The novel metallic states of the cuprates: “topological” Fermi liquids (FL*) and strange metals

PRL Conference on Condensed Matter Physics
Physical Research Laboratory, Ahmedabad
April 11, 2016

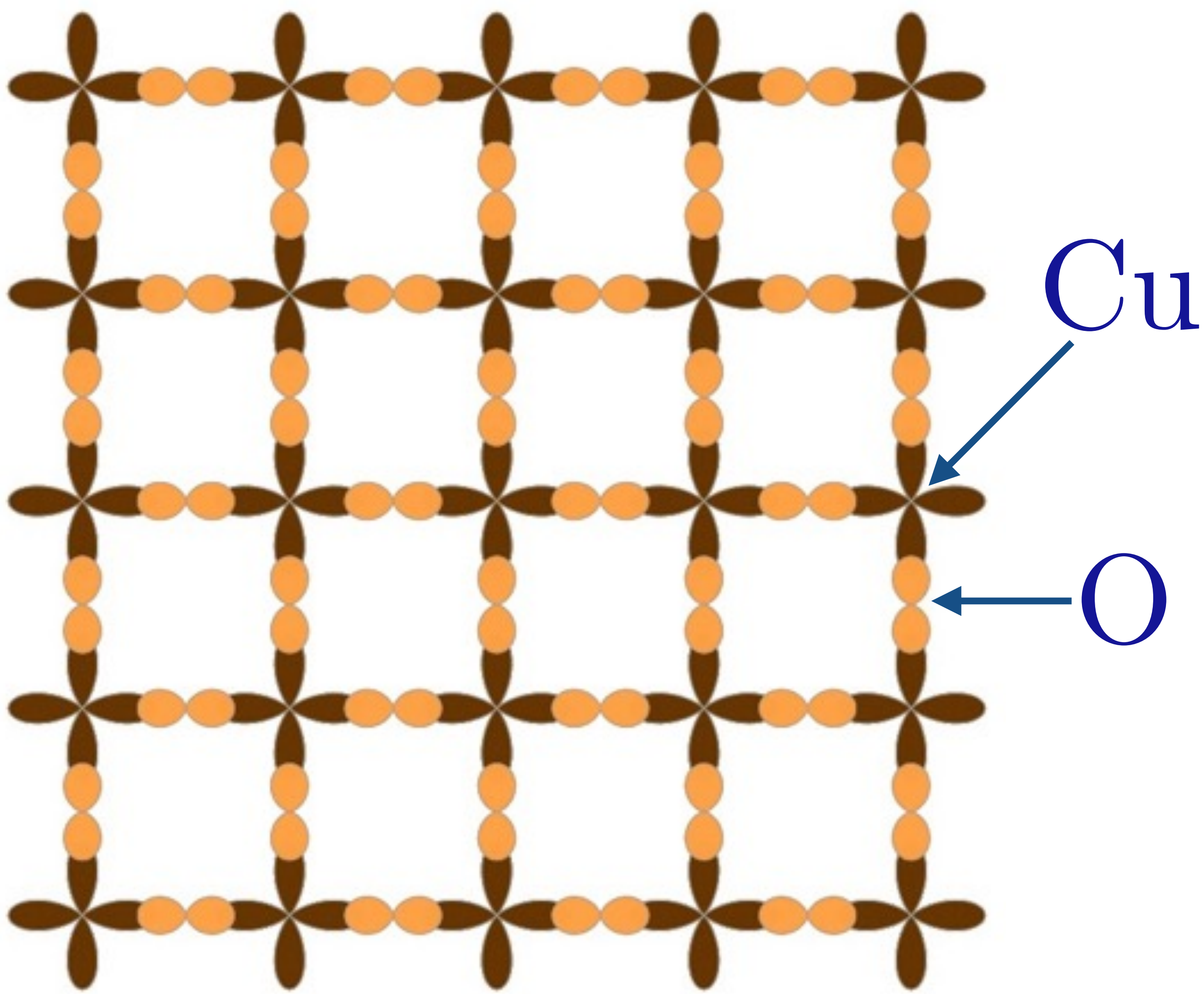
Subir Sachdev

Talk online: sachdev.physics.harvard.edu

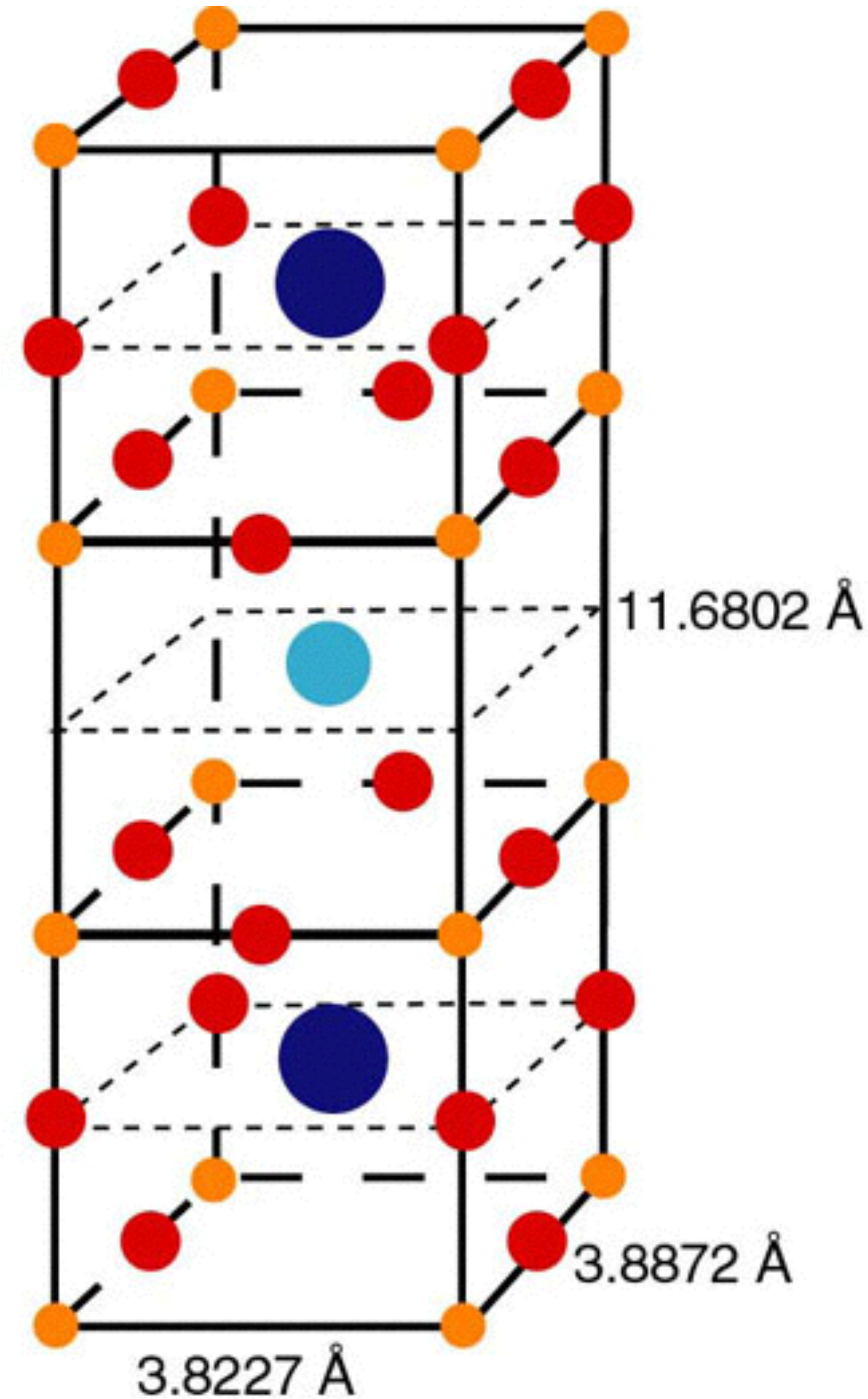
PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

PHYSICS HARVARD

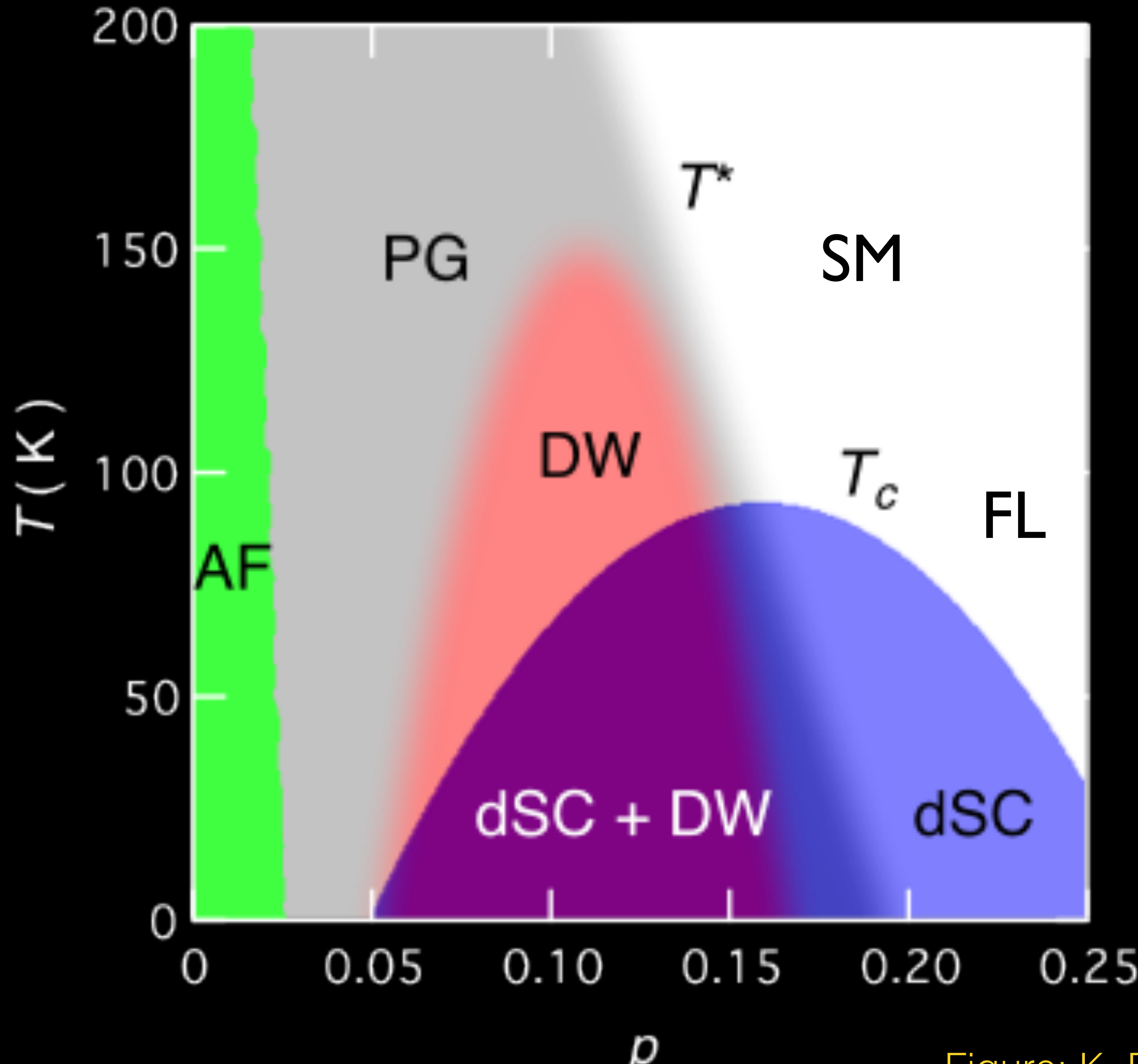
High temperature superconductors

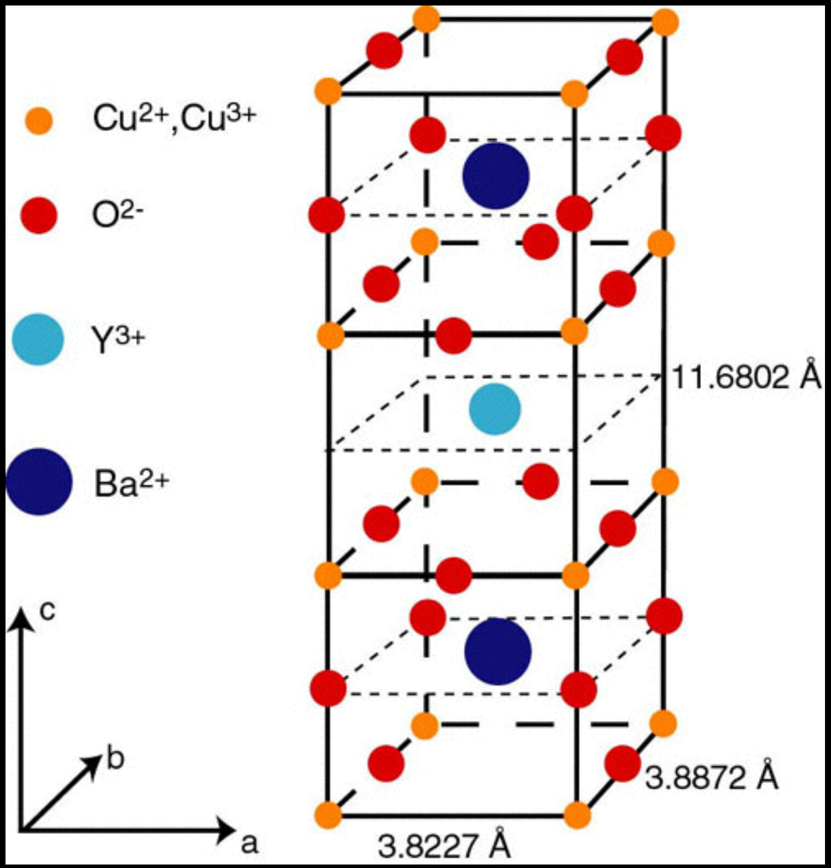


CuO_2 plane



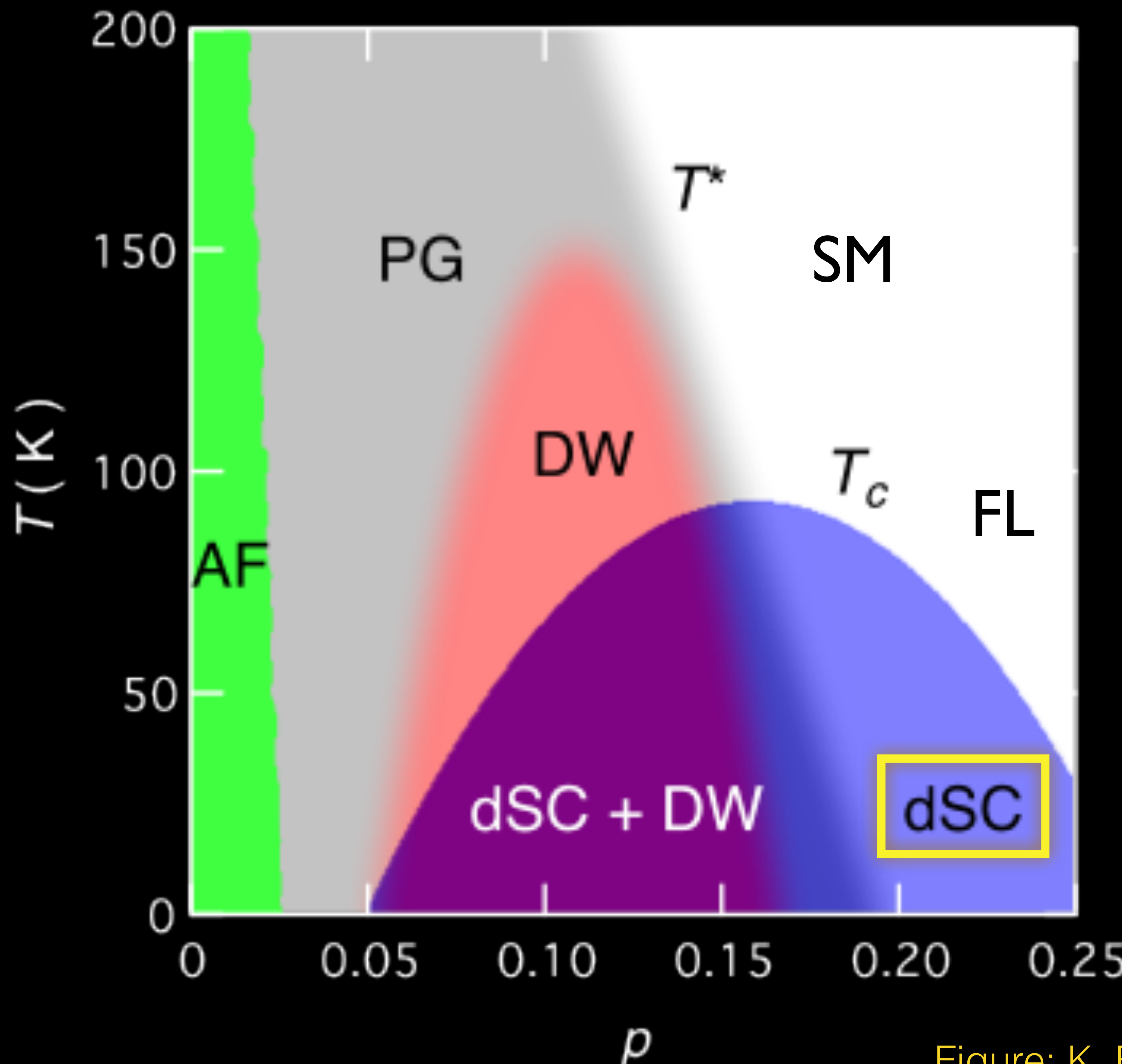
$YBa_2Cu_3O_{6+x}$

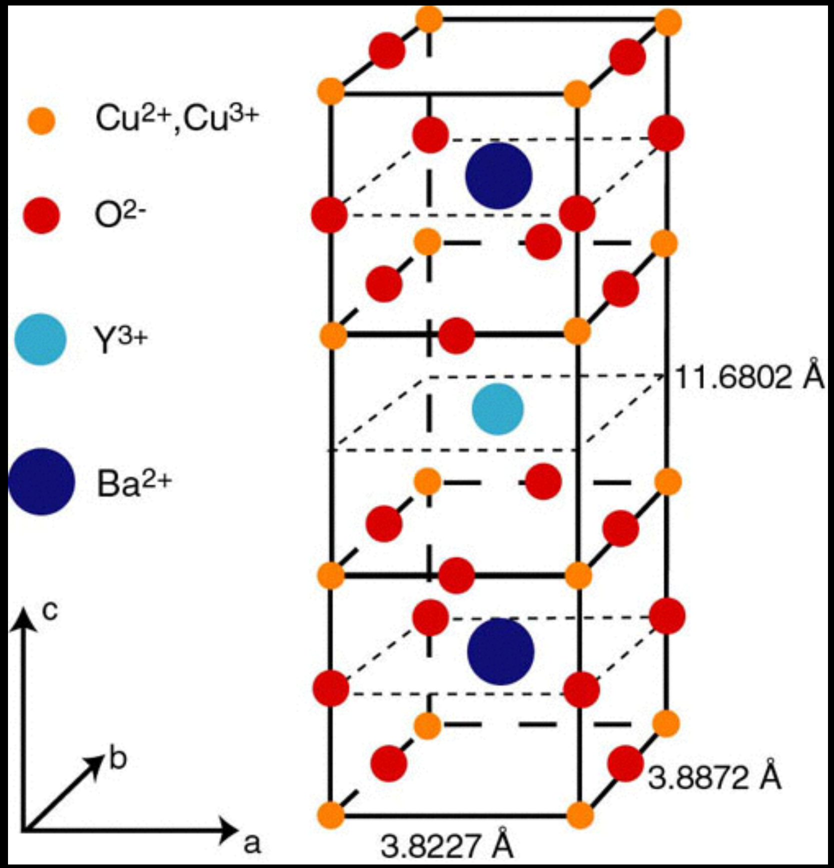




$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis





$YBa_2Cu_3O_{6+x}$

d-wave superconductor

Figure: K. Fujita and J. C. Seamus Davis

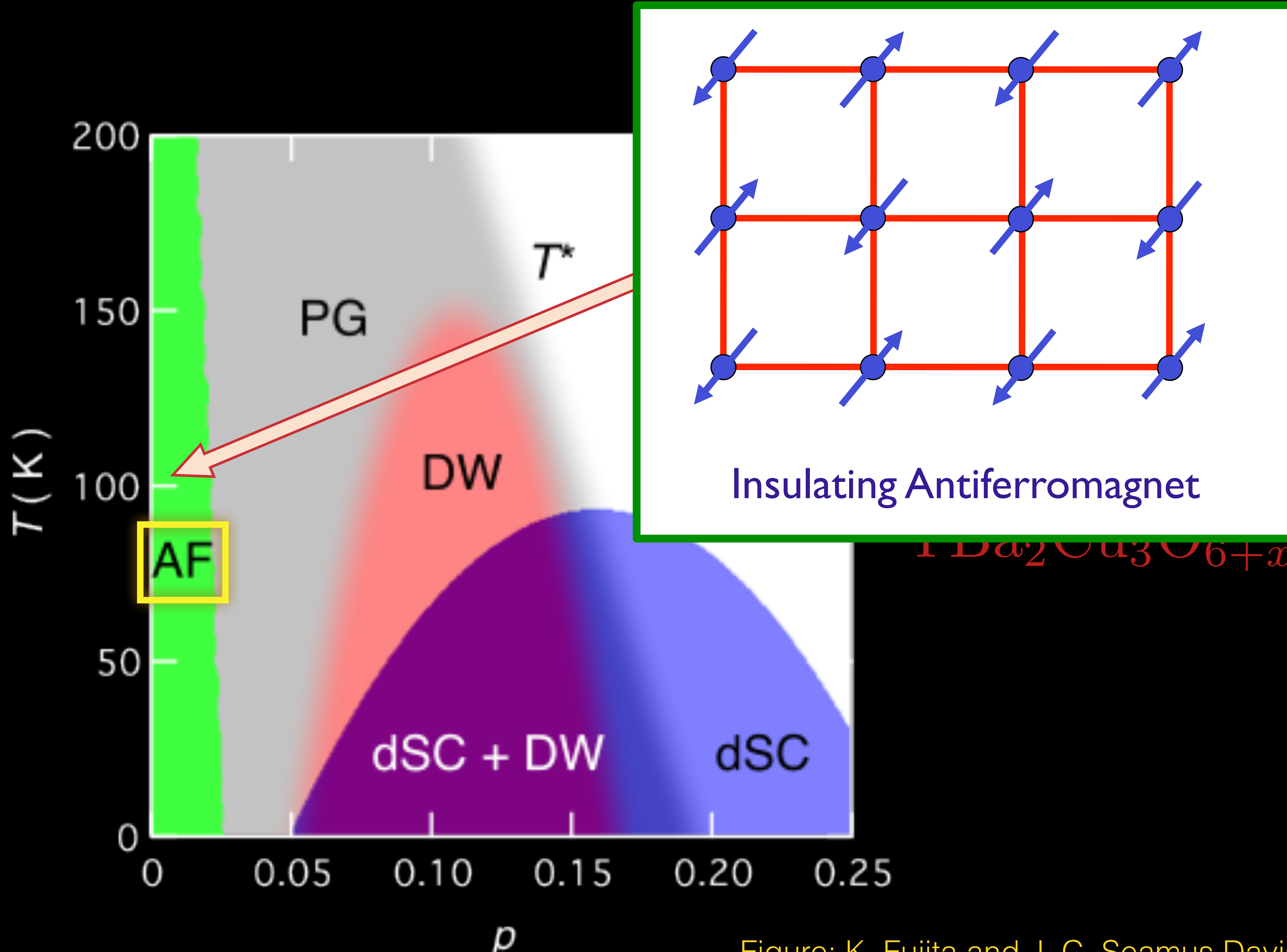
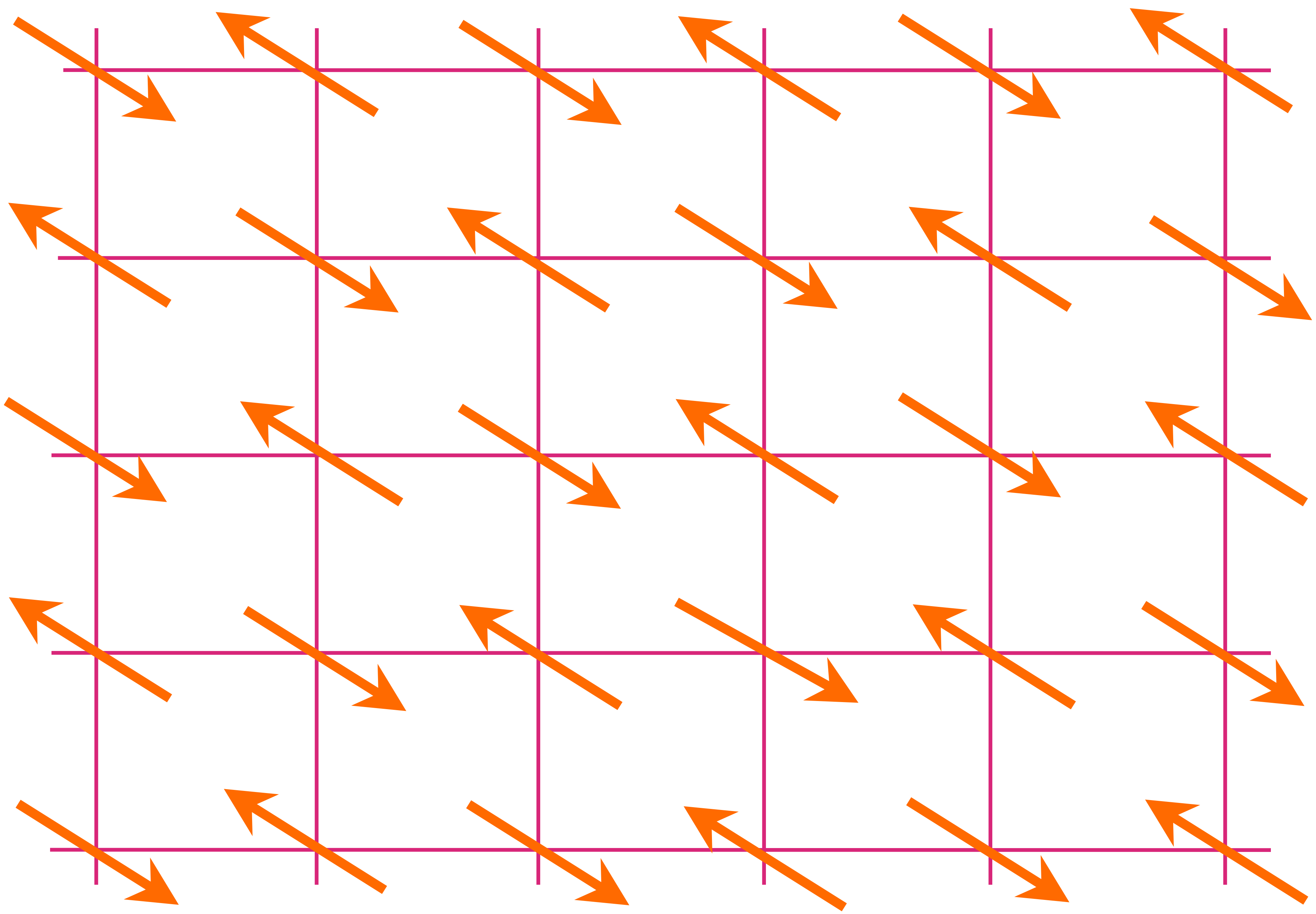
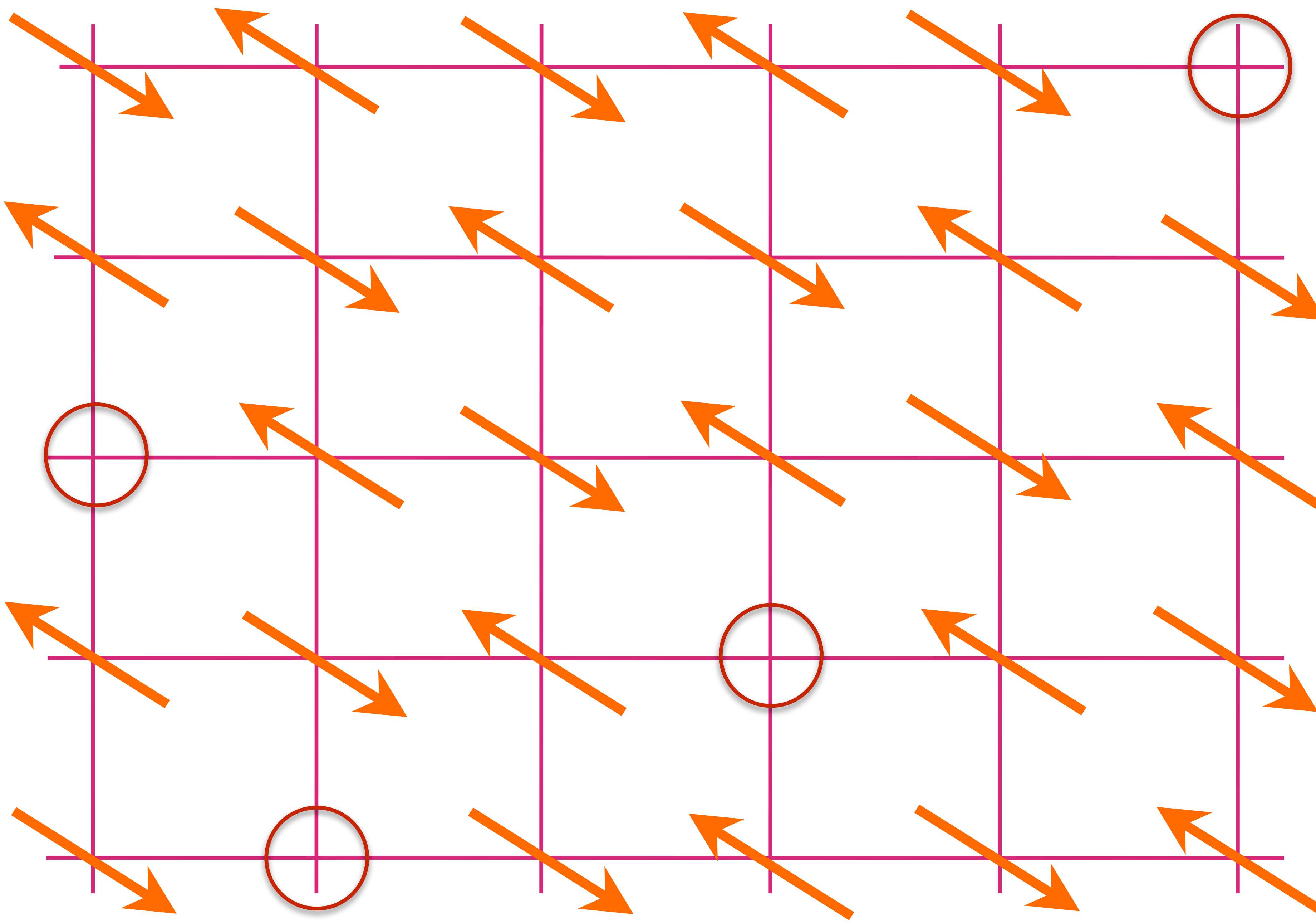


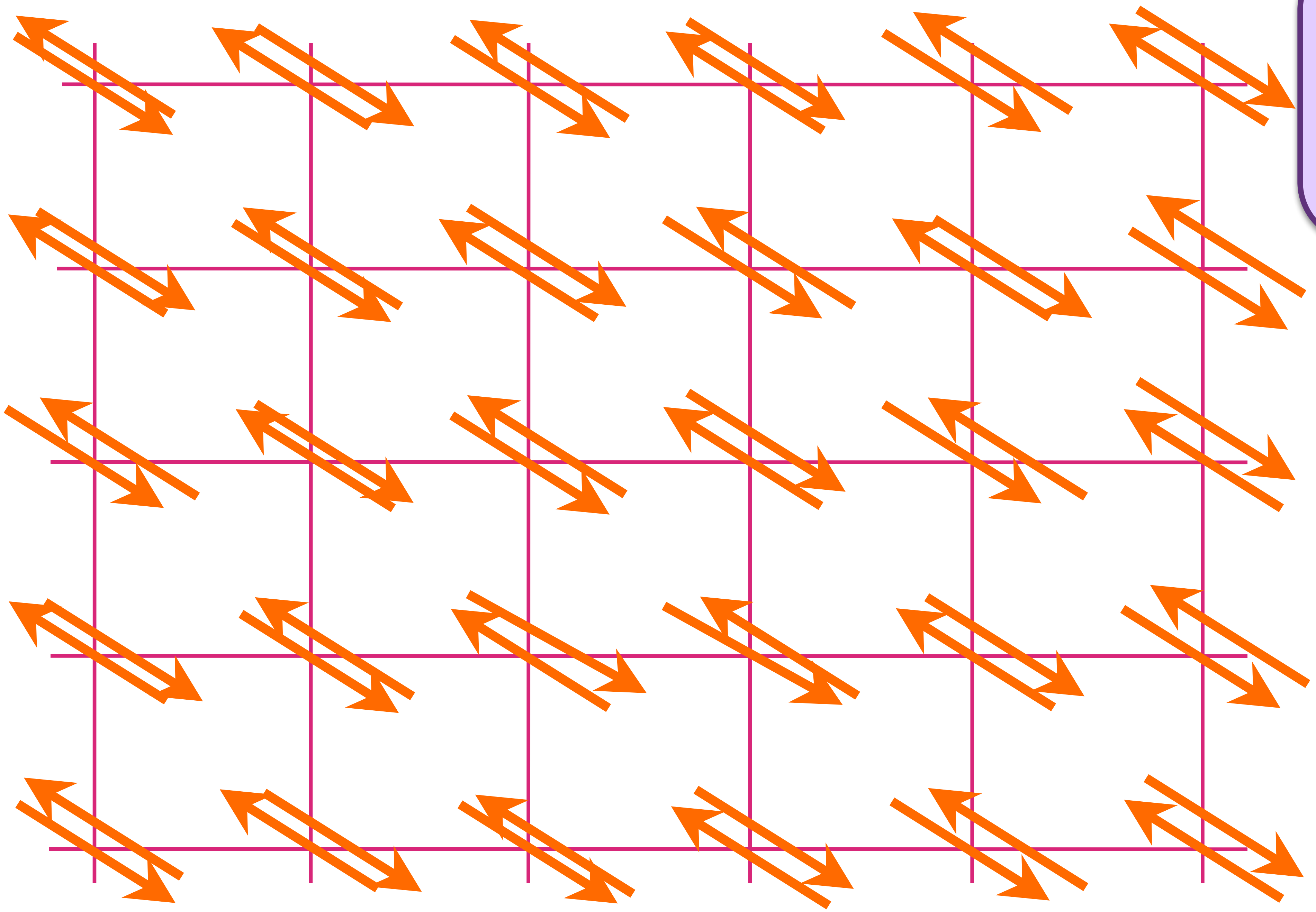
Figure: K. Fujita and J. C. Seamus Davis



