Electrical transport near a pair-breaking superconductor-metal quantum phase transition

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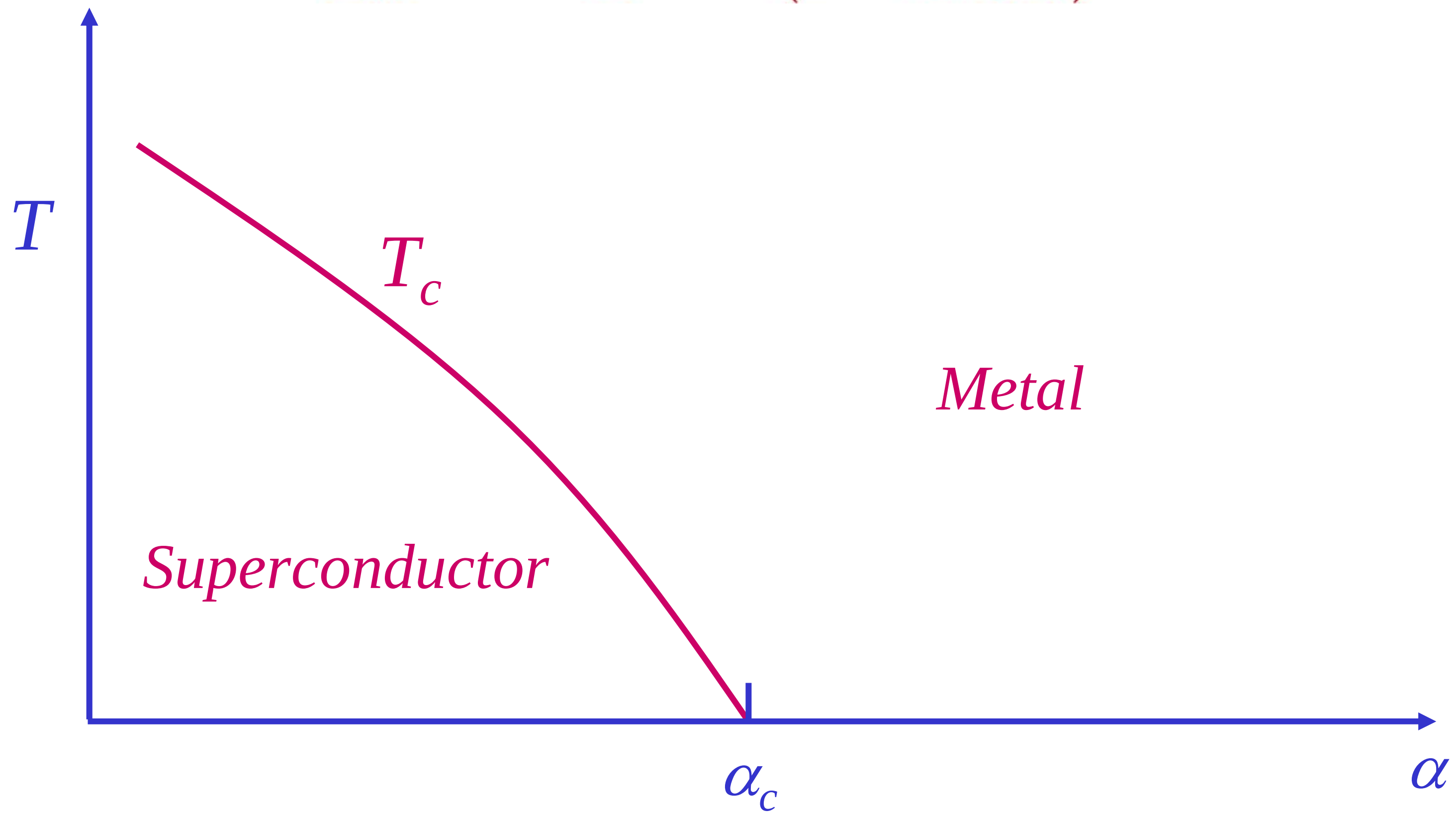
See also talk by Daniel Podolsky,
N38.00007, Wed 9:12 AM

Talk online at http://sachdev.physics.harvard.edu

Standard Abrikosov-Gorkov theory for the suppression of the mean-field BCS critical temperature, T_{c0}, of a superconductor by a pair-breaking frequency α:

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\hbar\alpha}{2\pi k_B T_c}\right)$$



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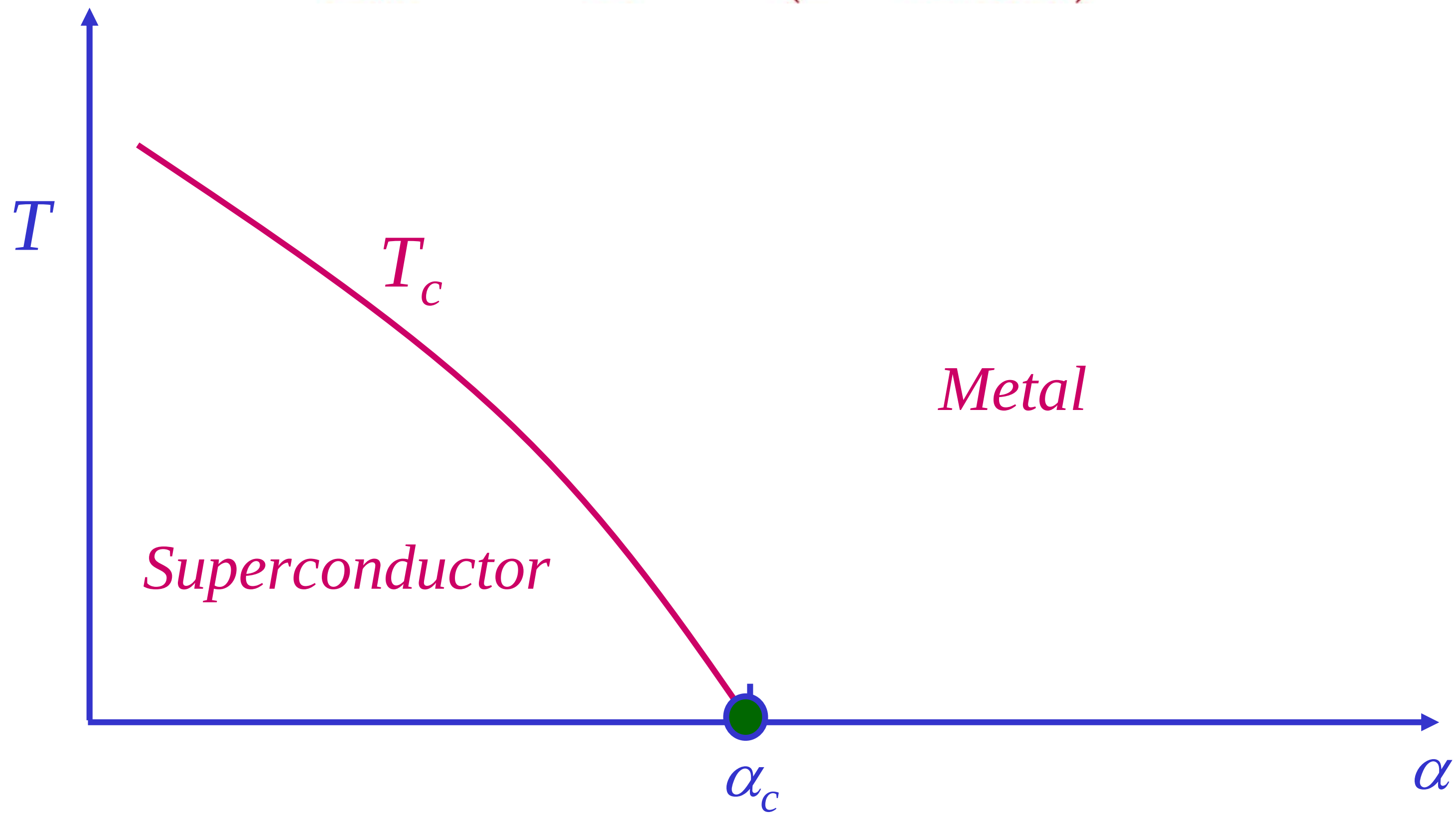


There is a critical $\alpha = \alpha_c$ such that $T_c = 0$ for $\alpha > \alpha_c$. We are interested in the nature of the crossovers near the quantum phase transition at $\alpha = \alpha_c$ especially in spatial dimensions $d = 1, 2$.

