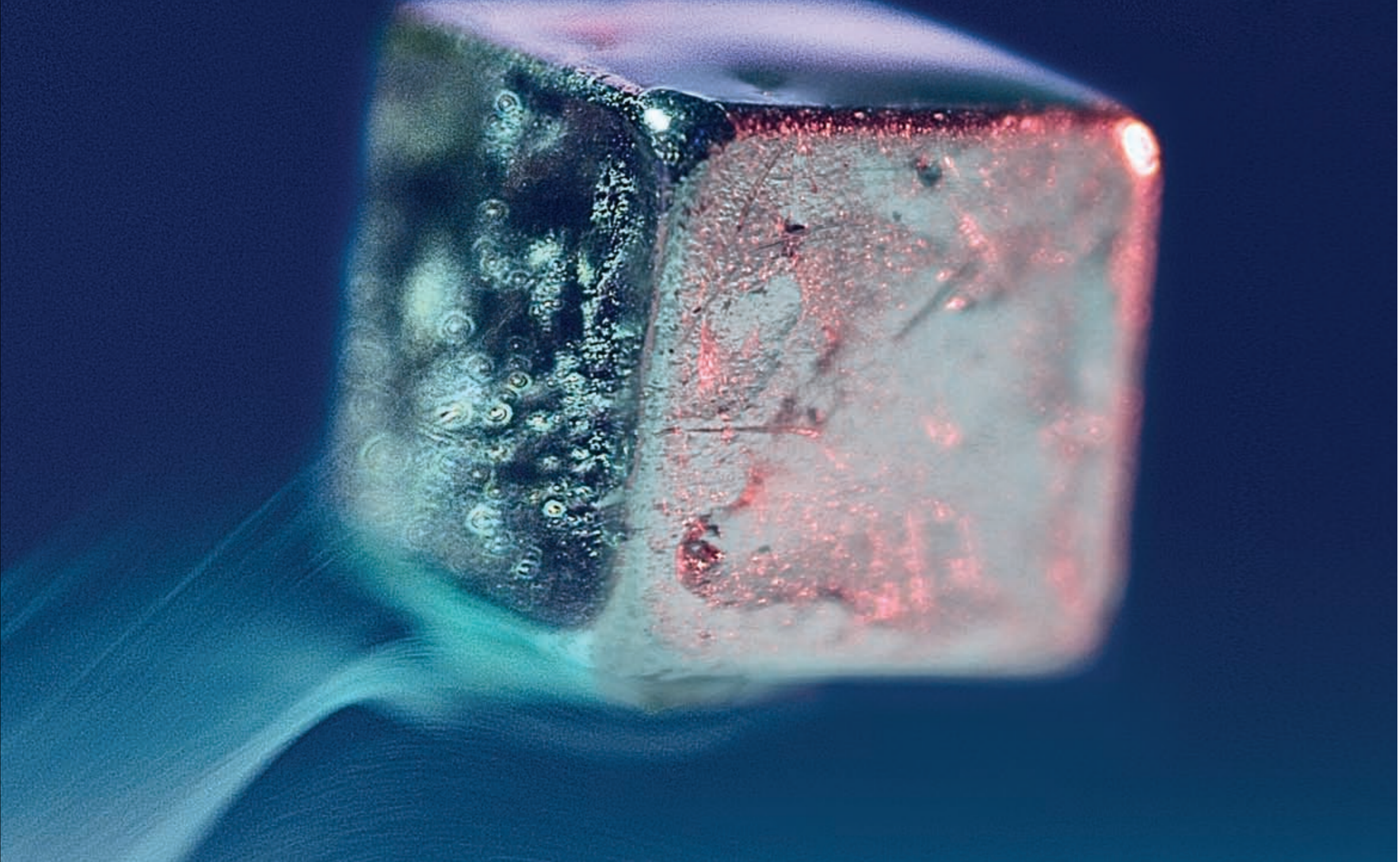
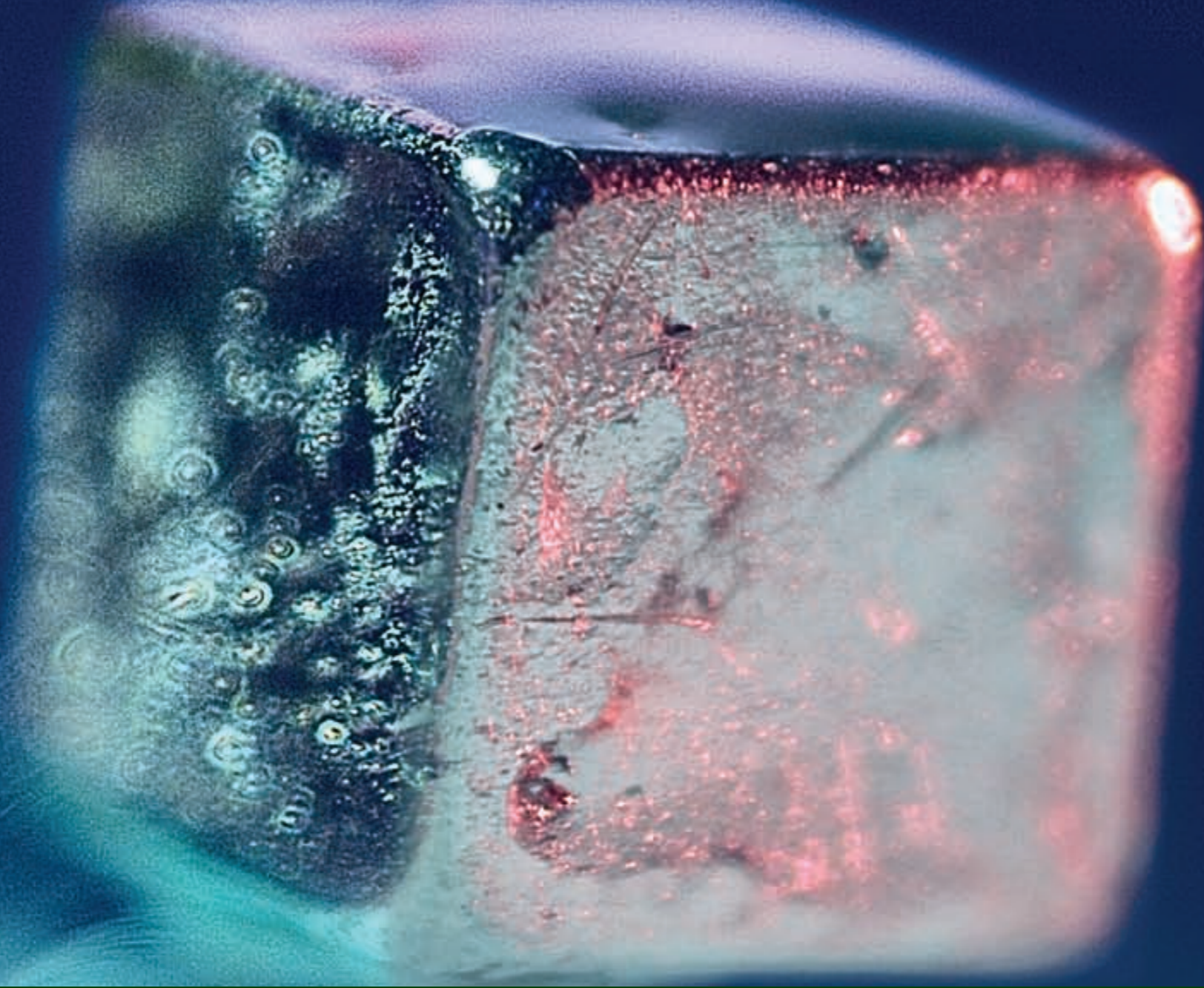


Quantum Entanglement and Superconductivity



Subir Sachdev, Harvard University

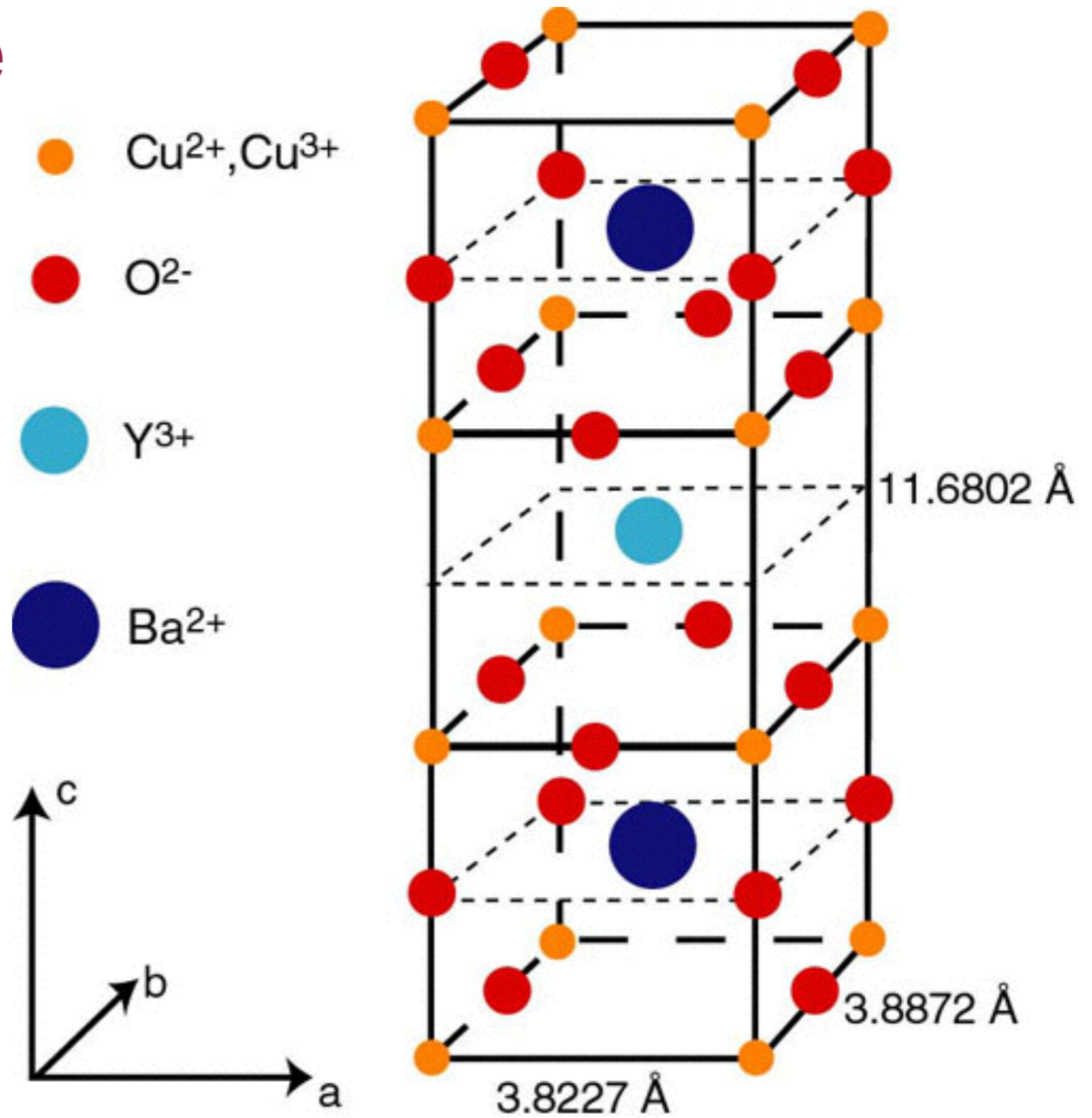
Quantum Entanglement and Superconductivity

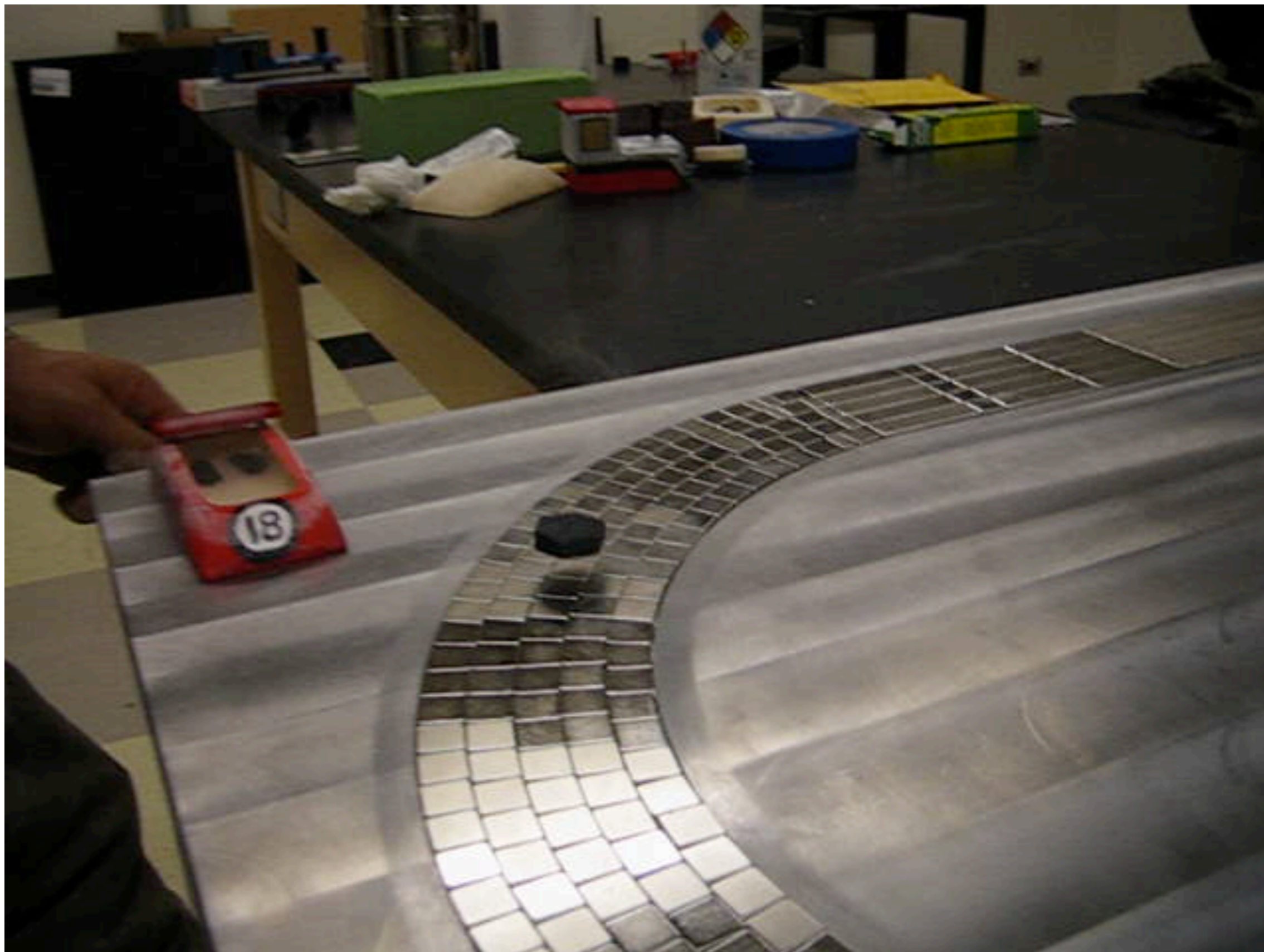


Superconductor, levitated by an unseen magnet, in which countless trillions of electrons form a vast interconnected quantum state.
Scientific American, January 2013

Subir Sachdev, Harvard University

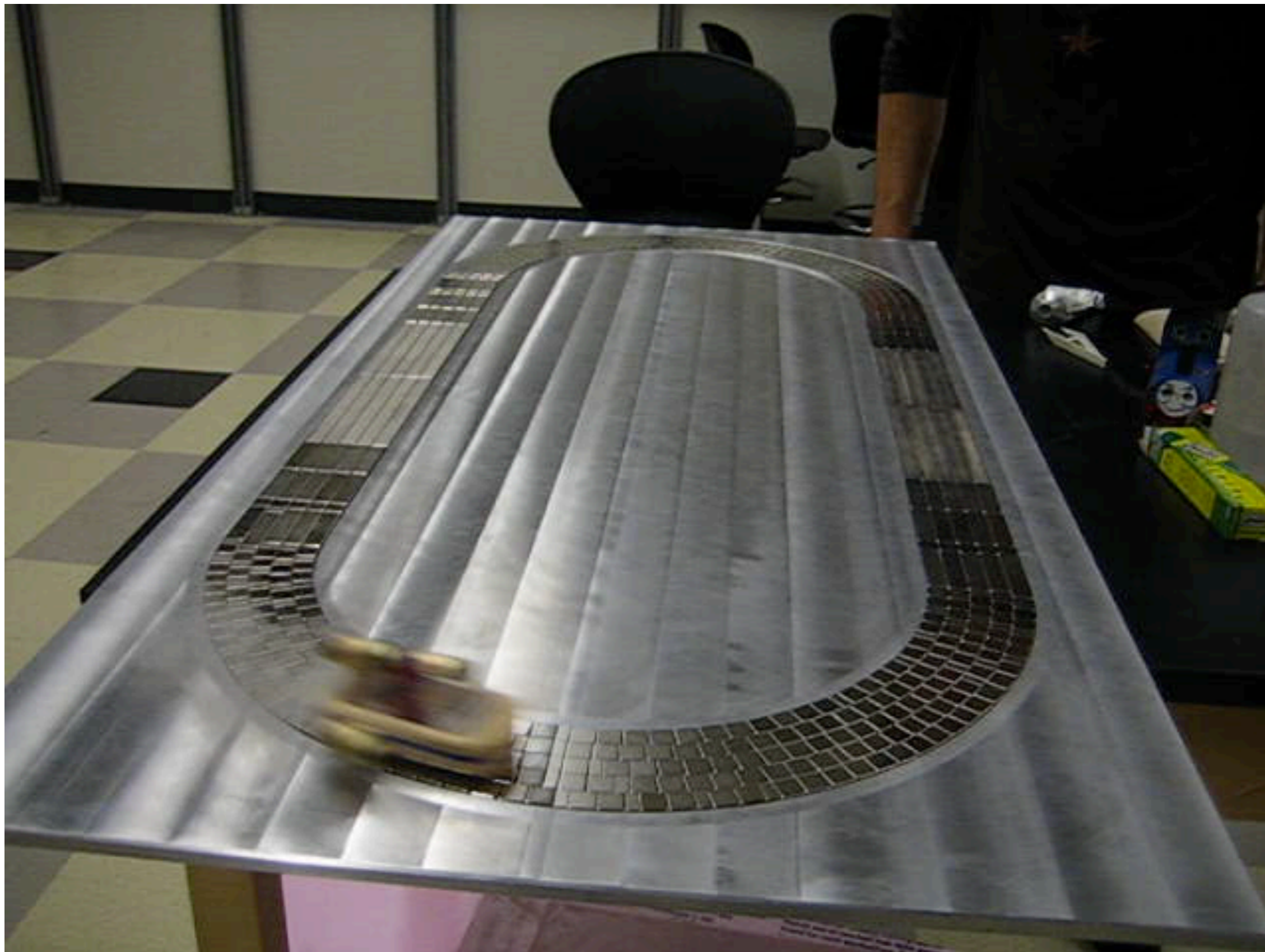
High temperature superconductors





Nd-Fe-B magnets, YBaCuO superconductor

Julian Hetel and Nandini Trivedi, Ohio State University



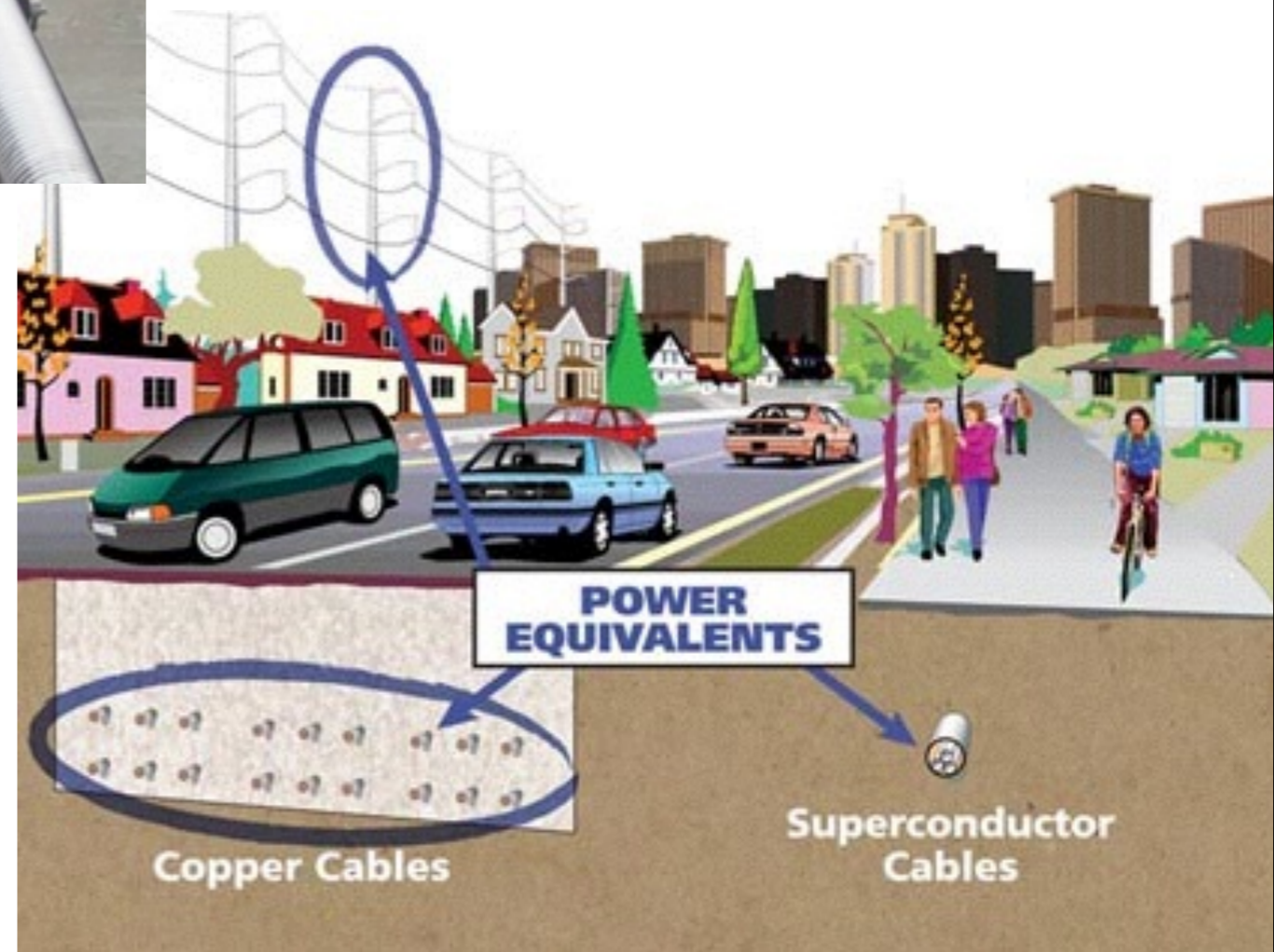
Nd-Fe-B magnets, YBaCuO superconductor

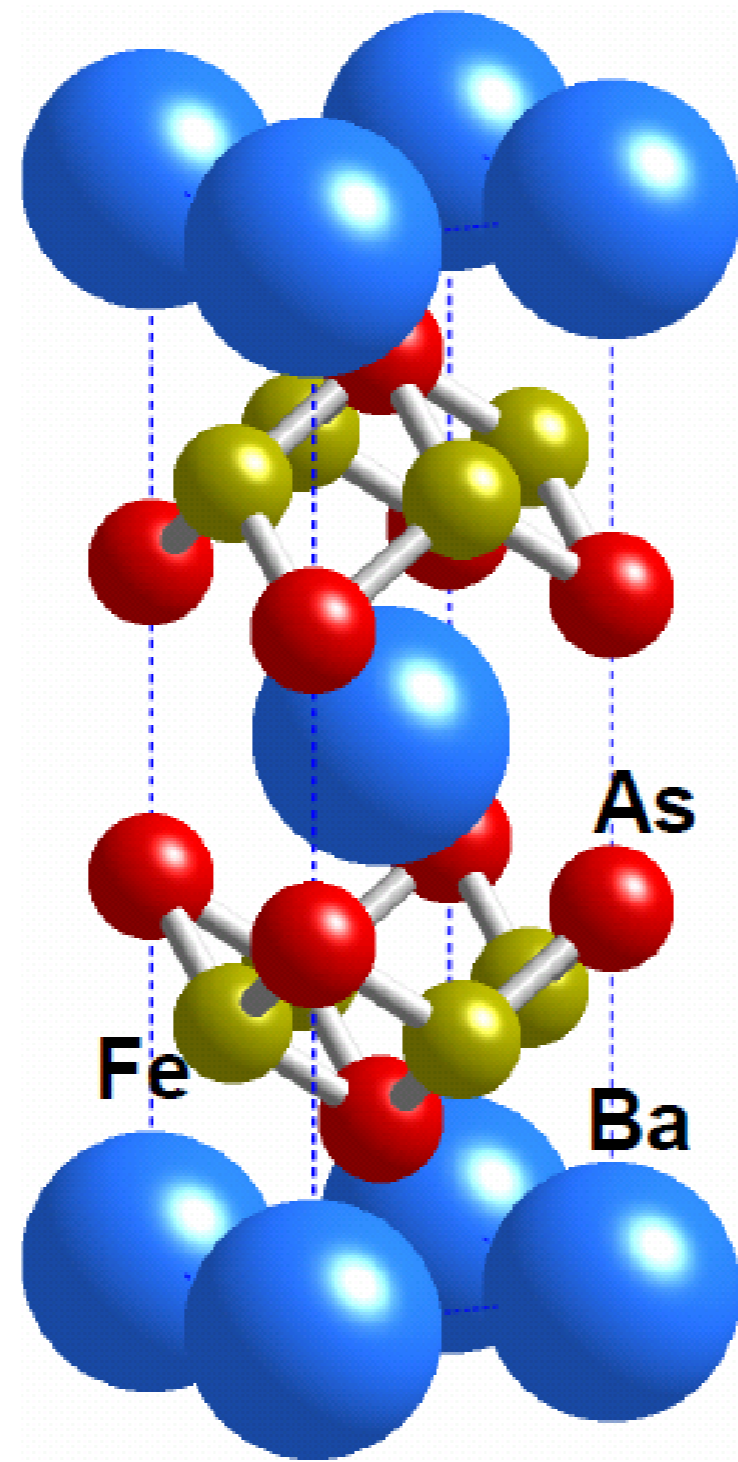
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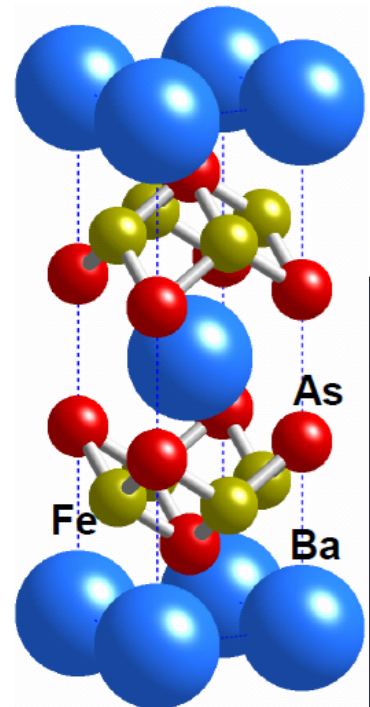


