# Equilibrium and non-equilibrium dynamics of SYK models

Strongly interacting conformal field theory
in condensed matter physics,
Institute for Advanced Study, Tsinghua University,
Beijing, June 25-27, 2017

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Talk online: sachdev.physics.harvard.edu

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## *Quantum matter with quasiparticles:*

- **Quasiparticles are additive excitations:**
  The low-lying excitations of the many-body system can be identified as a set $\{n_\alpha\}$ of quasiparticles with energy $\varepsilon_\alpha$

$$E = \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$$

  - **Note:** The electron liquid in one dimension and the fractional quantum Hall state both have quasiparticles; however, the quasiparticles do not have the same quantum numbers as an electron.

## *Quantum matter with quasiparticles:*

- Quasiparticles eventually collide with each other. Such collisions eventually leads to thermal equilibration in a chaotic quantum state, but the equilibration takes a long time. In a Fermi liquid, this time is of order $\hbar E_F/(k_B T)^2$ as $T \to 0$, where $E_F$ is the Fermi energy.

*Quantum matter without quasiparticles:*

- No quasiparticle decomposition of low-lying states
- Rapid thermalization

## Local thermal equilibration or phase coherence time, $\tau_\varphi$:

- There is an *lower bound* on $\tau_\varphi$ in all many-body quantum systems as $T \to 0$,

$$\tau_\varphi \geq C \frac{\hbar}{k_B T},$$

where $C$ is a $T$-independent constant.

- Systems *without* quasiparticles have

$$\tau_\varphi \sim \frac{\hbar}{k_B T},$$

K. Damle and S. Sachdev, PRB **56**, 8714 (1997)
S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)

# A simple model of a metal with quasiparticles

$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

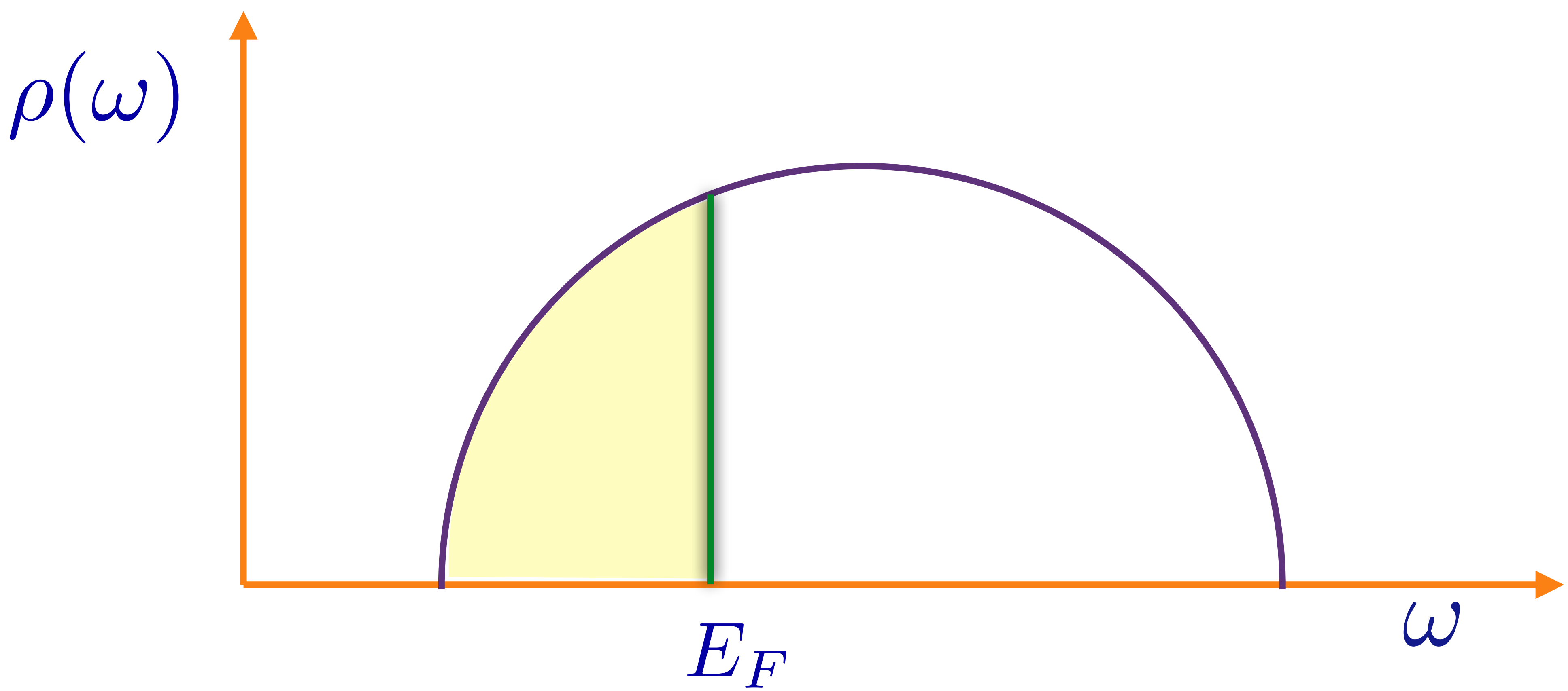
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

$t_{ij}$ are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

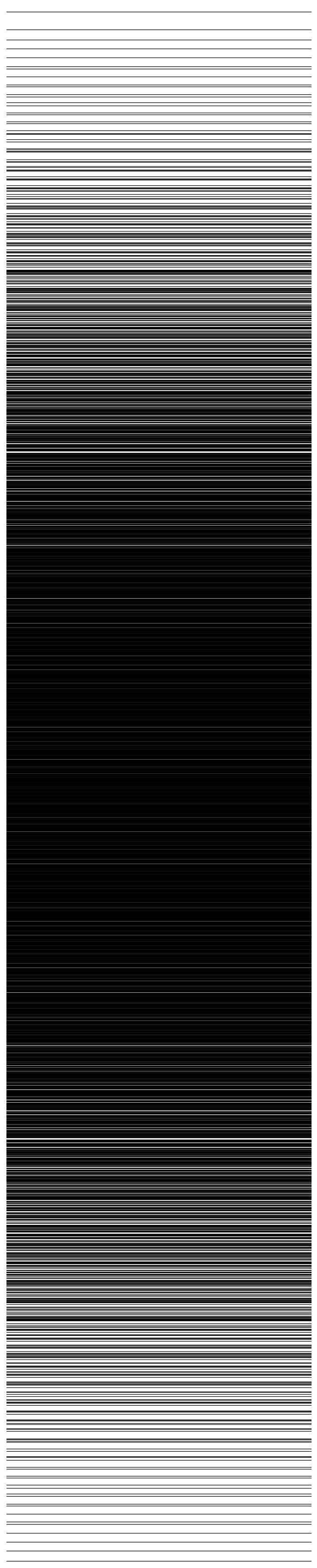
Fermions occupying the eigenstates of a *N* x *N* random matrix

# A simple model of a metal with quasiparticles

Let $\varepsilon_\alpha$ be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest $NQ$ eigenvalues, upto the Fermi energy $E_F$. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



# A simple model of a metal with quasiparticles



Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

There are $2^N$ many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of $E$ for a single cluster of size $N = 12$. The $\varepsilon_\alpha$ have a level spacing $\sim 1/N$.

# The Sachdev-Ye-Kitaev (SYK) model

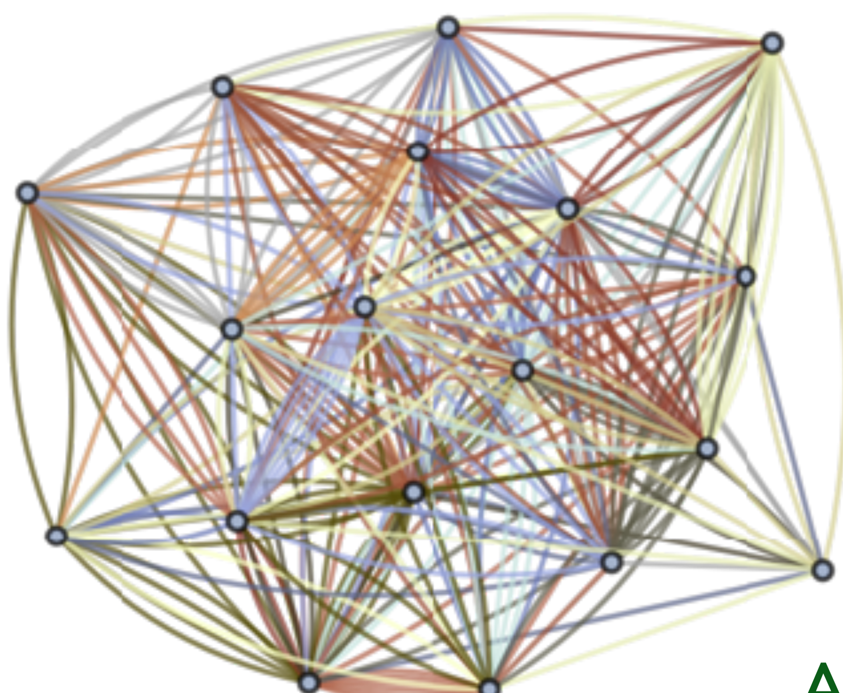
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large $N$ limit; T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

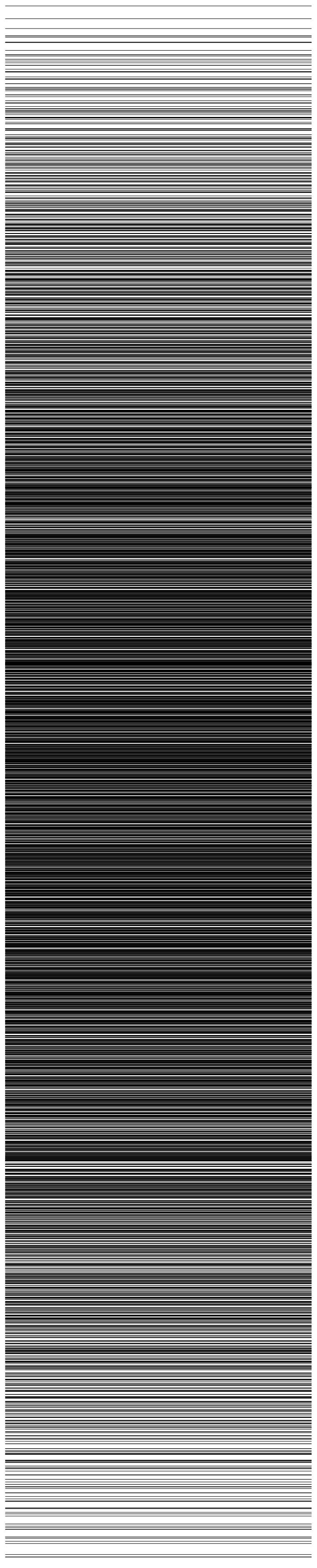
$J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$
$N \to \infty$ yields critical strange metal.



