The pseudogap metal
and
FL* (a “topological” Fermi liquid)

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Talk online: sachdev.physics.harvard.edu

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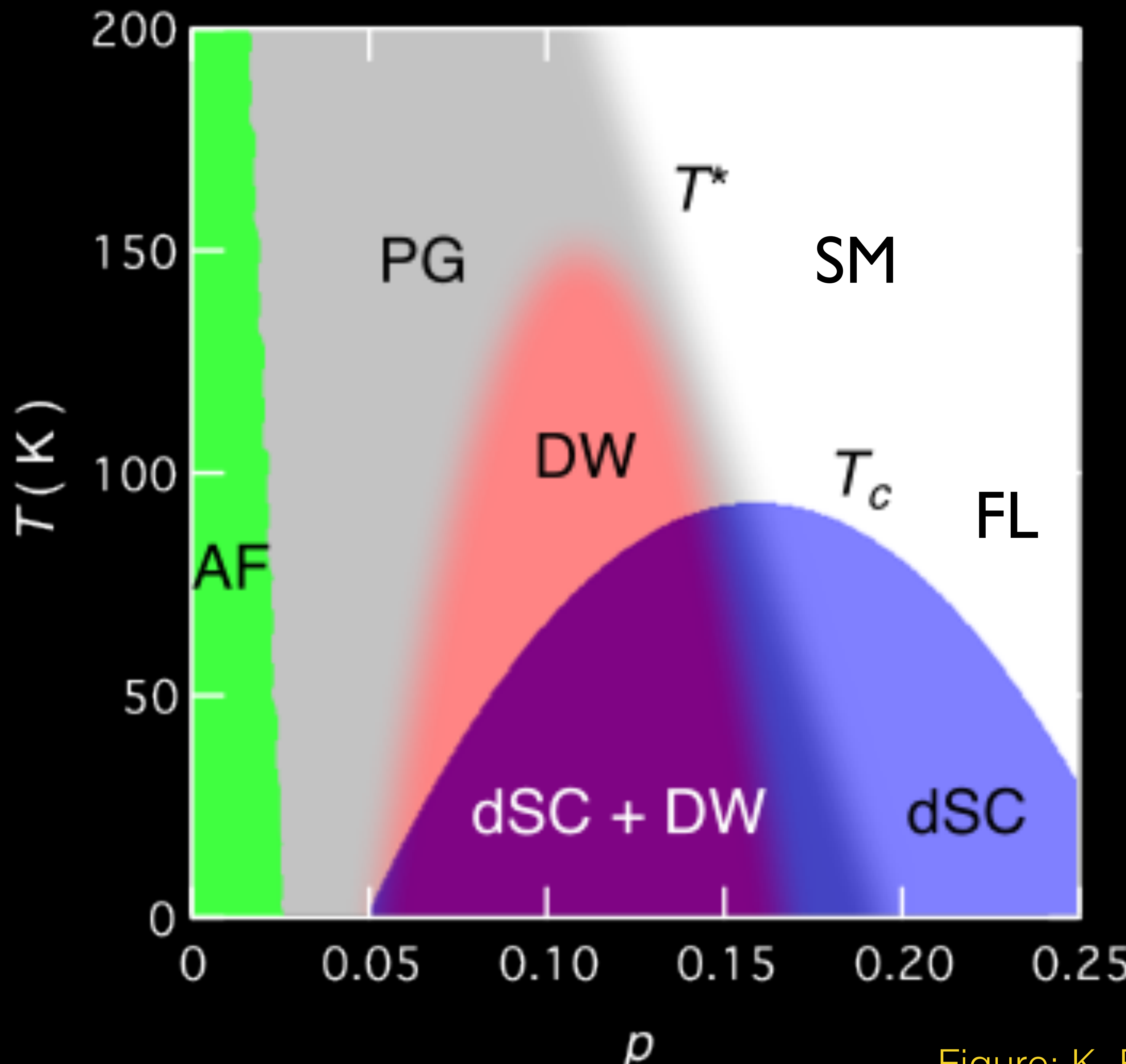
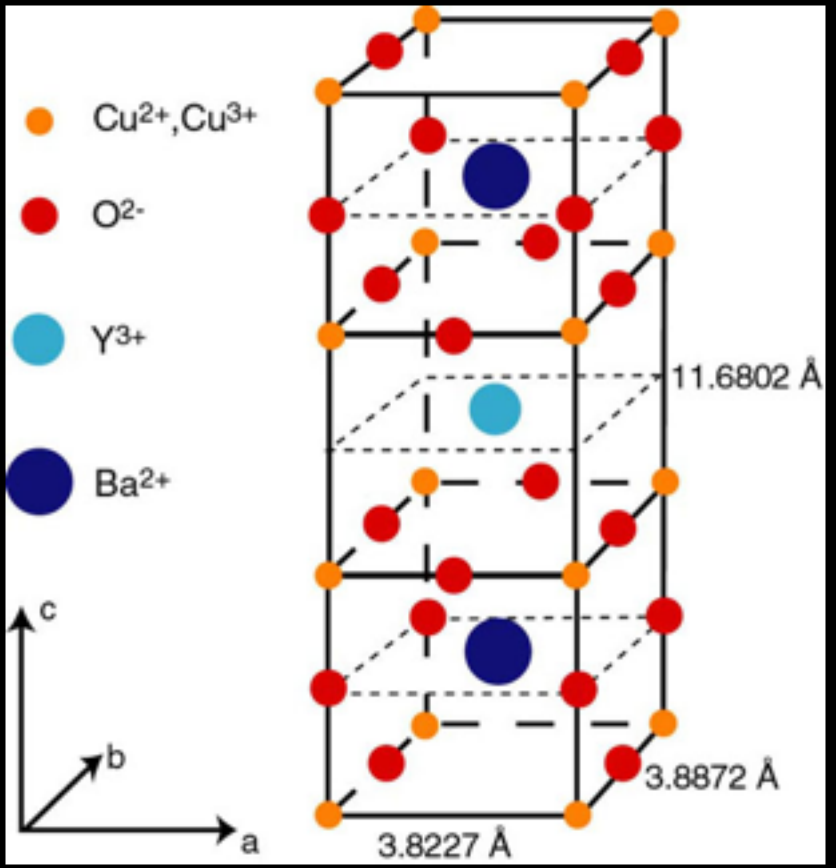
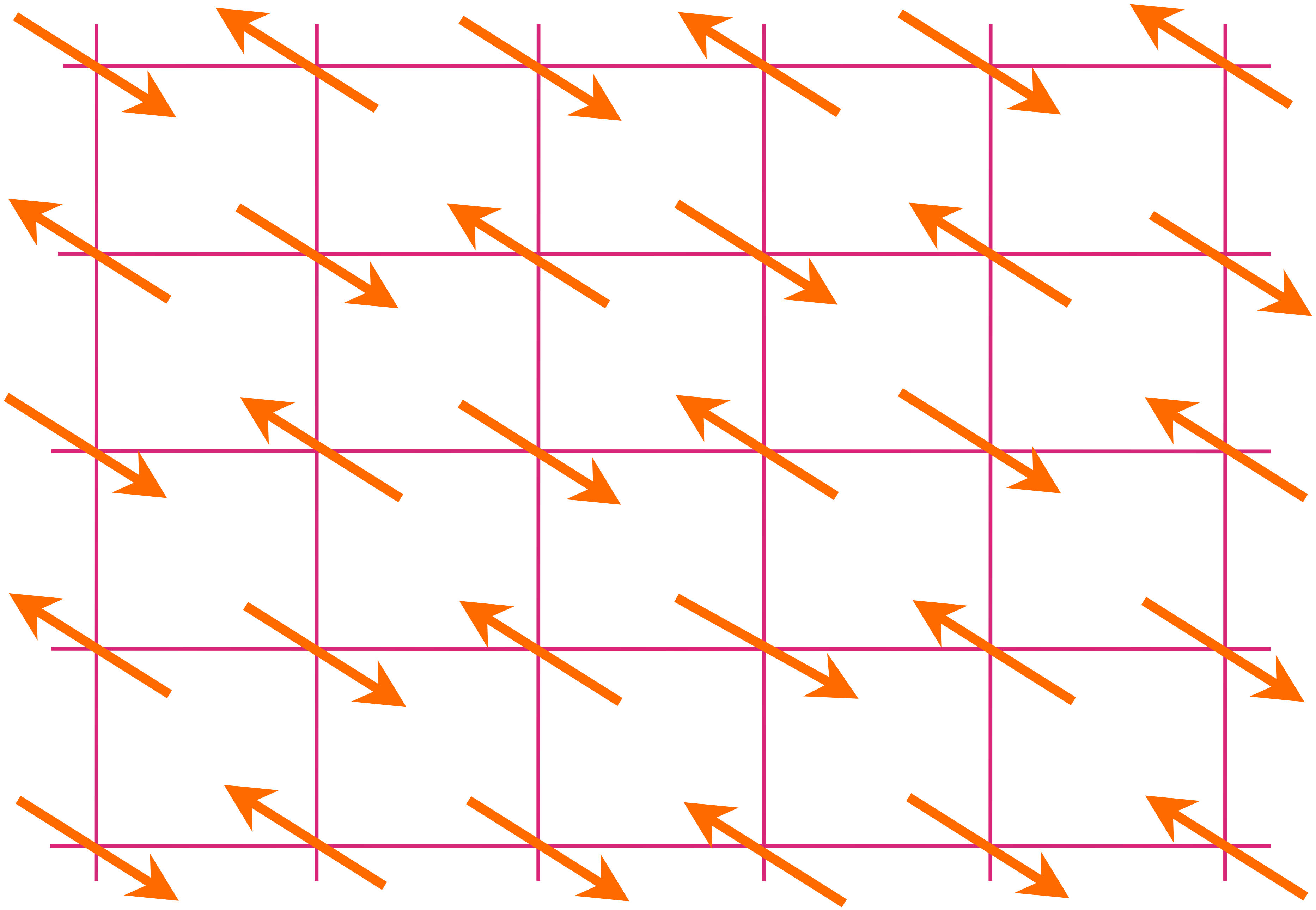


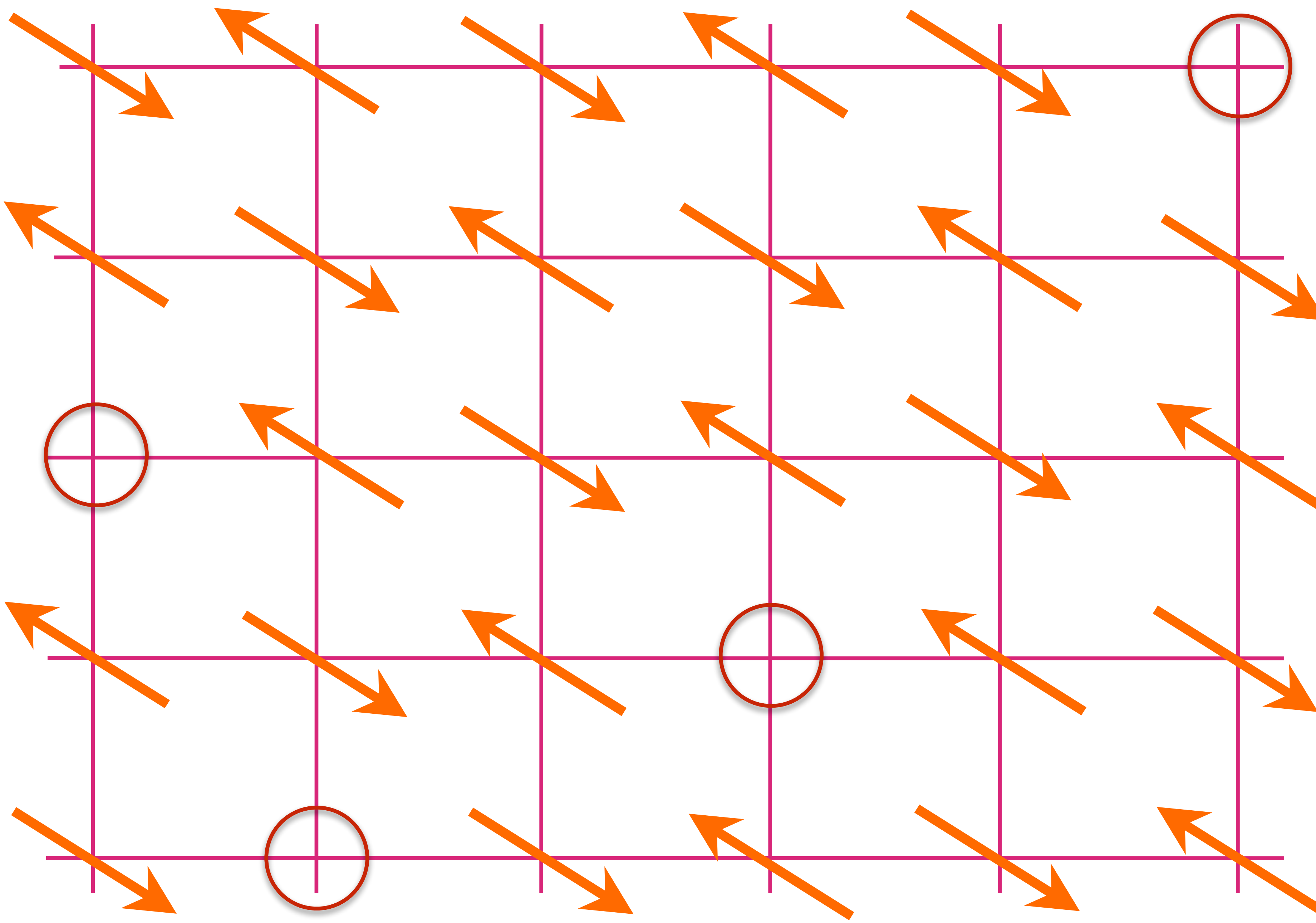


$YBa_2Cu_3O_{6+x}$

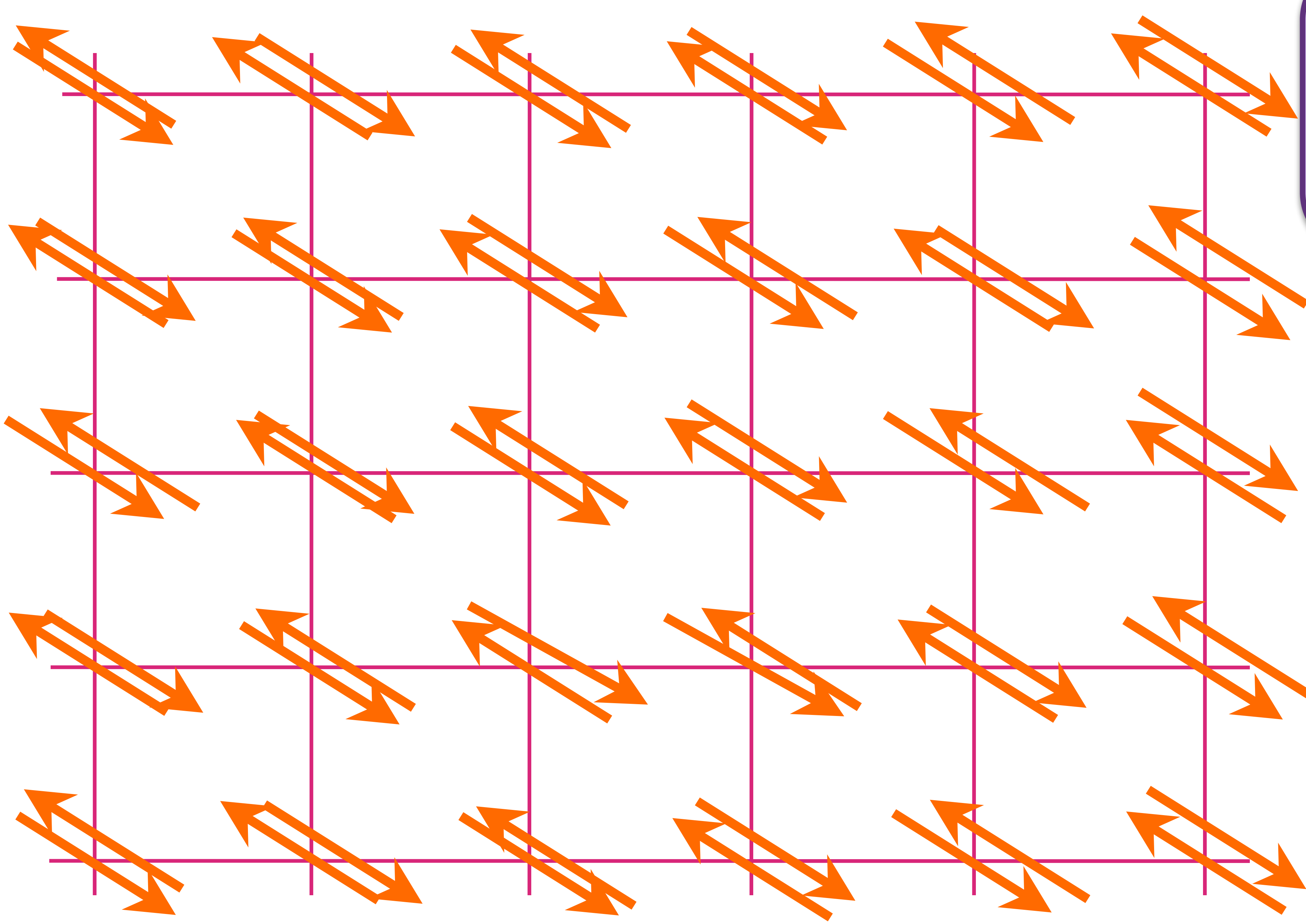
Figure: K. Fujita and J. C. Seamus Davis



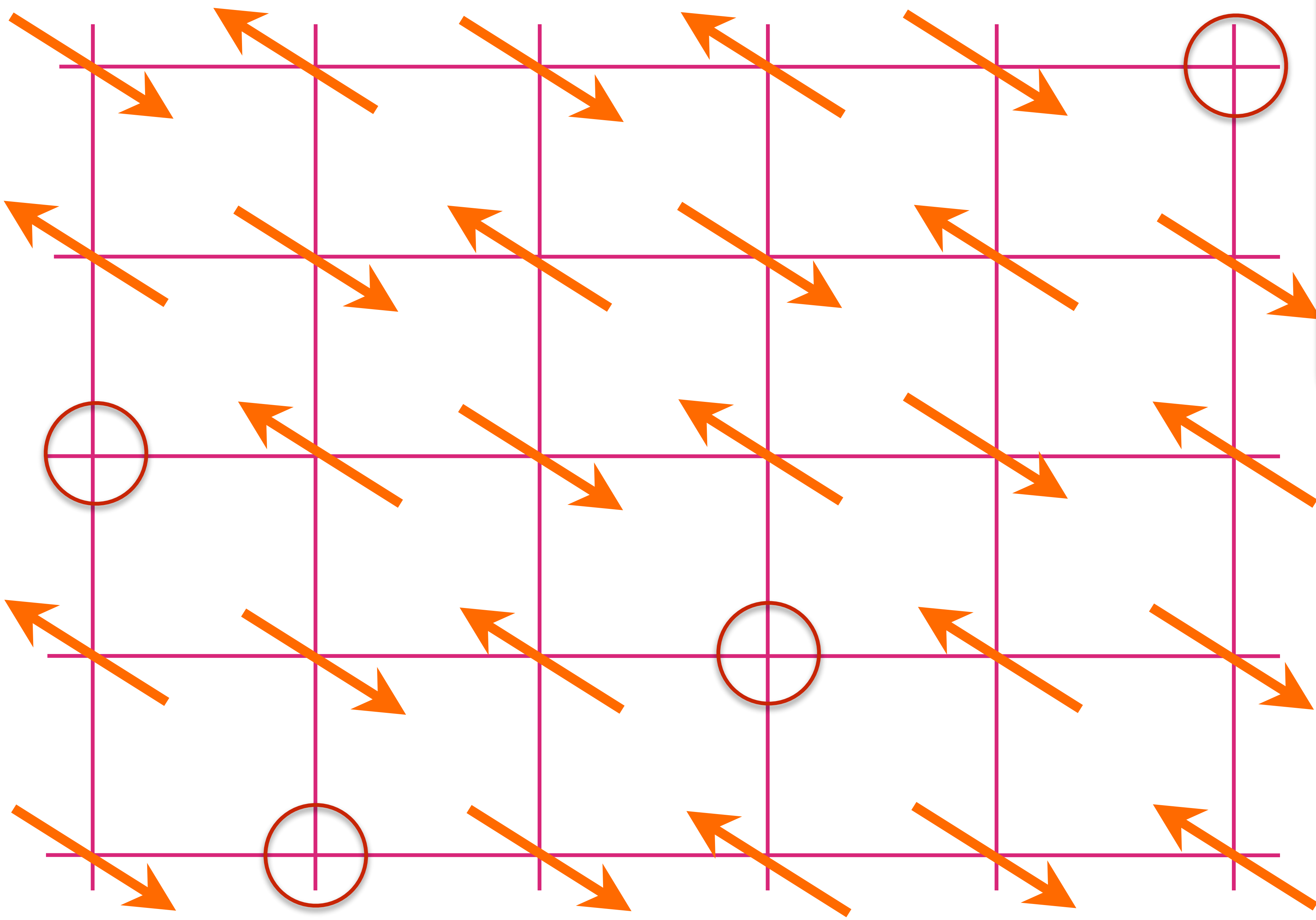
“Undoped” insulating anti-ferromagnet



Anti-
ferromagnet
with p mobile
holes
per square

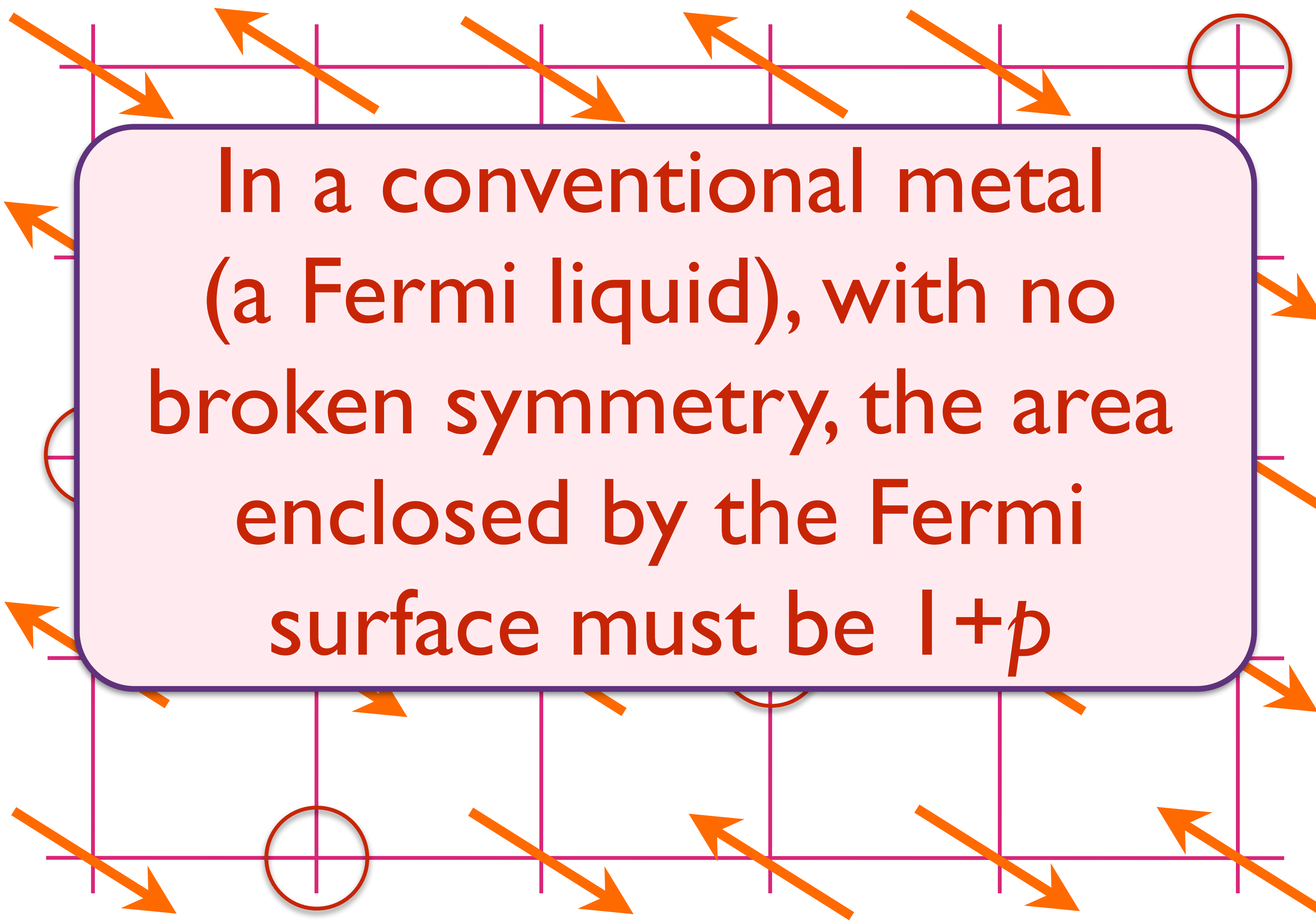


Filled
Band



Anti-ferromagnet with p mobile holes per square

But relative to the band insulator, there are $1+p$ holes per square

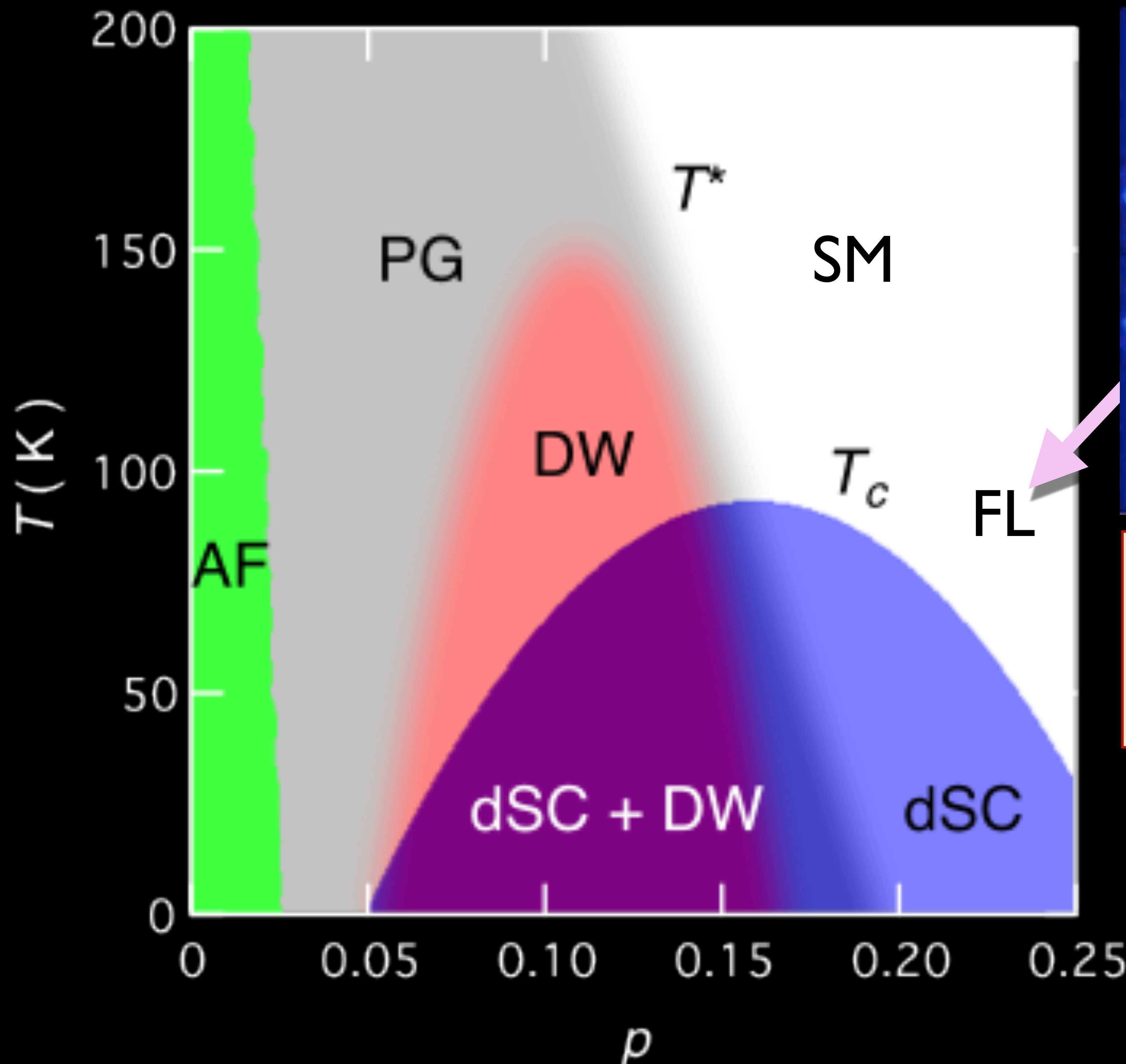


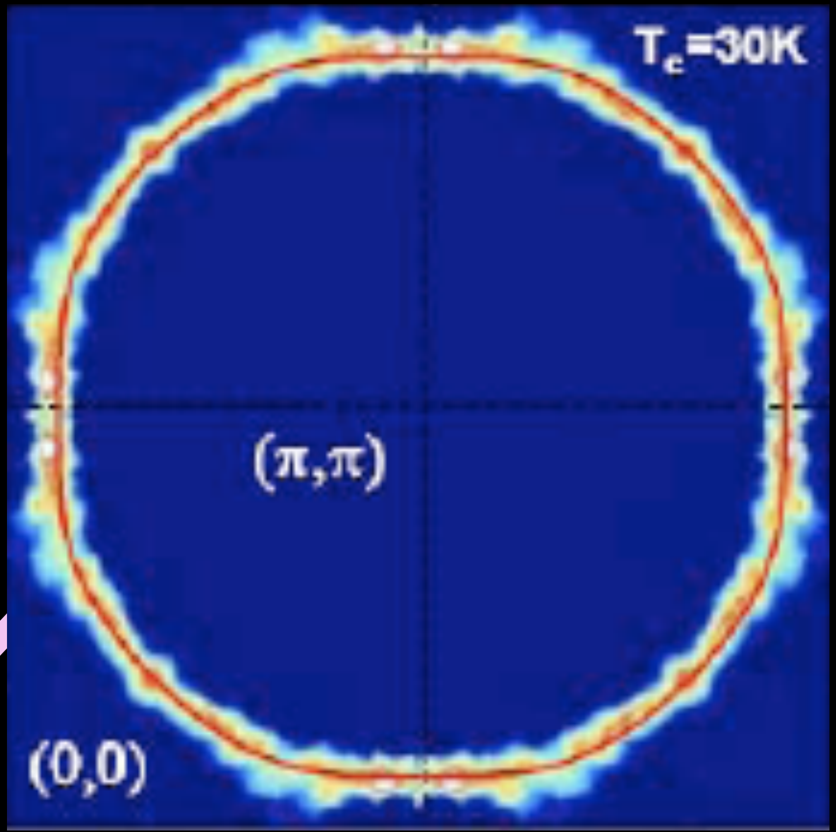
In a conventional metal (a Fermi liquid), with no broken symmetry, the area enclosed by the Fermi surface must be $1+p$

