Hydrodynamic transport near quantum critical points and the AdS/CFT correspondence

Particle theorists

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Dam Son, Washington

Condensed matter theorists

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Outline

1. Model systems

   (i) Superfluid-insulator transition of lattice bosons, (ii) graphene

2. Quantum-critical transport at integer filling, zero magnetic field, and with no impurities

   Collisionless-to-hydrodynamic crossover of CFT3s

3. Quantum-critical transport at integer generic filling, nonzero magnetic field, and with impurities

   Nernst effect and a hydrodynamic cyclotron resonance

4. The AdS/CFT correspondence

   Quantum criticality and dyonic black holes

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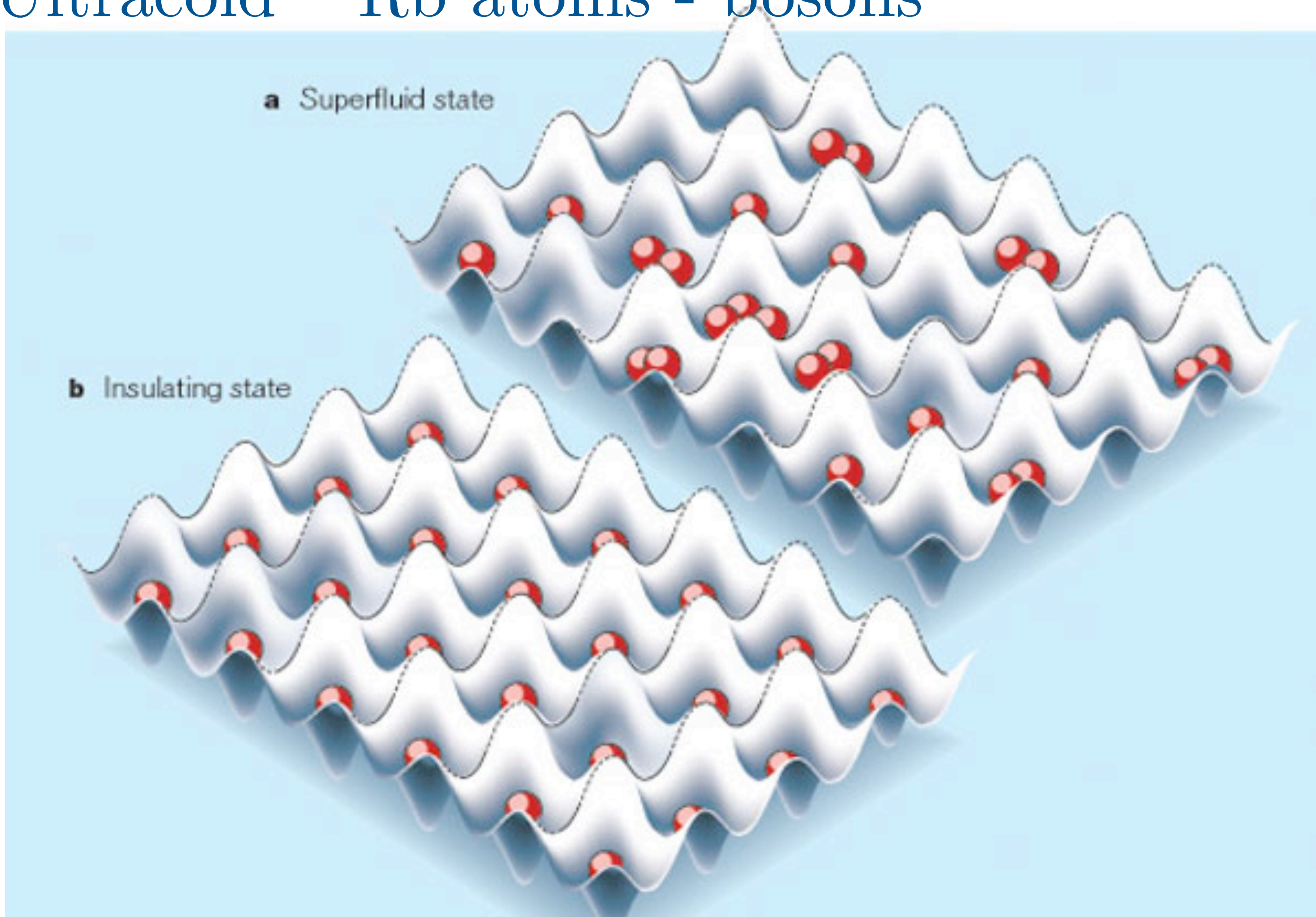
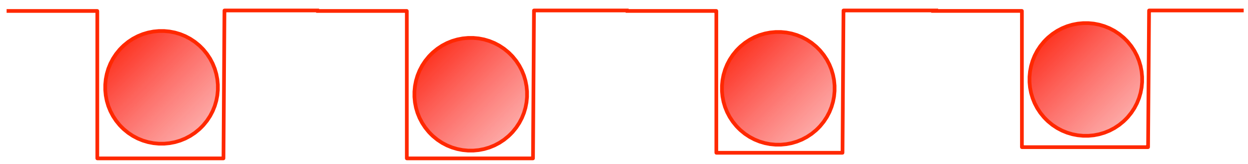
Ultracold ^{87}Rb atoms - bosons

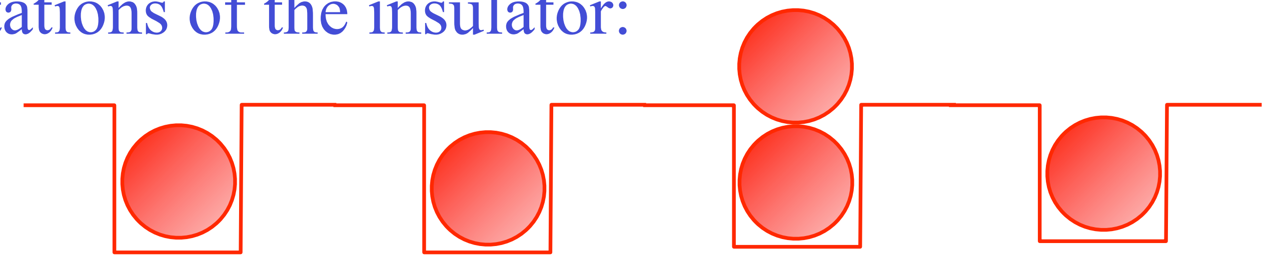


M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The insulator:



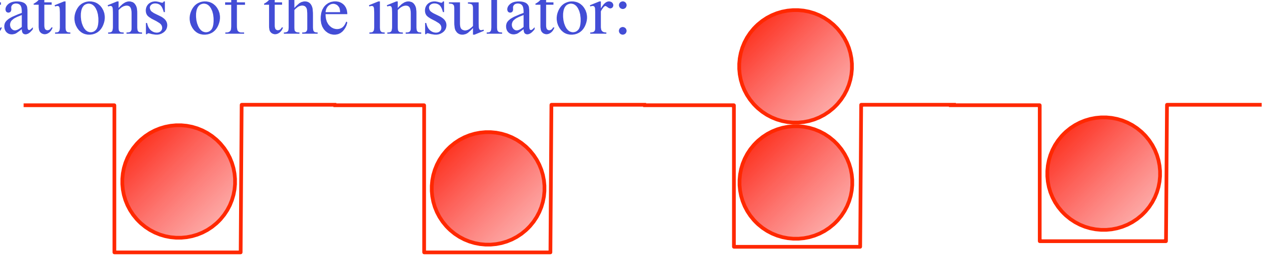
Excitations of the insulator:



Particles $\sim \psi^{\dagger}$

Holes $\sim \psi$

Excitations of the insulator:



Particles ~ $\psi^\dagger$

Holes ~ ψ

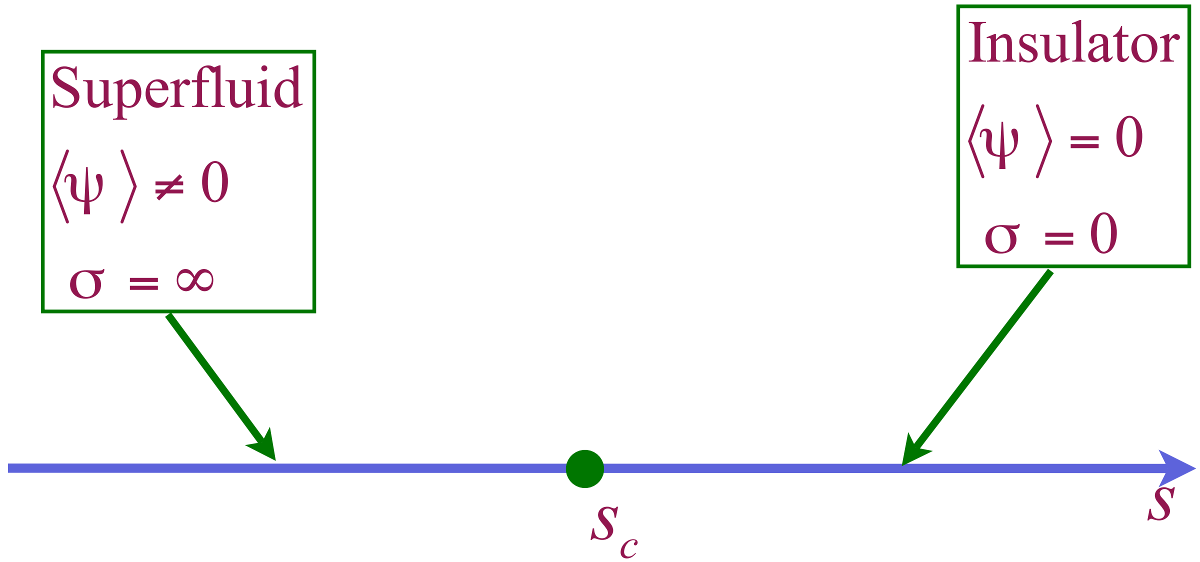
Density of particles = density of holes $\Rightarrow$

. "relativistic" field theory for ψ:

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla}\psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \quad \Leftrightarrow \quad \langle \psi \rangle = 0$$

$$\text{Superfluid} \quad \Leftrightarrow \quad \langle \psi \rangle \neq 0$$



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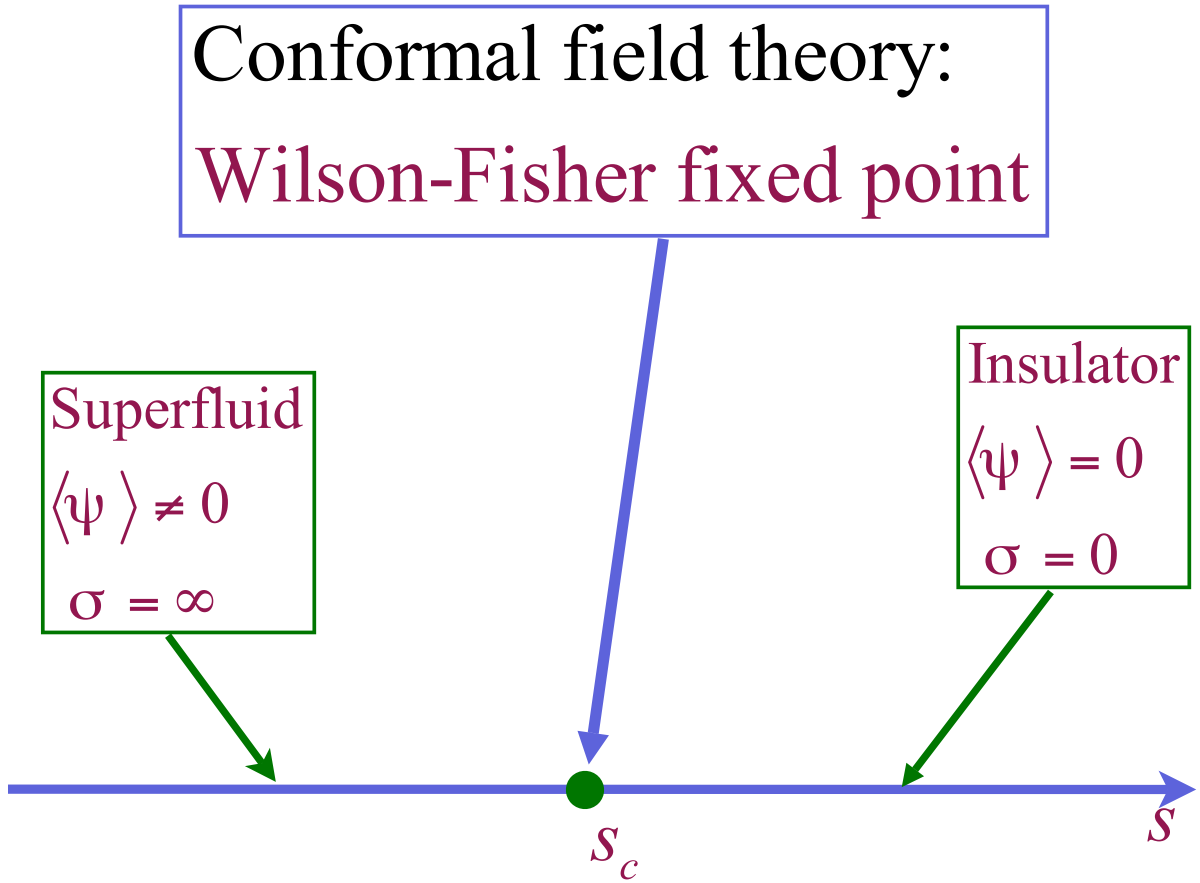
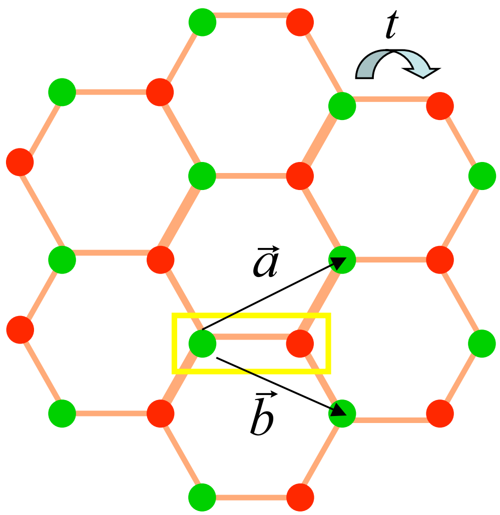
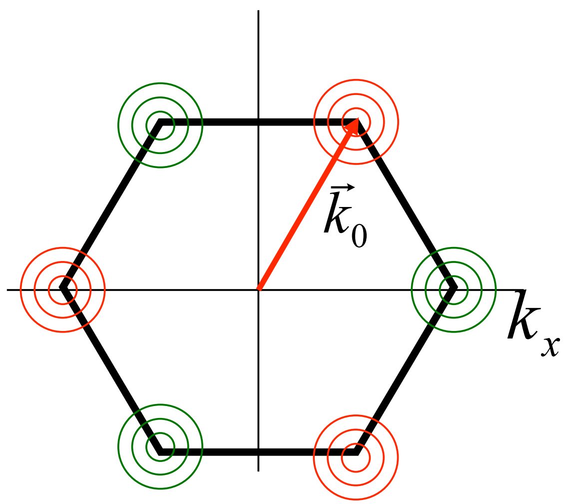
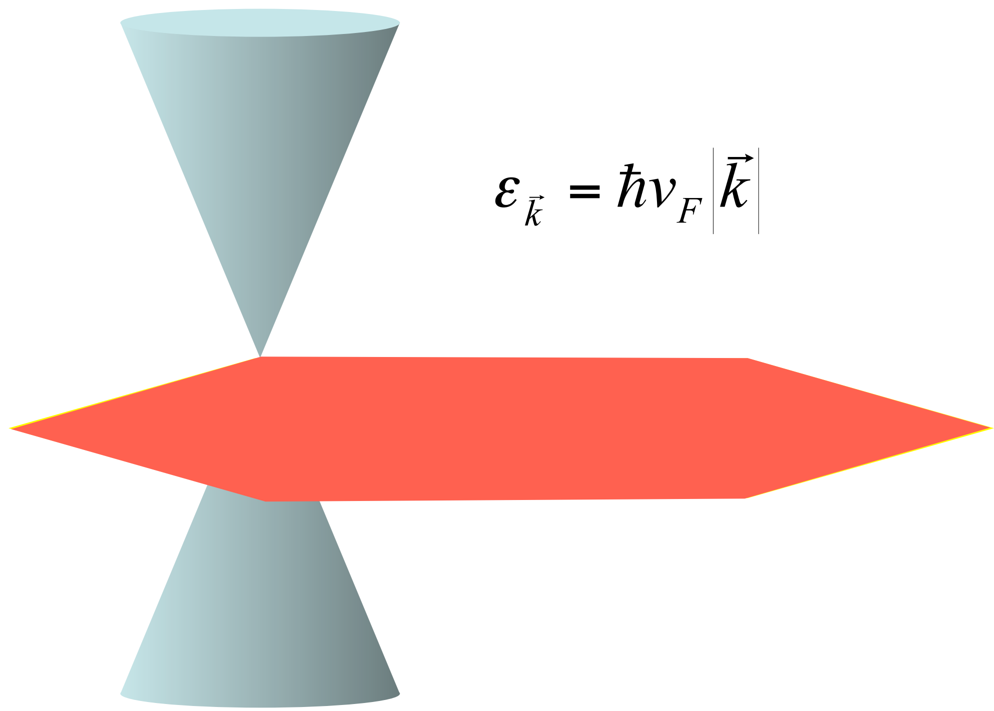


