What can string theory teach us about condensed matter physics?

Department of Physics, UC Berkeley, October 3, 2011

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Outline

- I. Quantum critical points and string theory Entanglement and emergent dimensions
- 2. Some difficult condensed matter questions and answers from string theory "Nearly-perfect" quantum fluids near the superfluid-insulator transition
- 3. High temperature superconductors and strange metals

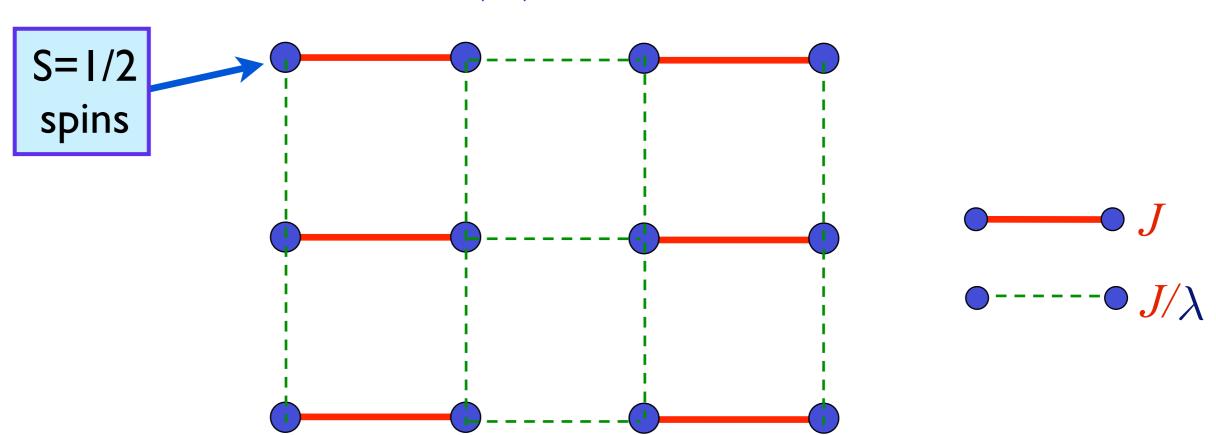
 Holography of compressible quantum phases

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 Holography of compressible quantum phases

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Examine ground state as a function of λ

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$J_{j} = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

At large λ ground state is a "quantum paramagnet" with spins locked in valence bond singlets

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$J$$

$$J$$

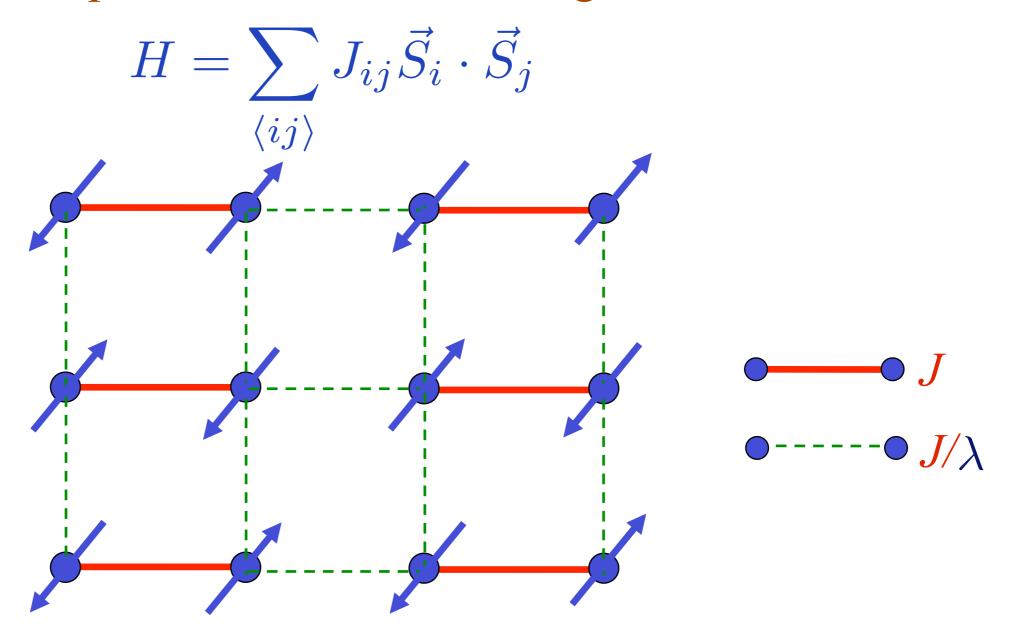
$$J/\lambda$$

$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Nearest-neighor spins are "entangled" with each other. Can be separated into an Einstein-Podolsky-Rosen (EPR) pair.

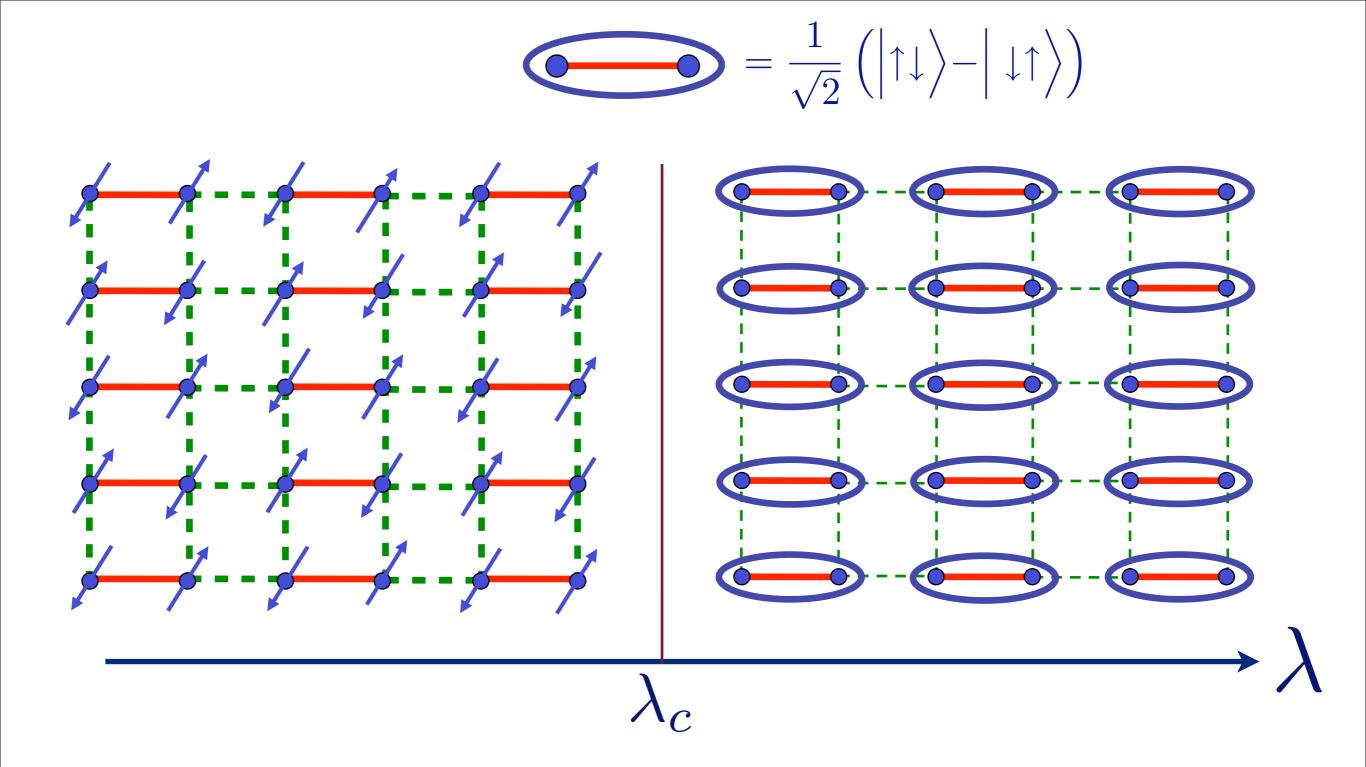
$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

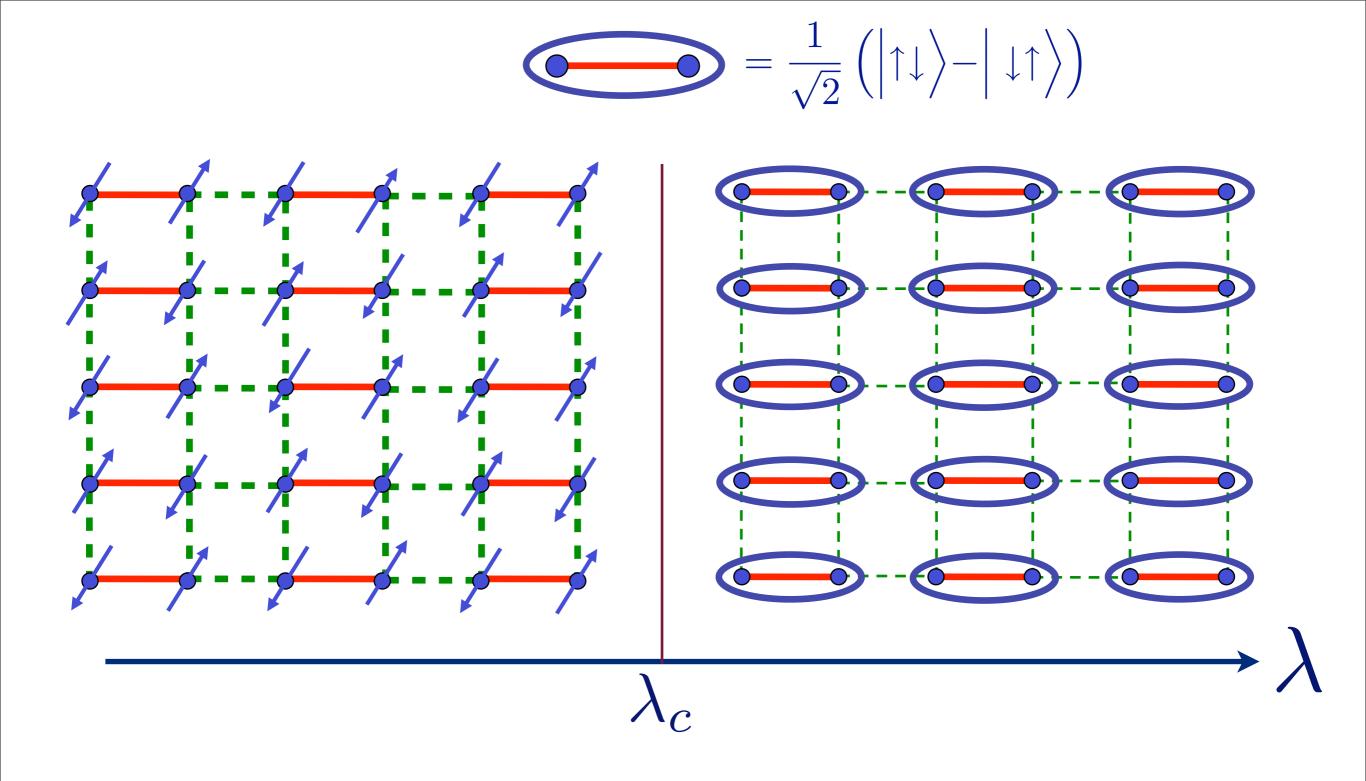
For $\lambda \approx 1$, the ground state has antiferromagnetic ("Néel") order, and the spins align in a checkerboard pattern



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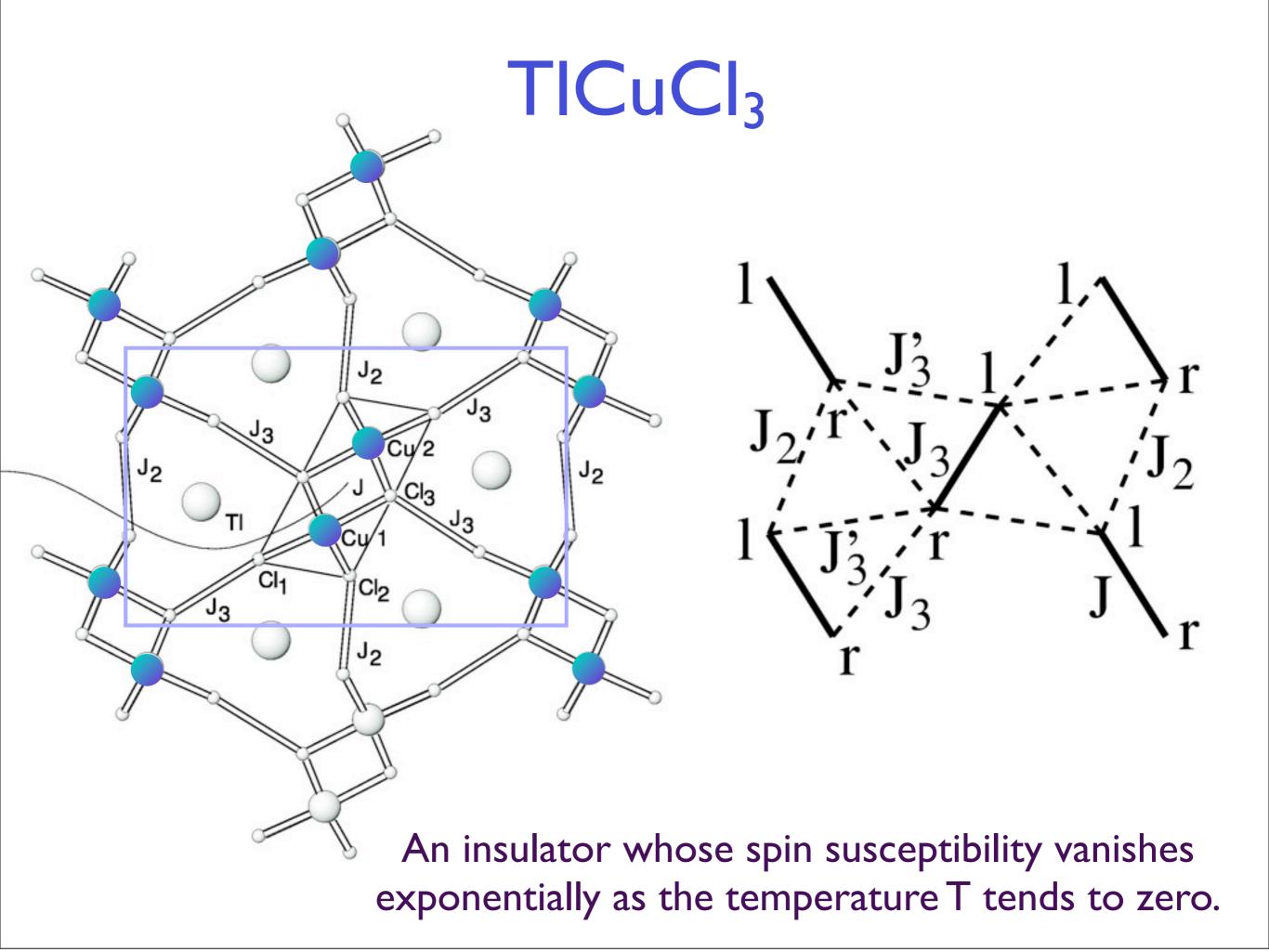
No EPR pairs

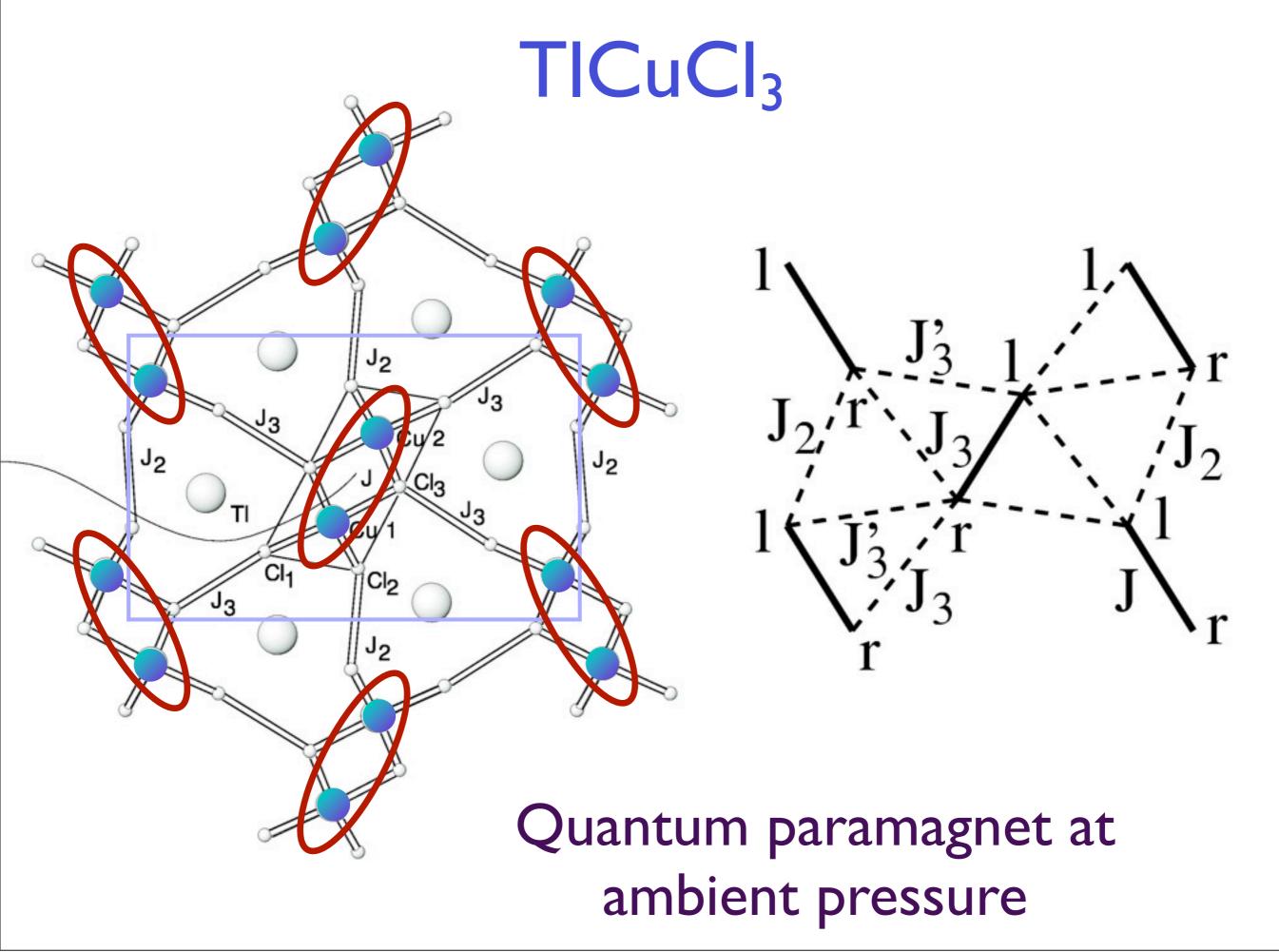


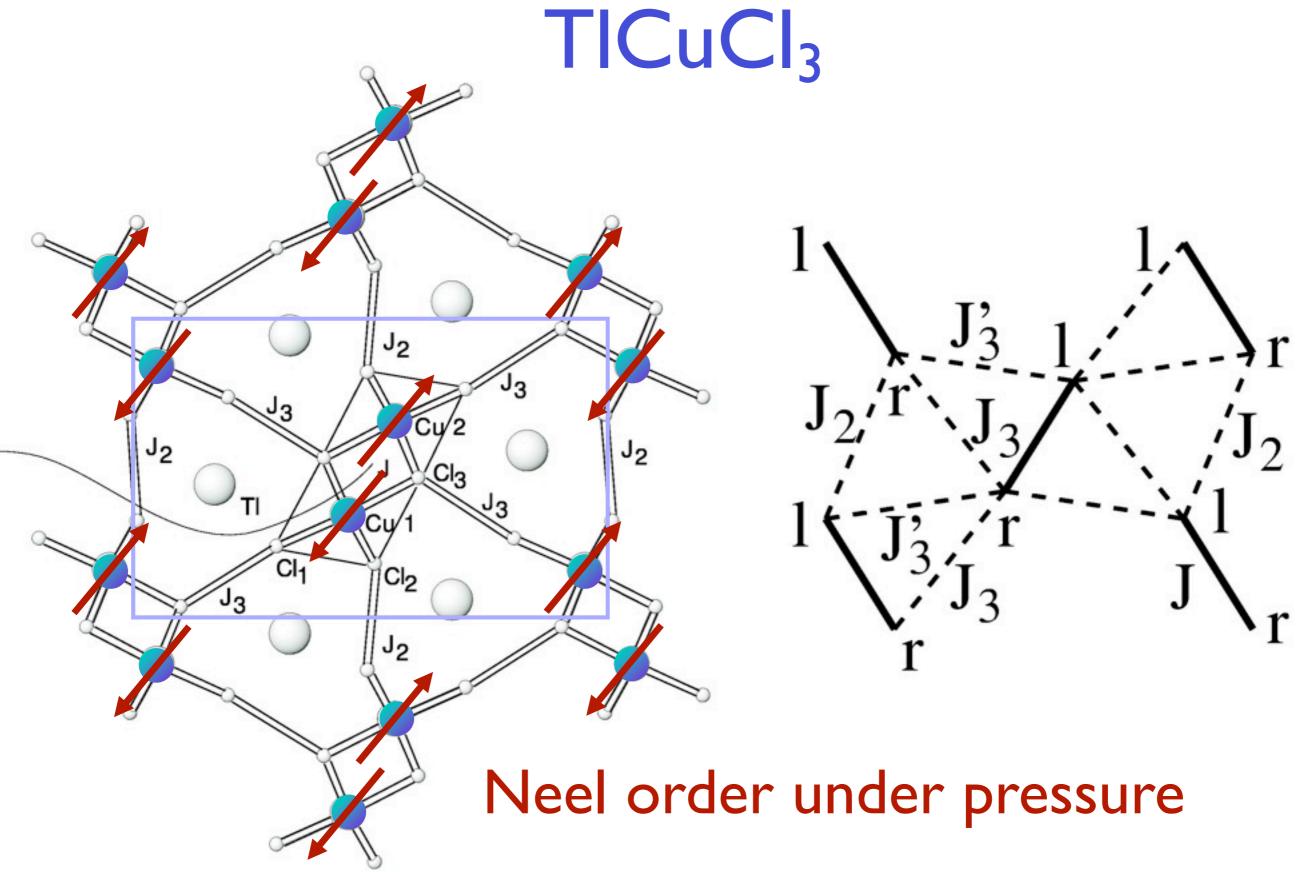




A. Oosawa, K. Kakurai, T. Osakabe, M. Nakamura, M. Takeda, and H. Tanaka, Journal of the Physical Society of Japan, 73, 1446 (2004).