Pairbreaking, α can be be due to a magnetic field, H, applied on a wire of radius r

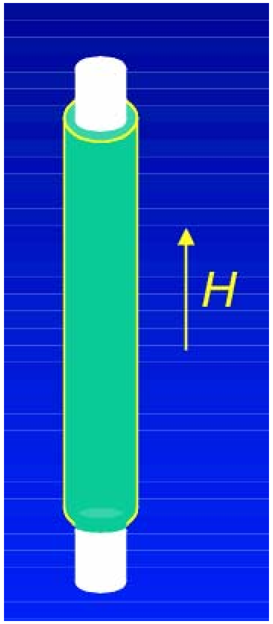
$$\alpha = D(eHr/c)^2/4,$$

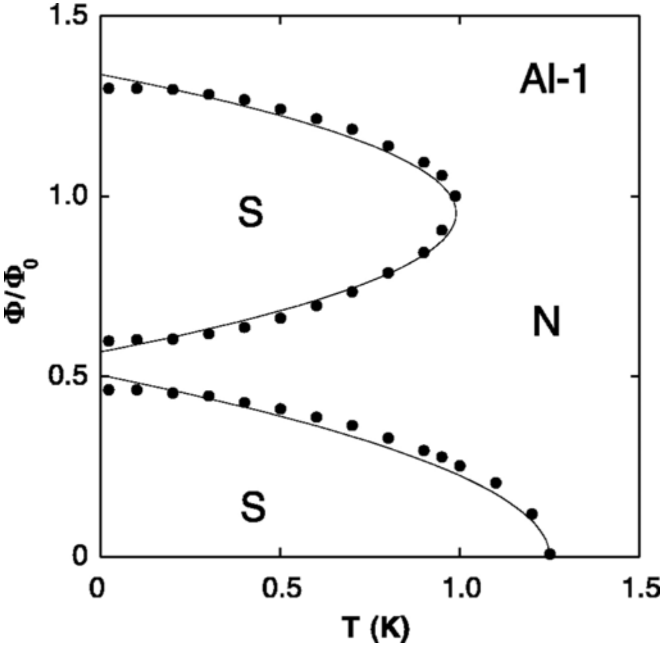
where D is the Cooperon diffusion constant.

On a hollow cylinder with radii r_1 and r_2:

$$\alpha = D\left[\frac{eH}{4c}\left[-4n + \frac{eH}{c}(r_1^2 + r_2^2)\right] + \frac{n^2 \ln(r_2/r_1)}{r_2^2 - r_1^2}\right]$$

where n is an integer. (A. V. Lopatin, N. Shah, and V. M. Vinokur, Phys. Rev. Lett. **94**, 037003 (2005)).





Y. Liu, Yu. Zadorozhny, M. M. Rosario, B. Y. Rock, P. T. Carrigan, and H. Wang, *Science* **294**, 2332 (2001).

Other sources of pairbreaking

- Parallel magnetic field, H, on a film of thickness t, $\alpha = D(eHt/c)^2/6$. Experiments by K. A. Parendo, K. H. Sarwa, B. Tan, and A. M. Goldman, cond-mat/0512704.

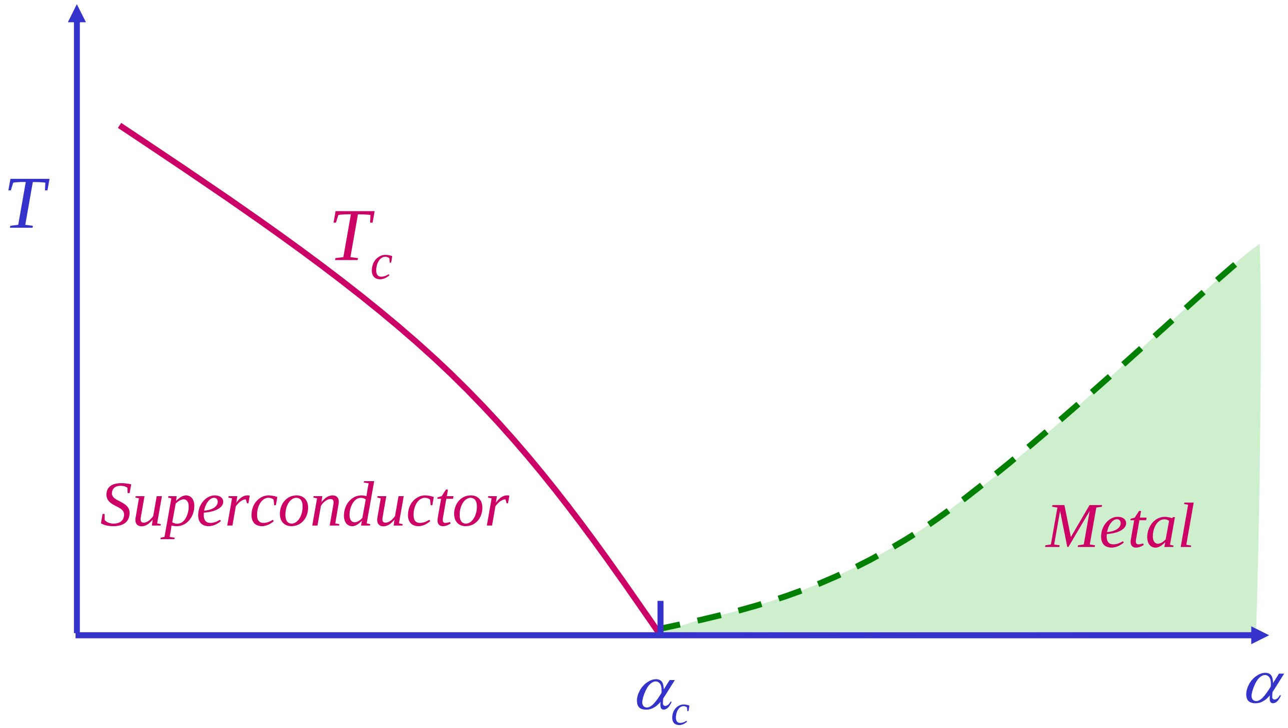
- Inhomogeneous pairing interaction (M. V. Feigelman and A. I. Larkin, Chem. Phys. **235**, 107 (1998); B. Spivak, A. Zyuzin, and M. Hruska, Phys. Rev. B **64**, 132502 (2001)).

- Impurities in a d-wave superconductor. (I. F. Herbut, Phys. Rev. Lett. **85**, 1532 (2000)).

- Magnetic impurities.

I. Theory for the superconductor-metal quantum phase transition

Computation of fluctuation conductivity in metal at low temperatures



At $T = 0$, the Maki-Thomson and density of states corrections to the conductivity, $\delta\sigma$, *increase* with increasing α (negative magnetoresistance):

$$\delta\sigma \sim (\alpha - \alpha_c)$$

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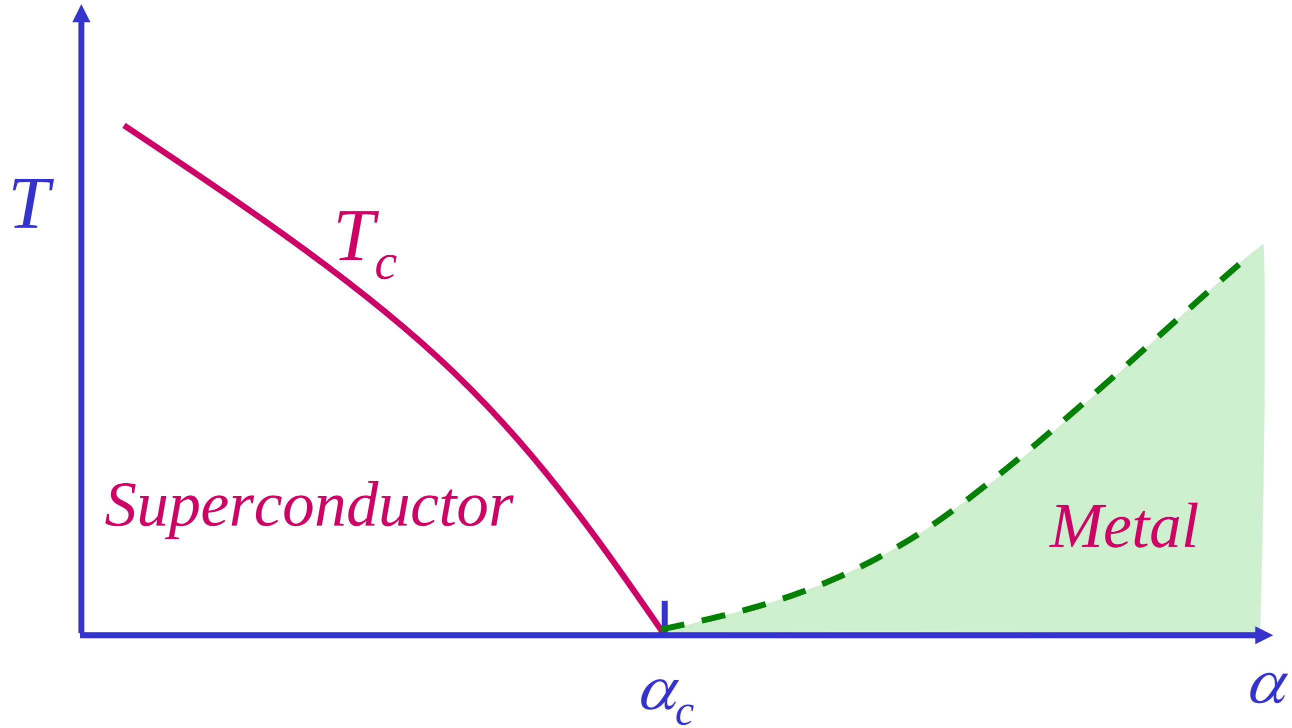