$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

Graphene







$$\varepsilon_{\vec{k}} = \hbar v_F \left|\vec{k}\right|$$

Graphene

Low energy theory has 4 two-component Dirac fermions, ψ_α, $\alpha = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$\mathcal{S} = \int d^2r d\tau \psi_\alpha^\dagger \left(\partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\alpha + \frac{e^2}{2} \int d^2r d^2r' d\tau \psi_\alpha^\dagger \psi_\alpha(r) \frac{1}{|r - r'|} \psi_\beta^\dagger \psi_\beta(r')$$

Dimensionless "fine-structure" constant $\lambda = e^2/(4\hbar v_F)$.
RG flow of α:

$$\frac{d\lambda}{d\ell} = -\lambda^2 + \ldots$$

Behavior is similar to a CFT3 with $\lambda \sim 1/\ln(\text{scale})$

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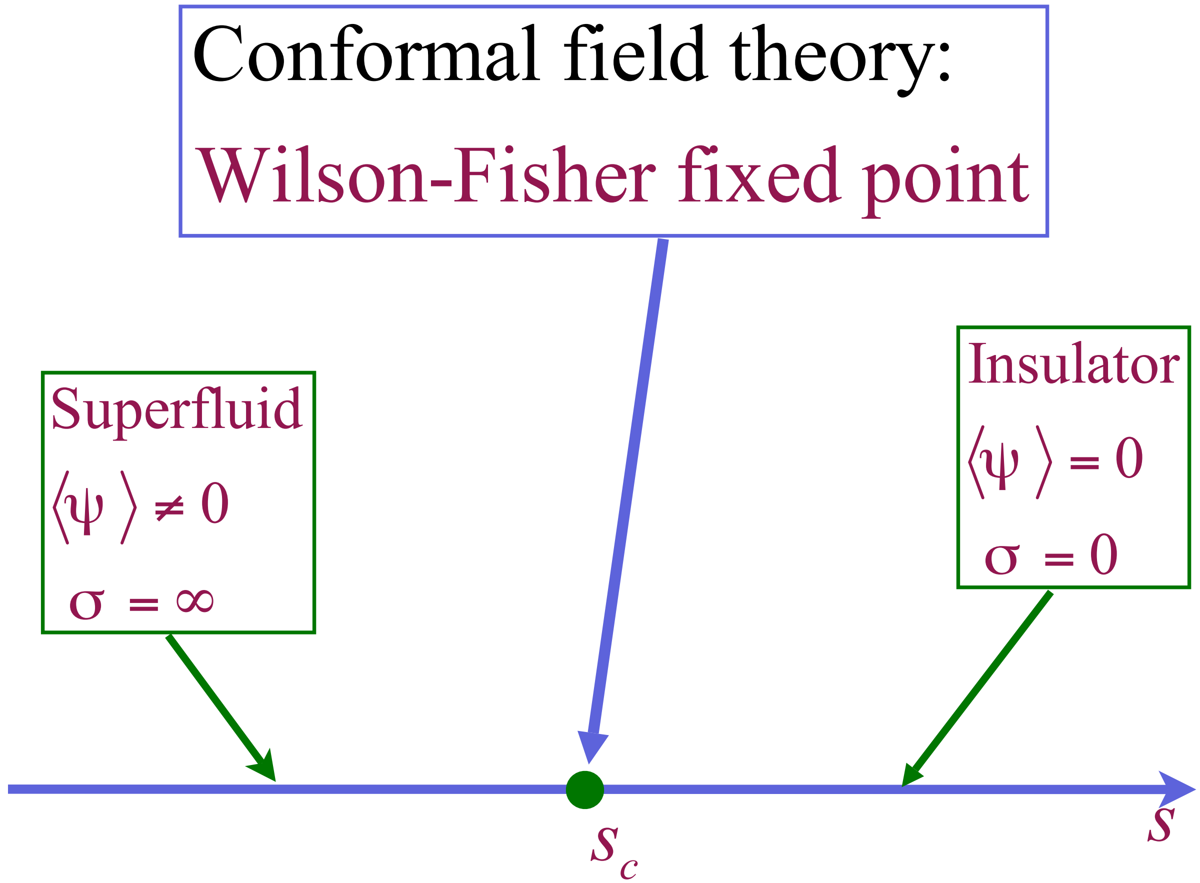
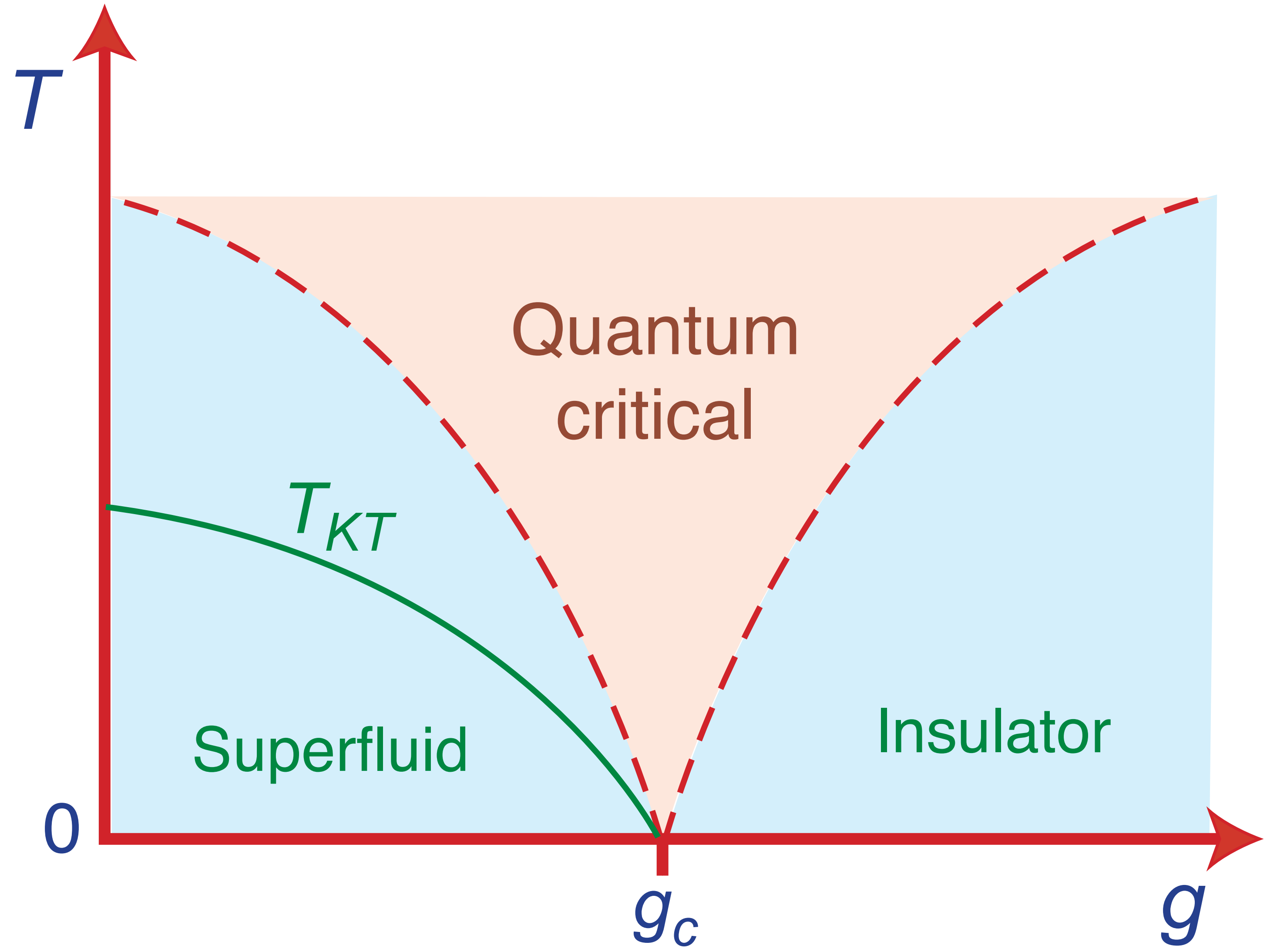
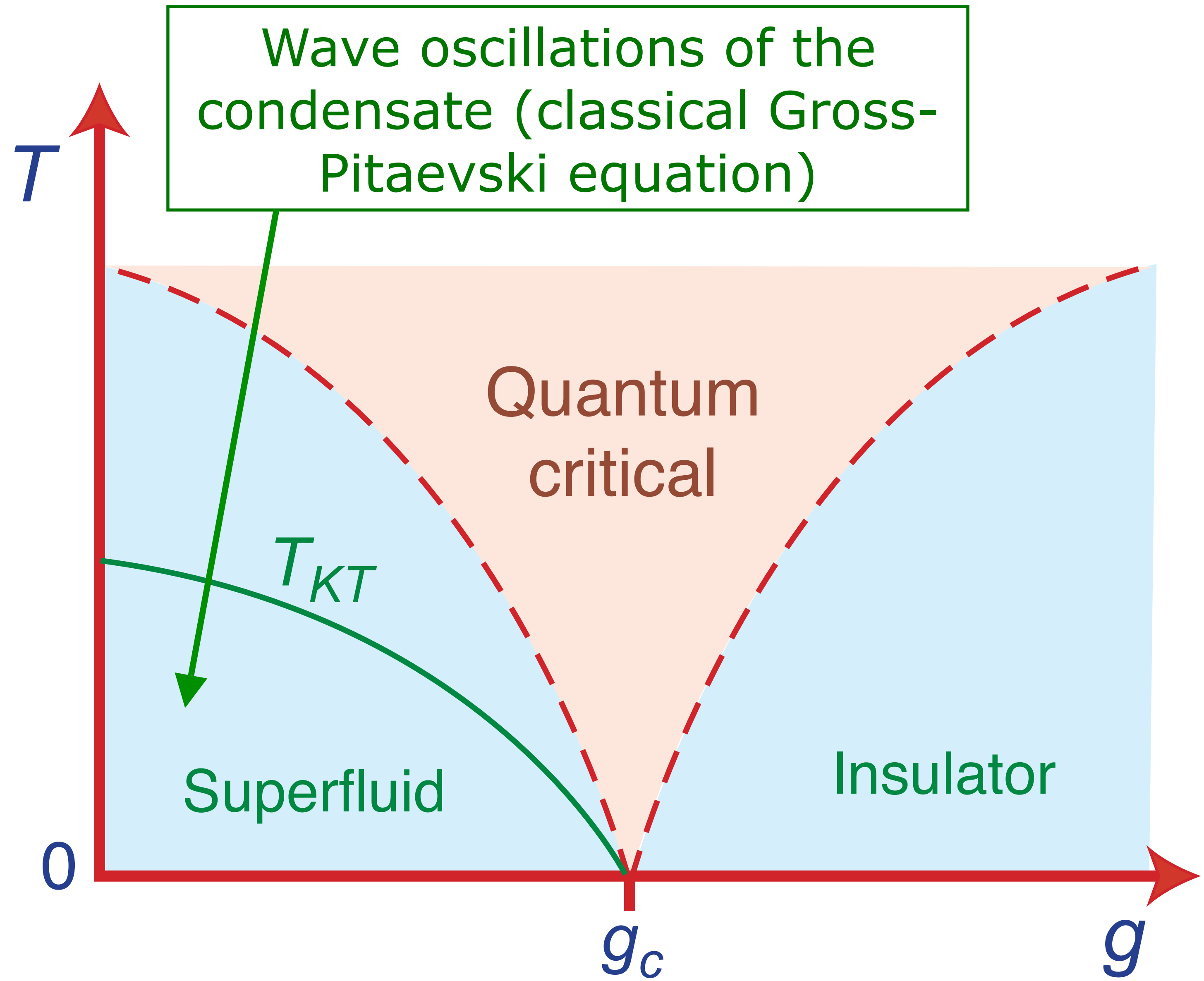
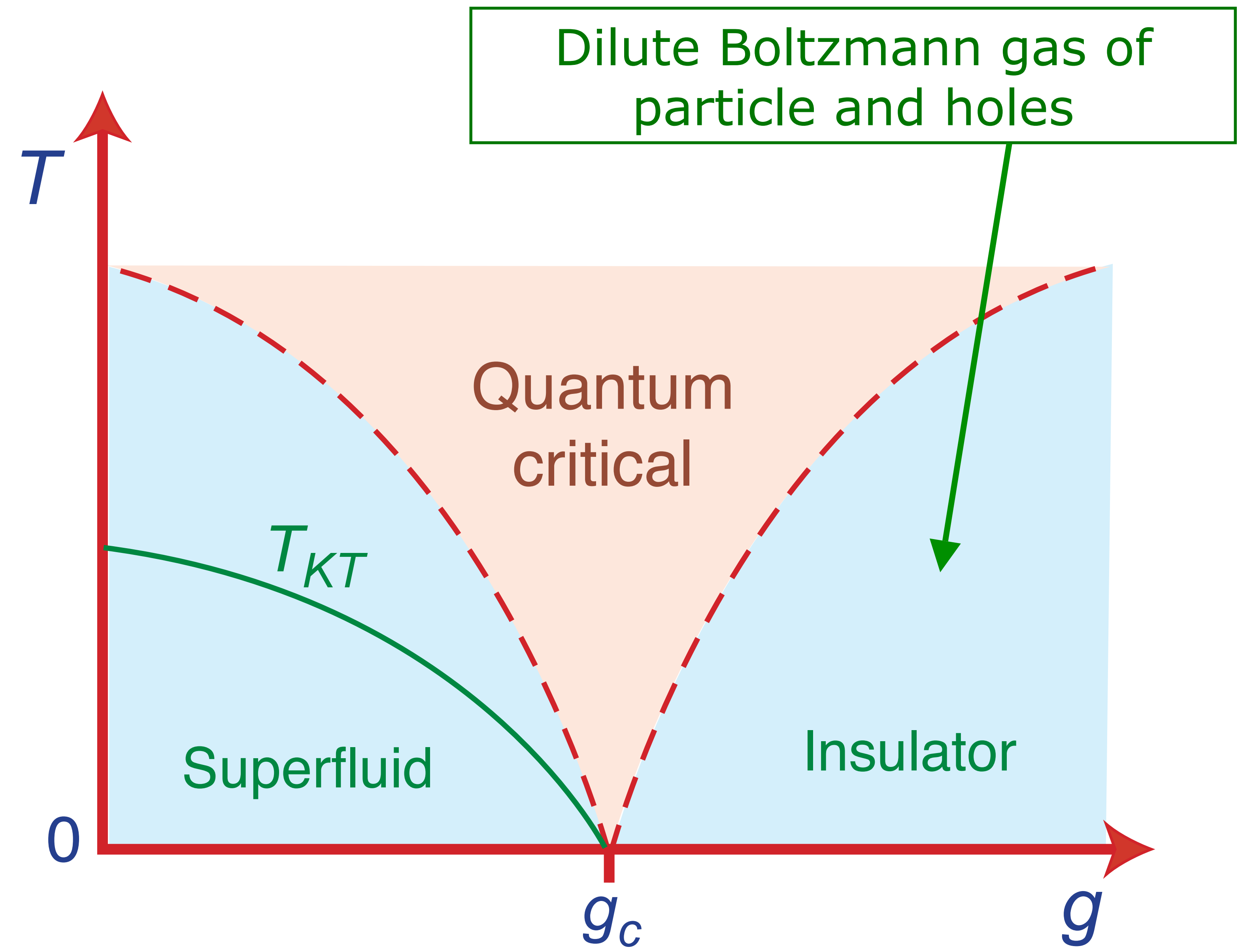