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# The Sachdev-Ye-Kitaev (SYK) model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-S_{GPS}}$

There are $2^N$ many body levels with energy $E$, which do not admit a quasiparticle decomposition. Shown are all values of $E$ for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS}$ with

$$\frac{S_{GPS}}{N} = \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots$$
$$< \ln 2$$

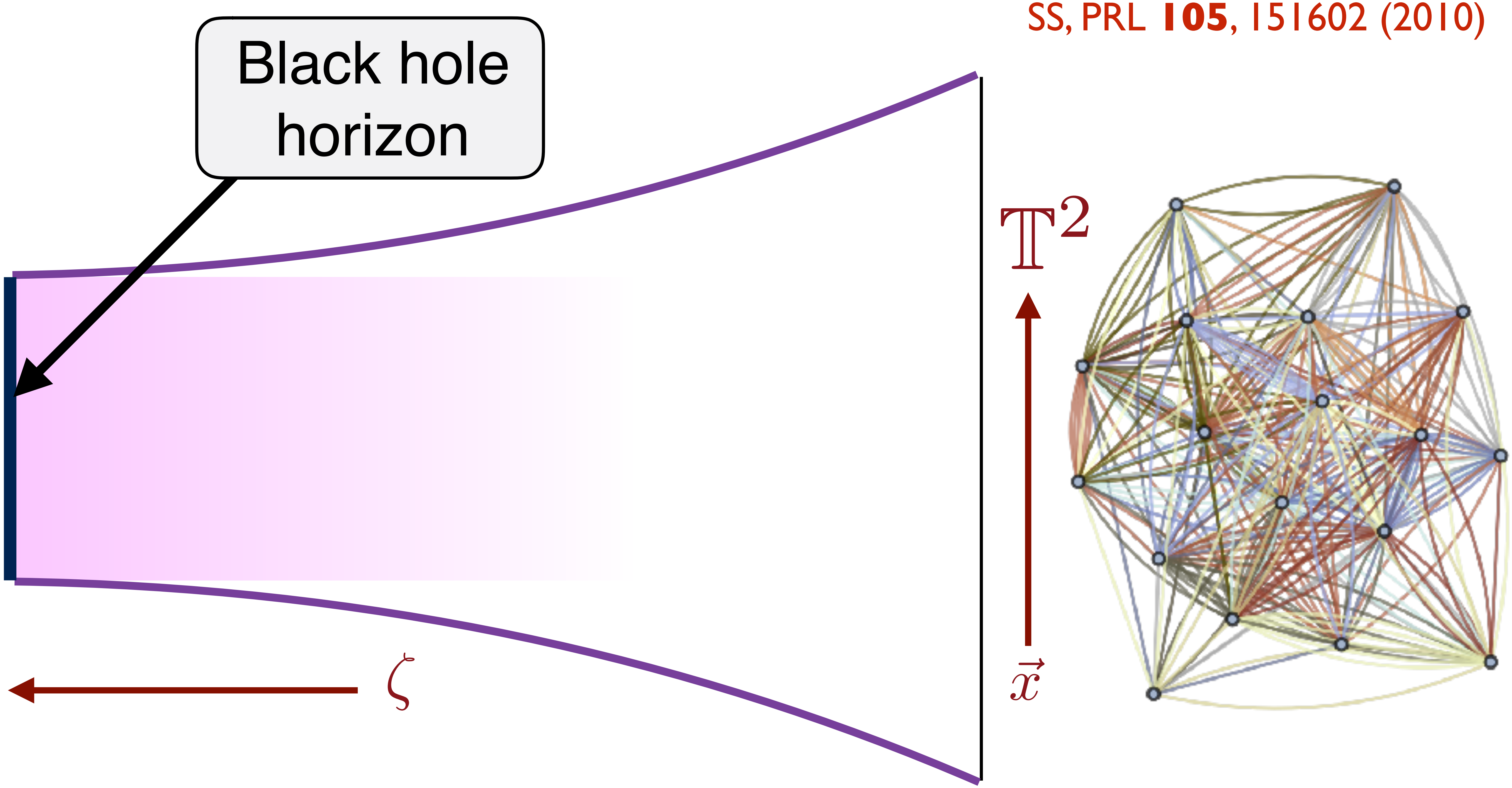
where $G$ is Catalan's constant, for the half-filled case $\mathcal{Q} = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

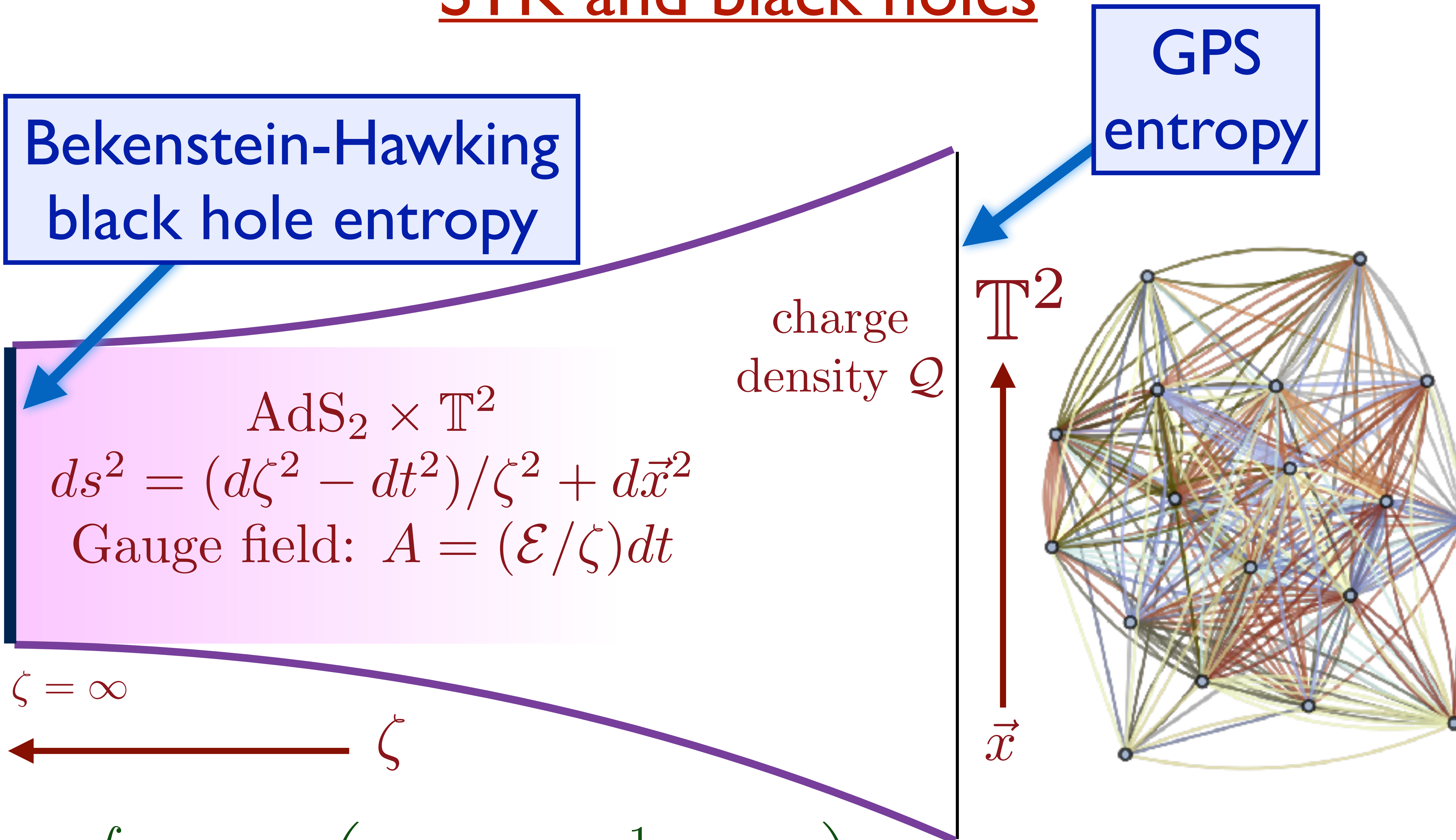
# SYK and black holes

SS, PRL **105**, 151602 (2010)




The SYK model has "dual" description
in which an extra spatial dimension, $\zeta$, emerges.
The curvature of this "emergent" spacetime is described
by Einstein's theory of general relativity