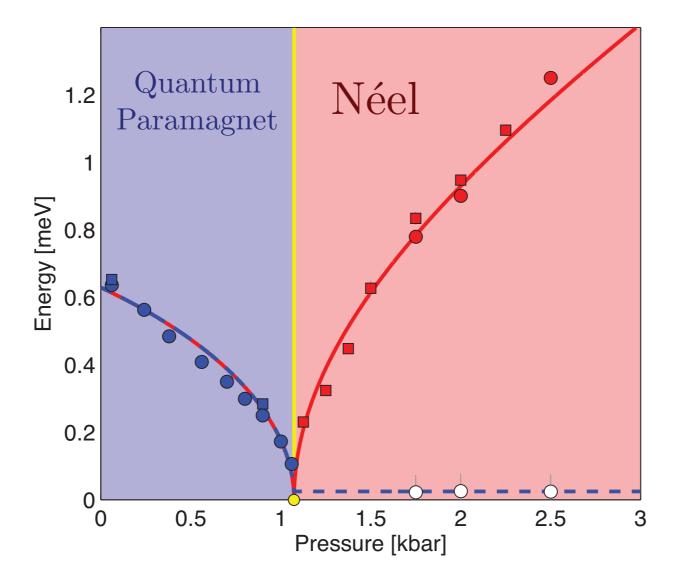






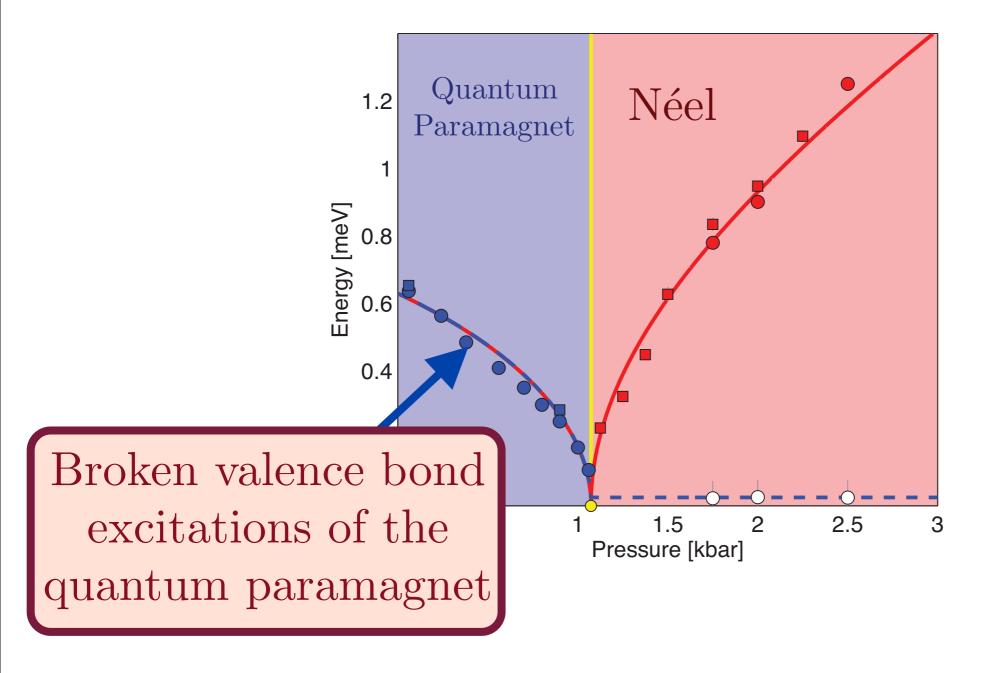
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Excitations of TICuCl₃ with varying pressure



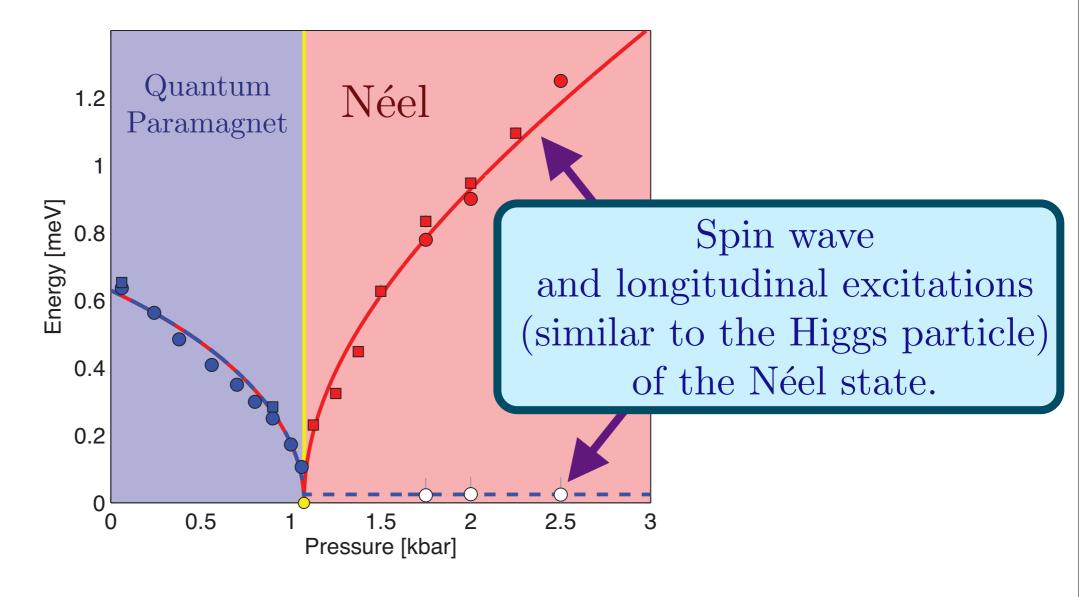
Christian Ruegg, Bruce Normand, Masashige Matsumoto, Albert Furrer, Desmond McMorrow, Karl Kramer, Hans-Ulrich Gudel, Severian Gvasaliya, Hannu Mutka, and Martin Boehm, *Phys. Rev. Lett.* **100**, 205701 (2008)

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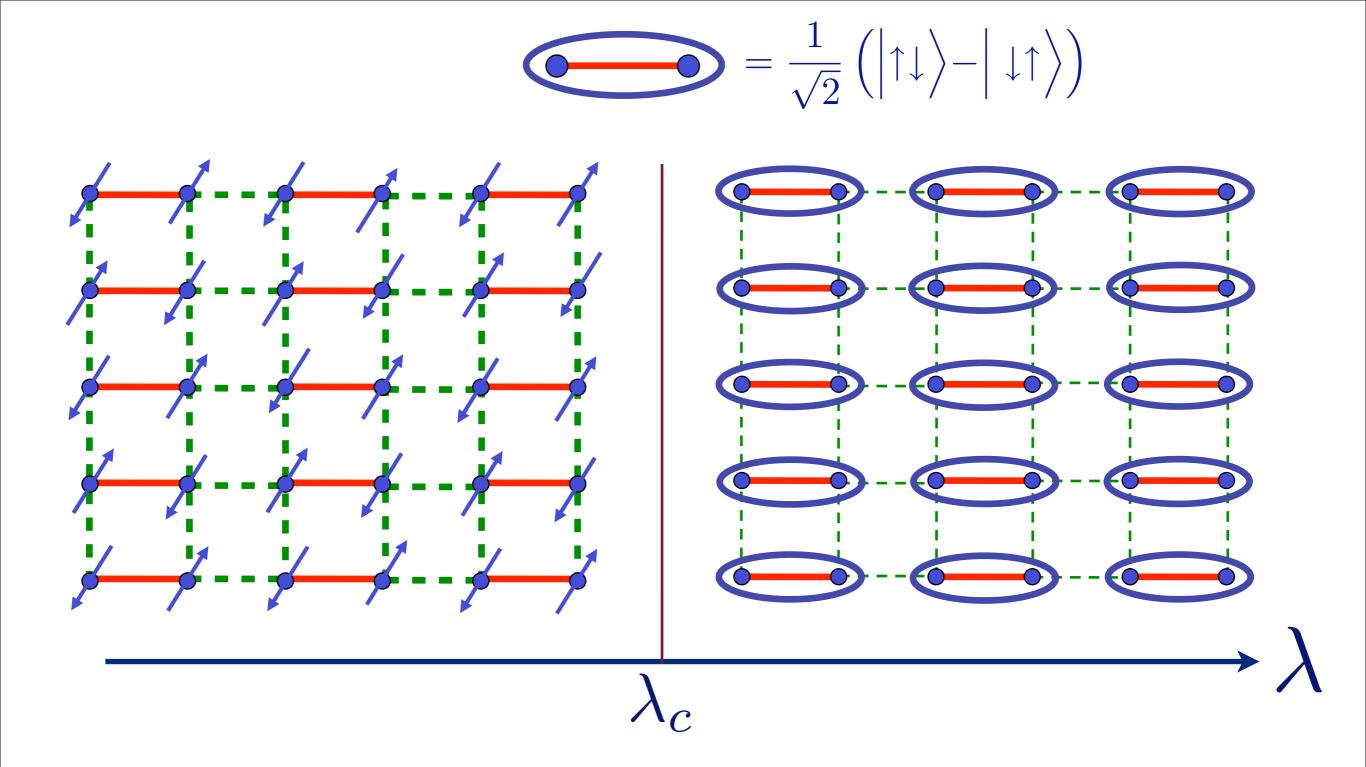


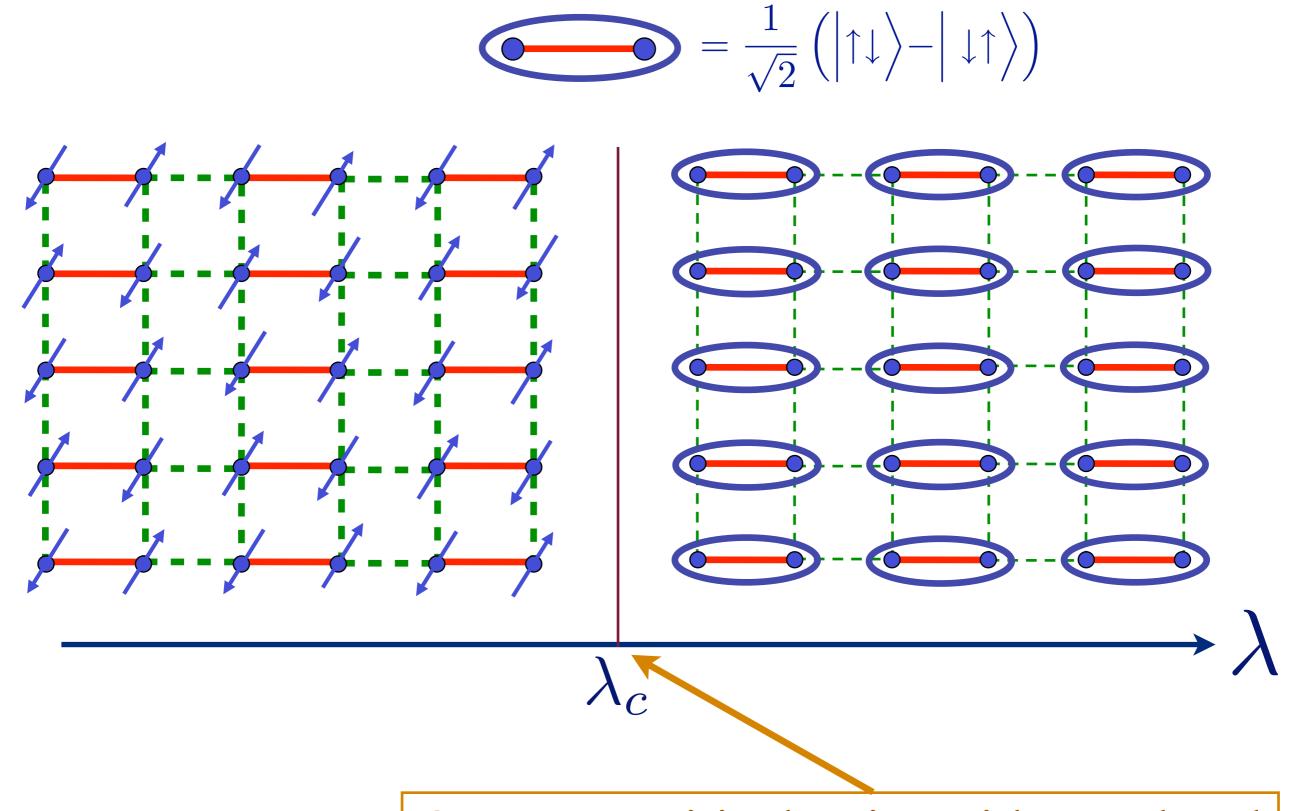
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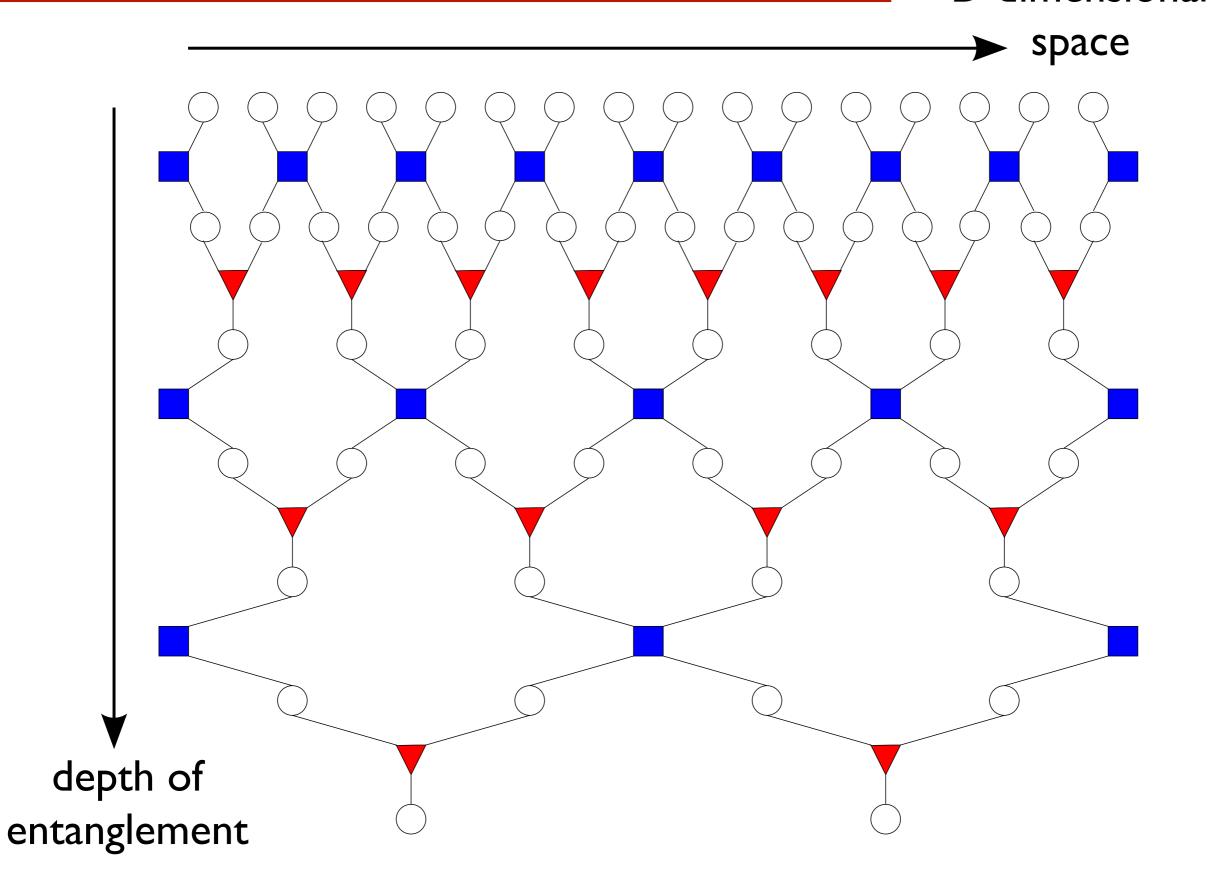




Quantum critical point with non-local entanglement in spin wavefunction

Tensor network representation of entanglement at quantum critical point

D-dimensional



Characteristics of quantum critical point

• Long-range entanglement

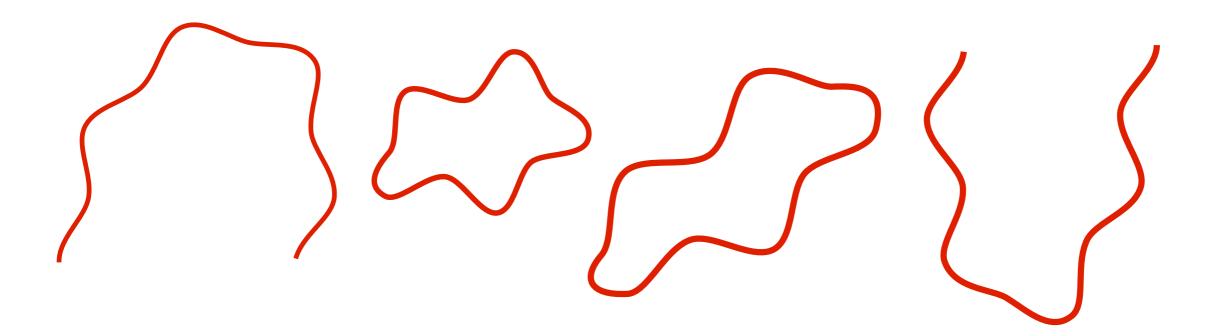
Characteristics of quantum critical point

- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").

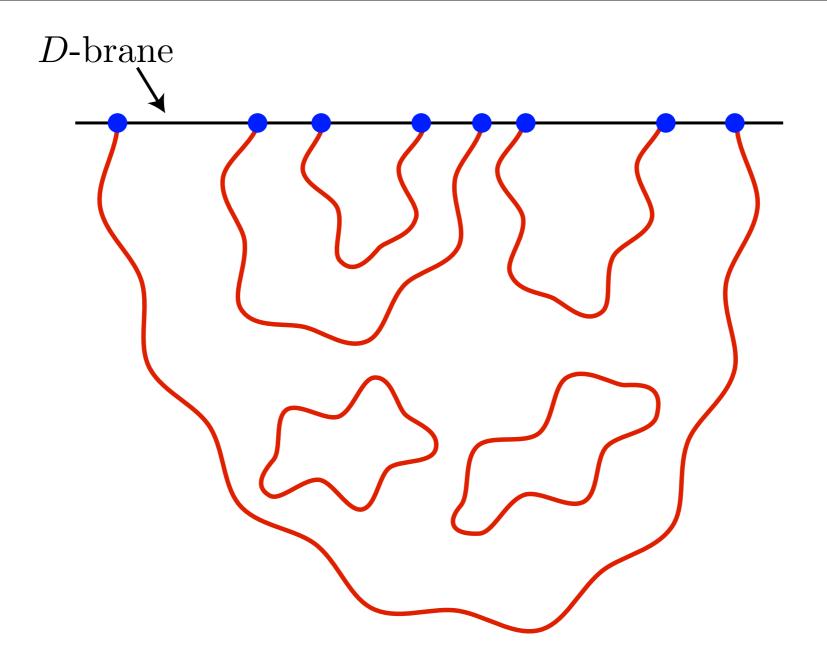
Characteristics of quantum critical point

- Long-range entanglement
- Long distance and low energy correlations near the quantum critical point are described by a quantum field theory which is relativistically invariant (where the spin-wave velocity plays the role of the velocity of "light").
- The quantum field theory is invariant under scale and conformal transformations at the quantum critical point: a **CFT3**

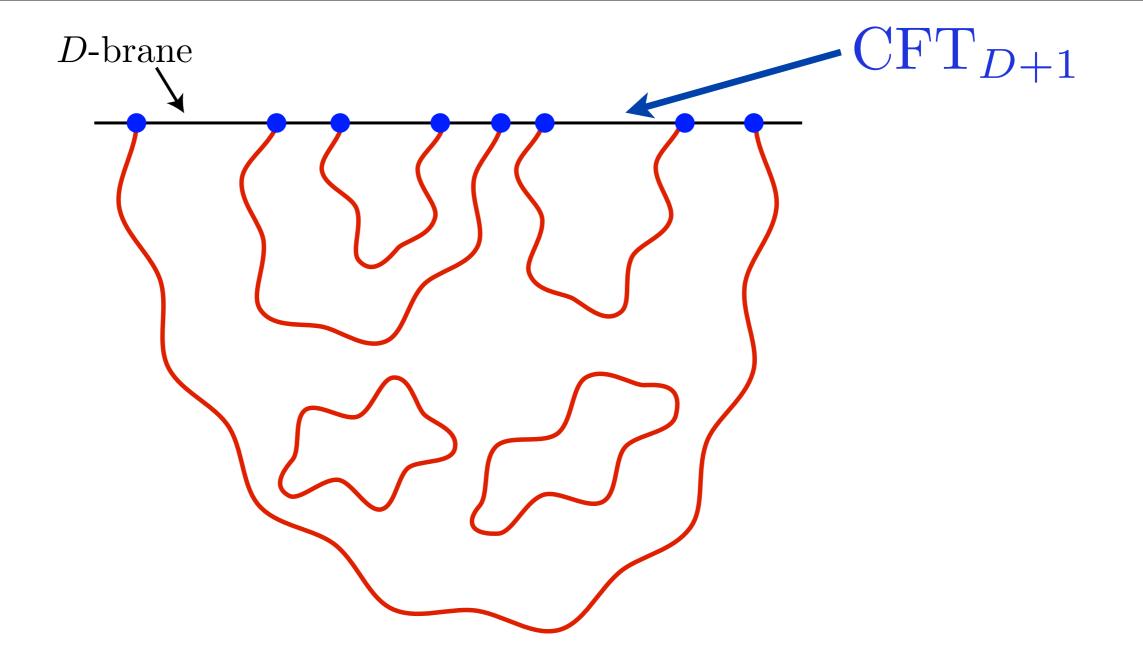
String theory



- Allows unification of the standard model of particle physics with gravity.
- Low-lying string modes correspond to gauge fields, gravitons, quarks . . .