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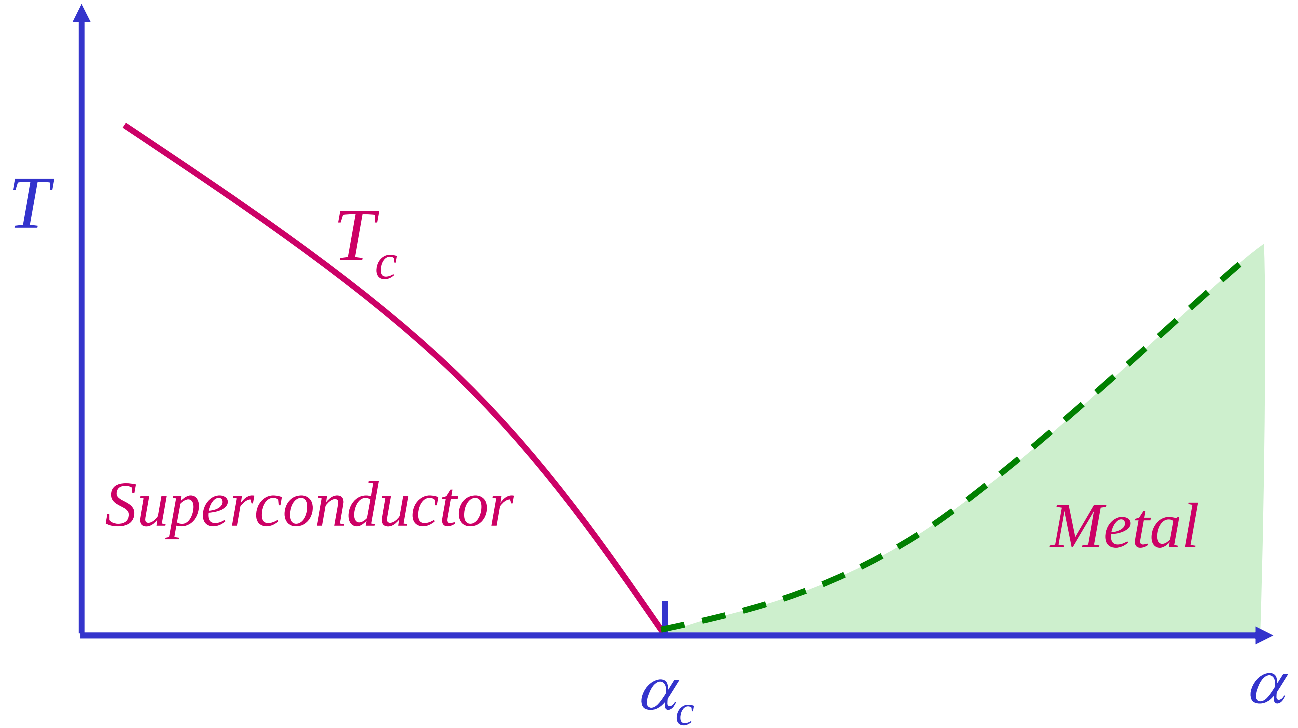
We will argue that these are corrections to scaling to the theory of the quantum critical point. These corrections are *dangerously irrelevant*, because they dominate at low T.

Computation of fluctuation conductivity in metal at low temperatures



At $T > 0$, Aslamazov-Larkin corrections lead to

$$\delta\sigma \sim \frac{4e^2}{h} \frac{D^{2-d}(k_B T/\hbar)^2}{(\alpha - \alpha_c)^{(6-d)/2}}$$

(A. V. Lopatin, N. Shah, and V. M. Vinokur, Phys. Rev. Lett. **94**, 037003 (2005))

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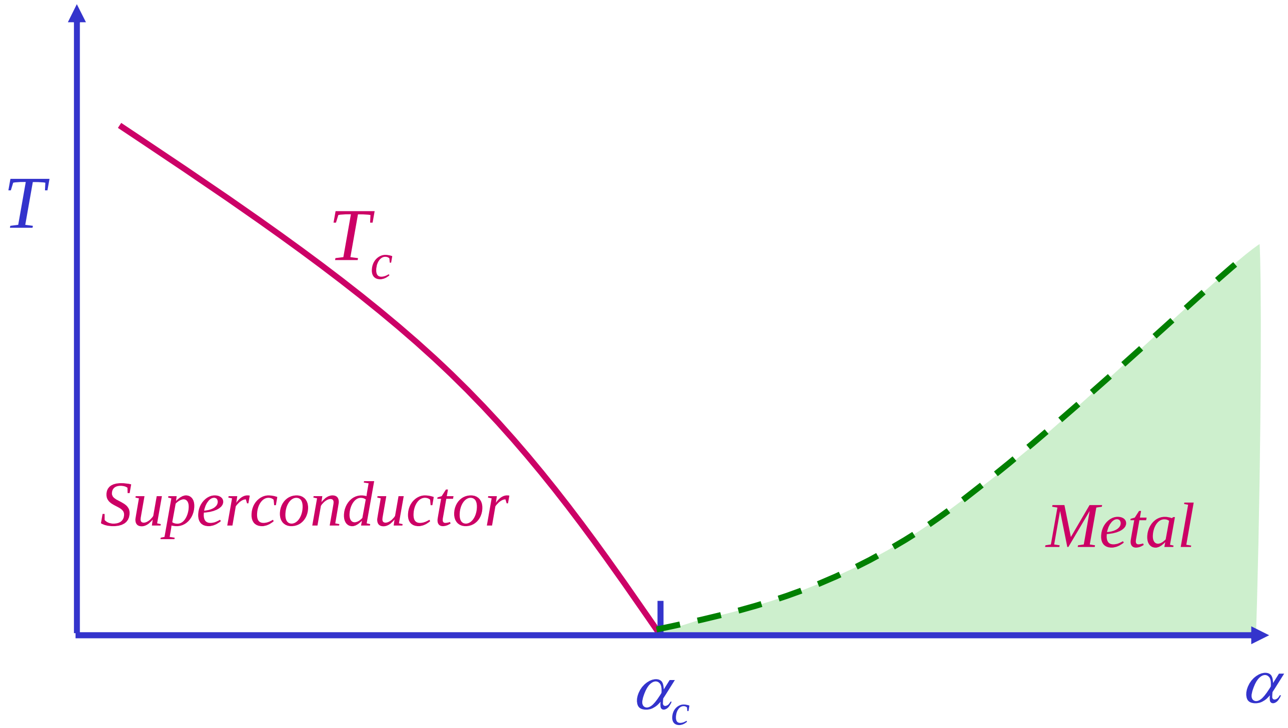

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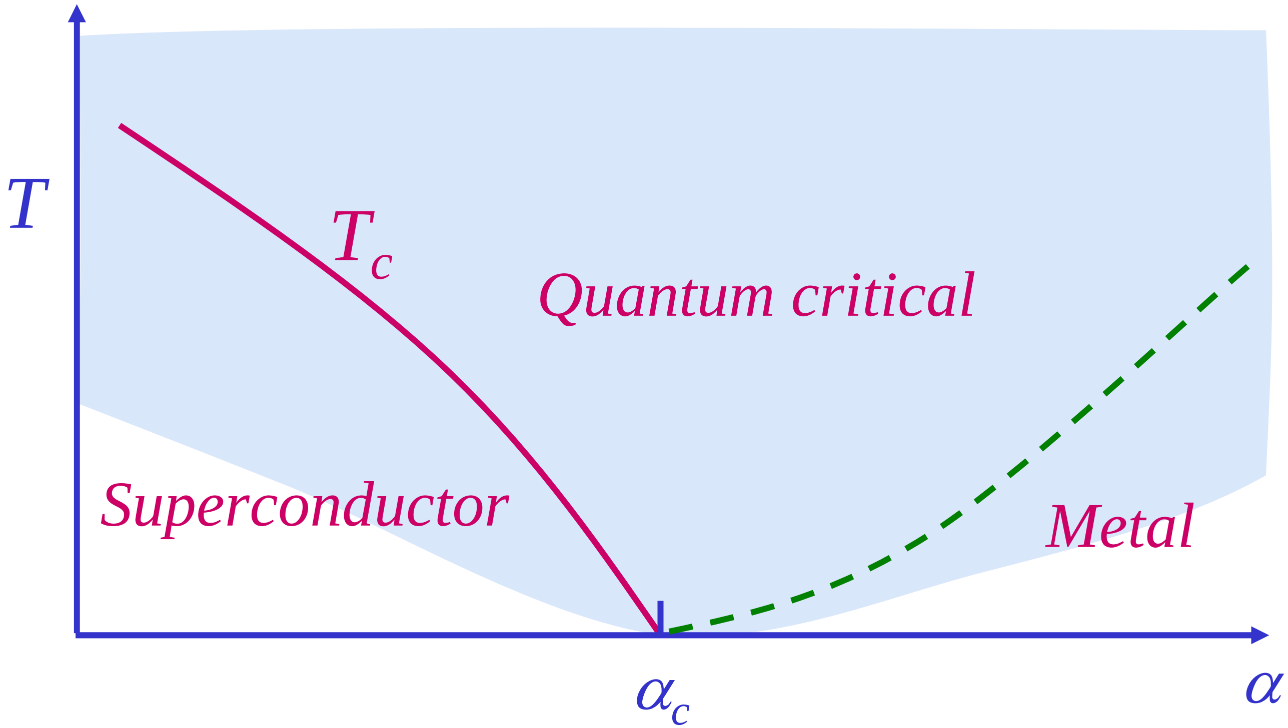
We will argue these are contained in the quantum critical theory. Note, however, the leading critical fluctuations vanish at $T = 0$ for $\alpha > \alpha_c$. This leads to a non-monotonic T dependence in critical theory.