YBCO cables

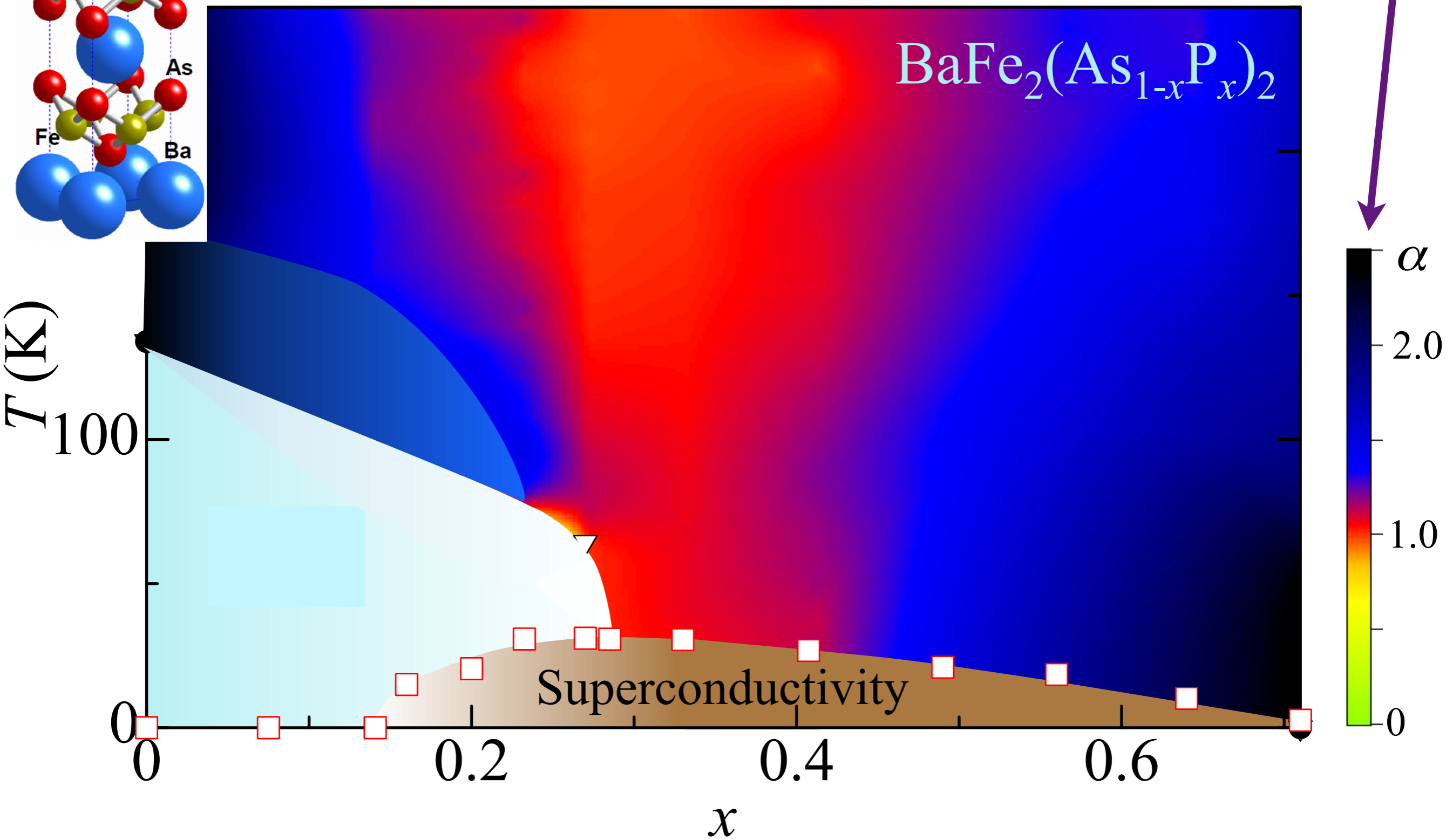
American
Superconductor
Corporation



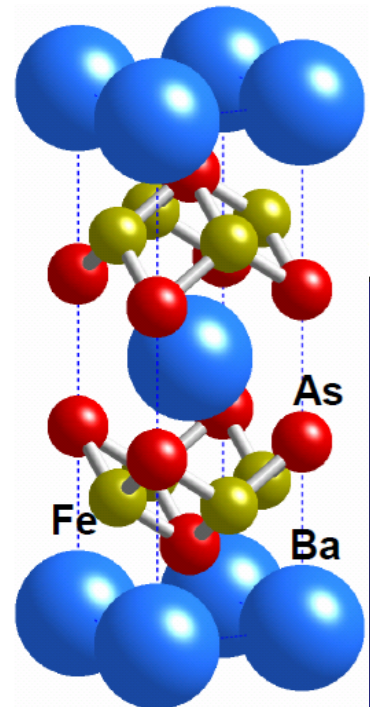




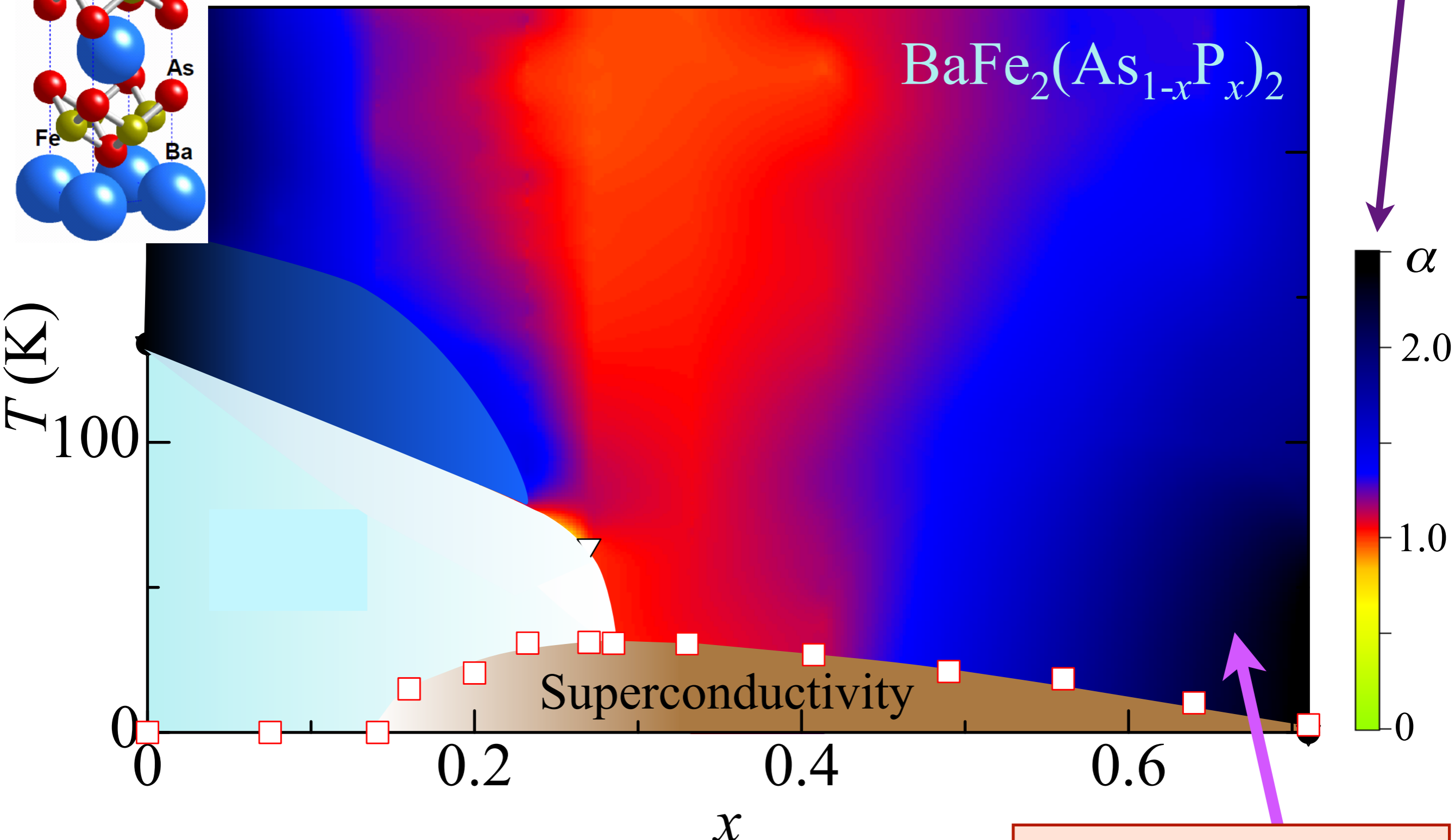
Resistivity
 $\sim \rho_0 + AT^\alpha$



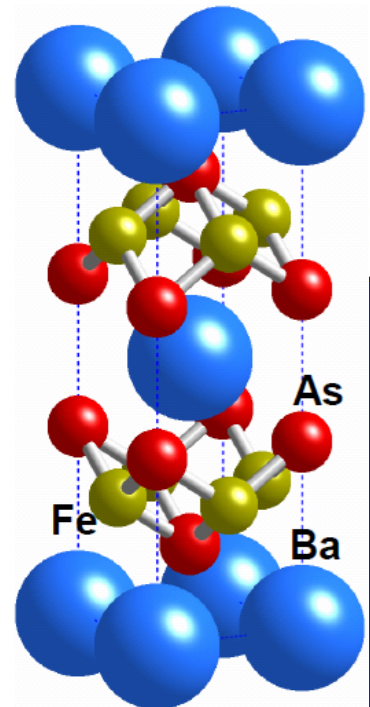
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



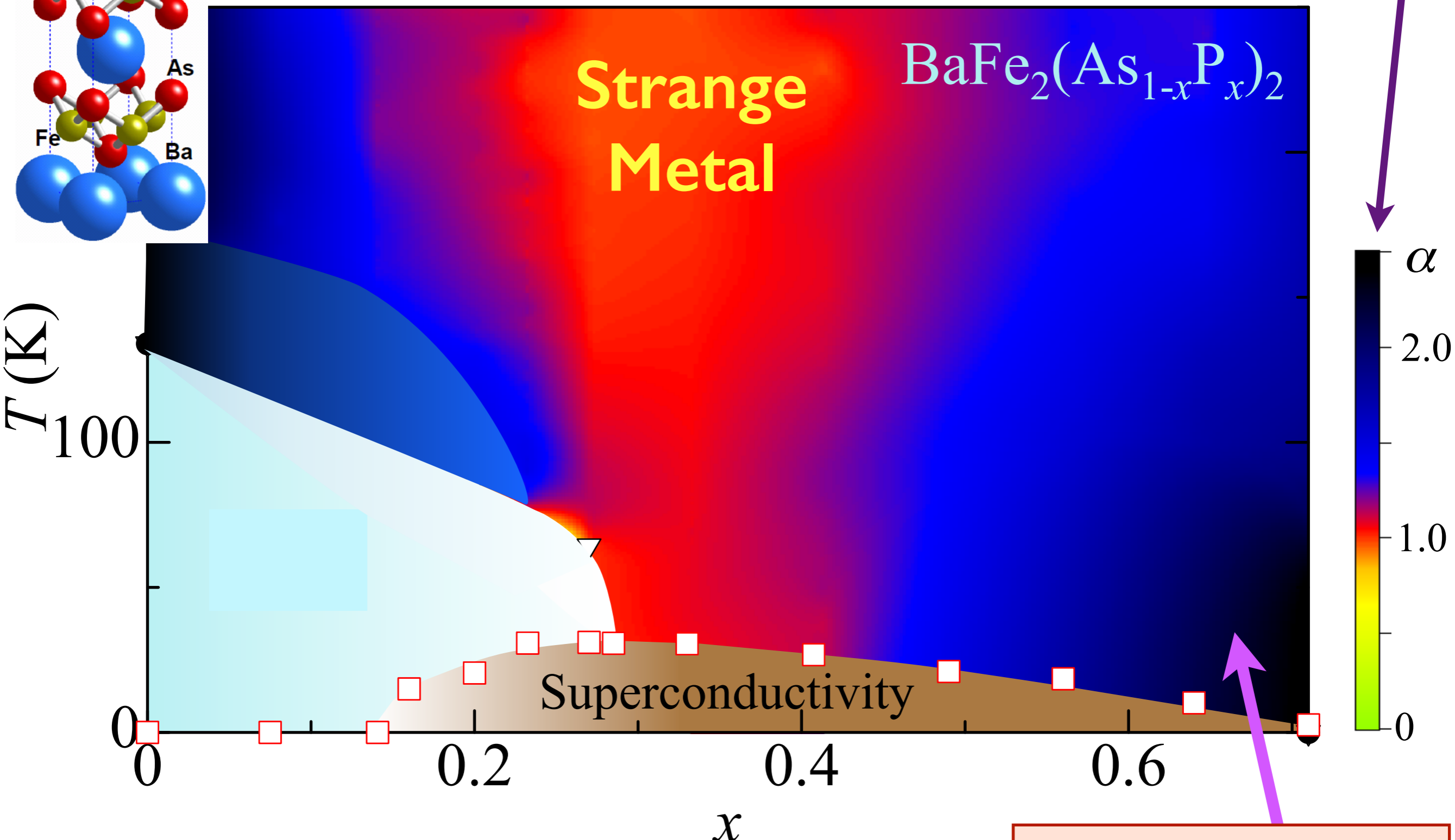
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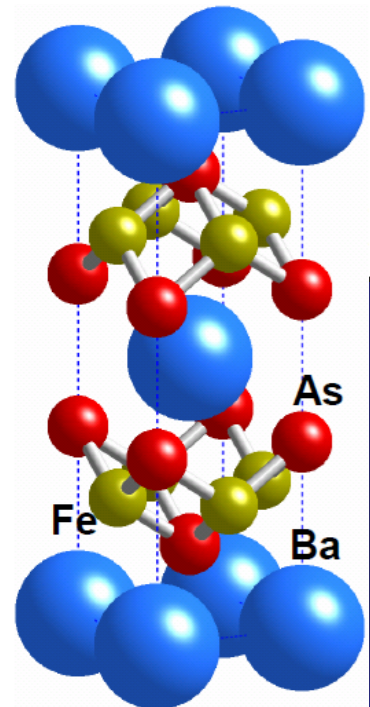
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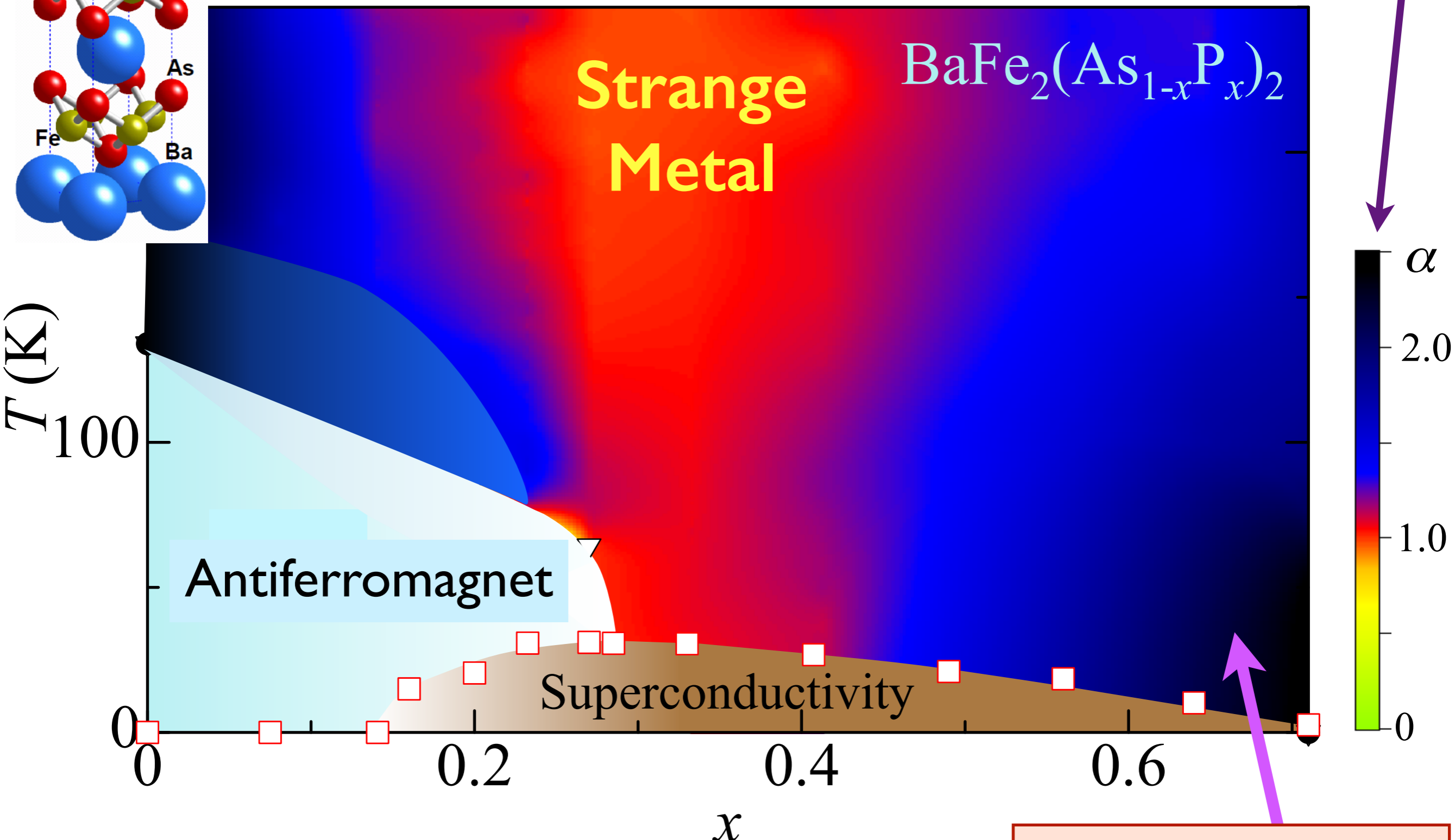
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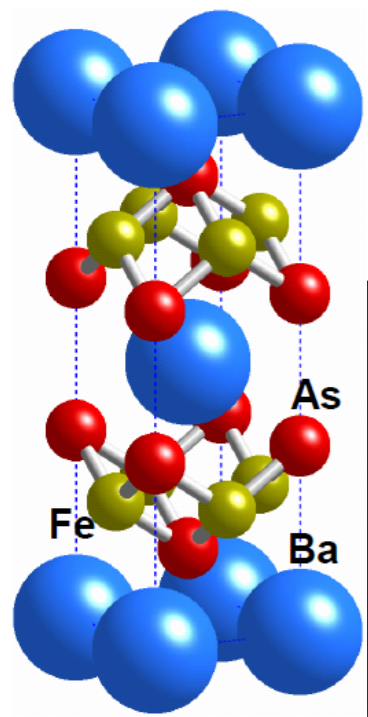
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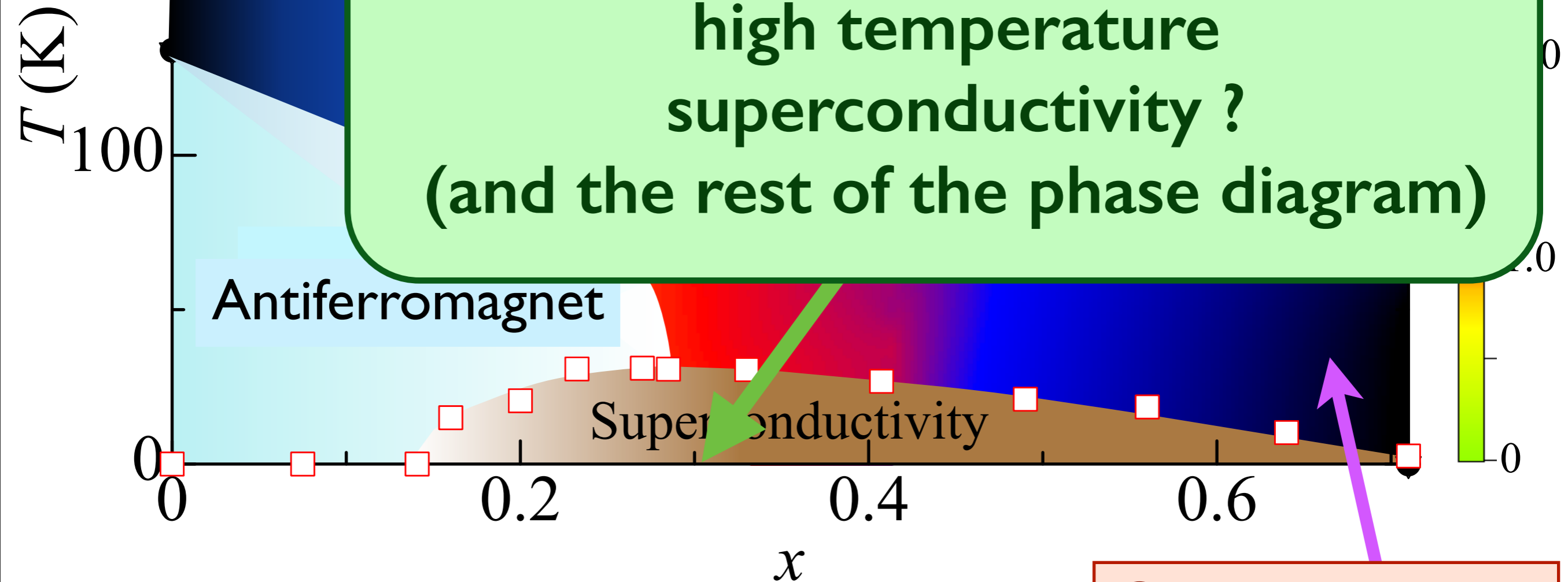


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“Quantum critical point”
with
“long-range quantum entanglement”.

Can this help us understand
high temperature
superconductivity ?
(and the rest of the phase diagram)

