From the SYK model, to a theory of the strange metal, and of quantum gravity in two spacetime dimensions

ARO MURI review,
University of Maryland
October 13, 2017

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Talk online: sachdev.physics.harvard.edu

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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Gapless spin-fluid ground state in a random quantum Heisenberg magnet

Authors Subir Sachdev, Jinwu Ye

Publication date 1993/5/24

Journal Physical review letters

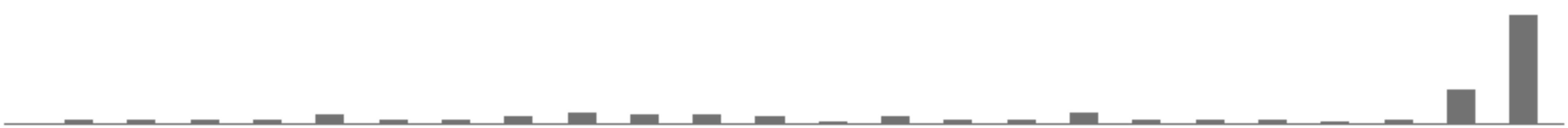
Volume 70

Issue 21

Pages 3339

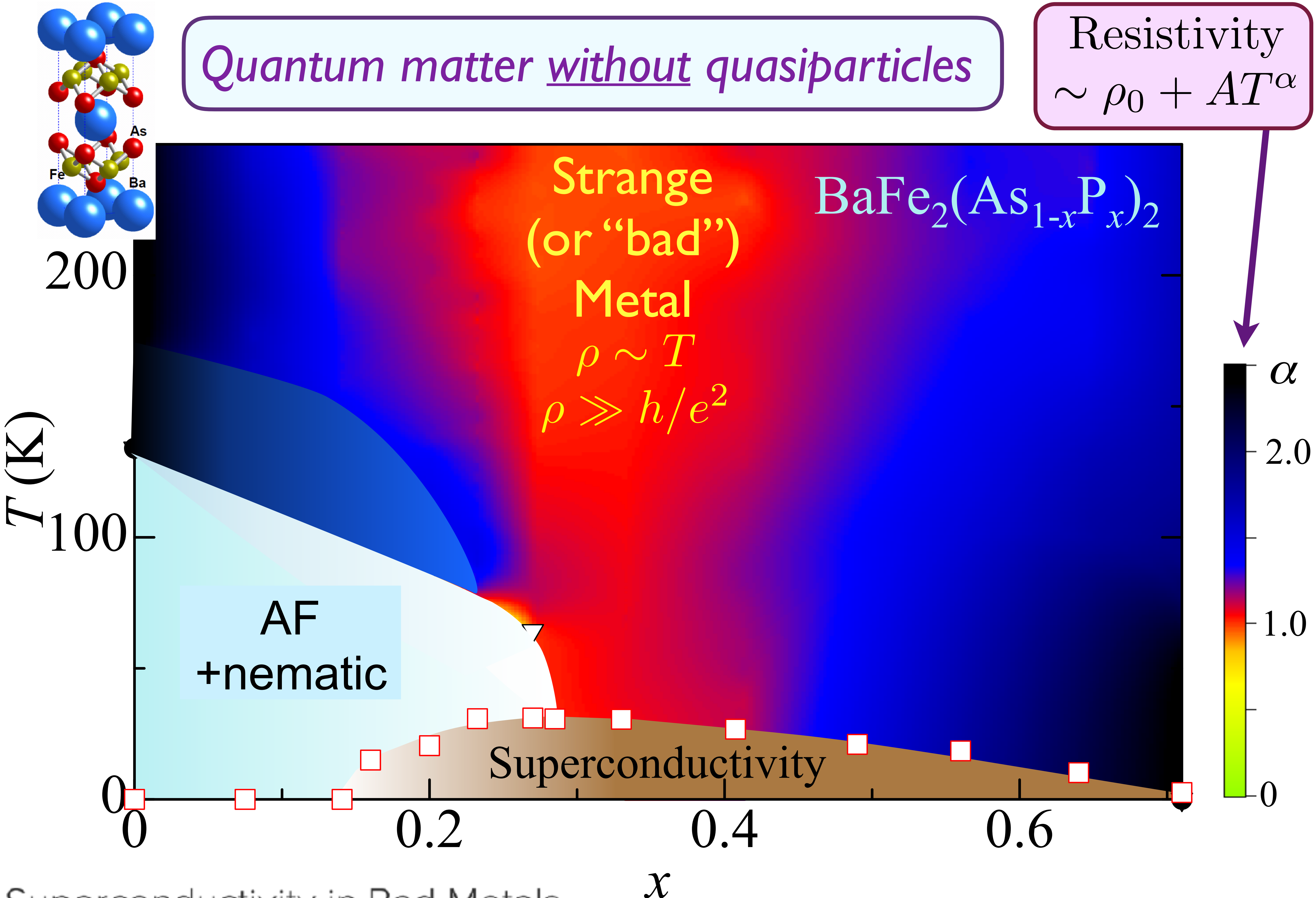
Publisher American Physical Society

Description Abstract We examine the spin-S quantum Heisenberg magnet with Gaussian-random, infinite-range exchange interactions. The quantum-disordered phase is accessed by generalizing to SU (M) symmetry and studying the large M limit. For large S the ground state is a spin glass, while quantum fluctuations produce a spin-fluid state for small S. The spin-fluid phase is found to be generically gapless—the average, zero temperature, local dynamic spin susceptibility obeys $\chi(\omega) \sim \ln(1/|\omega|) + i(\pi/2)\,\mathrm{sgn}(\omega)$ at low frequencies.



Quantum matter without quasiparticles

Resistivity $\sim \rho_0 + AT^\alpha$

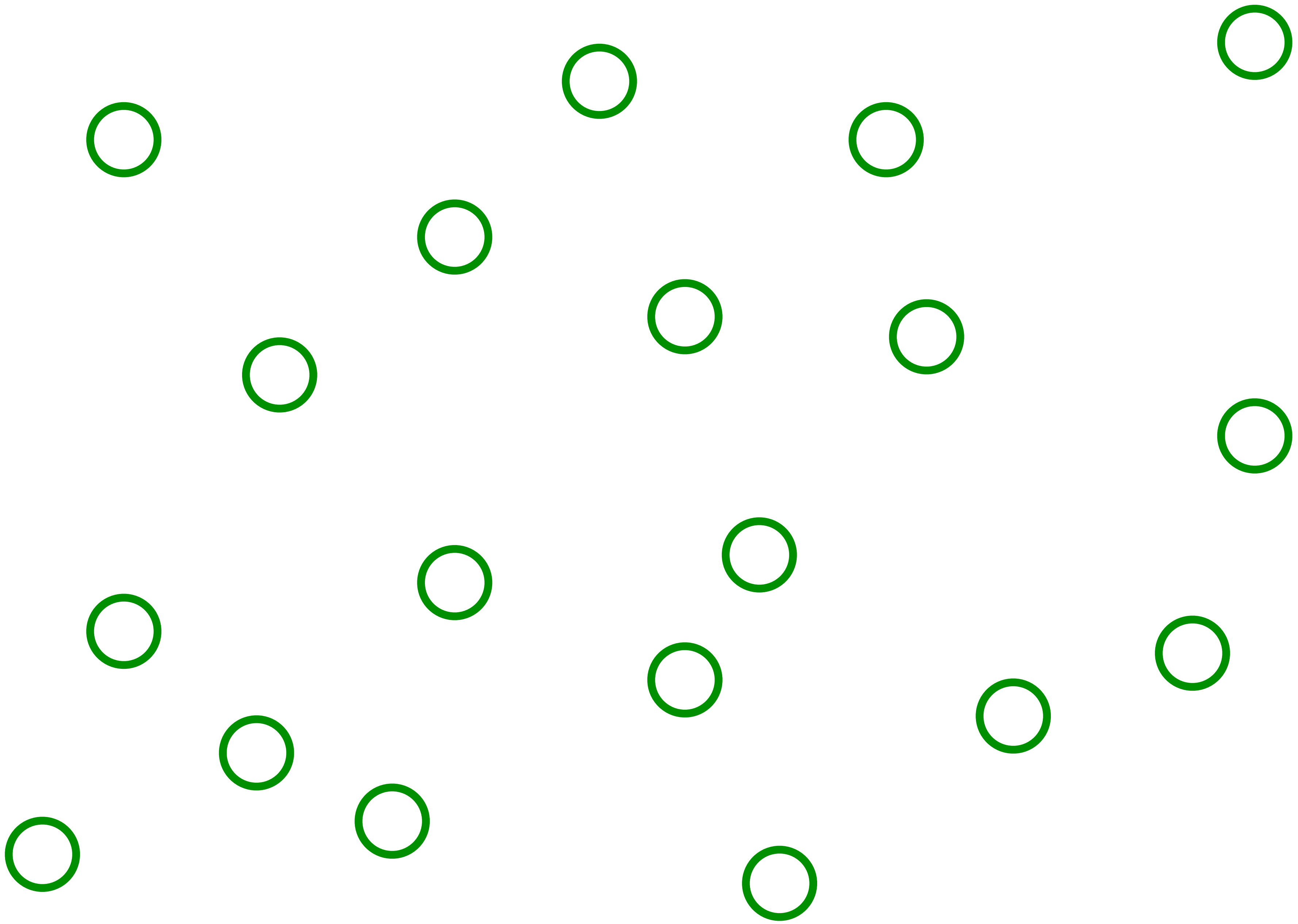


Superconductivity in Bad Metals

V. J. Emery and S. A. Kivelson
Phys. Rev. Lett. **74**, 3253 – Published 17 April 1995

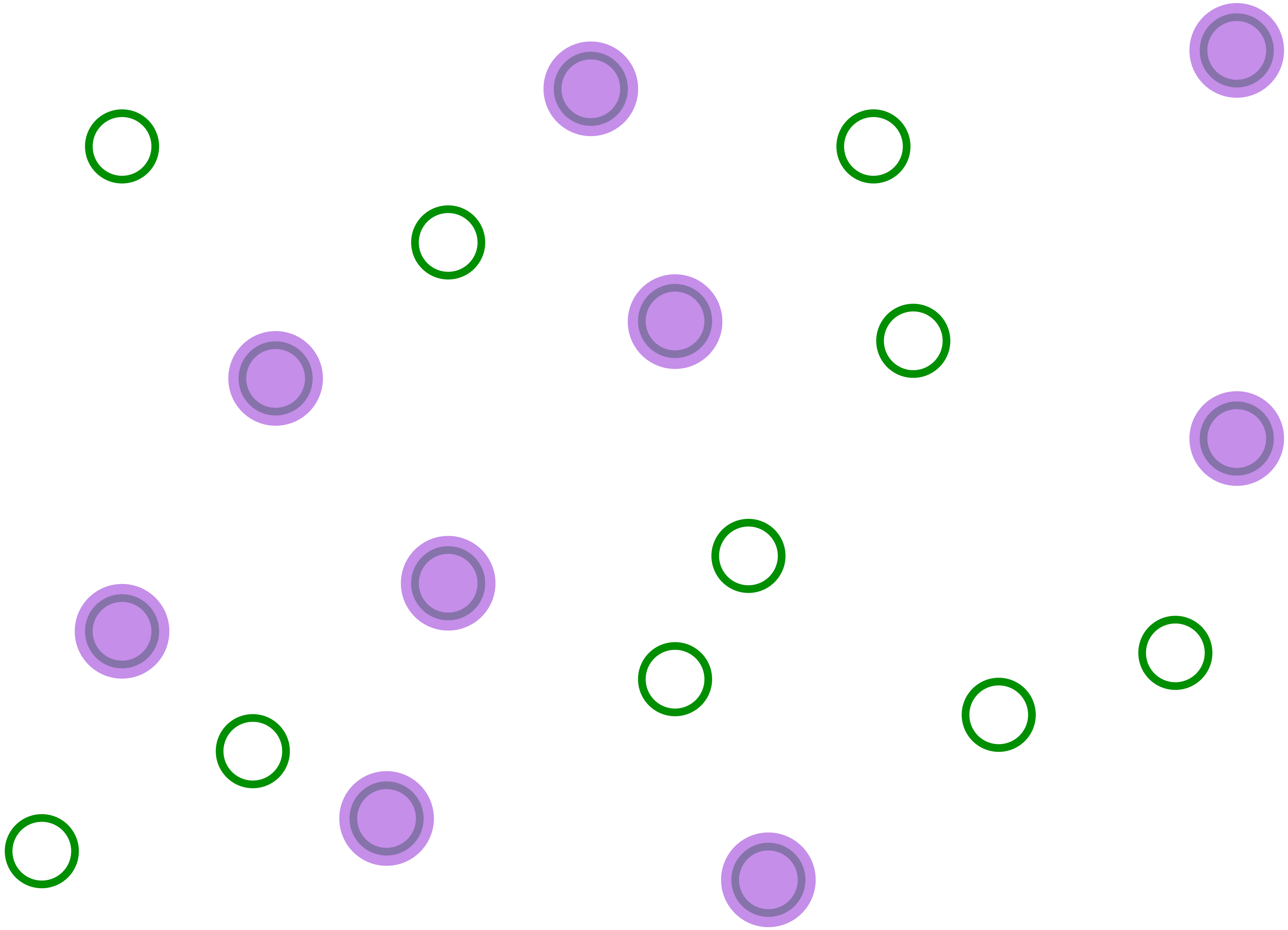
S. Kasahara, T. Shibauchi, K. Hashimoto, K. Ikada, S. Tonegawa, R. Okazaki, H. Shishido, H. Ikeda, H. Takeya, K. Hirata, T. Terashima, and Y. Matsuda, *PRB* **81**, 184519 (2010)