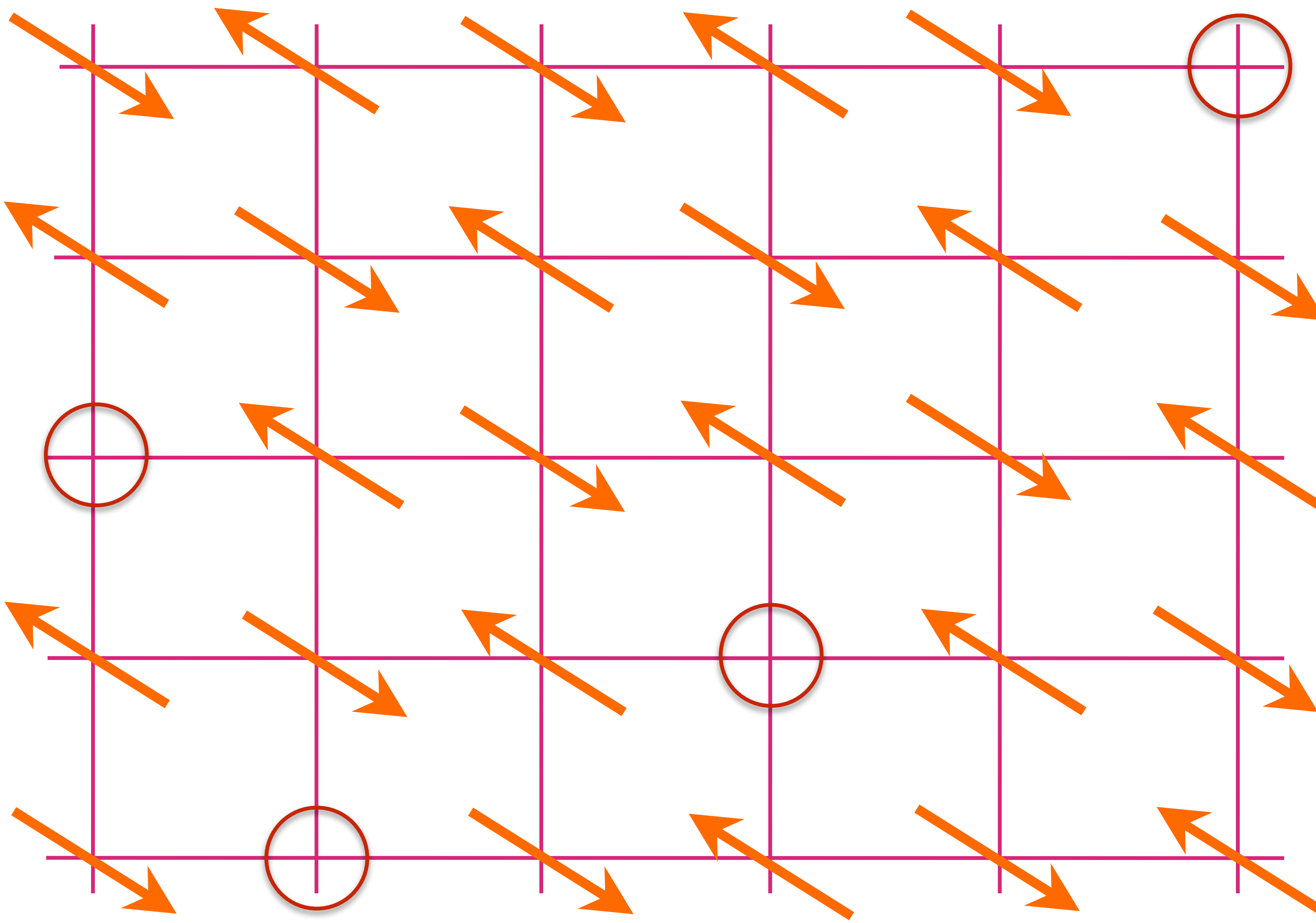
“Undoped” insulating anti-ferromagnet



Anti-ferromagnet with p mobile holes per square

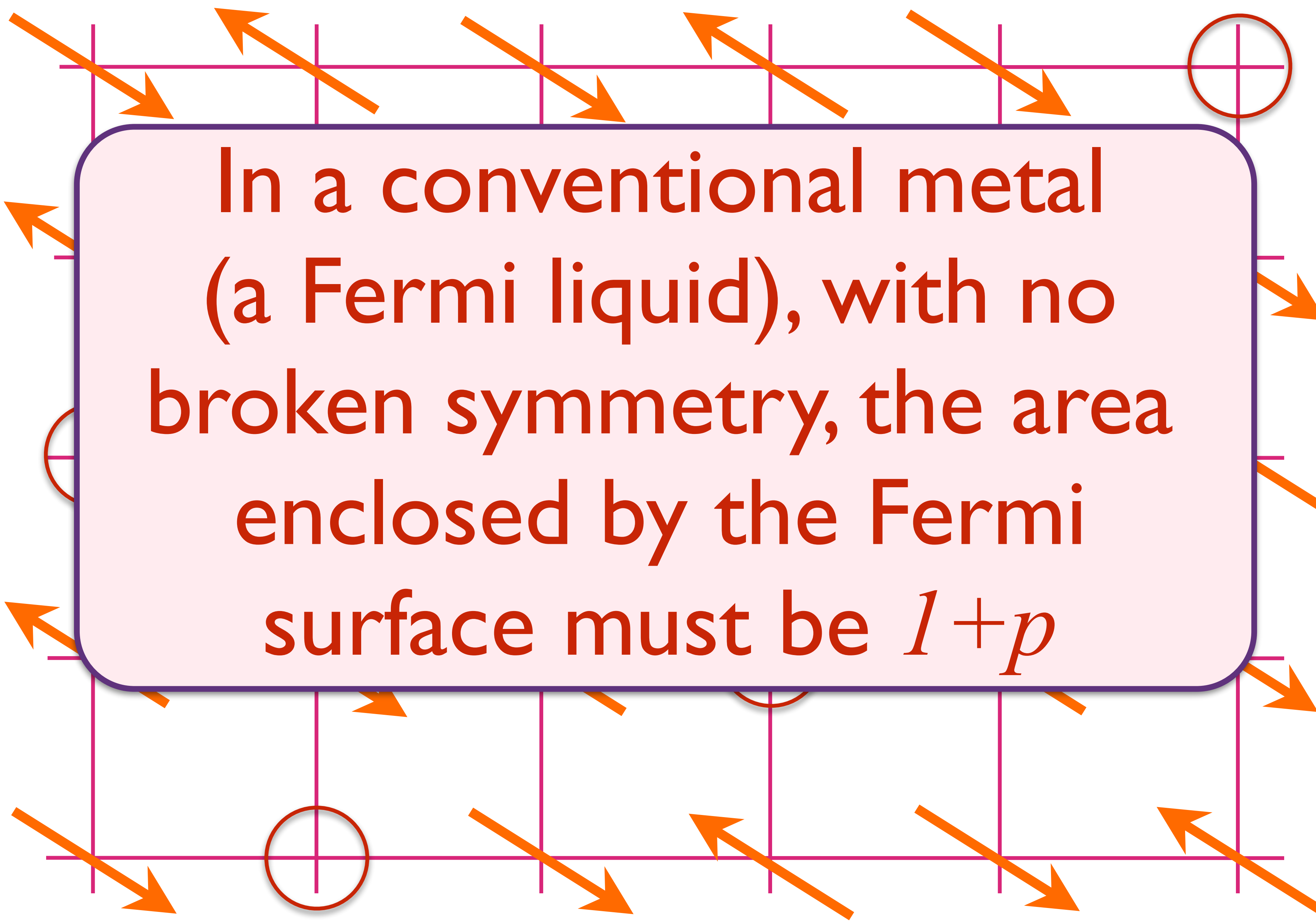


Filled
Band



Anti-ferromagnet with p mobile holes per square

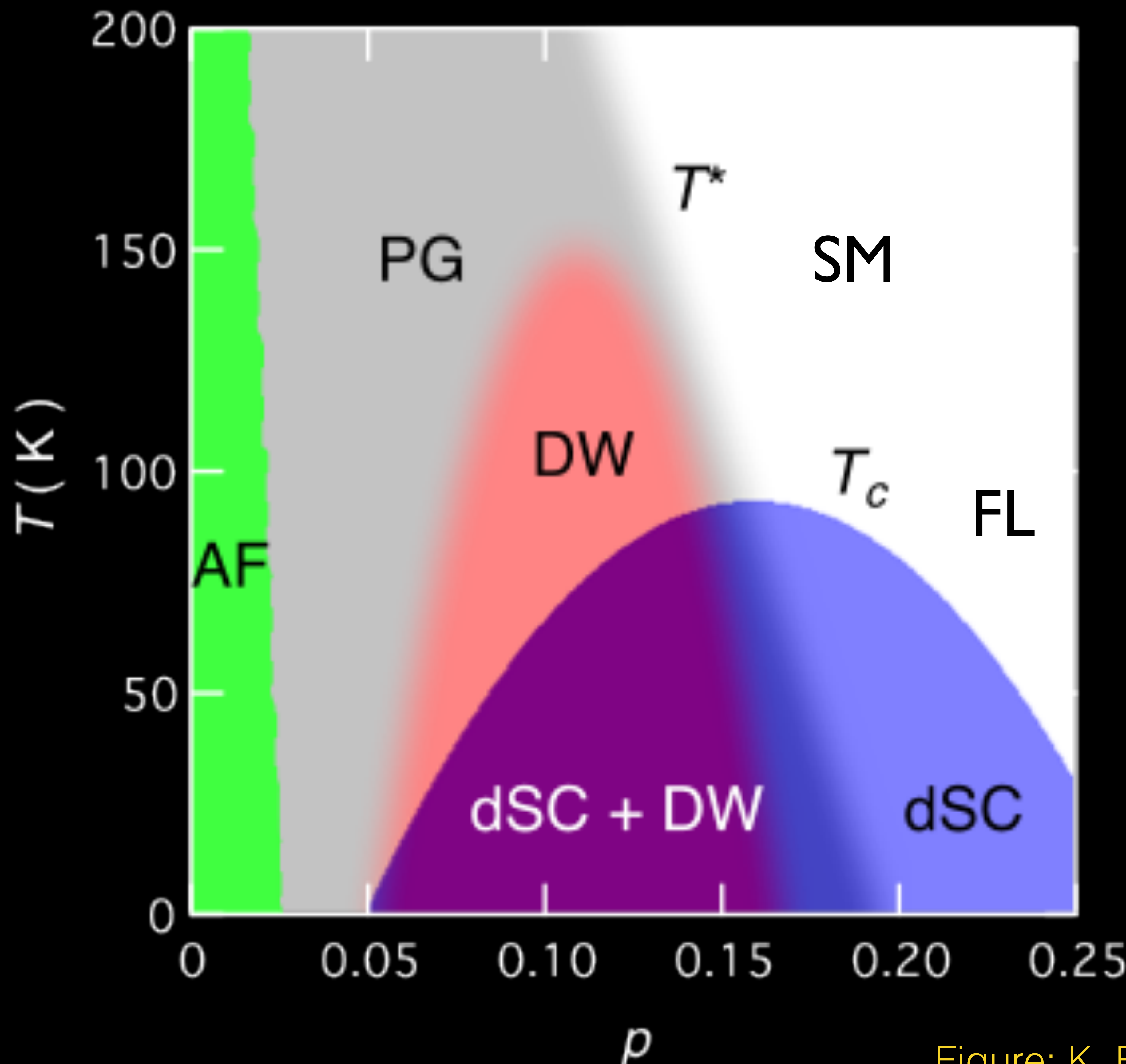
But relative to the band insulator, there are $1+p$ holes per square

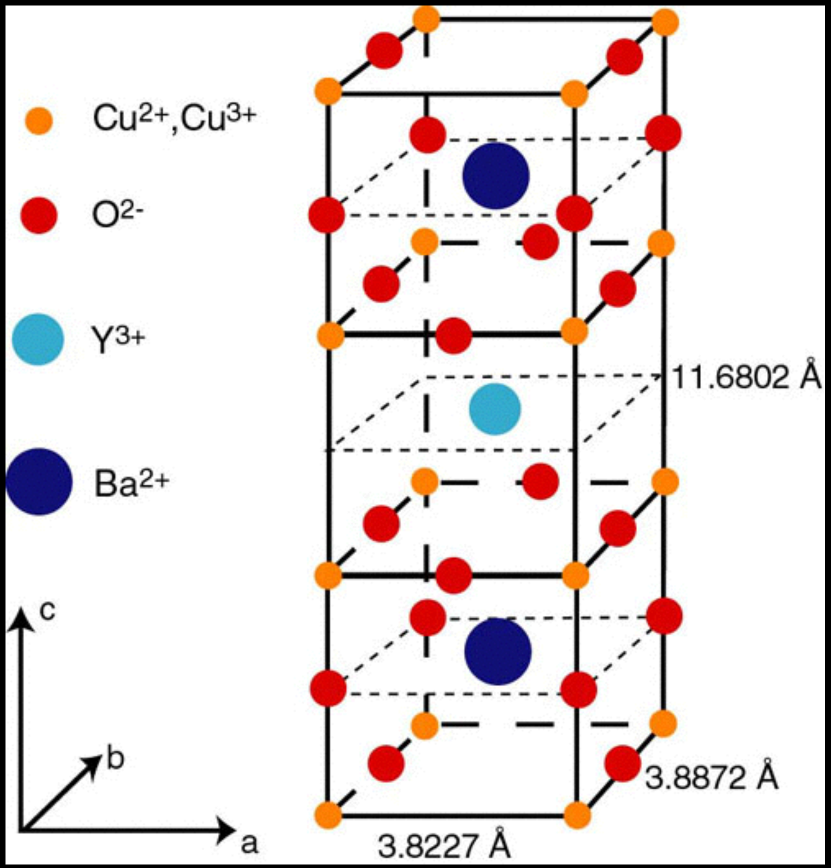


In a conventional metal (a Fermi liquid), with no broken symmetry, the area enclosed by the Fermi surface must be $1+p$

Anti-ferromagnet with p mobile holes per square

But relative to the band insulator, there are $1+p$ holes per square

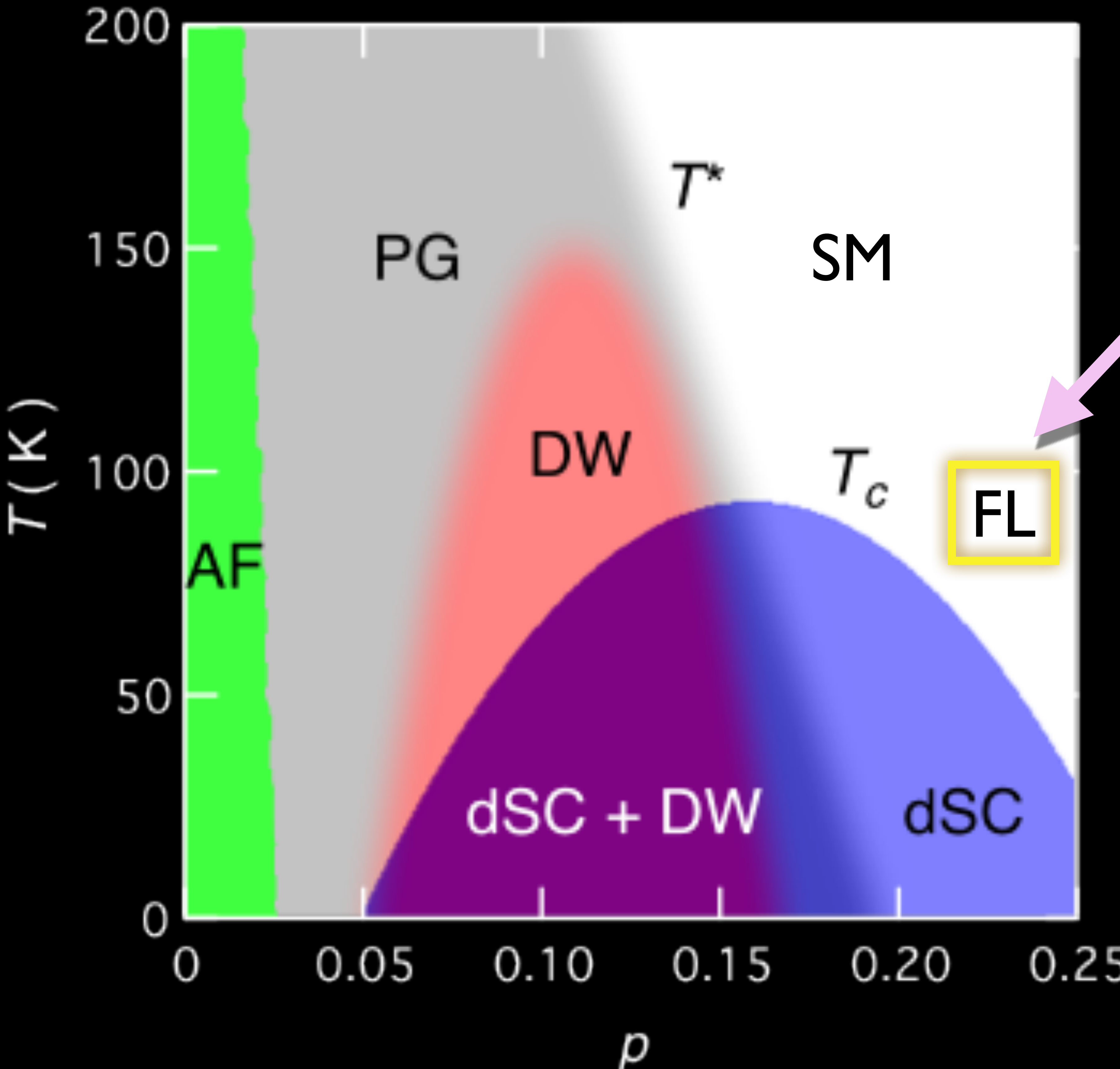


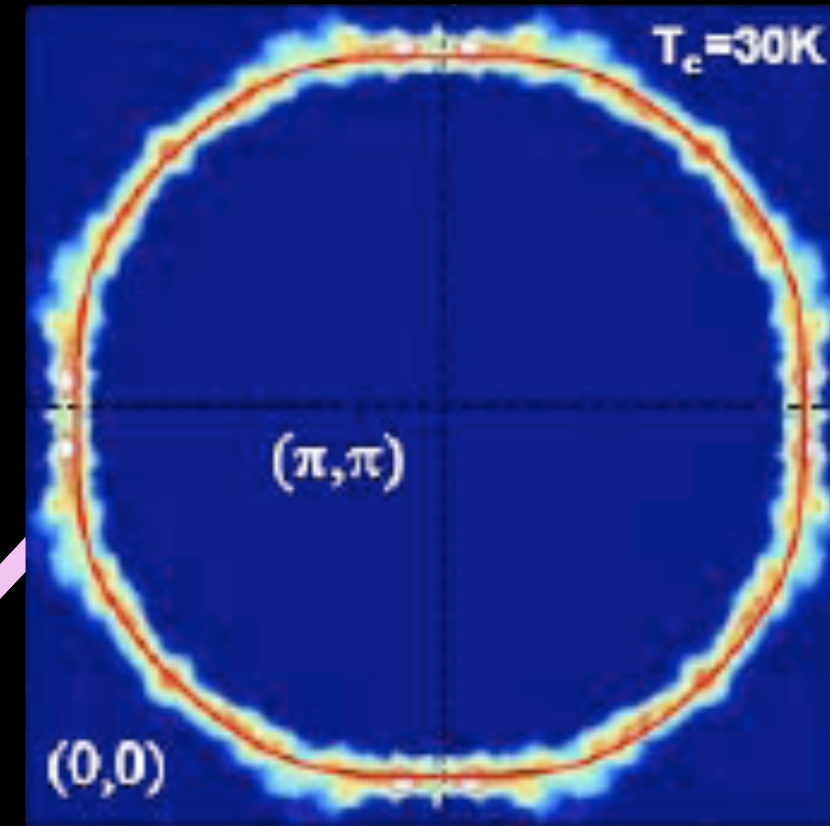


$YBa_2Cu_3O_{6+x}$

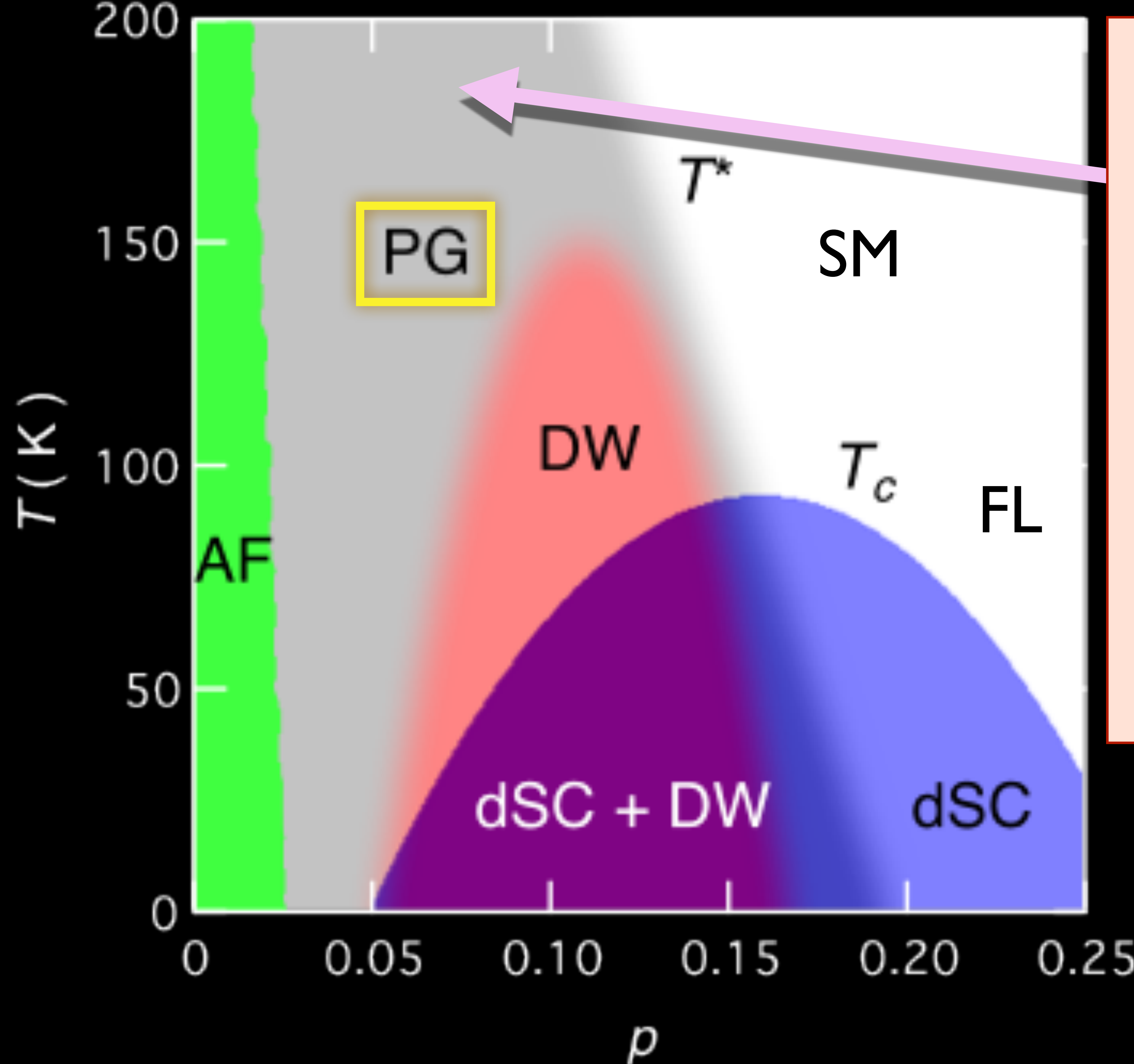
Figure: K. Fujita and J. C. Seamus Davis

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)





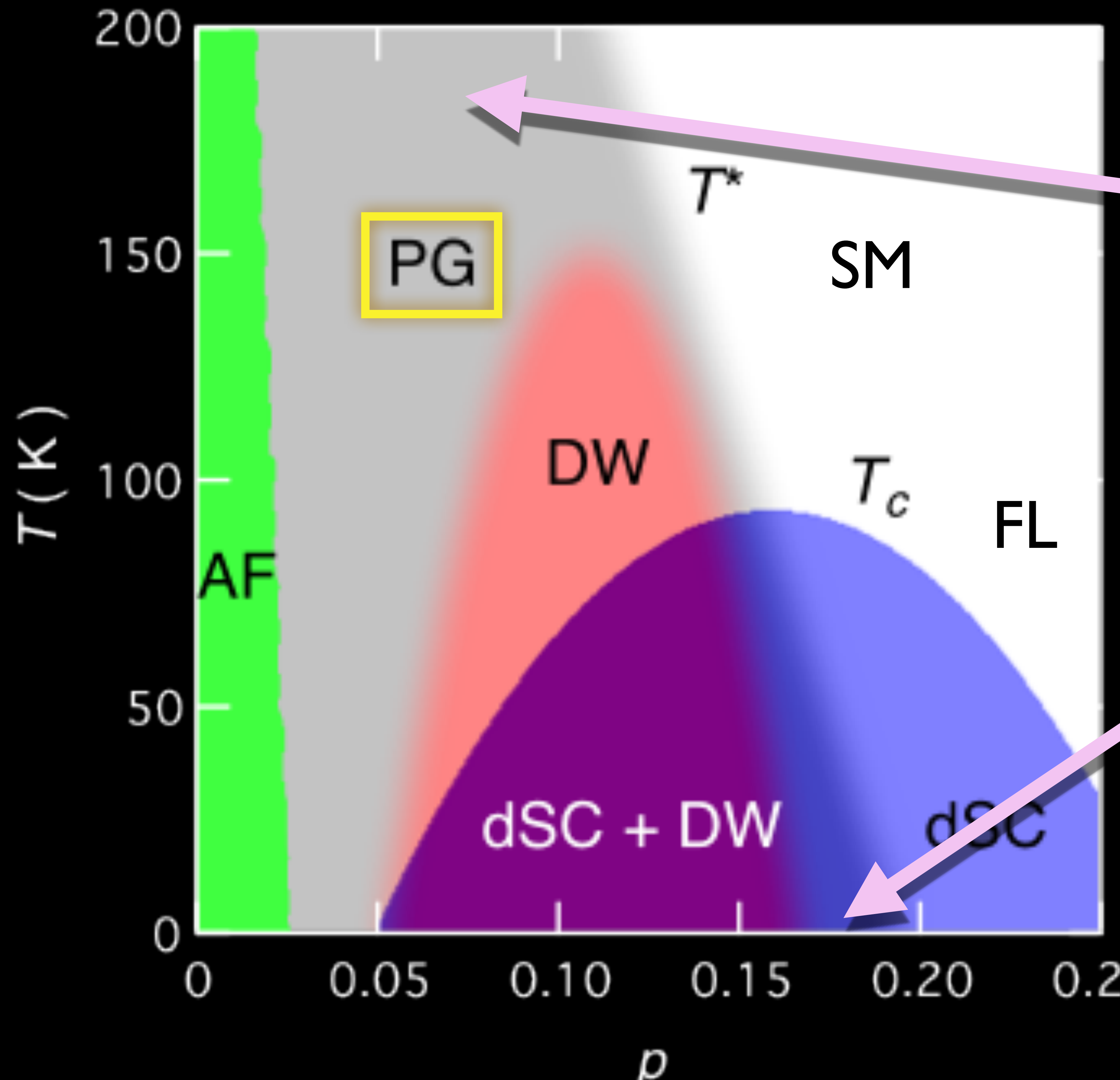
A conventional metal:
the Fermi liquid with Fermi surface of size *1+p*



Pseudogap metal at low p

Many experimental indications that this metal behaves like a Fermi liquid, but with Fermi surface size p and *not* $1+p$.

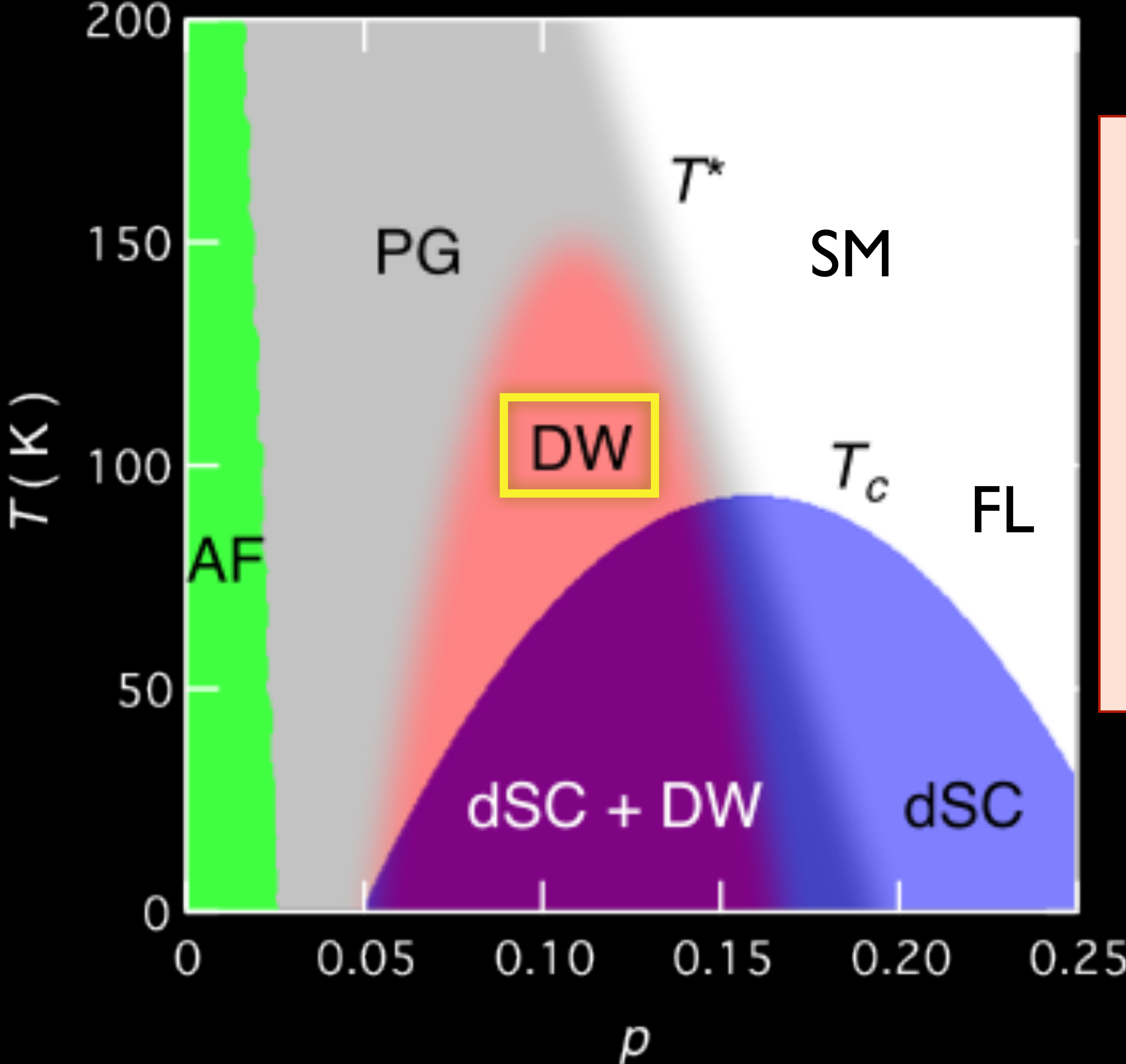
S. Badoux, W. Tabis, F. Laliberté, G. Grissonnanche, B. Vignolle, D. Vignolles, J. Béard, D.A. Bonn, W.N. Hardy, R. Liang, N. Doiron-Leyraud, L. Taillefer, and C. Proust, Nature **531**, 210 (2016).



Pseudogap metal at low *p*

Many experimental indications that this metal behaves like a Fermi liquid, but with Fermi surface size *p* and *not 1+p*.

Recent experiments show the PG metal is also present at low *T* in high magnetic field



DW is "(charge) density wave" order, which is a low *T* instability of the PG metal. It yields important clues on the nature of the PG metal, and will be discussed later.

Q: Can we have a metal with a Fermi surface of size p with electron-like (charge e, spin 1/2) quasiparticles?

Q: Can we have a metal with a Fermi surface of size p with electron-like (charge e, spin 1/2) quasiparticles?

A: Yes, but only if there are additional excitations described by emergent gauge fields. In the simplest case we shall discuss, these excitations are described by a topological quantum field theory (TQFT).

We shall call such a metal **FL***

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett* **90**, 216403 (2003)

1. Flux insertion on the torus, and momentum balance in *any* quantum state.

2. Fermi surface size in FL

3. Insulating spin liquids and topological quantum field theory (TQFT)

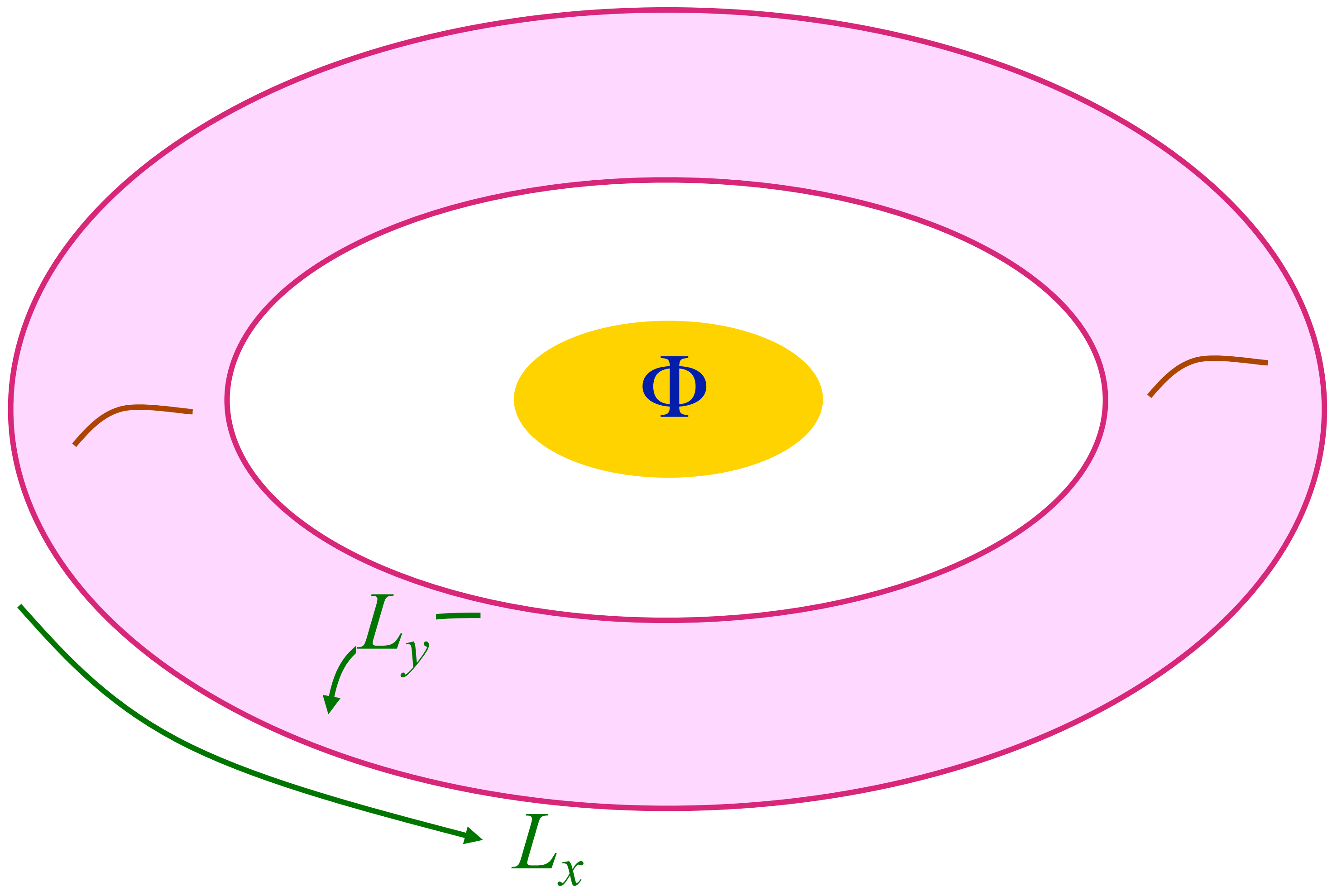
4. A one-band FL* model for the PG metal

1. Flux insertion on the torus, and momentum balance in *any* quantum state.

2. Fermi surface size in FL

3. Insulating spin liquids and topological quantum field theory (TQFT)

4. A one-band FL* model for the PG metal



We take N particles, each with charge Q, on a $L_x \times L_y$ lattice on a torus.
We pierce flux $\Phi = hc/Q$ through a hole of the torus.
An exact computation shows that the change in crystal momentum of *any* many-body quantum state due to flux piercing is

$$\Delta P_x \equiv P_{xf} - P_{xi} = \frac{2\pi N}{L_x} (\text{mod } 2\pi) = 2\pi n L_y (\text{mod } 2\pi)$$

where $n = N/(L_x L_y)$ is the density.