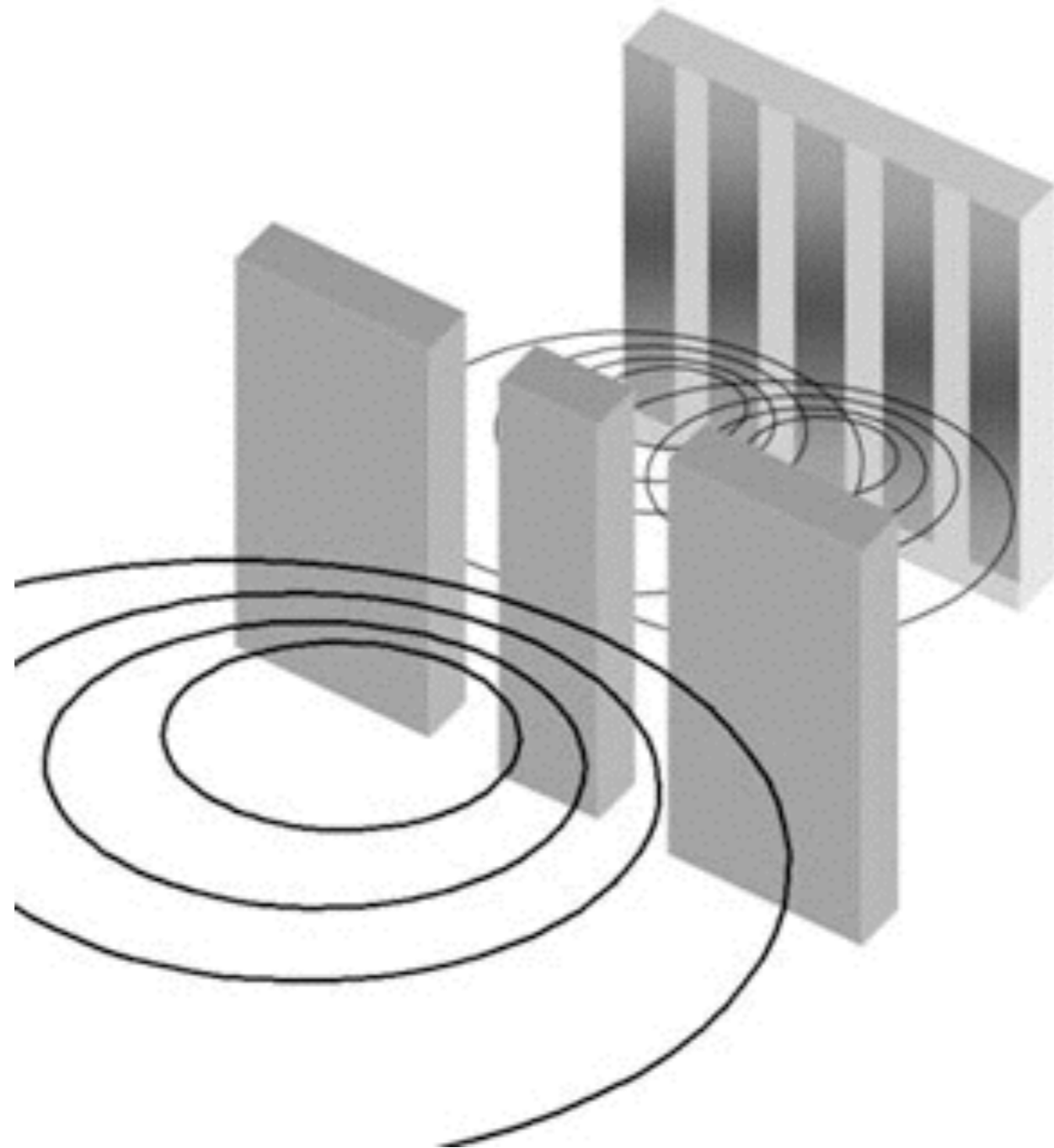


Ordinary metal

**Quantum
superposition and
entanglement**

Principles of Quantum Mechanics: I. Quantum Superposition

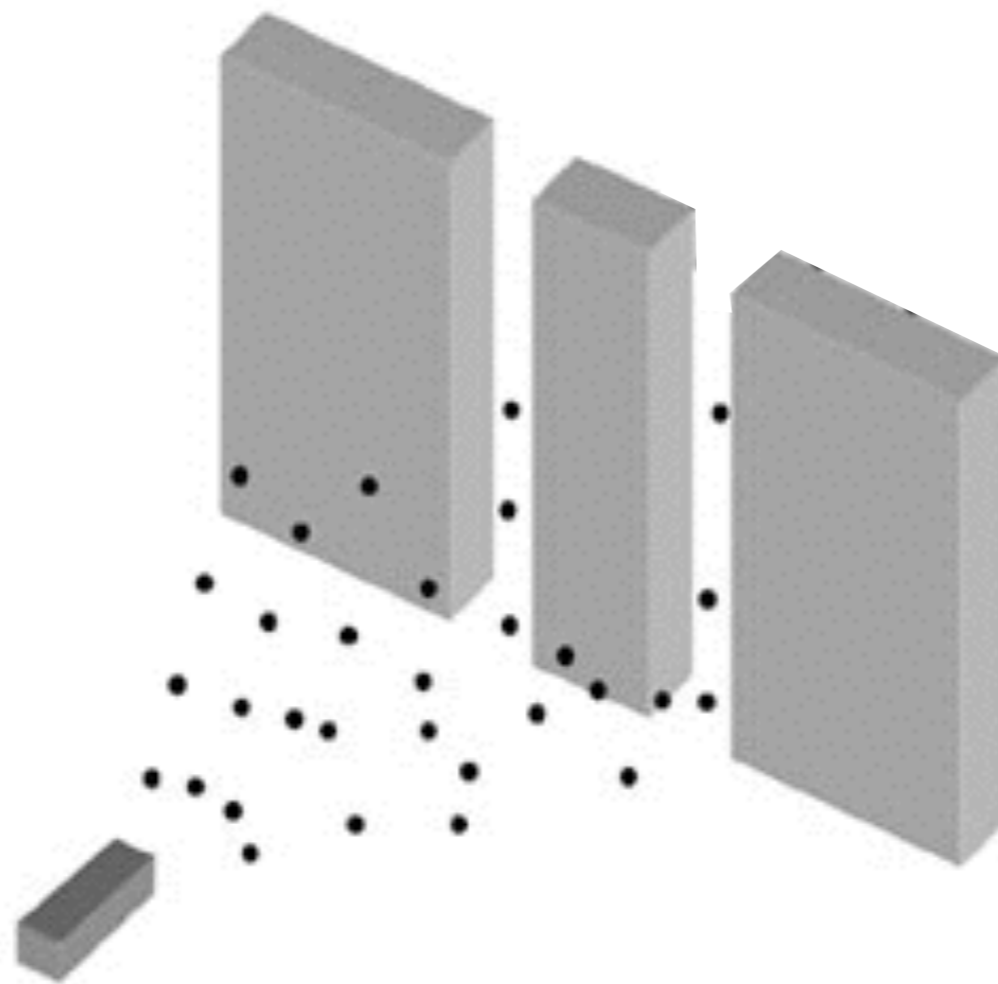
The double slit experiment



Interference of water waves

Principles of Quantum Mechanics: I. Quantum Superposition

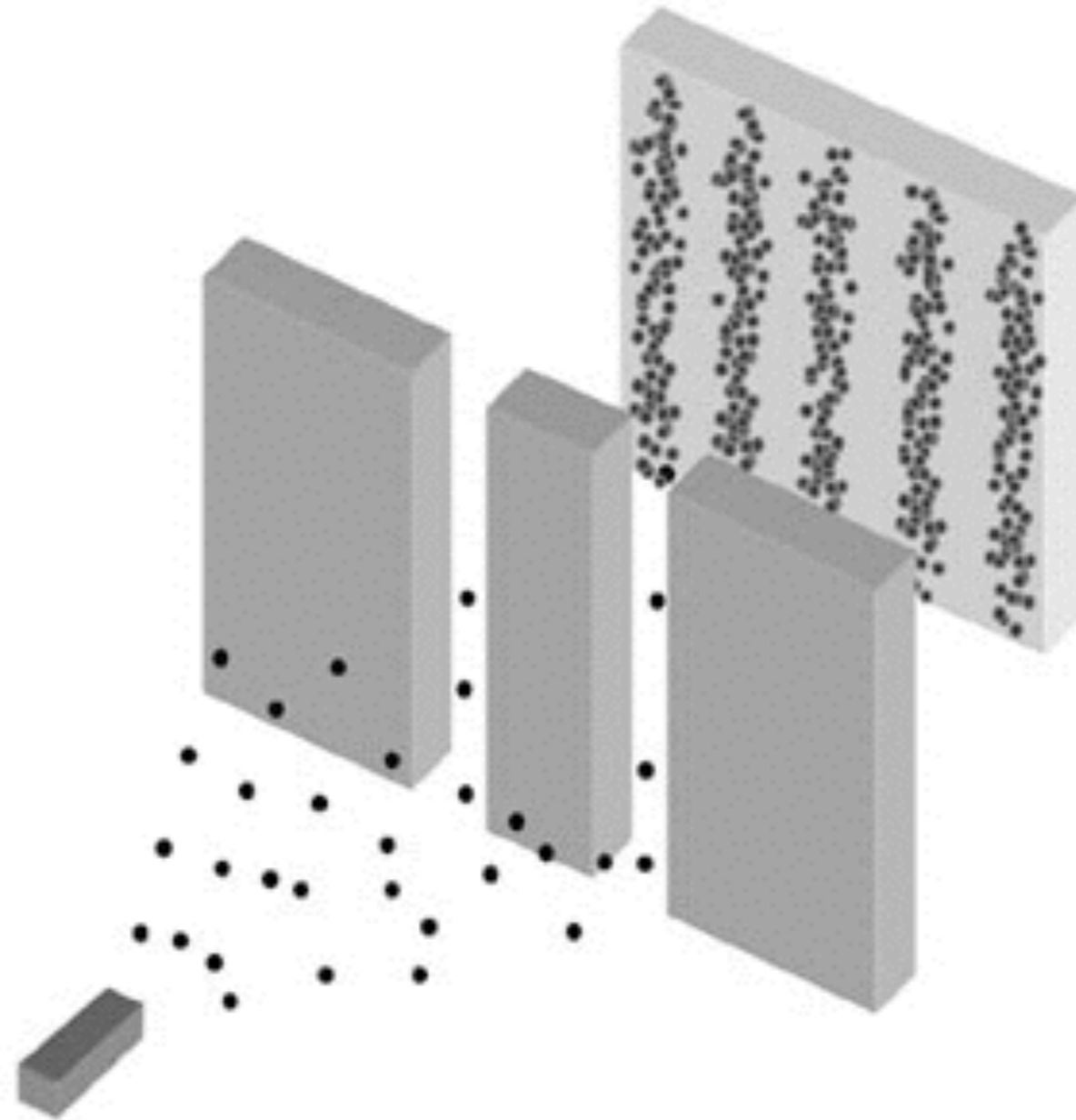
The double slit experiment



Send electrons through the slits

Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

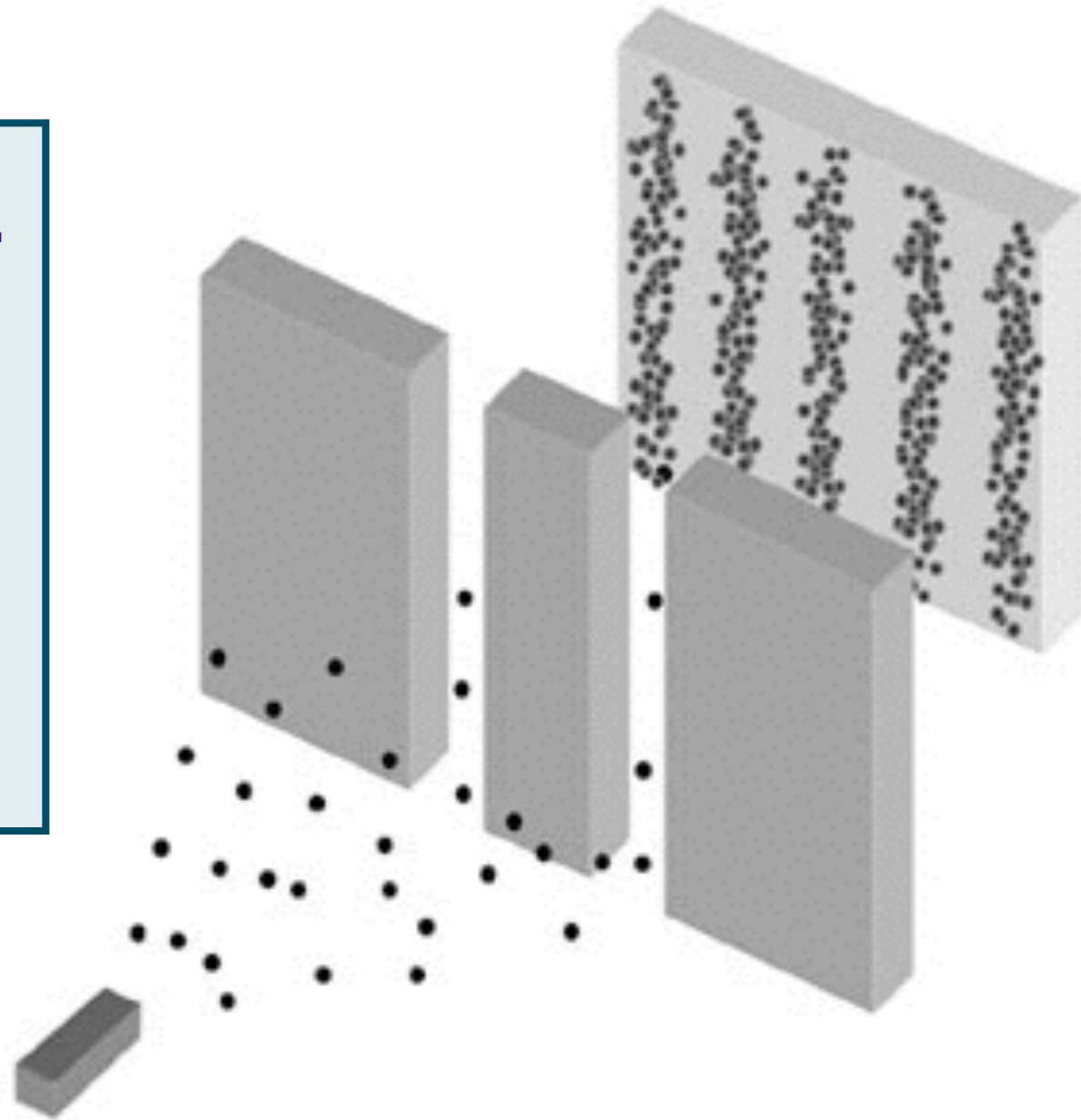


Interference of electrons

Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

Which slit
does an
electron
pass
through ?

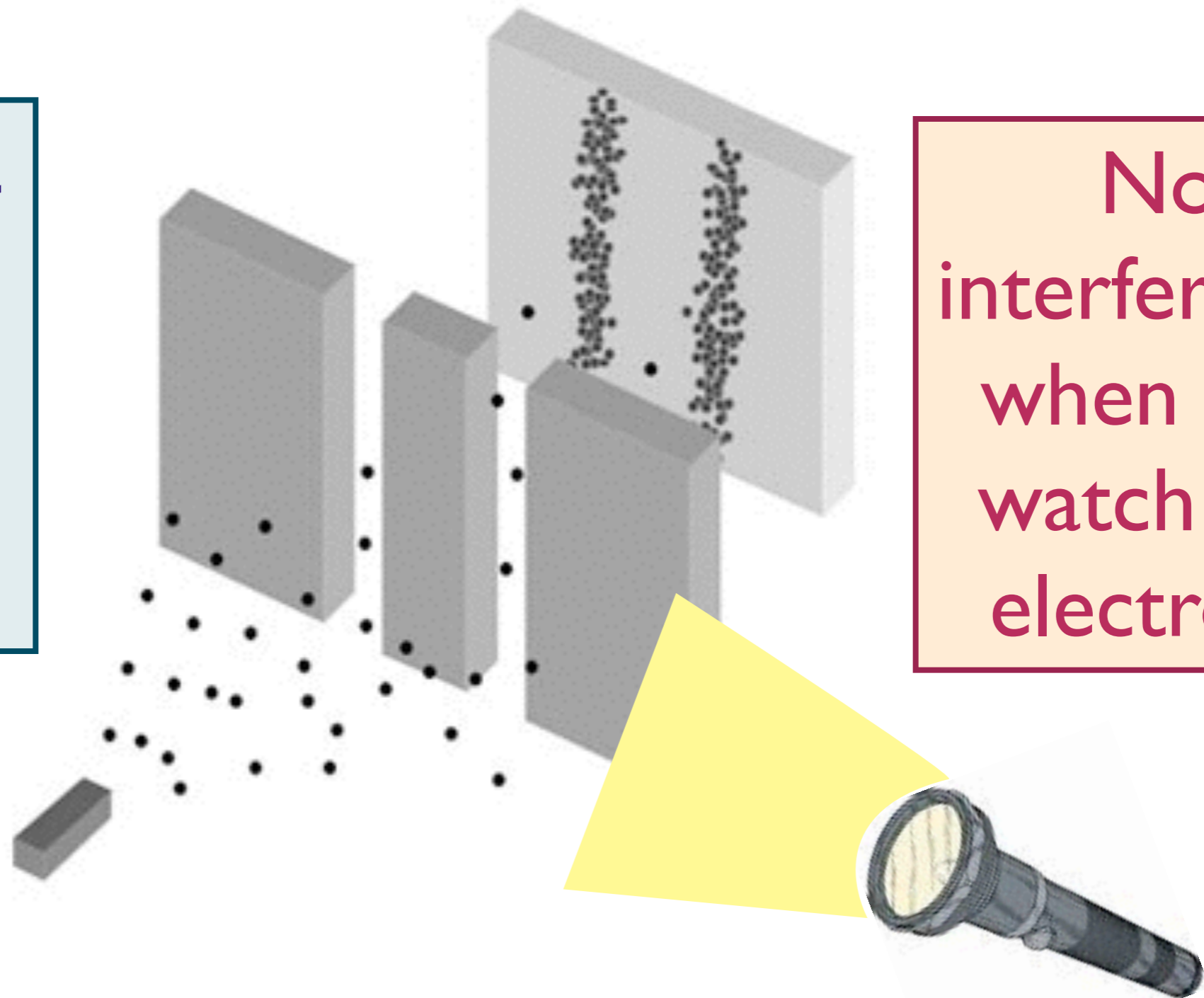


Interference of electrons

Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

Which slit
does an
electron
pass
through ?



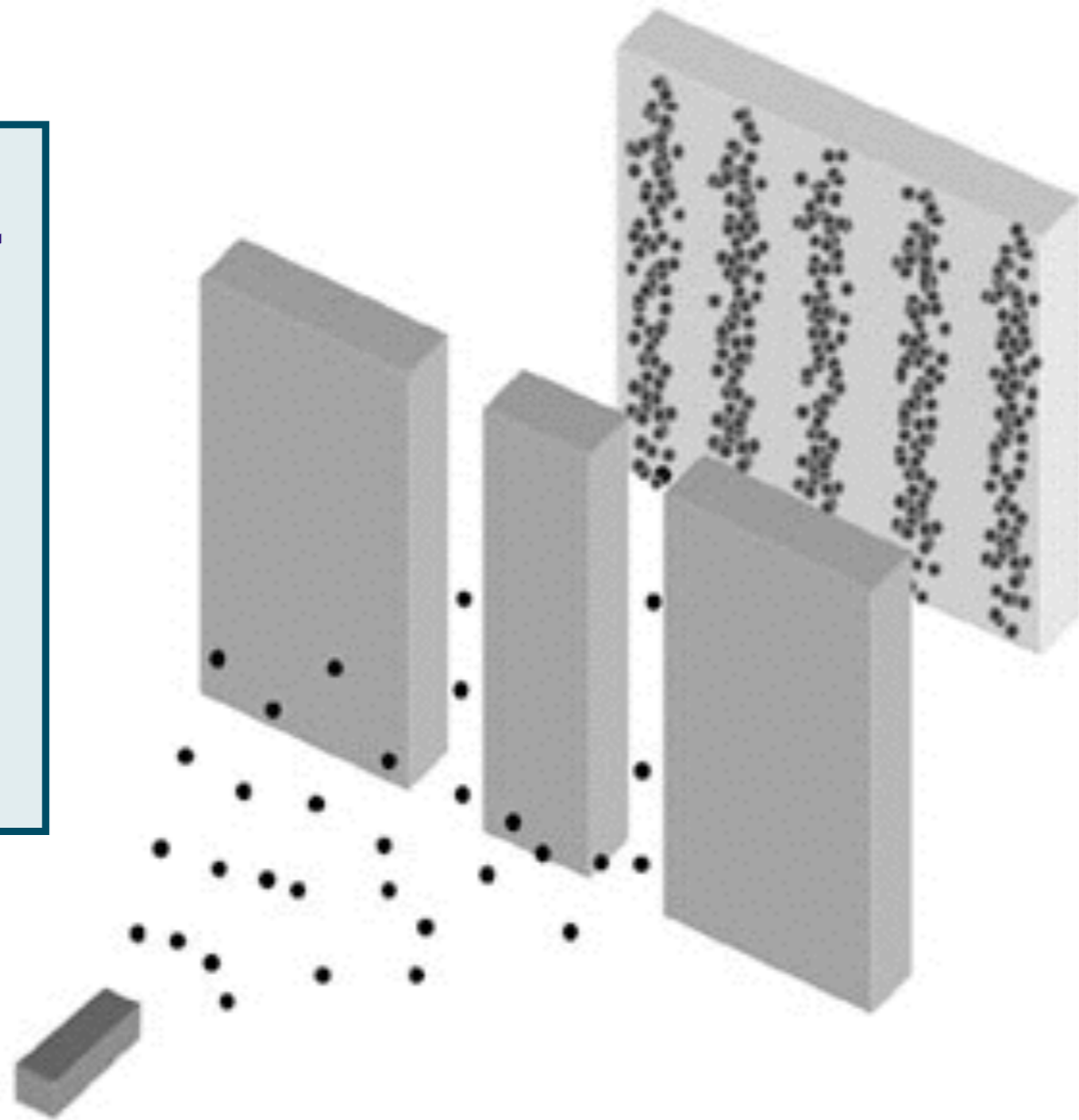
No
interference
when you
watch the
electrons

Interference of electrons

Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

Which slit
does an
electron
pass
through ?

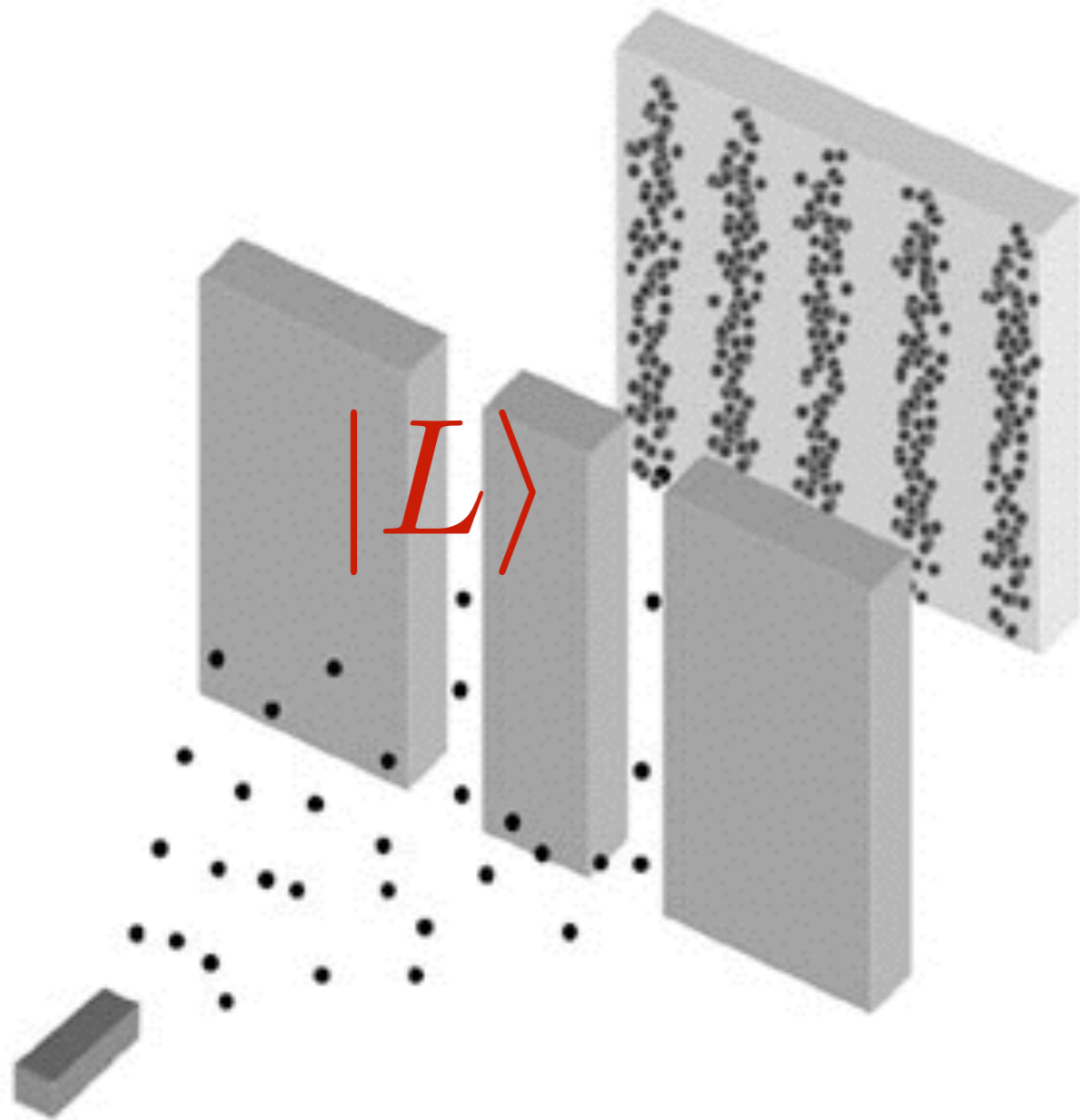


Each
electron
passes
through
both slits !

Interference of electrons

Principles of Quantum Mechanics: I. Quantum Superposition

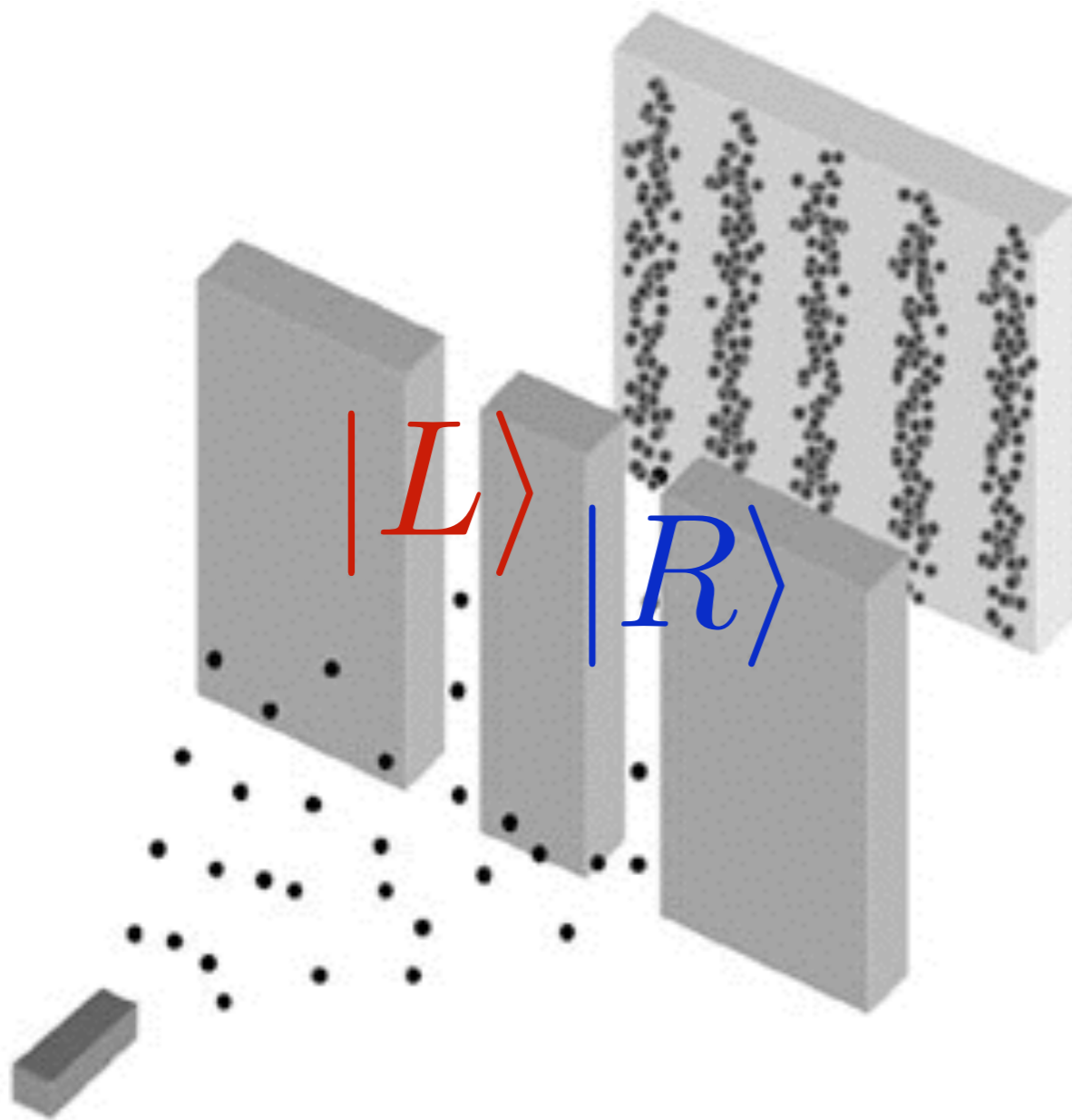
The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment

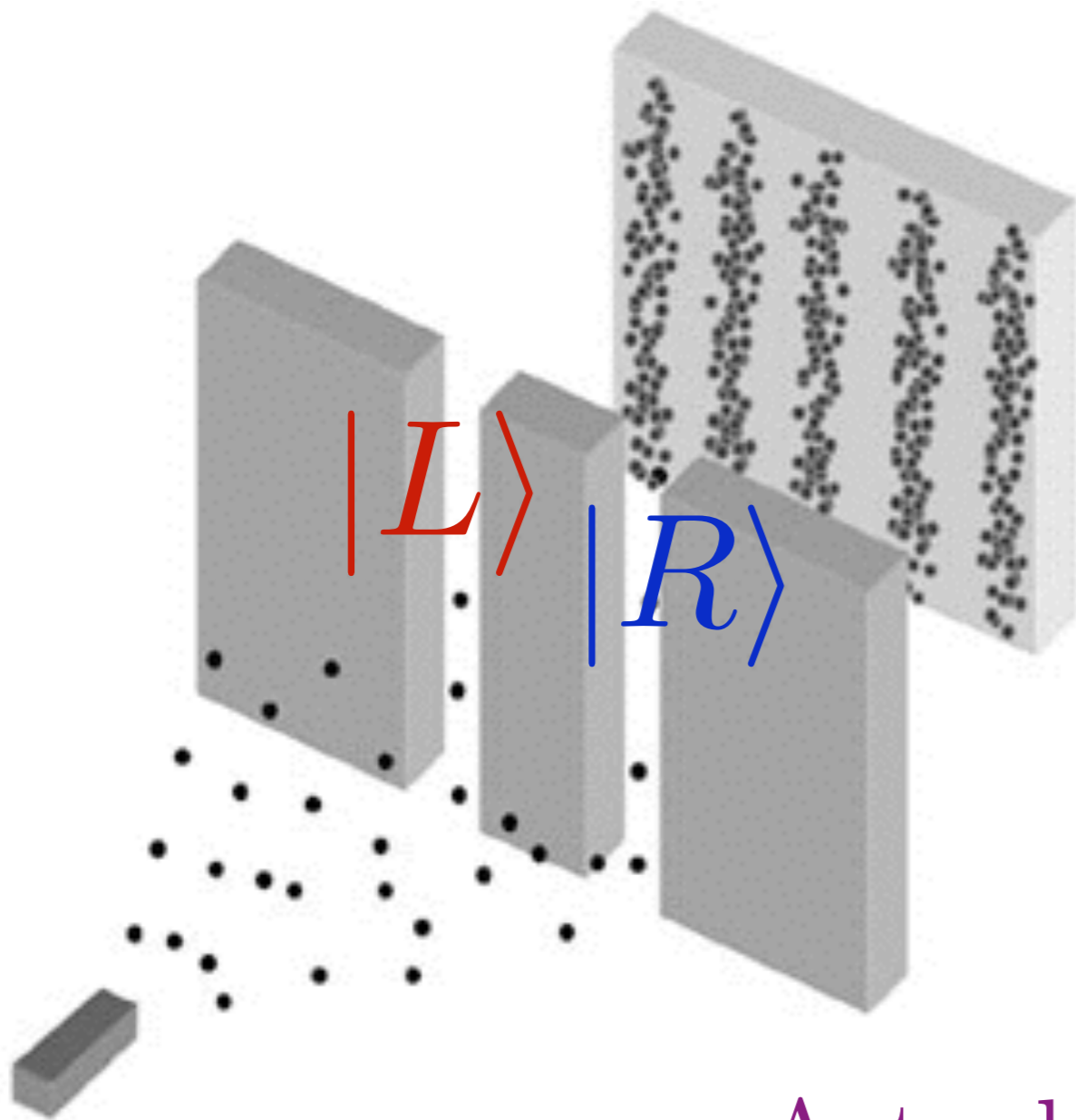


Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Principles of Quantum Mechanics: I. Quantum Superposition

