# Universal theory of strange metals from spatially random interactions

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arXiv:
2103.08615
2203.04990
2207.08841

Fermi surface coupled to a critical boson:
No spatial disorder
*A non-Fermi liquid but NOT a strange metal*

Fermi surface coupled to a critical boson:
Potential disorder
*A marginal Fermi liquid but NOT a strange metal*

Fermi surface coupled to a critical boson:
Interaction disorder
*A marginal Fermi liquid AND a strange metal*

## Fermi liquids and their cousins: (defined by single-particle properties)

- Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \ll |\varepsilon|$ and a density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.
- Non-Fermi liquids: No quasiparticles. Would-be fermionic quasiparticles have $1/\tau(\varepsilon) \gg |\varepsilon|$ and a density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.
- Marginal Fermi liquids: Fermionic quasiparticles with a lifetime obeying $1/\tau(\varepsilon) \sim |\varepsilon|$ and a density of states $N(\varepsilon) \sim$ constant as $|\varepsilon| \to 0$.

## Properties of a strange metal: (defined by transport and thermo)

- Resistivity $\rho(T) = \rho_0 + AT + \ldots$ as $T \to 0$ and $\rho(T) < h/e^2$ (in $d = 2$). Metals with $\rho(T) > h/e^2$ are bad metals.
- Specific heat $\sim T \ln(1/T)$ as $T \to 0$.

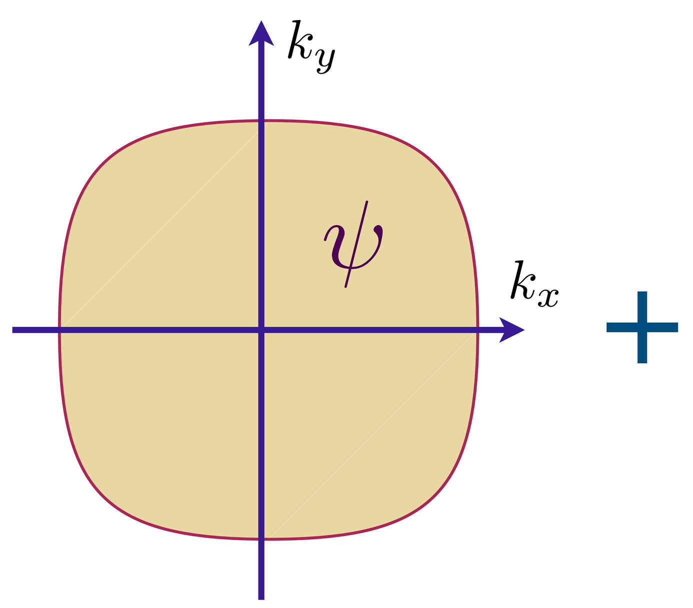
S.A. Hartnoll and A.P. MacKenzie, arXiv:2107.07802

- Optical conductivity

$$\sigma(\omega) = \frac{K}{\dfrac{1}{\tau_{\rm trans}(\omega)} - i\omega \dfrac{m^*(\omega)}{m}} \quad ; \quad \frac{1}{\tau_{\rm trans}(\omega)} = \frac{k_B T}{\hbar} G\left(\frac{\hbar\omega}{k_B T}\right)$$

B. Michon……A. Georges, arXiv:2205.04030

## 3 key ingredients of our universal theory of strange metals:




Fermi surface of fermions $\psi$

### 1. a critical boson $\phi$

One of

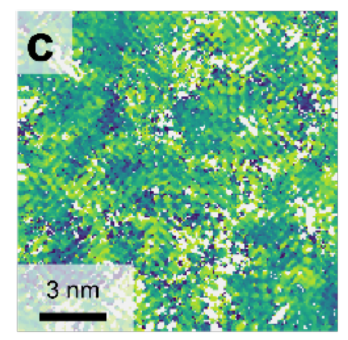
- Nematic order
- Ferromagnetic order
- Transverse component of abelian or non-abelian gauge field
- Antiferromagnetic order…

### 2. Spatially random interactions:

*e.g.* randomness in hopping $t_{ij}$, leads to randomness in exchange interactions $t_{ij}^2/U$. Decoupling such interactions with a $\phi^2$ term which is spatially uniform, we obtain a fermion ($\psi$) and boson ($\phi$) Yukawa coupling of the form

$$\int d^2r d\tau \Big[g + g'(r)\Big] \psi^\dagger(r,\tau)\psi(r,\tau)\phi(r,\tau)\,,$$

where $g$ is spatially uniform and $g'(r)$ is spatially random with zero average. There can also be potential disorder $\int d^2r d\tau\ v(r)\psi^\dagger(r,\tau)\psi(r,\tau)$ but this is not key.




Gap map in cuprate (Tromp et al. arXiv:2205.09740)

### 3. Fermion-boson drag:

For electron-phonon scattering in metals, we have "Bloch's law" (1931): a resistivity $\rho(T) \sim T^5$. However, Bloch's law ignores conservation of total momentum, or **phonon drag**.

In a non-Fermi liquid, we cannot separate the momenta carried by the fermions and the bosons, because neither of them exists at low energies! We must treat the combined system together: *extreme drag*. The analog of Bloch's law does not apply.