The Sachdev-Ye-Kitaev (SYK) model



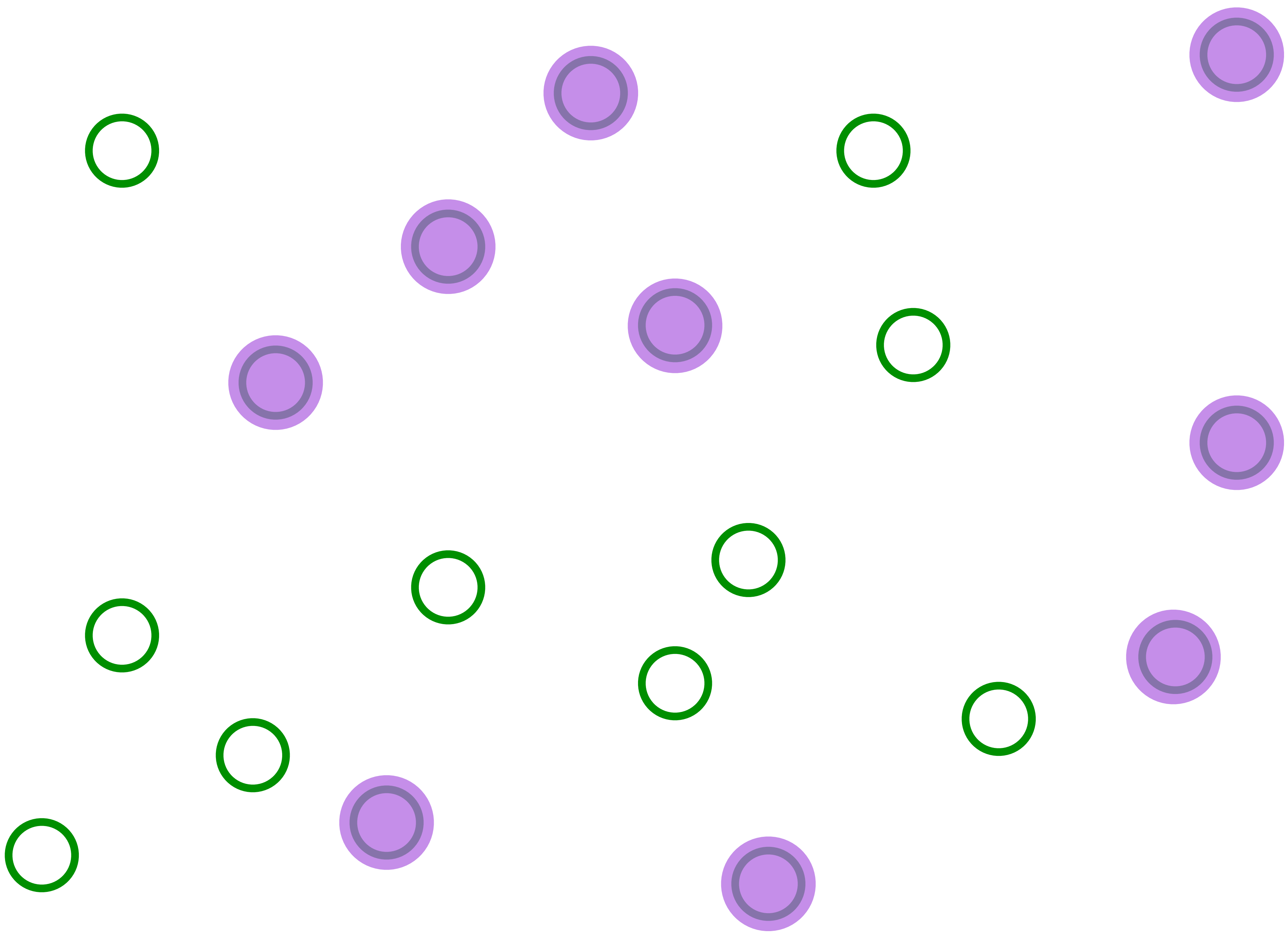
Pick a set of random sites/orbitals

The SYK model



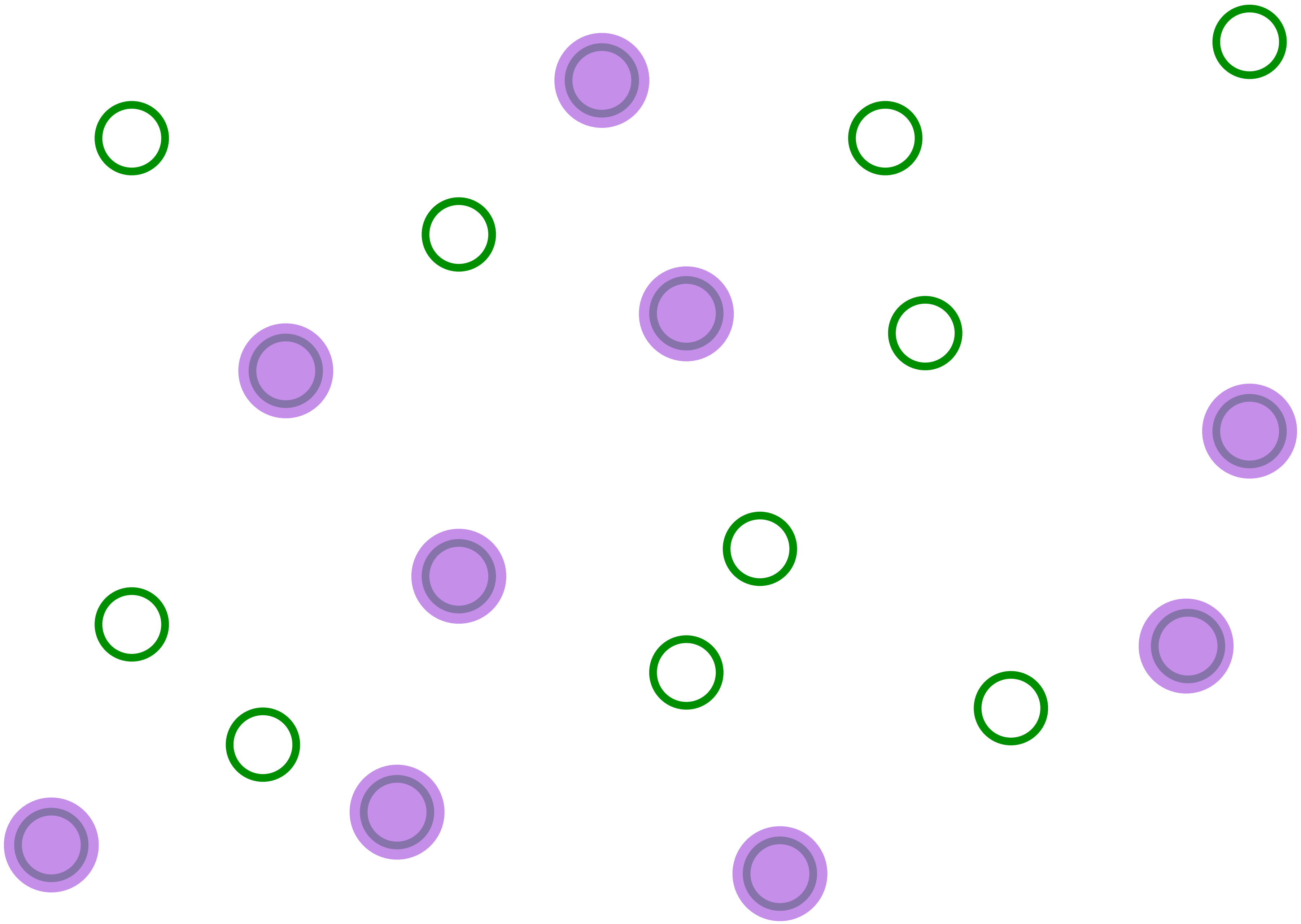
Place electrons randomly on some sites/orbitals

The SYK model



Entangle electrons pairwise randomly

The SYK model



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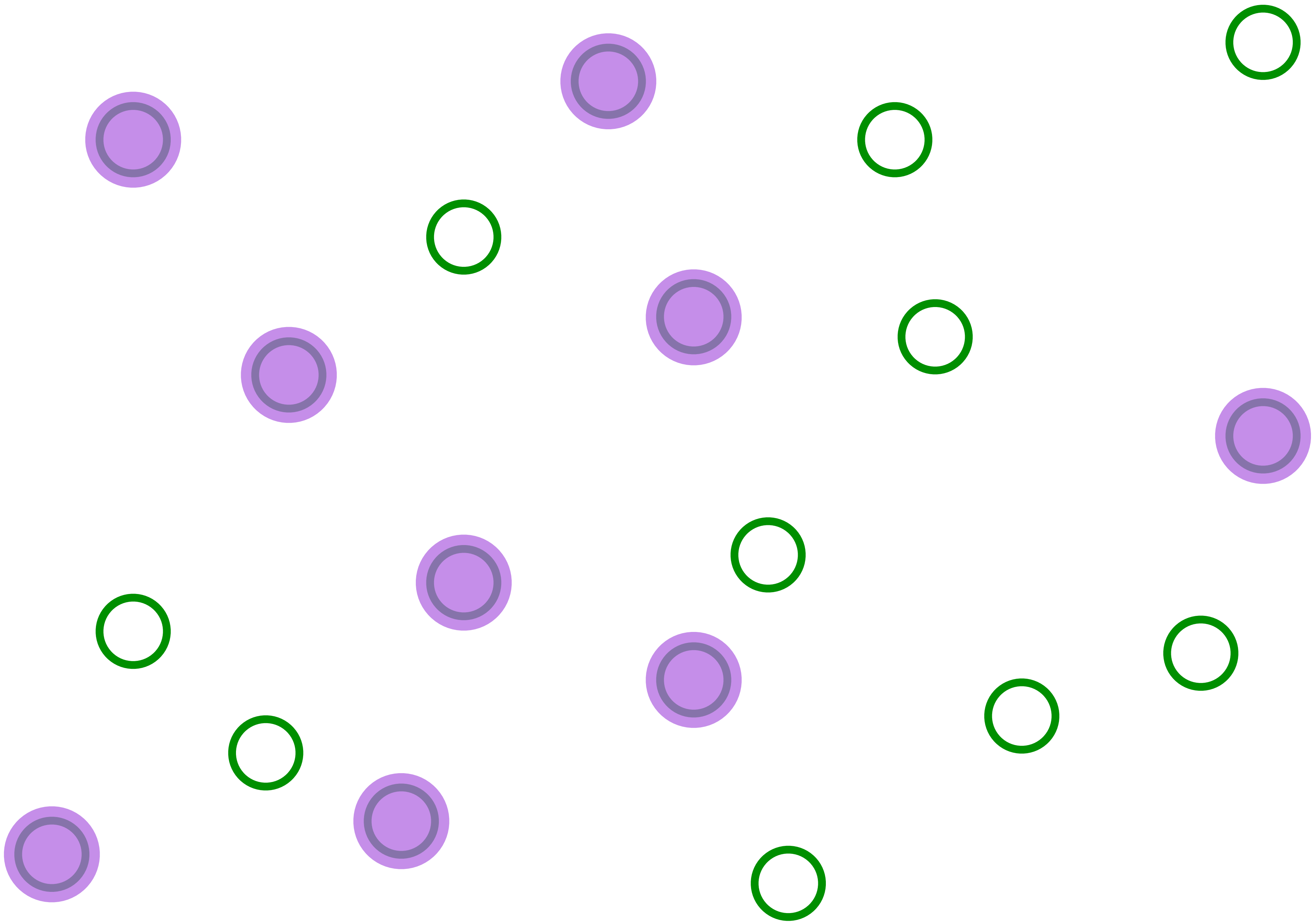

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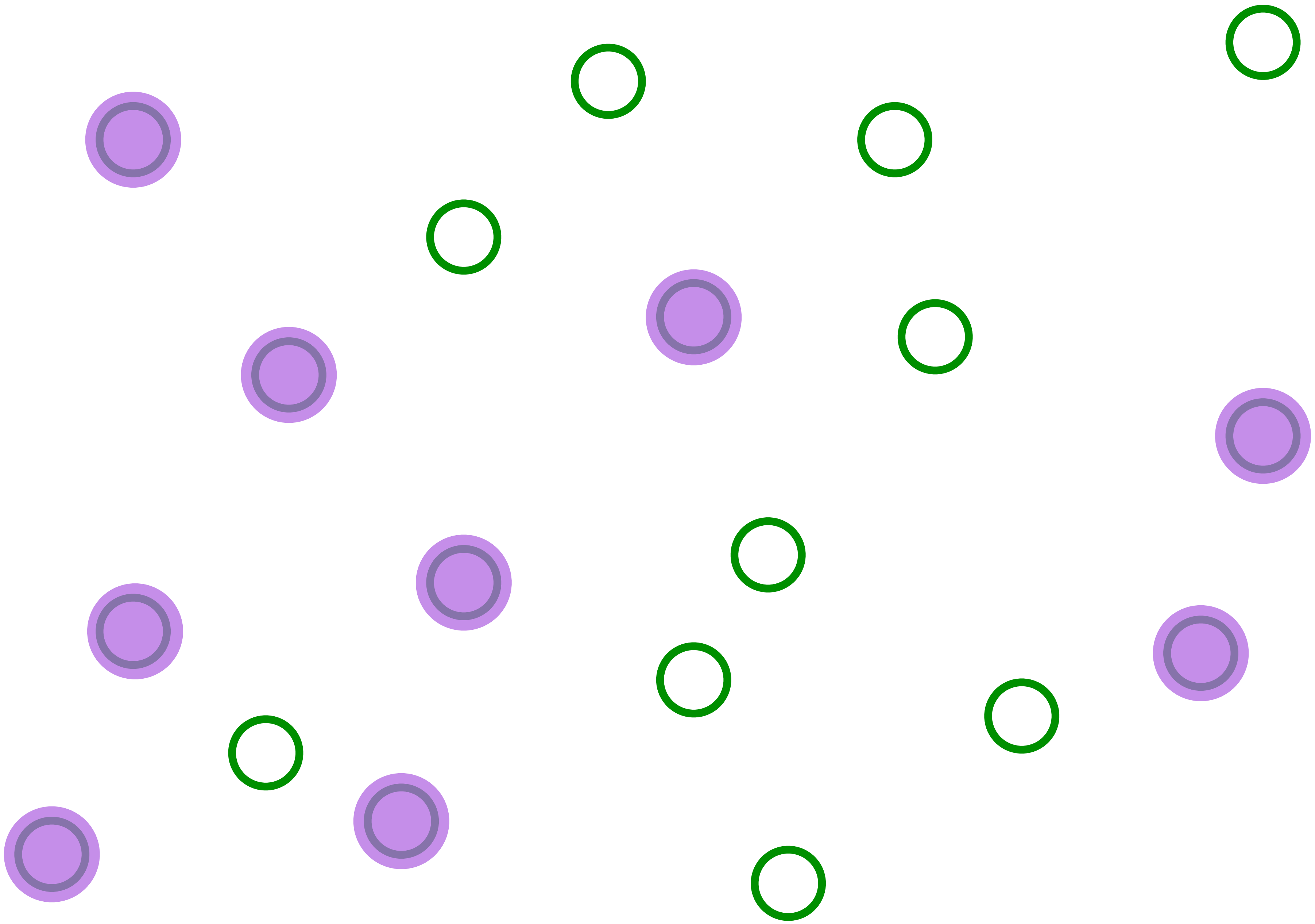
The SYK model



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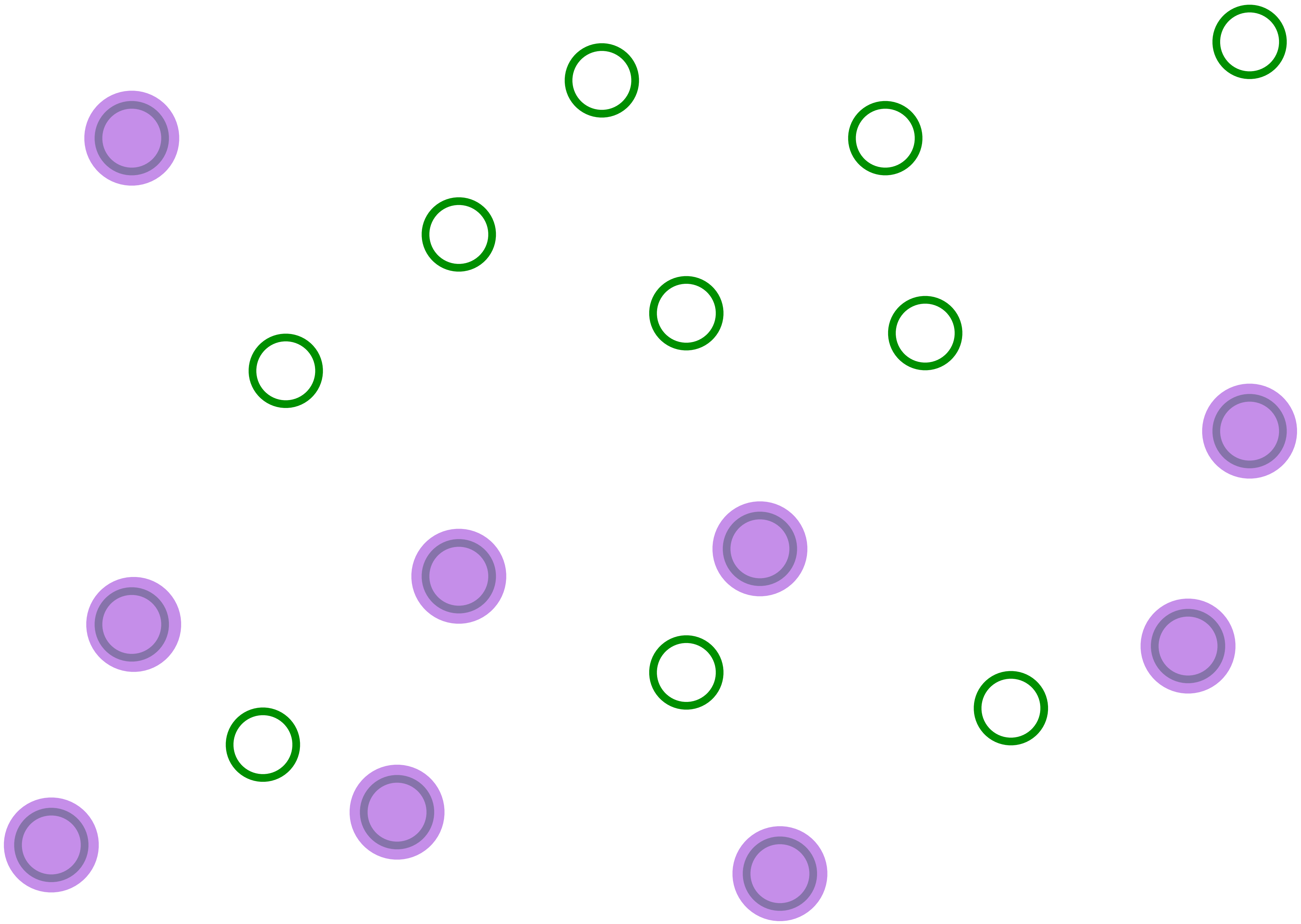

