A model of d-wave superconductivity, antiferromagnetism, and charge order on the square lattice

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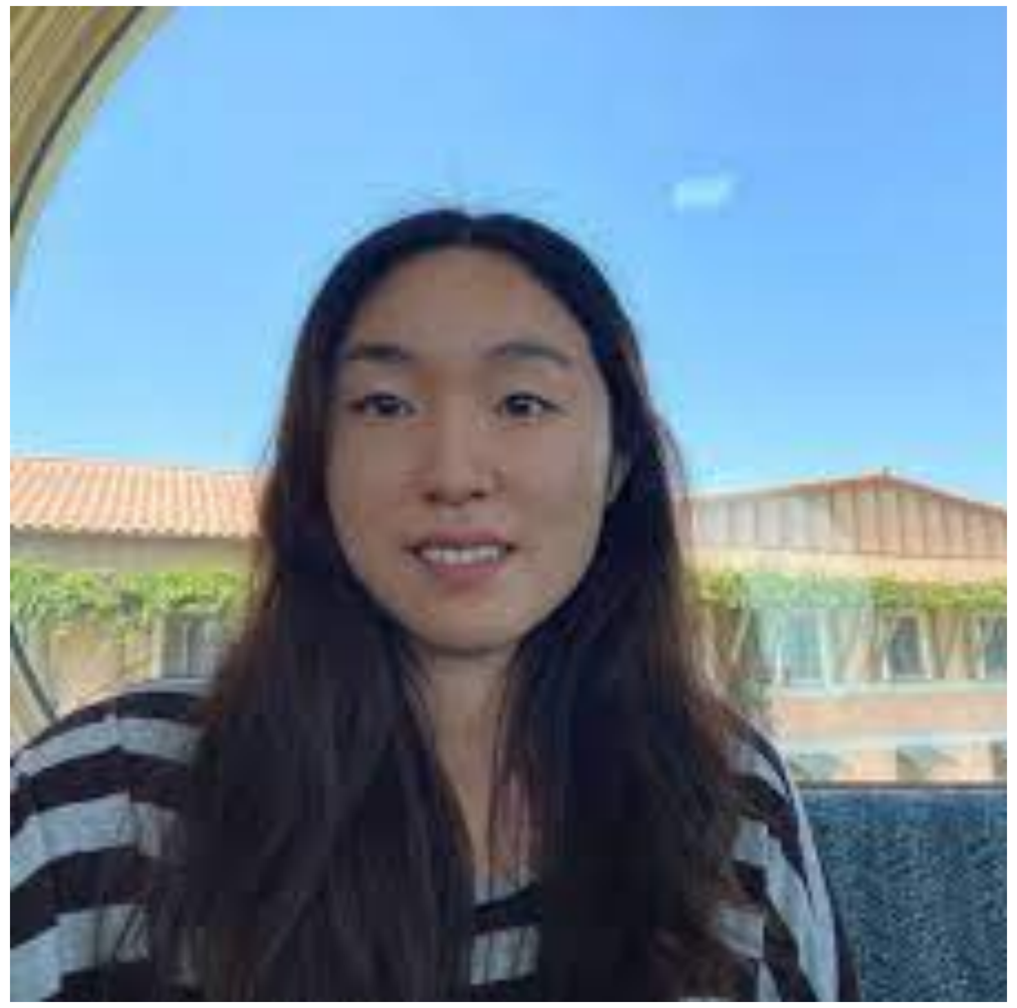
Subir Sachdev
F30.00008

Maine Christos, Zhu-Xi Luo, Henry Shackleton, Ya-Hui Zhang, Mathias Scheurer, and S. S., arXiv:2302.07885
Alexander Nikolaenko, Jonas v. Milczewski, Darshan G. Joshi, and S.S., arXiv:2211.10452

Talk online: sachdev.physics.harvard.edu

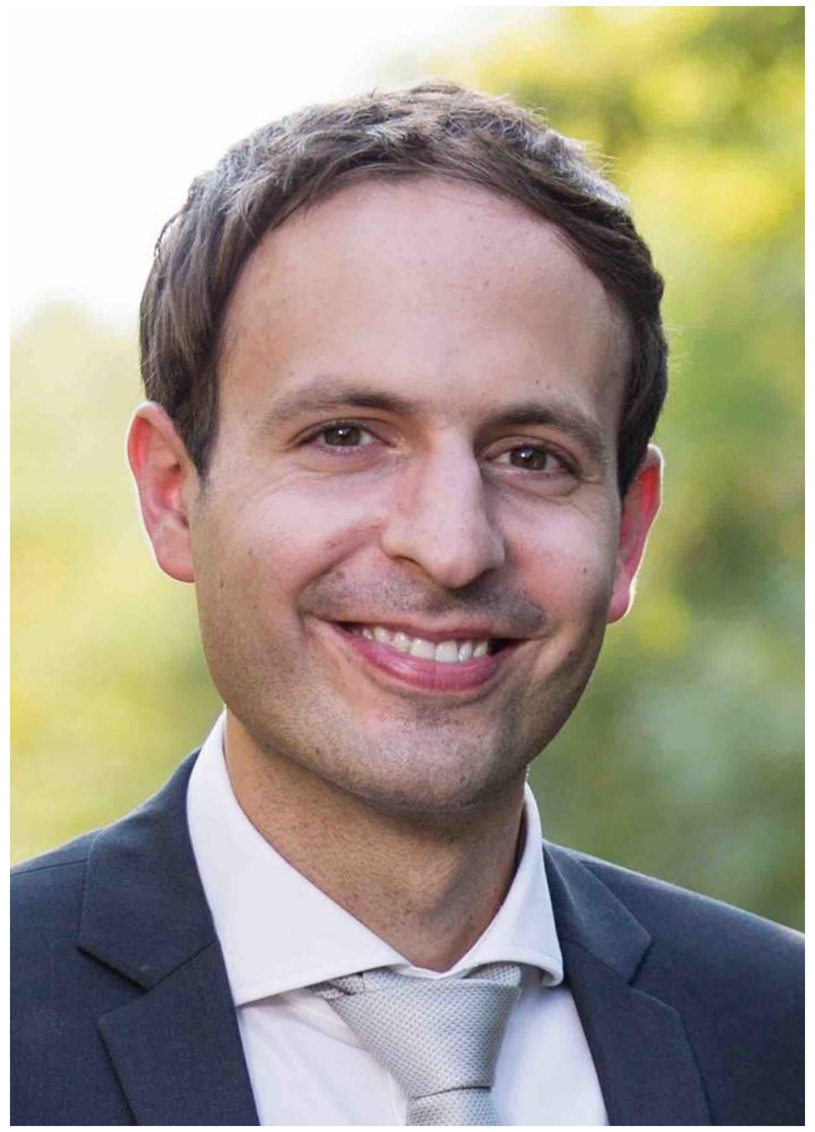
PHYSICS
HARVARD

Maine Christos

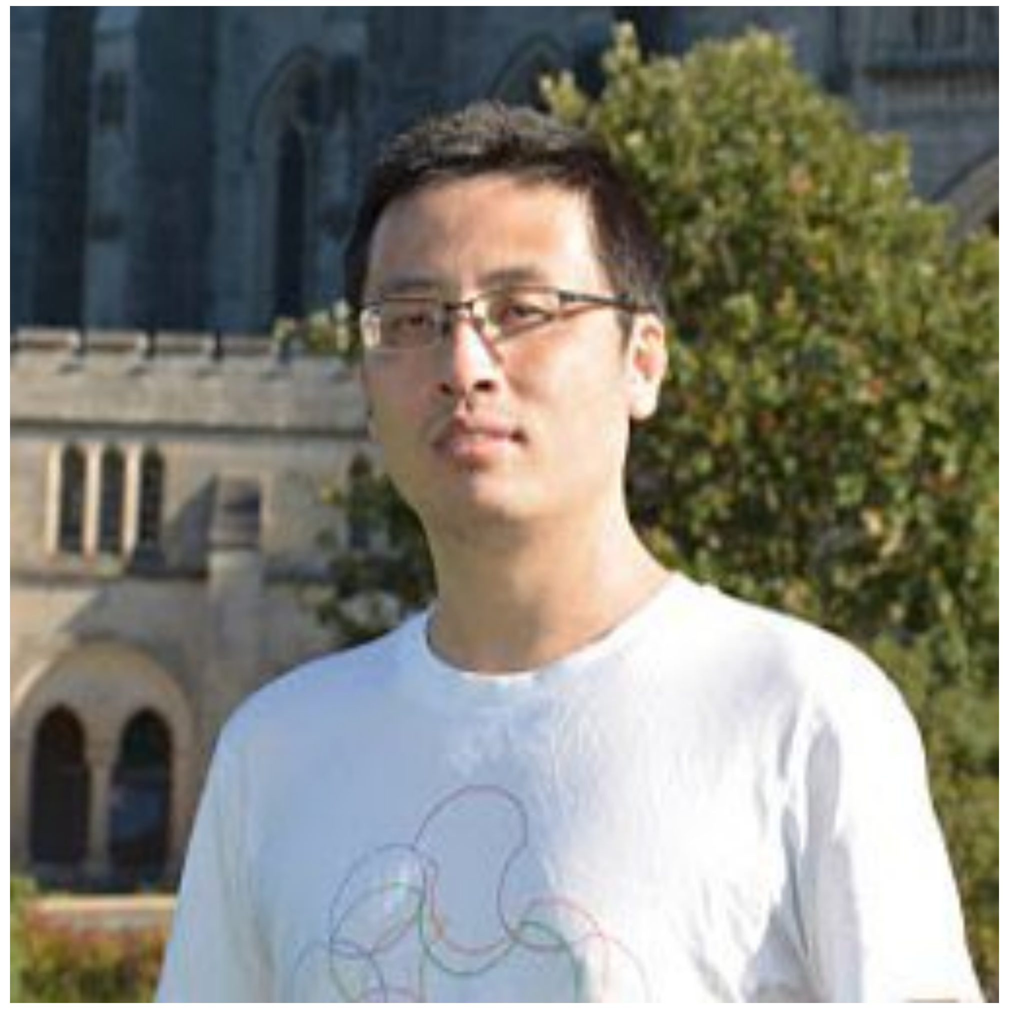


Zhu-Xi Luo

Henry Shackleton



Mathias Scheurer



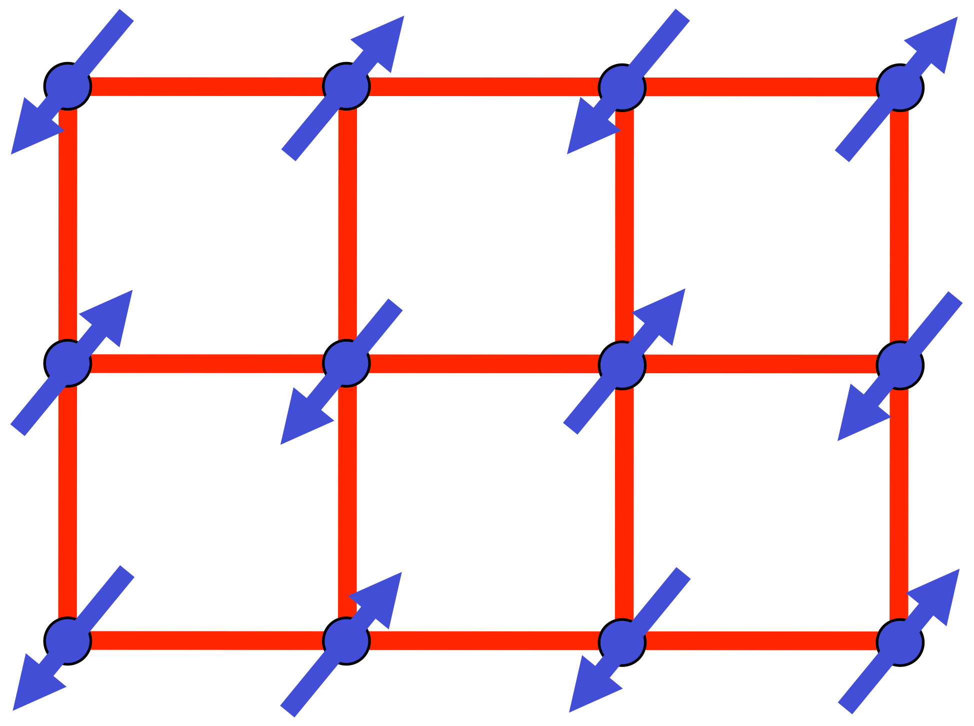
Ya-Hui Zhang

Alexander Nikolaenko

Darshan Joshi

Jonas von Milczewski

Insulating S=1/2 antiferromagnet



$$H = \sum_{i<j} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

Schwinger bosons

$$\boldsymbol{S}_i = \frac{1}{2} b_{i\alpha}^{\dagger} \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta} \,, \quad \sum_{\alpha=\uparrow,\downarrow} b_{i\alpha}^{\dagger} b_{i\alpha} = 1$$

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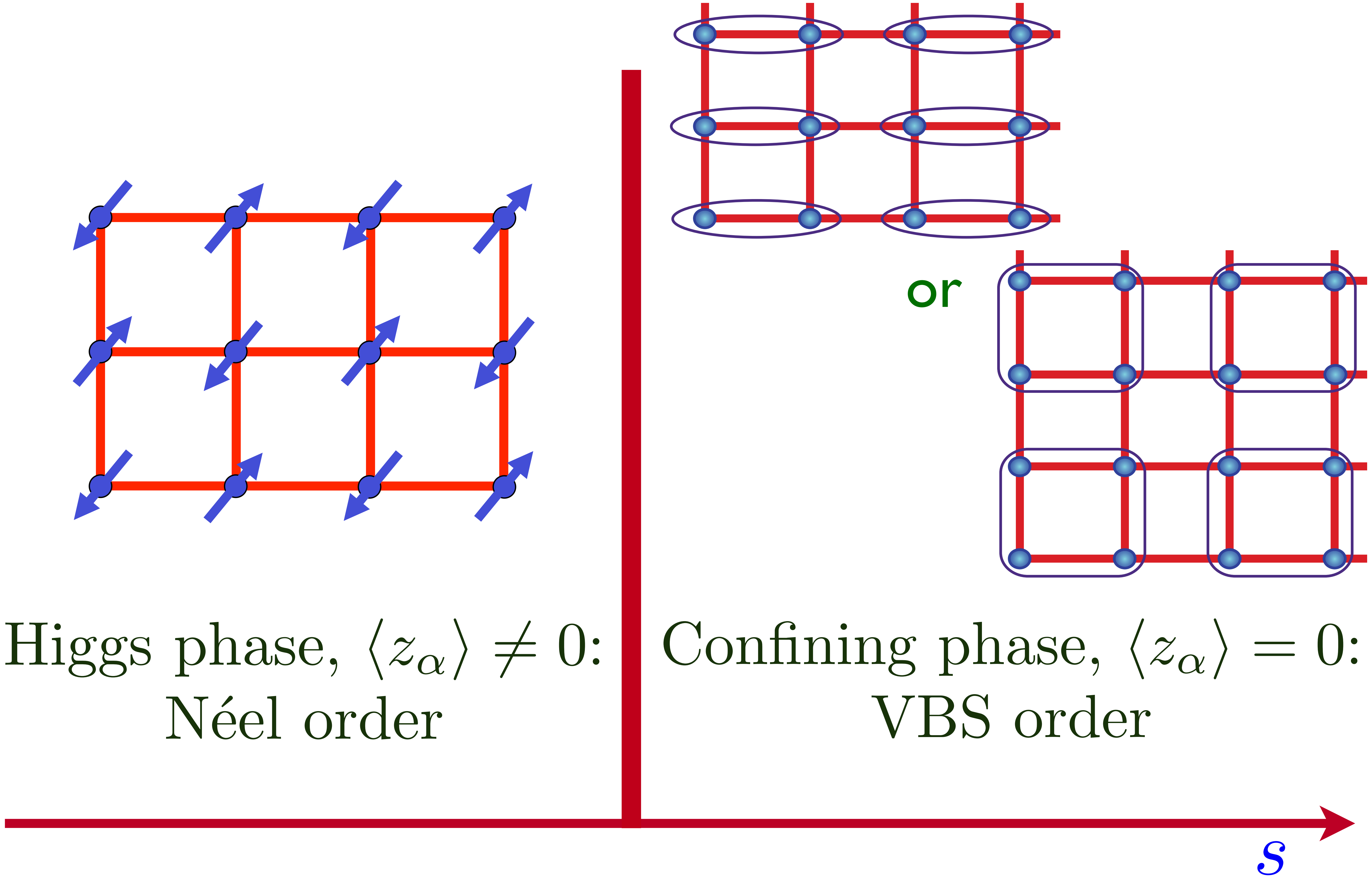