Anti-ferromagnet with p mobile holes per square

But relative to the band insulator, there are $1+p$ holes per square

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

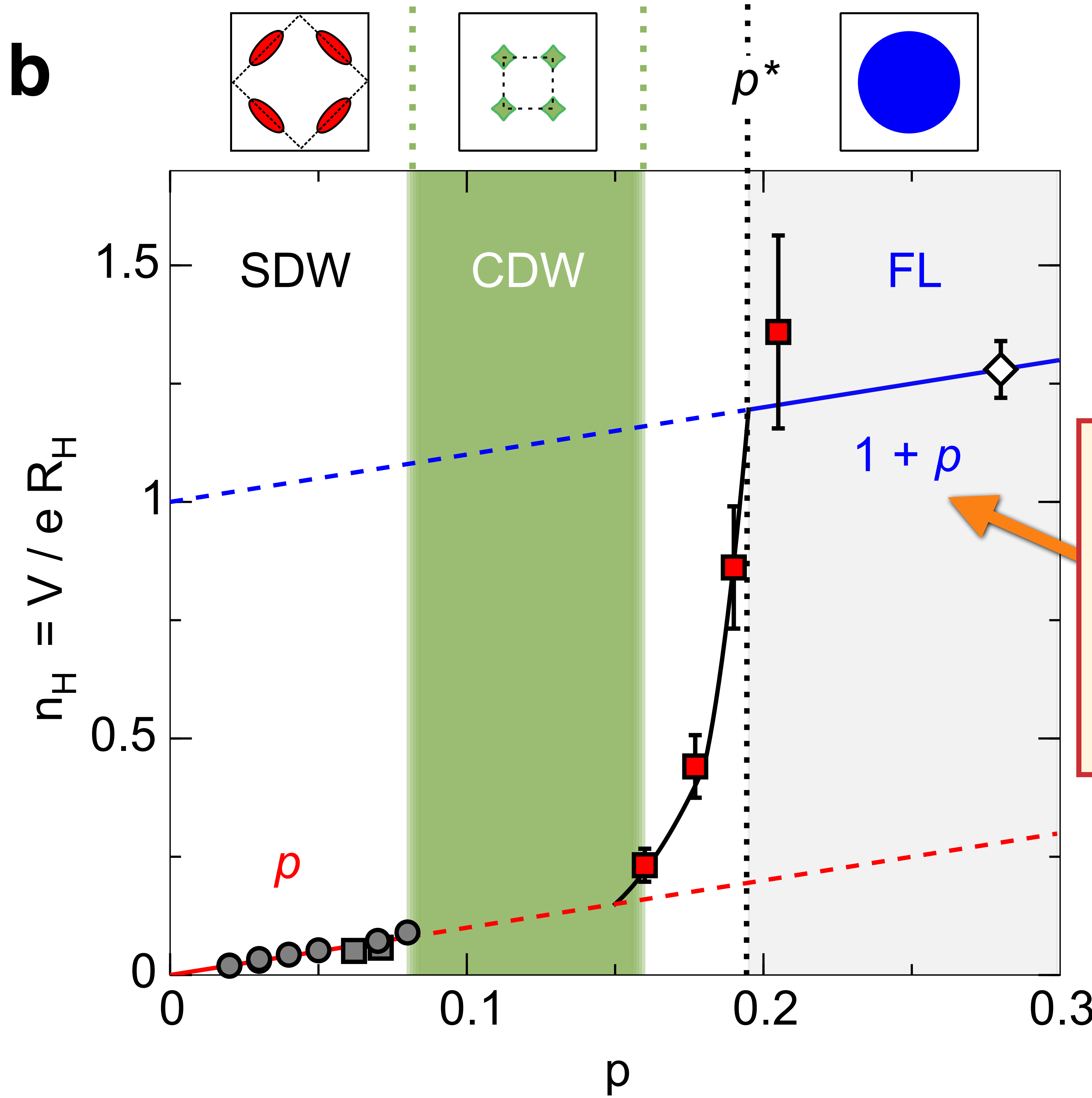




Fermi liquid

Area enclosed by Fermi surface $=1+p$

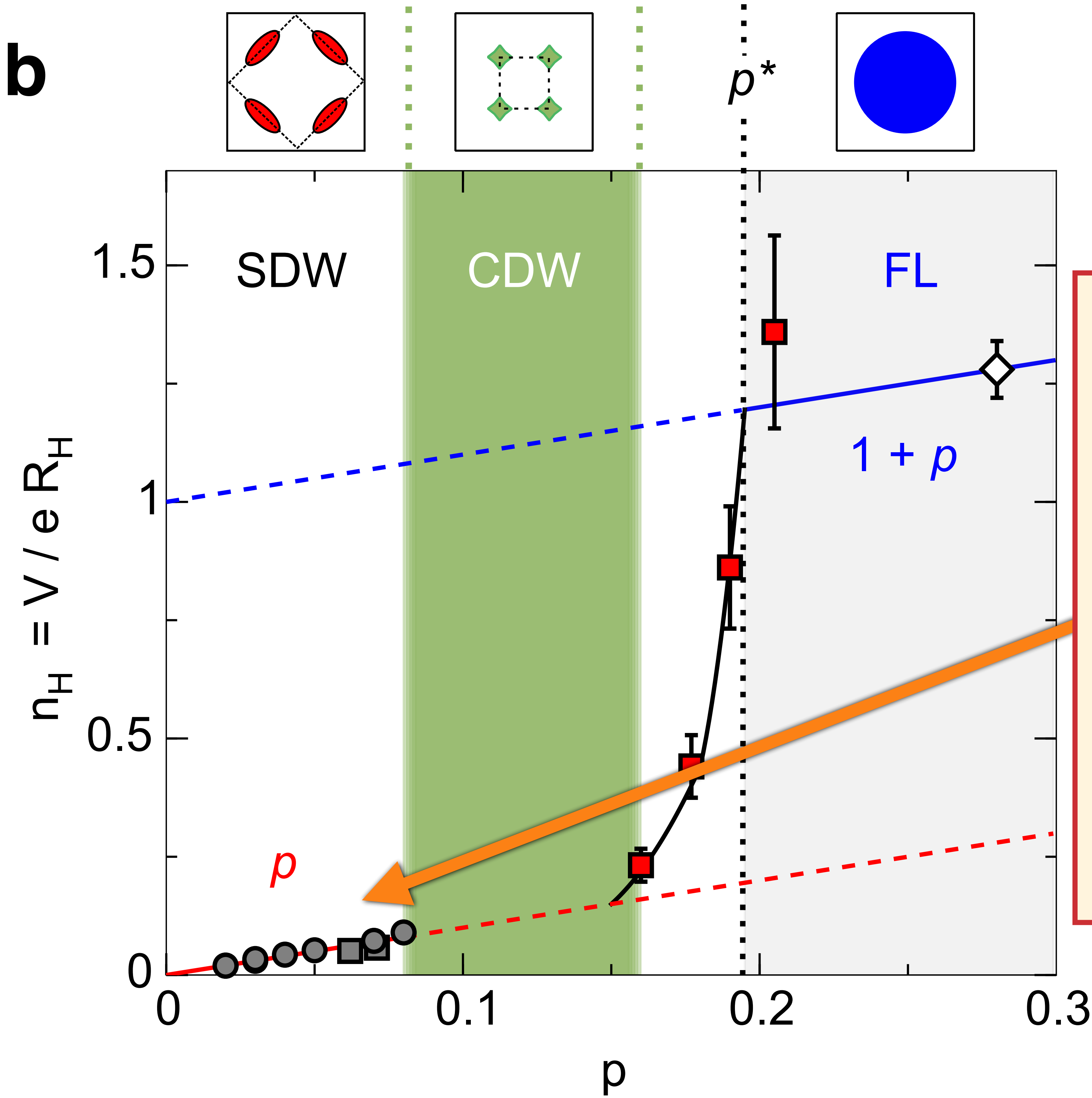
Hall effect measurements in YBCO



Fermi liquid (FL) with carrier density 1+p

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

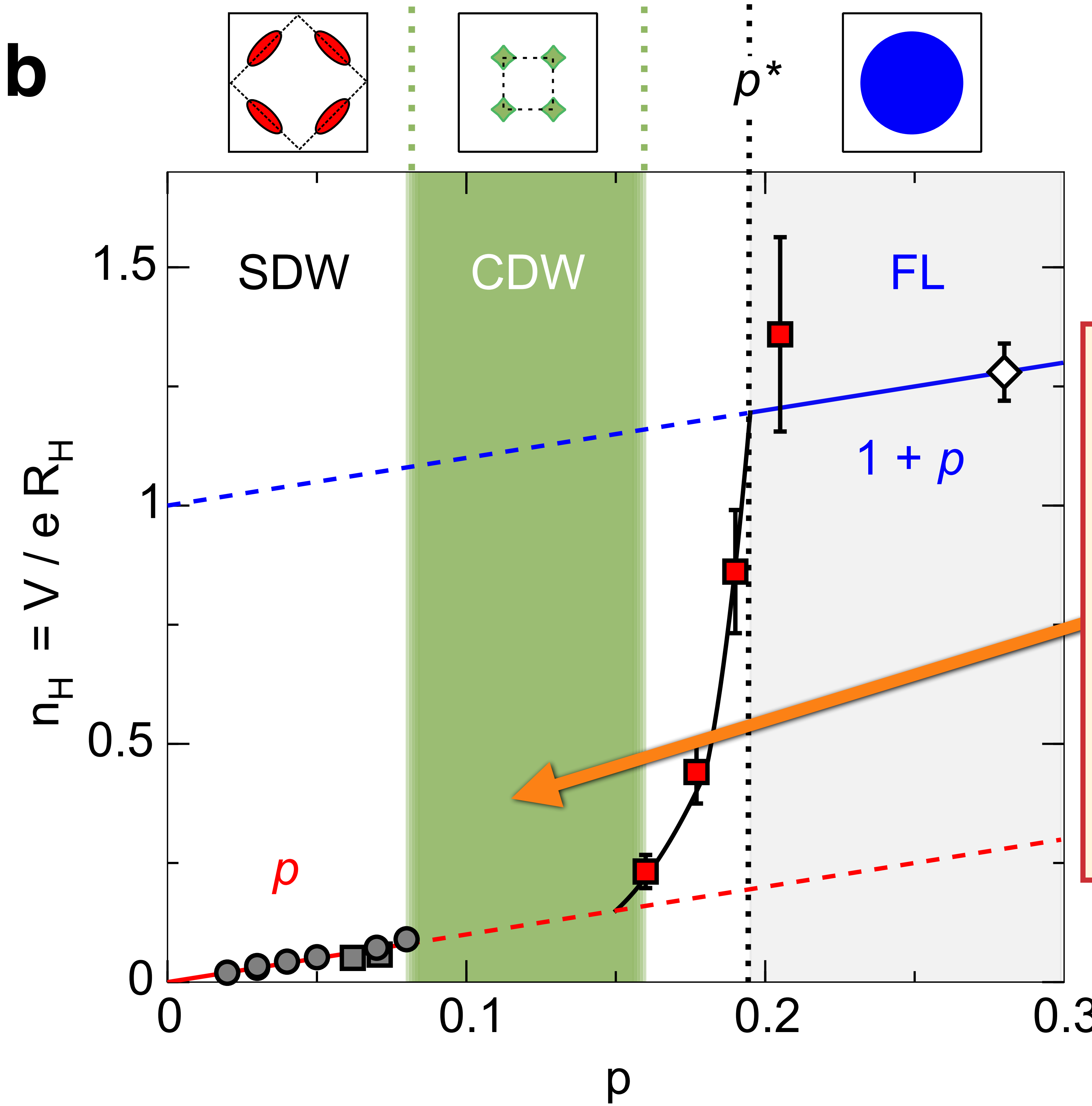
Hall effect measurements in YBCO



Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density *p*

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

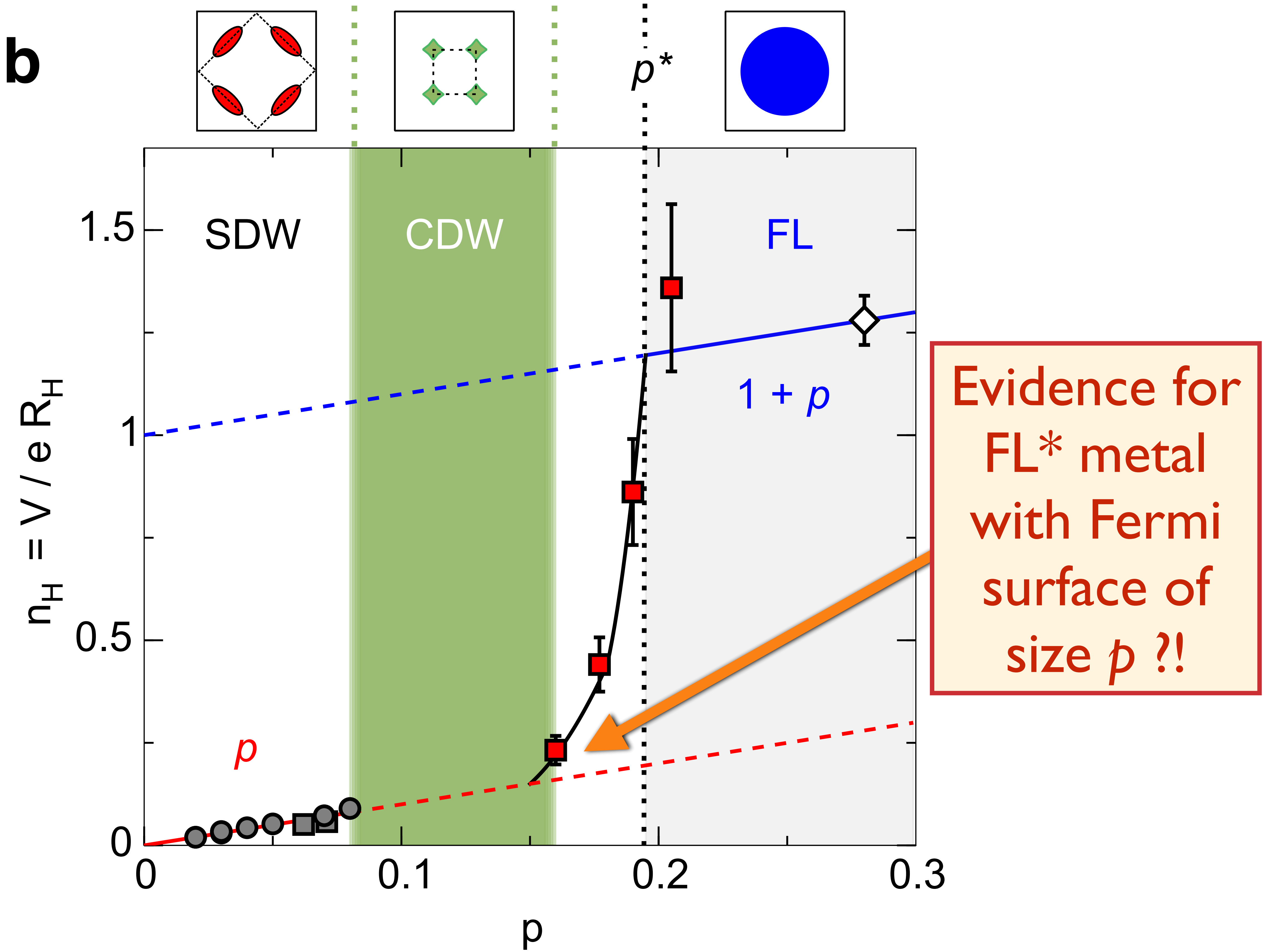
Hall effect measurements in YBCO



Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

Hall effect measurements in YBCO



Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

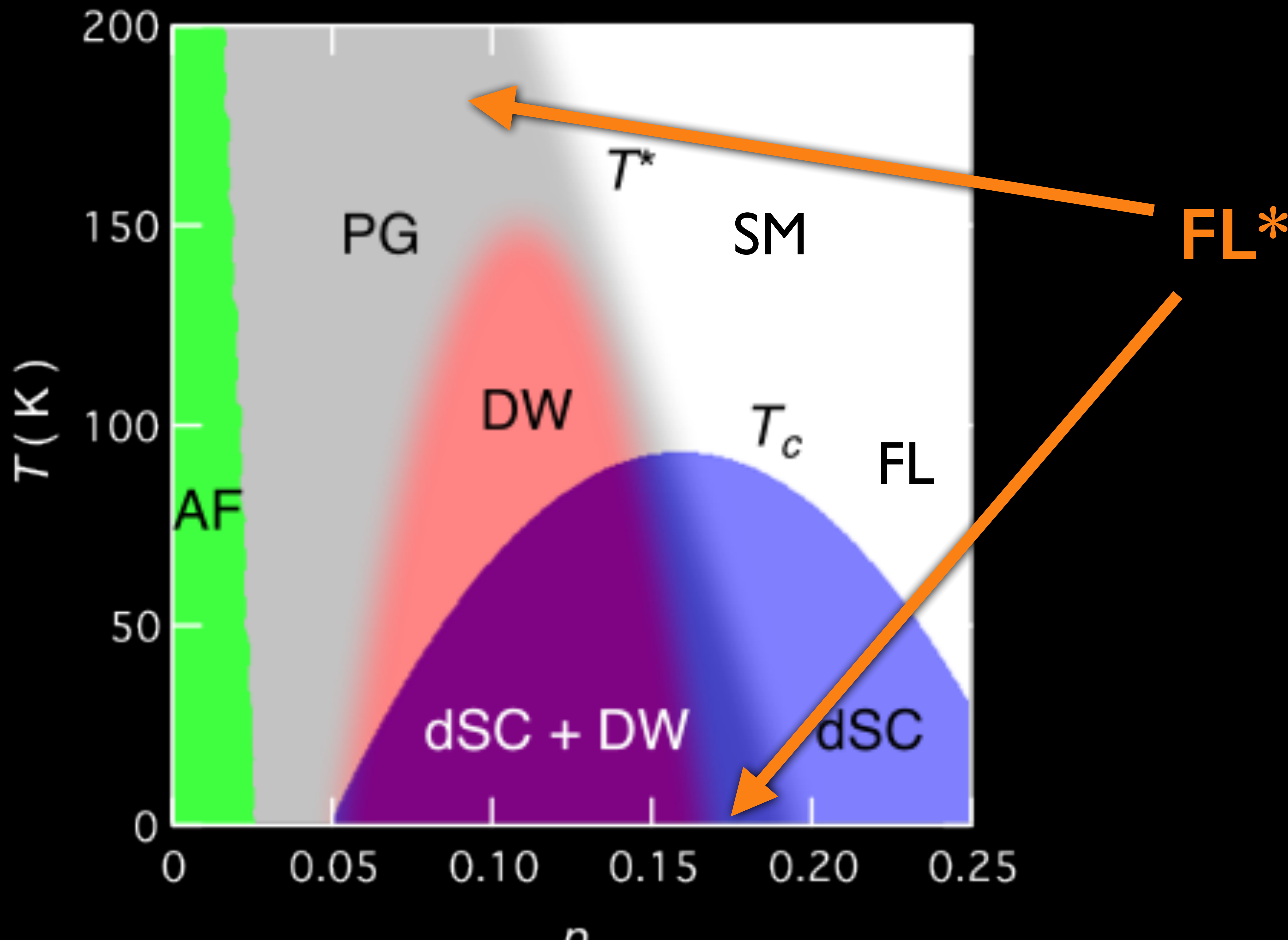


Figure: K. Fujita and J. C. Seamus Davis

FL*

A metal with:

- A Fermi surface of electrons enclosing volume p, and not the Luttinger volume of $1+p$
- Topological character described by emergent gauge fields

There is a general and fundamental relationship between these two features.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett* **90**, 216403 (2003)

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3. Confinement transitions out of Z_2-FL*

- Condensation of bosonic excitations of Z_2-FL*
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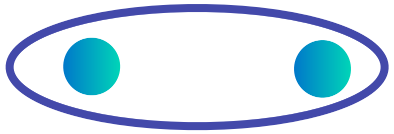
Insulating spin liquid



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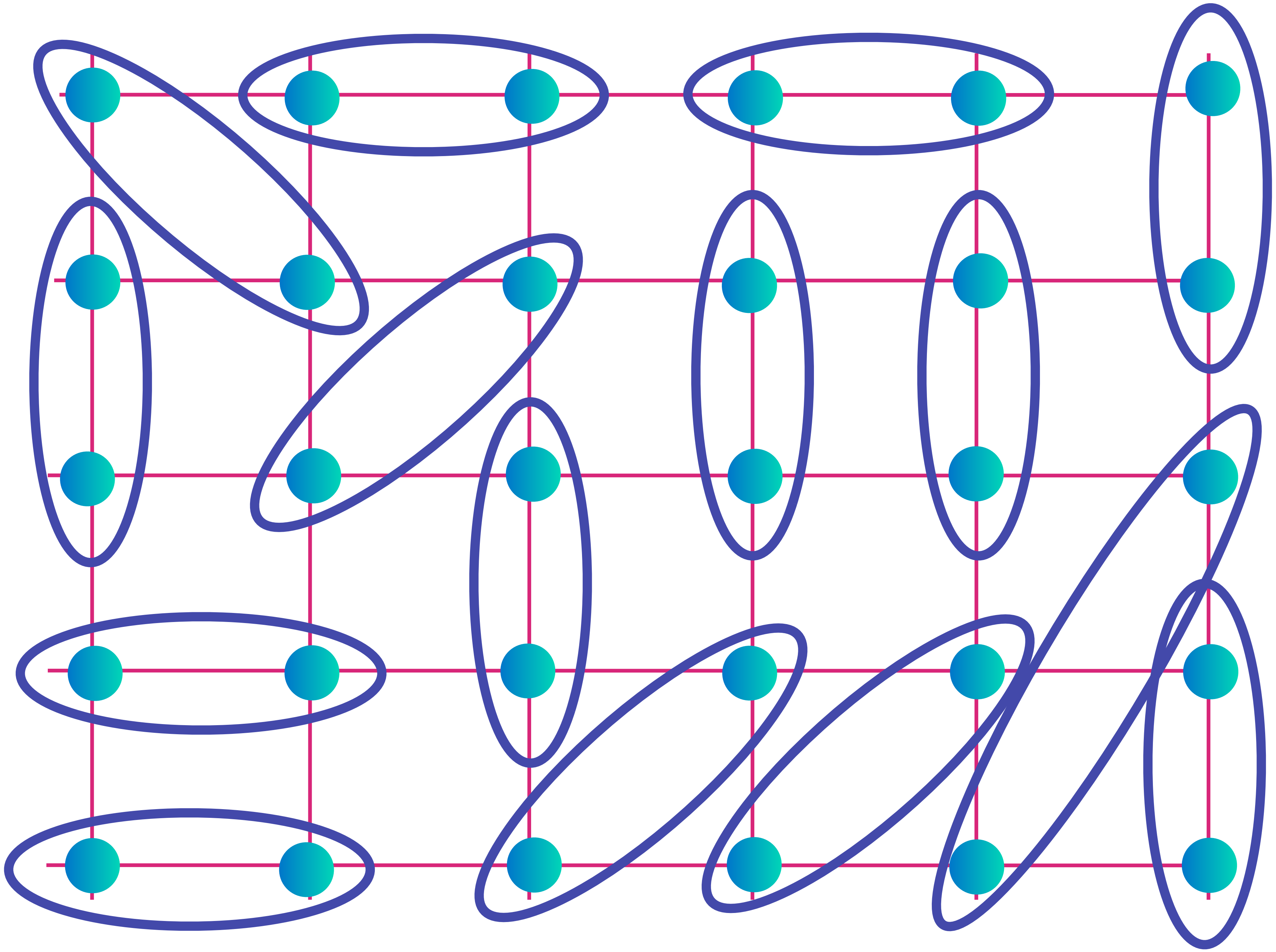

The first proposal of a quantum state with long-range entanglement

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)
P.W. Anderson, Materials Research Bulletin **8**, 153 (1973)

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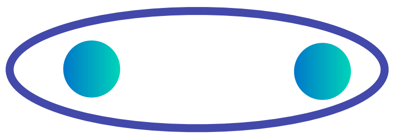

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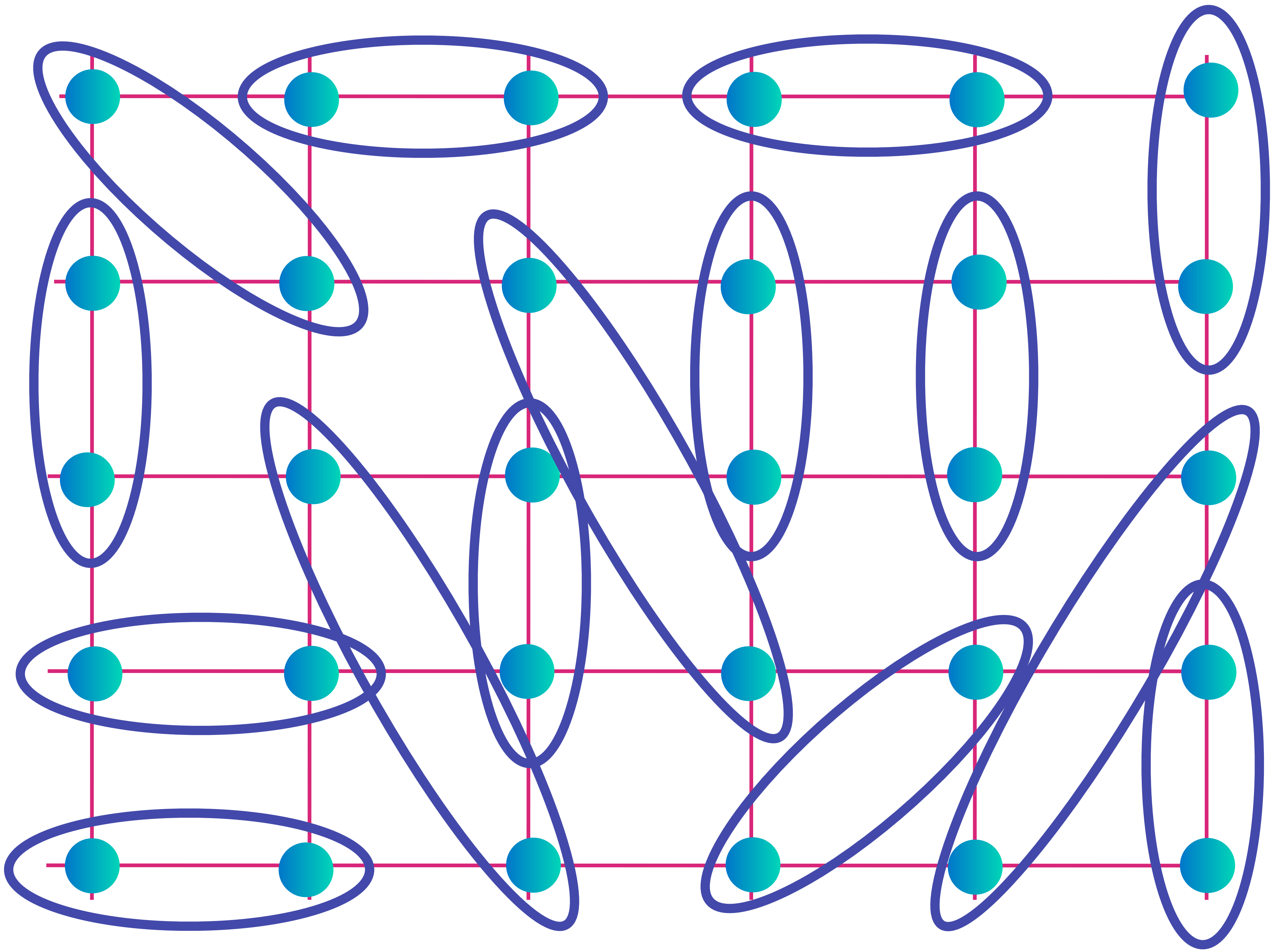

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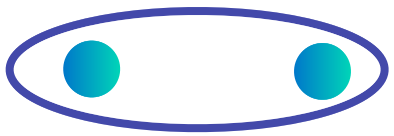
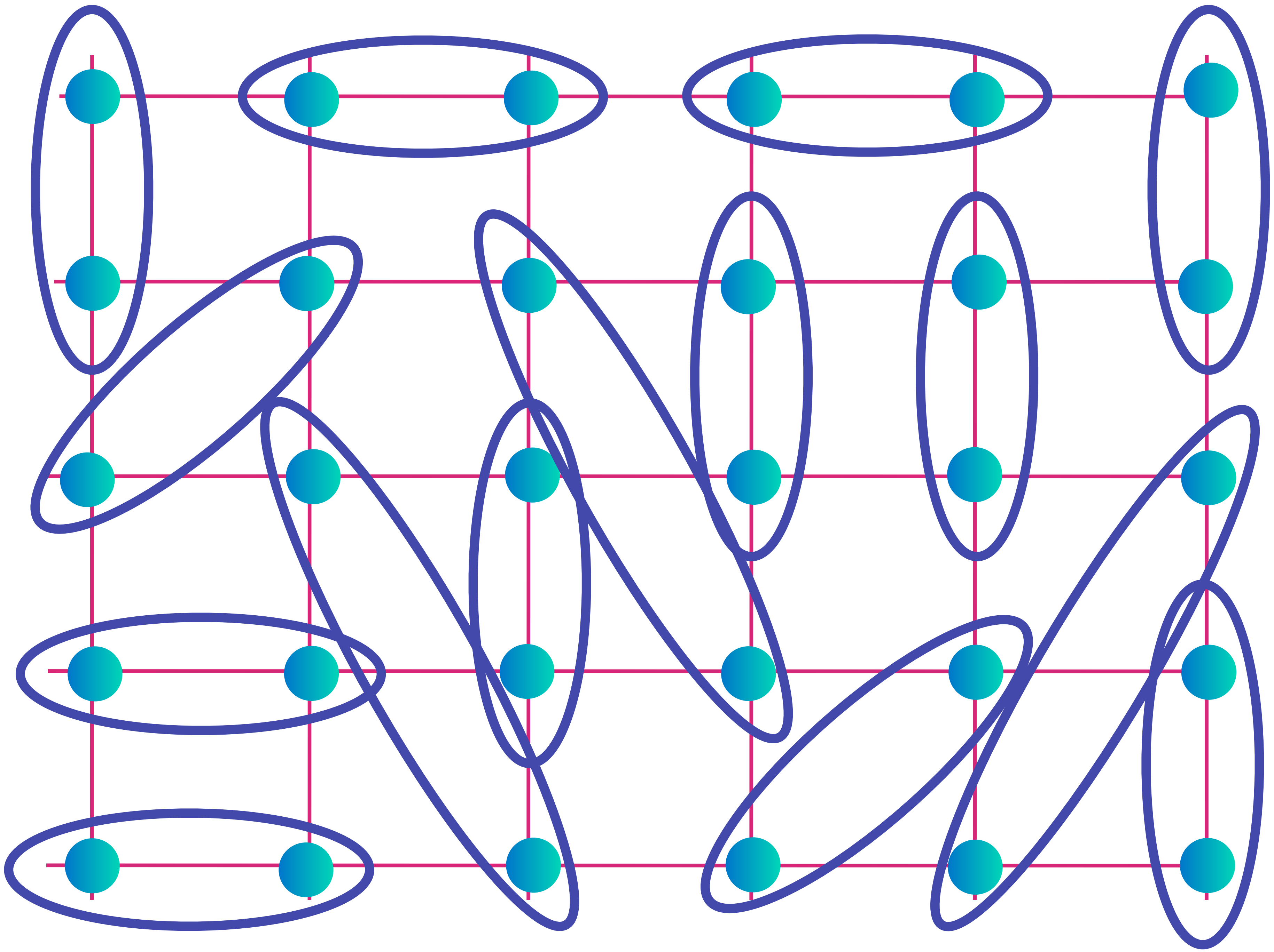
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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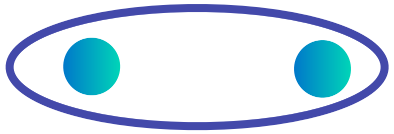

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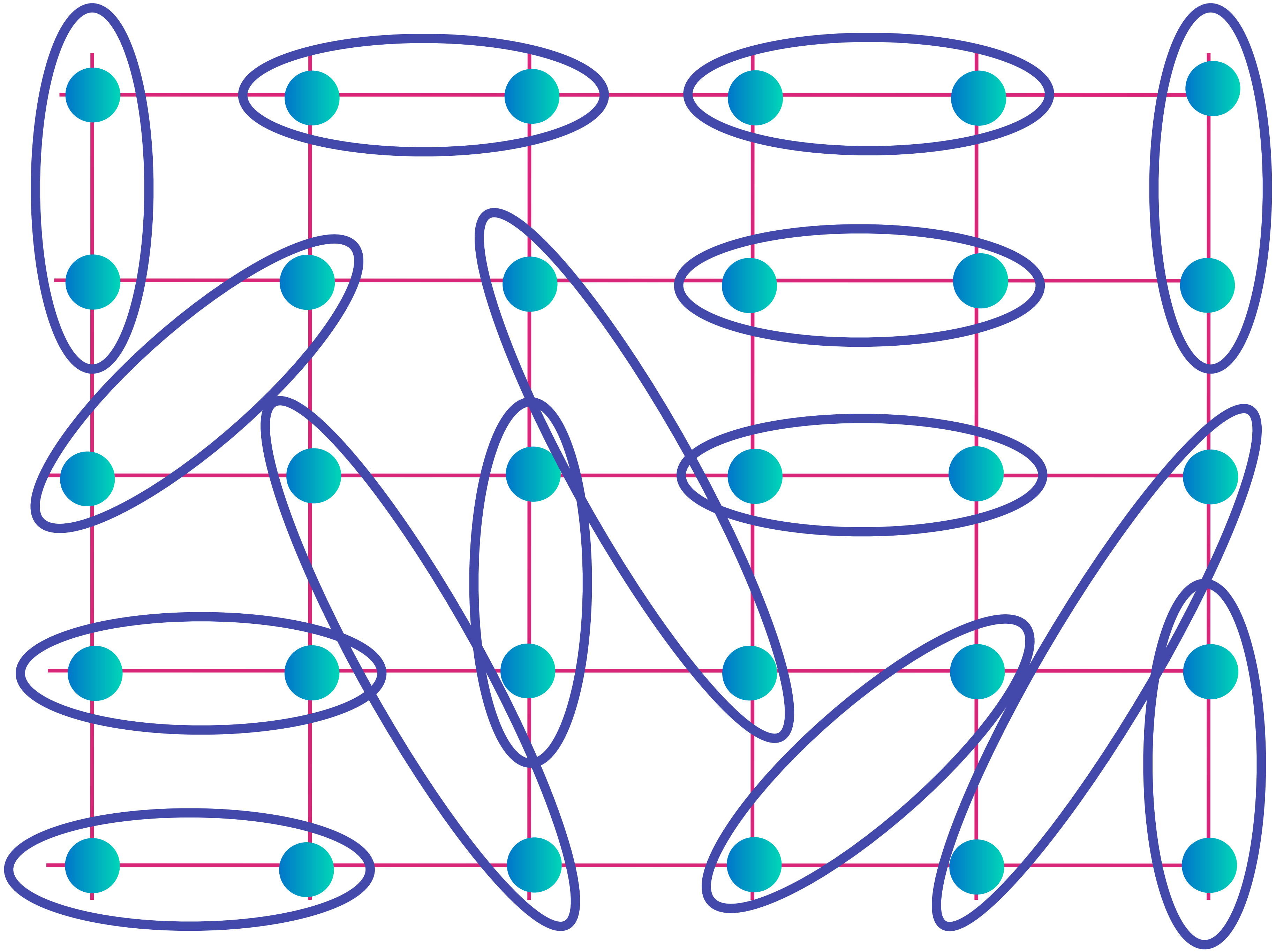

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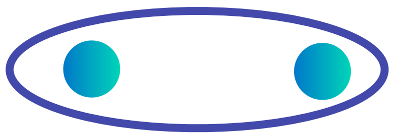
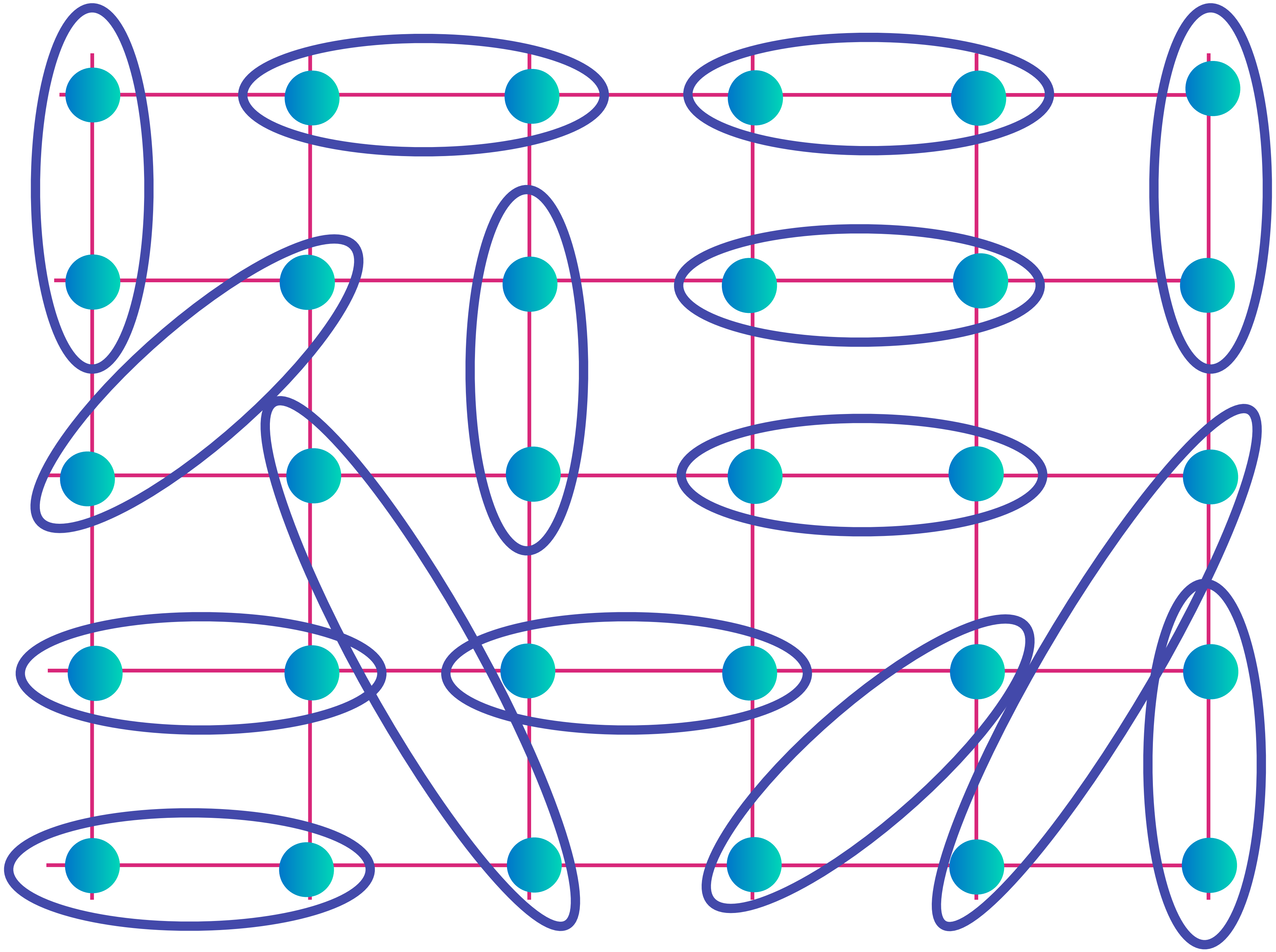
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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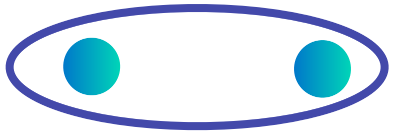
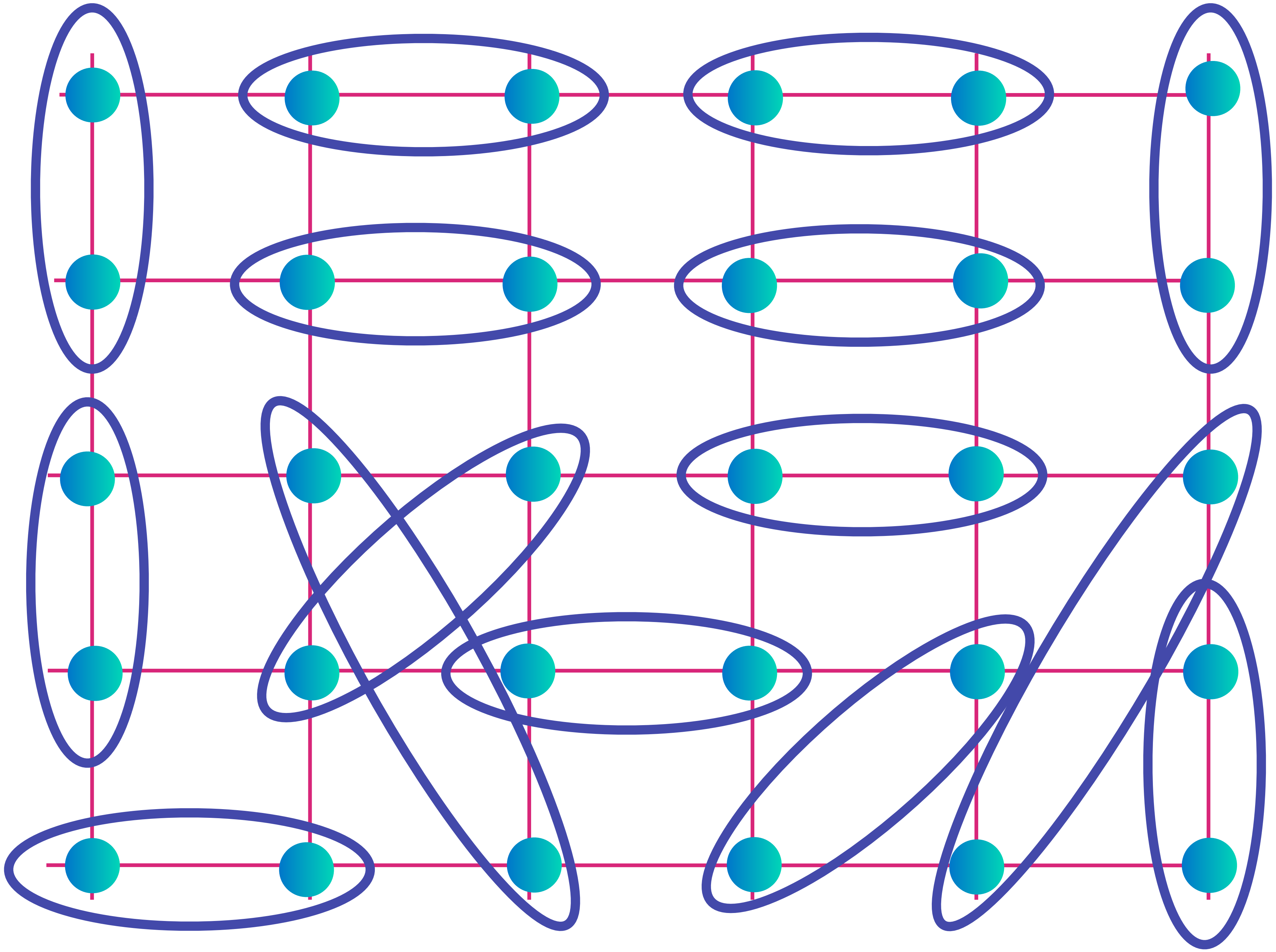
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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Insulating spin liquid

Modern description:

The $\mathbb{Z}_2$ spin liquid: Described by the simplest, non-trivial, topological field theory with time-reversal symmetry:

$$\mathcal{L} = \frac{1}{4\pi} K_{IJ} \int d^3x \, \epsilon^{\mu\nu\lambda} a^I_\mu \partial_\nu a^J_\lambda$$

where a^I, $I = 1, 2$ are U(1) gauge connections, and the K matrix is

$$K = \left(\begin{array}{cc} 0 & 2 \\ 2 & 0 \end{array} \right)$$

N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991)
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See also E. Fradkin and S. H. Shenker, "Phase diagrams of lattice gauge theories with Higgs fields," Phys. Rev. D **19**, 3682 (1979);
J. M. Maldacena, G. W. Moore and N. Seiberg, "D-brane charges in five-brane backgrounds," JHEP 0110, 005 (2001).