The double slit experiment



Let $|L\rangle$ represent the state with the electron in the left slit

And $|R\rangle$ represents the state with the electron in the right slit

Actual state of *each* electron is

$$|L\rangle + |R\rangle$$

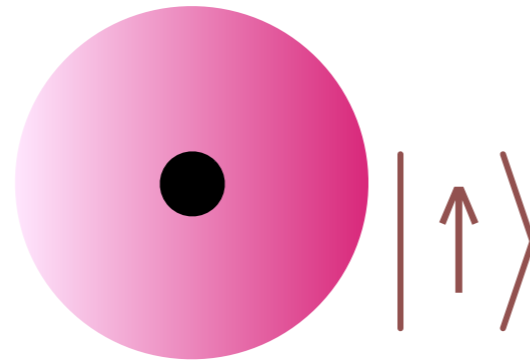
Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition
with more than one particle

Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle

Hydrogen atom:

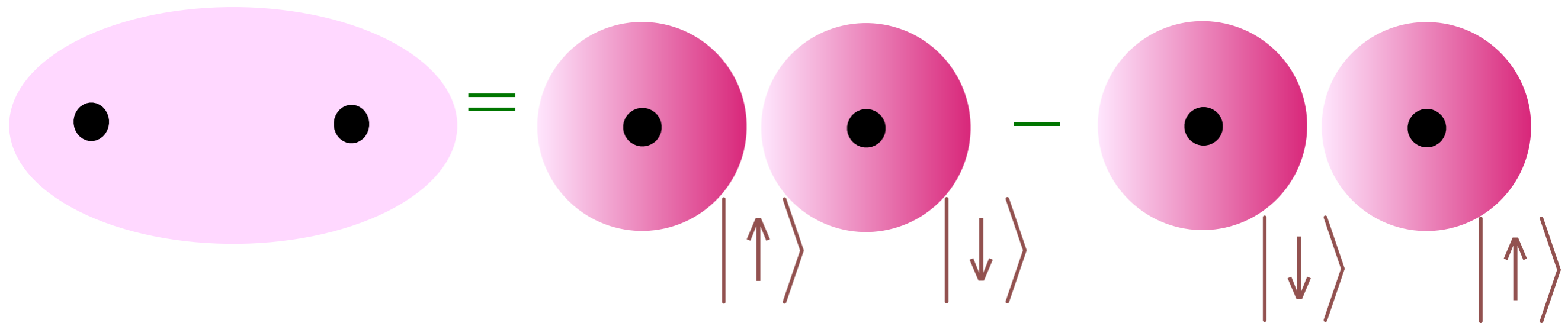


Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle



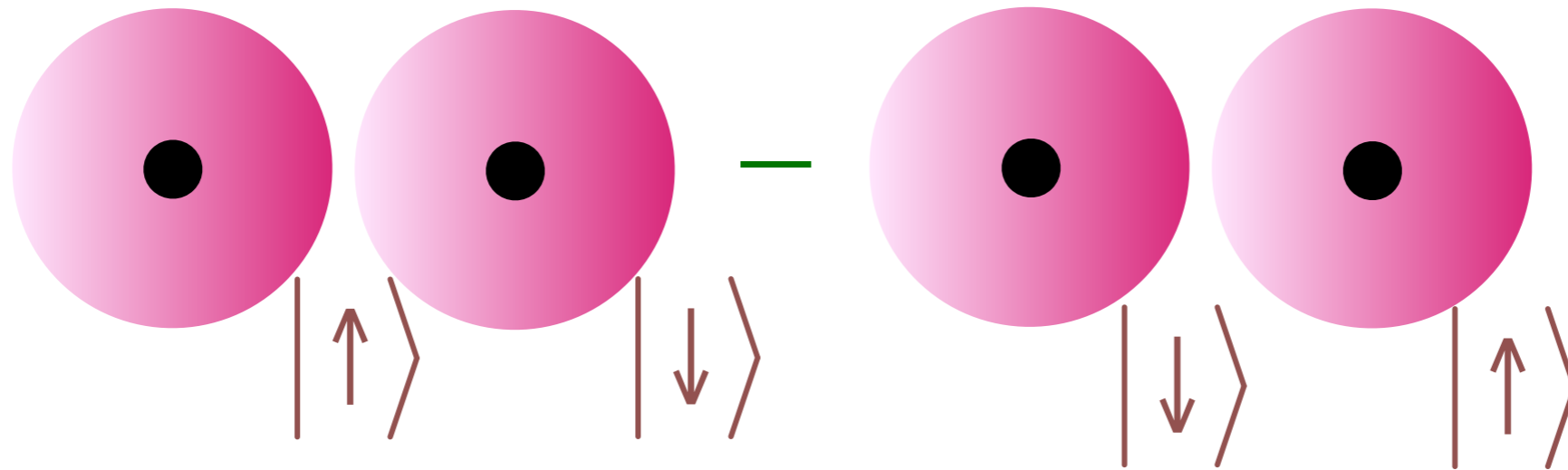
Hydrogen molecule:



$$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

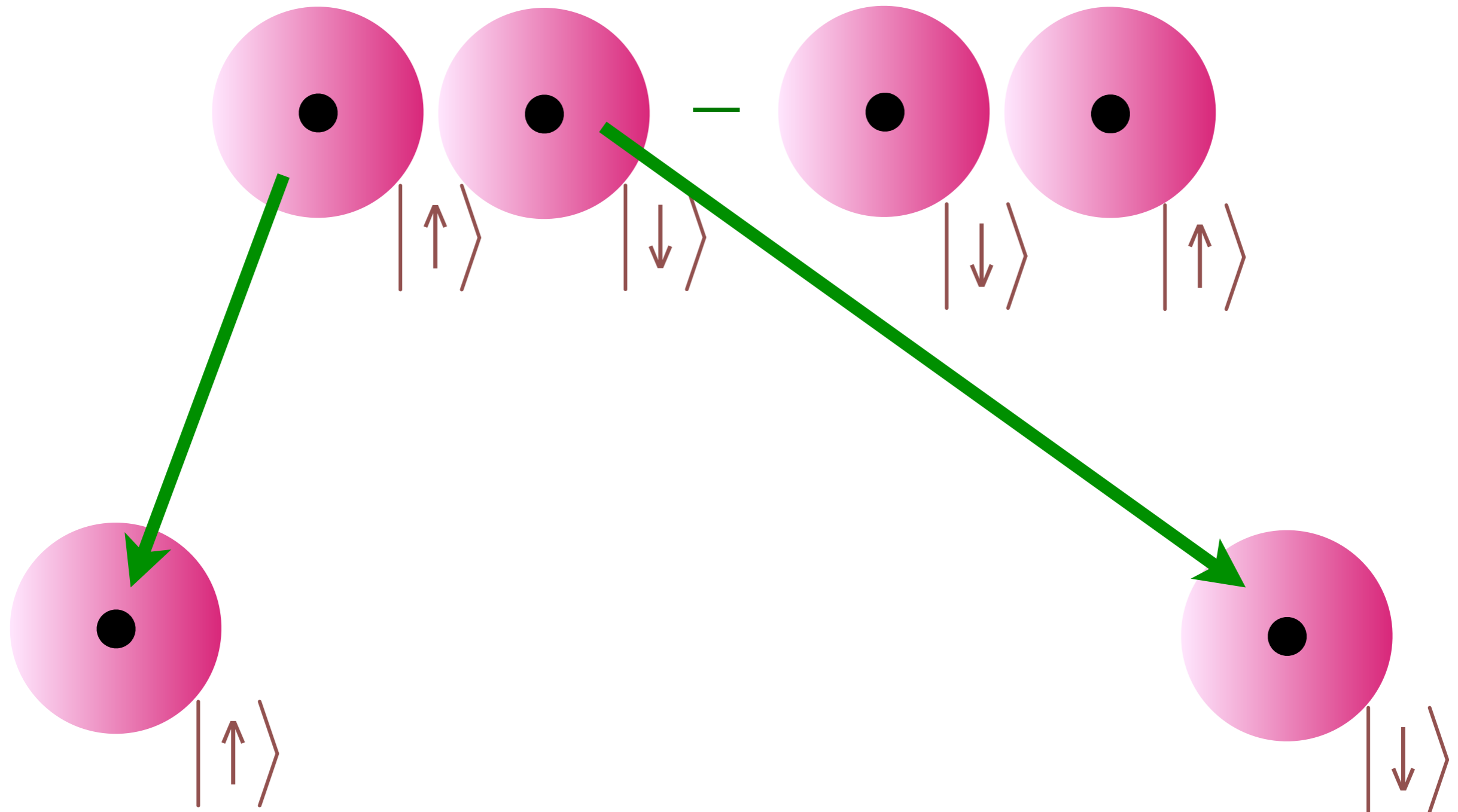
Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle



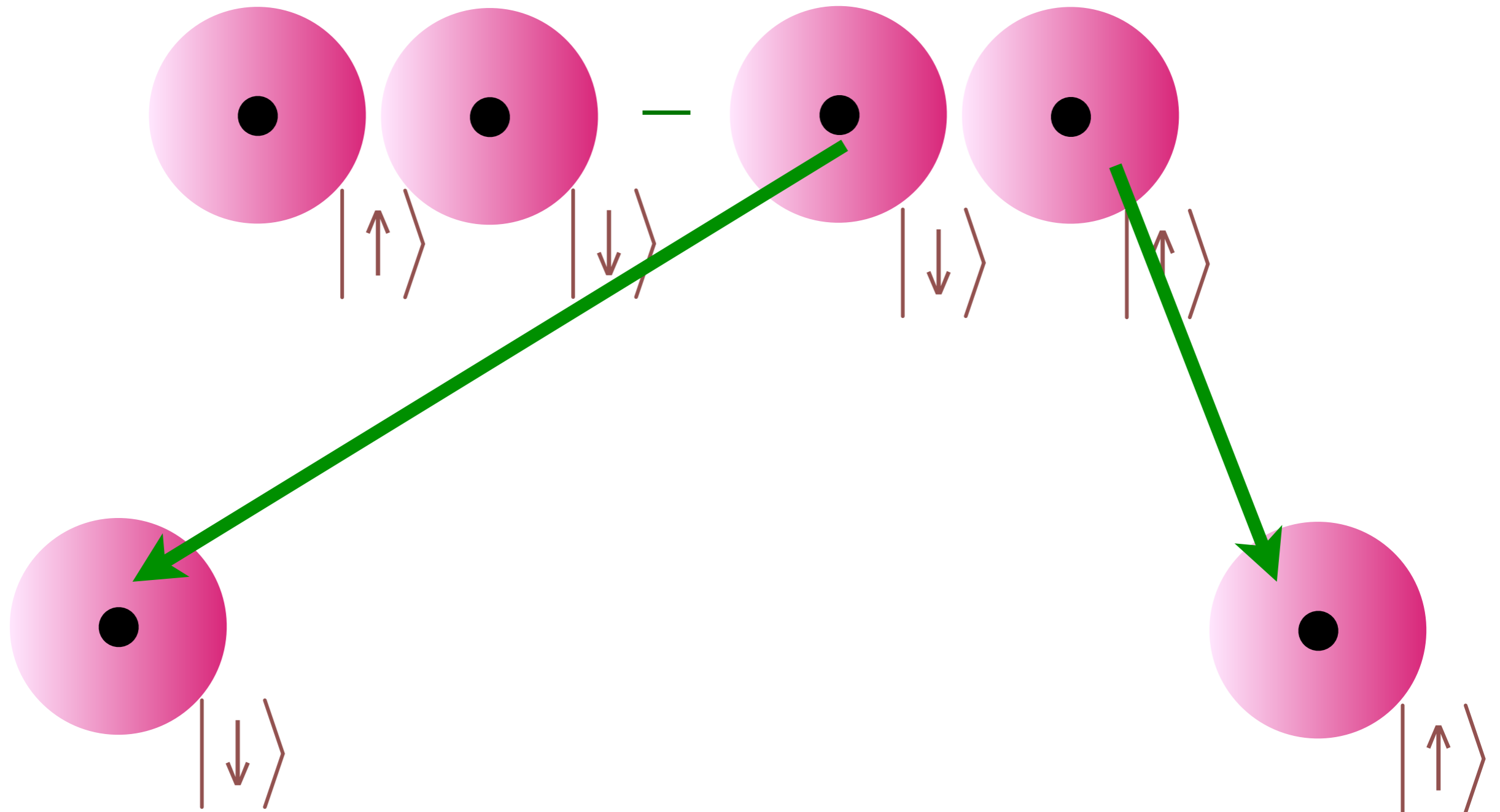
Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle



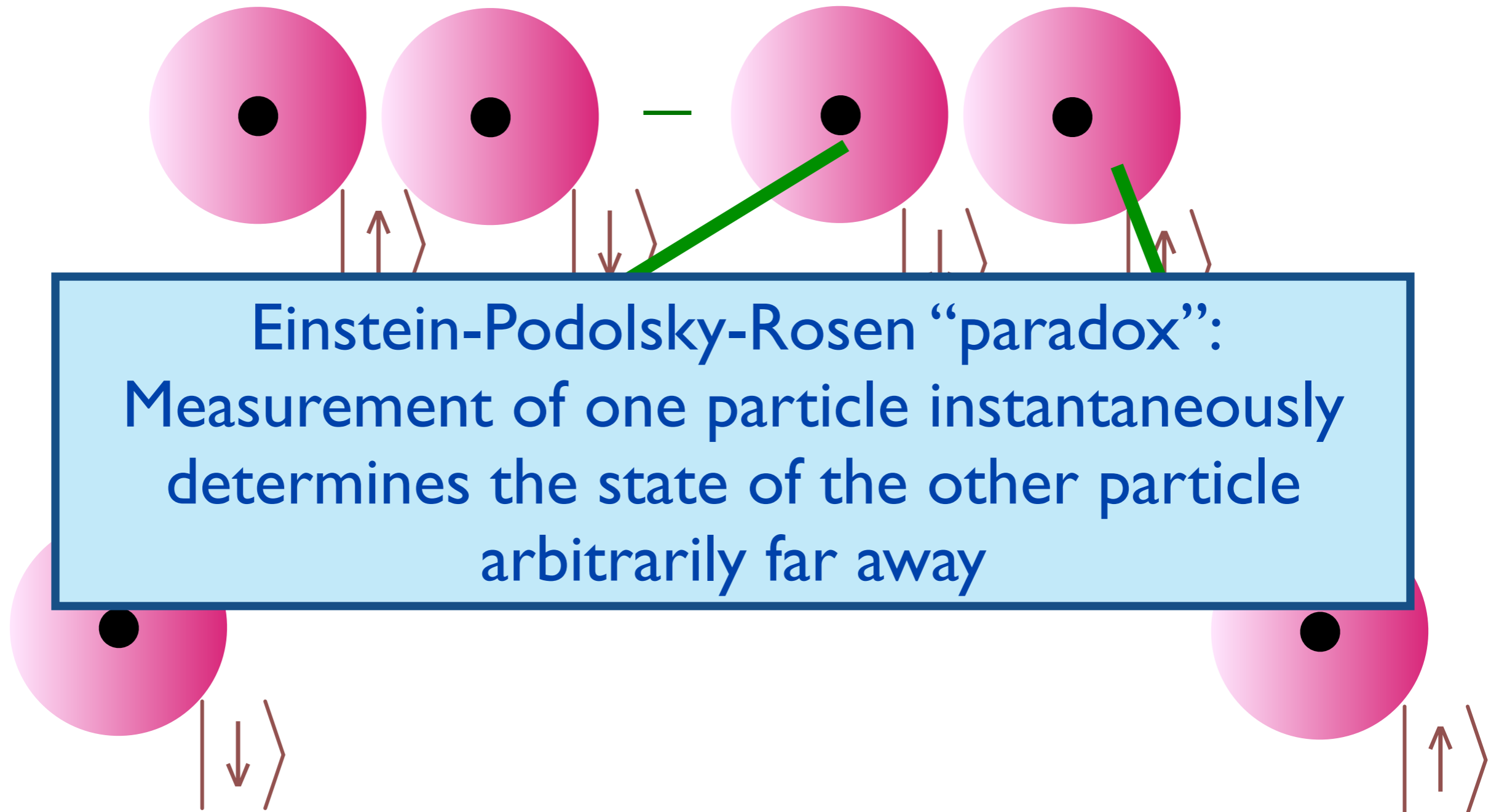
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Principles of Quantum Mechanics: II. Quantum Entanglement

Quantum Entanglement: quantum superposition with more than one particle



**Quantum
superposition and
entanglement**

**Quantum
superposition and
entanglement**

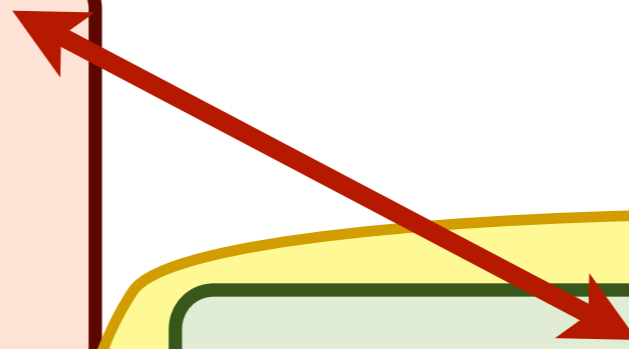
**String theory
and black holes**

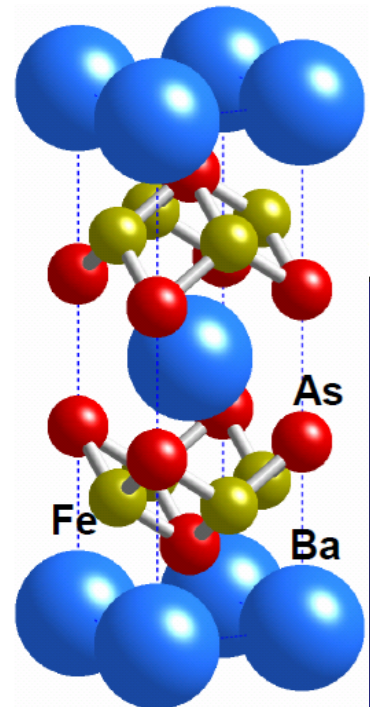
**Quantum critical
points
and long-range
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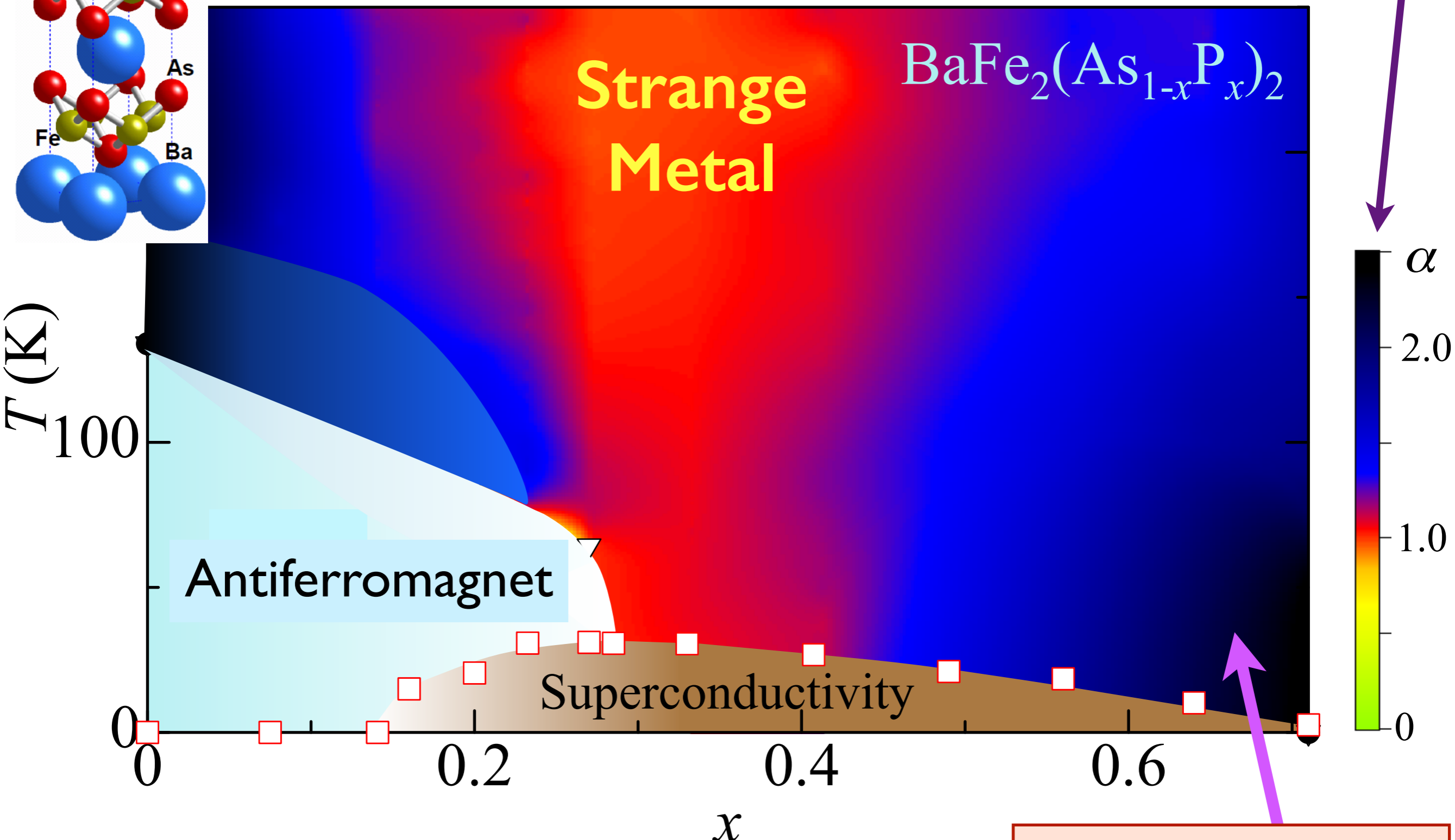
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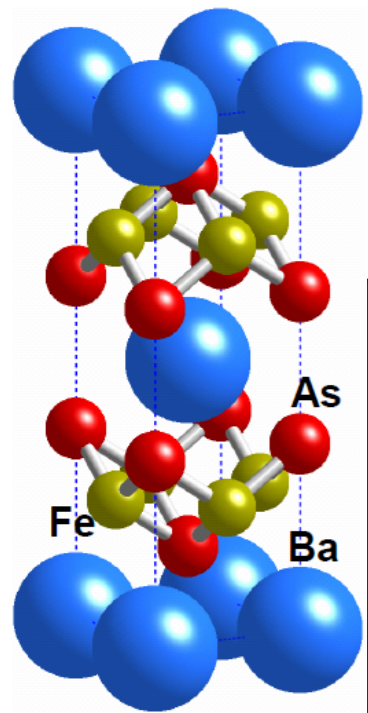




Resistivity
 $\sim \rho_0 + AT^\alpha$



S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *Physical Review B* **81**, 184519 (2010)



Electrons pair to form “Cooper pairs” which are *bosons*.

Bosons do not obey the exclusion principle. So all the Cooper pairs occupy the lowest energy state, and we obtain a “Bose-Einstein condensate.

This condensate is responsible for superconductivity.

$$\rho = \rho_0 + AT^\alpha$$

$(\chi)^2$

α

2.0

1.0

0

T (K)