1. Flux insertion on the torus, and momentum balance in *any* quantum state.

2. Fermi surface size in FL

3. Insulating spin liquids and topological quantum field theory (TQFT)

4. A one-band FL* model for the PG metal

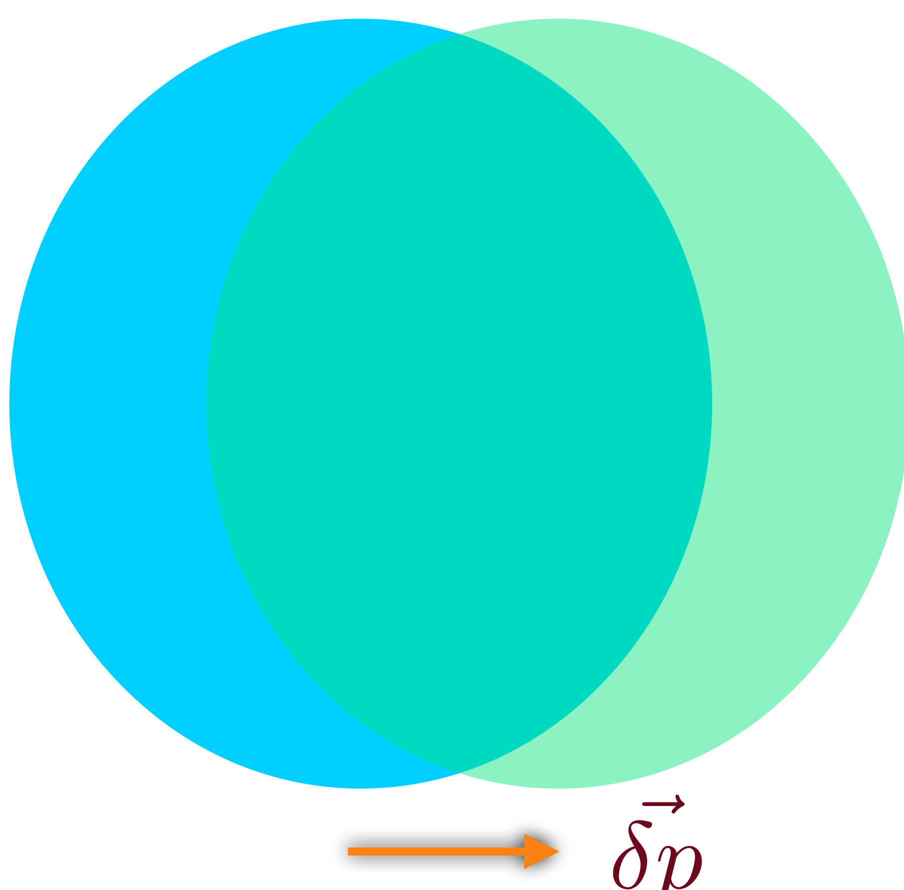
Flux insertion in FL

Now we compute the momentum balance *assuming* that the only low energy excitations are nearly-free quasiparticles near the Fermi surface, and each quasiparticle picks up a momentum $\vec{\delta p} \equiv (2\pi/L_x, 0)$. Then a simple geometric computation shows that

$$\Delta P_x = \left(\frac{2\pi}{L_x}\right) \frac{L_x L_y}{4\pi^2} V_{\rm FS} \pmod{2\pi}$$

where $V_{\rm FS}$ is the volume of the Fermi surface. Assuming this holds for all mutually prime $L_{x,y}$, and for spinful electrons we can establish Luttinger's theorem:

$$\frac{V_{\rm FS}}{4\pi^2} = n \equiv (1+p) \pmod{2}$$



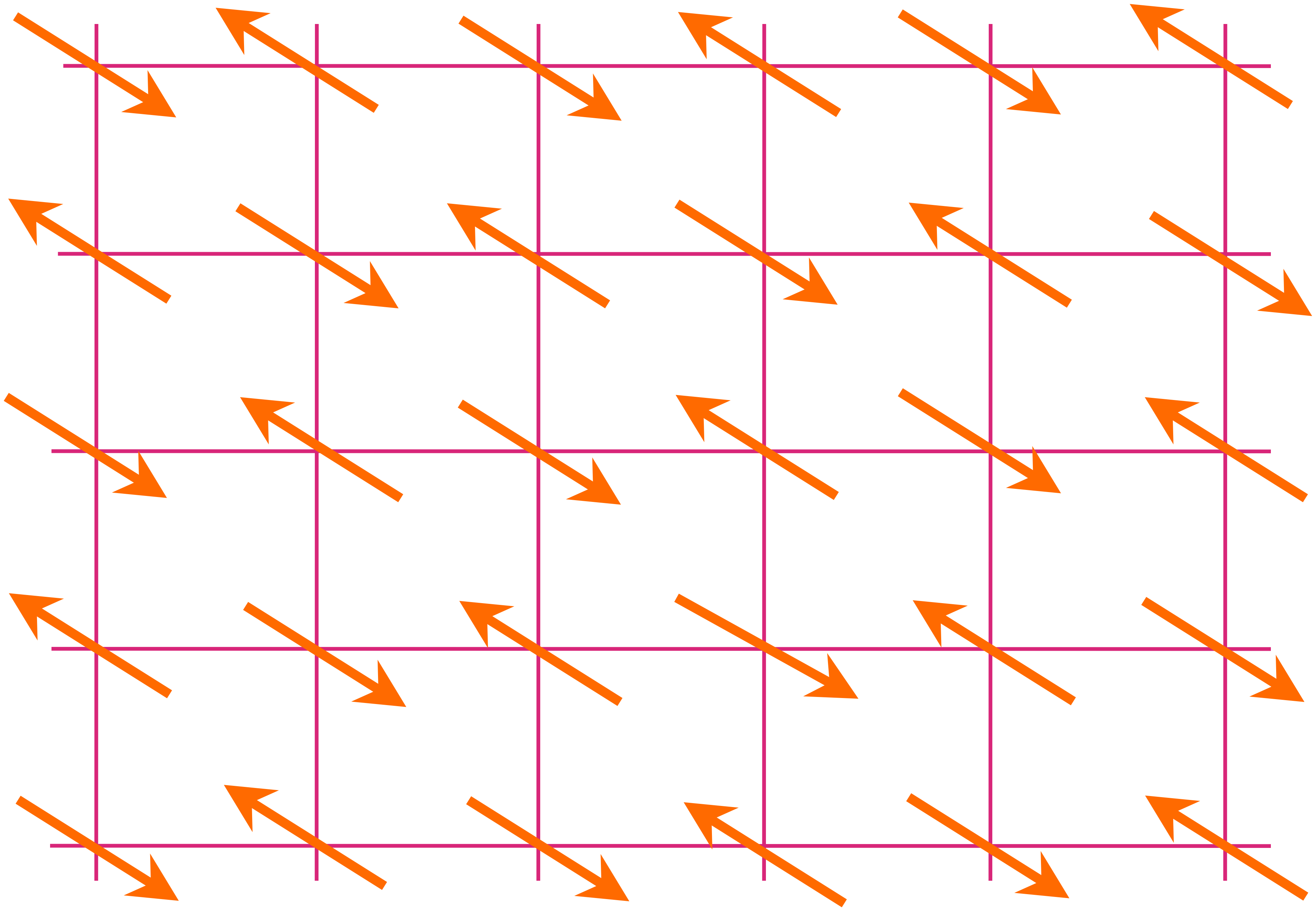
M. Oshikawa, PRL **84**, 3370 (2000)

1. Flux insertion on the torus, and momentum balance in *any* quantum state.

2. Fermi surface size in FL

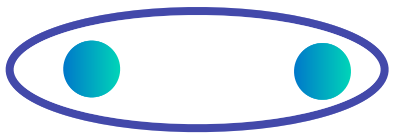
3. Insulating spin liquids and topological quantum field theory (TQFT)

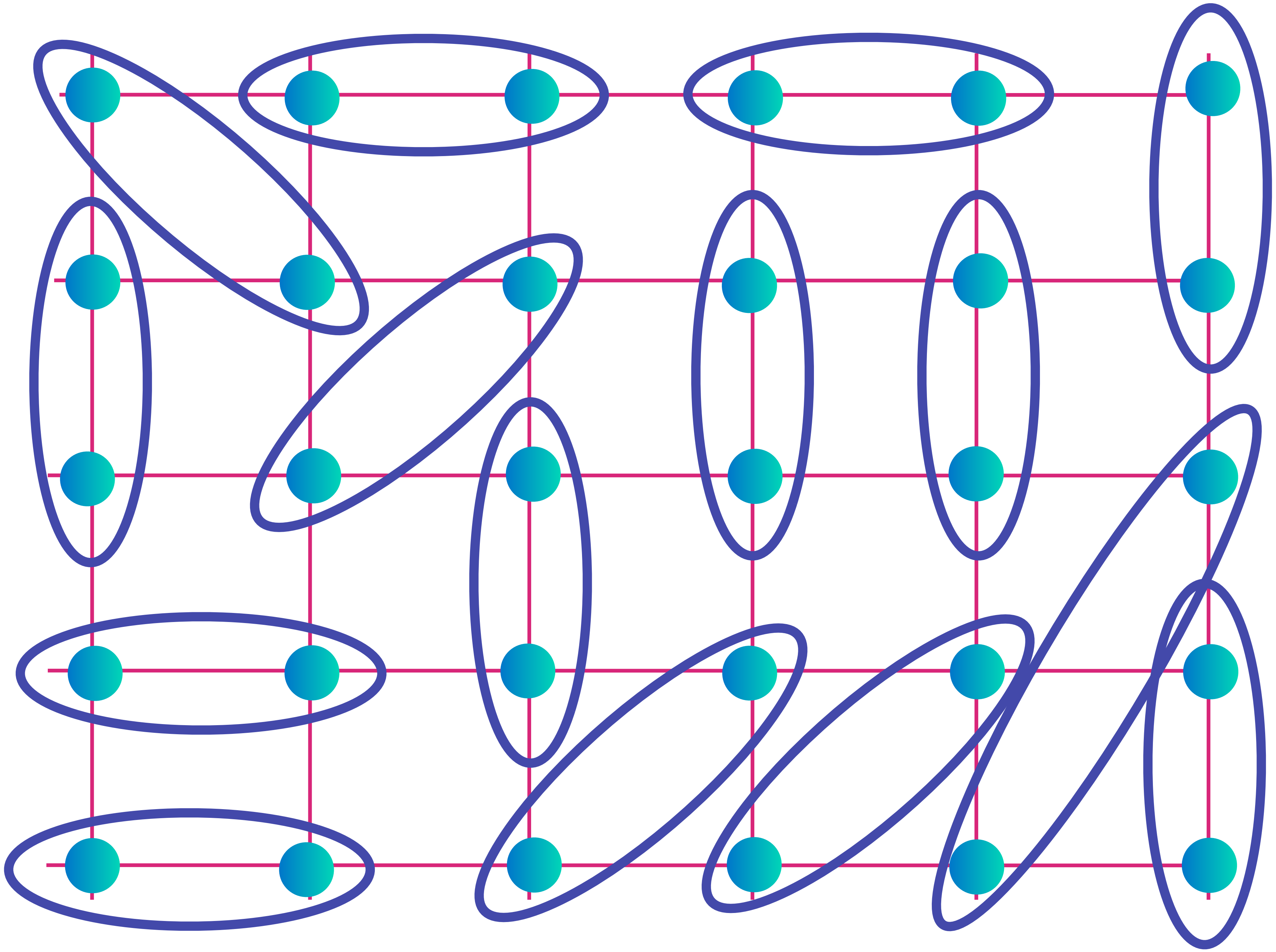
4. A one-band FL* model for the PG metal



“Undoped” Anti-ferromagnet

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



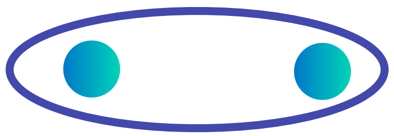
The first proposal of a quantum state described by a TQFT

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

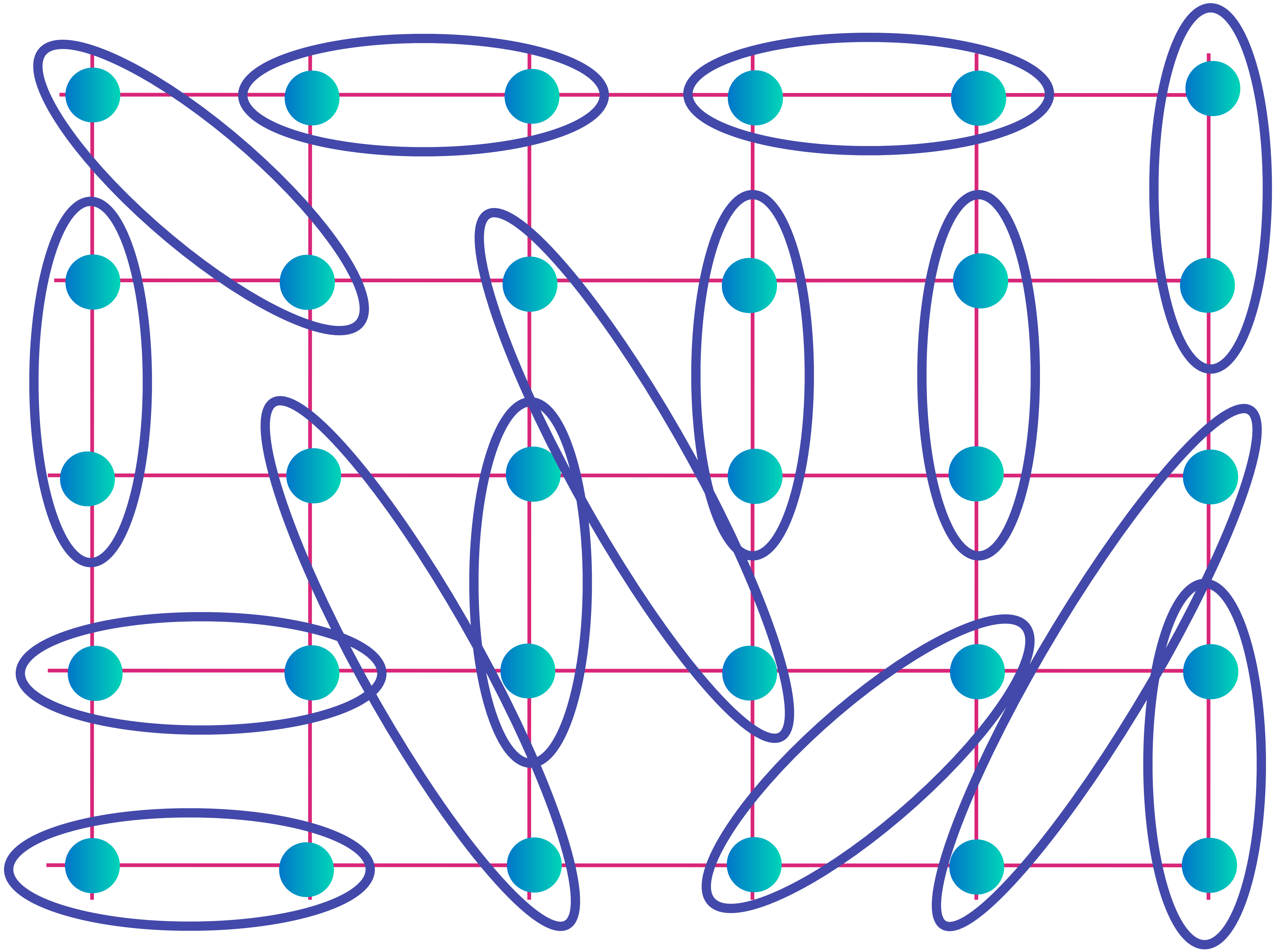
P. W. Anderson, Materials Research Bulletin **8**, 153 (1973)

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

Insulating spin liquid



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



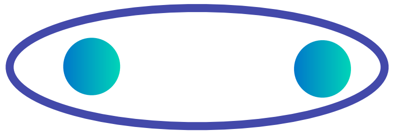
The first proposal of a quantum state described by a TQFT

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

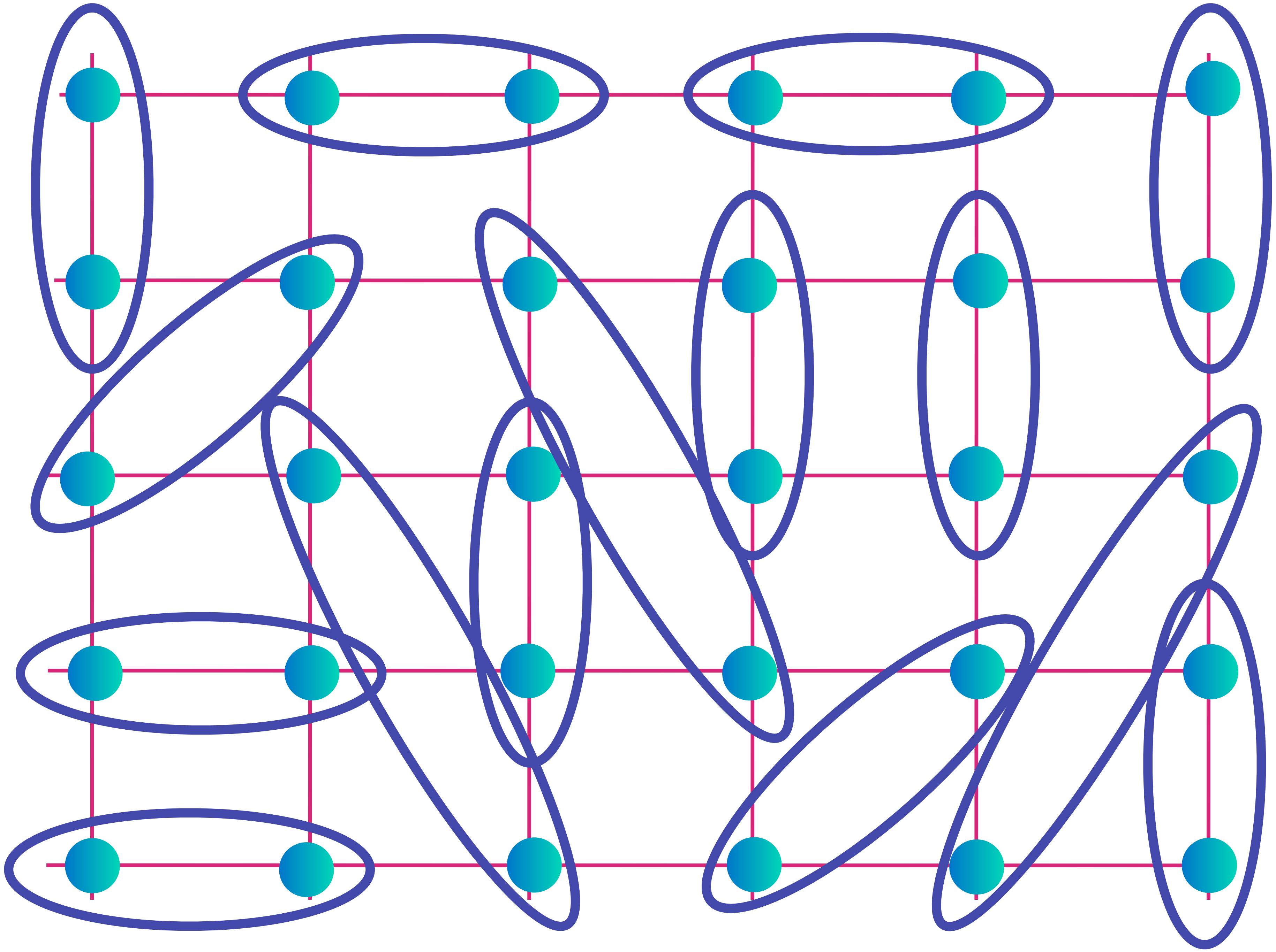
P. W. Anderson, Materials Research Bulletin **8**, 153 (1973)

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

Insulating spin liquid



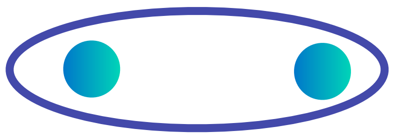
$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

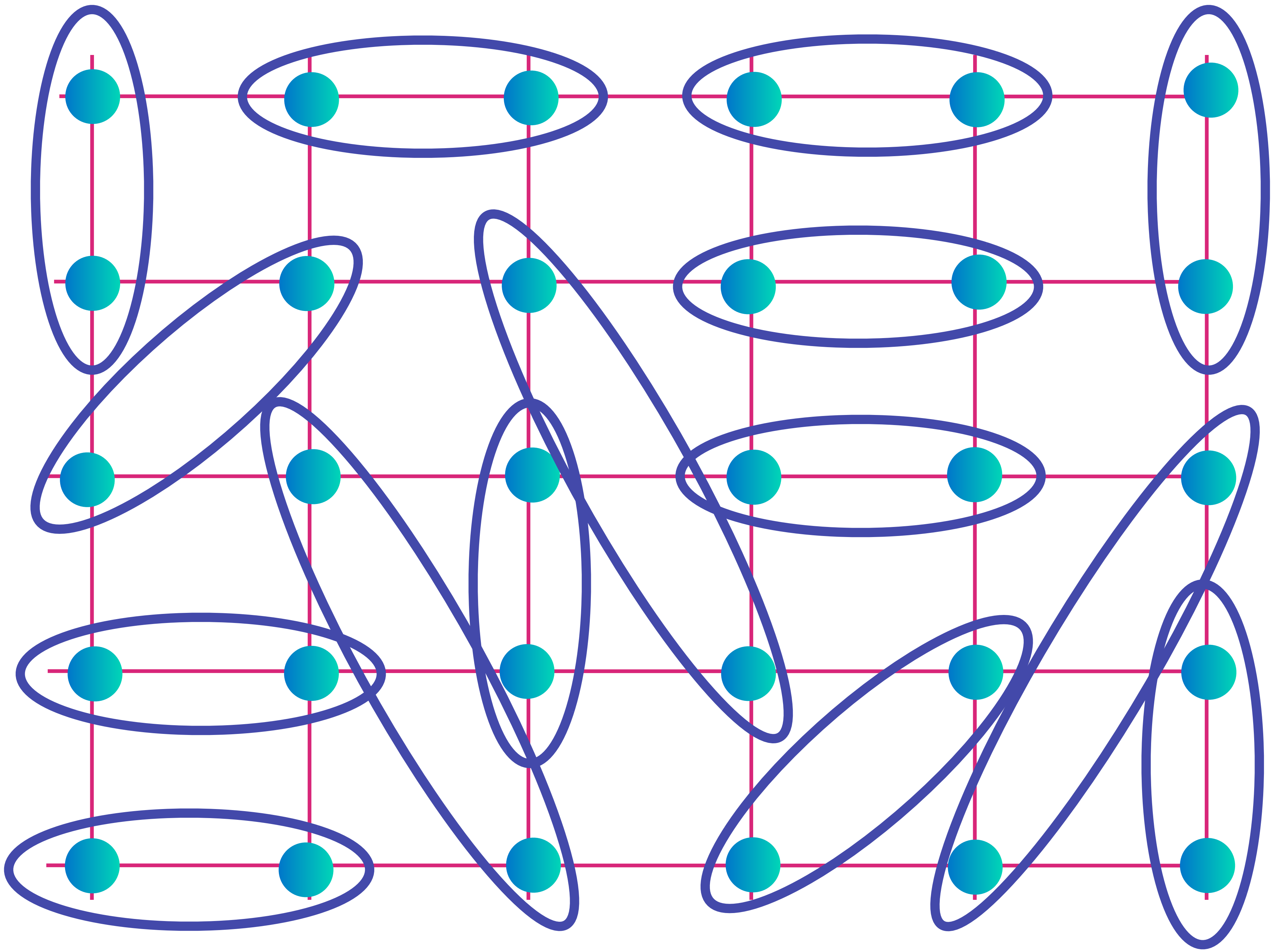


The first proposal of a quantum state described by a TQFT

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)
P. W. Anderson, Materials Research Bulletin **8**, 153 (1973)
G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



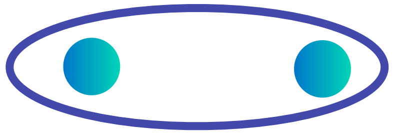
The first proposal of a quantum state described by a TQFT

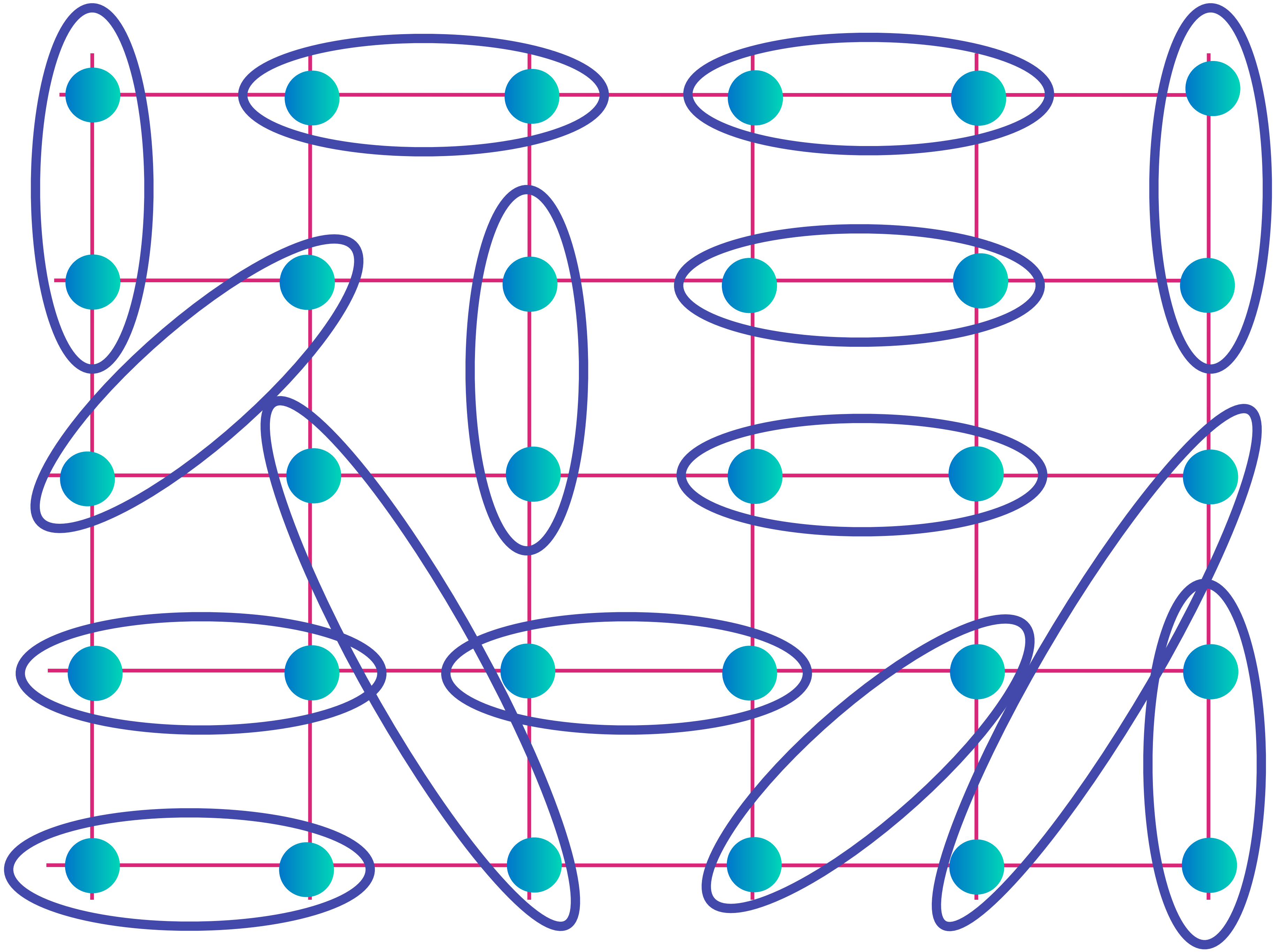
L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

P. W. Anderson, Materials Research Bulletin **8**, 153 (1973)

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



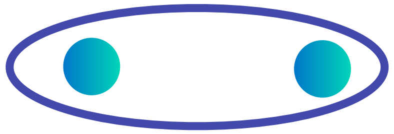
The first proposal of a quantum state described by a TQFT

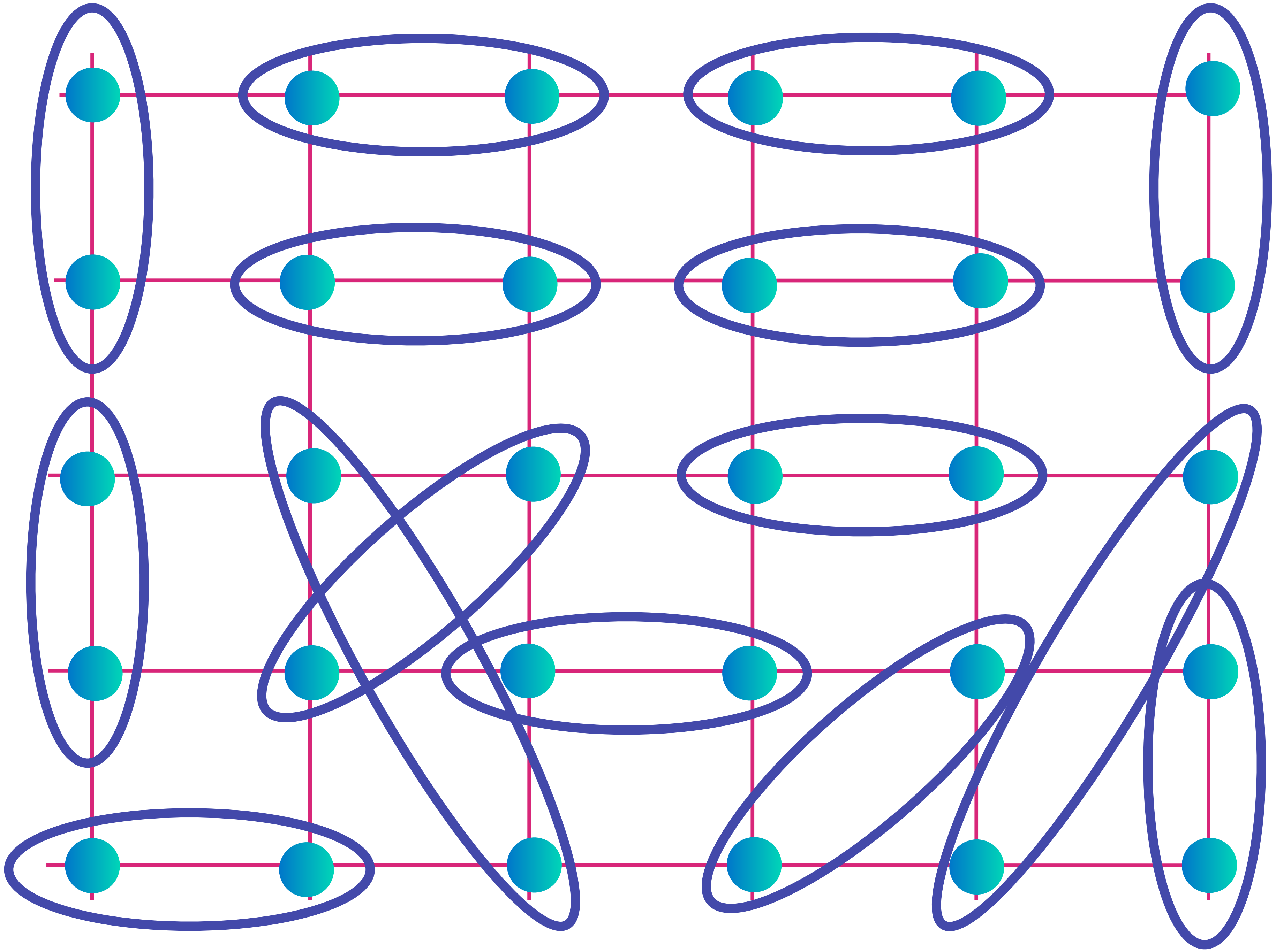
L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

P. W. Anderson, Materials Research Bulletin **8**, 153 (1973)

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

Insulating spin liquid

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



The first proposal of a quantum state described by a TQFT

L. Pauling, Proceedings of the Royal Society London A **196**, 343 (1949)

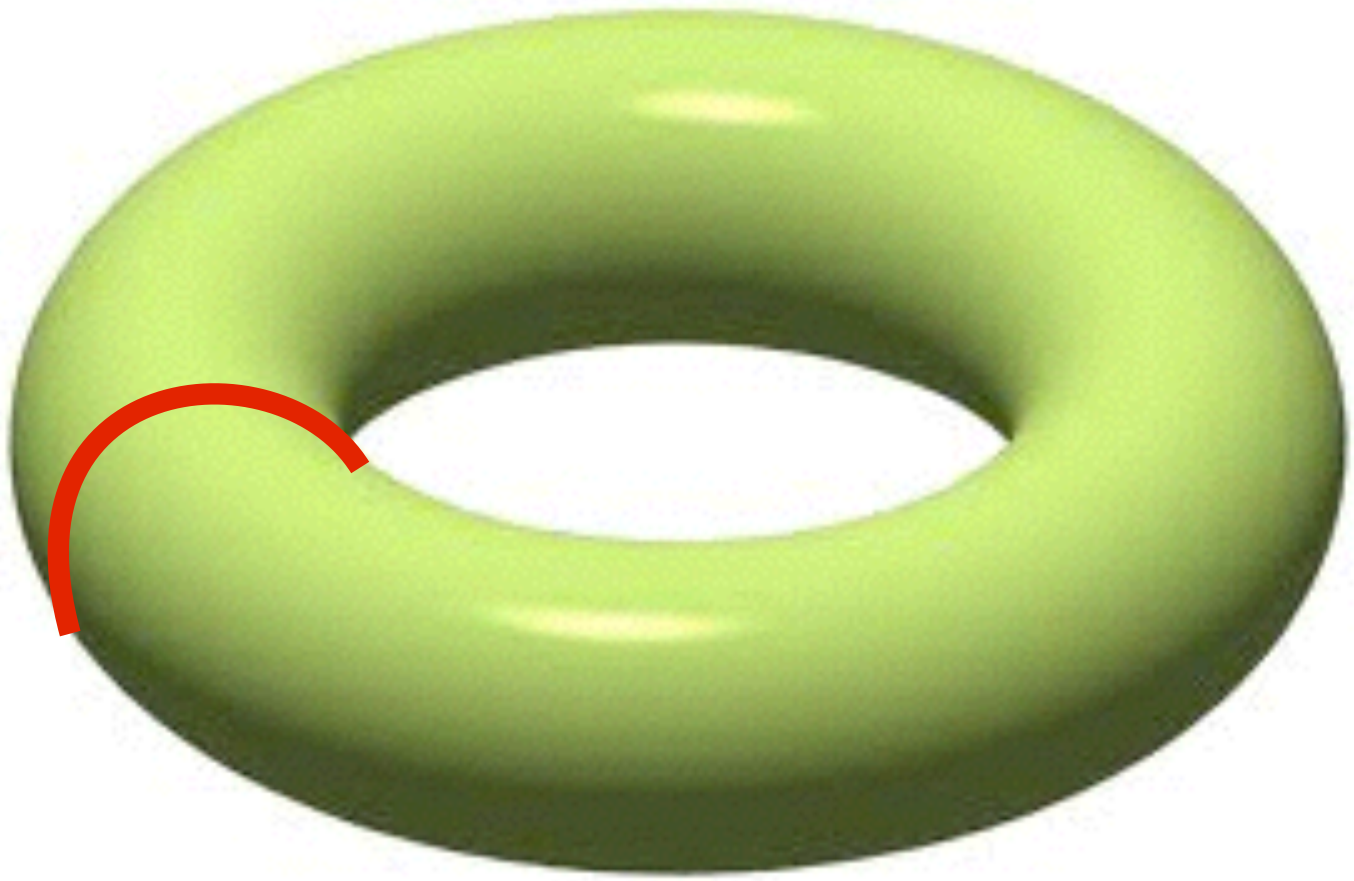
P. W. Anderson, Materials Research Bulletin **8**, 153 (1973)

G. Baskaran and P. W. Anderson, Phys. Rev. B **37**, 580(R) (1988)

Why is this a TQFT ?