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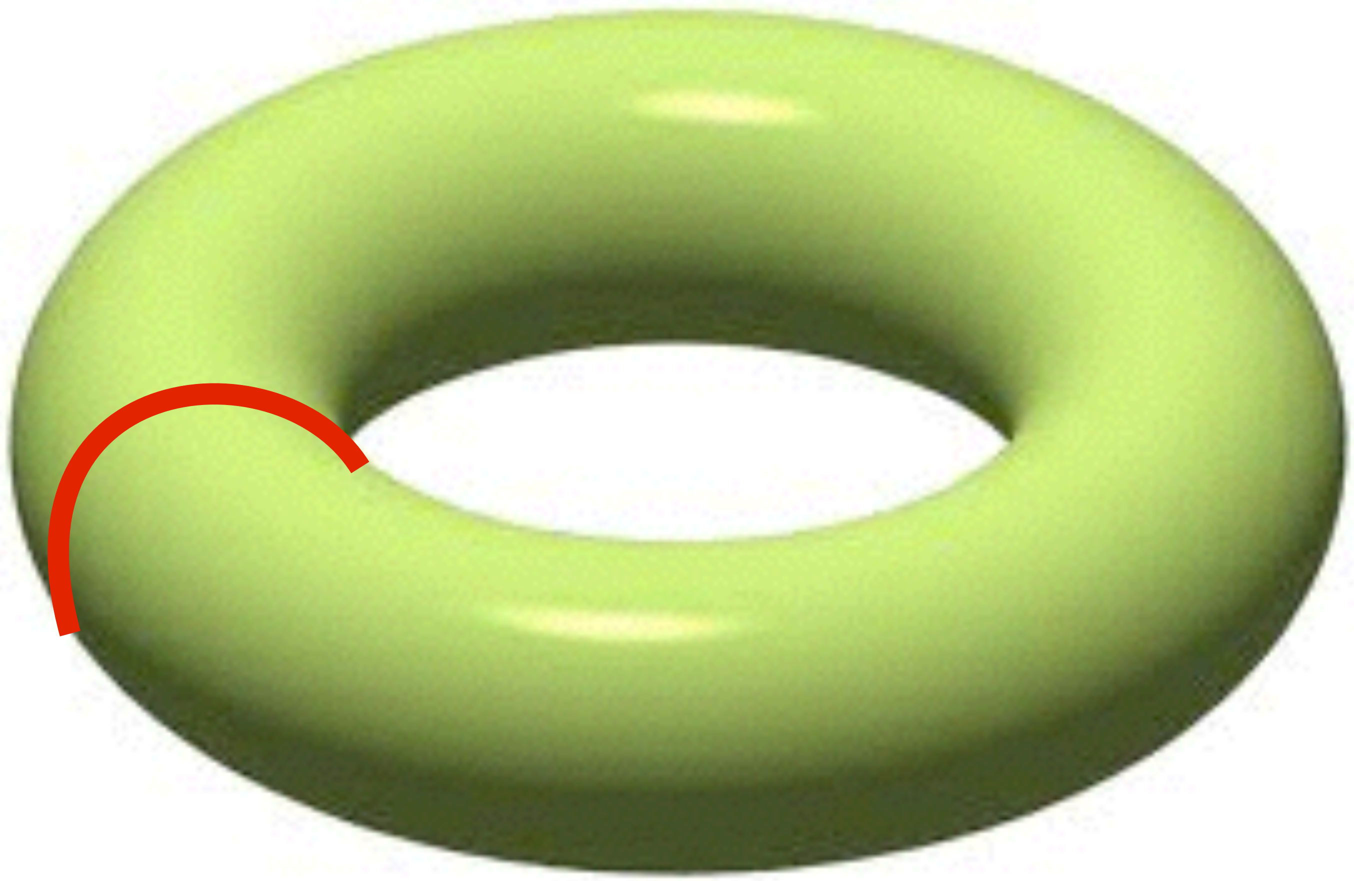
Ground state degeneracy

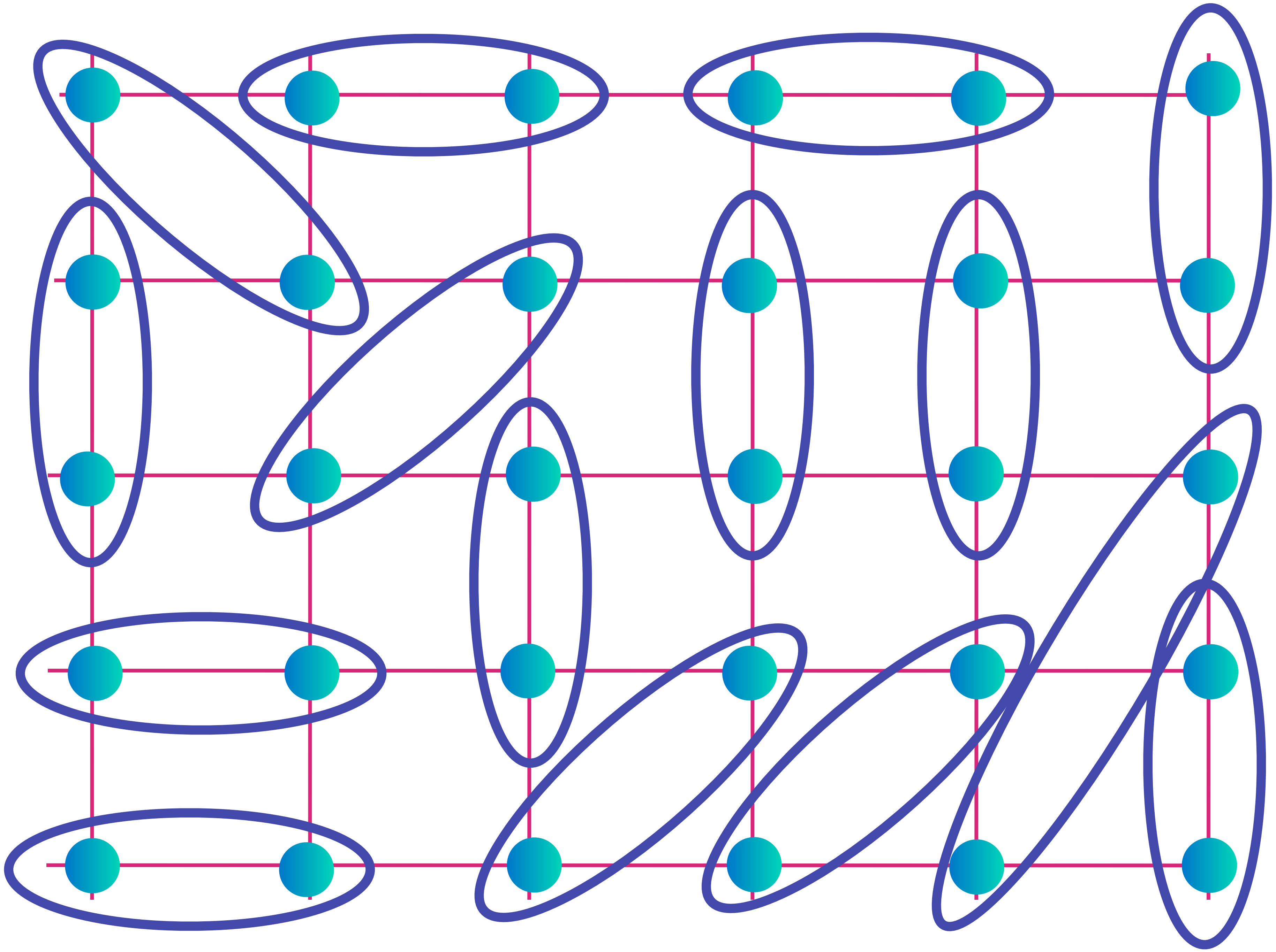


Place insulator on a torus:

Ground state becomes degenerate due to gauge fluxes enclosed by cycles of the torus

Ground state degeneracy

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$



Place insulator on a torus:

obtain "topological" states nearly degenerate with the ground state: number of dimers crossing red line is conserved modulo 2

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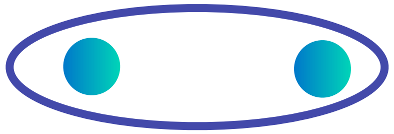
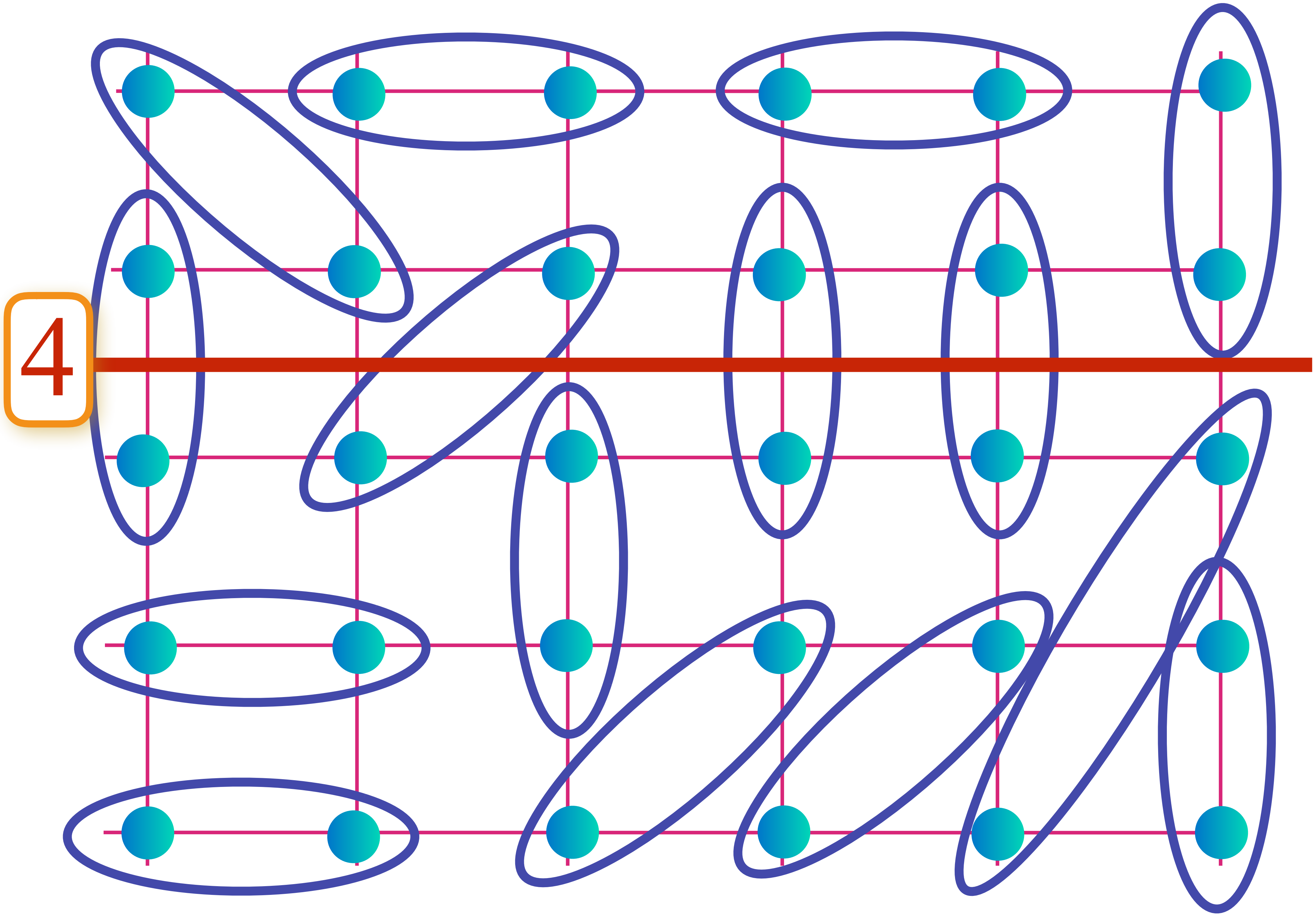
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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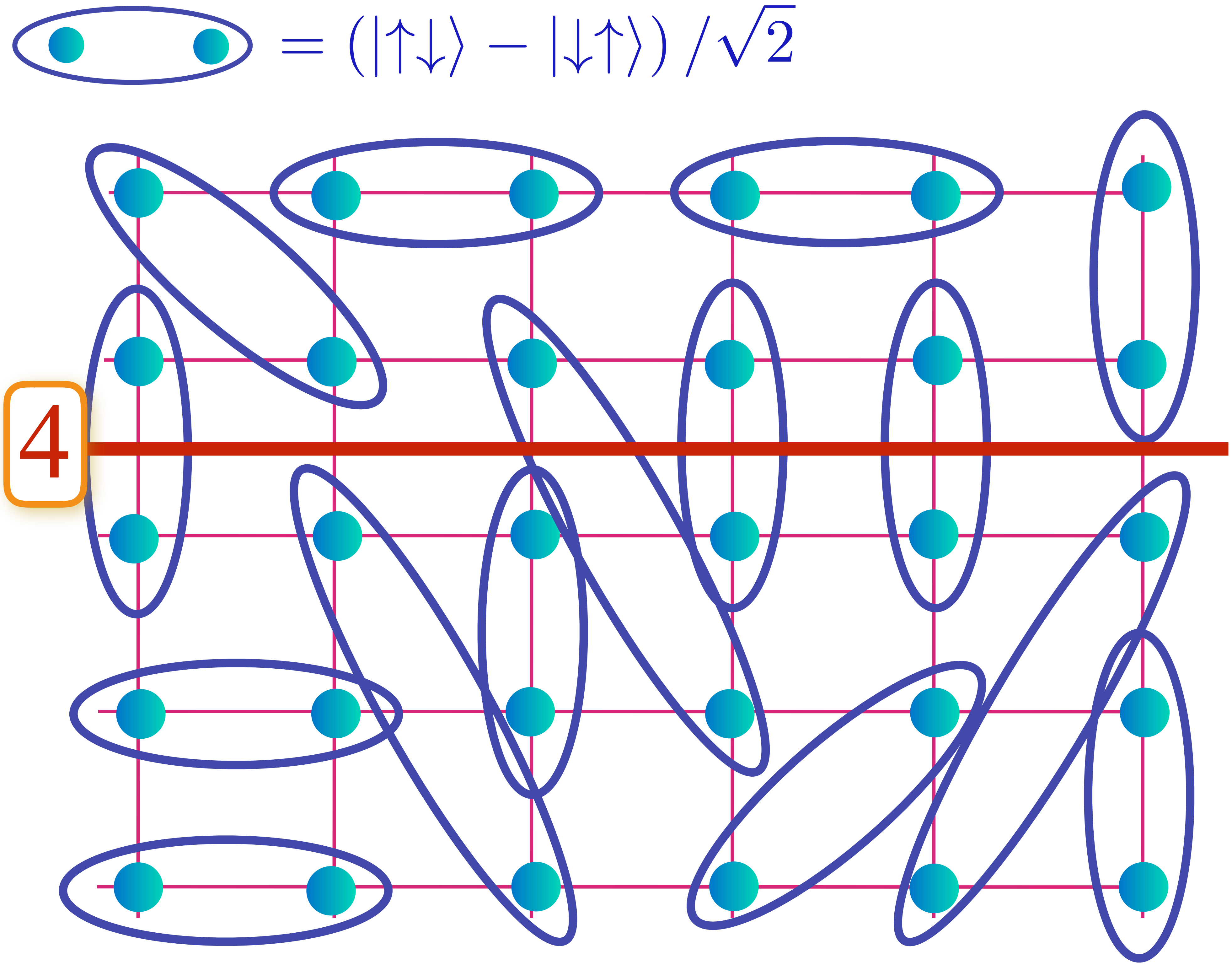

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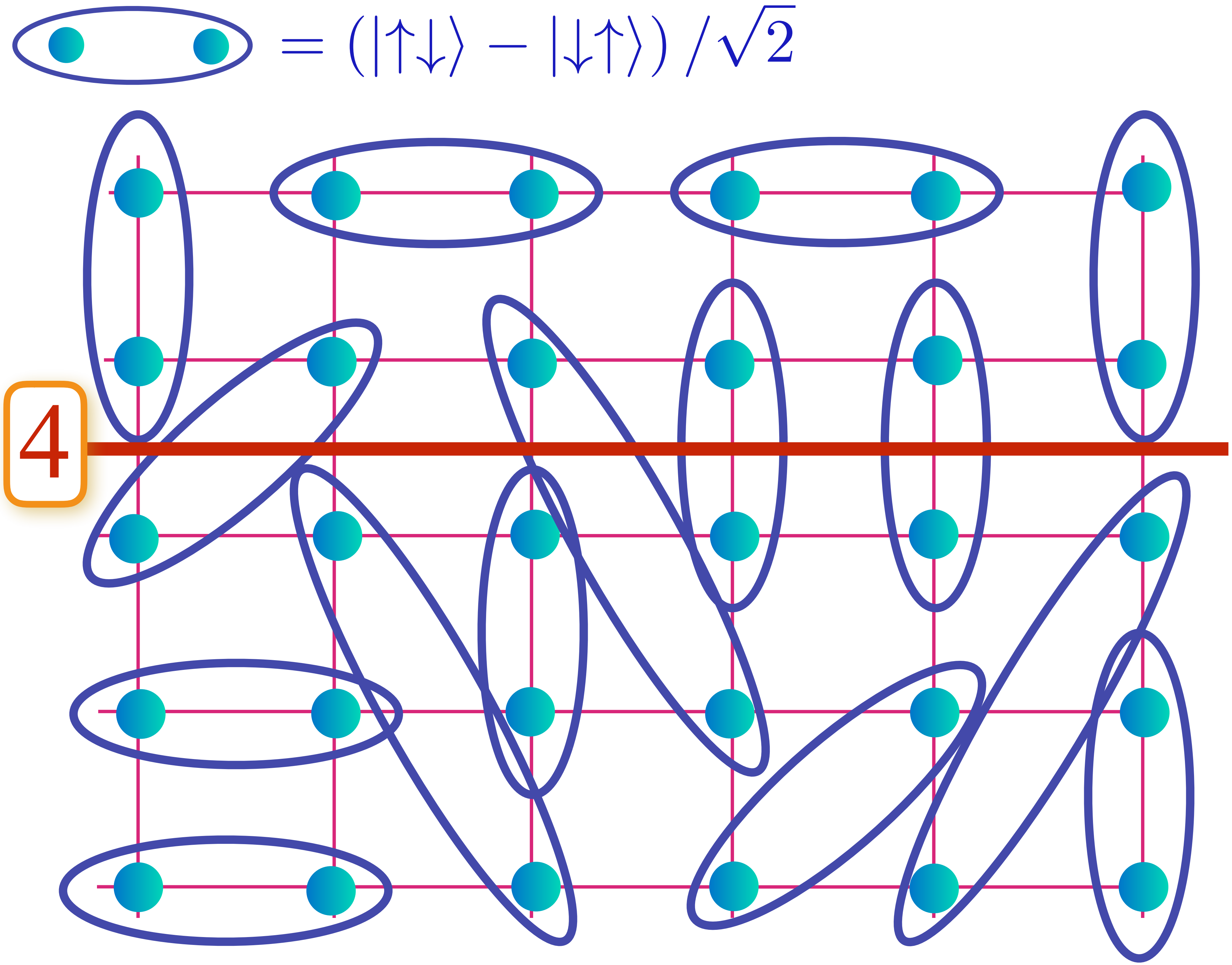

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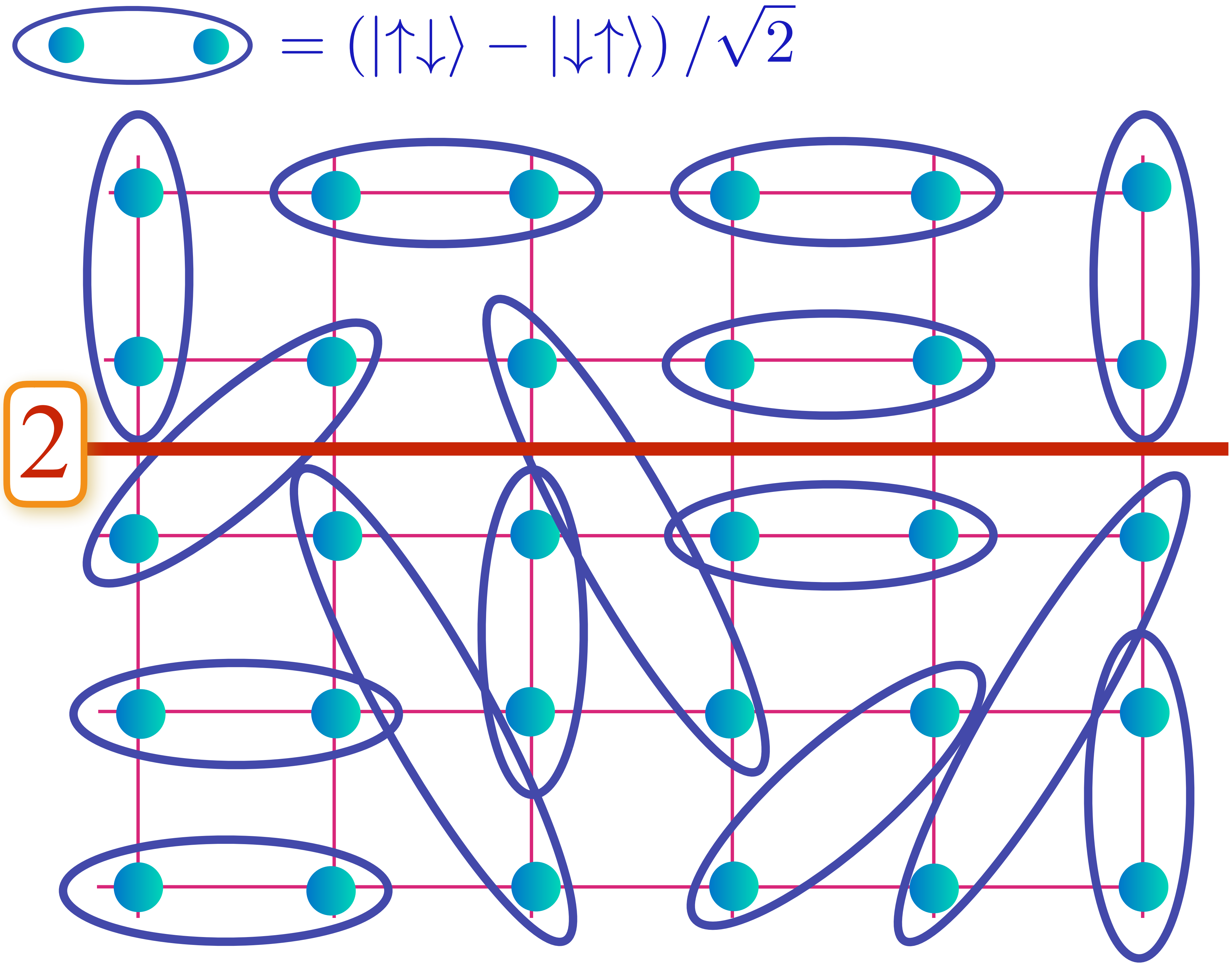

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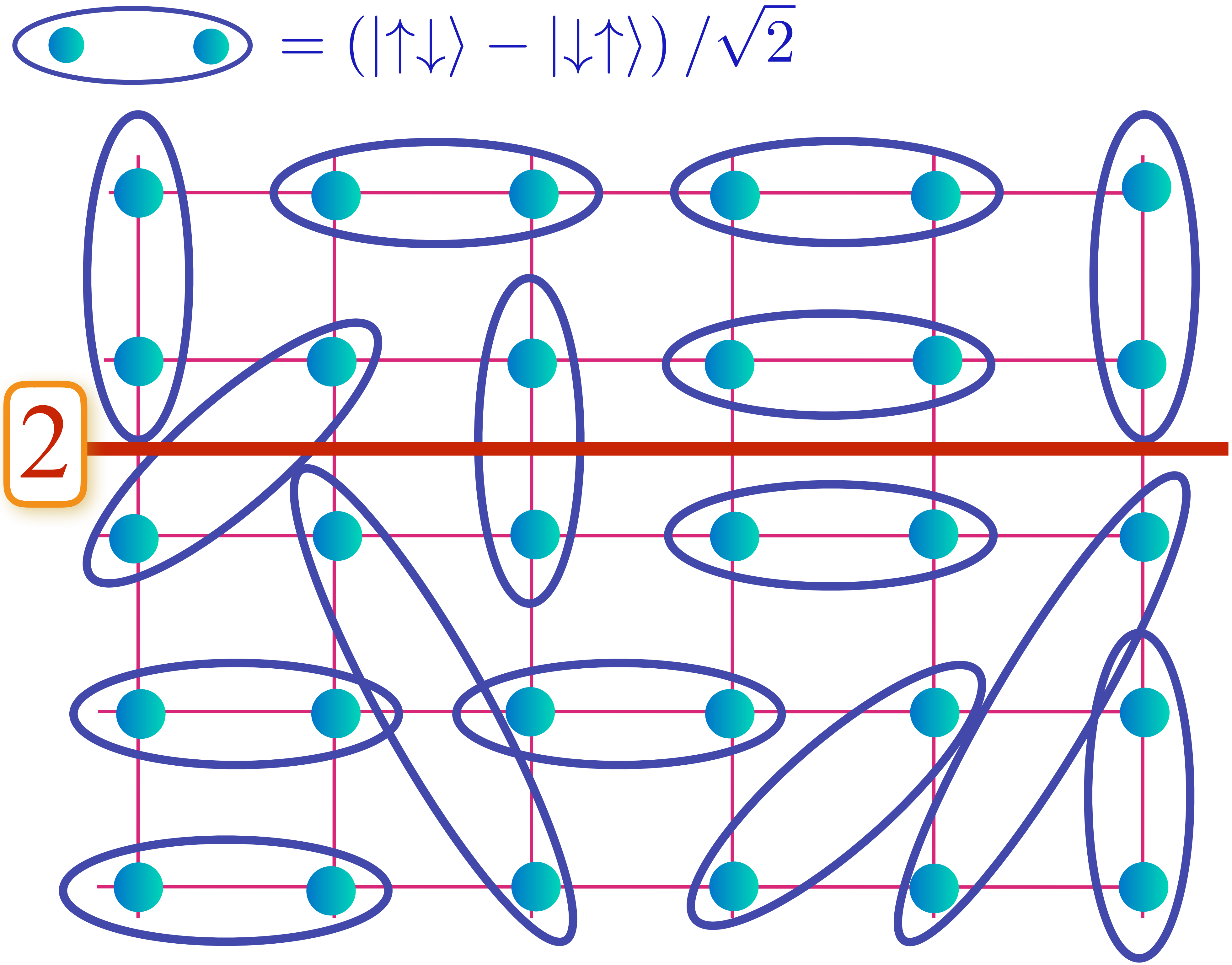

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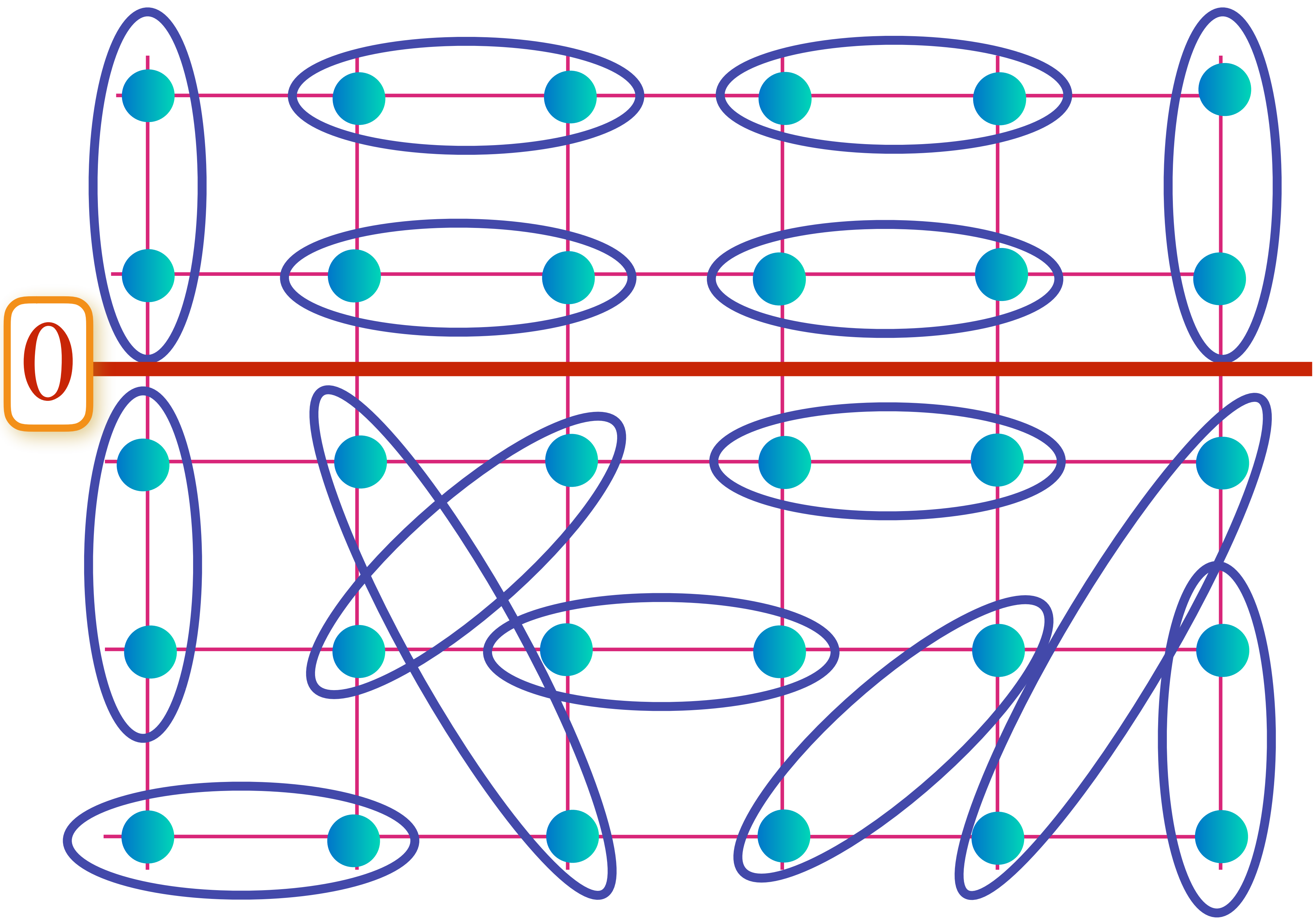

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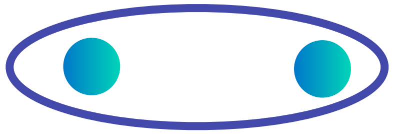
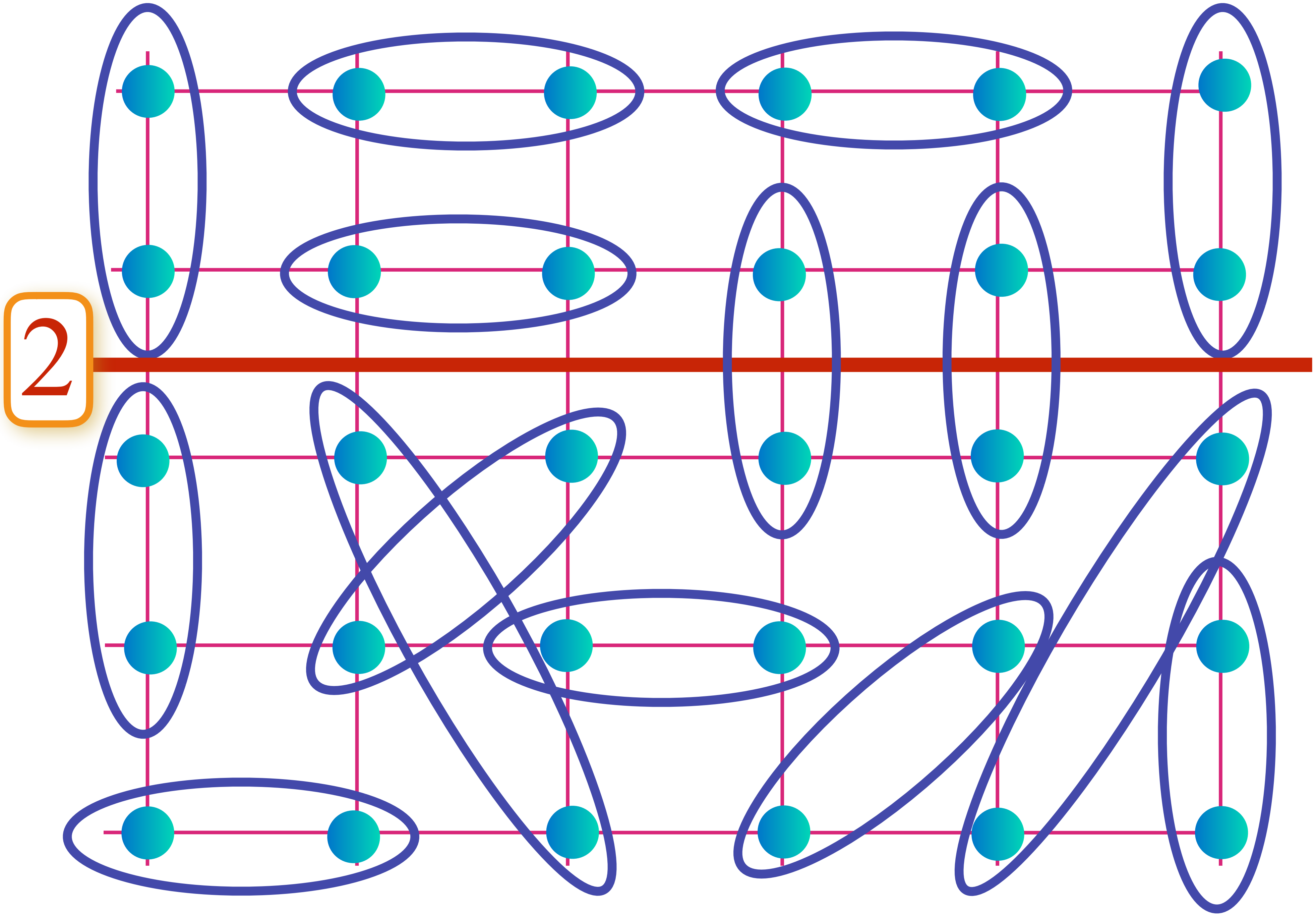
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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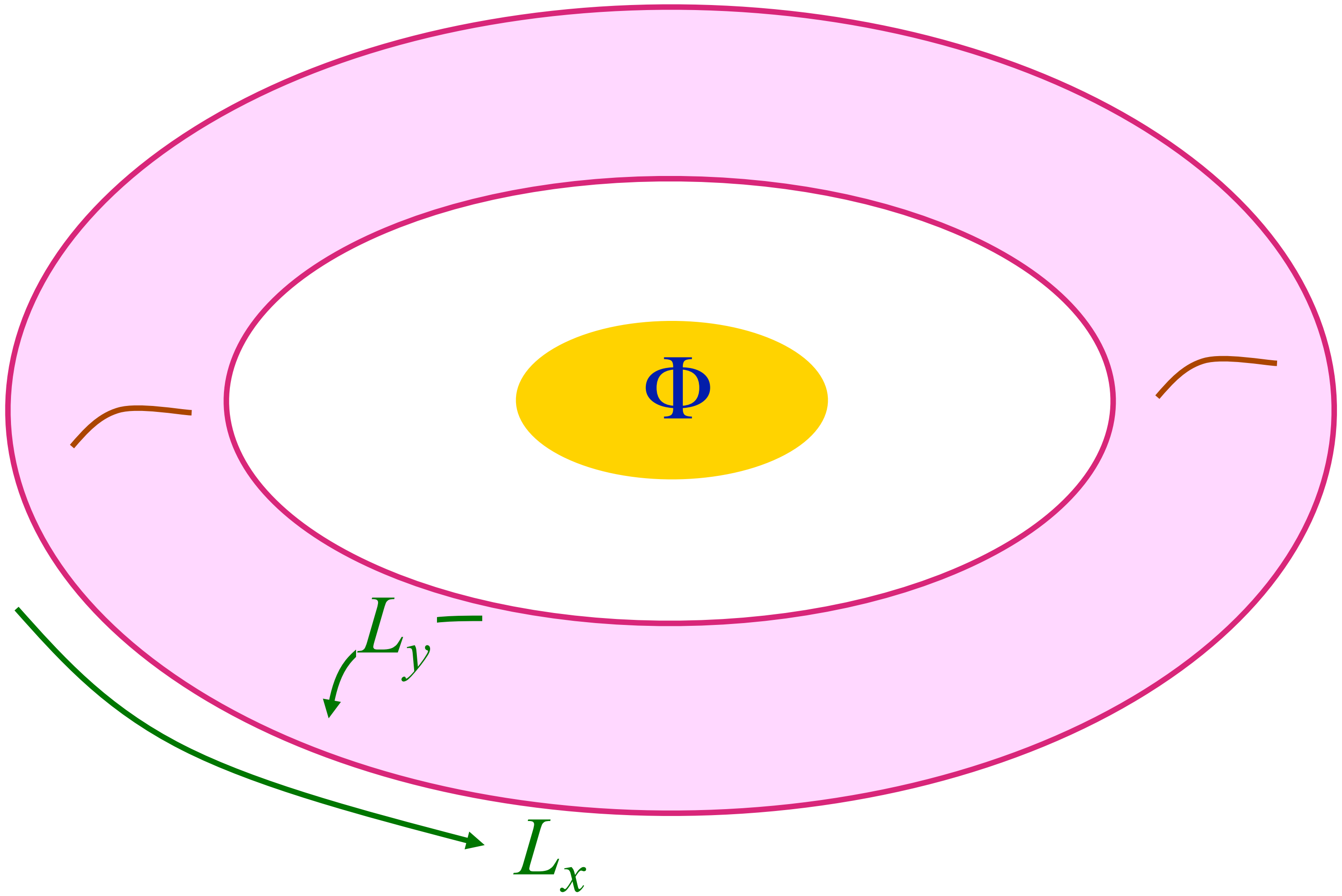
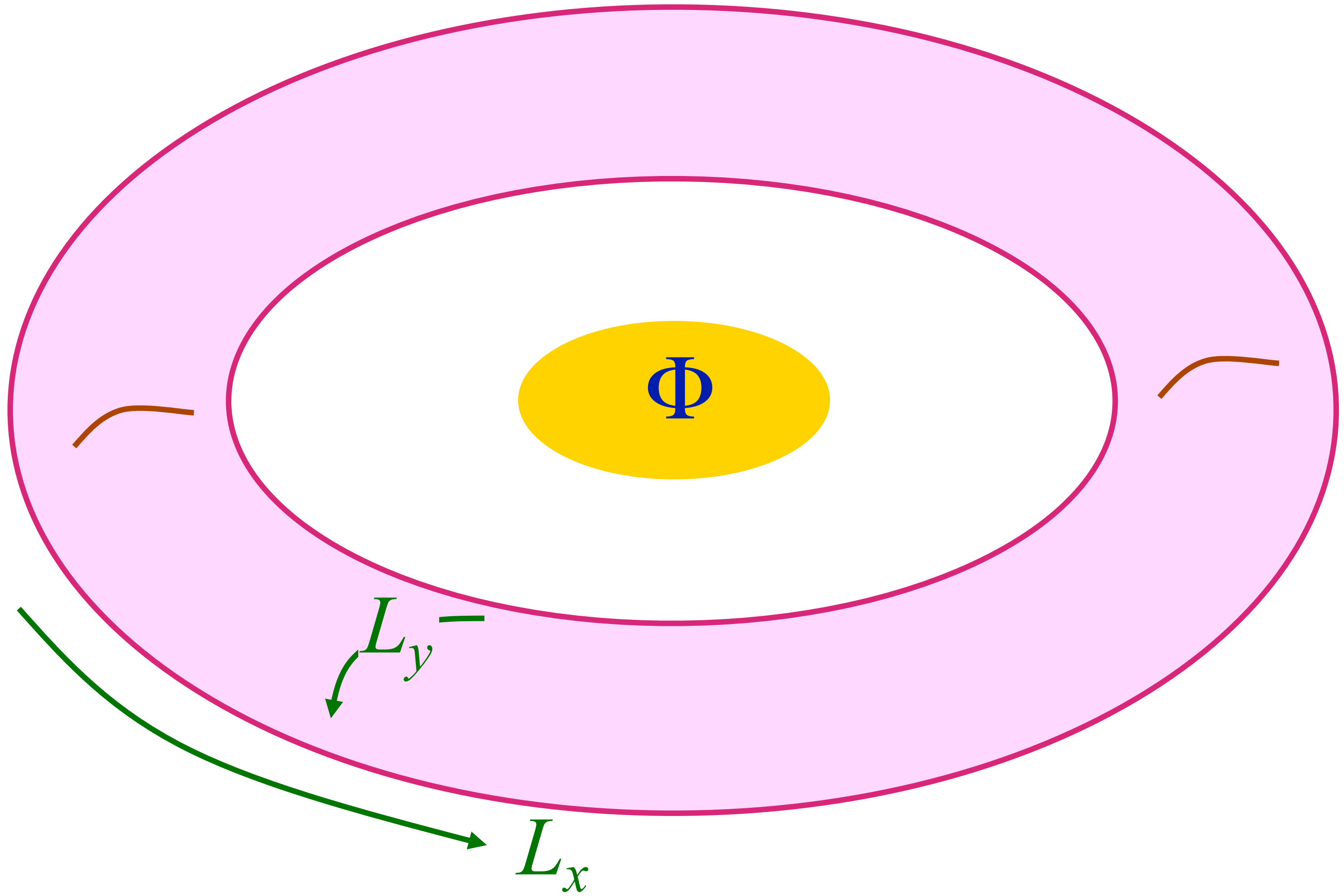
Topology and the Fermi surface size in <u>FL</u>



M. Oshikawa, PRL **84**, 3370 (2000)
A. Paramekanti and A. Vishwanath, PRB **70**, 245118 (2004)

We take N particles, each with charge Q, on a $L_x \times L_y$ lattice on a torus. We pierce flux $\Phi = hc/Q$ through a hole of the torus. An <u>exact</u> computation shows that the change in crystal momentum of the many-body state due to flux piercing is $\Delta P_x = 2\pi N/L_x (\text{mod}\, 2\pi)$. Alternatively, computing this momentum change from the quasi-particles around the Fermi surface we conclude that the Fermi surface encloses the Luttinger area $(2\pi^2)(1+p)$.