# SYK and black holes

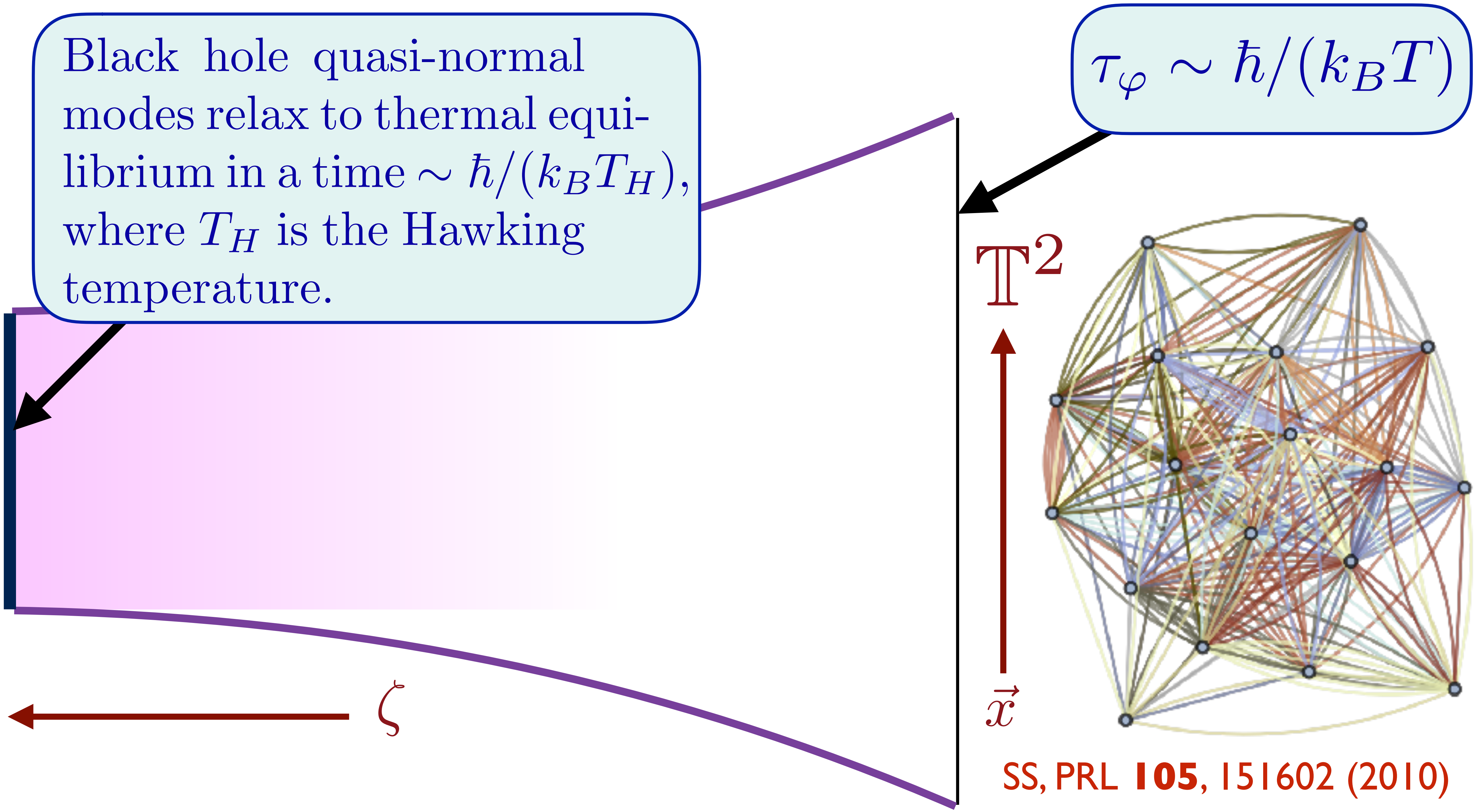



$$S = \int d^4x \sqrt{-\hat{g}} \left( \hat{\mathcal{R}} + 6/L^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right)$$

SS, PRL **105**, 151602 (2010)

The BH entropy is proportional to the size of $\mathbb{T}^2$, and hence the surface area of the black hole. Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$.

# SYK and black holes




The SYK model has "dual" description in which an extra spatial dimension, $\zeta$, emerges. The curvature of this "emergent" spacetime is described by Einstein's theory of general relativity

# Quantum Quench of the SYK model

Andreas
Eberlein

Julia
Steinberg

Valentin
Kasper

A. Eberlein, V. Kasper, S. Sachdev and J. Steinberg, arXiv:1706.xxxxx

Quench from
$pq$ fermion + $q$ fermion interactions for $t < 0$,
to only $q$ fermion interactions for $t > 0$.

$$H = f(t)\,(i)^{\frac{pq}{2}} \sum_{1\leq i_1<i_2<\ldots<i_{pq}\leq N} j_{i_1 i_2 \ldots i_{pq}} \psi_{i_1}\psi_{i_2}\ldots\psi_{i_{pq}}$$
$$+\, g(t)\,(i)^{\frac{q}{2}} \sum_{1\leq i_1<i_2<\ldots<i_q\leq N} j'_{i_1 i_2 \ldots i_q} \psi_{i_1}\psi_{i_2}\ldots\psi_{i_q}$$

for $t < 0$, $f(t) = 1$ and $g(t) = 1$;
for $t > 0$, $f(t) = 0$ and $g(t) = 1$.

$$\langle j^2_{i_1\ldots i_{pq}}\rangle = \frac{J_p^2 (pq-1)!}{N^{pq-1}} \quad , \quad J_p(t) = J_p f(t)$$
$$\langle j'^2_{i_1\ldots i_q}\rangle = \frac{J^2 (q-1)!}{N^{q-1}} \quad , \quad J(t) = J g(t)$$

# Kadanoff-Baym equations

$$iG^{>}(t_1,t_2) = \langle T_C\, \psi(t_1^-)\psi(t_2^+)\rangle \quad , \quad G^{<}(t_1,t_2) = -G^{>}(t_2,t_1)$$

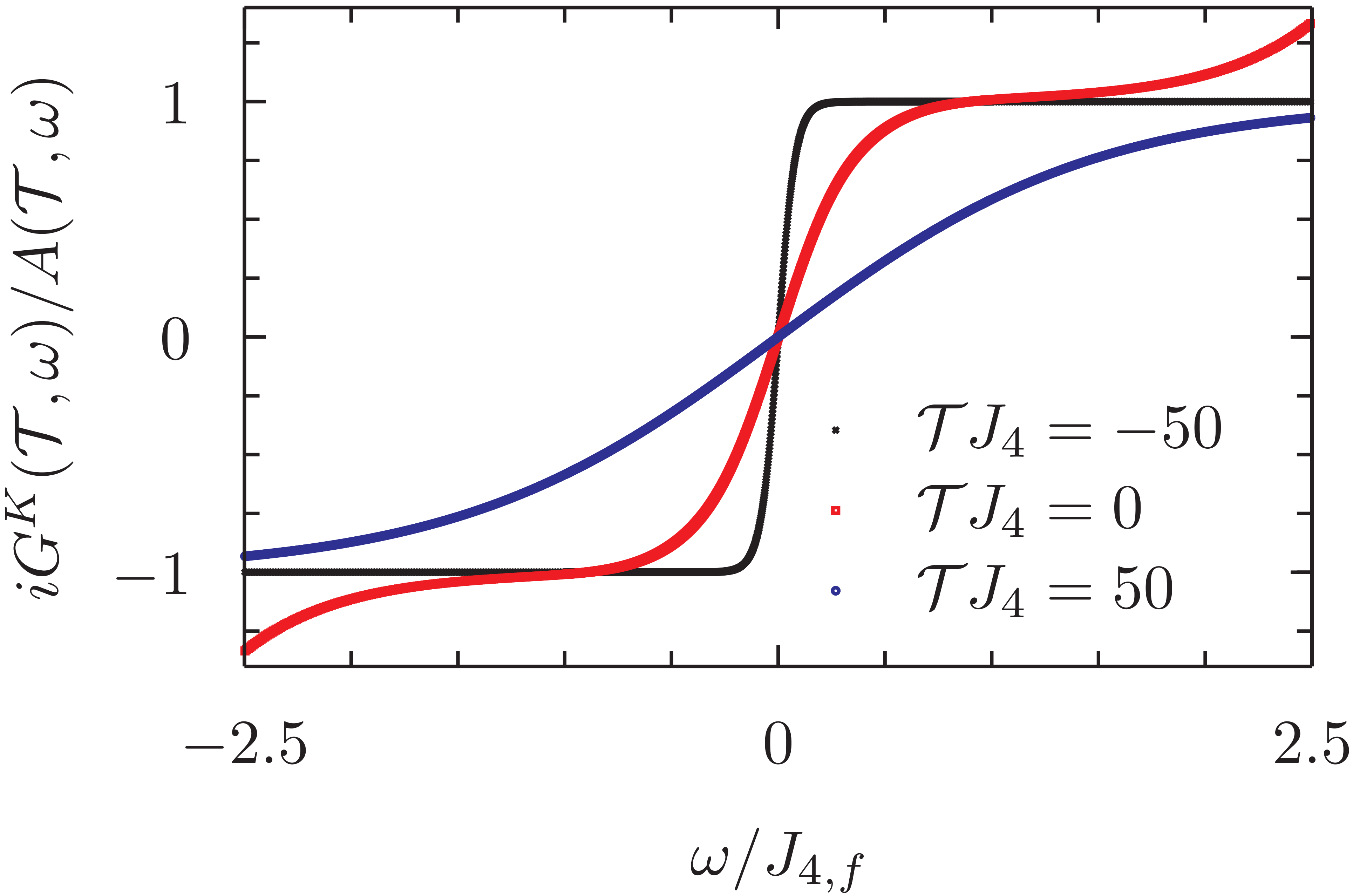
$$\begin{aligned}
i\frac{\partial}{\partial t_1}G^{>}(t_1,t_2) = &-i^q \int_{-\infty}^{t_1} dt_3\, J(t_1)J(t_3)\left[(G^{>})^{q-1}(t_1,t_3) - (G^{<})^{q-1}(t_1,t_3)\right] G^{>}(t_3,t_2) \\
&+ i^q \int_{-\infty}^{t_2} dt_3\, J(t_1)J(t_3)(G^{>})^{q-1}(t_1,t_3)\left[G^{>}(t_3,t_2) - G^{<}(t_3,t_2)\right] \\
&- i^{pq} \int_{-\infty}^{t_1} dt_3\, J_p(t_1)J_p(t_3)\left[(G^{>})^{pq-1}(t_1,t_3) - (G^{<})^{pq-1}(t_1,t_3)\right] G^{>}(t_3,t_2) \\
&+ i^{pq} \int_{-\infty}^{t_2} dt_3\, J_p(t_1)J_p(t_3)(G^{>})^{pq-1}(t_1,t_3)\left[G^{>}(t_3,t_2) - G^{<}(t_3,t_2)\right] \ ,
\end{aligned}$$

$$\begin{aligned}
-i\frac{\partial}{\partial t_2}G^{>}(t_1,t_2) = &-i^q \int_{-\infty}^{t_1} dt_3\, J(t_3)J(t_2)\left[G^{>}(t_1,t_3) - G^{<}(t_1,t_3)\right] (G^{>})^{q-1}(t_3,t_2) \\
&+ i^q \int_{-\infty}^{t_2} dt_3\, J(t_3)J(t_2)G^{>}(t_1,t_3)\left[(G^{>})^{q-1}(t_3,t_2) - (G^{<})^{q-1}(t_3,t_2)\right] \\
&- i^{pq} \int_{-\infty}^{t_1} dt_3\, J_p(t_3)J_p(t_2)\left[G^{>}(t_1,t_3) - G^{<}(t_1,t_3)\right] (G^{>})^{pq-1}(t_3,t_2) \\
&+ i^{pq} \int_{-\infty}^{t_2} dt_3\, J_p(t_3)J_p(t_2)G^{>}(t_1,t_3)\left[(G^{>})^{pq-1}(t_3,t_2) - (G^{<})^{pq-1}(t_3,t_2)\right] \ .
\end{aligned}$$