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Low energy $\mathbb{CP}^1$ U(1) gauge theory

$$z_\alpha \sim b_{A\alpha} + \varepsilon_{\alpha\beta} b_{B\beta}$$

$$\mathcal{L} = |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u|z_\alpha|^4 + \mathcal{L}_{\text{monopole}}$$

N. Read and S. Sachdev, Physical Review Letters **62**, 1694 (1989)

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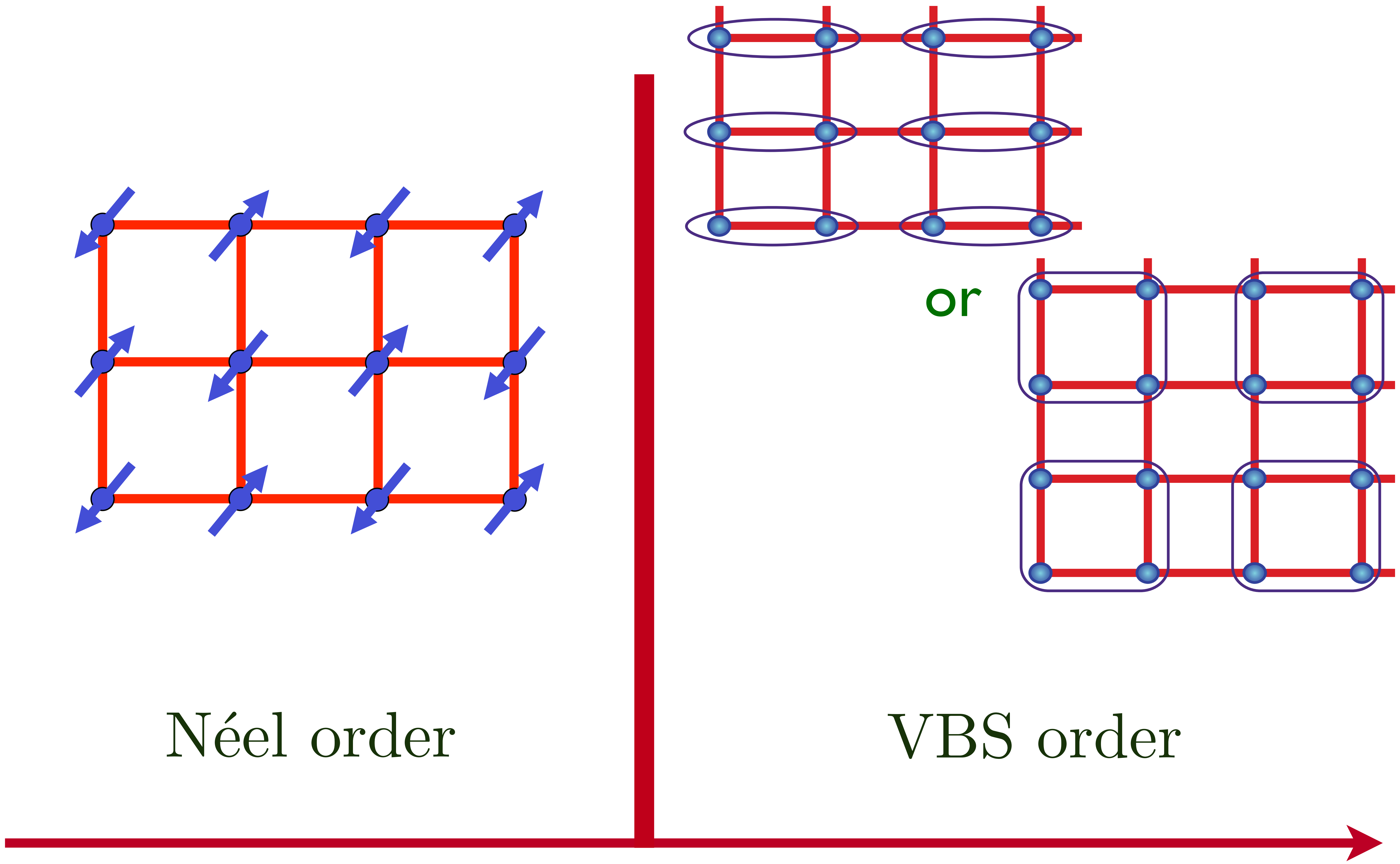

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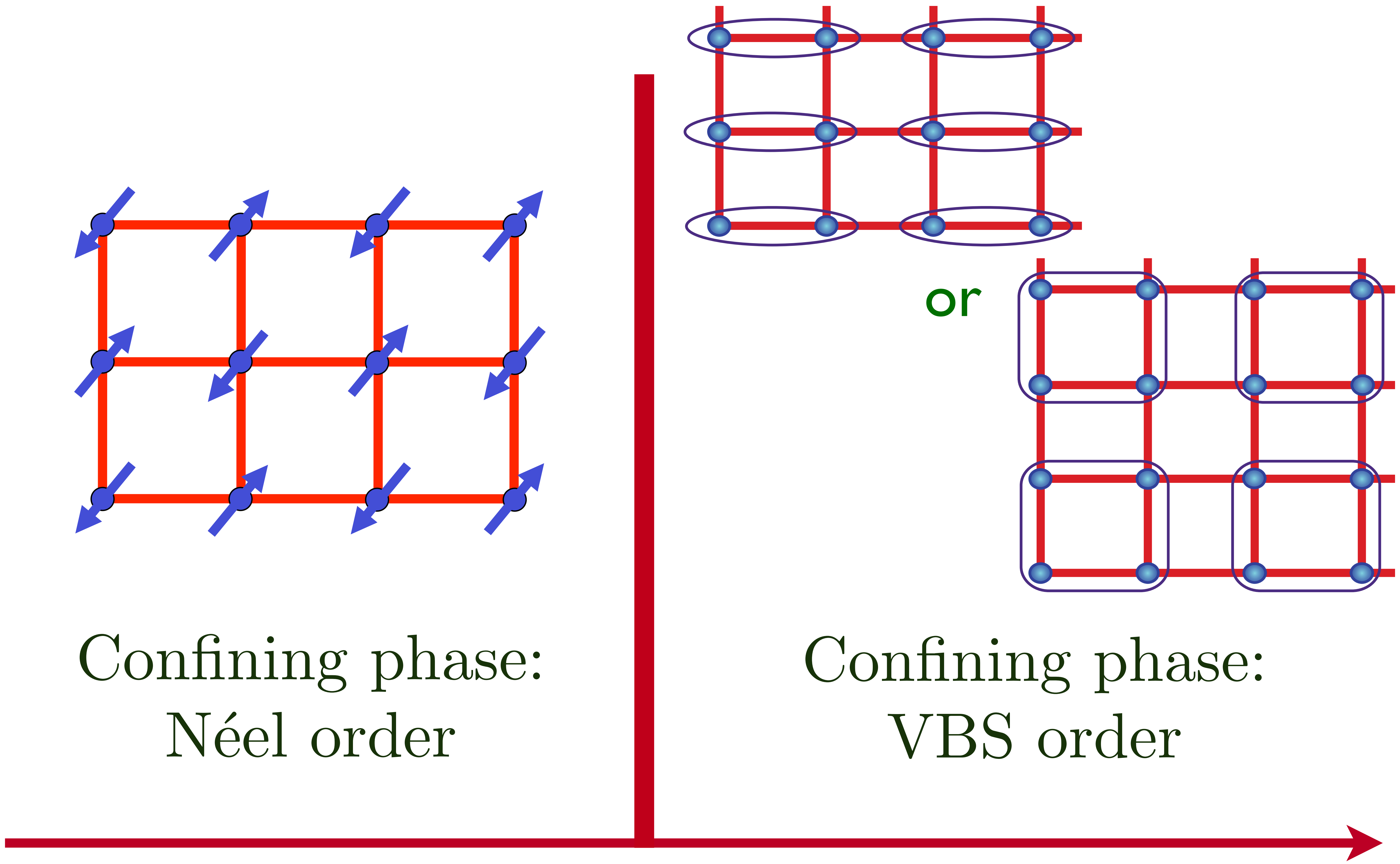

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π-flux mean-field leads to a low energy theory of $N_f = 2$ Dirac fermions Ψ_s coupled to an emergent SU(2) gauge field.

$$\mathcal{L} = i\overline{\Psi}_s D_\mu \Psi_s + \ldots$$

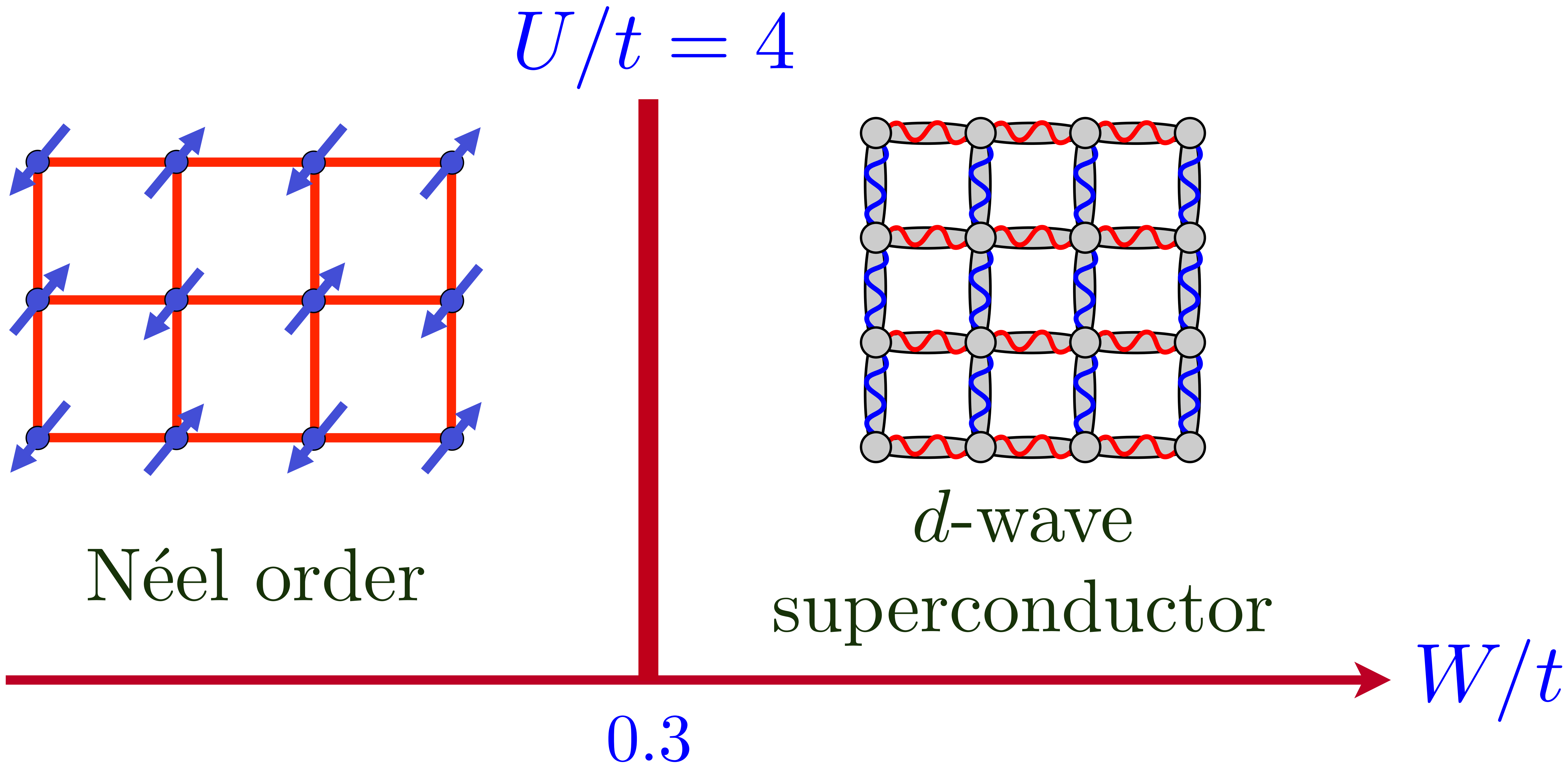
Dual to $\mathbb{CP}^1$ U(1) gauge theory!

C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, Physical Review X **7**, 031051 (2017)

Include charge fluctuations at half-filling: repulsive Hubbard model with pair-hopping

$$H = -\frac{t}{2}\sum_i K_i + U\sum_i (n_{i\uparrow} - \tfrac{1}{2})(n_{i\downarrow} - \tfrac{1}{2}) - W\sum_i K_i^2 \,,$$

$$K_i = \sum_{\hat{e}=\pm\hat{x},\pm\hat{y}} \left(c_{i\alpha}^\dagger c_{i+\hat{e},\alpha} + c_{i+\hat{e},\alpha}^\dagger c_{i\alpha} \right)$$

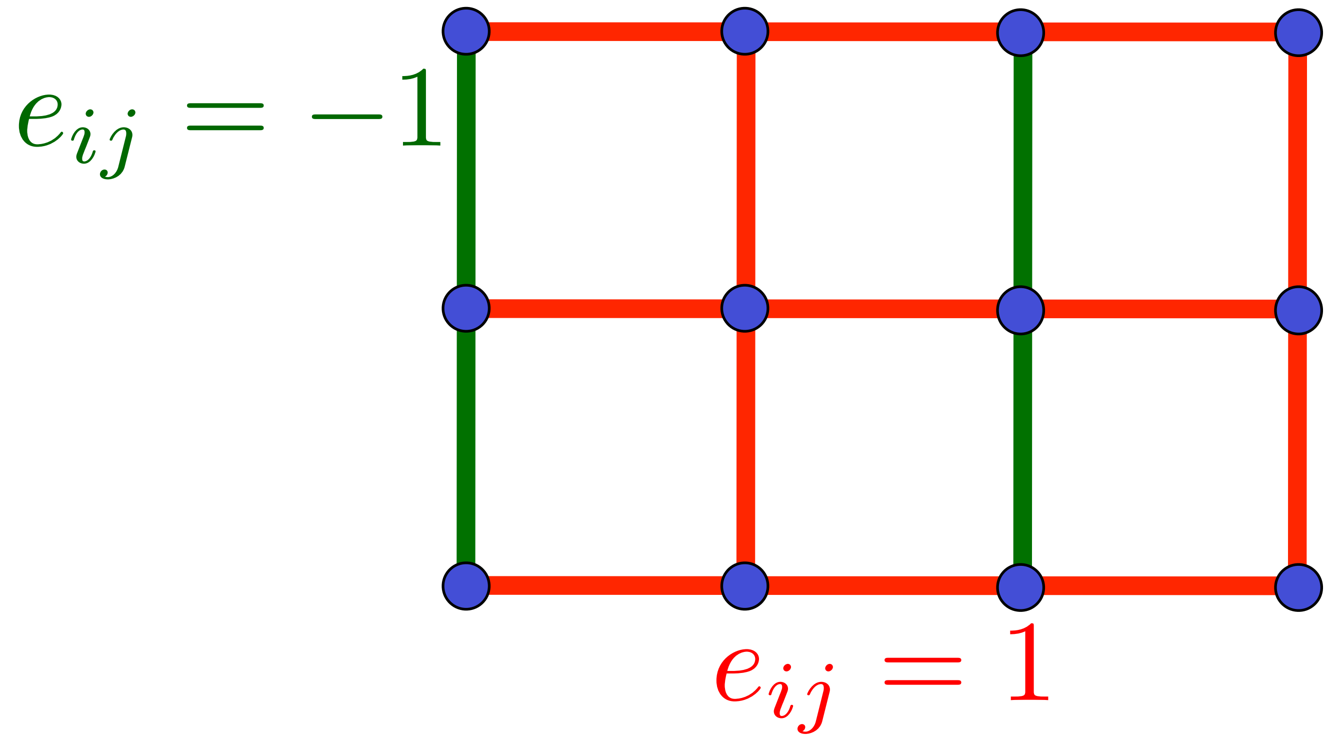


F. F. Assaad, M. Imada, and D. J. Scalapino, Physical Review Letters **77**, 4592 (1996)

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory

- Begin with the π-flux spin liquid in the fermionic spinon description.

$$H_f = iJ \sum_{\langle ij \rangle} e_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} - f_{j\alpha}^\dagger f_{i\alpha} \right)$$



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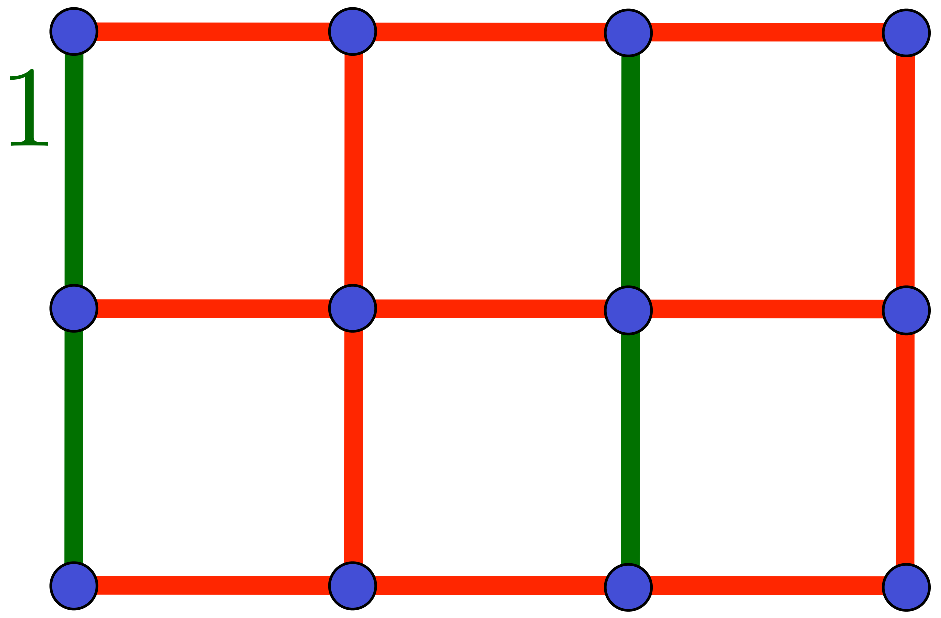

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H_f is invariant under distinct SU(2) rotations in spin and Nambu space.