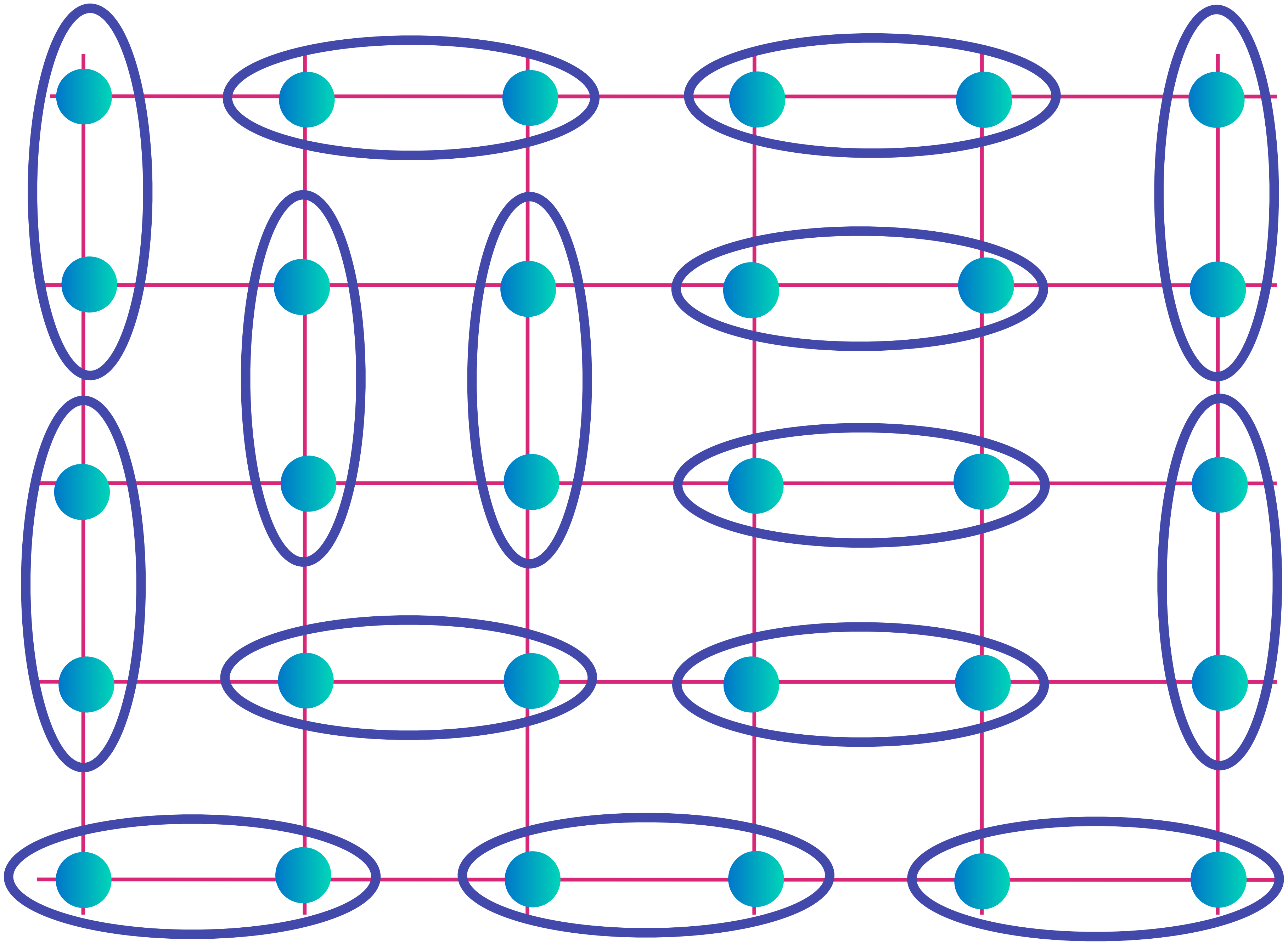
Topology and the Fermi surface size in FL*

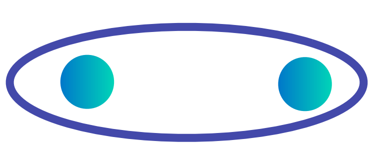


The exact momentum transfers $\Delta P_x = 2\pi(1+p)L_y(\text{mod}2\pi)$ and $\Delta P_y = 2\pi(1+p)L_x(\text{mod}2\pi)$ due to flux piercing arise from

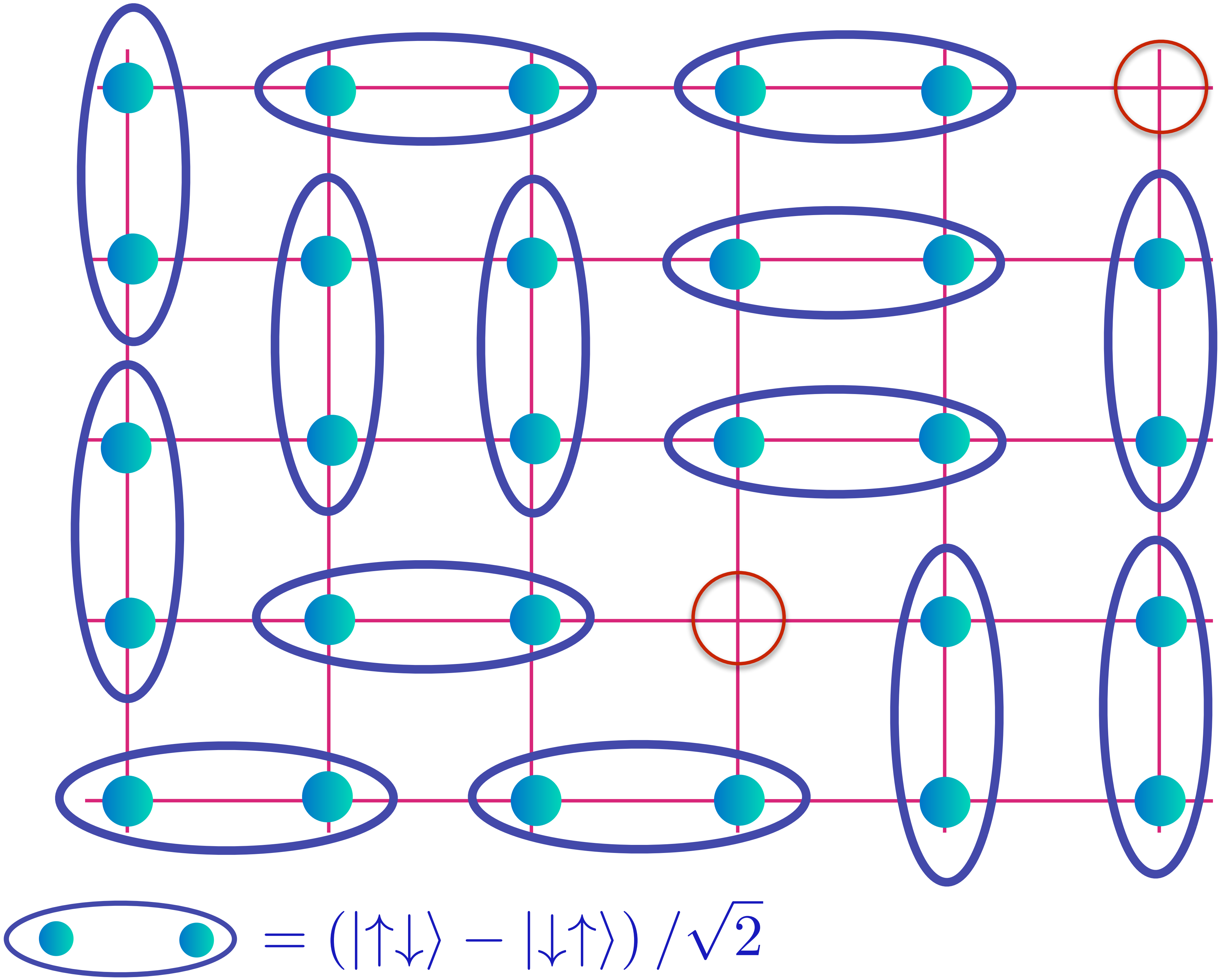
- A contribution $2\pi p L_{x,y}$ from the small Fermi surface of quasiparticles of size p.
- The remainder is made up by the topological sector: flux insertion creates a "vison" in the hole of the torus.

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

Start with a spin liquid and then remove electrons



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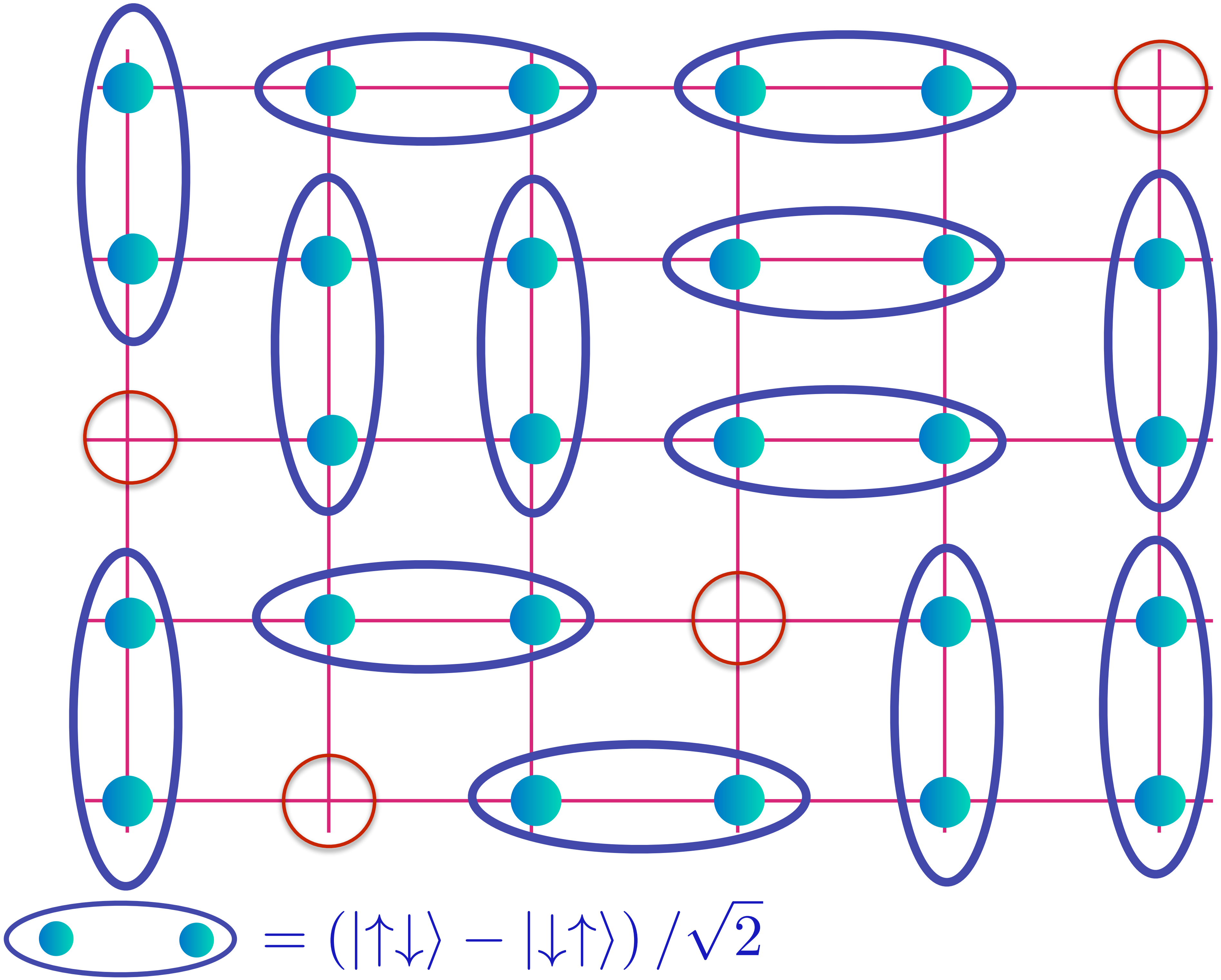

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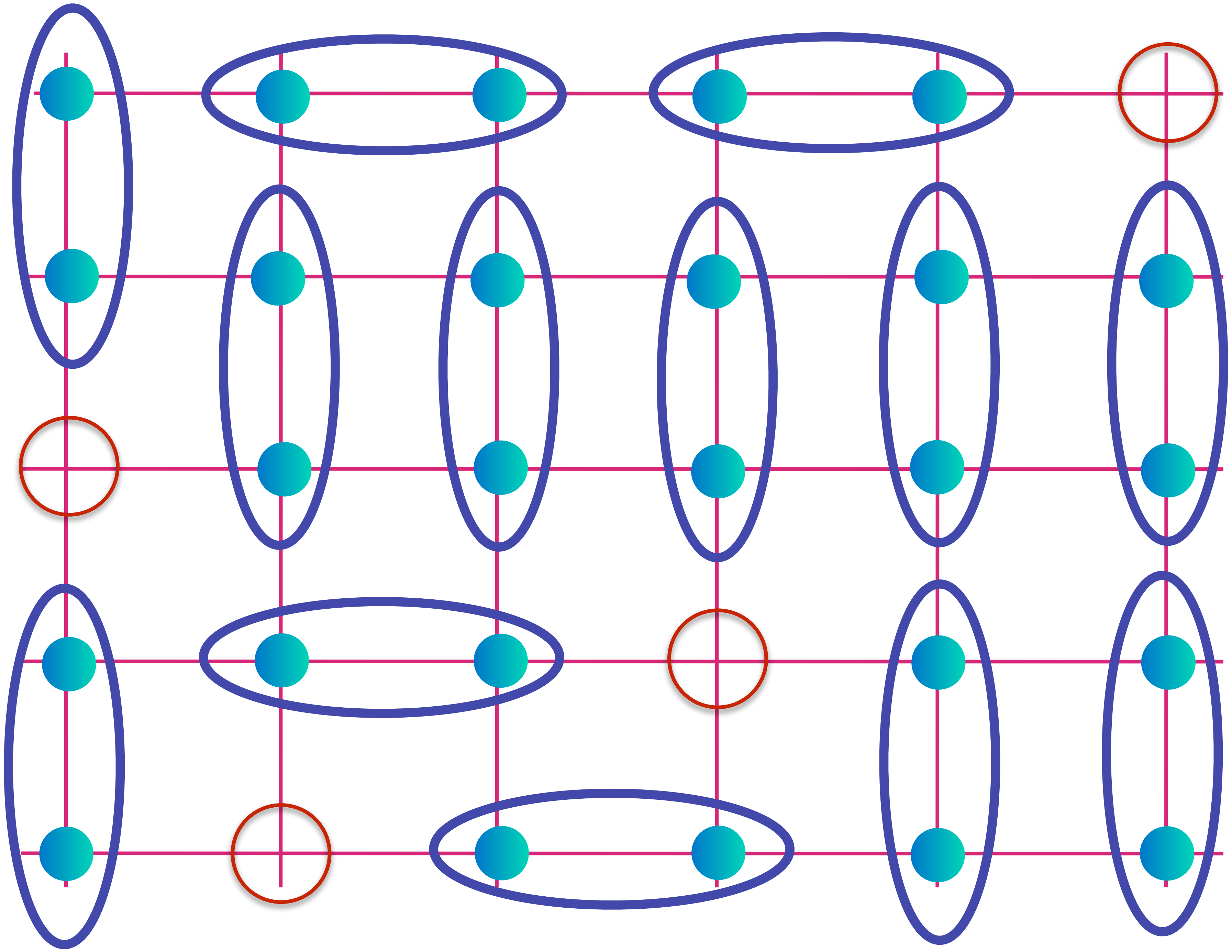

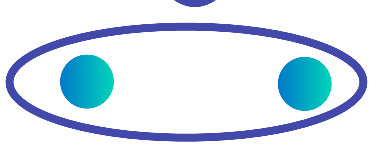 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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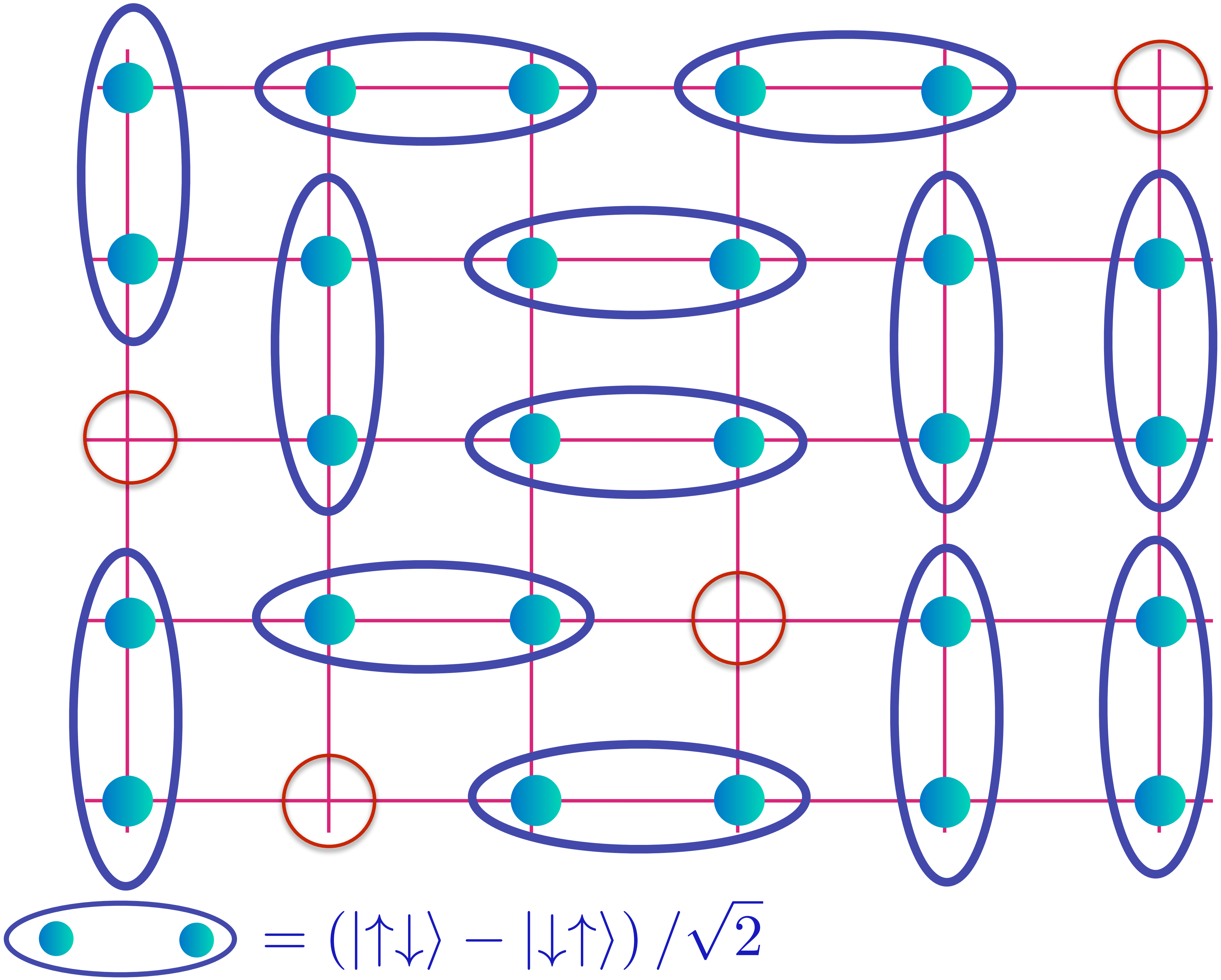
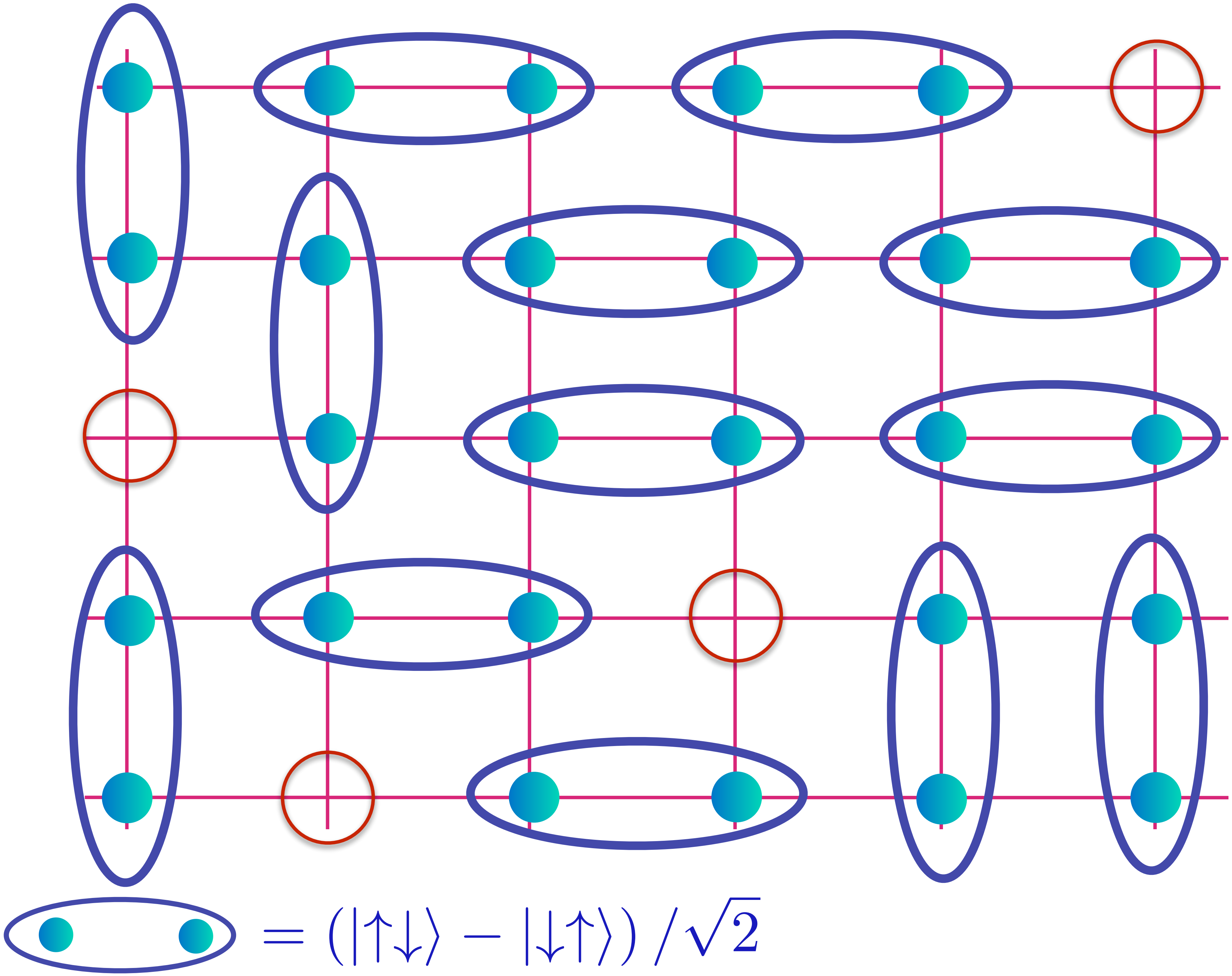


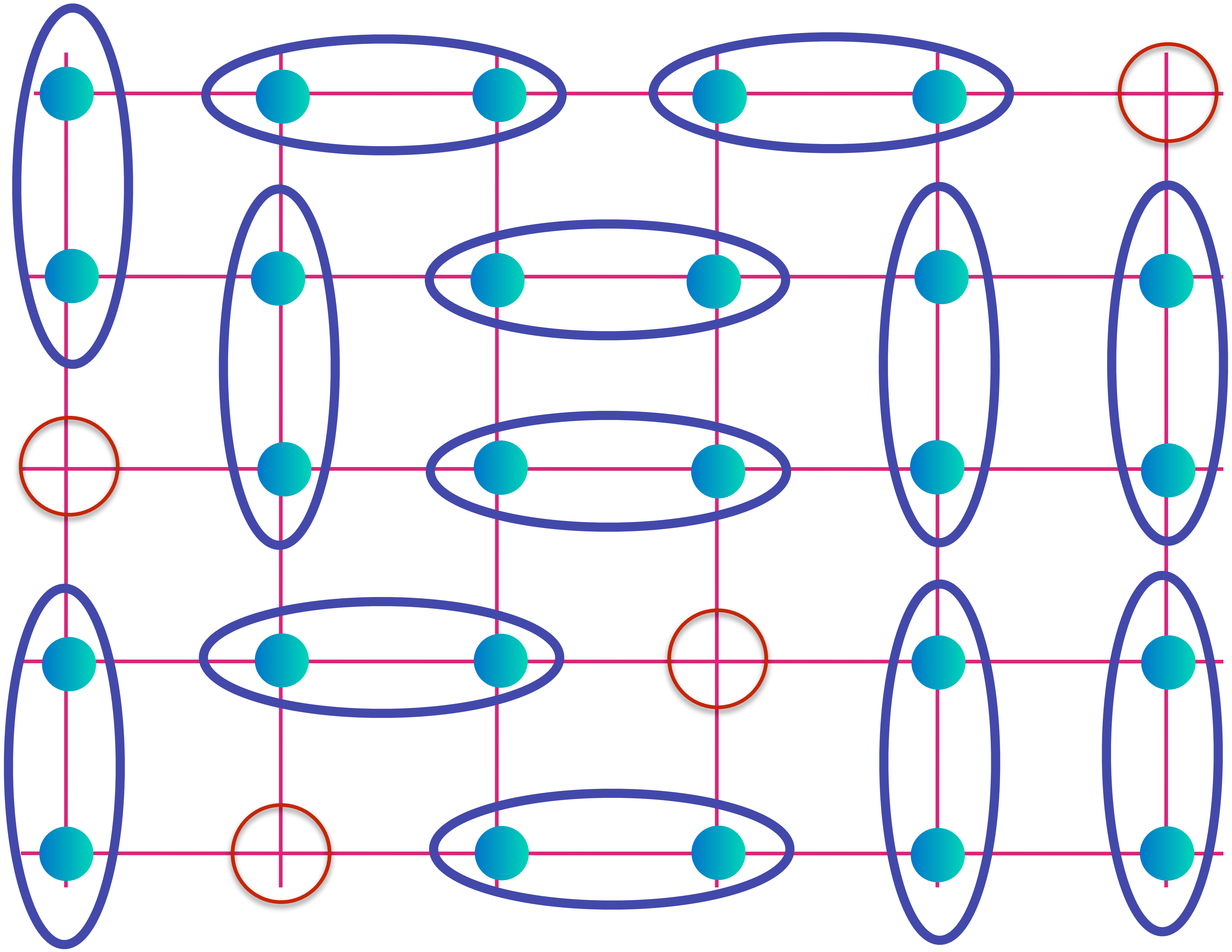
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

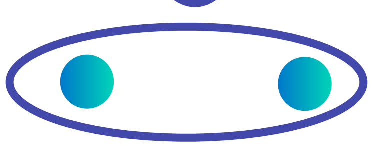
Start with a spin liquid and then remove electrons



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

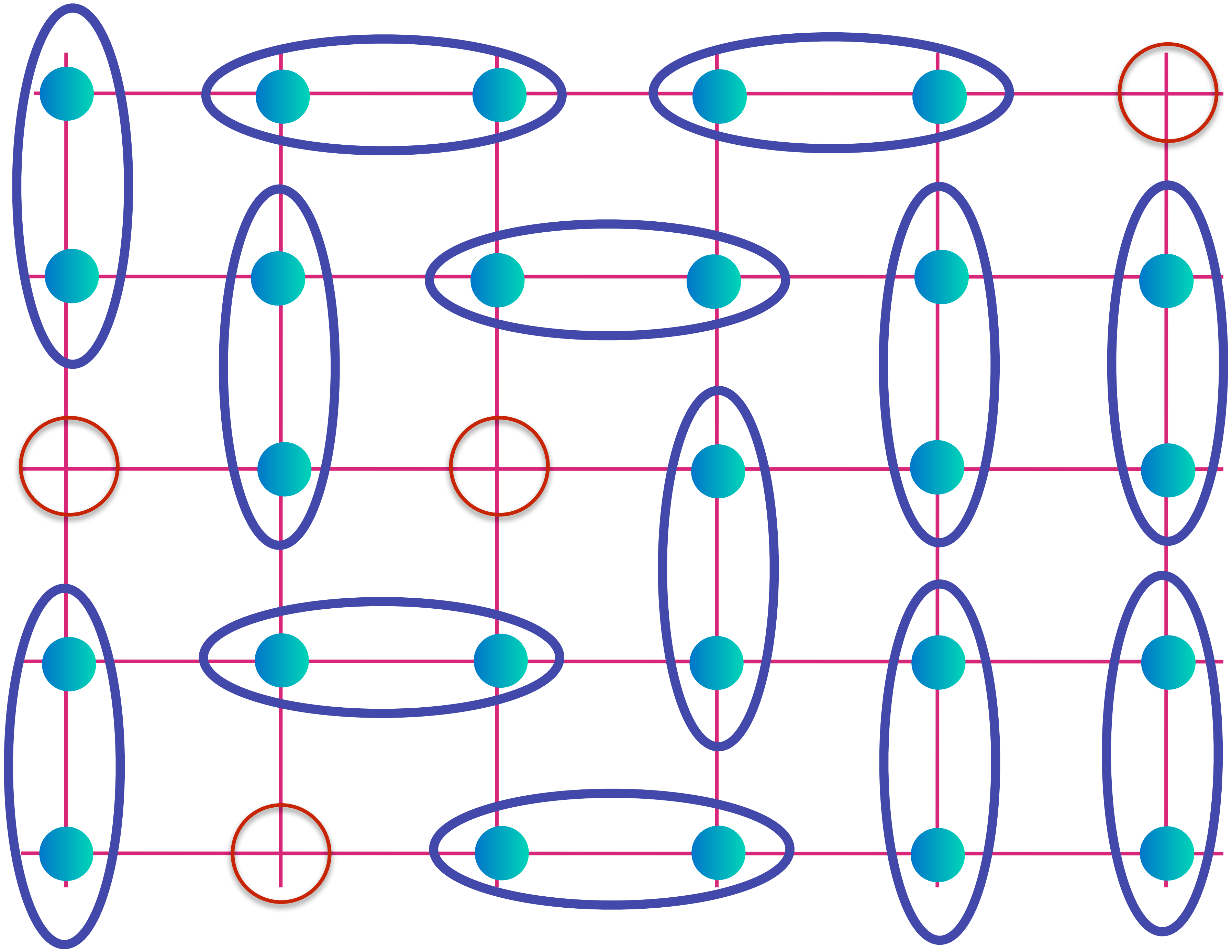
Start with a spin liquid and then remove electrons

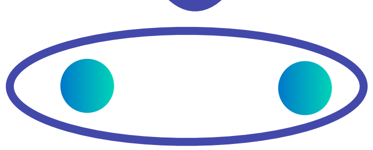




$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

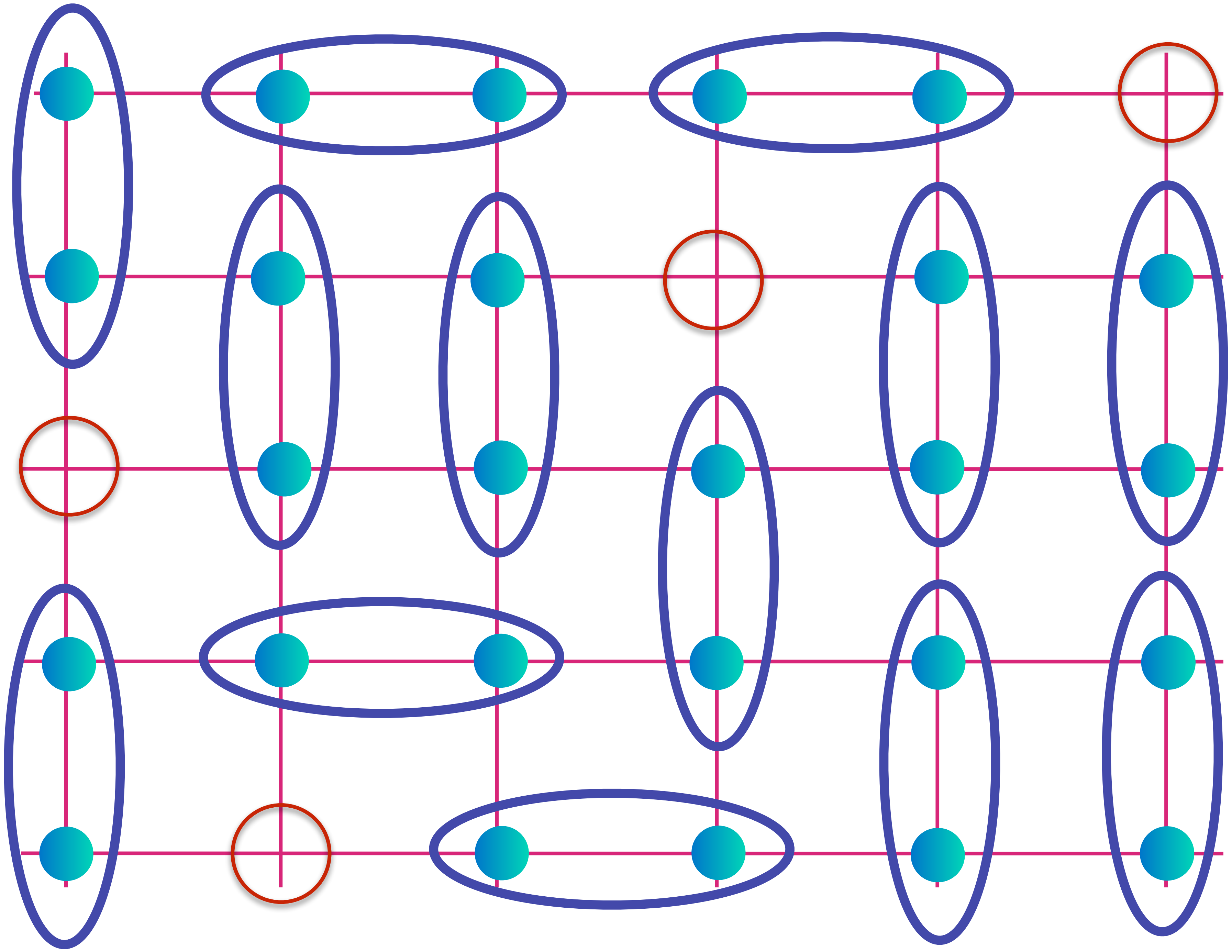
Start with a spin liquid and then remove electrons

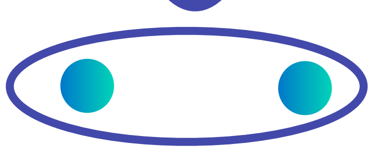




$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

A mobile charge +e, but carrying no spin

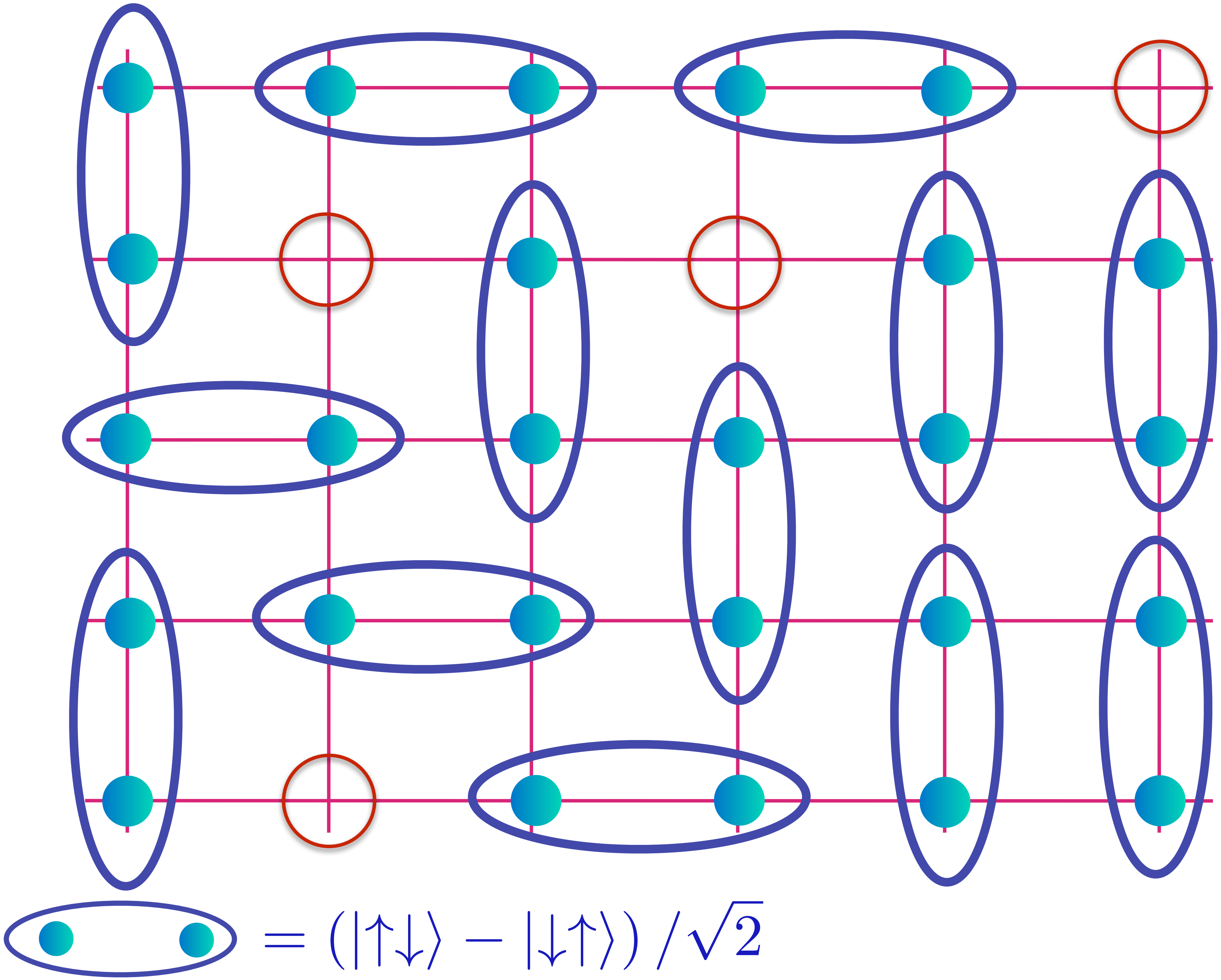


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

A mobile charge +e, but carrying no spin

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

N. Read and B. Chakraborty, PRB **40**, 7133 (1989)



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

Spin liquid with density *p* of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, PRB **35**, 8865 (1987)