$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla} \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

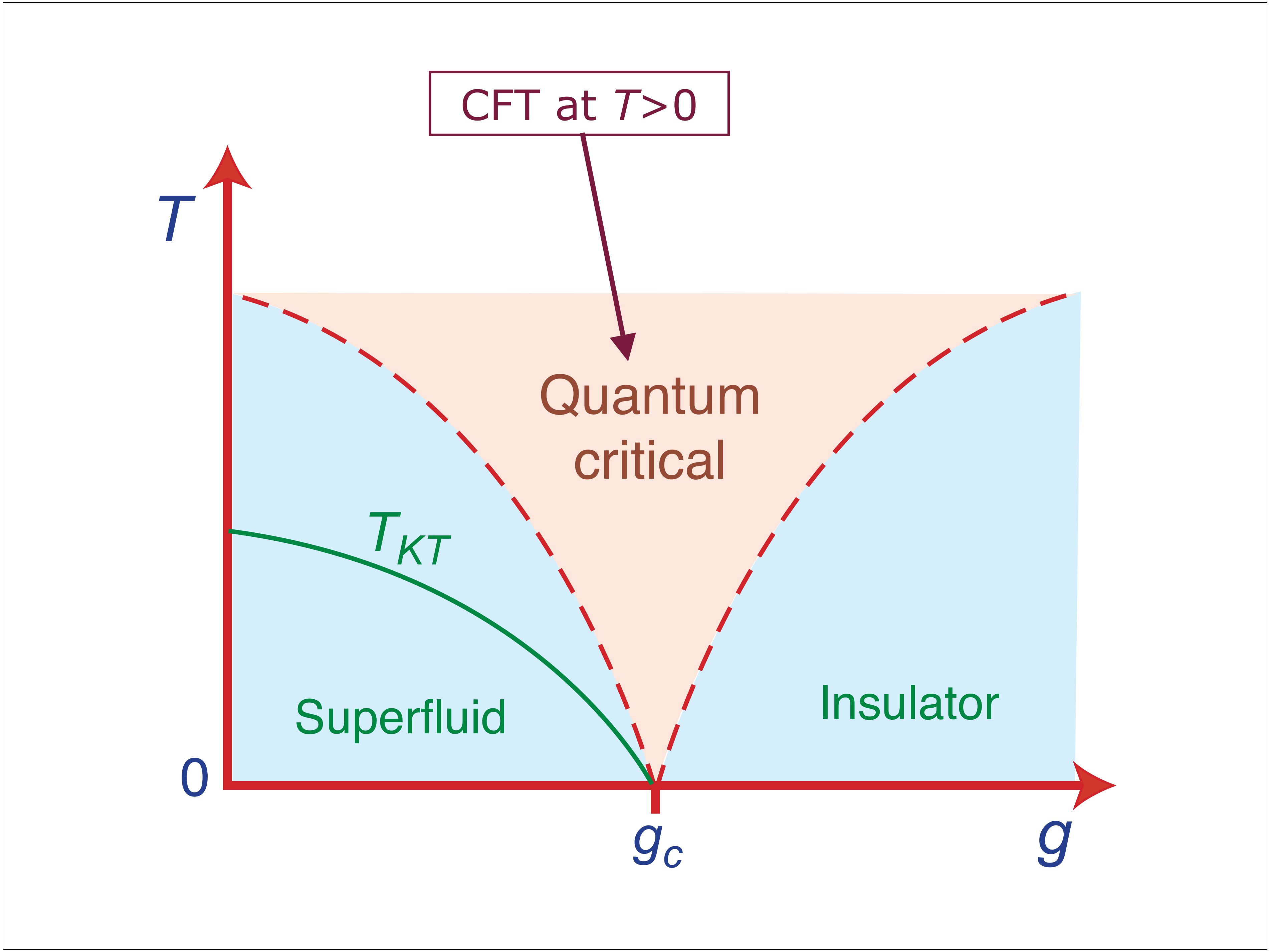

T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g



Wave oscillations of the condensate (classical Gross-Pitaevski equation)
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g



Dilute Boltzmann gas of particle and holes
T
Quantum critical
T_KT
Superfluid
Insulator
0
g_c
g


CFT at T>0
T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g

Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}}(T \to 0) = \infty$$
$$\sigma_{\text{Insulator}}(T \to 0) = 0$$
$$\sigma_{\text{Quantum critical point}}(T \to 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

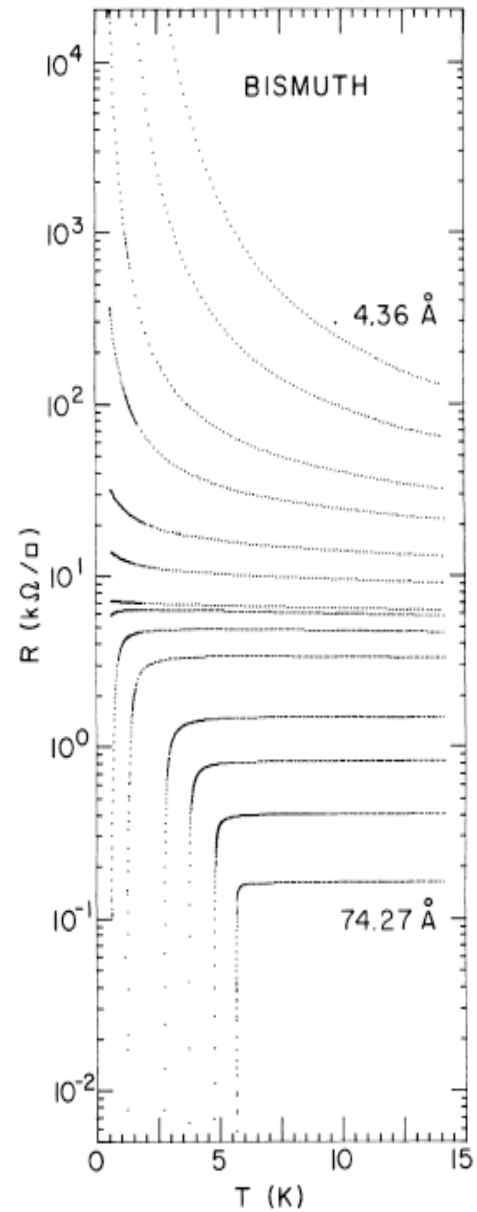


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k,\omega)$

For *all* CFT2s, at all $\hbar\omega / k_B T$

$$\chi(k,\omega) \quad = \quad \frac{4e^2}{h} K \frac{vk^2}{v^2k^2 - \omega^2} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} \frac{Kv}{-i\omega}$$

where K is a universal number characterizing the CFT2 (the level number), and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k, \omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} \chi(k, \omega)$

For *all* CFT3s, at $\hbar\omega \gg k_B T$

$$\chi(k, \omega) \quad = \quad \frac{4e^2}{h} K \frac{k^2}{\sqrt{v^2 k^2 - \omega^2}} \quad ; \quad \sigma(\omega) = \frac{4e^2}{h} K$$

where K is a universal number characterizing the CFT3, and v is the velocity of "light".

Density correlations in CFTs at $T > 0$

Two-point density correlator, $\chi(k,\omega)$

Kubo formula for conductivity $\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2}\chi(k,\omega)$

However, for *all* CFT3s, at $\underline{\hbar\omega \ll k_B T}$, we have the Einstein relation

$$\chi(k,\omega) \quad = \quad 4e^2\chi_c \frac{Dk^2}{Dk^2 - i\omega} \quad ; \quad \sigma(\omega) = 4e^2 D\chi_c = \frac{4e^2}{h}\Theta_1\Theta_2$$

where the **compressibility**, χ_c, and the **diffusion constant** D obey

$$\chi = \frac{k_B T}{(hv)^2}\Theta_1 \quad ; \quad D = \frac{hv^2}{k_B T}\Theta_2$$

with Θ_1 and Θ_2 universal numbers characteristic of the CFT3

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Density correlations in CFTs at $T>0$

In CFT3s collisions are "phase" randomizing, and lead to relaxation to local thermodynamic equilibrium. So there is a crossover from collisionless behavior for $\hbar\omega \gg k_B T$, to hydrodynamic behavior for $\hbar\omega \ll k_B T$.

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\[2ex] \dfrac{4e^2}{h} \Theta_1 \Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

and in general we expect $K \neq \Theta_1 \Theta_2$ (verified for Wilson-Fisher fixed point).