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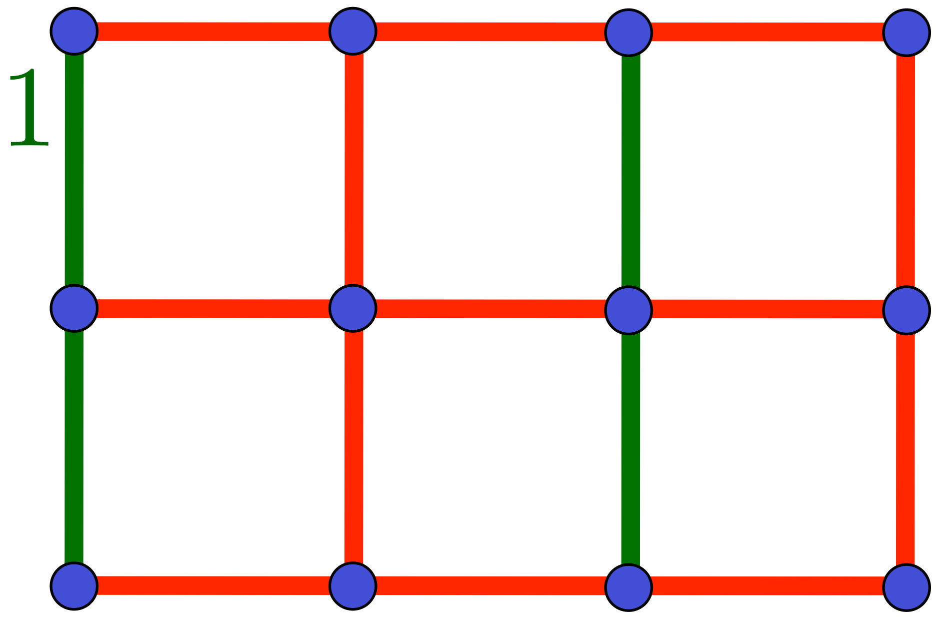

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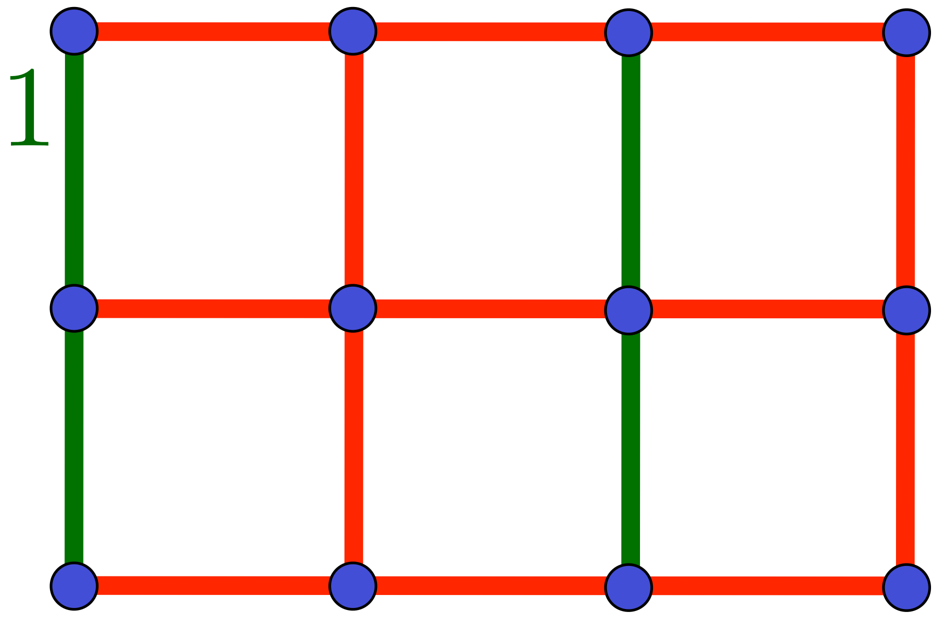

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- This uniquely identifies B_i as the 'chargon' of X.-G. Wen and P.A. Lee, PRL **76**, 503 (1996), related to the electrons $c_{i\sigma}$ by

$$B_i = \begin{pmatrix} B_{1i} \\ B_{2i} \end{pmatrix} \quad ; \quad \begin{pmatrix} c_{i\uparrow} \\ c^\dagger_{i\downarrow} \end{pmatrix} \sim \begin{pmatrix} B^*_{1i} & B^*_{2i} \\ -B_{2i} & B_{1i} \end{pmatrix} \begin{pmatrix} f_{i\uparrow} \\ f^\dagger_{i\downarrow} \end{pmatrix}$$

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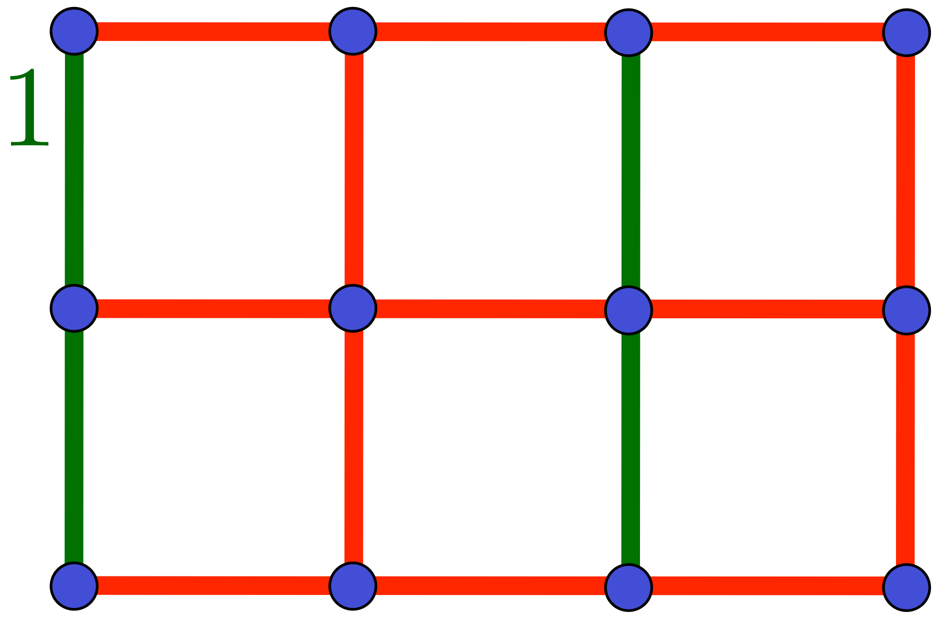

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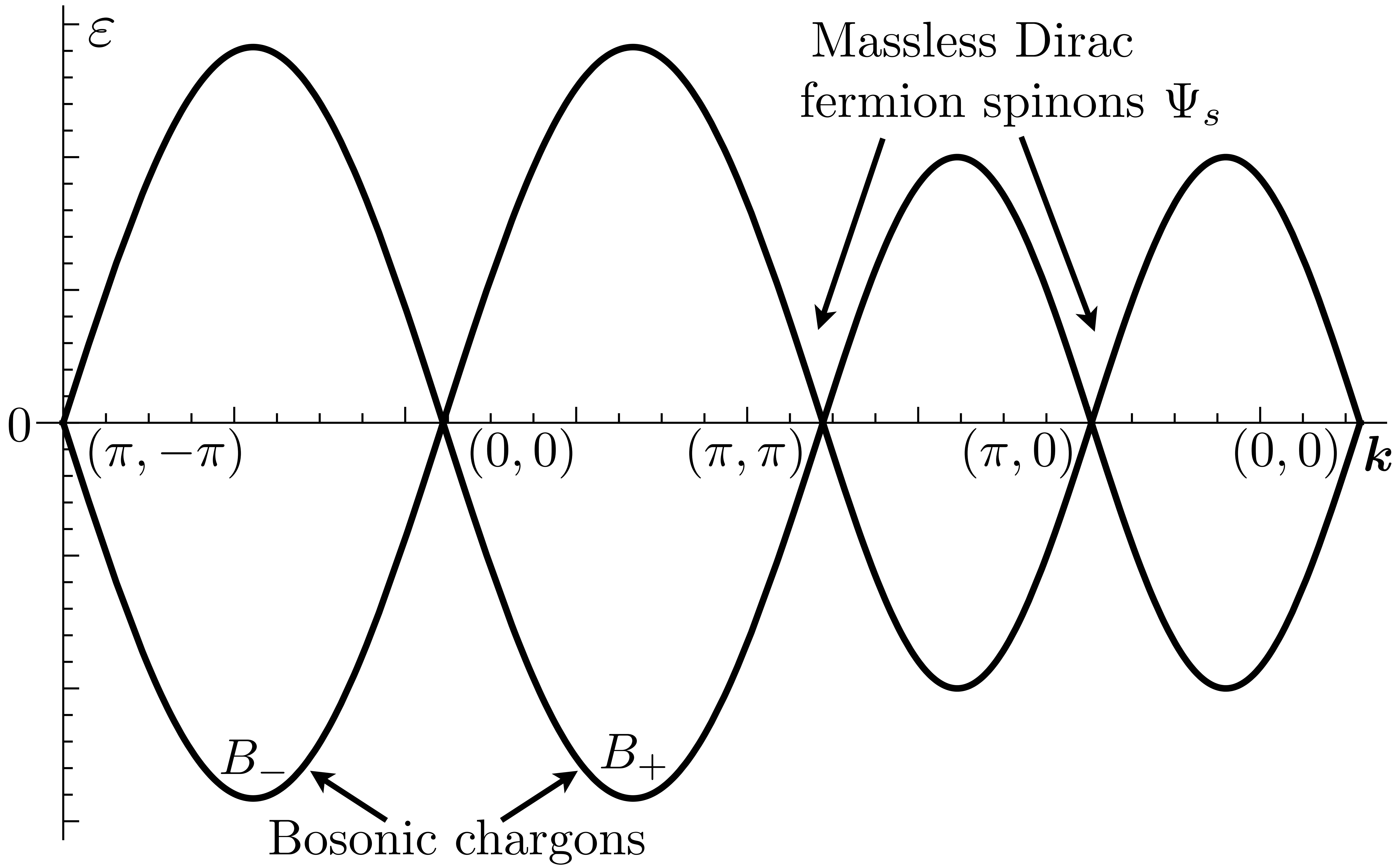
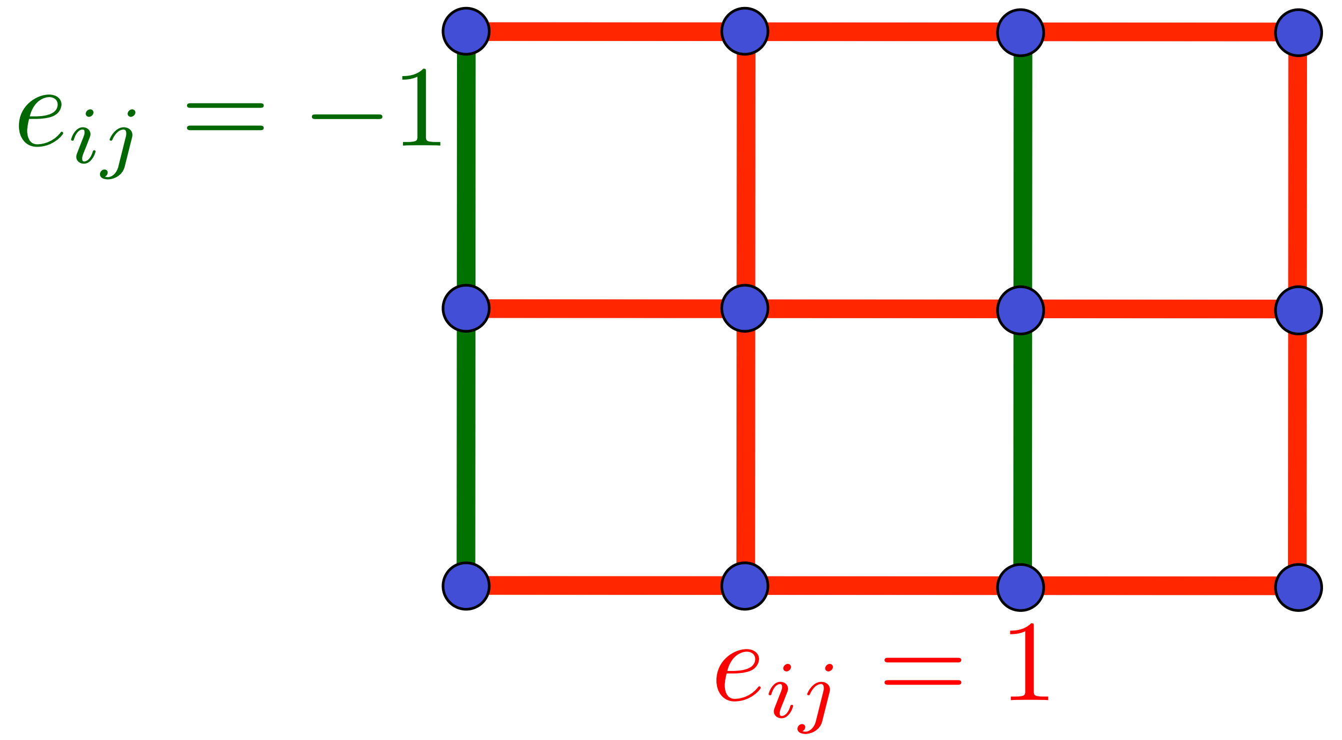
- Knowing the projective symmetry transformations of Ψ_i, we can deduce those of the B_i, and obtain the effective Hamiltonian for B_i

$$\boxed{H_B = r \sum_i B_i^\dagger B_i + iw \sum_{\langle ij \rangle} e_{ij} \left(B_i^\dagger B_j - B_j^\dagger B_i \right)} + \ldots$$