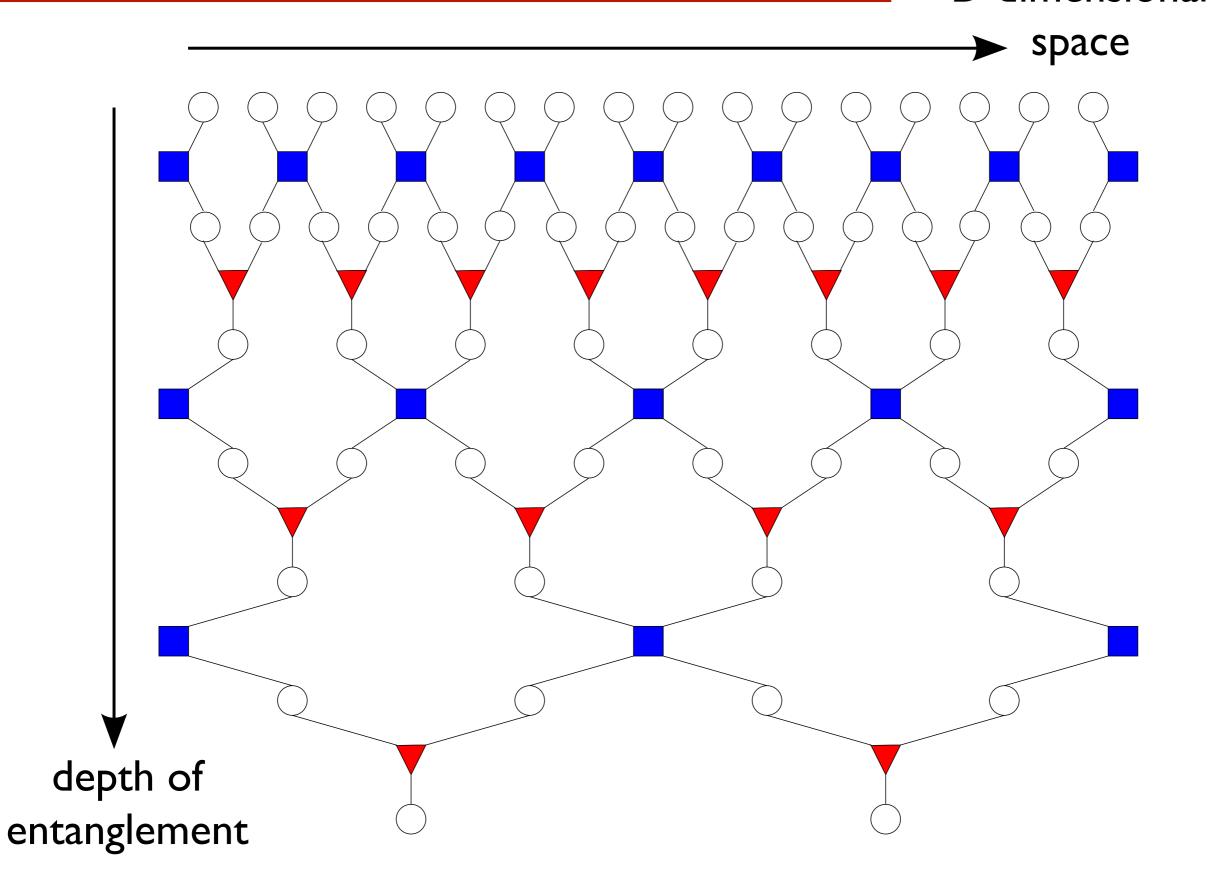
- A *D*-brane is a *D*-dimensional surface on which strings can end.
- The low-energy theory on a *D*-brane is an ordinary quantum field theory with no gravity.



- A D-brane is a D-dimensional surface on which strings can end.
- The low-energy theory on a D-brane is an ordinary quantum field theory with no gravity.
- In D=2, we obtain strongly-interacting **CFT3**s. These are "dual" to string theory on anti-de Sitter space: **AdS4**.

Tensor network representation of entanglement at quantum critical point

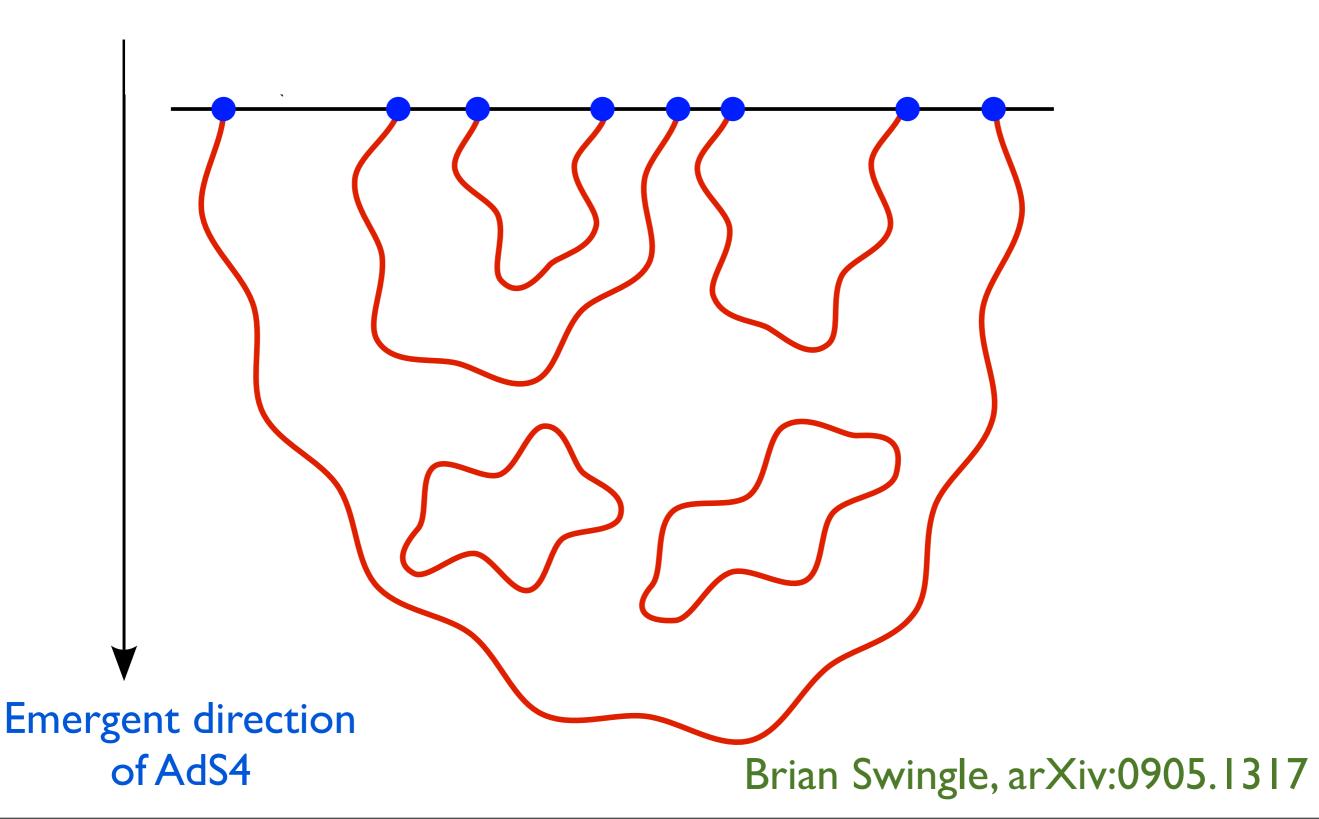
D-dimensional



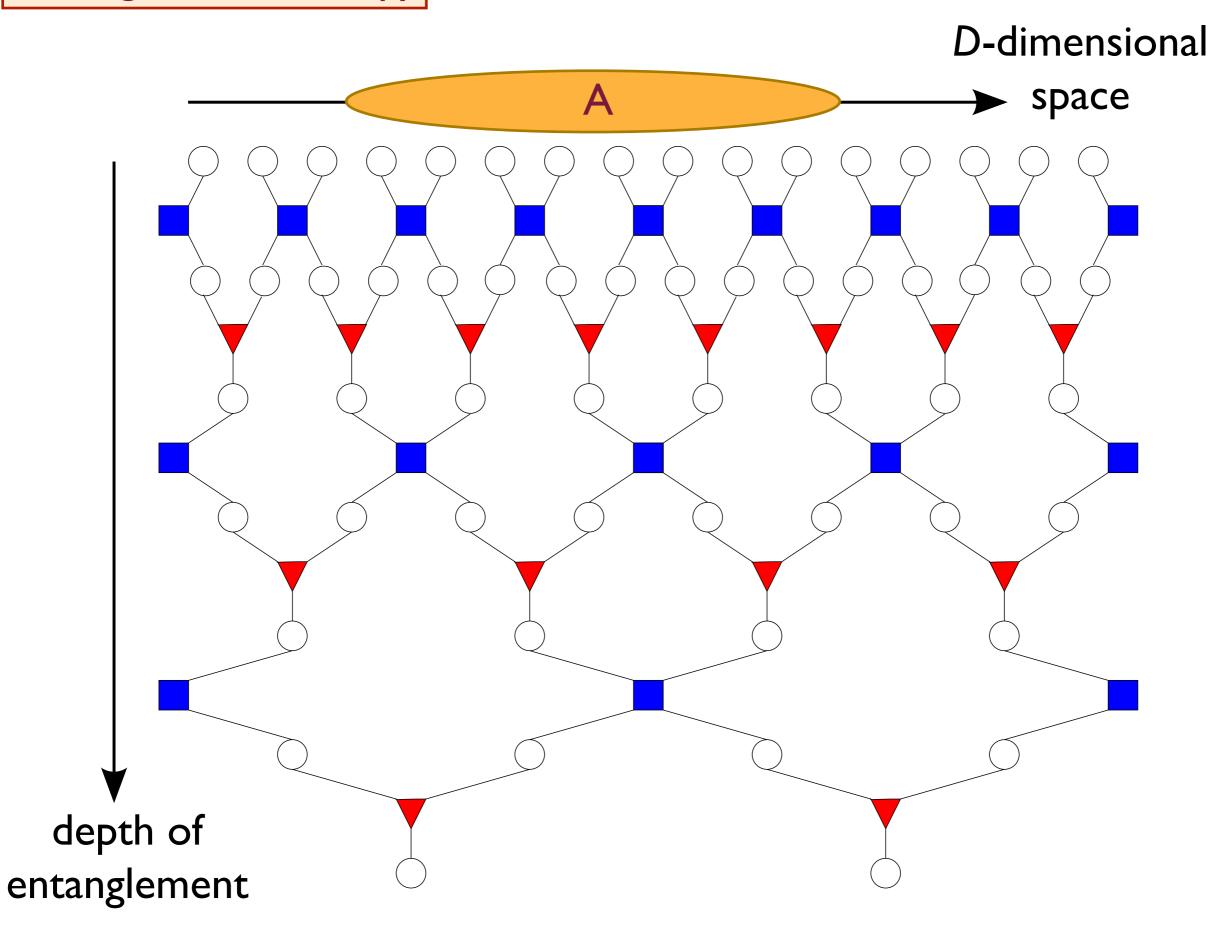
String theory near a D-brane

D-dimensional

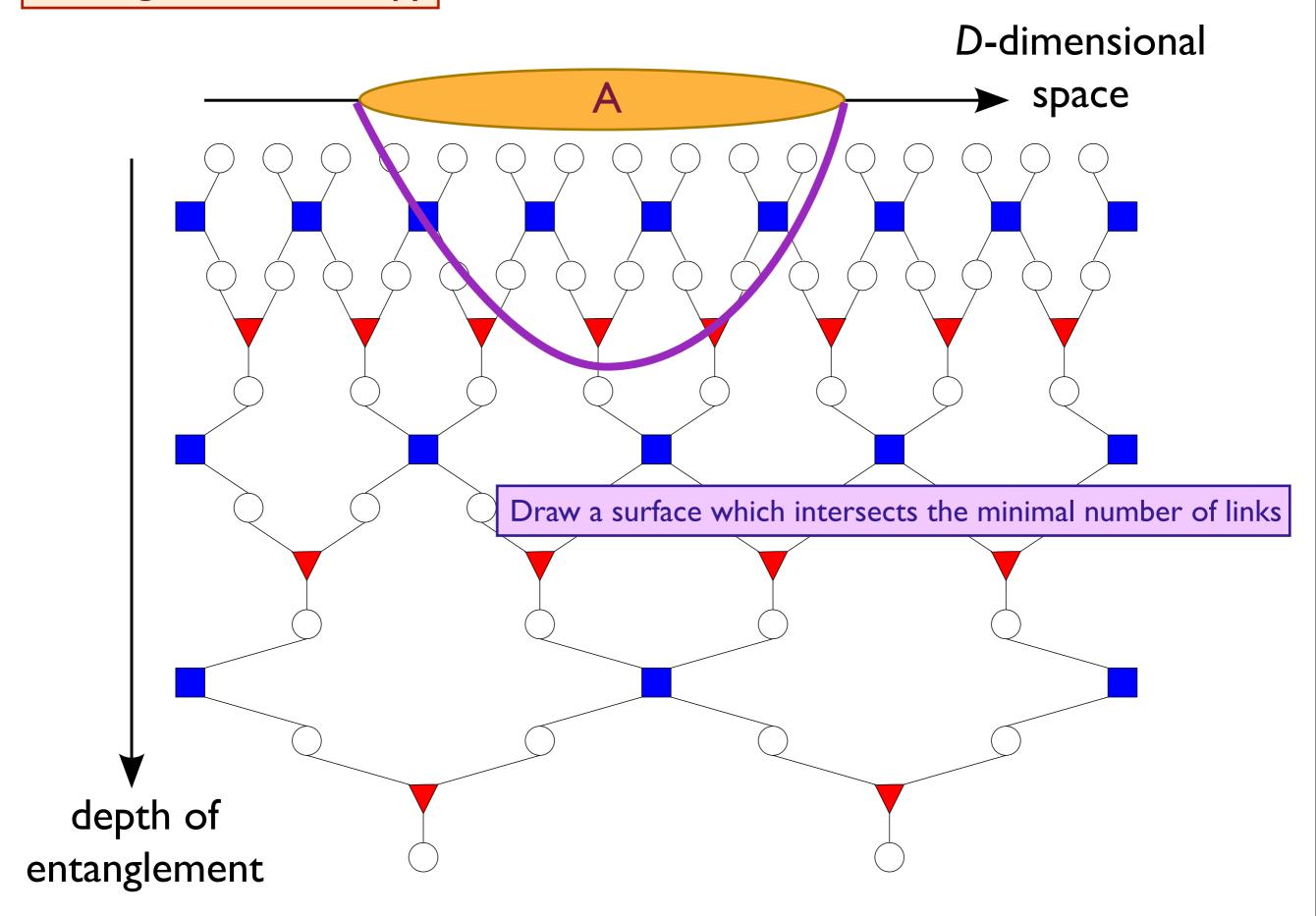
space



Entanglement entropy



Entanglement entropy



Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

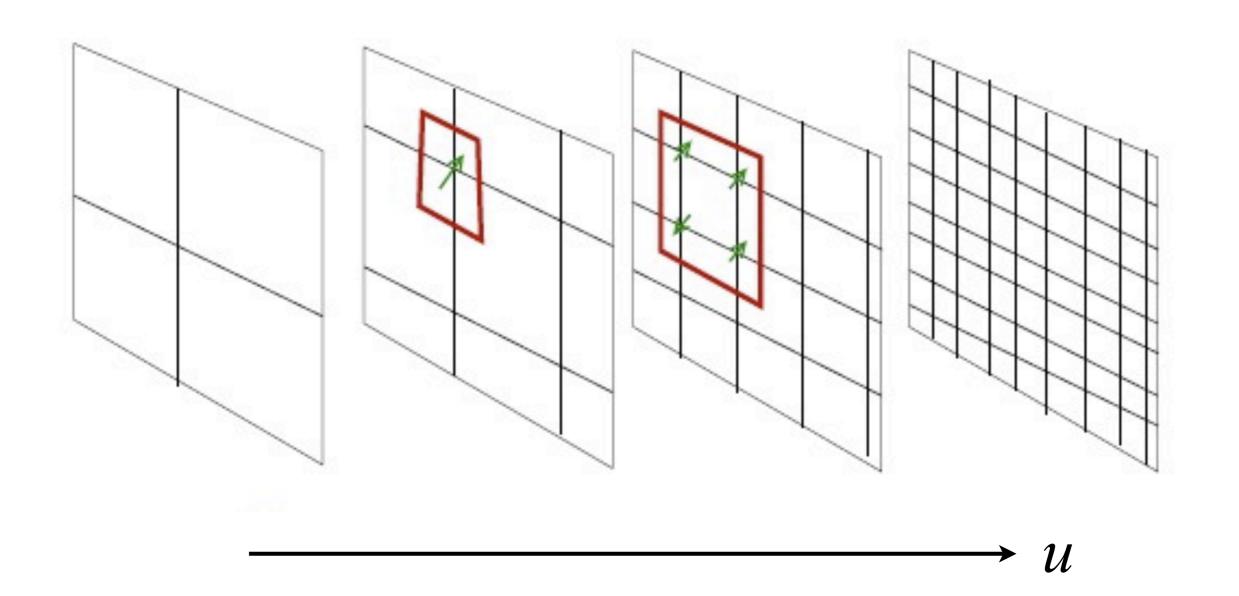
This can be seen both the string and tensor-network pictures

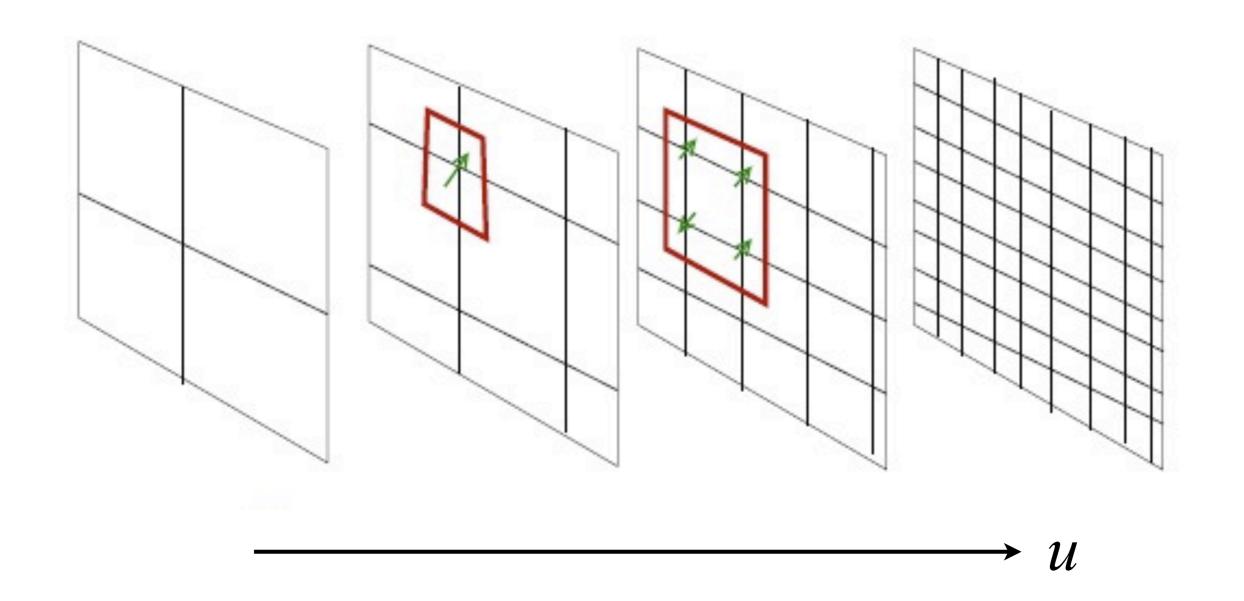
Swingle, Ryu, Takayanagi

Field theories in D+1 spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u\frac{dg}{du} = \beta(g)$$

where u is the energy scale. The RG equation is local in energy scale, i.e. the RHS does not depend upon u.





Key idea: \Rightarrow Implement u as an extra dimension, and map to a local theory in D+2 spacetime dimensions.