Theory for quantum-critical region, and beyond



Cooperon fluctuations have propagator $\sim 1/(Dq^2+|\omega|+\alpha)$. Self-interactions between such fluctuations are described by

$$\mathcal{S}_{\text{bulk}} = \int d^d x \left[\int \frac{d\omega}{2\pi} \left(D|\nabla_x \psi(x,\omega)|^2 + (|\omega| + \alpha)|\psi(x,\omega)|^2 \right) + \frac{u}{2} \int d\tau |\psi(x,\tau)|^4 \right],$$

R. Ramazashvili and P. Coleman, Phys. Rev. Lett. **79**, 3752 (1997); I. F. Herbut, Phys. Rev. Lett. **85**, 1532 (2000)); D. Dalidovich and P. Phillips, Phys. Rev. Lett. **84**, 737 (2000); B. Spivak, A. Zyuzin, and M. Hruska, Phys. Rev. B **64**, 132502 (2001))

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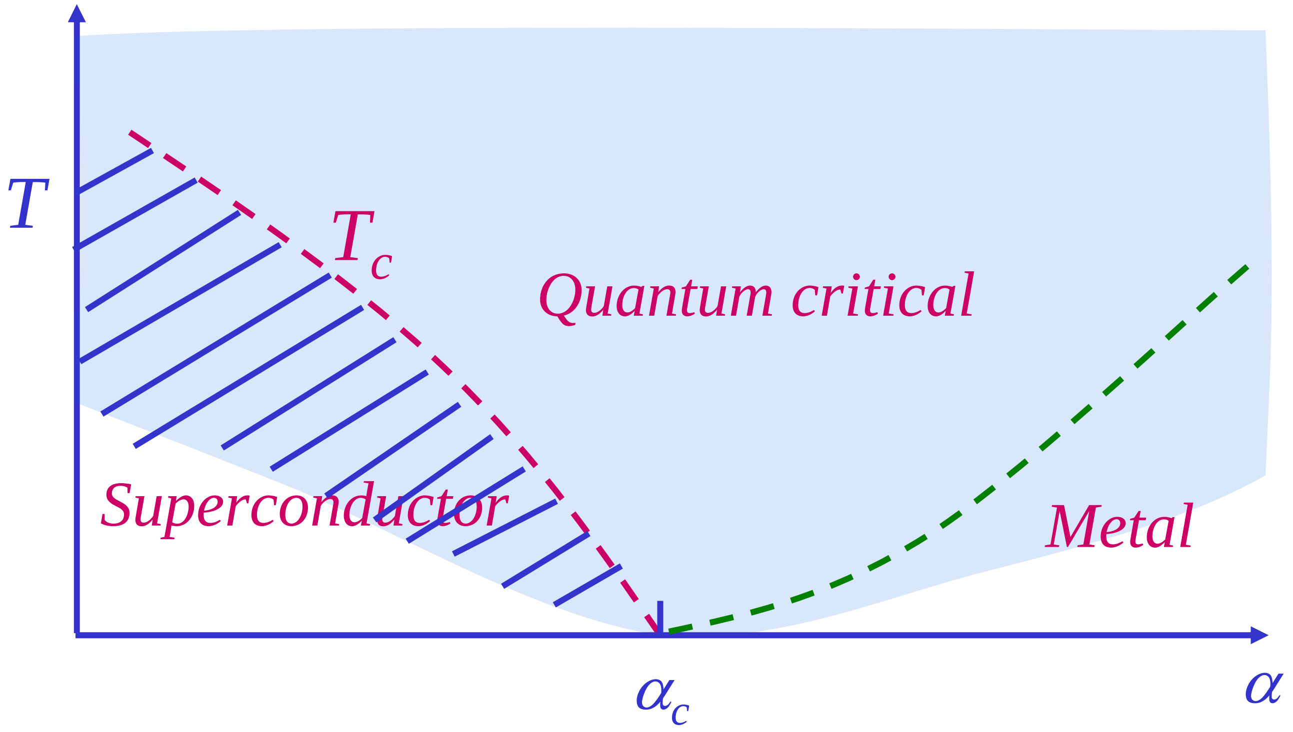

In one dimension, theory reduces to the Langer-Ambegaokar-McCumber-Halperin theory (Model A dynamics), near mean-field T_c

$$\frac{\partial \psi}{\partial t} = - \left[-D \partial_x^2 \psi + \alpha \psi + u |\psi|^2 \psi \right]$$

$$+ \text{thermal Langevin noise}$$

Role of charge conservation in quantum critical theory

(related to the question of why dissipation is not $|\omega| q^2$)

Dynamics of quantum theory (and model A) does not conserve total charge.

Analogous the Fermi-liquid/spin-density-wave transition (Hertz theory), where dynamics of critical theory does not conserve total spin.

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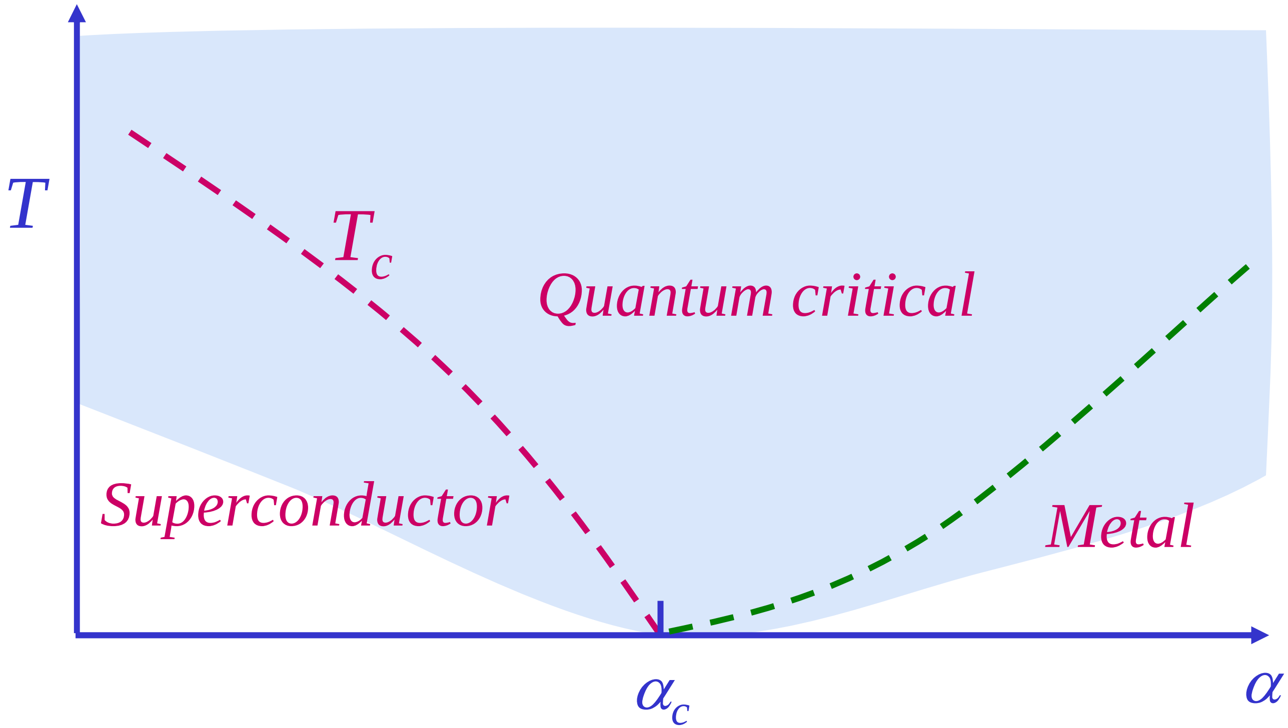
Conservation laws place strong constraints for $\omega/q \to \infty$, but can be ignored in the critical regime, where $\omega/q \to 0$.

L. B. Ioffe and A. J. Millis, Phys. Rev. B **51**, 16151 (1995)

Cooper pairs (SDW) fluctuations decay into fermionic excitations at a finite rate, before any appreciable phase precession due to changes in chemical potential (magnetic field).

