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HYDRODYNAMIC THEORY OF THERMOELECTRIC TRANSPORT IN WEYL SEMI-METALS

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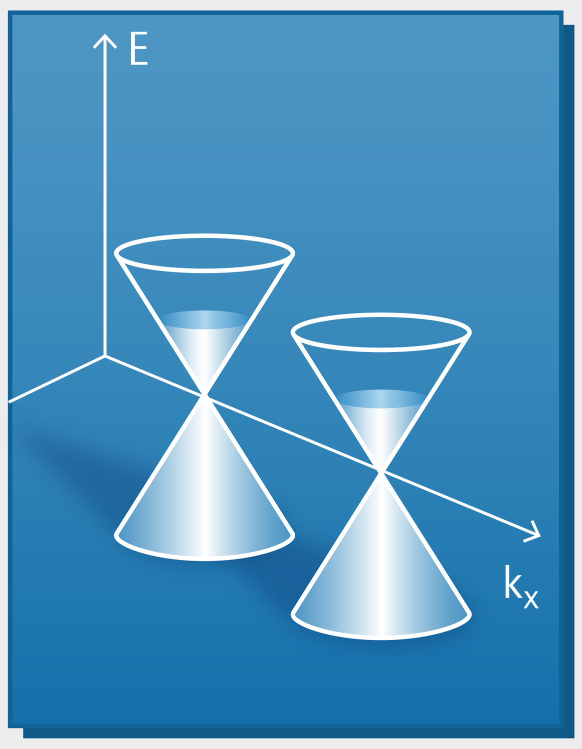
1 Weyl semi-metals

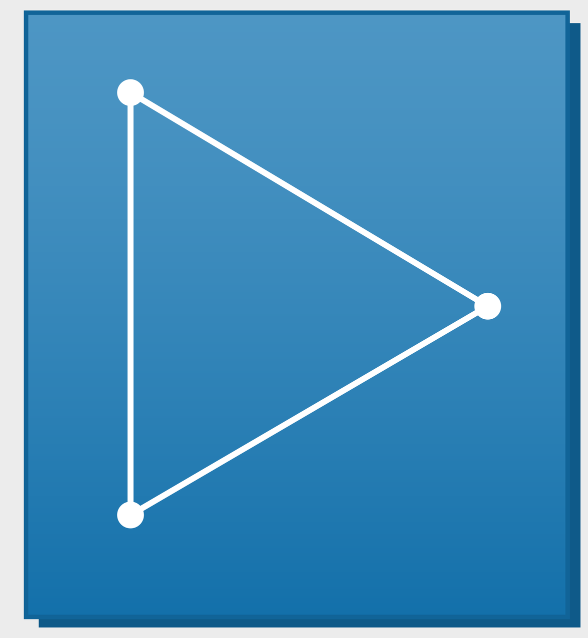


- Excitations near the Fermi energy are relativistic Weyl fermions.
- There are multiple Weyl nodes with no net chirality.
- In weak fields $B \ll T^2$, metals with Weyl quasiparticles exhibit negative longitudinal magnetoresistance

$$\sigma_{zz} = \frac{e^4 v_F^3}{4\pi^2 \hbar c^2 \mu^2} B^2 \tau$$

Son, Spivak (2012)

2 Anomalies



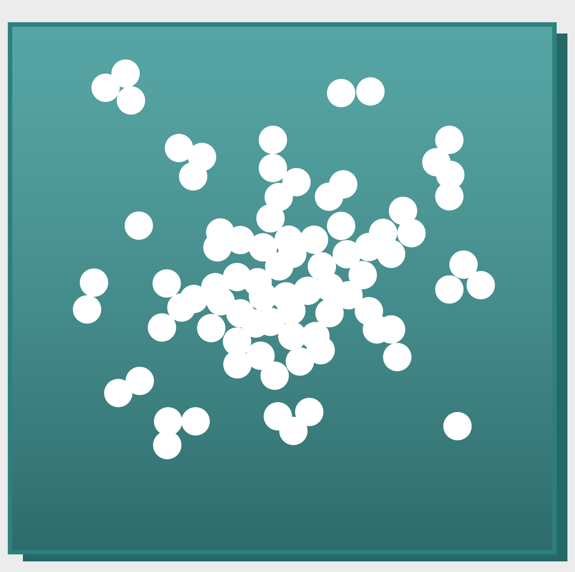
- In the language of QFT, Weyl fermions are anomalous.
- Charge conservation is violated by quantum effects:

$$\nabla_\mu J^\mu = -\frac{C}{8}\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} - \frac{G}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}R^{\alpha}{}_{\beta\mu\nu}R^{\beta}{}_{\alpha\rho\sigma}$$

axial anomaly — mixed axial-gravitational anomaly

- For a Weyl node with Berry flux k: $C = \frac{k}{4\pi^2}$, $G = \frac{k}{24}$

3 Hydrodynamics



- One simple limit is when the fastest timescale is the electron-electron scattering time.
- Opposite of the quasiparticle limit: the electrons behave as a hydrodynamic fluid.
- The fluid is described by a local temperature $T(x)$, chemical potential $\mu(x)$ and fluid velocity $u^\mu(x)$.
- Imposing symmetries, and the local 2nd law of thermodynamics, fixes J^μ and $T^{\mu\nu}$ in terms of these variables.