# Numerical solutions of Kadanoff-Baym equations

*p=1/2, q=4*

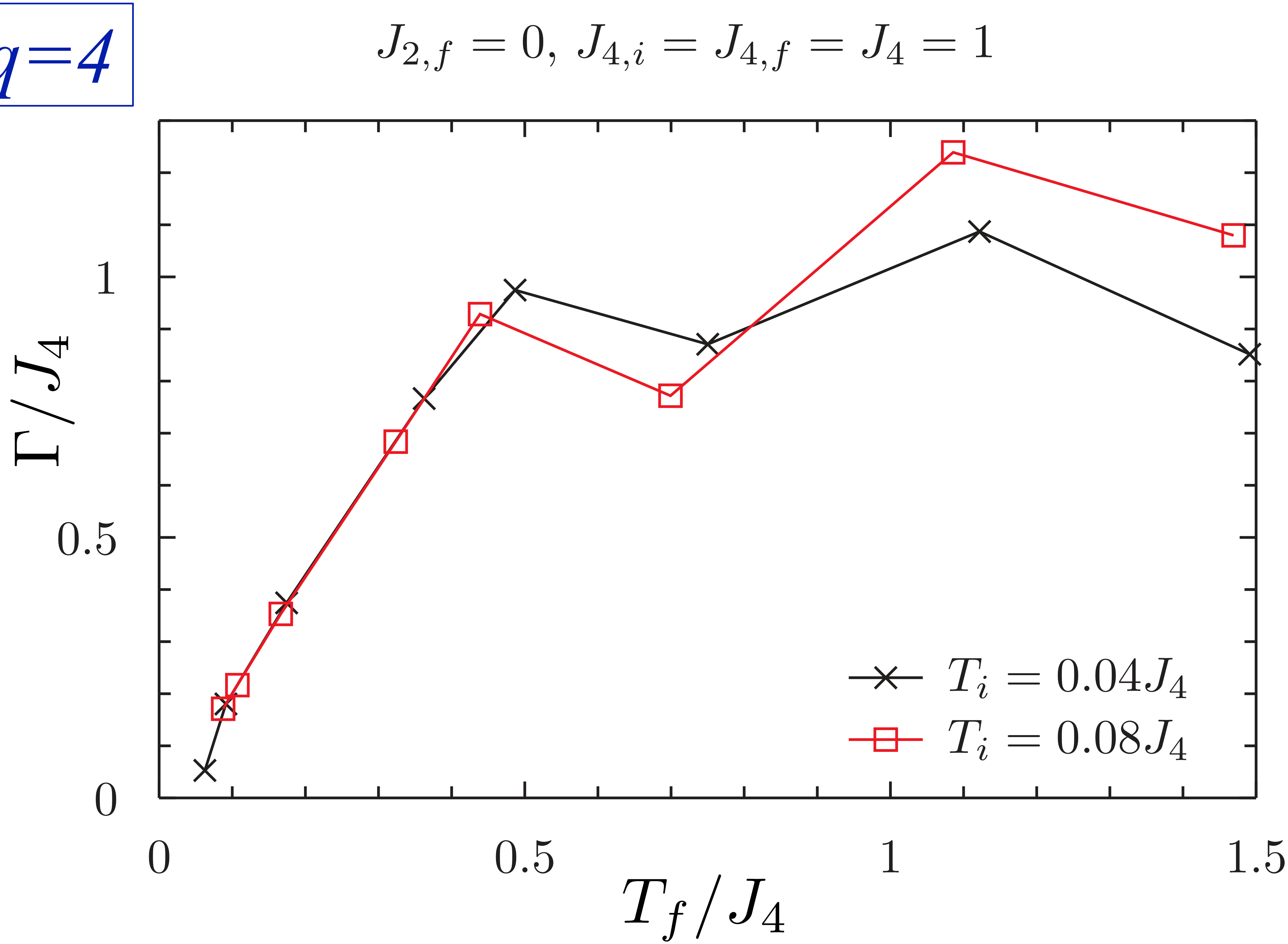
$J_{2,i} = 0.5,\ J_{2,f} = 0,\ J_{4,i} = J_{4,f} = 1,\ T_i = 0.04 J_4$




Determine an effective inverse temperature from such data, and then fit to $\beta_{\text{eff}}(\mathcal{T}) = \beta_f + \alpha \exp\left(-\Gamma\mathcal{T}\right)$, where $\mathcal{T} = (t_1 + t_2)/2$, and obtain the relaxation rate, $\Gamma$.

# Numerical solutions of Kadanoff-Baym equations

*p=1/2, q=4*




As $T_f \to 0$, the relaxation rate $\Gamma \sim T_f$

# $q \to \infty$ limit of Kadanoff-Baym equations

$$G^{>}(t_1,t_2) = -\frac{i}{2}\left[1 + \frac{1}{q} g(t_1,t_2) + \ldots\right]$$

$$\begin{aligned}
\frac{\partial}{\partial t_1} g(t_1,t_2) = {} & 2\int_{-\infty}^{t_2} dt_3\, \mathcal{J}(t_1)\mathcal{J}(t_3) e^{g(t_1,t_3)} - \int_{-\infty}^{t_1} dt_3\, \mathcal{J}(t_1)\mathcal{J}(t_3)\left[e^{g(t_1,t_3)} + e^{g(t_3,t_1)}\right] \\
& + 2\int_{-\infty}^{t_2} dt_3\, \mathcal{J}_p(t_1)\mathcal{J}_p(t_3) e^{pg(t_1,t_3)} - \int_{-\infty}^{t_1} dt_3\, \mathcal{J}_p(t_1)\mathcal{J}_p(t_3)\left[e^{pg(t_1,t_3)} + e^{pg(t_3,t_1)}\right] \\
\frac{\partial}{\partial t_2} g(t_1,t_2) = {} & 2\int_{-\infty}^{t_1} dt_3\, \mathcal{J}(t_3)\mathcal{J}(t_2) e^{g(t_3,t_2)} - \int_{-\infty}^{t_2} dt_3\, \mathcal{J}(t_3)\mathcal{J}(t_2)\left[e^{g(t_3,t_2)} + e^{g(t_2,t_3)}\right] \\
& + 2\int_{-\infty}^{t_1} dt_3\, \mathcal{J}_p(t_3)\mathcal{J}_p(t_2) e^{pg(t_3,t_2)} - \int_{-\infty}^{t_2} dt_3\, \mathcal{J}_p(t_3)\mathcal{J}_p(t_2)\left[e^{pg(t_3,t_2)} + e^{pg(t_2,t_3)}\right]
\end{aligned}$$

$$\mathcal{J}^2(t) = qJ^2(t)2^{1-q} \quad , \quad \mathcal{J}_p^2(t) = qJ_p^2(t)2^{1-pq}$$

These non-linear, partial, integro-differential equations are exactly solvable !

# $q \to \infty$ limit of Kadanoff-Baym equations

From a derivative of the Kadanoff-Baym equations, we obtain

$$\frac{\partial^2}{\partial t_1 \partial t_2} g(t_1, t_2) = 2\mathcal{J}(t_1)\mathcal{J}(t_2) e^{g(t_1,t_2)} + 2\mathcal{J}_p(t_1)\mathcal{J}_p(t_2) e^{pg(t_1,t_2)} \,.$$

For $t_1 > 0$ and $t_2 > 0$, this is the two-dimensional Liouville equation. The most general solution is of the form

$$g(t_1, t_2) = \ln \left[ \frac{-h_1'(t_1) h_2'(t_2)}{\mathcal{J}^2 (h_1(t_1) - h_2(t_2))^2} \right] \,.$$

A remarkable and significant feature of this expression is that it is exactly invariant under SL(2,C) transformations

$$h_\alpha(t) \to \frac{a\, h_\alpha(t) + b}{c\, h_\alpha(t) + d} \,,$$

where $\alpha = 1, 2$ and $a, b, c, d$ are arbitrary complex numbers.

# $q \to \infty$ limit of Kadanoff-Baym equations

Substituting back into the Kadanoff-Baym equations, we find that for generic initial conditions in the regime $t_1 < 0$ and $t_2 < 0$, the solutions for $h_1(t_1)$ and $h_2(t_2)$ at $t_1 > 0$ and $t_2 > 0$ can be written as

$$h_1(t) = \frac{a\, e^{i\theta} e^{\sigma t} + b}{c\, e^{i\theta} e^{\sigma t} + d} \quad , \quad h_2(t) = \frac{a\, e^{-i\theta} e^{\sigma t} + b}{c\, e^{-i\theta} e^{\sigma t} + d} \, .$$

The complex constants $a, b, c, d$, and the real constants $\sigma$, $\theta$ are determined by the initial conditions in the $t_1 < 0$ and $t_2 < 0$ quadrant of the $t_1$-$t_2$ plane. Substituting back into the SL(2,C) invariant Green's function, we obtain

$$g(t_1, t_2) = \ln \left[ \frac{-\sigma^2}{4\mathcal{J}^2 \sinh^2(\sigma(t_1 - t_2)/2 + i\theta)} \right] \quad , \quad \sigma = 2\mathcal{J} \sin(\theta) \, ,$$

which is (as expected) independent of $a$, $b$, $c$, $d$. The surprising feature is that depends only upon $t = t_1 - t_2$, and is independent of $\mathcal{T} = (t_1 + t_2)/2$.

# $q \to \infty$ limit of Kadanoff-Baym equations

Indeed, from the KMS condition, we find $g$ describes a state in thermal equilibrium at an inverse temperature

$$\beta_f = \frac{2(\pi - 2\theta)}{\sigma} \,.$$

As $T_f \to 0$, $\theta \to 0$, and $\sigma = 2\pi T_f$, the maximal chaos rate.