At the RG fixed point, $\beta(g) = 0$, the D + 1 dimensional field theory is invariant under the scale transformation

$$x^{\mu} \rightarrow x^{\mu}/b$$
 , $u \rightarrow b u$

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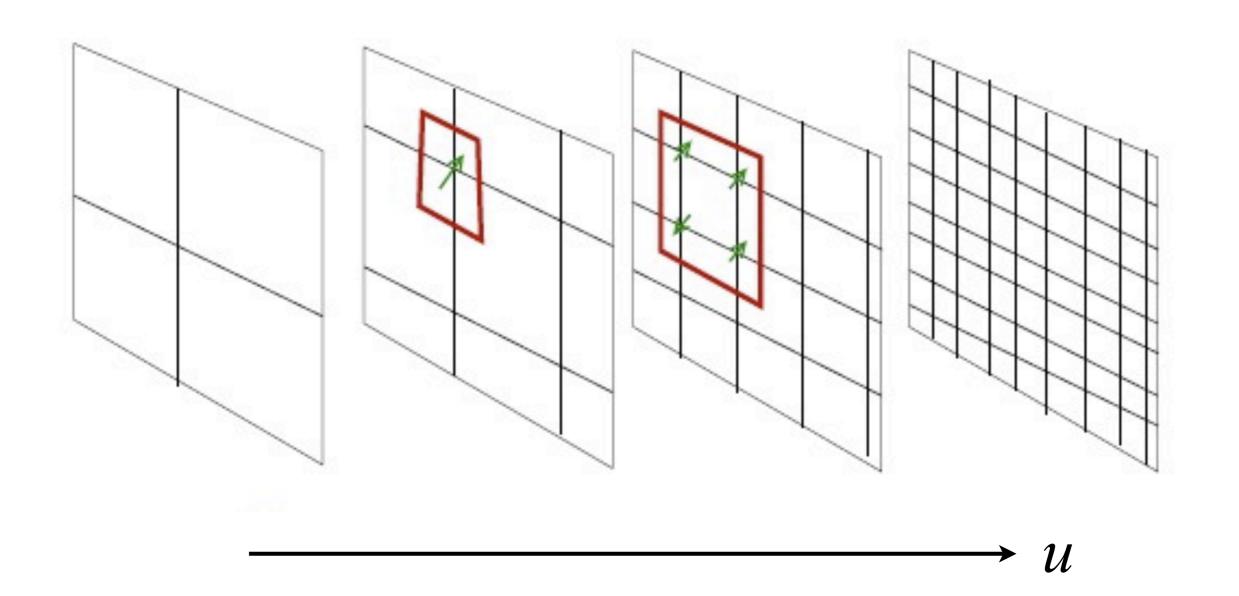
This is an invariance of the metric of the theory in D+2 dimensions. The unique solution is

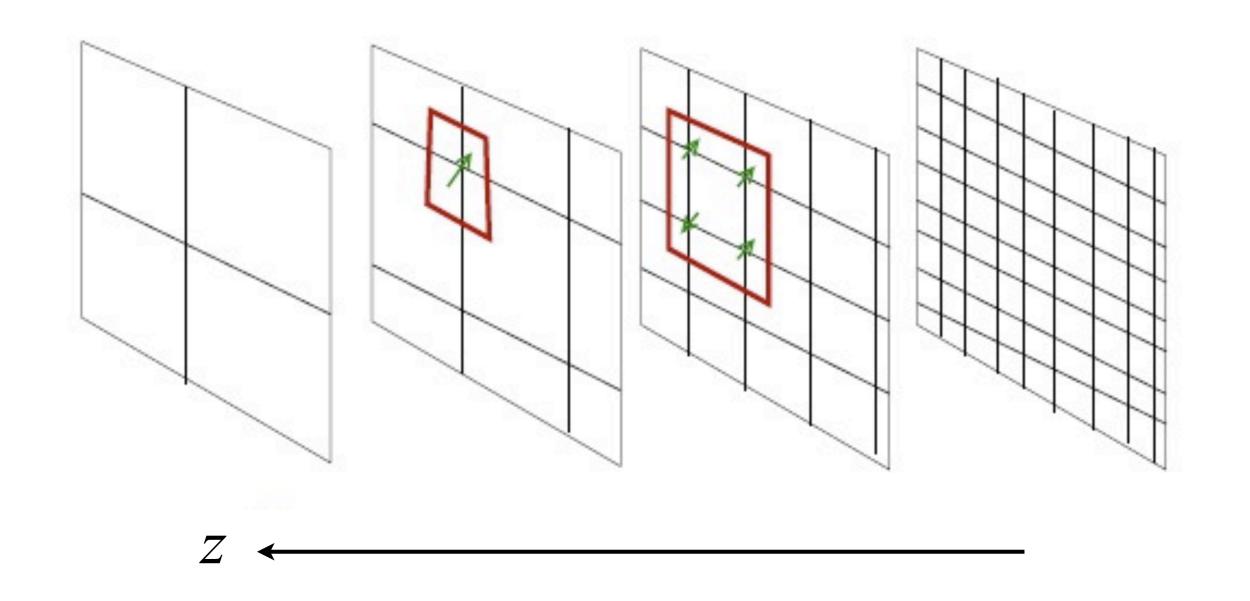
$$ds^{2} = \left(\frac{u}{L}\right)^{2} dx^{\mu} dx_{\mu} + L^{2} \frac{du^{2}}{u^{2}}.$$

Or, using the length scale $z = L^2/u$

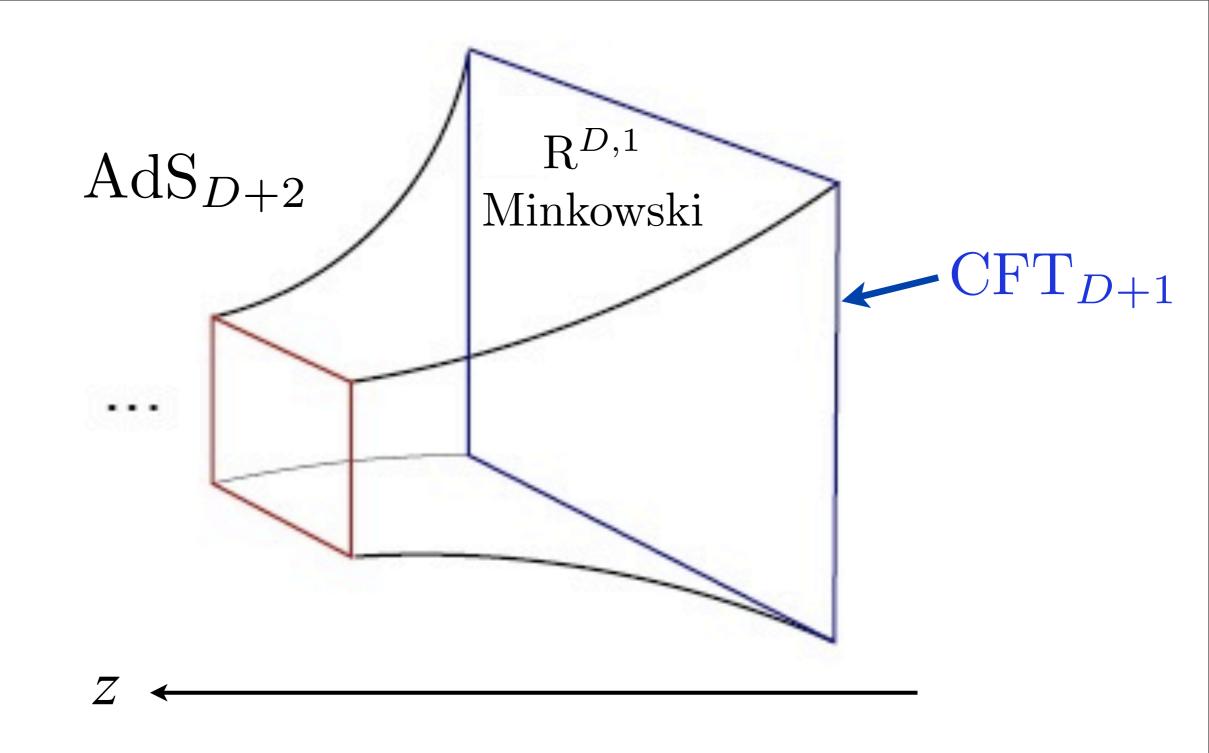
$$ds^{2} = L^{2} \frac{dx^{\mu} dx_{\mu} + dz^{2}}{z^{2}}.$$

This is the space AdS_{D+2} , and L is the AdS radius.





J. McGreevy, arXiv0909.0518



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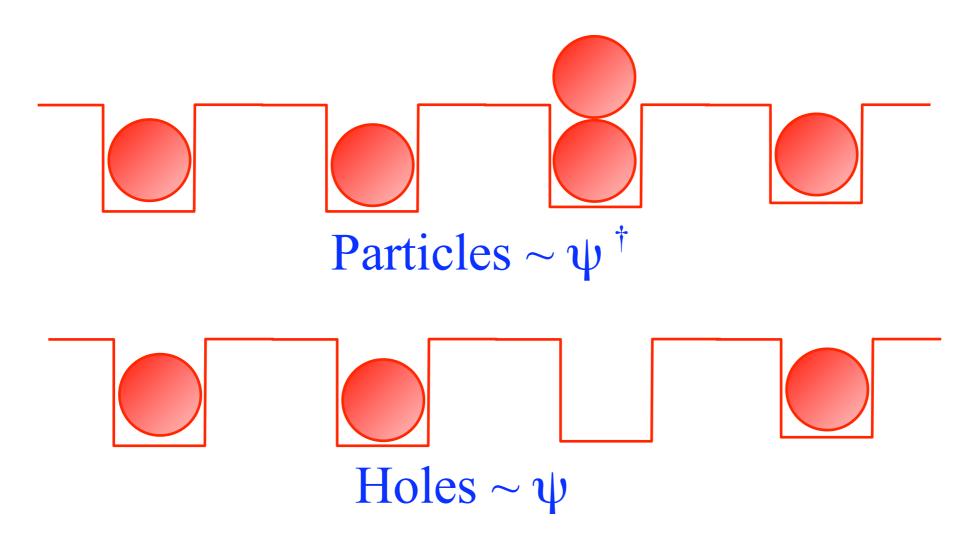
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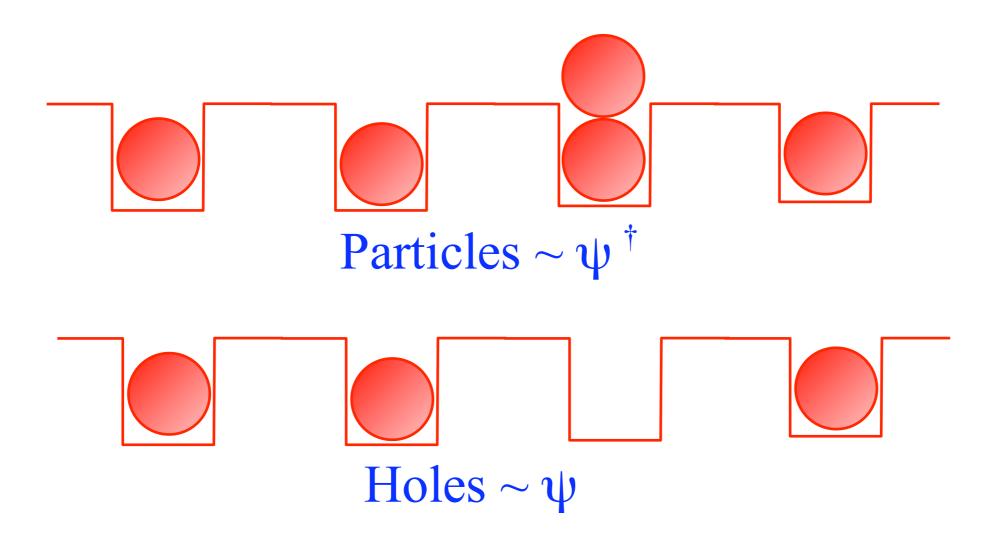
Superfluid-insulator transition Superfluid state b Insulating state Ultracold ⁸⁷Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* 415, 39 (2002).

Excitations of the insulator:

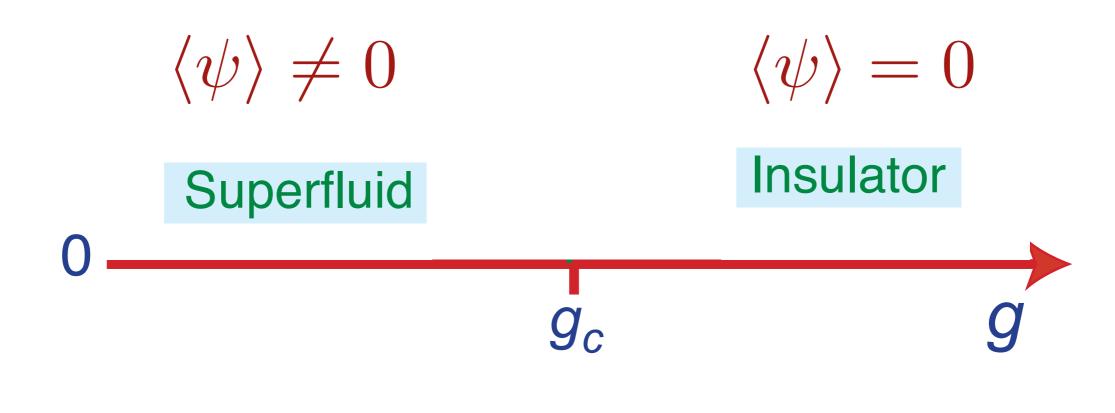


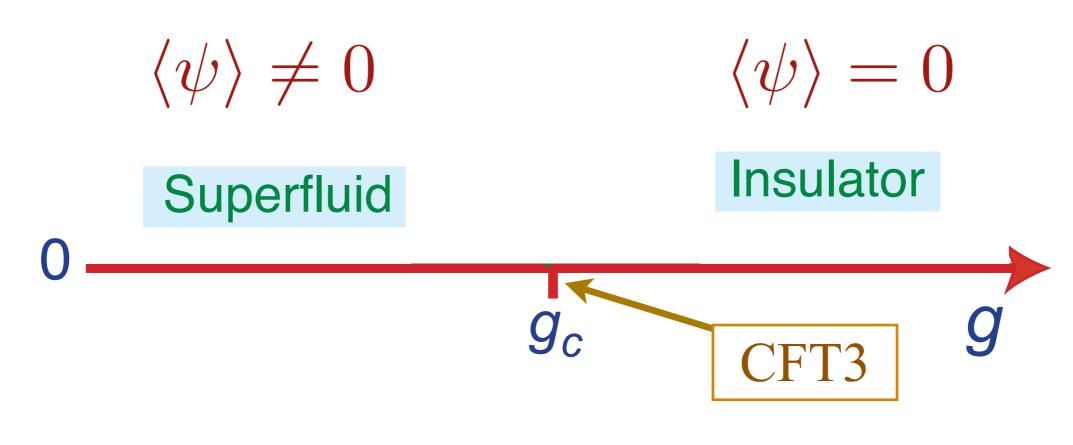
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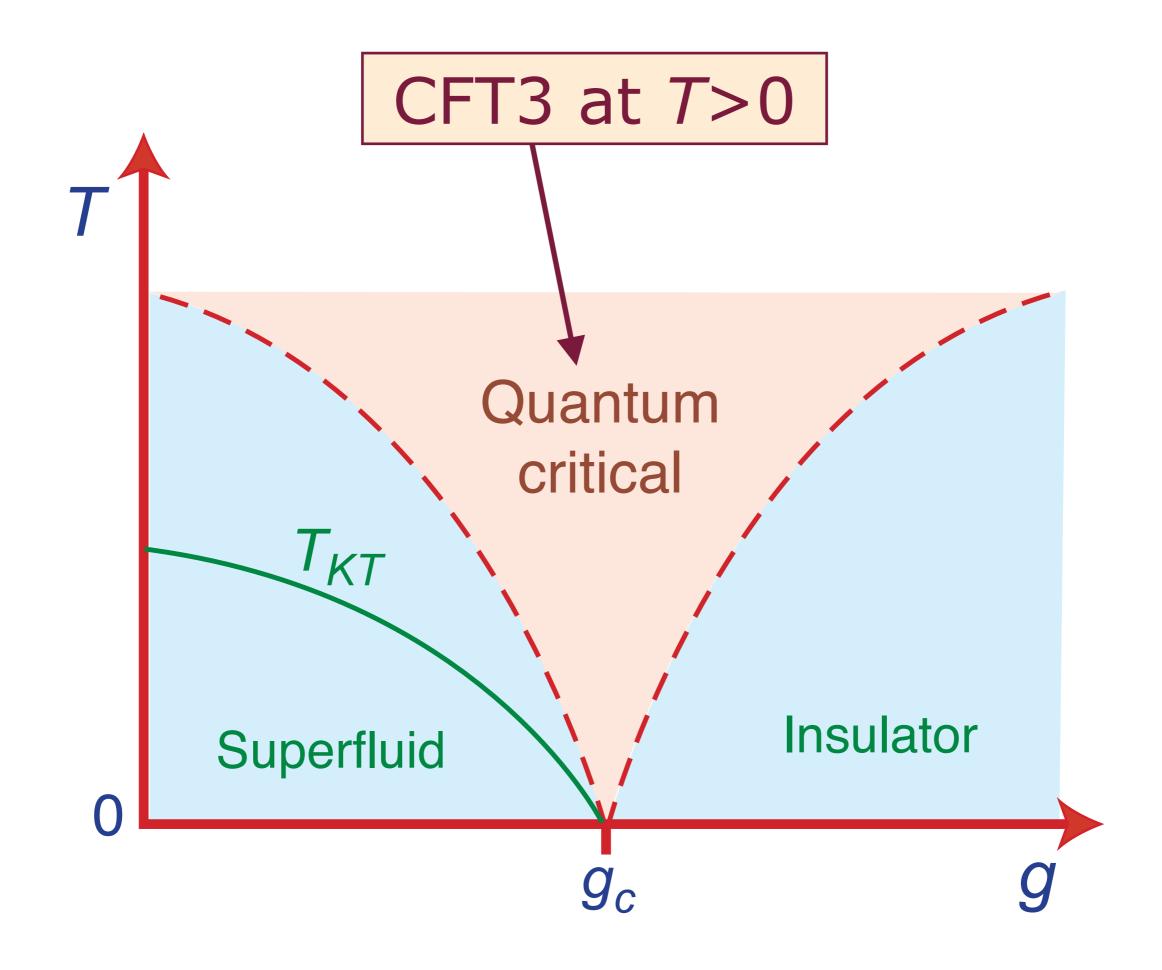


Density of particles = density of holes \Rightarrow "relativistic" field theory for ψ :

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, Phys. Rev. B 40, 546 (1989).







Quantum "nearly perfect fluid" with shortest possible equilibration time, $\tau_{\rm eq}$

$$au_{
m eq} = \mathcal{C} rac{\hbar}{k_B T}$$

where \mathcal{C} is a universal constant

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett.* **64**, 587 (1990) K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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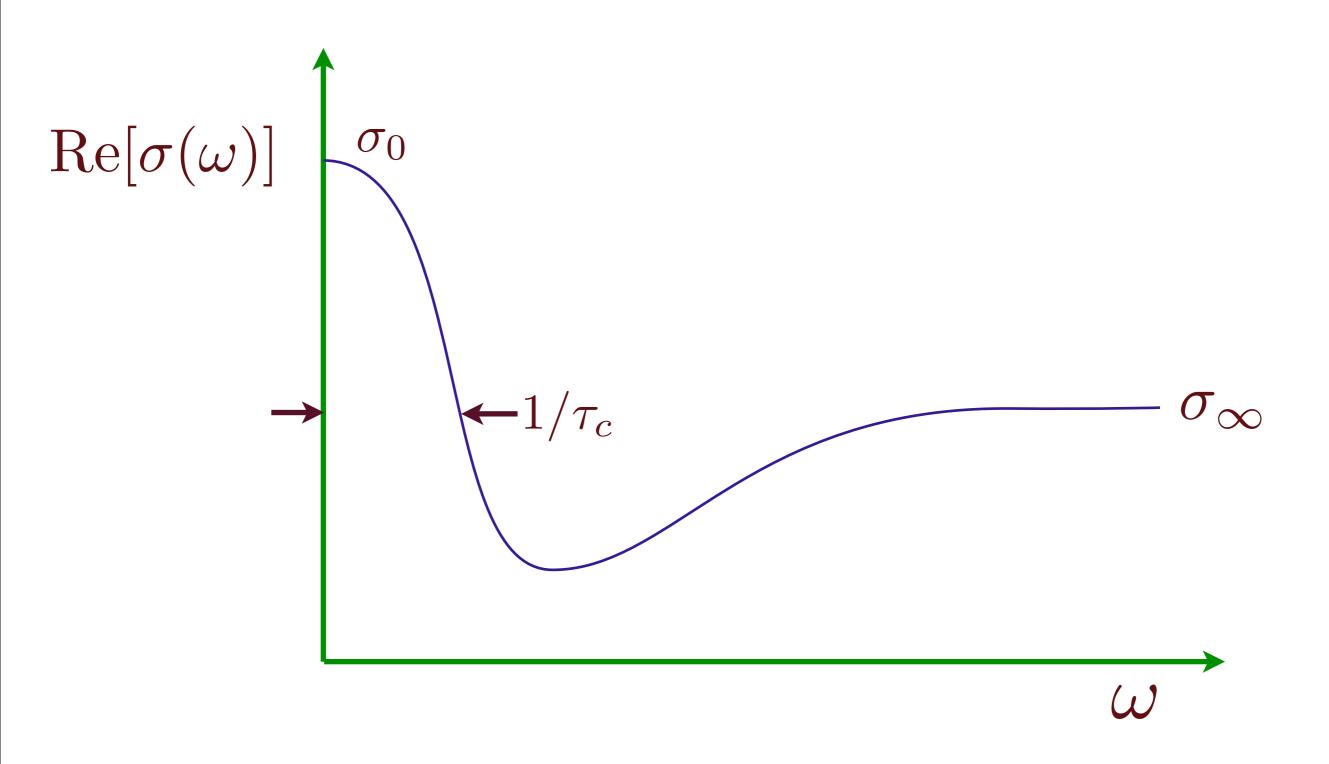
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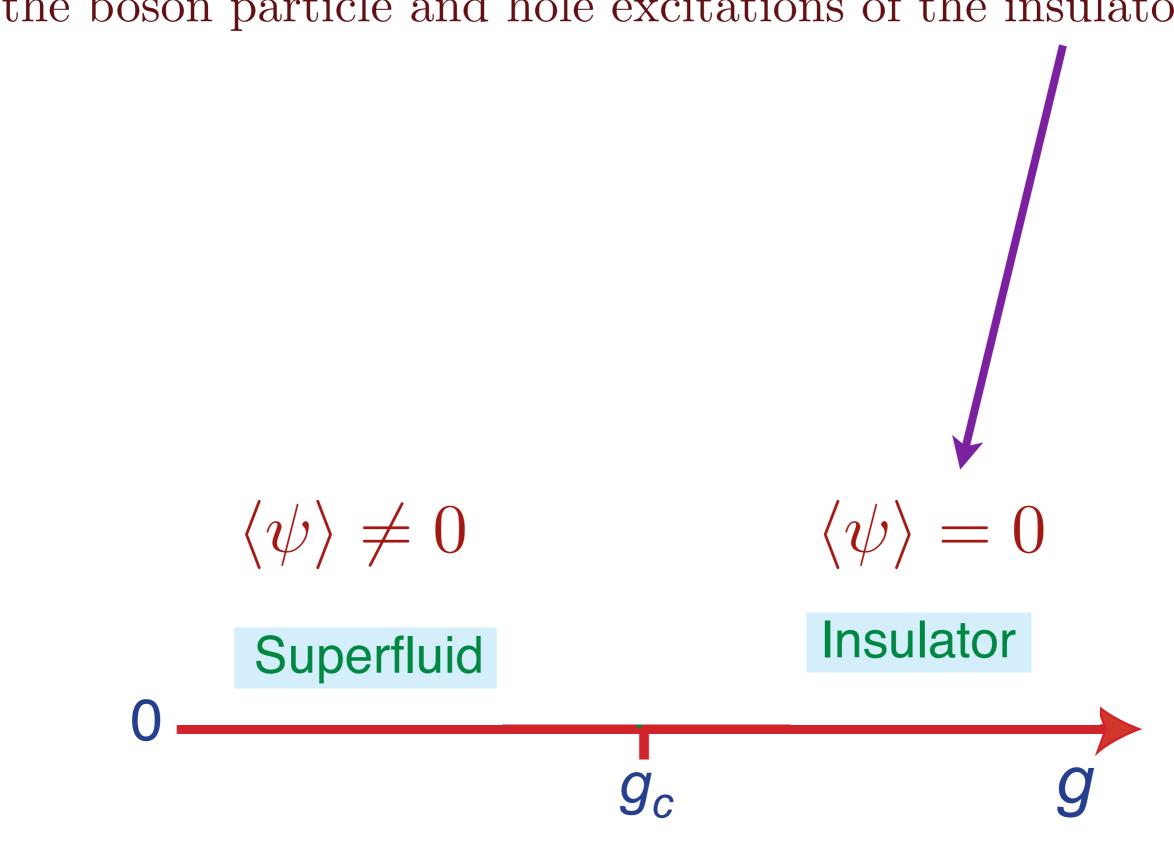
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \to \infty) = \sigma_{\infty}$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