Place insulator on a torus:

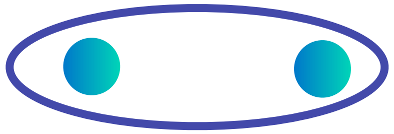
Why is this a TQFT ?



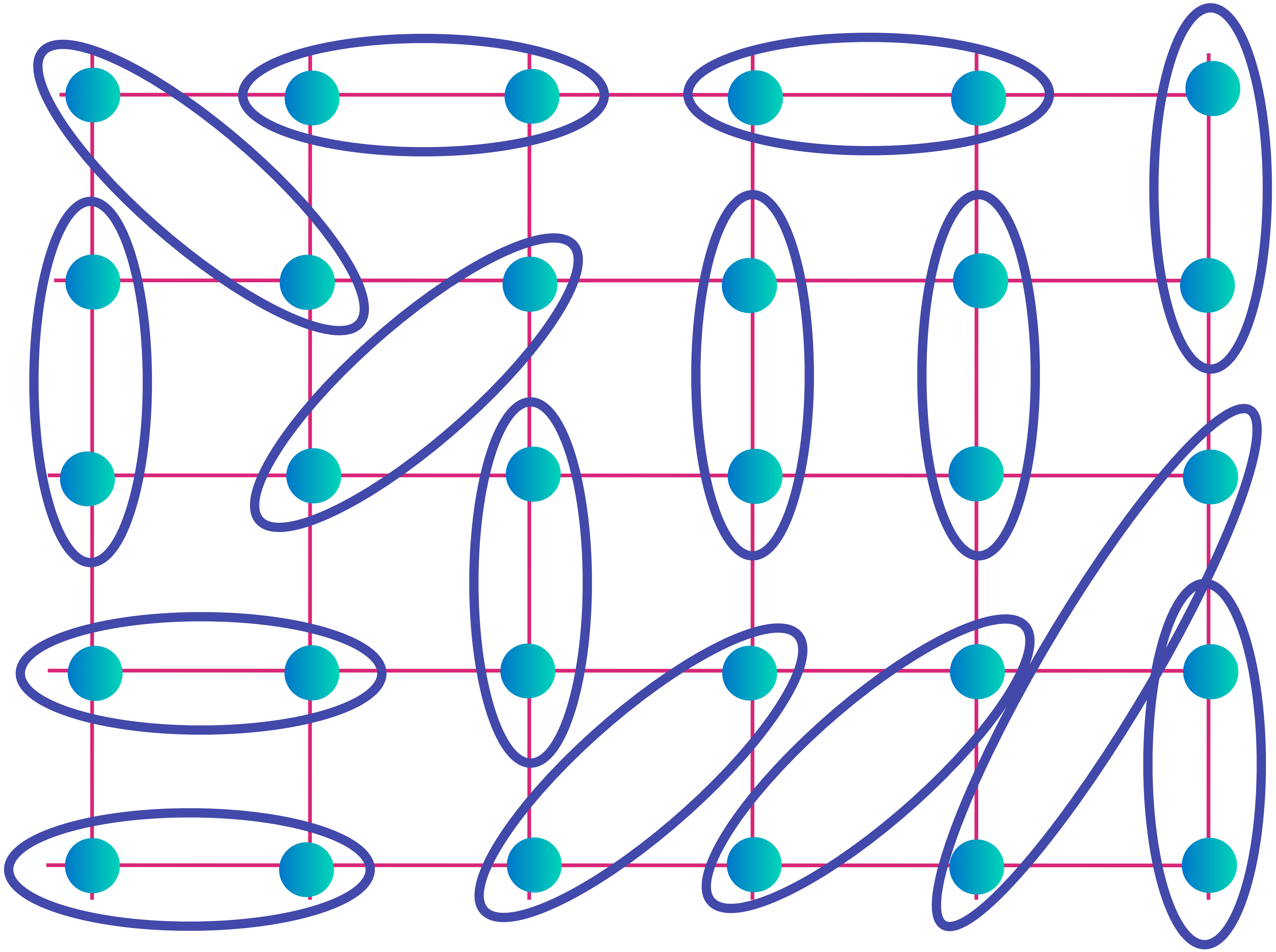
Place insulator on a torus:

Obtain a degenerate orthogonal state by modifying the wavefunction on a “branch-cut” encircling the torus.

Why is this a TQFT ?



$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



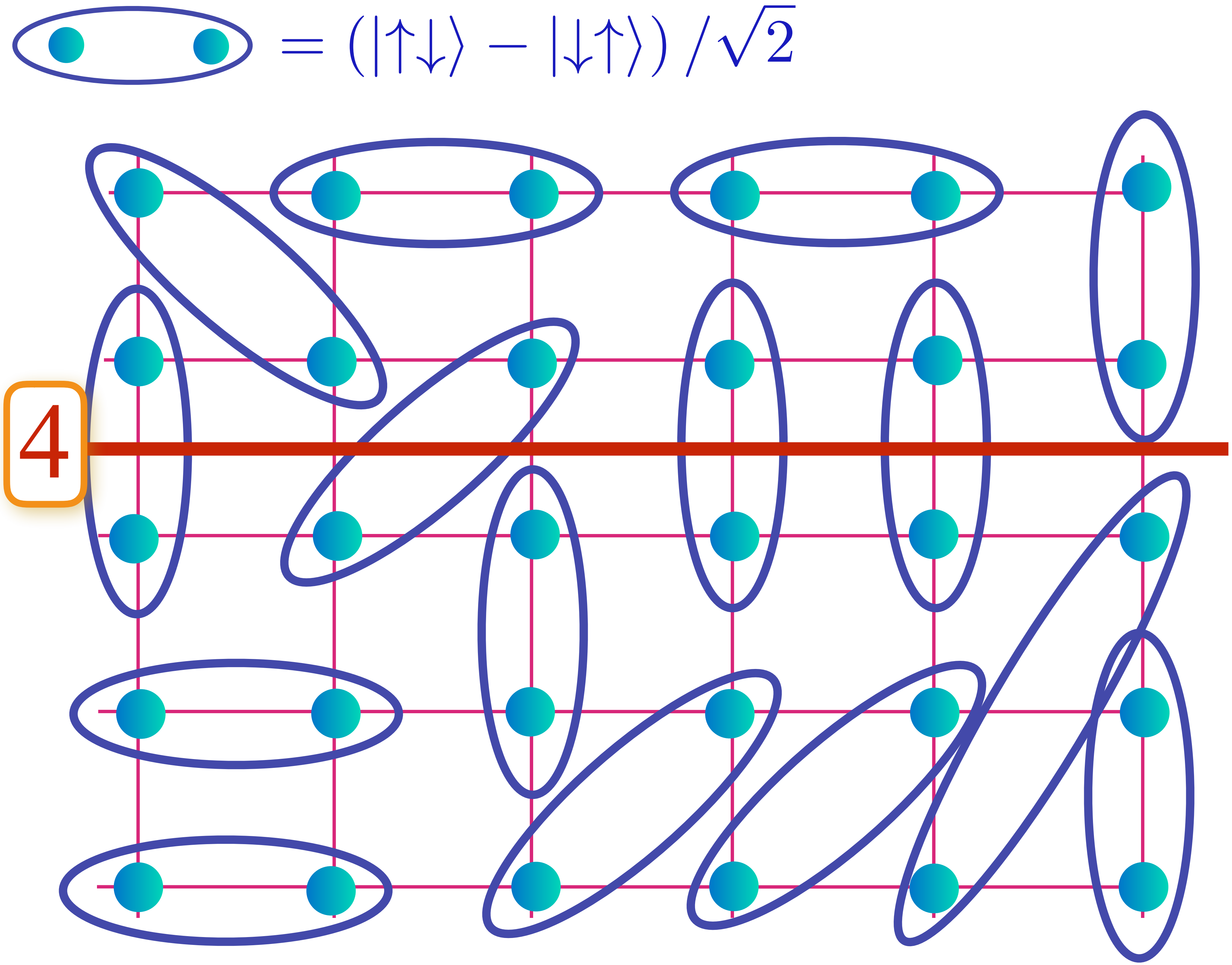
Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Why is this a TQFT ?

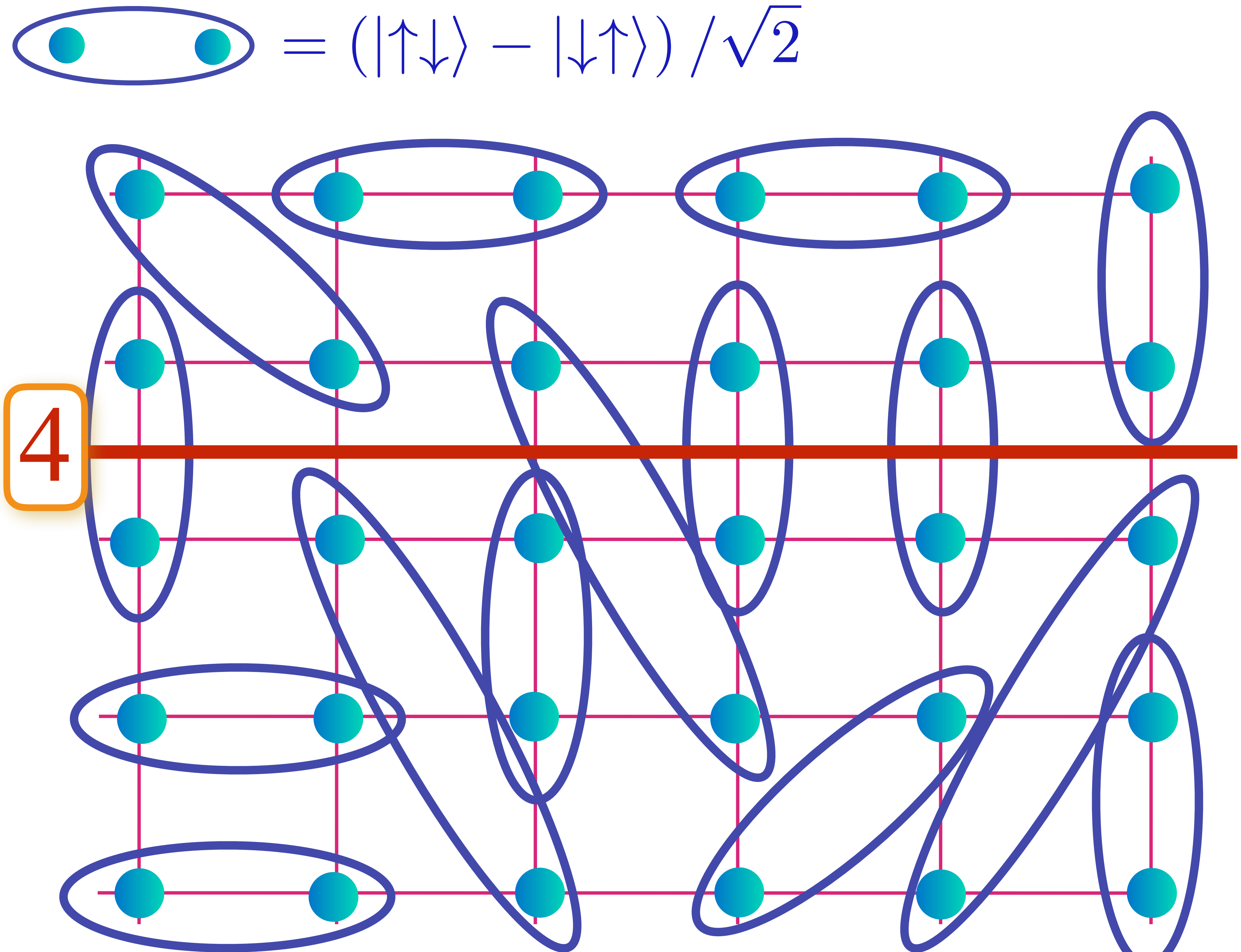


Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Why is this a TQFT ?

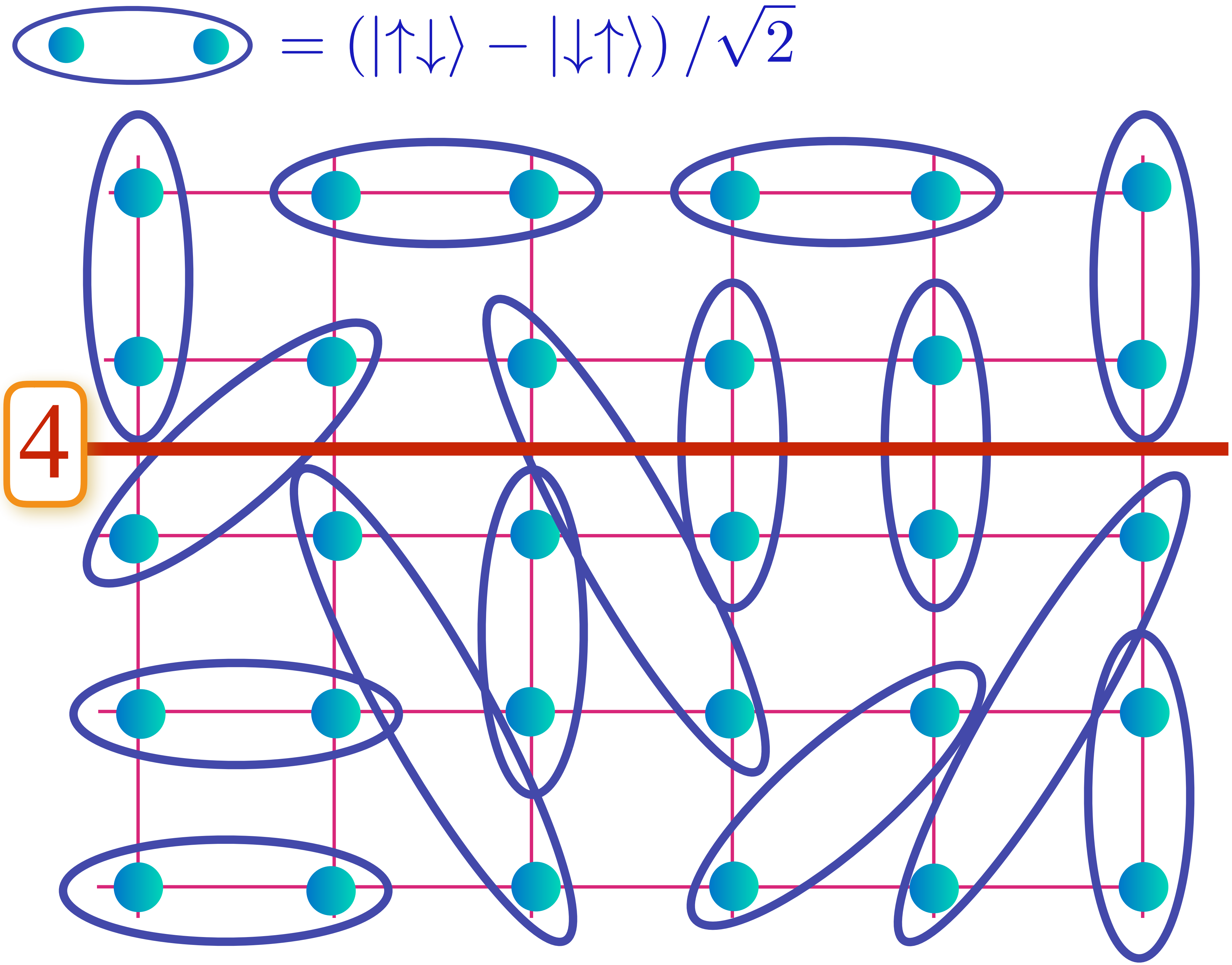


Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Why is this a TQFT ?

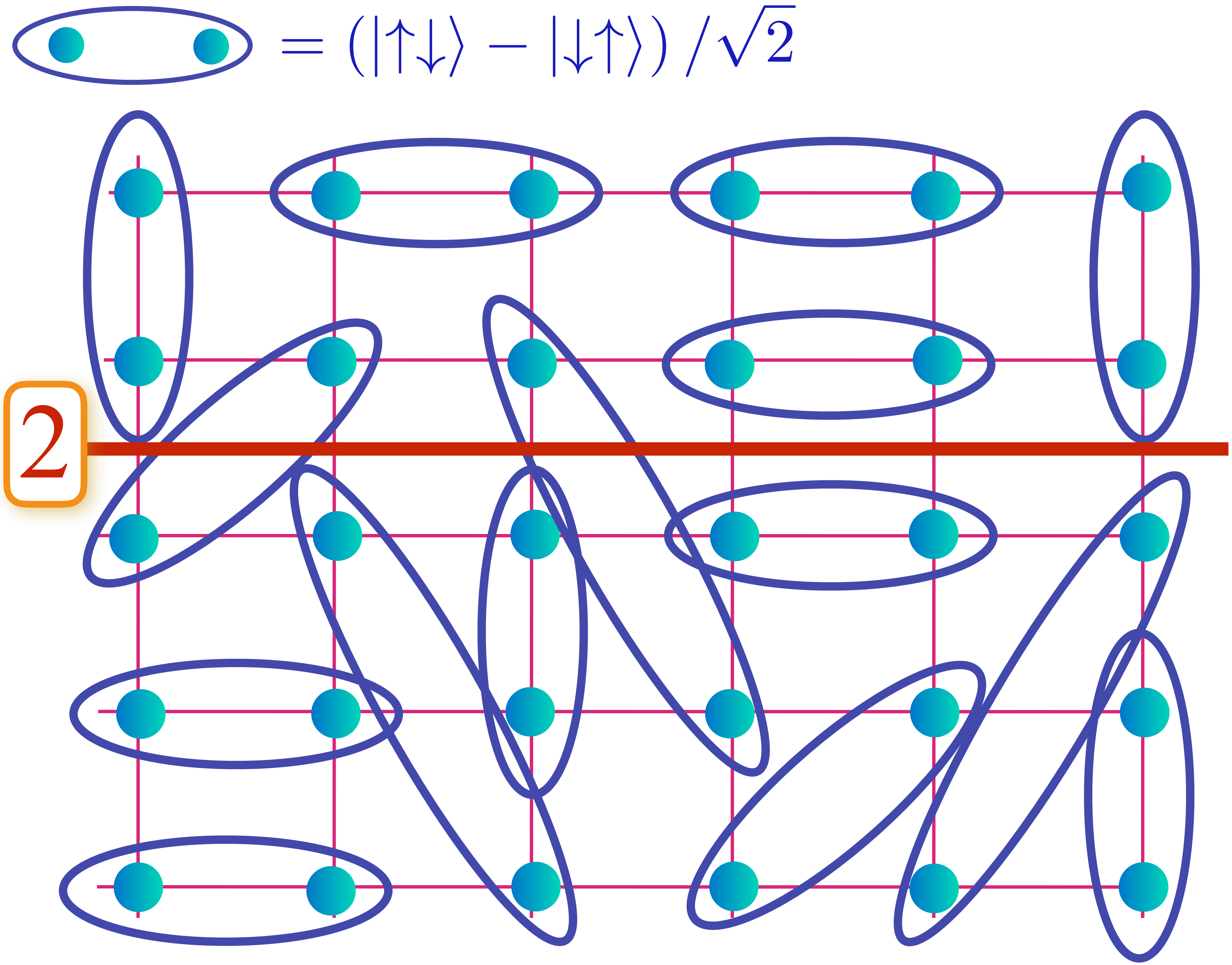


Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Why is this a TQFT ?



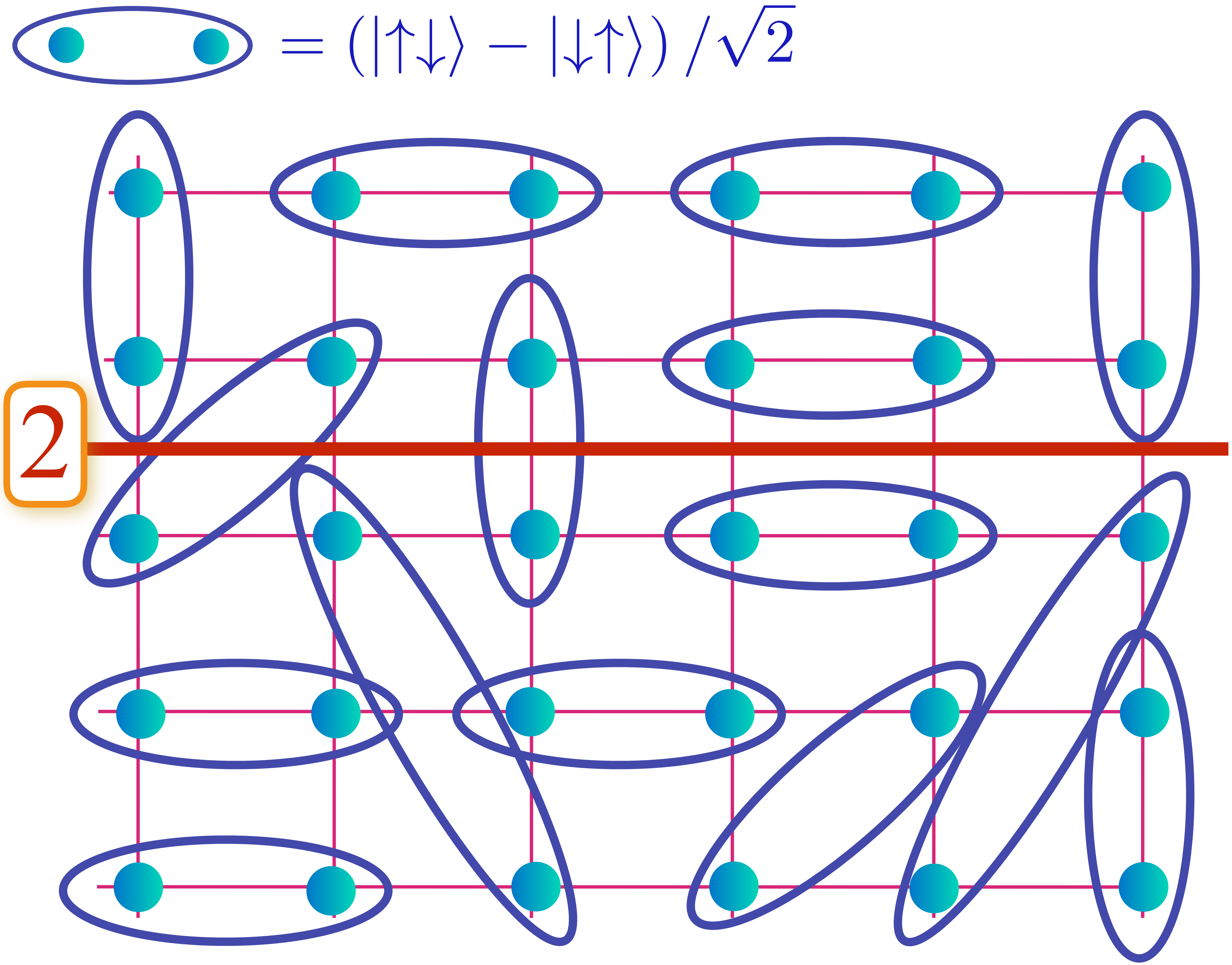
Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Why is this a TQFT ?

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

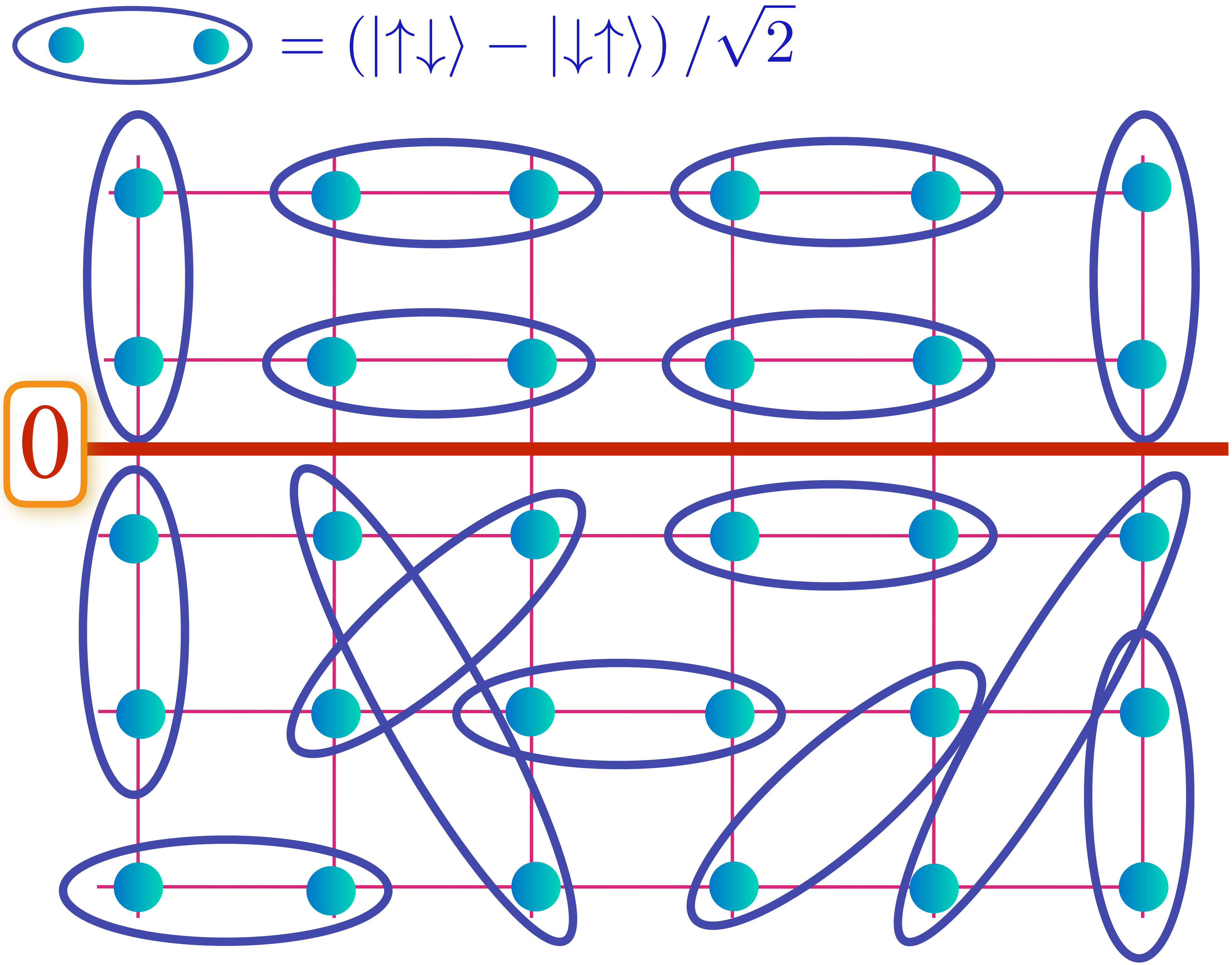


Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Why is this a TQFT ?

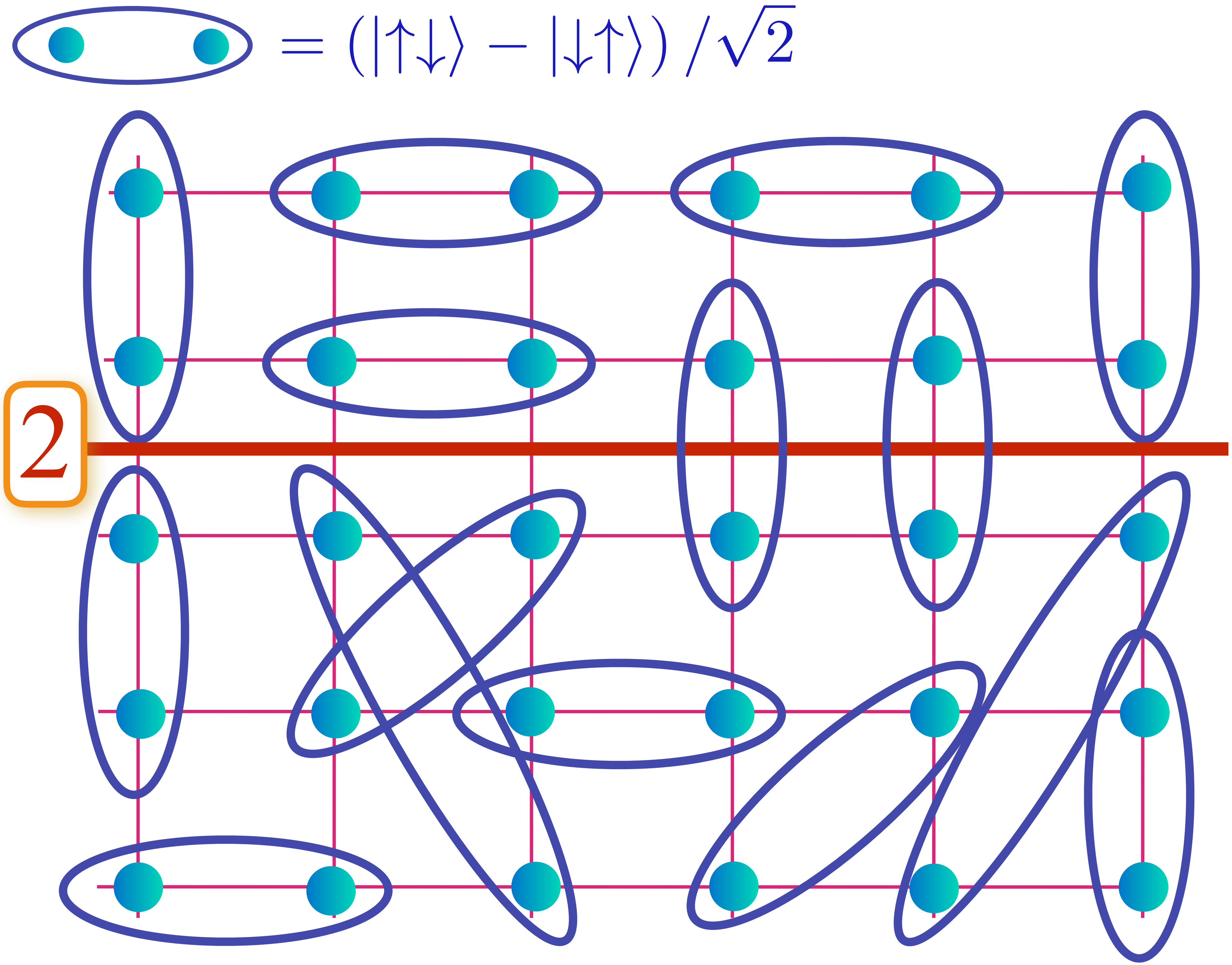


Place insulator on a torus:

Number of dimers crossing “branch-cut” is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)
S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

Why is this a TQFT ?



Place insulator on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

D.J. Thouless, PRB **36**, 7187 (1987)

S.A. Kivelson, D.S. Rokhsar and J.P. Sethna, Europhys. Lett. **6**, 353 (1988)

The TQFT

The $\mathbb{Z}_2$ spin liquid: Described by the simplest, non-trivial, topological field theory with time-reversal symmetry:

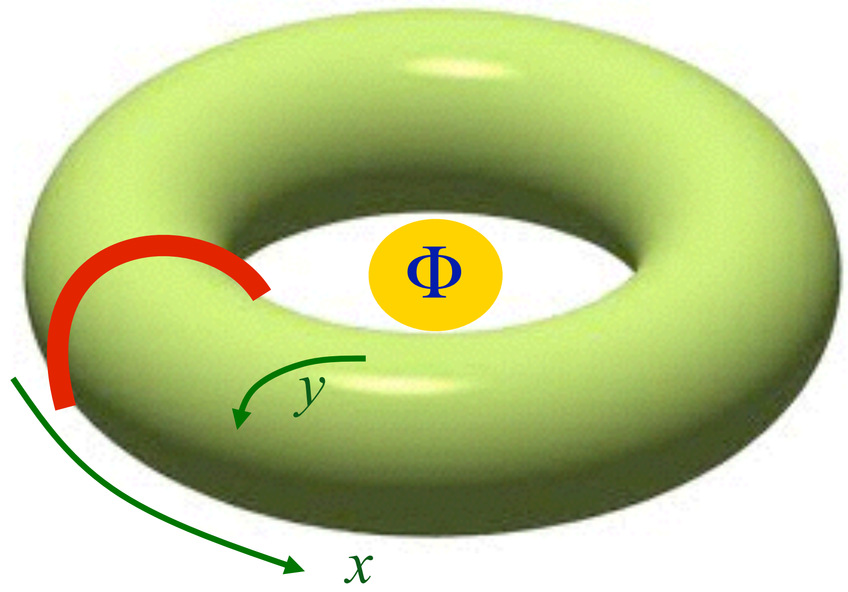
$$\mathcal{L} = \frac{1}{4\pi} K_{IJ} \int d^3x \, a^I \wedge da^J$$

where a^I, $I = 1, 2$ are U(1) gauge connections, and the K matrix is

$$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

N. Read and S. Sachdev, Phys. Rev. Lett. 66, 1773 (1991)
X.-G. Wen, Phys. Rev. B 44, 2664 (1991)
M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, Annals of Physics **310**, 428 (2004)

Momentum balance in the TQFT



R. A. Jalabert and S. Sachdev, Phys. Rev. B **44**, 686 (1991)

S. Sachdev and M. Vojta, J. Phys. Soc. Jpn **69**, Supp. B, 1 (1999)

T. Senthil and M. P. A. Fisher, PRB **62**, 7850 (2000)

The gauge field A, producing flux Φ, couples to one of the emergent gauge fields, a, of the TQFT by $\mathcal{L} = (1/(2\pi)) \int d^3x \, a \wedge dA$. Consequently, the flux insertion changes the quantum state $|\Psi\rangle$

$$|\Psi\rangle \Rightarrow W_y \, |\Psi\rangle \quad , \quad W_y \equiv \left(i \oint dy \, a_y \right)$$

The 'Wilson-loop' operator, W_y, is precisely the "branch-cut"! The Φ flux insertion therefore cycles between the degenerate states on the torus. Further, one can show that under a lattice translation, T_x, in a model with $S = 1/2$ spin in each unit cell, $T_x^{-1} W_y T_x = \exp(i\pi L_y) W_y$. Consequently, the momentum transfer (per spin) by the flux insertion is

$$\Delta P_x = \pi L_y = \left(\frac{2\pi}{L_x} \right) \frac{1}{2} \, L_x L_y$$

This is precisely the value expected for a quantum system at a density $n = 1/2$ (per spin) by the general argument.

1. Flux insertion on the torus, and momentum balance in *any* quantum state.

2. Fermi surface size in FL

3. Insulating spin liquids and topological quantum field theory (TQFT)

4. A one-band FL* model for the PG metal

Q: Can we have a metal with a Fermi surface of size p with electron-like (charge e, spin 1/2) quasiparticles?