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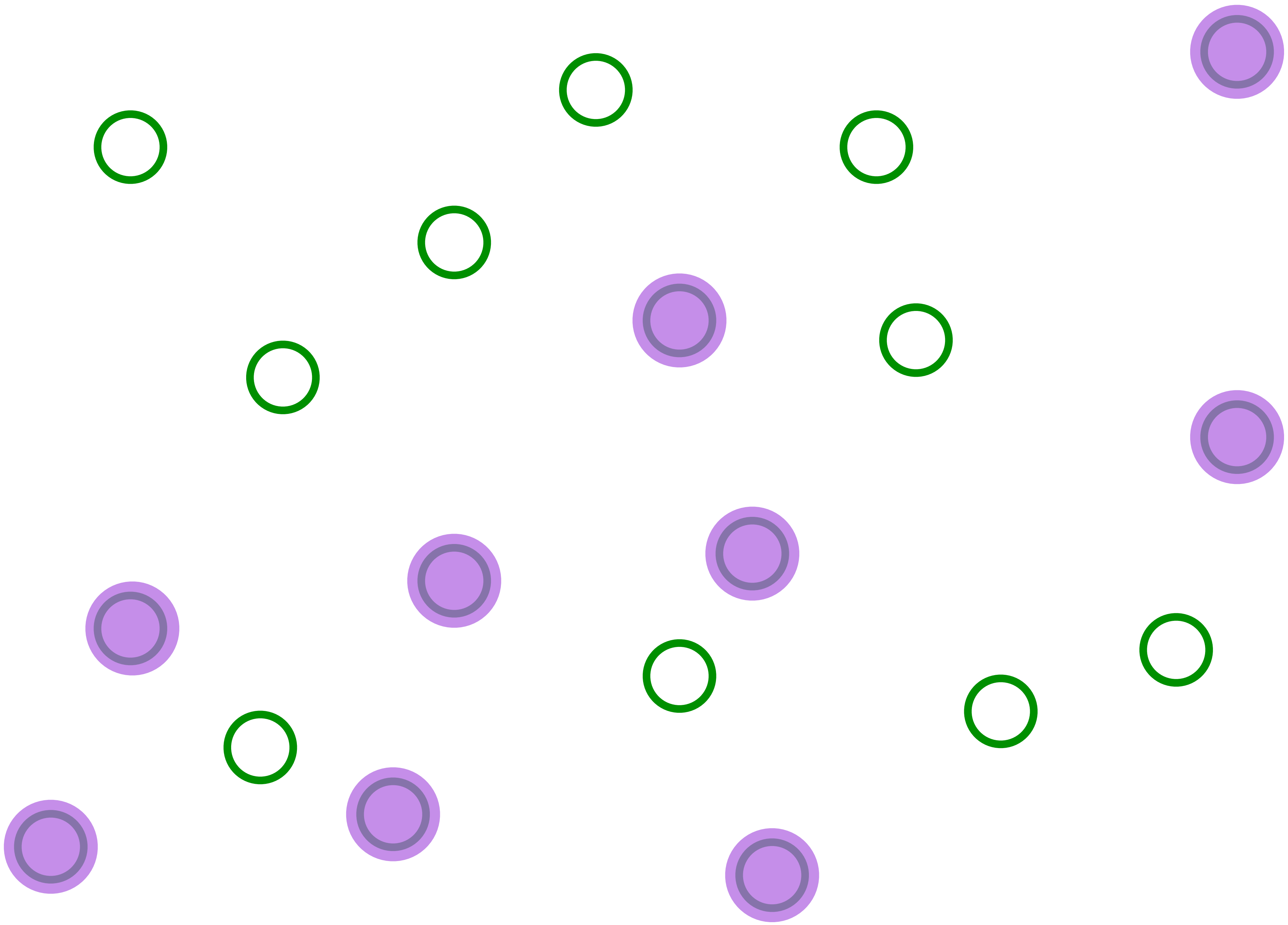

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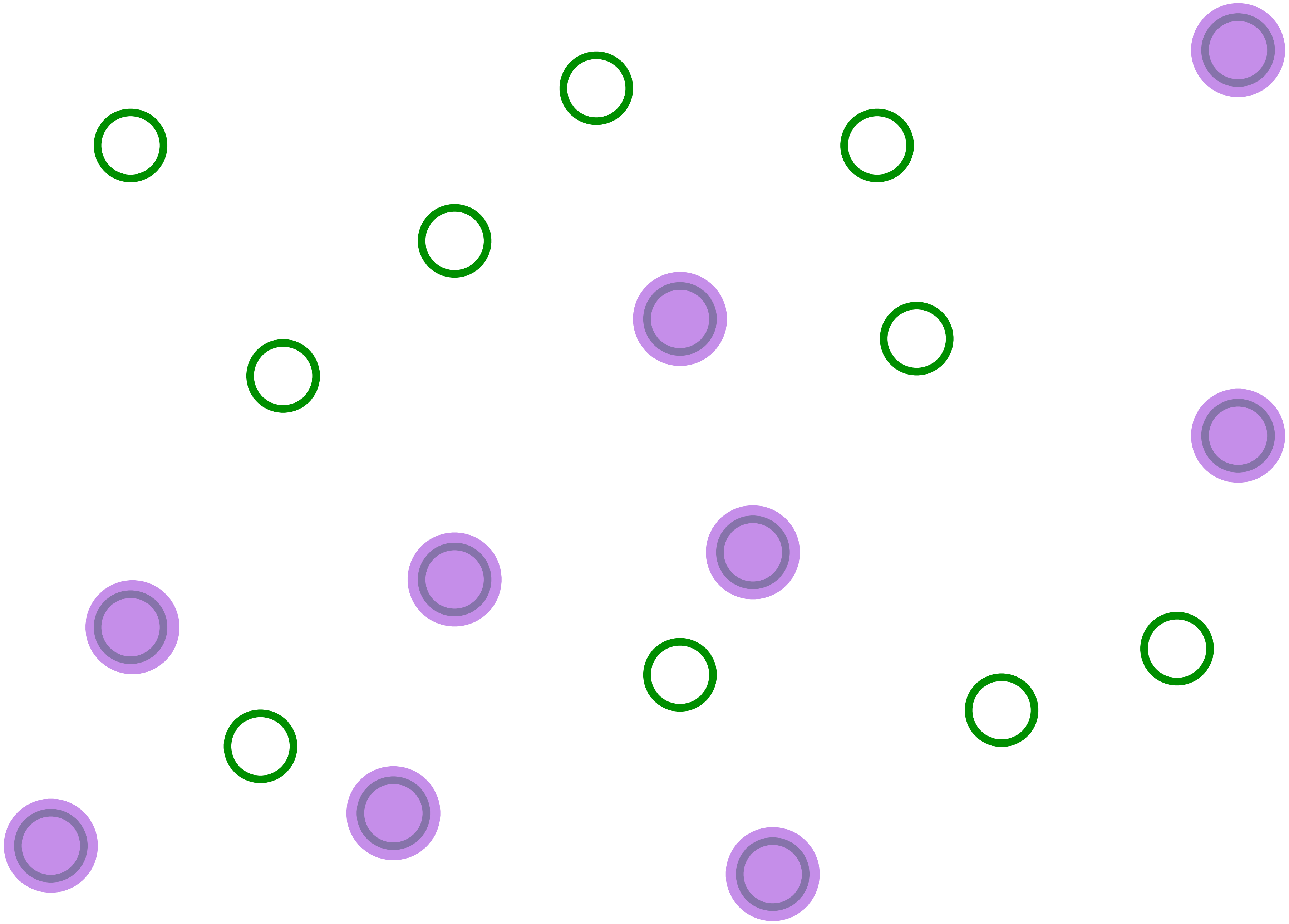


This describes both a strange metal and a black hole!

The SYK model

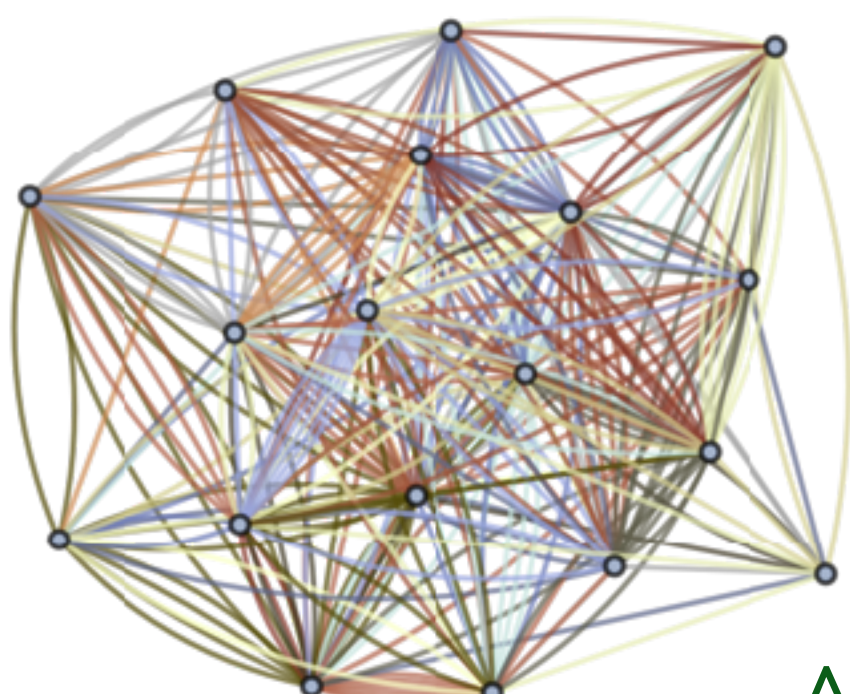
(See also: the "2-Body Random Ensemble" in nuclear physics; did not obtain the large *N* limit;
T.A. Brody, J. Flores, J.B. French, P.A. Mello, A. Pandey, and S.S.M. Wong, Rev. Mod. Phys. **53**, 385 (1981))

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} U_{ij;k\ell}\, c_i^\dagger c_j^\dagger c_k c_\ell - \mu \sum_i c_i^\dagger c_i$$

$$c_i c_j + c_j c_i = 0 \quad , \quad c_i c_j^\dagger + c_j^\dagger c_i = \delta_{ij}$$

$$\mathcal{Q} = \frac{1}{N} \sum_i c_i^\dagger c_i$$

$U_{ij;k\ell}$ are independent random variables with $\overline{U_{ij;k\ell}} = 0$ and $\overline{|U_{ij;k\ell}|^2} = U^2$
$N \to \infty$ yields critical strange metal.



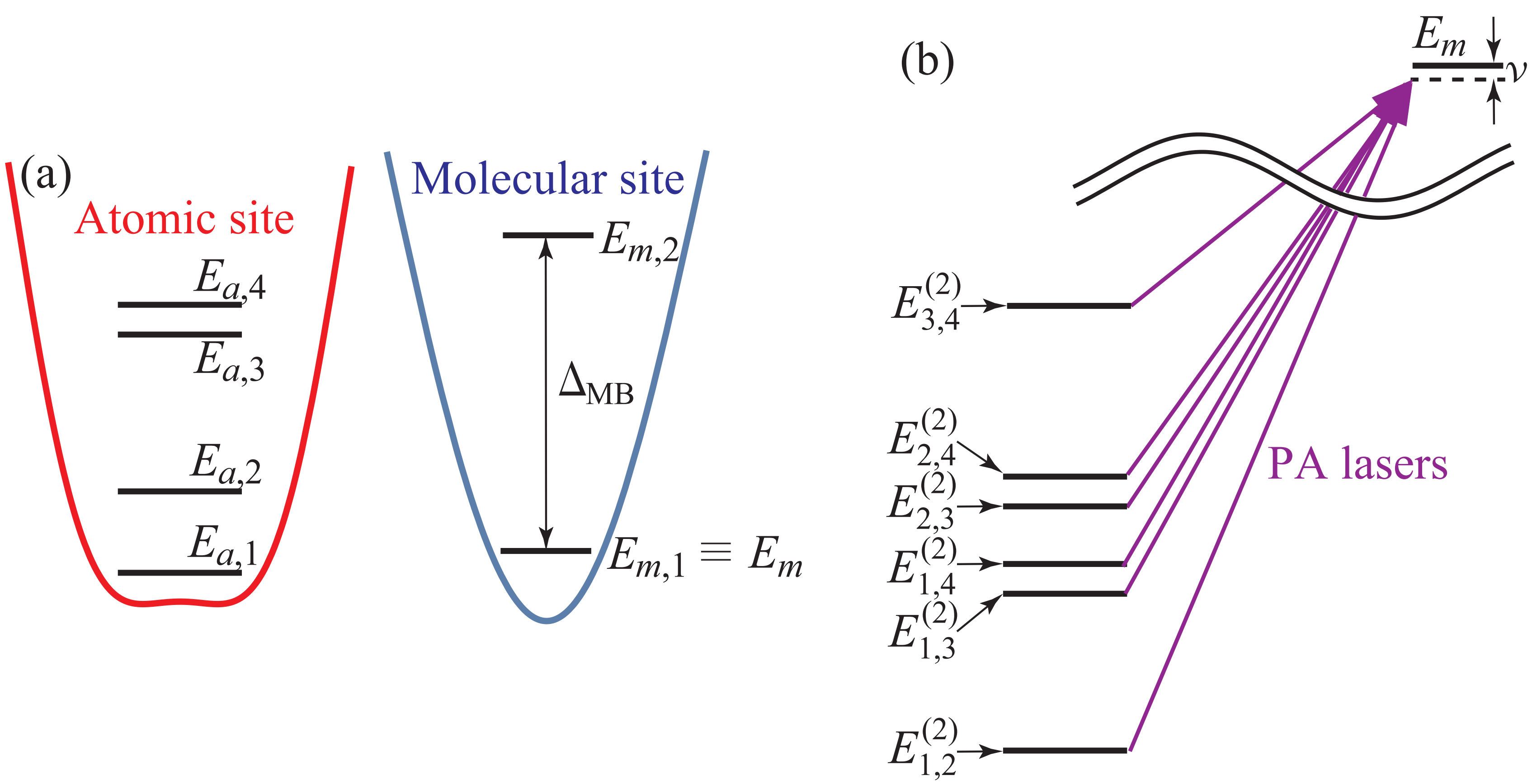
S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

