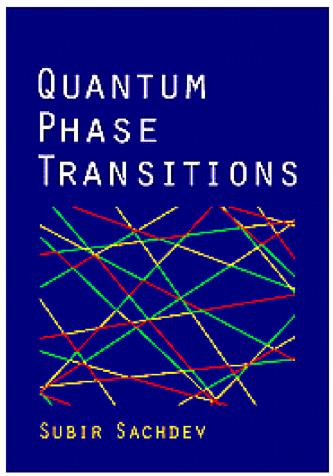
Quantum phase transitions in atomic gases and condensed matter

Subir Sachdev

Science **286**, 2479 (1999).



Quantum Phase Transitions
Cambridge University Press

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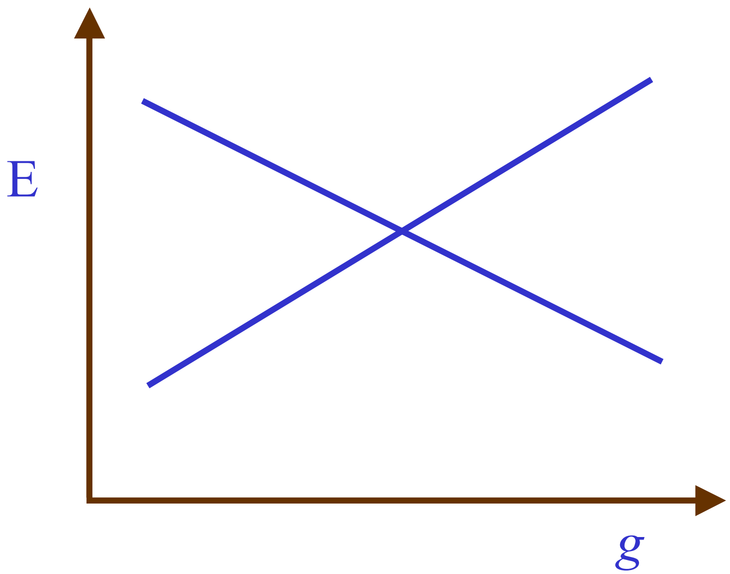
http://pantheon.yale.edu/~subir

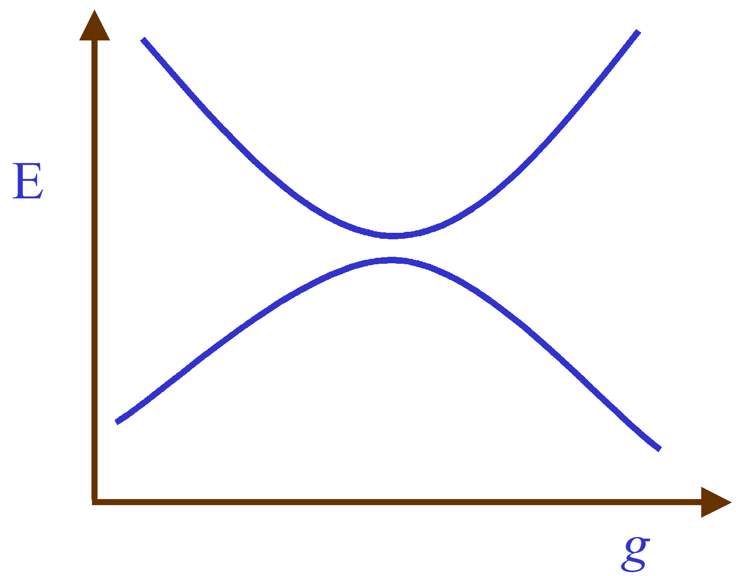
What is a quantum phase transition ?

Non-analyticity in ground state properties as a function of some control parameter *g*



True level crossing:

Usually a *first*-order transition



Avoided level crossing which becomes sharp in the infinite volume limit:

second-order transition

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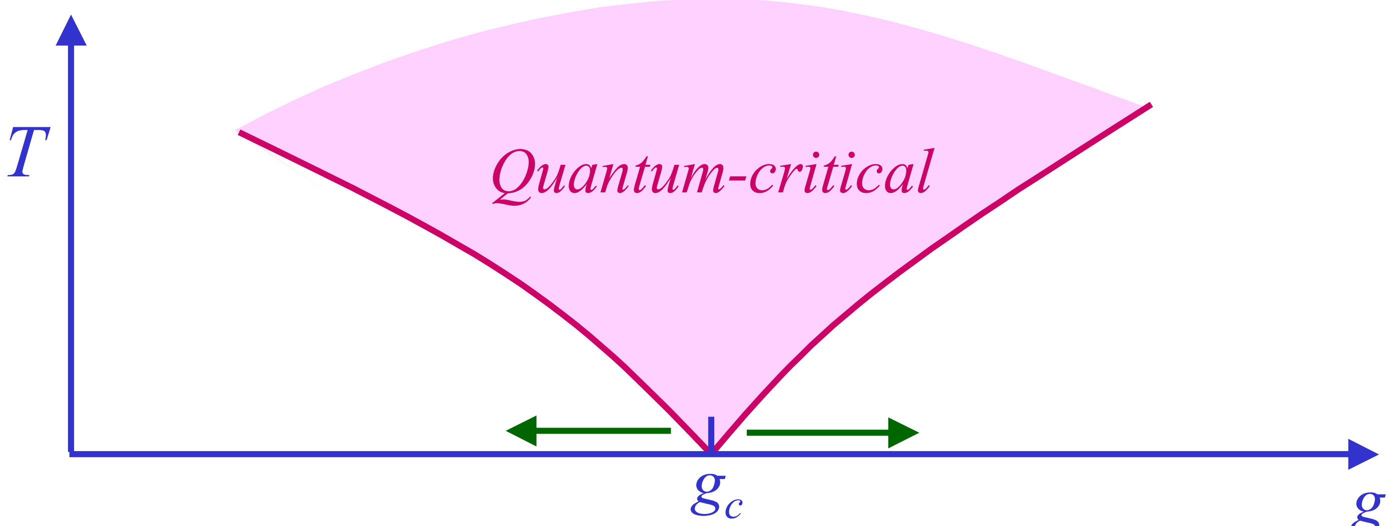
Why study quantum phase transitions ?



- Theory for a quantum system with strong correlations: describe phases on either side of g_c by expanding in deviation from the quantum critical point.

- Critical point is a novel state of matter without quasiparticle excitations

- Critical excitations control dynamics in the wide *quantum-critical* region at non-zero temperatures.

Important property of ground state at $g=g_c$:
temporal and spatial scale invariance;
characteristic energy scale at other values of g: $\Delta \sim \left|g - g_c\right|^{z\nu}$

Outline

I. The quantum Ising chain.

II. The superfluid-insulator transition

III. Quantum transitions without local order parameters: fractionalization.

IV. Conclusions

I. Quantum Ising Chain

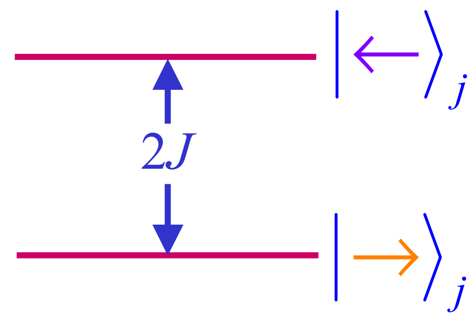
Degrees of freedom: $j = 1 \ldots N$ qubits, N "large"

$$\left|\uparrow\right\rangle_j, \left|\downarrow\right\rangle_j$$

or $$\left|\rightarrow\right\rangle_j = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_j + \left|\downarrow\right\rangle_j\right), \quad \left|\leftarrow\right\rangle_j = \frac{1}{\sqrt{2}}\left(\left|\uparrow\right\rangle_j - \left|\downarrow\right\rangle_j\right)$$

Hamiltonian of decoupled qubits:

$$H_0 = -J\sum_j \sigma_j^x$$



Coupling between qubits:

$$H_1 = -Jg\sum_j \sigma^z_j \sigma^z_{j+1}$$

$$\boxed{\Big(\left|\rightarrow\right\rangle_j\left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_j\left\langle\rightarrow\right|\Big)\Big(\left|\rightarrow\right\rangle_{j+1}\left\langle\leftarrow\right| + \left|\leftarrow\right\rangle_{j+1}\left\langle\rightarrow\right|\Big)}$$