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Collisionless-hydrodynamic crossover in graphene

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T / (\ln(\Lambda/T))^2 \\[2ex] \dfrac{4e^2}{h} (\ln(\Lambda/T))^2 \Theta_1 \Theta_2 & , \quad \hbar\omega \ll k_B T / (\ln(\Lambda/T))^2 \end{cases}$$

L. Fritz, M. Mueller, J. Schmalian and S. Sachdev, *to appear.*

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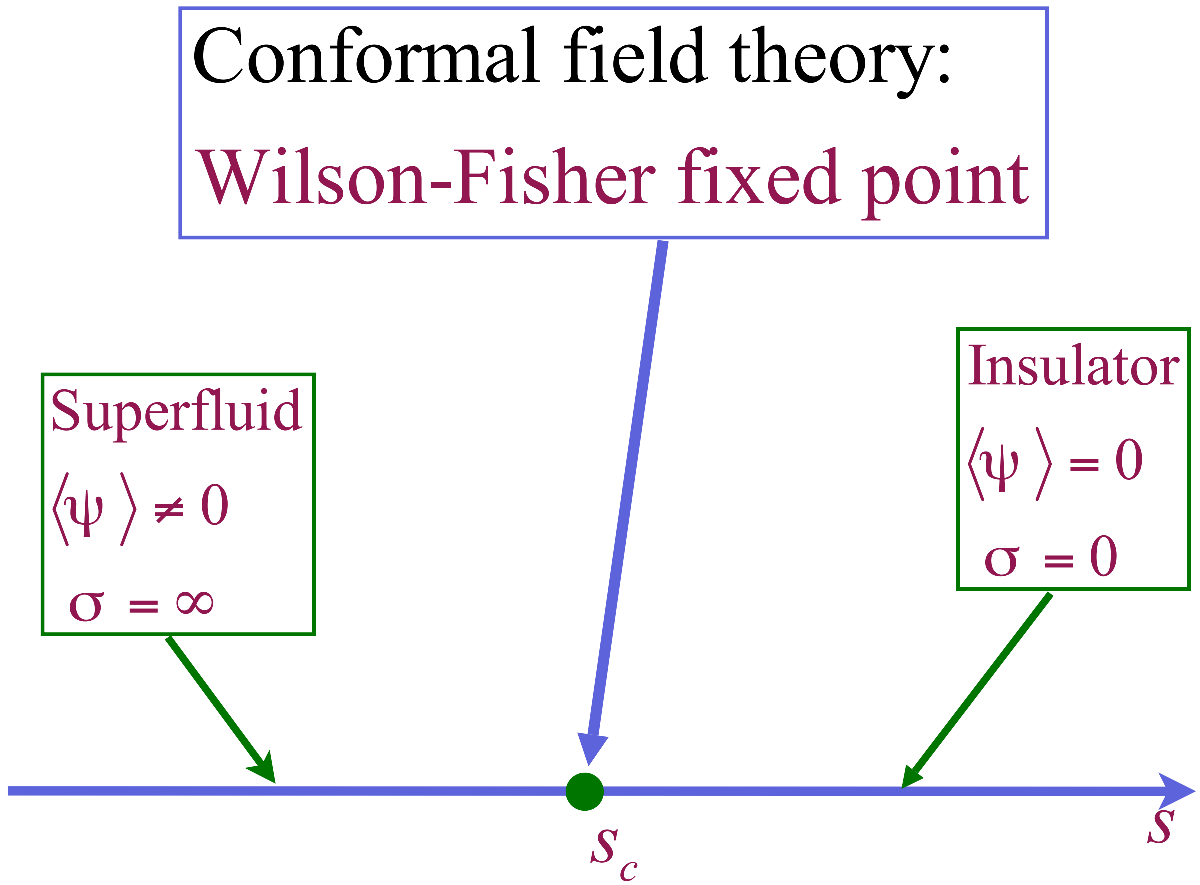
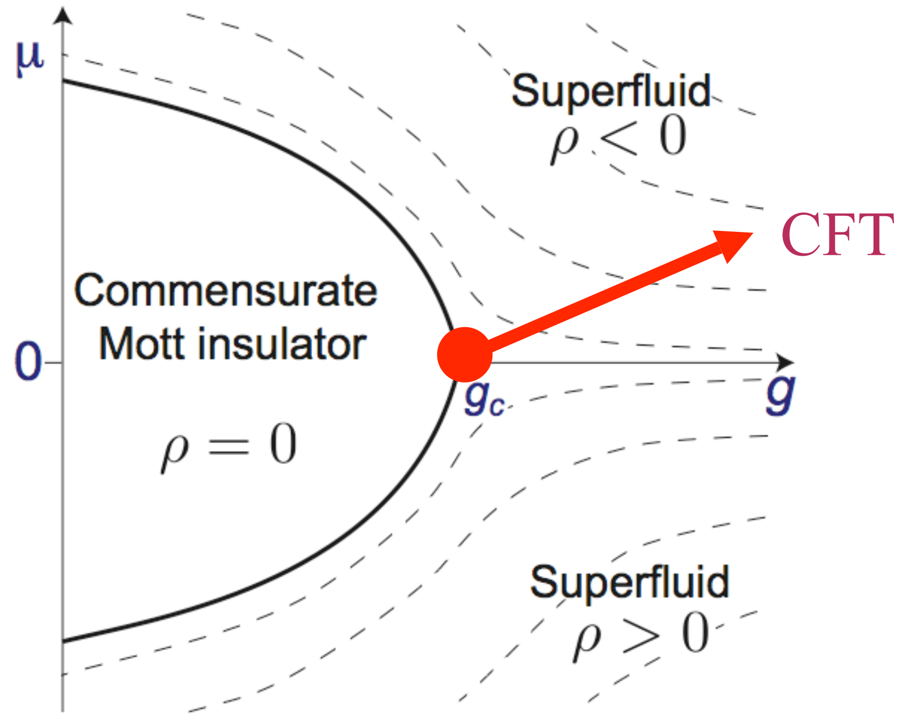


$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + c^2 |\vec{\nabla}\psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field *B*

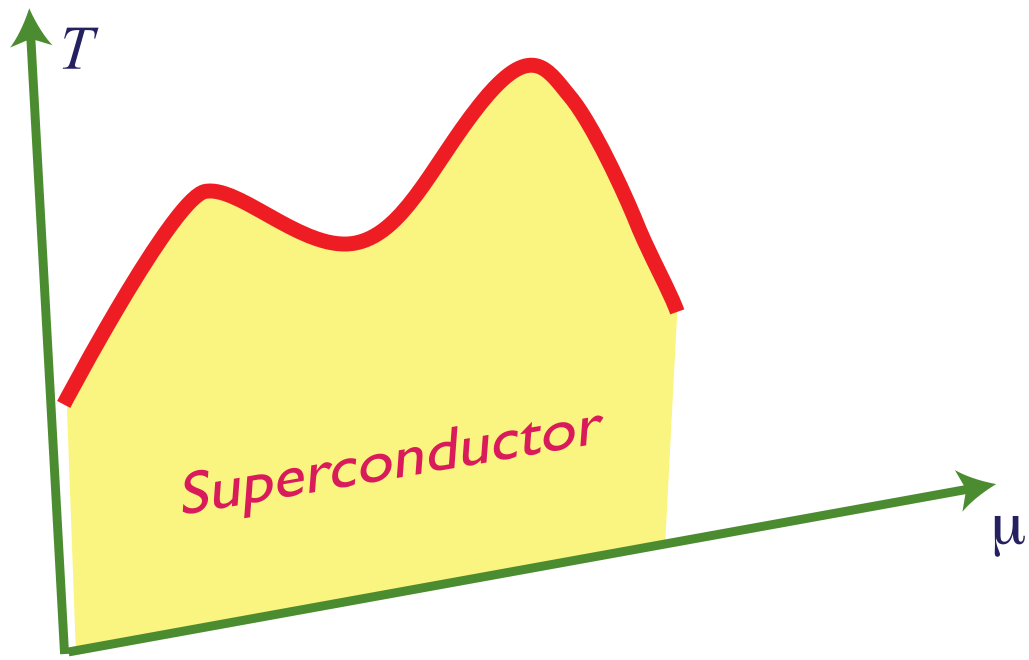


e.g.

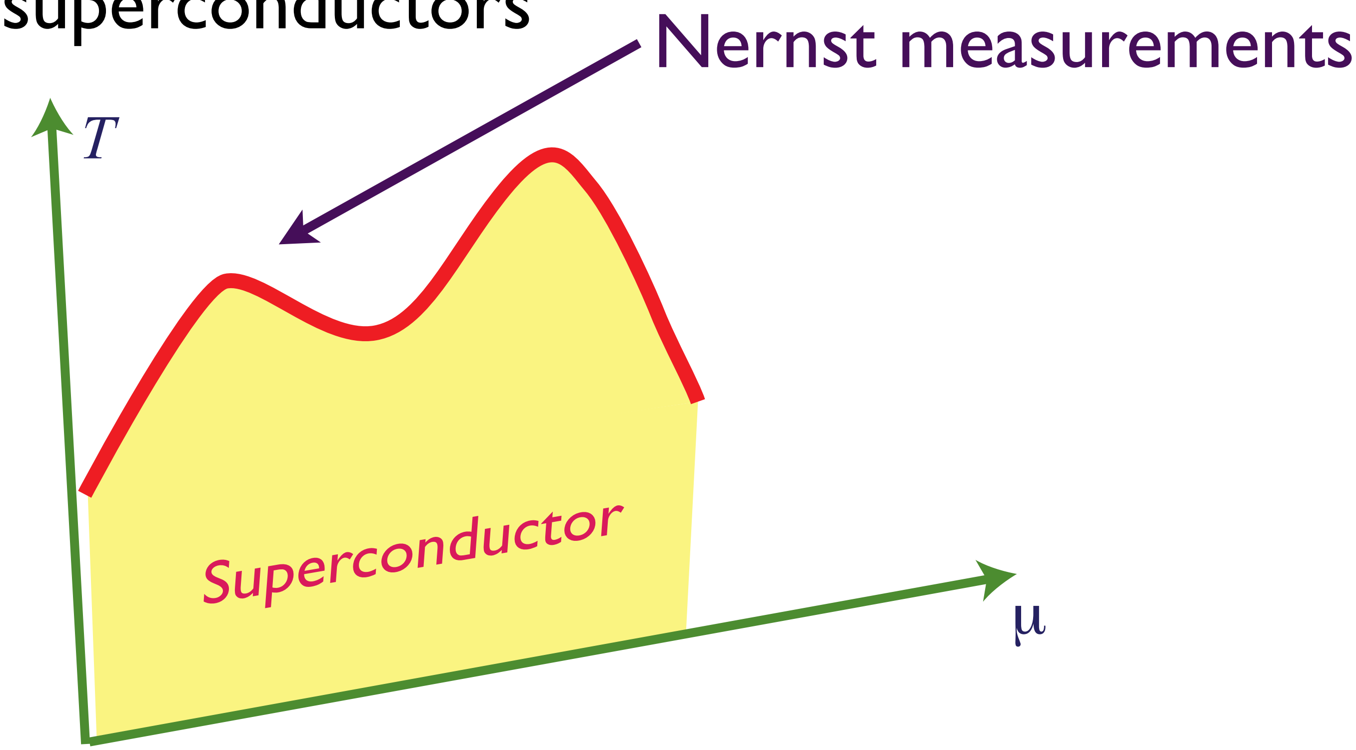
$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left| (\vec{\nabla} - i\vec{A})\psi \right|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