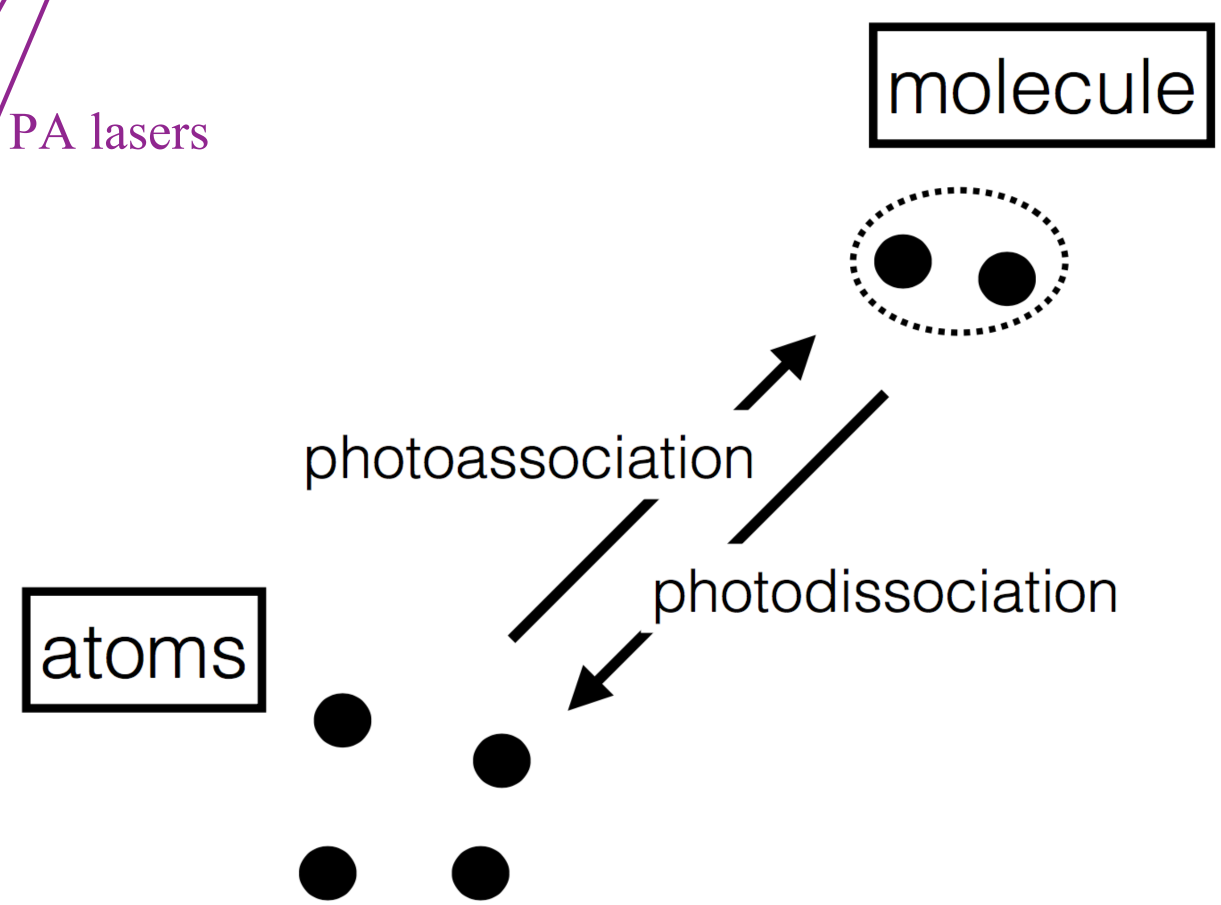
A. Kitaev, unpublished; S. Sachdev, PRX **5**, 041025 (2015)

Creating and probing the Sachdev-Ye-Kitaev model with ultracold gases: Towards experimental studies of quantum gravity

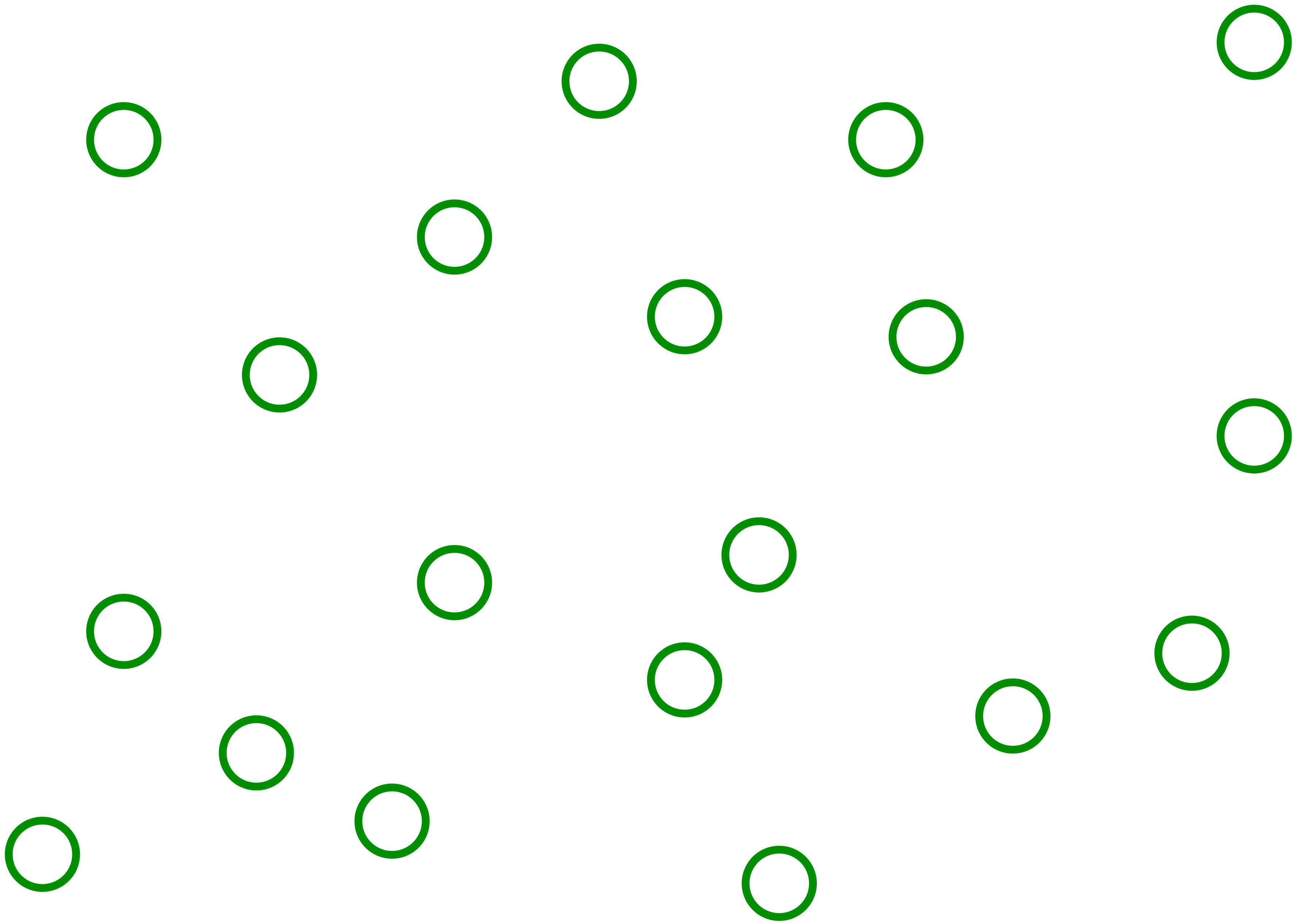
Prog. Theor. Exp. Phys. 2017, 083I01

Ippei Danshita[1,*], Masanori Hanada[1,2,3], and Masaki Tezuka[4]