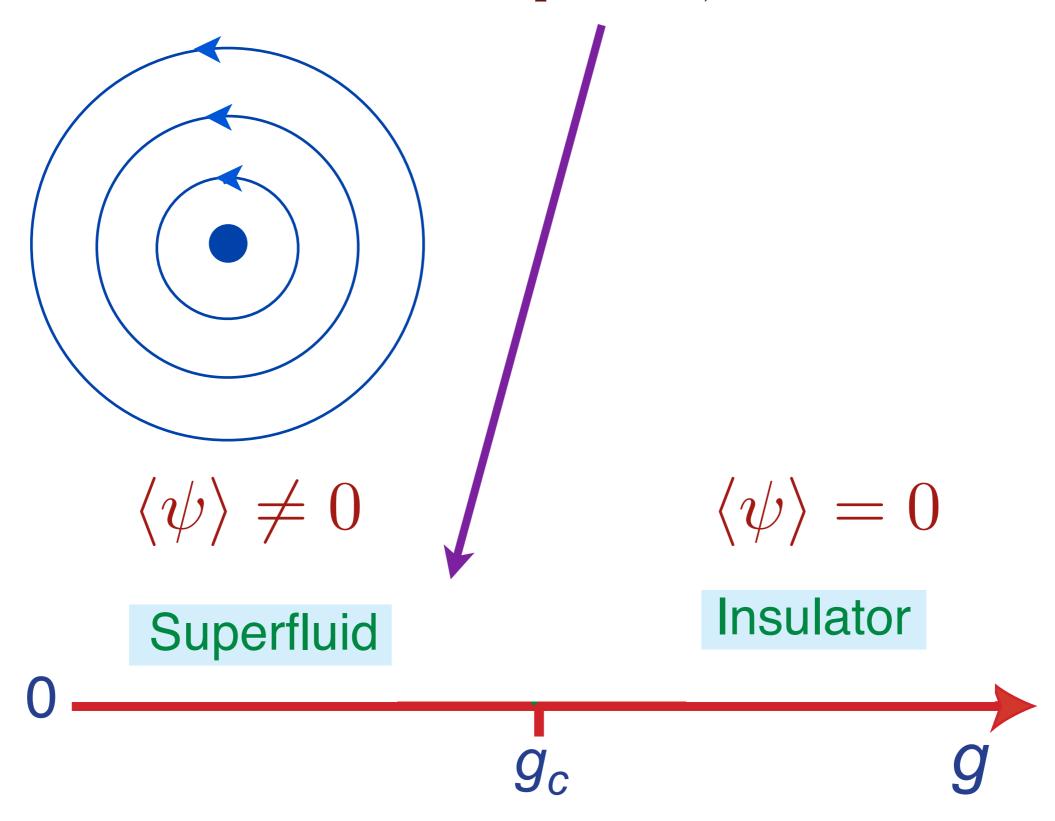
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



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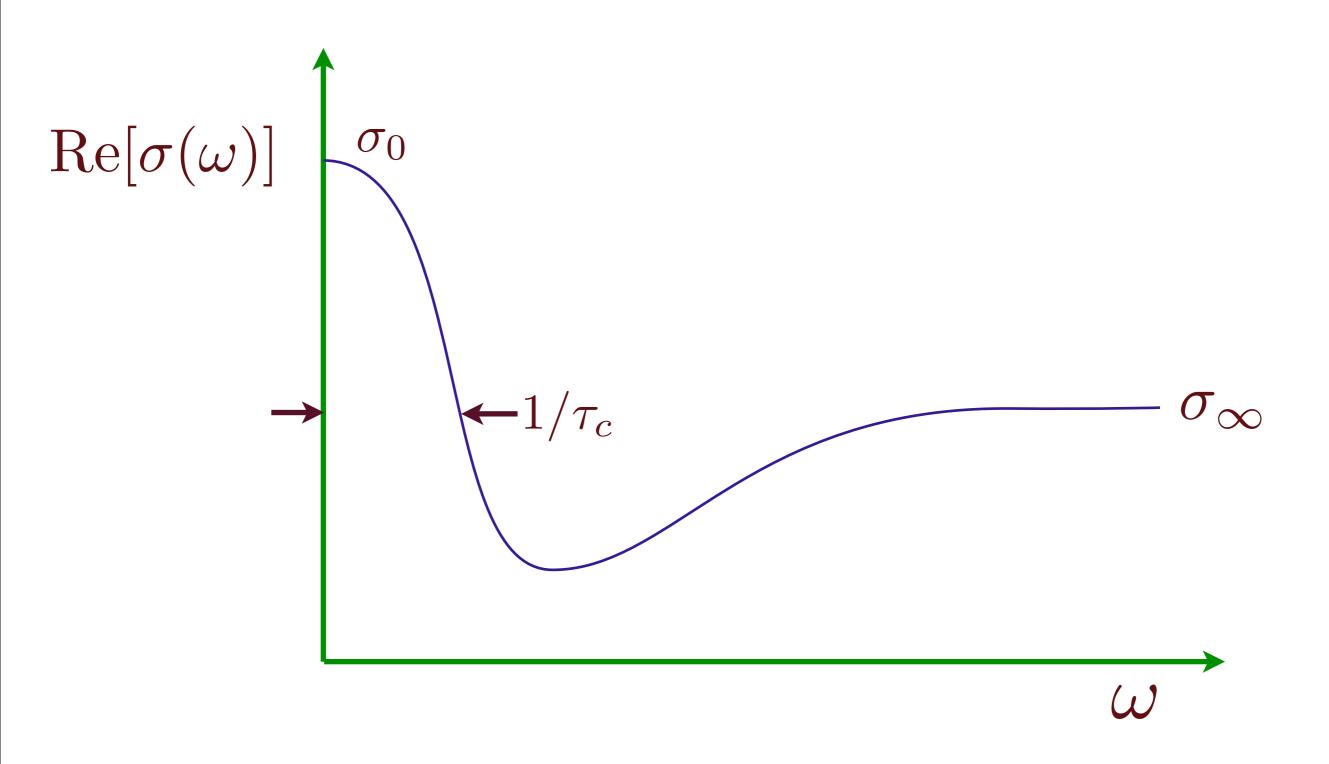
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) "dual" CFT3 with an emergent U(1) gauge field. Their T > 0 dynamics can also be described by a Boltzmann equation:

Conductivity = Resistivity of vortices

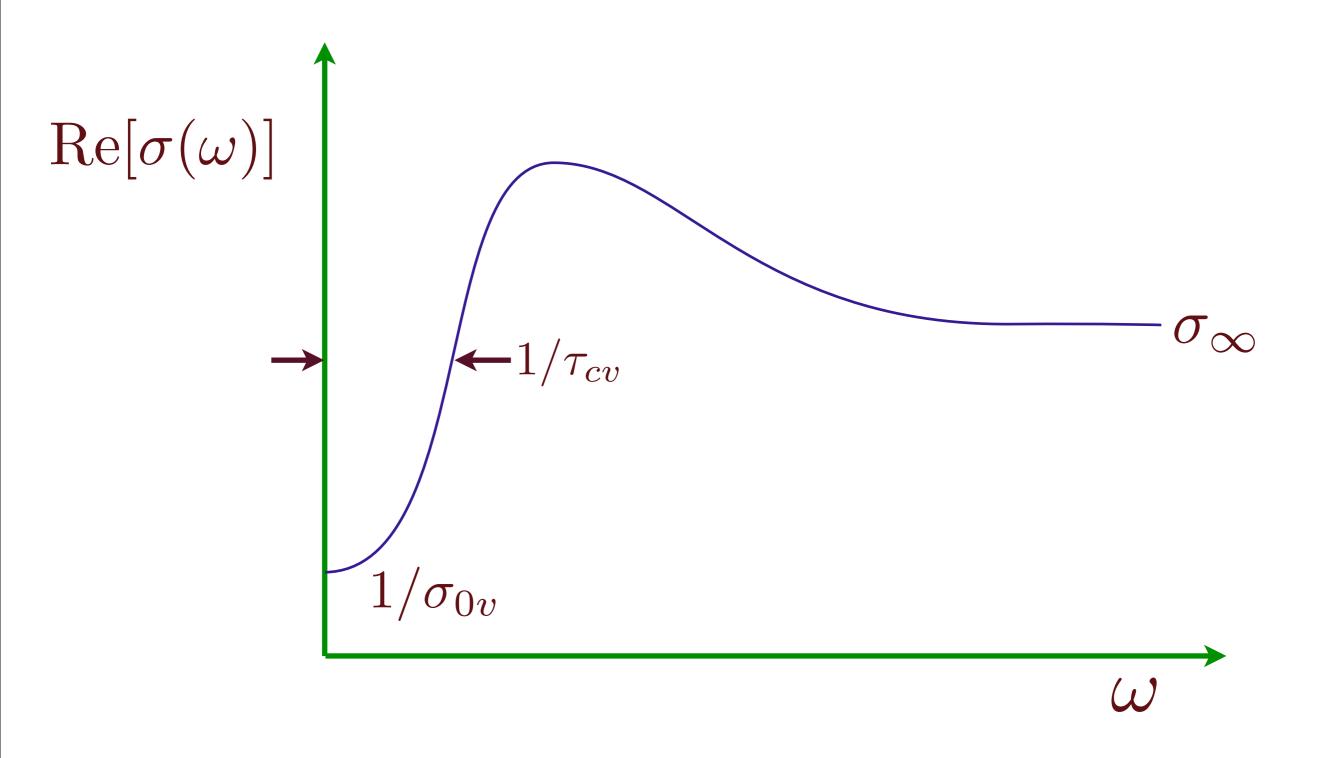
$$\langle \psi
angle
eq 0$$
 $\langle \psi
angle = 0$ Insulator g_c

M.P.A. Fisher, *Physical Review Letters* **65**, 923 (1990)

