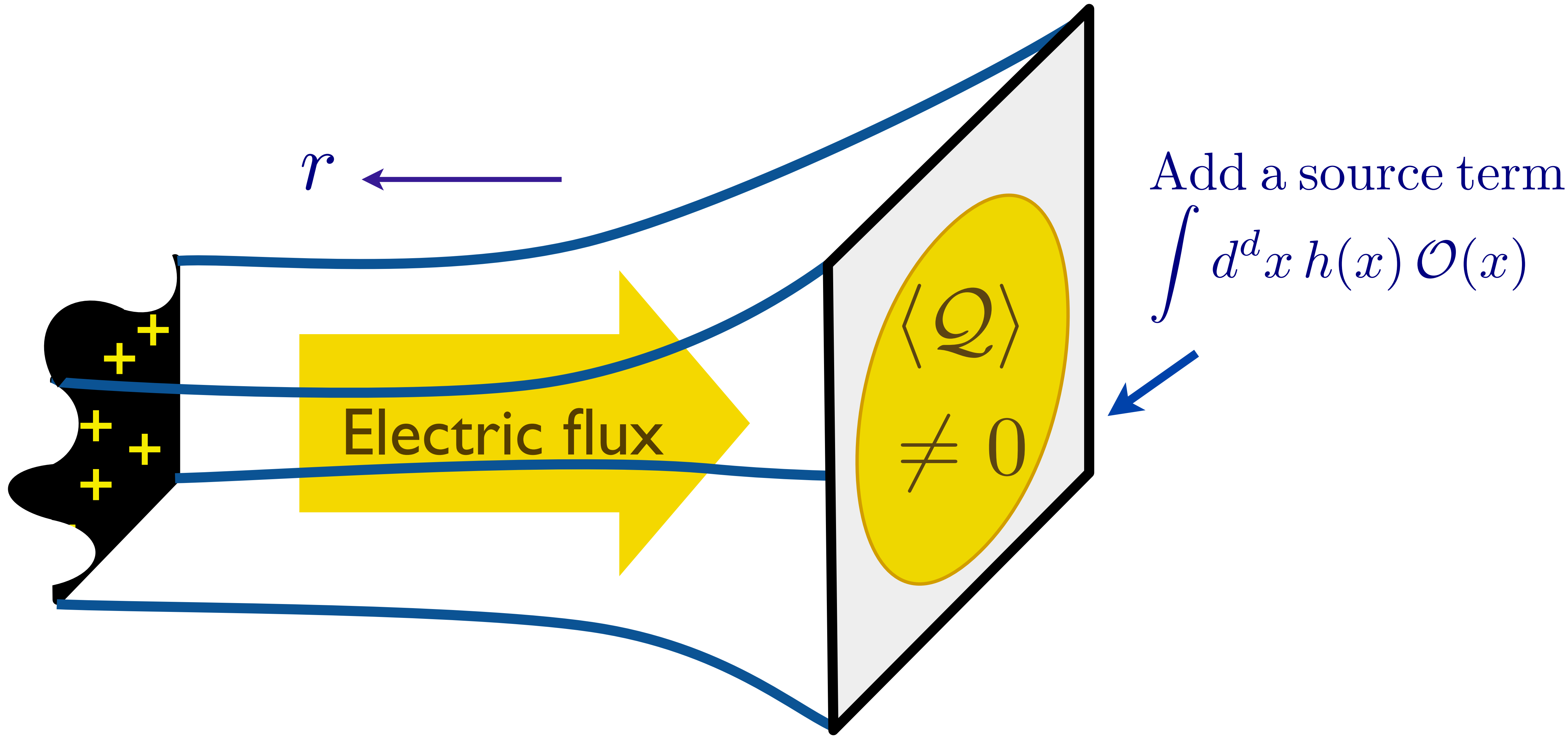
Quantum matter on the boundary with a variable charge density $\mathcal{Q}$ of a global U(1) symmetry.

More general theories have "hyperscaling violating metric"

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right) \quad \text{at } T=0$$

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, JHEP **1201**, 94 (2012).
L. Huijse, S. Sachdev, B. Swingle, Phys. Rev. B **85**, 035121 (2012)

Inhomogeneous charged black branes



Weakly disordered charged black branes yield results identical to those obtained from memory functions and holography

• G.T. Horowitz, J.E. Santos, and D. Tong, JHEP **1207**, 168 (2012), JHEP **1211**, 102 (2012). • D. Vegh, arXiv:1301.0537. • M. Blake, D. Tong, and D. Vegh, PRL **112**, 071602 (2013). • M. Blake and D. Tong, PRD **88**, 106004 (2013). • A. Lucas, S. Sachdev, and K. Schalm, PRD 89, 066018 (2014). • A. Lucas, JHEP **1503**, 071 (2015). • R. A. Davison and B. Goutéraux, arXiv:1505.05092; arXiv:1507.07137. • M. Blake, arXiv:1505.06992; arXiv:1507.04870.