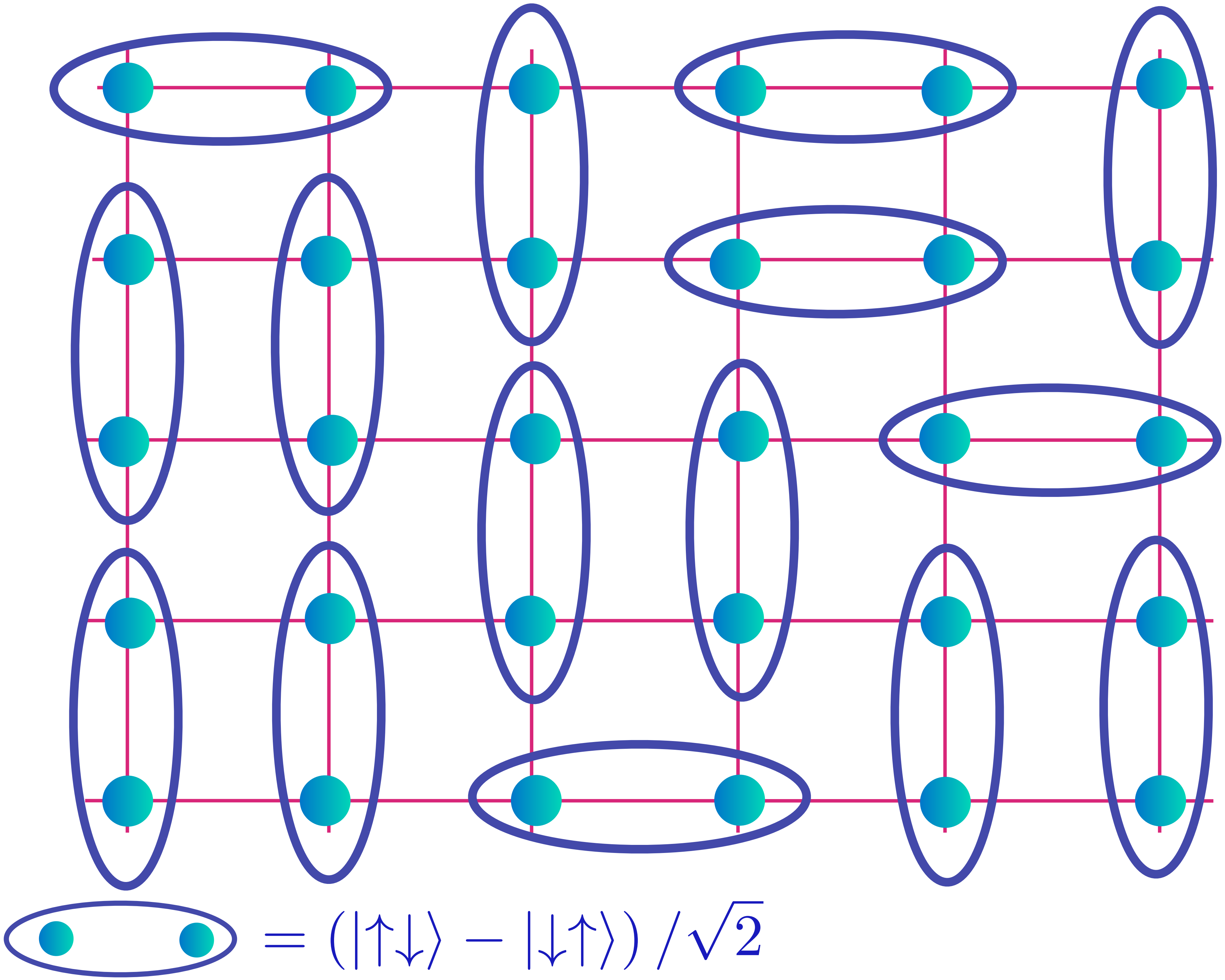
Q: Can we have a metal with a Fermi surface of size p with electron-like (charge e, spin 1/2) quasiparticles?

A: A Z_2-FL* metal has 2 classes of low energy excitations:

- Electron-like quasiparticles on a Fermi surface of size p
- A sector described by a TQFT

The classes combine to satisfy the momentum balance at a density $1+p$.

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett* **90**, 216403 (2003)

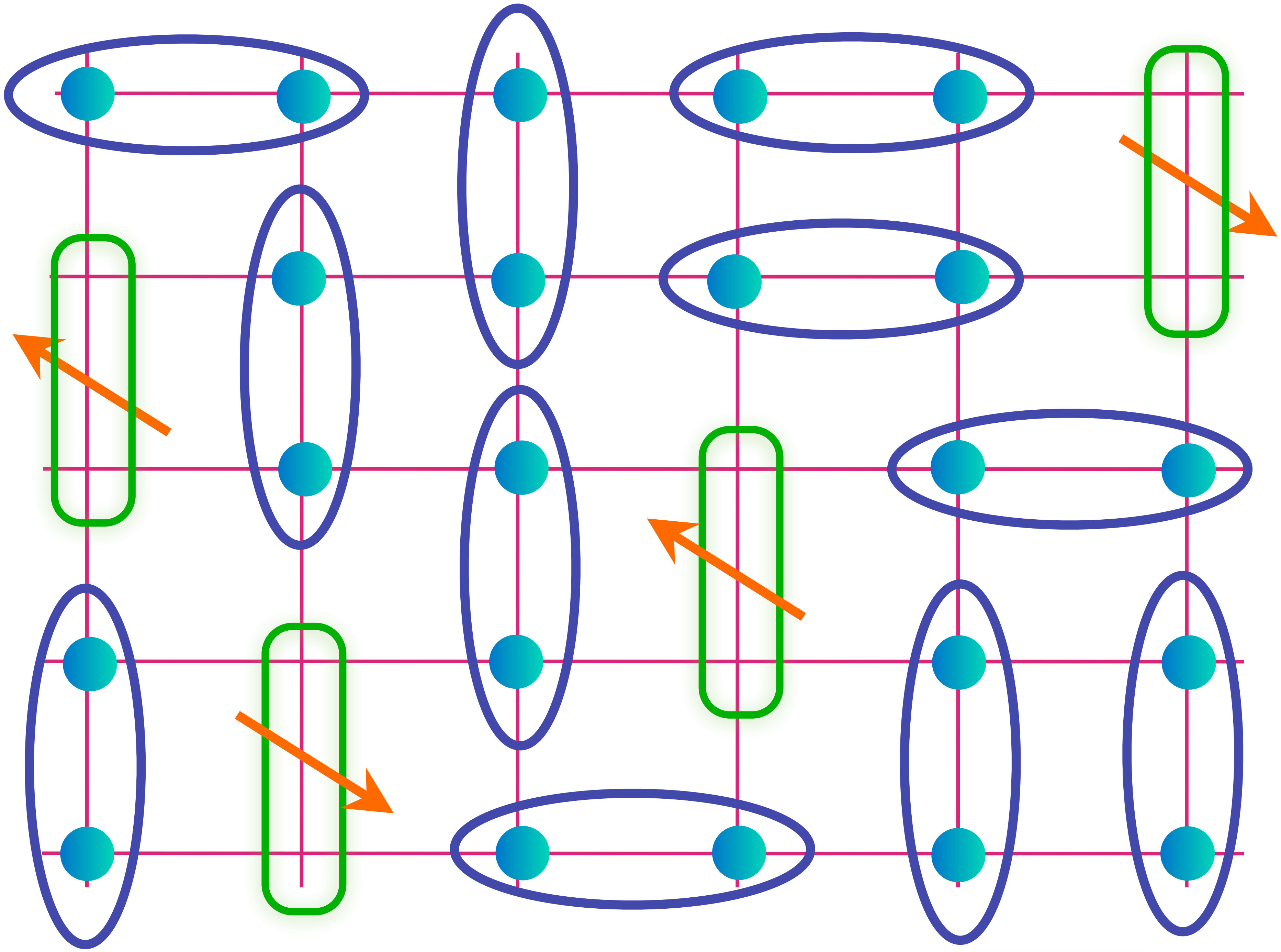


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

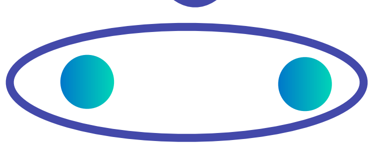
Start with a spin liquid and then remove electrons

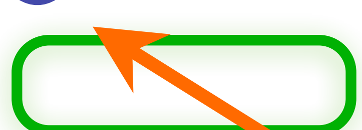
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

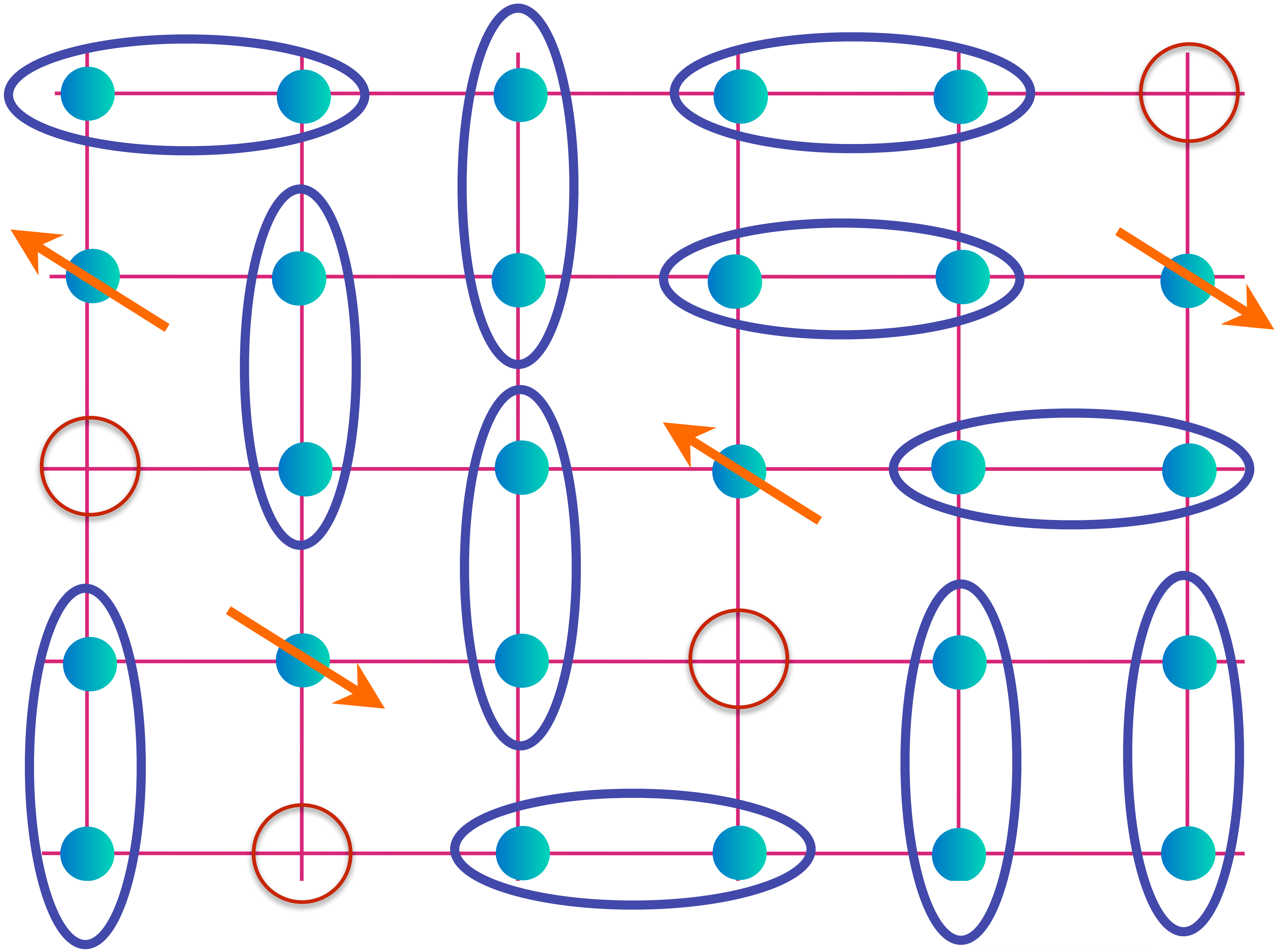
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

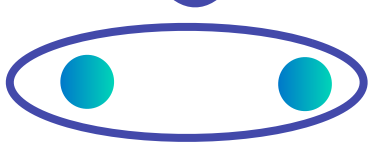
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

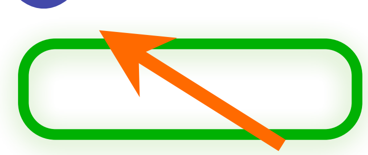
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

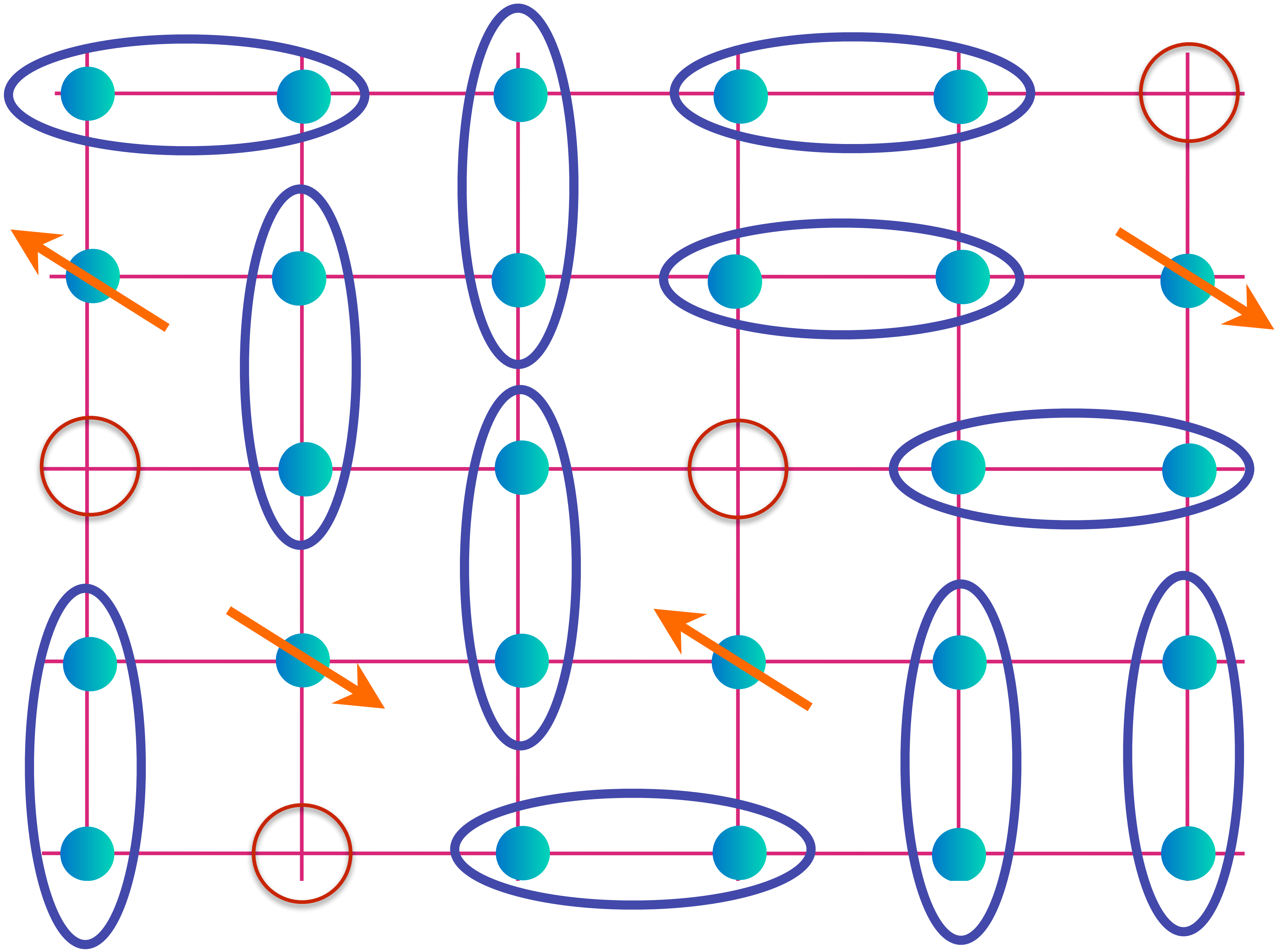
 $= \left(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle \right) / \sqrt{2}$

 $= \left(|{\uparrow \circ}\rangle + |{\circ \uparrow}\rangle \right) / \sqrt{2}$

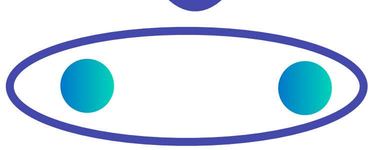
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

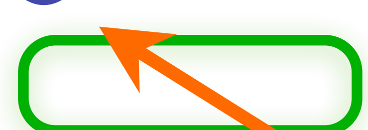
FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

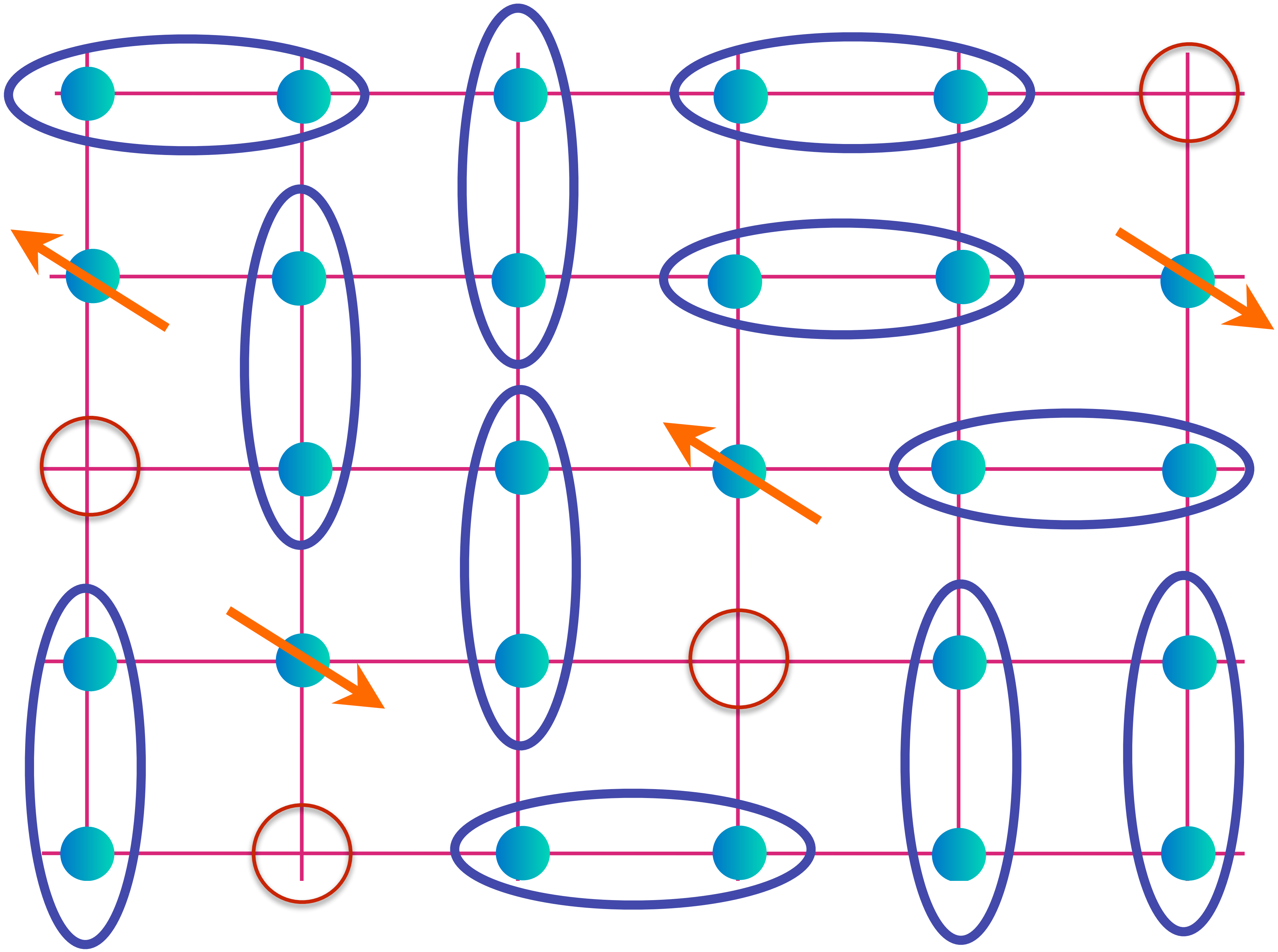
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$

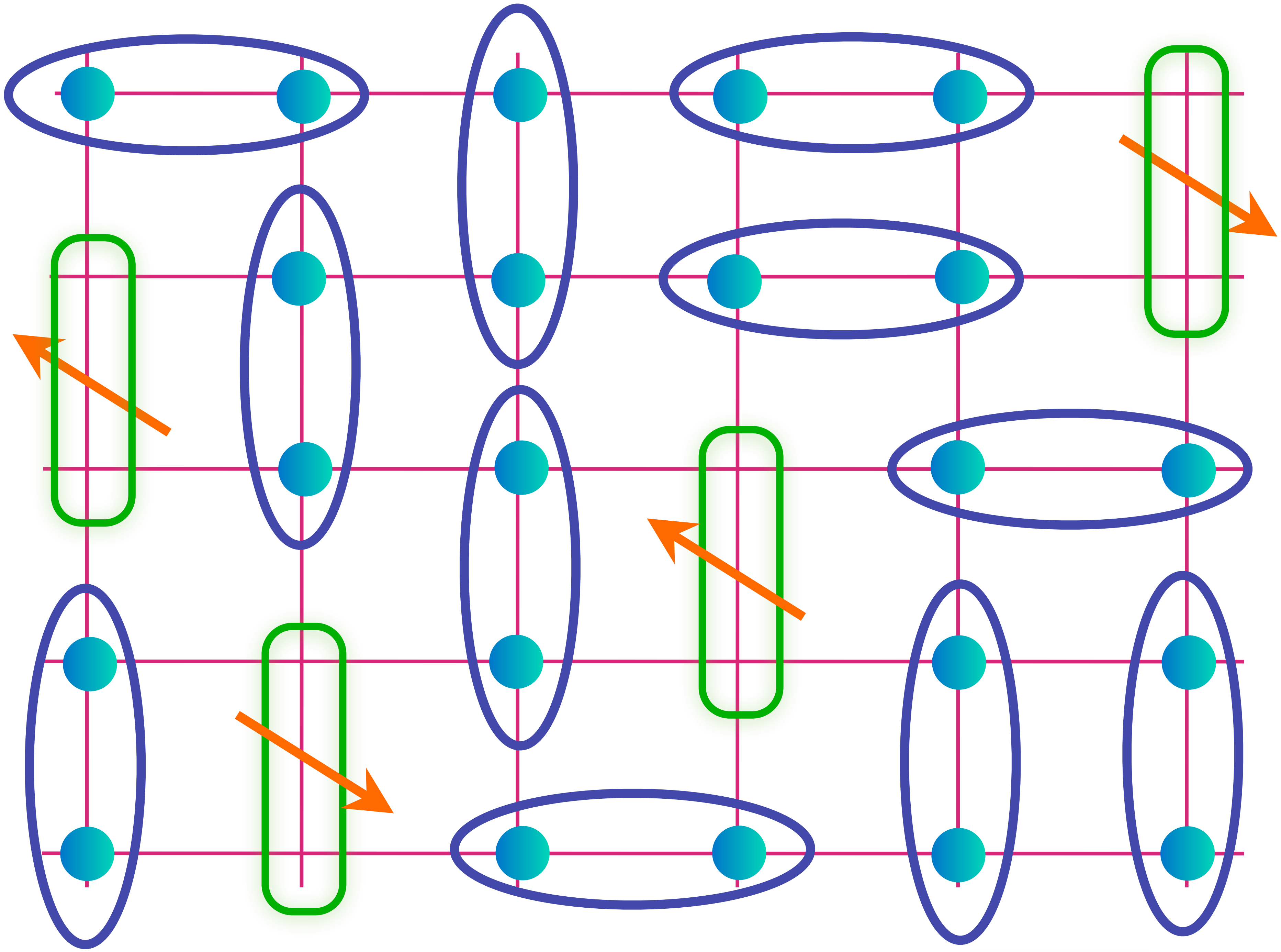
$$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

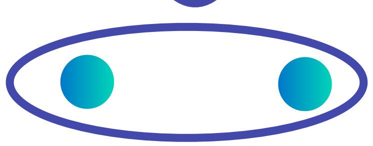
Quasiparticles of FL*

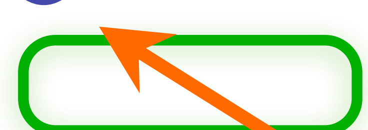
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)

R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Each green "dimer" is a "bound state" of a vacancy and a free spin

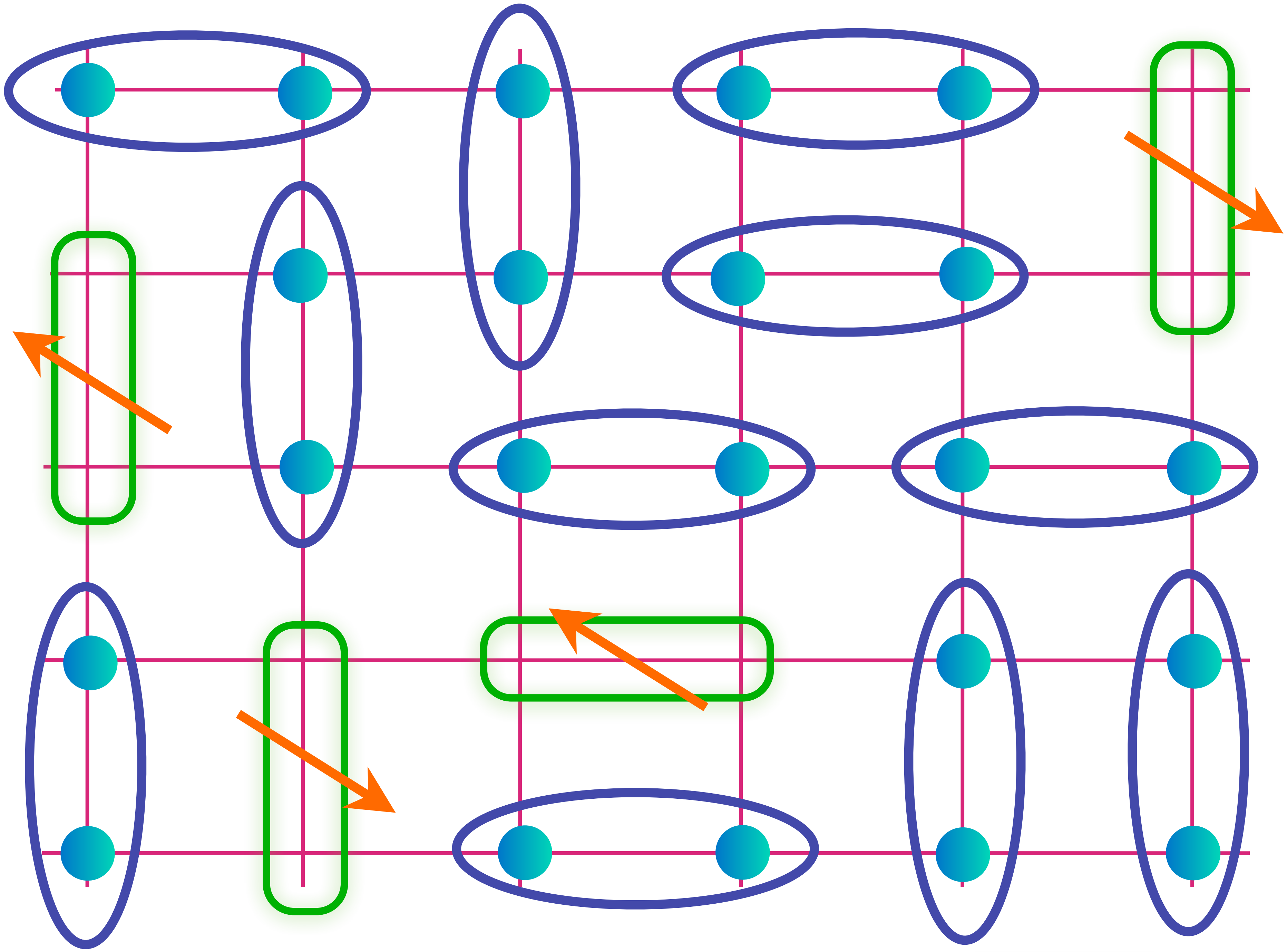
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

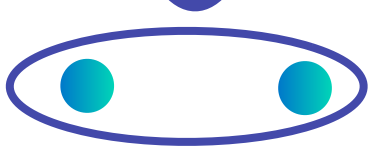
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

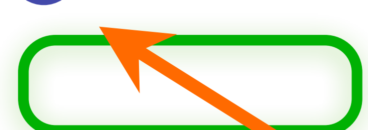
Quasiparticles of FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

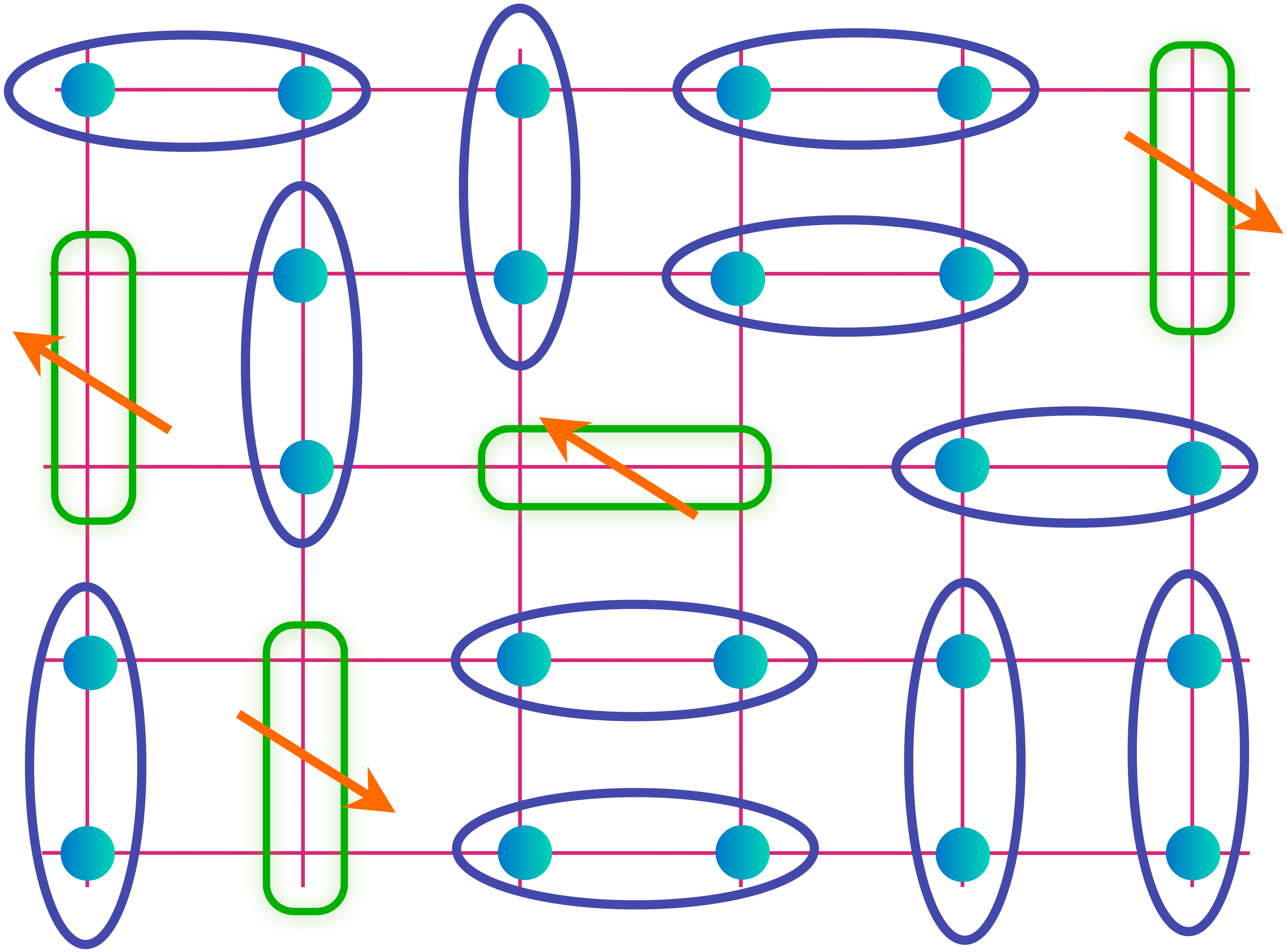
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

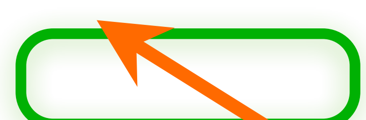
Quasiparticles of FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

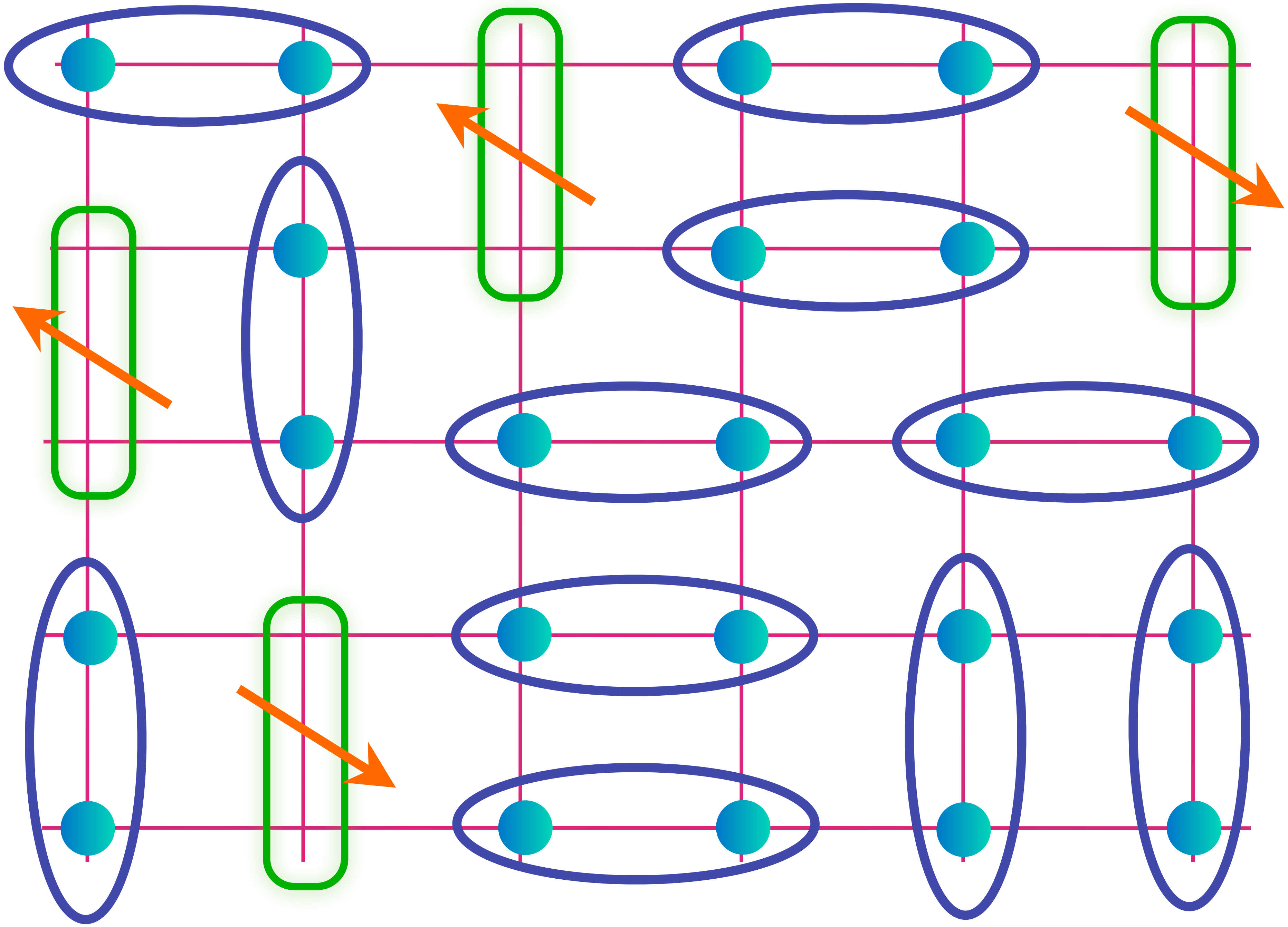
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

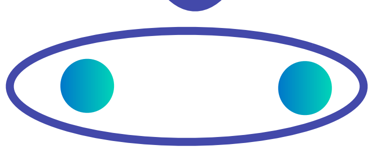
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

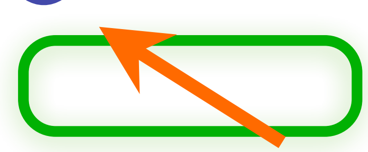
Quasiparticles of FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size *p* of electrons

