Quantum entanglement and the phases of matter

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PHYSICS



Foundations of quantum many body theory: I. Ground states <u>connected</u> adiabatically to independent electron states



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2. Boltzmann-Landau theory of quasiparticles



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Quantum entanglement:

EPR pair: Non-local correlations between quantum measurements due to superposition between many-electron states



Hydrogen molecule

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Modern phases of quantum matter:

I. Ground states <u>disconnected</u> from independent electron states: many-particle entanglement

2. Boltzmann-Landau theory of quasiparticles

Famous example:

The <u>fractional quantum Hall</u> effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge. Modern phases of quantum matter:

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Field theory: topological quantum field theory

<u>Modern phases of quantum matter:</u>

 I. Ground states <u>disconnected</u> from independent electron states: many-particle entanglement
2. No quasiparticles Quantum matter without quasiparticles: I. Ground states <u>disconnected</u> from independent electron states: many-particle entanglement 2. No quasiparticles

- Superfluid-insulator transition of ultracold bosonic atoms in an optical lattice
- Graphene
- Strange metals in high temperature superconductors
- Quark-gluon plasma
- Charged black hole horizons in anti-de Sitter space

Quantum matter without quasiparticles: I. Ground states <u>disconnected</u> from independent electron states: many-particle entanglement 2. No quasiparticles

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Field theory example: conformal field theory

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Superfluid-insulator transition



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002). Xibo Zhang, Chen-Lung Hung, Shih-Kuang Tung, and Cheng Chin, *Science* **335**, 1070 (2012)



Insulator (the vacuum) at large repulsion between bosons

$|\text{Ground state}\rangle = \prod_{i} b_{i}^{\dagger} |0\rangle$



























Superfluid at small repulsion between bosons





 $\Psi \rightarrow$ a complex field representing the Bose-Einstein condensate of the superfluid















K. Damle and S. Sachdev, PRB 56, 8714 (1997); S. Sachdev, Quantum Phase Transitions, Cambridge (1999)



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Philip Kim



Kin Chung Fong



Jesse Crossno



Andrew Lucas

Graphene



 k_y $\blacktriangleright k_x$

Same "Hubbard" model as for ultracold atoms, but for electrons on the honeycomb lattice





U/t


Graphene



 k_y $\blacktriangleright k_x$

Same "Hubbard" model as for ultracold atoms, but for electrons on the honeycomb lattice





Electron Fermi surface

Graphene



Hole Fermi surface Electron Fermi surface





Graphene



D. E. Sheehy and J. Schmalian, PRL 99, 226803 (2007)
M. Müller, L. Fritz, and S. Sachdev, PRB 78, 115406 (2008)
M. Müller and S. Sachdev, PRB 78, 115419 (2008)





<u>Fermi liquids</u>: quasiparticles moving ballistically between impurity (red circles) scattering events



<u>Fermi liquids</u>: quasiparticles moving ballistically between impurity (red circles) scattering events



Strange metals: electrons scatter frequently off each other, so there is no regime of ballistic quasiparticle motion. The electron "liquid" then "flows" around impurities

Thermal and electrical conductivity with quasiparticles

► Wiedemann-Franz law in a Fermi liquid:





G. S. Kumar, G. Prasad, and R.O. Pohl, J. Mat. Sci. 28, 4261 (1993)



Transport in Strange Metals

For a strange metal with a "relativistic" Hamiltonian, hydrodynamic, holographic, and memory function methods yield Lorentz ratio $L = \kappa/(T\sigma)$ $=\frac{v_F^2 \mathcal{H} \tau_{\rm imp}}{T^2 \sigma_Q} \frac{1}{\left(1+e^2 v_F^2 \mathcal{Q}^2 \tau_{\rm imp}/(\mathcal{H} \sigma_Q)\right)^2}$ $\mathcal{Q} \rightarrow$ electron density; $\mathcal{H} \rightarrow$ enthalpy density $\sigma_Q \rightarrow$ quantum critical conductivity $\tau_{\rm imp} \rightarrow$ momentum relaxation time from impurities. Note that for $\mathcal{Q} \neq 0, L \to 0$ as $\tau_{\rm imp} \to \infty$, while for $\mathcal{Q} = 0, L \to \infty$ as $\tau_{\rm imp} \to \infty$.

S. A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, PRB **76**, 144502 (2007) M. Müller and S. Sachdev, PRB **78**, 115419 (2008)











Red dots: data Blue line: value for $L = L_0$



Red dots: data Blue line: value for $L = L_0$



Red dots: data Blue line: value for $L = L_0$



Strange metal in graphene





Strange metal in graphene







Strange metal in graphene

Negative local resistance due to viscous electron backflow in graphene



L. Levitov and G. Falkovich, arXiv:1508.00836, Nature Physics online

Strange metal in graphene Science 351, 1055 (2016)

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin¹, I. Torre^{2,3}, R. Krishna Kumar^{1,4}, M. Ben Shalom^{1,5}, A. Tomadin⁶, A. Principi⁷, G. H. Auton⁵, E. Khestanova^{1,5}, K. S. Novoselov⁵, I. V. Grigorieva¹, L. A. Ponomarenko^{1,4}, A. K. Geim¹, M. Polini^{3,6}





Graphene:"a metal that behaves like water"



Quantum matter without quasiparticles:

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Proposed a SU(2) gauge theory for transtion for quantum-criticality at optimal doping, as the origin of strange metal (SM) behavior at higher T

Quark-gluon plasma



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G.Vidal, Phys. Rev. Lett. 99, 220405 (2007)







 $J_{ij;k\ell}$ are independent random variables with $\overline{J_{ij;k\ell}} = 0$ and $\overline{|J_{ij;k\ell}|^2} = J^2$ $N \to \infty$ yields critical strange metal.