II. Quantum criticality in d=1

Theory for quantum-critical region, and beyond in *d*=1

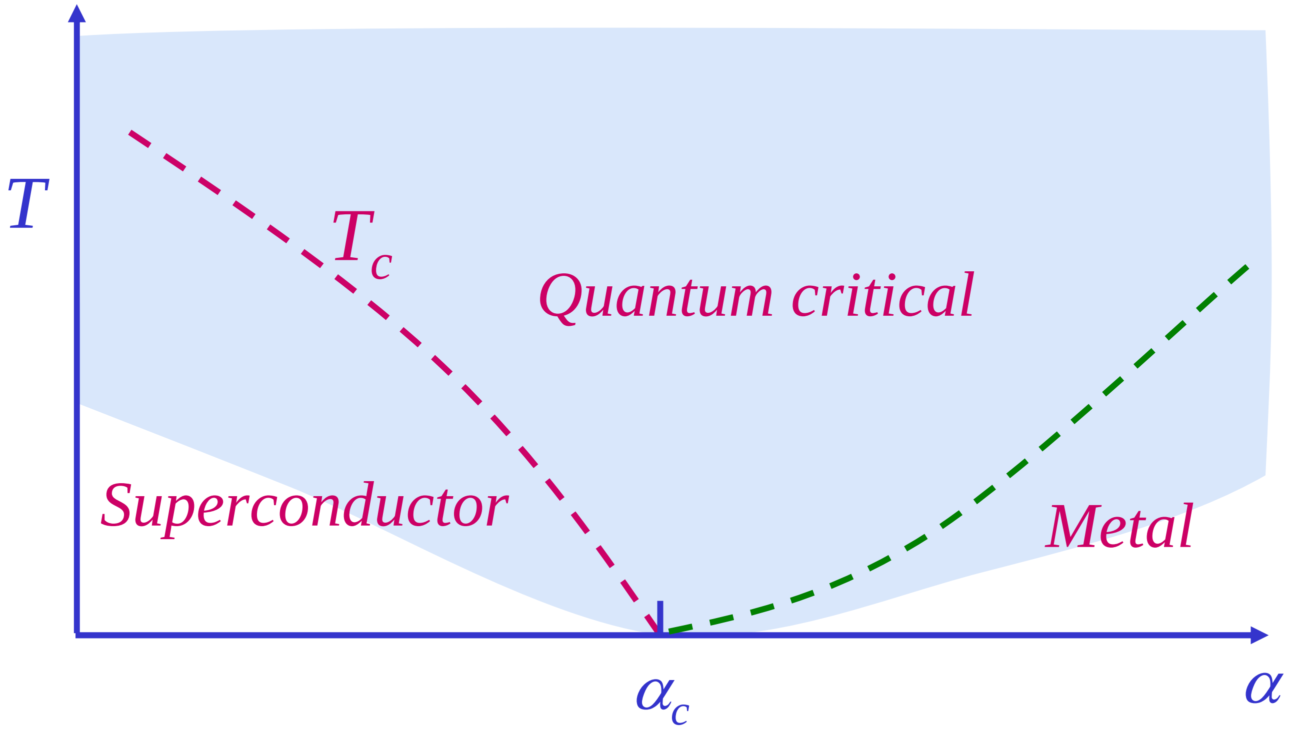


Quantum critical theory obeys strong hyperscaling properties in spatial dimensions $d < 2$. Exponents can be determined by an expansion in $\epsilon = 2 - d$ in a theory with n-component fields ($n = 2$ here).

$$
\begin{aligned}
z &= 2 - \eta \quad ; \quad \eta = \frac{(n+2)(12-\pi^2)}{4(n+8)^2}\epsilon^2 \\
\nu &= \frac{1}{2} + \frac{(n+2)}{4(n+8)}\epsilon + \frac{(n+2)(n^2 + (38 - 7\pi^2/6)n + 132 - 19\pi^2/3)}{8(n+8)^3}\epsilon^2
\end{aligned}
$$

Results at $\epsilon = 1$ in very good agreement with QMC simulations.

Theory for quantum-critical region, and beyond in d=1

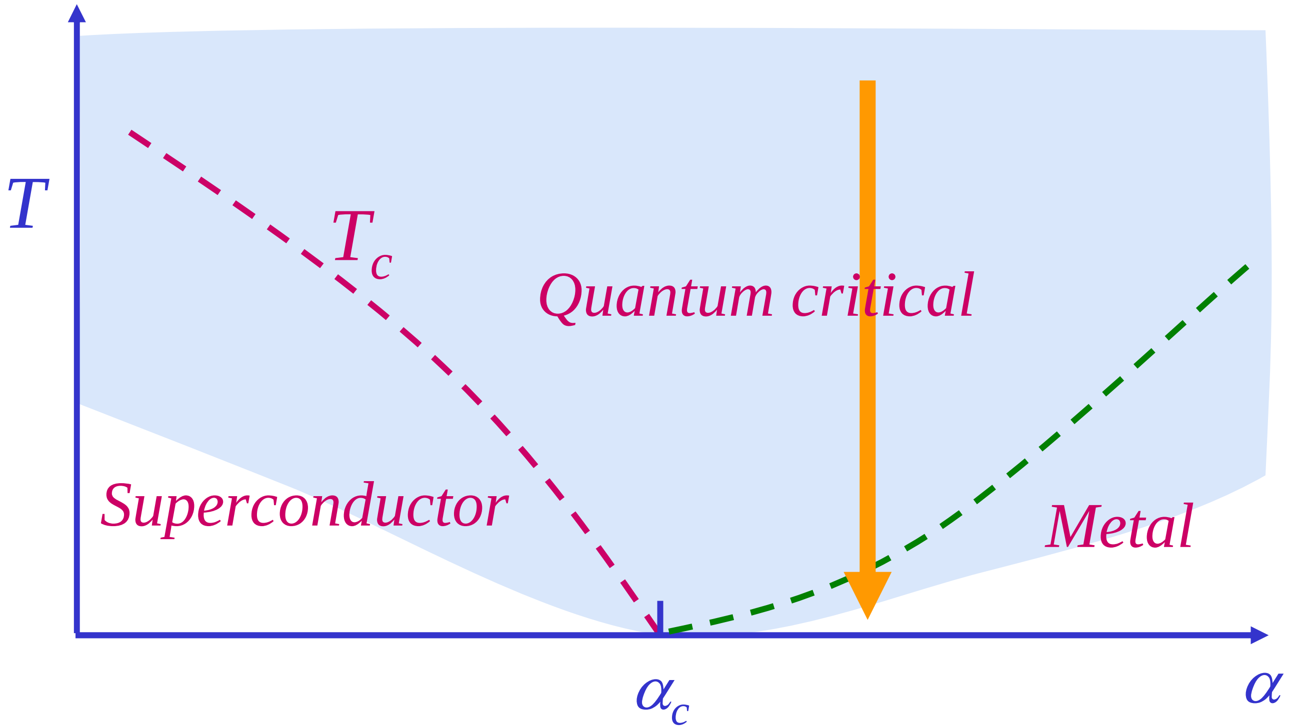


In $d = 1$, conductivity of critical theory obeys universal scaling form:

$$\delta\sigma = \frac{4e^2}{h}\left(\frac{\hbar D}{k_B T}\right)^{1/z} \Phi_\sigma\left(\frac{\alpha - \alpha_c}{T^{1/(z\nu)}}\right)$$

where Φ_σ is a scaling function.