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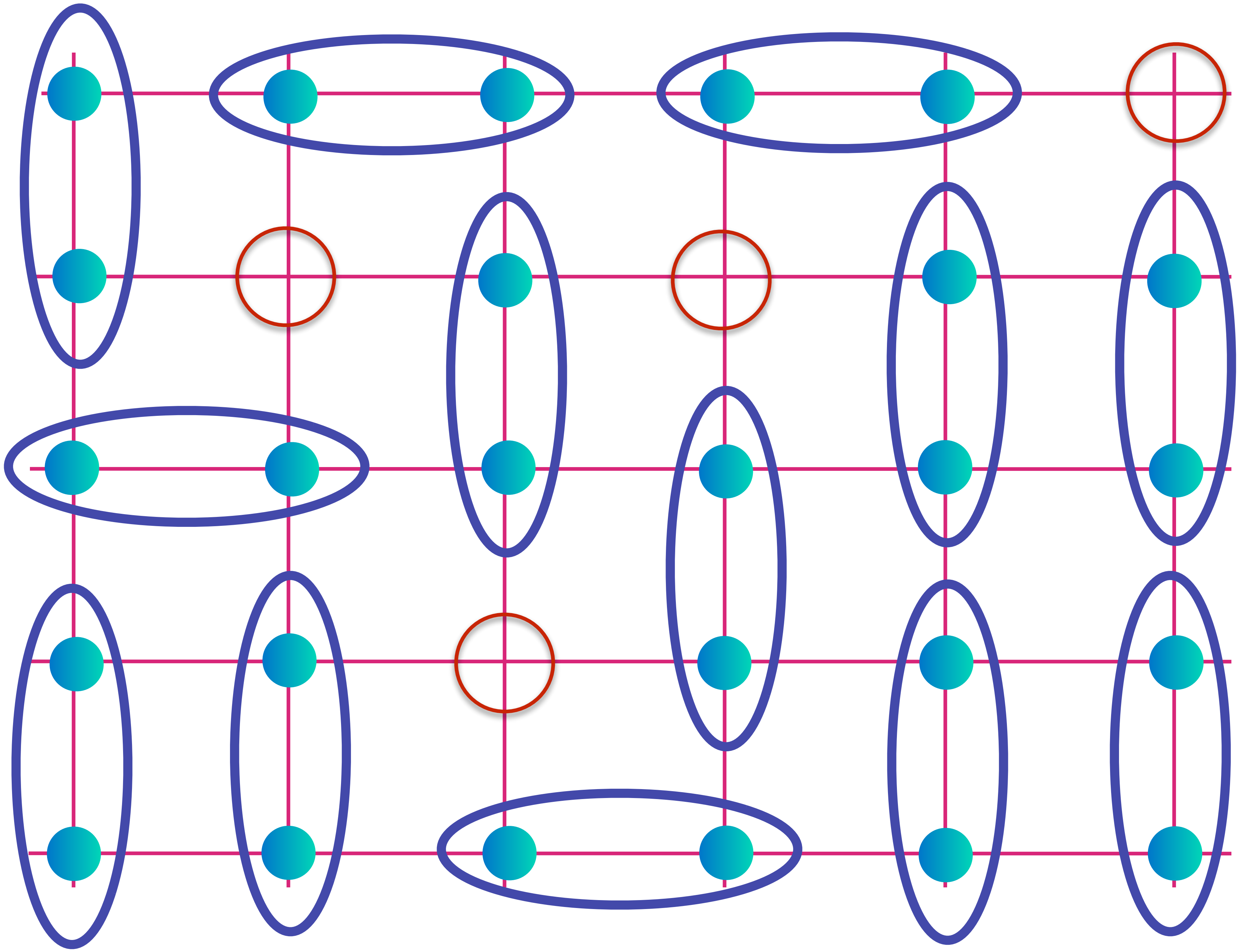

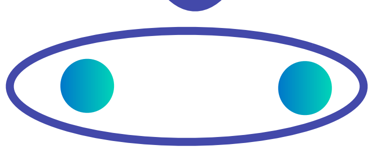 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

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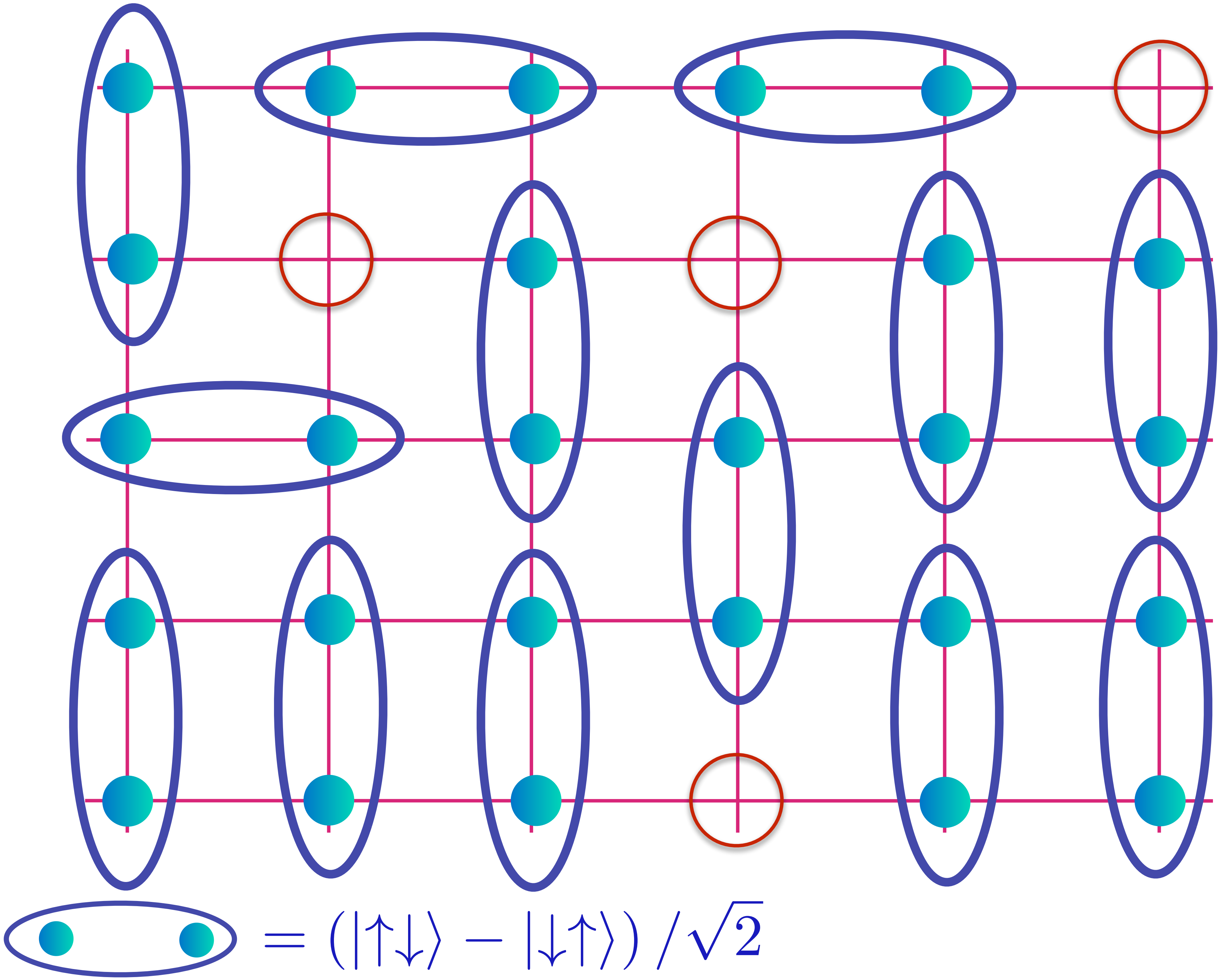

$$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$

Spin liquid with density *p* of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

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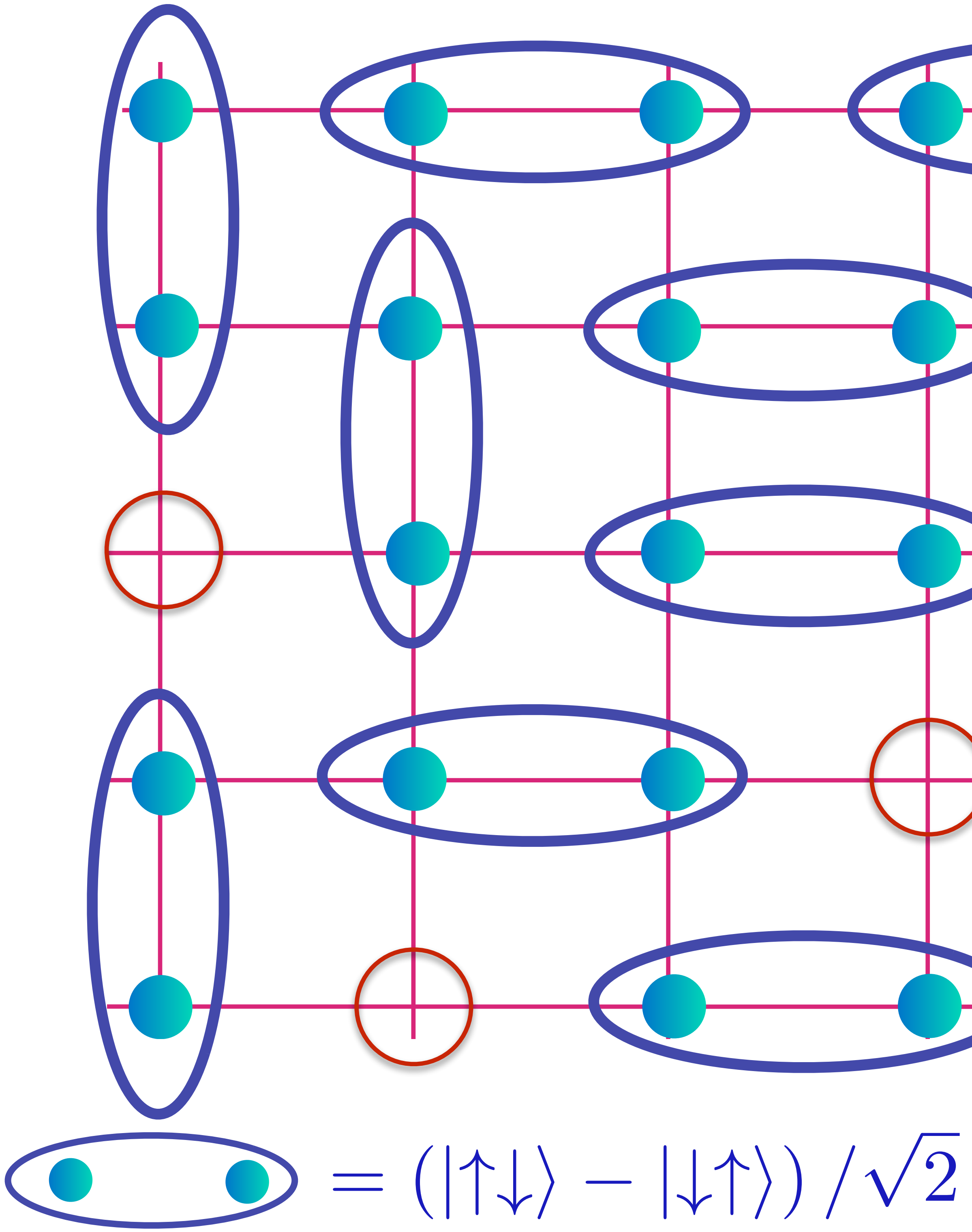
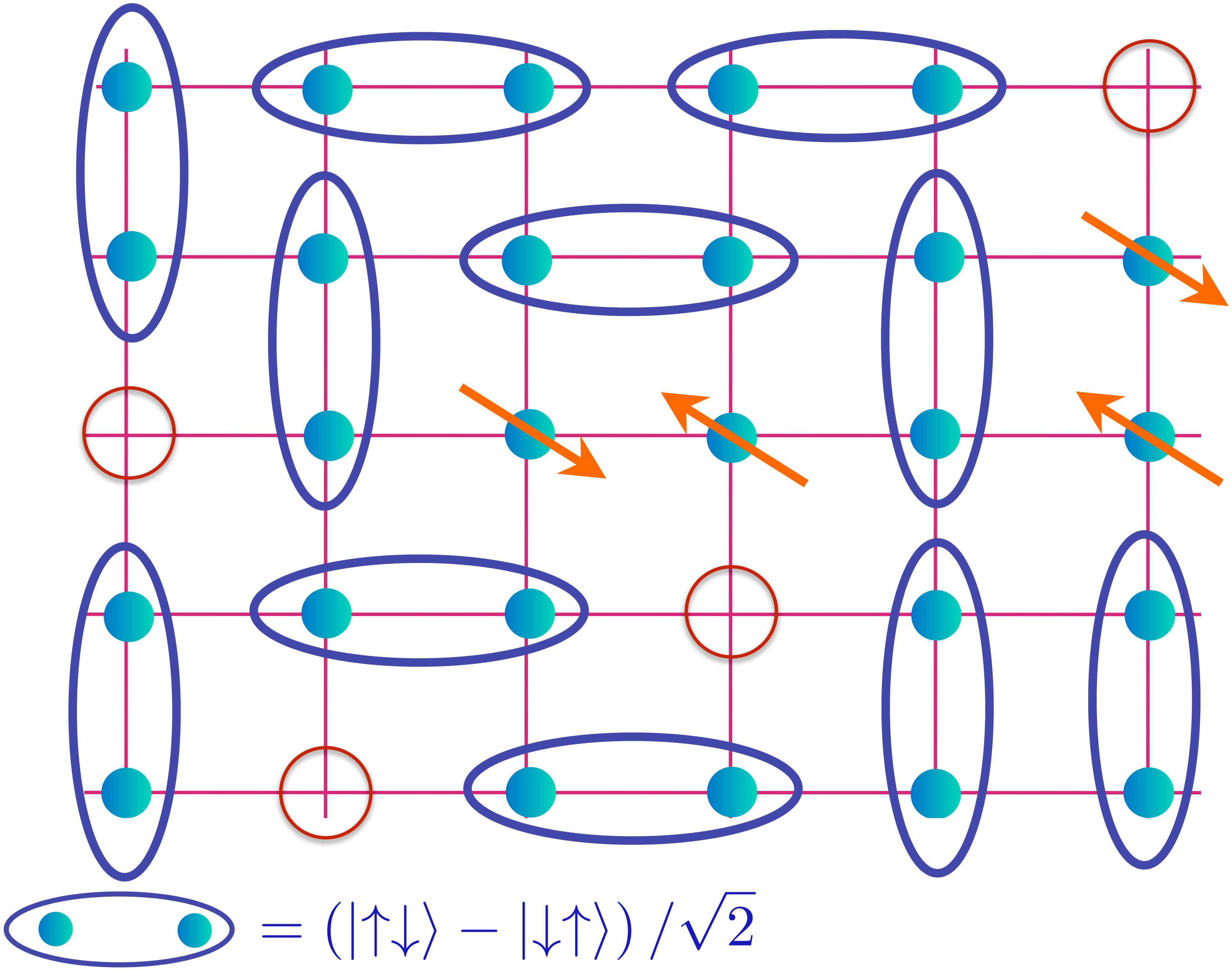
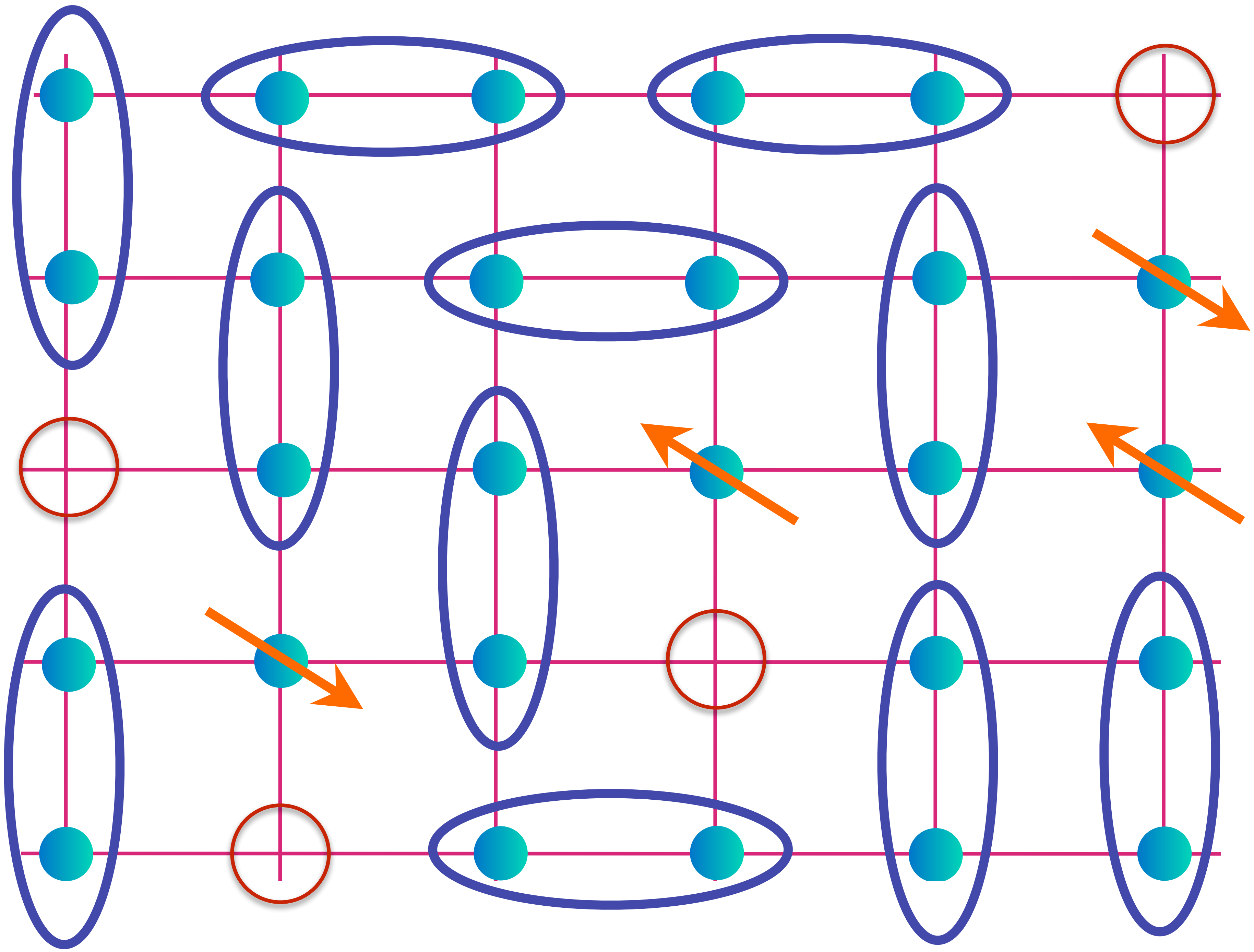
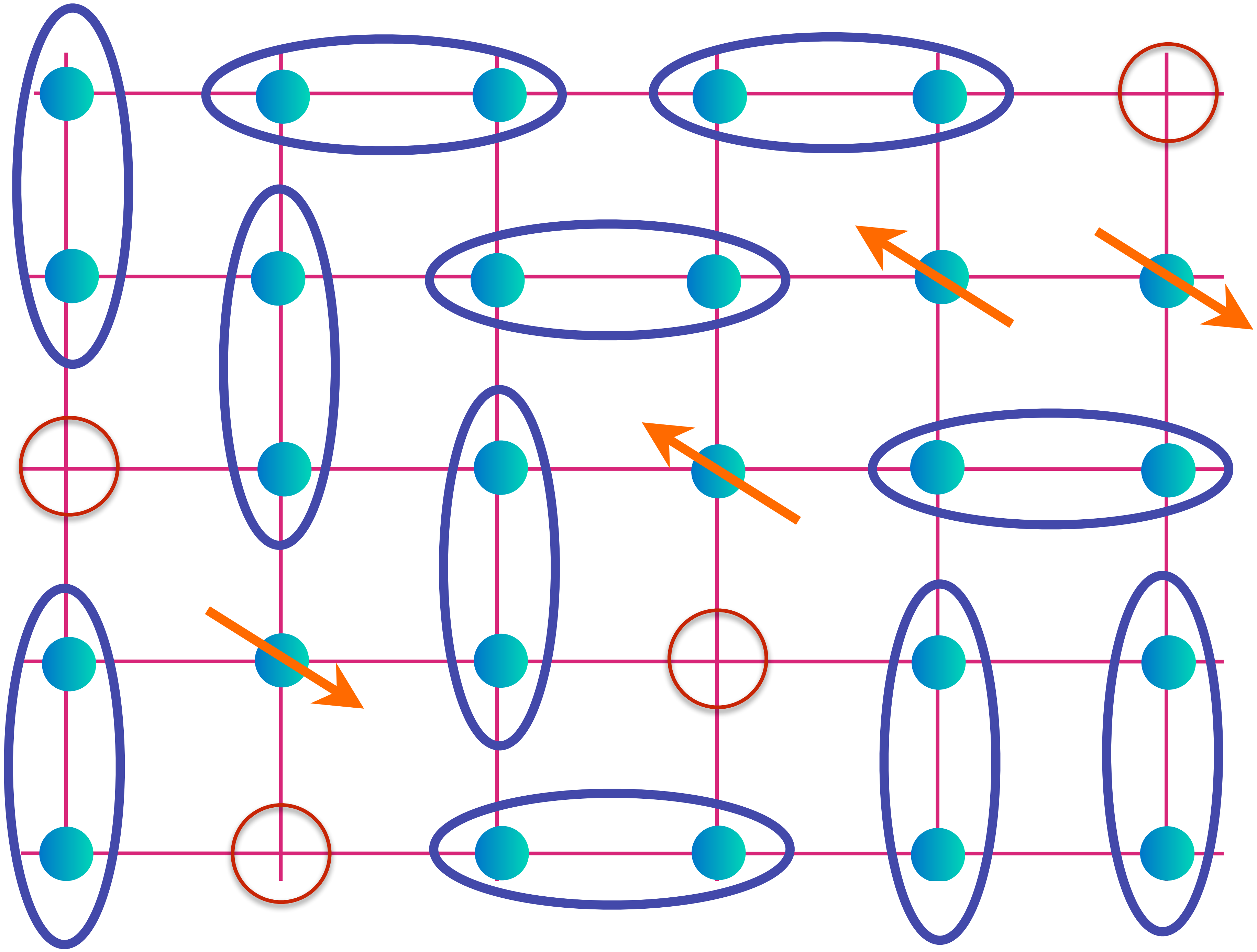


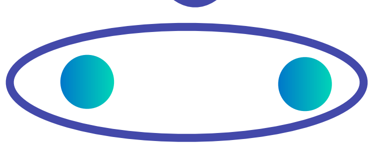
Spin liquid with density *p* of spinless, charge +e "holons". These can form a Fermi surface of size p, but not of electrons

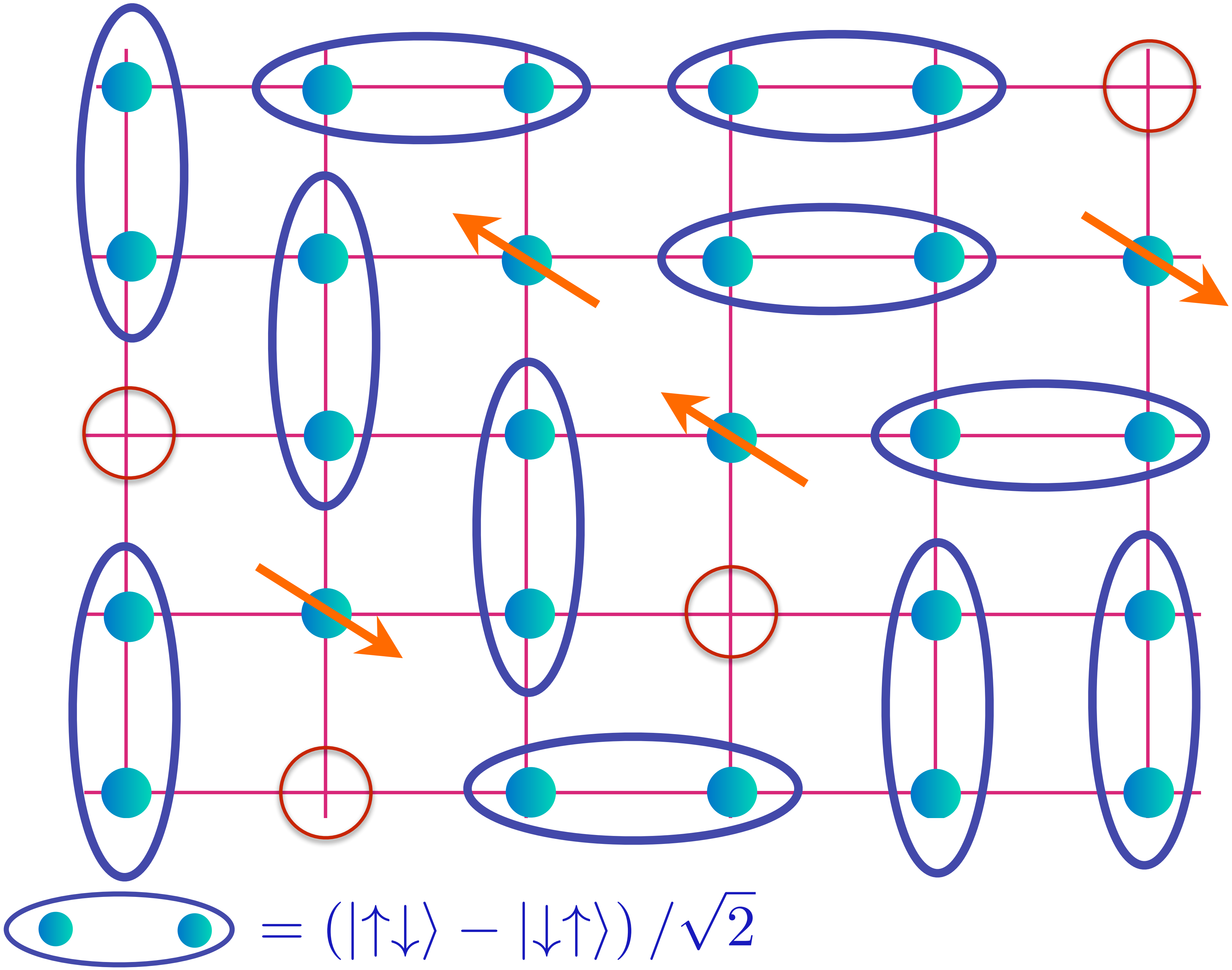

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

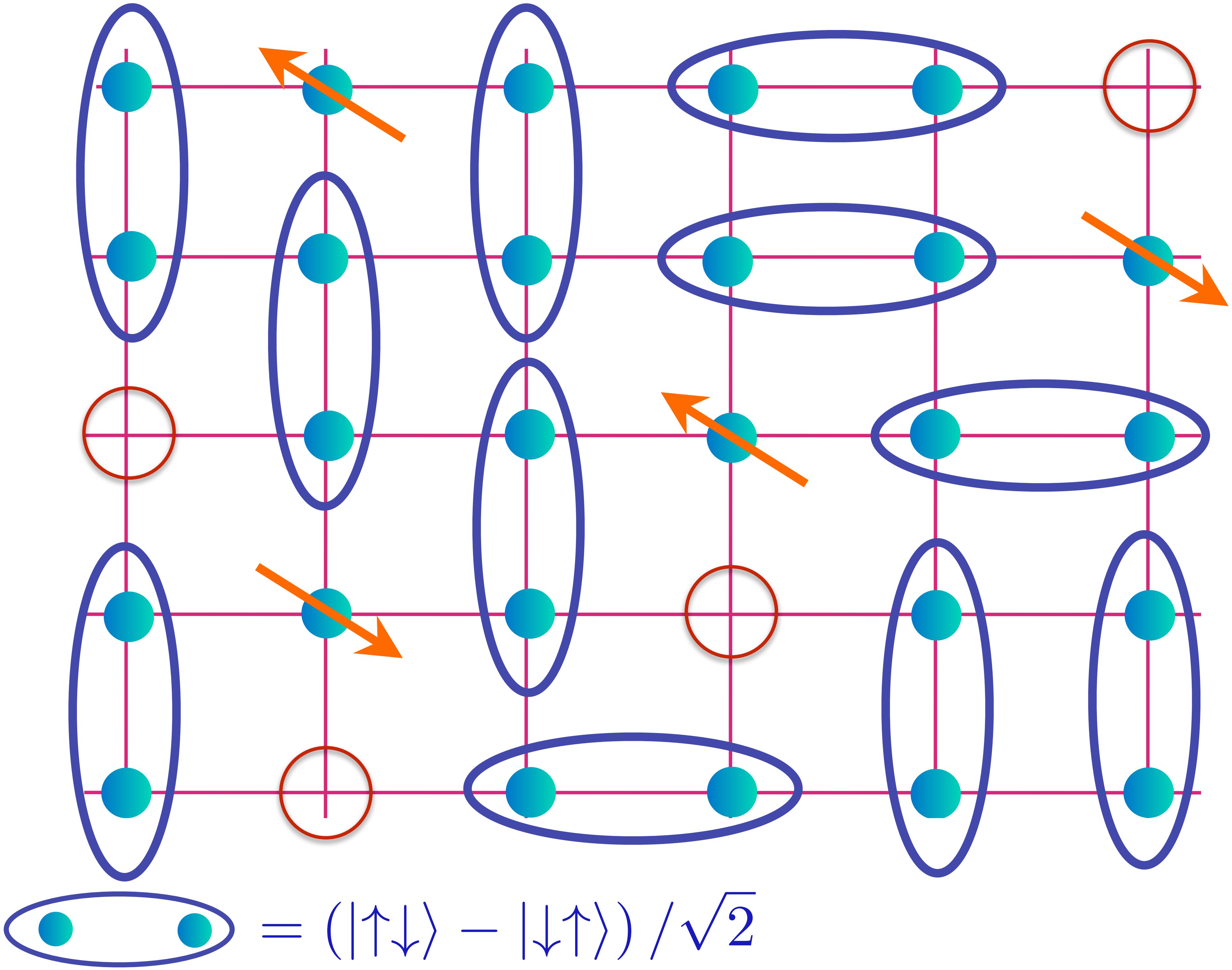


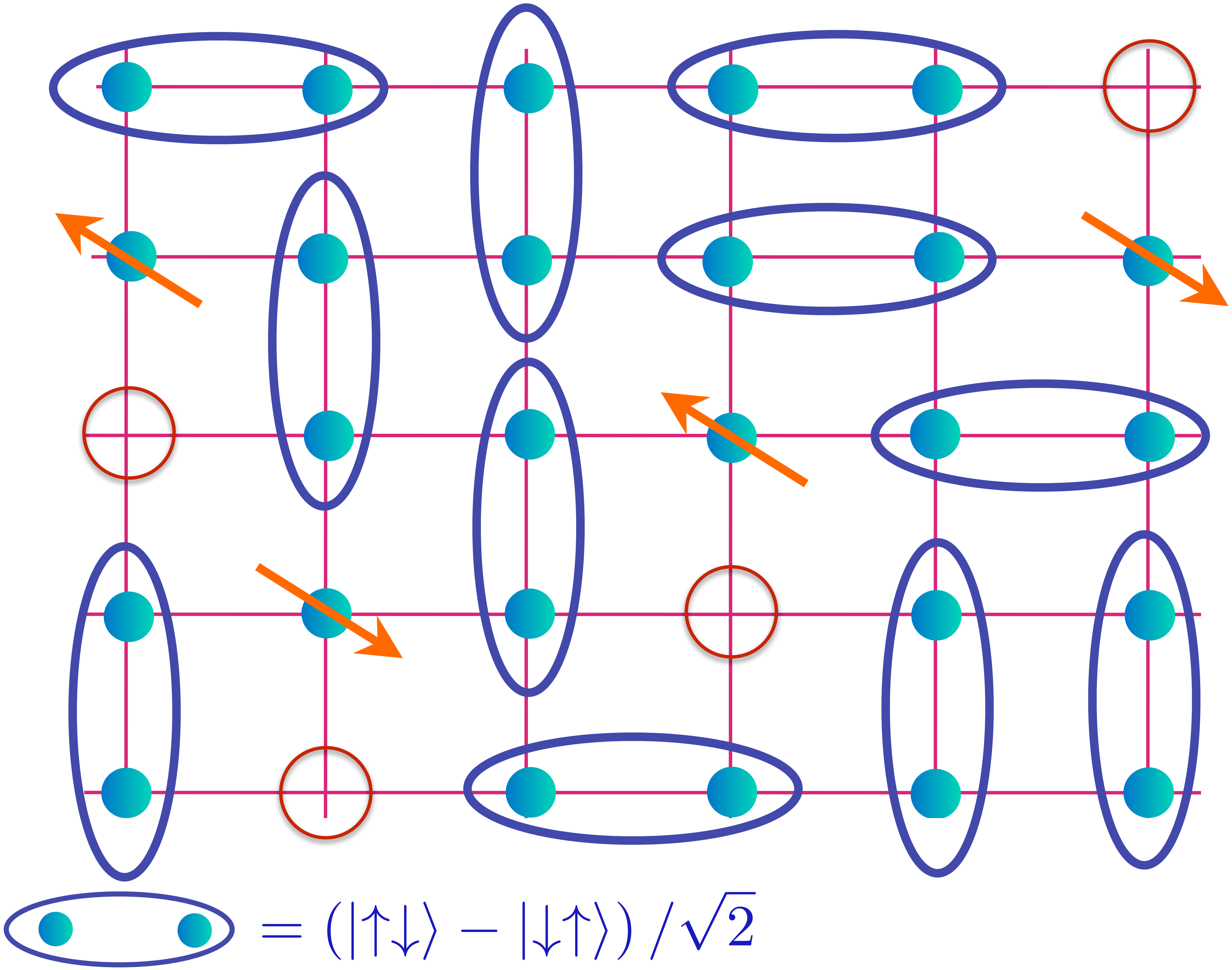
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$$

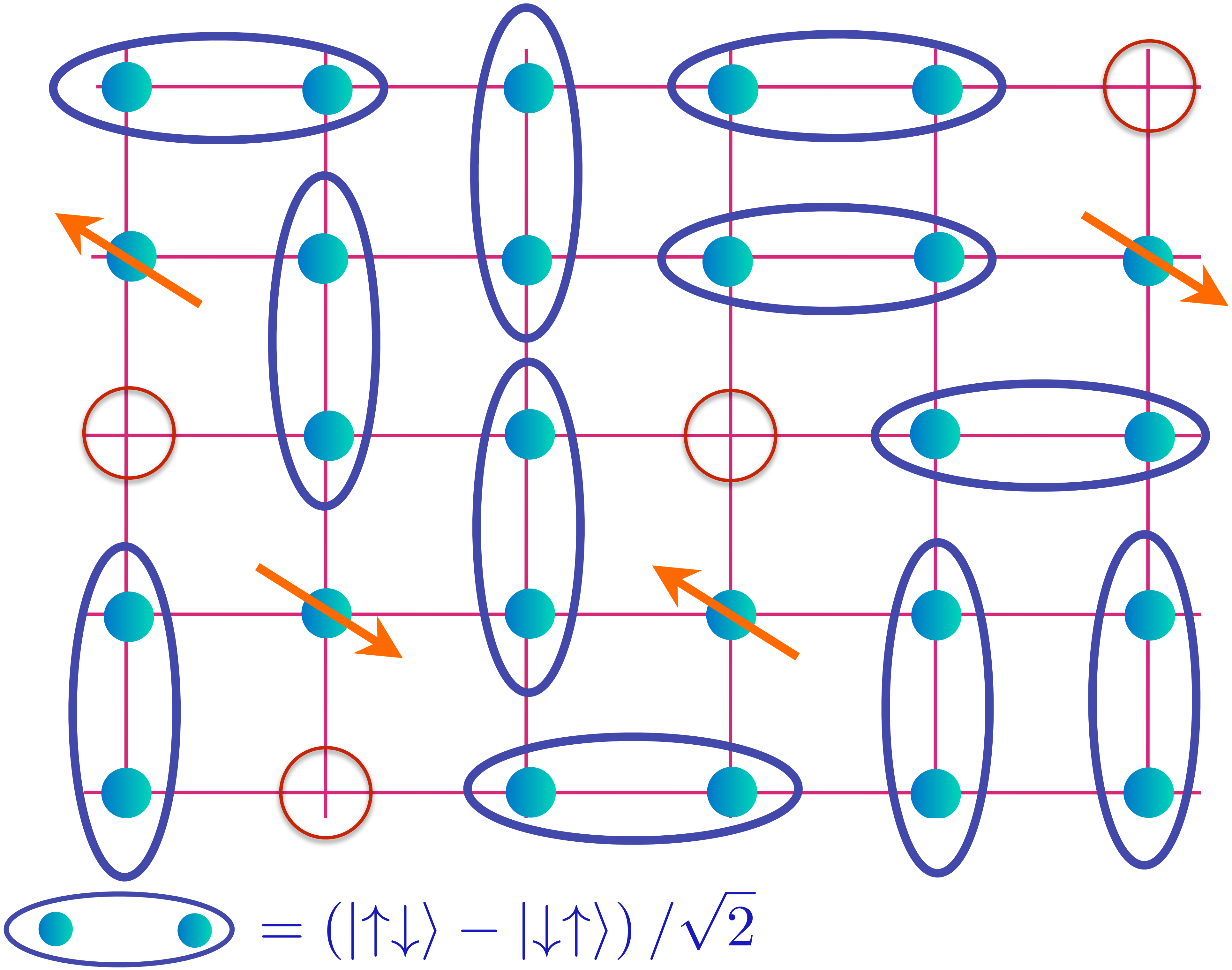


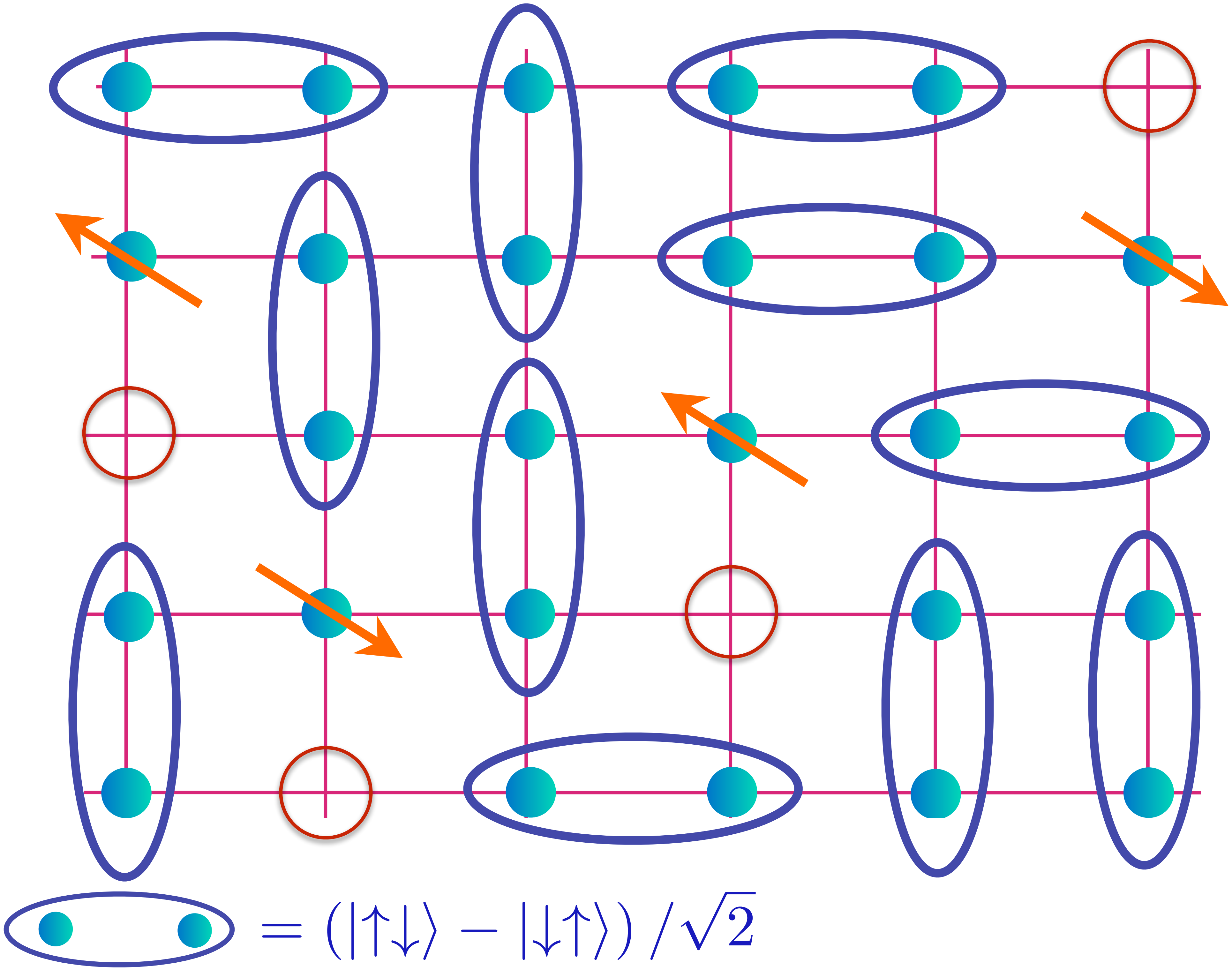
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