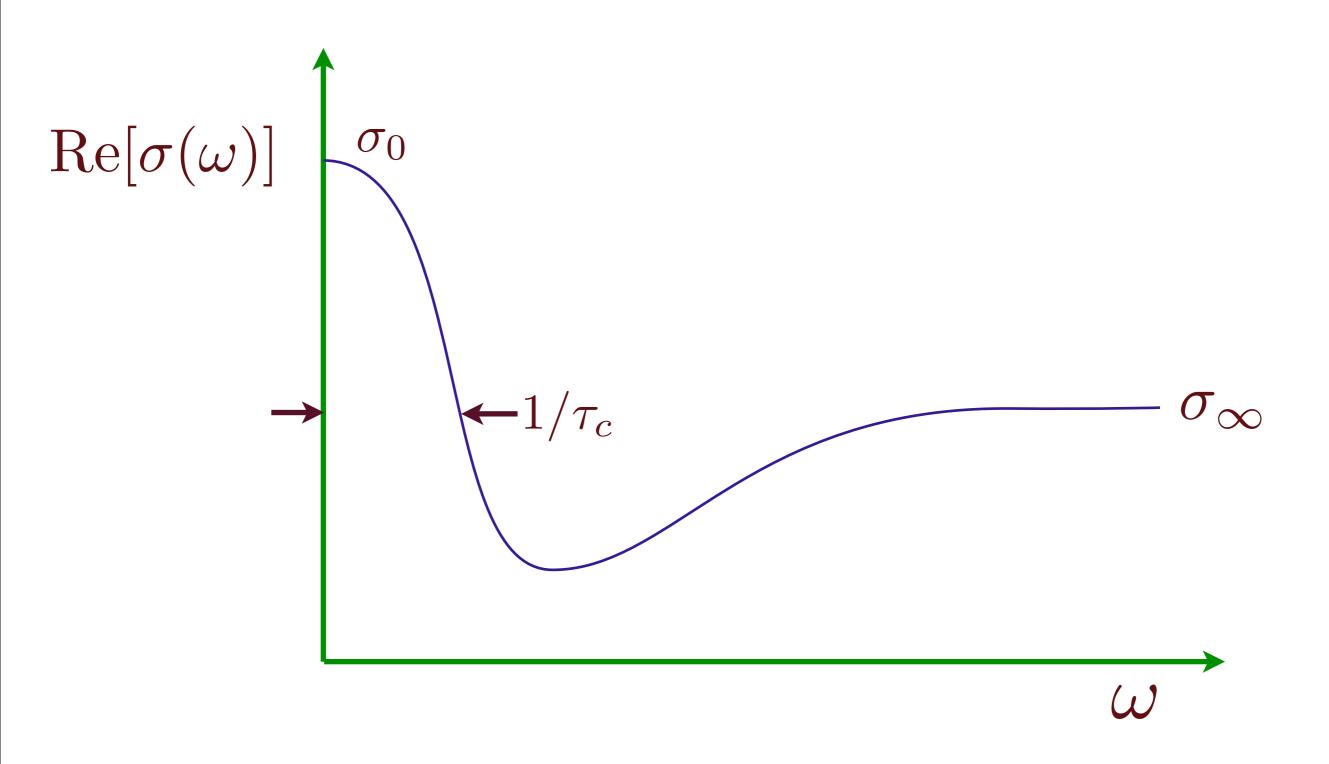
Boltzmann theory of bosons



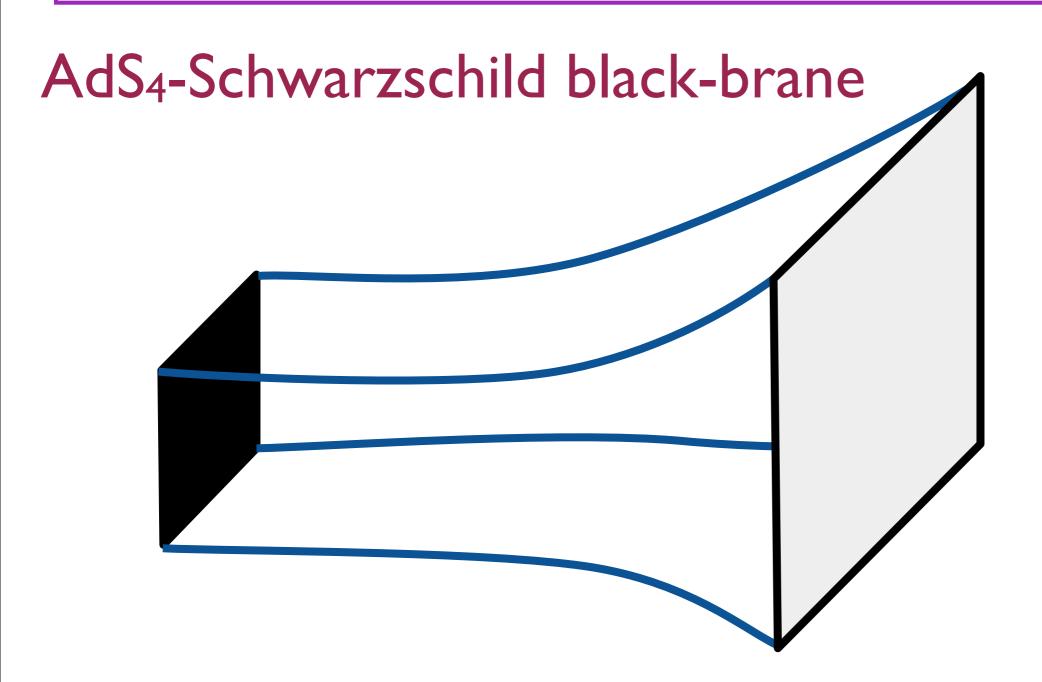
Boltzmann theory of vortices



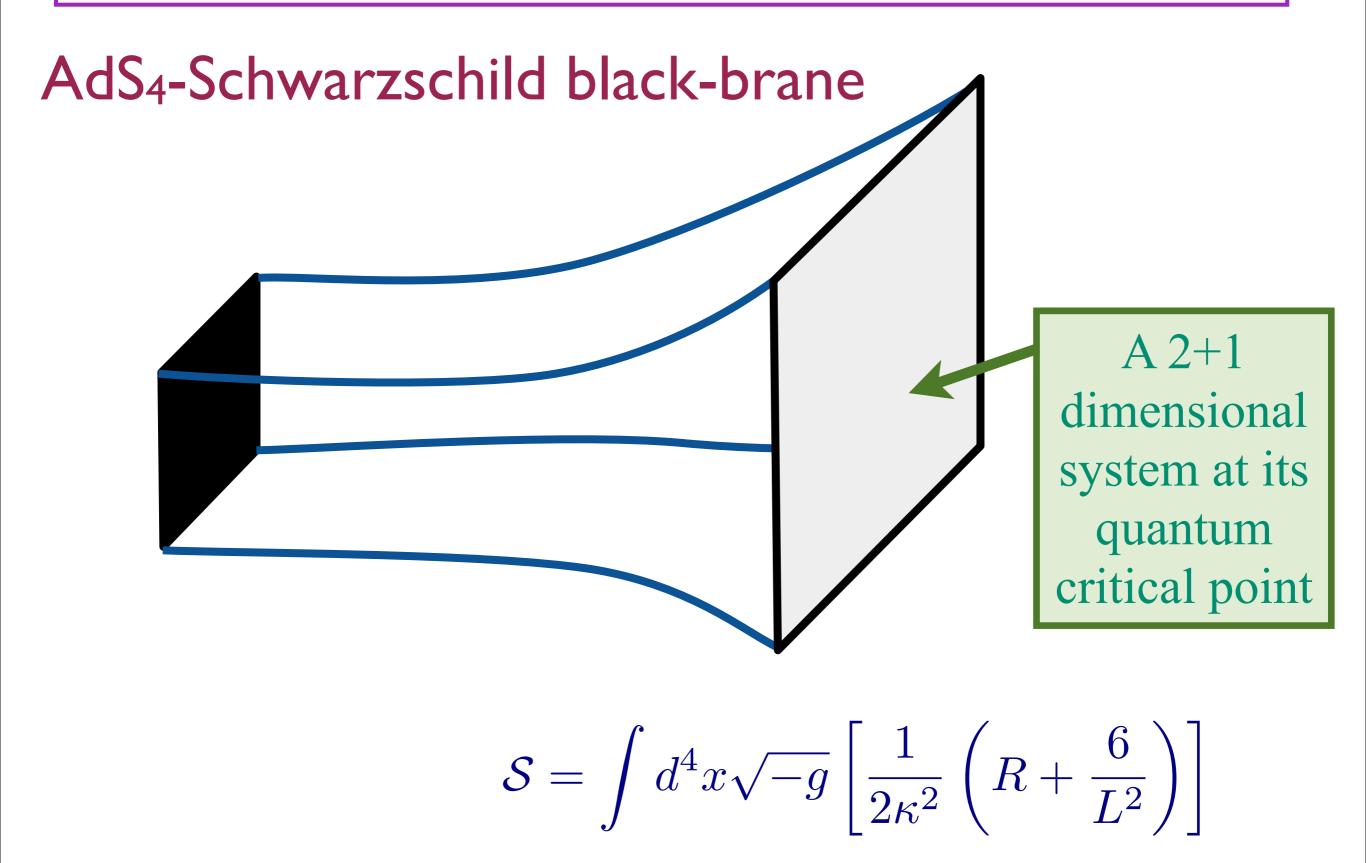
Boltzmann theory of bosons

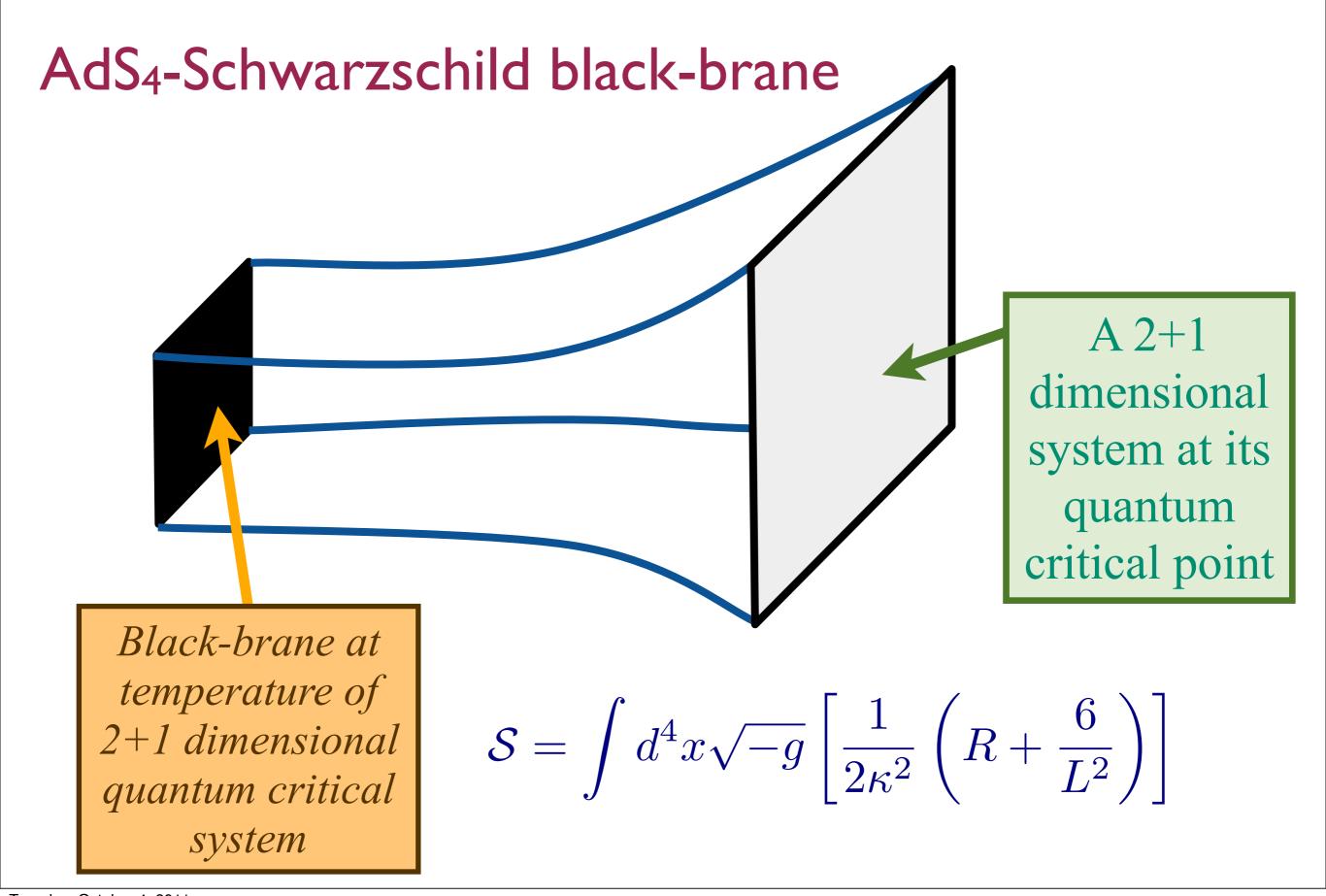


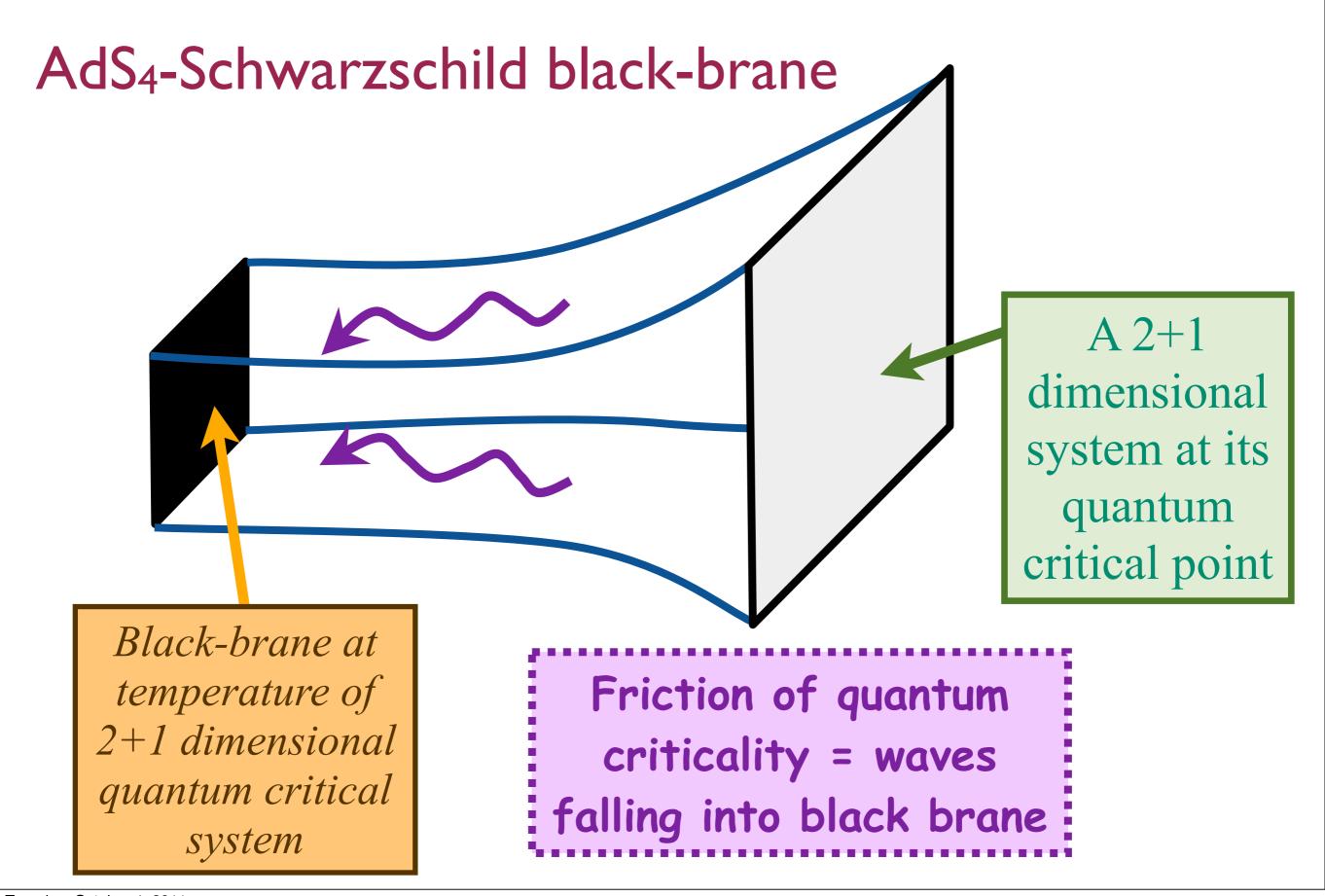
Answers from string theory



$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$







AdS4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS₄-Schwarzschild

$$S_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4 -Schwarzschild

We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS_4):

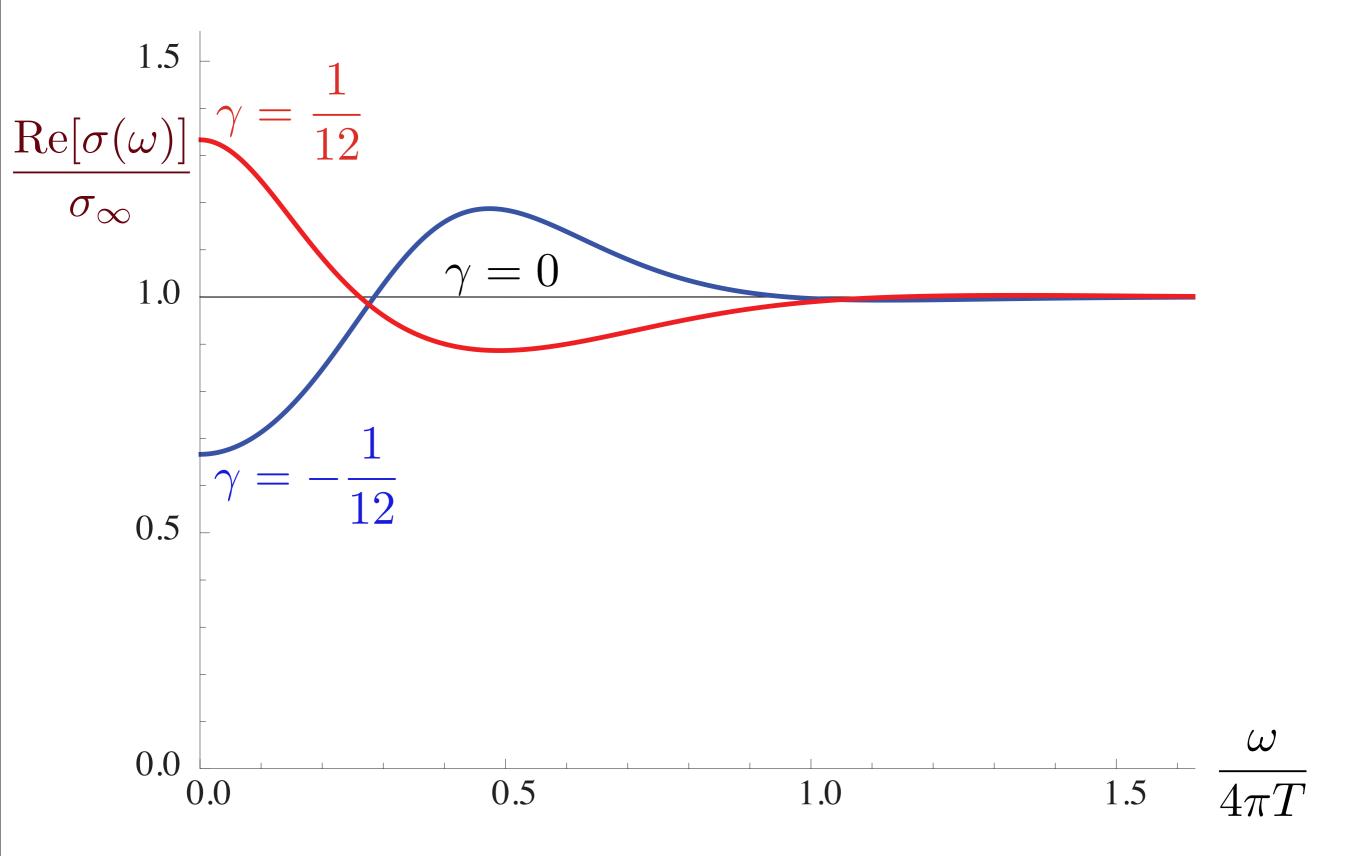
$$S_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} + \frac{\gamma L^2}{e^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.

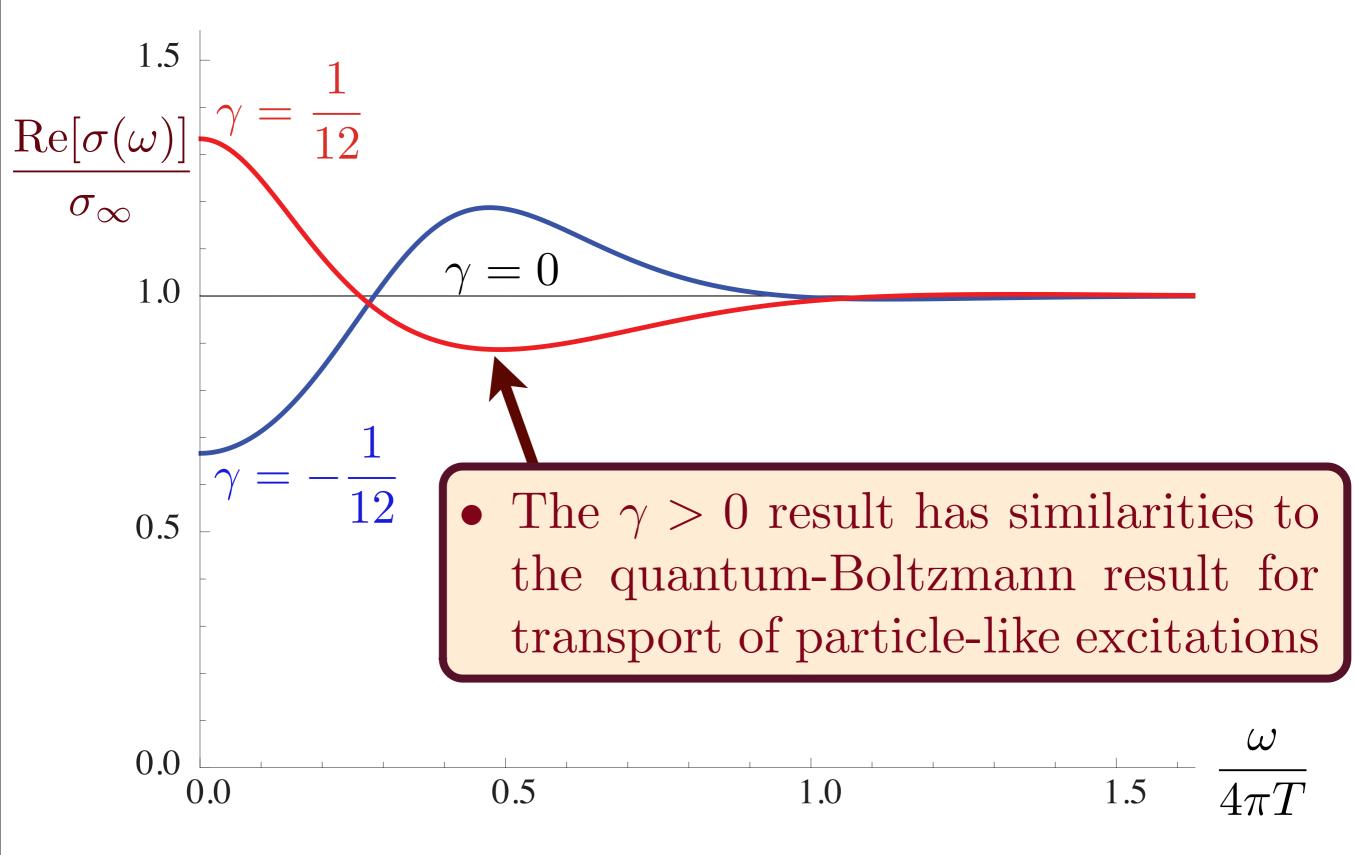
Stability and causality constraints restrict $|\gamma| < 1/12$.

The value of γ can be fixed by matching to a direct computation in the CFT3 at T=0.

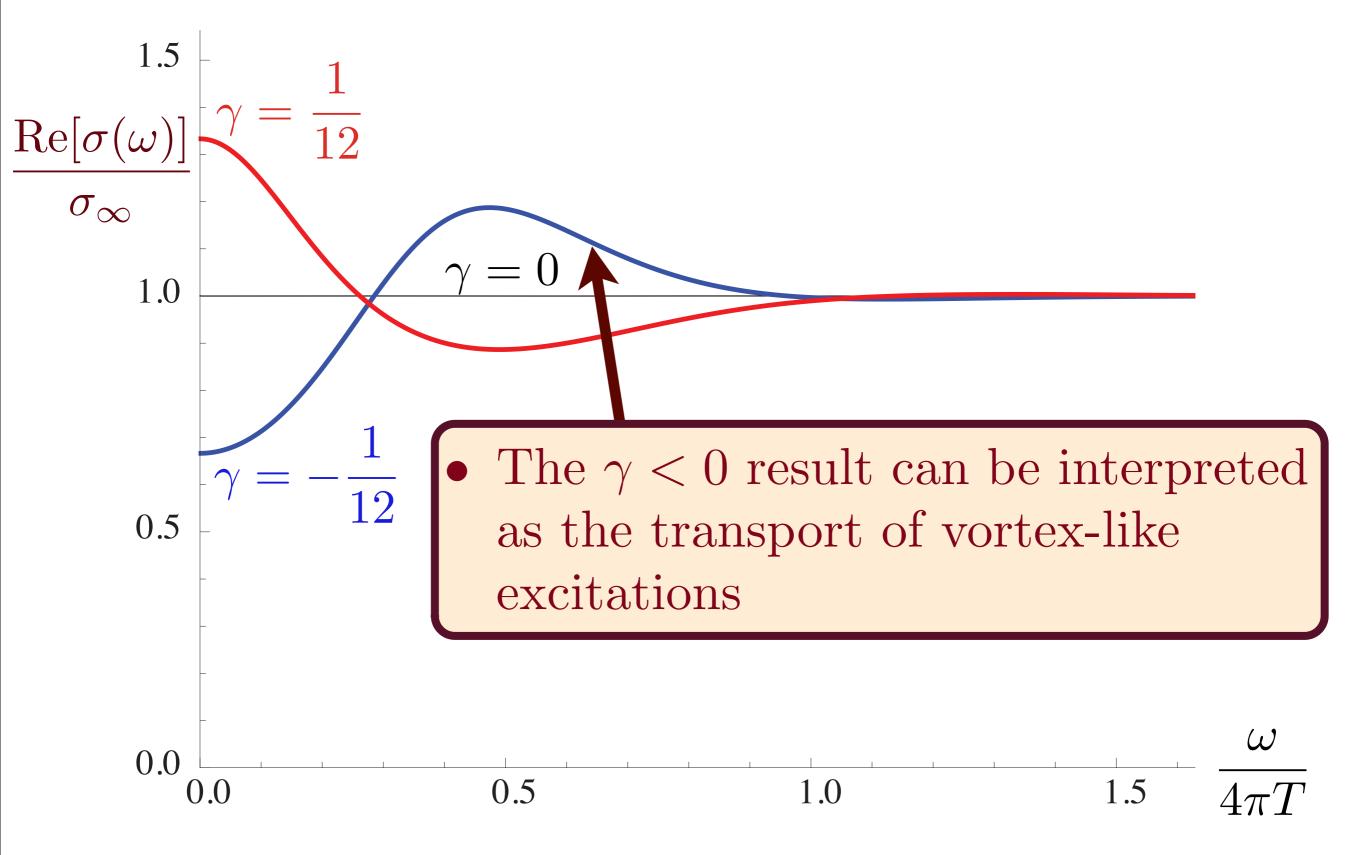
R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* 83, 066017 (2011)