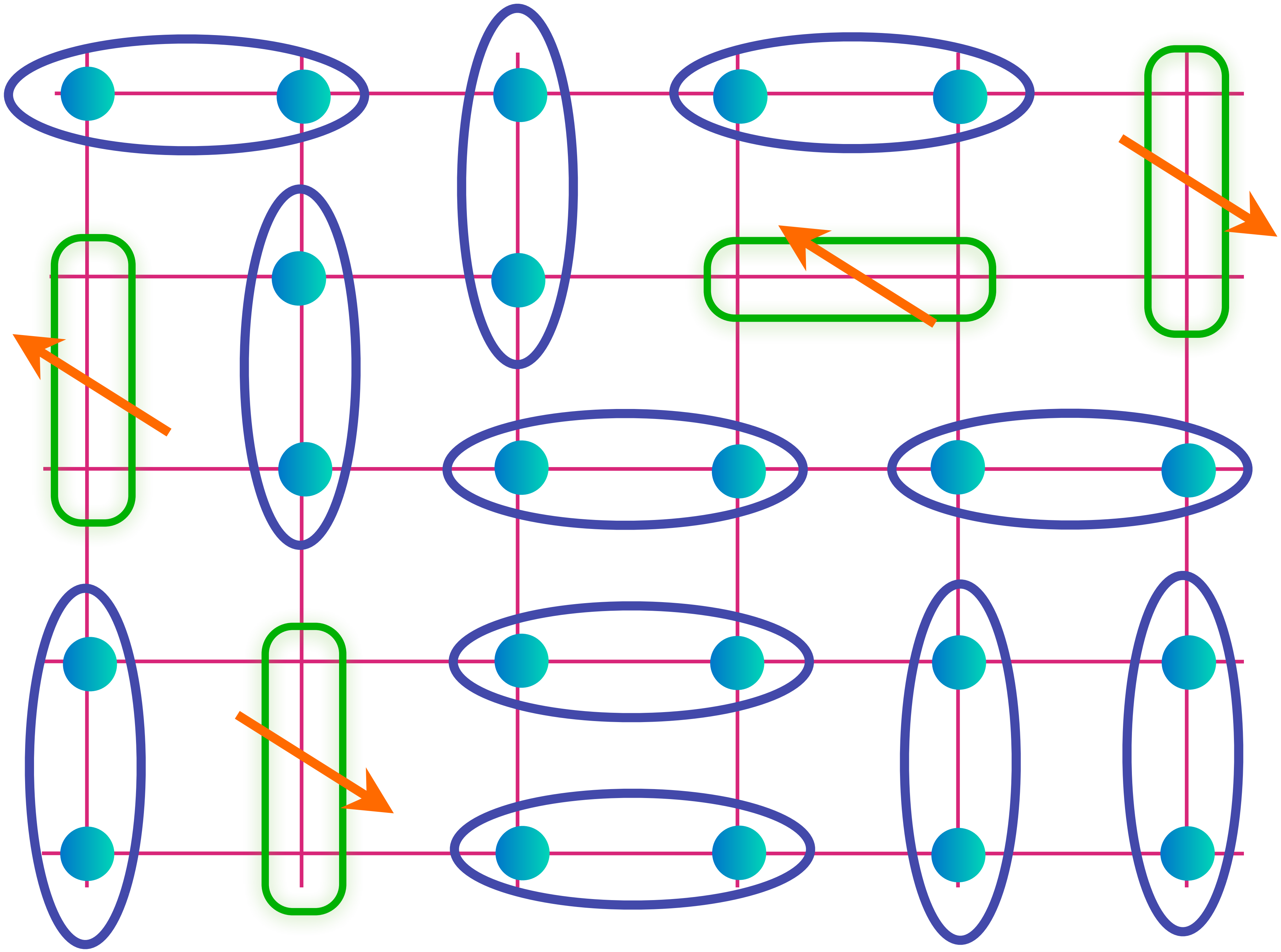
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

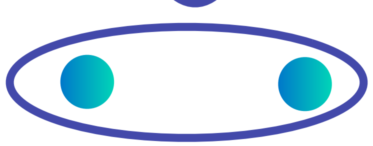
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Quasiparticles of FL*

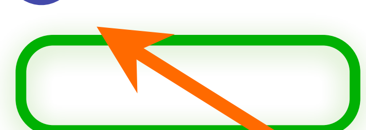
S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons



$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

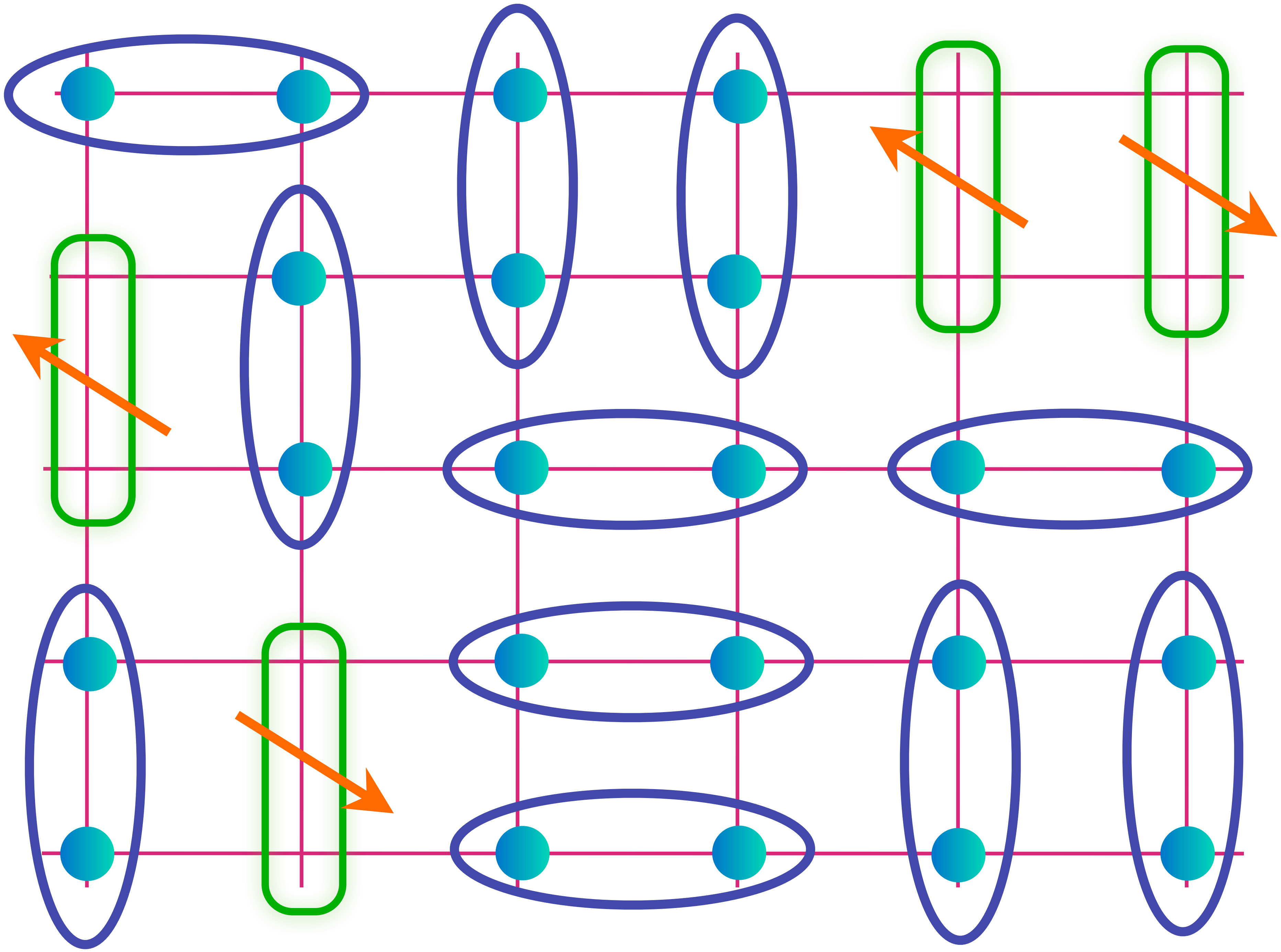


$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

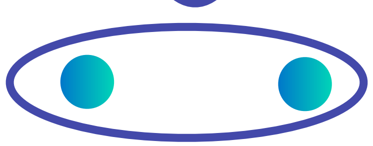
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

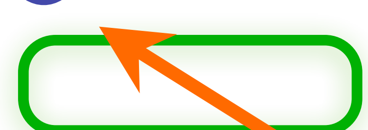
Quasiparticles of FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile $S=1/2$, charge $+e$ fermionic dimers: form a Fermi surface of size p of electrons

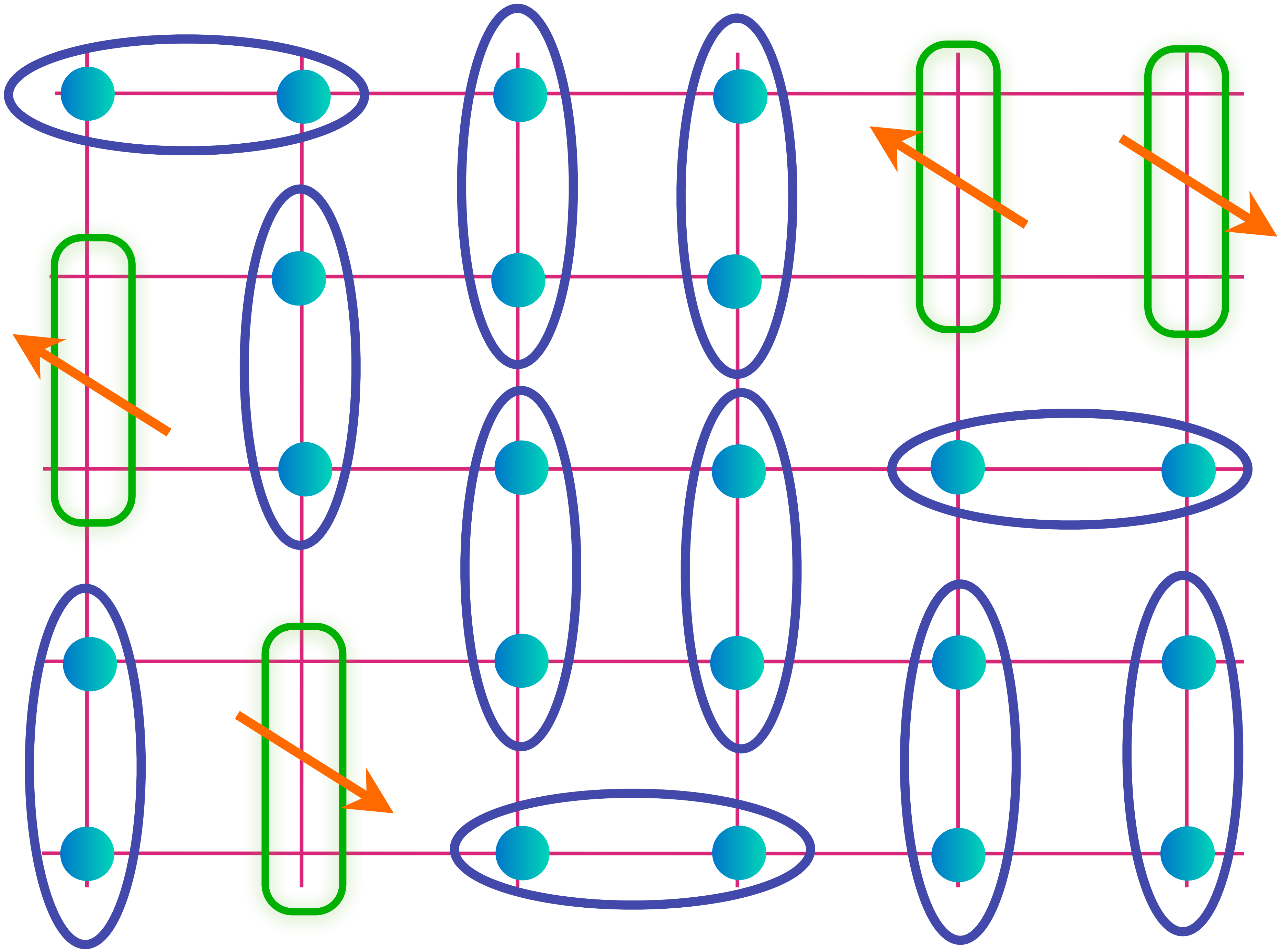
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

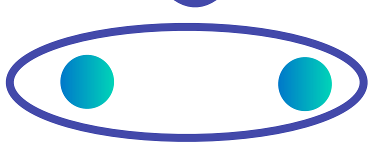
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

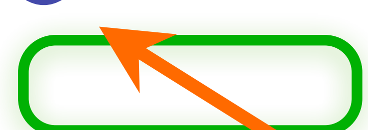
Quasiparticles of FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size p of electrons

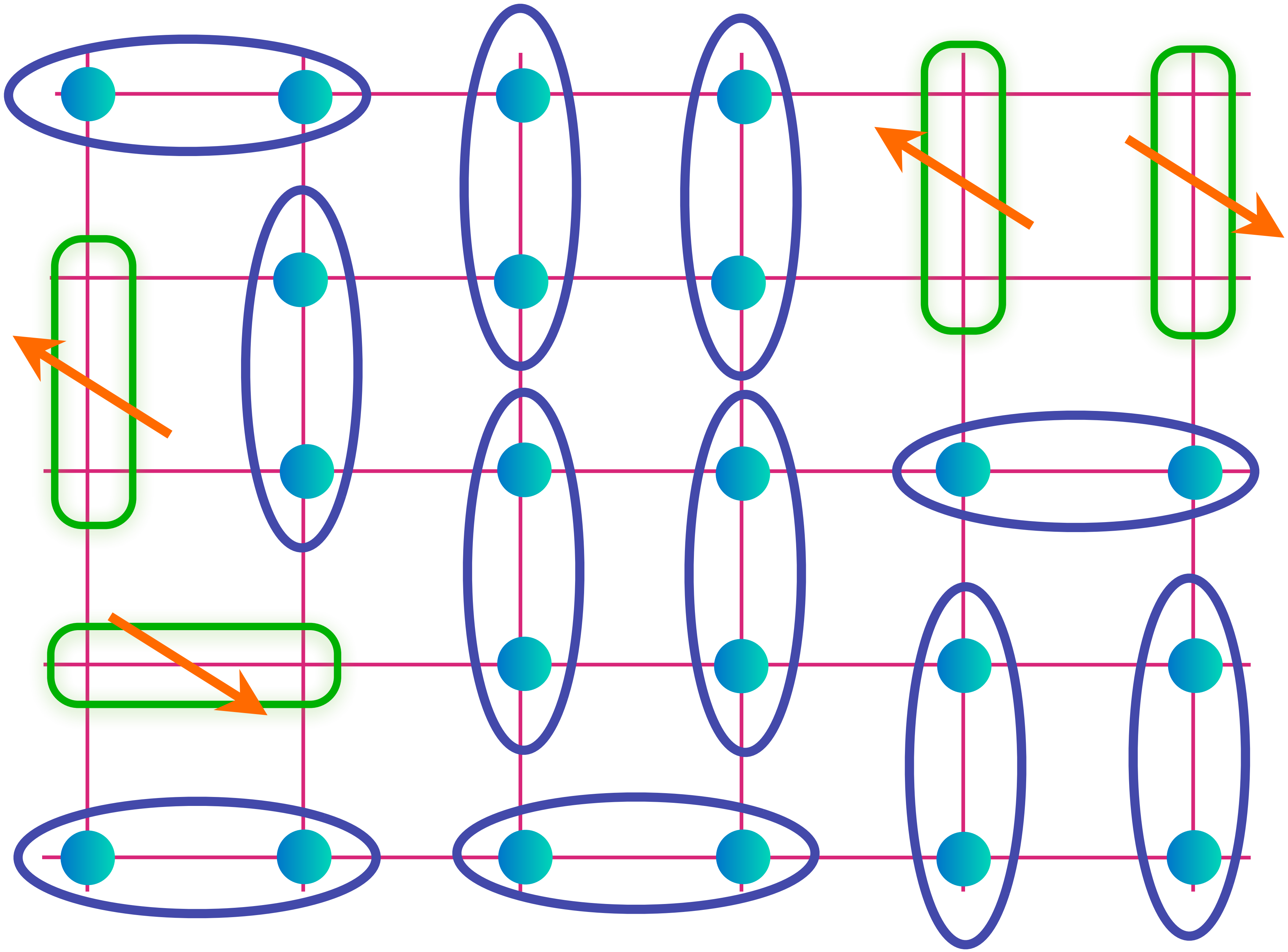
 $= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$

 $= (|\uparrow\circ\rangle + |\circ\uparrow\rangle)/\sqrt{2}$

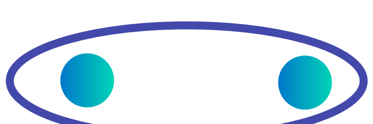
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

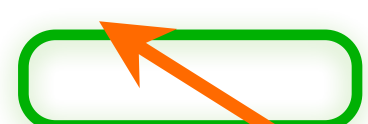
Quasiparticles of FL*

S. Sachdev PRB **49**, 6770 (1994); X.-G. Wen and P.A. Lee PRL **76**, 503 (1996)
R. K. Kaul, A. Kolezhuk, M. Levin, S. Sachdev, and T. Senthil, PRB **75**, 235122 (2007)



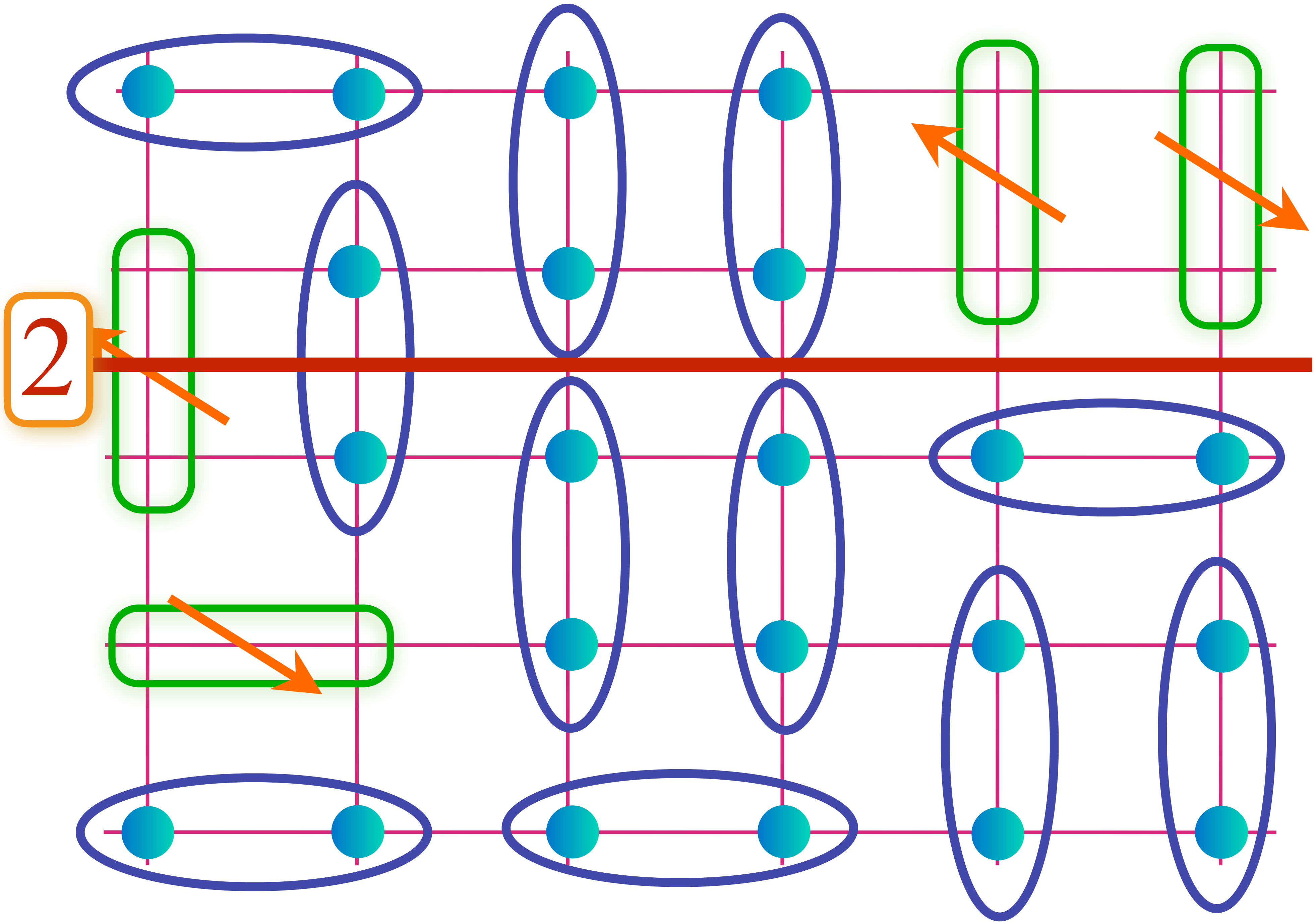
Mobile S=1/2, charge +e fermionic dimers: form a Fermi surface of size *p* of electrons

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

FL* as a TQFT



Place FL* on a torus:

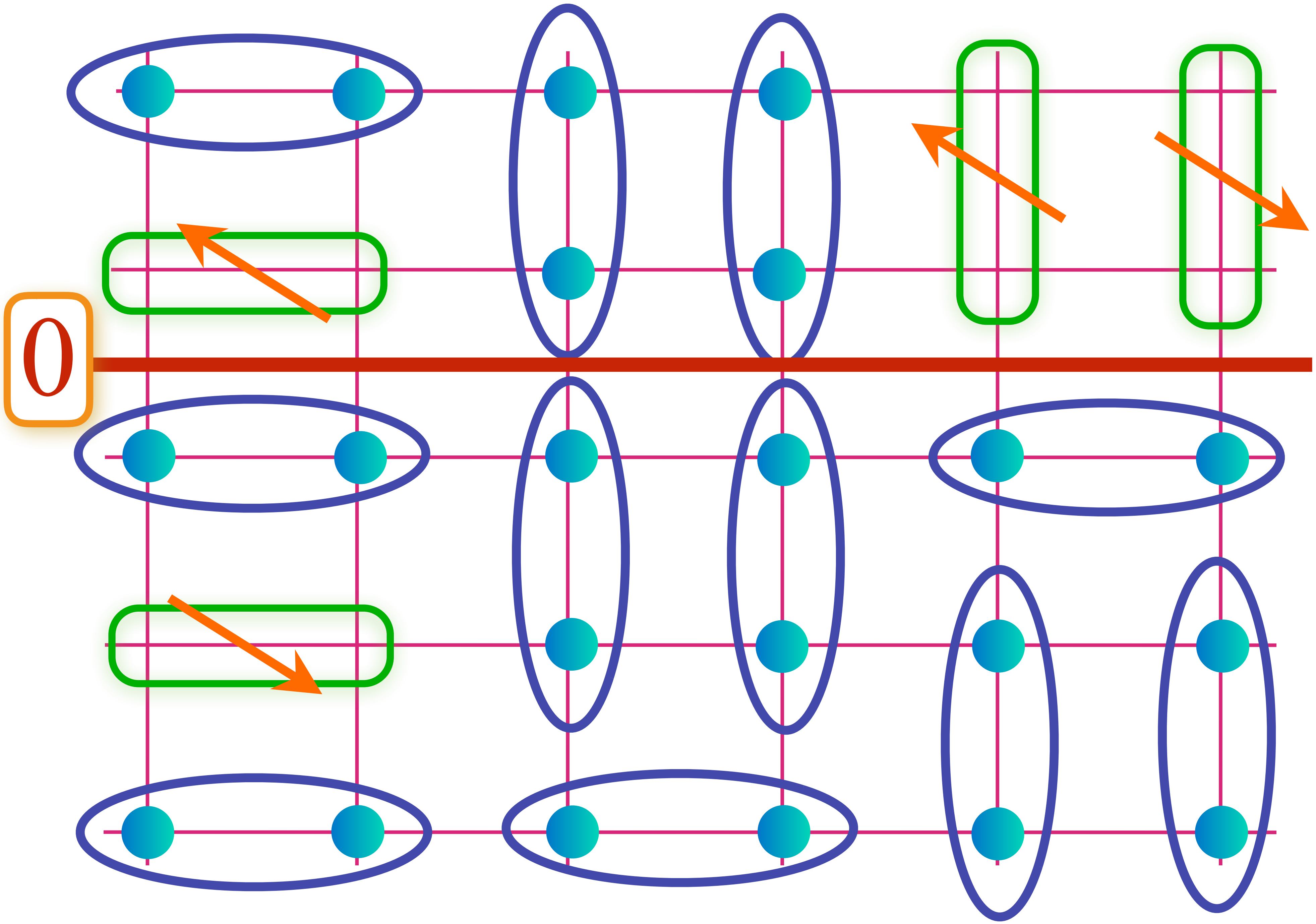
Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

(blue ellipse) $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

(green box) $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL* as a TQFT



Place FL* on a torus:

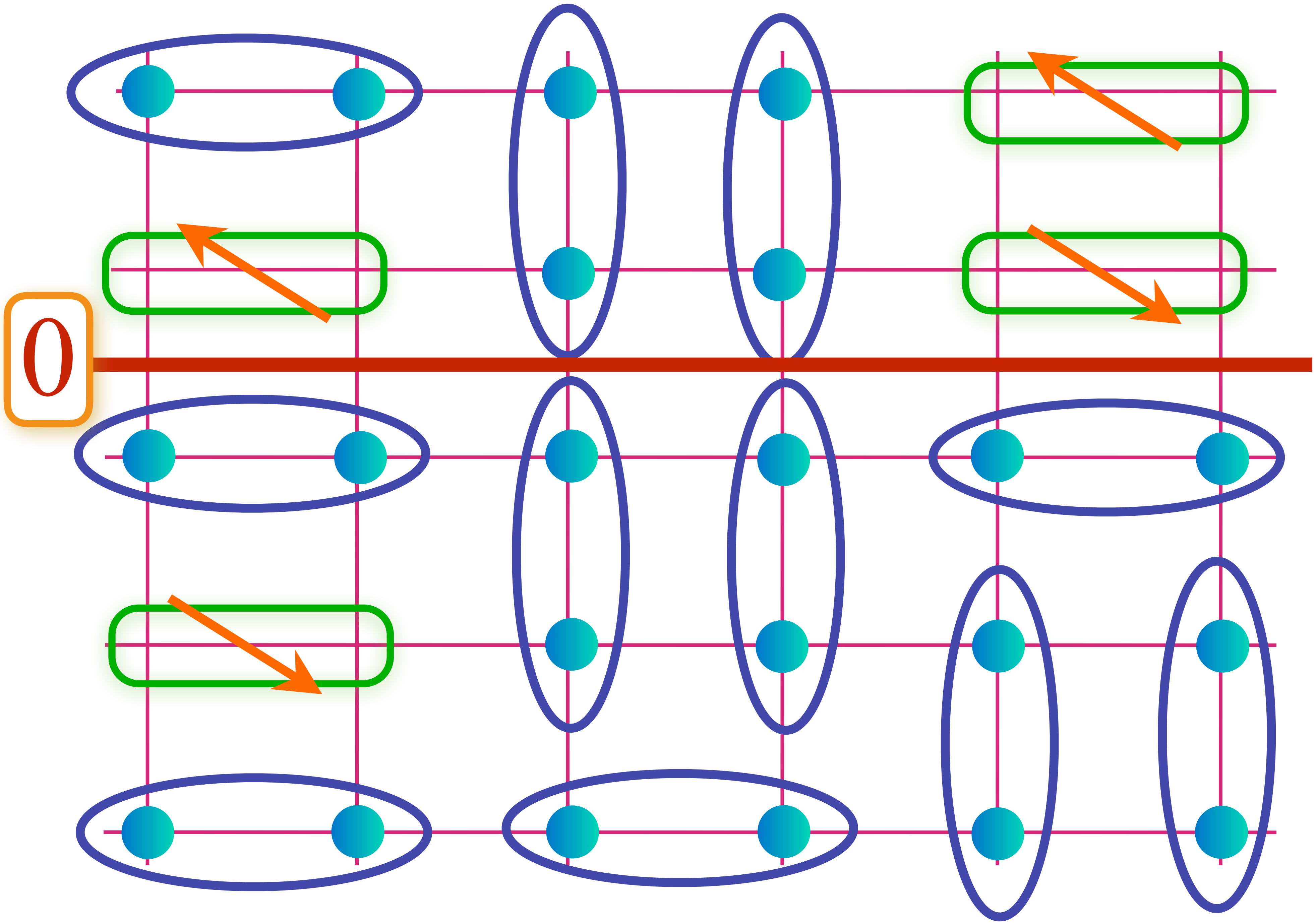
Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

$= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

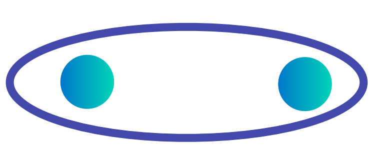
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

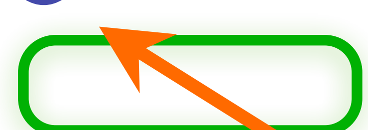
FL* as a TQFT



Place FL* on a torus:

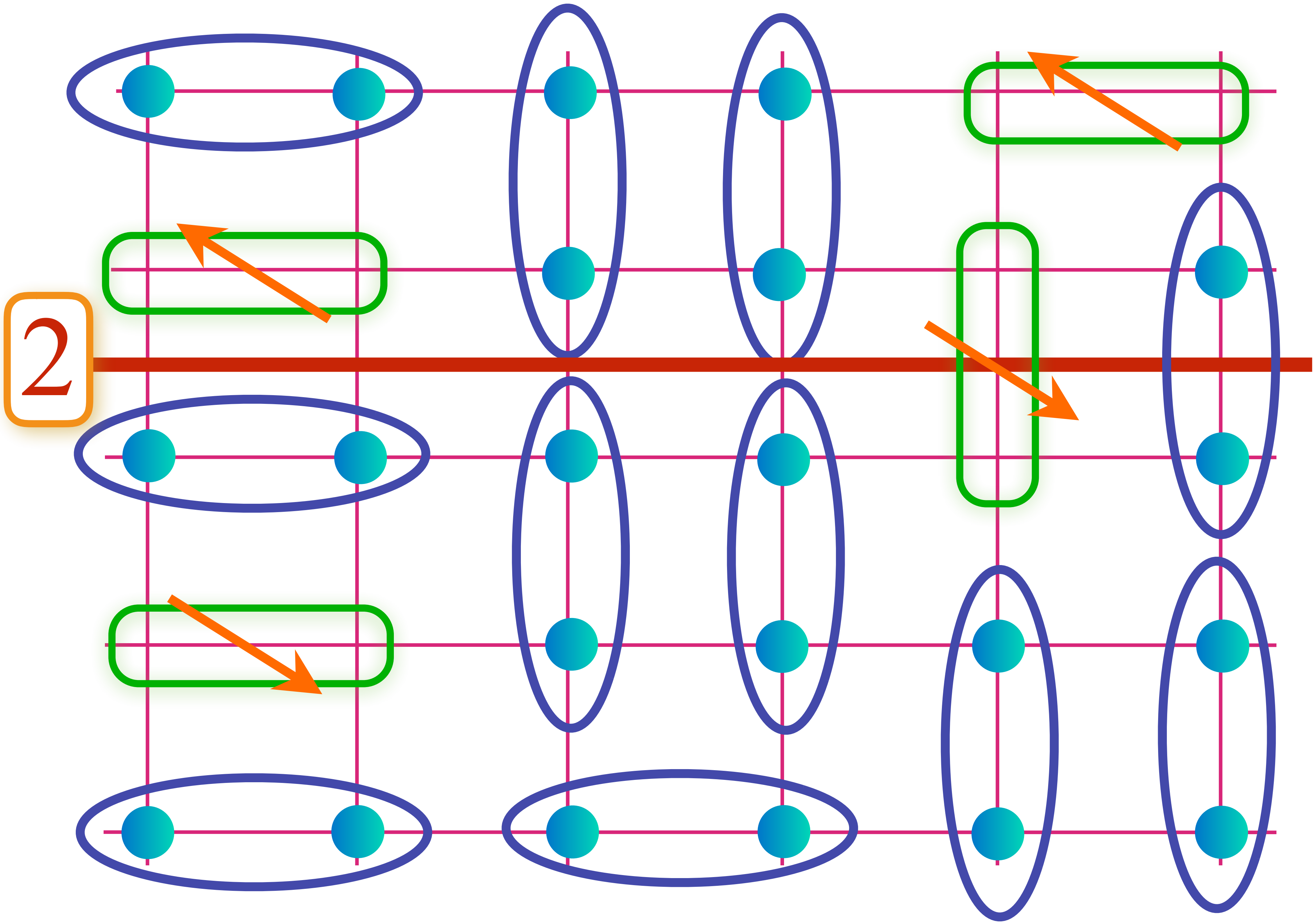
Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle \right) / \sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

FL* as a TQFT



Place FL* on a torus:

Number of dimers crossing "branch-cut" is conserved modulo 2: there are nearly degenerate states with odd and even dimer-cuts

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$ $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004)

1. Flux insertion on the torus, and momentum balance in *any* quantum state.

2. Fermi surface size in FL

3. Insulating spin liquids and topological quantum field theory (TQFT)

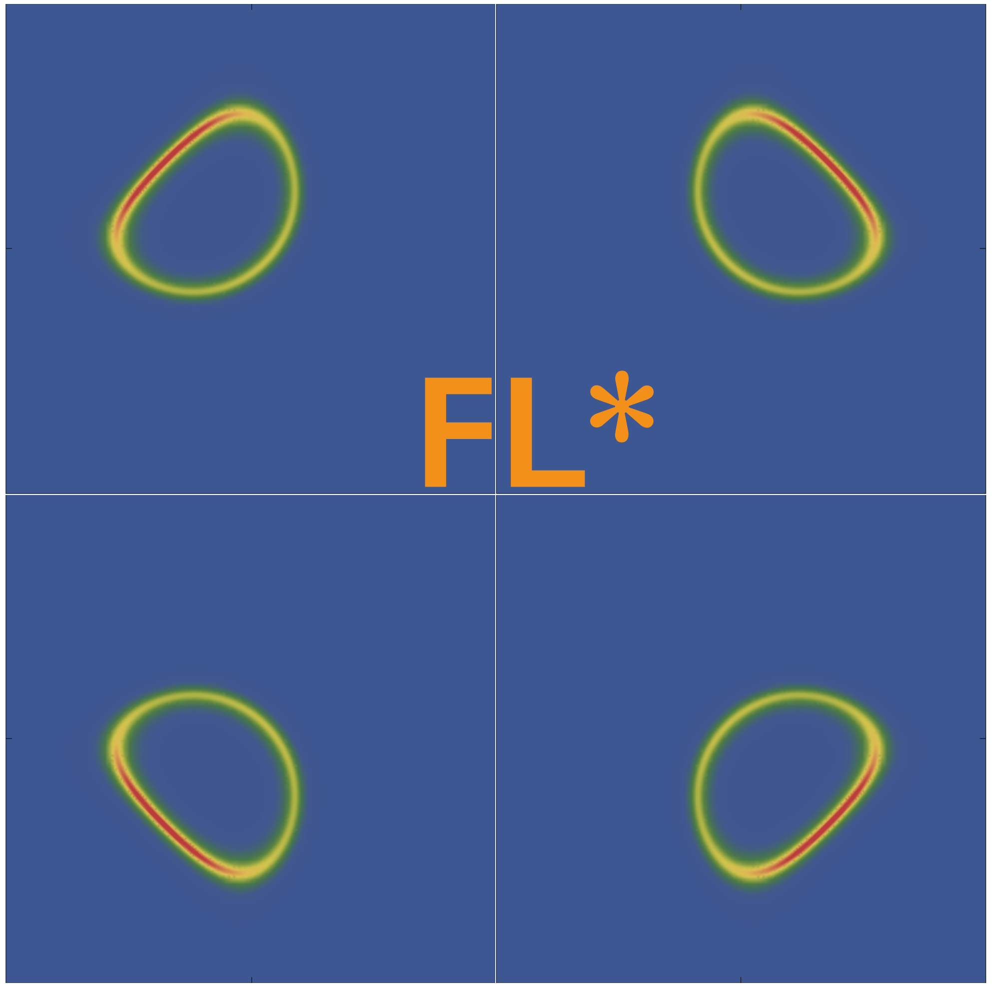
4. A one-band FL* model for the PG metal

Evidence for pseudogap metal as FL*

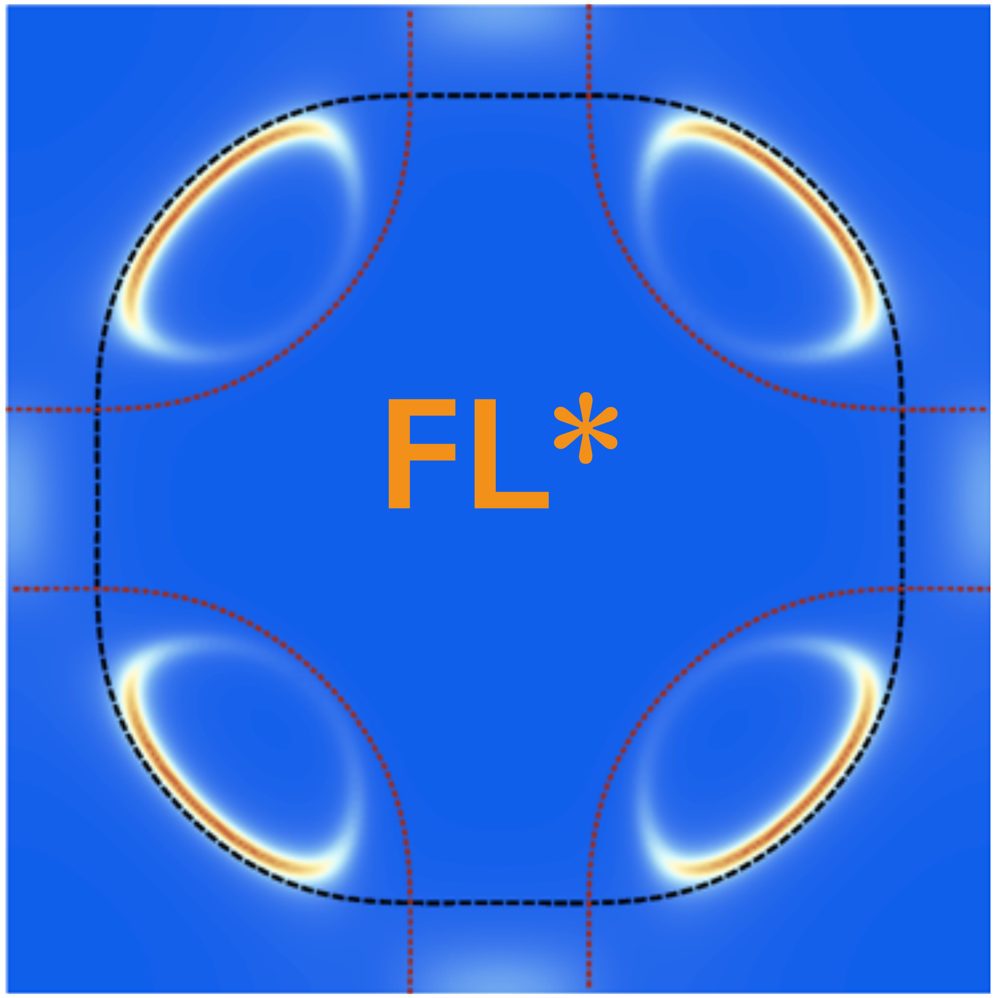
Evidence for pseudogap metal as FL*

- FL* Fermi pockets are compatible with photoemission at high T.