Prefers neighboring qubits

are *either* $\left|\uparrow\right\rangle_j \left|\uparrow\right\rangle_{j+1}$ *or* $\left|\downarrow\right\rangle_j \left|\downarrow\right\rangle_{j+1}$

(not entangled)

Full Hamiltonian

$$\boxed{H = H_0 + H_1}$$

leads to entangled states at g of order unity

Weakly-coupled qubits $\left(g \ll 1\right)$

Ground state:

$$\left|G\right\rangle = \left|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\right\rangle$$

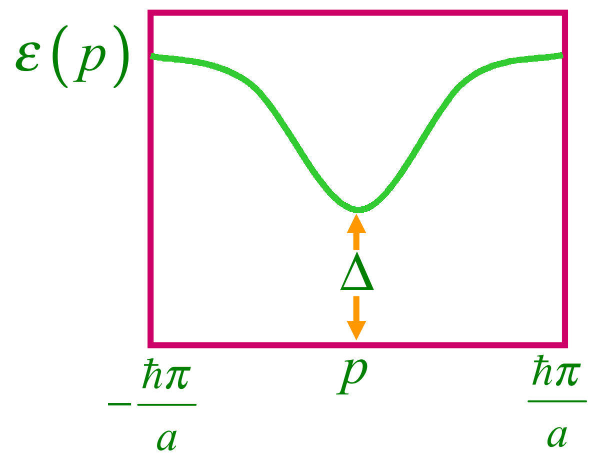
$$-\frac{g}{2}\left|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\leftarrow\leftarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\right\rangle - \cdots$$

Lowest excited states:

$$\left|\ell_j\right\rangle = \left|\cdots \rightarrow\rightarrow\rightarrow\rightarrow\underset{j}{\leftarrow}\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \cdots\right\rangle + \cdots$$

Coupling between qubits creates "flipped-spin" *quasiparticle* states at momentum p

$$\left|p\right\rangle = \sum_j e^{ipx_j/\hbar}\left|\ell_j\right\rangle$$



Excitation energy $\varepsilon\left(p\right) = \Delta + 4gJ\sin^2\left(\frac{pa}{2\hbar}\right) + O\left(g^2\right)$

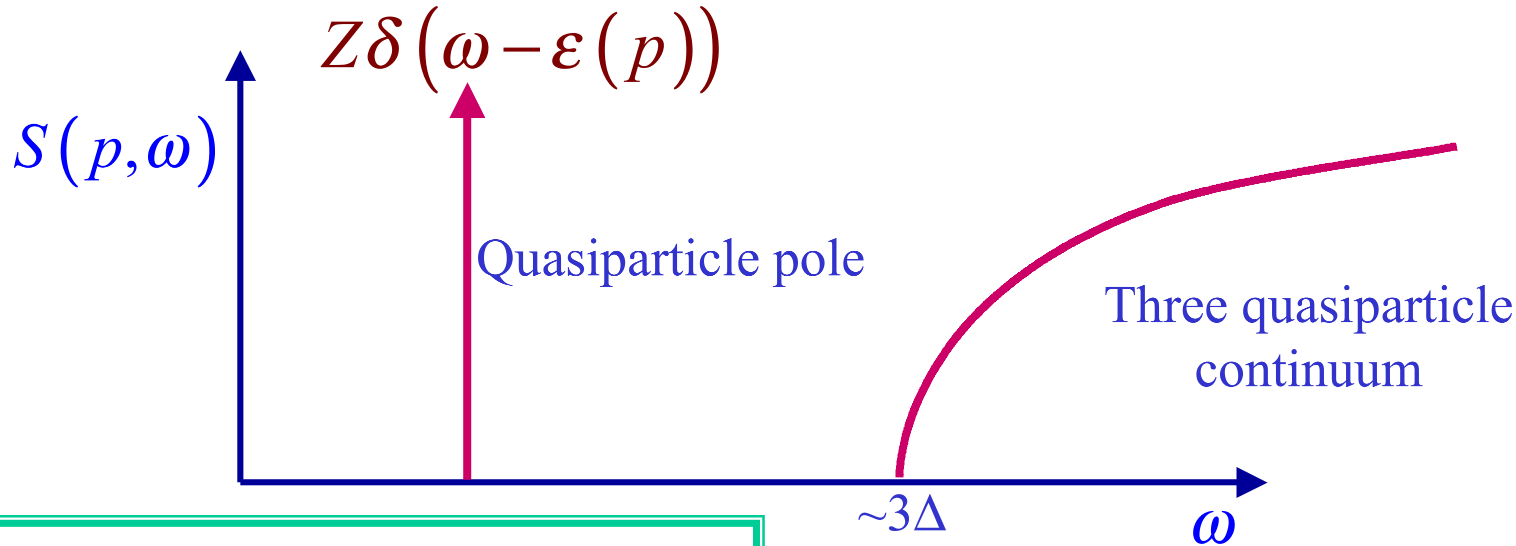
Excitation gap $\Delta = 2J - 2gJ + O\left(g^2\right)$

Entire spectrum can be constructed out of multi-quasiparticle states

Dynamic Structure Factor $S(p,\omega)$:

$\boxed{\text{Weakly-coupled qubits } (g \ll 1)}$

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



$\boxed{\text{Structure holds to all orders in } g}$

At $T > 0$, collisions between quasiparticles broaden pole to a Lorentzian of width $1/\tau_\varphi$ where the ***phase coherence time*** τ_φ is given by

$$\frac{1}{\tau_\varphi} = \frac{2k_B T}{\pi\hbar} e^{-\Delta/k_B T}$$

S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Strongly-coupled qubits $\left(g \gg 1\right)$

Ground states:

$$\left|G\uparrow\right\rangle = \left|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\right\rangle$$

$$-\frac{1}{2g}\left|\cdots\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\right\rangle - \cdots$$

Ferromagnetic moment

$N_0 = \left\langle G\right|\sigma^z\left|G\right\rangle \neq 0$

Second state $\left|G\downarrow\right\rangle$ obtained by $\uparrow \Leftrightarrow \downarrow$

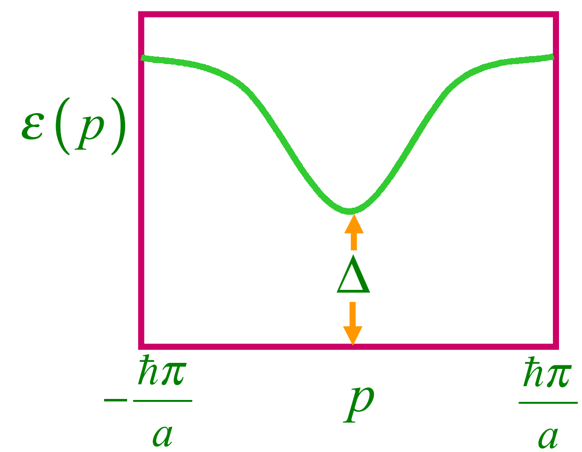
$\left|G\downarrow\right\rangle$ and $\left|G\uparrow\right\rangle$ mix only at order g^{-N}

Lowest excited states: domain walls

$$\left|d_j\right\rangle = \left|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow_j\downarrow\downarrow\downarrow\downarrow\downarrow\cdots\right\rangle + \cdots$$

Coupling between qubits creates new "domain-wall" *quasiparticle* states at momentum p