**The system is thermal at $t = 0^+$**

Actually, this may be a *pre-thermal* state (pointed out by Aavishkar Patel). The next correction

$$G^>(t_1, t_2) = -\frac{i}{2}\left[1 + \frac{1}{q} g(t_1, t_2) + \frac{1}{q^2} g_2(t_1, t_2) \ldots\right]$$

may lead to a $g_2$ which relaxes at a rate $\sim T_f$.

# $q \to \infty$ limit of Kadanoff-Baym equations

This result has a remarkable connection to the Schwarzian action. Consider the Euler-Lagrange equation of motion of a Lagrangian, $\mathcal{L}$, which is the Schwarzian of $h(t)$

$$\mathcal{L}[h(t)] = \frac{h'''(t)}{h'(t)} - \frac{3}{2}\left(\frac{h''(t)}{h'(t)}\right)^2 .$$

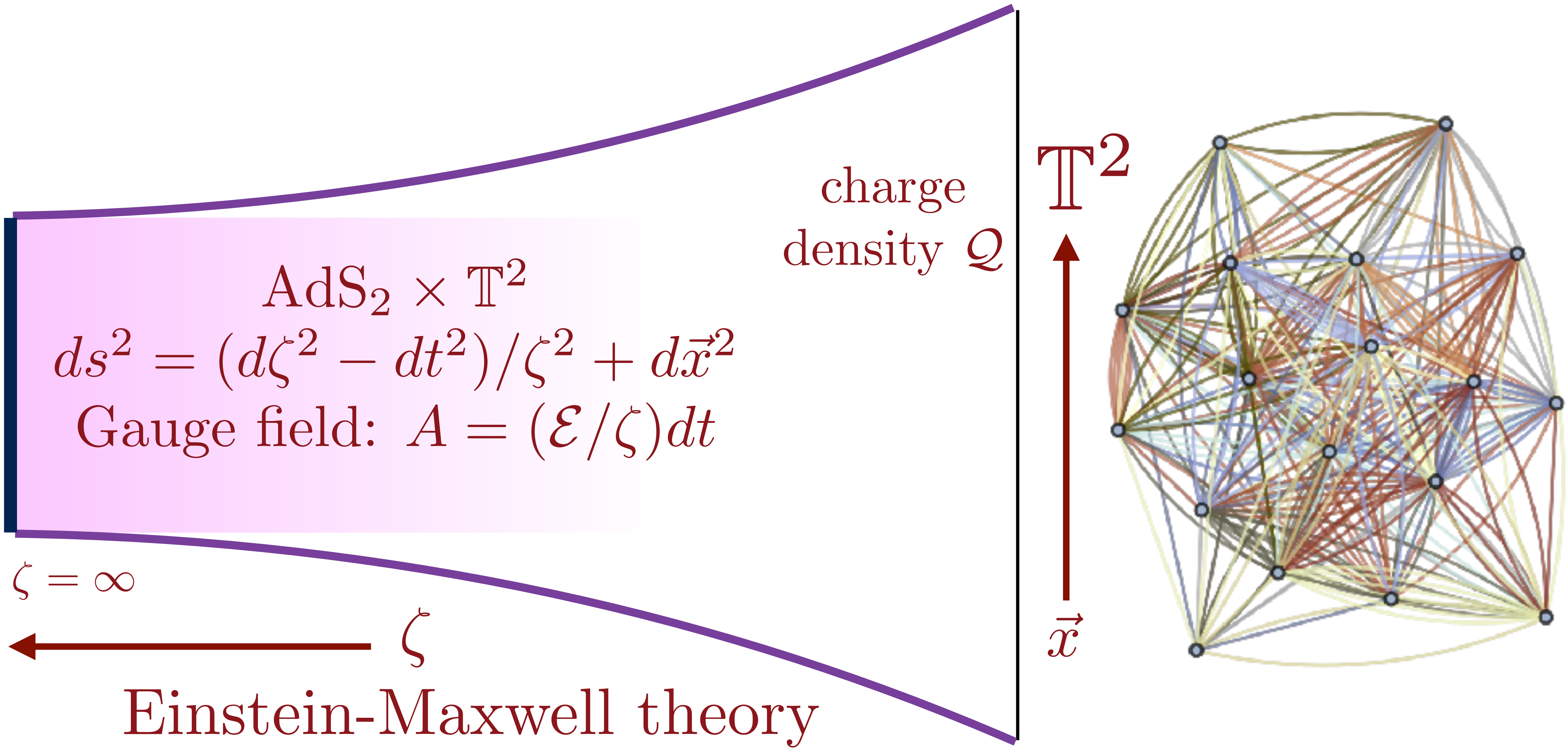
The equation of motion is

$$[h'(t)]^2\, h''''(t) + 3\,[h''(t)]^3 - 4h'(t)h''(t)h'''(t) = 0\,.$$

This is exactly solved by

$$h(t) = \frac{a\, e^{\sigma t} + b}{c\, e^{\sigma t} + d}\,.$$

# SYK and $AdS_2$



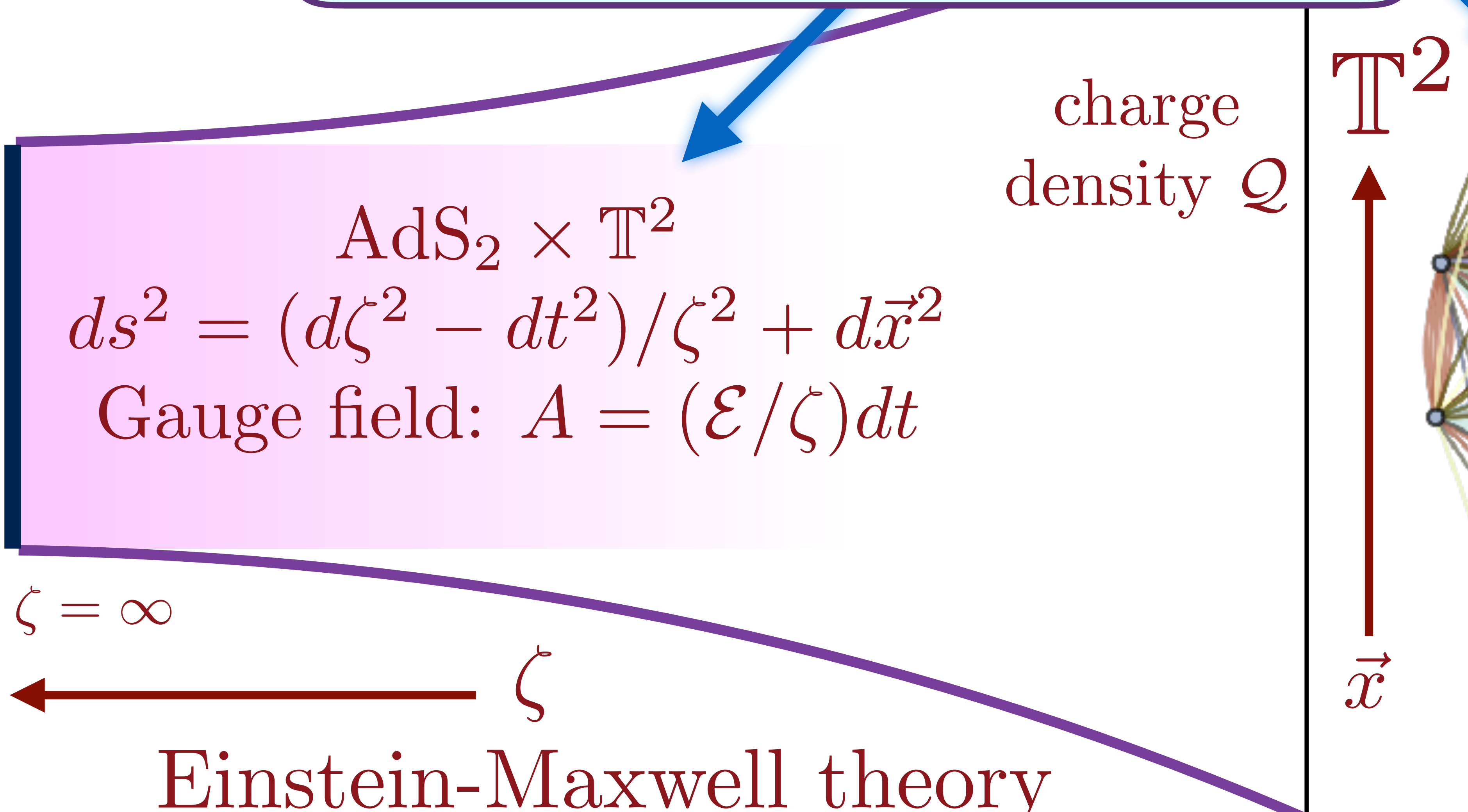

Einstein-Maxwell theory
+ cosmological constant

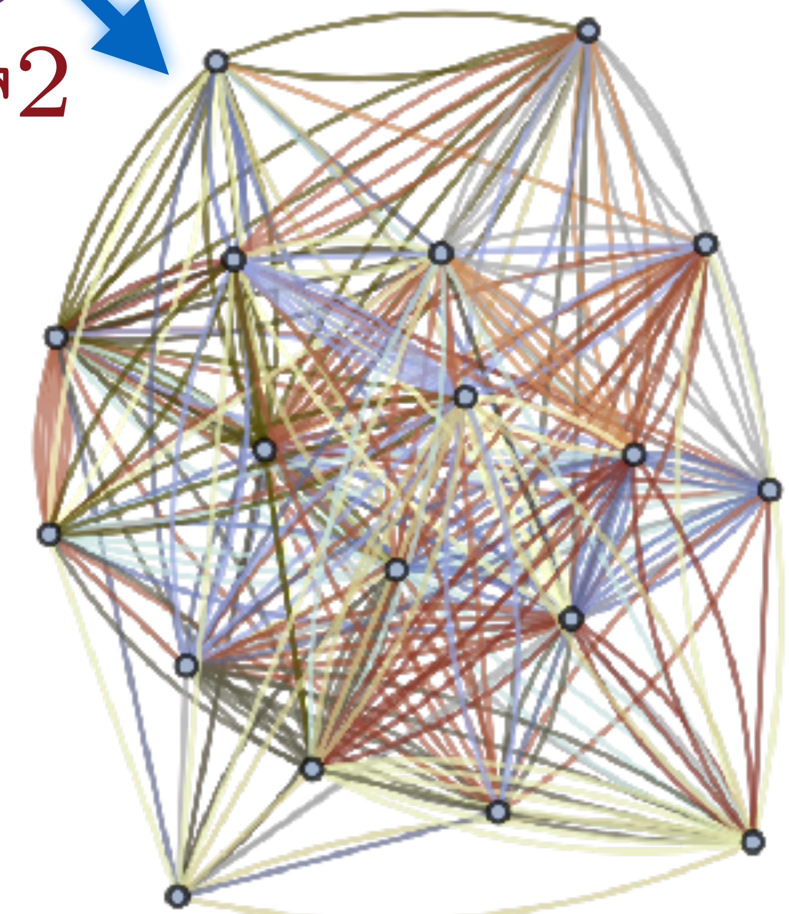
Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