Fermi surfaces in one-band models of FL*



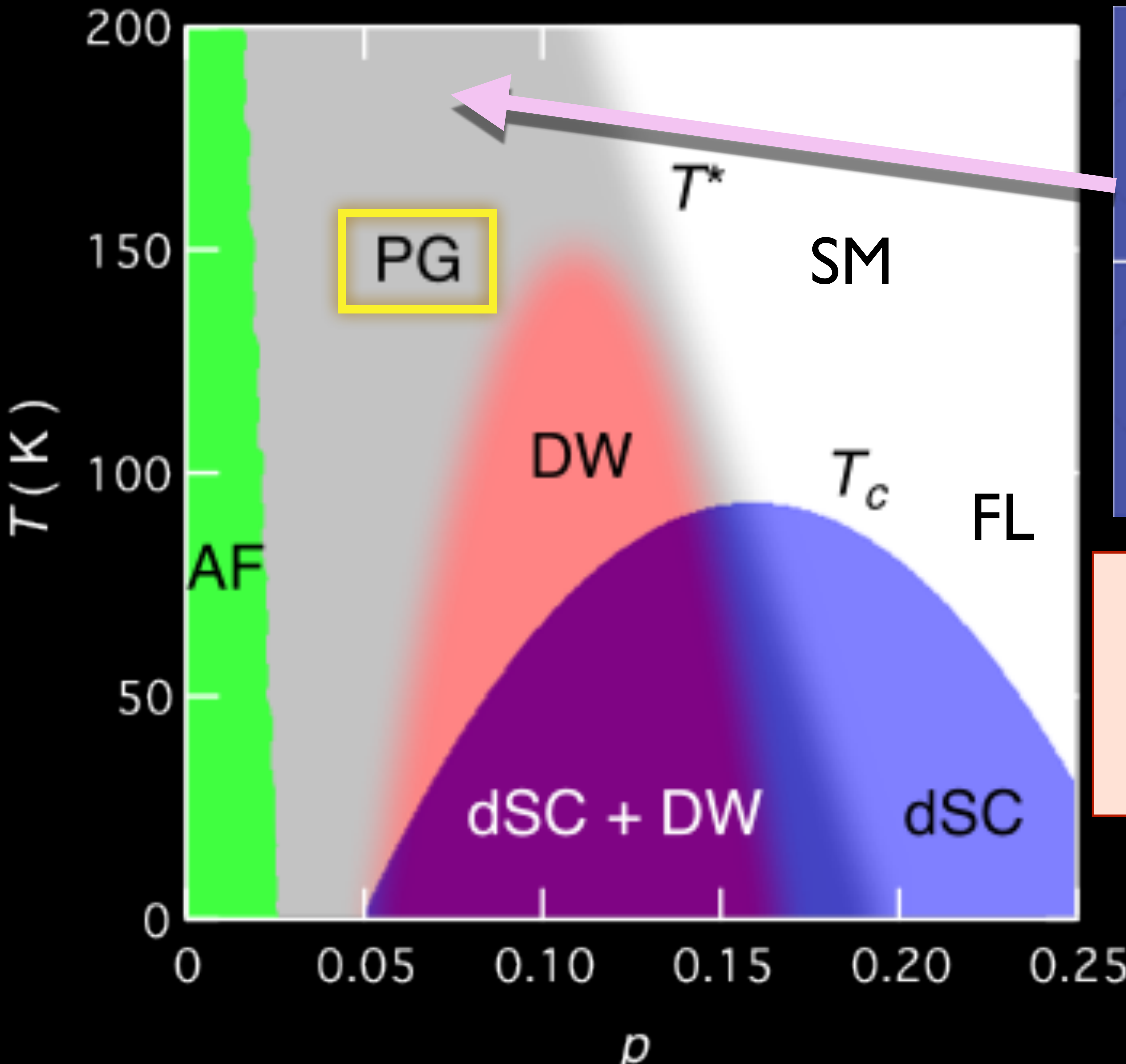
M. Punk, A. Allais, and S. Sachdev,
PNAS **112**, 9552 (2015)

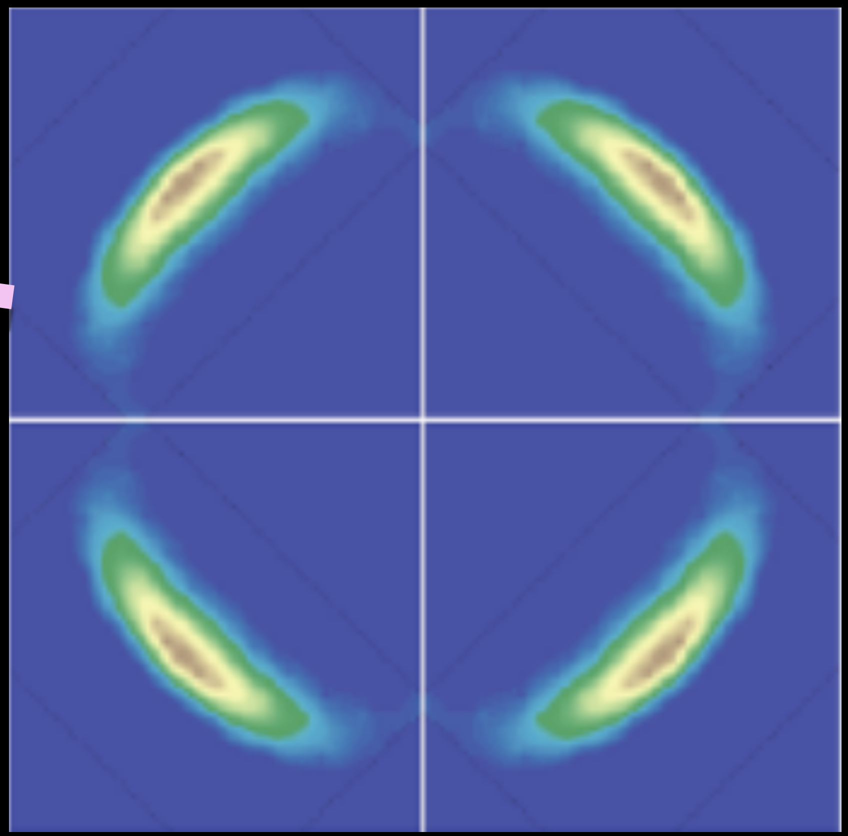


Y. Qi and S. Sachdev,
Phys. Rev. B **81**, 115129 (2010)

"Back side" of Fermi surface is suppressed for observables which change electron number in the square lattice

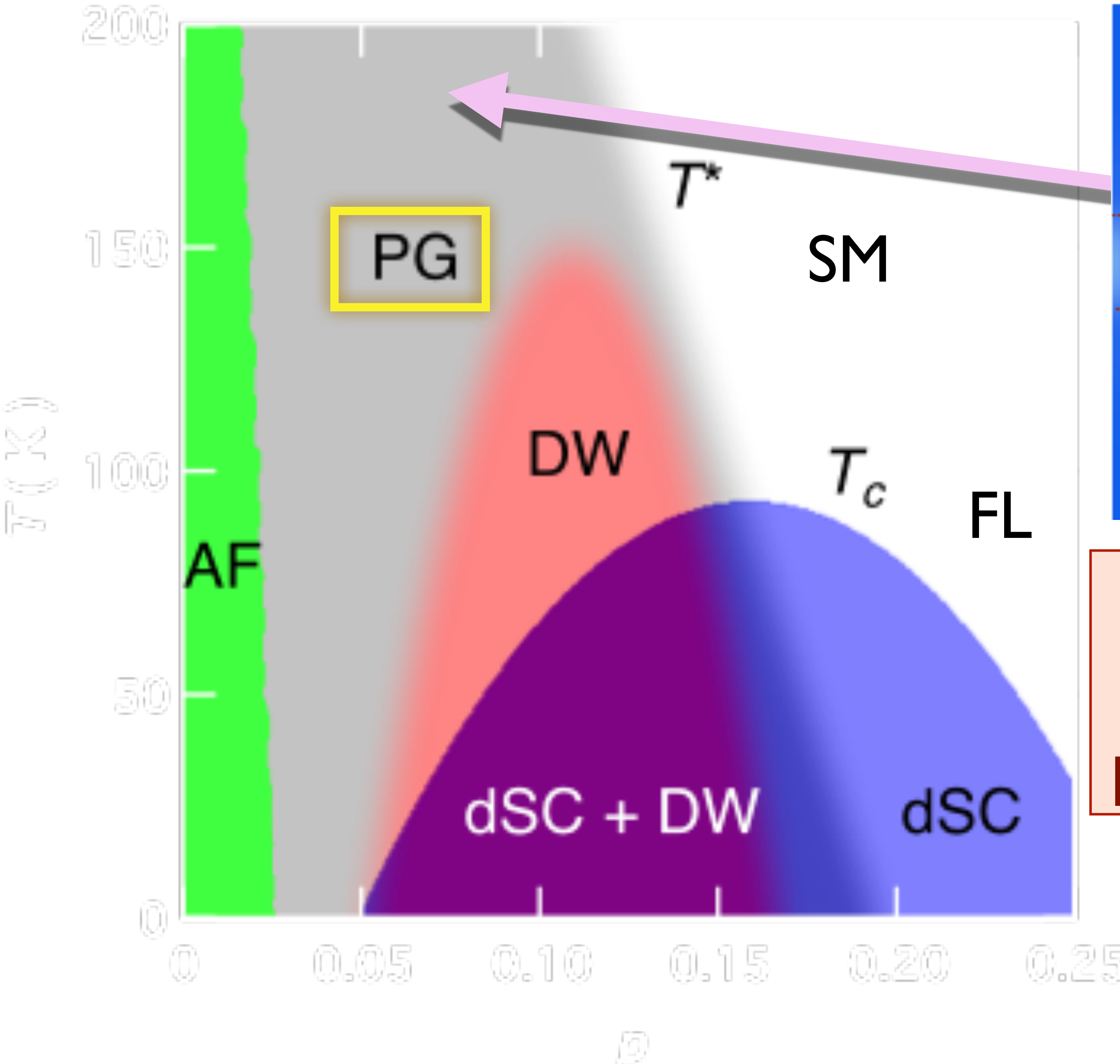
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

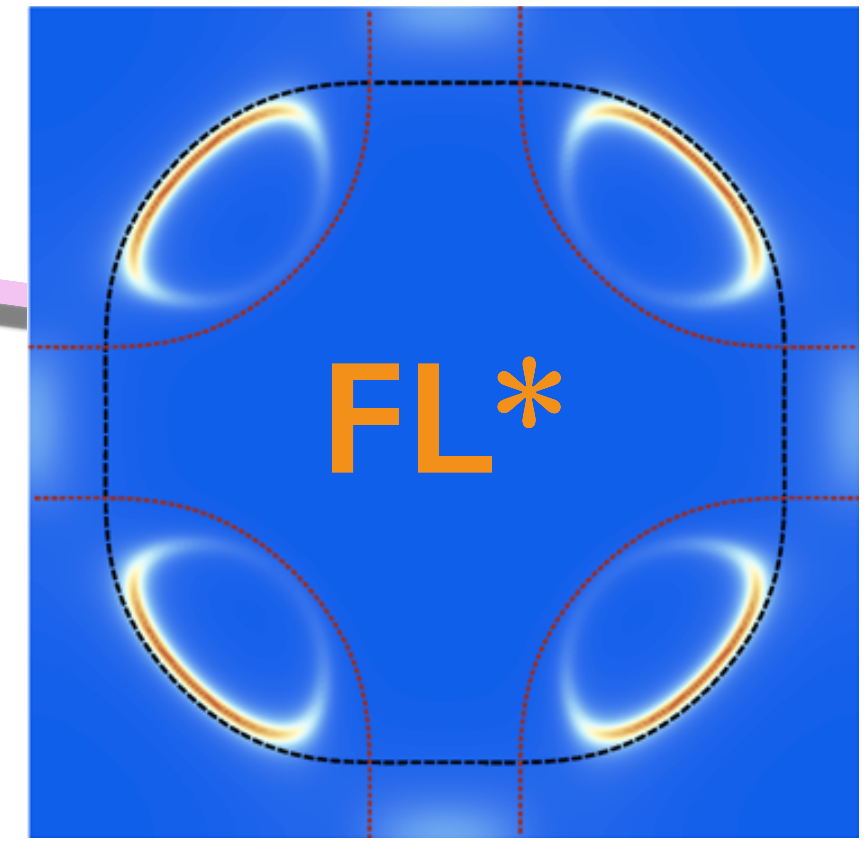




Photoemission in pseudogap metal

Y. Qi and S. Sachdev, Phys. Rev. B **81**, 115129 (2010)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)





Photoemission in FL* model of pseudogap metal

Evidence for pseudogap metal as FL*

- FL* Fermi pockets are compatible with photoemission at high T.

- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).

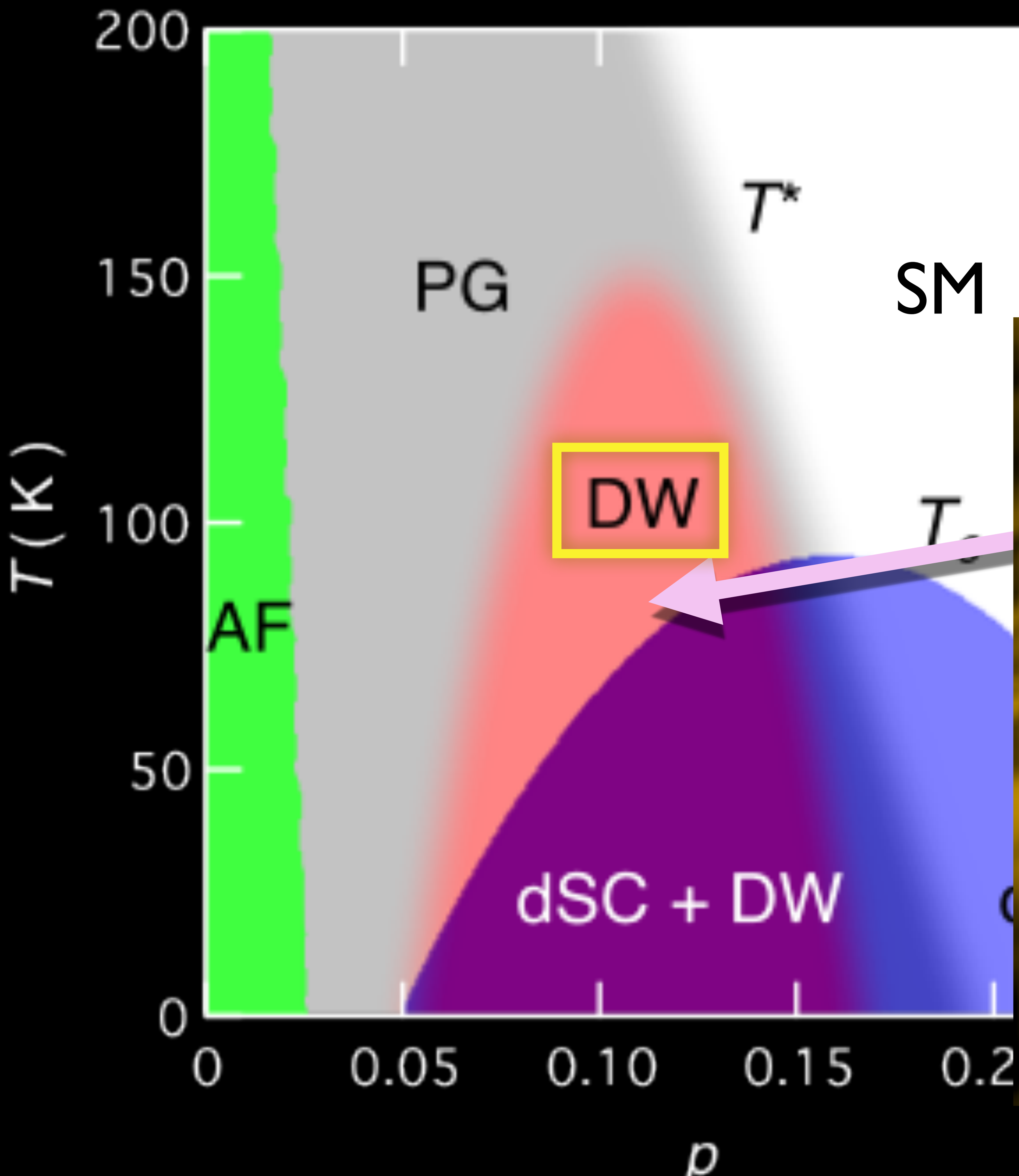
Evidence for pseudogap metal as FL*

- FL* Fermi pockets are compatible with photoemission at high T.
- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).
- Magnetoresistance $\rho_{xx} \sim \tau^{-1}\left(1 + aH^2T^2\right)$ with $\tau \sim T^{-2}$ (Chan *et al.*, PRL **113**, 177005 (2014)).

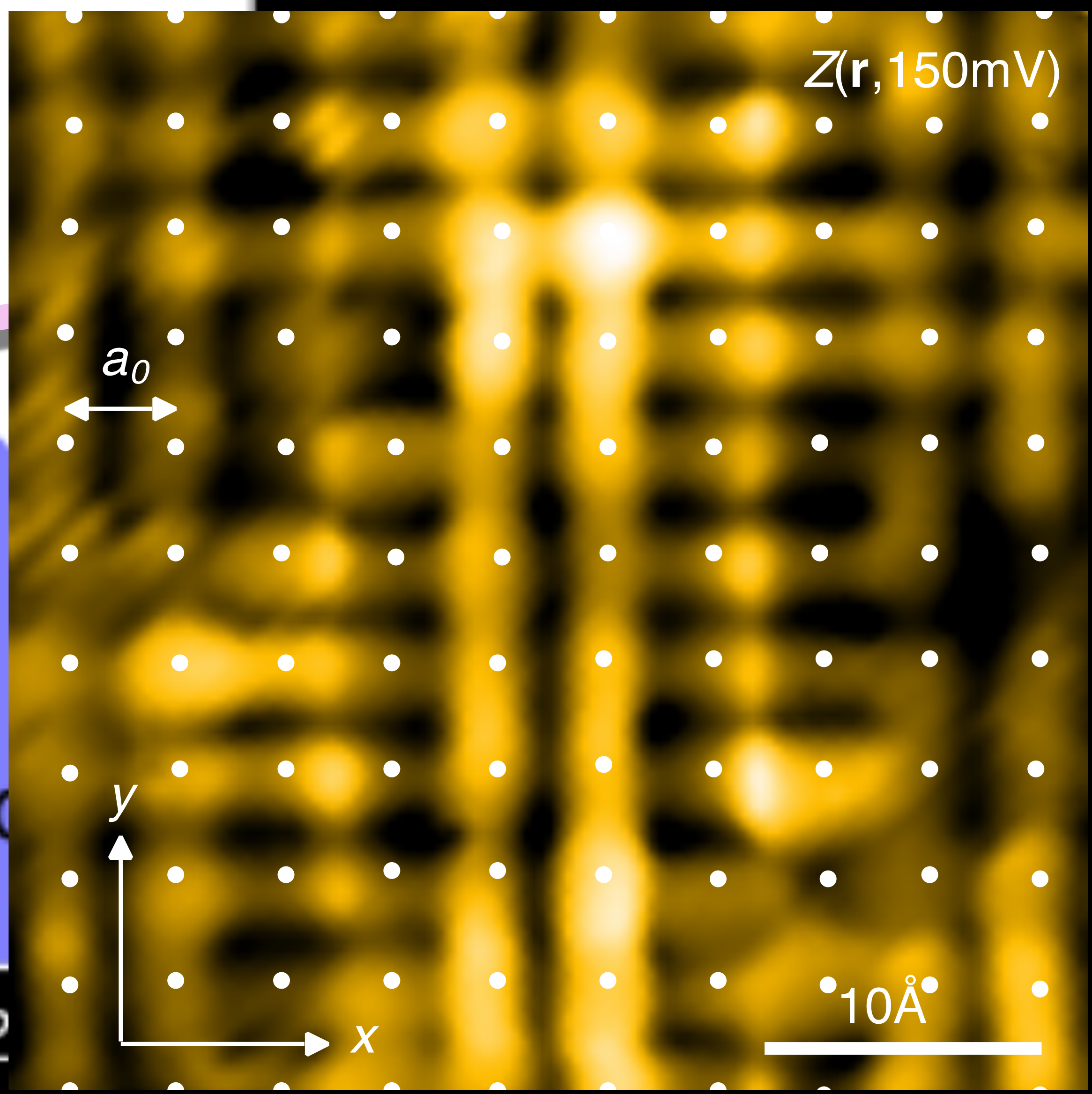
Evidence for pseudogap metal as FL*

- FL* Fermi pockets are compatible with photoemission at high T.
- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).
- Magnetoresistance $\rho_{xx} \sim \tau^{-1}\left(1 + aH^2T^2\right)$ with $\tau \sim T^{-2}$ (Chan *et al.*, PRL **113**, 177005 (2014).
- Charge density wave instabilities of FL* have wave vector and form-factors which agree with STM/X-ray observations in DW region (D. Chowdhury and S. Sachdev, PRB **90**, 245136 (2014)).

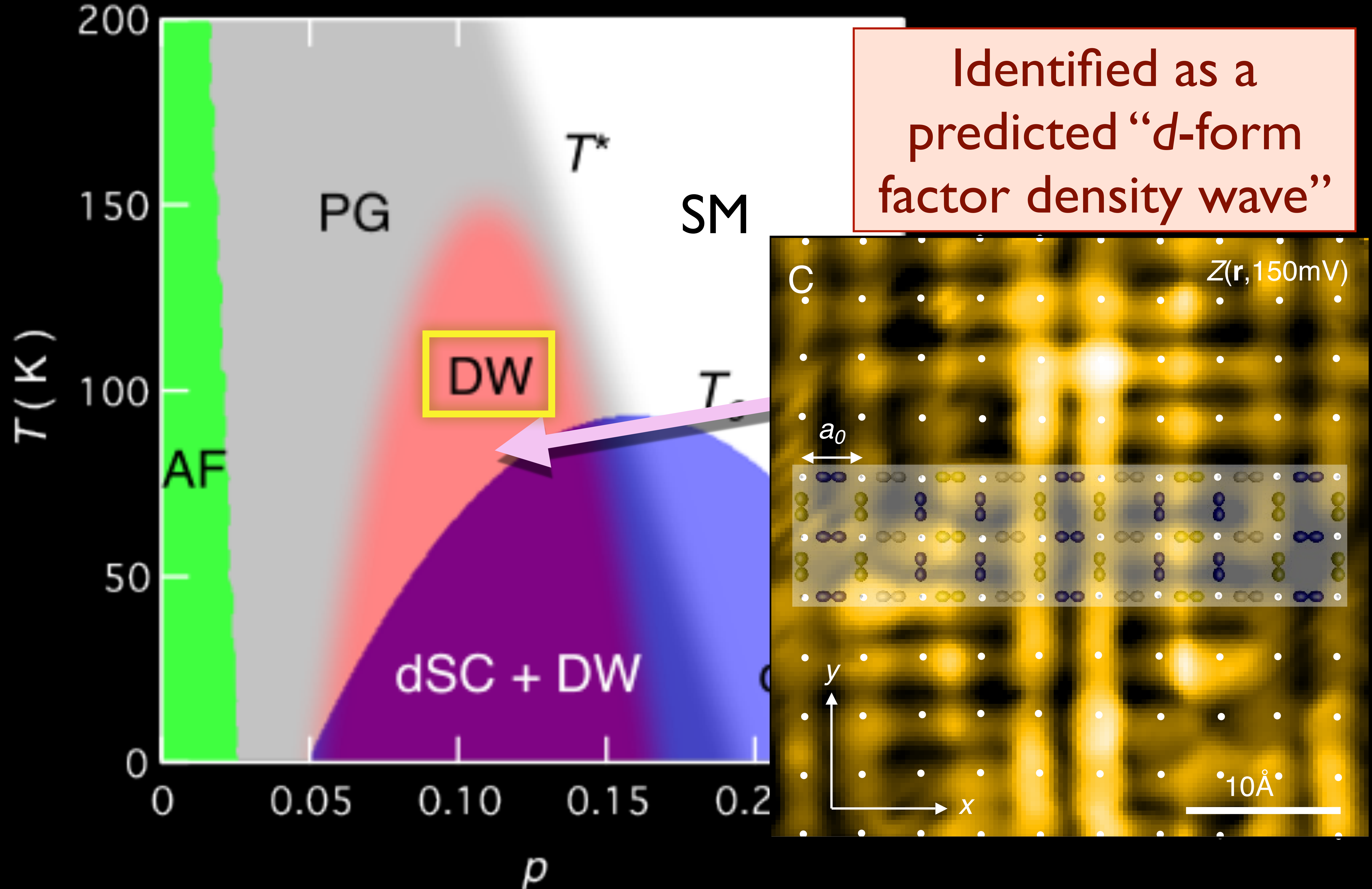
Y. Kohsaka *et al.*, SCIENCE **315**, 1380 (2007)
M. H. Hamidian *et al.*, NATURE PHYSICS **12**, 150 (2016)



Density wave (DW) order at low T and p



M. A. Metlitski and S. Sachdev, PRB **82**, 075128 (2010). S. Sachdev R. La Placa, PRL **111**, 027202 (2013).
K. Fujita, M. H Hamidian, S. D. Edkins, Chung Koo Kim, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, H. Eisaki, S. Uchida, A. Allais, M. J. Lawler, E.-A. Kim, S. Sachdev, and J. C. Davis, PNAS **111**, E3026 (2014)



Evidence for pseudogap metal as FL*

- FL* Fermi pockets are compatible with photoemission at high T.

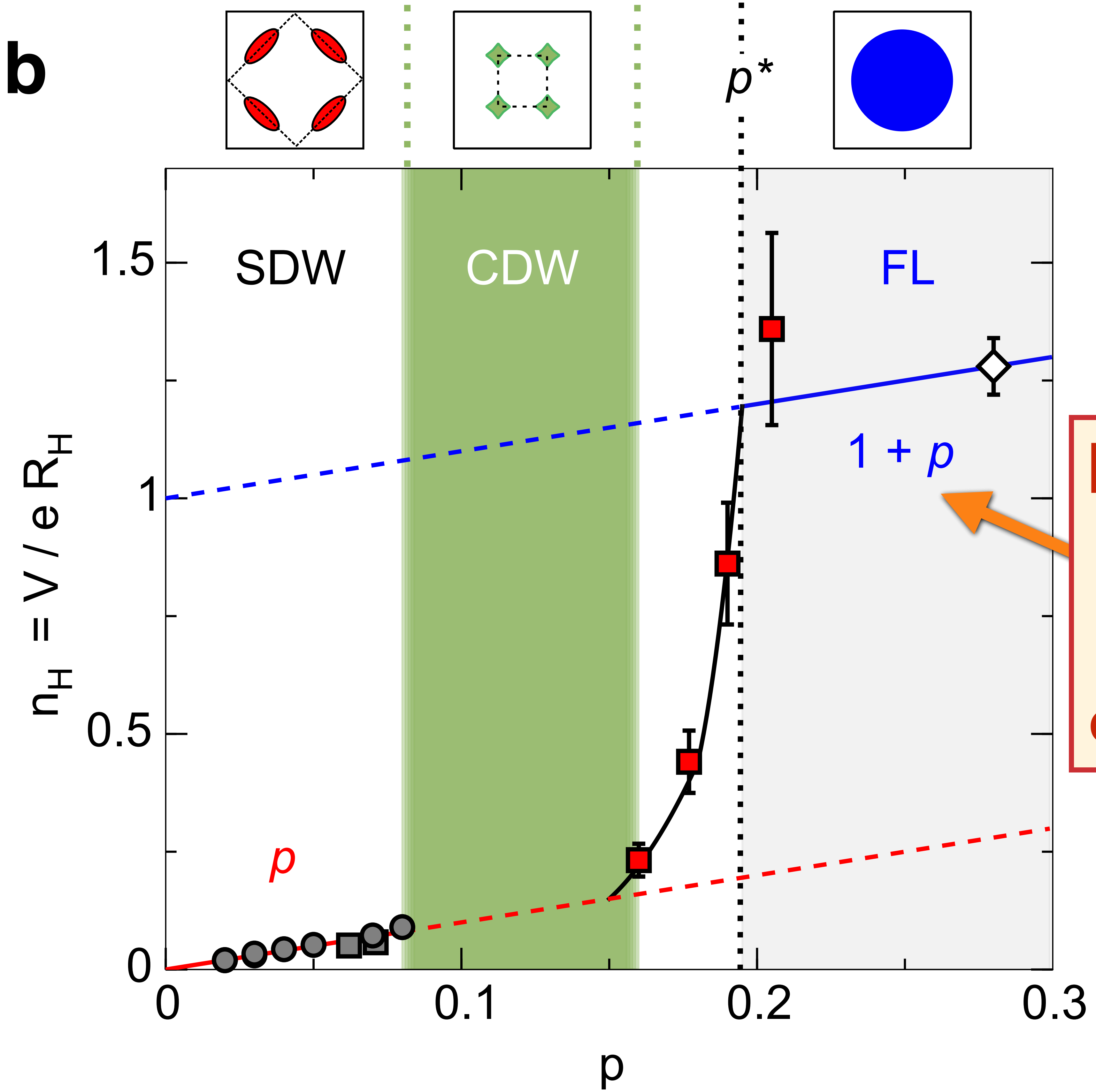
- Optical conductivity $\sim 1/(-i\omega + 1/\tau)$ with $1/\tau \sim \omega^2 + T^2$, with carrier density p (Mirzaei *et al.*, PNAS **110**, 5774 (2013)).

- Magnetoresistance $\rho_{xx} \sim \tau^{-1}\left(1 + aH^2T^2\right)$ with $\tau \sim T^{-2}$ (Chan *et al.*, PRL **113**, 177005 (2014).

- Charge density wave instabilities of FL* have wave vector and form-factors which agree with STM/X-ray observations in DW region (D. Chowdhury and S. Sachdev, PRB **90**, 245136 (2014)).

- T-independent positive Hall co-efficient, R_H, corresponding to carrier density p in the higher temperature pseudogap (Ando *et al.*, PRL **92**, 197001 (2004)) *and* in recent measurements at high fields, low T, and around $p \approx 0.16$ in YBCO (Badoux *et al.*, Nature **531**, 210 (2016)).