Theory for quantum-critical region, and beyond in *d*=1



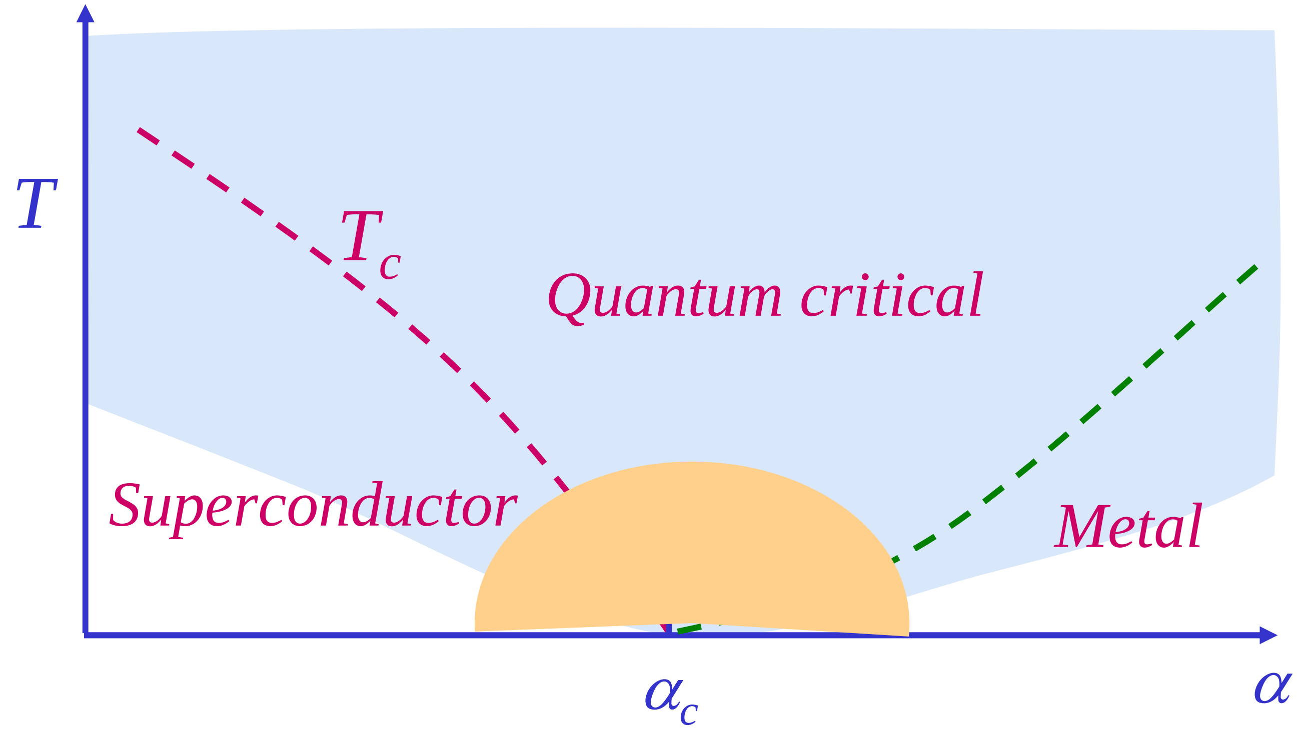
Quantum critical T dependence in $d = 1$:

$$\delta\sigma \sim \begin{cases} \dfrac{1}{T^{1/z}} & \text{for } T > (\alpha - \alpha_c)^{z\nu} \\[2ex] \dfrac{T^2}{(\alpha - \alpha_c)^{(2z+1)\nu}} & \text{for } T < (\alpha - \alpha_c)^{z\nu} \end{cases}$$

Non-monotonic dependence on T.

III. Nanowires near the superconductor-metal quantum critical point

Nanowires near the quantum critical point in *d*=1

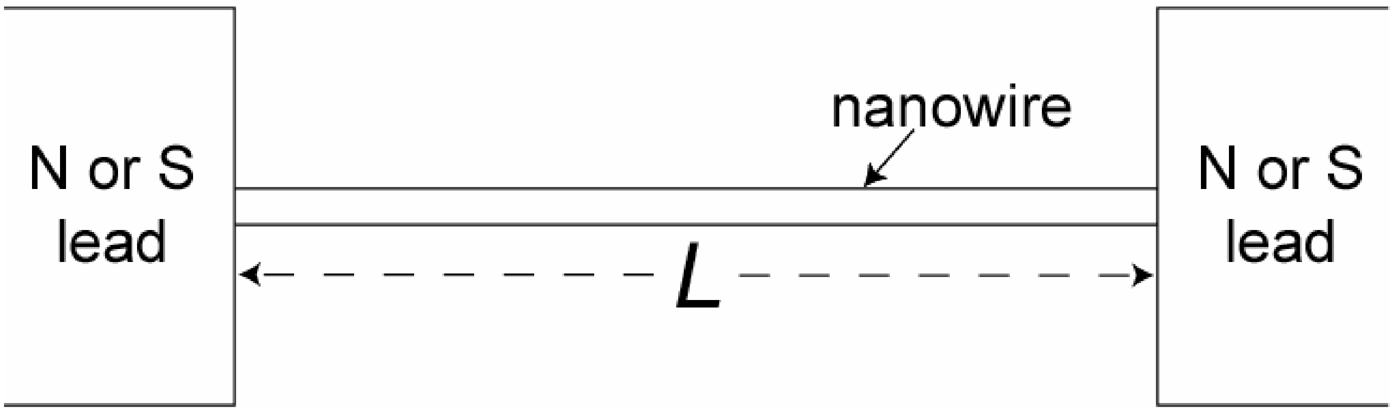


Now the *conductance*, g, of the wire is universal

$$g = \frac{4e^2}{h} F(\omega L^{1/z})$$

where L is the length of the wire, and $L < (\hbar D / k_B T)^{1/2}$.

Effect of the leads



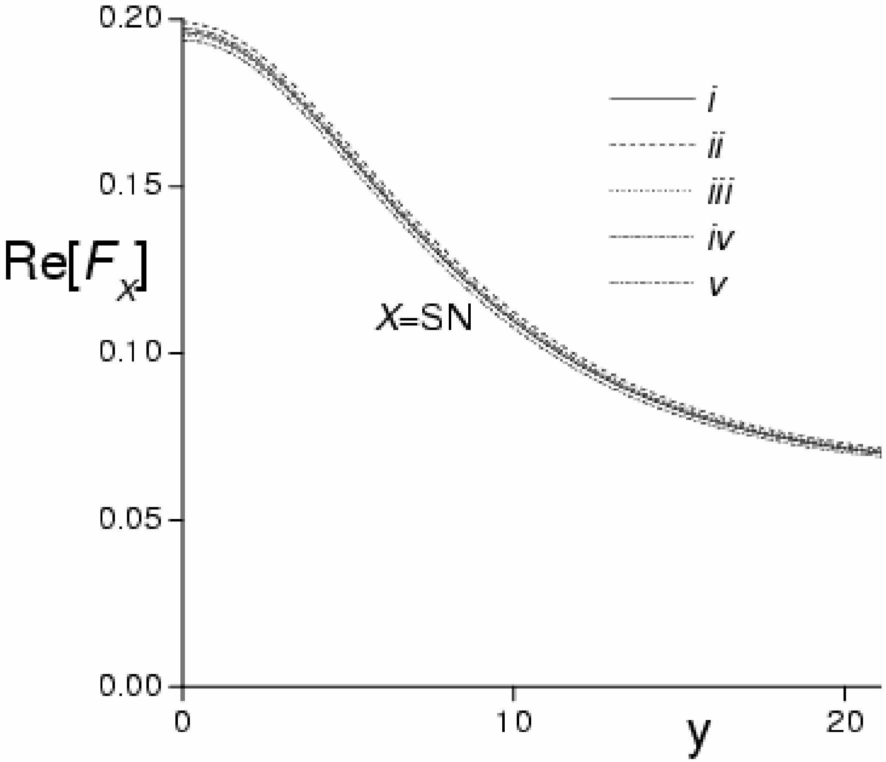
$$\mathcal{S}_{\text{lead}} = \int d\tau \left[-H^* \psi(0,\tau) - H \psi^*(0,\tau) + C |\Psi(0,\tau)|^2 \right]$$

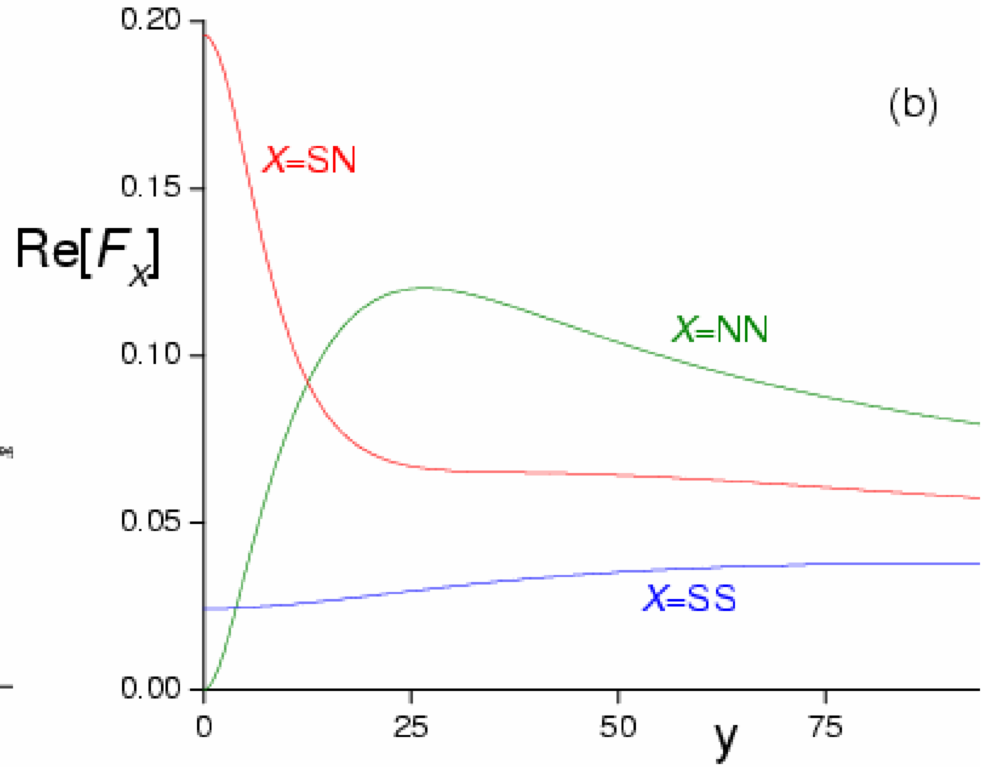
where $H \neq 0$ for a superconducting lead.