$$\left|p\right\rangle = \sum_j e^{ipx_j/\hbar}\left|d_j\right\rangle$$



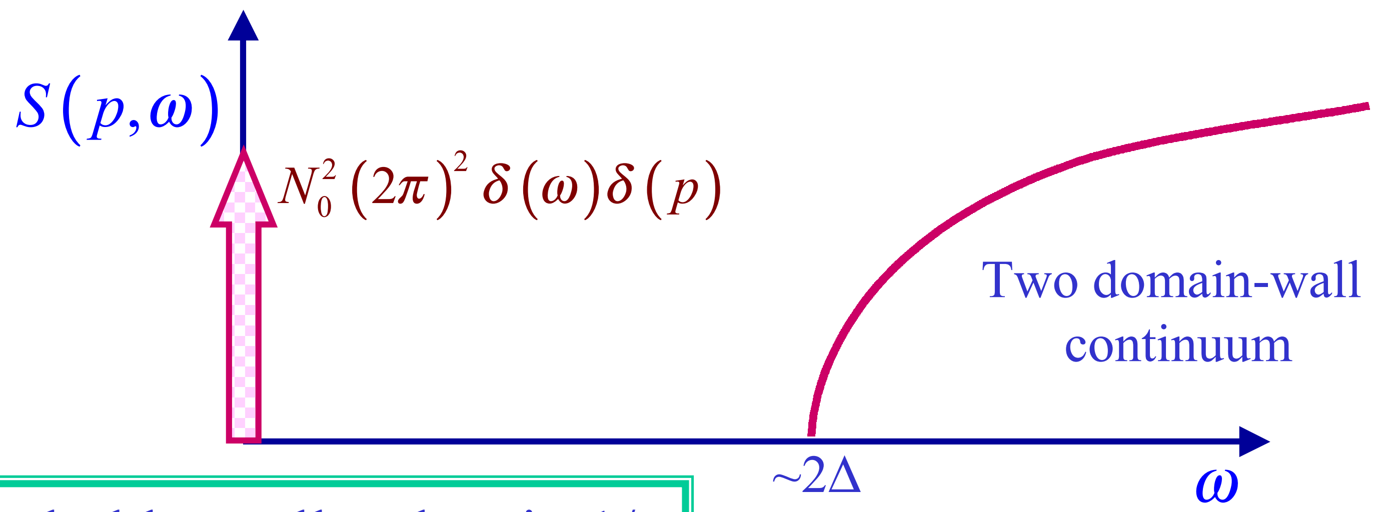
Excitation energy $\varepsilon(p) = \Delta + 4J\sin^2\left(\frac{pa}{2\hbar}\right) + O\left(g^{-1}\right)$

Excitation gap $\Delta = 2gJ - 2J + O\left(g^{-1}\right)$

Dynamic Structure Factor $S(p,\omega)$:

Strongly-coupled qubits $(g \gg 1)$

Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa) while transferring energy $\hbar\omega$ and momentum p



Structure holds to all orders in $1/g$

At $T > 0$, motion of domain walls leads to a finite ***phase coherence time*** τ_φ,

and broadens coherent peak to a width $1/\tau_\varphi$ where $$\frac{1}{\tau_\varphi} = \frac{2k_B T}{\pi \hbar} e^{-\Delta/k_B T}$$

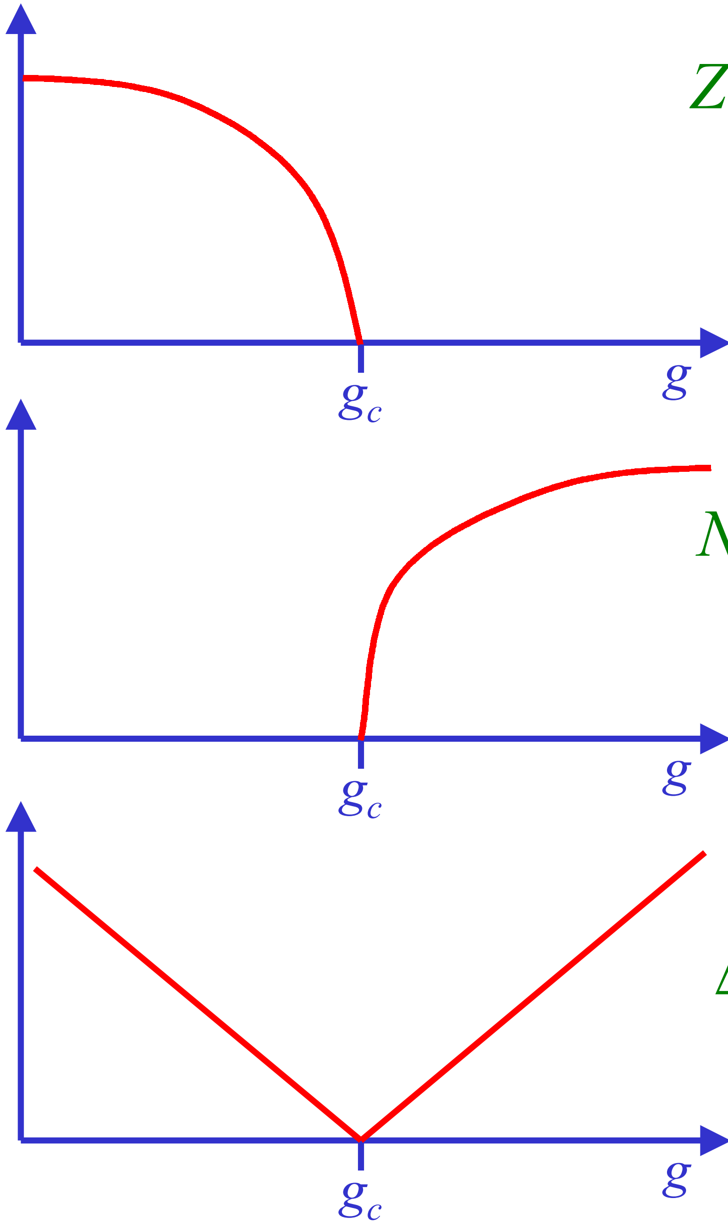
S. Sachdev and A.P. Young, *Phys. Rev. Lett.* **78**, 2220 (1997)

Entangled states at g of order unity

“Flipped-spin” Quasiparticle weight Z

Ferromagnetic moment N_0

Excitation energy gap Δ



$$Z \sim (g_c - g)^{1/4}$$

A.V. Chubukov, S. Sachdev, and J.Ye, *Phys. Rev.* B **49**, 11919 (1994)

$$N_0 \sim (g - g_c)^{1/8}$$

P. Pfeuty *Annals of Physics*, **57**, 79 (1970)

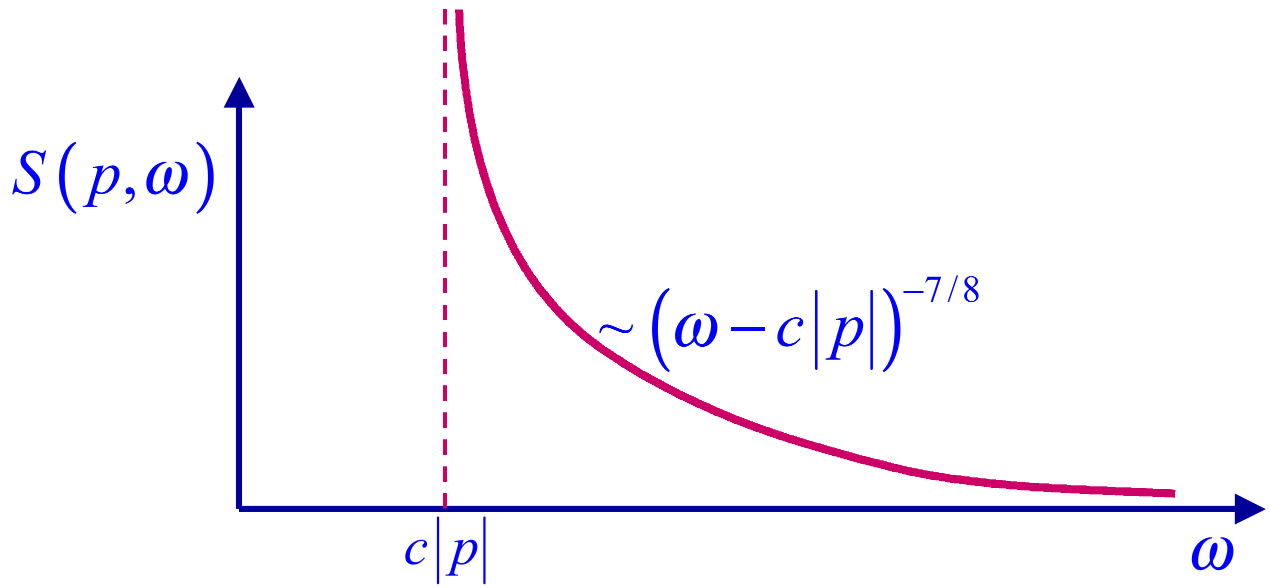
$$\Delta \sim |g - g_c|$$

Dynamic Structure Factor $S(p,\omega)$:

$\boxed{\text{Critical coupling } (g = g_c)}$

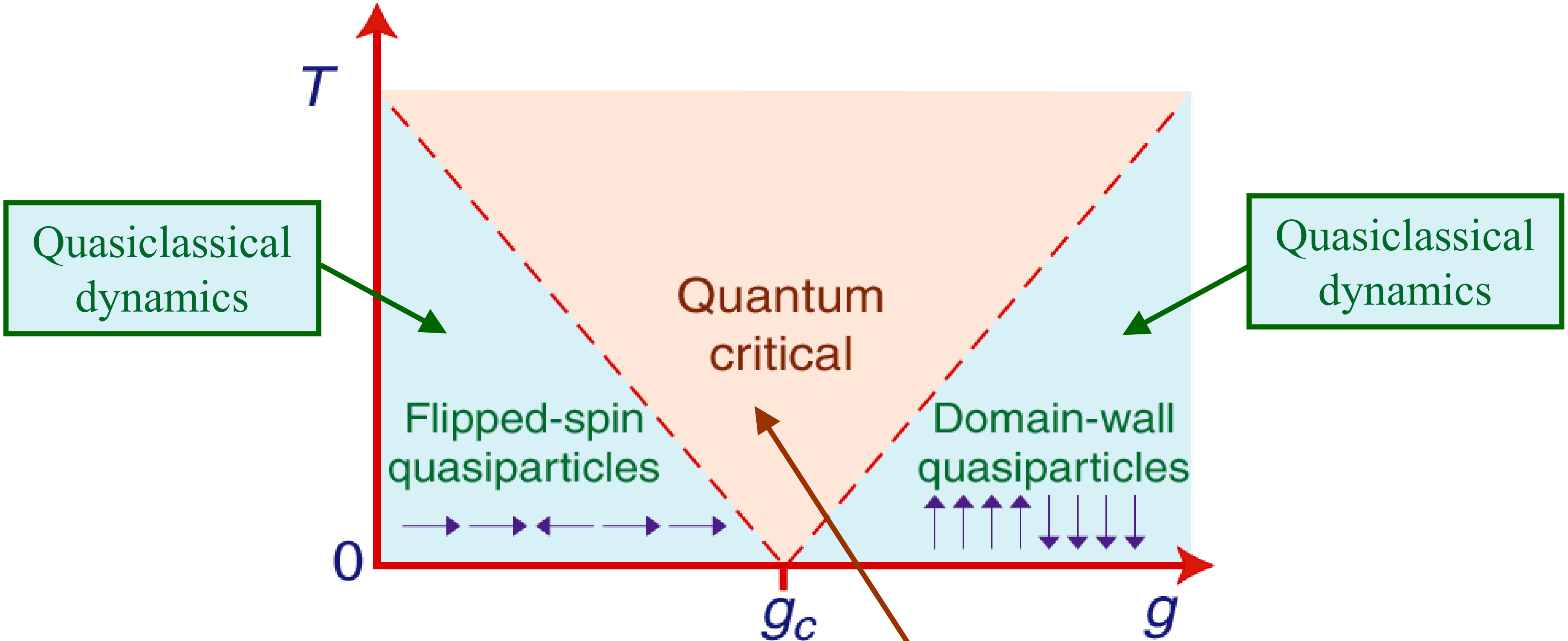
Cross-section to flip a $|\rightarrow\rangle$ to a $|\leftarrow\rangle$ (or vice versa)

while transferring energy $\hbar\omega$ and momentum p



No quasiparticles --- dissipative critical continuum

Crossovers at nonzero temperature



$$\chi(\omega) = \frac{i}{\hbar} \sum_k \int_0^\infty dt \left\langle \left[\sigma_j^z(t), \sigma_k^z(0) \right] \right\rangle e^{i\omega t}$$

$$= \frac{A}{T^{7/4}\left(1 - i\omega\tau_\varphi + \ldots\right)}$$

Phase coherence time τ_φ given by $\dfrac{1}{\tau_\varphi} = \left(2\tan\dfrac{\pi}{16}\right)\dfrac{k_B T}{\hbar}$

S. Sachdev and J. Ye,
Phys. Rev. Lett. **69**, 2411 (1992).
S. Sachdev and A.P. Young,
Phys. Rev. Lett. **78**, 2220 (1997).

II. The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu\sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

M.P.A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher *Phys. Rev. B* **40**, 546 (1989).

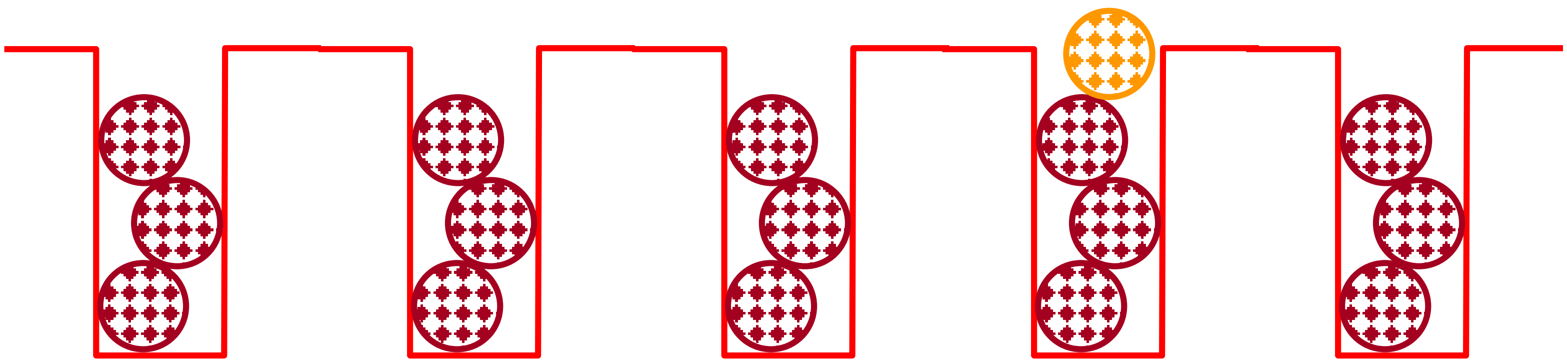
For small U/t, ground state is a superfluid BEC with

superfluid density $\approx$ density of bosons

What is the ground state for large *U/t* ?

Typically, the ground state remains a superfluid, but with

superfluid density $\ll$ density of bosons

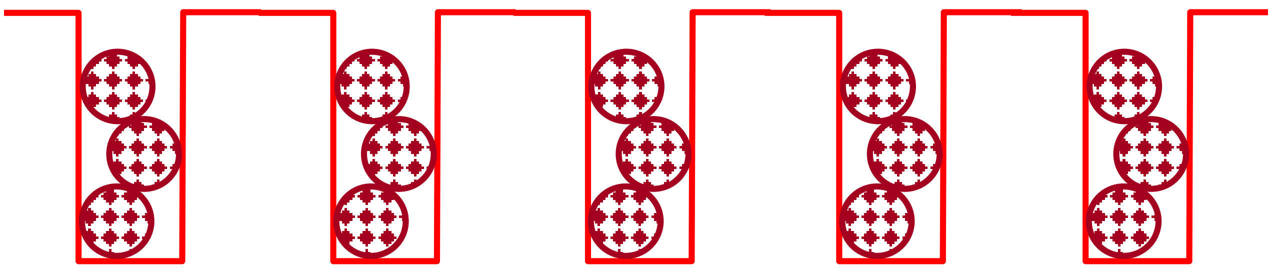


The superfluid density evolves smoothly from large values at small *U/t*, to small values at large *U/t*, and there is no quantum phase transition at any intermediate value of *U/t*.

(In systems with Galilean invariance and at zero temperature, superfluid density=density of bosons always, independent of the strength of the interactions)

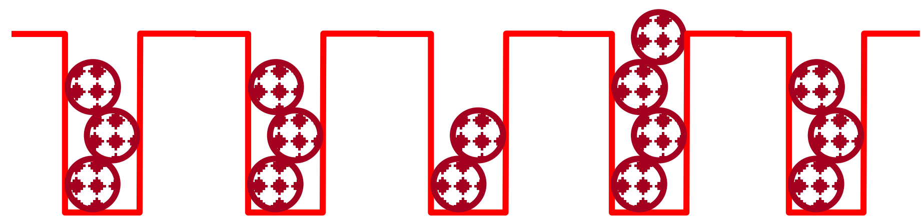
What is the ground state for large *U/t* ?