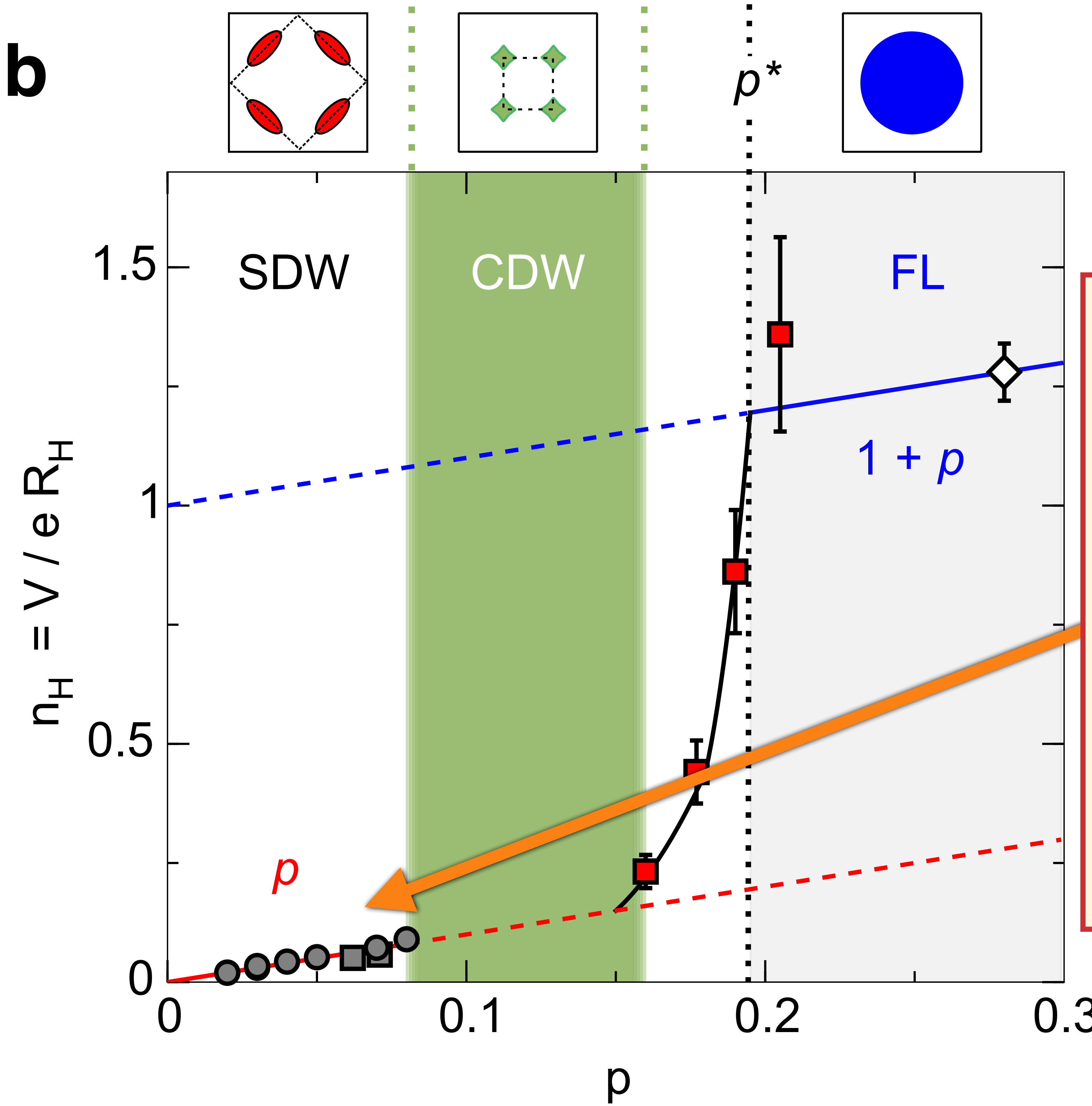
Hall effect measurements in YBCO



Fermi liquid (FL) with carrier density 1+*p*

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

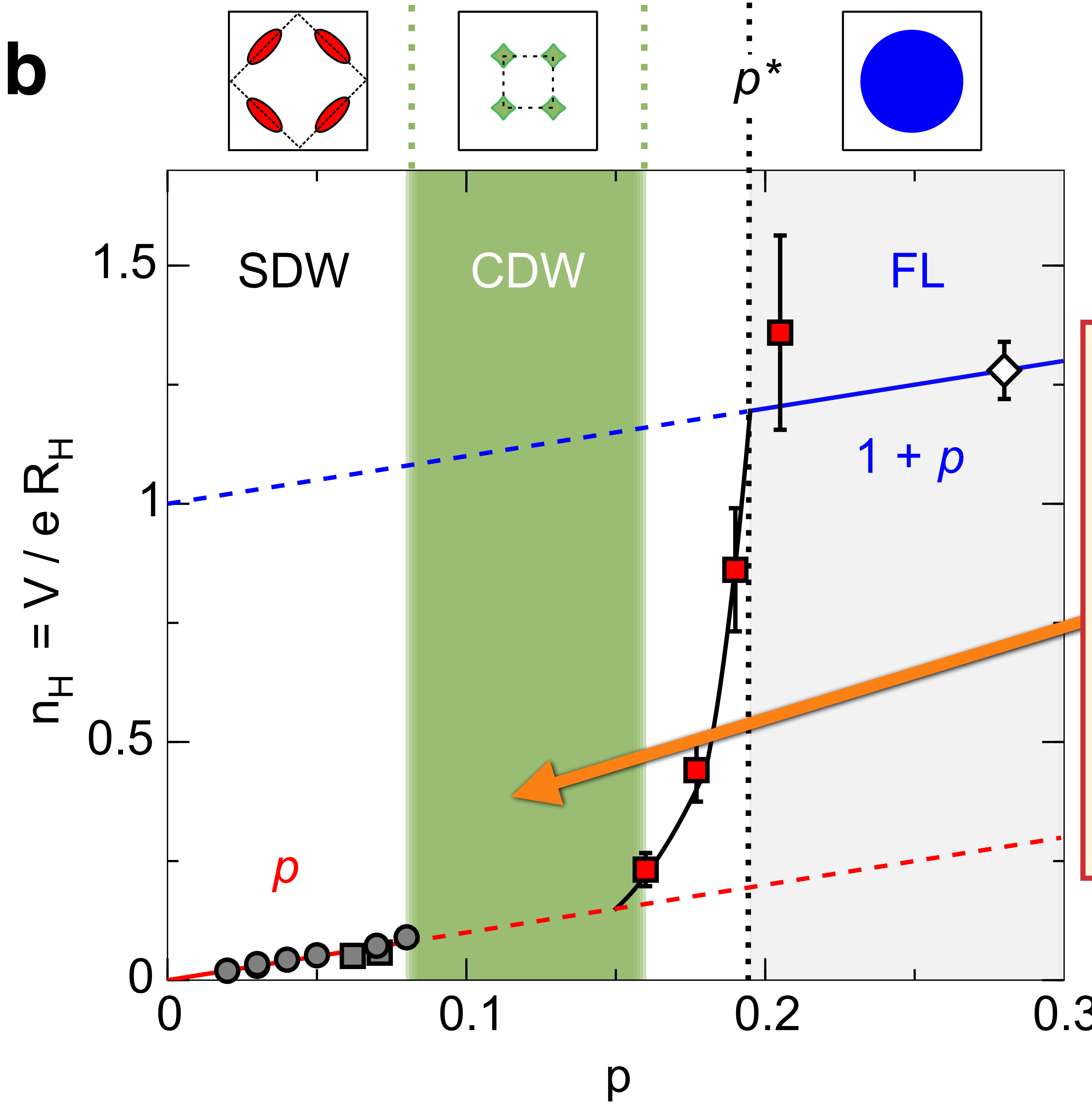
Hall effect measurements in YBCO



Spin density wave (SDW) breaks translational invariance, and the Fermi liquid then has carrier density *p*

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

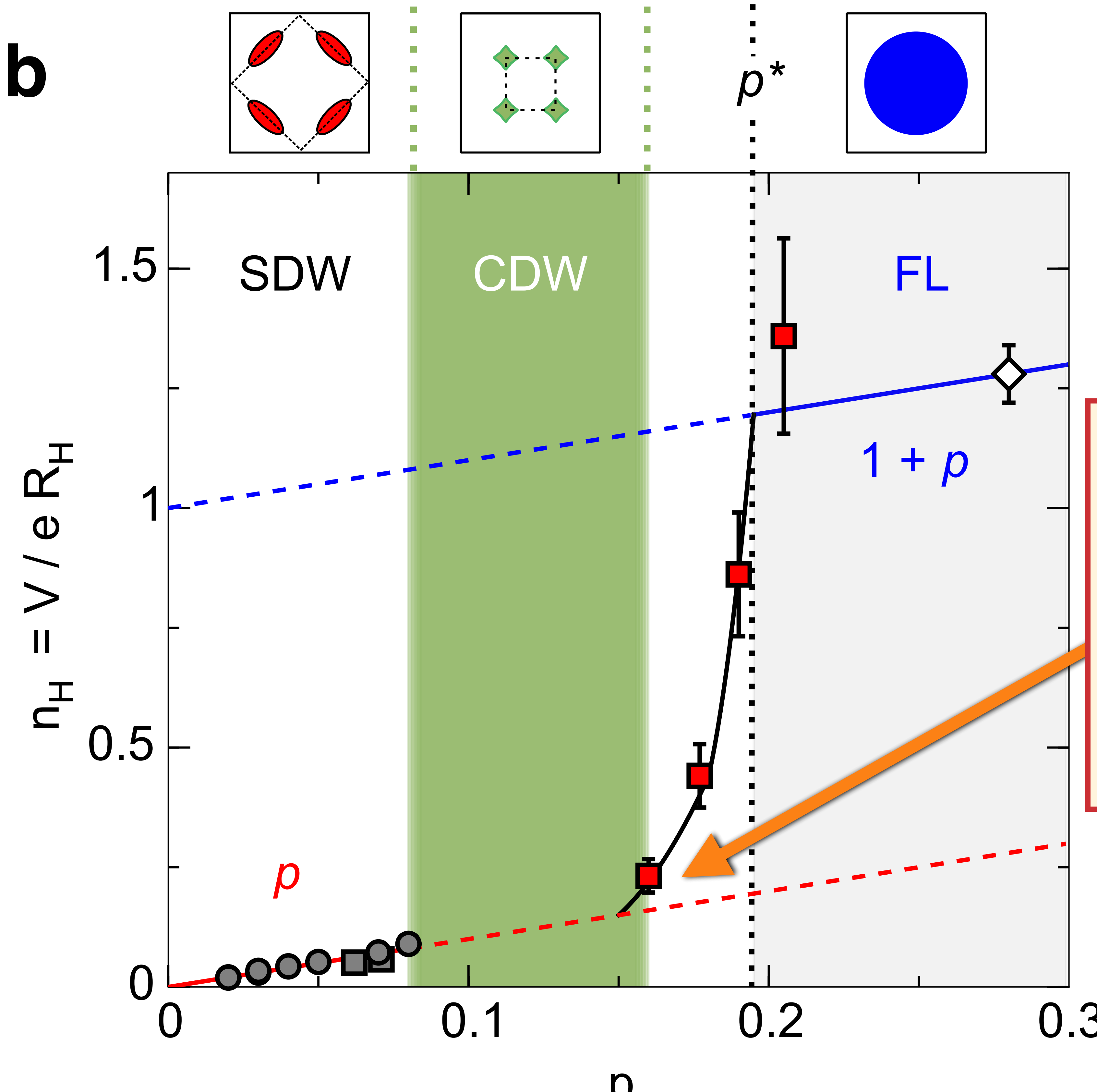
Hall effect measurements in YBCO



Charge density wave (CDW) leads to complex Fermi surface reconstruction and negative Hall resistance

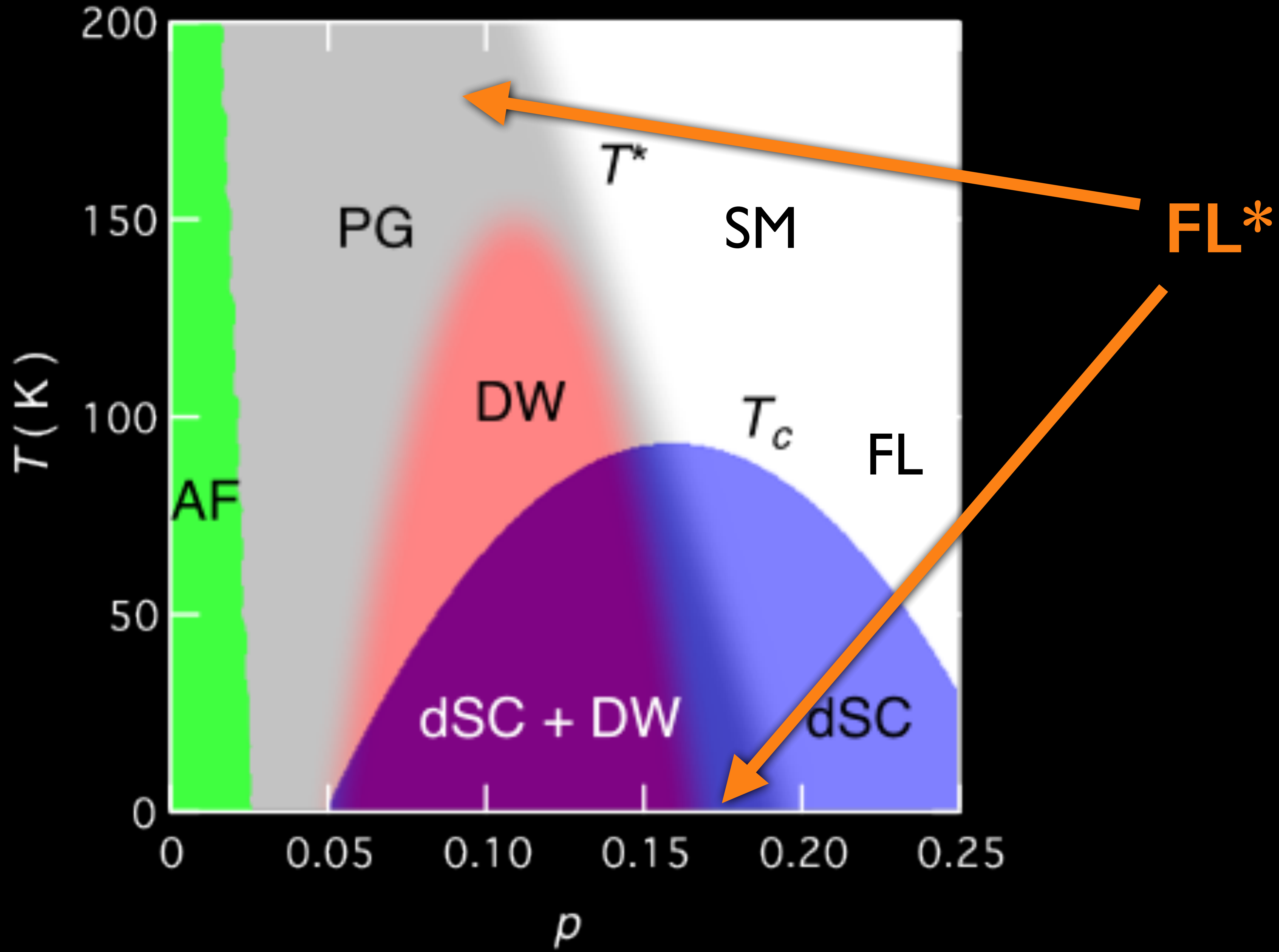
Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)

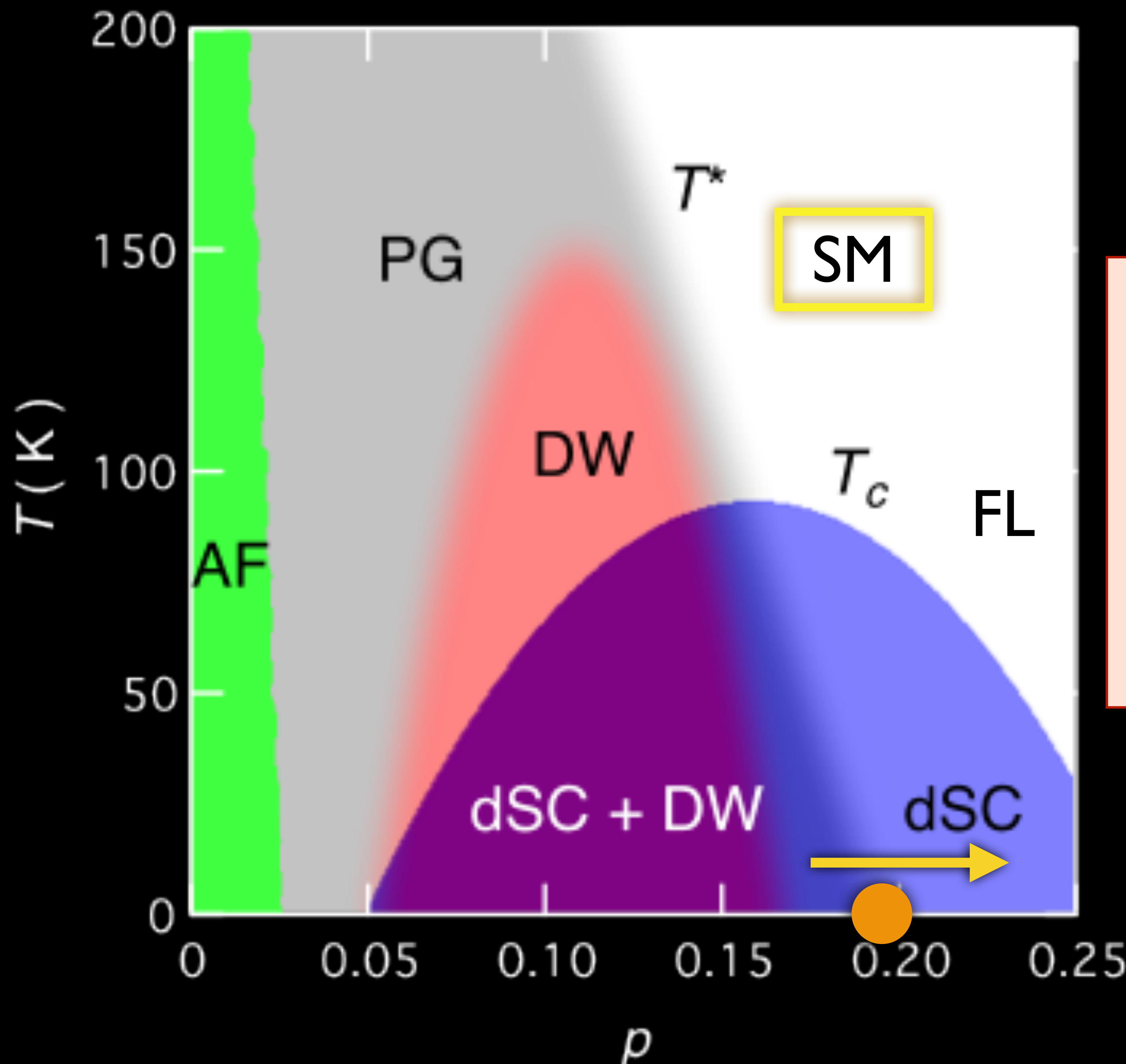
Hall effect measurements in YBCO



Evidence for FL* metal with Fermi surface of size p !

Badoux, Proust, Taillefer et al., Nature **531**, 210 (2016)


200
150
100
50
0
T (K)
0
0.05
0.10
0.15
0.20
0.25
p
AF
PG
T*
SM
DW
Tc
FL
dSC + DW
dSC
FL*



Transtion from Z_2-FL* to FL as a theory of the strange metal (SM)

Quantum critical point at optimal doping

- Transition is primarily "topological". Main change is in the size of the Fermi surface.

Quantum critical point at optimal doping

- Transition is primarily "topological". Main change is in the size of the Fermi surface.
- Symmetry-breaking and Landau order parameters appear to play a secondary role.

Quantum critical point at optimal doping

- Transition is primarily "topological". Main change is in the size of the Fermi surface.
- Symmetry-breaking and Landau order parameters appear to play a secondary role.
- The main symmetry breaking which appears co-incident with the transition is Ising-nematic ordering. But this symmetry cannot change the size of the Fermi surface; similar comments apply to time-reversal symmetry.

Quantum critical point at optimal doping

- Transition is primarily "topological". Main change is in the size of the Fermi surface.

- Symmetry-breaking and Landau order parameters appear to play a secondary role.

- The main symmetry breaking which appears co-incident with the transition is Ising-nematic ordering. But this symmetry cannot change the size of the Fermi surface; similar comments apply to time-reversal symmetry.

- Need a gauge theory for transition from "topological" to "confined" state.

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

- Spinless fermion ψ (the fermionic chargon) transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface, and carries electromagnetic charge

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

- Spinless fermion ψ (the fermionic chargon) transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface, and carries electromagnetic charge

- A SU(2) fundamental scalar z_α (the bosonic spinon), carrying electron-spin and electromagnetically neutral.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

- Spinless fermion ψ (the fermionic chargon) transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface, and carries electromagnetic charge

- A SU(2) fundamental scalar z_α (the bosonic spinon), carrying electron-spin and electromagnetically neutral.

- A SU(2) gauge boson.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

- Spinless fermion ψ (the fermionic chargon) transforming as a gauge SU(2) fundamental, with dispersion $\varepsilon_{\mathbf{k}}$ from the band structure, at a non-zero chemical potential: has a "large" Fermi surface, and carries electromagnetic charge

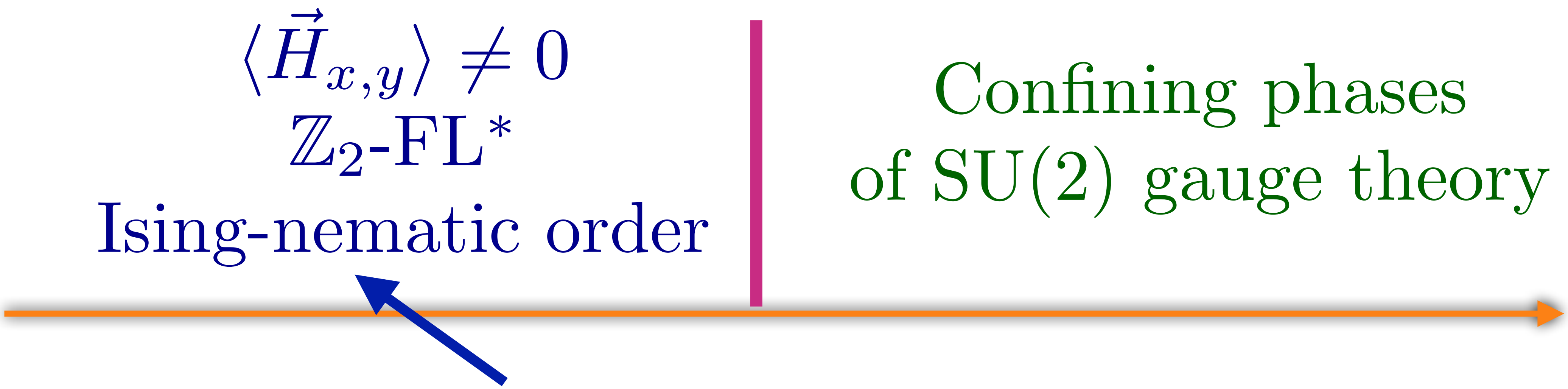
- A SU(2) fundamental scalar z_α (the bosonic spinon), carrying electron-spin and electromagnetically neutral.

- A SU(2) gauge boson.

- Two complex Higgs fields, $\vec{H}_x$ and $\vec{H}_y$, transforming as gauge SU(2) adjoints, and carrying non-zero lattice momentum.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

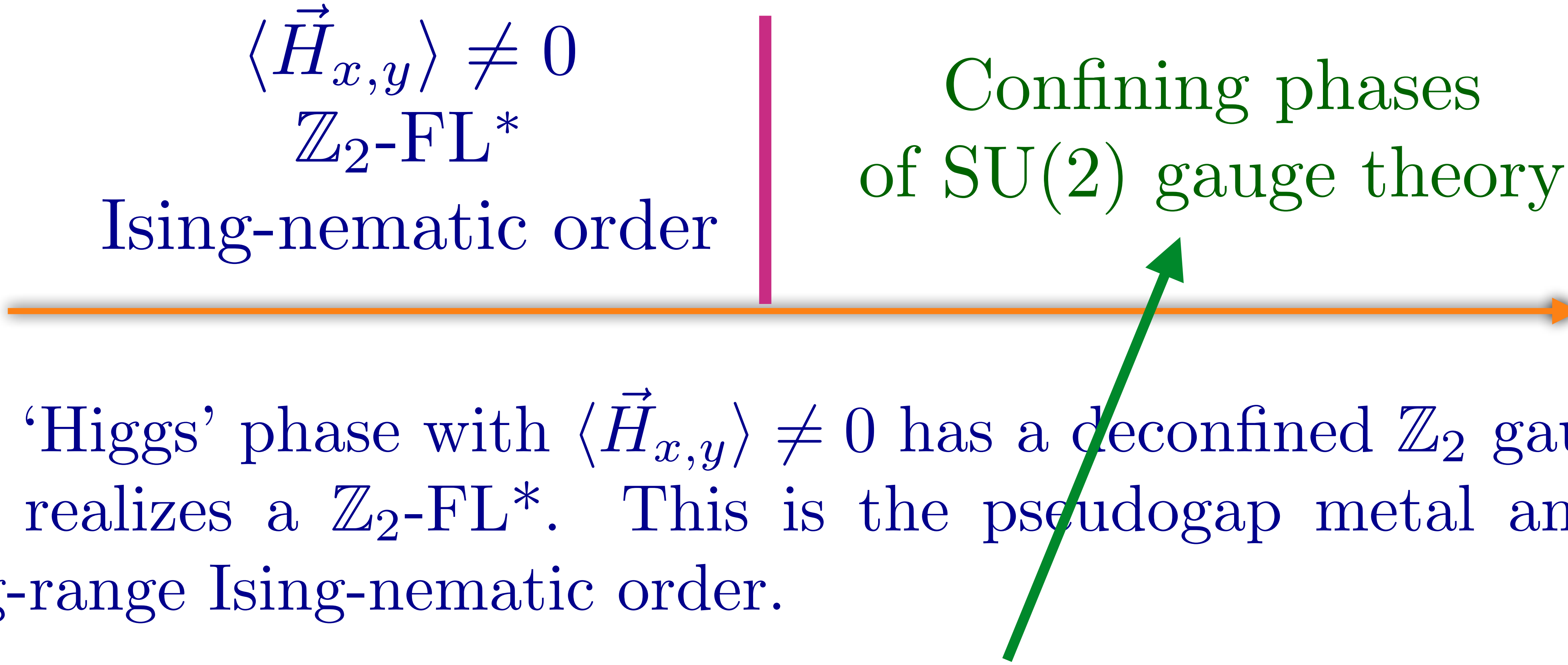
SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL



- The 'Higgs' phase with $\langle \vec{H}_{x,y} \rangle \neq 0$ has a deconfined $\mathbb{Z}_2$ gauge field, and realizes a $\mathbb{Z}_2$-FL*. This is the pseudogap metal and it has long-range Ising-nematic order.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

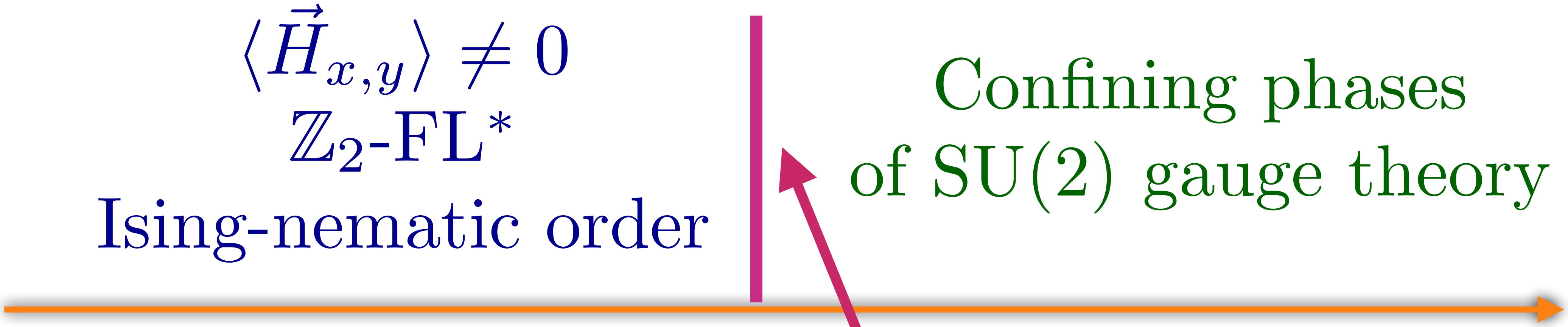


- The 'Higgs' phase with $\langle \vec{H}_{x,y} \rangle \neq 0$ has a deconfined $\mathbb{Z}_2$ gauge field, and realizes a $\mathbb{Z}_2$-FL*. This is the pseudogap metal and it has long-range Ising-nematic order.

- The 'confining' phase is a FL or a superconductor, and no Ising-nematic order. These are the conventional states at large p.

S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)

SU(2) gauge theory for transition between $\mathbb{Z}_2$-FL* and FL

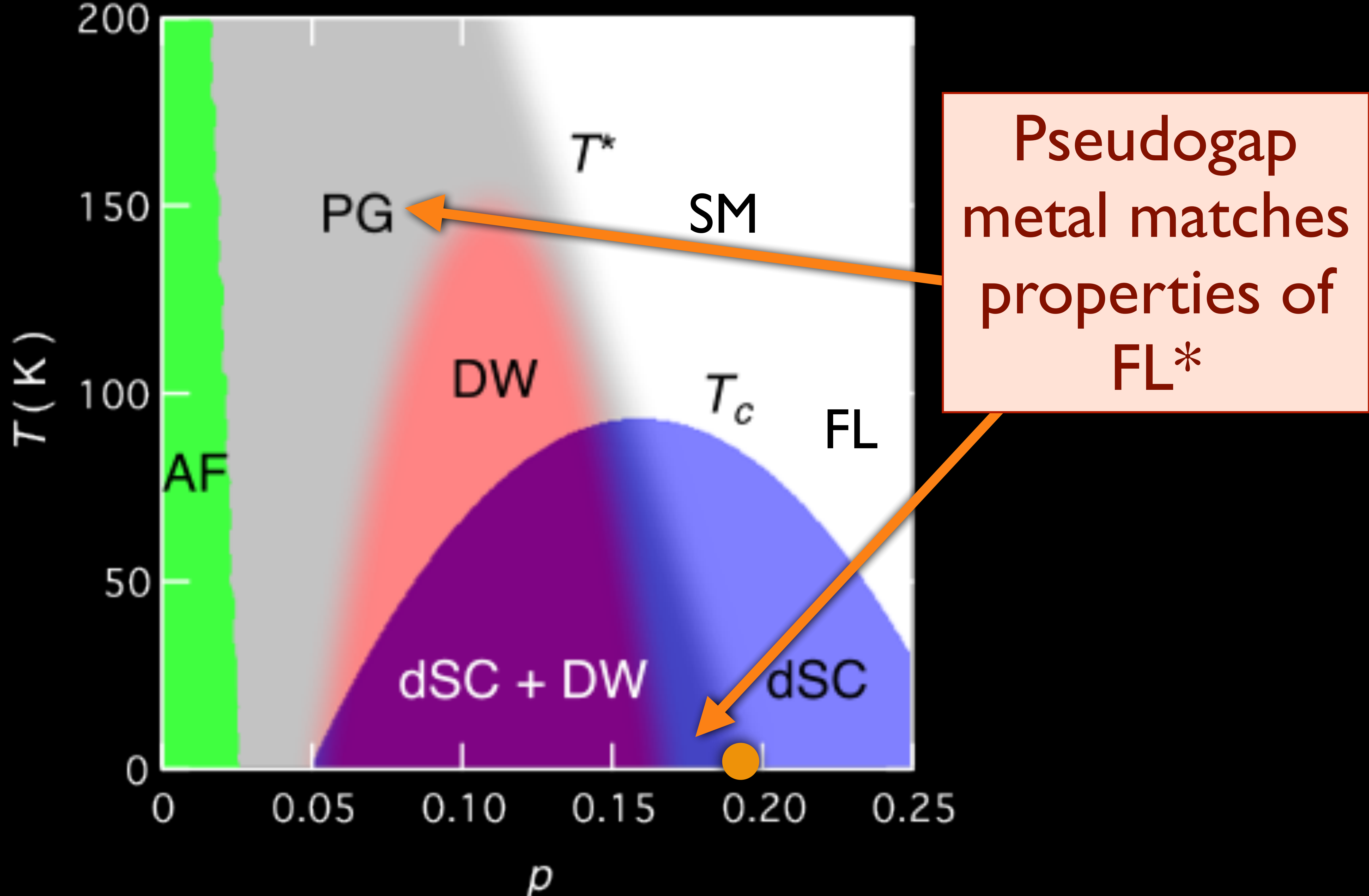


- The 'Higgs' phase with $\langle \vec{H}_{x,y} \rangle \neq 0$ has a deconfined $\mathbb{Z}_2$ gauge field, and realizes a $\mathbb{Z}_2$-FL*. This is the pseudogap metal and it has long-range Ising-nematic order.

- The 'confining' phase is a FL or a superconductor, and no Ising-nematic order. These are the conventional states at large p.

- When the Higgs potential is critical, we obtain a non-Fermi liquid of a ψ Fermi surface coupled to Landau-damped gauge bosons, and critical Landau-damped Higgs field. This is a candidate for describing the strange metal.

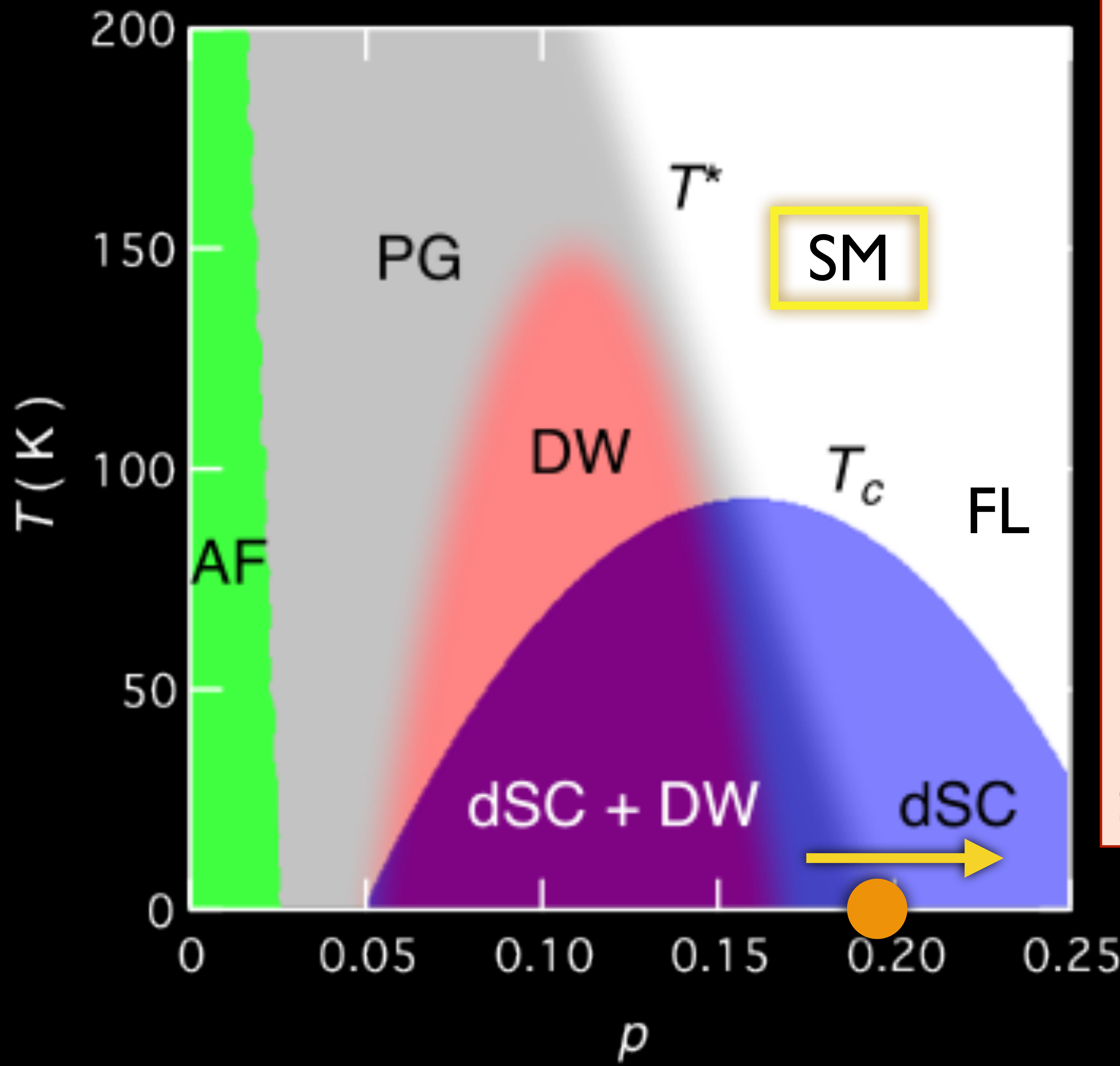
S. Sachdev, M. A. Metlitski, Y. Qi, and C. Xu, Phys. Rev. B **80**, 155129 (2009)
D. Chowdhury and S. Sachdev, PRB **91**, 115123 (2015)


200
150
100
50
0
T (K)
0
0.05
0.10
0.15
0.20
0.25
p
AF
PG
T*
SM
DW
Tc
FL
dSC + DW
dSC
Pseudogap metal matches properties of FL*

FL*

We have described a metal with:

- A Fermi surface of electrons enclosing volume p, and not the Luttinger volume of $1+p$
- The Z_2-FL* also has excitations described by a topological quantum field theory (TQFT) and this is essentially to modify the size of the Fermi surface in the momentum balance argument.



Proposed a SU(2) gauge theory for transtion from Z_2-FL* to FL. This phase transition is beyond the Landau-Ginzburg-Wilson paradigm, and is instead a Higgs-confinement transition in a SU(2) gauge theory