Include charge fluctuations at half-filling: confinement of SU(2) gauge theory





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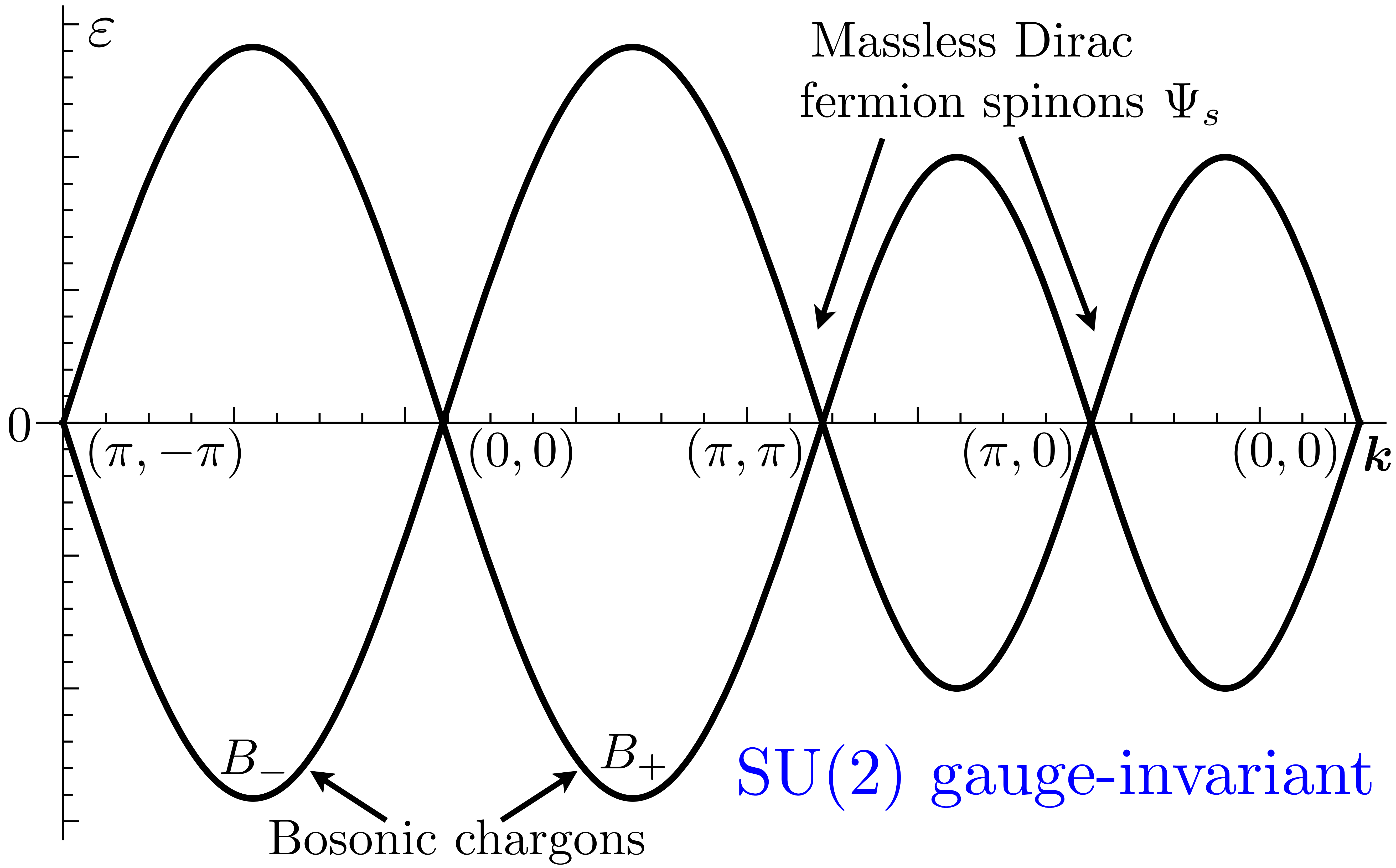

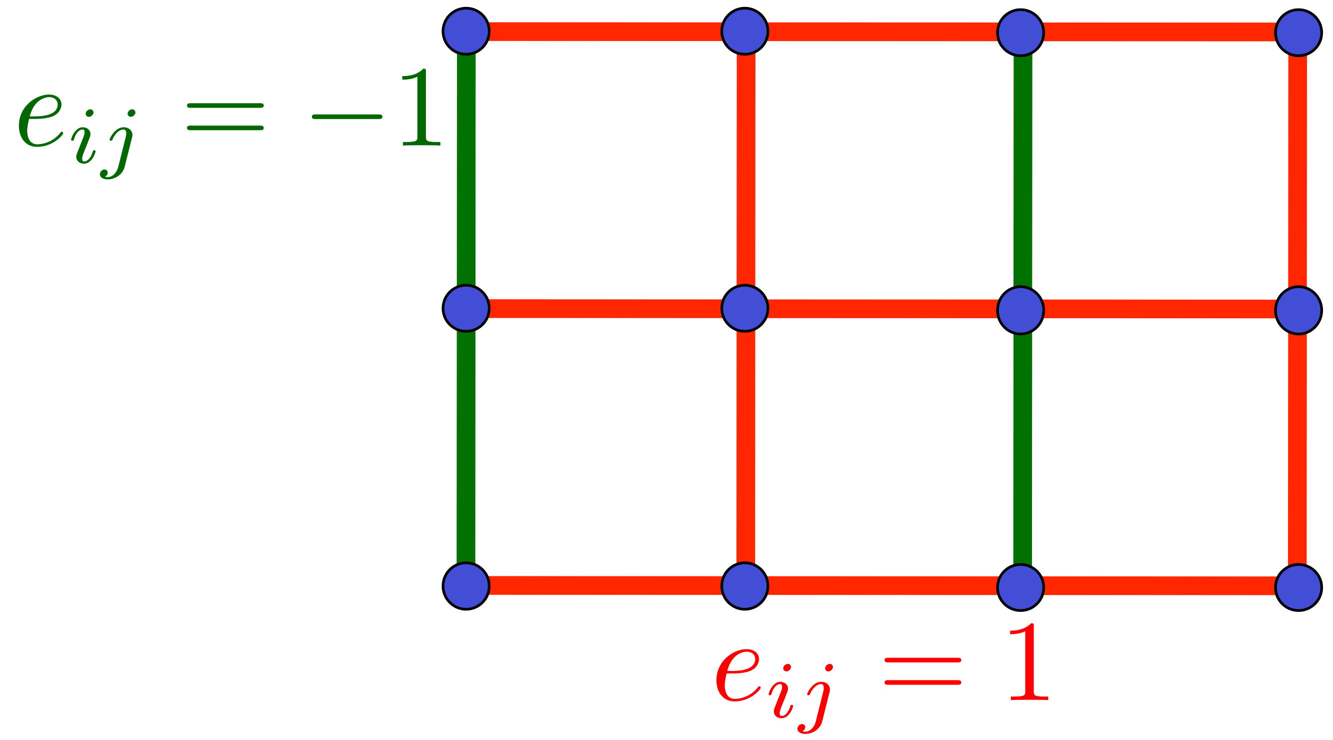

SU(2) gauge-invariant order parameters of Higgs phases:

$$
\begin{aligned}
x\text{-CDW}: &\quad \rho_{(\pi,0)} = B^*_{a+}B_{a+} - B^*_{a-}B_{a-} \\
y\text{-CDW}: &\quad \rho_{(0,\pi)} = B^*_{a+}B_{a-} + B^*_{a-}B_{a+} \\
d\text{-density wave}: &\quad D = i\left(B^*_{a+}B_{a-} - B^*_{a-}B_{a+}\right) \\
d\text{-wave superconductor}: &\quad \Delta = \varepsilon_{ab}B_{a+}B_{b-}
\end{aligned}
$$

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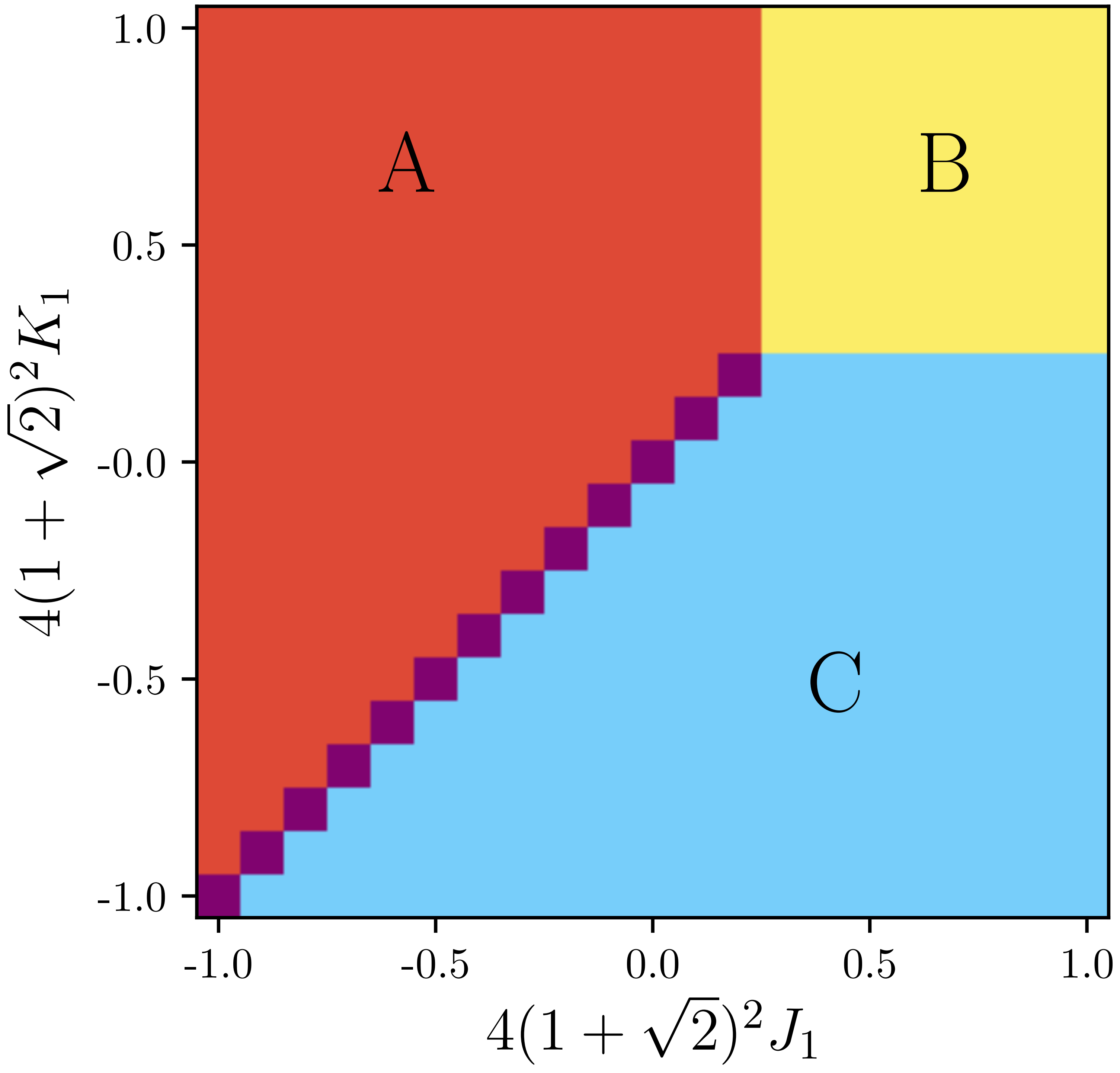
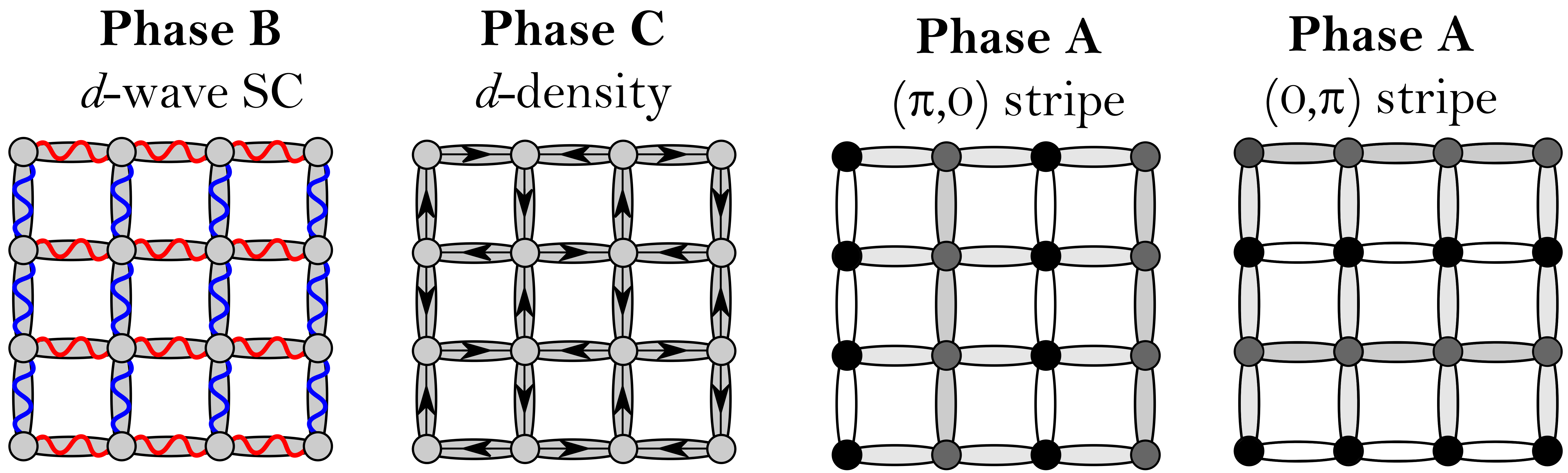


$$\langle B \rangle \neq 0$$



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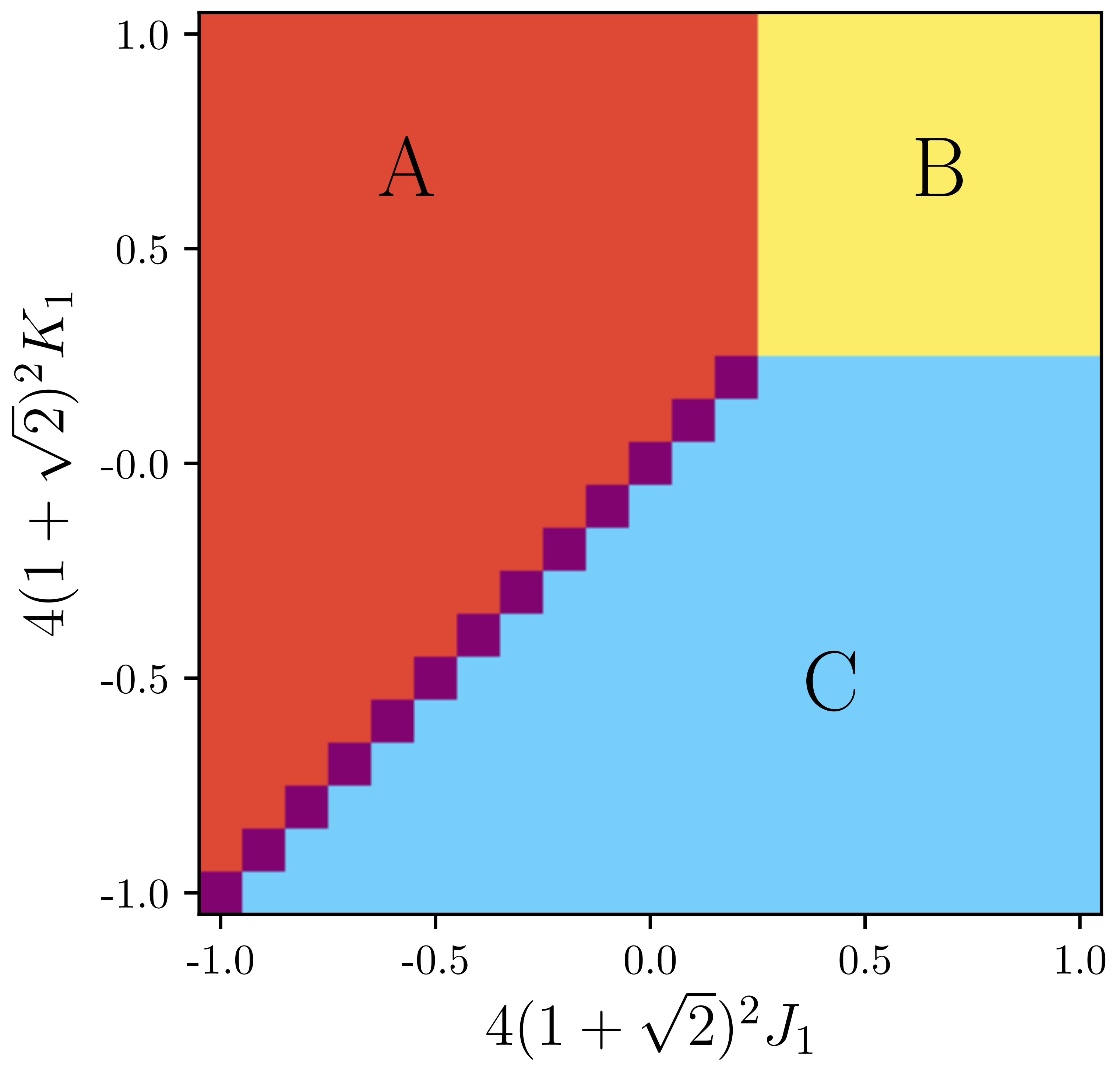