Both H and C scale to strong-coupling, and therefore we have Dirichlet boundary conditions ($\Psi = 0$) for a N lead, and Fixed boundary conditions for a S lead

Conductance is *independent* of the specific bare values of H and C.

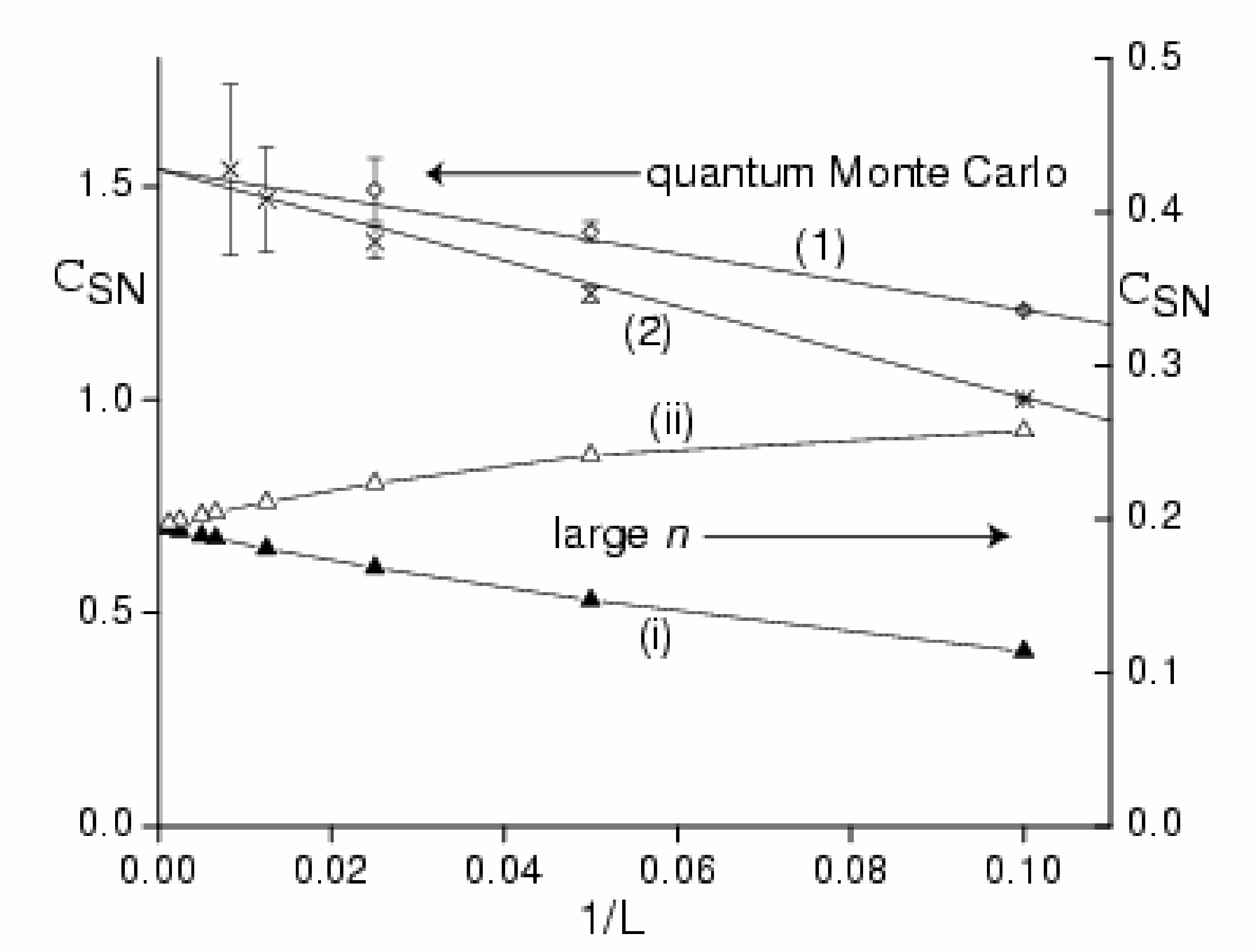
Large n computation of conductance





$$g = \frac{4e^2}{h} F_X(y) \quad ; \quad y = c_1 \omega L^z$$

Quantum Monte Carlo and large *n* computation of d.c. conductance



$$g = \frac{4e^2}{h} C_{SN}$$

IV. Quantum criticality in d=2

Theory for quantum-critical region, and beyond in *d*=2

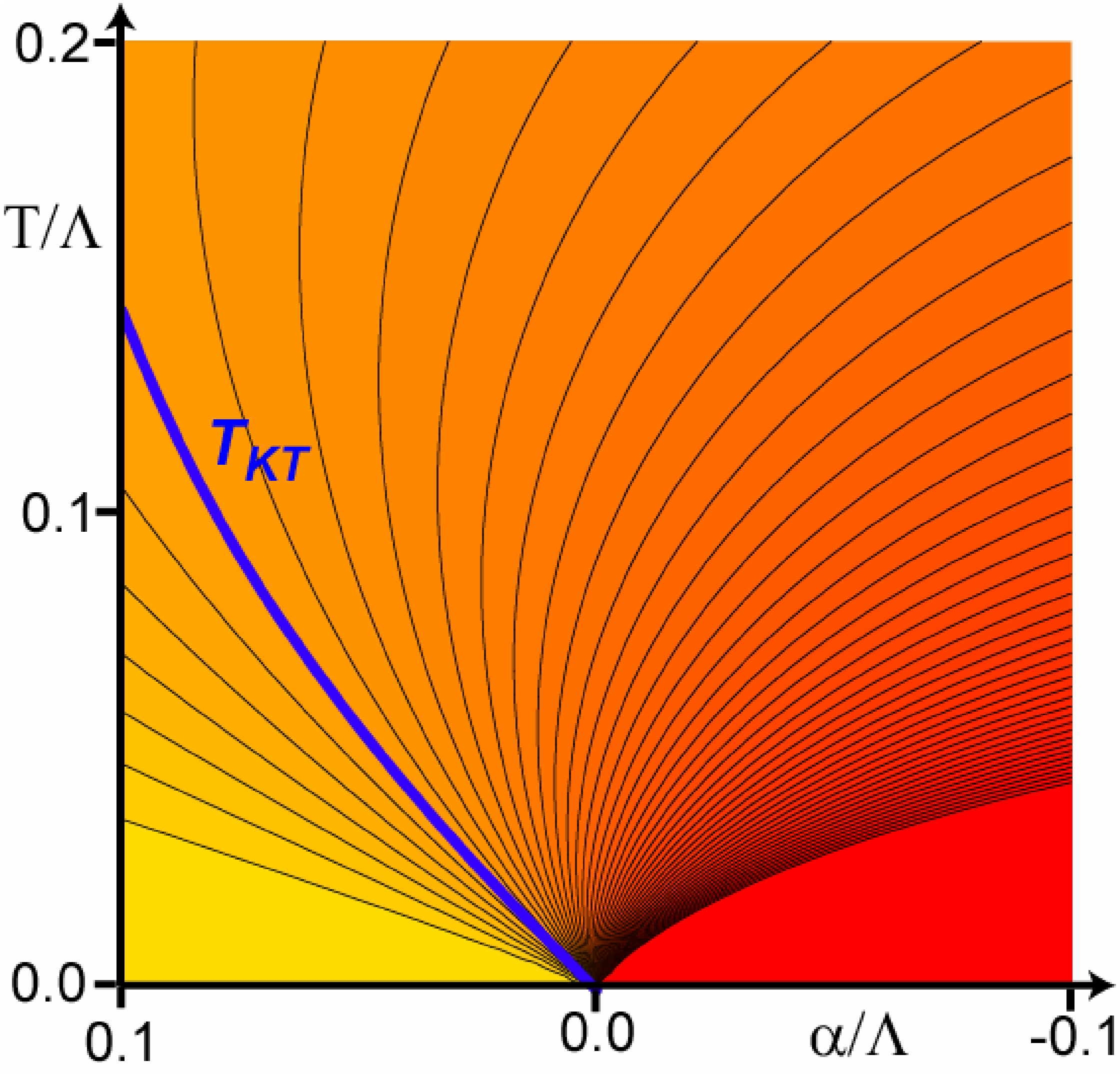
To leading logarithmic accuracy, (nearly) all physical properties can be expressed in terms of computable universal functions of two energy scales, R, and U. R measures distance from the quantum critical point, while U is a quartic self-coupling. These are parameters in a classical theory of equal-time correlations with free energy

$$F = \int d^2x \left[|\nabla \Psi|^2 + \widetilde{R}|\Psi|^2 + \frac{U}{2}|\Psi|^4 \right].$$

R and U depend upon the bare values of α, D, T, and logarithmically on a cutoff energy scale Λ, and are determined by solving a simple integral equation.