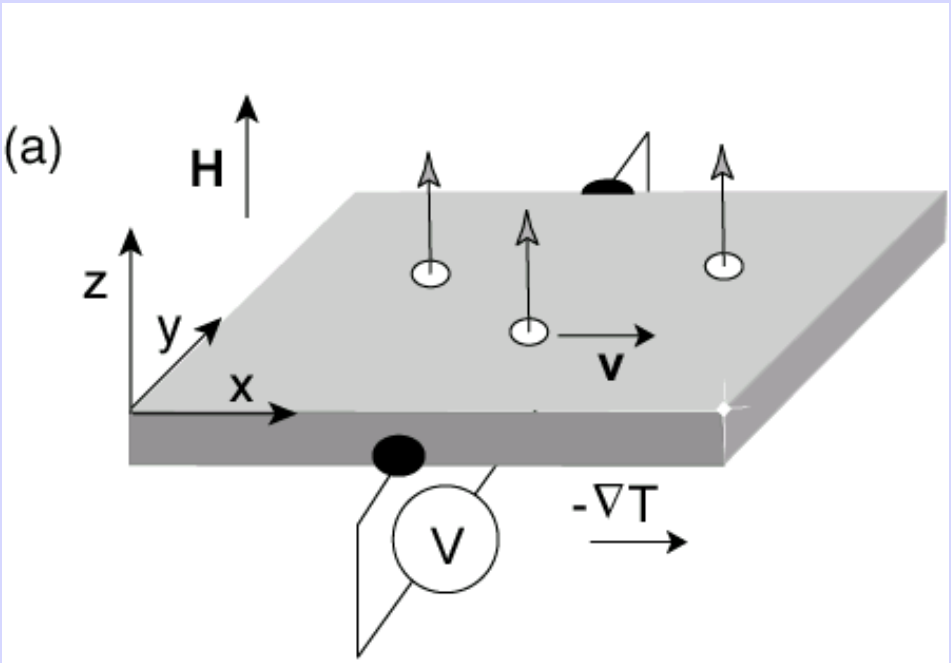
Cuprate superconductors

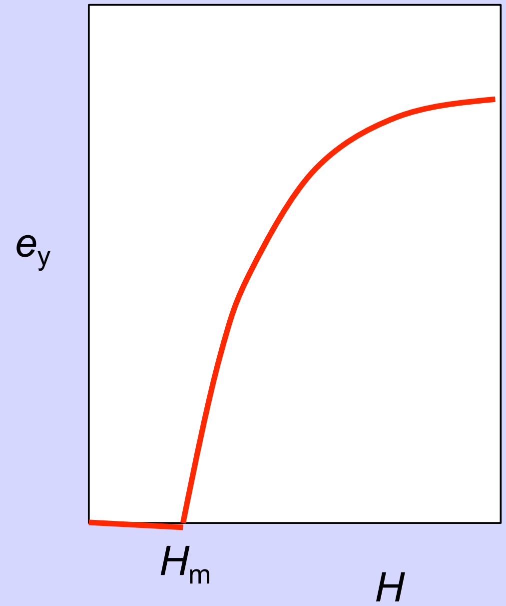


Cuprate superconductors

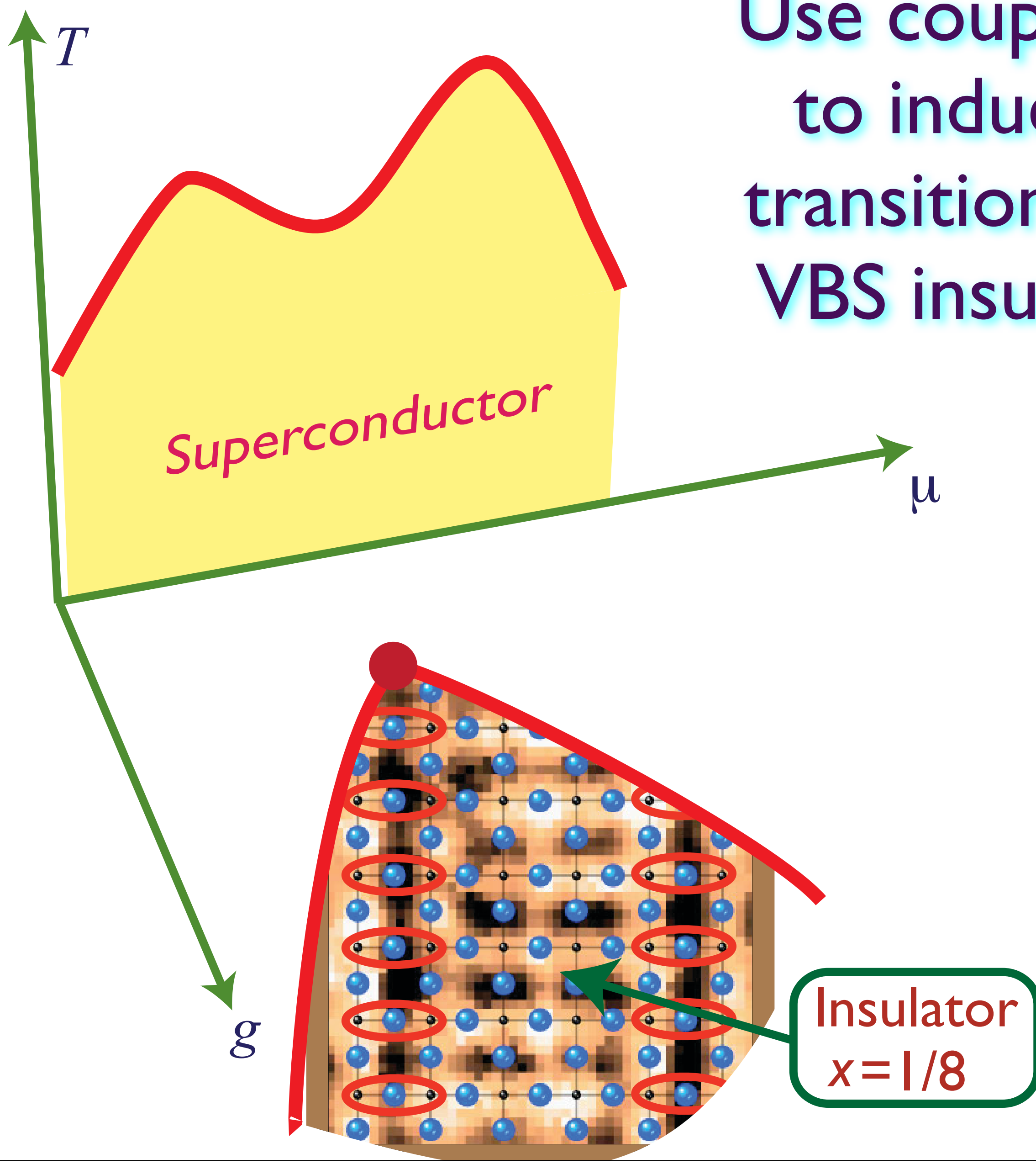


Nernst experiment



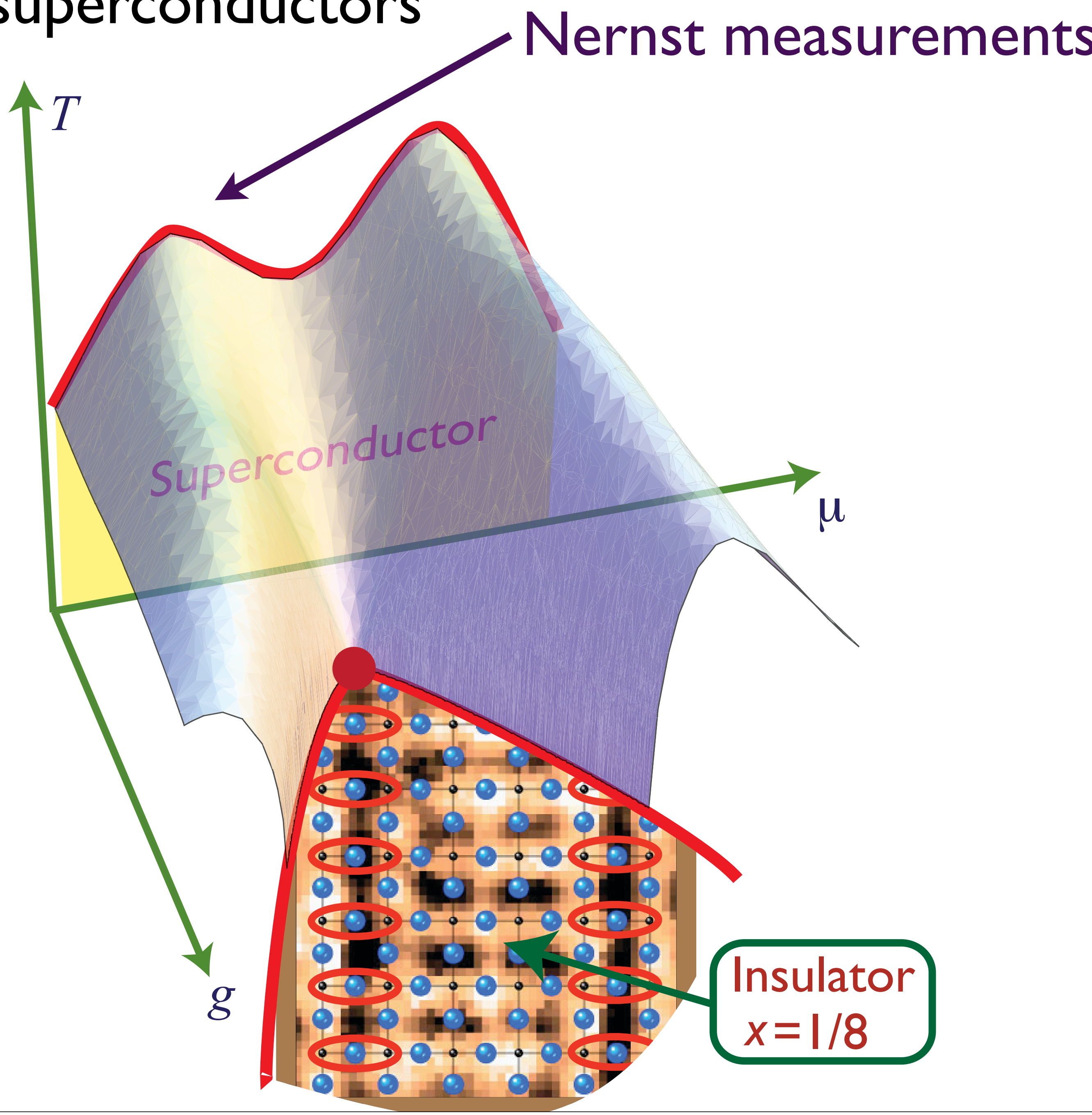


Cuprate superconductors



Use coupling *g* to induce a transition to a VBS insulator

Cuprate superconductors



In the regime $\hbar\omega \ll k_B T$, we can use the principles of hydrodynamics:

- Describe system in terms of local state variables which obey the equation of state
- Express conserved currents in terms of gradients of state variables using transport co-efficients. These are restricted by demanding that the system relaxes to *local equilibrium i.e.* entropy production is positive.
- The conservation laws are the equations of motion.

The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^{μ}, the current,
- ρ, the **difference** in density from the Mott insulator.
- ε, the local energy
- P, the local pressure,
- u^{μ}, the local velocity, and
- σ_Q, a universal conductivity, which is the **single transport co-efficient**.

The dependence of ε, P, σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu
\end{aligned}
$$

Conservation laws/equations of motion

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$
\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu
\end{aligned}
$$

Constitutive relations which follow from Lorentz transformation to moving frame

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\ T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\ J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda \right) + \mu \frac{\partial_\mu T}{T} \right] \end{aligned}$$

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient

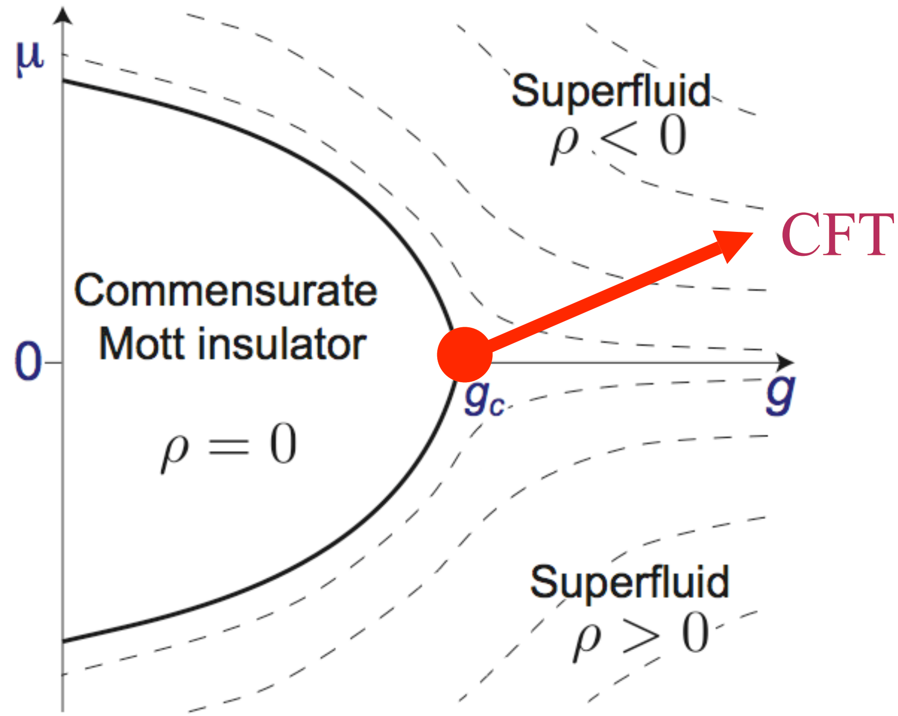
S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ

- A magnetic field B

- An impurity scattering rate $1/\tau_{\mathrm{imp}}$ (its T dependence follows from scaling arguments)



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 \left| (\vec{\nabla} - i\vec{A})\psi \right|^2 - g|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\mathrm{imp}}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned} \partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu + \frac{1}{\tau_{\mathrm{imp}}} (\delta^\mu_\nu + u^\mu u_\nu) T^{\nu\gamma} u_\gamma \\ T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\ J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda\right) + \mu \frac{\partial_\mu T}{T} \right] \end{aligned}$$

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T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[\left(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda\right) + \mu \frac{\partial_\mu T}{T} \right]
\end{aligned}$$

Solve initial value problem and relate results to response functions (Kadanoff+Martin)

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$= \sigma_Q + \frac{4e^2\rho^2 v^2}{(\varepsilon + P)} \frac{1}{(-i\omega + 1/\tau_{\text{imp}})} \quad \text{as } B \to 0$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Hall conductivity

$$\sigma_{xy} = -\frac{2e\rho c}{B}\left[\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\text{imp}}}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2}\right]$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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$$= \frac{2e\rho c}{B} \quad \text{as } \omega \to 0 \text{ and } \tau_{\rm imp} \to \infty$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

From these relations, we obtained results for the transport co-efficients, expressed in terms of a "cyclotron" frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Hall conductivity

$$\begin{aligned}\sigma_{xy} &= -\frac{2e\rho c}{B}\left[\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\text{imp}}}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2}\right] \\ &= B\left[\sigma_Q \frac{4e\rho v^2}{(\varepsilon + P)(1/\tau_{\text{imp}} - i\omega)} + \frac{8e^3\rho^3 v^4}{(\varepsilon + P)^2(1/\tau_{\text{imp}} - i\omega)^2}\right] \\ &\qquad \text{as } B \to 0\end{aligned}$$

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Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2}\right) \left(\frac{\varepsilon + P}{k_B T \rho}\right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right] \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2}\right] \end{aligned}$$

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$$= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B}\right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\rm imp})}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

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Nernst signal

$$e_N = \left(\frac{k_B}{2e}\right)\left(\frac{\varepsilon + P}{k_B T \rho}\right)\left[\frac{\omega_c/\tau_{\rm imp}}{(\omega_c^2/\gamma + 1/\tau_{\rm imp})^2 + \omega_c^2}\right]$$

$$\frac{k_B}{2e} = 43.086 \mu\mathrm{V/K}$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

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Transverse thermoelectric co-efficient

$$\left(\frac{h}{2ek_B}\right)\alpha_{xy} = \Phi_s \overline{B}\,(k_B T)^2 \left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2 \frac{\overline{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P}(k_B T)^3\,\hbar/2\pi\tau_{\rm imp}}{\Phi_{\varepsilon+P}^2 (k_B T)^6 + \overline{B}^2 \overline{\rho}^2 (2\pi\tau_{\rm imp}/\hbar)^2},$$