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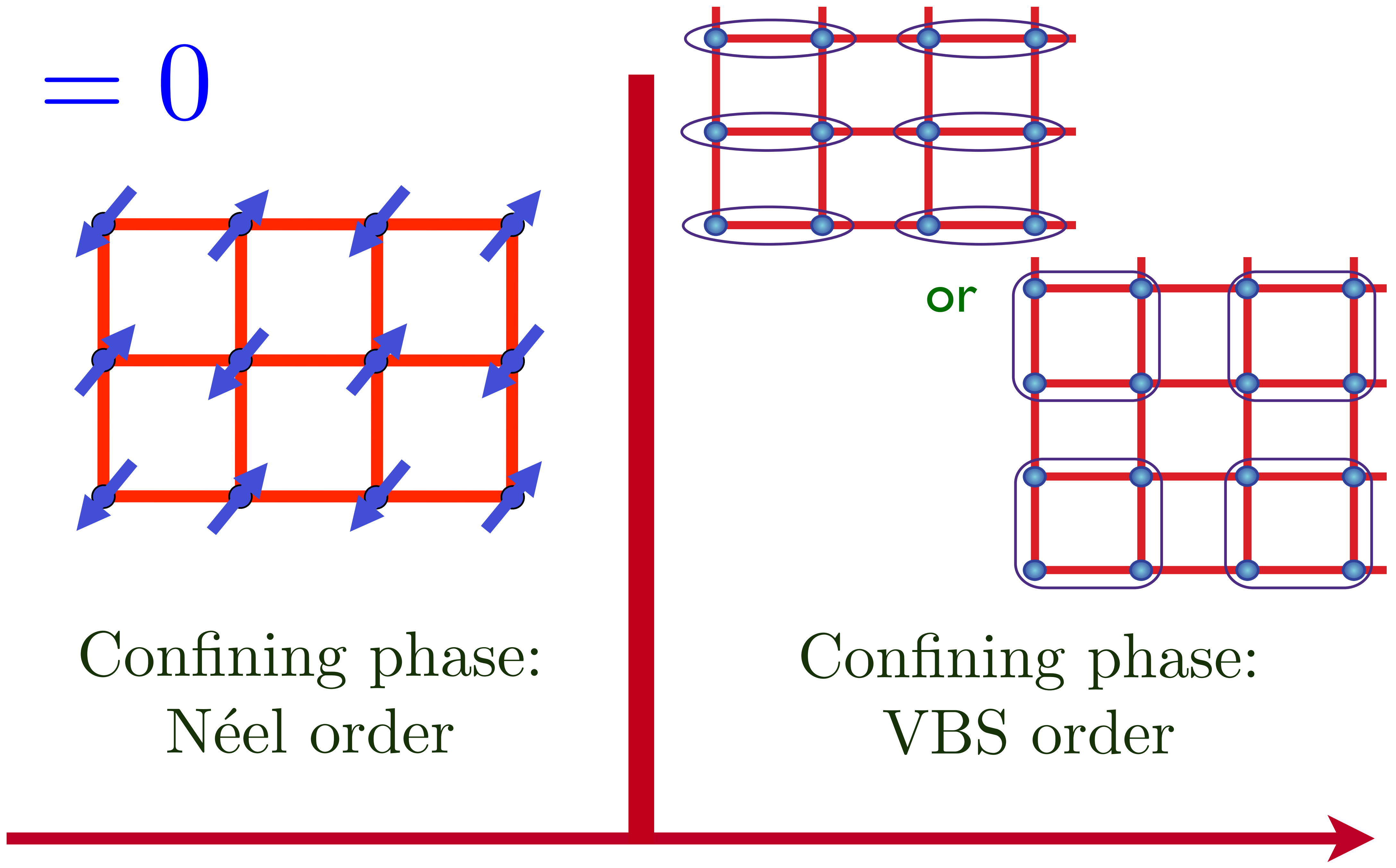

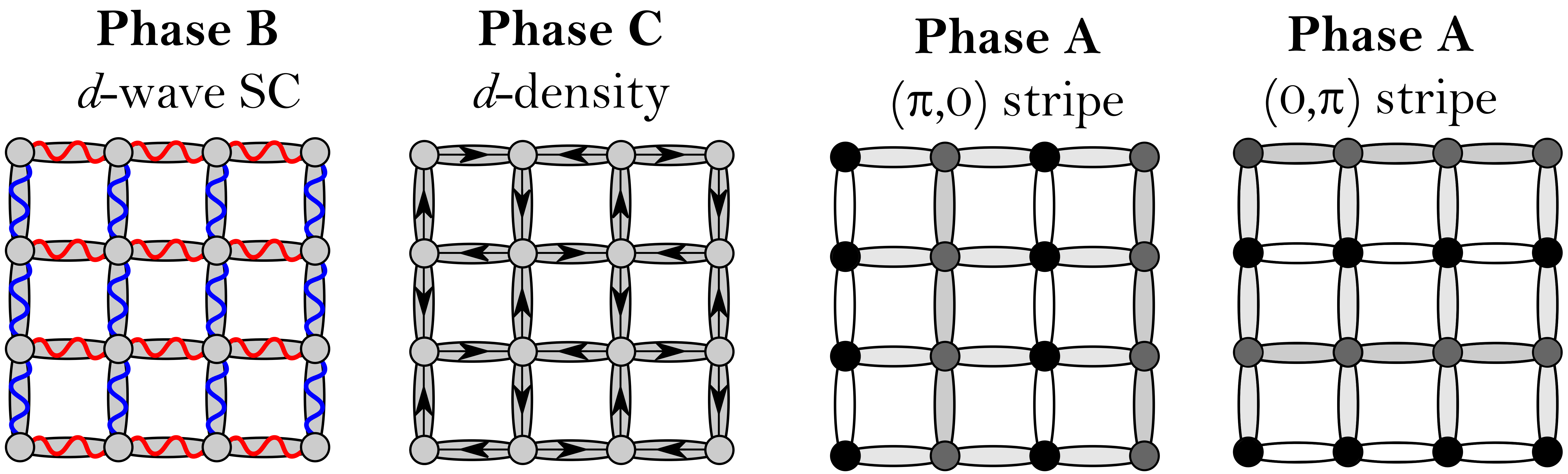

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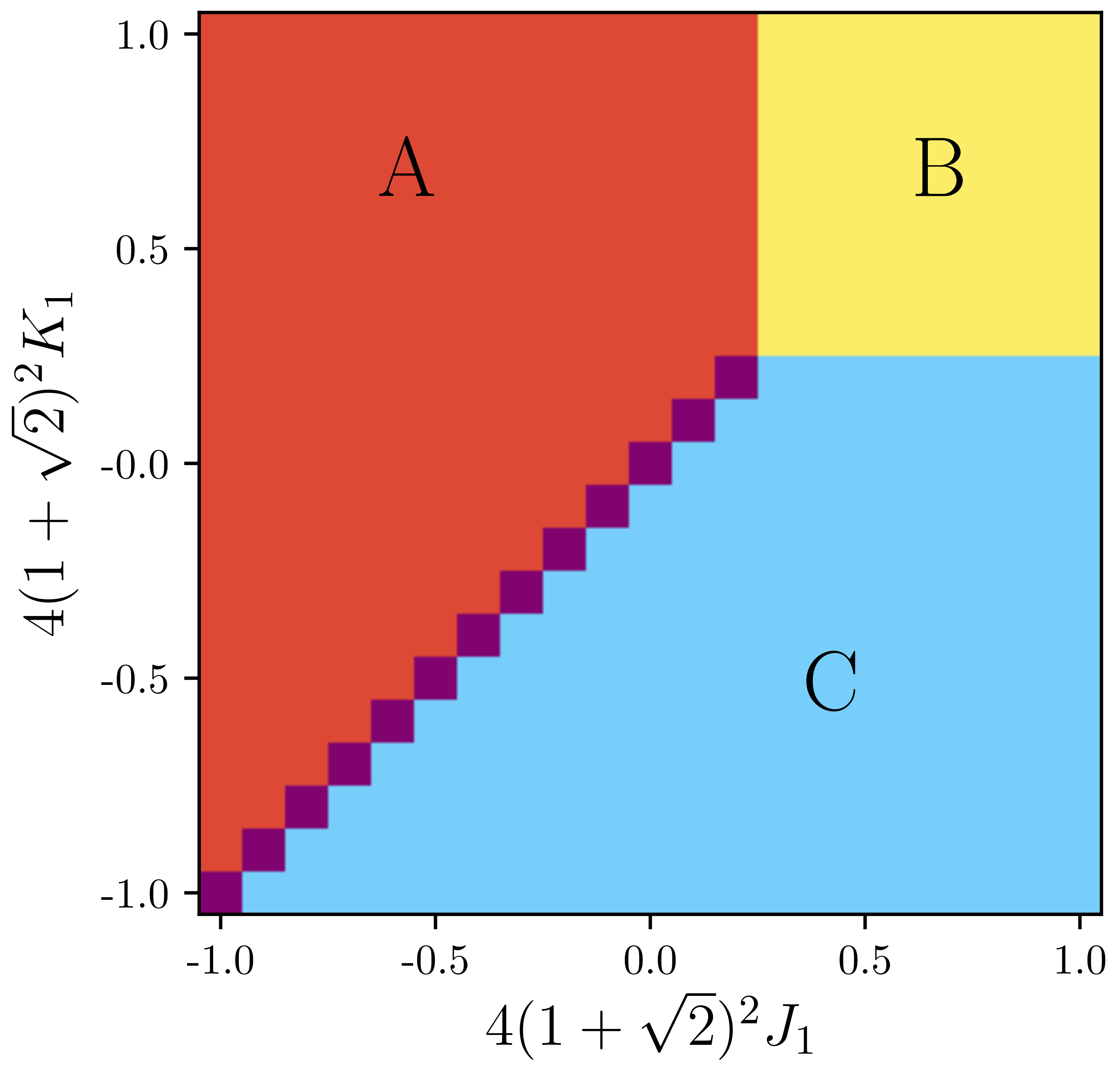

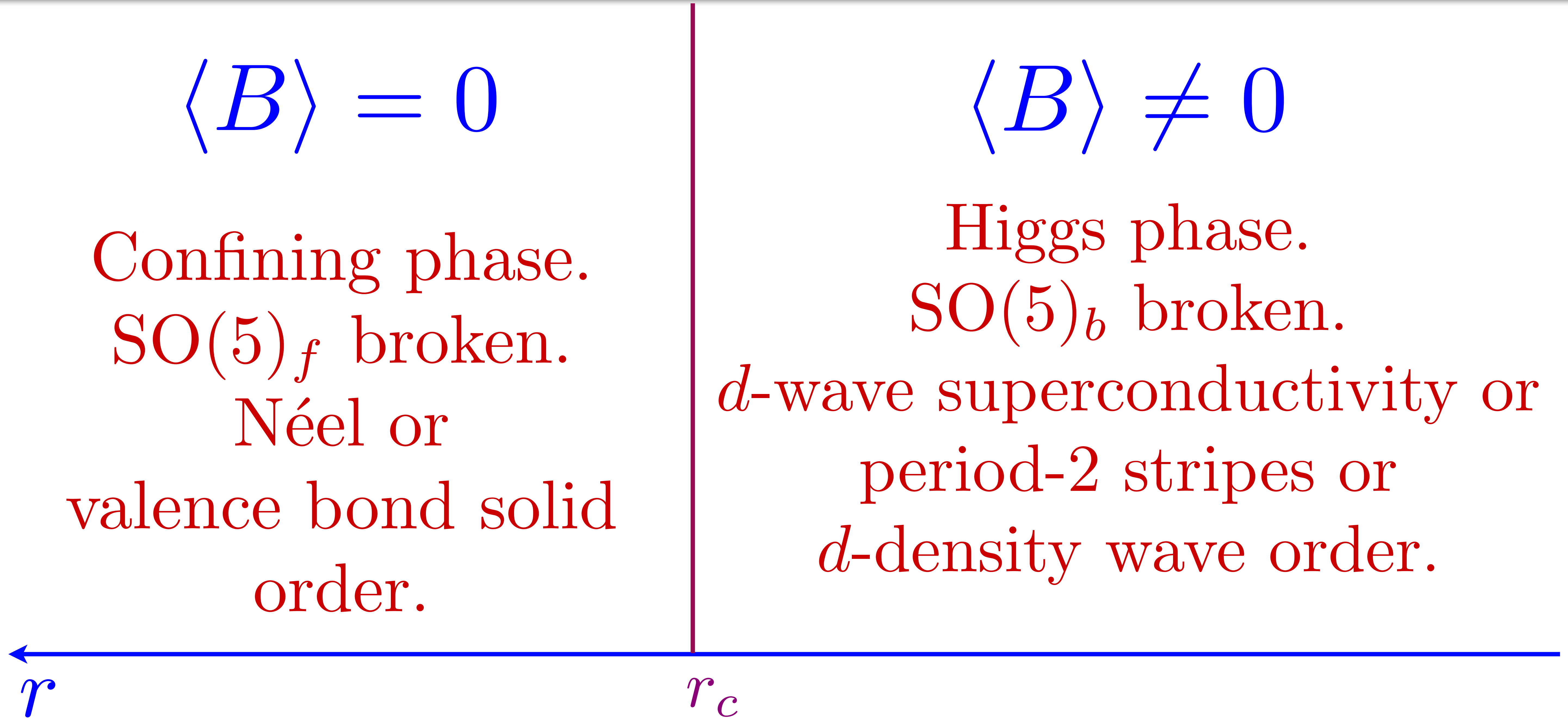

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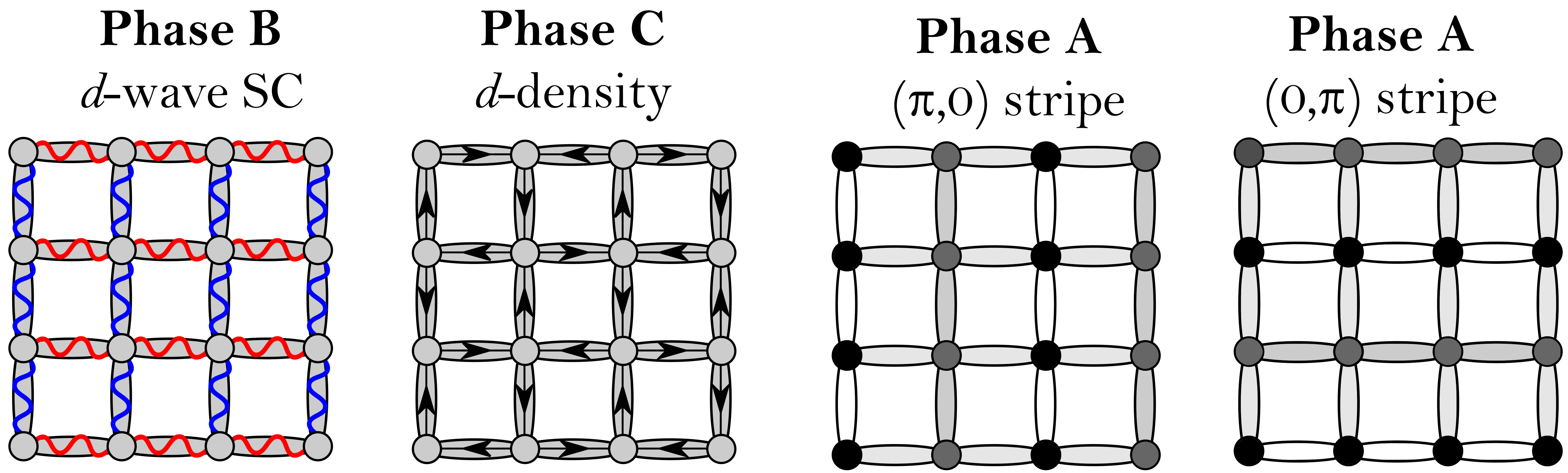