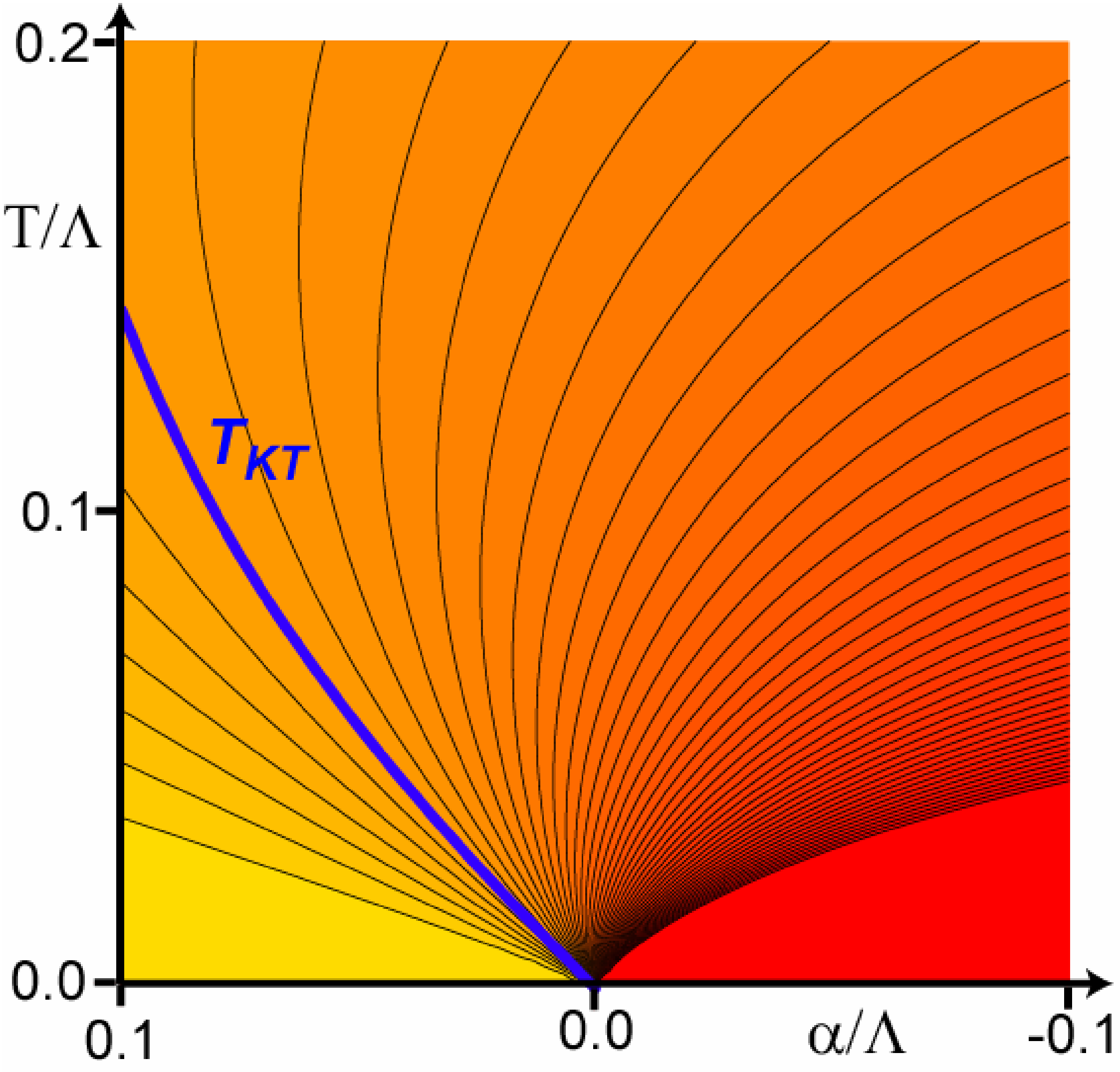
The loci of points with a fixed U/R has the same physical properties, upto a shift in the overall energy scale, R.

Locus of points with U/R constant



The Kosterlitz-Thouless transition occurs at $T = T_{KT}$, where $U/R \approx 34$ (a universal number).

Locus of points with U/R constant

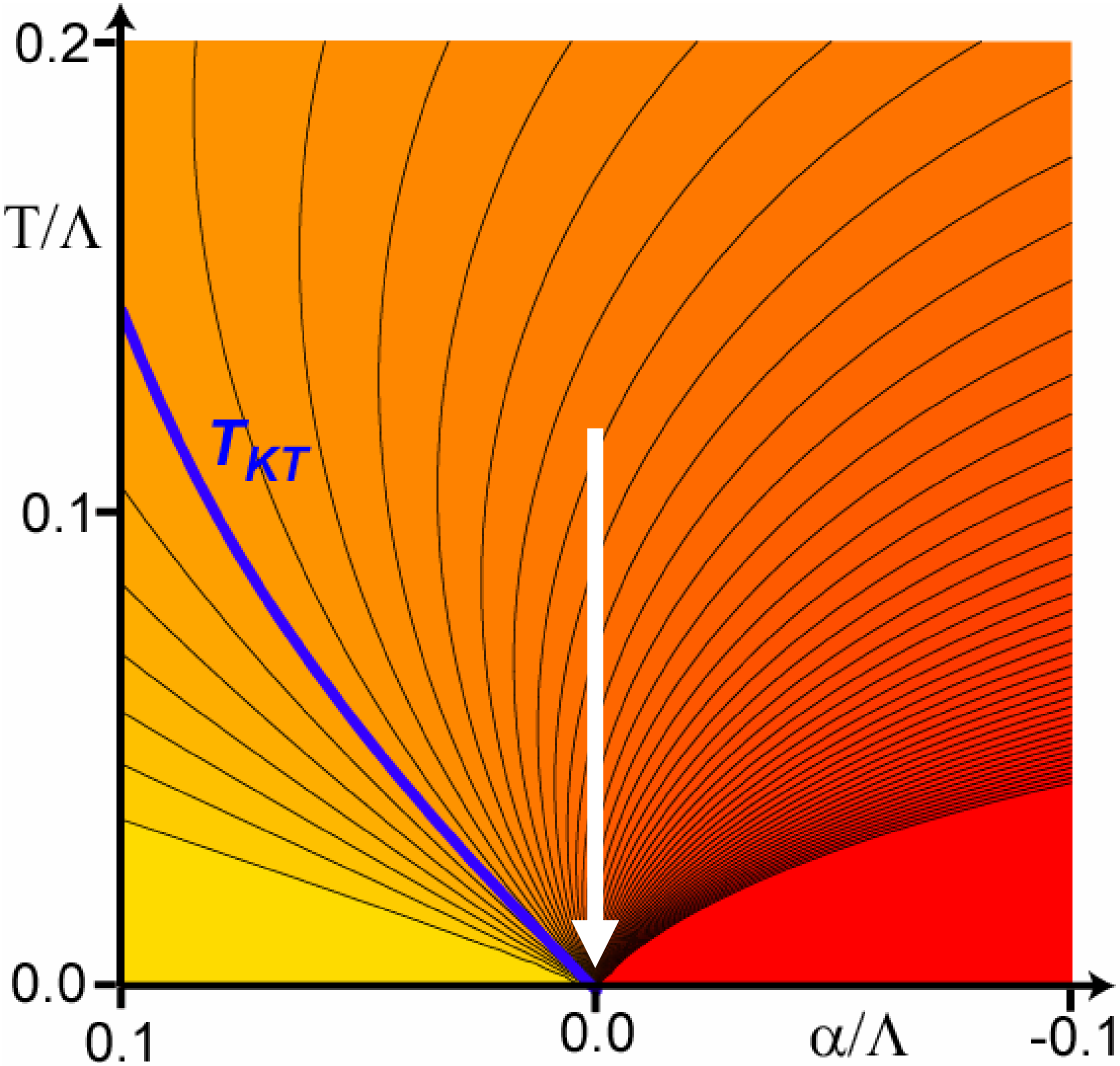


The conductivity obeys the scaling form

$$\sigma = \frac{4e^2}{h}\frac{k_B T}{R}\Phi_A\left(\frac{U}{R}\right)$$

where Φ_A is a completely universal function which can be (numerically) determined by a classical, continuum Model A theory.

Locus of points with U/R constant



At the quantum-critical point

$$R \sim \frac{k_B T}{\ln(\Lambda/(k_B T))}$$

$$\frac{U}{R} \sim \frac{1}{\ln\ln(\Lambda/(k_B T))}$$

$$\sigma \sim \frac{4e^2}{h}\ln(\Lambda/(k_B T))$$

Conclusions

- Universal transport in wires near the superconductor-metal transition
- Theory includes contributions from thermal and quantum phase slips ---- reduces to the classical LAMH theory at high temperatures
- Sensitivity to leads should be a generic feature of the ``coherent'' transport regime of quantum critical points.
- Complete computation of electrical transport in $d=2$ to leading logarithmic accuracy.