100

Antiferromagnet

Superconductivity

0

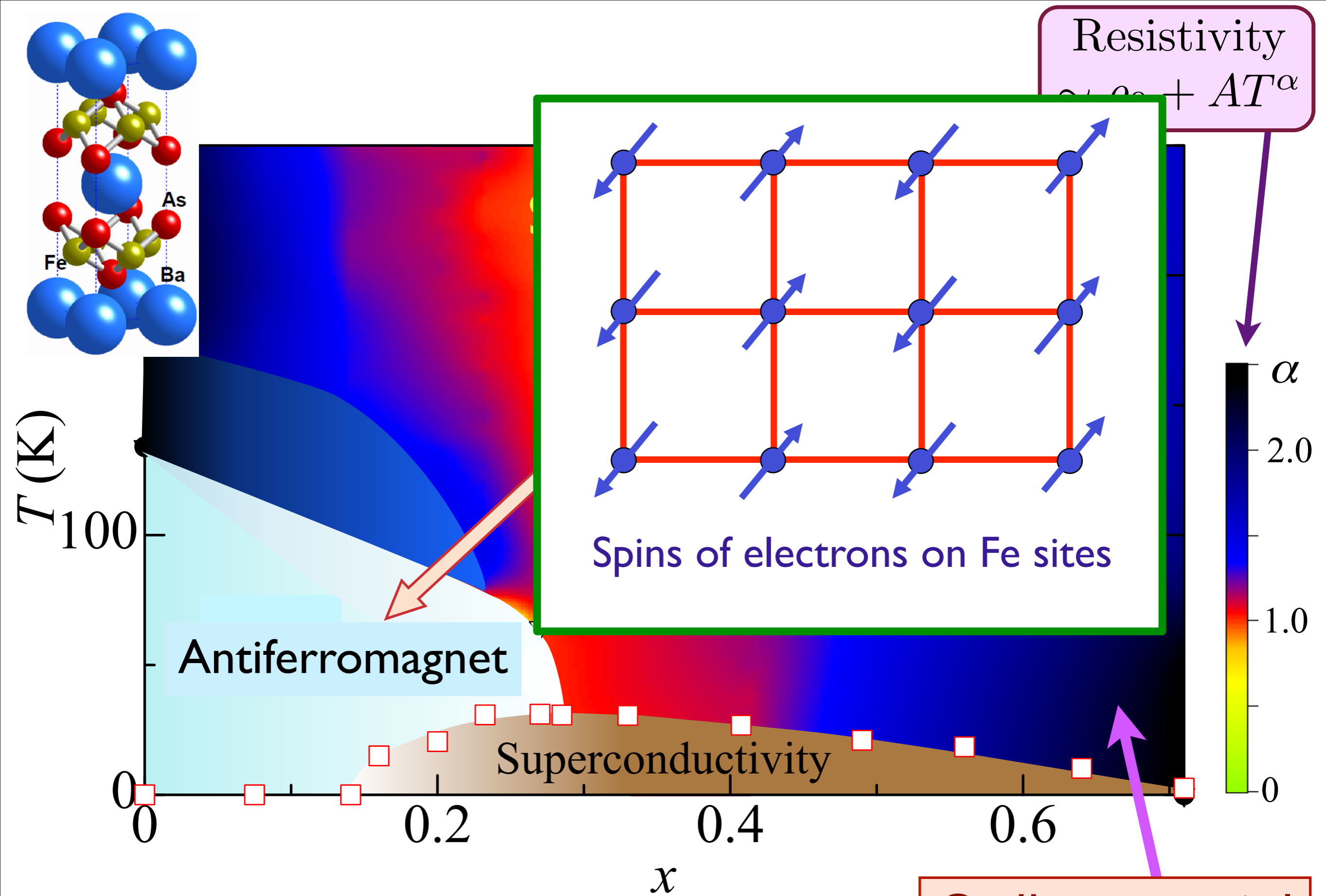
0.2

0.4

0.6

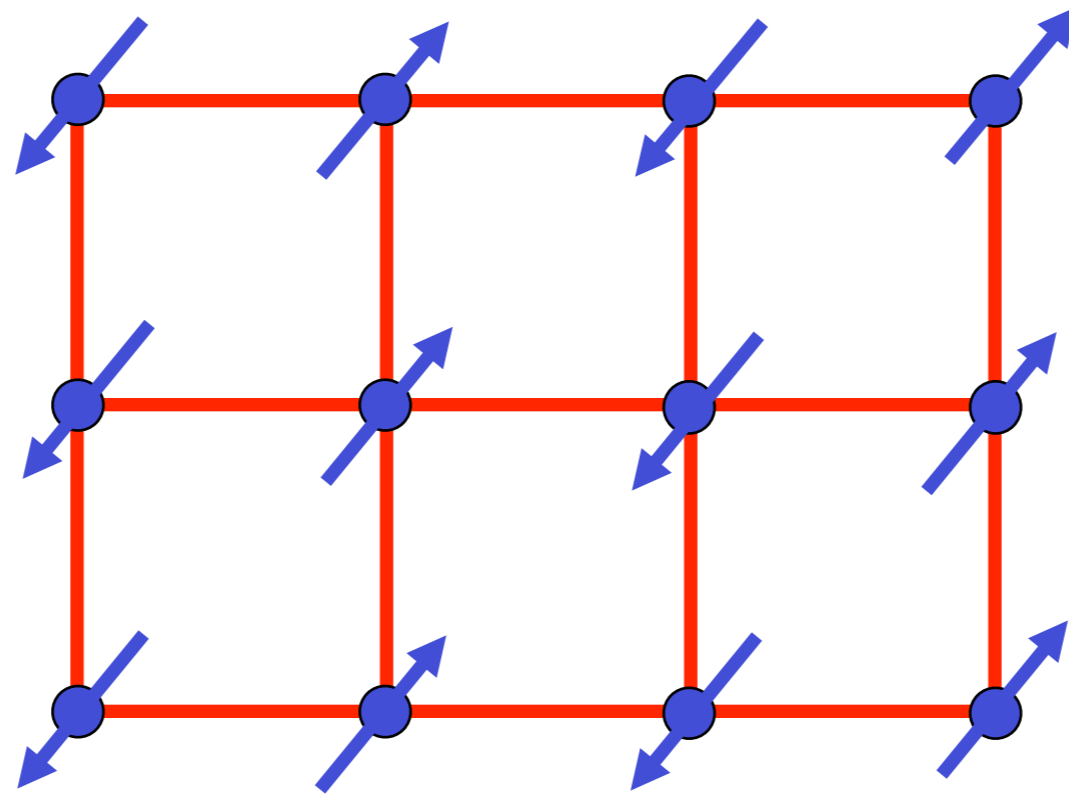
χ

Ordinary metal



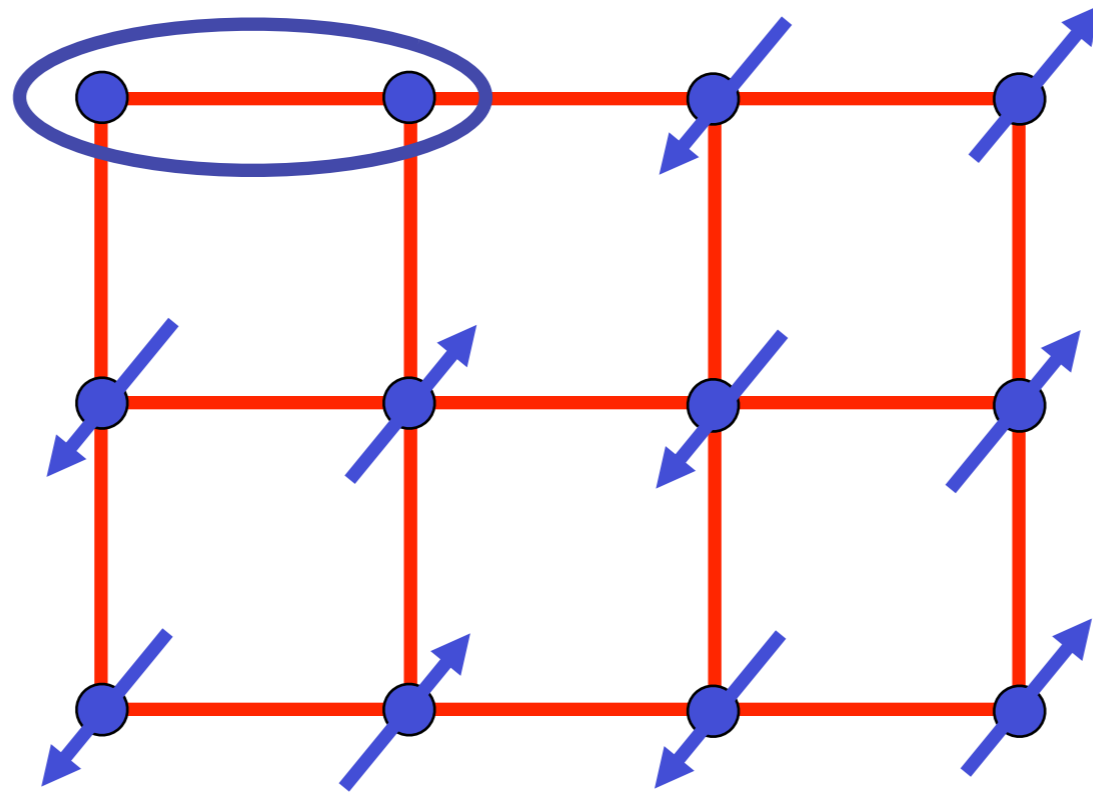
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Ordinary metal

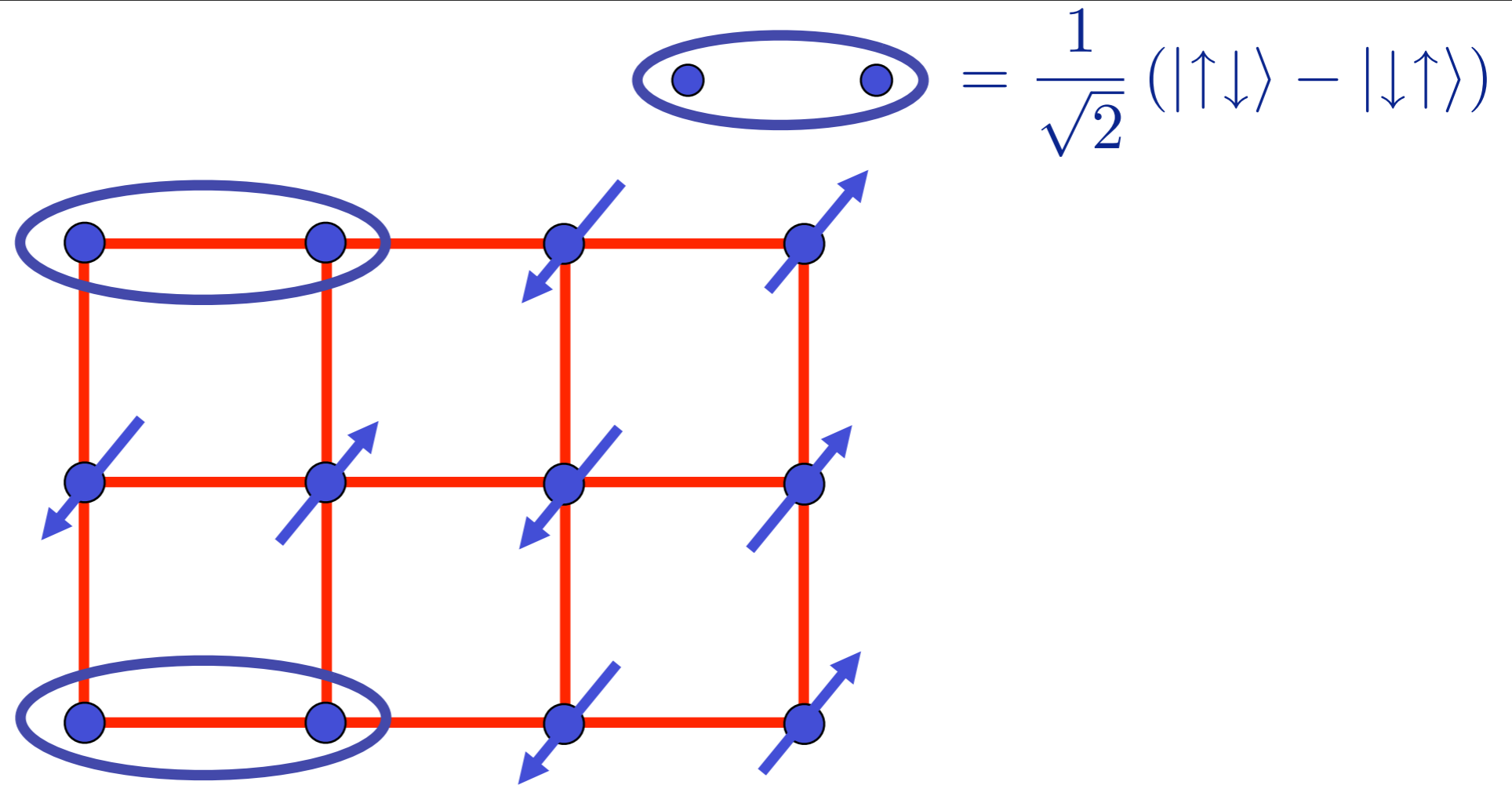


Spins of electrons on Fe sites

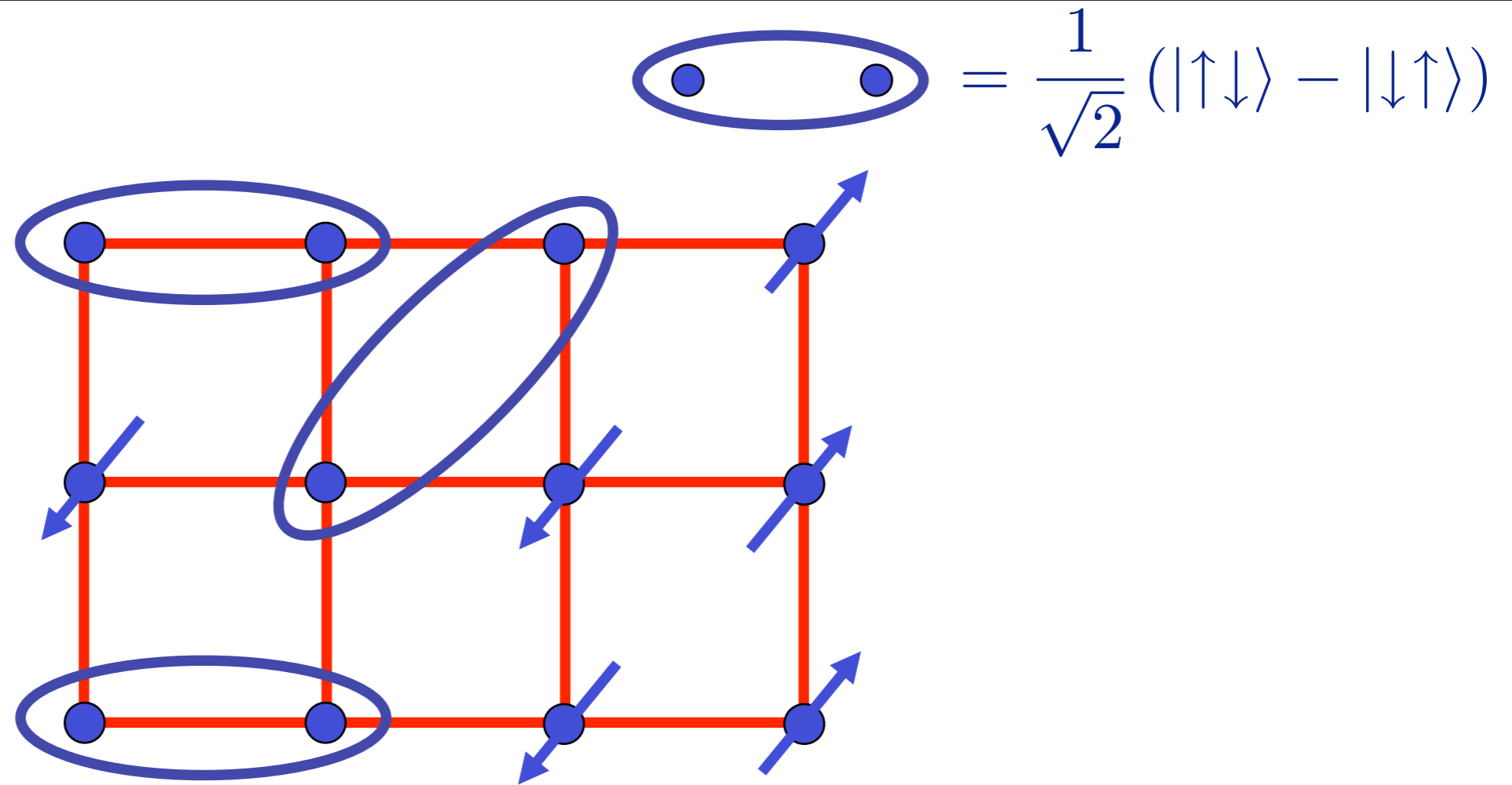
$$\text{Diagram of two dots in an oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



As we increase x , the electron spins entangle in pairs

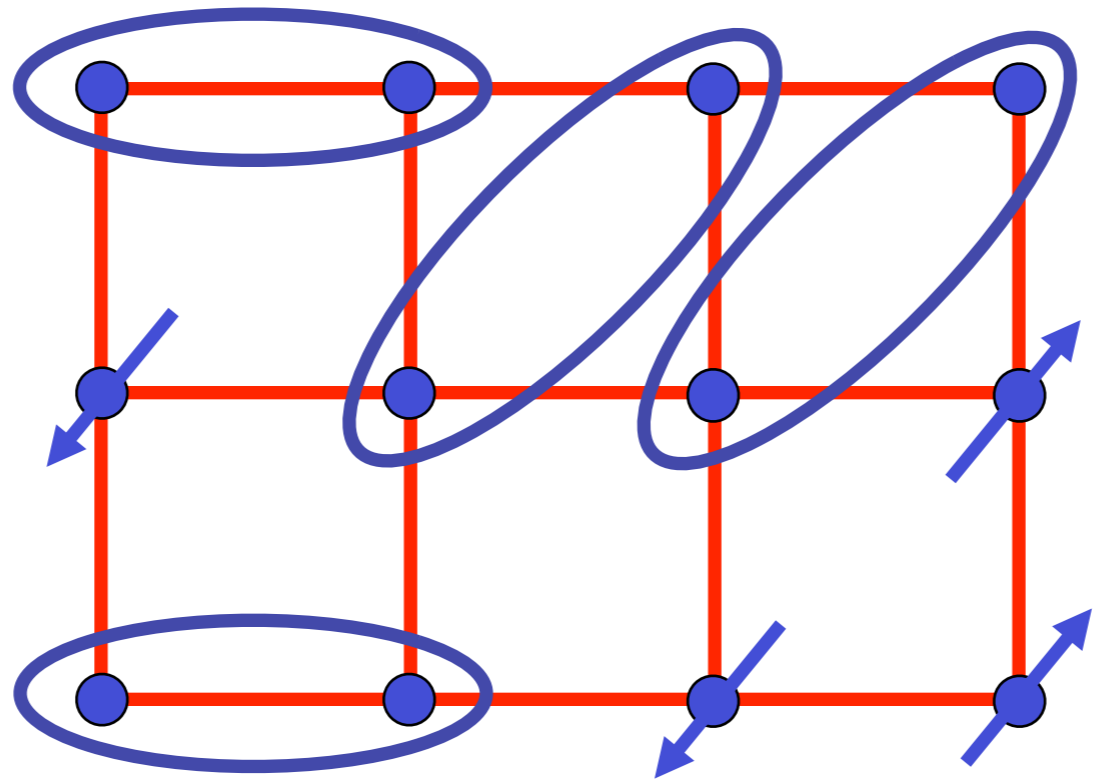


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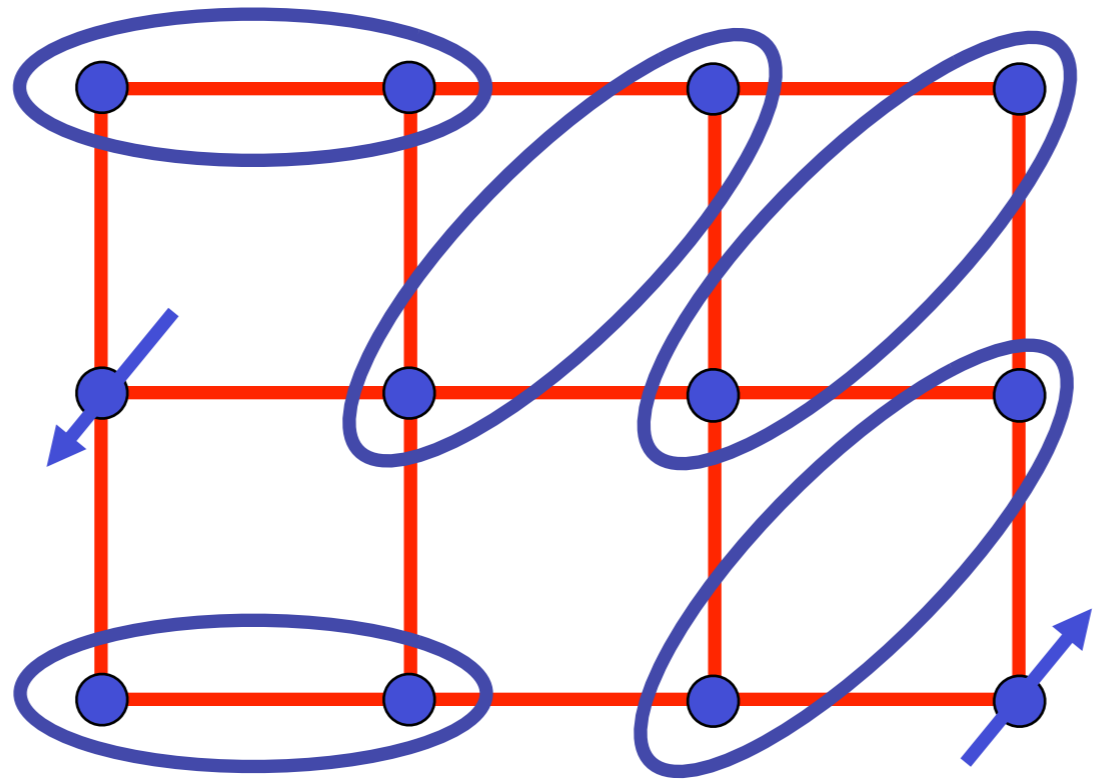
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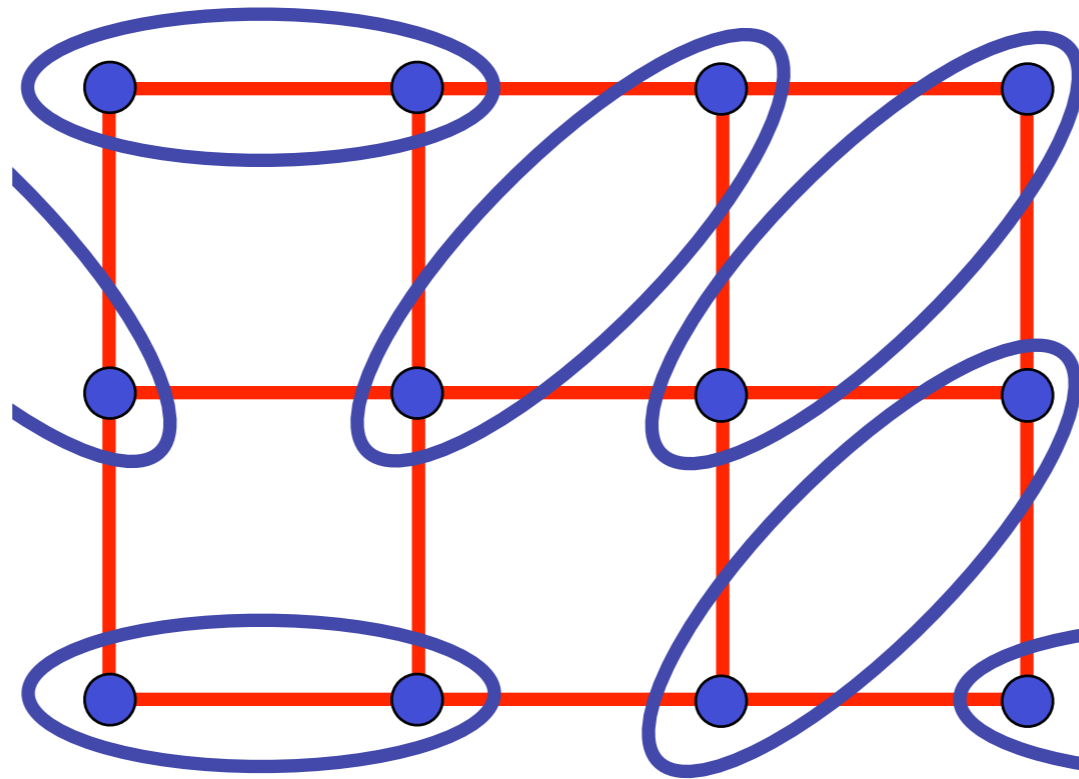
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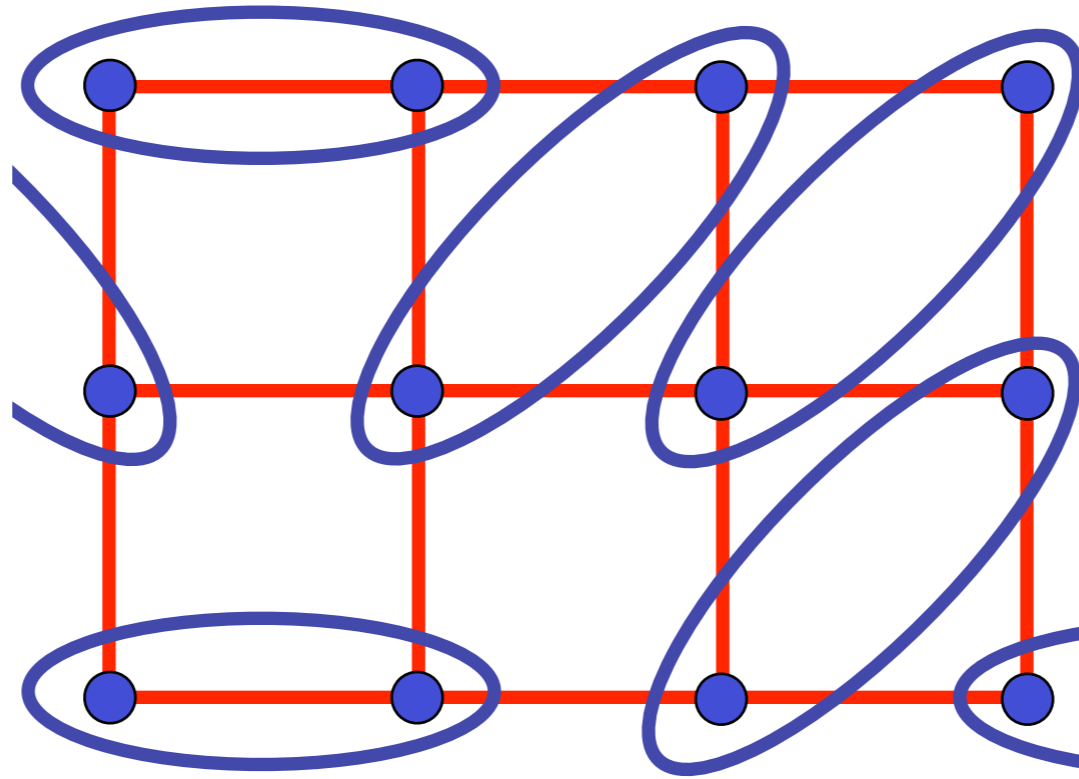
As we increase x , the electron spins entangle in pairs

$$\text{Diagram of two blue dots in an oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



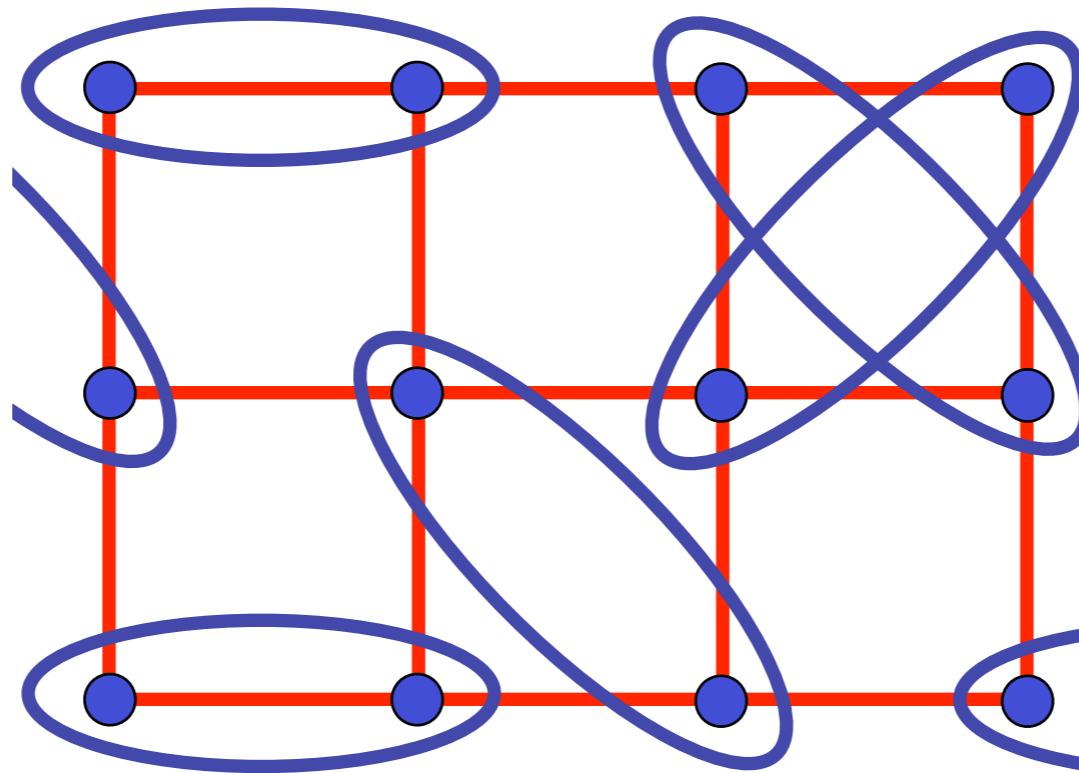
As we increase x , the electron spins entangle in pairs

$$\text{Diagram of two dots in an oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$



As we increase x , the electron spins entangle in pairs.
And then the pairs entangle with each other.