SS, PRL **105**, 151602 (2010)

# SYK and $AdS_2$

Same long-time effective action involving Schwarzian derivatives of a time reparameterization $\tau \to h(\tau)$



Einstein-Maxwell theory + cosmological constant

Mapping to SYK applies when temperature $\ll 1/(\text{size of } \mathbb{T}^2)$

J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857; K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

# Thermal diffusivity and chaos in metals without quasiparticles

Mike Blake

Aavishkar Patel

Wenbo Fu

Yingfei Gu

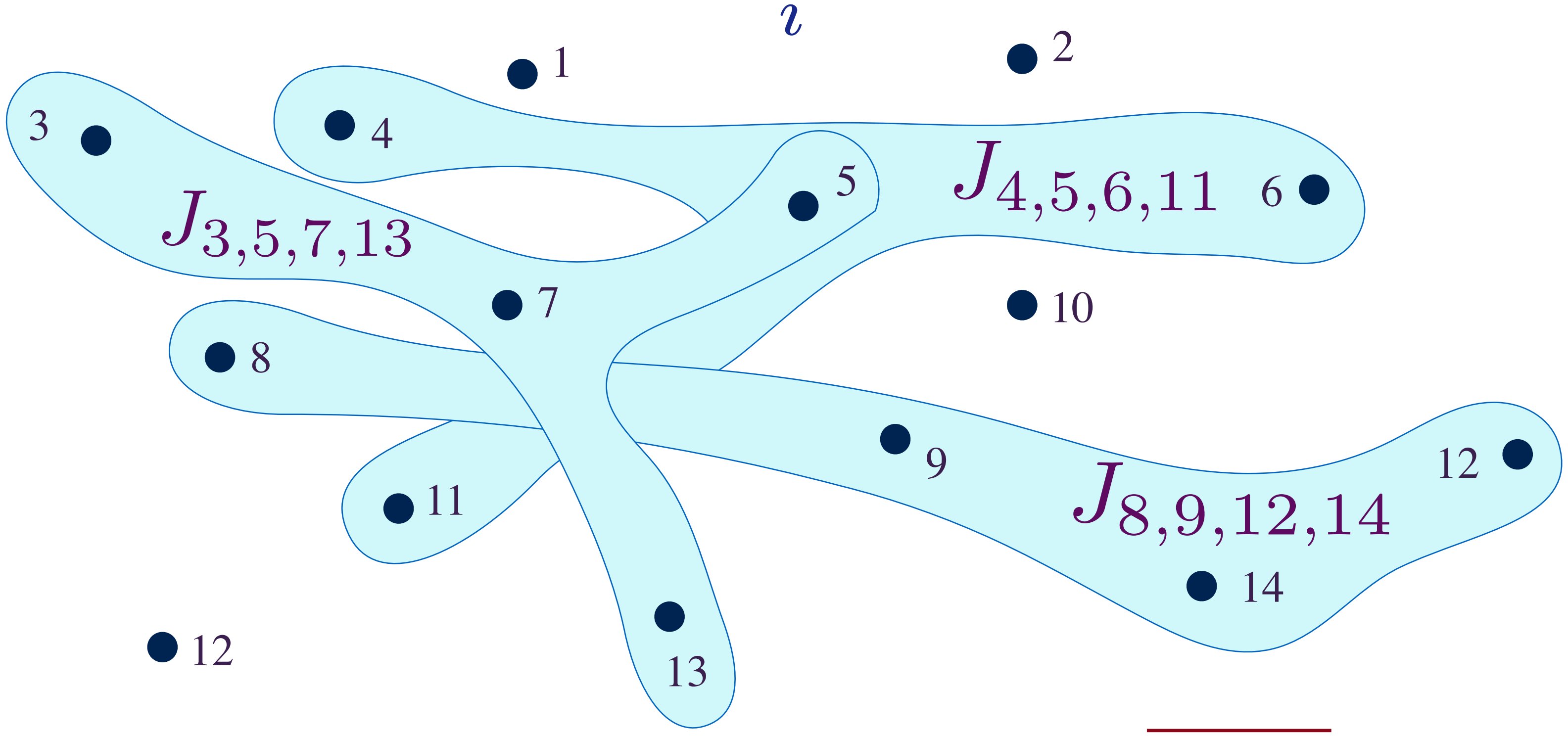
Richard Davison

# SYK model

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

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S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

# SYK and AdS$_2$

## Reparametrization and phase zero modes

We can write the path integral for the SYK model as

$$\mathcal{Z} = \int \mathcal{D}G(\tau_1, \tau_1)\mathcal{D}\Sigma(\tau_1, \tau_2)e^{-NS[G,\Sigma]}$$

for a known action $S[G, \Sigma]$. We find the saddle point, $G_s$, $\Sigma_s$, and only focus on the "Nambu-Goldstone" modes associated with breaking reparameterization and U(1) gauge symmetries by writing

$$G(\tau_1, \tau_2) = [f'(\tau_1)f'(\tau_2)]^{1/4} G_s(f(\tau_1) - f(\tau_2))e^{i\phi(\tau_1) - i\phi(\tau_2)}$$

(and similarly for $\Sigma$). Then the path integral is approximated by

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-NS_{\text{eff}}[f,\phi]}.$$

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

# SYK and AdS$_2$

$$\mathcal{Z} = \int \mathcal{D}f(\tau)\mathcal{D}\phi(\tau)e^{-NS_{\rm eff}[f,\phi]}.$$

Symmetry arguments, and explicit computations, show that the effective action is

$$S_{\rm eff}[f,\phi] = \frac{K}{2}\int_0^{1/T} d\tau(\partial_\tau\phi + i(2\pi\mathcal{E}T)\partial_\tau\epsilon)^2 - \frac{\gamma}{4\pi^2}\int_0^{1/T} d\tau\,\{\tan(\pi T(\tau+\epsilon(\tau),\tau\},$$

where $f(\tau) \equiv \tau + \epsilon(\tau)$, and we have used the Schwarzian:

$$\{g,\tau\} \equiv \frac{g'''}{g'} - \frac{3}{2}\left(\frac{g''}{g'}\right)^2.$$

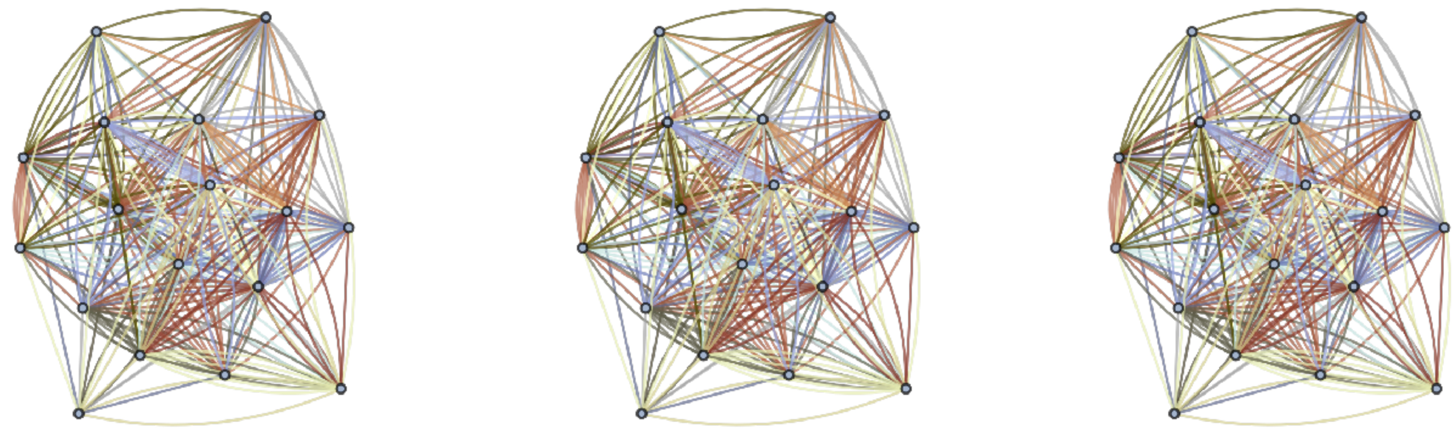
$K$ and $\gamma$ are related to the compressibility and the linear-$T$ specific heat, and is a derivative of the entropy:

$$K = \left(\frac{\partial\mathcal{Q}}{\partial\mu}\right)_T \quad , \quad S(T) = S_0 + \gamma T + \ldots \quad , \quad 2\pi\mathcal{E} = \frac{dS_0}{d\mathcal{Q}}$$

All these results apply unchanged to the gravitational theory on AdS$_2$.