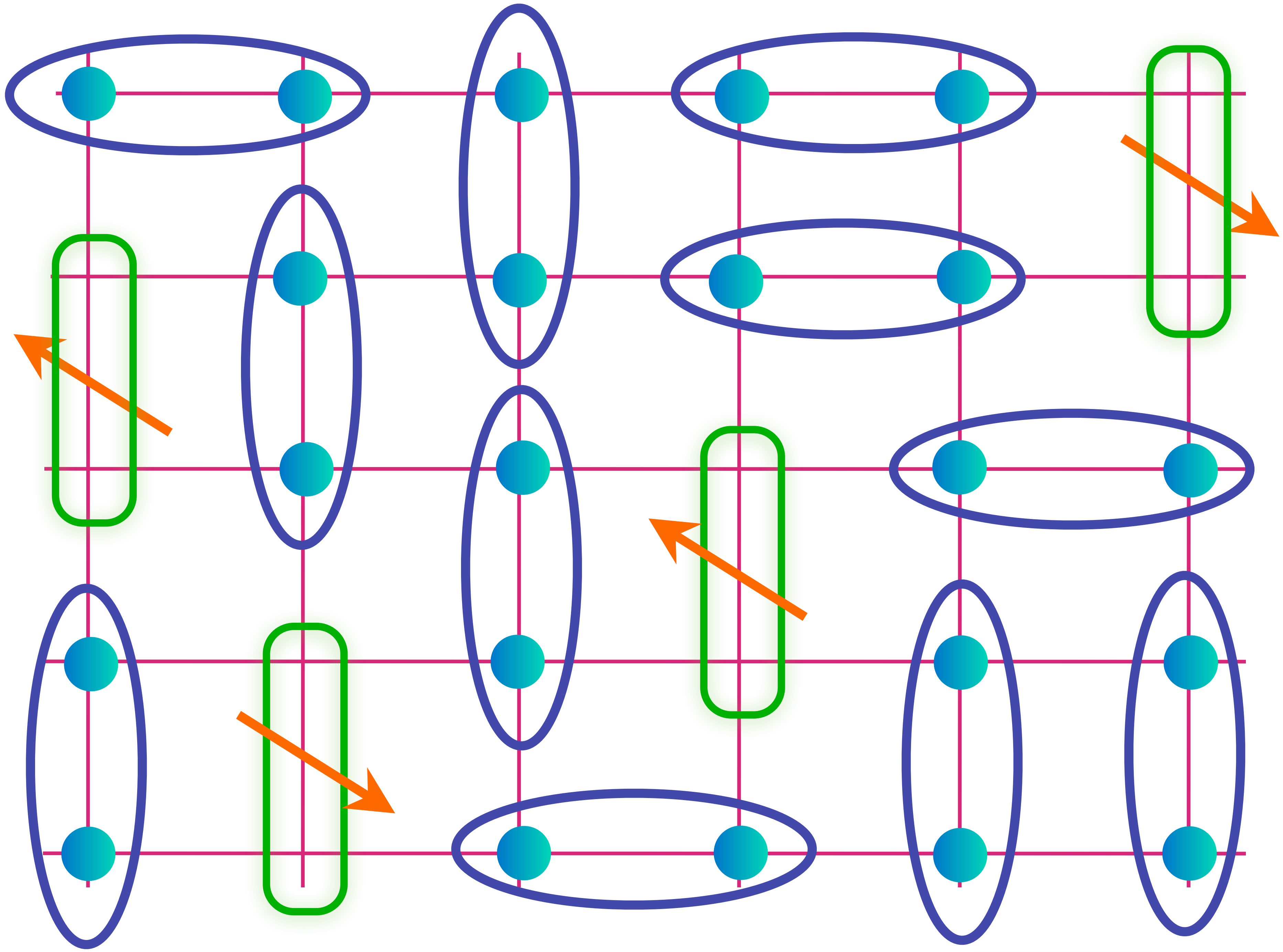

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$


$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

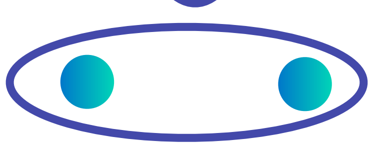

$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

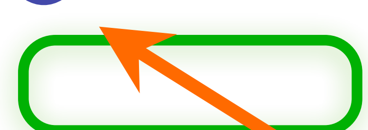
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

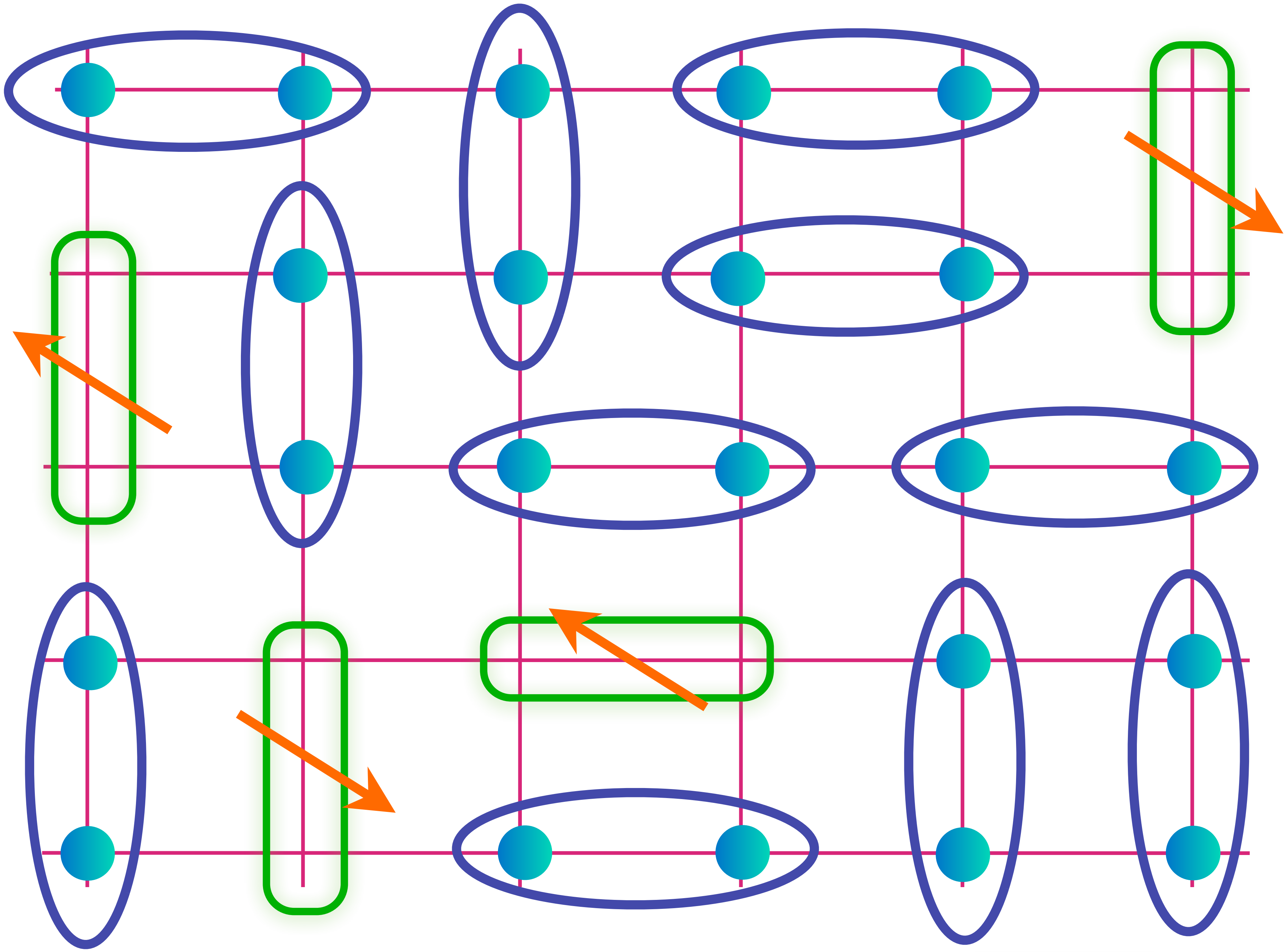
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

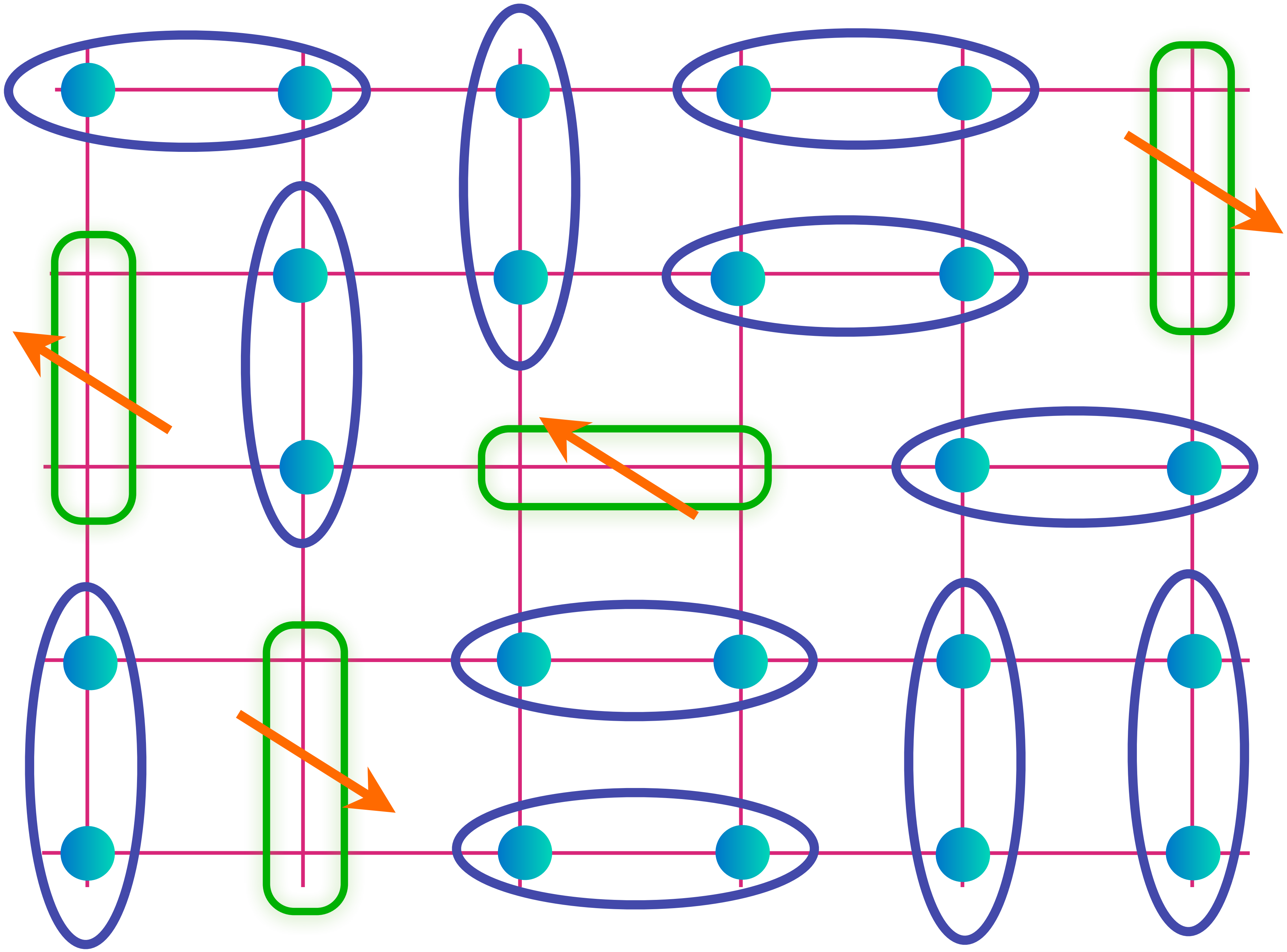
(blue ellipse) $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

(green box) $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

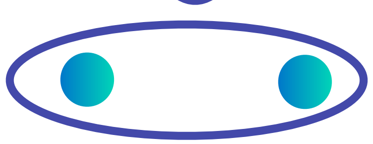
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

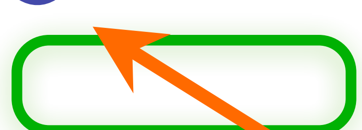
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

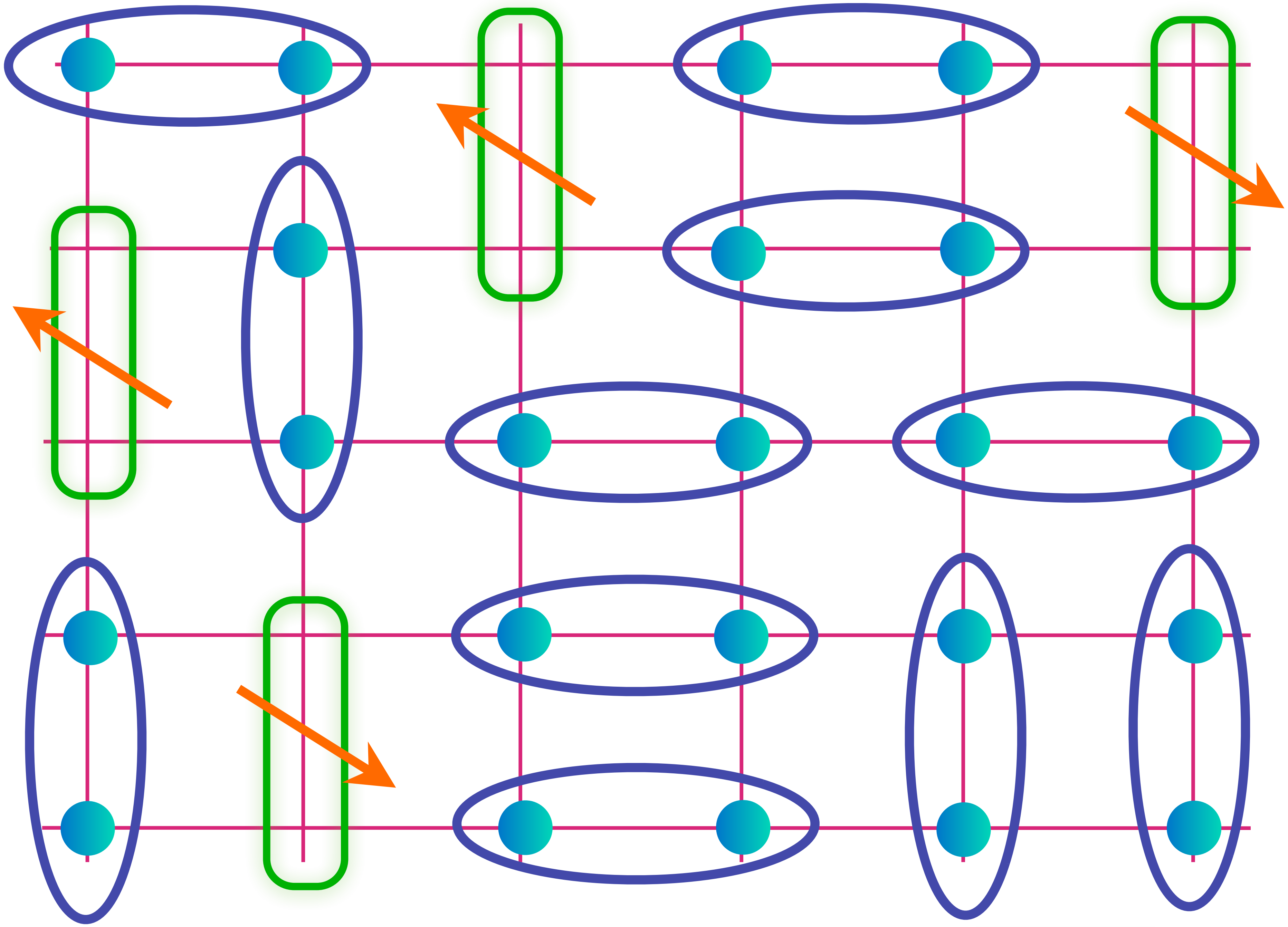
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
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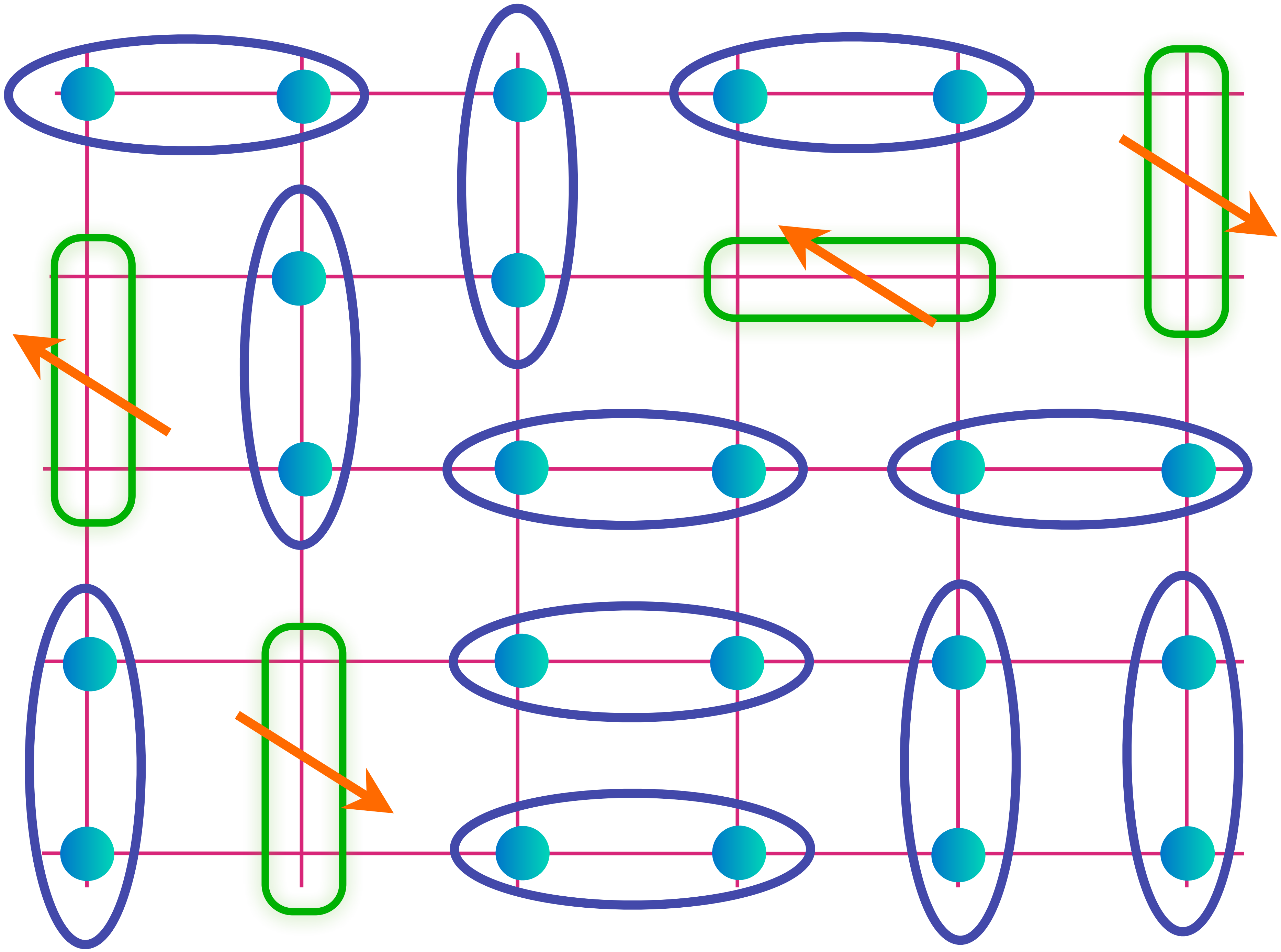
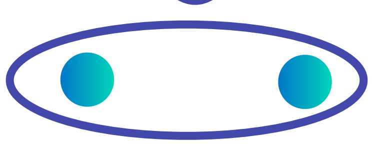
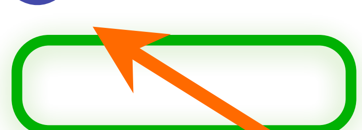


Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
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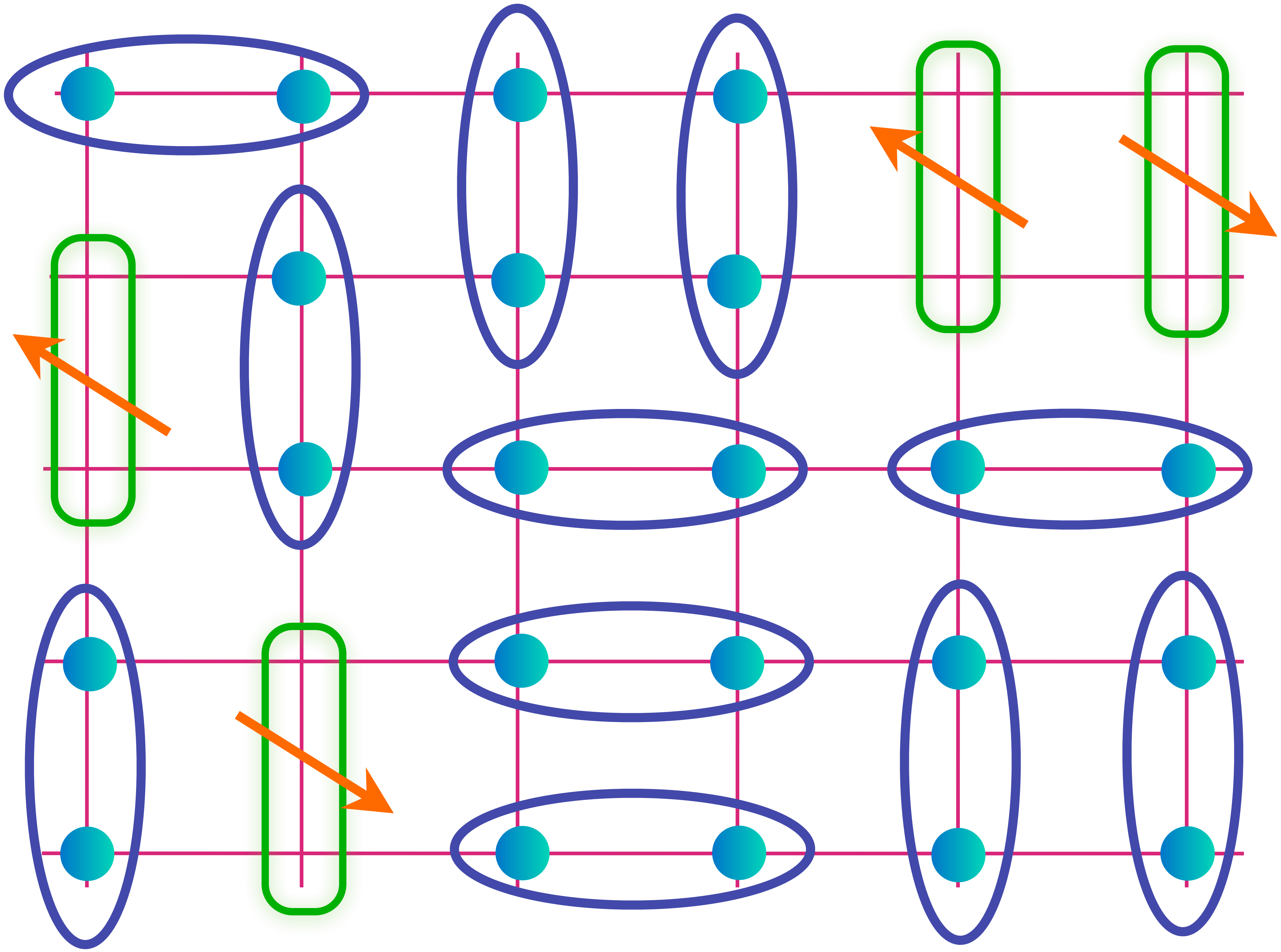
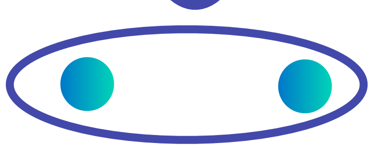
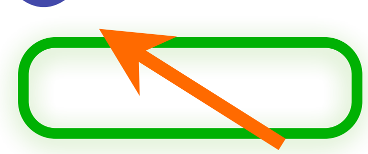


Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
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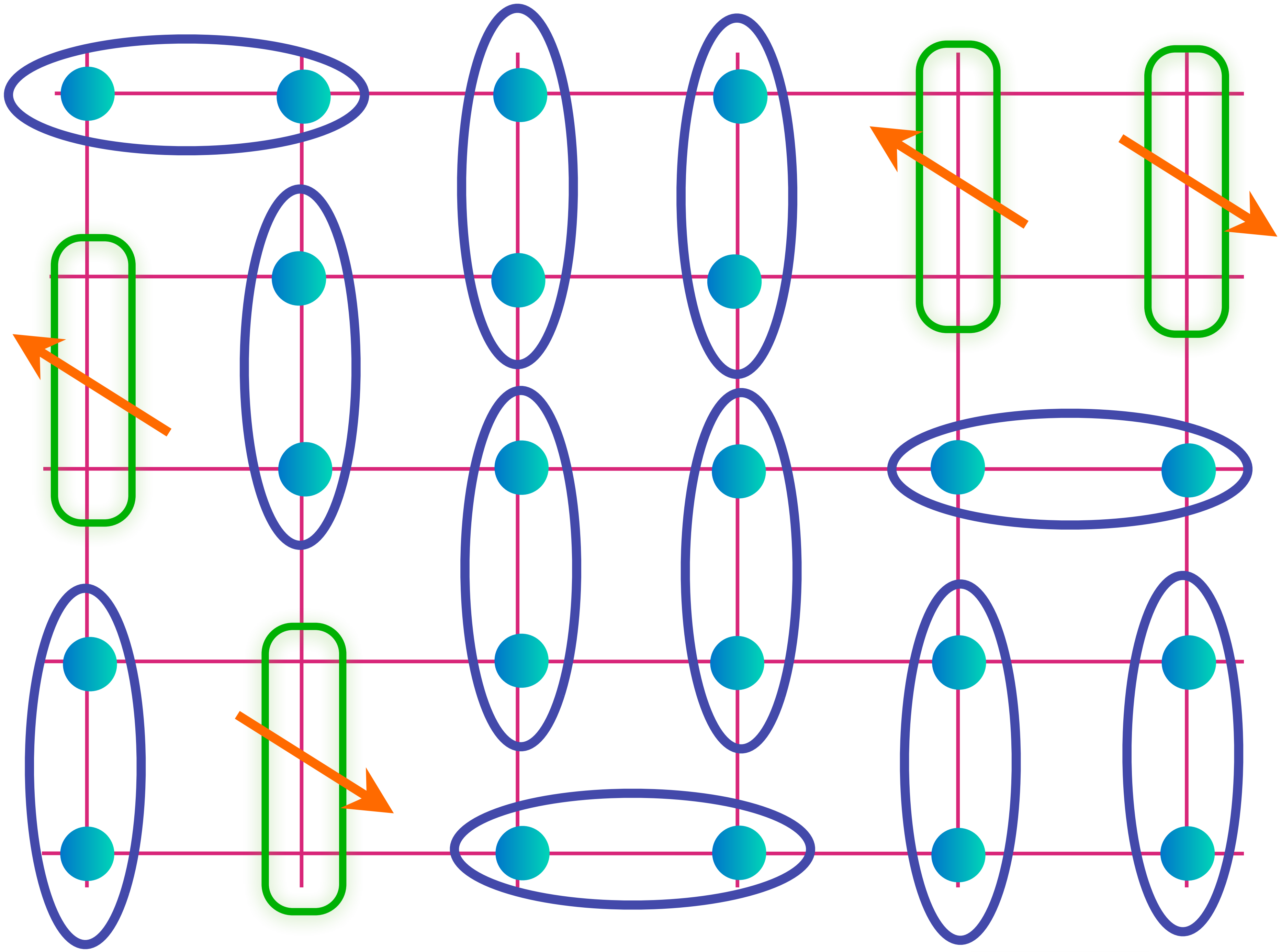

Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size *p* of electrons

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

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S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
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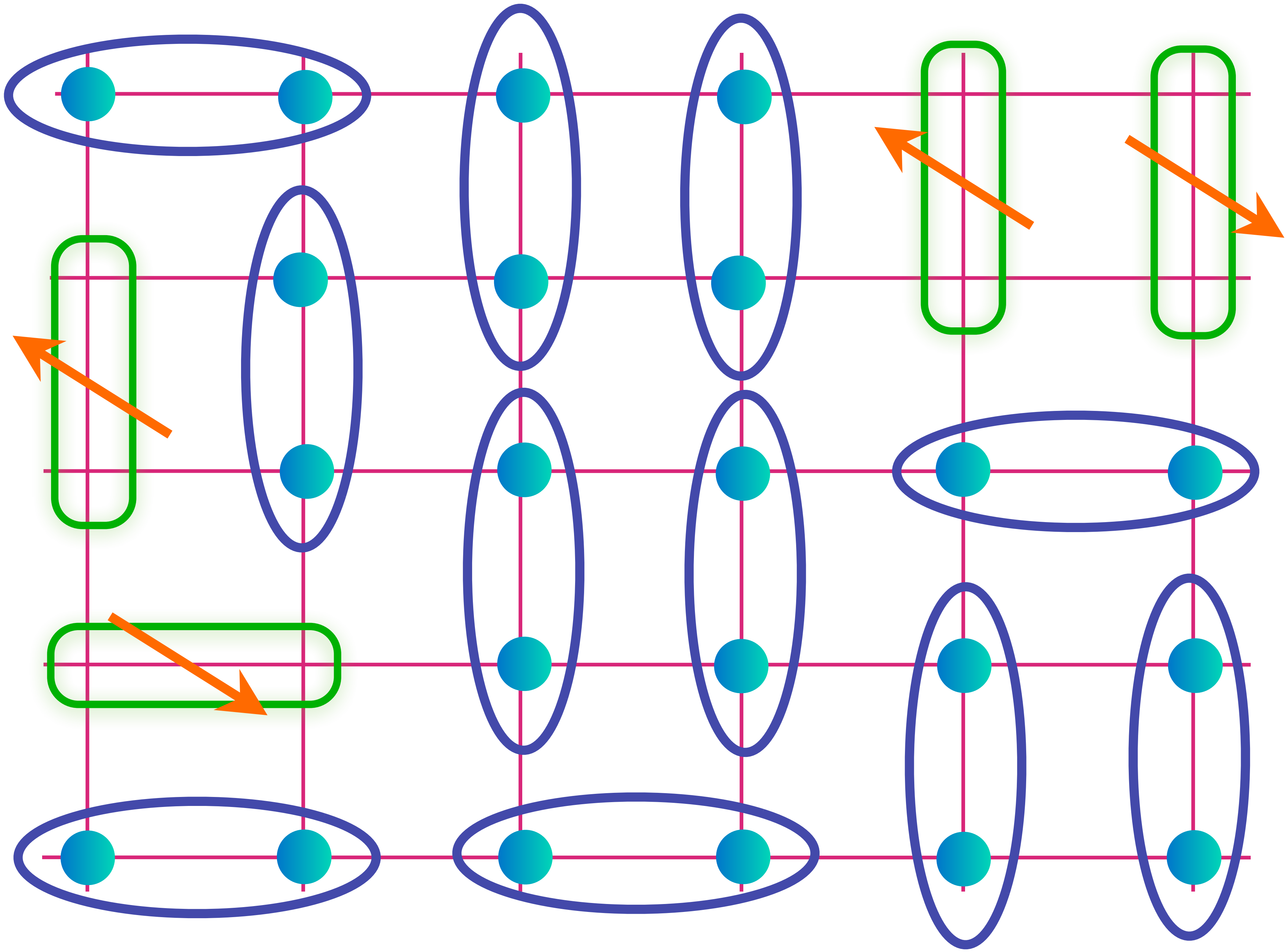
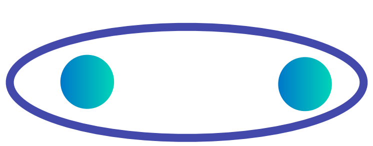
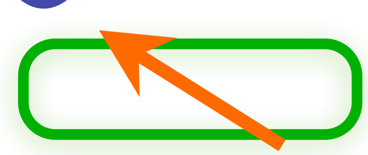


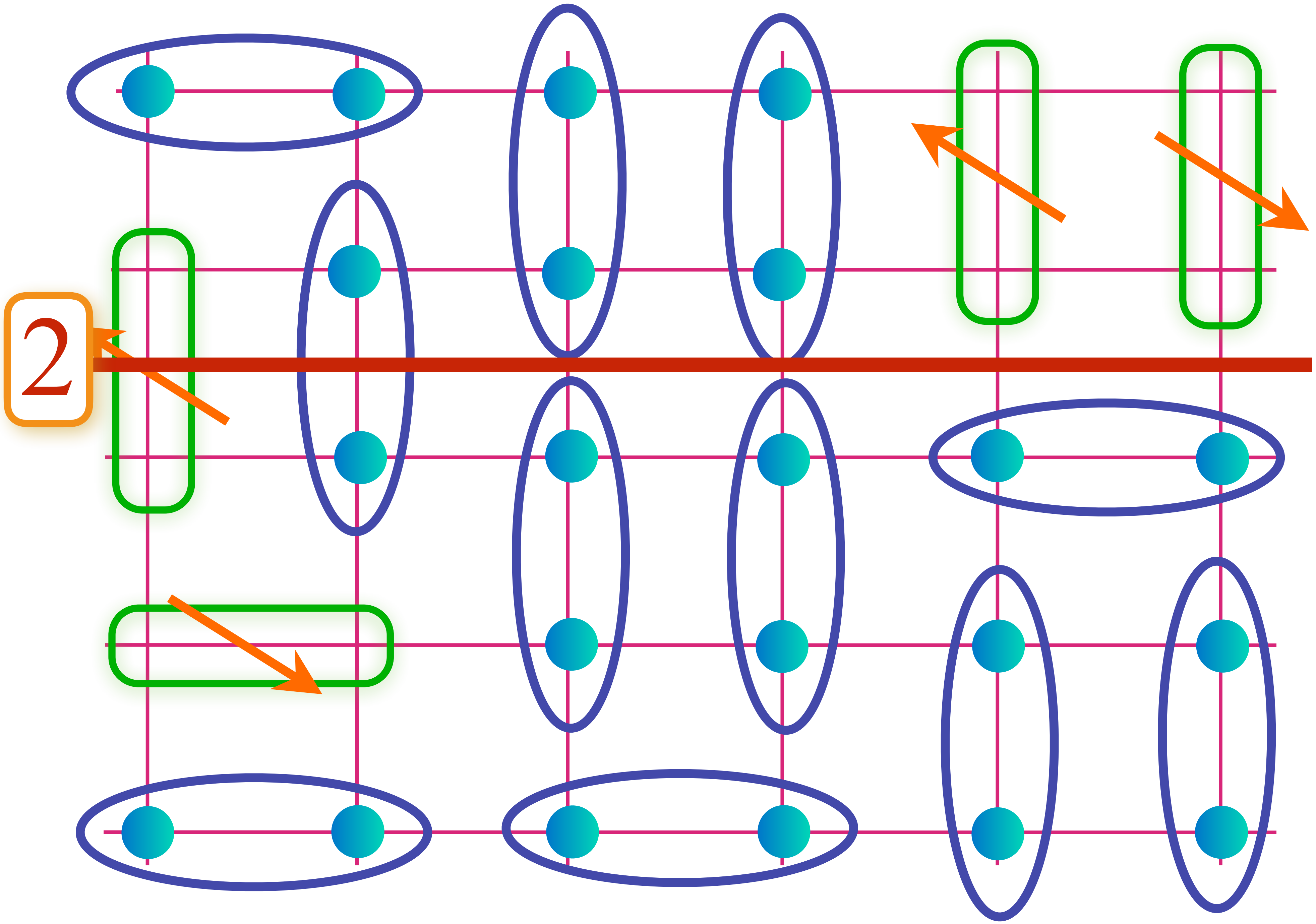
Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

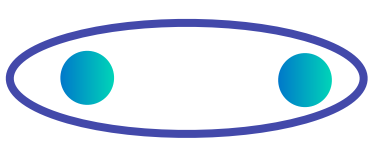
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

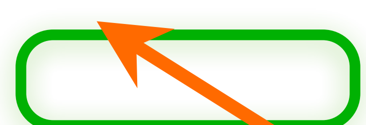
FL*



Place FL* on a torus:

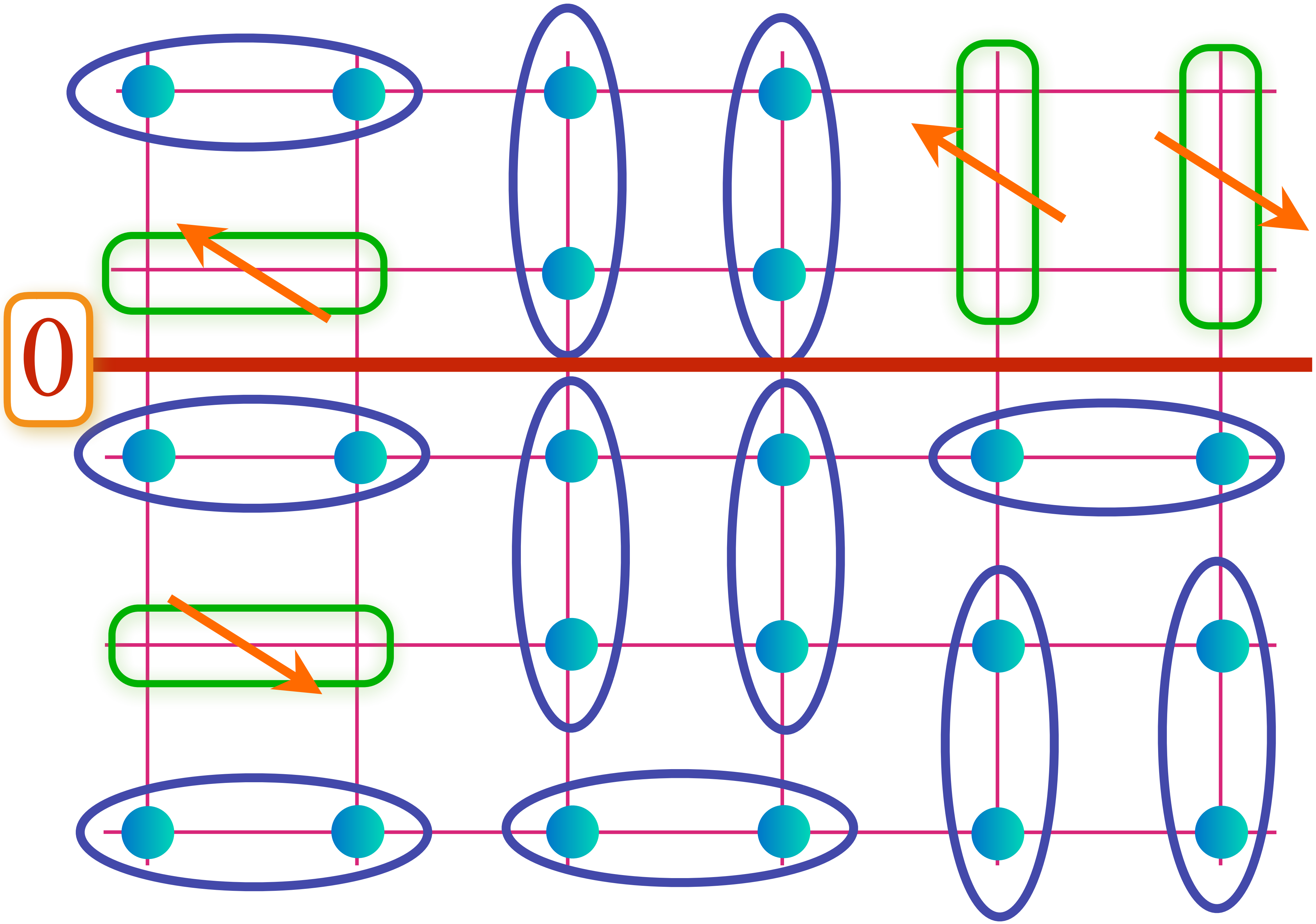
obtain "topological" states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL*



Place FL* on a torus:

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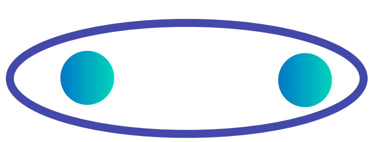
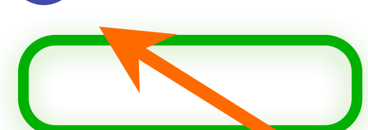


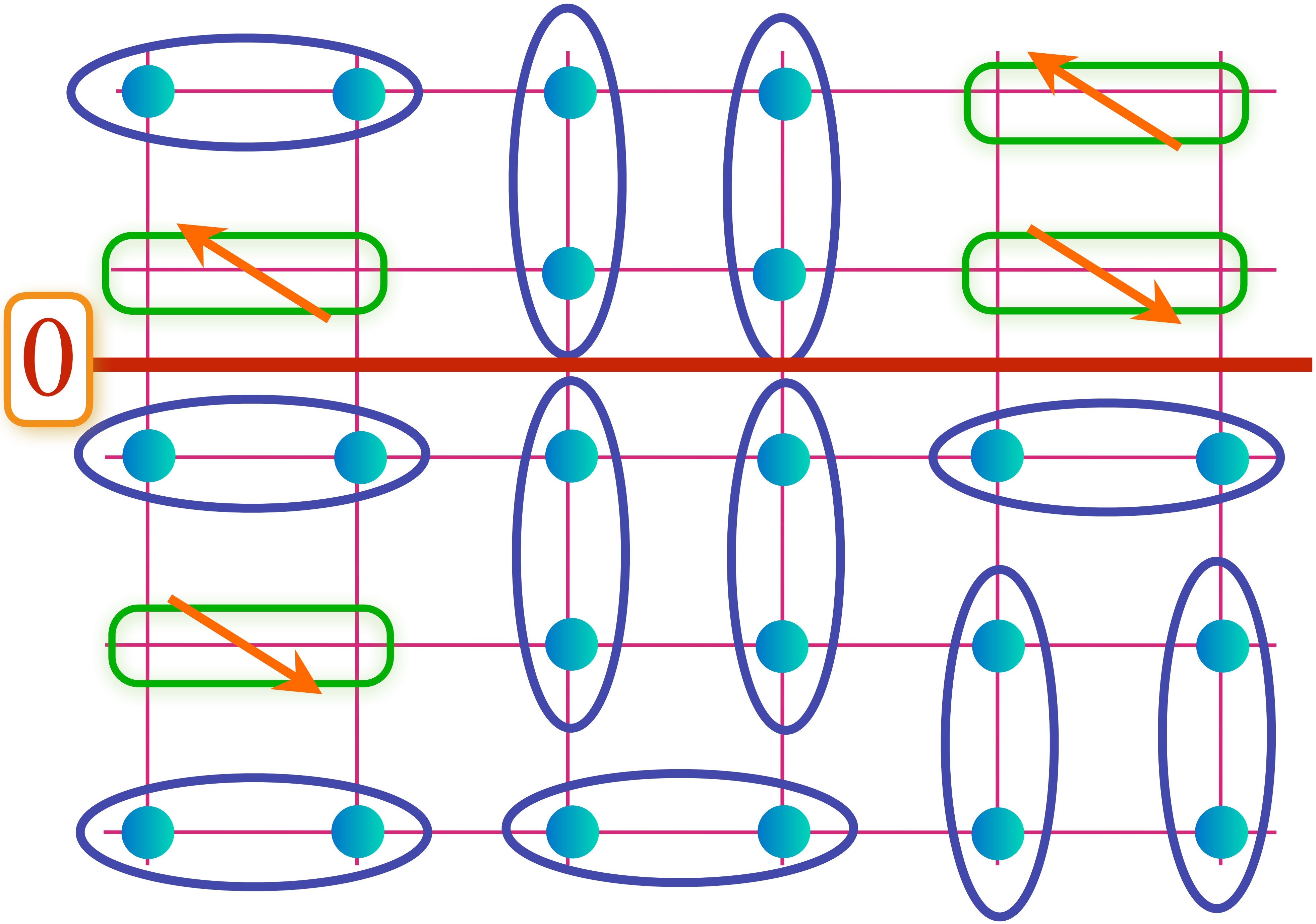
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

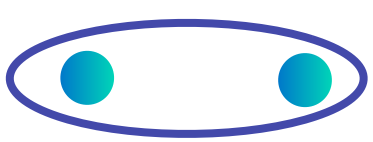
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL*

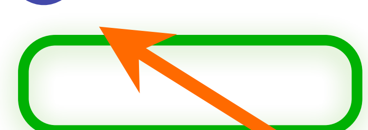


Place FL* on a torus:

obtain "topological" states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2



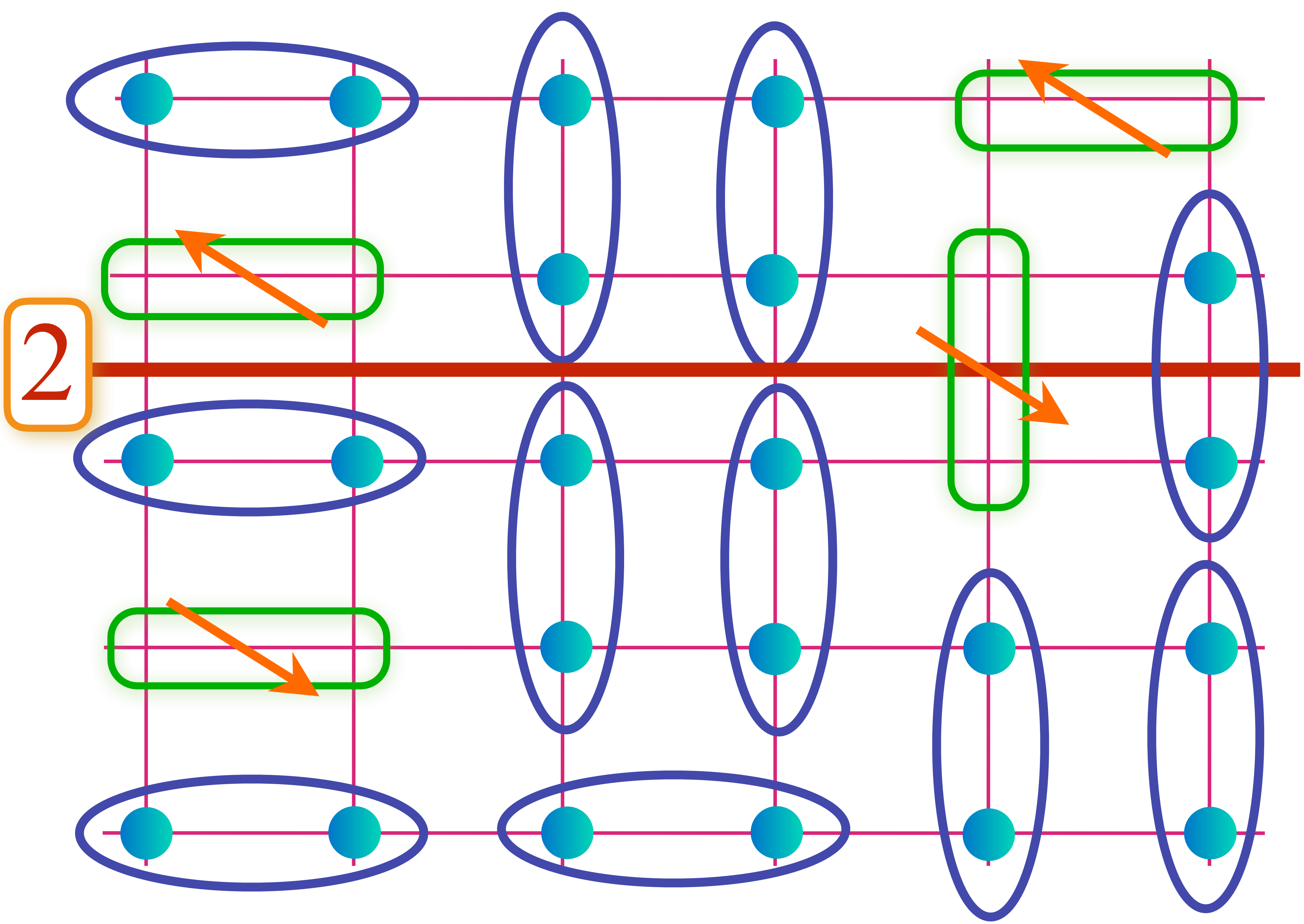
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$



$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

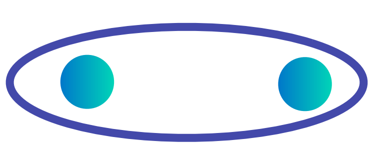
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

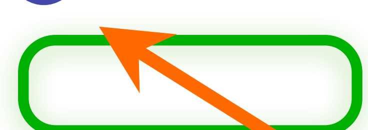
FL*



Place FL* on a torus:

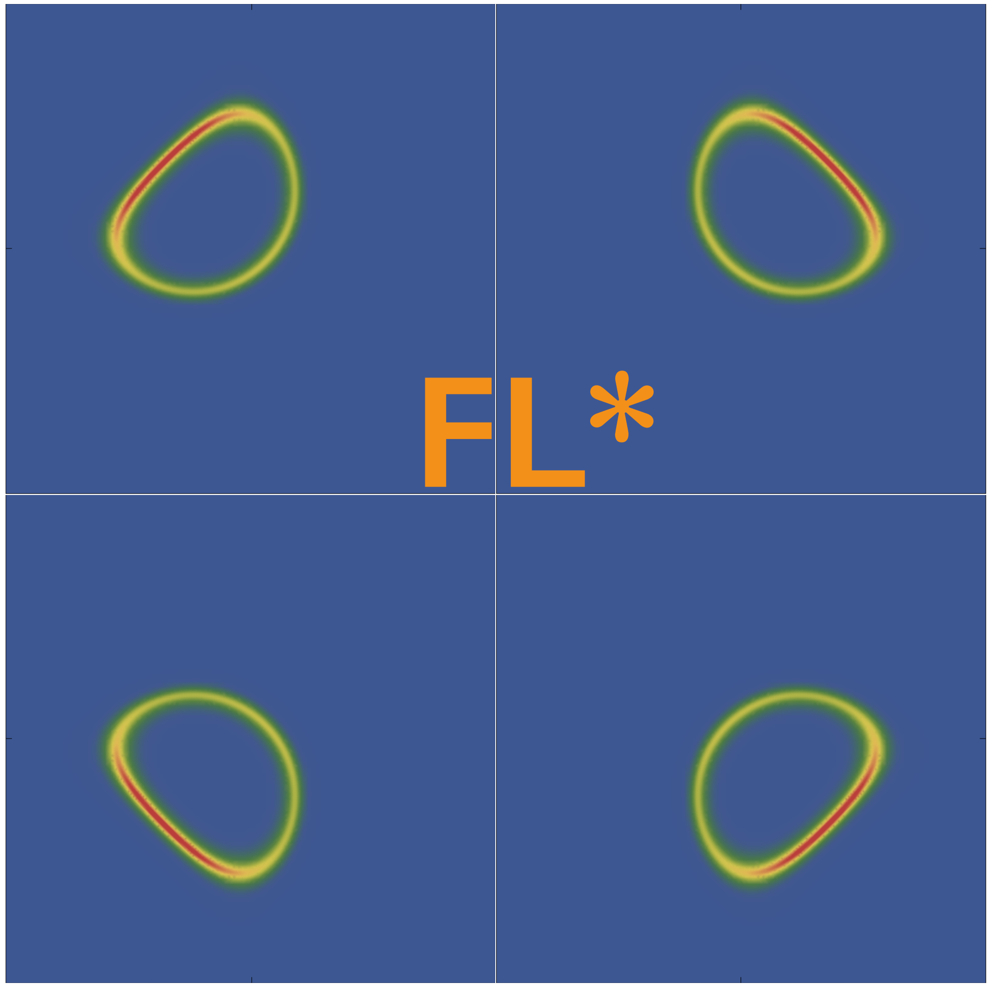
obtain "topological" states nearly degenerate with quasiparticle states: number of dimers crossing red line is conserved modulo 2

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

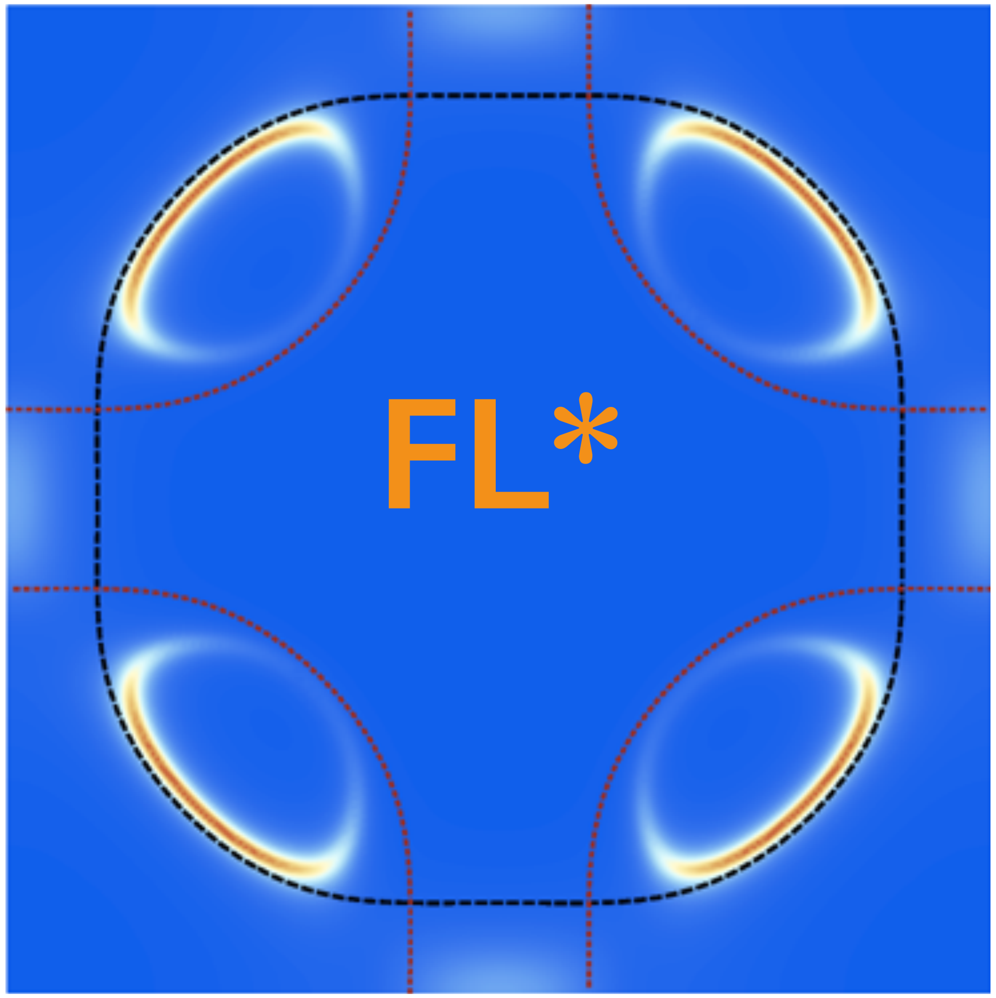
 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

Fermi surfaces in models of FL*



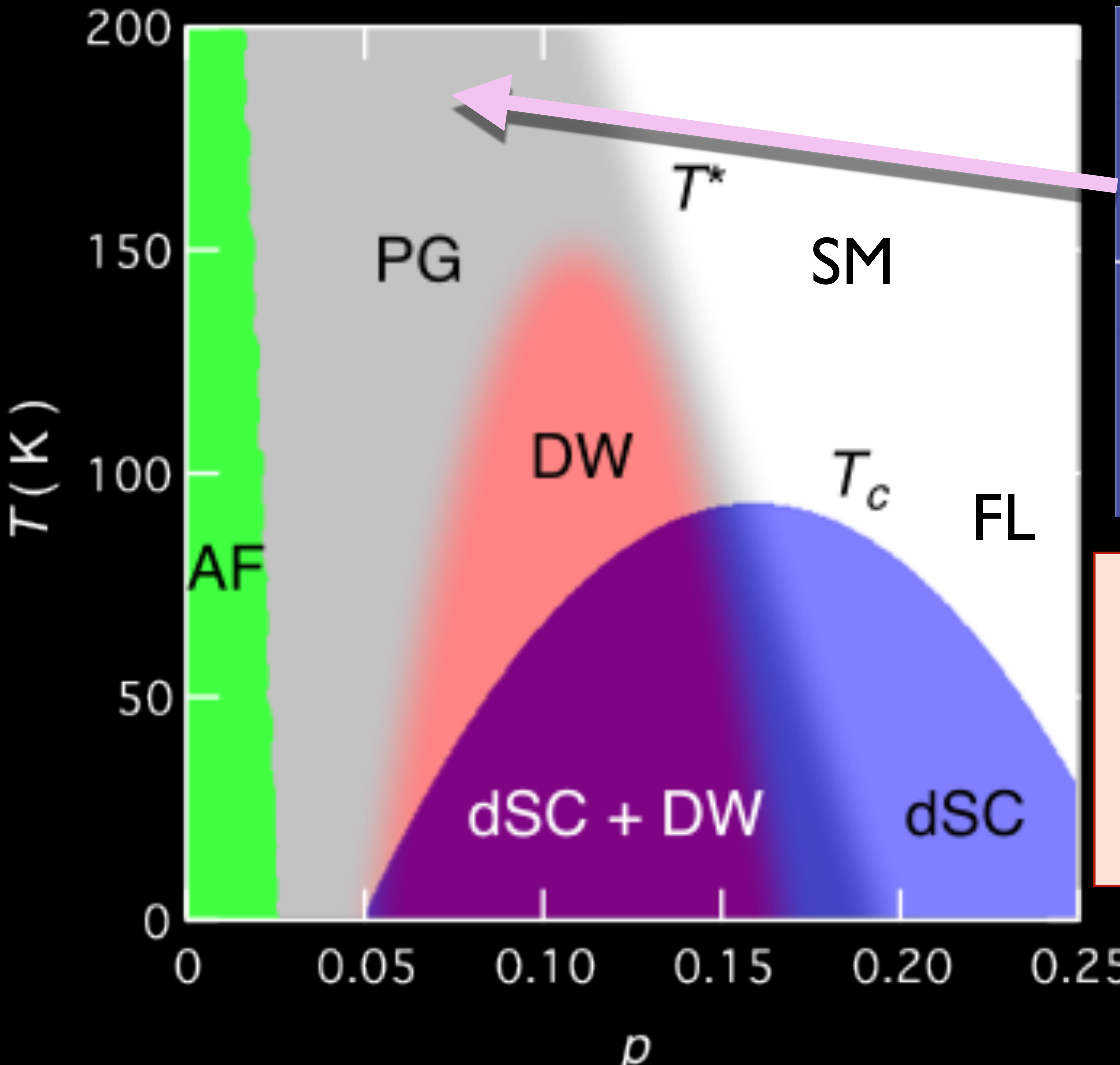
M. Punk, A. Allais, and S. Sachdev,
PNAS **112**, 9552 (2015)

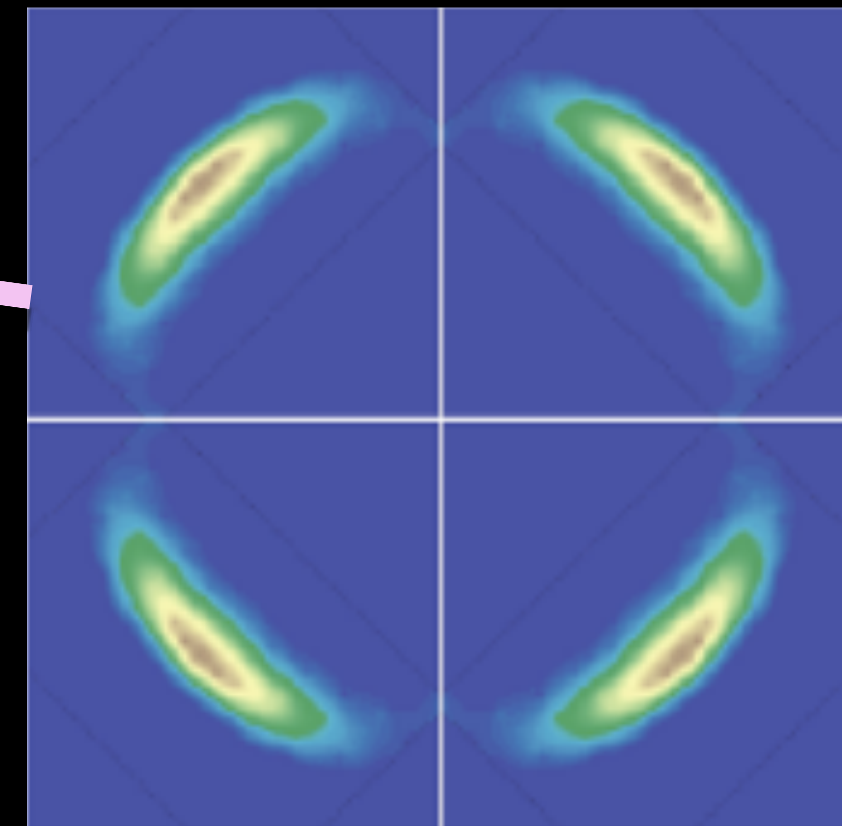


Y. Qi and S. Sachdev,
Phys. Rev. B **81**, 115129 (2010)

"Back side" of Fermi surface is suppressed for observables which change electron number in the square lattice

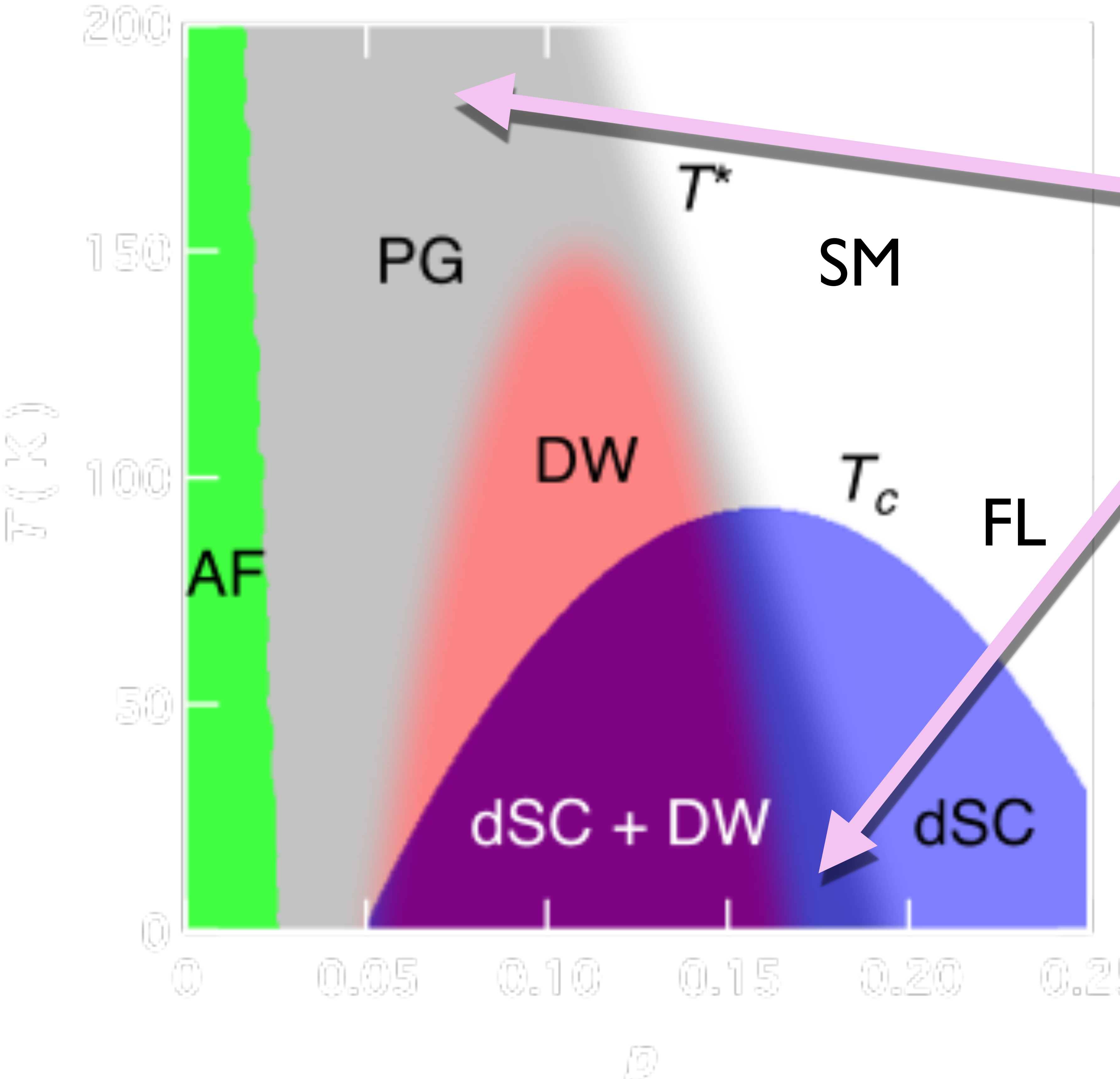
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

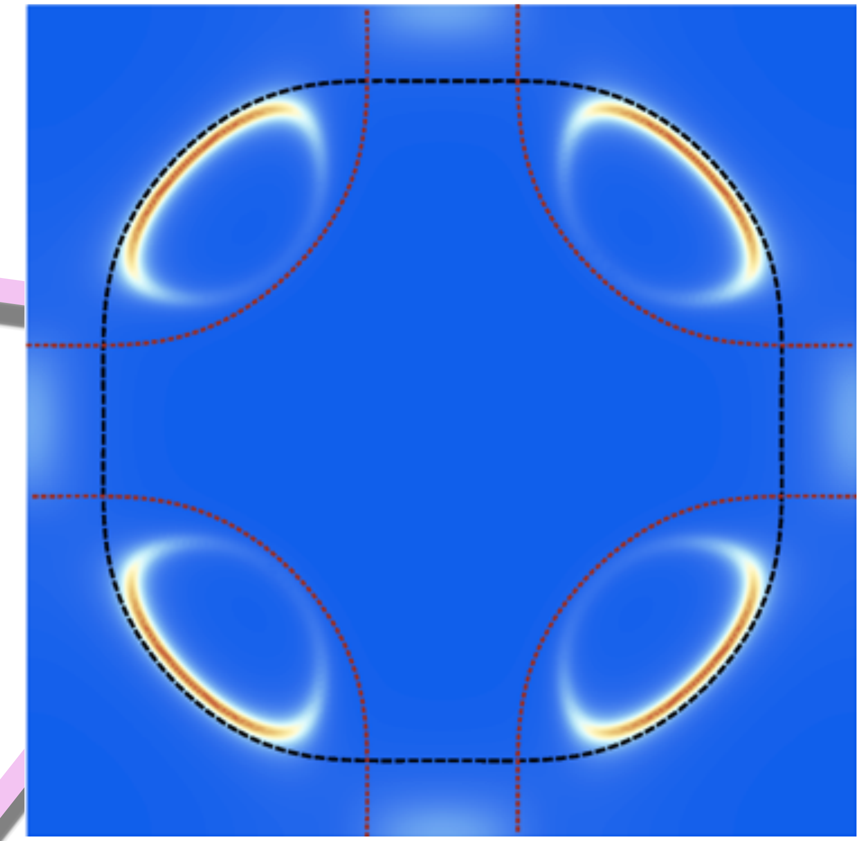




Pseudogap metal at low p

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)





A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size p

1. The insulating spin liquid and topological field theory

2. Topology and the size of the Fermi surface

3. Confinement transitions out of Z_2-FL*

- Condensation of bosonic excitations of Z_2-FL*
- Confinement via a deconfined critical point with a SU(2) gauge field

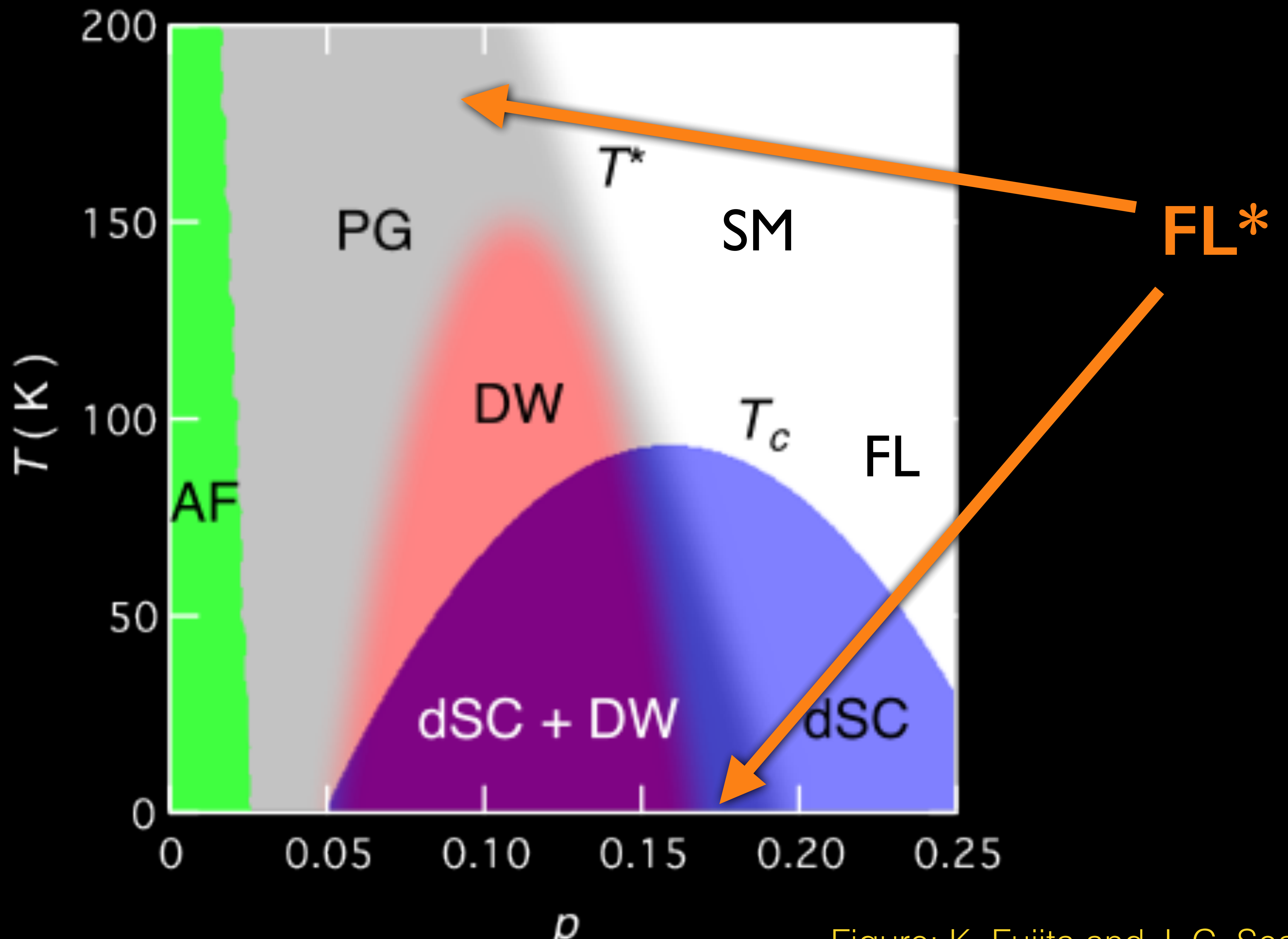
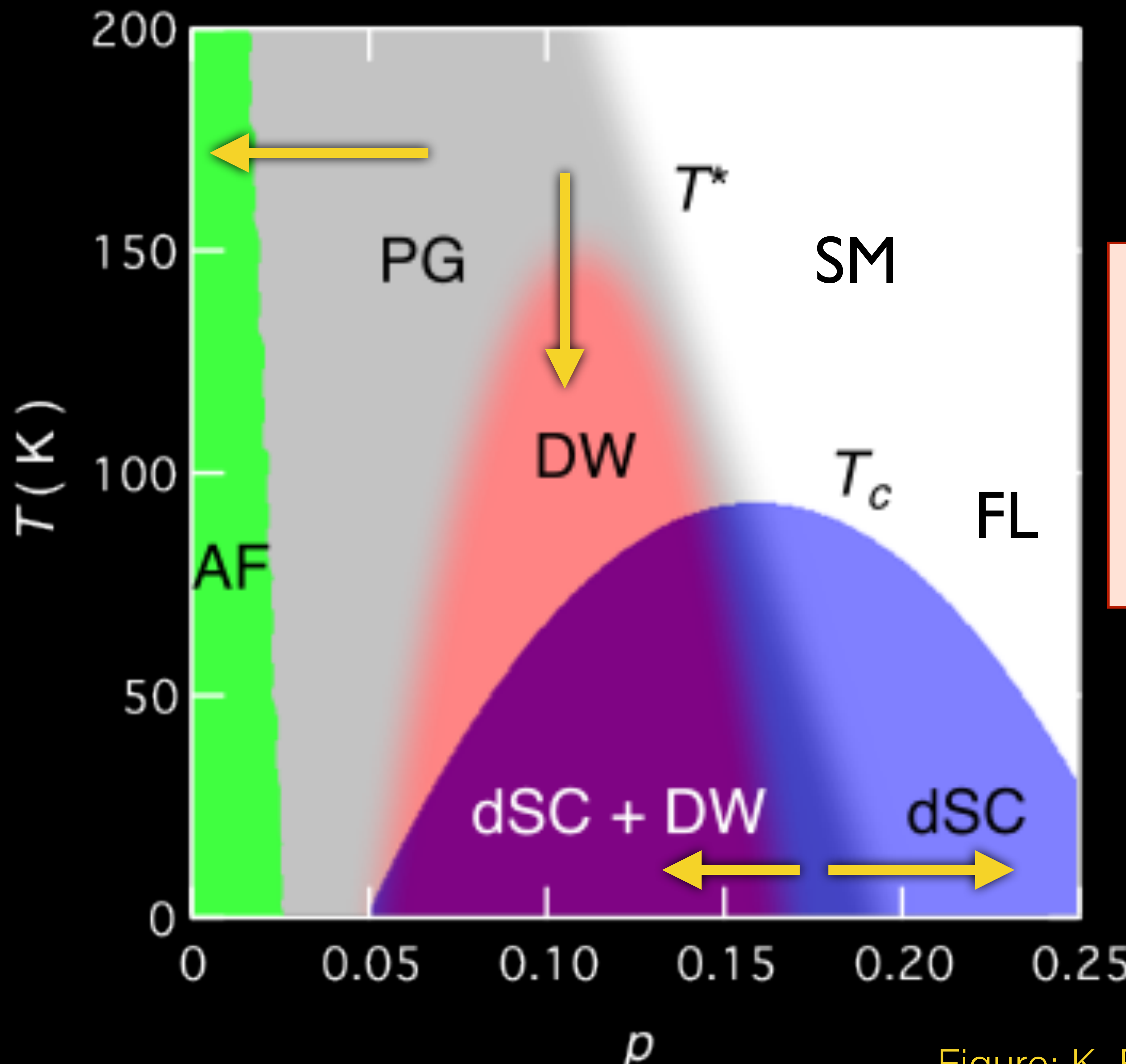


Figure: K. Fujita and J. C. Seamus Davis



Possible confinement transitions from Z_2-FL*

Figure: K. Fujita and J. C. Seamus Davis

Hilbert space sectors of a Z_2 spin liquid

	1	e	m	ϵ
S	0	$1/2$	0	$1/2$
Statistics	boson	boson	boson	fermion
Mutual semions	$-$	$m,\ \epsilon,\ m_c,\ \epsilon_c$	$e,\ \epsilon,\ e_c,\ \epsilon_c$	$e,\ m,\ e_c,\ m_c$
Q	0	0	0	0
Field operator	$-$	b	ϕ	f