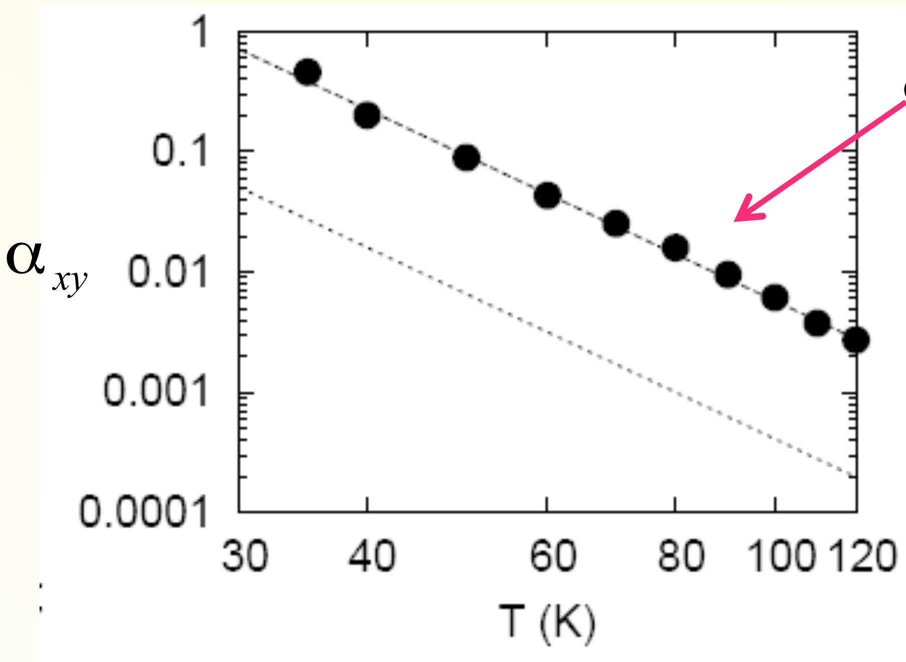
where

$$B = \overline{B}\phi_0/(\hbar v)^2 \quad ; \quad \rho = \overline{\rho}/(\hbar v)^2.$$

S.A. Hartnoll, P.K. Kovtun, M. Müller, and S. Sachdev, *Phys. Rev.* B **76** 144502 (2007)

LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xx} e_N$



Y. Wang et al., Phys. Rev. B ***73****, 024510 (2006).*

$$\alpha_{xy} \propto \frac{1}{T^4}$$

$$\alpha_{xy} \propto \frac{BT^2\left(\#\rho^2\tau_{imp} + \#T^3\right)}{T^6 + \#B^2\rho^2\tau_{imp}^2}$$

(T small)

$$\frac{\alpha_{xy}}{B}(B \to 0) \approx \left(\frac{2ek_B}{h\phi_0}\right)\frac{\Phi_s}{\Phi_{\varepsilon+P}^2}\left(\frac{2\pi\tau_{\rm imp}}{\hbar}\right)^2\frac{\rho^2(\hbar v)^6}{(k_BT)^4}$$

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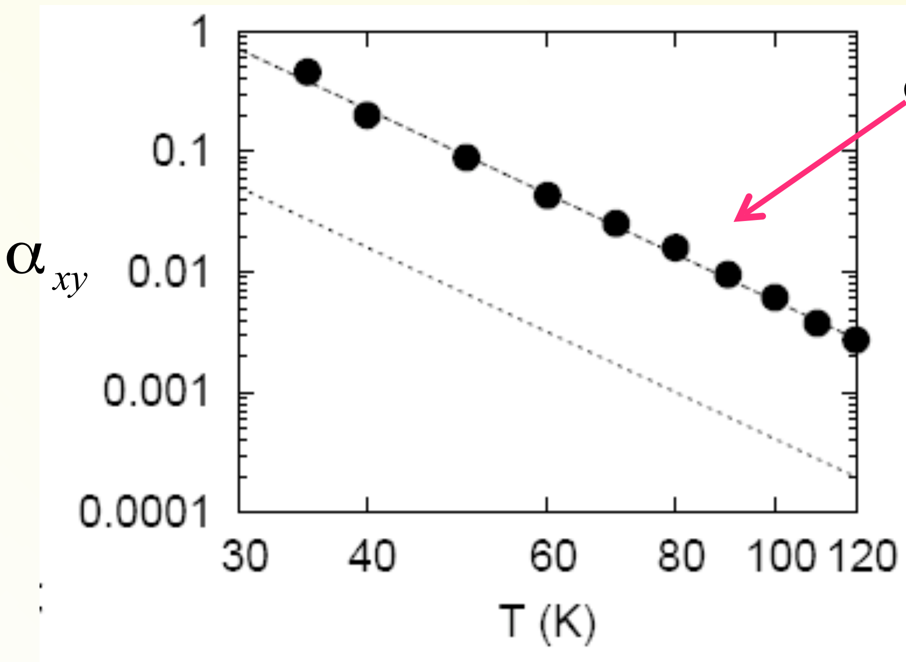

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$$\hbar v \approx 47 \text{meV} \, \mathring{A}$$

$$v \approx 2.5 \times 10^{-5} c$$

$$\tau_{imp} \approx 10^{-12} \text{s}$$

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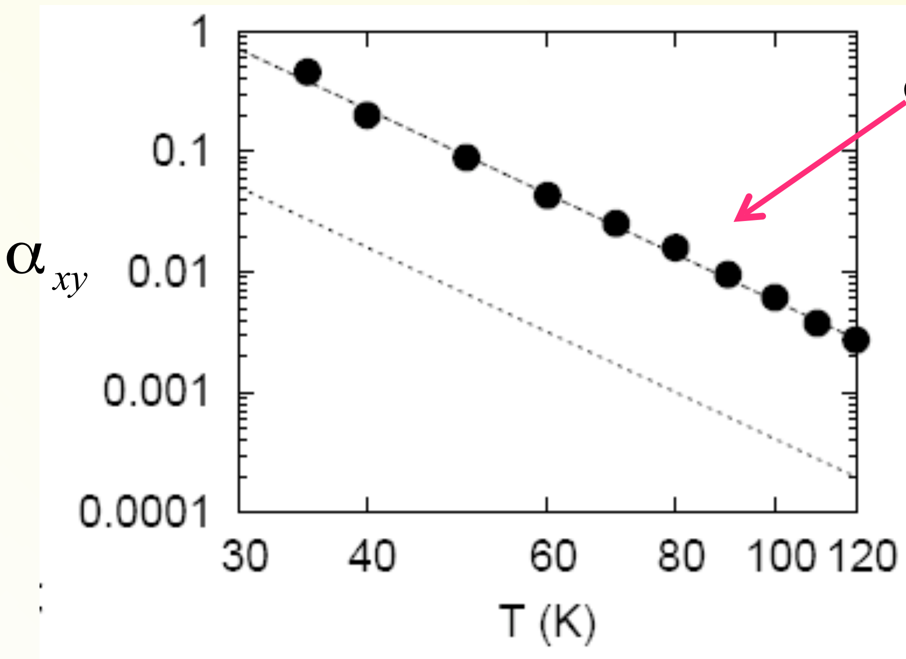

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$\hbar v \approx 47\,\text{meV}\,\text{Å}$

$v \approx 2.5 \times 10^{-5} c$

$\tau_{imp} \approx 10^{-12}\,\text{s}$

$\to$ Prediction for ω_c:

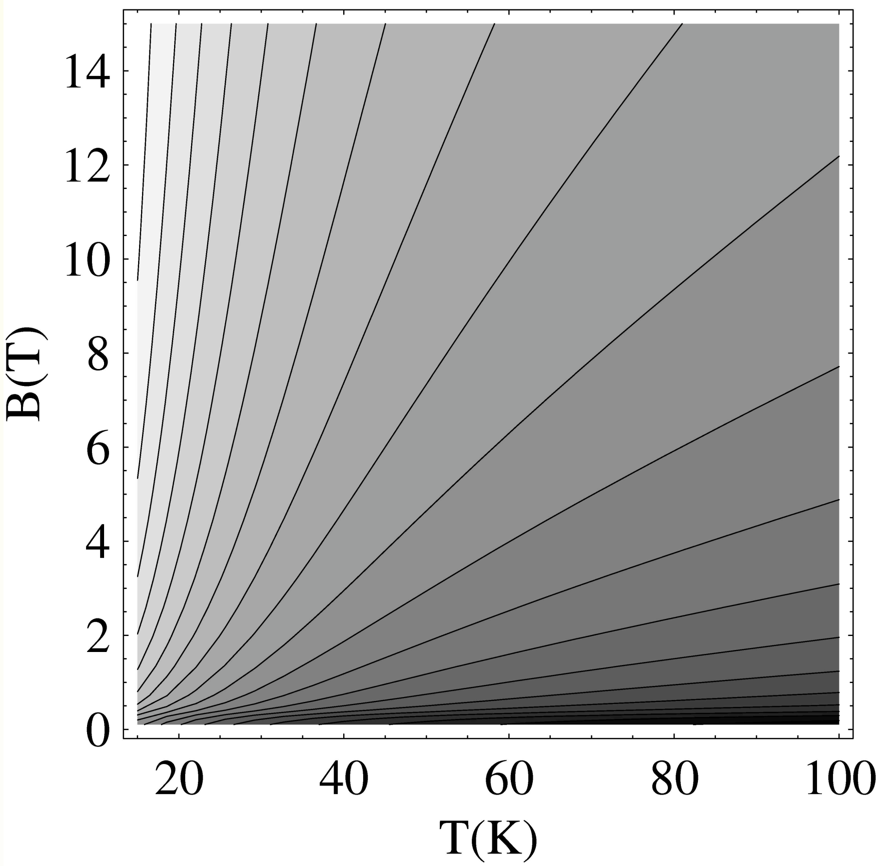
$$\omega_c = 6.2\ \text{GHz}\frac{B}{1\ \text{T}}\left(\frac{35\ \text{K}}{T}\right)^3$$

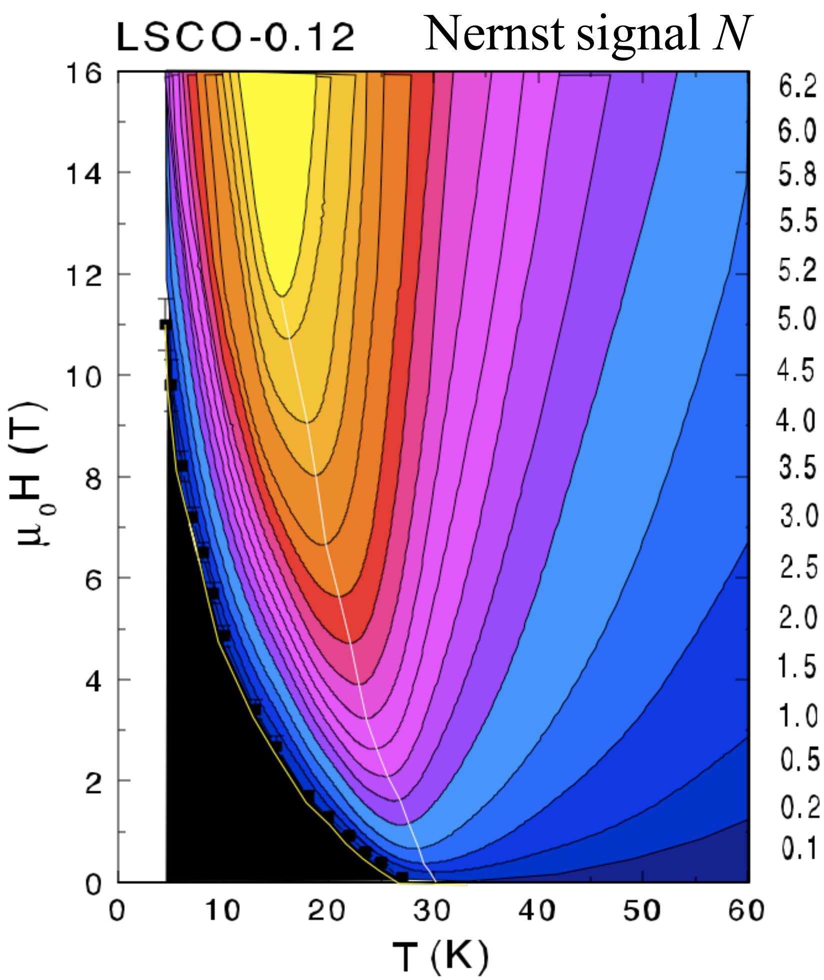
- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at T=35 K)
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} \leq \omega_c$