J. Maldacena and D. Stanford, arXiv:1604.07818;
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849;
S. Sachdev, PRX **5,** 041025 (2015); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857;
K. Jensen, arXiv:1605.06098; J. Engelsoy, T.G. Mertens, and H. Verlinde, arXiv:1606.03438

# Coupled SYK models



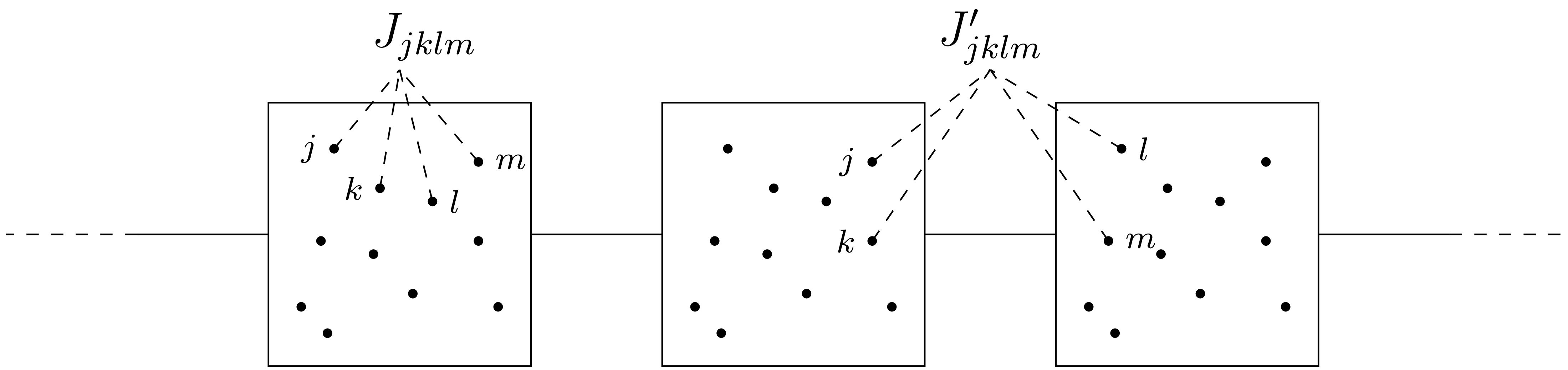
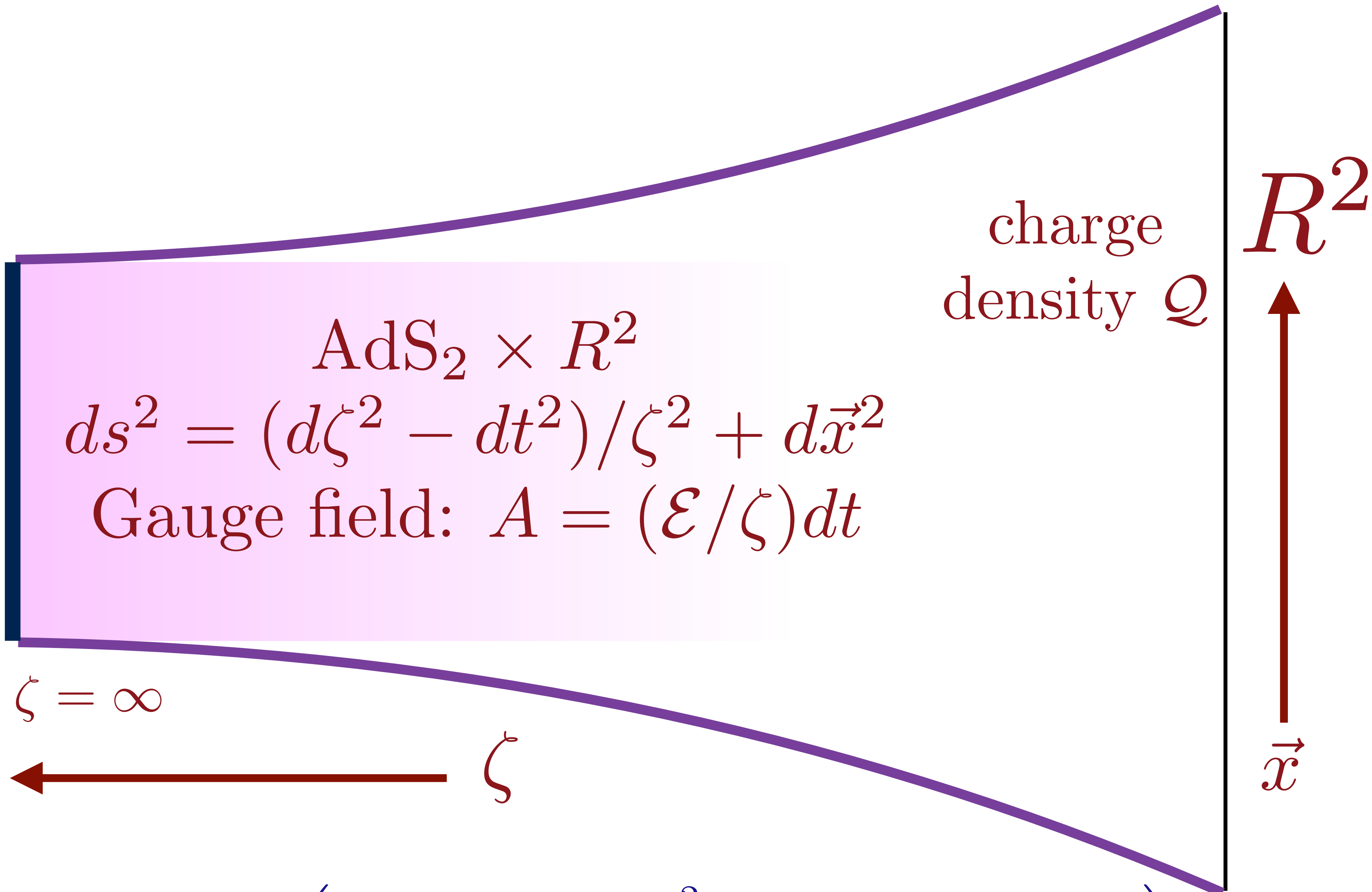
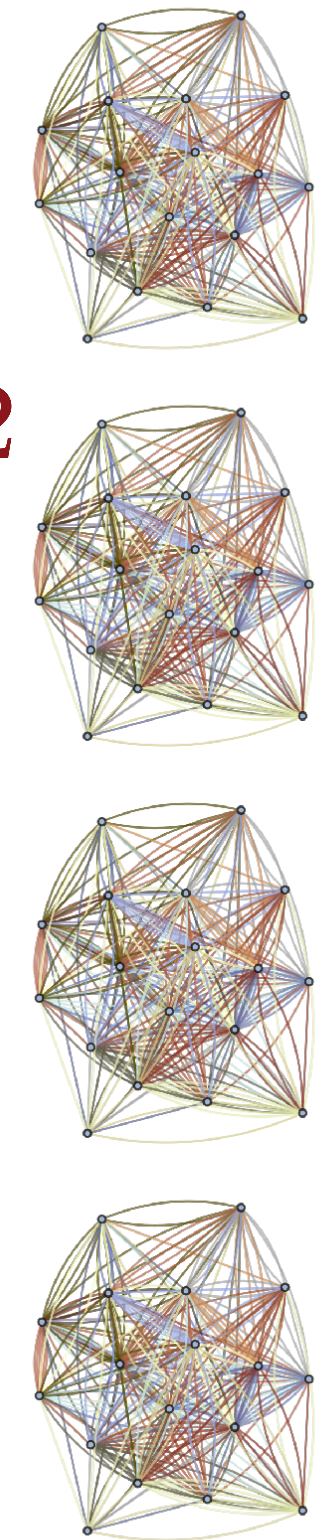



Figure 1: A chain of coupled SYK sites: each site contains $N \gg 1$ fermion with SYK interaction. The coupling between nearest neighbor sites are four fermion interaction with two from each site.


Yingfei Gu, Xiao-Liang Qi, and D. Stanford, arXiv:1609.07832
R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

# Coupled SYK and AdS₄



$$S = \int d^4x \sqrt{-\hat{g}} \left( \hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = k x_i$ leading to momentum disspation

# Coupled SYK and $AdS_4$

The response functions of the density, $\mathcal{Q}$, and the energy, $E$ exhibit diffusion

$$\begin{pmatrix} \langle \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} / T \\ \langle E - \mu \mathcal{Q}; \mathcal{Q} \rangle_{k,\omega} & \langle E - \mu \mathcal{Q}; E - \mu \mathcal{Q} \rangle_{k,\omega} / T \end{pmatrix} = \left[ i\omega(-i\omega + D k^2)^{-1} + 1 \right] \chi_s$$

where the diffusivities are related to the thermoelectric conductivities by the Einstein relations

$$D = \begin{pmatrix} \sigma & \alpha \\ \alpha T & \overline{\kappa} \end{pmatrix} \chi_s^{-1}.$$

The Seebeck co-efficient (thermopower), $\alpha/\sigma$, is given exactly by a thermodynamic derivative

$$\frac{\alpha}{\sigma} = \frac{\partial S_0}{\partial \mathcal{Q}}$$

The coupled-SYK and $AdS_4$ models realize
a disordered metal with no quasiparticle excitations.
(a "strange metal")

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

## Quantum chaos:

- The growth of chaos is characterized by

$$\left\langle \left| \{c(x,t), c^\dagger(0,0)\} \right|^2 \right\rangle \sim \exp\left(\frac{1}{\tau_L}\left(t - \frac{|x|}{v_B}\right)\right)$$

in terms of the Lyapunov time $\tau_L \geq \hbar/(2\pi k_B T)$ and the BUTTERFLY VELOCITY $v_B$.