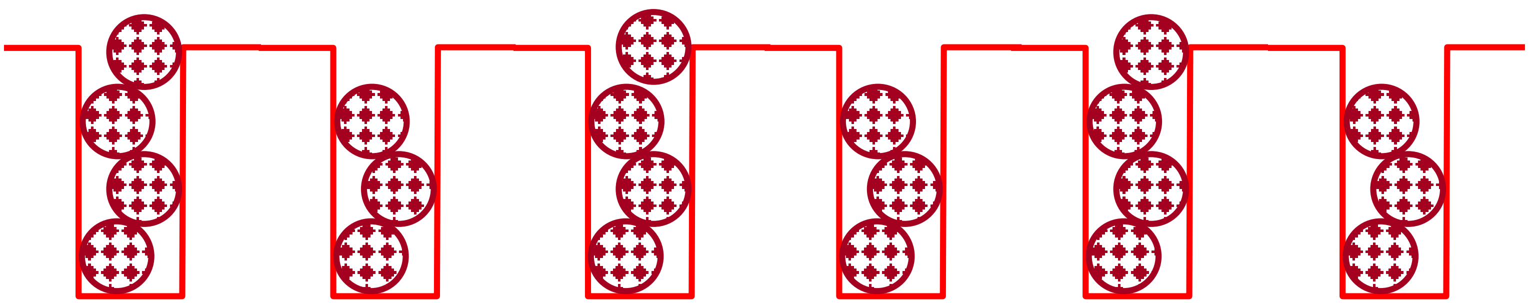
Incompressible, insulating ground states, with zero superfluid density, appear at special commensurate densities



$\boxed{\langle n_j \rangle = 3}$

$-\dfrac{t}{U}$

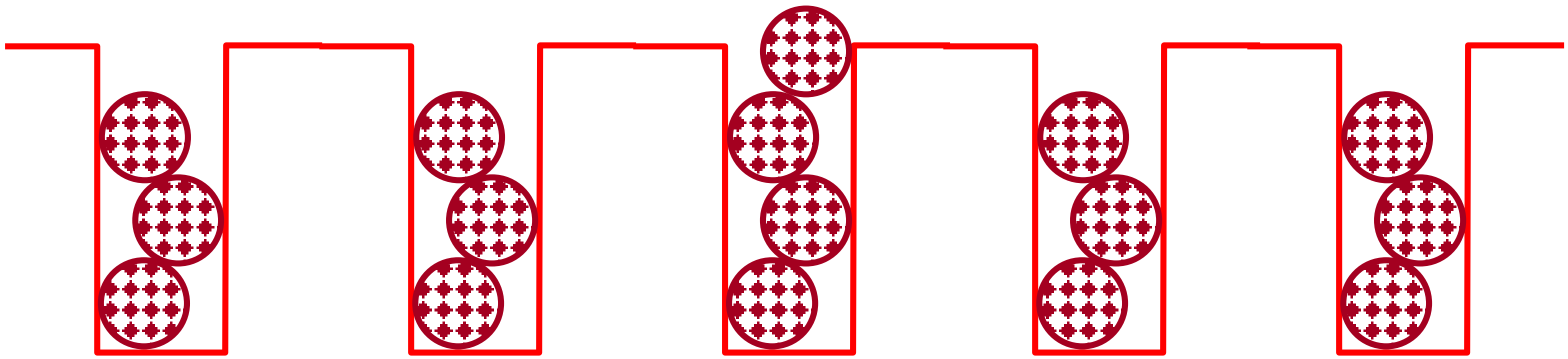




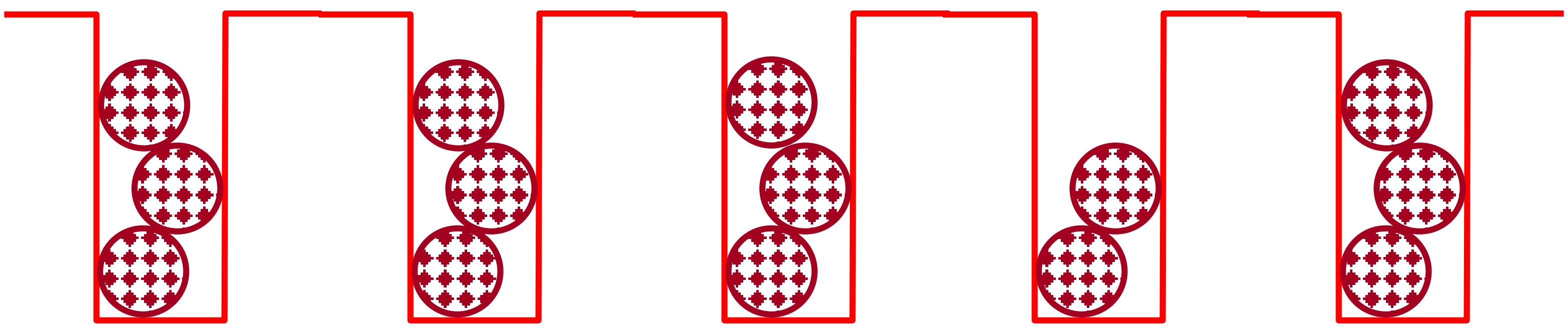
$\boxed{\langle n_j \rangle = 7/2}$

Ground state has "density wave" order, which spontaneously breaks lattice symmetries

Excitations of the insulator: infinitely long-lived, finite energy *quasiparticles* and *quasiholes*



Energy of quasi-particles/holes: $\varepsilon_{p,h}(p) = \Delta_{p,h} + \frac{p^2}{2m^*_{p,h}}$

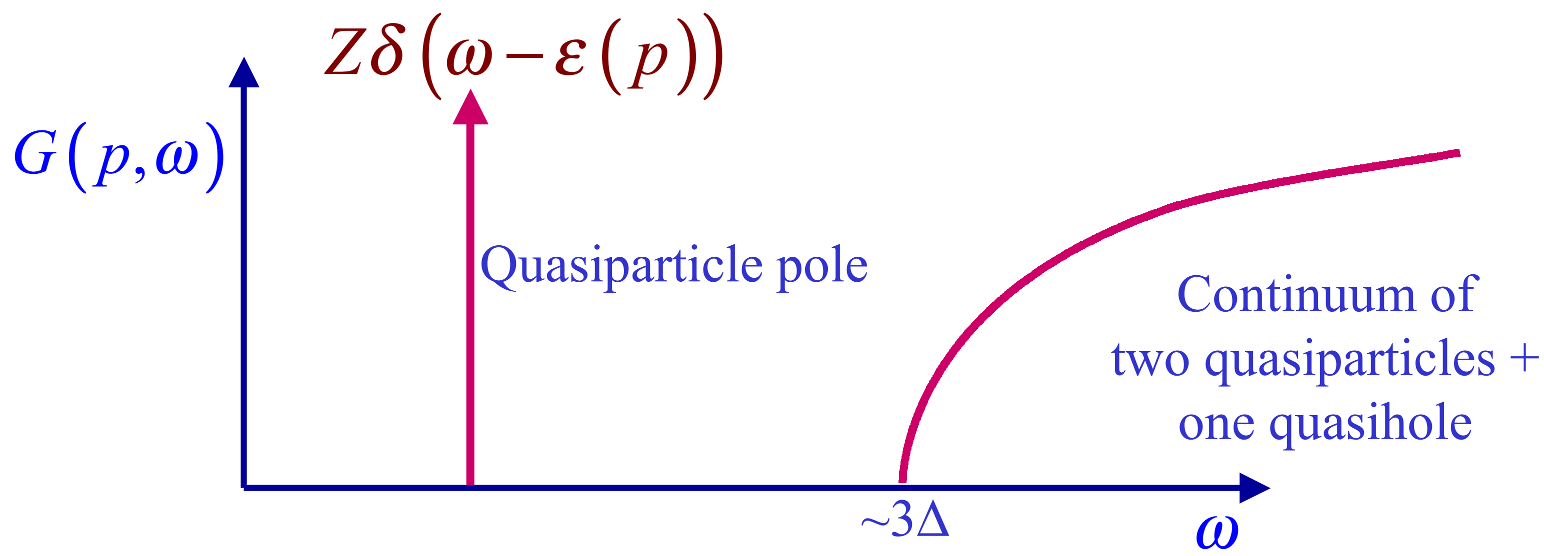


Boson Green's function $G(p,\omega)$:

Insulating ground state

Cross-section to add a boson

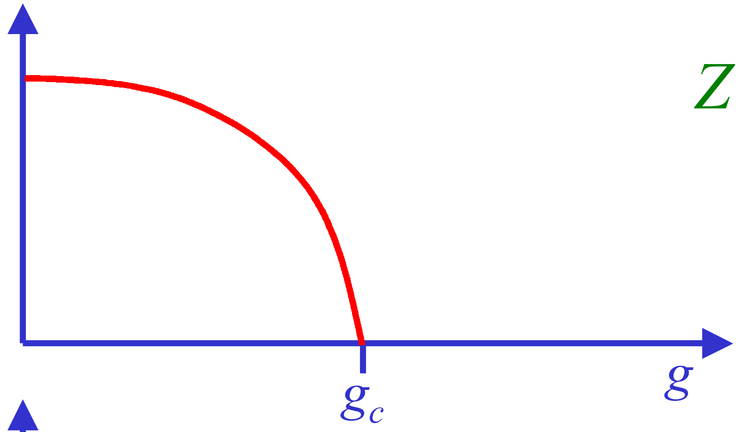
while transferring energy $\hbar\omega$ and momentum p



Similar result for quasi-hole excitations obtained by removing a boson

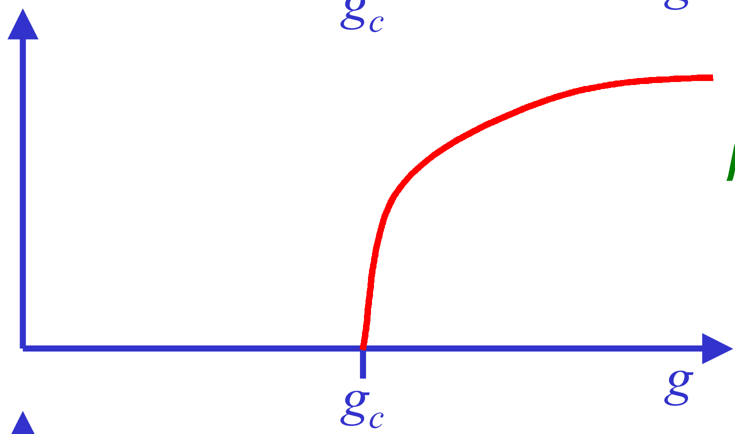
Entangled states at $g \equiv t/U$ of order unity

Quasiparticle weight Z



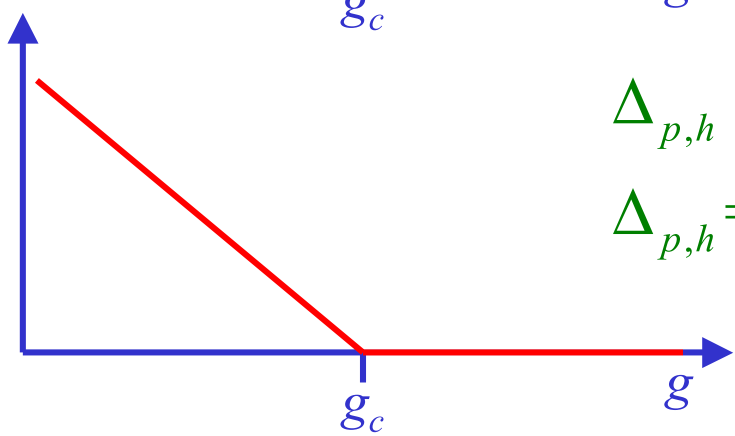
$$Z \sim (g_c - g)^{\eta\nu}$$

Superfluid density ρ_s



$$\rho_s \sim (g - g_c)^{(d+z-2)\nu}$$

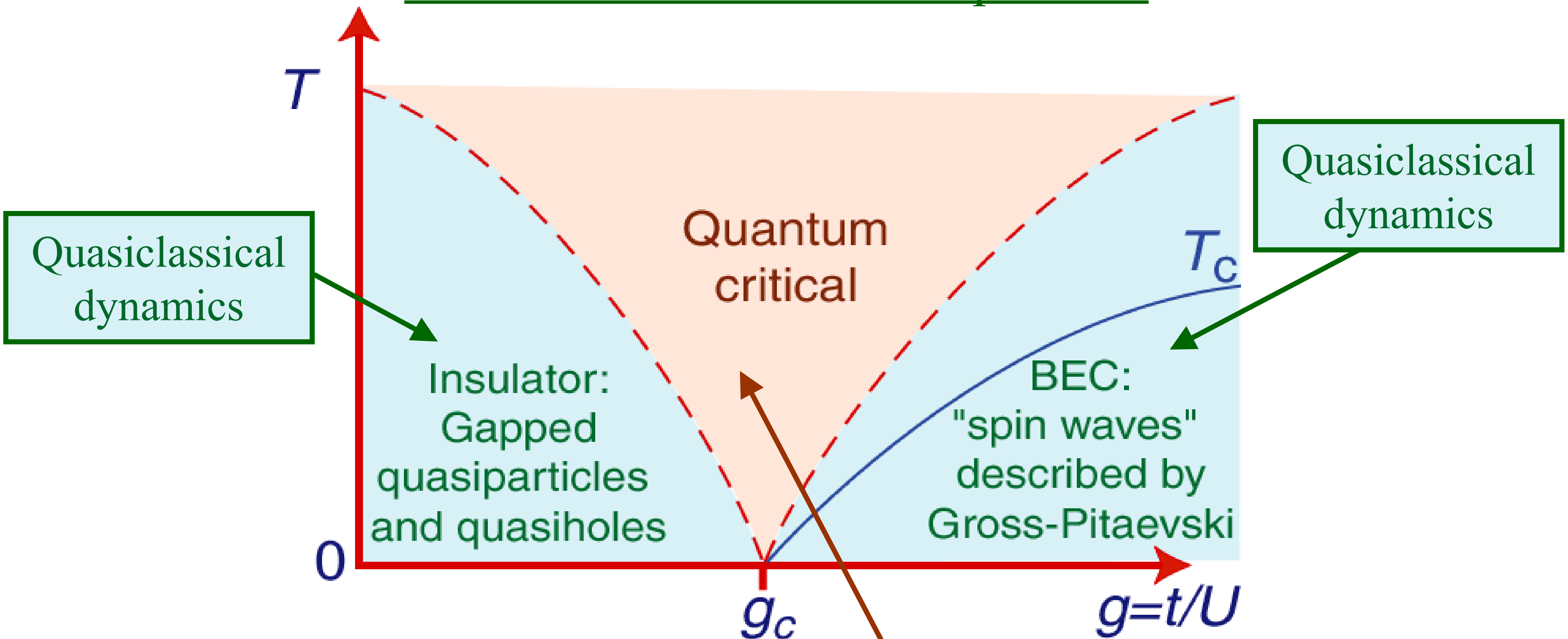
Excitation energy gap Δ



$$\Delta_{p,h} \sim (g_c - g)^{\nu} \text{ for } g < g_c$$

$$\Delta_{p,h} = 0 \text{ for } g > g_c$$

Crossovers at nonzero temperature



Relaxational dynamics ("Bose molasses") with phase coherence/relaxation time τ_φ given by

$$\frac{1}{\tau_\varphi} = \left(\text{Universal number}\right)\frac{k_B T}{\hbar} \quad \left(1\mu\text{K} = 20.9\text{kHz}\right)$$