LSCO Experiments

B, T-dependence

Theory for $\alpha_{xy} \approx \sigma_{xx} N$

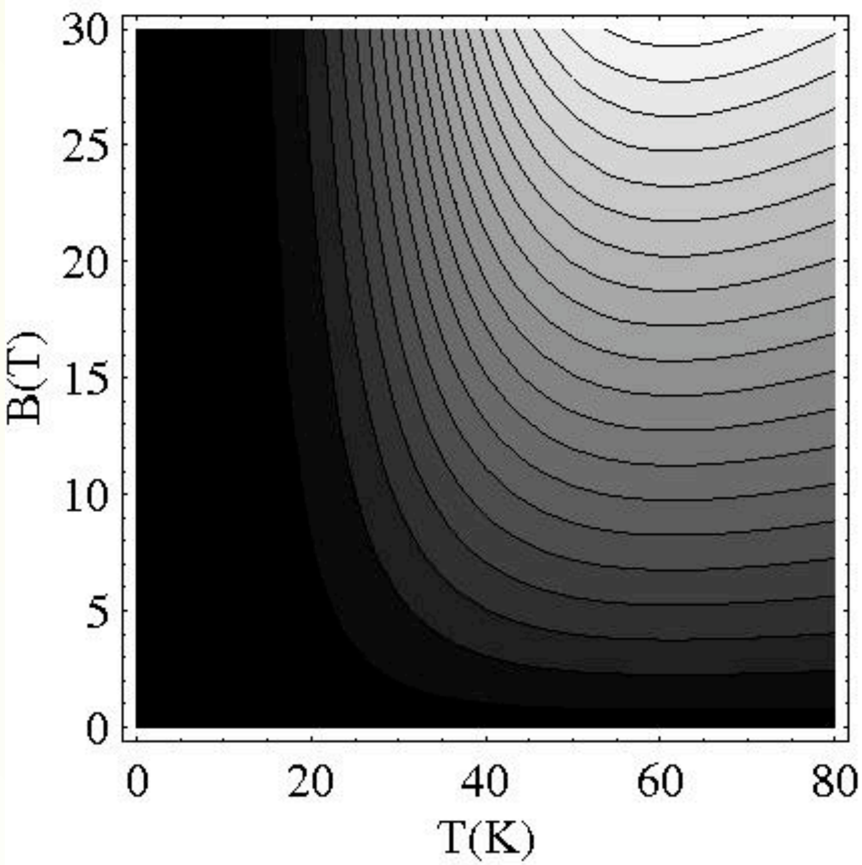




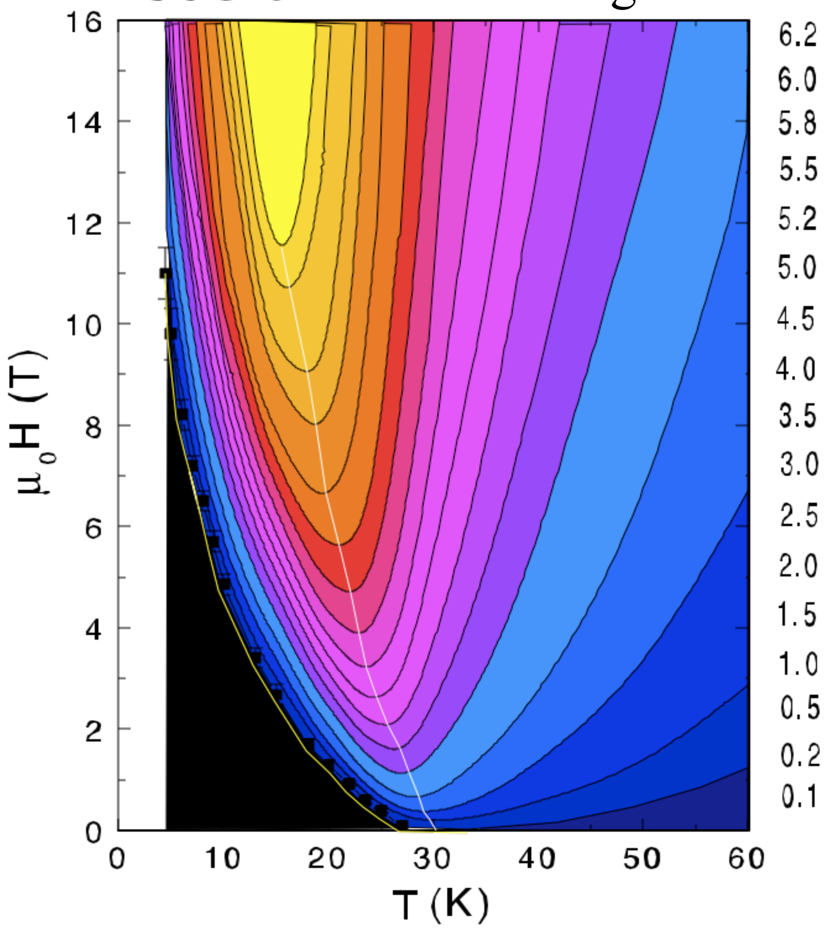
Y. Wang, L. Li, and N. P. Ong, Phys. Rev. B ***73****, 024510 (2006).*

LSCO Experiments

Theory for N



LSCO-0.12 Nernst signal N



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Outline

1. Model systems

   (i) Superfluid-insulator transition of lattice bosons, (ii) graphene

2. Quantum-critical transport at integer filling, zero magnetic field, and with no impurities

   Collisionless-to-hydrodynamic crossover of CFT3s

3. Quantum-critical transport at integer generic filling, nonzero magnetic field, and with impurities

   Nernst effect and a hydrodynamic cyclotron resonance

4. The AdS/CFT correspondence

   Quantum criticality and dyonic black holes

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Black Holes

Objects so massive that light is gravitationally bound to them.

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The region inside the black hole **horizon** is causally disconnected from the rest of the universe.

$$\text{Horizon radius } R = \frac{2GM}{c^2}$$

Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \dfrac{k_B A}{4\ell_P^2}$

where A is the area of the horizon, and

$\ell_P = \sqrt{\dfrac{G\hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$

Black Hole Thermodynamics

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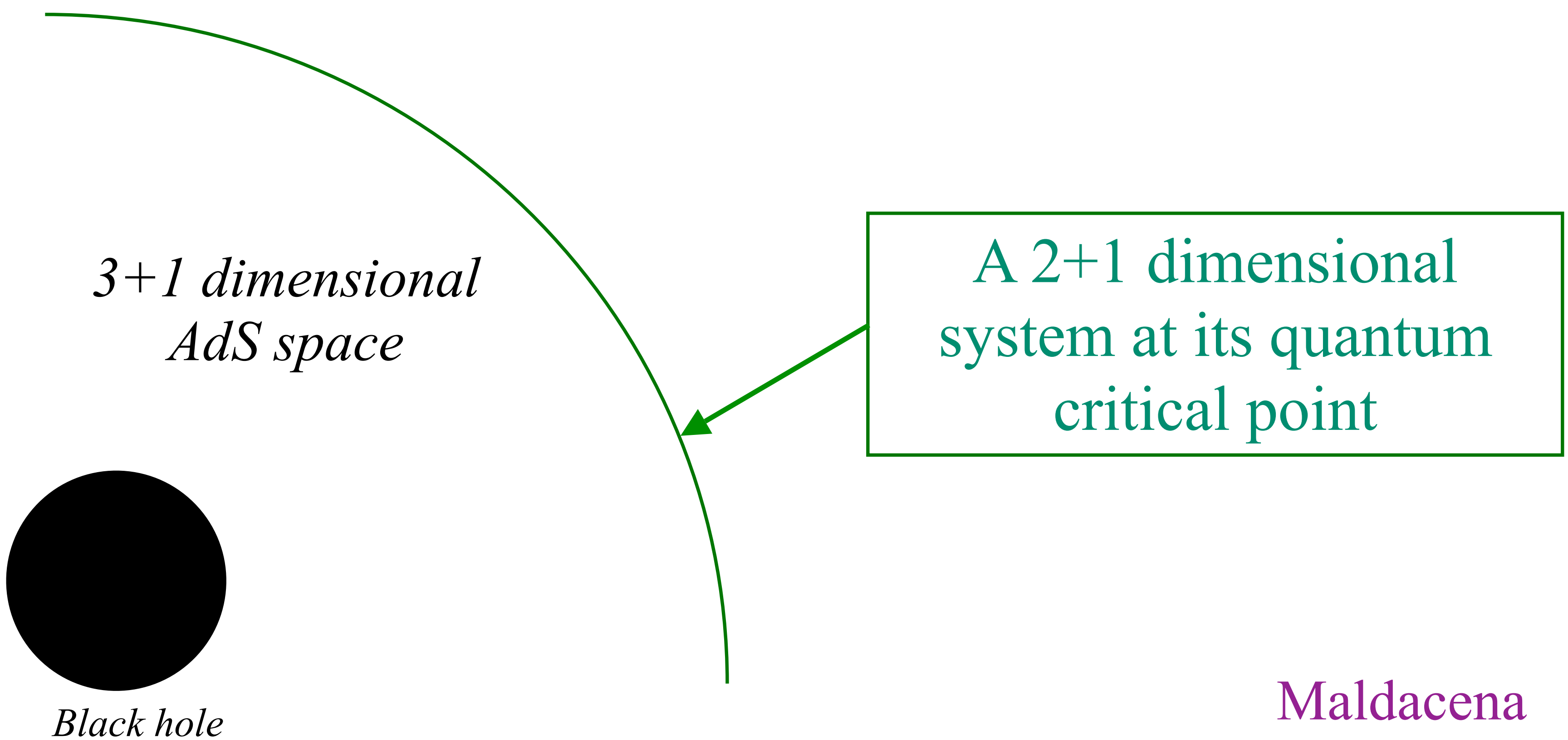
$$\text{Horizon temperature: } 4\pi k_B T = \frac{\hbar^2}{2M\ell_P^2}$$

AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions



Maldacena

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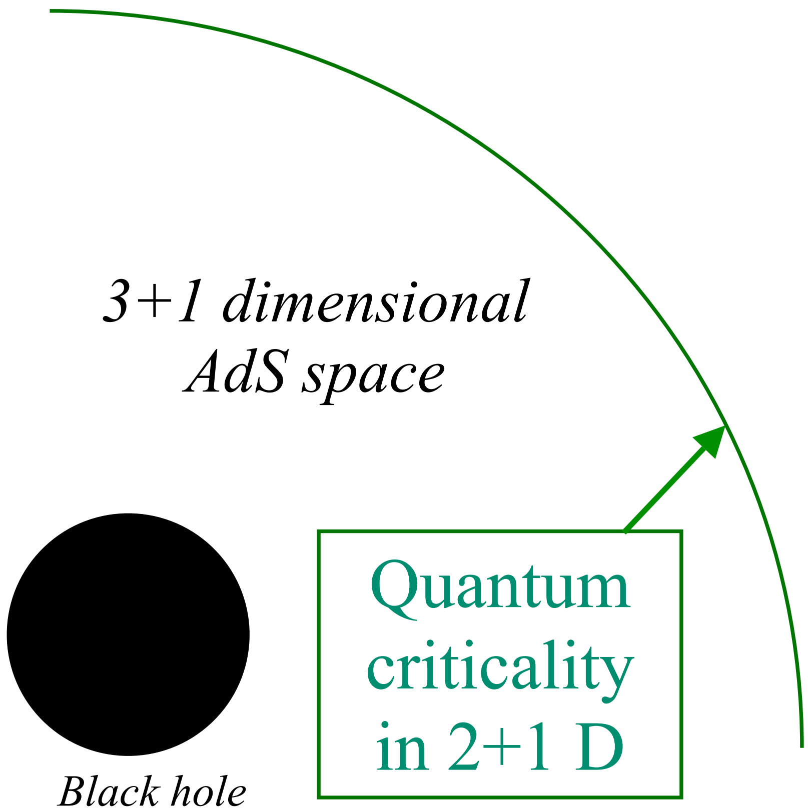

Black hole temperature = temperature of quantum criticality

Strominger, Vafa

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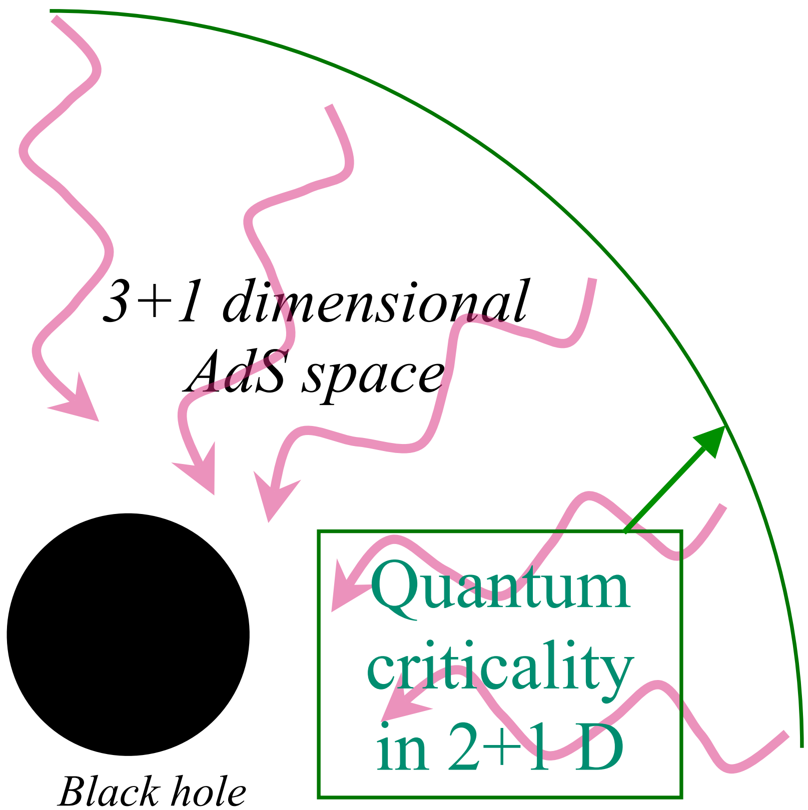

Dynamics of quantum criticality = waves in curved gravitational background

Maldacena

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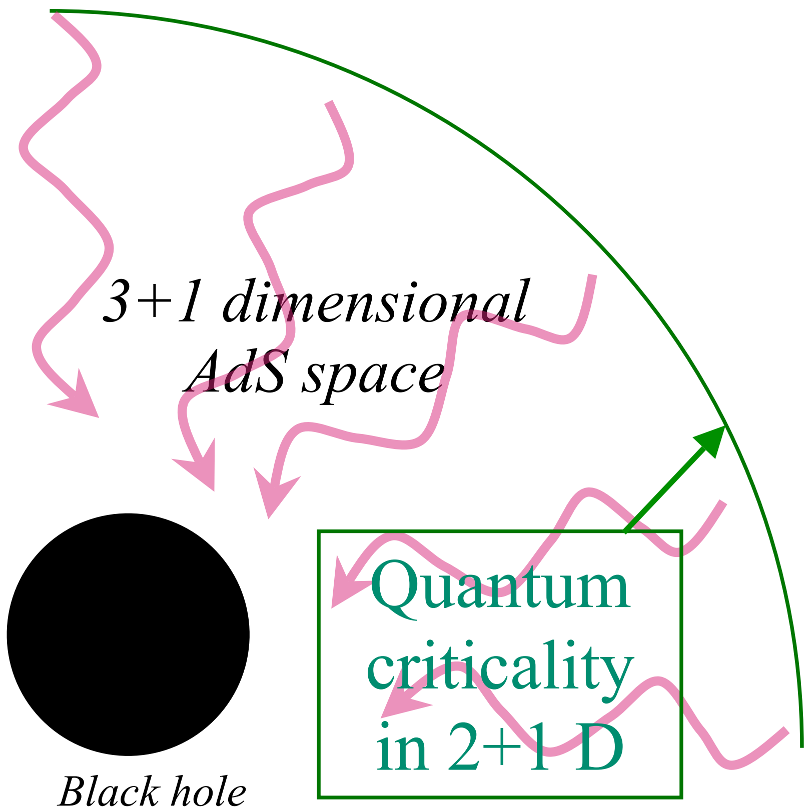

"Friction" of quantum critical dynamics = black hole absorption rates

Son

Application of the AdS/CFT correspondence

Can obtain the exact ω and k dependence of the quantum critical density correlation functions of many supersymmetric CFT3s (which are similar to supersymmetric generalizations of critical spin liquid theories). There are dual to black hole solutions of 11-dimensional supergravity.