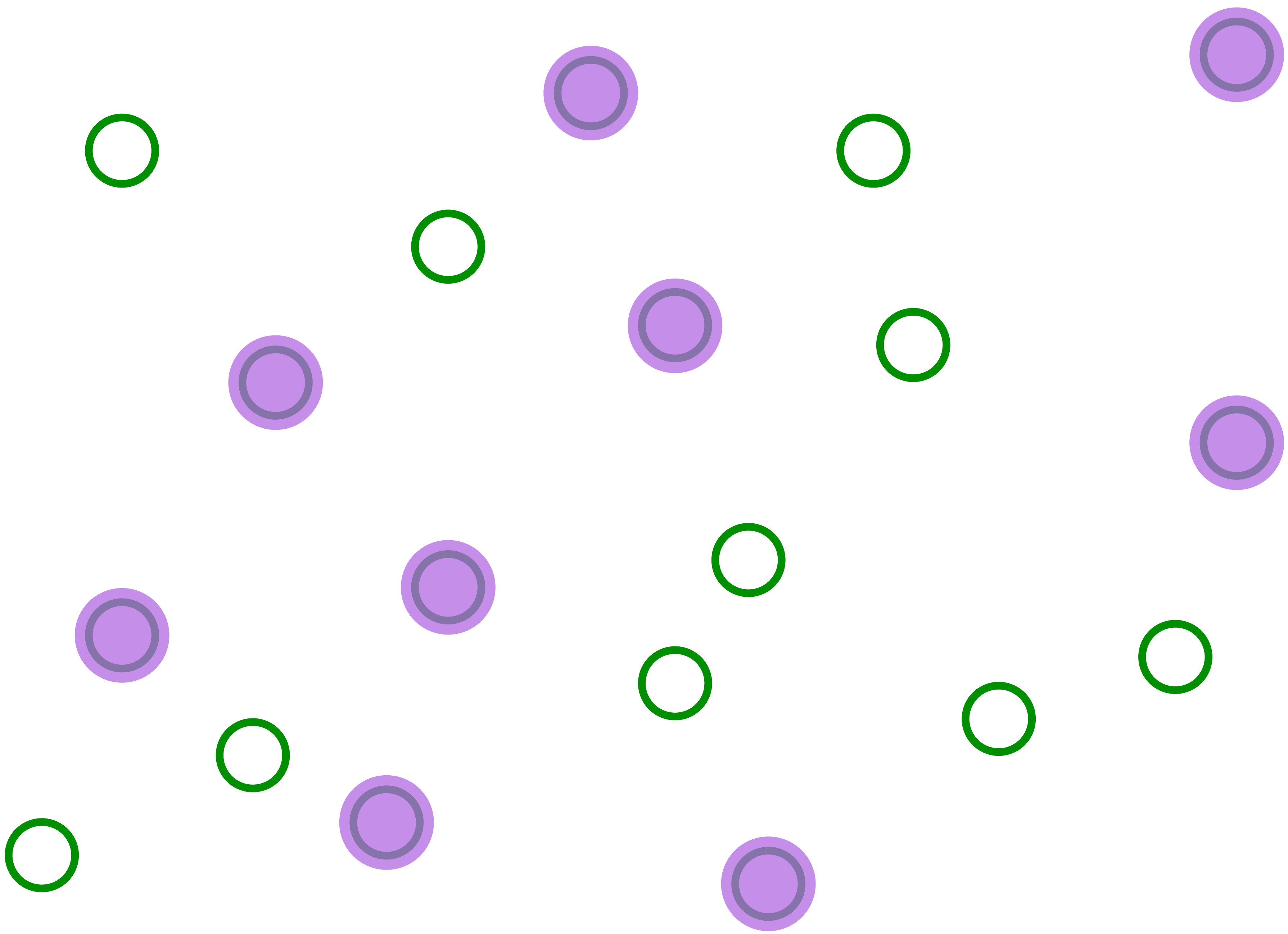


A simple model of a metal with quasiparticles



Pick a set of random positions

A simple model of a metal with quasiparticles



Place electrons randomly on some sites

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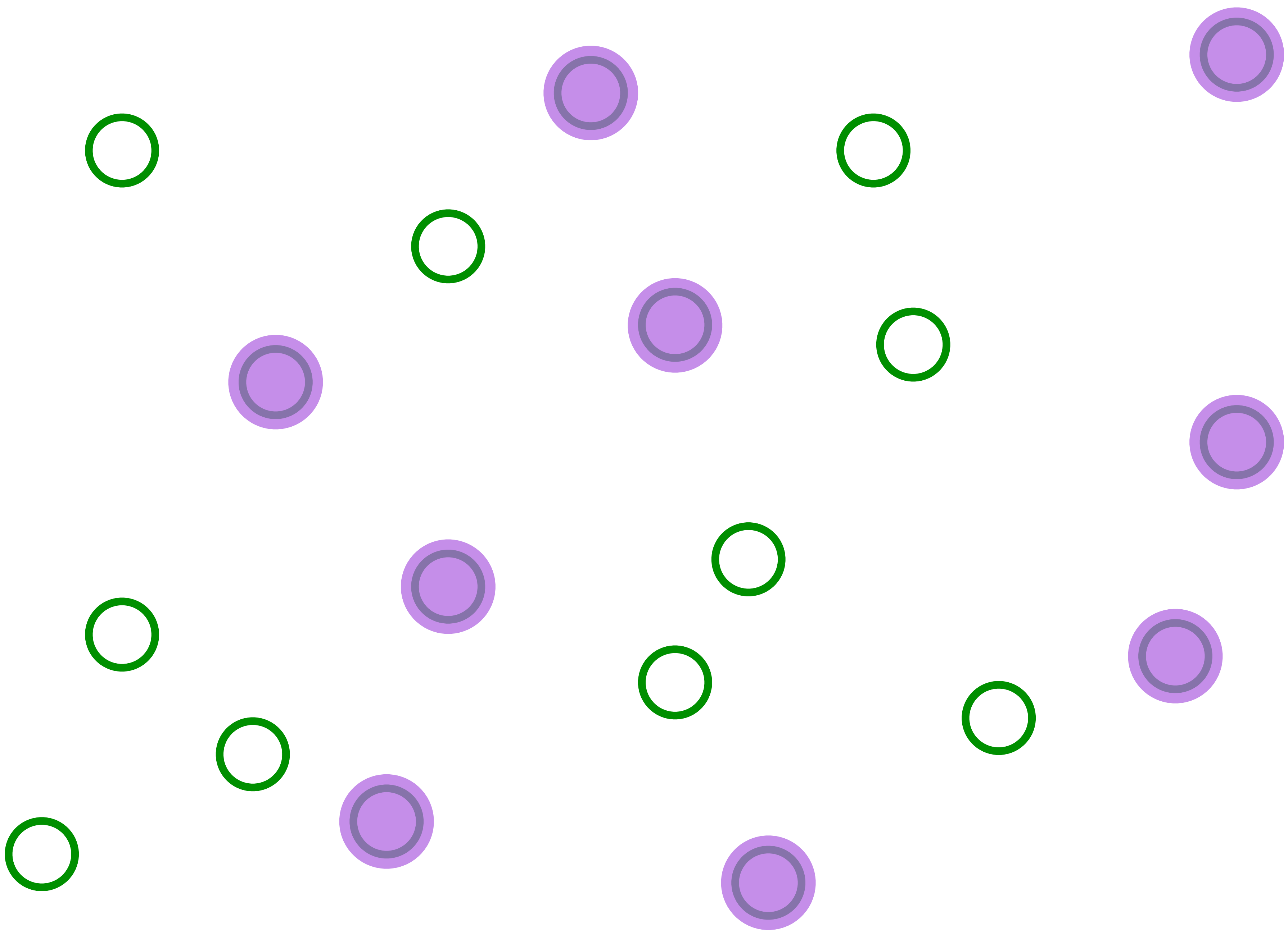
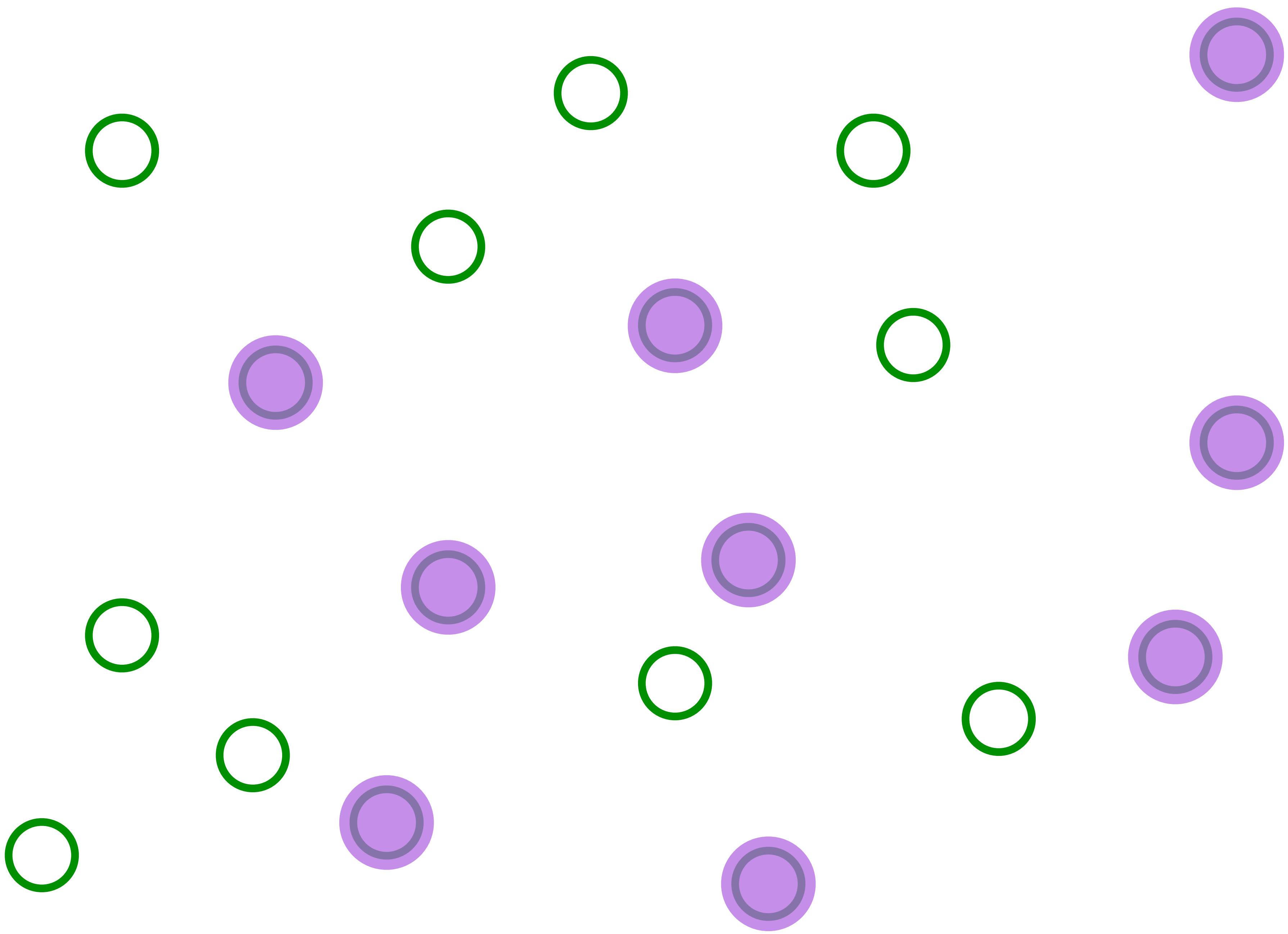


Electrons move one-by-one randomly

A simple model of a metal with quasiparticles



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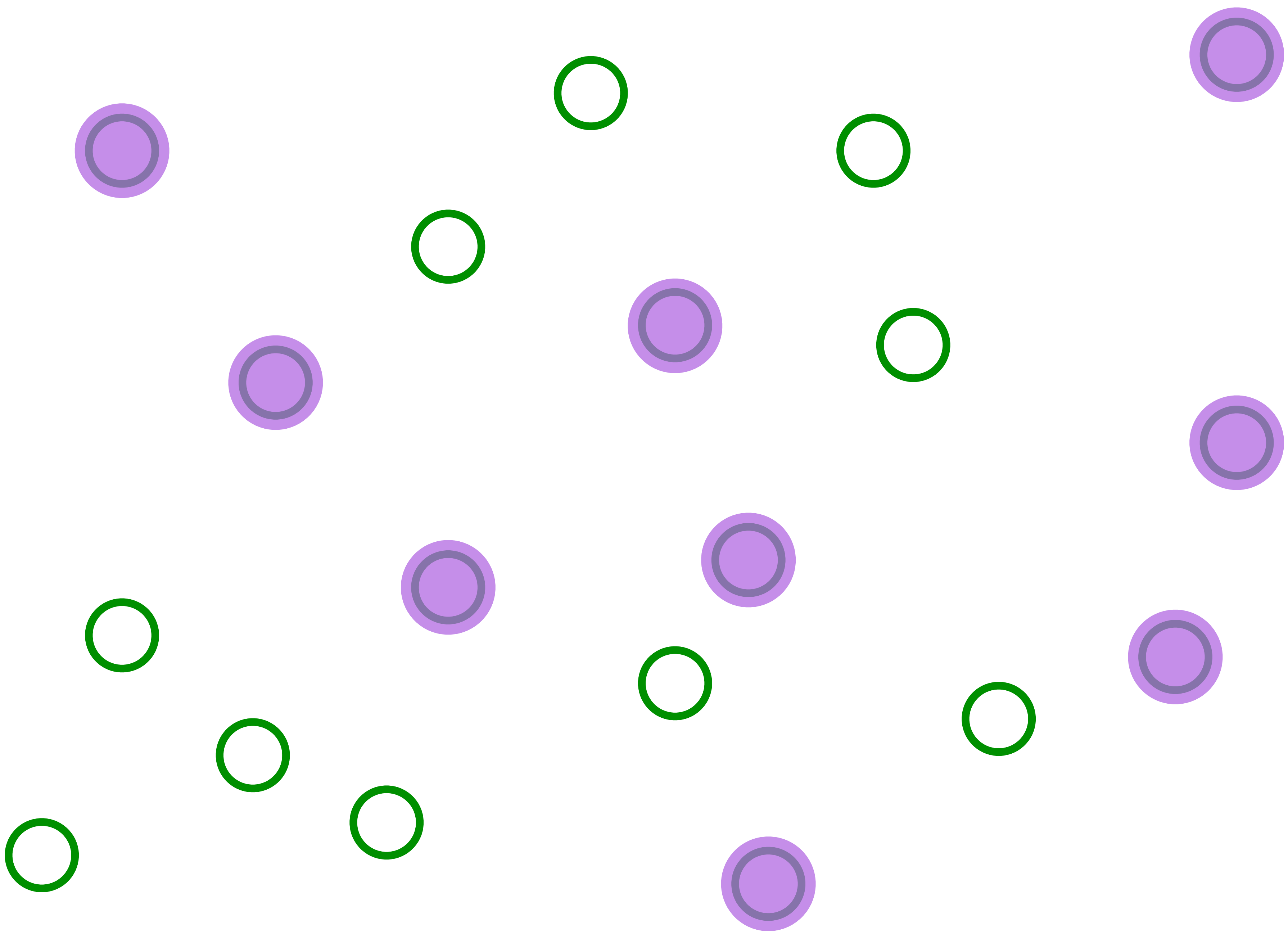

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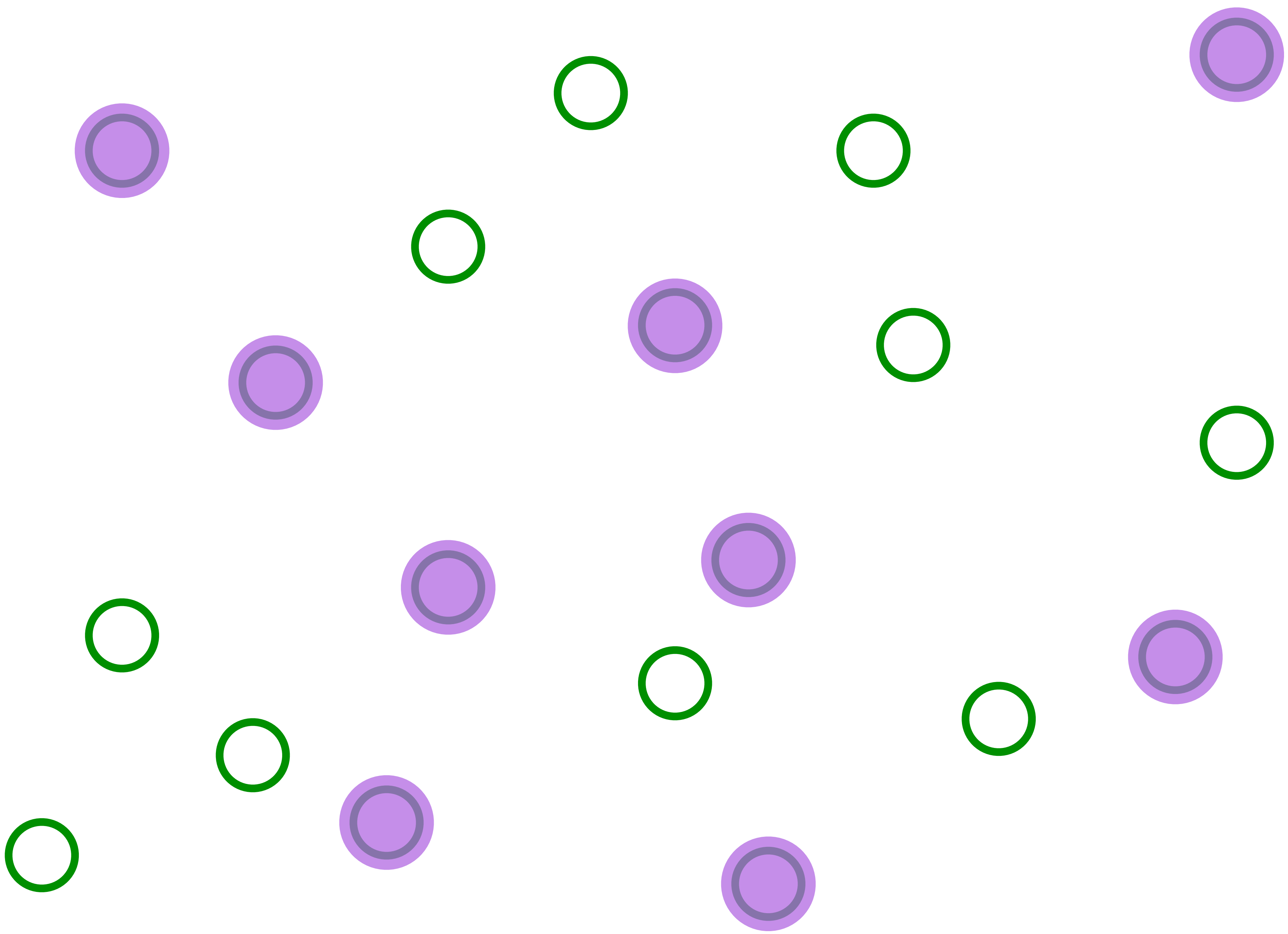

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$$H = \frac{1}{(N)^{1/2}} \sum_{i,j=1}^{N} t_{ij} c_i^\dagger c_j + \ldots$$

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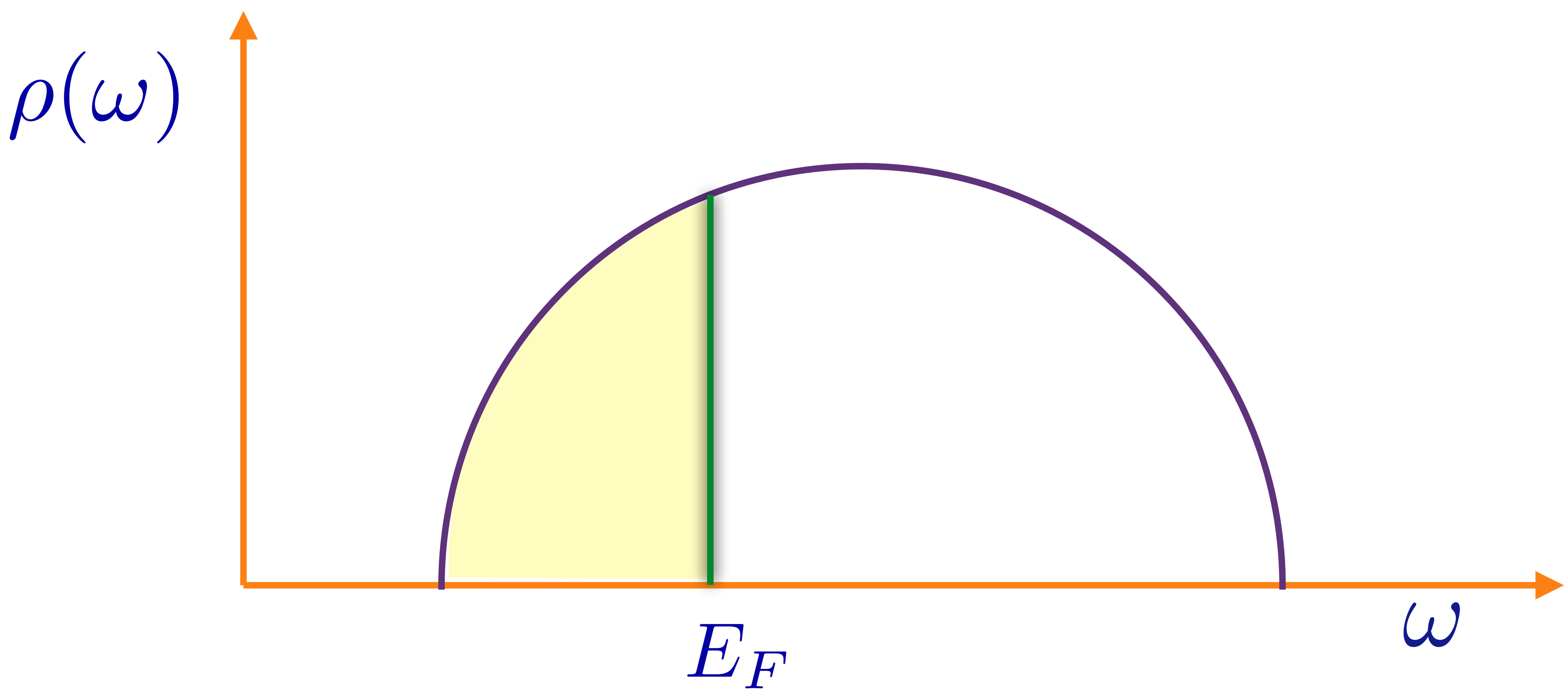
$$\frac{1}{N} \sum_i c_i^\dagger c_i = \mathcal{Q}$$

t_{ij} are independent random variables with $\overline{t_{ij}} = 0$ and $\overline{|t_{ij}|^2} = t^2$

Fermions occupying the eigenstates of a
N x *N* random matrix

A simple model of a metal with quasiparticles

Let ε_α be the eigenvalues of the matrix $t_{ij}/\sqrt{N}$. The fermions will occupy the lowest NQ eigenvalues, upto the Fermi energy E_F. The density of states is $\rho(\omega) = (1/N)\sum_\alpha \delta(\omega - \varepsilon_\alpha)$.