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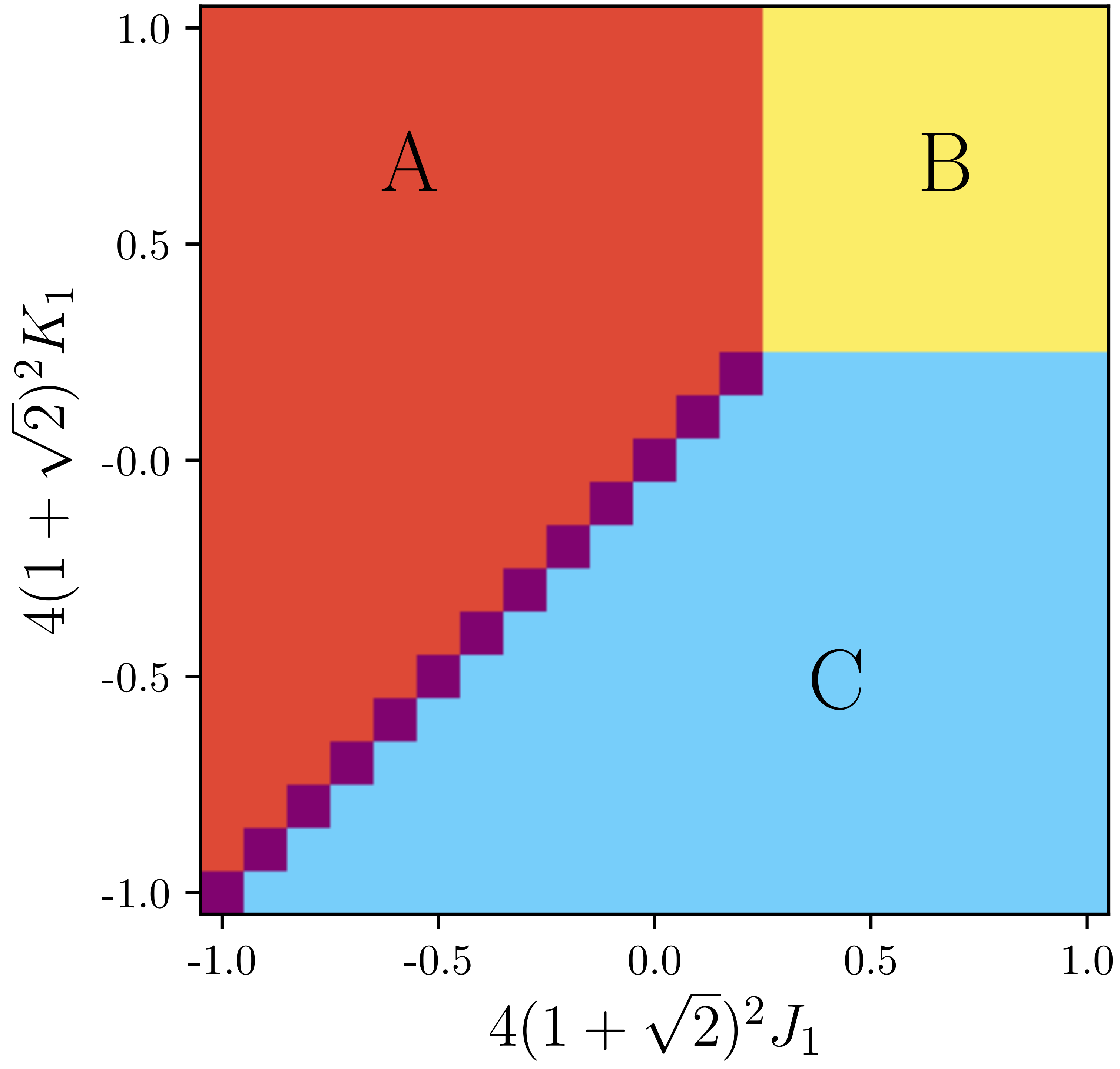

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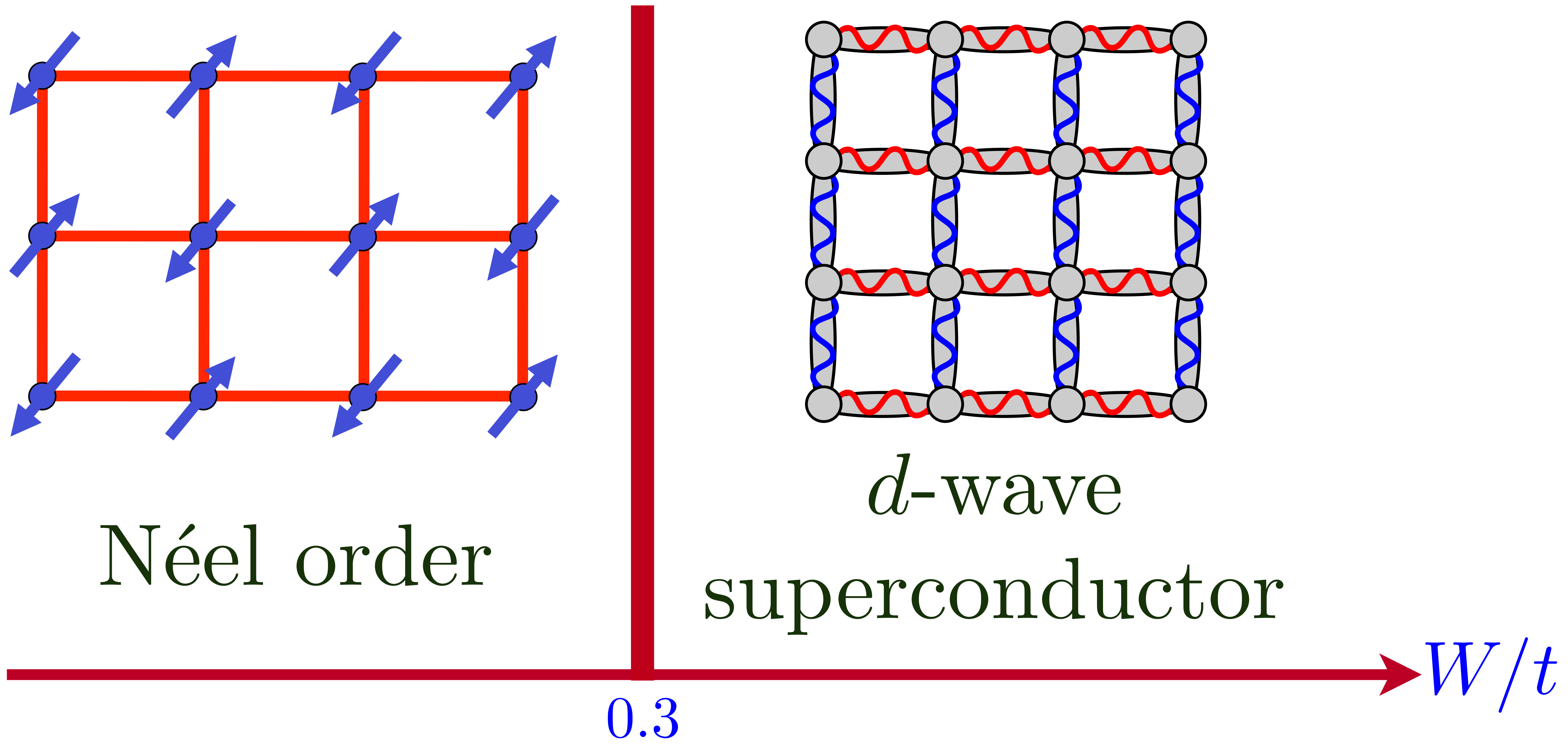

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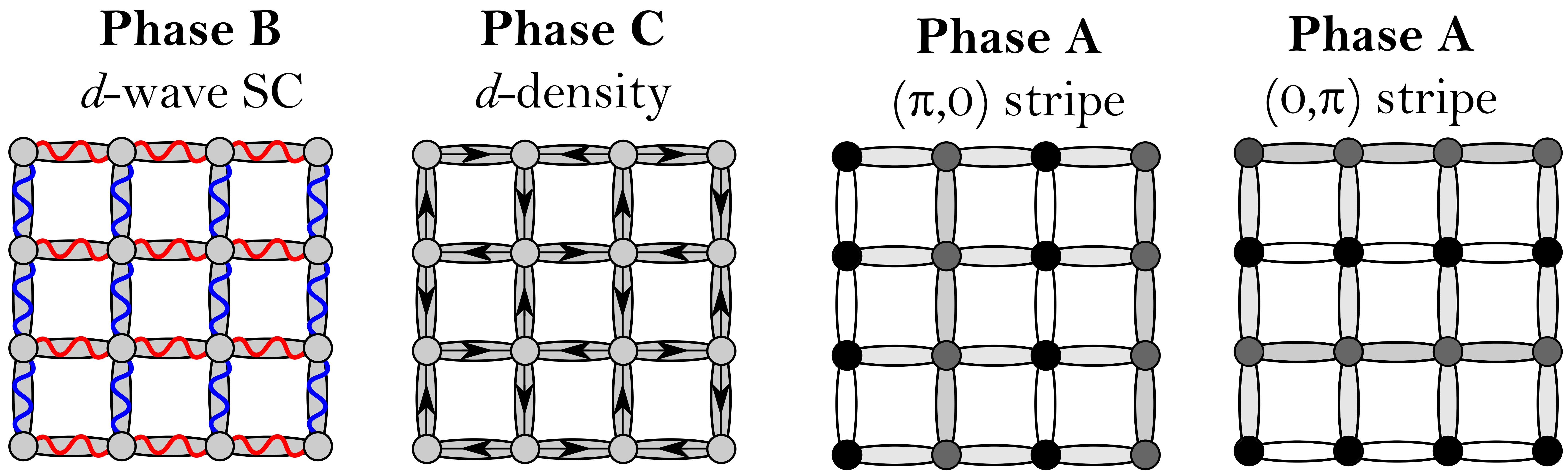

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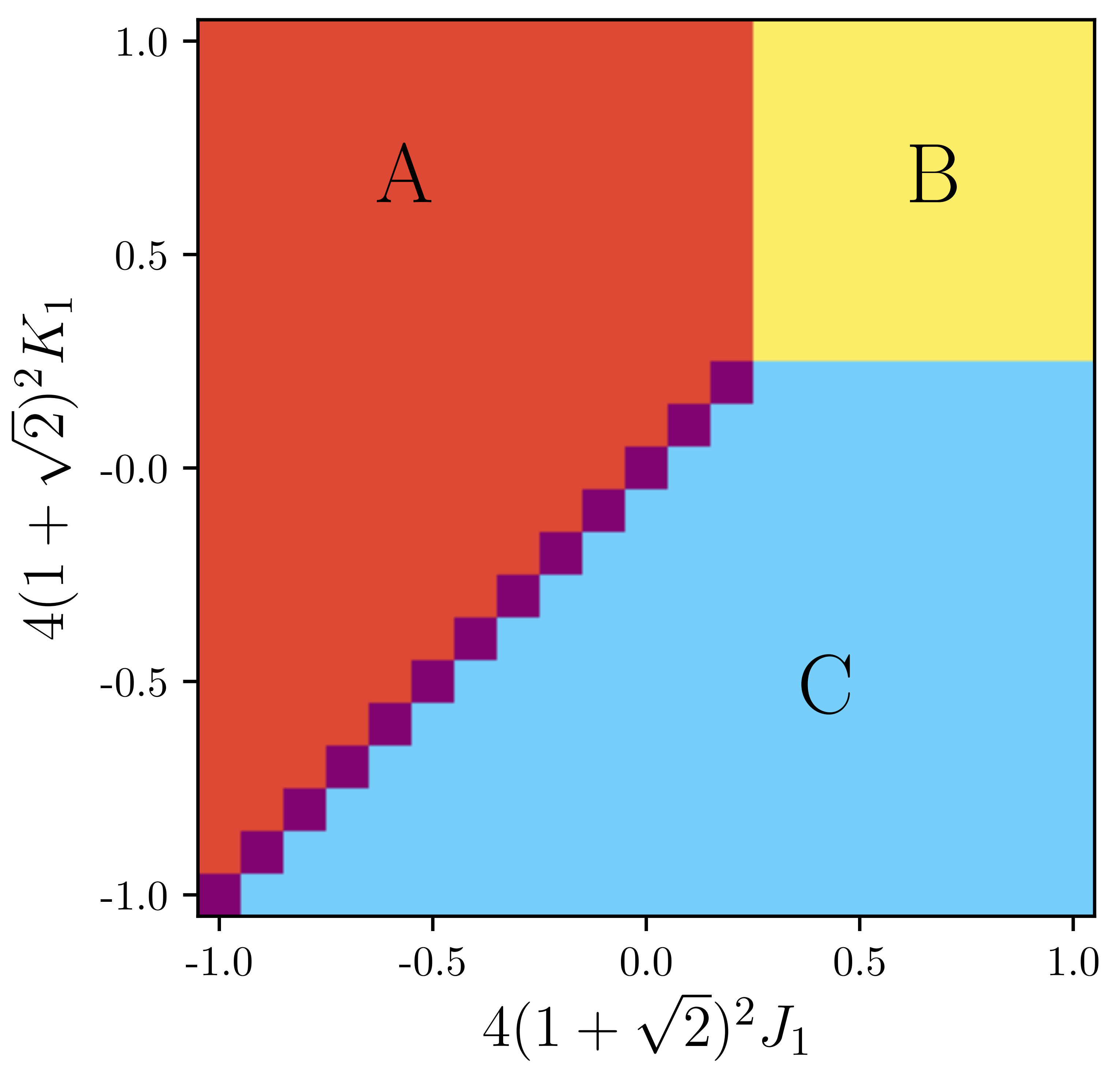

$\langle B \rangle = 0$

Confining phase.
$SO(5)_f$ broken.
Néel or
valence bond solid
order.