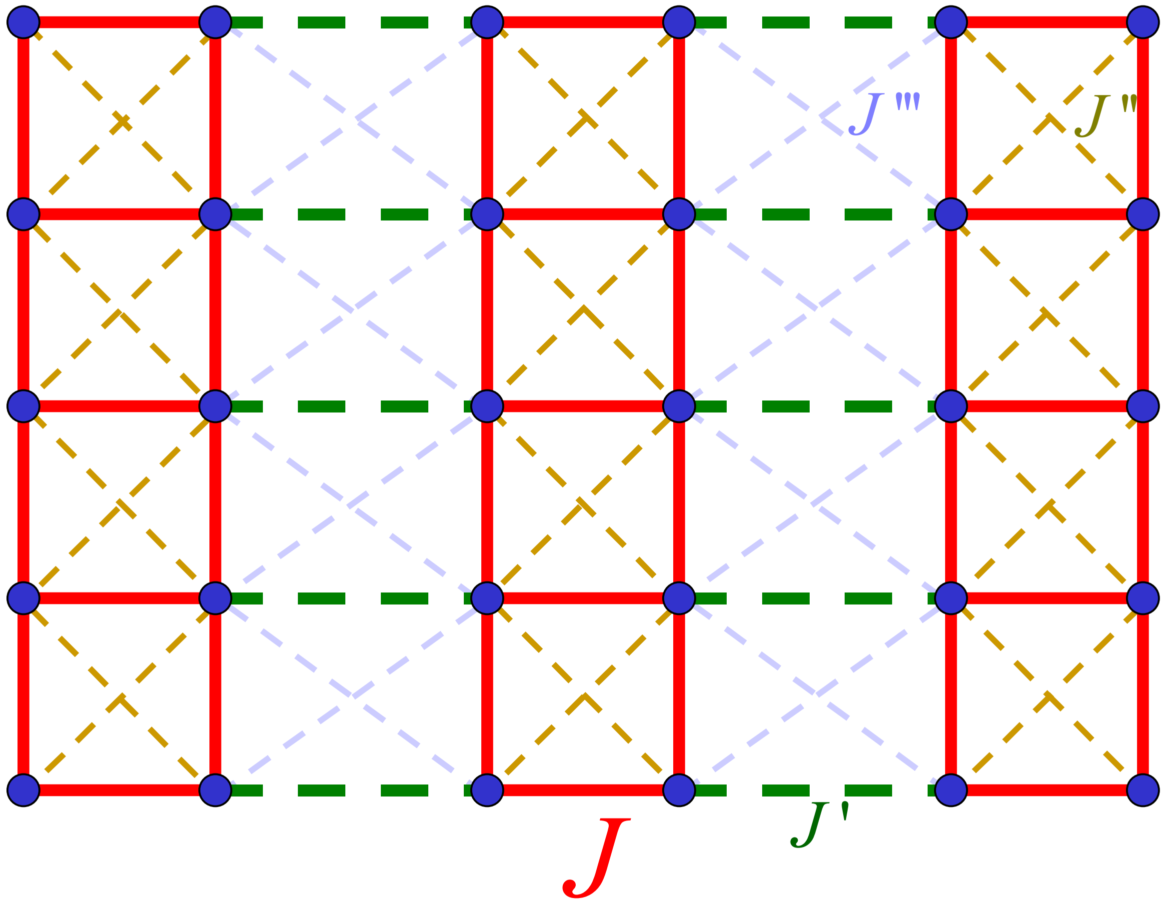
S. Sachdev and J. Ye,
Phys. Rev. Lett. **69**, 2411 (1992).
K. Damle and S. Sachdev
Phys. Rev. B **56**, 8714 (1997).

Conductivity (in d=2) $= \dfrac{Q^2}{h}\Sigma\left(\dfrac{\hbar\omega}{k_B T}\right)$ $\quad \Sigma \rightarrow$ universal function

M.P.A. Fisher, G. Girvin, and G. Grinstein, *Phys. Rev. Lett.* **64**, 587 (1990).
K. Damle and S. Sachdev *Phys. Rev.* B **56**, 8714 (1997).

II. Quantum transitions without local order parameters: fractionalization

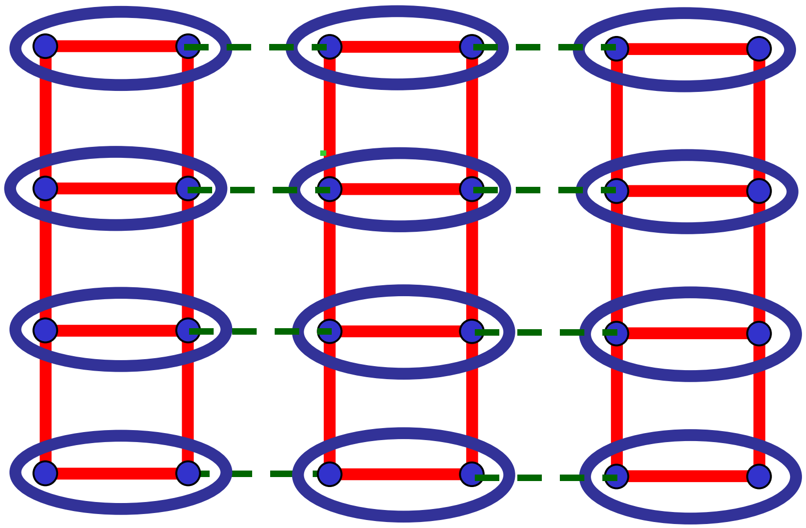
S=1/2 spins on coupled 2-leg ladders

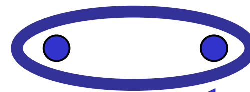


$$H = \sum_{<ij>} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

e.g. $SrCu_2O_3$

Ground state for J large

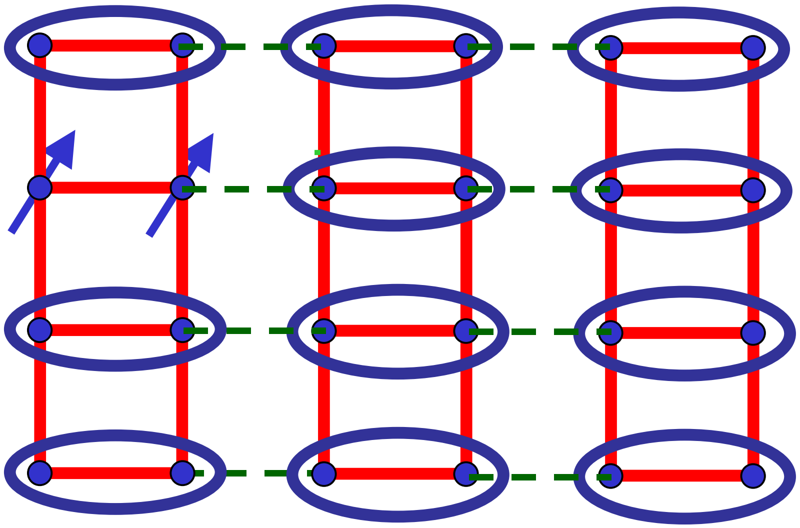


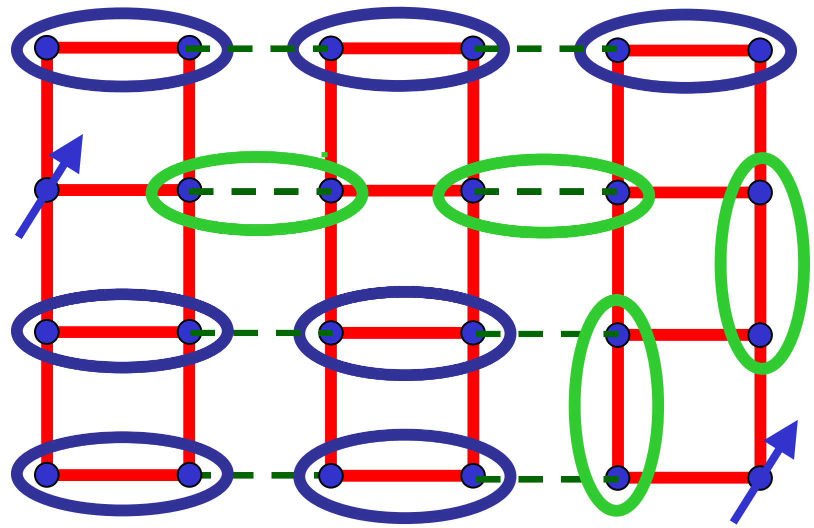


$$= \frac{1}{\sqrt{2}}\left(\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle \right)$$