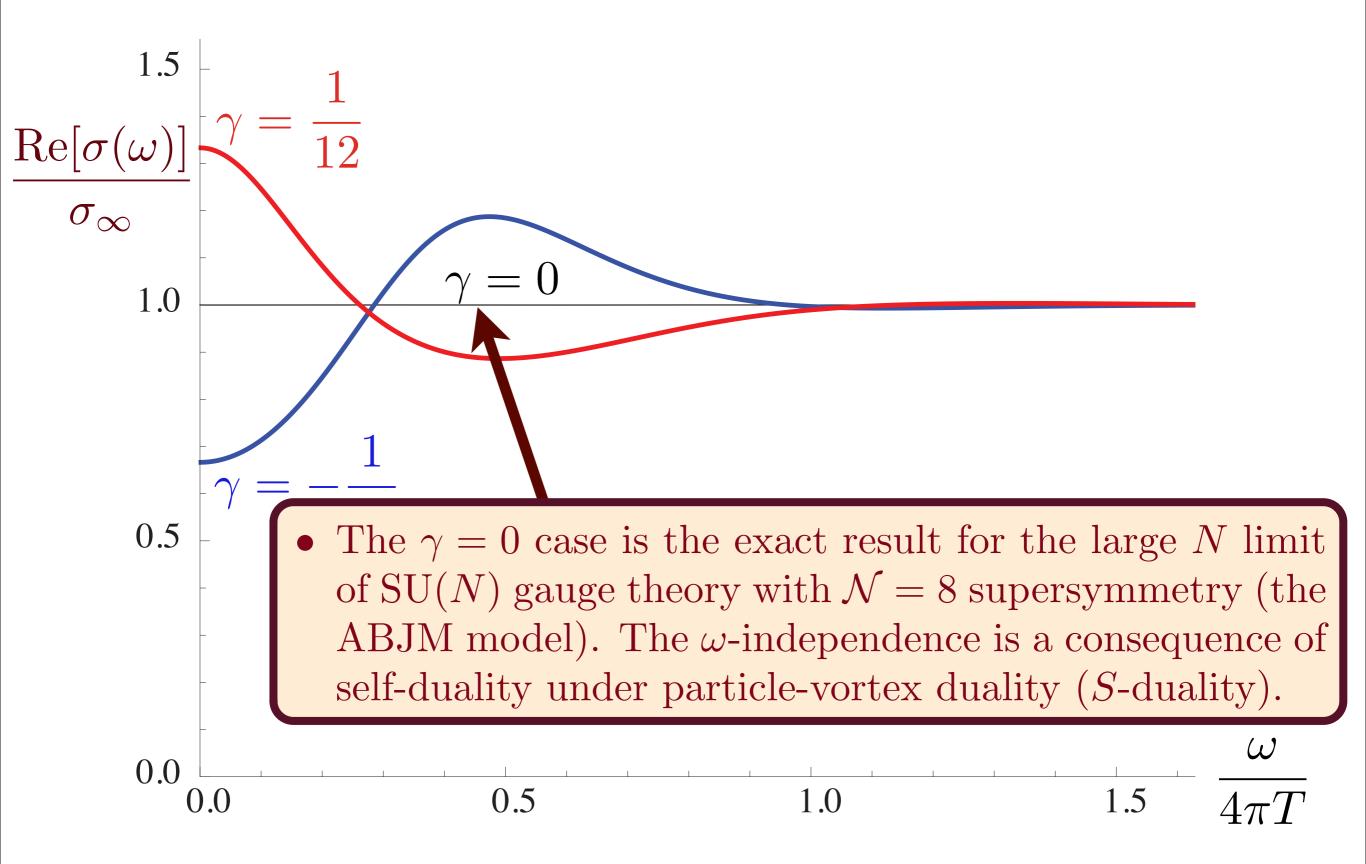
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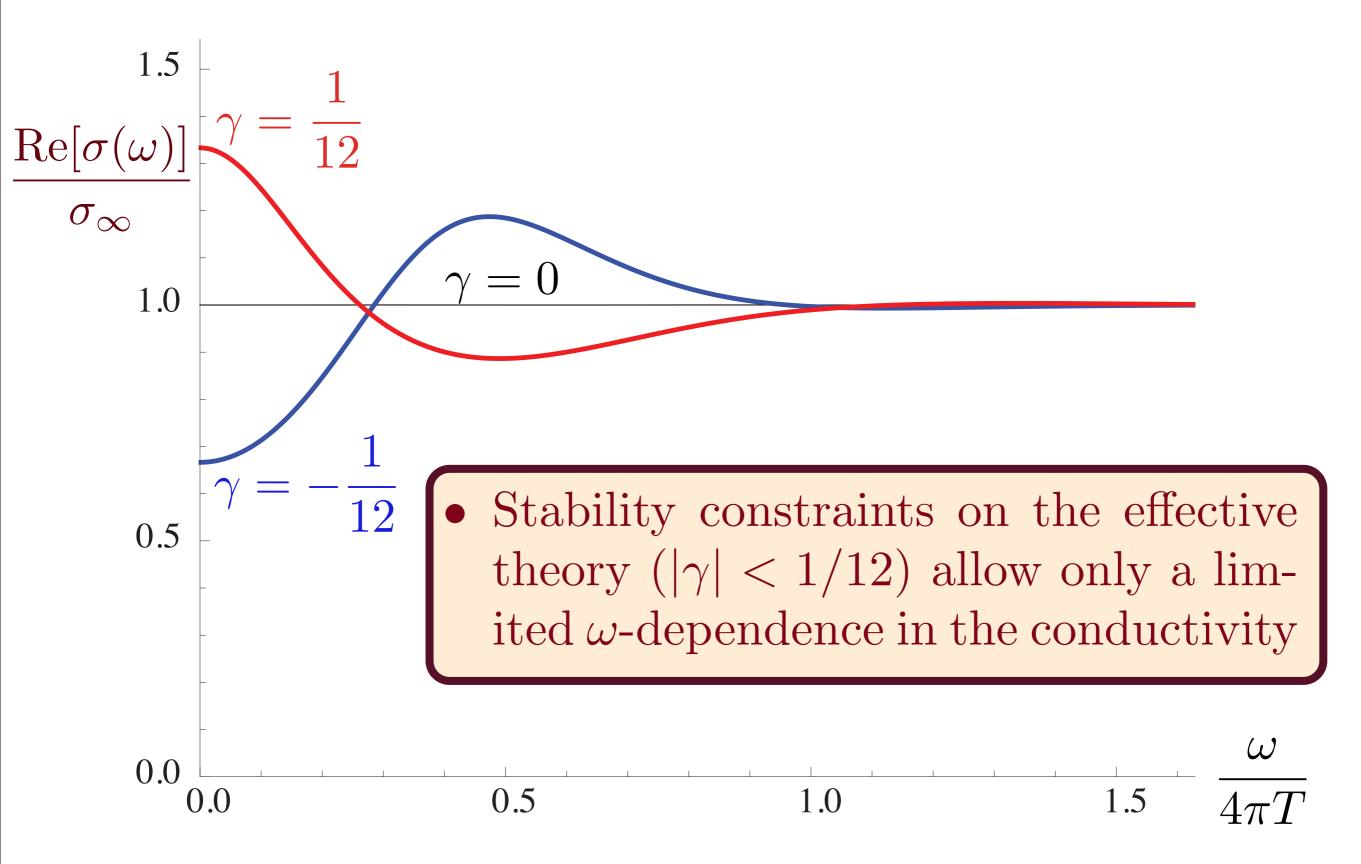
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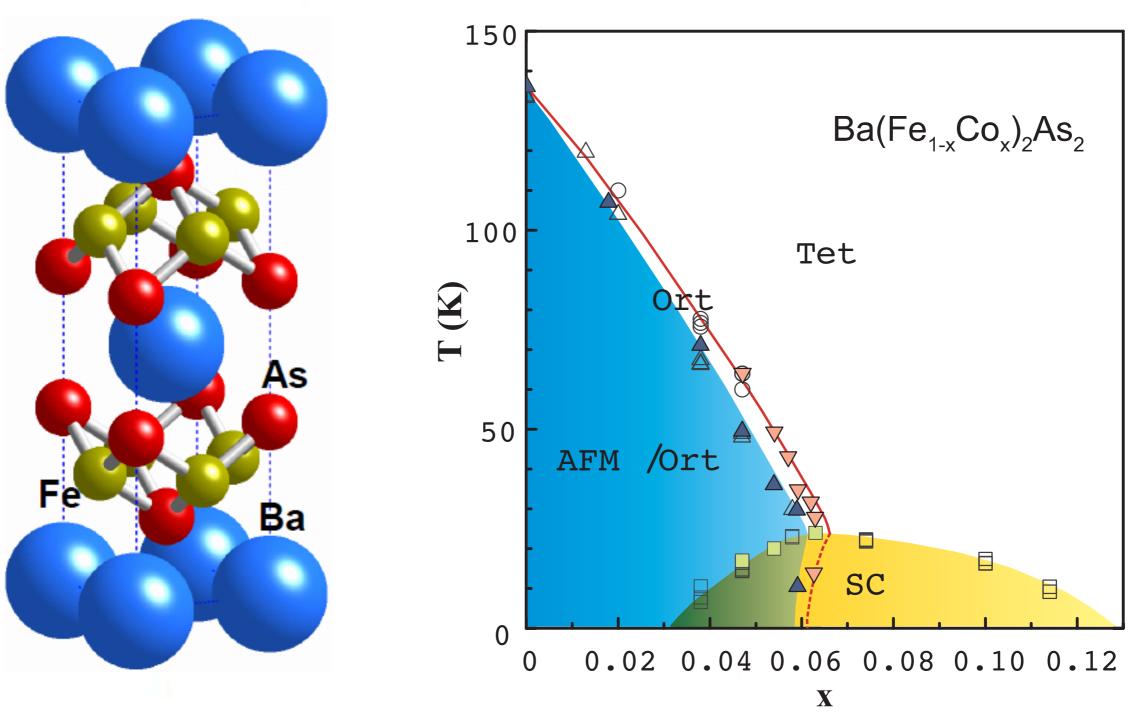
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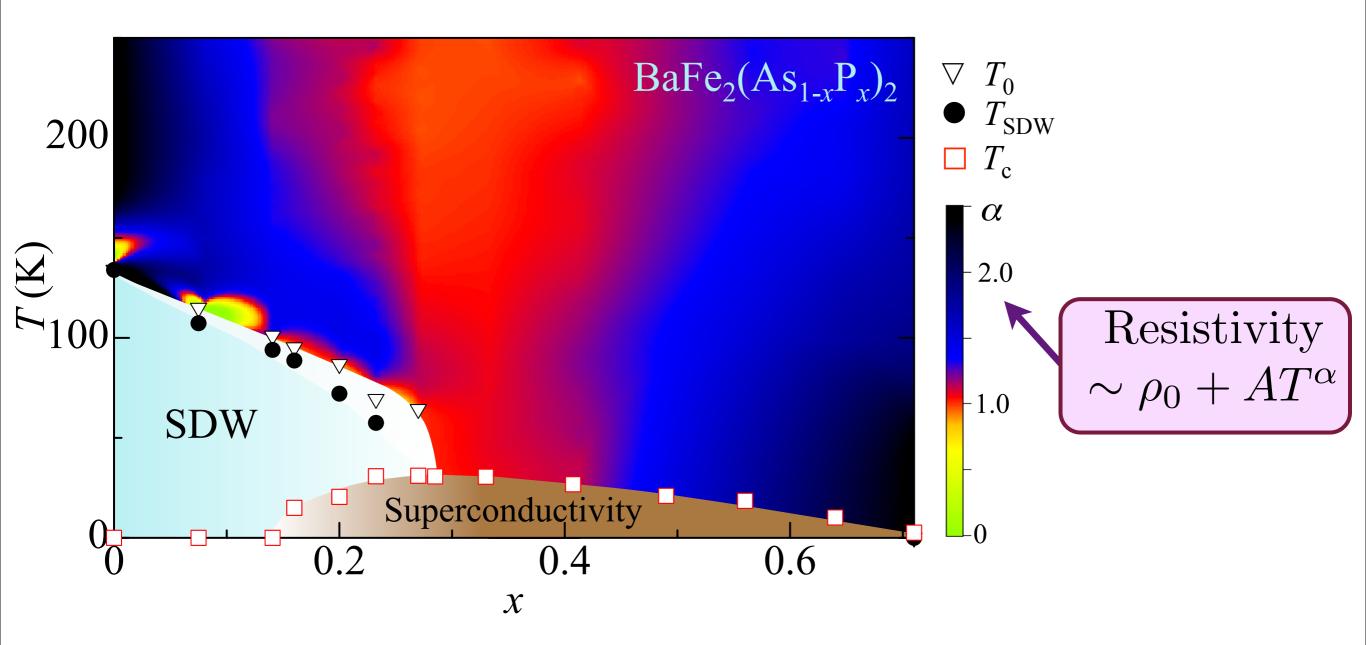
Iron pnictides:

a new class of high temperature superconductors



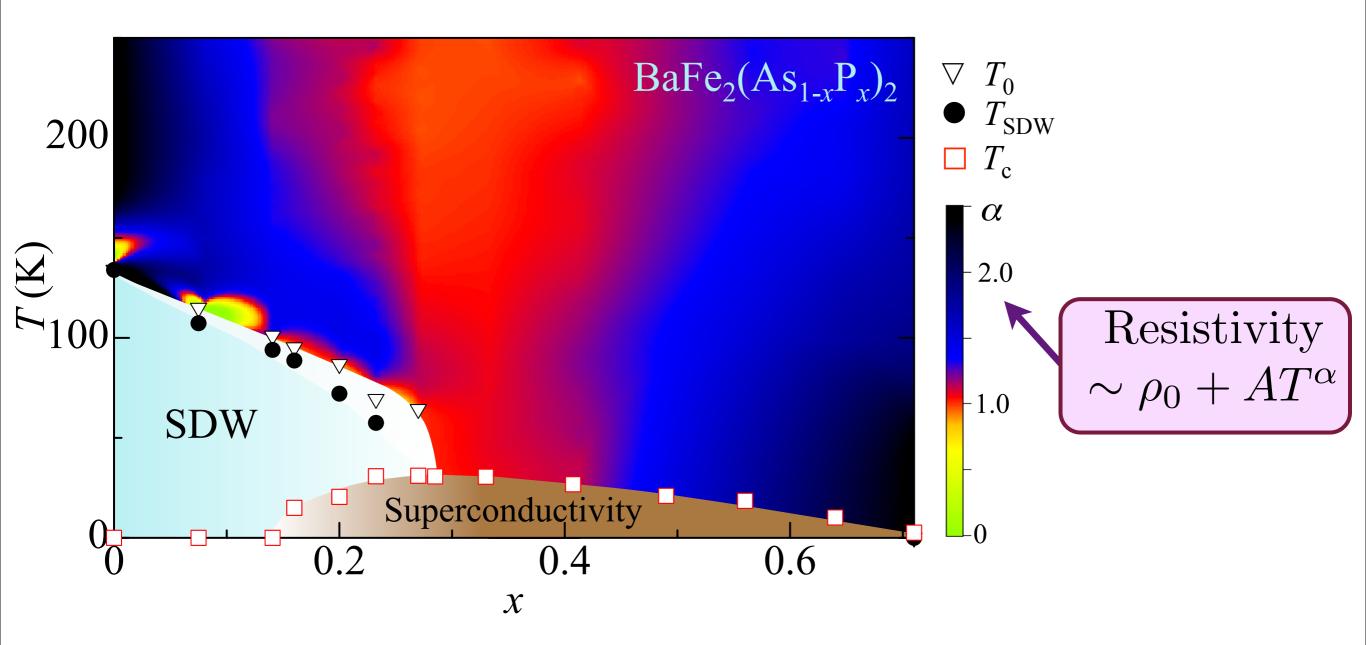
S. Nandi, M. G. Kim, A. Kreyssig, R. M. Fernandes, D. K. Pratt, A. Thaler, N. Ni, S. L. Bud'ko, P. C. Canfield, J. Schmalian, R. J. McQueeney, A. I. Goldman, *Physical Review Letters* **104**, 057006 (2010).

Temperature-doping phase diagram of the iron pnictides:



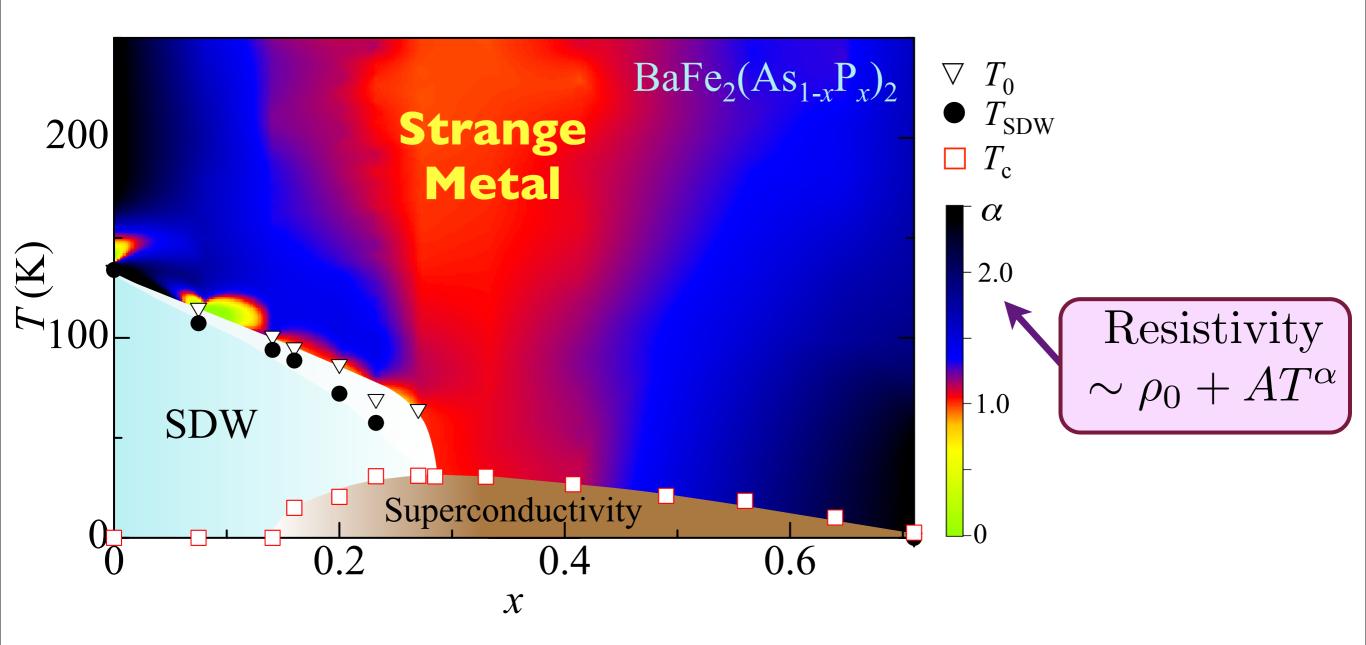
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* 81, 184519 (2010)

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None of these phases are CFTs

Their electron densities are variable, i.e. they are compressible, and they are electrical conductors.

While finding such phases is simple at high temperatures, there are only a few possible compressible quantum phases...

• Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge Q (the "electron density") in spatial dimension d > 1.

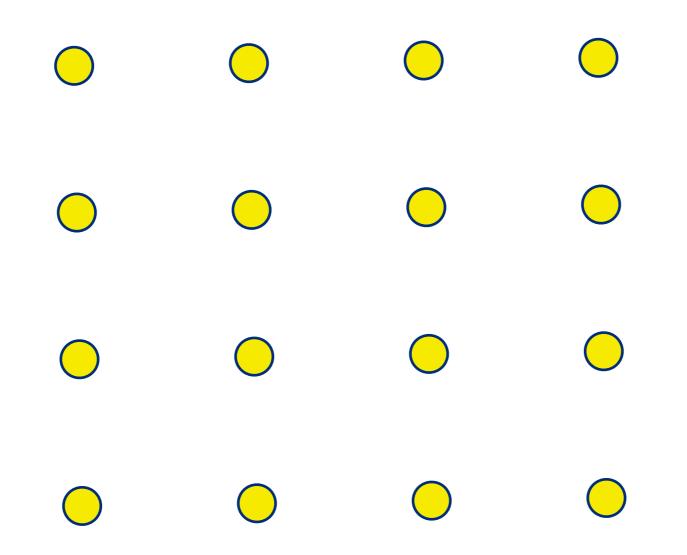
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- Describe <u>zero temperature</u> phases where $d\langle \mathcal{Q} \rangle/d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H \mu \mathcal{Q}$.

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- Describe <u>zero temperature</u> phases where $d\langle Q \rangle/d\mu \neq 0$, where μ (the "chemical potential") which changes the Hamiltonian, H, to $H \mu Q$.
- Compressible systems must be gapless.
- Conformal systems are compressible in d = 1, but not for d > 1.

One compressible state is the **solid** (or "Wigner crystal" or "stripe").

This state breaks translational symmetry.

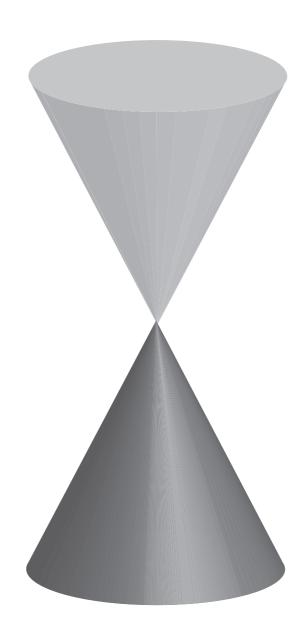


Another familiar compressible state is the **superfluid**.

This state breaks the global U(I) symmetry associated with Q

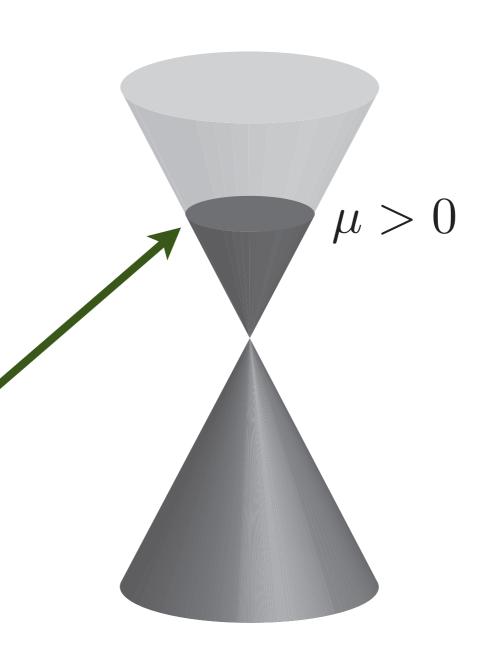


Condensate of fermion pairs

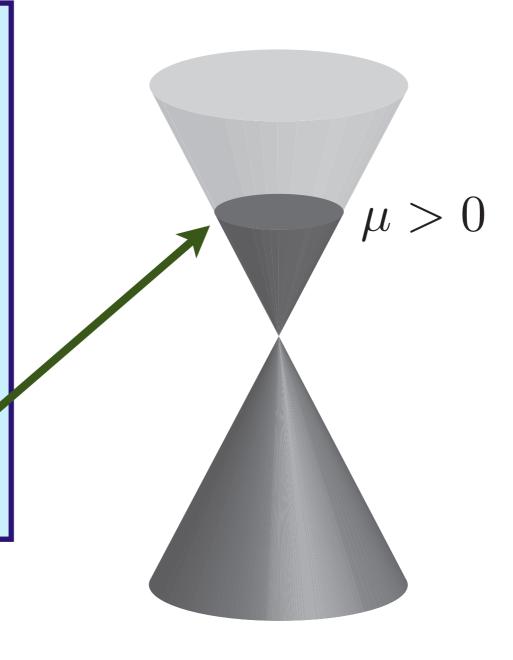


Graphene

The only other familiar compressible phase is a Fermi Liquid with a Fermi surface

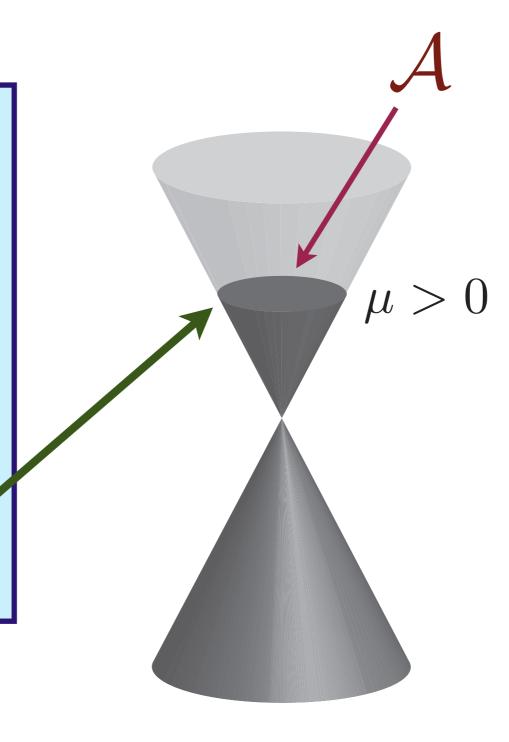


The only other familiar compressible phase is a Fermi Liquid with a Fermi surface



• The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

The only other familiar compressible phase is a Fermi Liquid with a Fermi surface



• Luttinger relation: The total "volume (area)" \mathcal{A} enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

Classify states of compressible quantum matter in continuum theories which preserve translational invariance.

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Are there any other compressible phases?

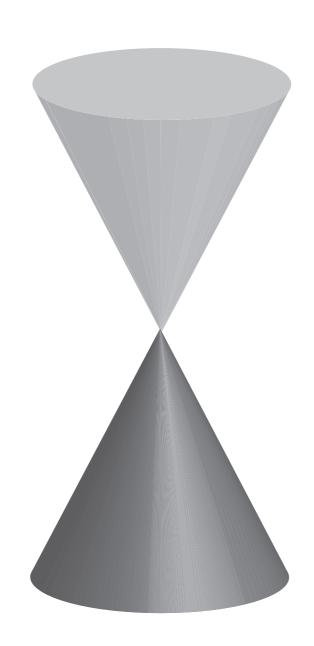
Classify states of compressible quantum matter in continuum theories which preserve translational invariance.