$\langle B \rangle \neq 0$

Higgs phase.
$SO(5)_b$ broken.
d-wave superconductivity or
period-2 stripes or
d-density wave order.

r $\longleftarrow$ r_c

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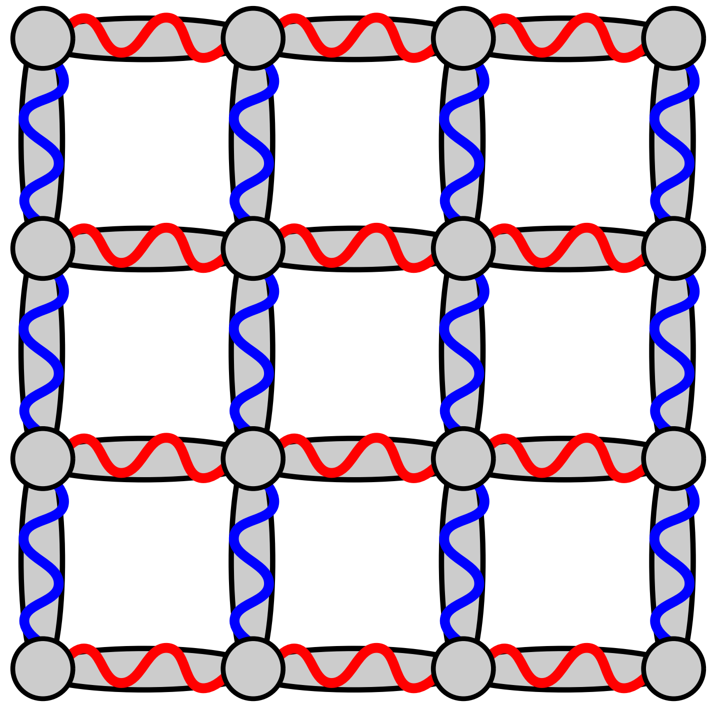
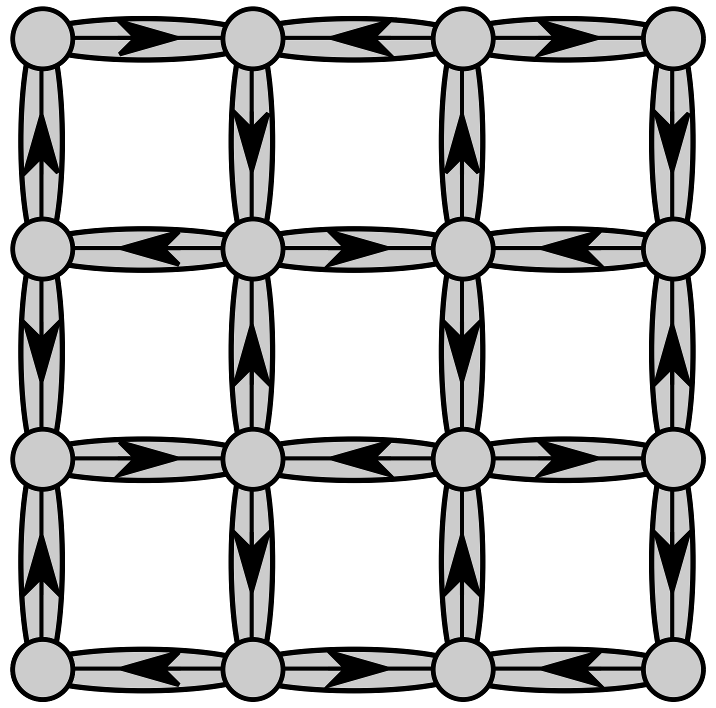
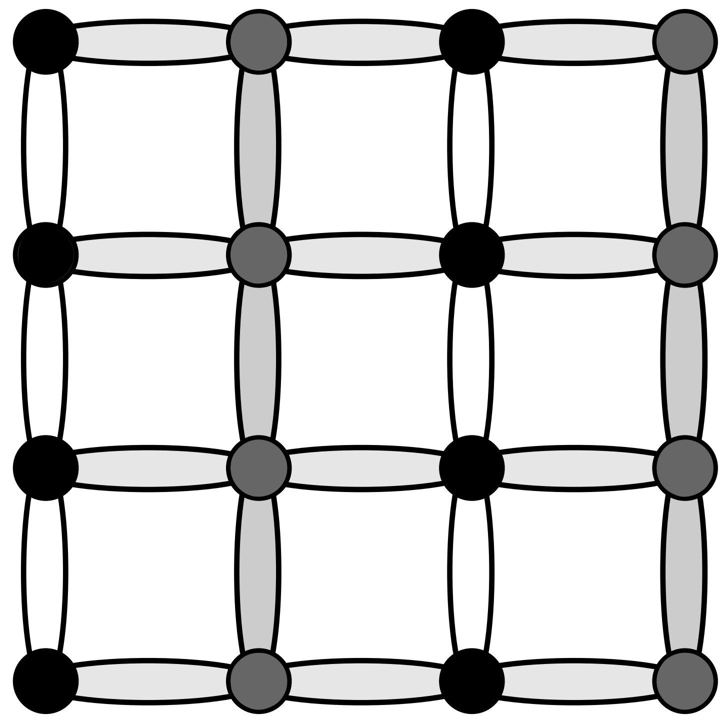
Phase B
d-wave SC



Phase C
d-density



Phase A
(π,0) stripe



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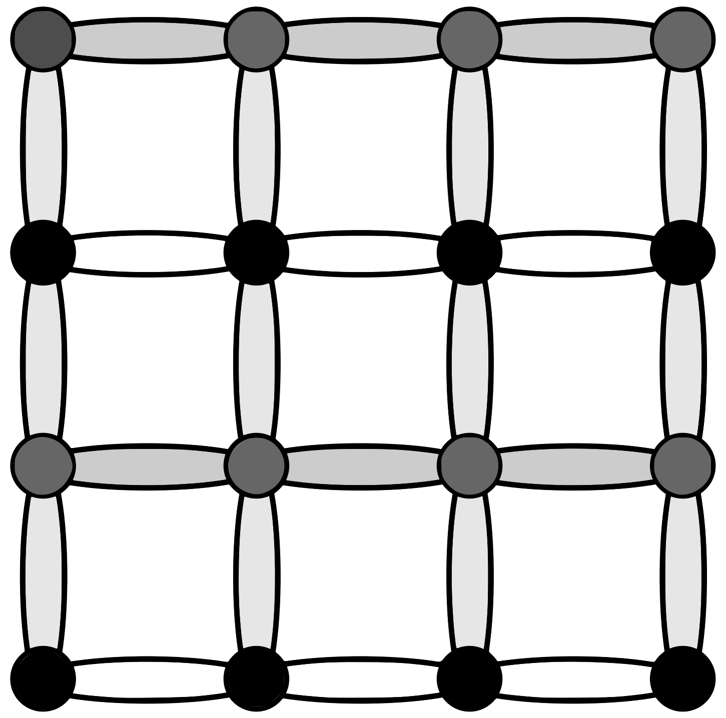
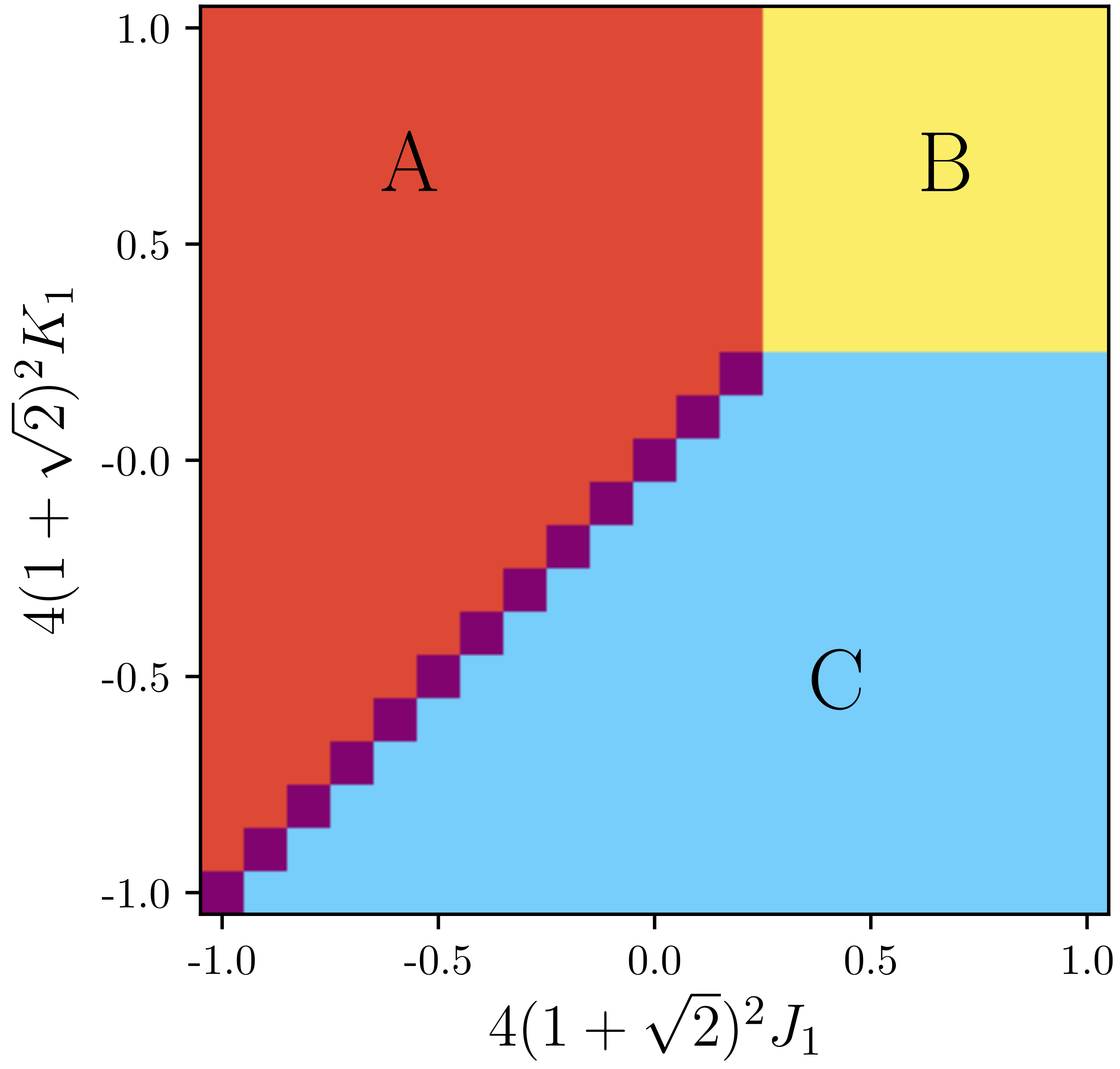


Include charge fluctuations at half-filling: confinement of SU(2) gauge theory



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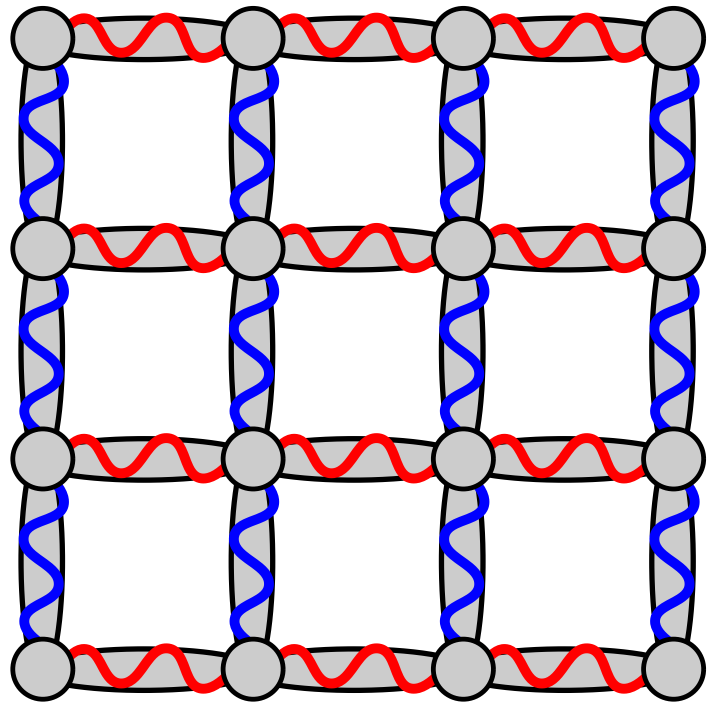
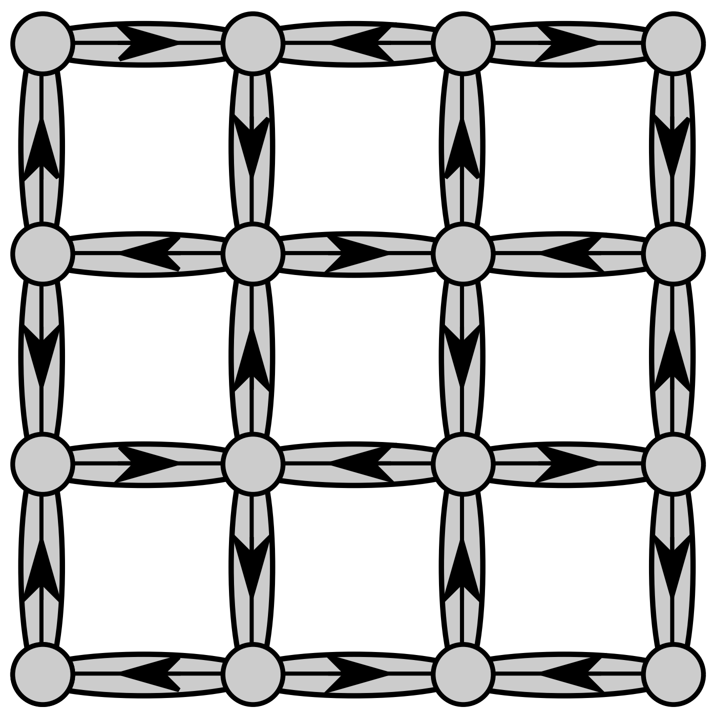
Possible CFT.
DQCP with
$\mathrm{SO}(5)_f \times \mathrm{SO}(5)_b$
symmetry.

Phase B
d-wave SC



Phase C
d-density



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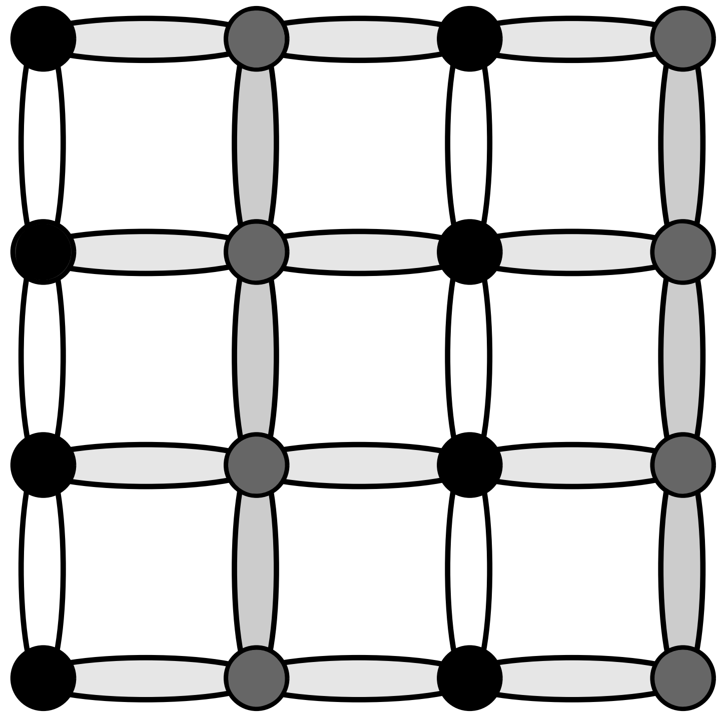