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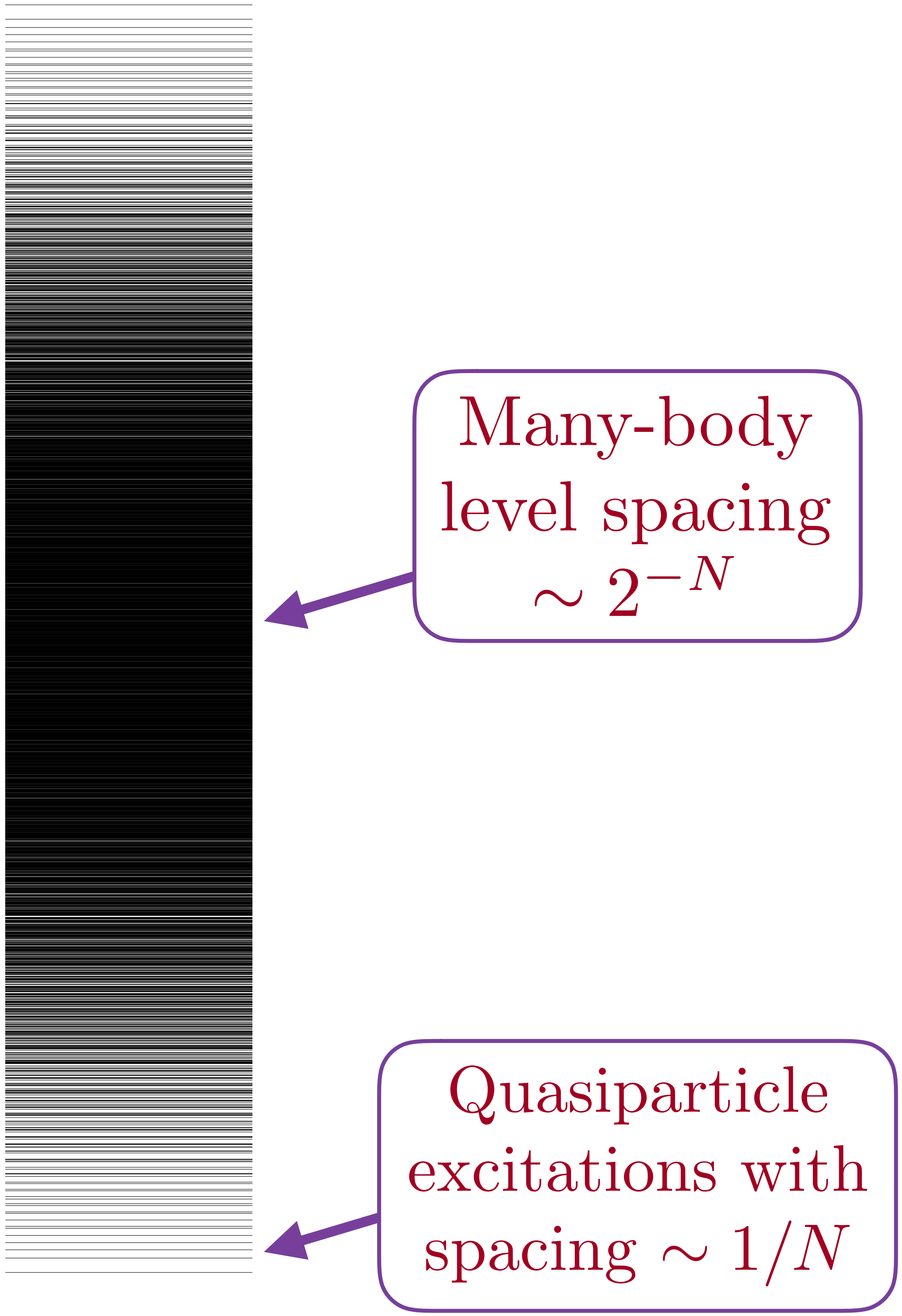

There are 2^N many body levels with energy

$$E = \sum_{\alpha=1}^{N} n_\alpha \varepsilon_\alpha,$$

where $n_\alpha = 0, 1$. Shown are all values of E for a single cluster of size $N = 12$. The ε_α have a level spacing $\sim 1/N$.

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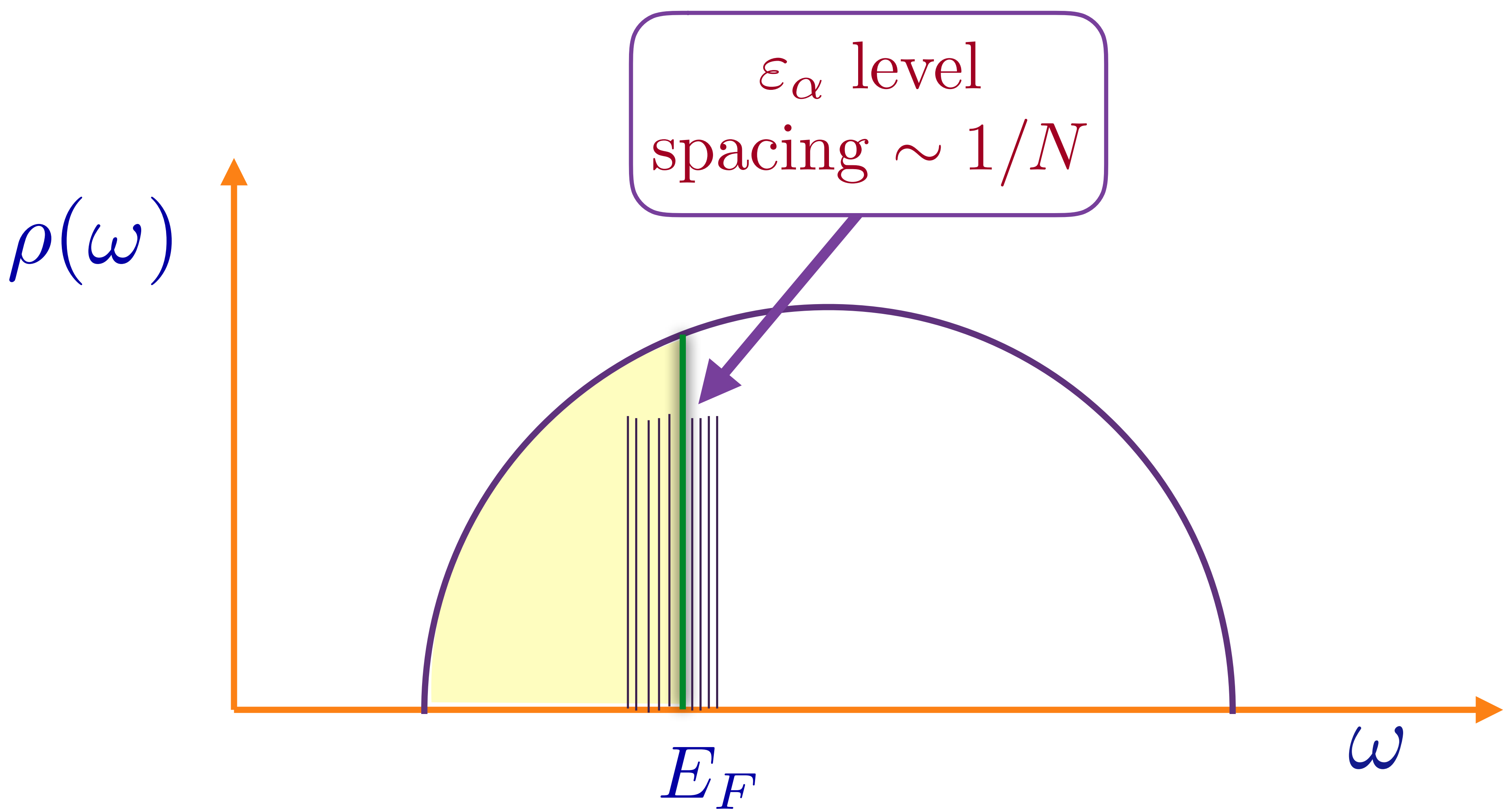

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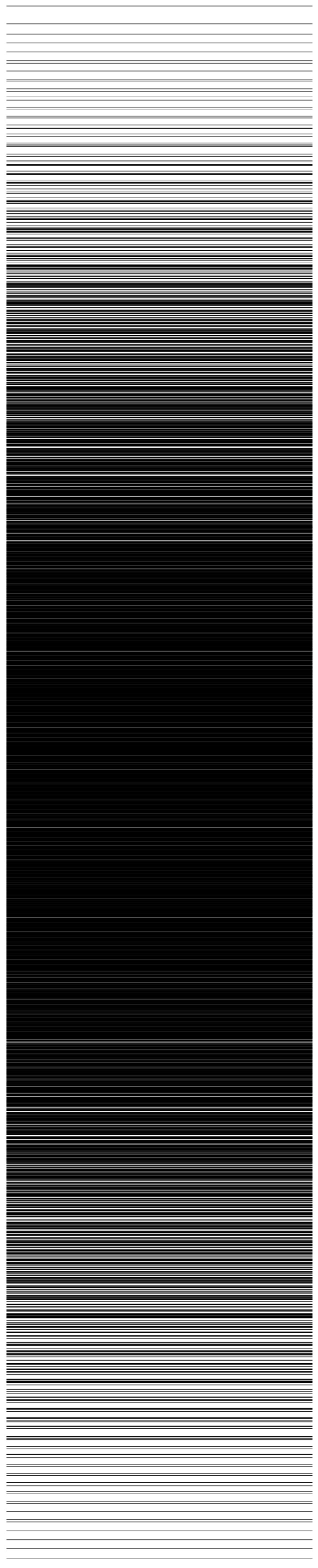

Many-body level spacing $\sim 2^{-N}$

Quasiparticle excitations with spacing $\sim 1/N$

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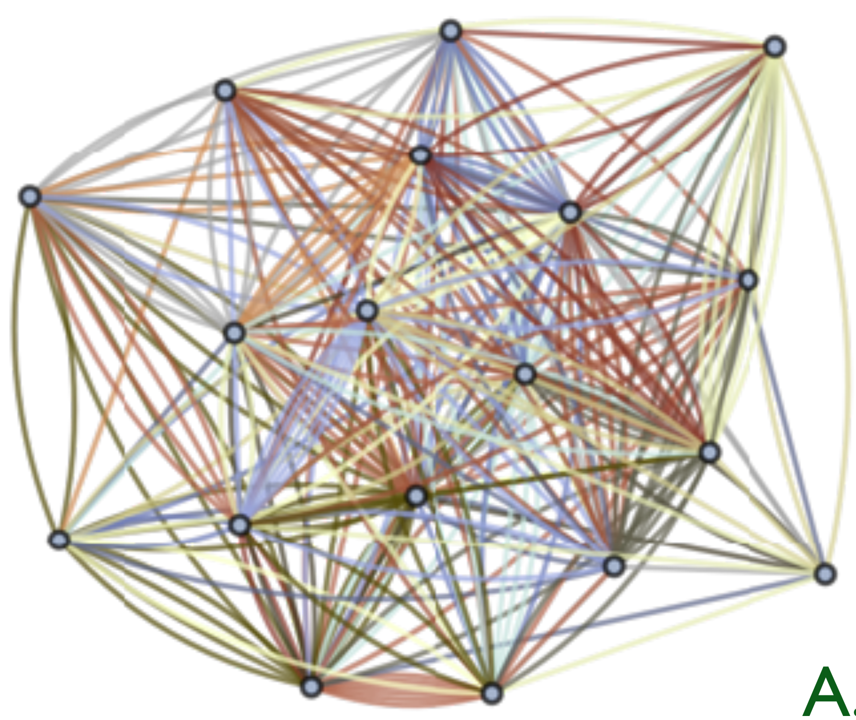

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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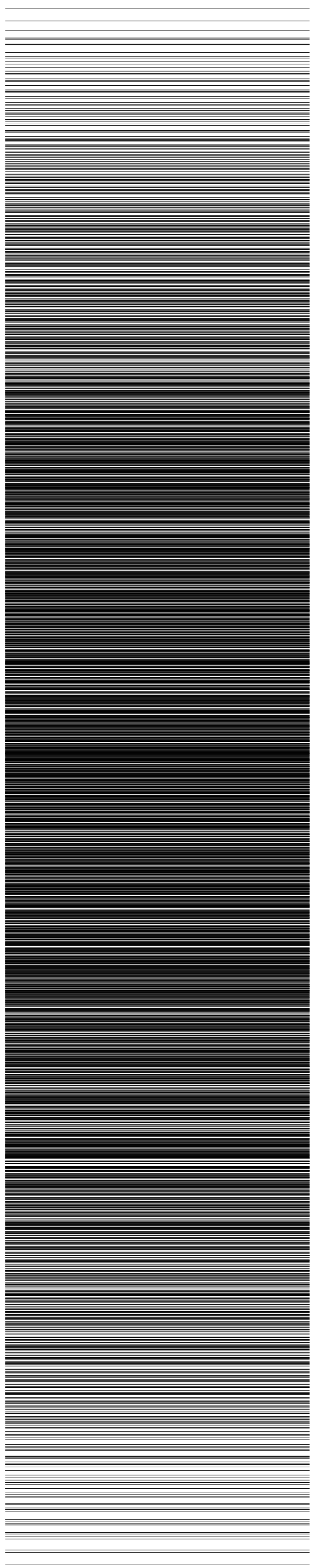
The SYK model



Many-body level spacing $\sim 2^{-N} = e^{-N \ln 2}$

Non-quasiparticle excitations with spacing $\sim e^{-N s_0}$

There are 2^N many body levels with energy E, which do not admit a quasiparticle decomposition. Shown are all values of E for a single cluster of size $N = 12$. The $T \to 0$ state has an entropy $S_{GPS} = N s_0$ with

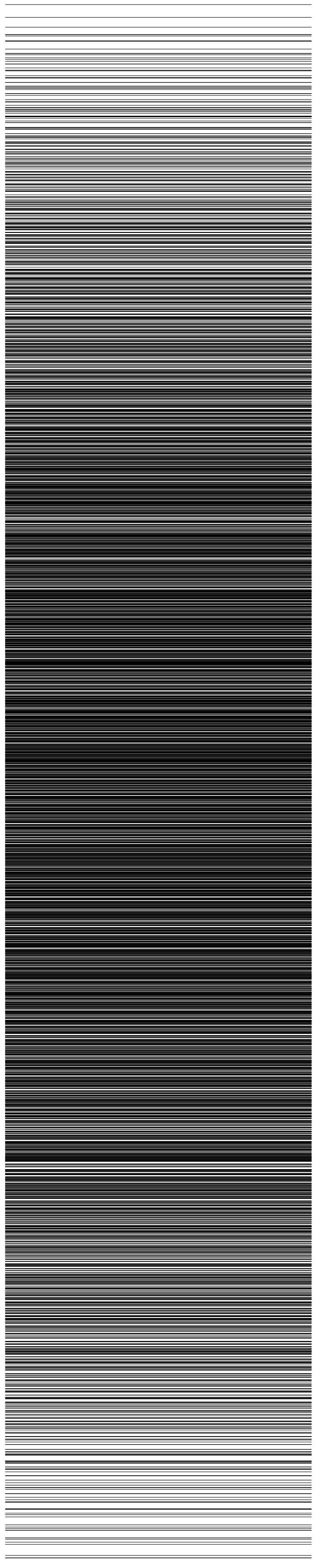
$$
\begin{aligned}
s_0 &= \frac{G}{\pi} + \frac{\ln(2)}{4} = 0.464848\ldots \\
&< \ln 2
\end{aligned}
$$

where G is Catalan's constant, for the half-filled case $\mathcal{Q} = 1/2$.