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Statistics	boson	boson	boson	fermion
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Q	0	0	0	0
Field operator	–	b	ϕ	f

spinon,
Schwinger boson

Hilbert space sectors of a Z_2 spin liquid

	1	e	m	ϵ
S	0	1/2	0	1/2
Statistics	boson	boson	boson	fermion
Mutual semions	–	$m, \epsilon, m_c, \epsilon_c$	$e, \epsilon, e_c, \epsilon_c$	e, m, e_c, m_c
Q	0	0	0	0
Field operator	–	b	ϕ	f

spinon,
Schwinger fermion

Hilbert space sectors of a Z_2 spin liquid

	1	e	m	ϵ
S	0	$1/2$	0	$1/2$
Statistics	boson	boson	boson	fermion
Mutual semions	$-$	$m,\ \epsilon,\ m_c,\ \epsilon_c$	$e,\ \epsilon,\ e_c,\ \epsilon_c$	$e,\ m,\ e_c,\ m_c$
Q	0	0	0	0
Field operator	$-$	b	ϕ	f

vison

Additional sectors of Z_2-FL*

	1_c	e_c	m_c	ϵ_c
S	$1/2$	0	$1/2$	0
Statistics	fermion	fermion	fermion	boson
Mutual semions	$-$	$m, \epsilon, m_c, \epsilon_c$	$e, \epsilon, e_c, \epsilon_c$	e, m, e_c, m_c
Q	1	1	1	1
Field operator	c	$-$	$-$	B

Additional sectors of Z_2-FL*

	1_c	e_c	m_c	ϵ_c
S	$1/2$	0	$1/2$	0
Statistics	fermion	fermion	fermion	boson
Mutual semions	–	$m,\ \epsilon,\ m_c,\ \epsilon_c$	$e,\ \epsilon,\ e_c,\ \epsilon_c$	$e,\ m,\ e_c,\ m_c$
Q	1	1	1	1
Field operator	c	–	–	B

electron

Additional sectors of Z_2-FL*

	1_c	e_c	m_c	ϵ_c
S	$1/2$	0	$1/2$	0
Statistics	fermion	fermion	fermion	boson
Mutual semions	–	m, ϵ, m_c, ϵ_c	e, ϵ, e_c, ϵ_c	e, m, e_c, m_c
Q	1	1	1	1
Field operator	c	–	–	B

fermionic chargon

Additional sectors of Z_2-FL*

	1_c	e_c	m_c	ϵ_c
S	$1/2$	0	$1/2$	0
Statistics	fermion	fermion	fermion	boson
Mutual semions	—	$m,\ \epsilon,\ m_c,\ \epsilon_c$	$e,\ \epsilon,\ e_c,\ \epsilon_c$	$e,\ m,\ e_c,\ m_c$
Q	1	1	1	1
Field operator	c	—	—	B

bosonic chargon

Additional sectors of Z_2-FL*

	1_c	e_c	m_c	ϵ_c
S	$1/2$	0	$1/2$	0
Statistics	fermion	fermion	fermion	boson
Mutual semions	–	$m,\ \epsilon,\ m_c,\ \epsilon_c$	$e,\ \epsilon,\ e_c,\ \epsilon_c$	$e,\ m,\ e_c,\ m_c$
Q	1	1	1	1
Field operator	c	–	–	B

vison + electron

Hilbert space sectors of a Z_2 spin liquid

	1	e	m	ϵ
S	0	$1/2$	0	$1/2$
Statistics	boson	boson	boson	fermion
Mutual semions	–	$m, \epsilon, m_c, \epsilon_c$	$e, \epsilon, e_c, \epsilon_c$	e, m, e_c, m_c
Q	0	0	0	0
Field operator	–	b	ϕ	f

spinon, Schwinger boson: Condensation leads to incommensurate spin density wave order, and small Fermi surfaces

A.V. Chubukov, T. Senthil and S. Sachdev, PRL **72**, 2089 (1994)

Hilbert space sectors of a Z_2 spin liquid

	1	e	m	ϵ
S	0	$1/2$	0	$1/2$
Statistics	boson	boson	boson	fermion
Mutual semions	$-$	$m,\ \epsilon,\ m_c,\ \epsilon_c$	$e,\ \epsilon,\ e_c,\ \epsilon_c$	$e,\ m,\ e_c,\ m_c$
Q	0	0	0	0
ator	$-$	b	ϕ	f

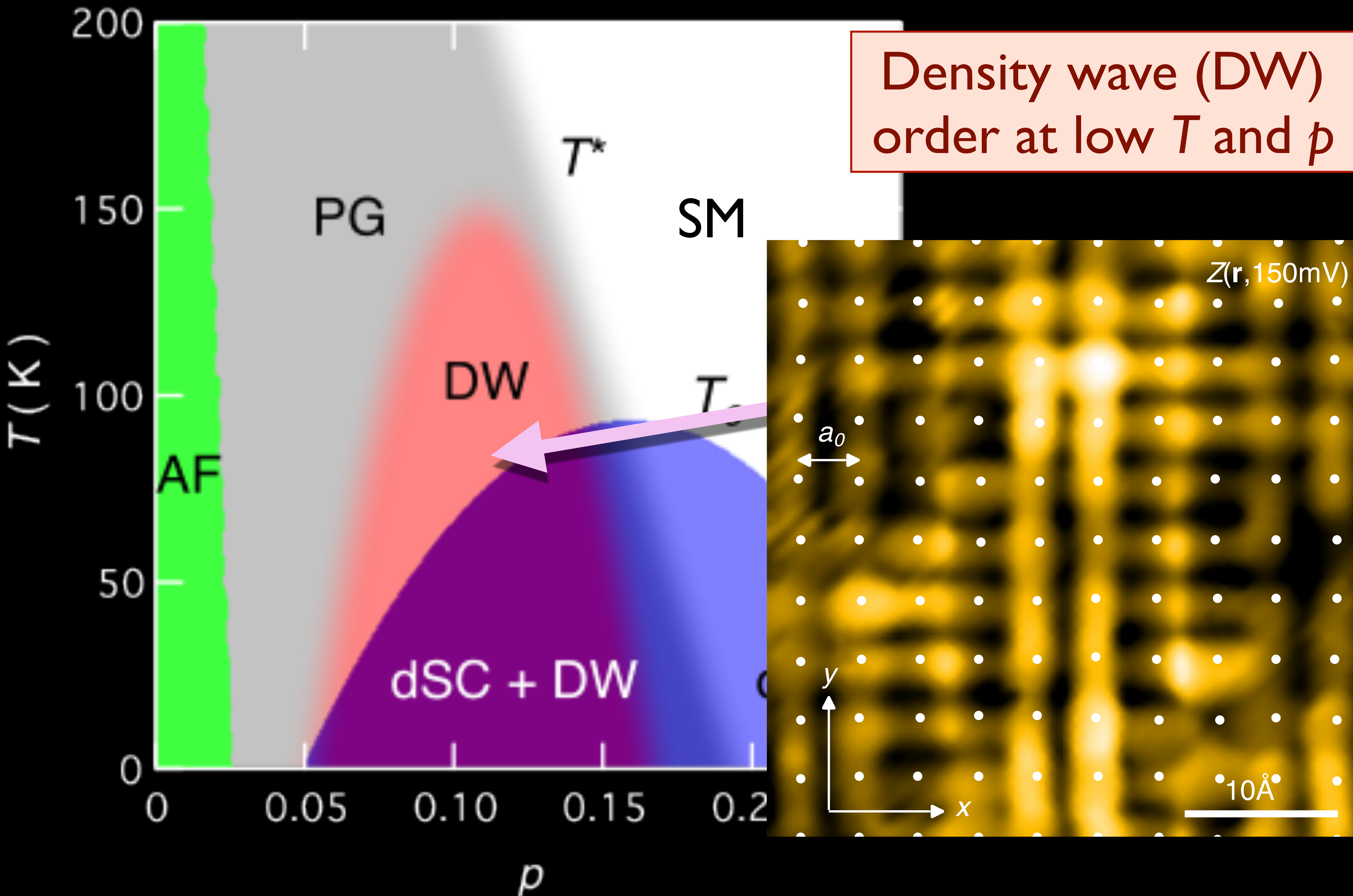
vison:
Condensation leads to
bond density wave order,
including cases with *d*-form factor

Aavishkar Patel
Talk Y27.00008, Friday, 12:39 PM

A.A. Patel, D. Chowdhury, A. Allais, and S. Sachdev, arXiv:1602.05954

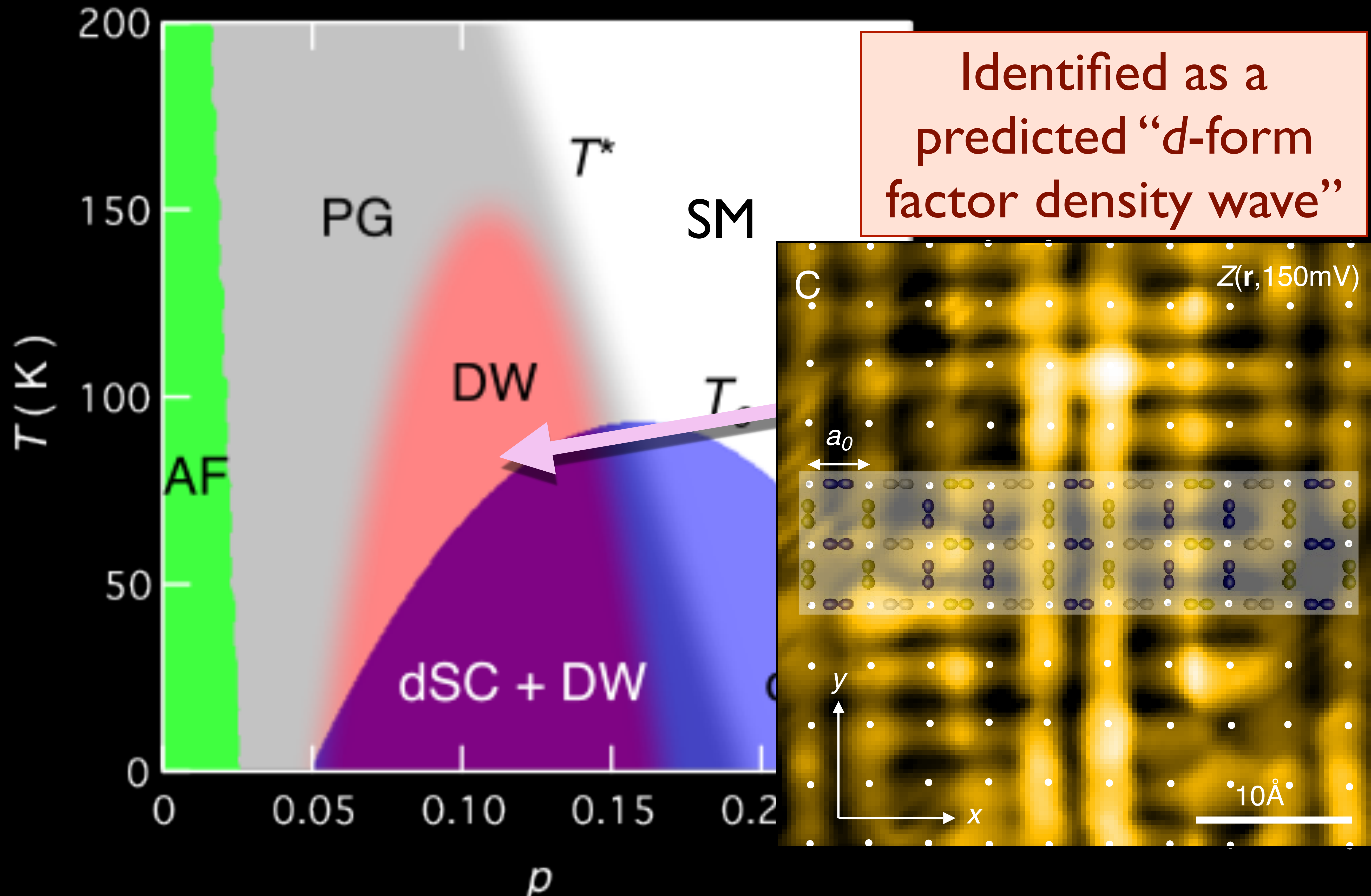
Y. Kohsaka *et al.*, Science **315**, 1380 (2007)
M. H. Hamidian *et al.*, Nature Physics **12**, 150 (2016)

Density wave (DW) order at low T and p

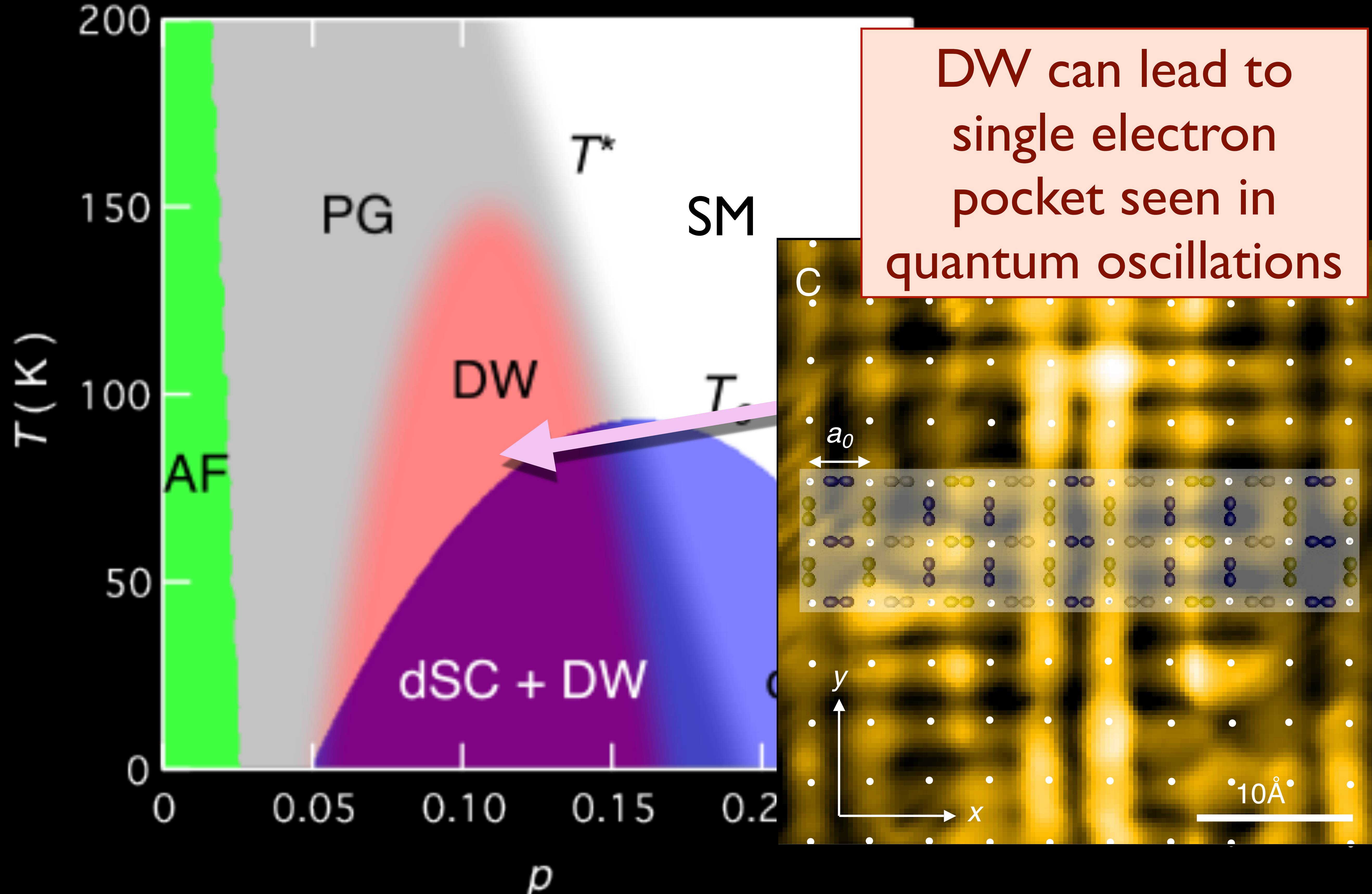


M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi,
H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)

Identified as a predicted "*d*-form factor density wave"



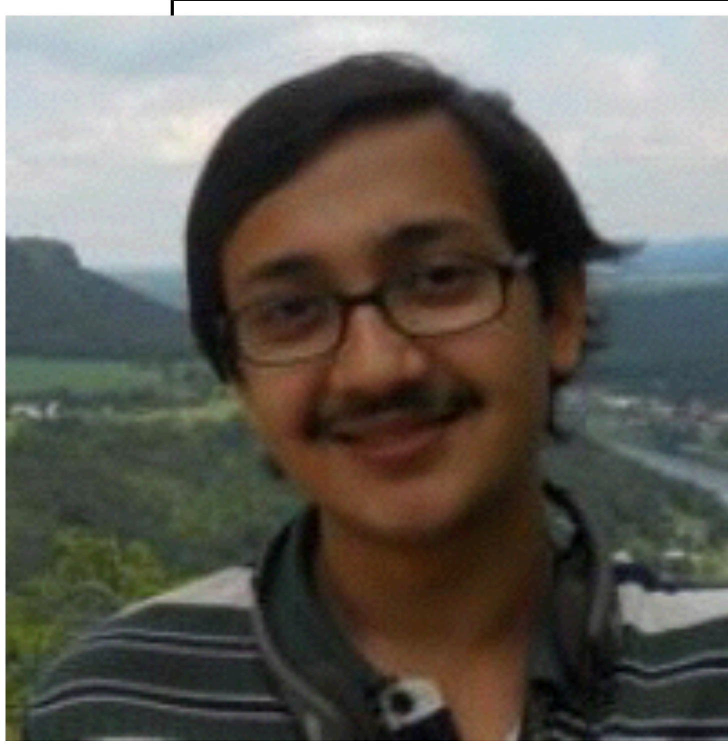
M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



DW can lead to single electron pocket seen in quantum oscillations

Additional sectors of Z_2-FL*

	1_c	e_c	m_c	ϵ_c
S	$1/2$	0	$1/2$	0
Statistics	fermion	fermion	fermion	boson
Mutual semions	–	$m,\ \epsilon,\ m_c,\ \epsilon_c$	$e,\ \epsilon,\ e_c,\ \epsilon_c$	$e,\ m,\ e_c,\ m_c$
Q	1	1	1	1
	c	–	–	B



bosonic chargon:
Condensation (usually)
leads to pair density wave order

Shubhayu Chatterjee
Talk H5.00004, Tuesday, 3:06 PM

S. Chatterjee, Y. Qi, S. Sachdev.
and J. Steinberg, arXiv:1603.03041

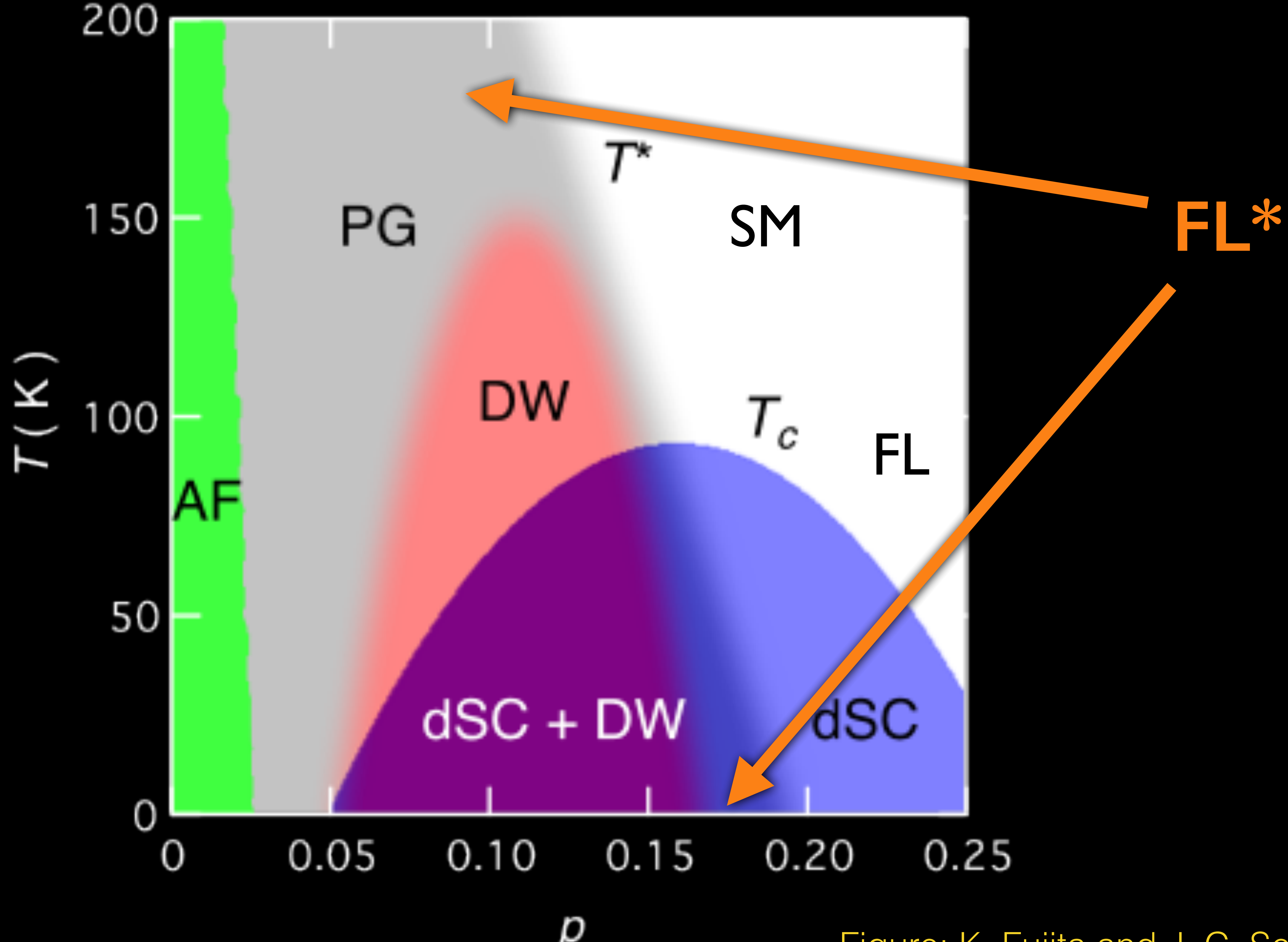
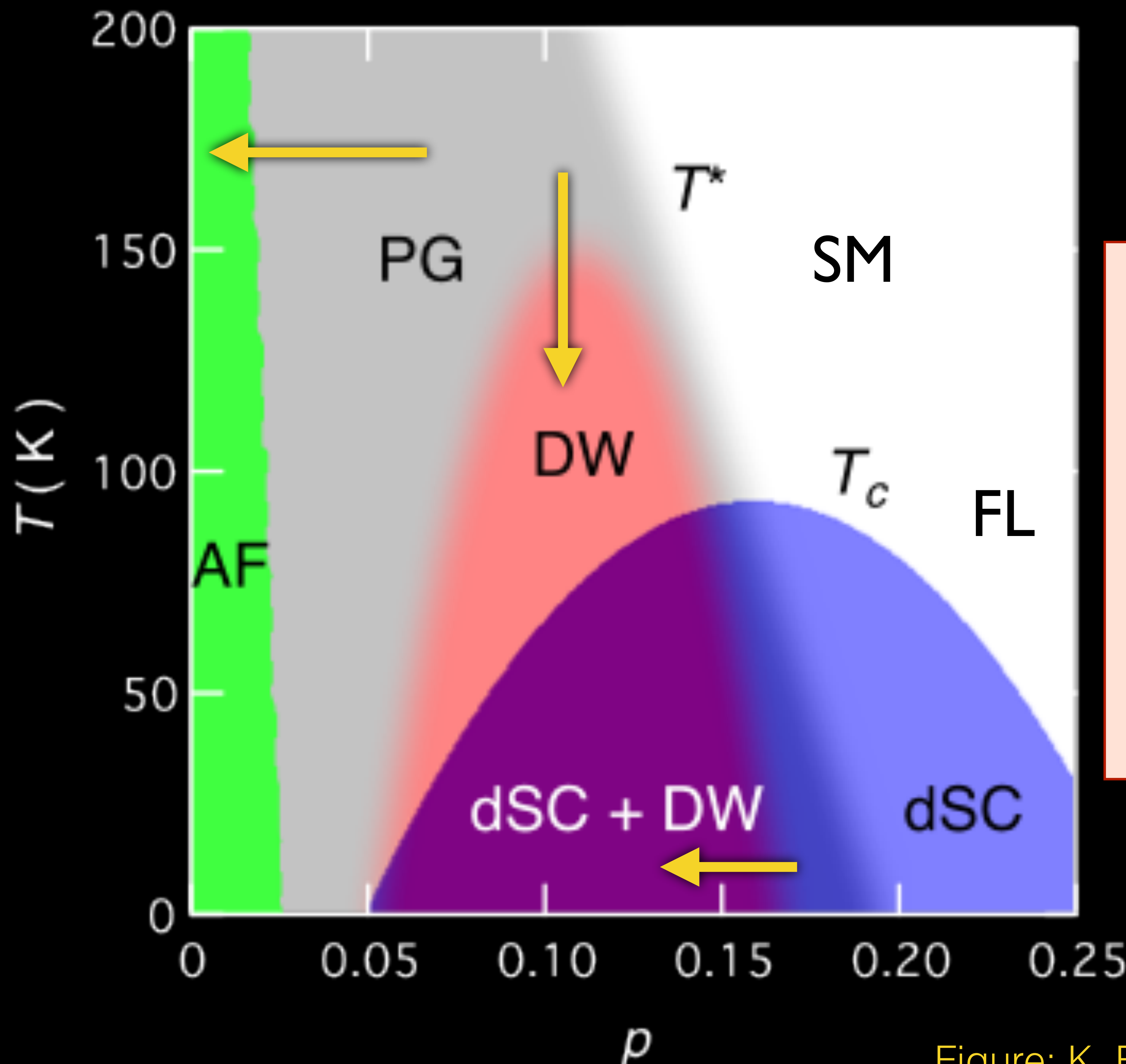
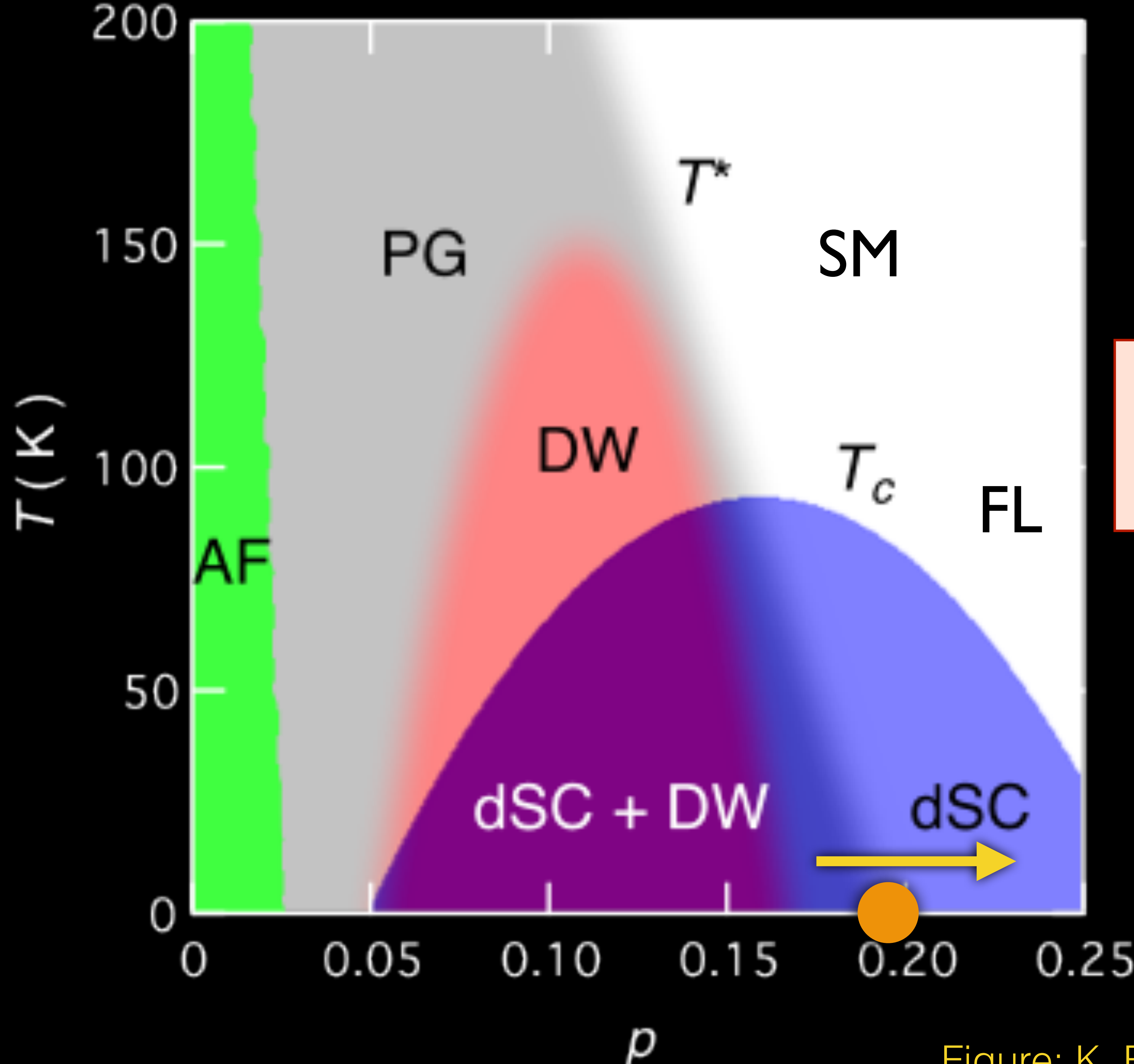


Figure: K. Fujita and J. C. Seamus Davis



3 confinement transitions from condensation of bosons in Z_2-FL*

Figure: K. Fujita and J. C. Seamus Davis



Transtion from Z_2-FL* to FL ?

Figure: K. Fujita and J. C. Seamus Davis

1. The insulating spin liquid and topological field theory

2. Topology and the size of the Fermi surface

3. Confinement transitions out of Z_2-FL*

- Condensation of bosonic excitations of Z_2-FL*
- Confinement via a deconfined critical point with a SU(2) gauge field

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

- Spinless fermion ψ (the fermionic chargon) transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface, and carries electromagnetic charge

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

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- A SU(2) fundamental scalar z_α (the bosonic spinon), carrying electron-spin and electromagnetically neutral.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
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- A SU(2) gauge boson.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

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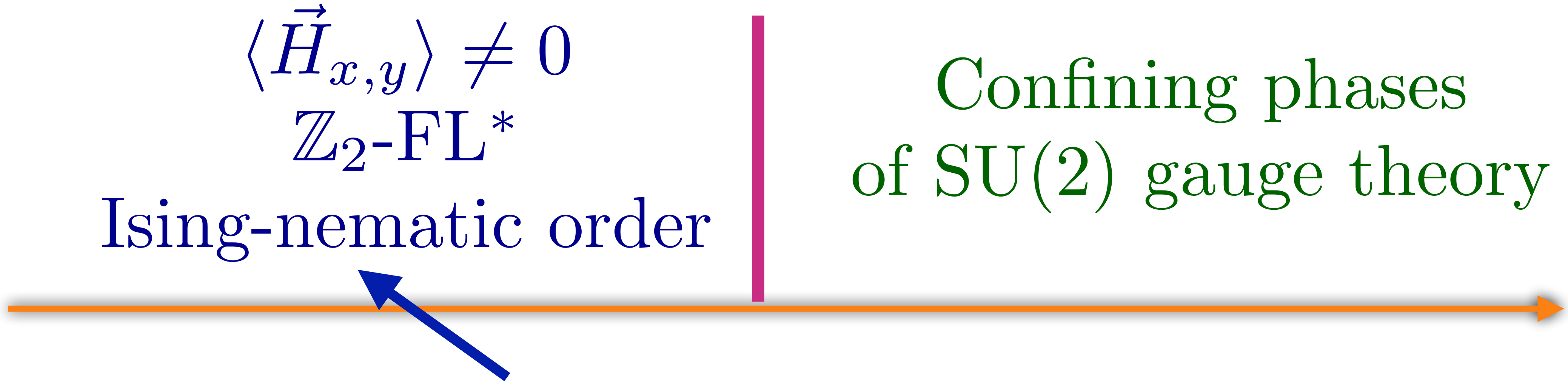
- A SU(2) gauge boson.

- Two complex Higgs fields, $\vec{H}_x$ and $\vec{H}_y$, transforming as gauge SU(2) adjoints, and carrying non-zero lattice momentum.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL



- The 'Higgs' phase with $\langle \vec{H}_{x,y} \rangle \neq 0$ has a deconfined $\mathbb{Z}_2$ gauge field, and realizes a $\mathbb{Z}_2$-FL*. This is the pseudogap metal and it has long-range Ising-nematic order.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

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- The 'confining' phase is a FL or a superconductor, and no Ising-nematic order. These are the conventional states at large p.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
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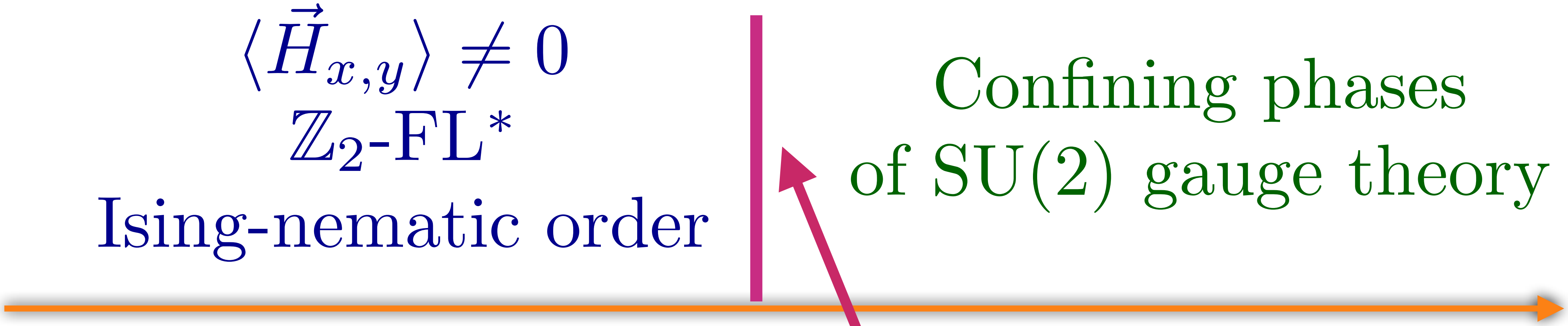
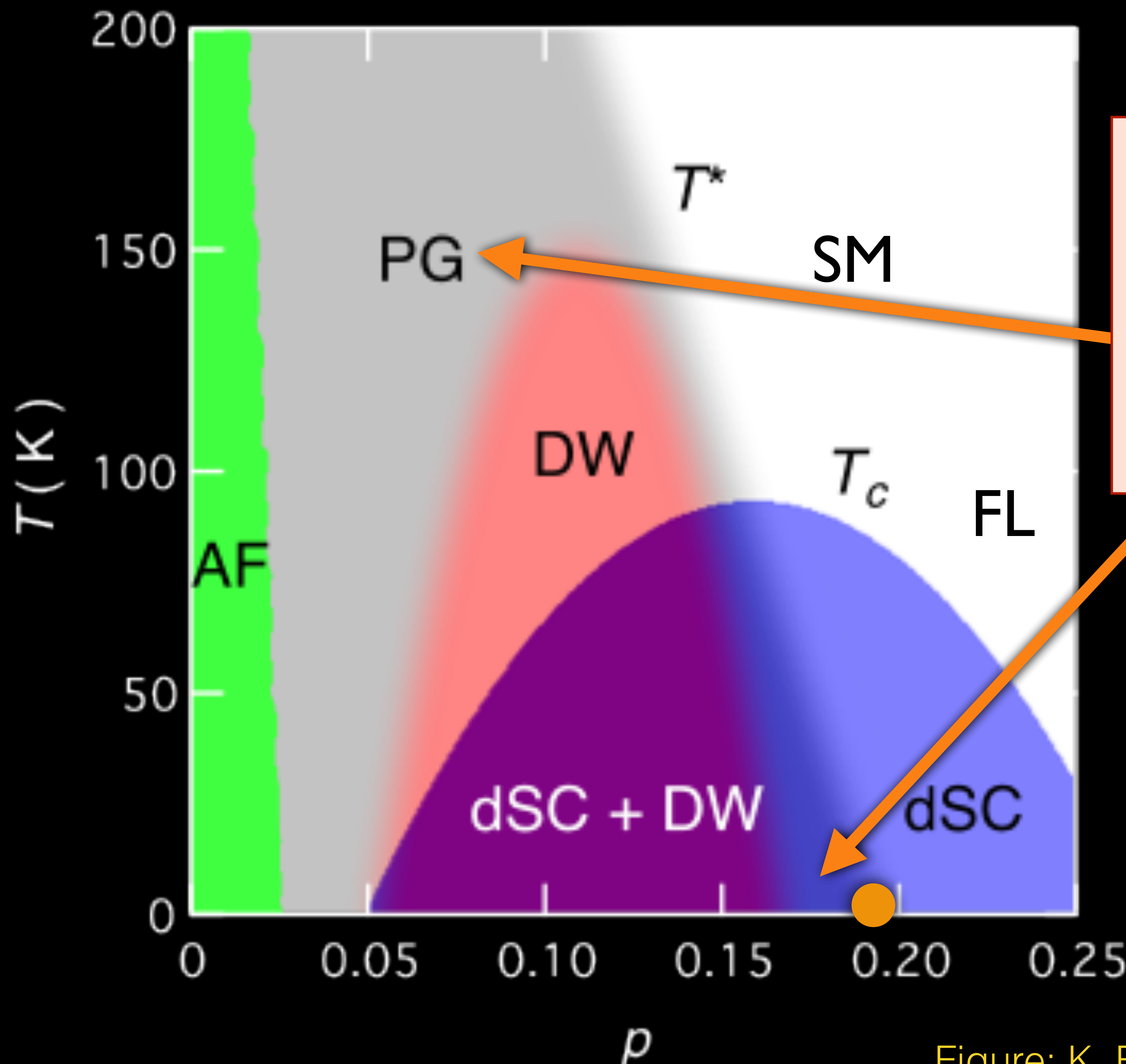


- The 'Higgs' phase with $\langle \vec{H}_{x,y} \rangle \neq 0$ has a deconfined $\mathbb{Z}_2$ gauge field, and realizes a $\mathbb{Z}_2$-FL*. This is the pseudogap metal and it has long-range Ising-nematic order.

- The 'confining' phase is a FL or a superconductor, and no Ising-nematic order. These are the conventional states at large p.

- When the Higgs potential is critical, we obtain a non-Fermi liquid of a ψ Fermi surface coupled to Landau-damped gauge bosons, and critical Landau-damped Higgs field. This is a candidate for describing the strange metal.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)



Pseudogap metal matches properties of Z_2-FL* phase

Figure: K. Fujita and J. C. Seamus Davis

FL*

We have described a metal with:

- A Fermi surface of electrons enclosing volume p, and not the Luttinger volume of $1+p$
- Topological character leads to emergent gauge fields and additional low energy quantum states on a torus not associated with quasiparticle excitations

There is a general and fundamental relationship between these two features.
Promising indications that such a metal describes the pseudogap of the cuprate superconductors