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$$L = \frac{v_F^2 \mathcal{H} \tau_{\text{imp}}}{T^2 \sigma_Q} \left(1 + \frac{e^2 v_F^2 \mathcal{Q}^2 \tau_{\text{imp}}}{\mathcal{H}\sigma_Q}\right)^{-2},$$

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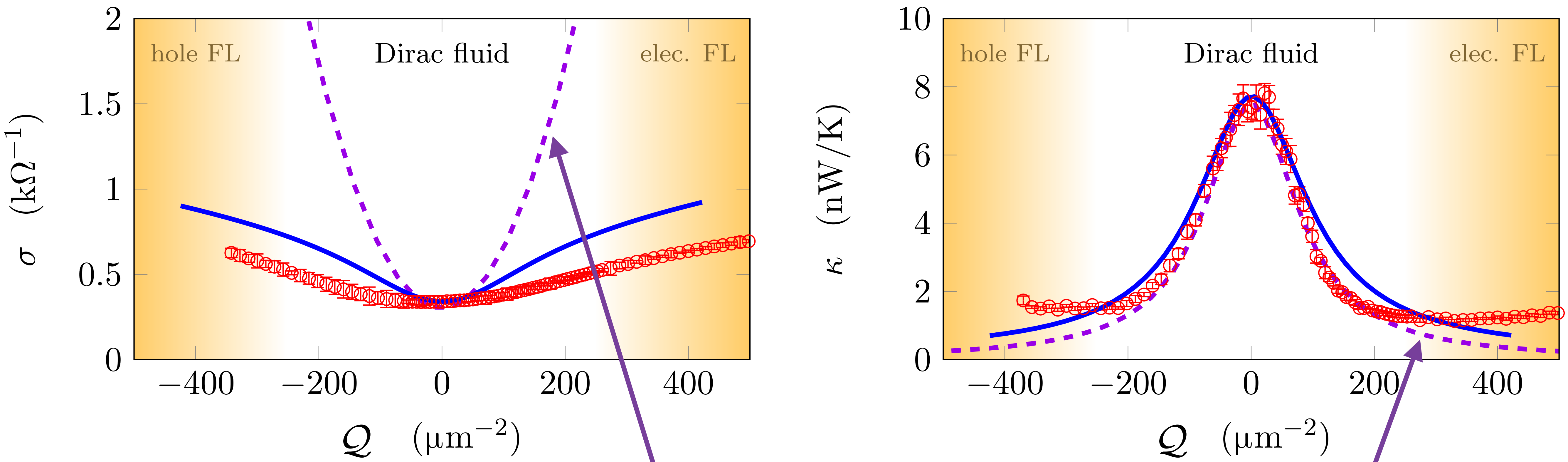
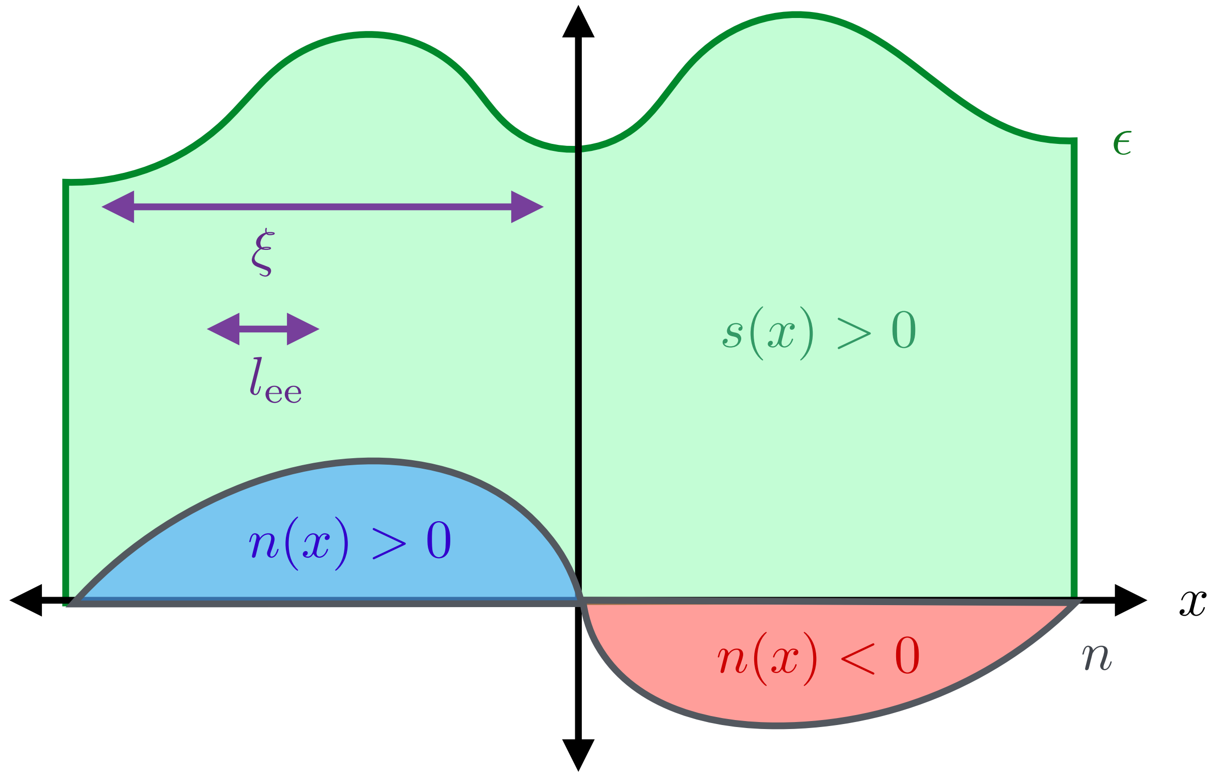
S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007)
M. Müller and S. Sachdev, PRB **78**, 115419 (2008)