GPS: A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model



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$$< \ln 2$$

No quasiparticles !

$E \neq \sum_\alpha n_\alpha \varepsilon_\alpha$

$+ \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$

ev,

PRB **63**, 134406 (2001)

W. Fu and S. Sachdev, PRB **94**, 035135 (2016)

The SYK model

- Low energy, many-body density of states $\rho(E) \sim e^{Ns_0} \sinh(\sqrt{2(E-E_0)N\gamma})$

D. Stanford and E. Witten, 1703.04612
A. M. Garica-Garcia, J.J.M. Verbaarschot, 1701.06593
D. Bagrets, A. Altland, and A. Kamenev, 1607.00694
A. Georges, O. Parcollet, and S. Sachdev, PRB **63**, 134406 (2001)

- Low temperature entropy $S = Ns_0 + N\gamma T + \ldots$.

A. Kitaev, unpublished
J. Maldacena and D. Stanford, 1604.07818

- $T = 0$ fermion Green's function $G(\tau) \sim \tau^{-1/2}$ at large τ. (Fermi liquids with quasiparticles have $G(\tau) \sim 1/\tau$)

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- $T > 0$ Green's function has conformal invariance $G \sim (T/\sin(\pi k_B T\tau/\hbar))^{1/2}$

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A. Georges and O. Parcollet PRB **59**, 5341 (1999)

- Study of non-equilibrium quench dynamics shows $\tau_{\rm eq} \sim \hbar/(k_B T)$.

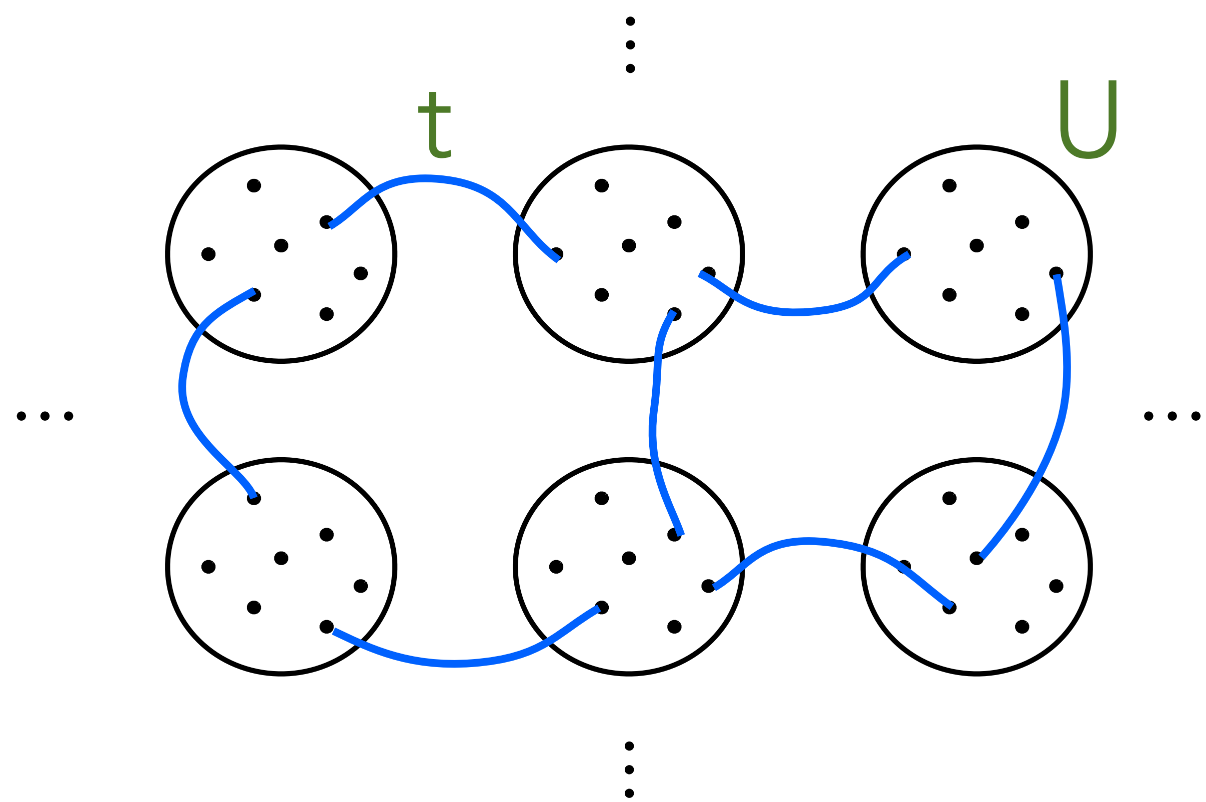
A. Eberlein, V. Kasper, S. Sachdev, and J. Steinberg, arXiv:1706.07803

See also A. Georges and O. Parcollet PRB **59**, 5341 (1999)

arXiv:1705.00117

Title: A strongly correlated metal built from Sachdev-Ye-Kitaev models

Authors: Xue-Yang Song, Chao-Ming Jian, Leon Balents



$$H = \sum_x \sum_{i<j,k<l} U_{ijkl,x} c^\dagger_{ix} c^\dagger_{jx} c_{kx} c_{lx} + \sum_{\langle xx' \rangle} \sum_{i,j} t_{ij,xx'} c^\dagger_{i,x} c_{j,x'}$$

$$\overline{|U_{ijkl}|^2} = \frac{2U^2}{N^3} \qquad \overline{|t_{ij,x,x'}|^2} = t_0^2/N$$

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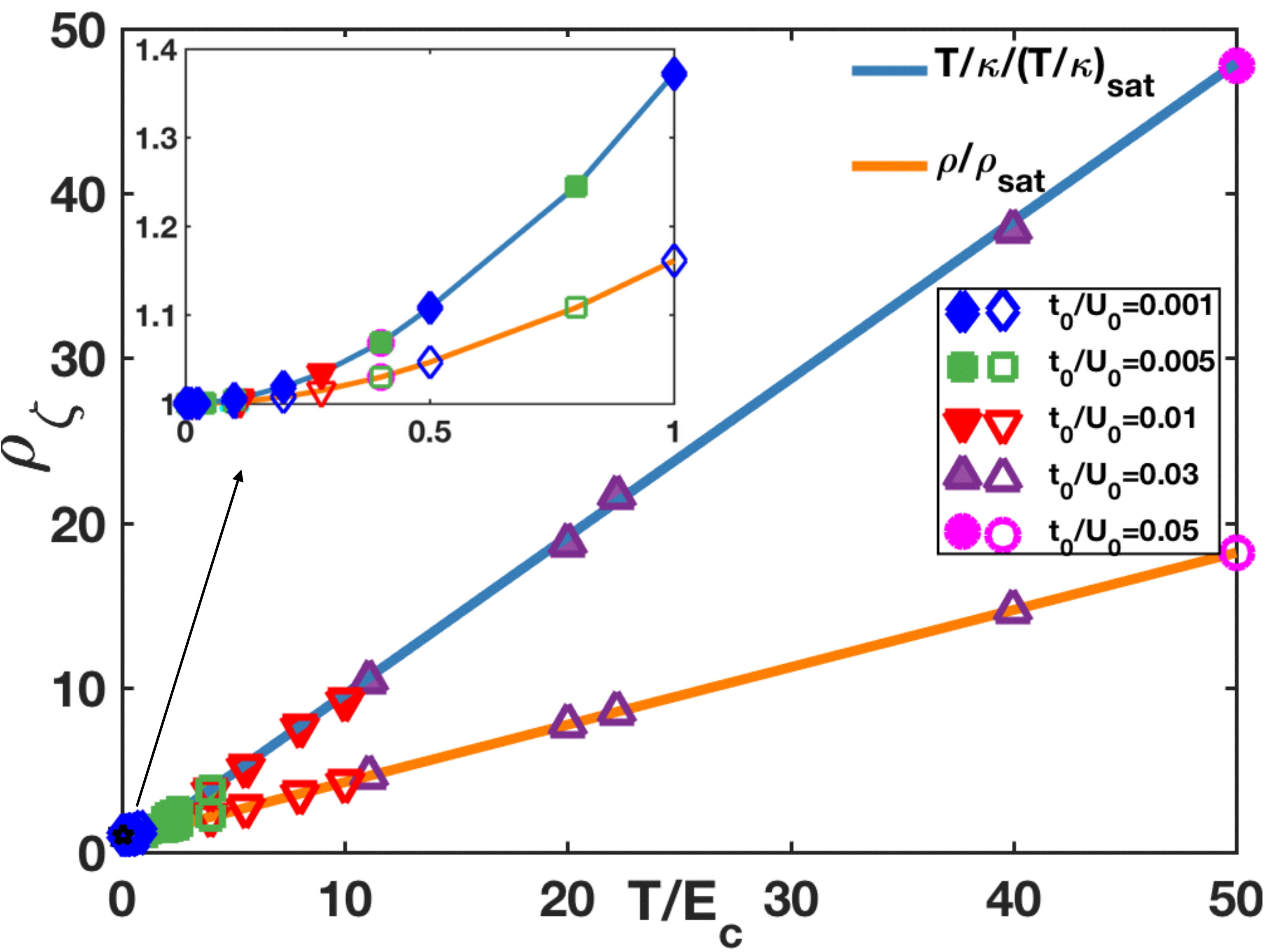
Low 'coherence' scale



$$E_c \sim \frac{t_0^2}{U}$$

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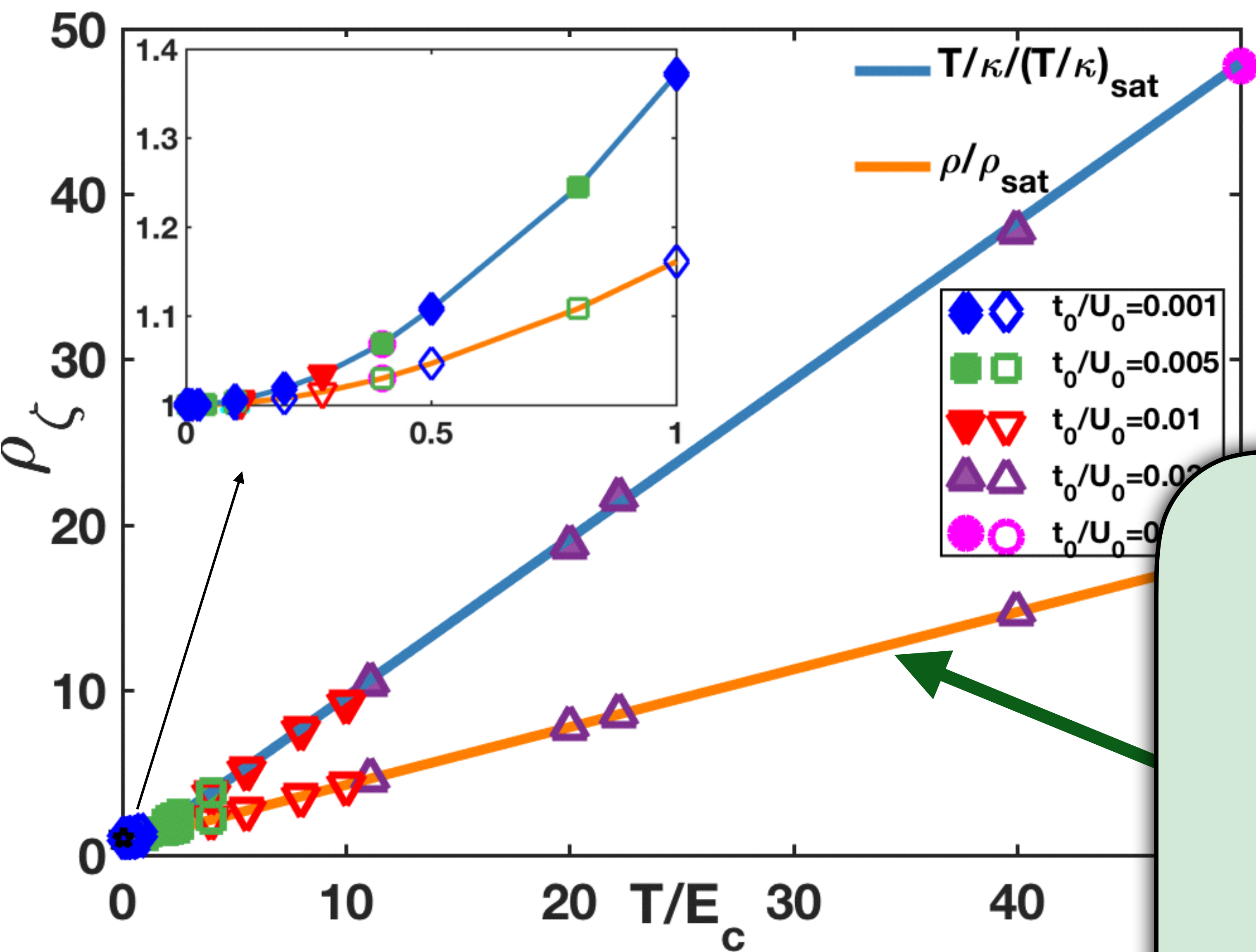

For $E_c < T < U$, the resistivity, ρ, and entropy density, s, are

$$\rho \sim \frac{h}{e^2}\left(\frac{T}{E_c}\right), \quad s = s_0$$

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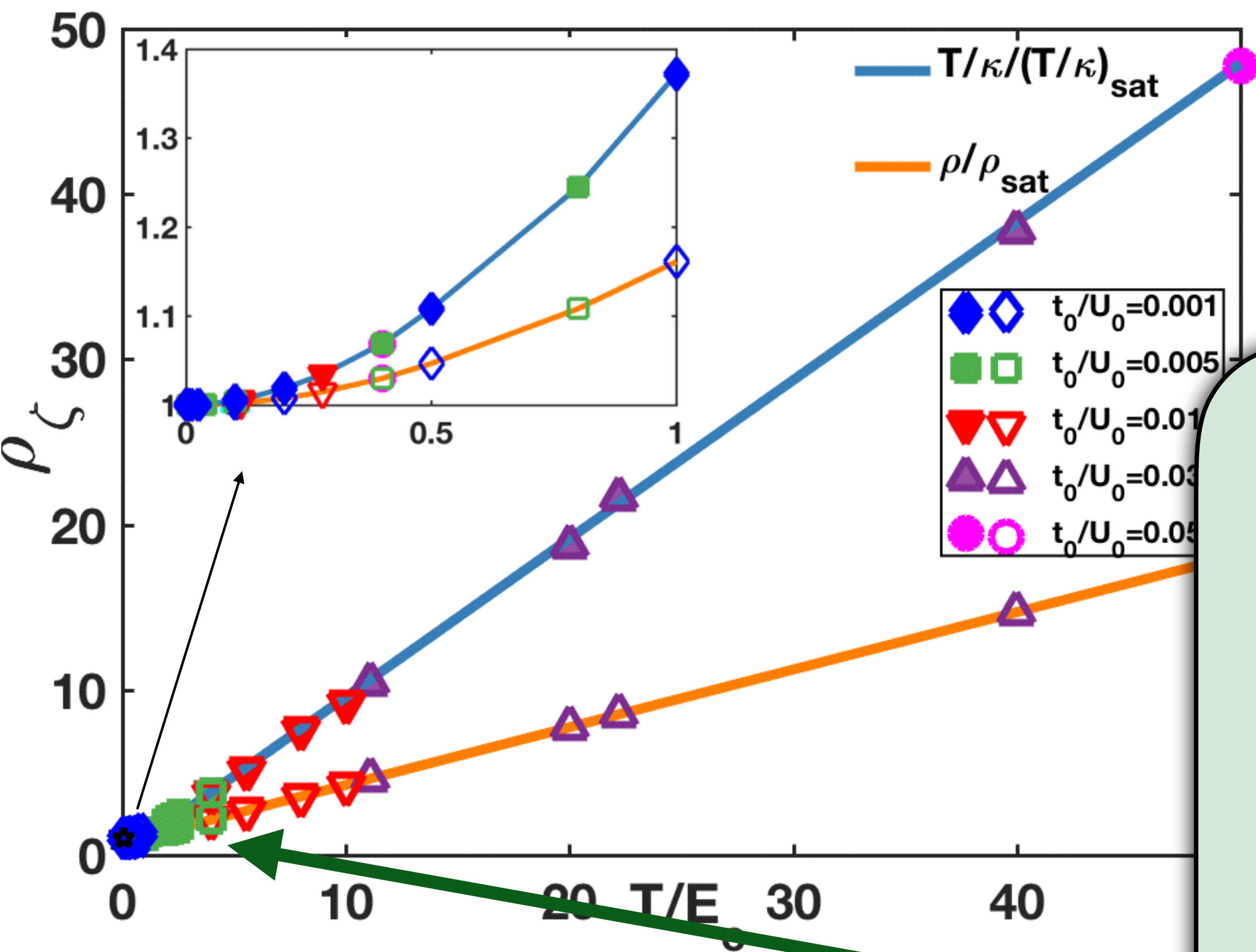