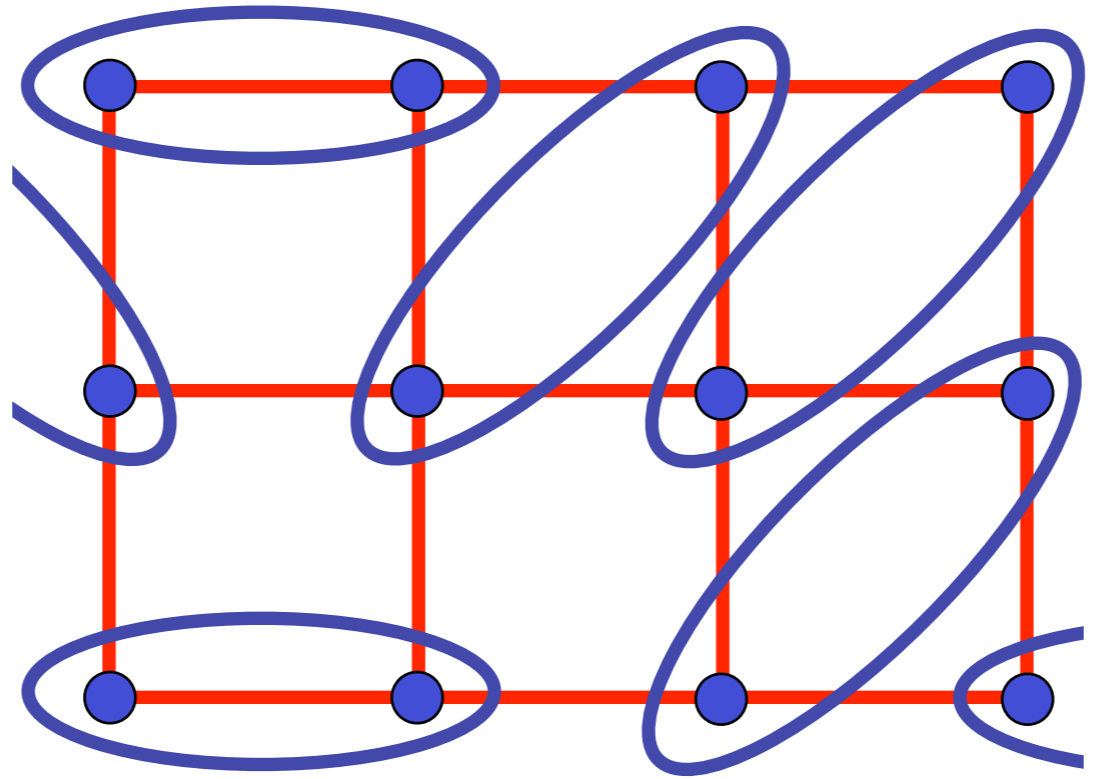
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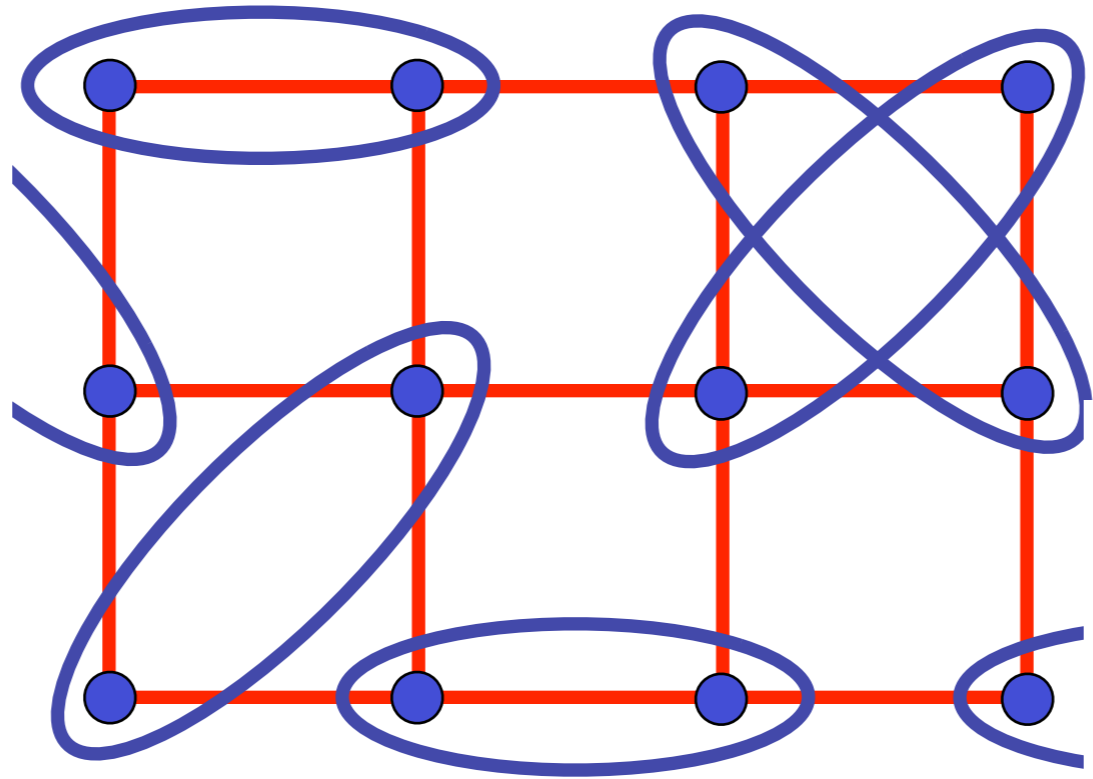
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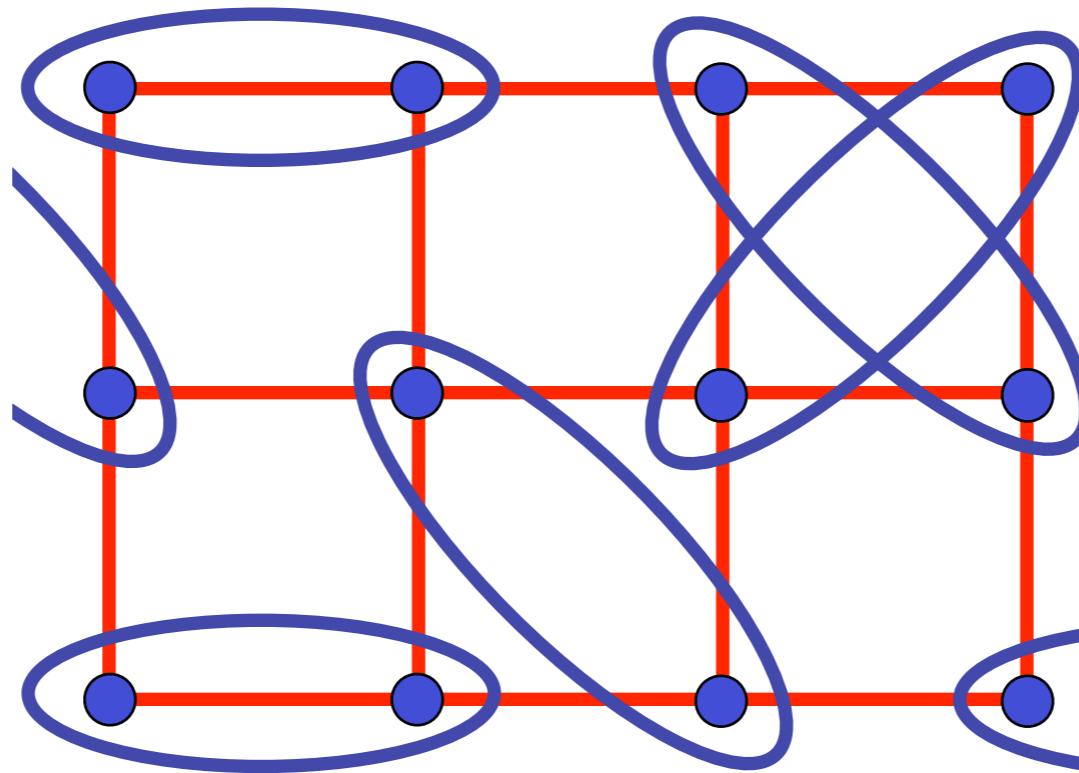
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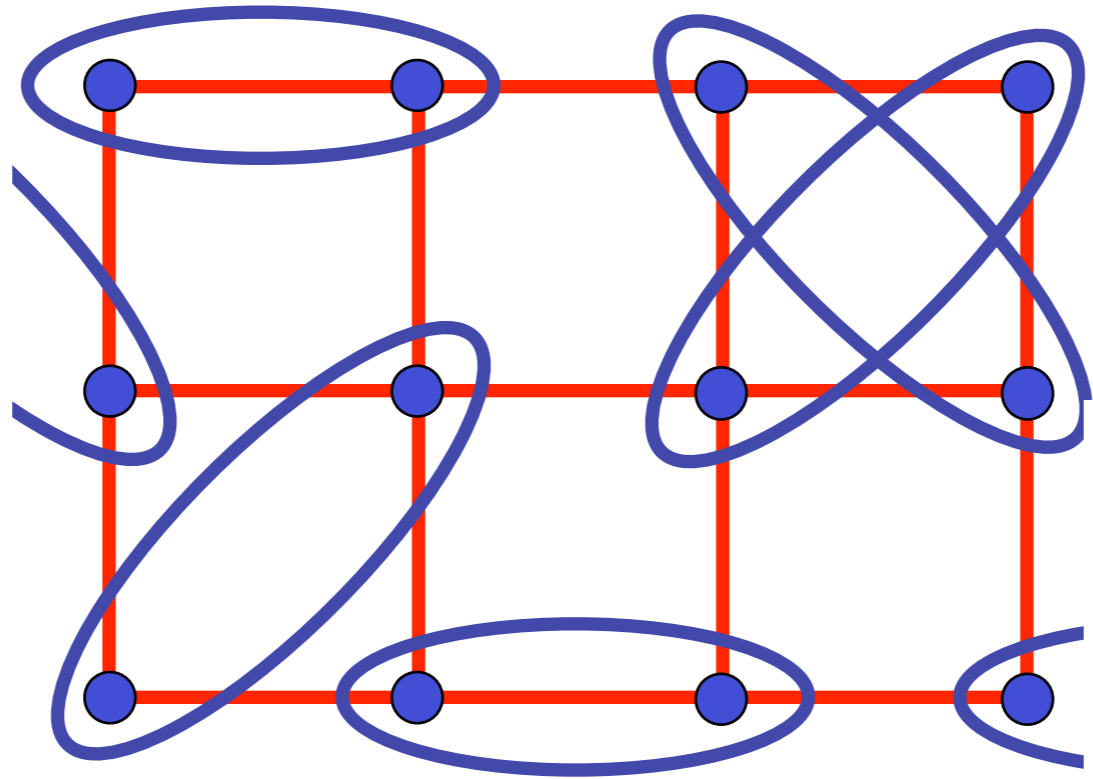
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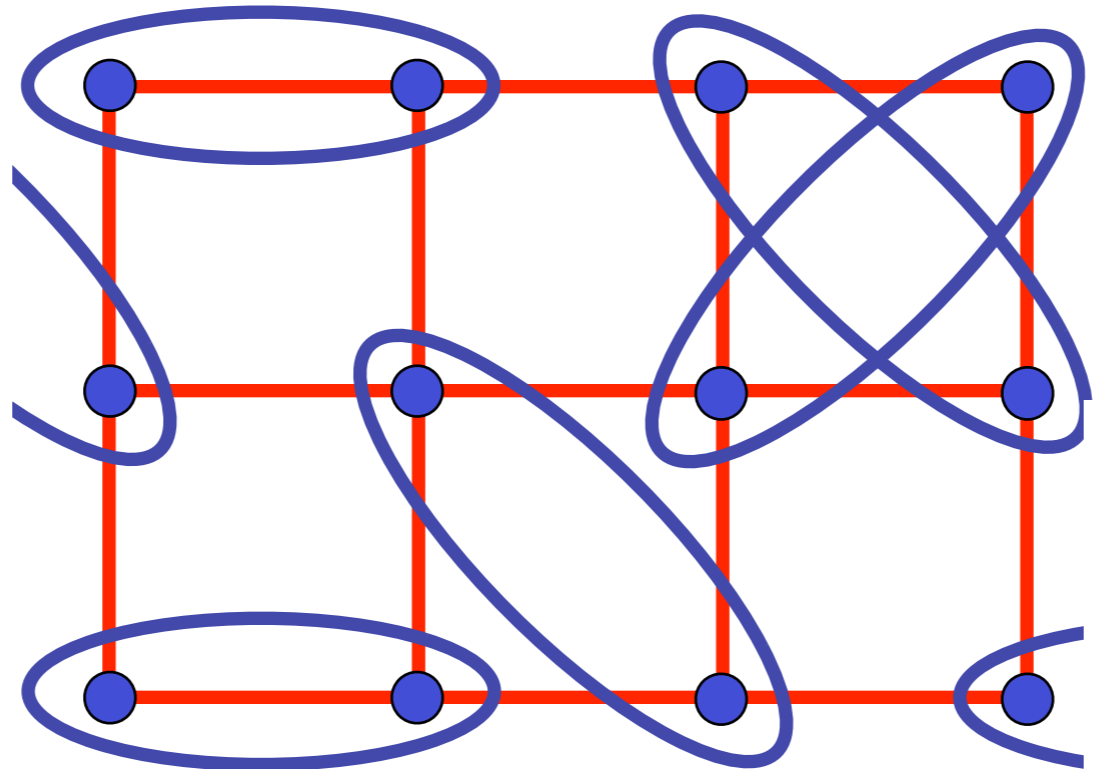
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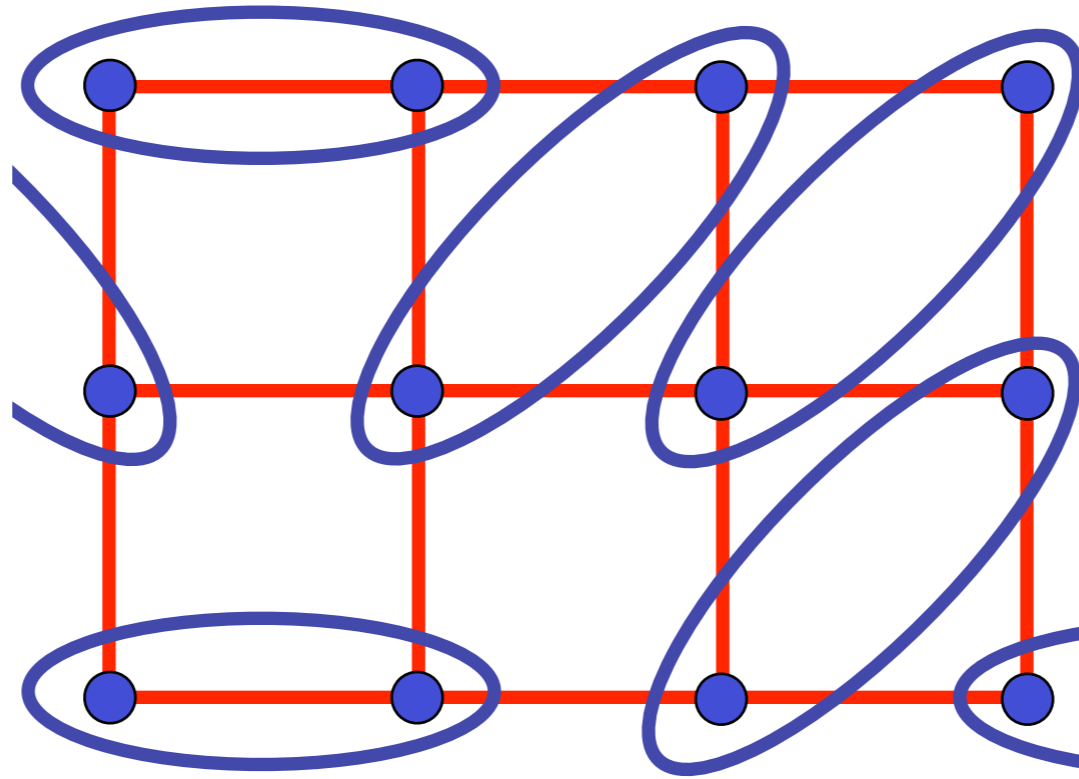
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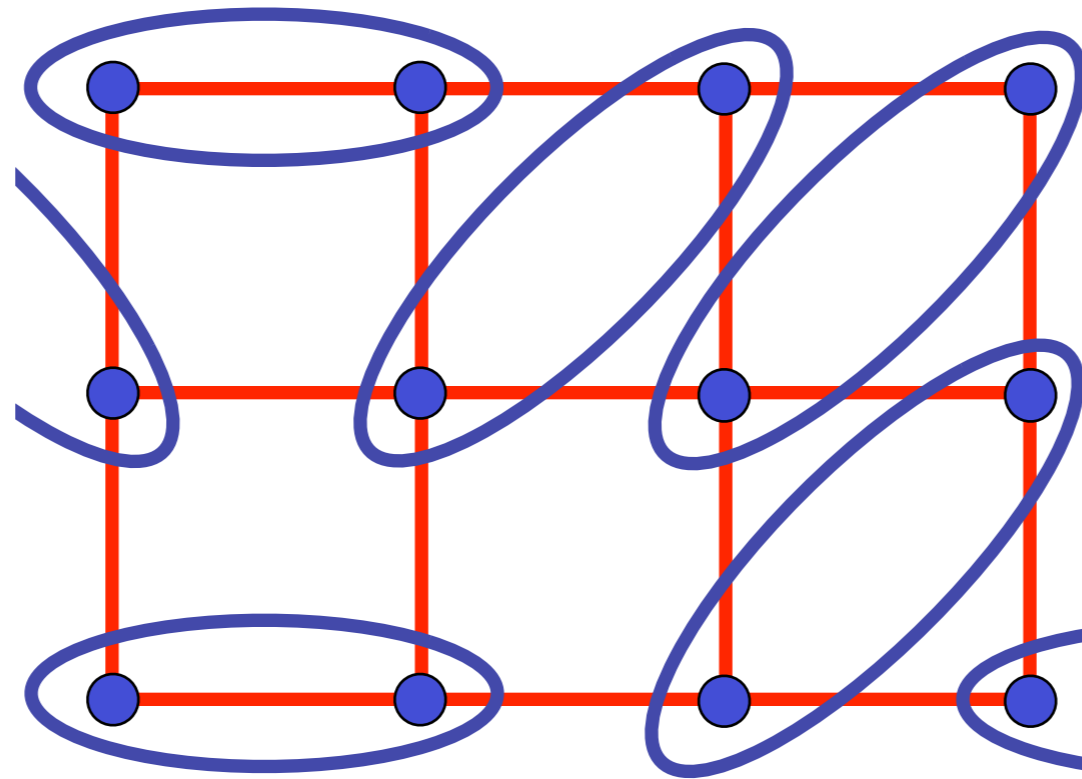
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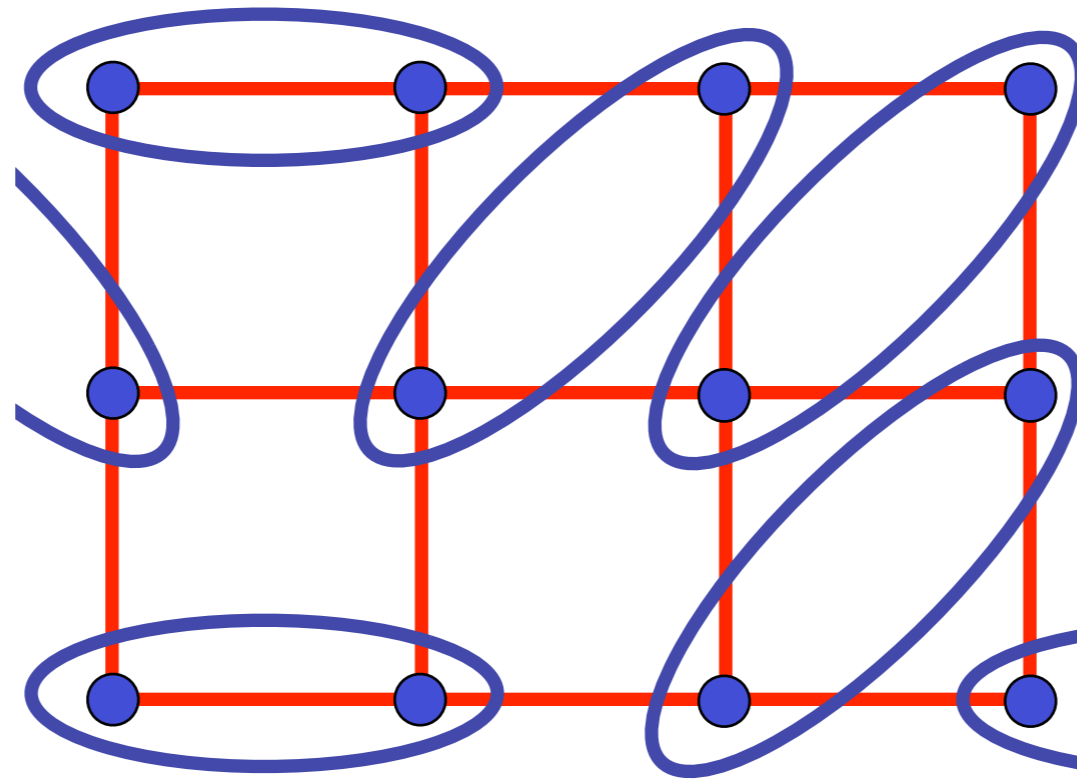
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As we increase x , the electron spins entangle in pairs.

And then the pairs entangle with each other.
And then pairs of pairs entangle, and so on....

$$\text{Diagram of two dots in an oval} = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

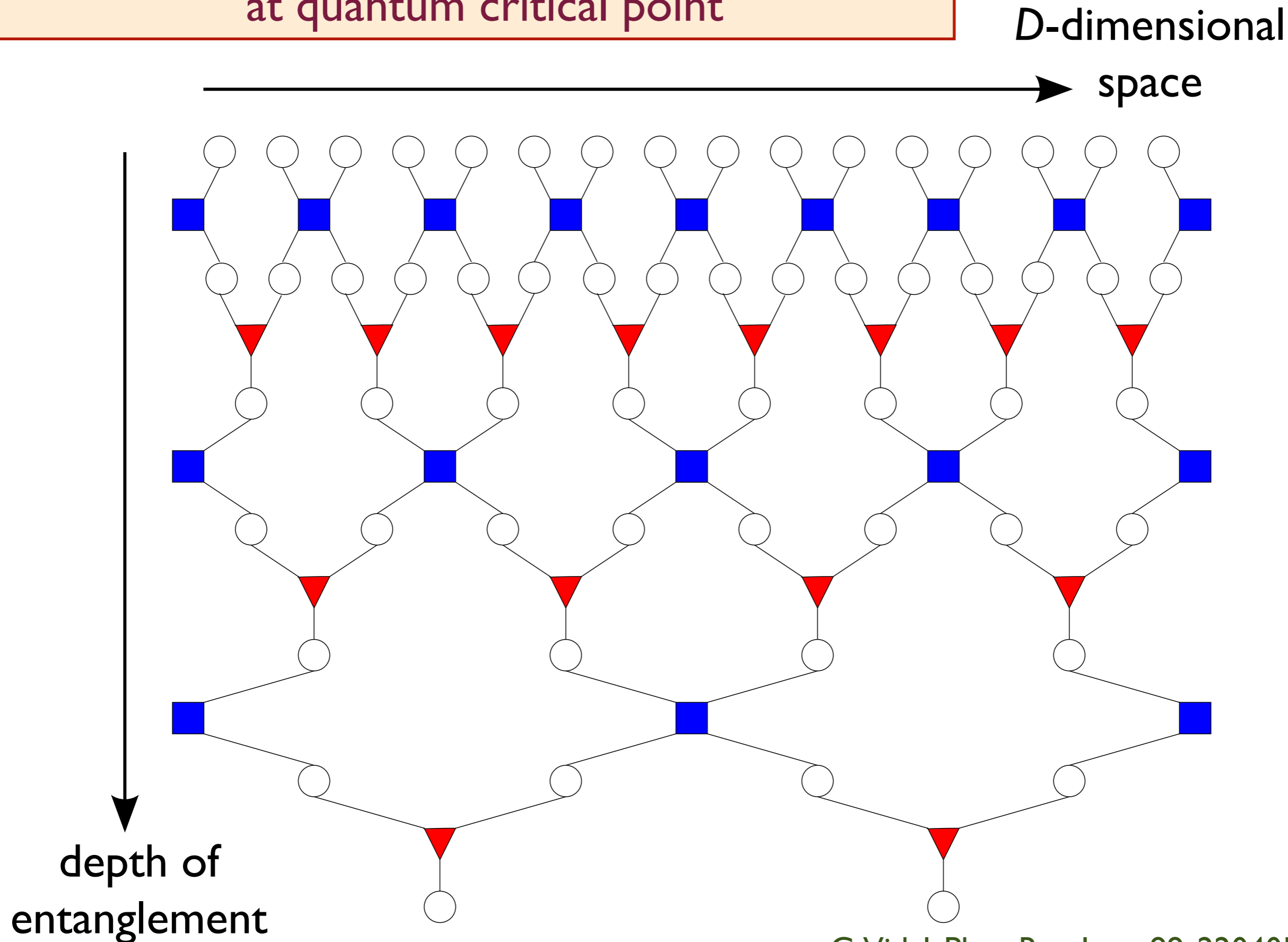


As we increase x , the electron spins entangle in pairs.

And then the pairs entangle with each other.
And then pairs of pairs entangle, and so on....

This goes on ad-infinitum at the quantum critical point, leading to long-range quantum entanglement