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Yes

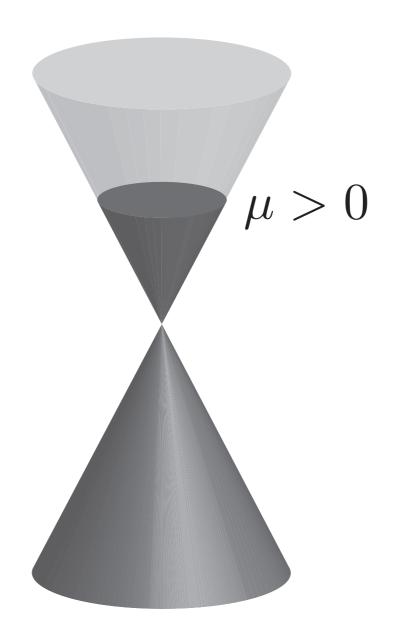
Are there any other compressible phases? Yes....

Begin with a strongly-coupled CFT

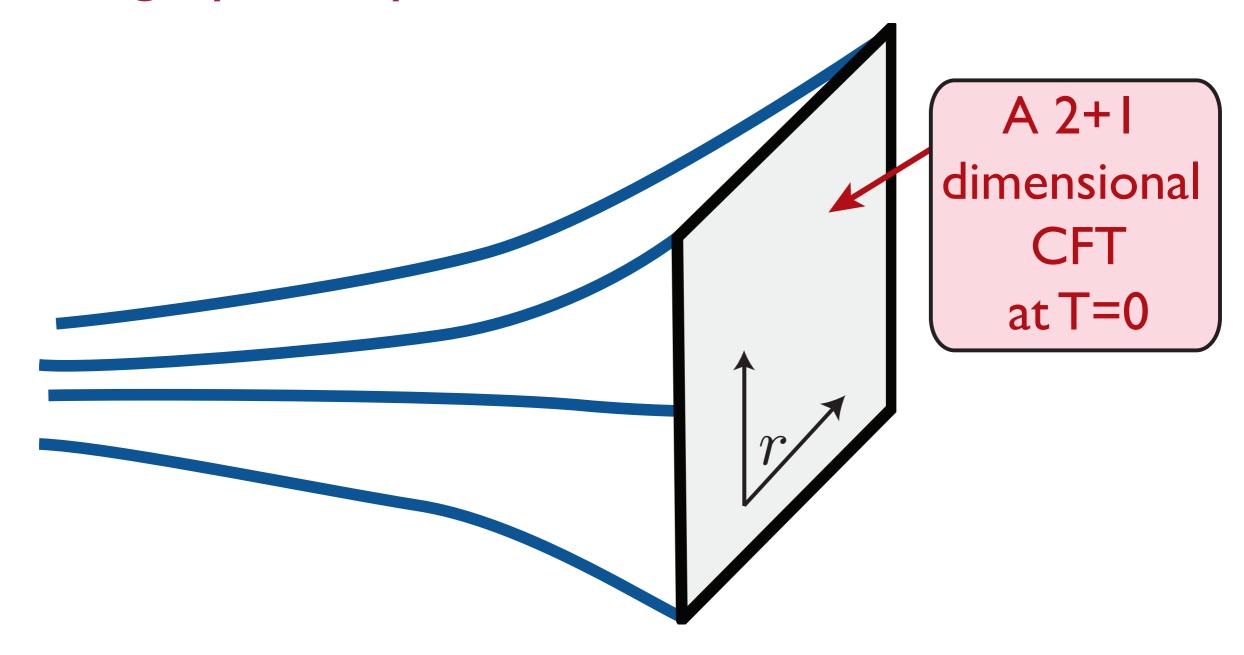


Dirac fermions + gauge field +

Are there holographic theories describing the appearance of superfluid or Fermi liquid ground states when a chemical potential is applied?

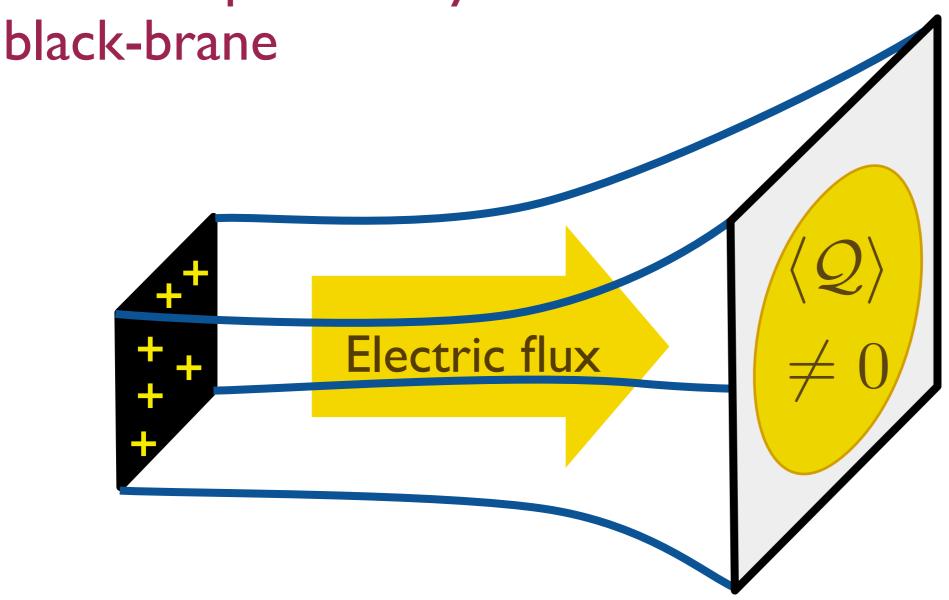


Holographic representation: AdS₄

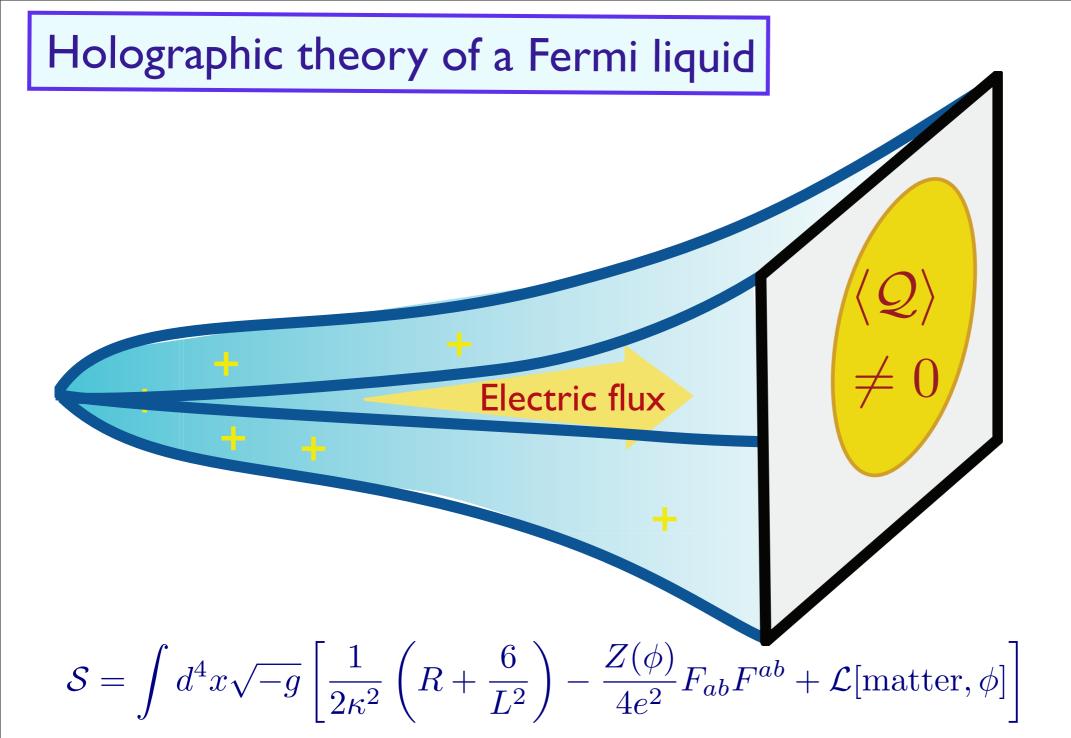


$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

The Maxwell-Einstein theory of the applied chemical potential yields a AdS₄-Reissner-Nordtröm

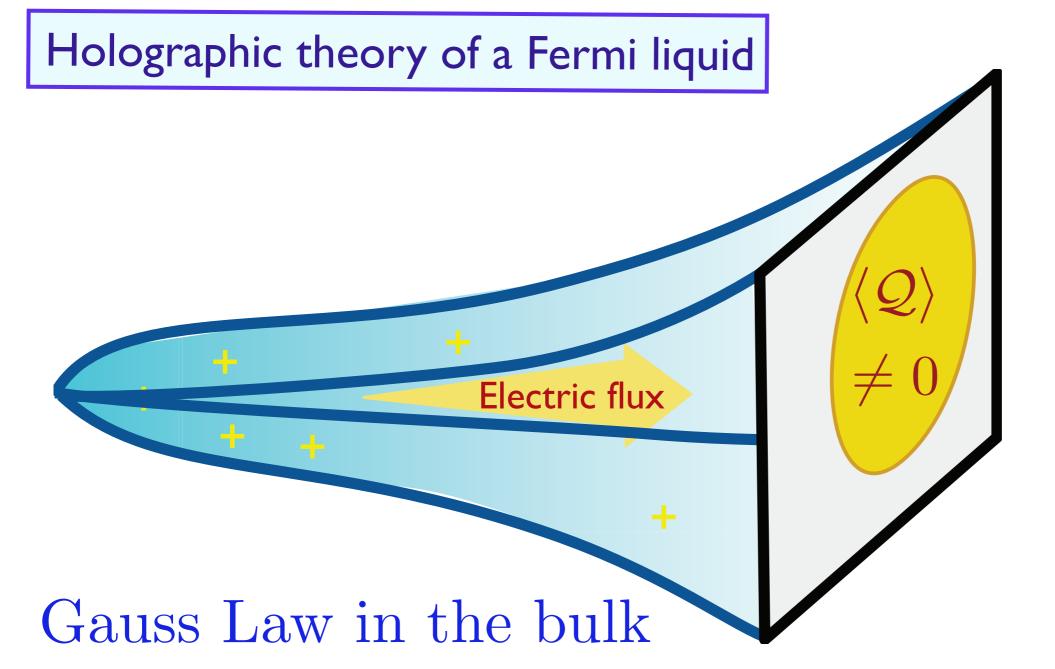


$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4e^2} F_{ab} F^{ab} \right]$$



A Fermi liquid is a "confining" phase, in which all the low energy excitations are gauge-neutral.

In such a confining phase, the horizon disappears, there is charge density delocalized in the bulk spacetime.



⇔ Luttinger theorem on the boundary

A Fermi liquid is a "confining" phase, in which all the low energy excitations are gauge-neutral.

In such a confining phase, the horizon disappears, there is charge density delocalized in the bulk spacetime.

Such a holographic theory describes a quantum state which agrees with Landau's Fermi liquid theory in all respects.

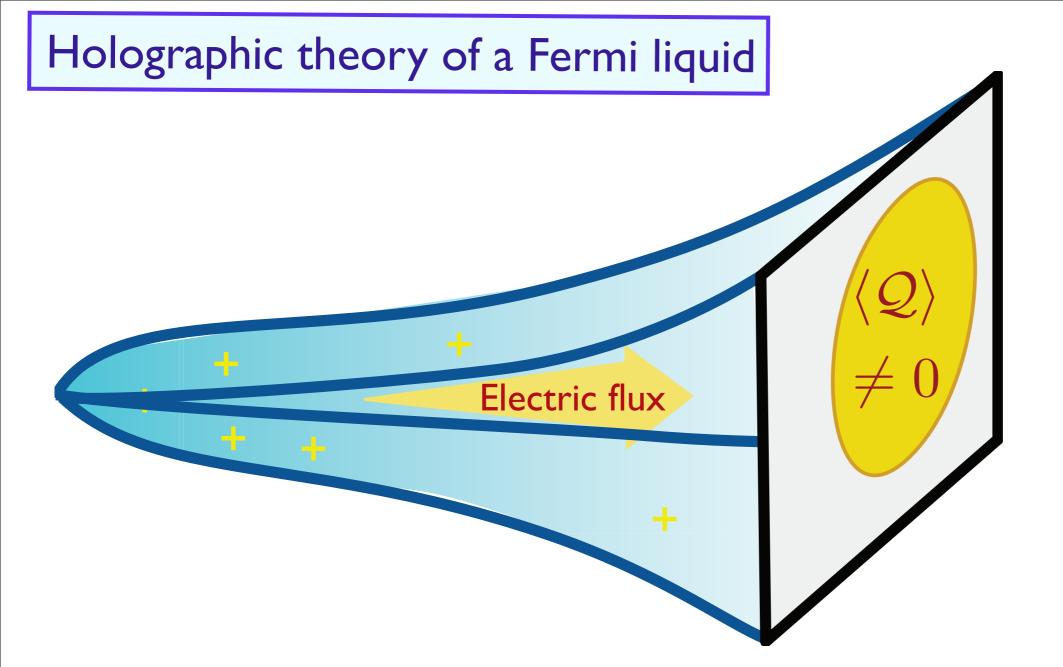
A similar description can be obtained for superfluids using bosonic matter in the bulk.

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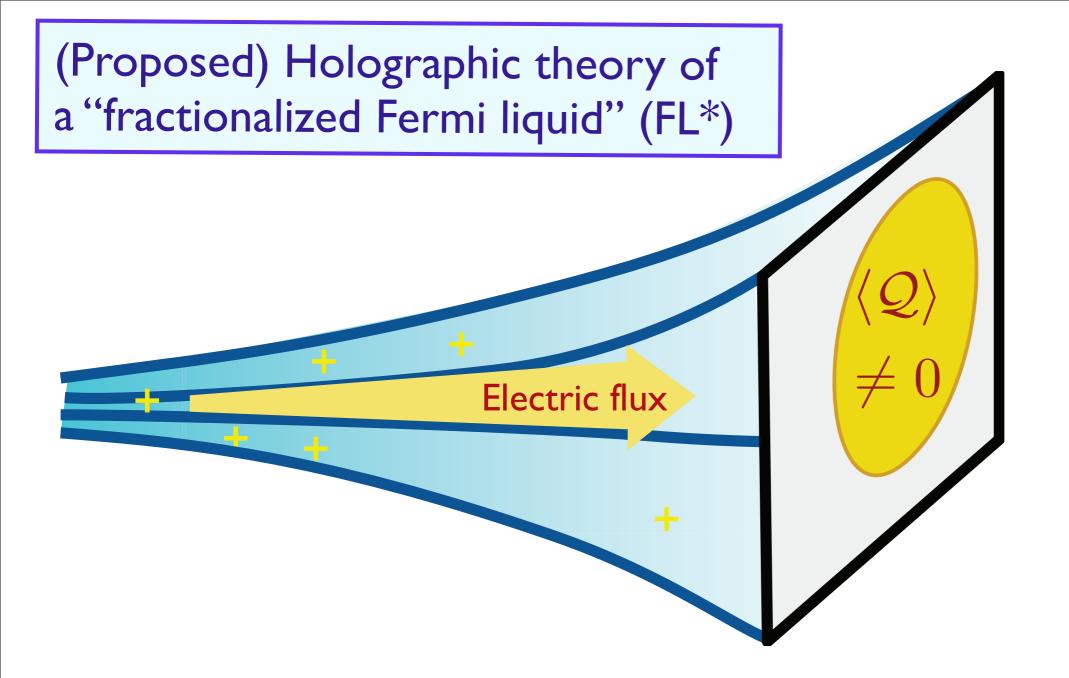
A similar description can be obtained for superfluids using bosonic matter in the bulk.

How about more exotic compressible states (in the hopes of describing the strange metal)?

S. Sachdev arXiv:1107.5321

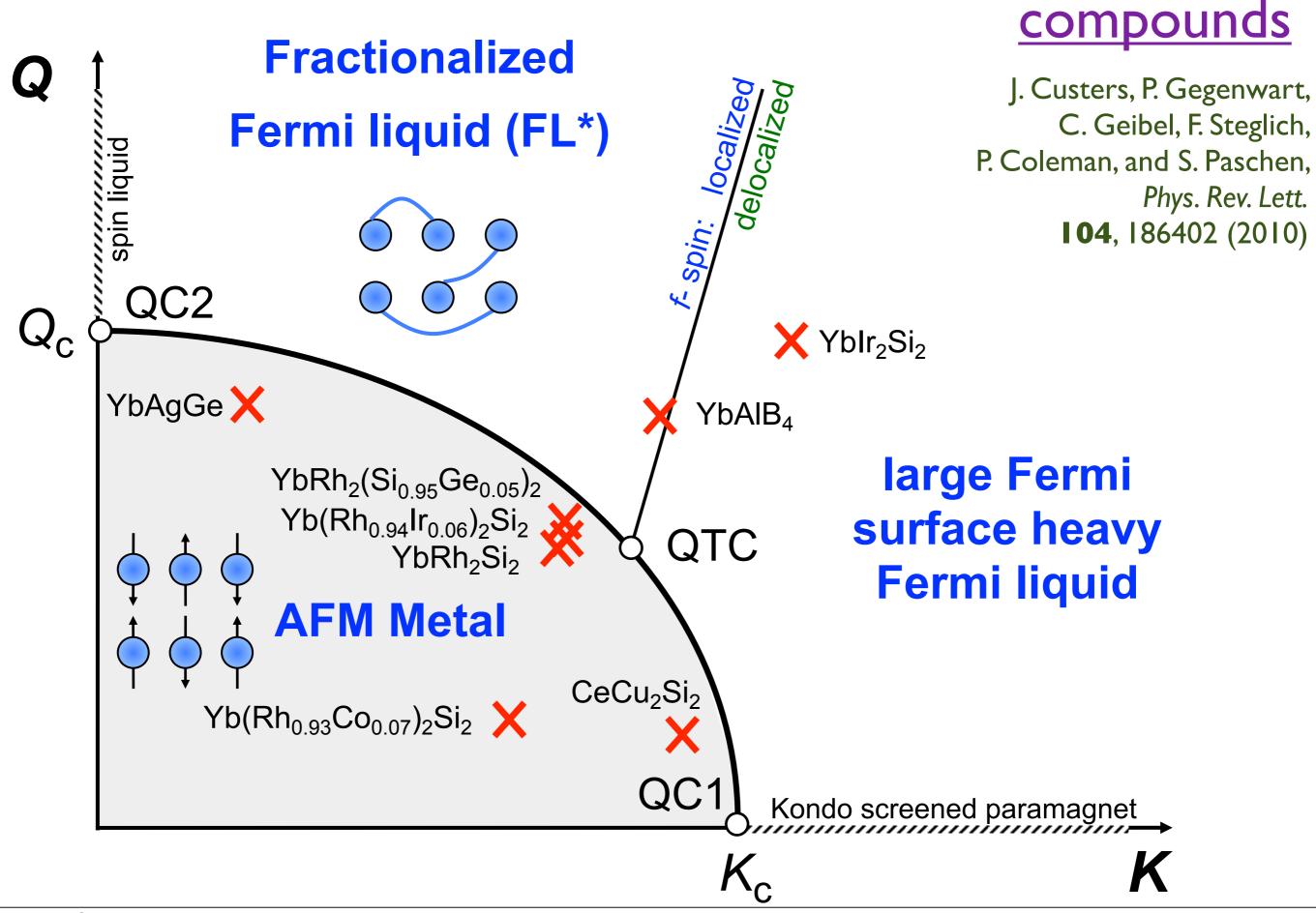


In a confining FL phase, the metric terminates, the bulk charge equals the boundary charge, and the electric flux vanishes in the IR.

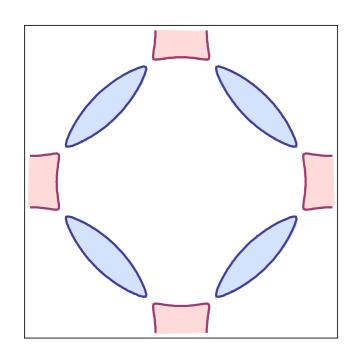


In a deconfined FL* phase, the metric extends to infinity (representing critical IR modes), and part of the electric flux "leaks out".

Experimental phase diagram of the heavy-fermion



Separating onset of SDW order and Fermi surface reconstruction in the cuprates



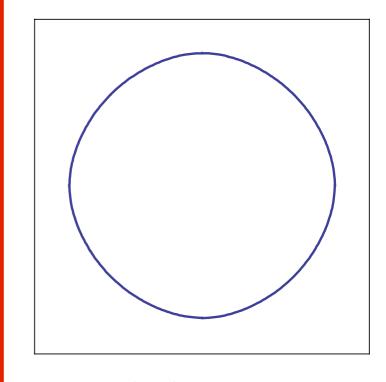
$$\langle \vec{\varphi} \rangle \neq 0$$

Metal with electron and hole pockets

Electron and/or hole
Fermi pockets form in
"local" SDW order, but
quantum fluctuations
destroy long-range
SDW order

$$\langle \vec{\varphi} \rangle = 0$$

Fractionalized Fermi liquid (FL*) phase with no symmetry breaking and "small" Fermi surface



$$\langle \vec{\varphi} \rangle = 0$$

Metal with "large" Fermi surface

Y. Qi and S. Sachdev, *Physical Review B* 81, 115129 (2010); M. Punk and S. Sachdev, to appear; see also T. C. Ribeiro and X.-G. Wen, *Physical Review B* 74, 155113 (2006)

Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points
- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.
- Prospects non-linear, and non-equilibrium transport

Conclusions

- Presented a holographic model of a Fermi liquid
- Fractionalized Fermi liquid (FL*), appears in deconfined gauge theories, holographic models, and lattice theories of the heavy-fermion compounds and cuprates superconductors.
- Numerous plausible sightings of the FL* phase in recent experiments