For $T < E_c$, the resistivity, ρ, and entropy density, s, are

$$\rho = \frac{h}{e^2}\left[c_1 + c_2\left(\frac{T}{E_c}\right)^2\right]$$

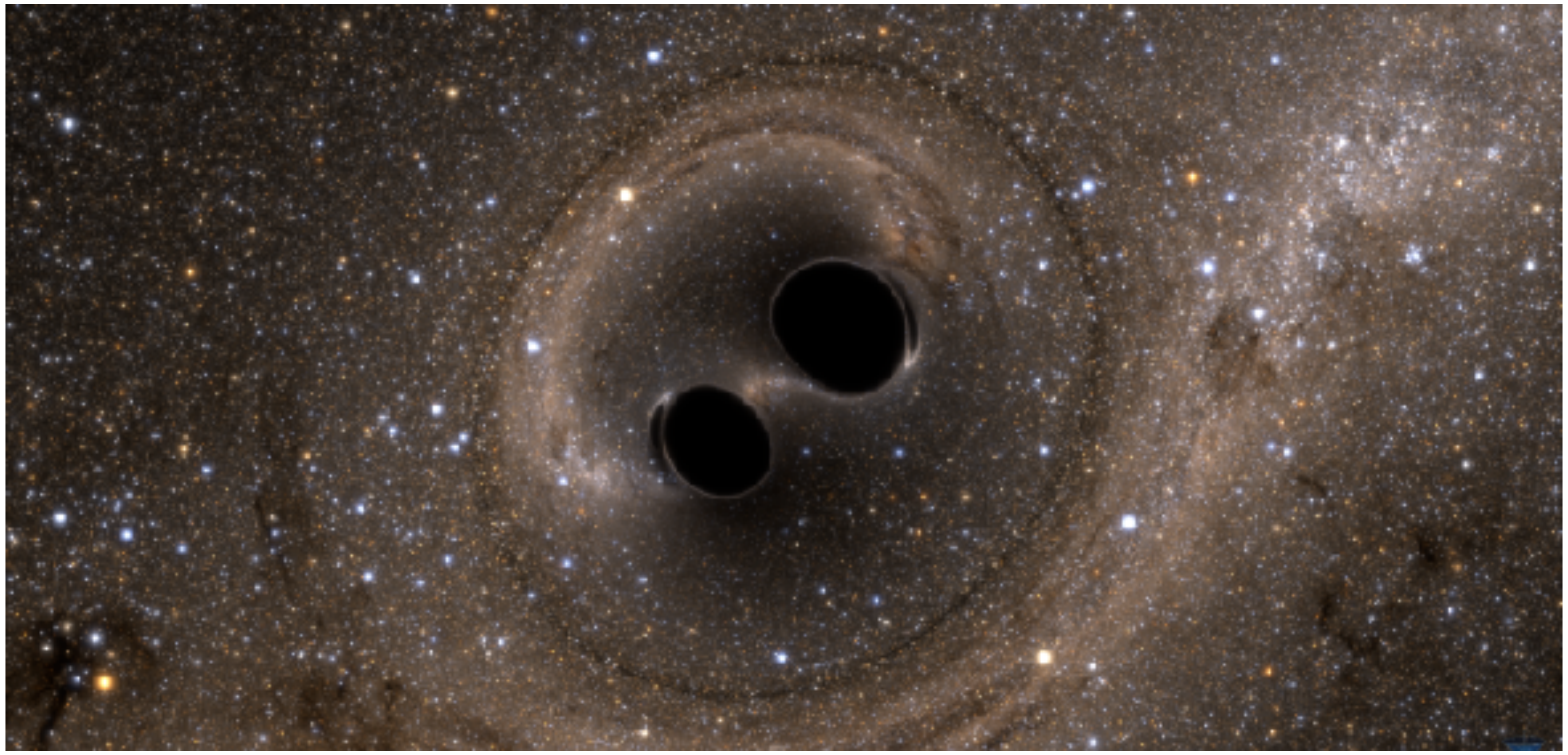
$$s \sim s_0\left(\frac{T}{E_c}\right)$$

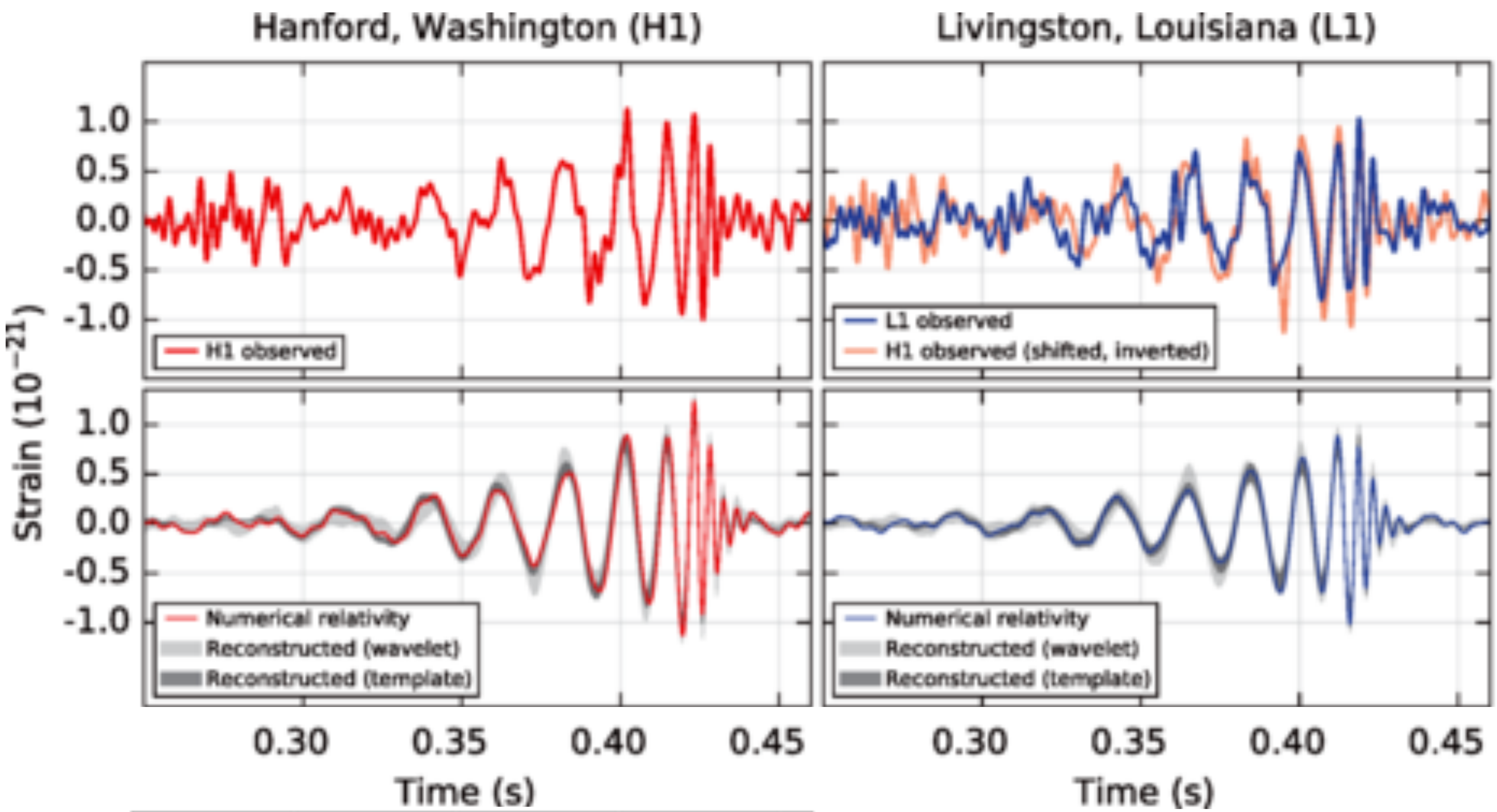
Quantum matter without quasiparticles:

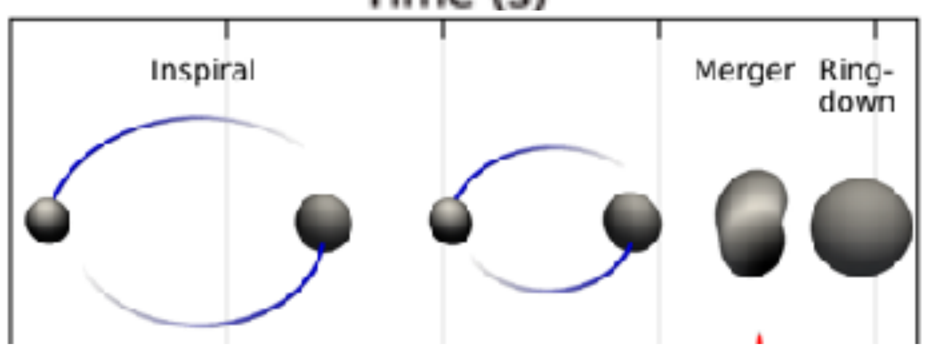
- No quasiparticle decomposition of low-lying states: $E \neq \sum_\alpha n_\alpha \varepsilon_\alpha + \sum_{\alpha,\beta} F_{\alpha\beta} n_\alpha n_\beta + \ldots$
- Thermalization and many-body chaos in the shortest possible time of order $\hbar/(k_B T)$.
- These are also characteristics of black holes in quantum gravity.

Black holes

- Black holes have an entropy and a temperature, T_H.
- The entropy is proportional to their surface area.
- They relax to thermal equilibrium in a time $\sim \hbar/(k_B T_H)$.





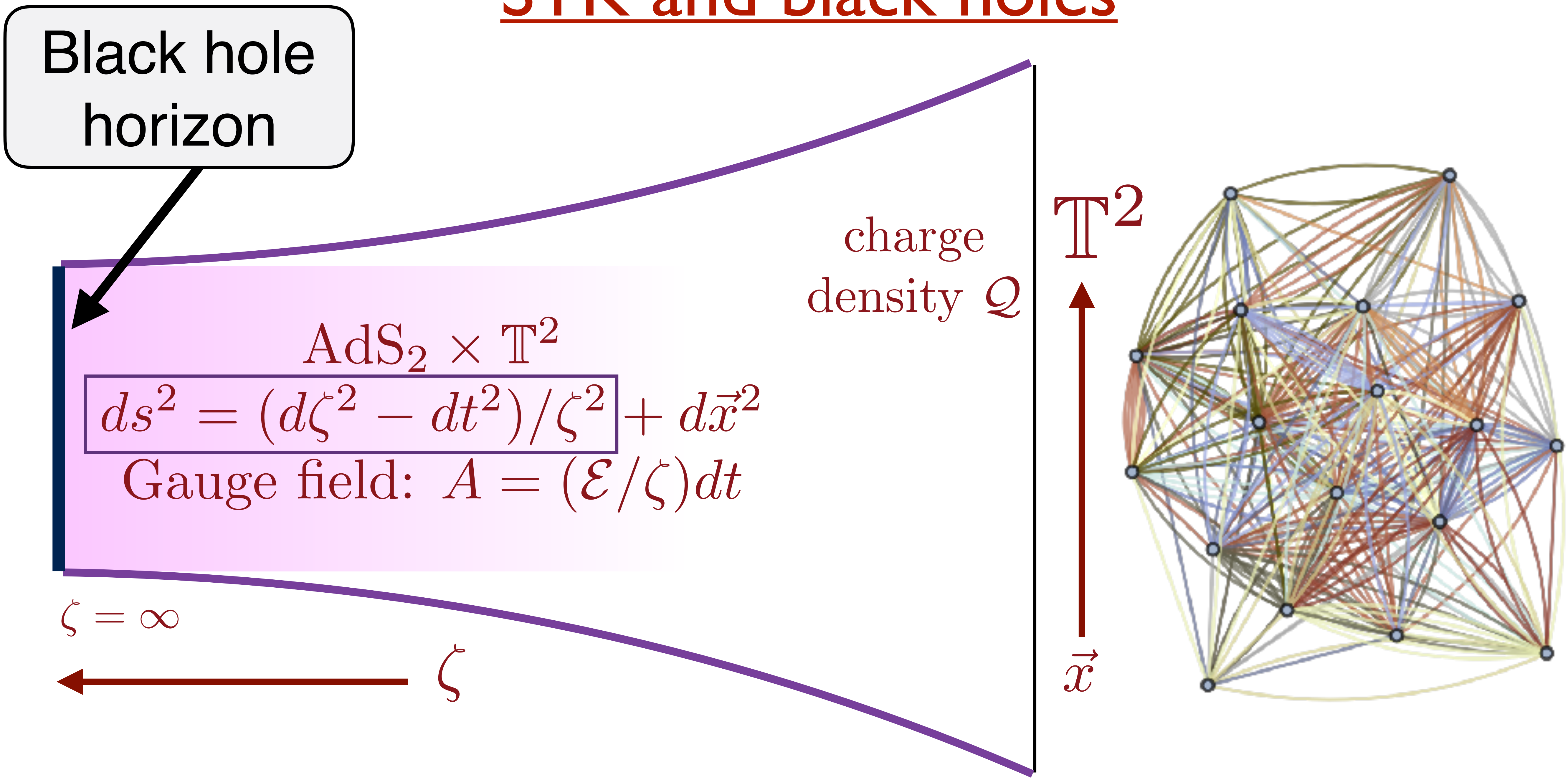


LIGO

September 14, 2015

- The Hawking temperature, T_H influences the radiation from the black hole at the very last stages of the ring-down (not observed so far). The ring-down (approach to thermal equilibrium) happens very rapidly in a time $\sim \dfrac{\hbar}{k_B T_H} = \dfrac{8\pi G M}{c^3} \sim 8$ milliseconds.

SYK and black holes



Quantum gravity on the 1+1 dimensional spacetime AdS_2 (when embedded in AdS_4) is holographically matched to the 0+1 dimensional SYK model

S. Sachdev, PRL **105**, 151602 (2010); A. Kitaev (unpublished); J. Maldacena, D. Stanford, and Zhenbin Yang, arXiv:1606.01857