Tensor network representation of entanglement at quantum critical point



**Quantum
superposition and
entanglement**

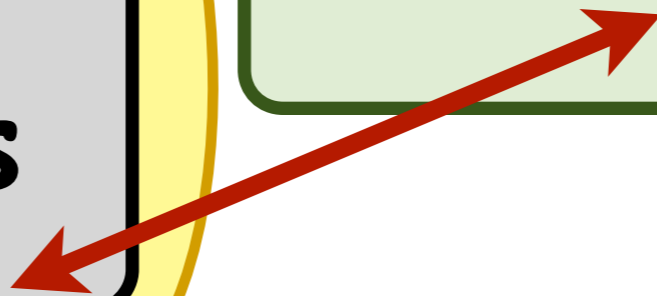
**String theory
and black holes**

**Quantum critical
points
and long-range
entanglement of
electrons
in crystals**

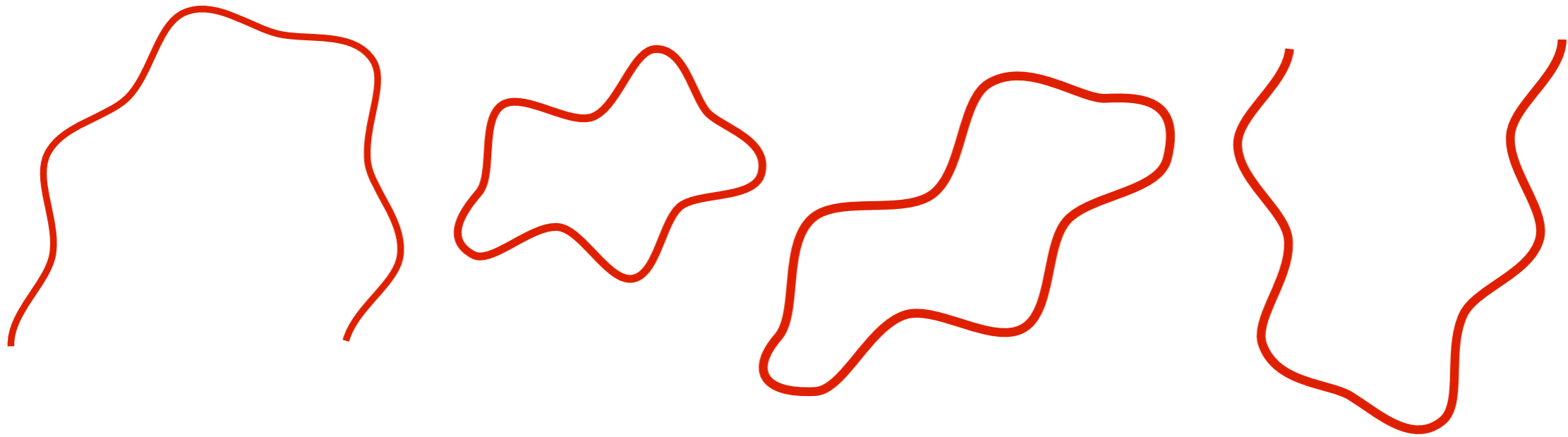
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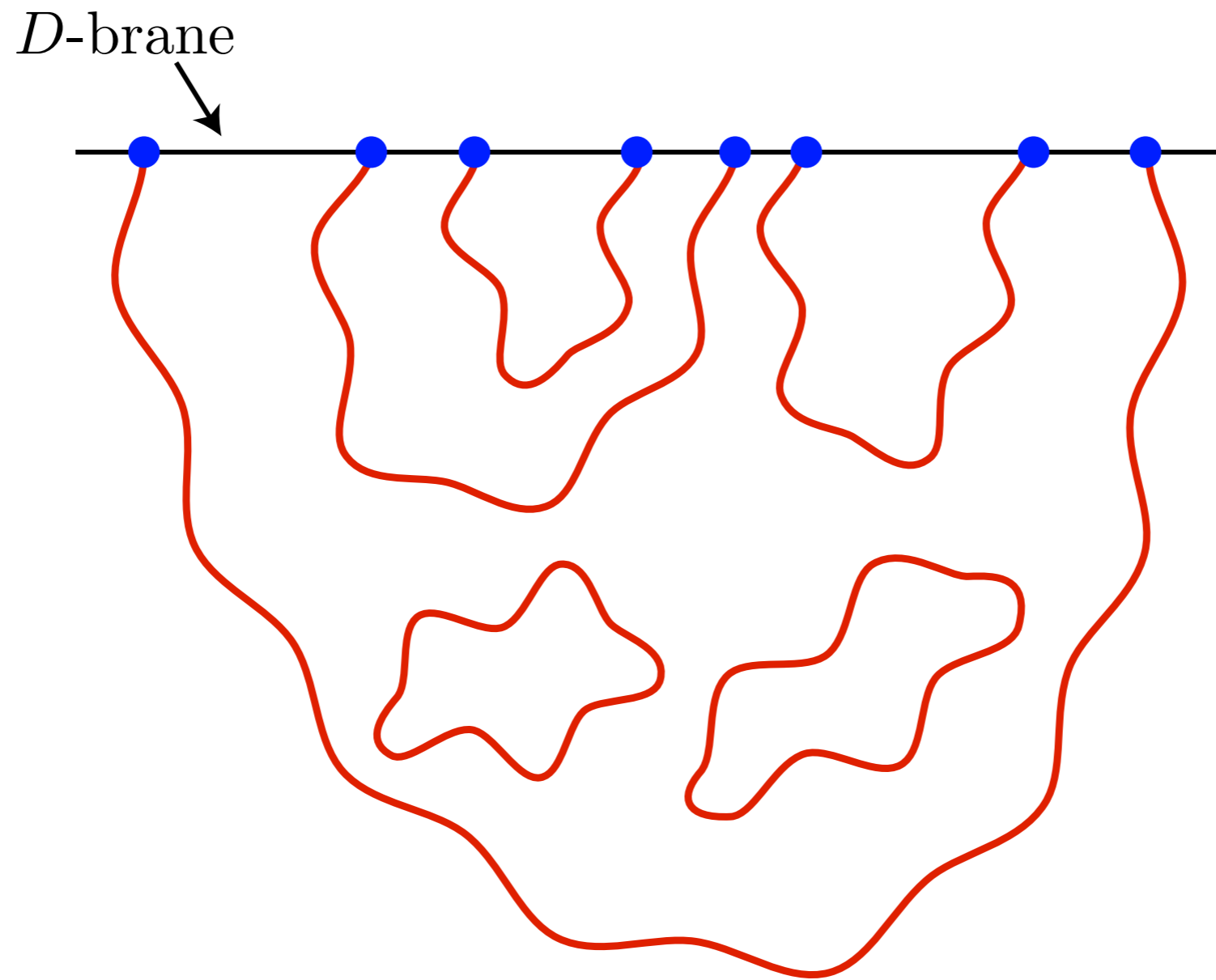
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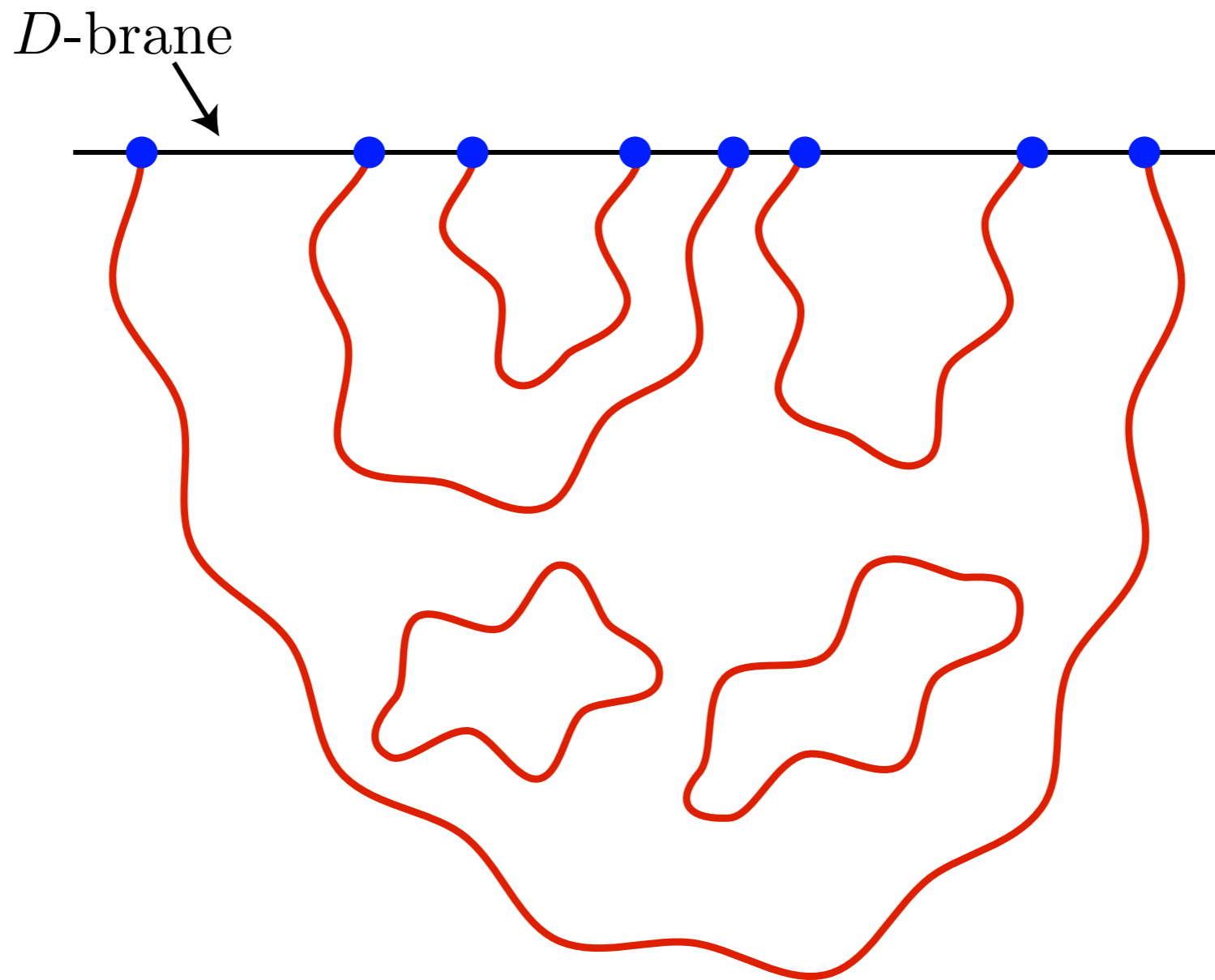
String theory



- Allows unification of the standard model of particle physics with Einstein's theory of gravitation (general relativity).
- Vibrations of a string (its “musical notes”) correspond to quarks, gravitons, the Higgs boson, photons, gluons

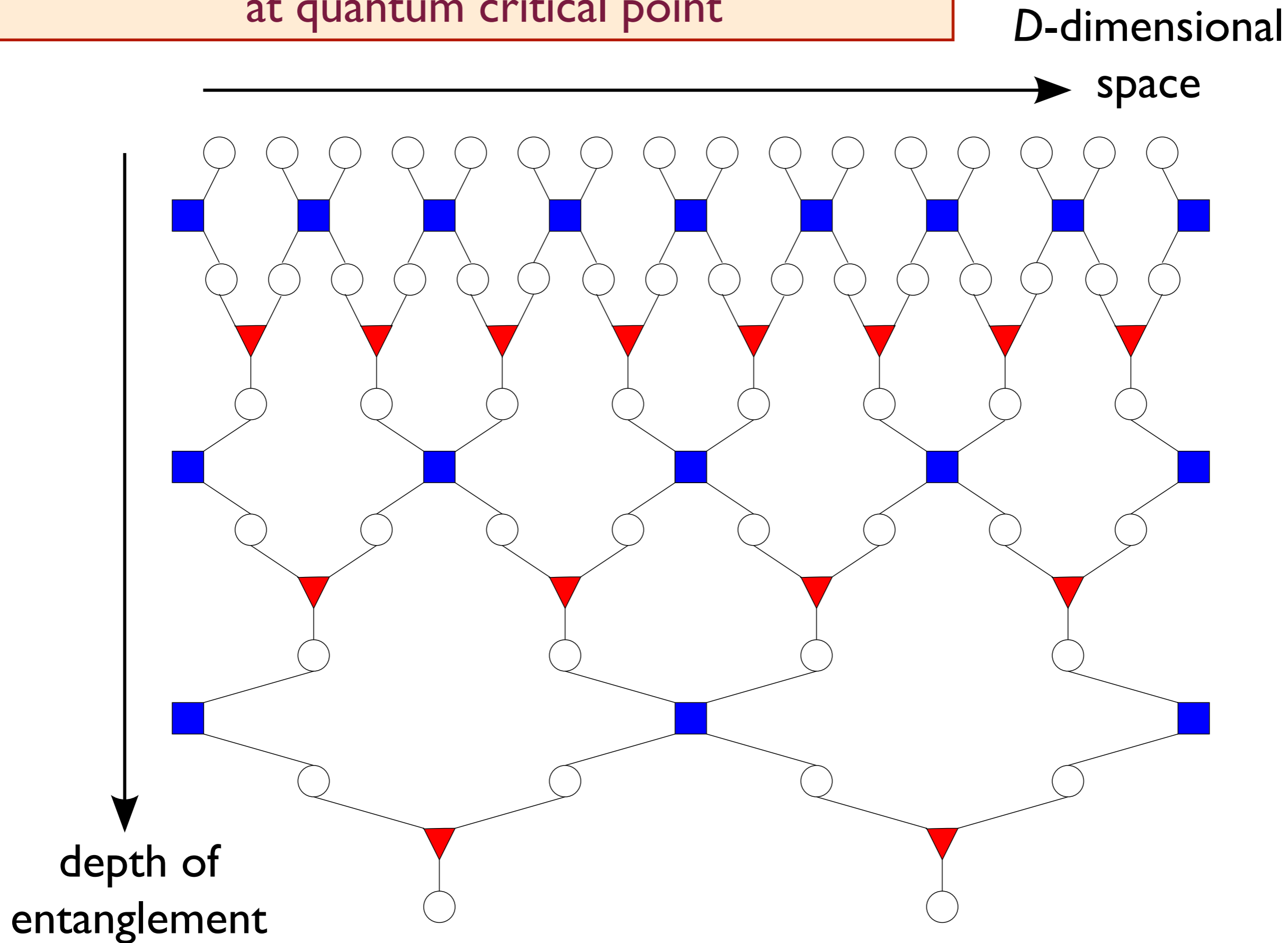


- A D -brane is a D -dimensional surface on which strings can end.



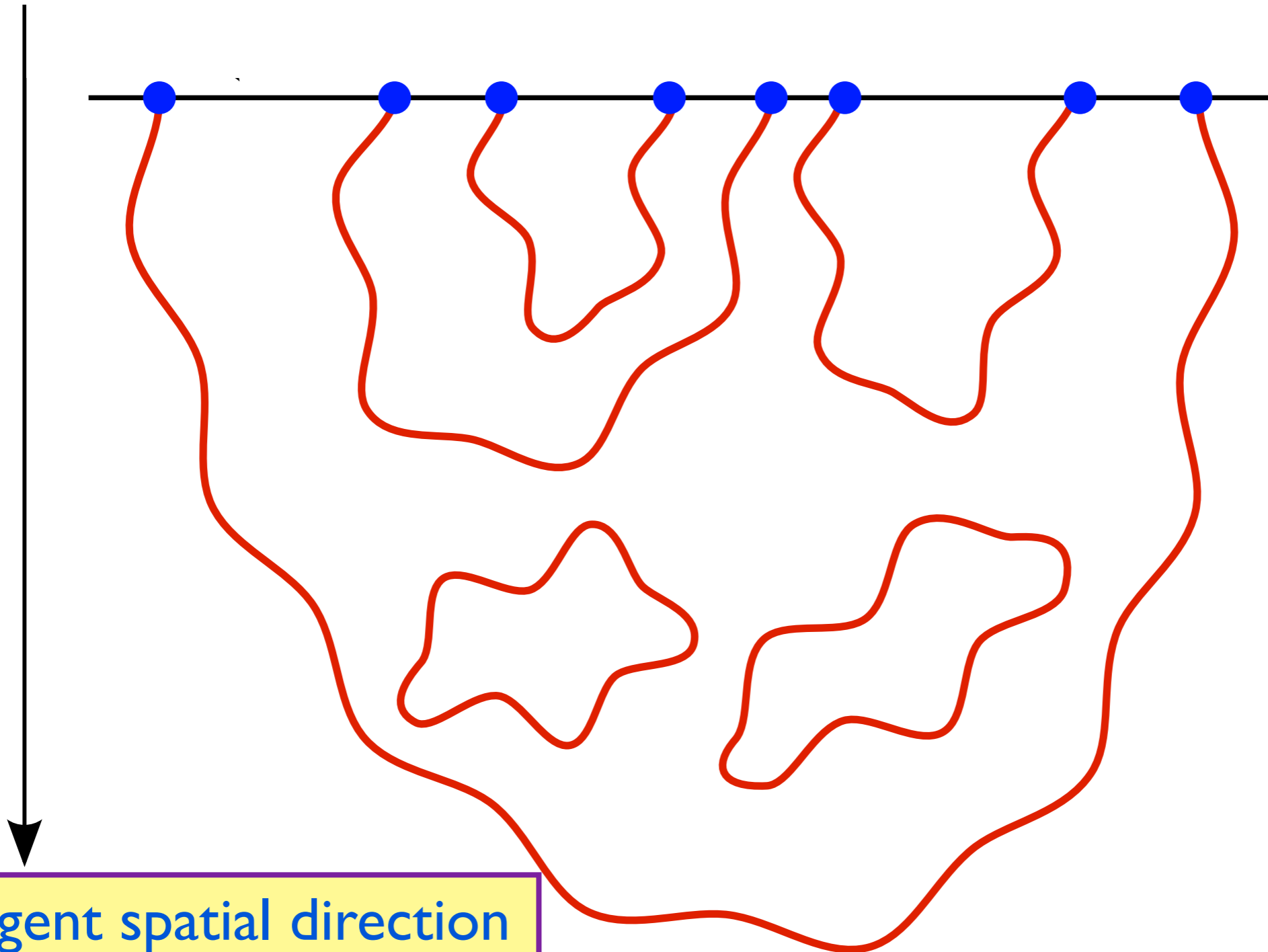
- A D -brane is a D -dimensional surface on which strings can end.
- If we focused only on the blue points on the D -dimensional surface, they would appear to us to have long-range quantum entanglement !

Tensor network representation of entanglement at quantum critical point



String theory near
a D-brane

D -dimensional
space

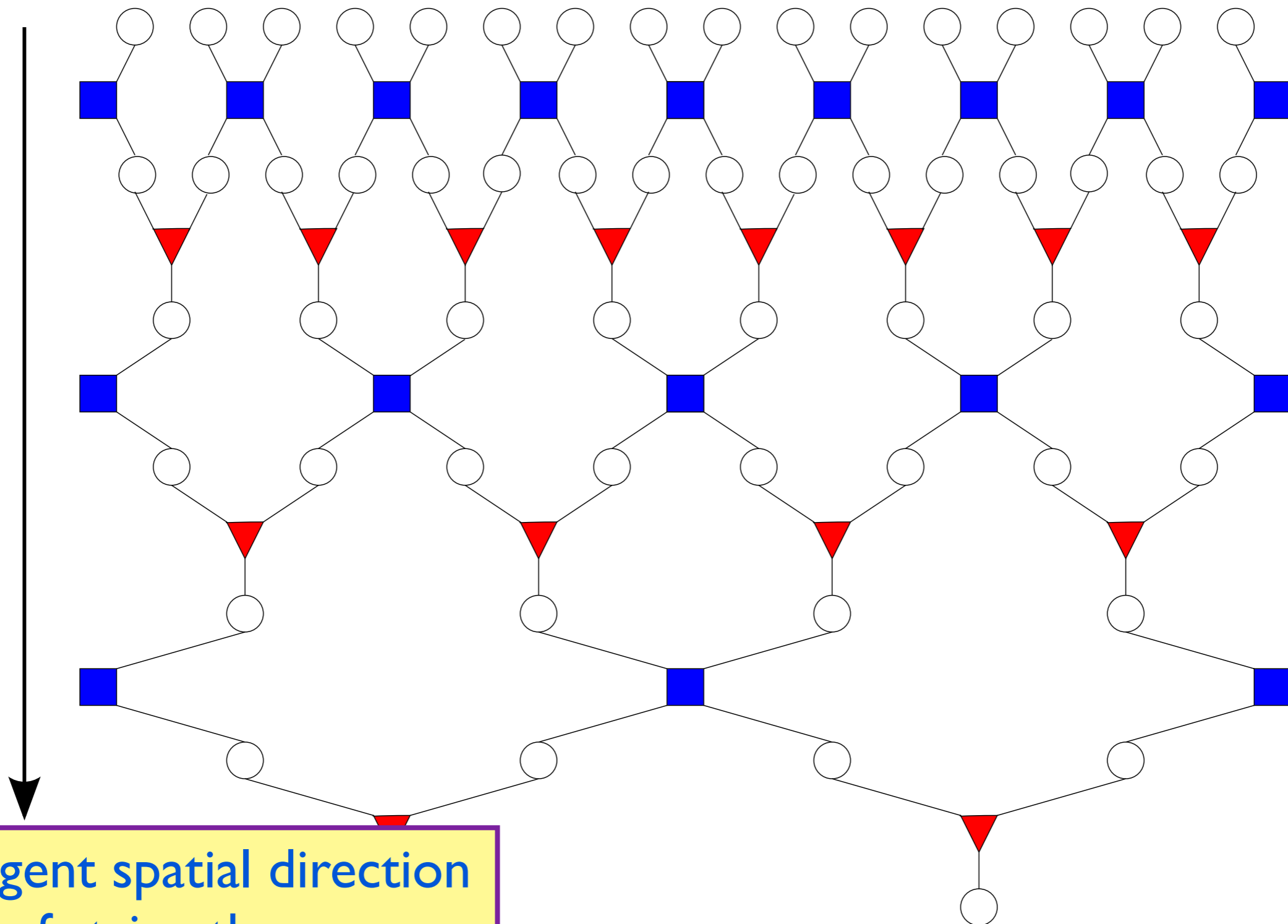


Emergent spatial direction
of string theory

Tensor network representation of entanglement at quantum critical point

D -dimensional

space



Emergent spatial direction
of string theory

States of matter with
long-range quantum entanglement
in D dimensions



String theory and
Einstein's *General Relativity*
in $D+1$ dimensions

States of matter with
long-range quantum entanglement
in D dimensions

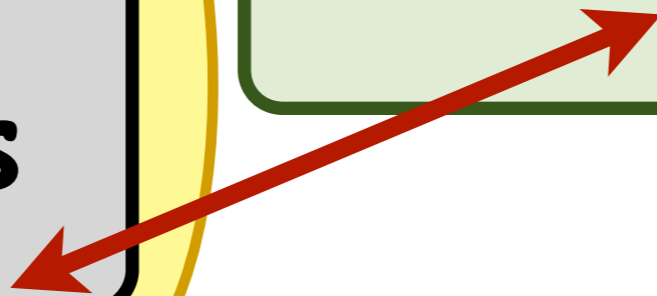
Are there solutions of Einstein's General
Relativity in $D+1$ dimensions which correspond
to superconductors and "strange metals" ?

String theory and
Einstein's General Relativity
in $D+1$ dimensions

**Quantum
superposition and
entanglement**

**Quantum critical
points
and long-range
entanglement of
electrons
in crystals**

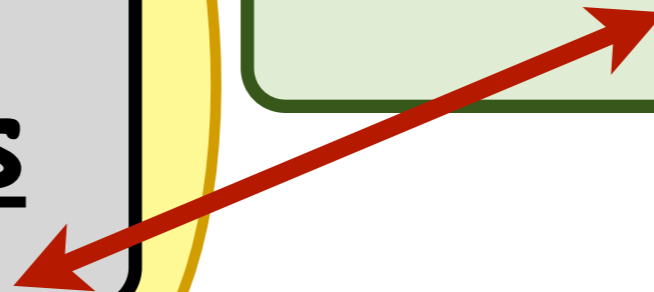
**String theory
and black holes**



Quantum
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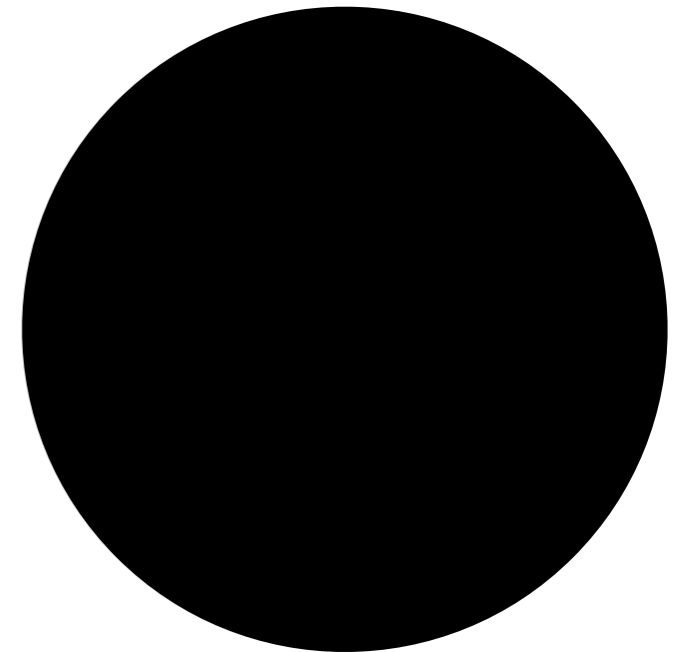
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Black Holes

Objects so massive that light is gravitationally bound to them.

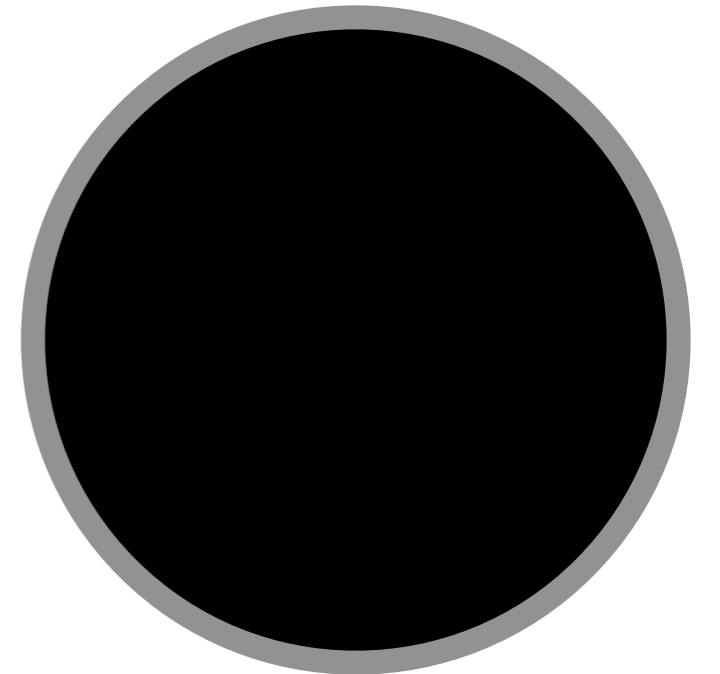


Black Holes

Objects so massive that light is gravitationally bound to them.

In Einstein's theory, the region inside the black hole **horizon** is disconnected from the rest of the universe.

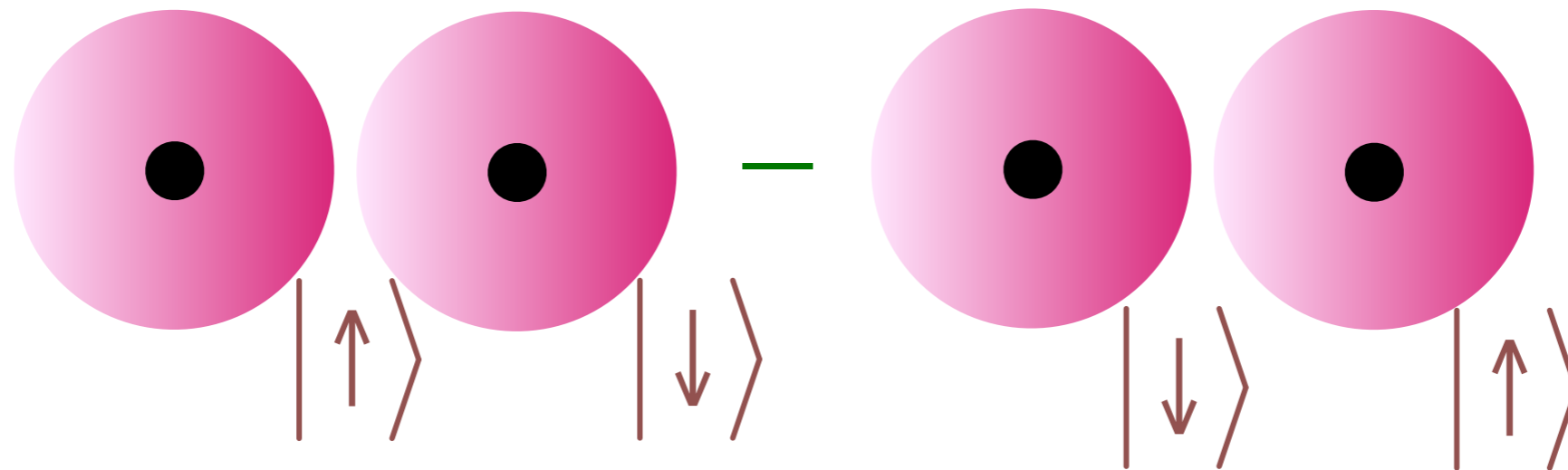
$$\text{Horizon radius } R = \frac{2GM}{c^2}$$



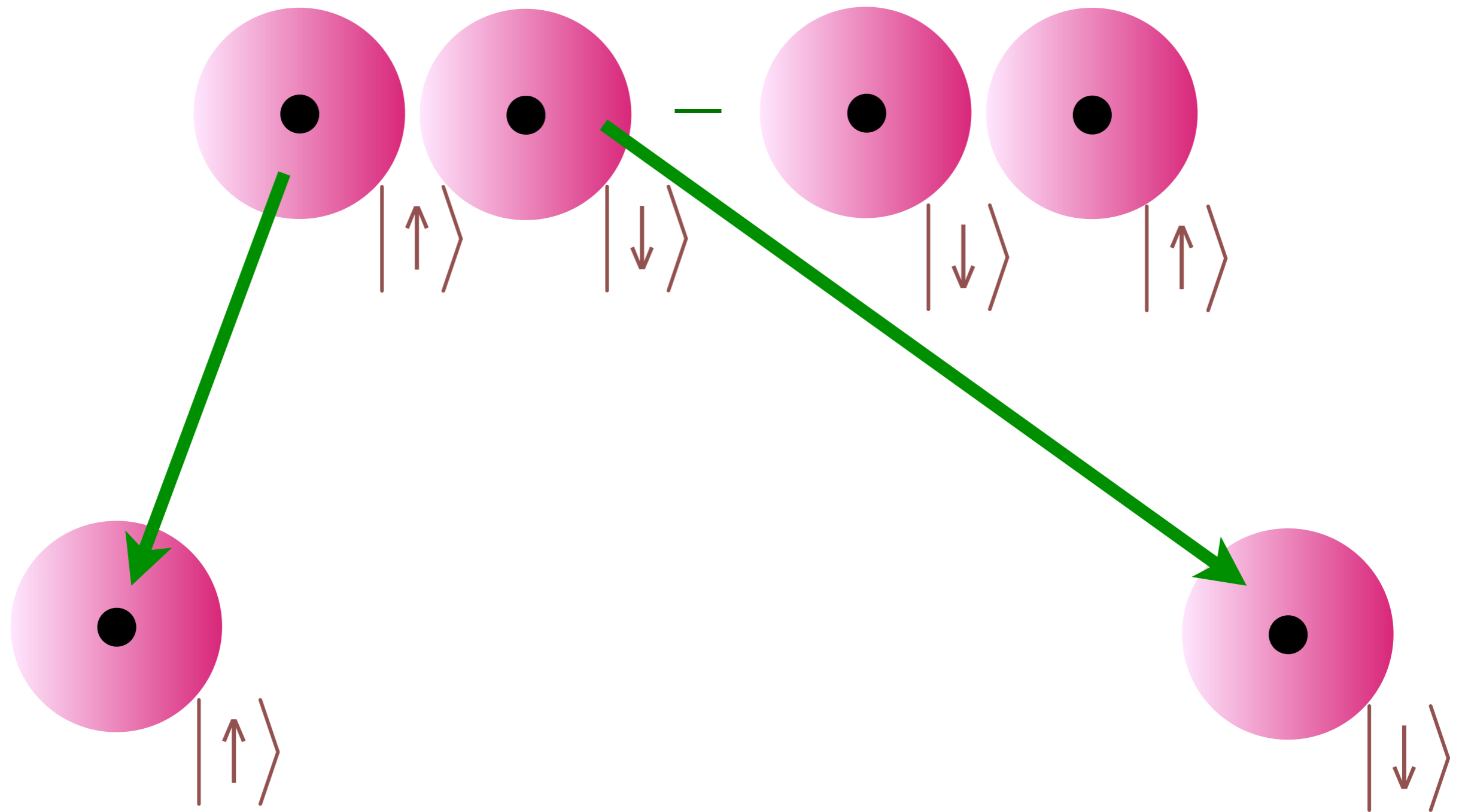
Black Holes + Quantum theory

Around 1974, Bekenstein and Hawking showed that the application of the quantum theory across a black hole horizon led to many astonishing conclusions