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- The solutions exhibit the predicted collisionless-to-hydrodynamic crossover.
- Exact values are obtained for transport co-efficients (these are the first such results for a clean, interacting many-body system in dimensions $d > 1$.)
- Adding μ and B to the CFT3 corresponds to adding electric and magnetic charges to the black hole. Solutions of the Einstein-Maxwell equations in this dyonic black hole background yield thermoelectric response functions which agree perfectly with *all* hydrodynamic results presented earlier.

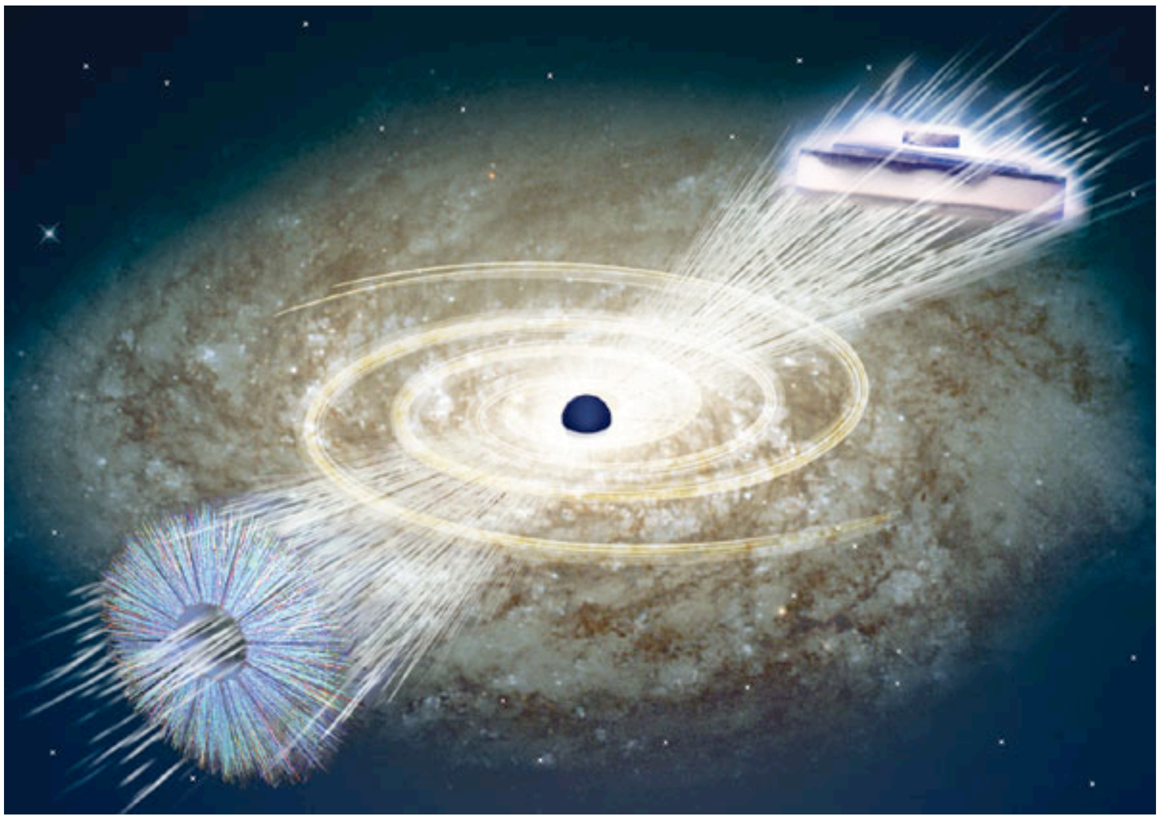
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

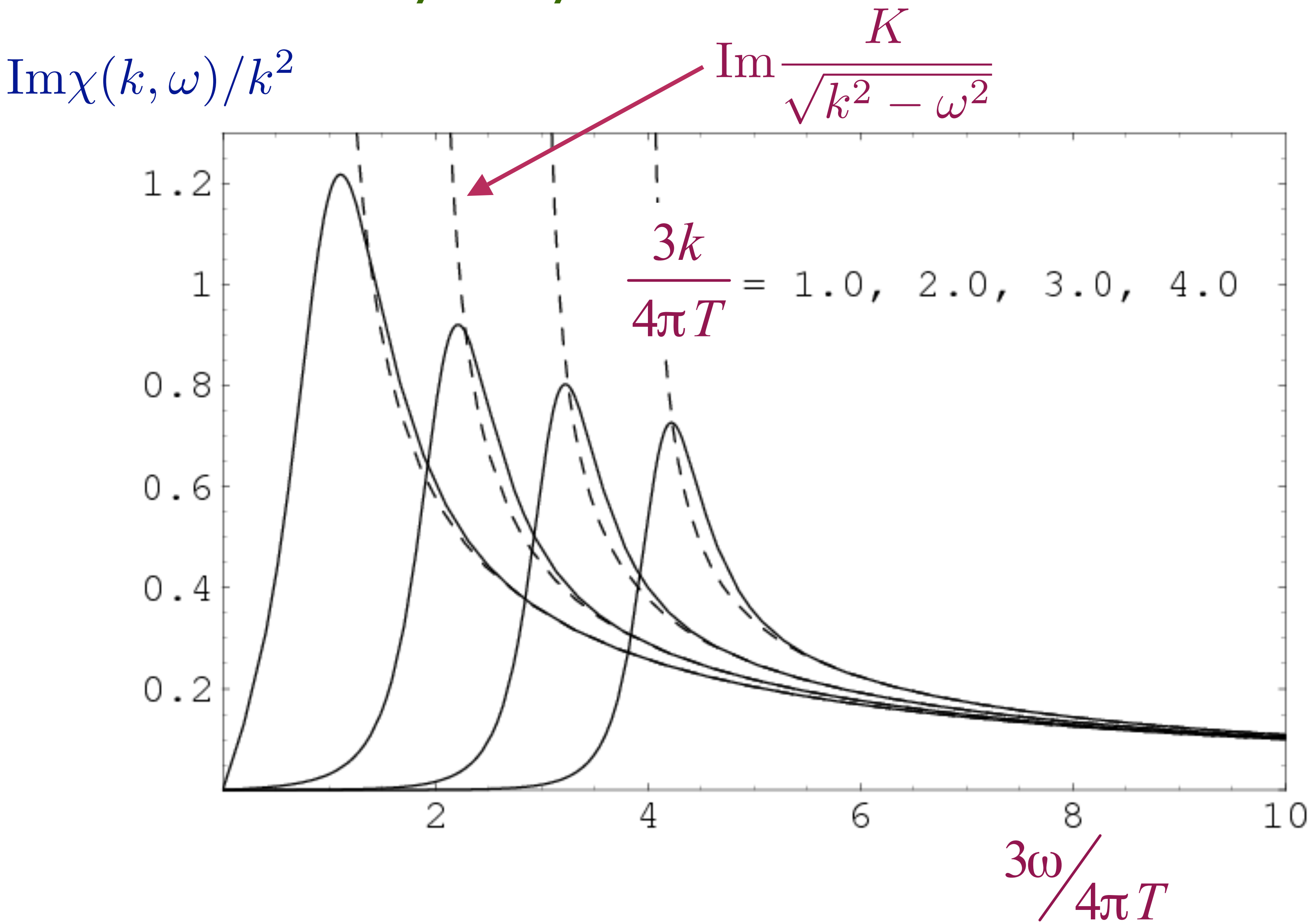
NATURE|Vol 448|30 August 2007



Conclusions

- Hydrodynamic theory for thermoelectric response functions of quantum critical systems
- Applications to the cuprates and graphene.
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.

Collisionless to hydrodynamic crossover of SYM3



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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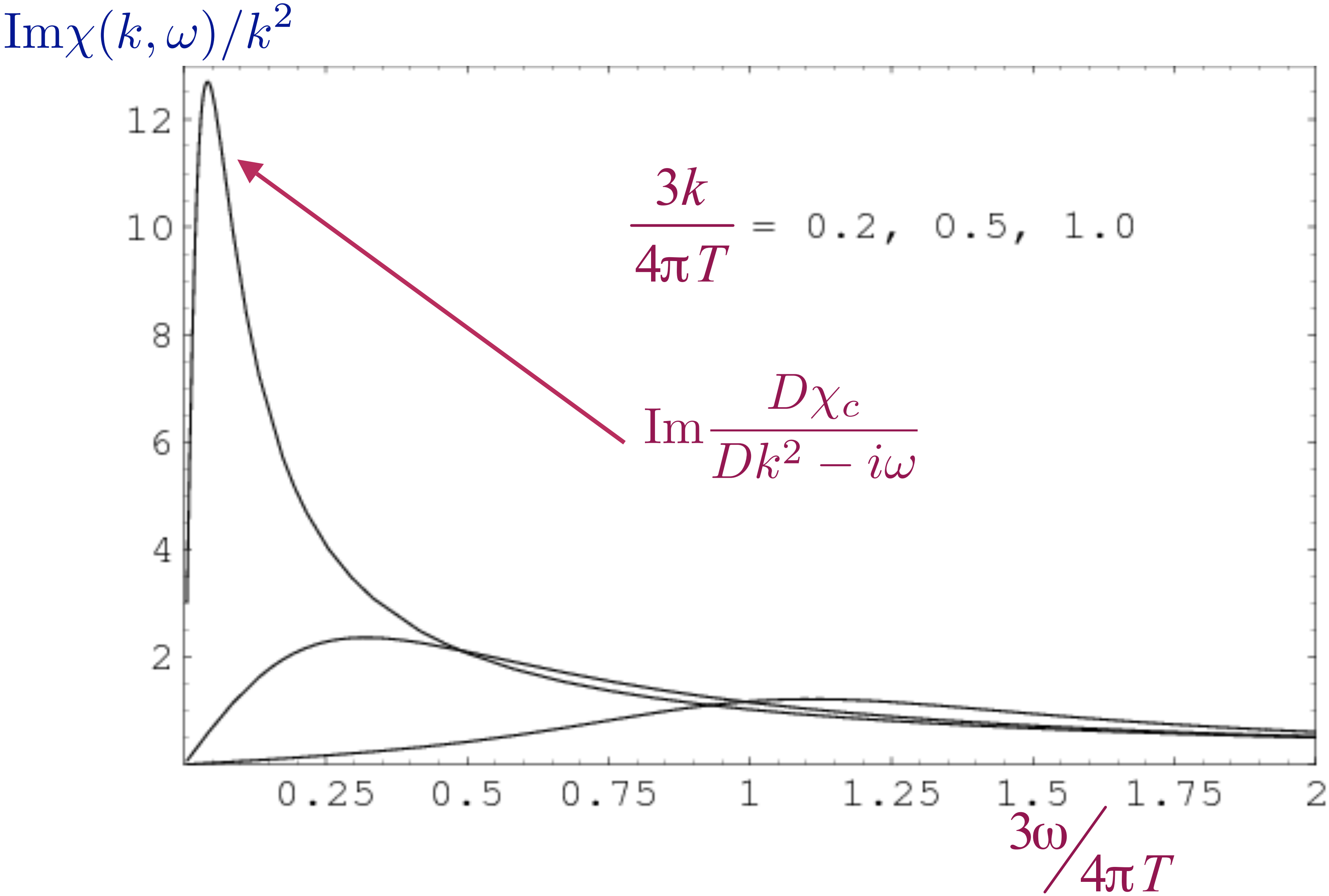

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Universal constants of SYM3

$$\chi_c = \frac{k_B T}{(hv)^2}\Theta_1$$

$$D = \frac{hv^2}{k_B T}\Theta_2$$

$$\sigma(\omega) = \begin{cases} \dfrac{4e^2}{h} K & , \quad \hbar\omega \gg k_B T \\ \dfrac{4e^2}{h}\Theta_1\Theta_2 & , \quad \hbar\omega \ll k_B T \end{cases}$$

$$K = \frac{\sqrt{2}N^{3/2}}{3}$$

$$\Theta_1 = \frac{8\pi^2\sqrt{2}N^{3/2}}{9}$$

$$\Theta_2 = \frac{3}{8\pi^2}$$

C. Herzog, JHEP **0212**, 026 (2002)

P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

Electromagnetic self-duality

- Unexpected result, $K = \Theta_1 \Theta_2$.
- This is traced to a *four*-dimensional electromagnetic self-duality of the theory on AdS_4. In the large N limit, the SO(8) currents decouple into 28 U(1) currents with a Maxwell action for the U(1) gauge fields on AdS_4.
- This special property is not expected for generic CFT3s.
- Open question: Does $K = \Theta_1 \Theta_2$ hold beyond the $N \to \infty$ limit ? In other words, does this "self-duality" survive in the full M theory.