Comparison to theory with a single momentum relaxation time τ_{imp}. Best fit of density dependence to thermal conductivity does not capture the density dependence of electrical conductivity

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear

Non-perturbative treatment of disorder

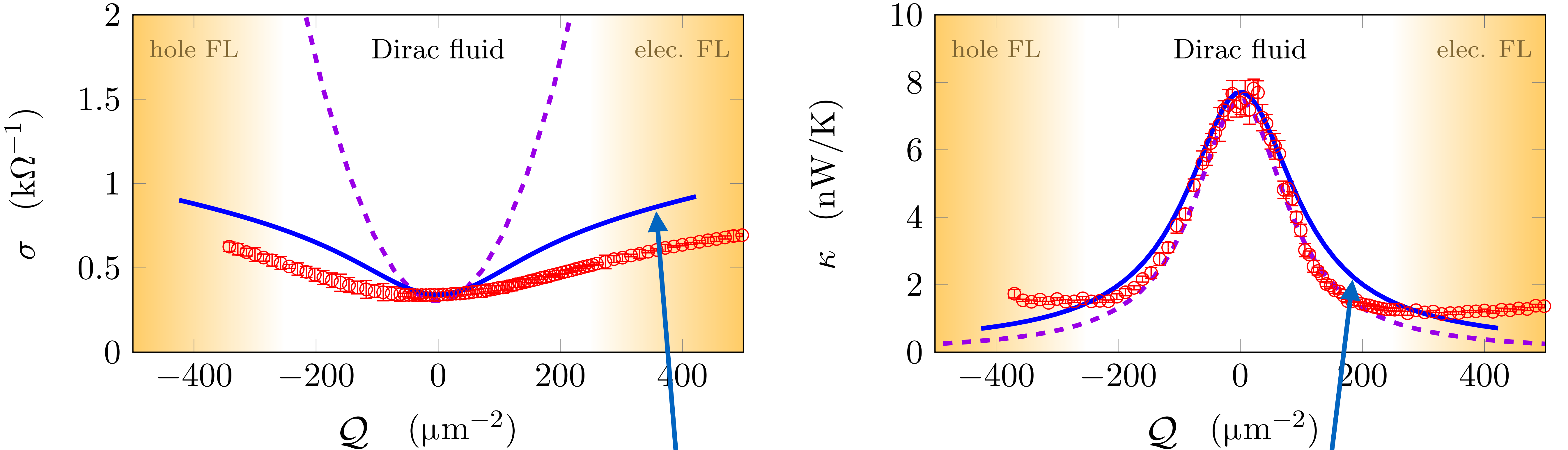


Note
$n \equiv \mathcal{Q}$

Figure 3: A cartoon of a nearly quantum critical fluid where our hydrodynamic description of transport is sensible. The local chemical potential $\mu(\mathbf{x})$ always obeys $|\mu| \ll k_{\rm B}T$, and so the entropy density $s/k_{\rm B}$ is much larger than the charge density $|n|$; both electrons and holes are everywhere excited, and the energy density ϵ does not fluctuate as much relative to the mean. Near charge neutrality the local charge density flips sign repeatedly. The correlation length of disorder ξ is much larger than $l_{\rm ee}$, the electron-electron interaction length.

Numerically solve the hydrodynamic equations in the presence of a x-dependent chemical potential. The thermoelectric transport properties will then depend upon the value of the shear viscosity, η.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear



Solution of the hydrodynamic equations in the presence of a space-dependent chemical potential.
Best fit of density dependence to thermal conductivity now gives a better fit to the density dependence of the electrical conductivity (for $\eta/s \approx 10$). The T dependencies of other parameters also agree well with expectation.

A. Lucas, J. Crossno, K.C. Fong, P. Kim, and S. Sachdev, arXiv:1510.01738, PRB to appear

Quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \dfrac{\hbar}{k_B T}$
- Continuously variable density, $\mathcal{Q}$
  (conformal field theories are usually at fixed density, $\mathcal{Q} = 0$)
- Theory built from hydrodynamics/holography
  /memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.