## Large $N$ theory

"Yukawa" coupling: $\dfrac{g_{ij\ell}}{N}\displaystyle\int d^2r d\tau\, \psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

Random potential: $+\dfrac{1}{\sqrt{N}}\displaystyle\int d^2r d\tau\, v_{ij}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)$

Random interactions: $+\dfrac{1}{N}\displaystyle\int d^2r d\tau\, g'_{ijl}(r)\psi_i^\dagger(r,\tau)\psi_j(r,\tau)\phi_l(r,\tau)$

$$\overline{g_{ijl}} = 0 \quad , \quad \overline{g_{ijl}^* g_{abc}} = g^2\, \delta_{ia}\delta_{jb}\delta_{lc}$$

$$\overline{v_{ij}(r)} = 0 \quad , \quad \overline{v_{ij}^*(r) v_{lm}(r')} = v^2\, \delta(r-r')\delta_{il}\delta_{jm}$$

$$\overline{g'_{ijl}(r)} = 0 \quad , \quad \overline{g'^*_{ijl}(r) g'_{abc}(r')} = g'^2\, \delta(r-r')\delta_{ia}\delta_{jb}\delta_{lc}$$

Indices $i, j, \ldots = 1 \ldots N$. Obtain SYK-like '$G$-$\Sigma$' theory

Related model with $g' \neq 0$, but $g = v = 0$, studied in E. E. Aldape, T. Cookmeyer, A. A. Patel, E. Altman, arXiv:2012.00763

Boson self energy: $\Pi = \Pi_g + \Pi_{g'}$

$$\Pi_g(i\Omega) \sim -\frac{g^2}{v^2}|\Omega|, \qquad \Pi_{g'}(i\Omega) \sim -g'^2|\Omega|$$

Boson propagator: $D(q, i\Omega) = \dfrac{1}{q^2 + \gamma|\Omega|}$

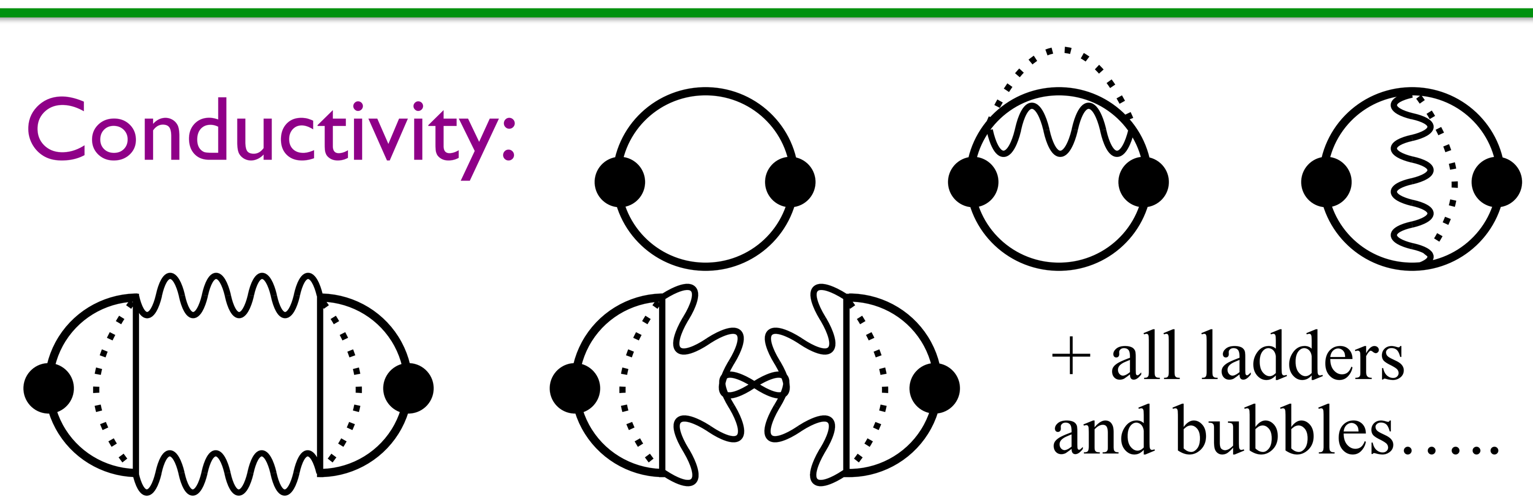
Fermion self energy: $\Sigma = \Sigma_v + \Sigma_g + \Sigma_{g'}$

$$\Sigma_v(i\omega) \sim -iv^2 {\rm sgn}(\omega), \quad \Sigma_g(i\omega) \sim -i\frac{g^2}{v^2}\omega \ln(1/|\omega|),$$

$$\Sigma_{g'}(i\omega) \sim -ig'^2\omega \ln(1/|\omega|)$$

2 sources of marginal Fermi liquid self energy. The $g^2$ log term does not contribute to transport but the $g'^2$ log term does!

## Conductivity:




Conductivity: $\sigma(\omega) \sim [1/\tau_{\rm trans}(\omega) - i\omega\, m^*(\omega)/m]^{-1}$

$$\frac{1}{\tau_{\rm trans}(\omega)} \sim v^2 + g'^2|\omega| \quad ; \quad \frac{m^*(\omega)}{m} \sim \frac{2g'^2}{\pi}\ln(\Lambda/\omega)$$

Residual resistivity is determined by $v^2$;

Linear-in-$T$ resistivity determined by $g'^2$.