- In the SYK and $AdS_2$ holographic models we find $\tau_L = \hbar/(2\pi k_B T)$ and a non-universal $v_B \sim T^{1/2}$. However the thermal diffusivity (which controls the diffusion of temperature)

$$D_T = \frac{\kappa}{c_\rho},$$

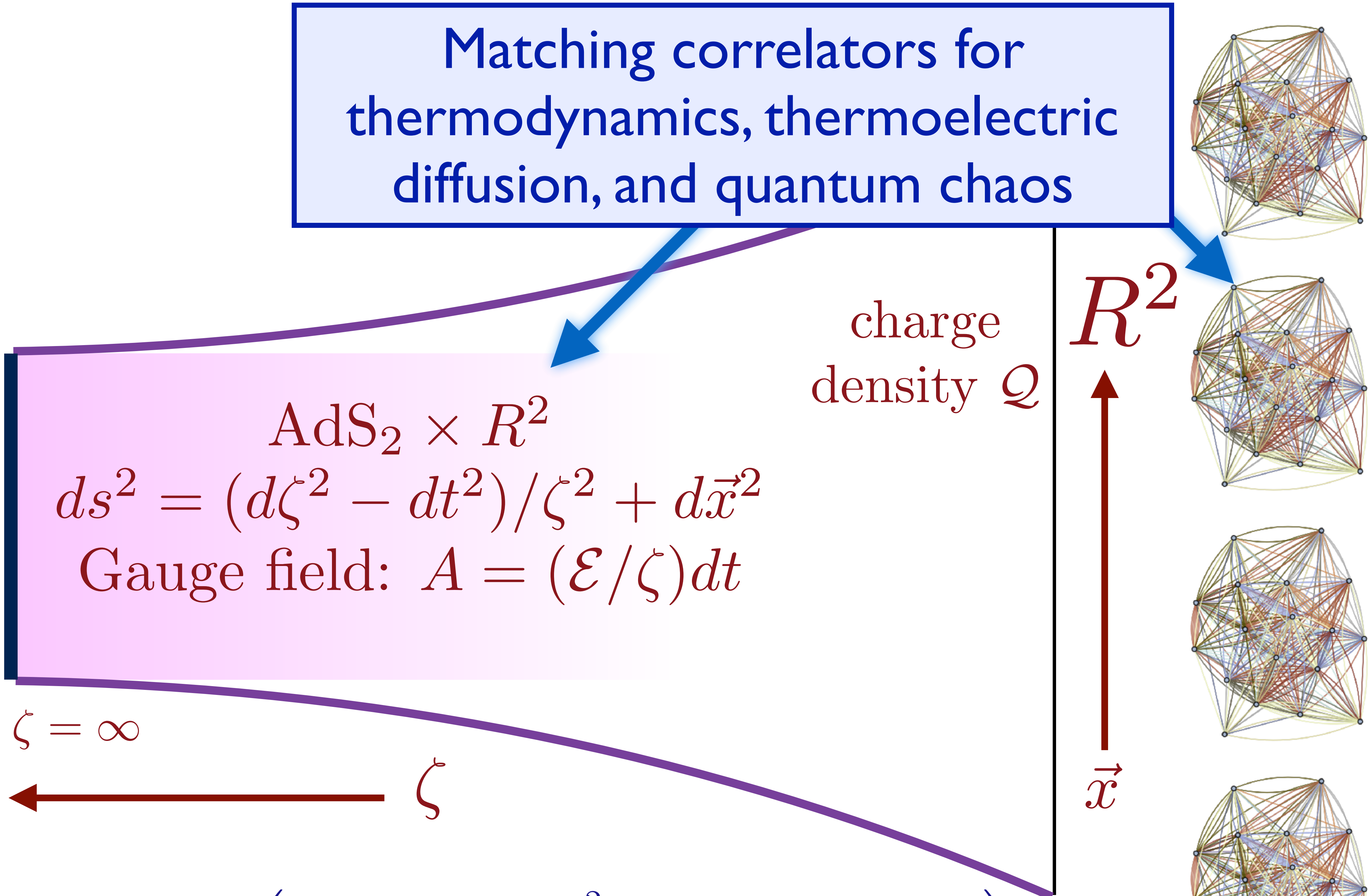
where $c_\rho$ is the specific heat at fixed density, is given exactly by

$$D_T = v_B^2 \tau_L.$$

There is no universal relationship between the charge diffusivity, $D_c$, and $v_B$.

R. Davison, Wenbo Fu, A. Georges, Yingfei Gu, K. Jensen, S. Sachdev, arXiv.1612.00849

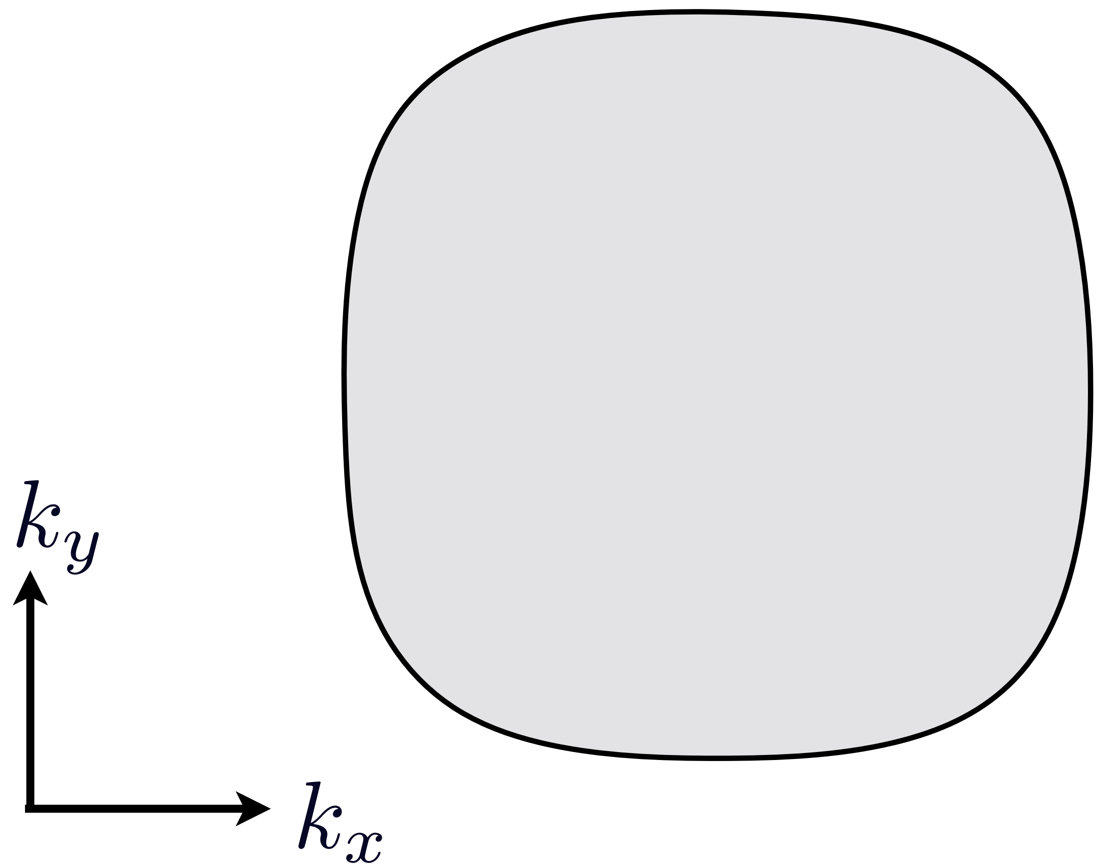
# Coupled SYK and $AdS_4$




$$S = \int d^4x \sqrt{-\hat{g}} \left( \hat{\mathcal{R}} + 6/L^2 - \frac{1}{2} \sum_{i=1}^{2} (\partial \hat{\varphi}_i)^2 - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \right),$$

Einstein-Maxwell-axion theory with saddle point $\hat{\varphi}_i = kx_i$ leading to momentum disspation

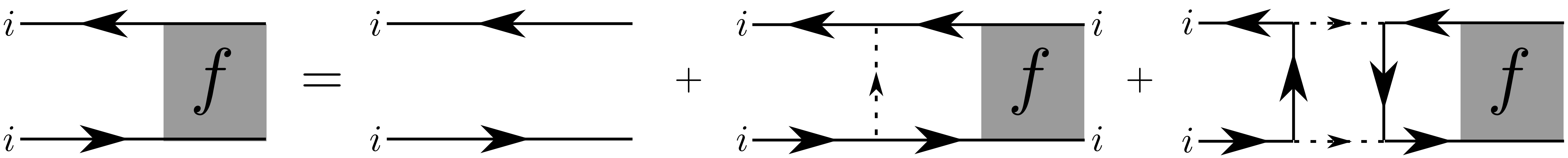
# Fermi surface coupled to a gauge field




$$\mathcal{L}[\Psi, a] = \Psi^\dagger \left( \partial_\tau - i a_\tau - \frac{(\nabla - i\vec{a})^2}{2m} - \mu \right) \Psi + \frac{1}{2g^2} (\nabla \times \vec{a})^2$$

# Fermi surface coupled to a gauge field

## Compute out-of-time-order correlator to diagnose quantum chaos




Strongly-coupled theory with no quasiparticles and fast scrambling:

$$\tau_L \approx \frac{\hbar}{2.48\, k_B T}$$

$$v_B \approx 4.1\, \frac{N T^{1/3}}{e^{4/3}} \frac{v_F^{5/3}}{\gamma^{1/3}}$$

$$D_T \approx 0.42\, v_B^2 \tau_L$$

$N$ is the number of fermion flavors, $v_F$ is the Fermi velocity, $\gamma$ is the Fermi surface curvature, $e$ is the gauge coupling constant.

A.A. Patel and S. Sachdev, arXiv:1611.00003

## Quantum chaos:

- We examined chaos and transport in a wide-class of holographic fixed points. In general, chaos and transport can be extracted from the fixed point solution, while the $AdS_2$ geometries (and SYK models) required consideration of the leading irrelevant operator. For the holographic fixed points, we find

$$
\begin{aligned}
\tau_L &= \frac{\hbar}{2\pi k_B T} \\
v_B &\sim T^{1-1/z} \\
D_T &= \frac{z}{2z-2}\, v_B^2 \tau_L \,,
\end{aligned}
$$

where $z$ is the dynamic critical exponent. There is no general relation to chaos for other transport co-efficients.

M. Blake, R.A. Davison, and S. Sachdev, arXiv.1612.00849

## Quantum chaos:

- We define the chaos parameters by

$$\left\langle \left| \{c(x,t), c^\dagger(0,0)\} \right|^2 \right\rangle \sim \exp\left( \frac{1}{\tau_L} \left( t - \frac{|x|}{v_B} \right) \right)$$

and the thermal diffusivity close to local equilibrium by

$$\frac{\partial T}{\partial t} = D_T \nabla^2 T \,.$$

Then we find the general relation

$$D_T = C\, v_B^2 \tau_L$$

where $C$ is a numerical constant of order unity, independent of short-distance details.

- Quantum chaos is intimately linked to the loss of phase coherence from electron-electron interactions. As the time derivative of the local phase is determined by the local energy, phase fluctuations and chaos are linked to interaction-induced energy fluctuations, and hence thermal diffusivity.

M. Blake, R.A. Davison, and S. Sachdev, arXiv.1612.00849