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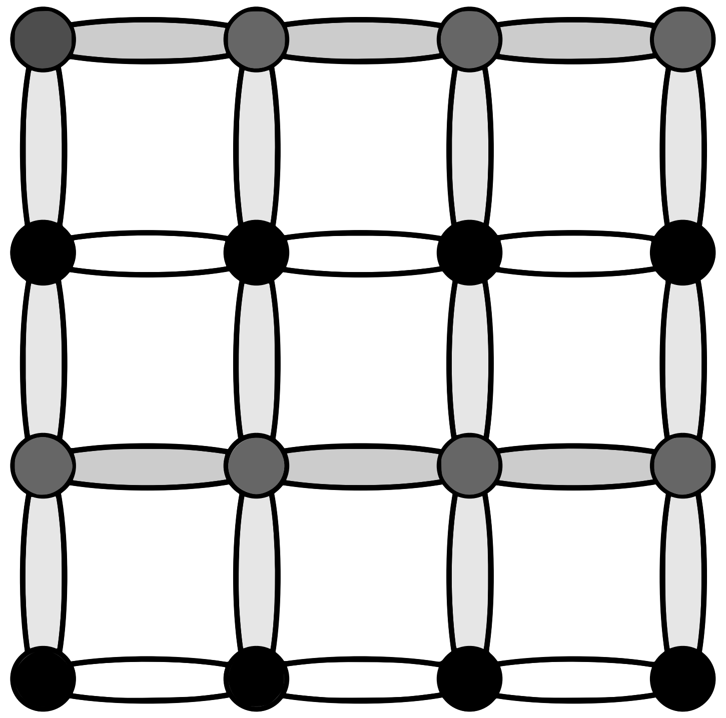
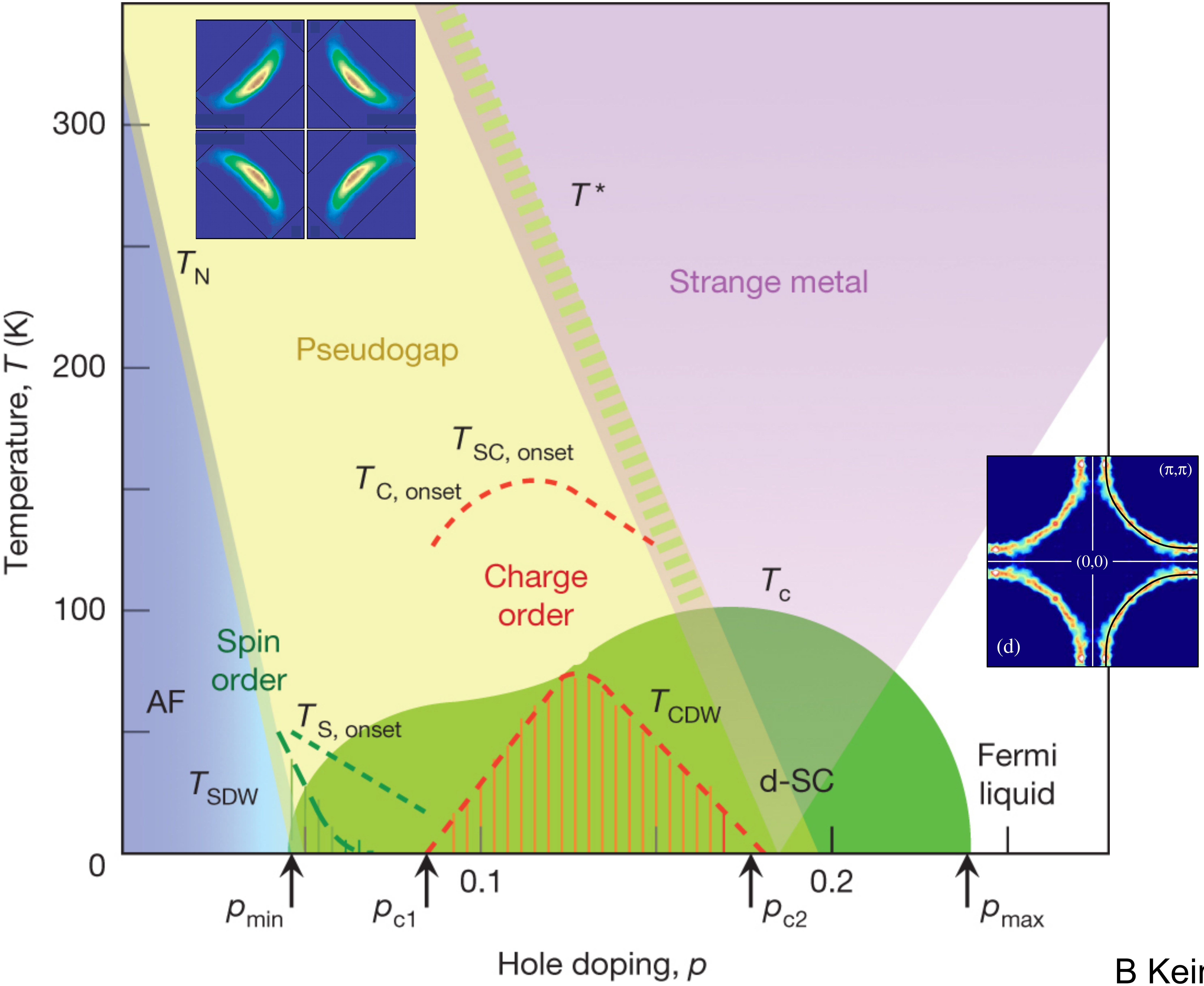
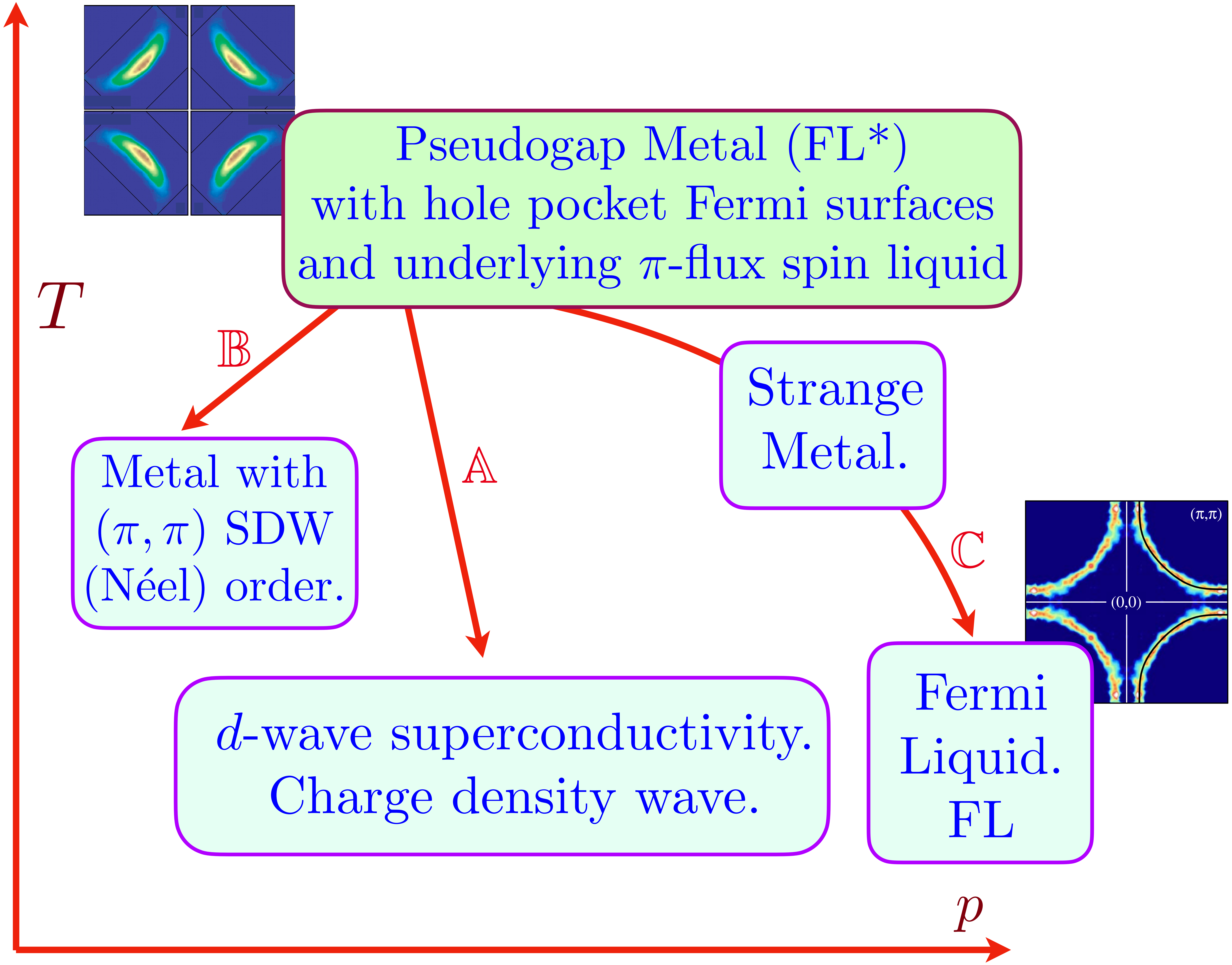
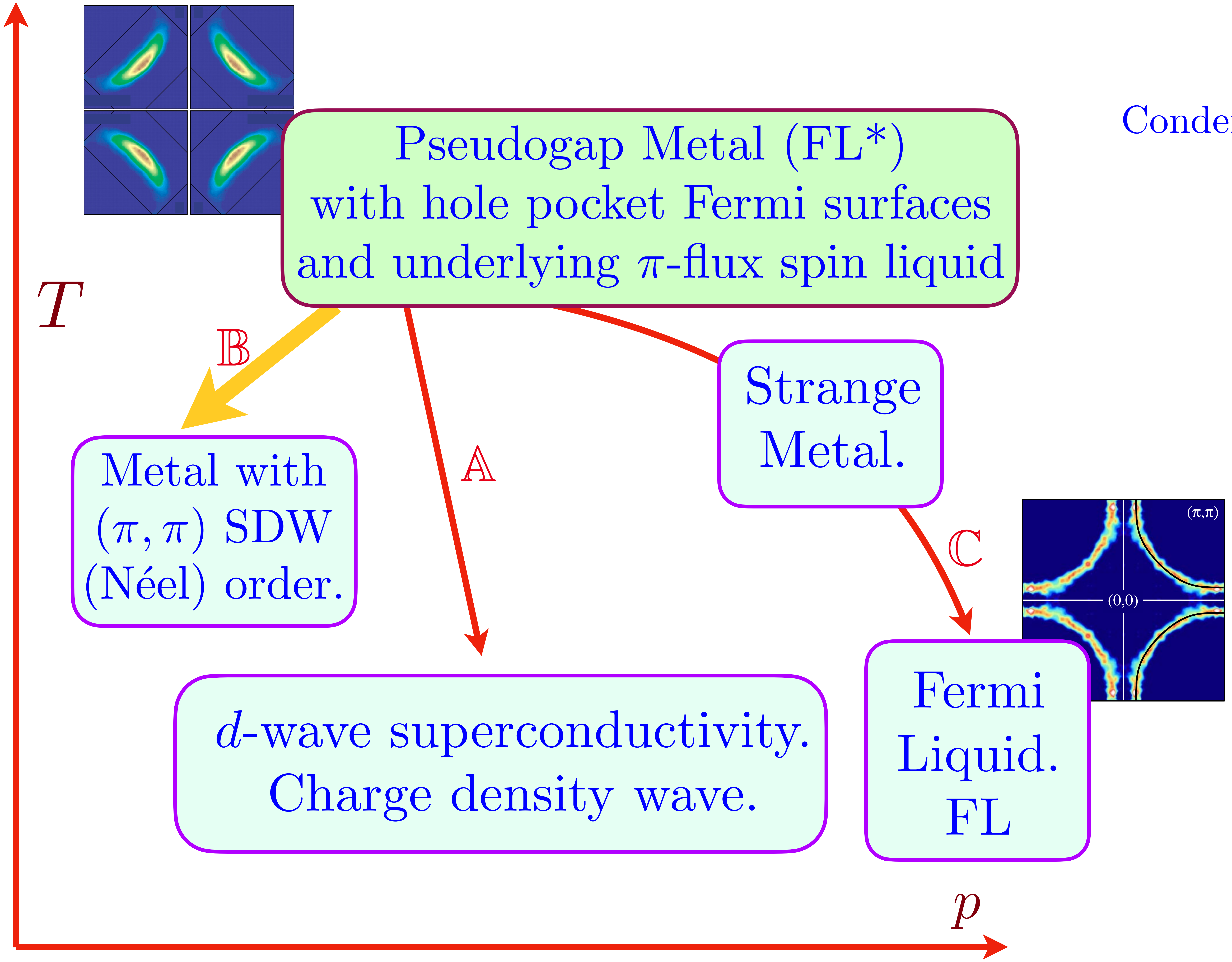




B Keimer *et al.* *Nature* **518**, 179-186 (2015)



Pseudogap Metal (FL*)
with hole pocket Fermi surfaces
and underlying π-flux spin liquid
T
$\mathbb{B}$
$\mathbb{A}$
$\mathbb{C}$
Strange Metal.
Metal with (π, π) SDW (Néel) order.
d-wave superconductivity.
Charge density wave.
Fermi Liquid. FL
(π,π)
(0,0)
p



Arrow $\mathbb{B}$

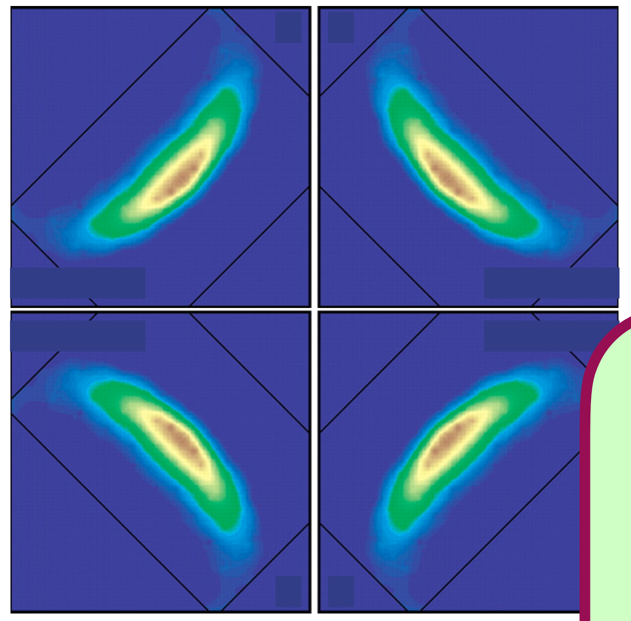
Condensation of z_α in dual $\mathbb{CP}^1$ U(1) gauge theory.


Arrow 𝔸
Condensation of B in SU(2) gauge theory.
Longer-range couplings in H_B can lead to charge order with other periods
Pseudogap Metal (FL*) with hole pocket Fermi surfaces and underlying π-flux spin liquid
T
𝔹
𝔸
ℂ
Metal with (π, π) SDW (Néel) order.
Strange Metal.
d-wave superconductivity. Charge density wave.
Fermi Liquid. FL
p
(π,π)
(0,0)
(a)
4.0
3.2
2.4
1.6
0.8
0.0
$V_{2,2}$
0.0
0.32
0.64
0.96
1.28
1.6
$V_{1,1}$
F
G
H
C
A_1
(b)
Phase F
Period 4 bond centered stripe + d-density
Phase G
Period 4 site centered stripe + d-density

Arrow $\mathbb{C}$

SU$(2)_s\times$U$(1)_a$ gauge theory in ancilla model

Ya-Hui Zhang and S. Sachdev,
Physical Review Research **2**, 023172 (2020);
Physical Review B **102** , 155124 (2020)



Pseudogap Metal (FL*)
with hole pocket Fermi surfaces
and underlying π-flux spin liquid

T

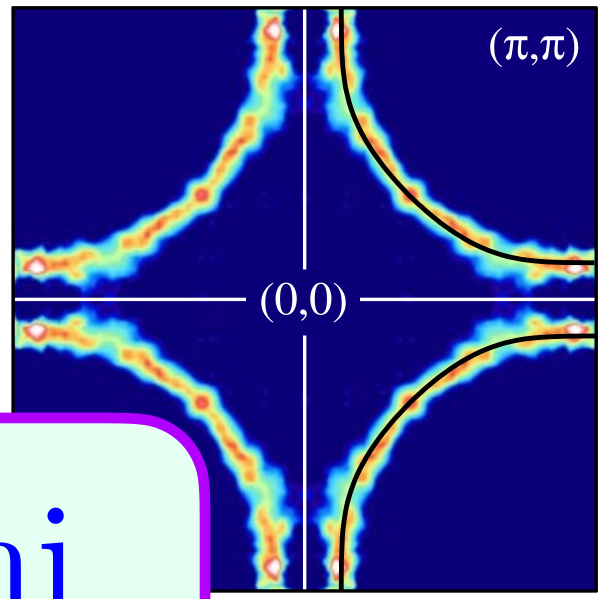
$\mathbb{B}$

$\mathbb{A}$

Metal with
(π,π) SDW
(Néel) order.

Strange
Metal.

$\mathbb{C}$



d-wave superconductivity.
Charge density wave.

Fermi
Liquid.
FL

p


Arrow $\mathbb{A}$
Condensation of B in SU(2) gauge theory.
Longer-range couplings in H_B can lead to charge order with other periods
Pseudogap Metal (FL*) with hole pocket Fermi surfaces and underlying π-flux spin liquid
T
$\mathbb{B}$
$\mathbb{A}$
$\mathbb{C}$
Strange Metal.
Metal with (π, π) SDW (Néel) order.
d-wave superconductivity. Charge density wave.
Fermi Liquid. FL
p
(π,π)
(0,0)
(a)
4.0
3.2
2.4
1.6
0.8
0.0
$V_{2,2}$
0.0
0.32
0.64
0.96
1.28
1.6
$V_{1,1}$
F
G
H
C
A_1
(b)
Phase F
Period 4 bond centered stripe + d-density
Phase G
Period 4 site centered stripe + d-density