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)



A fermion can move only by entangling with another fermion: the Hamiltonian has "nothing but entanglement".

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)

Infinite-range strange metals

Local fermion density of states

$$\rho(\omega) = -\operatorname{Im} G(\omega) \sim \begin{cases} \omega^{-1/2} , \, \omega > 0 \\ e^{-2\pi\mathcal{E}} |\omega|^{-1/2}, \, \omega < 0. \end{cases}$$

 ${\mathcal E}$ encodes the particle-hole asymmetry

While \mathcal{E} determines the *low* energy spectrum, it is determined by the *total* fermion density \mathcal{Q} :

$$\mathcal{Q} = \frac{1}{4} (3 - \tanh(2\pi\mathcal{E})) - \frac{1}{\pi} \tan^{-1} \left(e^{2\pi\mathcal{E}} \right).$$

Analog of the relationship between \mathcal{Q} and k_F in a Fermi liquid.

S. Sachdev and J.Ye, Phys. Rev. Lett. **70**, 3339 (1993) A. Georges, O. Parcollet, and S. Sachdev Phys. Rev. B **63**, 134406 (2001)





$$\mathcal{Q} = \frac{1}{N} \sum_{i} \left\langle c_i^{\dagger} c_i \right\rangle.$$

Local fermion density of states

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Known 'equation of state' determines \mathcal{E} as a function of \mathcal{Q}

Microscopic zero temperature
entropy density,
$$S$$
, obeys
 $\frac{\partial S}{\partial Q} = 2\pi \mathcal{E}$
Holographic gravity theory

Start with simplest theory of Einstein gravity and Maxwell electromagnetism

$$\mathcal{S}_{EM} = \int d^{d+2}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(\mathcal{R} + \frac{d(d+1)}{L^2} - \frac{R^2}{g_F^2} F^2 \right) \right]$$

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Solve equations of motion in the the presence of a d-dimensional flat boundary with charge density Q

Electric flux

AdS-Reissner-Nordstrom

Quantum matter on the boundary with a variable charge density Q of a global U(1) symmetry.

A. Chamblin, R. Emparan, C.V. Johnson, and R. C. Myers, PRD 60, 064018 (1999)

Charged black branes



Realizes a strange metal: a state with an unbroken global U(1) symmetry with a continuously variable charge density, Q, at T = 0 which does not have any quasiparticle excitations.



 ${\mathcal E}$ encodes the particle-hole asymmetry

T. Faulkner, Hong Liu, J. McGreevy, and D. Vegh, PRD 83, 125002 (2011)





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$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell} c_i^{\dagger} c_j^{\dagger} c_k c_{\ell}$$



$$\mathcal{Q} = \frac{1}{N} \sum_{i} \left\langle c_i^{\dagger} c_i \right\rangle.$$

Local fermion density of states

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Known 'equation of state' determines \mathcal{E} as a function of \mathcal{Q}

Microscopic zero temperature entropy density, S, obeys $\frac{\partial S}{\partial Q} = 2\pi \mathcal{E}$ Einstein-Maxwell theory + cosmological constant

Horizon area \mathcal{A}_h ; $\mathrm{AdS}_2 \times R^d$ $ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2$ Gauge field: $A = (\mathcal{E}/\zeta)dt$

 $\zeta = \infty$

Boundary area \mathcal{A}_b ; charge density \mathcal{Q}

 \vec{x}

 $\mathcal{L} = \overline{\psi} \Gamma^{\alpha} D_{\alpha} \psi + m \overline{\psi} \psi$ Local fermion density of states

$$p(\omega) \sim \begin{cases} \omega^{-1/2}, \ \omega > 0\\ e^{-2\pi \mathcal{E}} |\omega|^{-1/2}, \ \omega < 0. \end{cases}$$

'Equation of state' relating \mathcal{E} and \mathcal{Q} depends upon the geometry of spacetime far from the AdS₂

Black hole thermodynamics (classical general relativity) yields $\frac{\partial S_{\rm BH}}{\partial Q} = 2\pi \mathcal{E}$

A. Sen hep-th/0506177; S. Sachdev PRX 5, 041025 (2015)

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell} c_i^{\dagger} c_j^{\dagger} c_k c_\ell$$

Einstein-Maxwell theory

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;k\ell} c_i^{\dagger} c_j^{\dagger} c_k c_\ell$$

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S. Sachdev, PRL 105, 151602 (2010); PRX 5, 041025 (2015)

A bound on quantum chaos:

• The "Lyapunov exponent" for chaos, λ_L , is given by out-of-time-order correlators, and for quantum systems near equilibrium, it obeys the bound $\lambda_L \leq 2\pi k_B T/\hbar$.

J. Maldacena, S. H. Shenker and D. Stanford, arXiv: 1503.01409

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S. H. Shenker and D. Stanford, arXiv: 1306.0622

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S. H. Shenker and D. Stanford, arXiv:1306.0622

• The bound is also saturated by the SYK model

A. Kitaev, unpublished J. Polchinski and V. Rosenhaus, arXiv:1601.06768

Entangled quantum matter without quasiparticles

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Entangled quantum matter without quasiparticles

- No quasiparticle excitations
- Shortest possible "collision time", or more precisely, fastest possible local equilibration time $\sim \frac{\hbar}{k_B T}$
- Continuously variable density, Q(conformal field theories are usually at fixed density, Q = 0)
- Theory built from hydrodynamics/holography /memory-functions/strong-coupled-field-theory
- Exciting experimental realization in graphene.
- Future work: detection of hydrodynamic flow in other strange metals