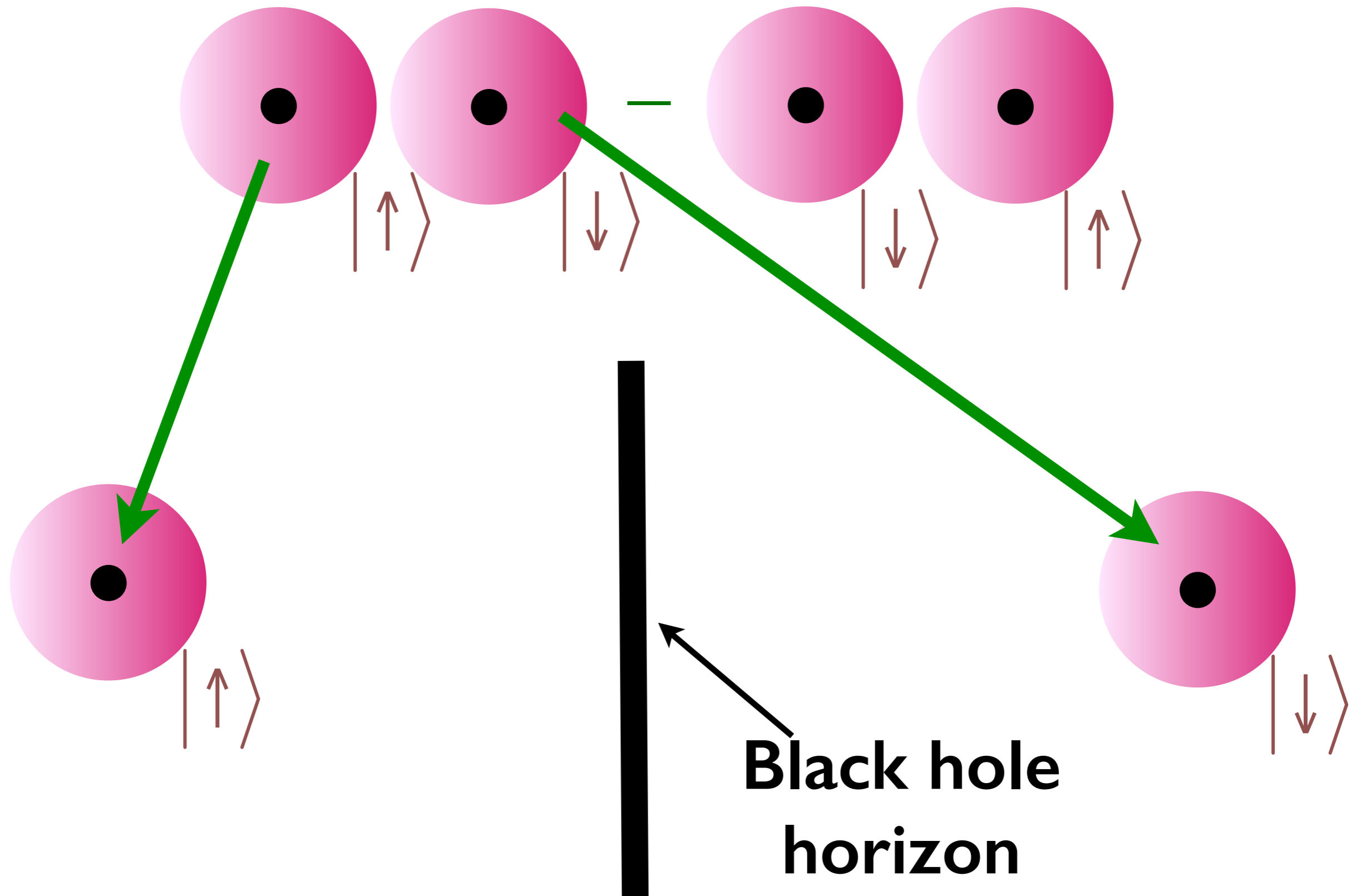
Quantum Entanglement across a black hole horizon



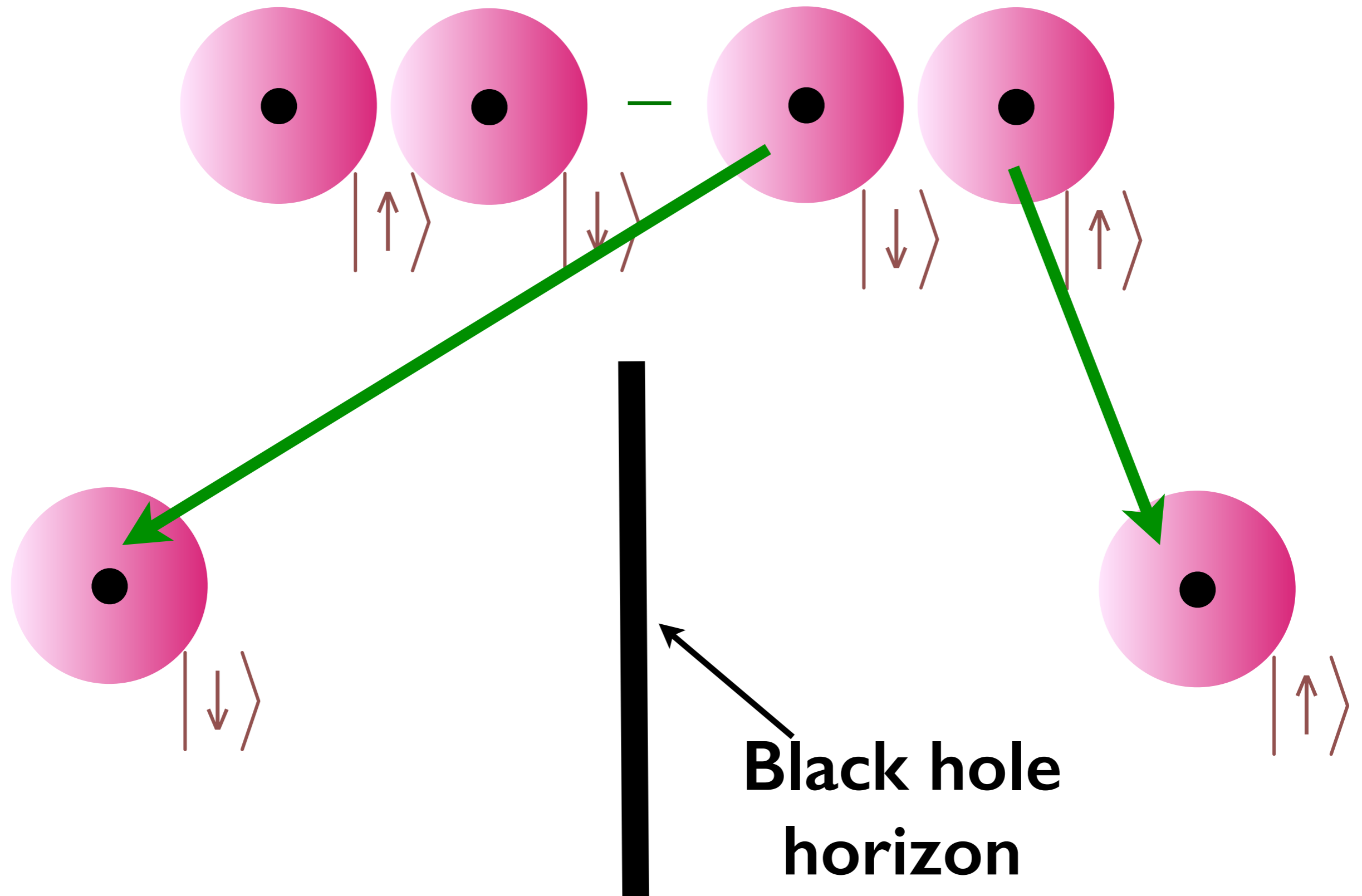
Quantum Entanglement across a black hole horizon



Quantum Entanglement across a black hole horizon

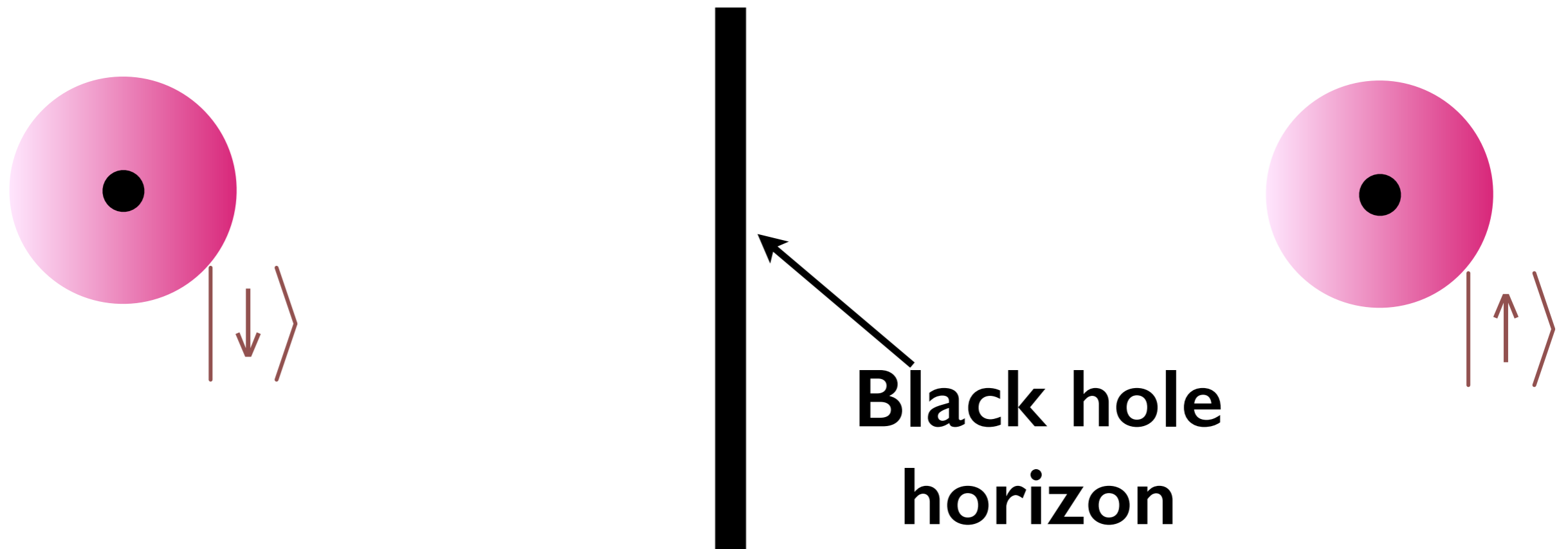


Quantum Entanglement across a black hole horizon



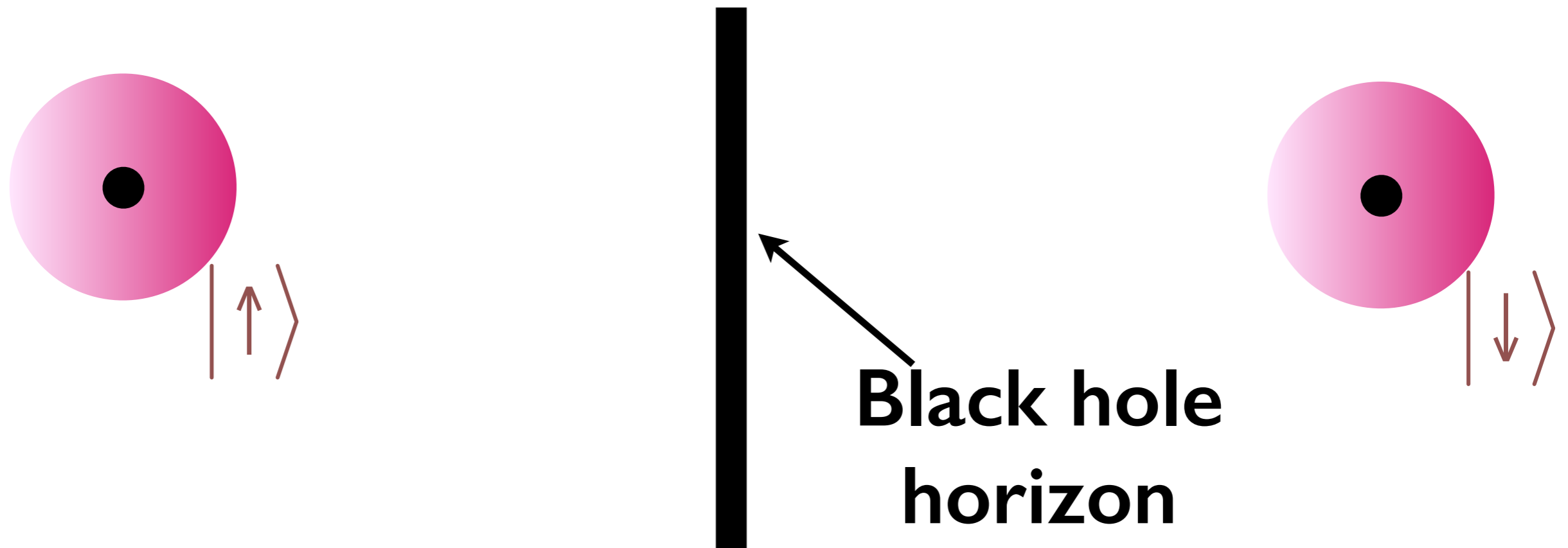
Quantum Entanglement across a black hole horizon

There is long-range quantum entanglement between the inside and outside of a black hole



Quantum Entanglement across a black hole horizon

There is long-range quantum entanglement between the inside and outside of a black hole



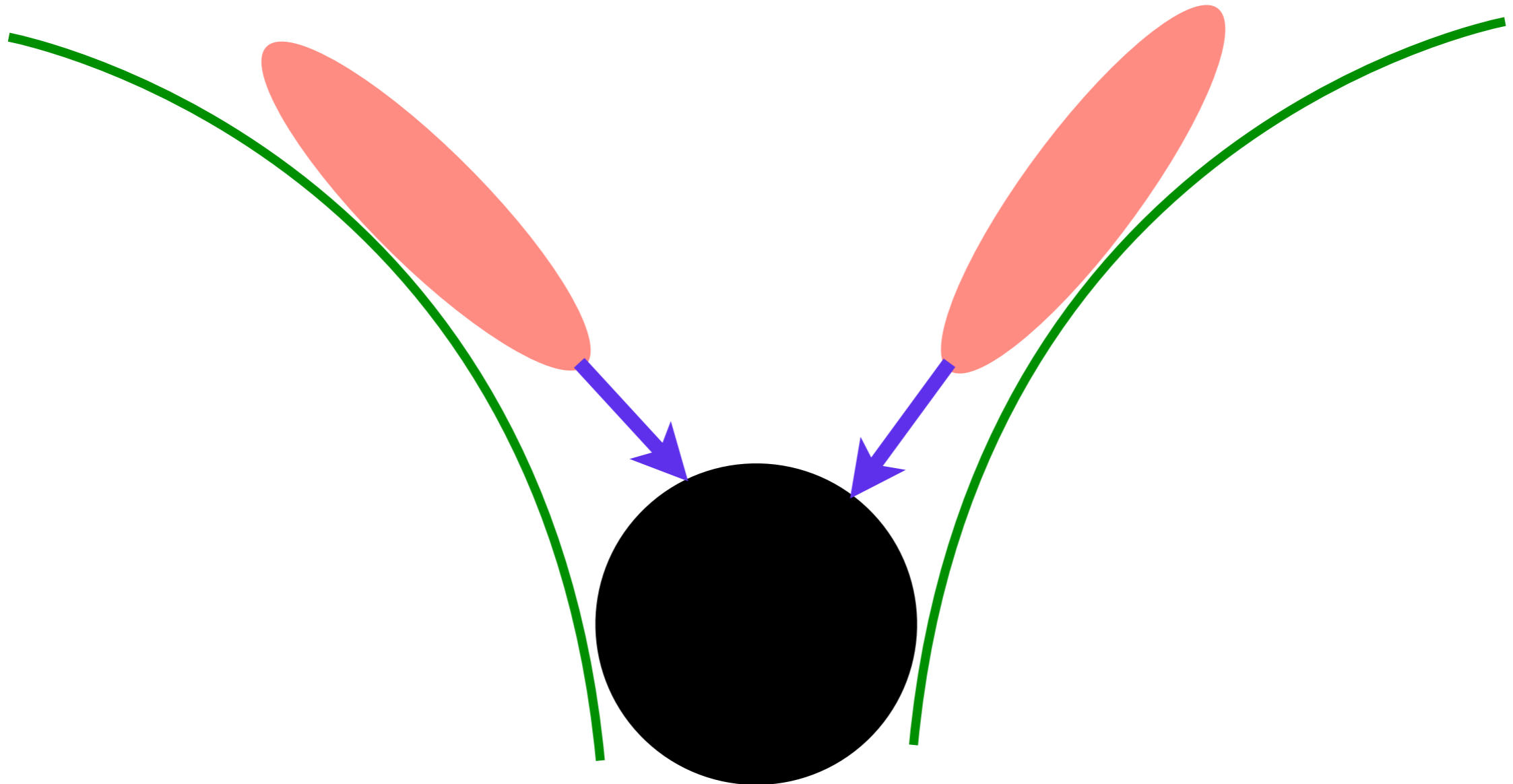
Quantum Entanglement across a black hole horizon

There is long-range quantum entanglement between the inside and outside of a black hole

This entanglement leads to a black hole temperature (the Hawking temperature) and a black hole entropy (the Bekenstein entropy)

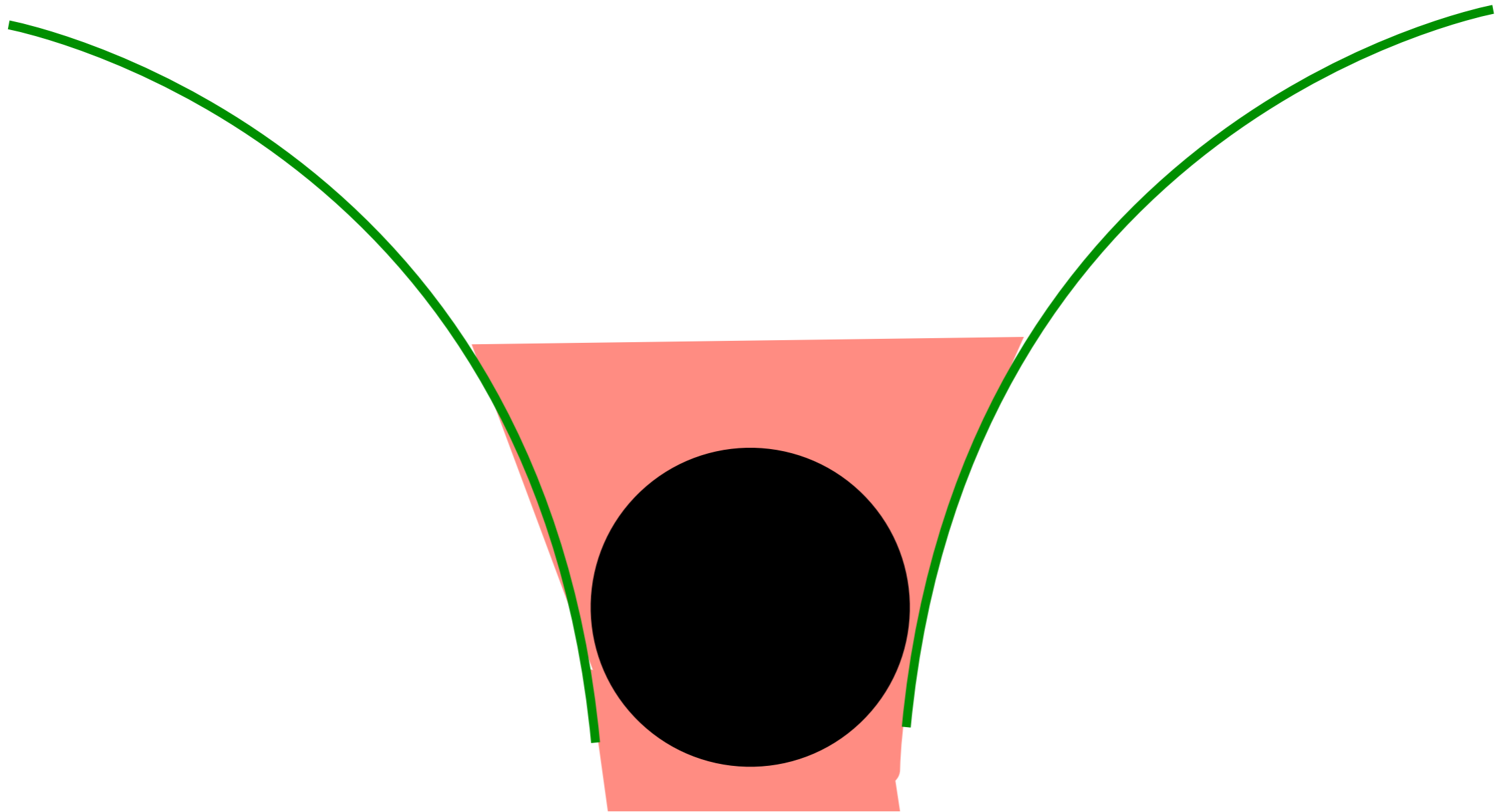
States of quantum matter and black holes

Add electrical charge to a black hole in a curved spacetime: initially the charges fall past the horizon into the black hole



States of quantum matter and black holes

However, eventually there is a balance between the gravitational forces pulling the charges into the black hole, and the repulsive electrical forces which push them out, and a stable state is obtained



States of quantum matter and black holes

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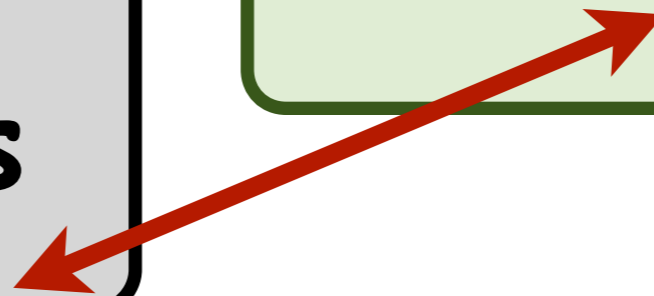
These black hole states of General Relativity in $D+1$ dimensions correspond to (and allow us to compute the properties of) superconductors and strange metals in D dimensions

**Quantum
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entanglement**

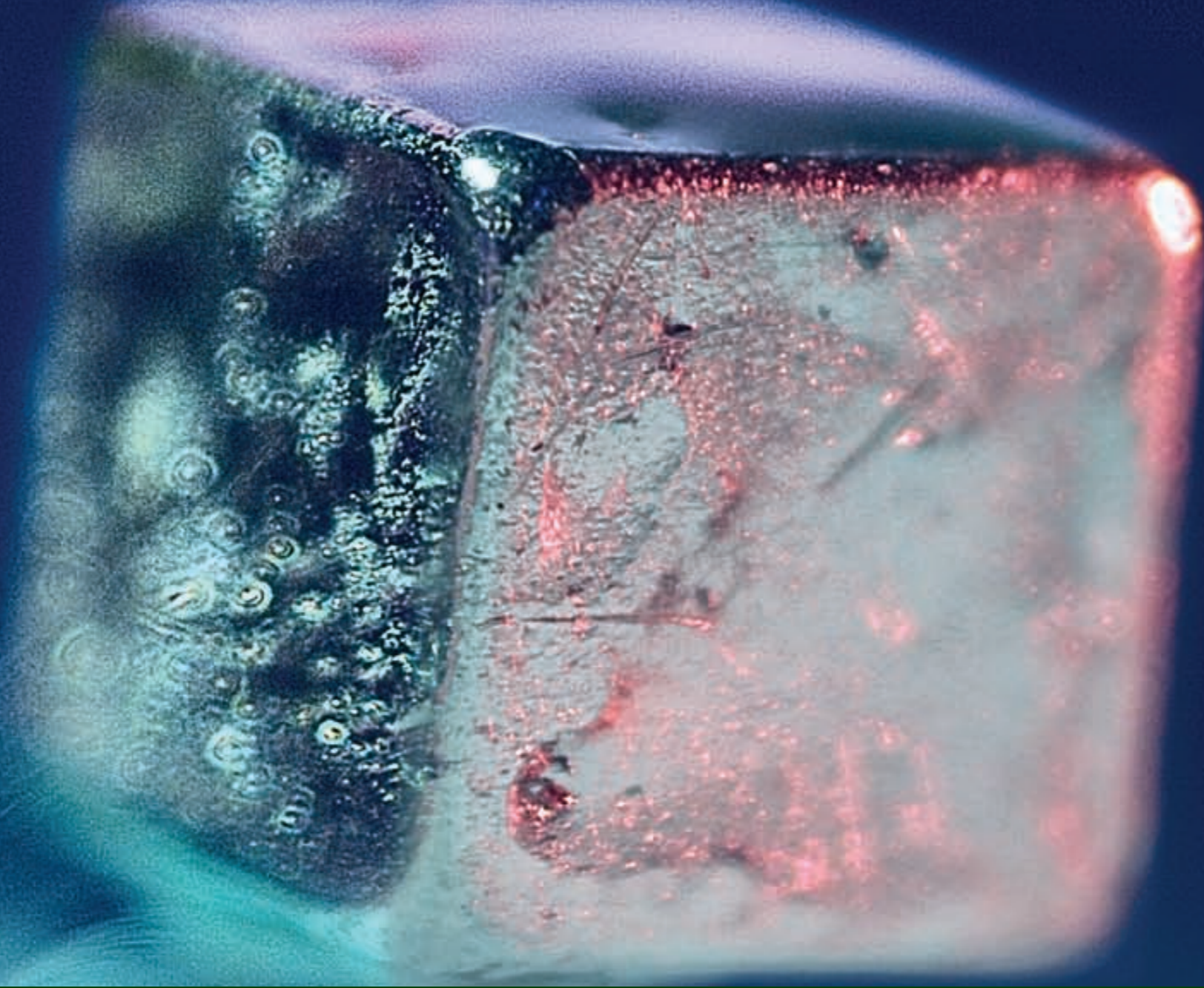
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Quantum Entanglement and Superconductivity



Superconductor, levitated by an unseen magnet, in which countless trillions of electrons form a vast interconnected quantum state.
Scientific American, January 2013

Subir Sachdev, Harvard University