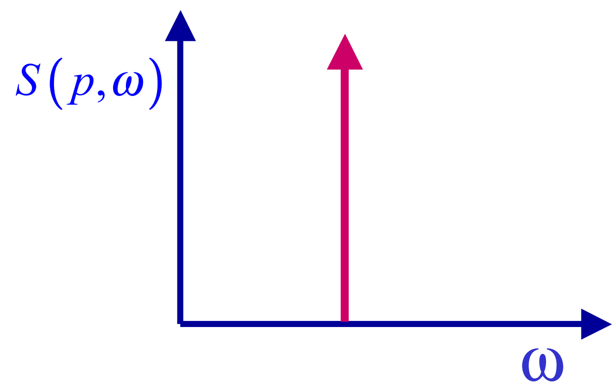
S=0 quantum paramagnet

Elementary excitations of paramagnet

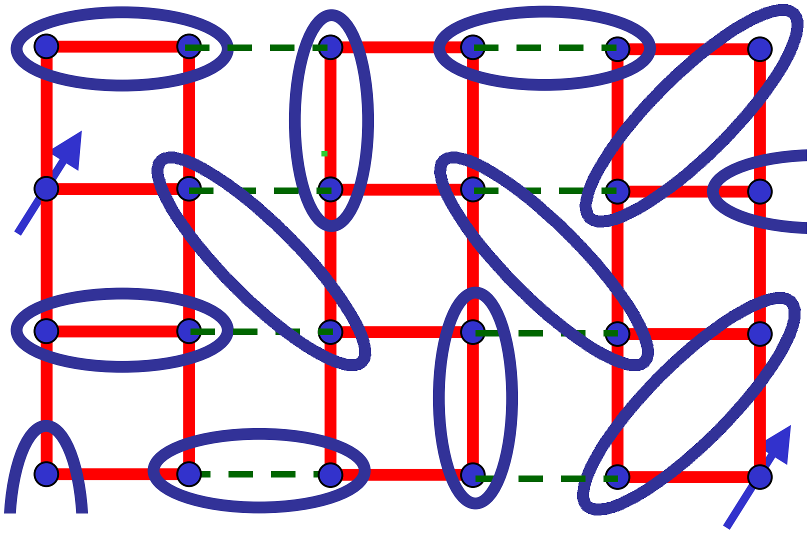


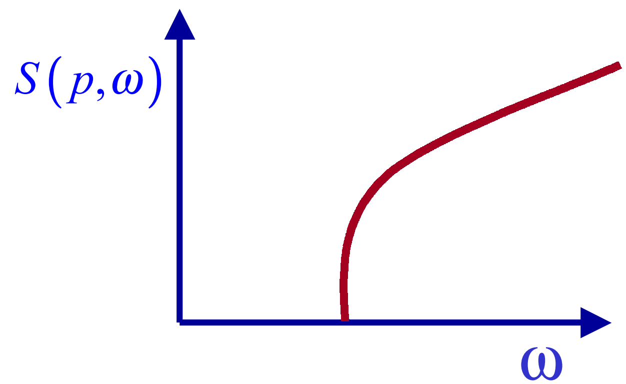


For large J, there is a stable $S=1$, neutral, quasiparticle excitation: its two $S=1/2$ constituent spins are confined by a linear attractive potential



Elementary excitations of paramagnet





P.W. Anderson, *Science* **235**, 1196 (1987).

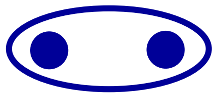
For smaller J, there can be a confinement-deconfinement transition at which the $S=1/2$ spinons are liberated: these are neutral, $S=1/2$ quasiparticles

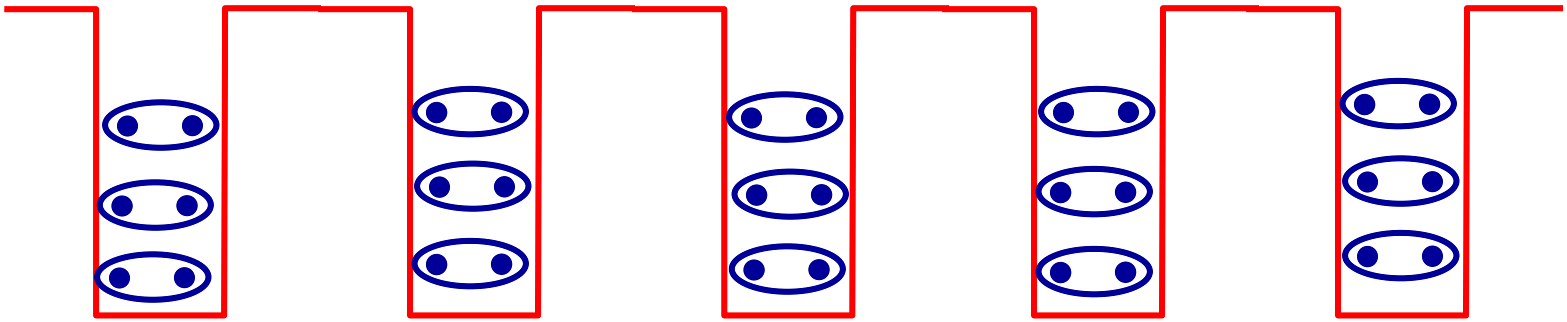
The gap to all excitations with non-zero S remains finite across this transition, but the gap to spin singlet excitations vanishes. There is no local order parameter and the transition is described by a Z_2 gauge theory

N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991).
X.G. Wen, *Phys. Rev.* B **44**, 2664 (1991).
T. Senthil and M. P. A. Fisher *Phys. Rev.* B **62**, 7850 (2000).

Fractionalization in atomic gases

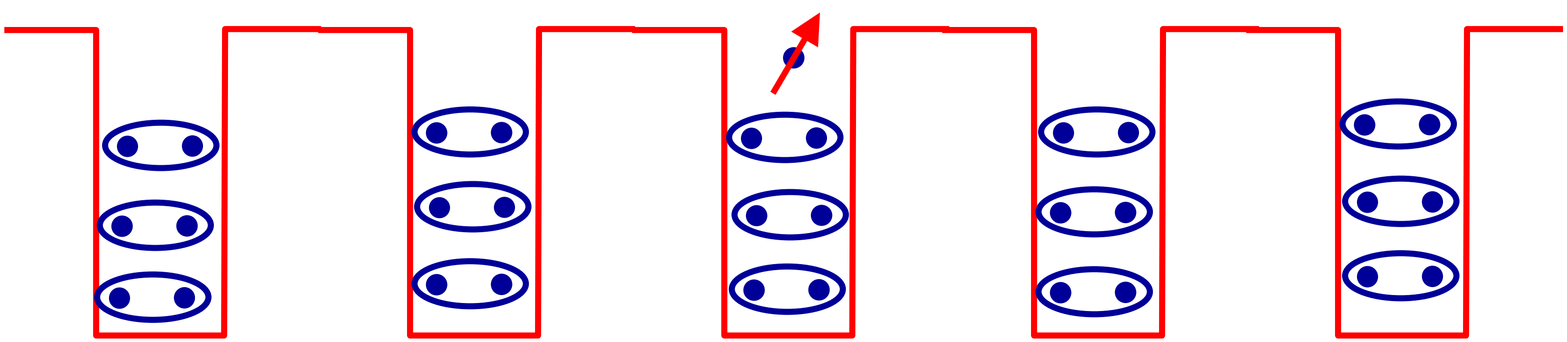
E. Demler and F. Zhou,
cond-mat/0104409

 = Two $F=1$ atoms in a spin singlet pair



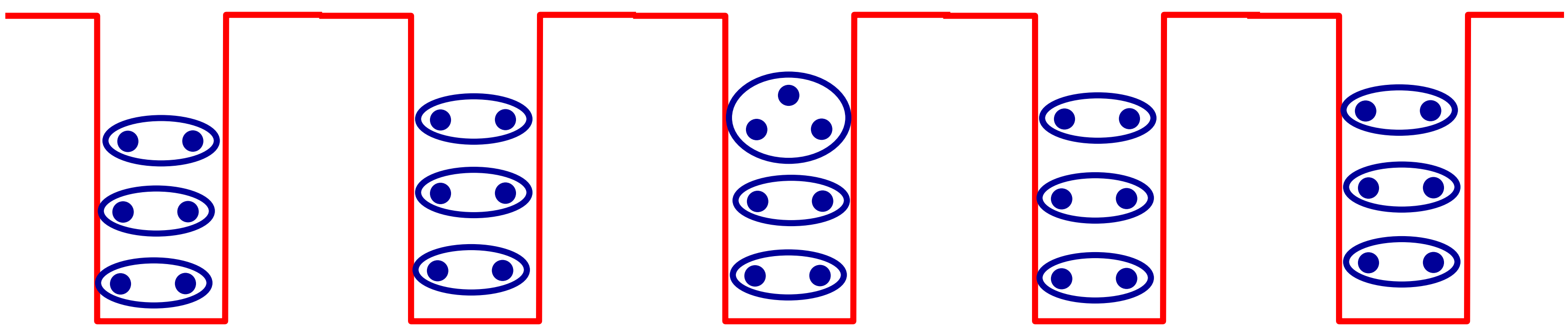
"Ordinary" spin-singlet insulator

Quasiparticle excitation



Quasiparticle carries both spin and "charge"

Quasiparticle excitation in a fractionalized spin-singlet insulator



Quasiparticle carries “charge” but no spin

Spin-charge separation

Conclusions

I. Study of quantum phase transitions offers a controlled and systematic method of understanding many-body systems in a region of strong entanglement.

II. Atomic gases offer many exciting opportunities to study quantum phase transitions because of ease by which system parameters can be continuously tuned.

III. Promising outlook for studying quantum systems with “fractionalized” excitations (only observed so far in quantum Hall systems in condensed matter).