4 Anomalous hydrodynamics

- The axial anomaly causes extra current flow via the chiral vortical effect and the chiral magnetic effect:

$$J^\mu \supset \frac{C\mu^2}{2}\left(1 - \frac{2}{3}\frac{n\mu}{\epsilon + P}\right)\varepsilon^{\mu\nu\rho\sigma}u_\nu\nabla_\rho u_\sigma + \frac{C\mu}{2}\left(1 - \frac{1}{2}\frac{n\mu}{\epsilon + P}\right)\varepsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}$$

Son, Surowka (2009)

- This is required for the local 2nd law of thermodynamics to be satisfied.
- This mixed chiral-gravitational anomaly also has an effect:

$$J^\mu \supset -\frac{4G\mu n T^2}{\epsilon + P}\varepsilon^{\mu\nu\rho\sigma}u_\nu\nabla_\rho u_\sigma - \frac{GT^2 n}{2(\epsilon + P)}\varepsilon^{\mu\nu\rho\sigma}u_\nu F_{\rho\sigma}$$

Jensen, Loganayagam, Yarom (2012)

- These effects are present even in the absence of electromagnetic fields or spacetime curvature.

5 Longitudinal magnetoresistance in the thermoelectric conductivities

- Treat each Weyl node as a hydrodynamic fluid, with slow inter-node scattering

$$\nabla_\mu J^\mu_a = -\frac{C_a}{8}\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} - \frac{G_a}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma}R^{\alpha}{}_{\beta\mu\nu}R^{\beta}{}_{\alpha\rho\sigma} - \sum_b\left(\mathcal{R}_{ab}\frac{\mu_b}{T_b} + \mathcal{S}_{ab}\frac{1}{T_b}\right)$$

$$\nabla_\mu T^{\mu\nu}_a = F^{\nu\mu}J_{\mu\ a} - \frac{G_a}{16\pi^2}\nabla_\mu\left(\varepsilon^{\rho\sigma\alpha\beta}F_{\rho\sigma}R^{\nu\mu}{}_{\alpha\beta}\right) + u^\nu_a\sum_b\left(\mathcal{U}_{ab}\frac{\mu_b}{T_b} + \mathcal{V}_{ab}\frac{1}{T_b}\right)$$

- Turn on a weak background magnetic field and weakly disordered chemical potential

$$F = B\, dx \wedge dy + \partial_i\mu_0\, dx_i \wedge dt$$

- In the presence of small electric fields and thermal gradients, each current has two components

$$\delta J^i_a = n_a\delta v^i_a + B_i C_a \delta\mu_a$$
$$\delta Q^i_a = T s_a \delta v^i_a + 2B_i G_a T\delta T_a$$

- There are two contributions to each conductivity

$$\sigma_{zz} = \sum_a \frac{n_a^2}{\Gamma_a} + \mathfrak{s}B^2 \qquad \bar{\kappa}_{zz} = \sum_a \frac{T s_a^2}{\Gamma_a} + \mathfrak{h}B^2 \qquad \alpha_{zz} = \sum_a \frac{n_a s_a}{\Gamma_a} + \mathfrak{a}B^2$$

- The first is relaxation of the fluid velocity at a rate Γ_a due to disorder.
- The second is due to anomalous charge and heat transport in a B field.

$$\mathfrak{s} = T\begin{pmatrix} C_a & C_a\mu\end{pmatrix}\begin{pmatrix}\mathcal{R}_{ab} & -\mathcal{S}_{ab}\\ -\mathcal{U}_{ab} & \mathcal{V}_{ab}\end{pmatrix}^{-1}\begin{pmatrix}C_b\\ C_b\mu\end{pmatrix}$$

$$\mathfrak{h} = 4T^4\begin{pmatrix} 0 & G_a\end{pmatrix}\begin{pmatrix}\mathcal{R}_{ab} & -\mathcal{S}_{ab}\\ -\mathcal{U}_{ab} & \mathcal{V}_{ab}\end{pmatrix}^{-1}\begin{pmatrix}0\\ G_b\end{pmatrix}$$

$$\mathfrak{a} = 2T^2\begin{pmatrix} 0 & G_a\end{pmatrix}\begin{pmatrix}\mathcal{R}_{ab} & -\mathcal{S}_{ab}\\ -\mathcal{U}_{ab} & \mathcal{V}_{ab}\end{pmatrix}^{-1}\begin{pmatrix}C_b\\ C_b\mu\end{pmatrix}$$

- Longitudinal magnetoresistance in α or $\bar{\kappa}$ requires a mixed axial-gravitational anomaly.
- The conductivity matrix is Onsager reciprocal and positive semi-definite.
- The WF law is violated in the hydrodynamic limit.

Summary

- Weyl semi-metals exhibit the physics of anomalous, relativistic QFTs.
- There are two distinct anomalies in relativistic QFTs: the axial anomaly C and the mixed axial-gravitational anomaly G.
- Each anomaly produces longitudinal magnetoresistance in a specific component of the thermoelectric conductivity matrix.
- A measurement of these effects would be the first experimental observation of the axial-gravitational anomaly in any branch of physics.